

Supplement Formulas of the Fine-structure Constant and Explanations to Some Recent Measurements

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the previous two papers “Chen’s Formulas of the Fine-structure Constant” (viXra:2002.0203) and “Supplement Formulas of the Fine-structure Constant α , New Formulas of Euler Number e and Their Relationships with Nuclides” (viXra:2008.0020) for giving some supplement formulas and explanations to some recent measurements. In the previous papers, many formulas of the fine-structure constant (α) based on the key numbers 112, 173, 137, 83, 29 *et al* had been given, and some fractional number formulas of α based on 56 and 81 had also been given. In this paper, some new fractional number formulas of α based on 56 and 81 were supplemented, and some explanations to recent (the last decade) measurements of α were presented, especially the latest most accurate measurement of α (reported by *Nature* on Dec. 2, 2020) was commented.

Keywords: formulas of the fine-structure constant, measurements of the fine-structure constant, explanations and comments.

1. Introduction

As Richard Feynman supposed, the biggest atomic number Z_{\max} would be the 137 because the velocity of ground state electron in a hydrogen-like atom (with just one electron) would reach the speed of light when the atomic number increases to 137 according to one of the definitions of the fine structure constant α as follows.

$$\alpha = \frac{v_e}{c} = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{v_{e-zH}}{Zc} \approx \frac{1}{Z_{\max}} = \frac{1}{137}$$

- v_e : the velocity of ground state electron in hydrogen atom
- ${}_Z H$: a hydrogen-like atom with Z protons and just one electron
- $v_{e-{}_Z H}$: the velocity of ground state electron in a hydrogen like atom
- Z_{\max} : the supposed maximum atomic number 137 (${}_{137}\text{Fy}$)

This also implies that the fine-structure constant α relates to atomic numbers or even nucleon numbers of elements. Consequently, we had constructed many formulas of the fine-structure constant based on the atomic numbers 112, 173, 137, 83, 29 *et al.* For examples, the two most typical and concise formulas are listed as follows¹.

$$\alpha_1 = \frac{36}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{279}{278}\right)^{557}}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

The nucleon number 137 appears just once in all 286 primordial nuclides, i. e., ${}_{56}\text{Ba}_{81}$, so modification to $137=56+81$ should be the other way to construct formulas of the fine-structure constant. The method and the most typical formulas are listed as follows².

$${}^{130,132,134,136,137,138}_{56}\text{Ba}_{74,76,78,79,80,81,82}$$

$$137 = 56 + 81, \quad 1/\alpha \approx 137.035999 = 56 + 81 + \delta, \quad \delta = f(56|81)$$

$$\alpha_1 = \frac{1}{56 + 81 + \frac{1}{28 - \delta_{1-28}}} = \frac{1}{56 + 81 + \frac{1}{27 + \delta_{1-27}}} = 1/137.035999037435$$

$$\alpha_2 = \frac{1}{56 + 81 + \frac{1}{28 - \delta_{2-28}}} = \frac{1}{56 + 81 + \frac{1}{27 + \delta_{2-27}}} = 1/137.035999111818$$

$$\alpha_1 = \frac{1}{56 + 81 + \frac{1}{28 - \frac{13 \cdot (2 \cdot 56 \cdot 11 - 1)}{3 \cdot 5 \cdot (2 \cdot 56 \cdot 43 + 1)}}} = 1/137.035999037435$$

$$\alpha_2 = \frac{1}{56 + 81 + \frac{1}{28 - \frac{2 \cdot (16 \cdot 27 - 1)}{3 \cdot (16 \cdot 81 + 1)}}} = 1/137.035999111818$$

In this paper, we give some supplement formulas of the fine-structure constant which had been omitted in the two previous papers^{1,2}, and give some explanations to recent (the last decade) measurements of the fine-structure constant.

2. The Fine-structure Constant Expressed as Fractional Numbers

$$\alpha_1 = \frac{1}{137 + \frac{1}{27 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{16 + \frac{1}{15 + \frac{1}{10 + \frac{1}{1 + \frac{1}{2}}}}}}}}}}} = \frac{1}{56 + 81 + \frac{1}{27 + \frac{56252}{72255}}}$$

$$= \frac{1}{56 + 81 + \frac{1}{27 + \frac{4 \cdot 7^3 \cdot 41}{3 \cdot 5 \cdot (112 \cdot 43 + 1)}}}$$

²⁷Al₁₄ ^{28,29,30}Si_{14,15,16} ³¹P₁₆ ^{46,47,48,49,50}Ti_{24,25,26,27,28} ^{54,56}Fe_{28,30} ^{58,60}Ni_{30,32} ^{79,81}Br_{44,46}
^{82,83,84}Kr_{46,47,48} ^{85,87}Rb_{48,50} ^{97,98,99}Tc*₄₃ ^{54,55,56} ^{113,115}In_{64,66} ^{135,136,137,138}Ba₅₆ ^{79,80,81,82}Ce₅₈ ¹⁴⁰Ce₈₂
^{145,3-49}Pm*₆₁ ^{84,86} ^{162,164}Dy₆₆ ^{96,98} ^{203,5-41}Tl₈₁ ^{122,124} ²⁰⁸Pb₈₂ ²⁰⁹Bi*₈₃ ²⁰⁹Po*₈₄ ²²²Rn*₈₆ ¹³⁶ ²⁸⁵Cn*₁₁₂ ¹⁷³
^{8-43,346,348}Fy^{ie}_{136,137,138} ^{208,209,210} ⁴¹⁶Ch*₁₆₄ ²⁵²

$$\alpha_2 = \frac{1}{137 + \frac{1}{27 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{17 + \frac{1}{2 + \frac{1}{5 + \frac{1}{2}}}}}}}}} = \frac{1}{56 + 81 + \frac{1}{27 + \frac{3029}{3891}}}$$

$$= \frac{1}{56 + 81 + \frac{1}{27 + \frac{13 \cdot (2 \cdot 9 \cdot 13 - 1)}{3 \cdot (16 \cdot 81 + 1)}}} = \frac{1}{56 + 81 + \frac{1}{27 + \frac{13 \cdot (8 \cdot 29 + 1)}{3 \cdot (16 \cdot 81 + 1)}}}$$

²⁷Al₁₄ ^{28,29,30}Si_{14,15,16} ^{50,52,53,54}Cr_{26,28,29,30} ^{54,56,57,58}Fe_{28,30,31,32} ⁵⁹Co₃₂ ^{63,65}Cu_{34,36} ¹⁰⁰Ru₄₄ ⁵⁶
¹⁰⁴Pd₄₆ ¹¹²Cd₄₈ ¹³²Xe₅₄ ⁷⁸ ^{5-27,136,137,138}Ba₅₆ ^{79,80,81,82} ¹⁷⁴Yb₇₀ ¹⁰⁴ ¹⁹⁴Pt₇₈ ¹¹⁶ ^{344,346,12-29}Fy^{ie}_{136,137,138} ^{16-13,209,210}

3. The Speed of Light in Atomic Unites Expressed as Fractional Numbers

$$c_{au} = 56 + 81 + \frac{1}{28 - \delta_{c-28}} = 56 + 81 + \frac{1}{27 + \delta_{c-27}} = 137.035999074626$$

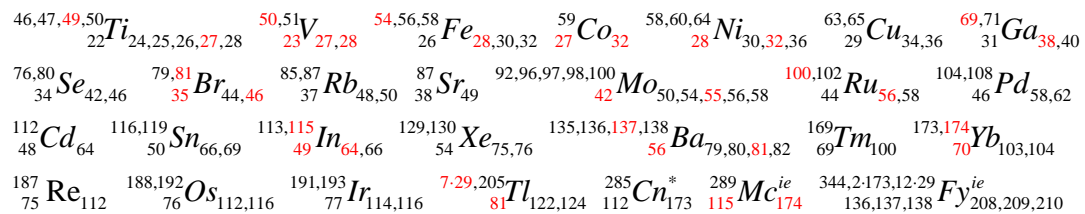
$$c_{au} = \frac{1}{\alpha_c} = 56 + 81 + \frac{1}{27 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{16 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3}}}}}}}}}}}}$$

$$= 56 + 81 + \frac{1}{27 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{16 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{4 + \frac{1}{9 \cdot 23}}}}}}}}}}}} = 56 + 81 + \frac{1}{27 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{16 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{8 \cdot 19}}}}}}}}}}}}$$

$$= 56 + 81 + \frac{1}{27 + \frac{25076}{32211}}$$

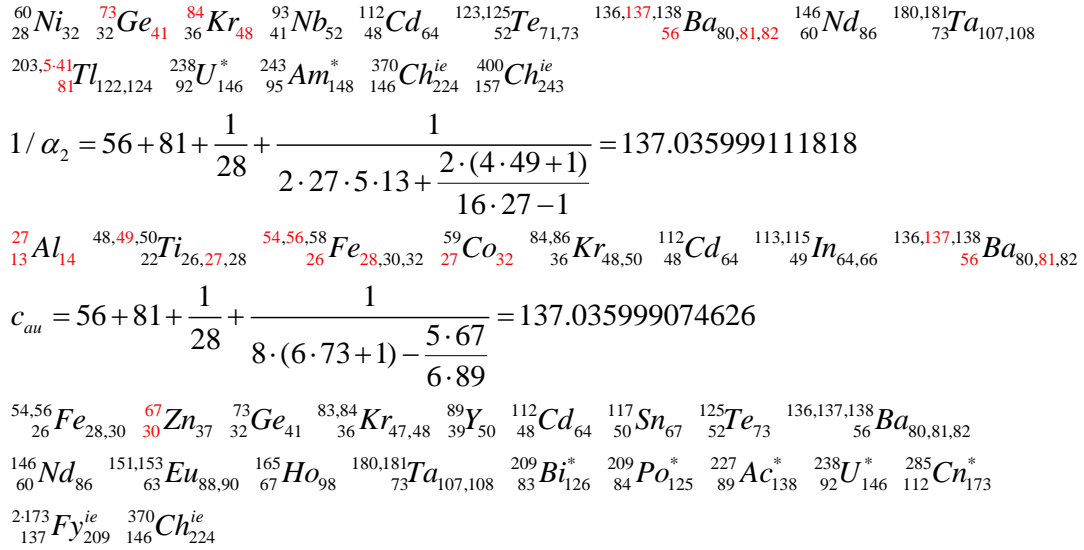
$$= 56 + 81 + \frac{1}{27 + \frac{4 \cdot (4 \cdot (2 \cdot 27 \cdot 29 + 1) + 1)}{2 \cdot 5 \cdot (4 \cdot 5 \cdot 7 \cdot 23 + 1) + 1}} = 56 + 81 + \frac{1}{27 + \frac{4 \cdot (4 \cdot (32 \cdot 49 - 1) + 1)}{2 \cdot 5 \cdot (4 \cdot 5 \cdot 7 \cdot 23 + 1) + 1}}$$

$$= 137.035999074626$$



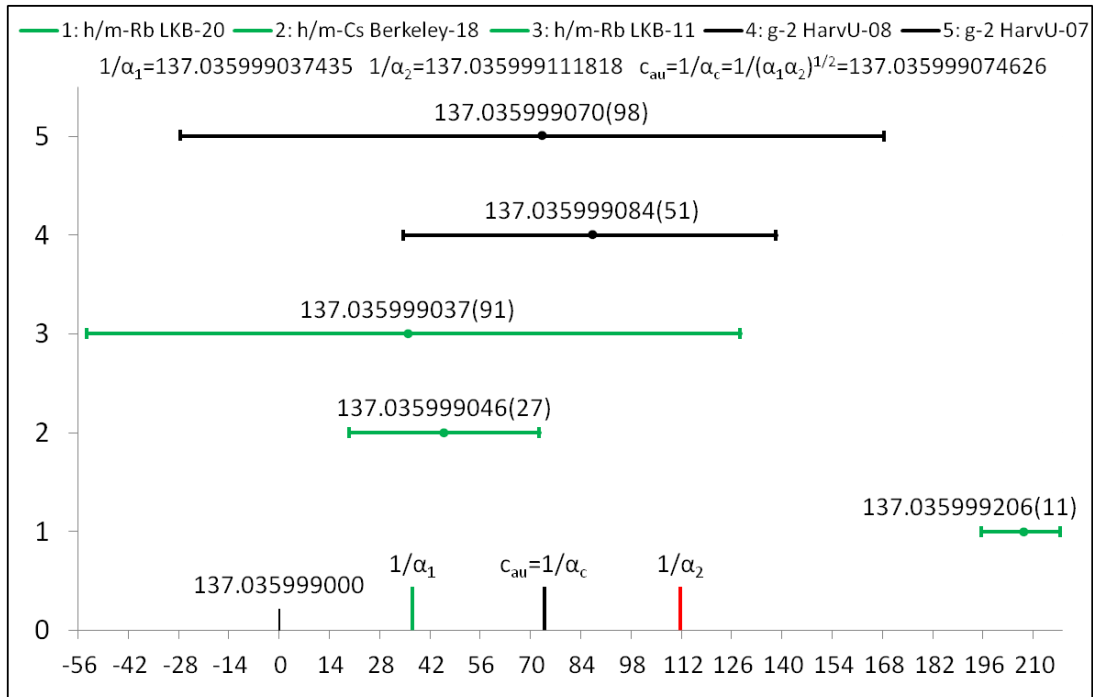
4. The Meanings of Numerical Values of α and c_{au}

$$1/\alpha_1 = 56 + 81 + \frac{1}{28} + \frac{1}{8 \cdot (6 \cdot 73 + 1) - \frac{41}{3 \cdot 81}} = 137.035999037435$$



5. Evaluation of the Latest and Previous Reported Values of Measurements

The latest reported value of α determined by the most accurate measurement is $1/137035999206(11)^3$. However, this result is quite different from those of the two previous measurements, i.e., $1/137.035999046(27)^4$ and $1/137.035999037(91)^5$. Here, we try to give explanations for this very strange contradiction (**Fig. 1** and **Table 1**).



Comparison of Calculated and Measured Values of $1/\alpha$
 Gang Chen, Tianman Chen and Tianyi Chen (2020/9/4-5,10-11, 12/7)

Fig. 1

Table 1. Evaluation of the measured values of α_1 by comparison to the calculated value.

Measurement	Measured Value of $1/\alpha_1$	Calculated Value of $1/\alpha_1$	Error	Accuracy	Ratio
StanfU-2002 ⁶	137.036000(10)		1	10	0.1
LKB-2011 ⁵	137.035999037(91)	137.035999037435	0	91	0
Berkeley-2018 ⁴	137.035999046(27)		9	27	0.33
LKB-2020 ³	137.035999206(11)		169	11	15.4

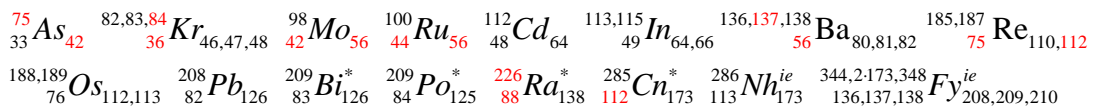
Note: Suppose the calculated value is correct and precise; Error is the difference between measured value and the calculated value; Ratio means error to accuracy ratio.

The following are some typical examples of our formulas of the fine-structure constant (α), in which there are many coincidences with themselves and with nuclides. For examples, there is one per infinite coincidence of $(2\pi)_{112}$ and 112 in Formula (1) and there are marvelous coincidences of 56 and 81 in Formula (3) and (4). So we believe our formulas and the two calculated values of α are correct and precise. And hence, four relatively accurate measurement results (with the same measuring method) are evaluated as in **Fig. 1** and **Table 1**. It seems the measurements got to a critical point, after which the more accurate the measurement is, the less correct the result is. This would be the reason why the latest measurement result $(1/137.035999206(11))$ ³ is so strange and contradict the previous measurement results. If this reason is true, the measurement result in 2011 $(1/137.035999037(91))$ ⁵ would be the best correct although less accurate, and we should trust it the most. The three green flat lines in **Fig. 5** almost prove the above explanation, however, to verify it further, it would be better that experimental physicist give additional measurement results, for example, $1/\alpha=137.0359991mn(27-11)$ with $1mn$ between 046 to 216.

$$(2\pi)_k = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}$$

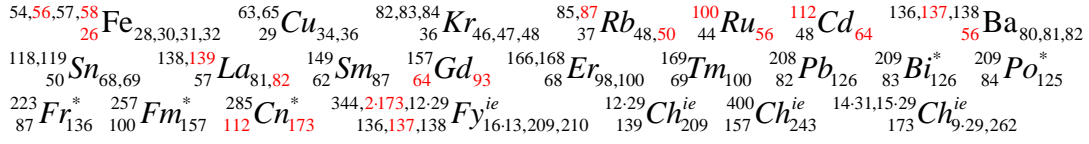
$$(1): \alpha_1 = \frac{36}{7 \cdot (2\pi)_{112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

Note: $7 \cdot (2\pi)_{112} \approx 44$



$$(2): \alpha_2 = \frac{13 \cdot (2\pi)_{278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

Note: $13 \cdot (2\pi)_{278} \approx 81.73 \approx 82$

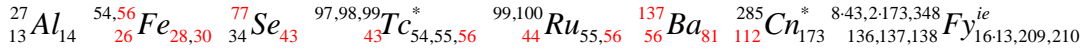


$$(3): c_{au} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \sqrt{112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)})}$$

$$= 137.035999074626$$



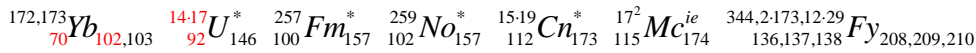
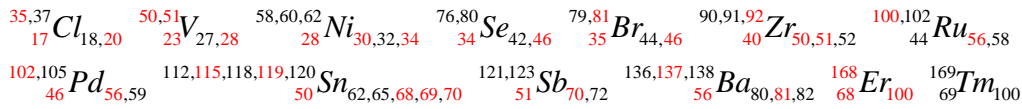
$$(4): 1/\alpha_1 = 56 + 81 + \frac{1}{28 - \frac{13 \cdot (2 \cdot 56 \cdot 11 - 1)}{3 \cdot 5 \cdot (2 \cdot 56 \cdot 43 + 1)}} = 137.035999037435$$



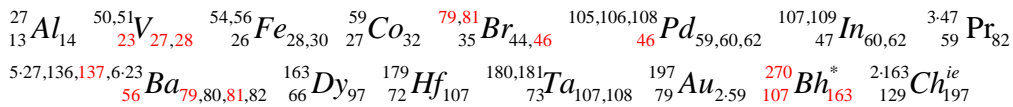
$$(5): 1/\alpha_2 = 56 + 81 + \frac{1}{28 - \frac{2 \cdot (16 \cdot 27 - 1)}{3 \cdot (16 \cdot 81 + 1)}} = 137.035999111818$$



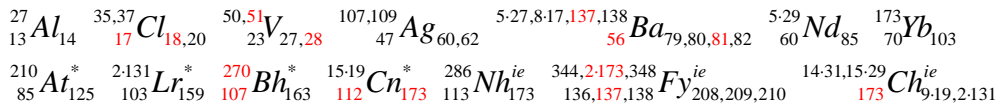
$$(6): c_{au} = \frac{1}{\alpha_c} = 56 + 81 + \frac{1}{28 - \frac{5 \cdot (4 \cdot 3 \cdot 7 \cdot 17 - 1)}{2 \cdot 5 \cdot (4 \cdot 5 \cdot 7 \cdot 23 + 1) + 1}} = 137.035999074626$$



$$(7): 1/\alpha_1 = 56 + 81 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{4 \cdot 79 \cdot (2 \cdot 23 \cdot 163 + 1)} = 137.035999037435$$



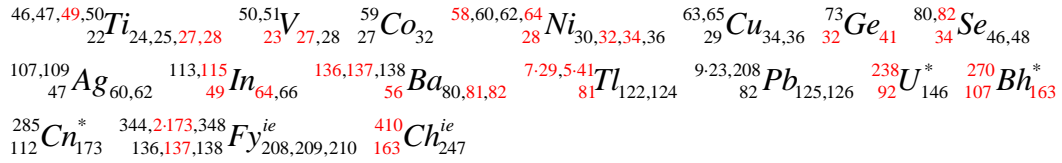
$$(8): 1/\alpha_2 = 56 + 81 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{5 \cdot 17 \cdot 137 \cdot 173} = 137.035999111818$$



$$(9): c_{au} = 56 + 81 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{5 \cdot 23 \cdot 29 \cdot (4 \cdot 163 + 1)}$$

$$= 56 + 81 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{4 \cdot 49 \cdot 41 \cdot (16 \cdot 17 - 1)}$$

$$= 56 + 81 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{4 \cdot 49 \cdot 41 \cdot (270 + 1)} = 137.035999074626$$



6. CODATA Recommended Values of the Fine-structure Constant

Table 2. CODATA recommended values of $1/\alpha$.

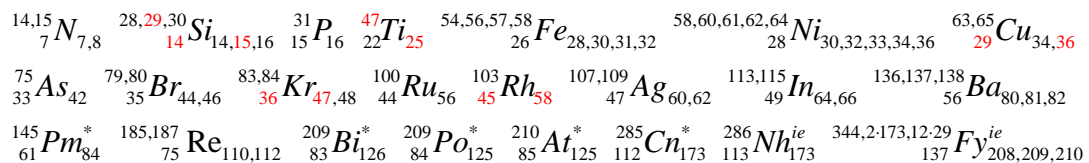
Year	CODATA Recommended Value of $1/\alpha$
2006	137.035999680(93)
2010	137.035999074(44)
2014	137.035999139(31)
2018	137.035999084(21)
2022	?

Table 2 indicates that since 2010 CODATA recommended values of the fine-structure constant have been fluctuating according to experiment results, and this embarrassing situation would continue for a long time. Some questions are what value CODATA would recommend in 2022, whether 137.035999206 would be OK and so on. So it should be the time to provide theoretical criteria such as our formulas and values.

7. Formulas of α according to the Latest Most Accurate Measurement

The Latest most accurate measurement gave a value of $1/\alpha$ of $137.035999206(11)^3$. If the value and the accuracy of measurement correlated each other, this result should be somewhat reasonable and we could construct formula based on it as follows. However, the following formula is less beautiful and concise than the original one.

$$\begin{aligned}
 \alpha_1 &= \frac{36}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{9,25}}} \frac{1}{112 + \frac{1}{4 \cdot 5 \cdot (2 \cdot 3 \cdot 47 - 1) + \frac{29}{9 \cdot 5}}} \\
 &= \frac{36}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{9,25}}} \frac{1}{112 + \frac{1}{4 \cdot 5 \cdot (8 \cdot 5 \cdot 7 + 1) + \frac{29}{9 \cdot 5}}} \\
 &= 1/137.035999205994
 \end{aligned}$$



References

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Appendix I: Research and Writing History

Section	Page	Date	Remarks
1	1-3	2020/12/13	
2	3	2020/12/5-6	
3	4	2020/12/5-6	
4	4-5	2020/12/8	
5	5-8	2020/12/5-8	
6	8	2020/12/8	
7	8	2020/12/9-10	
References	9		
Acknowledgement	9		
Appendix I	10		
Preparing this paper	1-10	2020/12/5-14	

Notes: Dates were recorded according to Beijing Time;
ie means ideal extended elements.