On Planck's Spectrum as Function of Wavelength

Sjaak Uitterdijk
SjaakenLutske@hetnet.nl

Abstract-Scrutinizing Planck's spectra as function of frequency and as function of wavelength learns that the last mentioned one leads to baffling results.

Introduction

Planck's book about this subject, originally written in 1913, has been translated to English as shown in [1]. He presents two types of spectra for the so-called black body radiation, one as function of frequency the other as function of wavelength. The second one turns out to be a scientific disaster.

The black body power density spectrum as function of frequency resp. wavelength

Planck presented the following two spectra, supplemented with his commentary in Italics:

$$K_\nu = h\nu^3 c^{-2}/(\exp(h\nu/kT) - 1) \qquad \text{W/m}^2\text{/Hz}$$

"This is the specific intensity of a monochromatic plane polarized ray of the frequency ν which is emitted from a black body at the temperature T into vacuum in a direction perpendicular to the surface."

$$E_\lambda = (hc^2\lambda^{-5})/(\exp(hc/k\lambda T) - 1) \qquad \text{W/m}^2\text{/m}$$

"This is the specific intensity of a monochromatic ray not to the frequency ν but, as is usually done in experimental physics, to the wavelength λ…"

The spectrum E_λ is incorrect for the following 3 reasons:

1. the maximum of K_ν is not at the same frequency as of E_λ
2. K_ν and E_λ show a completely incomprehensible relationship
3. the dimension of E_λ is meaningless/unphysical

ad 1 The maximum of K_ν is found for $dK_\nu/d\nu = 3\nu^2\cdot(e^{a\nu} -1)^{-1} - \nu^3\cdot(e^{a\nu} -1)^{-2}\cdot e^{a\nu}\cdot a = 3 - a\nu/(1 - e^{-a\nu}) = 0$

Approximating $1 - e^{-a\nu}$ by $a\nu - a^2\nu^2/2$ leads to $\nu = (4/3)\cdot kT/h$ Hz

Approximating $1 - e^{-a\nu}$ by $a\nu - a^2\nu^2/2 + a^3\nu^3/6$ leads to $\nu = 4\cdot kT/h$ Hz

Approximating $(e^{a\nu} -1)^{-1}$ by $e^{-a\nu}$ directly in K_ν leads to $\nu = 3\cdot kT/h$ Hz

The numerical calculation of K_ν shows that the latter approximation is accurately close to reality.
This approximation applied to E_λ and replacing $1/\lambda$ by y, leads to $E_y = hc^2y^5\cdot e^{-by}$, with $b = hc/kT$.
$dE_y/dy = 5y^4\cdot e^{-by} + y^5\cdot(-b)\cdot e^{-by} = 5 - y\cdot b = 0$, so the maximum of E_λ is found at $\nu = 5\cdot kT/h$.

ad 2 The cause of the deviation from $\nu = 3\cdot kT/h$ is *only* the power 5 of λ in E_λ.
Writing blindly hc^2/λ^3 instead of hc^2/λ^5 would lead to the dimension W/m instead of W/m²/m of E_λ.
The solution to this problem has to be found in the introduction of a constant with dimension m^{-2}, instead of the introduction, as Planck did, of λ^{-2}. However such a constant does not exist.
In order to show the mutual completely incomprehensible relationship between K_ν and E_λ there maximum values are compared.

Applying $\nu = 3\cdot kT/h$ in K_ν results in $K_{\nu max} = 9.5\cdot10^{-20}\cdot T^3$ W/m²/Hz

Applying $\nu = 5\cdot kT/h$ in E_λ results in $E_{\lambda max} = 2.0\cdot10^{-6}\cdot T^5$ W/m²/m

These results show their mutual completely incomprehensible relationship and that E_λ has to be rejected.

Ad3 The correct expression for E_λ is found when ν^3 in K_ν is be replaced by c^3/λ^3 and E_λ ritten as E_ν:

$$E_\nu = (hc/\lambda^3)/(\exp(hc/k\lambda T) - 1) \qquad \text{W/m}^2\text{/Hz}$$

The integration of this spectrum has of course to be done w.r.t. the frequency. In a numerical situation, where λ is taken as the primary variable, $\Delta\lambda = \lambda_n - \lambda_{n-1}$ has to be replaced by $\Delta\nu = c\cdot(1/\lambda_{n-1} - 1/\lambda_n)$.
This result forces us to conclude that the dimension W/m²/m of Planck's spectrum E_λ has to be rejected.

Given the surprising accuracy of the simplified spectra the graphs of K_ν and E_ν have been drawn for both the original and the simplified situation.

Regarding the outcome of the integral of the spectra: the original, carried out by extremely esoteric mathematics, see reference [2], leads to $\boldsymbol{\pi^4/15}\cdot h^{-3} c^{-2} k^4 \cdot T^4$, the simple one to $\mathbf{6}\cdot h^{-3} c^{-2} k^4 \cdot T^4$ W/m².

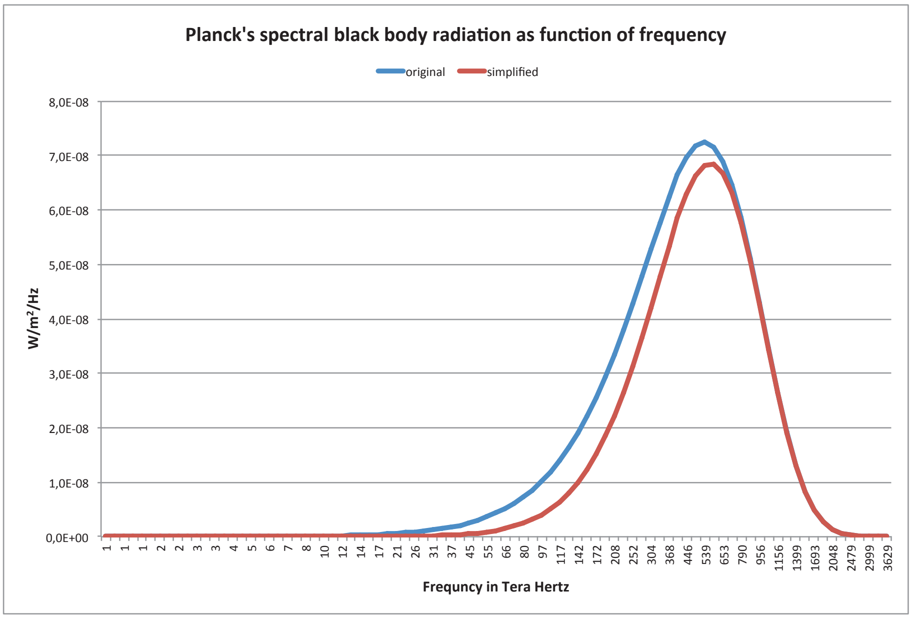

Figure of K_ν

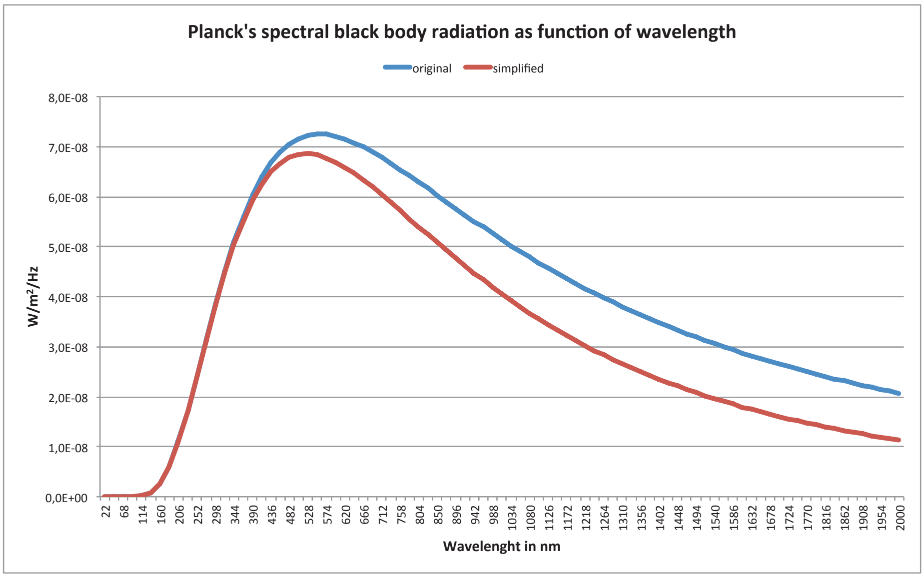

Figure of E_ν

Conclusion

The originally by Planck proposed spectrum as function of wavelength has to be rejected and replaced by the one as function of frequency, in which the variable frequency is replaced by *c* divided by wavelength.

References

[1] Planck M. The theory of heat radiation. P. Blakiston's Son & Co., Philadelphia, PA, 1914, free available at: http://www.gutenberg.org/zipcat2.php/40030/40030-pdf.pdf

[2] Stefan-Boltzmann Constant Incorrect by a Factor of 2π https://vixra.org/abs/1909.0647