Photon Induced Low Energy Nuclear Reactions

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Abstract

We propose a new mechanism for inducing low energy nuclear reactions (LENRs). The process is initiated by a perturbation which we assume is caused by an external photon. The initial two body nuclear state absorbs the photon and forms an intermediate state which makes a transition into the final nuclear state with emission of a light particle which in the present paper is taken to be a photon. We need to sum over all energies of the intermediate state. Since the energy of this state is unconstrained we get contributions from very high energies for which the barrier penetration factor is not too small. The contribution from such high energy states is typically suppressed due to the large energy denominators and its matrix element with the initial state. Furthermore the process is higher order in perturbation theory in comparison to the standard fusion process. However these factors are relatively mild compared to the strong suppression due to the barrier penetration factor at low energies. By considering a specific reaction we find that its cross section is higher than the cross section of the standard process by a factor of 10^{41} or more. This enhancement makes LENRs observable in laboratory even for relatively low energies. Hence we argue that LENRs are possible and we provide a theoretical set up which may explain some of the experimental claims in this field.

1 Introduction

The phenomenon of low energy nuclear reactions (LENRs) has now been studied for many decades. A useful summary is provided in the papers [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] and in the collection of articles in [14]. Theoretically it has been a challenge to understand how such reactions can occur due to the large Coulomb barrier [15, 16]. Several theoretical proposals exist that try to invoke screening effects in medium [17, 18, 19, 20], formation of correlated states

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in a nonstationary potential [21, 22], clusterization effects [23], time independent perturbation [24], deuterium evolution reaction model [25], electroweak interactions [26, 27] etc. A detailed study of screening has been performed in [18] with the conclusion that by itself it is unable to explain the enhanced cross sections even in the energy range of 1 KeV [17]. In [22] it has been suggested that the incident particle may be in a superposition of several states and due to destructive interference the reflection coefficient becomes significantly smaller than unity leading to considerable enhancement in transmission. It has also been proposed that the nuclear particles may form clusters due to enhanced electron screening which may lead to smaller Coulomb barrier [23]. However the LENR phenomena are still not understood theoretically.

Experimentally it is quite clear that there is indeed an enhancement of cross sections at low energies [17, 18, 28, 29, 30, 31, 32, 33, 34]. It has been argued [35] that the experimental data is better characterized in terms of production of nuclear particles rather than excess heat [36, 37]. While the experimental results at very low energies require careful further analysis, the results in the energy range of order of a few keV have consistently shown enhanced cross sections in condensed medium. Some of these experiments involve a beam of high energy deuterons impinging on a solid medium [33]. The lower energy experiments use electrolysis, heating, diffusion, electric discharge etc. [14]. The community has slowly come to an agreement that the nuclear fusion cross sections in this energy regime are much higher than expected theoretically and the ratio of experimental to theoretically predicted cross sections increases rapidly with decrease in energy. There have been many attempts to explain this behaviour by a suitable generalization of the theoretical analysis.

We point out that there exist well known situations in which a particle is able to tunnel through a high potential barrier with a rather high probability. We consider a text book example of a double hump potential (see page 129 of [38], third edition). Here we assume that the potential barrier is much larger than the energy of the incident particle. Using the WKB approximation, one finds that although the transmission for such a potential is generally small, there exist some special values of energy for which the transmission can be very large. Whether such a mechanism is really realized in nuclear fusion reactions in condensed matter is not clear. Here we use it only to illustrate that high potential does not always mean low transmission. It is also well known that the nuclear fusion rates are rather large if the reaction proceeds by resonance. This arises when the energy of the incident particle is equal or close to one of the nuclear states. In the present paper, however, we shall not consider resonant reactions.

We are interested in explaining the phenomena of LENRs, the energies being of order eV. We point out that LENRs have also been seen experimentally at energies as low as 30-40 meV [39]. In this paper we propose a new process which has so far not been considered in the literature and may be relevant for LENRs. In this case the reaction proceeds by a perturbation in the initial state. This perturbation may be in the form of a real photon or a virtual photon. The virtual photon may be exchanged by an incident flux of electrons, or other charged particles, with the nucleus under consideration. The perturbation leads to formation of a mixed state which can be expressed as a superposition of all the eigenstates of the unperturbed Hamiltonian. Each of these eigenstates then contribute to the fusion process. As prescribed by the uncertainty principle, each of these states can exist for a short time interval during which they can undergo fusion. Due to the fact that there is no restriction on the energy of these states it is possible that the barrier penetration factor may not lead to a strong suppression. Here we develop the formalism for such reactions and provide estimates for some simple cases. Similar ideas have been proposed earlier [21, 22, 24], however the precise process we consider has not been discussed in the literature in this context. As we shall see the process under consideration is rather simple and has close similarity to standard processes discussed in graduate level Physics textbooks [38, 40].

Let us now briefly review the standard fusion rate calculations [41, 42, 43, 44, 45]. Consider the nuclear fusion reaction in which a light nucleus a is incident on target nucleus B,

$$a + B \to C + d \tag{1}$$

Here C is the final nucleus and d is another particle, which may be a photon, neutron or a pair of particles, such as a neutrino and a positron. We use the center of mass and relative coordinates and convert the initial two body system into an effective one body problem by introducing the concept of reduced mass. The particle a may be a proton or a deuteron or some other light nuclei. We assume that the kinetic energy of the initial state is E when the two particles are at large distances from one another. Let Z_1 and Z_2 be the atomic numbers of the two initial state nuclei and A_1 and A_2 be their atomic mass numbers. The potential energy diagram may be schematically represented as in Fig. 1. We consider the non-resonant case in which there is no nuclear bound state with energy equal to the incident two particle energy. The important factors which contribute to the reaction rate are the tunneling probability and rate of decay of this state into a nuclear state by emission of particle d. The cross section for this process may be expressed as [41, 42]

$$\sigma(E) = \frac{S(E)}{E}\; B(E) \tag{2}$$

where

$$B(E) = \exp(-b/\sqrt{E}) \tag{3}$$

and b is a constant. If we ignore screening the factor b is given by [41]

$$b = 31.28 Z_1 Z_2 \sqrt{A} \quad \mathrm{keV}^{1/2} \tag{4}$$

where $A = A_1 A_2/(A_1+A_2)$ is the reduced atomic weight. Here the exponential factor represents the probability for barrier penetration and the remaining factors depend on the amplitude for production of particle d. Hence for the case of photon production the factor $S(E)$ will depend on the electromagnetic coupling. For the case of non-resonant scattering, $S(E)$ is a slowly varying function of E. For the case of no screening, i.e. just Coulomb repulsion between two nuclei, the barrier penetration can be shown to take the form given in the exponent in Eq. 3 to a very good approximation. In general when screening due to electrons is taken into account, the form may be somewhat more complicated. The screening effects can, however, be incorporated approximately by adding a screening energy E_s to the energy factor E in the exponent in Eq. 3.

2 Standard Fusion Process

The fusion reaction typically involves emission of some particle, such as a photon, a light nuclei or a pair of particles such as, a positron and a neutrino. Hence besides involving strong interactions, it may also involve electromagnetic and/or weak interactions. Here we briefly review

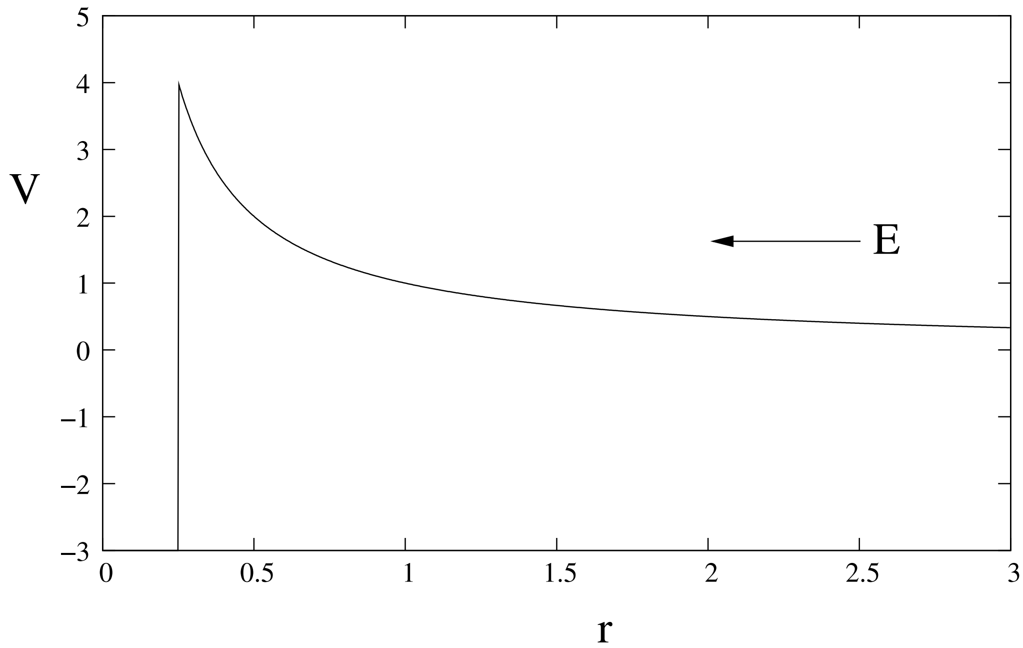

Figure 1: A schematic representation of the nuclear potential experienced by a particle at energy E incident from infinity. The rise in potential at short distances represents the Coulomb barrier.

the calculation of a fusion reaction rate. To be specific we will focus on an electromagnetic transition and consider the reaction

$$^{1}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma(\omega_f) \tag{5}$$

i.e. fusion of a proton and a deuteron to form a He-3 with emission of a photon. However our analysis will be applicable to all two body reactions which involve emission of a photon. For other final states the same analysis will work but with a suitable choice of perturbation Hamiltonian. In this work we shall mostly use atomic units.

Let the initial state two body wave function be denoted by $|i\rangle$. We assume that the two particles are free initially. The reaction rate will involve the overlap of this wave function with the He-3 wave function which will depend on the quantum tunneling amplitude. The two nuclei in the initial state are treated as an equivalent one particle by introducing the concept of reduced mass. The electrons surrounding the nuclei, which may be either bound or free play the role of modifying the effective potential experienced by the two nuclei, i.e. they lead to screening of the Coulomb potential. The Hamiltonian of the system can be written as

$$H = H_0 + H_I \tag{6}$$

where H_0 is the unperturbed Hamiltonian and H_I is a time dependent perturbation. The unperturbed Hamiltonian is given by

$$H_0 = K_1 + K_2 + \mathcal{V}(r) \tag{7}$$

where K_1 and K_2 are the kinetic energies of the two nuclei and $\mathcal{V}(r)$ is the effective potential. We express H_0 in terms of center of mass and relative coordinates and ignore center of mass motion. The effective potential $\mathcal{V}(r)$ is obtained after integrating over the contributions from electrons and hence contains screening potential as well as terms arising from orbital angular momentum.

The reaction proceeds by emission of a photon. The electromagnetic field can be written as,

$$\vec{A}(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k}}\sum_{\beta} c\sqrt{\frac{\hbar}{2\omega}}\left[a_{\vec{k},\beta}(t)\vec{\epsilon}_\beta e^{i\vec{k}\cdot\vec{r}} + a^\dagger_{\vec{k},\beta}(t)\vec{\epsilon}_\beta e^{-i\vec{k}\cdot\vec{r}}\right] \tag{8}$$

At leading order the reaction rate can be computed by considering the following time dependent perturbation (see, for example [38, 40])

$$H_I(t) = \frac{\xi e}{\mu c}\vec{A}(\vec{r},t)\cdot\vec{p} \tag{9}$$

where μ is the reduced mass of the two particle system, $\vec{A}$ is the vector potential, $\vec{p}$ the momentum operator and ξ is given by,

$$\xi = \frac{m_2 - m_1}{m_2 + m_1} \tag{10}$$

This factor is derived by using relative and center of mass coordinates and is applicable for the process under consideration for which $Z_1 = Z_2 = 1$. Here the center of mass motion is not relevant and we focus on the relative coordinates. Furthermore, for simplicity, we have made the standard approximation of ignoring the $\vec{r}$ dependence of electromagetic field, i.e. $e^{-i\vec{k}\cdot\vec{r}} \approx 1$ and $e^{i\vec{k}\cdot\vec{r}} \approx 1$ where $\vec{k}$ is the wave vector of the photon. This is valid in the present case since we will only be interested in electric dipole transitions. For the process under consideration, the final state wave function gets dominant contribution only over nuclear length scales and the wave length of the emitted photon is expected to be much larger than this scale. For our system m_2 and m_1 are the masses of deuteron and Hydrogen respectively and hence $\xi = 1/3$. We need to compute the transition amplitude, given by,

$$\langle f|T(t_0,t)|i\rangle = \left(-\frac{i}{\hbar}\right)\int_{t_0}^{t} dt'\langle f|H_I(t')|i\rangle e^{i(E_f-E_i)t'/\hbar} \tag{11}$$

Here $T(t_0,t)$ is the time translation operator. It is implicitly assumed that there also exists a photon of frequency ω_f and wave vector $\vec{k}_f$ in the final state. We have not explicitly shown this in the equation above. Inserting the vector potential and acting with the creation operator, we obtain

$$\langle f|T(t_0,t)|i\rangle = -\frac{ie\xi}{\hbar\mu}\sqrt{\frac{\hbar}{2\omega_f V}}\int_{t_0}^{t} dt'\langle f|\vec{\epsilon}_\beta\cdot\vec{p}\,|i\rangle e^{i(E_f-E_i+\hbar\omega_f)t'/\hbar} \tag{12}$$

where $V = L^3$ is the quantization volume, $\vec{\epsilon}_\beta$ is the photon polarization vector and E_i and E_f are the energies of the initial and final states respectively.

We next perform the time integral and take the limit $t_0 \to -\infty$ and $t \to \infty$ to obtain

$$|\langle f|T(t_0,t)|i\rangle|^2 = \frac{e^2\xi^2\pi\Delta T}{\mu^2\omega_f V}\delta(E_f - E_i + \hbar\omega_f)|\langle f|\vec{\epsilon}_\beta\cdot\vec{p}\,|i\rangle|^2 \tag{13}$$

where $\Delta T = t - t_0$ is the total time. Let ρ_γ be the density of photon states for emission into solid angle $d\Omega$, given by,

$$\rho_\gamma = \frac{V\omega_f^2}{(2\pi)^3}\frac{d\Omega}{\hbar c^3} \tag{14}$$

Using this and integrating over $E_{\gamma f} = \hbar\omega_f$, we obtain the transition probability per unit time $dP_{d\Omega}/dt$ for solid angle $d\Omega$. It is given by,

$$\frac{dP_{d\Omega}}{dt} = \int dE_{\gamma f}\rho_\gamma \frac{|\langle f|T(t_0,t)|i\rangle|^2}{\Delta T} \tag{15}$$

Here we have ignored the small nuclear recoil. The transition probability dP/dt integrated over solid angle can then be expressed as

$$\frac{dP}{dt} = \frac{e^2\xi^2\omega_f}{8\pi^2\hbar c^3\mu^2}\int d\Omega \sum_\beta |\langle f|\vec{\epsilon}_\beta \cdot \vec{p}\ |i\rangle|^2 \tag{16}$$

where we have summed over the final state polarizations. It is convenient to replace the operator $\vec{p}$ by using

$$\vec{p} = i\mu[H_0, \vec{r}\ ]/\hbar \tag{17}$$

The matrix element can then be expressed as

$$\langle f|\vec{\epsilon}_\beta \cdot \vec{p}\ |i\rangle = \frac{i\mu}{\hbar}(E_f - E_i)\langle f|\vec{\epsilon}_\beta \cdot \vec{r}\ |i\rangle \tag{18}$$

We next perform the angular integration in Eq. 16. We point out that we are considering a dipole transition from the initial state $l = 0$ to the final state $l' = 1$. The initial state has $m = 0$ and the final state can have $m' = 0, \pm 1$ and we need to sum over m'. The result for this is already given in standard texts (see, for example, [38, 40]). The sum over photon polarizations and the angular integral in Eq. 16 gives an overall factor of $8\pi/3$. The final result after summing over m' is

$$\frac{dP}{dt} = \frac{e^2\xi^2\omega_f}{3\pi\hbar^3 c^3}(E_f - E_i)^2\ |I_{fi}|^2 \tag{19}$$

where I_{fi} is the integral over the radial wave functions, $R_f(r)$ and $R_i(r)$

$$I_{fi} = \int_0^\infty dr r^3 R_f^*(r) R_i(r) \tag{20}$$

In order to obtain the cross section for the process we divide dP/dt by the number N_2 of target particles in the volume V and the incident flux $F(E) = n_1 v$, where v is the relative velocity and n_1 is the number density of incident particles. Here we assume that the incident wave corresponding to the initial state nuclei is a plane wave propagating in the z direction, normalized over a volume $V = L^3$. The cross section can be expressed as,

$$\sigma = \frac{1}{N_2 F(E)}\frac{dP}{dt} \tag{21}$$

The factors involving the length parameter L would cancel the normalization of the wave function to give the final observable cross section σ. We have chosen a normalization such that $N_2 = 1$

and $F(E_i) = v/V$, where $v = \sqrt{2E_i/\mu}$ is the relative velocity between the two particles. We may express the cross section in terms of the standard expression $\sigma = S(E_i)B(E_i)/E_i$ (see Eq. 2). Here $S(E_i)$ is assumed to be a slowly varying function of E_i. Its value at $E_i = 0$ for the process in Eq. 5 is given by $S(0) = 2.5 \times 10^{-4}$ keV barn [41]. We should point out that this is valid for low energies where the barrier penetration factor dominates. However once the energies become higher than the Coulomb barrier, i.e. of order MeV, it may no longer be reasonable to drop the energy dependence of $S(E_i)$.

In order to proceed further we need the amplitude I_{fi} in Eq. 19. This involves the unknown nuclear wave function $R_f(r)$ and one may compute it using a model nuclear potential. Since we know the cross section for this process we may also extract its form from Eqs. 2 and 19. The cross section contains an overall factor of $1/E_i$ besides the exponential suppression factor. Hence we expect the matrix element would be proportional to $1/E_i^{1/4}$. This is because in the cross section there is an additional factor $1/\sqrt{E_i}$ arising due to the flux factor. We assume that the amplitude does not show rapid oscillation with E_i in the overlap region and hence we can assume its energy dependence to be of the form

$$I_{fi} \propto \frac{\sqrt{B(E_i)}}{E_i^{1/4}} \tag{22}$$

A more detailed, first principles, procedure would involve solving the Schrodinger equation using a model nuclear potential. We may get a reasonable idea by using the Coulomb wave functions obtained in Ref. [46]. These have also been used to compute the proton fusion reaction rate [47] and lead to exactly the same energy dependence as given in Eq. 22. This also agrees with the overall energy dependence of the amplitude that can be extracted from the WKB analysis [41].

We may now extract the full form of this amplitude from Eqs. 2 and 19. We point out that the factor ω_f in Eq. 16 is the frequency of the final state photon and will involve a typical nuclear energy scale. This will have a relatively slow dependence on E_i. We, therefore, propose the following model for I_{fi}:

$$I_{fi} = \frac{\sqrt{3}c}{2\xi\sqrt{\alpha\omega_f}} \frac{\hbar}{|E_f - E_i|} \sqrt{\frac{S(E_i)}{E_i} B(E_i) F(E_i) N_2} \tag{23}$$

where we have arbitrarily set the overall sign of the amplitude to be positive. This will cancel and does not contribute to the final result. Here we have used the fine structure constant $\alpha = e^2/(4\pi\hbar c)$. The extra factor of 4π in the denominator (in comparison to atomic units) arises since we follow the conventions used in [40]. This model is reasonable at low energies. Once the energy becomes of order MeV this will break down. This is because it may no longer be reasonable to assume that $S(E_i)$ is a slowly varying function of E_i and furthermore the amplitude may also show oscillations with E_i. For such energies it may be better to extract the form of this amplitude by a model calculation. The precise energy where it breaks down depends on the Coulomb barrier, which is higher for nuclei of larger atomic number. As mentioned above, a more detailed analysis can be performed by using a model nuclear potential, which we postpone to future work.

We should point out that here we have considered the fusion process from an initial state with $l = 0$ for which the Coulomb barrier is smallest. This is expected to be applicable at low energies. The final state has to be an excited state of ^{3}He with $l = 1$. The calculation above can be easily generalized for different initial and final states. The detailed form obtained in this section will be used in the calculation of the proposed process in the next section. However most of the factors will cancel out and the most important factors that will contribute are those contained in Eq. 2. We should also mention that here we are considering this process only as an illustration. This is applicable to all processes with a photon in the final state. Furthermore the calculation can be easily generalized for different final states.

3 Photon Induced Fusion

In this section we propose a specific higher order process which may give a large contribution at low energies inside a medium. We are specifically interested in a medium which is being driven by some external agent, as an electrochemical reaction [48]. Our proposed process basically relies on the existence of an incident flux of photons, which may be real or virtual, although for our analysis here we shall consider them to be real. Hence a medium is not absolutely required as long as we create conditions such that this flux exists. We load the system with a large number of light nuclei, such as protons or deuterons, perhaps through an electrochemical process and hence the system is also constantly changing with time. These particles may become bound to other particles and some may be in quasi-free state inside matter. The potential is shown schematically in Fig. 2 with the particle being in a bound state at energy E. The potential may be split into two parts the molecular (or atomic) and the nuclear potential, each of which hosts a tower of states, which may be termed molecular and nuclear states respectively. By molecular potential we mean the potential energy of two nuclei shielded by electrons when they are far away from one another such that the nuclear interaction is negligible. The nuclear potential refers to the potential experienced by the nuclei when they are within a few fermi distance from one another. These are illustrated in Fig. 2 by the potential wells at large and small distances respectively from the origin. The wave functions for these two eigenstates peak in the two respective potential wells and decay rapidly in the adjacent well.

The first order calculation of the fusion process is reviewed in the previous section. The main change in this calculation in a medium would be that at very low energies the two particle system would be in a bound state. For somewhat higher energies we may take it to be a quasi-free state within the medium. By this we mean that its wave function would be modified by the medium and the particles cannot have infinitely large separations. However compared to the typical scale of the nuclear force we may treat the incident wave almost as free. We point out that this is being done only for convenience. A more accurate analysis would solve the Schrodinger equation in medium in order to determine the wave functions of such particles and use them directly for the computation of the proposed process.

The proposed reaction is higher order in comparison to the reaction in the previous section and normally it will be suppressed in comparison. However at low energies, the cross section corresponding to the first order process falls very rapidly. Hence, it is possible that as the energy of the system goes down a higher order process starts to dominate. Here we examine such a

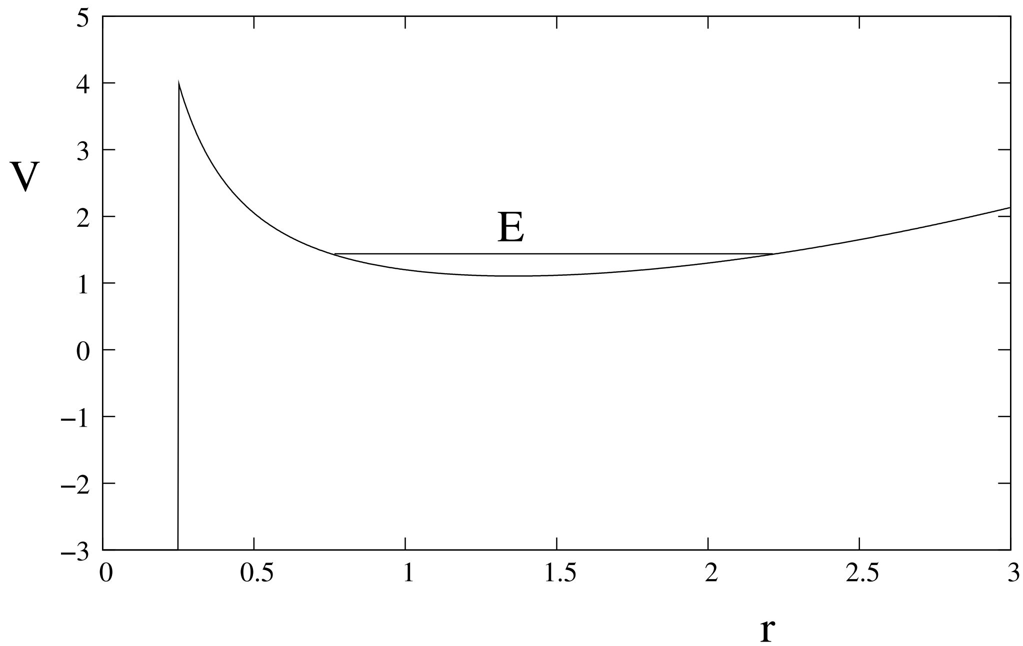

Figure 2: A schematic representation of the potential experienced by nucleus 1 at energy E inside a medium. The potential is centered at the position of another nucleus (2) in the medium. At very short distances the particle 1 experiences the nuclear force due to particle 2. At large distances it experiences the Coulomb force due to interactions with all particles in the medium. The potential levels off at large distances, not shown in the figure.

possibility in detail.

Let us assume that the initial state nucleus interacts with a photon. This may be a free photon or a virtual photon due to scattering from a charged particle. Here we will assume it to be free. This acts as a perturbation on the system and now the entire process proceeds within the framework of second order perturbation theory. The specific process we are considering may be written as

$$^1\mathrm{H} + {}^2\mathrm{H} + \gamma(\omega_i) \to {}^3\mathrm{He} + \gamma(\omega_f) \tag{24}$$

The process involves three particles in the initial state and would depend on the incident photon flux. Alternatively we may also have reactions induced by an incident flux of electrons and other charged particles which interacts with the nuclei by exchange of a photon.

We may write the second order contribution to the transition amplitude as

$$\langle f|T(t_0,t)|i\rangle = \left(-\frac{i}{\hbar}\right)^2 \sum_n \int_{t_0}^{t} dt' e^{i(E_f-E_n)t'/\hbar}\langle f|H_I(t')|n\rangle \int_{t_0}^{t'} dt'' e^{i(E_n-E_i)t''/\hbar}\langle n|H_I(t'')|i\rangle \tag{25}$$

with $H_I(t)$ given by Eq. 9. We point out that in the present case we have a photon present in the initial state also. We take its frequency to be ω_i whereas the frequency of the final state photon is taken to be ω_f. We obtain two contributions to the amplitude [40]. In one of these the initial state photon first gets annihilated and later the final state photon is emitted. In the second the time order of these photons is reversed.

We start by considering the first process. Since we need to annihilate the photon in the initial state, the time dependence of $H_I(t'')$ is $e^{-i\omega_i t''}$. The t'' integral then gives

$$\begin{aligned}\int_{t_0}^{t'} dt'' e^{i(E_n-E_i)t''/\hbar}\langle n|H_I(t'')|i\rangle &= -\frac{i\hbar e\xi}{\mu}\sqrt{\frac{\hbar}{2\omega_i V}}\langle n|\vec{\epsilon}_\beta\cdot\vec{p}\,|i\rangle \\ &\times \frac{e^{i(E_n-E_i-\hbar\omega_i)t'/\hbar} - e^{i(E_n-E_i-\hbar\omega_i)t_0/\hbar}}{E_n-E_i-\hbar\omega_i}\end{aligned} \tag{26}$$

In order to compute the transition matrix element in Eq. 25 we substitute the expression for the integral over t'' given in Eq. 26. We see from Eq. 26 that it involves two separate terms. The second term depends on the arbitrary parameter t_0 which needs to be set equal to $-\infty$ at the end of the calculation. The integral over t' for this term will set energy $E_f = E_n - \hbar\omega_f$ where ω_f is the frequency of the photon emitted in this process. The process under consideration would have $E_f + \hbar\omega_f = E_i + \hbar\omega_i$ where $E_i = E$ is the energy of the initial two particle system. Hence this term will give a contribution smaller than the standard leading order term discussed in section 2 and will be dropped. In any case we expect that the perturbation would go to zero as $t_0 \to -\infty$ and hence this term should be set to zero in any case.

We next evaluate the transition amplitude Eq. 25. The transition matrix element can be written as

$$\langle f|T(t,t_0)|i\rangle = i\frac{e^2\xi^2}{2\mu^2 V}\sqrt{\frac{1}{\omega_i\omega_f}}\int_{t_0}^{t} dt' e^{i(E_f-E_i-\hbar\omega_i+\hbar\omega_f)t'/\hbar}\sum_n \frac{\langle f|\vec{\epsilon}_{\beta'}\cdot\vec{p}\,|n\rangle\langle n|\vec{\epsilon}_\beta\cdot\vec{p}\,|i\rangle}{E_n-E_i-\hbar\omega_i} \tag{27}$$

To proceed further we replace the operator $\vec{p}$ in the second matrix element in terms of the commutator of H_0 and $\vec{r}$ using Eq. 18. We obtain

$$\langle f|T(t,t_0)|i\rangle = -\frac{e^2\xi^2}{2\mu\hbar V}\sqrt{\frac{1}{\omega_i\omega_f}}\int_{t_0}^{t} dt' e^{i(E_f-E_i-\hbar\omega_i+\hbar\omega_f)t'/\hbar}\mathcal{M} \tag{28}$$

where

$$\mathcal{M} = \sum_n \left[\frac{\langle f|\vec{\epsilon}^{\,\prime}_{\beta'}\cdot\vec{p}\,|n\rangle\langle n|\vec{\epsilon}_{\beta}\cdot\vec{r}\,|i\rangle}{E_n - E_i - \hbar\omega_i} + \frac{\langle f|\vec{\epsilon}_{\beta}\cdot\vec{p}\,|n\rangle\langle n|\vec{\epsilon}^{\,\prime}_{\beta'}\cdot\vec{r}\,|i\rangle}{E_n - E_i + \hbar\omega_f}\right](E_n - E_i) \tag{29}$$

Here we have also added the second contribution in which the time order of the photons is reversed, as mentioned earlier. However, it is clear that the second term is likely to be suppressed. This is because the denominator in this term involves the energy of the emitted photon which is generally quite large, of the order of 1 MeV. Hence it will always be much larger than the denominator in the first term except when E_n becomes comparable to $\hbar\omega_f$, i.e. of order MeV. However for such high energies, at which the Coulomb barrier is no longer relevant, it may not be reasonable to drop the energy dependence of the factor $S(E)$ in Eq. 2 for the fusion cross section. Furthermore the integral I_{fi} may no longer have a simple dependence on E_i and Eq. 23 may not be valid for such high energies. For this reason we shall terminate the sum over n at energies of order 500 KeV. For energies smaller than these the second term in Eq. 29 is expected to give a much smaller contribution than the first. Since here we are interested only in an order of magnitude estimate of the effect it is reasonable to just focus on the first term in the amplitude given in Eq. 29. The precise upper cutoff on the energy scale would depend on the reaction under consideration and is expected to be higher for larger values of the atomic number due to increase in the Coulomb barrier.

Using this we obtain the transition rate as before,

$$\frac{d\tilde{P}}{dt} = \frac{1}{\Delta T}\int dE_{\gamma f}\rho_\gamma|\langle f|T(t_0,t)|i\rangle|^2 \tag{30}$$

where $E_{\gamma f} = \hbar\omega_f$ is the final state photon energy and ρ_γ is the number density of photon states at energy $E_{\gamma f}$, given by Eq. 14. In this case the time integral will be proportional to $\Delta T\delta(E_f - E_i - \hbar\omega_i + \hbar\omega_f)$ where, as in the last section, $\Delta T = t - t_0$ is the total time. This transition rate corresponds to incident photon flux density of c/V. Hence we should divide by this flux density and multiply by the experimental flux density per unit frequency interval F_γ and integrate over frequency ω_i. We should also sum over the final photon polarizations and average over the initial photon polarizations. This leads to

$$\begin{aligned} \frac{dP}{dt} &= \frac{1}{\Delta T}\int d\omega_i F_\gamma \frac{V}{c}\int dE'_\gamma\rho_\gamma|\langle f|T(t_0,t)|i\rangle|^2 \\ &= \frac{\alpha^2\xi^4}{\mu^2c^2}\int d\Omega' d\omega_i F_\gamma\frac{\omega_f}{\omega_i}\frac{1}{2}\sum_{\beta\beta'}|\mathcal{M}|^2 \end{aligned} \tag{31}$$

Using this we can obtain the cross section by using Eq. 21.

The calculation involves a matrix element $\langle n|\vec{\epsilon}_\alpha\cdot\vec{r}|i\rangle$ which depends on the wave functions inside the medium. This is an important amplitude which would require detailed modelling of particle wave functions in medium. Here we shall make an estimate of this amplitude by assuming that both initial and final states behave approximately as free particle states within the medium. We expect that this approximation would be valid over a certain length scale beyond which the wave function will fall rapidly. This length scale is also expected to increase with energy. For our calculation we shall assume a value for this length scale that is much larger than the typical interatomic distance. For a more reliable estimate we would need a detailed form of the wave functions which we do not pursue in this paper and simply assume free particle wave functions over a certain distance scale. This should give a reliable order of magnitude estimate. The free particle is taken to be moving in the z direction. We can expand its wave function in terms of Legendre polynomials $P_l(\cos\theta)$, such that

$$e^{ikz} = \sum_{l=0}^{\infty} A_l j_l(kr) P_l(\cos\theta) \tag{32}$$

where j_l are the spherical Bessel functions and $A_l = i^l(2l+1)$. Here we shall take the initial particle to be in the $l = 1$ state and final (or intermediate) particle corresponding to state $|n>$, to be in $l = 0$ state. Hence we only need to consider these terms in the expansion. The overall normalization of the wave function A is set equal to $1/\sqrt{V}$, as usual.

We take the final state ($|n\rangle$) in the matrix element under consideration to be the s-wave state since fusion process is maximal for this state, although it is possible that our calculated cross section may not have as strong a dependence on l as the leading order process. In any case, this particular choice is being made for a sample calculation. In general we may assume states different from the ones taken here. The initial state is taken to have $l = 1$. We have taken the initial wave to be propagating in the z direction and are considering a transition from $l = 1, m = 0$ to $l = m = 0$ state. Hence we can replace $\vec{\epsilon}_\beta \cdot \vec{r}$ by $z\ \vec{\epsilon}_\beta \cdot \hat{z}$. Since the initial photon is unpolarized we need to average over the two polarization vectors. We obtain

$$\langle n|\vec{\epsilon}_\beta \cdot \vec{r}|i\rangle = \vec{\epsilon}_\beta \cdot \hat{z} \int d^3r\ \psi_n^* z \psi_i = i\frac{4\pi}{V} I\ \vec{\epsilon}_\beta \cdot \hat{z} \tag{33}$$

where

$$I = \int_0^L dr r^3 j_0(k'r) j_1(kr) \tag{34}$$

Here the factor $1/V$ arises due to the overall normalization factors $A = 1/\sqrt{V}$ in the two wave functions, and the functions $j_1(kr)$ and $j_0(k'r)$ due to the initial and intermediate state ($|n\rangle$) wave functions respectively. Furthermore k and $k' = k_n$ are the wave numbers associated with the initial and intermediate states respectively. The Bessel functions are given by

$$\begin{aligned} j_0(k') &= \frac{\sin k'r}{k'r} \\ j_1(kr) &= \left[\frac{\sin kr}{k^2r^2} - \frac{\cos kr}{kr}\right] \end{aligned} \tag{35}$$

With this we obtain

$$I = \frac{L}{2k'k}\left[\frac{\cos(k'+k)L}{k'+k} + \frac{\cos(k'-k)L}{k'-k}\right] \tag{36}$$

This is valid for $k' > k$. We will use this only in the limit $k' >> k$. In this limit we may drop the factor k in both the denominators. Hence

$$I \approx \frac{L}{2k'^2 k} \, \left[\cos(k'+k)L + \cos(k'-k)L\right] \tag{37}$$

We emphasize that the wave functions being used represent a simple model within the medium. We shall require wave functions of relatively high energy within condensed matter medium. A detailed study of such wave functions is postponed to future research. It may be reasonable to consider some generalizations in order to determine how they might affect our result. For example, at large distance in comparison to nuclear size, the dominant change from the free particle wave functions is expected to be an energy dependent phase shift. We have performed preliminary calculations of such phase shifts by using a reasonable nuclear potential and a Coulomb potential including electron screening effects. We find that the final results are same up to a factor of order unity. Hence we do not expect such generalizations to change our result in a fundamental manner.

We next obtain the cross section. As in the case of leading order calculation, we divide the transition rate in Eq. 31 with the number of target particles N_2 and the incident flux $F = vn_1$ in order to obtain the fusion cross section. Here v is the relative velocity between the two particles. This will essentially involve a division by a factor v/V where v is the relative velocity corresponding to energy E_i. The final nuclear state with quantum numbers (l', m') has $l' = 1$ and we also need to sum over m', as in the previous section. We obtain

$$\sigma^{(2)} = \frac{16\pi^2\alpha^2\xi^4}{\mu^2 c^2 V^2} \int d\Omega' d\omega_i F_\gamma \frac{\omega_f}{\omega_i} \frac{1}{2} \sum_{\beta\beta' m'} \left| \sum_n \frac{E_n - E_i}{E_n - E_i - \hbar\omega_i} I \langle f | \vec{\epsilon}'_{\beta'} \cdot \vec{p}\, | n\rangle \right|^2 \frac{\vec{\epsilon}_\beta \cdot \hat{z} \vec{\epsilon}_\beta \cdot \hat{z}}{N_2 F(E_i)} \tag{38}$$

The factor 1/2 before the sum over polarizations β, β' is due to averaging over the initial polarizations. We next average over the initial polarization vector orientations. This involves integral over $d\Omega$ corresponding to the angles of the initial photon polarization vector and division by 4π. This leads to an overall factor of 1/3 for each $\beta = 1, 2$. Hence we obtain

$$\sigma^{(2)} = \frac{16\pi^2\alpha^2\xi^4}{3\mu^2 c^2 V^2} \int d\Omega' d\omega_i F_\gamma \frac{\omega_f}{\omega_i} \sum_{\beta' m'} \left| \sum_n \frac{E_n - E_i}{E_n - E_i - \hbar\omega_i} I \langle f | \vec{\epsilon}'_{\beta'} \cdot \vec{p}\, | n\rangle \right|^2 \frac{1}{N_2 F(E_i)} \tag{39}$$

We next need the matrix element $\langle f | \epsilon'_{\beta'} \cdot \vec{p} | n\rangle$ which involves an overlap with a nuclear state. Using Eq. 18 we can express this as,

$$\langle f | \vec{\epsilon}'_{\beta'} \cdot \vec{p}\, | n\rangle = \frac{i\mu}{\hbar}(E_f - E_n) \langle f | \vec{\epsilon}'_{\beta'} \cdot \vec{r}\, | n\rangle \tag{40}$$

The matrix element on the right hand side can be expressed as

$$\langle f | \vec{\epsilon}'_{\beta'} \cdot \vec{r}\, | n\rangle = \int d^3 r \psi_f^* \vec{\epsilon}'_{\beta'} \cdot \vec{r} \psi_n = N \left(\int_0^\infty dr r^3 R_f^* R_n \right) \, I_\Omega \tag{41}$$

where N is a normalization factor and I_Ω is the angular integral, given by,

$$I_\Omega = \int d\Omega \left(Y_1^{m'}\right)^* \vec{\epsilon}'_{\beta'} \cdot \hat{r} Y_0^0 \tag{42}$$

Here Y_l^m are the spherical harmonics. The important point is that the angular part decouples from the radial part. It does not depend on the energy E_n and hence can be taken out of the sum over n in Eq. 39. The angular integrals and the sums over β' and m' can now be performed exactly as in the previous section and lead to an overall factor of $8\pi/3$. We, therefore, obtain

$$\sigma^{(2)} = \frac{8\pi}{3}\frac{16\pi^2\alpha^2\xi^4}{3\hbar^2c^2V^2}\int d\omega_i F_\gamma \frac{\omega_f}{\omega_i}\left|\sum_n \frac{(E_n - E_i)(E_f - E_n)}{E_n - E_i - \hbar\omega_i} I I_{fn}\right|^2 \frac{1}{N_2 F(E_i)} \tag{43}$$

The factor I_{fn} is the radial integral

$$I_{fn} = \left(\int_0^\infty dr r^3 R_f^* R_n\right) \tag{44}$$

The integral I_{fn} can be extracted from the leading order analysis described in the previous section by using Eqs. 22 and 23. In making this identification we note that we require the matrix element in a kinematic regime different from that in Eq. 23. In obtaining Eq. 23 we had $E_i = E_f + \hbar\omega_f$ whereas in the present case $E_n \neq E_f + \hbar\omega_f$. We shall assume that extrapolation to a different kinematic regime is allowed. This is reasonable since the main change is in the energy of the emitted photon which would be different in the two cases. However this energy is large in comparison to the incident energy and can have only a weak dependence on the incident energy. We also need to take care of the fact that in the present case the incident flux factor $F(E_i)$ corresponds to energy E_i whereas this matrix element involves the initial state of energy E_n. Hence the factor $1/E_i$ in Eq. 23 would be replaced by $1/\sqrt{E_iE_n}$. Essentially the matrix element will involve a factor $1/E_n^{1/4}$. It is important to note that the barrier penetration amplitude that arises in this matrix element depends on unperturbed energy eigenvalue E_n. Hence, if we take $E_n >> E_i$, we expect that the barrier penetration factor would be much larger in the present case. Similar ideas have been proposed earlier [21, 22, 24].

Let us now explain why we expect an enhancement in the second order result at low energies. We should point out that the energy E_n can take any value. We expect that as E_n becomes large the contribution will be suppressed since the intermediate state can only be short lived. This suppression is essentially represented by the factor $(E_n - E_i - \hbar\omega_i)$ in the denominator in Eq. 39. The process is also suppressed due to the fact that it is a higher order process. Furthermore the cross section would also depend on the incident photon flux, which may not be very small. As we shall see the integral I does not lead to very significant suppression. The most important factor, namely the barrier tunnelling amplitude is contained in the amplitude in Eq. 40. Our main point is that this factor is controlled by the energy E_n rather than E_i. The energy E_n can take a rather large value and hence the barrier tunnelling suppression factor may not be too small.

Let us now extract the amplitude I_{fn} from the calculation in the previous section, as explained above. Using Eq. 23 we can write this as

$$I_{fn} = \frac{\sqrt{3}c}{2\xi\sqrt{\alpha\omega_f}}\ \frac{\hbar}{|E_f - E_n|}\sqrt{\frac{S(E_n)}{E_n}B(E_n)F(E_n)N_2} \tag{45}$$

We then obtain

$$\sigma^{(2)} = \frac{32\pi^3\xi^2\alpha}{3V^2}\int d\omega_i \frac{F_\gamma}{\omega_i}\left|\sum_n \frac{E_n - E_i}{E_n - E_i - \hbar\omega_i} I \sqrt{\frac{S(E_n)B(E_n)F(E_n)}{E_n F(E_i)}}\right|^2 \tag{46}$$

We convert the sum over n into an integral using the standard density of states. We obtain

$$\sigma^{(2)} = \frac{32\pi^3\xi^2\alpha}{3V^2}\int d\omega_i \frac{F_\gamma}{\omega_i}\left|\int dE' \rho(E') \frac{E' - E_i}{E' - E_i - \hbar\omega_i} I \sqrt{\frac{S(E')B(E')F(E')}{E' F(E_i)}}\right|^2 \tag{47}$$

where

$$\rho(E) = g\frac{L^3\mu^{3/2}\sqrt{E}}{\sqrt{2}\pi^2\hbar^3} \tag{48}$$

Here g is the internal degeneracy factor of the state and we have replaced the discrete energy values E_n by the symbol E'. We next assume that $S(E')$ is a slowly varying function of energy and set its value at $E' = 0$. In general $S(E')$ shows a slow increase with energy. Including this increase will lead to a larger value for the cross section. The factor $F(E')/F(E) = \sqrt{E'/E}$. The cross section now becomes

$$\sigma^{(2)} = \frac{16\pi^2\xi^2\alpha\mu^3 g^2}{3\pi^3\hbar^6}\frac{S(0)}{E^{1/2}}\int d\omega_i \frac{F_\gamma}{\omega_i}\left|\int dE' \frac{E' - E_i}{E' - E_i - \hbar\omega_i} I \ (E')^{1/4}[B(E')]^{1/2}\right|^2 \tag{49}$$

Inserting the expression for I from Eq. 37 we obtain

$$\sigma^{(2)} = \frac{\xi^2\alpha g^2}{6\pi}\frac{S(0)}{E^{3/2}}\int d\omega_i \frac{F_\gamma}{\omega_i}\left|L\int dE' \frac{E' - E_i}{E' - E_i - \hbar\omega_i}\frac{[B(E')]^{1/2}}{(E')^{3/4}}[\cos(k'+k)L + \cos(k'-k)L]\right|^2 \tag{50}$$

This is our final result for the cross section for the reaction given in Eq. 24. With minor modifications it can be applied to similar reactions with different nuclei, with photons in the initial and final states. For a given incident flux of photons the cross section shows an increase with decrease in photon frequency. This may appear counter intuitive from the point of view of barrier penetration but is perfectly reasonable if one realizes that larger wavelengths generally lead to an increase in cross section. The barrier penetration factor occurs inside the integral over E' for which the upper limit is ∞. For large values of E' it need not be very small. In the next section we shall perform the integral over E' and evaluate this cross section by imposing an upper limit on energy.

We point out that analysis in this section has been performed for a particular process. However it is applicable for all processes which proceed by emission of a photon. We just need to use the factors $S(0)$ and $B(E)$ relevant for the process. It can also be easily generalized for other final states in which the reaction proceeds purely by strong interactions or by weak interactions by making a suitable change in the perturbation Hamiltonian. Furthermore in the initial state we have so far assumed that the reaction is induced by a free photon. As mentioned earlier a possible generalization is that the reaction may proceed by exchange of a virtual photon with an incident electron. We also emphasize that the inverse process in which a nucleus may break up into two daughter nuclei can also be induced by this process. This means that the reaction given in Eq. 24 can proceed in both directions. In general the inverse reaction will require a photon of very high energy. However we may conceive of situations in which a nucleus is in a state that it has enough energy to break up. In that case a low energy incident photon may affect its rate through this process.

In our analysis we have considered the initial state wave functions to be free particle wave functions. The analysis can easily be generalized to the case when the initial state is bound. We do not expect any major deviations in our result for such a case and postpone this to future work. A brief discussion on this is given in section 4. Furthermore the main purpose of our analysis in to obtain an order of magnitude estimate that demonstrates that such reactions can be important at low energies. For this purpose our approximation is reasonable. More refined analysis is postponed for future research.

3.1 Estimate of the cross section

Next, we estimate the cross section for the process in Eq. 24. The cross section may be expressed as

$$\sigma^{(2)} = \frac{\xi^2 \alpha g^2}{6\pi} \frac{S(0)}{E^{3/2}} \int d\omega_i \frac{F_\gamma}{\omega_i} \, |I_1|^2 \tag{51}$$

where

$$I_1 = 2L \int_{y_l}^{y_u} dy' \, \sqrt{\frac{B(y')}{y'}} \, \left[\cos(y'+y)\tilde{L} + \cos(y'-y)\tilde{L}\right] \tag{52}$$

$\tilde{L} = \sqrt{2\mu} L/\hbar$, $y = \sqrt{E_i}$ and $y' = \sqrt{E'}$. Here we have assumed that $E' >> E_i, \hbar\omega_i$ and set $E' - E_i \approx E'$ and $E' - E_i - \hbar\omega_i \approx E'$. The integral I_1 is expected to show oscillatory dependence on L. As explained earlier the length L has a physical interpretation. It is the distance scale over which the wave function acts approximately as a free particle. Beyond this distance it will decay rapidly within the medium. This physical picture would be valid only for energies smaller than an upper limit. Once the energy becomes very large, the wave function may spread over the entire medium since the Coulomb repulsion may no longer suppress it. For very high energies of order MeV, we may not be able to ignore the energy dependence of $S(E)$ in the basic formula for the cross section (Eq. 2). Hence we will impose an upper limit of about 500 KeV on this integral. The precise upper limit would depend on the reaction under consideration since the Coulomb repulsion is more effective for a high Z nucleus. Furthermore we will average the integral squared $|I_1|^2$ over a certain range of length scales. This is justified by the fact that we may have a range of initial state energies and we should average over them. For different energies we may need to choose slightly different values of L. Furthermore the medium may not be homogeneous and wave function may depend on position. Hence it is reasonable to add over contributions from a range of cut off length scales. We choose the range $20a_0$ to $20.2a_0$, where a_0 is the Bohr radius, in order to average over L. The value of $|I_1|^2$ is not very sensitive to these precise values provided it is averaged over a sufficiently large range.

In order to get an order of magnitude estimate we assume an experiment with an incident current of 1 ampere/cm^2 on some target medium. This corresponds to an electron flux of $(1.6 \times 10^{-19})^{-1}$ sec$^{-1} \cdot$ cm^{-2}. Assuming a potential difference of 10 eV, we expect maximum energy of electrons to be 10 eV. As these electrons collide with particles in the medium photons will be produced. Typically these photons will have energies significantly smaller than that of electrons. The electrons are expected to undergo multiple collisions, loosing energy at each collision. Hence we expect a significant flux of photons with energies substantially lower than 10 eV. Here we do not perform a detailed modelling of this process and assume that this leads to a photon flux of similar order as that of electrons with frequencies of photons in the infrared range. This is also reasonable since we do not expect the temperature in the medium to rise above 1000 K which will correspond to infrared frequencies. Furthermore we point out that the photon flux that we have assumed is also of same order of magnitude as obtained for a blackbody at room temperature of roughly 300 K. Hence we set $\omega_i = 2\pi\nu_i$ with $\nu_i = 5 \times 10^{13}$ Hz. This leads to

$$\int d\omega_i \frac{F_\gamma}{\omega_i} = \frac{1}{1.6 \times 10^{-19}} \; \frac{1}{2\pi \times 5 \times 10^{13}} \text{ cm}^{-2} \tag{53}$$

We set $g = 2$ assuming that the fusion proceeds by an initial state of spin 1/2. The initial state has $l = 0$ and the spins of proton and deuteron can combine to lead to total angular momentum of 1/2 and 3/2. Here we consider the contribution from the spin 1/2 state. The value of $S(0)$ for the reaction given in Eq. 5 is 2.5×10^{-4} keV barn. We take the initial state energy of $E_i = 10$ eV corresponding to a potential difference of 10 V.

We compute the integral $|I_1|^2$ by setting the lower limit on energy E' equal to 20 eV. We first compute it by ignoring screening effect, which will be included later. The integral fluctuates with L and hence we average it over a range of length scales for reasons discussed earlier. Its value as a function of the upper limit on energy E' after averaging over $L = 20a_0$ to $20.2a_0$ is shown in Fig. 3. We also show the result for the range $L = 20$ to 40 a_0 and for $L = 50$ to 60 a_0. We see that the three results agree reasonably well and hence find that the answer is not too sensitive to precise values of L chosen for averaging. We have also checked this calculation for a relatively low value of $L = 2$ to 2.2 a_0. This also gives a similar result. We find that $(I_1)^2$ increases with energy and then shows saturation beyond $E' = 300$ KeV. Actually at high energies it shows small fluctuations about a value of roughly 3.8×10^{-6}. As explained above our formalism may require modifications for sufficiently high energies and it is encouraging that we find approximate saturation near the upper limit shown in Fig. 3. We conservatively take the value of $(I_1)^2$ to be of order 3×10^{-6} in atomic units. For the values given we obtain

$$\sigma^{(2)} = 1.4 \times 10^{-25} \; I_1^2 \tag{54}$$

in atomic units. This leads to

$$\sigma^{(2)} = 9 \times 10^{-28} \; I_1^2 \approx 1.3 \times 10^{-47} \text{ cm}^2 \tag{55}$$

This exceeds the result obtained from Eq. 2 for $E = 10$ eV by an extremely large factor if we use the standard barrier penetration factor [41] given in Eq. 3,

$$B = \exp[-b/\sqrt{E}] \tag{56}$$

In the above estimate we have ignored screening. We now take these effects into account. For this purpose we use the approximate fit given in Ref. [15]. It finds that the barrier penetration factor may be approximated as $\exp(-4.13\sqrt{\mu})$. This is likely to give a reasonable order of magnitude estimate for the present case of $E = 10$ eV. More refined estimation is postponed to

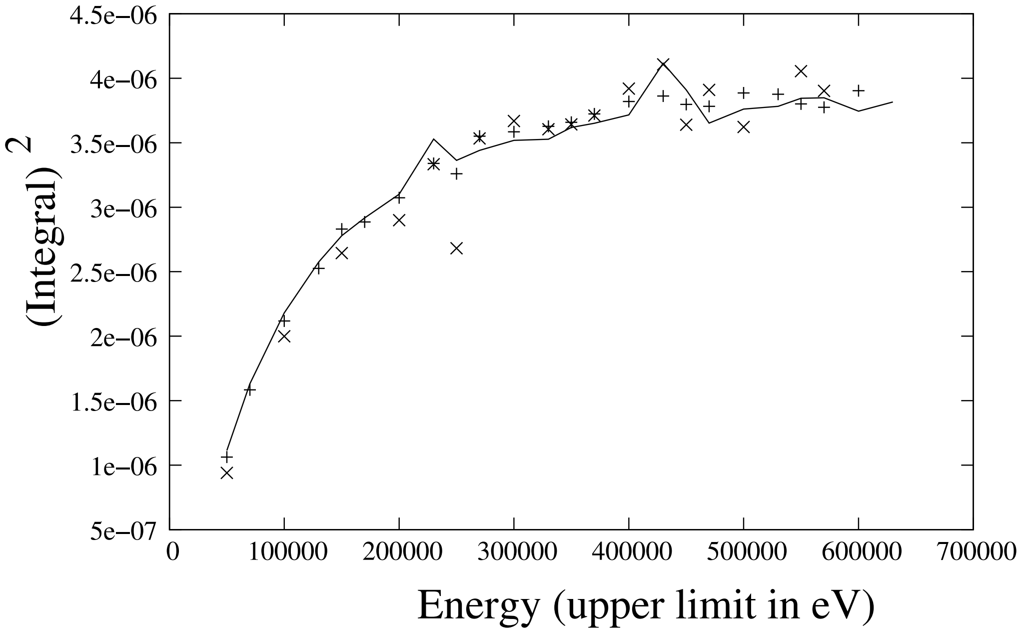

Figure 3: The square (I_1^2) of the integral I_1 in atomic units as a function of the upper limit on energy. The solid line is obtained by averaging over L in the range 20-20.2 a_0 while the points (+) and ($\times$) are obtained by averaging over $20 - 40$ a_0 and $50 - 60$ a_0 respectively.

future research. For the reaction under consideration this leads to an effective screening energy of approximately $E_s = 21$ eV, i.e. the barrier penetration factor in Eq. 56 agrees with this fit for energy equal to $E + E_s = 31$ eV. Using this screening energy we obtain the standard cross section for this process to be roughly equal to 10^{-88} cm^2 for $E = 10$ eV. Hence we find an enhancement of approximately 41 orders of magnitude. This is sufficiently large and makes low energy nuclear reactions observable in laboratory at conditions close to ambient temperature and pressure, but with electromagnetic perturbations analyzed in this work. In fact this is large enough to suggest that some of the experimental claims made on low energy nuclear reactions or cold fusion are reliable. We point out that our result does not show a very significant dependence on the screening energy.

In the estimate above we have ignored the energy dependence of the factor $S(E)$. This factor is expected to show a slow increase with energy which will lead to a further enhancement of the cross section and the reaction rate.

In our calculation above we have assumed the incident particle of 10 eV. It is interesting to see how our results would change if we took a much lower value of energy, such as 1 eV or less than an eV. In this case we would need to terminate our integral I_1 at much smaller values of the length scale L since now it will act as a bound state with a length scale of few a_0. As we have already remarked the values of $|I_1|^2$ even in this case are very similar to what is obtained earlier. Hence we do not expect any significant suppression even in this case due to barrier penetration factor and rates are likely to be very similar to what we have obtained in this section. Essentially it seems that this process does not have a very strong dependence on energy in comparison to what is seen in the leading order process discussed in section 2.

Our analysis can easily be generalized to other processes which may or may not have photon in the final state. Here we briefly comment on what may be expected in the case of photon induced fusion of a light nuclei, such as proton, with a high Z nuclei with the production of a photon. For this, our formalism may be applied directly with a suitable change in some parameters, such as, the reduced mass and the exponent in the barrier penetration factor $B(E)$. We have made some preliminary estimates of the integral I_1 for the case of $Z_1 = 1$, $Z_2 = 20$, $A_1 = 1$ and $A_2 = 40$. We find that in this case also $|I_1|^2$ is not very small, at most few orders of magnitude smaller than that in Fig. 3 obtained for the process given in Eq. 24. In some of these cases the factor $S(E)$ may be much larger than the current process and hence we expect that fusion with even high Z nuclei may occur at observable rates in a laboratory at relatively low energies and temperatures.

3.2 Cross section at Solar energies

We next evaluate the contribution of our proposed reaction at solar energies. We compare the cross section for the reaction given in Eq. 5 with the cross section for the reaction in Eq. 24. Using the standard formula Eq. 2 we find that at energy of 1 keV, relevant to solar interior, the cross section for Eq. 5 is roughly 10^{-39} cm^2. In order to estimate the cross section for Eq. 24 we consider the flux of blackbody photons at temperature of 10^7 K. The photon flux coming out of a spherical surface of small radius in solar interior, centered at the center of the Sun, is found to be approximately 3×10^{32} s$^{-1} \cdot$ cm^{-2}. Here we have assumed a mean energy of photons of order 1 keV. Dividing by the angular frequency ω for such photons we find that the flux factor leads to an enhancement factor of roughly 6×10^9 in comparison to Eq. 53. The energy denominator in Eq. 51 leads to a suppression factor of $1/(100)^{3/2}$. Taking all this into account we find the cross section for Eq. 24 to be roughly 10^{-41} cm^2 about 2 orders of magnitude lower than the cross section for the standard reaction. The lower value is obtained mainly because the cross section for our mechanism shows a very mild dependence on the initial energy $E = E_i$ whereas the standard reaction rises very sharply with E. It is clear that at solar energies our process gives a relatively small contribution and the standard process dominates. The difference is only about two orders of magnitude and hence we expect that somewhat below solar energies our proposed mechanism will start to play an important role. Hence it may play a role in stars of mass smaller than Sun, which can be investigated in future research.

4 Fusion from a bound initial state

In our analysis so far we have assumed that the initial two particle state can be approximated as a free particle state. As mentioned above this is only an approximation and for most energies the wave function of this state inside a medium will decay beyond a certain length scale. Hence it may be considered as a bound state whose spatial extent is likely to be large compared to a typical molecular bound state. For accurate calculations we will require such high energy wave functions in the energy range 10 eV to few MeV inside the condensed matter medium. The formalism developed in sections 2 and 3 can be easily generalized to handle such bound states.

Let us first consider the basic process Eq. 5 discussed in section 2. We now consider the initial state $|i\rangle$ as a bound state. The calculation of the transition rate dP/dt goes through exactly as earlier and we obtain Eq. 16. The transition rate in this case is conveniently expressed in terms of the rate constant A [49], whose expression can directly be extracted from Eq. 16 in terms of the nuclear wave function. The cross section can again be written as in Eq. 21 and we shall choose the wave function normalization such that $N_2 = 1$. However the flux factor $F(E)$ is now more complicated since we are not dealing with a plane wave. Here we do not need to go into such details since all we require is its energy dependence which is same as before. Hence it is again equal to the velocity v divided by a factor which has dimensions of volume. Its dependence on energy again goes as $\sqrt{E}$. With this understanding the final equation for the amplitude, Eq. 23, remains unchanged.

We next consider the photon induced reaction Eq. 24. In this case the calculation proceeds exactly as in section 3 up to Eq. 31. Using this we determine the cross section $\sigma'^{(2)}$ by the relation

$$\sigma'^{(2)} = \frac{dP}{dt}\frac{1}{N_2 F(E_i)} \tag{57}$$

We again focus on only the first term in the matrix element $\mathcal{M}$ in Eq. 29. Substituting this into Eq. 57 and using the amplitude in Eq. 45 we obtain

$$\sigma'^{(2)} = 2\pi\alpha\xi^2 S(0)\int d\omega_i \frac{F_\gamma}{\omega_i}\frac{1}{2}\sum_\beta\left|\sum_n \sqrt{\frac{B(E_n)}{E_n}}\left(\frac{E_n}{E_i}\right)^{1/4}\langle n|\vec{\epsilon}_\beta\cdot\vec{r}|i\rangle\frac{E_n - E_i}{E_n - E_i - \hbar\omega_i}\right|^2 . \tag{58}$$

Here we have summed over the final state photon polarizations as in section 3 and set the ratio $F(E_n)/F(E_i) = \sqrt{E_n/E_i}$. The remaining matrix element in the above equation can be expressed as

$$\langle n|\vec{\epsilon}_\beta\cdot\vec{r}|i\rangle = \int d^3r Y_0^0 R_{00}^*(E_n)\vec{\epsilon}_\beta\cdot\vec{r}R_{10}(E_i)Y_1^m \tag{59}$$

Here we have set $l = 1$ and $l = 0$ for the initial state and intermediate states ($|n\rangle$) respectively and $R_{10}(E_i)$ and $R_{00}(E_n)$ represent the corresponding radial wave functions. By a suitable choice of coordinates we can take the initial state to be $m = 0$ state. We then obtain

$$\langle n|\vec{\epsilon}_\beta\cdot\vec{r}|i\rangle = \frac{1}{\sqrt{3}}\vec{\epsilon}_\beta\cdot\hat{z}I_1 \tag{60}$$

where

$$I_1 = \int dr R_{00}^*(E_n)r^3 R_{10}(E_i) \tag{61}$$

Substituting back in Eq. 58 and averaging over the initial state polarization orientations and summing over β we obtain

$$\sigma'^{(2)} = \frac{2\pi\alpha\xi^2 S(0)}{9}\int d\omega_i \frac{F_\gamma}{\omega_i}\left|\sum_n \sqrt{\frac{B(E_n)}{E_n}}\left(\frac{E_n}{E_i}\right)^{1/4} I_1\frac{E_n - E_i}{E_n - E_i - \hbar\omega_i}\right|^2 . \tag{62}$$

We can now solve the Schrodinger equation inside the medium in order to obtain the radial wave functions R_{00} and R_{10} for a wide range of energies. We expect that the initial state wave function R_{10} would be required for a relatively low energy of order 10 eV, while the final wave function R_{00} would be required for a rather large range of energies extending up to order of MeV. For low energies we may obtain the bound state eigenfunctions while for high energies it may be convenient to use continuum eigenfunctions. In any case we see that the calculation is well defined once we know the precise eigenfunctions.

5 Discussion and Conclusions

We have introduced a new mechanism for nuclear fusion which may play an important role at low energies. We propose that the reaction takes place by a perturbation. In most of our analysis we have assumed that this perturbation is caused by a flux of real photons. However it may also take other forms. For example, a flux of electrons or other charged particles may also induce these reactions by exchange of virtual photons. The fusion reaction then proceeds by forming a virtual state whose energy is unrestricted and we need to consider contributions from all energies. Due to the high energy possible for these states the barrier penetration probability is not very small and it does not lead to a strong suppression of the cross section. The dominant suppression in the current case arises from the incident flux factor of photons or other particles, the use of relatively high energy intermediate states and the fact that the process involves second order in perturbation theory. The barrier penetration factor also leads to a suppression but it is much milder in comparison to the standard process.

We have performed a detailed calculation of the Hydrogen and deuteron fusion reaction to form He-3. Assuming a realistic incident photon flux at infrared frequencies we find that the cross section for this reaction is at least 41 orders of magnitude higher than the standard fusion reaction at incident energy of 10 eV. It is sufficiently large to make such fusion reactions observable in laboratory at low energies under conditions much milder than what is required for the conventional thermonuclear fusion. Furthermore this also provides an explanation for many of the claims made in the area of low energy nuclear reactions. However a detailed calculation is needed for each experimental situation in order to evaluate its applicability. The formalism is applicable to a wide range of processes. Remarkably the Coulomb barrier penetration factor shows a relatively mild dependence on the charge of the nuclei in comparison to the standard reaction. Hence even fusion with high Z nuclei corresponding to lattice sites is possible at low energies at observable rates. This probably explains the large number of new elements seen in many low energy nuclear reaction experiments.

We require wave functions for the two body nuclear system in a medium for which we have assumed that the free particle wave functions provide a good approximation. This can be improved in future for more accurate analysis but is sufficient for our order of magnitude estimates. A more refined treatment is postponed to future research. This will be required in order to make precision comparison with experimental data. It may also be interesting to design a controlled experiment whose outcome may be compared with theoretical predictions in order to test our proposal.

It seems possible that low energy nuclear reactions are happening through our proposed mechanism on a very wide scale as is often claimed by scientists working in this field. The rates are expected to be low but observable even at low energies and low temperature. The process does not show a very strong dependence on energy of the initial state nuclei but does depend on the flux of incident photons. The precise dependence needs to be determined by going beyond our order of magnitude estimate and performing a more detailed analysis. Remarkably, for fixed flux, the cross section actually increases with decrease in the energy of the incident photon [1]. Hence, for a given flux, the process is actually enhanced with decrease in the photon frequency. Due to the frequency dependence, it will be interesting to reevaluate the effect of low frequency electromagnetic waves on biological media taking our proposed reactions into consideration. Our proposal may also be used in order to design a low energy experiment or device which may provide optimal yield of fusion products. Given that the process can happen at ambient conditions it is possible that such processes have been happening slowly throughout the evolution of the Universe and may have some effect on the relative abundance of elements observed today. Such reactions may also be taking place on the surface of the Earth as well as in the interior and it will be interesting to study their implications. Hence it will be very interesting to apply our formalism to the large number of experimental claims in this area. We emphasize that with a suitable choice of interaction Hamiltonian our mechanism is applicable to all LENRs. Hence we hope our work will initiate an exciting era in the study and applications of low energy nuclear reactions eventually leading this field into precision science.

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