Helium - Beryllium Quasi – Bound System and Hoyle State

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Abstract

The strong nuclear force emerges because of a mass dependent inter-nucleon Yukawa potential energy. The same potential energy operates among clusters of nucleons such as alpha-alpha clusters. Here we show that a quasi-bound system of ^{4}He cluster and ^{8}Be has an excited state that matches with the Hoyle state.

The nucleus ^8_4Be is highly unstable and decays into two ^4_2He nuclei in a very short time. The half - life for this alpha decay is 8.9×10^{-17} sec. Within that time if a third alpha particle fuses with the Beryllium-8 nucleus it produces an excited resonance state of Carbon-12 nucleus. This excited resonance state is called the Hoyle state, which nearly always decays back into three alpha particles, but once in about 2500 times releases energy and changes into the stable base form of Carbon-12. The excited Hoyle state is 7.656 MeV. The strong interaction among nucleons are the reason why they are bound together. The same strong interaction enables clusters of nucleons to form and these clusters in turn exert the same type of strong interaction on each other. For example, the nucleus ^4_2He is a very stable structure consisting of two protons and two neutrons and this alpha cluster exerts strong force on another cluster as though it is itself a particle. The nucleus ^8_4Be can be explained as though it is made up of two interacting clusters of alpha particles. As and when these two alpha particles find enough energy to separate by a distance where the short-range nuclear force is zero, the Coulomb repulsion causes them to split apart and the Beryllium nucleus decays into two alpha particles. Sometimes these clusters can cling together temporarily to form a quasi-bound system. Such a quasi-bound system does have excited states. These excited states arise while solving the Schrödinger equation. Here we show that alpha-beryllium clusters form a quasi-bound system whose first excited state is the Hoyle-state.

The simplest nucleus is Deuteron. The potential energy is given by, [1],

$$V(r) = -G\frac{M_P^2}{M_0^2}\frac{m_p m_n e^{-ar}}{r}, \quad (1)$$

Where, (G is universal constant of gravitation),

$M_P^2 = \frac{g^2\hbar c}{G}$, and $g^2 = e^2\frac{1}{0.2254}$, and $e^2 = \frac{1}{137}$ is the fine structure constant and 0.2254 is the Weinberg mixing parameter of the electroweak model. The parameter M_0^2 is specific to each product of the masses that appear in Eq. (1). The exponential factor is what is called the Yukawa factor. It restricts the range of interaction by the exchange of Pions. For neutron proton interaction the parameter M_0^2 is given by, [1],

$$M_0^2 = 0.931826 \times 10^{-48} gm^2 . \quad (2)$$

While solving the Schrödinger equation with the potential energy (1) for the neutron-proton case we set the Yukawa factor equal to 1 as we do not know the solution of the differential equation with the Yukawa factor. The structure of the nucleus ${}_1^3H$ can be explained with the strong potential energy that is similar to Eq. (1). For this nucleus the potential energy is given by,

$$V(r) = -\frac{g^2\hbar c}{M_0^2}\frac{m_c m_n}{r} . \quad (3)$$

Here again the Yukawa factor is set equal to one. There is a neutron outside a cluster of a proton and neutron. The mass of the cluster is given by,

$$m_c = (m_n + m_p) = 3.347534 \times 10^{-24} gm . \quad (4)$$

The energy spectrum for the central potential (3) is given by,

$$E_{nl} = -\frac{\mu}{2\hbar^2}\frac{1}{n^2}\left[\frac{g^2\hbar c m_c m_n}{M_0^2}\right]^2 = -\frac{8.4817}{n^2} \text{ MeV}. \quad (5)$$

The reduced mass $\mu = \frac{m_c m_n}{m_c + m_n} = 1.116357 \times 10^{-24} gm$. (6)

The quantum numbers n, l etc are identical to those of the Hydrogen atom. The principal quantum number n= 1, 2, 3…The binding energy is compared to Eq. (5) when n=1, $and\ M_0^2$ is evaluated:

$$M_0^2 = 1.103243 \times 10^{-48} gm^2 \ . \tag{7}$$

It should be noted that the above value is different from Eq. (2). The binding energy of the triton nucleus is calculated from the mass defect. It also gives the ground state energy of this nucleus. The few principal energy levels of ${}^3_1H$ nucleus is given by,

$$E_1 = -8.4817\ MeV,\ E_2 = -2.1204\ MeV,\ E_3 = -0.9424\ MeV.\text{ etc} \tag{8}$$

To drive the importance of the cluster nucleon strong interaction we consider the structure of the nucleus ${}^3_2He$ .The potential energy is given by,

$$V(r) = -\frac{g^2\hbar c}{M_0^2}\frac{m_c m_p}{r} + \frac{e^2\hbar c}{r}\ , \tag{9}$$

where there is a proton outside a cluster of a neutron and proton. The mass of the cluster is same as in the case of the nucleus ${}^3_1H$. The energy spectrum for this nucleus is given by,

$$E_{nl} = -\frac{\mu}{2\hbar^2}\left[\frac{g^2\hbar c m_c m_p}{M_0^2} - e^2\hbar c\right]^2 \frac{1}{n^2} = -\frac{7.1780}{n^2}\ MeV. \tag{10}$$

In Eq. (10), the reduced mass $\mu = \frac{m_c m_p}{m_c + m_p} = 1.115332 \times 10^{-24} gm$.

The binding energy of this nucleus is 7.1780 MeV. The ground state energy for this nucleus is $-7.1780\ MeV$. From this value the parameter M_0^2 is obtained.

$$M_0^2 = 1.142021 \times 10^{-48} gm^2. \tag{11}$$

It should be noted that this is different from Eq. (7). The two nuclei have equal number of nucleons and their energy spectra should be noted. $E_1 = -7.1780\ MeV,\ E_2 = -1.7945\ MeV, and\ E_3 = -0.7976\ MeV$, etc.

The structure of the very strongly built nucleus ${}^4_2He$ is shown to be due to the strong potential energy between a proton and a cluster of two neutrons and a proton, [2,3]. Nucleons are also clusters of quarks. The nucleus ^{8}Be is made up of two clusters of ^{4}He nuclei. The ground state energy is 58.6 MeV, which is twice the ground state energy of the ^{4}He nucleus. If beryllium nucleus is made up of two clusters of alpha particles then the separation energy of an alpha cluster from the ^{8}Be nucleus is 28.3 MeV. [2,3]. The binding energy of ^{12}C is 92.16175 MeV. If the nucleus ^{12}C is made up of three clusters of alpha particles then the separation energy for each alpha particle from a Carbon nucleus is about 30.7 MeV. In the case of ^{16}O nucleus the ground state energy is 127 MeV. Again, if the Oxygen nucleus is made up of four alpha clusters the separation energy for an alpha cluster from the Oxygen nucleus is about 31.75 MeV. Let us assume that there is a cluster of $^{8}_{4}Be$ nucleus at which an alpha particle is fired with energy that is more than 30 MeV. This energy is more than the separation energy of an alpha particle from a bound Carbon -12 nucleus. If the Strong potential energy upon the fired ^{4}He due to the cluster ^{8}Be is just enough to overcome the Coulomb repulsion then it results in a quasi-bound system of $^{4}_{2}He + ^{8}_{4}Be$.The system is bound, but the ground state energy is not that of a Carbon nucleus but it is just equal to the separation energy of an alpha cluster from the Beryllium nucleus. The separation energy of an alpha cluster is about 30 MeV. It is something like the work function of a material that one encounters in the case of photo-electric effect. But when we solve the Schrödinger equation for the separate alpha cluster it will have energy levels or energy spectra. The potential energy that an alpha cluster experiences due to the Beryllium cluster is given by,

$$V(r) = -\frac{g^2\hbar c}{M_0^2}\frac{M_\alpha M_{Be}}{r} + \frac{8e^2\hbar c}{r}, \tag{12}$$

where $^{4}_{2}He$ and $^{8}_{4}Be$ masses are M_α and M_{Be} and M_0^2 is specific for this product. The first expression in the above potential contains a Yukawa factor but it is set equal to one. The time independent Schrödinger equation with the above potential energy is given by,

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right]\Psi(r,\vartheta,\varphi) = E\Psi(r,\vartheta,\varphi) \quad , \tag{13}$$

where the reduced mass ,

$$\mu = \frac{M_\alpha M_{Be}}{M_\alpha + M_{Be}} = 4.430809 \times 10^{-24} gm. \tag{14}$$

The potential energy is a central potential and as in the case of Hydrogen atom the full solution is readily given by,

$$\Psi(r,\vartheta,\varphi) = R_{nl}(r)Y_{lm}(\vartheta,\varphi) \tag{15}$$

The radial function, R_{nl} is given by,

$$R_{nl}(r) = Ae^{-\rho/2}\rho^{l}L_{n+l}^{2l+1}(\rho) \ , \tag{16}$$

$$\rho = \frac{2r}{na_0} \text{ , and } \qquad A = \sqrt{\frac{2^3}{n^3 a_0^3}\frac{1}{2n}\left\{\frac{(n-l-1)!}{([n+l]!)^3}\right\}} \ , \tag{17}$$

and,

$$a_0 = \frac{\hbar^2}{\mu}\frac{1}{\left[\frac{g^2\hbar c M_\alpha M_{Be}}{M_0^2} - 8e^2\hbar c\right]} \ . \tag{18}$$

The parameter a_0 can be readily computed with the following data:

$M_\alpha = 6.6461 \times 10^{-24} gm$ and $M_{Be} = 13.2928810^{-24} gm$ and $g^2\hbar c = 0.10239 \times 10^{-17} erg.cm$ $8e^2\hbar c = 0.18462 \times 10^{-17} erg.cm$ The most crucial factor M_0^2 is obtained by demanding that the ground state energy is equal in magnitude to the separation energy of an alpha cluster from the quasi-bound system of ${}^4_2He$ *and* ${}^8_4Be$. It is exactly like the work function of a material irradiated by light in the case of photo-electric effect. The material holds the electron with that energy called the work functions. The incident photon can knock out the electron only when its energy is more than the work function. We can say that the electron is quasi-bound to the material and its ground state energy is equal to the work function. If there is a potential energy which gives rise to this ground state energy it must also allow excited states for the electron because of quantum nature. Precisely this is what is done here. The energy eigen values for the interacting clusters ${}^4_2He$ and ${}^8_4Be$ with the potential energy Eq. (12) is given by,

$$E_{nl} = -\frac{\mu}{2\hbar^2}\left(\frac{g^2\hbar c M_\alpha M_{Be}}{M_0^2} - 8e^2\hbar c\right)^2 \times \frac{1}{n^2} \ . \tag{19}$$

Here 'n' is the principal quantum number which can take integer values as in the case of Hydrogen atom. As mentioned earlier the choice of the parameter M_0^2 is dictated by the requirement that the quasi-bound system has a ground state energy that is enough to knock out an intact alpha cluster from the ground state of the bound system. Hence,

$$M_0^2 = 13.284735 \times 10^{-48} gm^2 \ . \tag{20}$$

With this choice,

$$E_{nl} = -\frac{30.624}{n^2} MeV. \qquad (21)$$

The first two energy levels are given by,

$$E_{10} = -30.624 MeV, \qquad (22)$$

And,

$$E_{20} = -7.656 MeV\ . \qquad (23)$$

It is this first excited state of the quasi-bound system of the two clusters ${}^{4}_{2}He\ and\ {}^{8}_{4}Be$ is what is known as the Hoyle-state. It is a 0^{+} first excited state of the quasi-bound clusters ${}^{4}_{2}He + {}^{8}_{4}Be$ and this subsequently settles down to a ${}^{12}_{6}C$ configuration with of course a very small probability. This happens because of the strong force among the clusters. The wave functions for the ground state and the first excited state are given below:

$$\Psi_{100}(r, \vartheta, \phi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \quad . \qquad (24)$$

$$\Psi_{200} = \frac{1}{\sqrt{32\pi a_0^3}} \left[2 - \frac{r}{a_0}\right] e^{-r/2a_0}. \qquad (25)$$

The quasi- bound Helium-Beryllium system can be experimentally achieved by firing alpha particles of energy, say, 40 MeV at highly rarified Beryllium-8 nucleus [4,5,6].

References:

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