New notation in series of functions.

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0- Abstract:

In this paper we will see how it is possible to establish a precise and complete notation for operators that describe a series of functions.

1- Introduction:

As we saw earlier in my articles "new nomenclature in operators"[1] and "notation for operators based on Knuth's up arrow"[2], the notion of serial operator can be expanded not only to summation and productory but also to restory, divisory, exponentory and rootory with different notations. It can even be generalized for any operators with n-th iterations. In this article I am going to be based on arrow notation, using "up arrow" for positive operators and "down arrow" for negative operators. I will also use the concept of interval introduced in previous articles.

2- Operator definitions:

2.1 – Positive operators:

In this part we will see the operators that make the function grow positive, here the sum, the product, the exponent and all the positive operations that go beyond the exponential have a place. To unify these operators in a logical way we will use the up arrow, with a unit power for the exponent as is done in the classical way and different powers depending on whether it is an operation of more or less force than the exponential.

2.1.1- Summation:

Simple notation:

(1) $$\sum_{n=a(c)}^{b} f(x) = \underset{n=a(c)}{\overset{b}{\mathrm{A}_1}} f(x) = +f(a)+f(a+c)+f(a+2c)+\ldots+f(b-2c)+f(b-c)+f(b)$$

1 “New Nomenclature in Operators” - Juan Elias Millas Vera https://vixra.org/abs/2010.0077

2 “Notation for Operators Based in Knuth’s Up Arrow” – Juan Elias Millas Vera https://vixra.org/abs/2010.0099

Arrow notation:

(2) $$\underset{n=a(c)}{\overset{b}{A_1}} f(x) = \uparrow^{-1} f(a) \uparrow^{-1} f(a+c) \uparrow^{-1} f(a+2c) \uparrow^{-1} \ldots \uparrow^{-1} f(b-2c) \uparrow^{-1} f(b-c) \uparrow^{-1} f(b)$$

2.1.2- Productory:

Simple notation:

(3) $$\prod_{n=a(c)}^{b} f(x) = \underset{n=a(c)}{\overset{b}{A_2}} f(x) = f(a) \cdot f(a+c) \cdot f(a+2c) \cdot \ldots \cdot f(b-2c) \cdot f(b-c) \cdot f(b)$$

Arrow notation:

(4) $$\underset{n=a(c)}{\overset{b}{A_2}} f(x) = f(a) \uparrow^{0} f(a+c) \uparrow^{0} f(a+2c) \uparrow^{0} \ldots \uparrow^{0} f(b-2c) \uparrow^{0} f(b-c) \uparrow^{0} f(b)$$

2.1.3- Exponentory:

Simple notation:

(5) $$\underset{n=a(c)}{\overset{b}{\Theta}} f(x) = \underset{n=a(c)}{\overset{b}{A_3}} f(x) = ((((((f(a))^{f(a+c)})^{f(a+2c)})^{\ldots})^{f(b-2c)})^{f(b-c)})^{f(b)}$$

Arrow notation:

(6) $$\underset{n=a(c)}{\overset{b}{A_3}} f(x) = f(a) \uparrow f(a+c) \uparrow f(a+2c) \uparrow \ldots \uparrow f(b-2c) \uparrow f(b-c) \uparrow f(b)$$

2.1.4- Tetratory:

Arrow notation:

(7) $$\underset{n=a(c)}{\overset{b}{A_4}} f(x) = f(a) \uparrow^{2} f(a+c) \uparrow^{2} f(a+2c) \uparrow^{2} \ldots \uparrow^{2} f(b-2c) \uparrow^{2} f(b-c) \uparrow^{2} f(b)$$

2.1.5- Pentatory:

Arrow notation:

(8) $$\underset{n=a(c)}{\overset{b}{\mathrm{A}_5}} f(x)=f(a)\uparrow^3 f(a+c)\uparrow^3 f(a+2c)\uparrow^3 \ldots \uparrow^3 f(b-2c)\uparrow^3 f(b-c)\uparrow^3 f(b)$$

2.1.4- Generalization for all n of up arrow:

Unique notation:

(9)

$$\underset{n=a(c)}{\overset{b}{\mathrm{A}_n}} f(x)=f(a)\uparrow^{(n-2)} f(a+c)\uparrow^{(n-2)} f(a+2c)\uparrow^{(n-2)} \ldots \uparrow^{(n-2)} f(b-2c)\uparrow^{(n-2)} f(b-c)\uparrow^{(n-2)} f(b)$$

2.2- Negative operators:

In this part we can see the operators that make the function grow negative, here we will see the subtraction, division, roots and negative operations that go beyond the roots. In this case we will use the down arrow. We will assign the exponents to the arrow notation in such a way that it is analogous to the notation of the positive operators.

2.2.1- Restory:

Simple notation:

(10) $$\underset{n=a(c)}{\overset{b}{\mathrm{P}}} f(x)= \underset{n=a(c)}{\overset{b}{\Omega_1}} f(x)=-f(a)-f(a+c)-f(a+2c)-\ldots-f(b-2c)-f(b-c)-f(b)$$

Arrow notation:

(11) $$\underset{n=a(c)}{\overset{b}{\Omega_1}} f(x)=\downarrow^{-1} f(a)\downarrow^{-1} f(a+c)\downarrow^{-1} f(a+2c)\downarrow^{-1} \ldots \downarrow^{-1} f(b-2c)\downarrow^{-1} f(b-c)\downarrow^{-1} f(b)$$

2.2.2- Divisory:

Simple notation:

(12) $$\underset{n=a(c)}{\overset{b}{\Delta}} f(x)= \underset{n=a(c)}{\overset{b}{\Omega_2}} f(x)=f(a)\div f(a+c)\div f(a+2c)\div \ldots \div f(b-2c)\div f(b-c)\div f(b)$$

Arrow notation:

(13) $$\underset{n=a(c)}{\overset{b}{\Omega_2}} f(x)=f(a)\downarrow^0 f(a+c)\downarrow^0 f(a+2c)\downarrow^0 \ldots\downarrow^0 f(b-2c)\downarrow^0 f(b-c)\downarrow^0 f(b)$$

2.2.3- Rootory:

Simple notation:

(14) $$\underset{n=a(c)}{\overset{b}{Z}} f(x)=\underset{n=a(c)}{\overset{b}{\Omega_3}} f(x)=\sqrt[f(b)]{ANS}\sqrt[f(b-c)]{ANS}\sqrt[f(b-2c)]{ANS}\cdots\sqrt{ANS}\sqrt[f(a+2c)]{ANS}\sqrt[f(a+c)]{f(a)}$$

*Where ANS is the result of the inner root above

Arrow notation:

(15) $$\underset{n=a(c)}{\overset{b}{\Omega_3}} f(x)=f(a)\downarrow f(a+c)\downarrow f(a+2c)\downarrow\ldots\downarrow f(b-2c)\downarrow f(b-c)\downarrow f(b)$$

2.2.4- Anti-tetratory:

Arrow notation:

(16) $$\underset{n=a(c)}{\overset{b}{\Omega_4}} f(x)=f(a)\downarrow^2 f(a+c)\downarrow^2 f(a+2c)\downarrow^2 \ldots\downarrow^2 f(b-2c)\downarrow^2 f(b-c)\downarrow^2 f(b)$$

2.2.5- Anti-pentatory:

Arrow notation:

(17) $$\underset{n=a(c)}{\overset{b}{\Omega_5}} f(x)=f(a)\downarrow^3 f(a+c)\downarrow^3 f(a+2c)\downarrow^3 \ldots\downarrow^3 f(b-2c)\downarrow^3 f(b-c)\downarrow^3 f(b)$$

2.2.4- Generalization for all n of down arrow:

Unique notation:

(18)

$$\underset{n=a(c)}{\overset{b}{\Omega_n}} f(x)=f(a)\downarrow^{(n-2)} f(a+c)\downarrow^{(n-2)} f(a+2c)\downarrow^{(n-2)} \ldots\downarrow^{(n-2)} f(b-2c)\downarrow^{(n-2)} f(b-c)\downarrow^{(n-2)} f(b)$$

3- Notes:

For the correct work of the operators, the positive sign in the summation and the negative symbol in the restory in front of the first function of the series must be ensured.
For the intervals to be complete, the following must be true:

(19) $$\frac{b-a}{c}=n \rightarrow n \in \mathbb{Z}$$

4- Conclusions:

With this complete tool for generating operators we can go beyond exponents and beyond roots in series of functions and therefore also in numerical series by simply assigning f (x) = x. This is in my opinion a powerful tool for calculation and it was necessary to introduce it.