Introduction to mathematical series.

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0- Abstract:

Euler, Leibniz or Ramanujan are some names who have developed mathematical series. In this paper I want to introduce some series of these famous mathematicians and contribute some of my own open series.

1 – Classic series.

In this section we are going to see some examples of closed series.

1.1 – Euler series:

$$(1) \qquad 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+...=e$$

$$(2) \qquad \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+...=\frac{\pi^2}{6}$$

1.2- Leibniz-Madhava serie:

$$(3) \qquad 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...=\frac{\pi}{4}$$

1.3- Maclaurin and Tylor series:

$$(4) \qquad 1+x+x^2+x^3+...=\frac{1}{1-x}$$

$$(5) \qquad 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...=e^x$$

$$(6) \qquad x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...=\sin(x)$$

$$(7)\quad 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+...=\cos(x)$$

$$(8)\quad x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+...=\arctan(x)$$

$$(9)\quad x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...=\ln(1+x)$$

1.4- Ramanujan series:

$$(10)\quad \frac{1}{1^3}\cdot(\frac{1}{2})+\frac{1}{2^3}\cdot(\frac{1}{2^2})+\frac{1}{3^3}\cdot(\frac{1}{2^3})+\frac{1}{4^3}\cdot(\frac{1}{2^4})+...=\frac{1}{6}(\log(2))^3-\frac{\pi^2}{12}\log 2+(\frac{1}{1^3}+\frac{1}{3^3}+\frac{1}{5^3}...)$$

$$(11)\quad 1+9(\frac{1}{4})^4+17(\frac{1\cdot 5}{(4\cdot 8)})^4+25(\frac{1\cdot 5\cdot 9}{(4\cdot 8\cdot 12)})^4+...=\frac{(2\sqrt{2})}{(\sqrt{\pi}(\Gamma\frac{3}{4})^2)}$$

$$(12)\quad 1-5(\frac{1}{2})^3+9(\frac{1\cdot 3}{(2\cdot 4)})^3-...=\frac{3}{\pi}$$

$$(13)\quad \frac{1^{13}}{(e^{(2\pi)}-1)}+\frac{2^{13}}{(e^{(4\pi)}-1)}+\frac{3}{(e^{(6\pi)}-1)}+...=\frac{1}{24}$$

$$(14)\quad \frac{(\cot(\pi))}{1^7}+\frac{(\cot(2\pi))}{2^7}+\frac{(\cot(3\pi))}{3^7}+...=\frac{(19\pi^7)}{56700}$$

$$(15)\quad \frac{1}{(1^5\cos(\frac{\pi}{2}))}-\frac{1}{(3^5\cos(3\frac{\pi}{2}))}+\frac{1}{(5^5\cos(5\frac{\pi}{2}))}-...=\frac{\pi^5}{768}$$

2- New series.

Here I am going to present new series invented by me.

$$(16)\quad 1+\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+...$$

$$(17)\quad 1-\frac{1}{1!}+\frac{1}{3!}-\frac{1}{5!}+...$$

$$(18)\quad 1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+...$$

(19) $1-\frac{1}{2!}+\frac{1}{4!}-\frac{1}{6!}+\ldots$

(20) $1-\frac{1}{(1!)^2}+\frac{1}{(2!)^2}-\frac{1}{(3!)^2}+\ldots$

(21) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\ldots$

(22) $1-3\left(\frac{1}{2}\right)^x+5\left(\frac{1\cdot 3}{(2\cdot 4)}\right)^x-7\left(\frac{1\cdot 3\cdot 3}{(2\cdot 4\cdot 6)}\right)^x+\ldots$

(23) $1-2\left(\frac{1}{2}\right)^x+4\left(\frac{1\cdot 3}{(2\cdot 4)}\right)^x-6\left(\frac{1\cdot 3\cdot 3}{(2\cdot 4\cdot 6)}\right)^x+\ldots$

(24) $1-3\left(\frac{2}{1}\right)^x+5\left(\frac{2\cdot 4}{(1\cdot 3)}\right)^x-7\left(\frac{2\cdot 4\cdot 6}{(1\cdot 3\cdot 5)}\right)^x+\ldots$

(25) $1-2\left(\frac{2}{1}\right)^x+4\left(\frac{2\cdot 4}{(1\cdot 3)}\right)^x-6\left(\frac{2\cdot 4\cdot 6}{(1\cdot 3\cdot 5)}\right)^x+\ldots$

(26) $x-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\ldots$

(27) $x-\frac{x^2}{3}+\frac{x^4}{5}-\frac{x^6}{7}+\ldots$

(28) $x-\frac{x^2}{2}+\frac{x^4}{4}-\frac{x^6}{6}+\ldots$

(29) $\frac{1}{1^3}\cdot\left(\frac{1}{2}\right)+\frac{1}{3^3}\cdot\left(\frac{1}{2^2}\right)+\frac{1}{5^3}\cdot\left(\frac{1}{2^3}\right)+\frac{1}{7^3}\cdot\left(\frac{1}{2^4}\right)+\ldots$

(30) $\frac{1}{1}\cdot\left(\frac{1}{2^2}\right)+\frac{1}{3^3}\cdot\left(\frac{1}{4^4}\right)+\frac{1}{5^5}\cdot\left(\frac{1}{6^6}\right)+\frac{1}{7^7}\cdot\left(\frac{1}{8^8}\right)+\ldots$

(31) $\frac{1}{3}-\frac{2}{4}+\frac{3}{5}-\frac{4}{6}+\ldots$

(32) $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\frac{4}{5!}+\ldots$

(33) $x\div\left(\frac{x^2}{2}\right)\cdot\left(\frac{x^3}{3}\right)\div\left(\frac{x^4}{4}\right)\cdot\ldots$

(34) $x\div\left(\frac{x^3}{3}\right)\cdot\left(\frac{x^5}{5}\right)\div\left(\frac{x^7}{7}\right)\cdot\ldots$

$$(35)\quad x \div \left(\frac{x^3}{3!}\right) \cdot \left(\frac{x^5}{5!}\right) \div \left(\frac{x^7}{7!}\right) \cdot \ldots$$

$$(36)\quad x \div \left(\frac{x^2}{2!}\right) \cdot \left(\frac{x^4}{4!}\right) \div \left(\frac{x^6}{6!}\right) \cdot \ldots$$

$$(37)\quad \frac{x}{e^{(2\pi)}} - \frac{x^2}{e^{(4\pi)}} + \frac{x^3}{e^{(6\pi)}} - \frac{x^4}{e^{(8\pi)}} + \ldots$$

$$(38)\quad \frac{1}{(1\sin(\frac{\pi}{2}))} - \frac{1}{(3\sin(3\frac{\pi}{2}))} + \frac{1}{(5\sin(5\frac{\pi}{2}))} - \frac{1}{(7\sin(7\frac{\pi}{2}))} + \ldots$$

$$(39)\quad 1 - \frac{1}{(3\log(3))} + \frac{1}{(5\log(5))} - \frac{1}{(7\log(7))} + \ldots$$

$$(40)\quad \frac{(\tan(\pi))}{1!} - \frac{(\tan(2\pi))}{2!} + \frac{(\tan(3\pi))}{3!} - \frac{(\tan(4\pi))}{4!} + \ldots$$

$$(41)\quad \frac{2}{e} + \frac{2^2}{e^2} + \frac{2^3}{e^3} + \frac{2^4}{e^4} + \ldots$$

$$(42)\quad \frac{1}{(2\sin(\frac{\pi}{2}))} - \frac{1}{(4\sin(3\frac{\pi}{2}))} + \frac{1}{(6\sin(5\frac{\pi}{2}))} - \frac{1}{(8\sin(7\frac{\pi}{2}))} + \ldots$$

3- Conclusion.

As we could see in this paper, is possible to introduce new original open series with variations in the form of them. I have made a combination of natural numbers with special look in the even and odd series. I have also introduced trigonometric series and some exponential variations.