Quantum algorithm of Dempster combination rule

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Abstract

Dempster combination rule is widely used in many applications such as information fusion and decision making. However, the computational complexity of Dempster combination rule increases exponentially with the increase of frame of discernment. To address this issue, we propose the quantum algorithm of Dempster combination rule based on quantum theory. The algorithm not only realizes most of the functions of Dempster combination rule, but also effectively reduces the computational complexity of Dempster combination rule in future quantum computer. Meanwhile, we carried out a simulation experiment on the quantum cloud platform of IBM, and the experimental results showed that the algorithm is reasonable.

Keywords: Dempster combination rule, quantum algorithm of Dempster combination rule, computational complexity

1. Introduction

The Dempster-Shafer evidence theory was first proposed by Dempster and later extended by Shafer[1, 2]. It meets the weaker condition than Bayesian probability theory and has the ability to directly express uncertainty and unknowns. Then, because of its advantages, researchers constantly propose new

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theories to improve it. Such as uncertainty measure theory[3, 4], evidential reasoning[5, 6], quantum model of mass function[7], complex evidence theory[8] and etc. These theories are applied in decision-making[9, 10], pattern recognition[11, 12], risk assessment[13, 14] and other hybrid fields[15, 16].

On the other hand, there are some problems in evidence theory, such as evidence conflict and exponential explosion of calculation. To resolve the evidence conflict, researchers work from two perspectives: (a) Modify the Dempster combination rules to accomplish the redistribution of conflicts. (b) Dempster combination rule remains unchanged, and preprocess conflicting data before data combination. These measures effectively solve the problem of evidence conflict. For exponential explosion of calculation, with the increase of frame of discernment, the computational complexity also increases, it will be more serious. However, there is no reasonable approach to solve this problem.

Based on this gap, according to the idea of parallel computation in quantum theory[17], we propose the quantum algorithm of Dempster combination rule. The algorithm mainly consists of four steps. The first step is to convert the mass function data from the classical state to the quantum state. Then, combine mass function through the tensor product. The third step is to use projection operator to measure the probability of occurrence of basis vectors in the tensor product. Finally, the combined results are obtained by performing basic operations on the measured values. The complexity of the proposed algorithm is analyzed, and compared with the classical Dempster combination rule, its complexity is reduced. Meanwhile, we carried out experiments through IBM's quantum cloud platform. The experimental results indicate that the proposed algorithm is reasonable.

The rest of the paper is organized as follows. The relevant knowledge of evidence theory and quantum theory are introduced in Section 2. Section 3 proposes the quantum algorithm of Dempster combination rule, analyzes the complexity of the algorithm and carries out experimental verification. We summarize this work in Section 4.

2. Preliminaries

In the section, the basic knowledge of Dempster-Shafer evidence theory and quantum theory is introduced.

2.1. Dempster-Shafer evidence theory

Definition 1. *(Frame of discernment)*

Suppose Θ is a set of mutually exclusive and complete elements, Θ is defined as

$$\Theta = \{\theta_1, \theta_2, \cdots, \theta_i, \cdots, \theta_n\} \tag{1}$$

where θ_i represents a element or event.

The power set of Θ composed of 2^N elements is indicated as follows :

$$2^\Theta = \{\phi, \{\theta_1\}, \{\theta_2\}, \cdots, \{\theta_1, \theta_2\}, \cdots, \Theta\} \tag{2}$$

Definition 2. *(Mass Function)*

For a frame of discernment $\Theta = \{\theta_1, \theta_2, \cdots, \theta_i, \cdots, \theta_n\}$, the mass function m is defined as follows.

$$m : 2^\Theta \to [0, 1] \tag{3}$$

which satifies

$$m(\phi) = 0 \tag{4}$$

$$\sum_{F_i \subseteq \Theta} m(F_i) = 1 \tag{5}$$

where $m(F_i)$ represents the degree of supports F_i, F_i is the subset of Θ, so $m(F_i)$ is also called the basic belief assignment function (BBA).

Definition 3. *(Dempster combination rule)*

There are two BBAs, m_1 and m_2, Dempster combination rule is denoted as $m = m_1 \bigoplus m_2$, and it is defined as follows:

$$m(F_k) = \begin{cases} 0, & F_k = \emptyset \\ \frac{\sum_{F_i \cap F_j = F_k} m_1(F_i) m_2(F_j)}{1-K}, & F_k \neq \emptyset \end{cases} \tag{6}$$

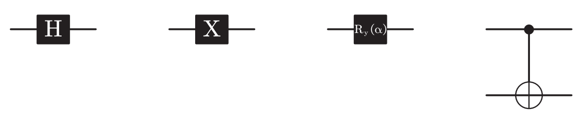
where $K = \sum_{F_i \cap F_j = \emptyset} m_1(F_i) m_2(F_j)$, K is the conflict coefficient, which represents the degree of conflict between different evidence bodies.

2.2. Quantum theory

Quantum mechanics is the basic theory describing the structure, motion and change of microscopic particles. It was jointly established by physicists such as Heisenberg, Schrodinger, Born, Dirac and et al[17]. In classical physics, the state of a system is described by the value of some mechanical quantity or its probability distribution. However, in quantum theory, the superposition state is used to describe the system based on the superposition principle. For example, the polarization of a photon can be expressed as the following superposition state[18, 19]:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{7}$$

where $|0\rangle$ is the horizontal polarization of the photon, the $|1\rangle$ is the vertical polarization of the photon, and $\alpha^2 + \beta^2 = p_0 + p_1 = 1$. Obviously, the p_0 and p_1 are the probabilities that the photons are horizontally polarized and vertically polarized after being measured, respectively. In quantum theory, the transformation of a system from one state to another is achieved through quantum logic gates. Some common quantum logic gates are shown in Figure 1.



$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} \cos\frac{\alpha}{2} & -\sin\frac{\alpha}{2} \\ \sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure 1: The common quantum logic gates

3. The quantum algorithm of Dempster combination rule

In this section, we proposed quantum algorithm of Dempster combination rule to reduce the computational complexity of Dempster combination rule.in

Dempster-Shafer evidence theory. The algorithm consists of four steps, as shown in Figure 2.

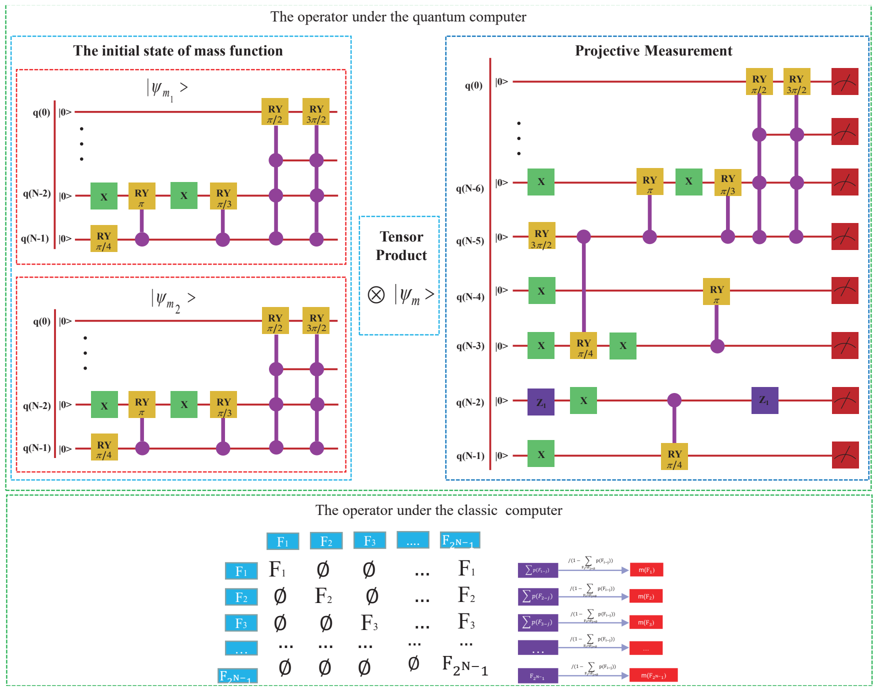


Figure 2: The flow of quantum algorithms

- Step 1: Suppose there is a frame of discernment $\Theta = \{\theta_1, \theta_2, \cdots, \theta_i, \cdots, \theta_n\}$, we map the power set of Θ to the basis vector of the quantum state, as shown in Table 1. The quantum state of the mass function can be expressed as follows:

$$|\psi_m\rangle = \sum_{i=1}^{2^{|\Theta|}} p_i \, |F_i\rangle \tag{8}$$

where p_i is the amplitude, the square of it is the degree of support for proposition F_i, and $\sum_{i=0}^{2^{|\Theta|}} {p_i}^2 = 1$. Therefore, it is obvious from Eq (8) that N basis vectors in the quantum state can reasonably represent 2^N propositions in the mass function.

Table 1: The quantum state corresponding to the mass function

$\emptyset$	θ_1	θ_2	θ_3	F_i	Θ
$\|000\cdots000\rangle$	$\|000\cdots001\rangle$	$\|000\cdots010\rangle$	$\|000\cdots011\rangle$	$\|\cdots\rangle$	$\|111\cdots111\rangle$
$\|\emptyset\rangle$	$\|\theta_1\rangle$	$\|\theta_2\rangle$	$\|\theta_3\rangle$	$\|F_i\rangle$	$\|\Theta\rangle$

Now, we construct the quantum state of the mass function, that is, the classical data of the mass function is prepared as the quantum state $|\psi_m\rangle$. Based on the idea of dichotomy[19], the quantum state of mass functions $|\psi_m\rangle$ is prepared as shown in Figure. 3. As can be seen from Figure. 3, the amplitude of the base vector is the product of nodes in the branch tree corresponding to the base vector. When $\alpha_{2^{N-1}} = \frac{\pi}{2}$, the amplitude of $|\emptyset\rangle$ is 0, that is to say, the remaining quantum states correspond to the propositions in the mass function. Based on Figure 3, the quantum circuit of BBAs are designed through the revolving gate and control gate, as shown in Figure. 4.

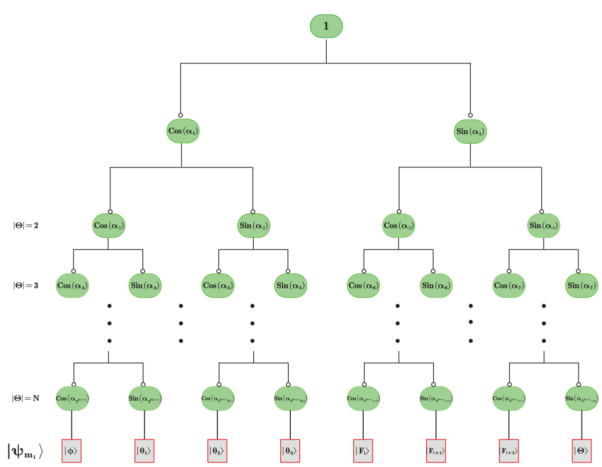


Figure 3: The calculation of quantum state amplitudes

- Step 2: Suppose there are two BBAs on the frame of discernment, corresponding quantum states are $|\psi_{m_1}\rangle$ and $|\psi_{m_2}\rangle$. The tensor product of $|\psi_{m_1}\rangle$ and $|\psi_{m_2}\rangle$ is denoted as $|\psi_m\rangle = |\psi_{m_1}\rangle \bigotimes |\psi_{m_2}\rangle$.

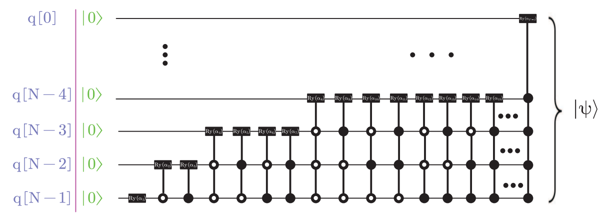


Figure 4: Prepare the initial state quantum circuit

- Step 3: In $|\psi_m\rangle$, only part of the quantum states need to be measured based on Dempster combination rule. The probability of $|F_i\rangle$ is measured by the following equation.

$$p\left(F_{i-j}\right) = \langle\psi_m|\ M_{F_{i-j}}^{\dagger} M_{F_{i-j}} \Big|\ \psi_m\rangle \tag{9}$$

where M_{F_k} is measurement operators, and $M_{F_{i-j}} = |F_i^1\rangle \otimes |F_j^2\rangle \left(\langle F_i^1| \otimes \langle F_j^2|\right)$, $|F_i^1\rangle$ and $|F_j^2\rangle$ are the basis vectors in $|\psi_{m_1}\rangle$ and $|\psi_{m_2}\rangle$, respectively.

- Step 4: The combined BBA $m\left(F_k\right)$ can be obtained by normalizing $p\left(F_k\right)$ with the following equation:

$$m\left(F_k\right) = \frac{\sum_{F_i \cap F_i = F_k} p\left(F_{i-j}\right)}{1 - \sum_{F_i \cap F_i = \emptyset} p\left(F_{i-j}\right)} \tag{10}$$

In Dempster-Shafer evidence theory, $2^{|\Theta|}-1$ selection operations, $\left(2^{|\Theta|}-1\right)^2$ multiplication operations and $2^{|\Theta|}-1$ division operations are required for m_1 and m_2 combinations. In quantum algorithm of Dempster combination rule, the combination of $|\psi_{m_1}\rangle$ and $|\psi_{m_2}\rangle$ requires only $2^{|\Theta|}-1$ selections, 1 tensor product operation, and $2^{|\Theta|}-1$ divisions. Obviously, the number of operations required to combine evidence is the sum of these operations. Therefore, the operation times of basic operations in the classical Dempster combination rule and the quantum algorithm of Dempster combination rule vary with the frame of discernment Θ, as shown in Figure 5. As can be seen from Figure 5, with the increase of Θ, the number of operations of Dempster combination rule in the quantum algorithm of Dempster combination rule is far less than that of classical

Dempster combination rule. The larger the Θ, the larger the computational advantage in the quantum algorithm of Dempster rule.

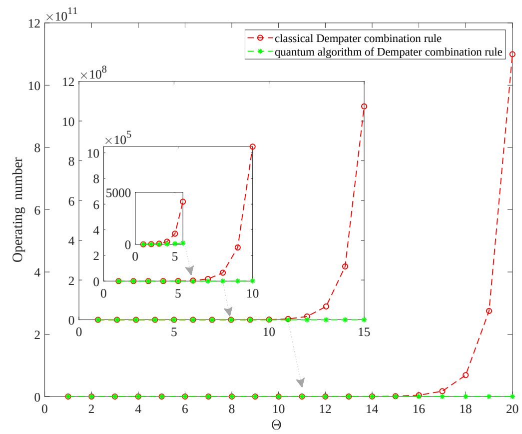


Figure 5: The computational complexity of different algorithms

Meanwhile, a verification experiment is designed on the quantum computer of IBM. The experimental results are shown in Table 2. It can be seen from the table 2 that with the increase of the frame of discernment, the running time of the algorithm on the quantum computer increases, which is reasonable. With the increase of the frame of discernment, the basis vectors of $|\psi_m\rangle$ also increase, and the measurement time also increases (each basis vector is measured 1024 times). Therefore, the running time will increase. In addition, current quantum computers are based on quantum logic gates. The operation process of quantum logic gate and the preparation process of quantum state are not idealized processes, and these operations have corresponding error rates. Taking IBMQX4 chip as an example, the error rates of single-bit logic gate and controlled non-gate are 0.00134 and 0.02992 respectively. These faulty operations may result in an inaccurate combination result.

Table 2: three-line table

$\|\Theta\|$	2	3	4	5	6
$Time(s)$	6.2	6.3	6.5	6.7	7.6

4. Conclusion

In the paper, in order to reduce the computational complexity of Dempster combination rule, according to the idea of parallelism in quantum theory, we propose the quantum algorithm of Dempster combination rule. This algorithm can not only combine any two evidence bodies, but also reduce the computational complexity. Meanwhile, the algorithm is implemented on IBM's quantum computer and verified by experiments. The experimental results show that with the increase of the frame of discernment, the execution time on the quantum computer also increases, which is reasonable. It is worth expecting that with the development of quantum theory, the computing power of quantum computer will be gradually improved, and the operation time of quantum algorithm of Dempster combination rule will be continuously shortened.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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