A new solvable quintic equation of the shape $x^5 + ax^2 + b = 0$

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Abstract

So far, there are in all five solvable quintics of the shape $x^5 + ax^2 + b = 0$. We have found one more. In this paper, we give that equation and its solution.

It is known, up to scaling of the variable, there are exactly five solvable quintics of the shape $x^5 + ax^2 + b = 0$, which are (where s is a scaling factor) [1]; [2] :

$$x^5 - 2s^3x^2 - \frac{s^5}{5}$$
$$x^5 - 100s^3x^2 - 1000s^5$$
$$x^5 - 5s^3x^2 - 3s^5$$
$$x^5 - 5s^3x^2 + 15s^5$$
$$x^5 - 25s^3x^2 - 300s^5$$

However, we have found a new one, it is also solvable.

$$x^5 - 5s^3x^2 + 2s^5 = 0$$

$$x = \frac{1}{2}(1 \pm \sqrt{5 + \frac{20}{r_0} + 2rr_0})s$$

$$r^2 = 5 + \frac{20}{r_0} - r_0^2$$

$$r_0^2 = (\sqrt[3]{\frac{1475}{27} + i\sqrt{\frac{180074375}{729}}} + \sqrt[3]{\frac{1475}{27} - i\sqrt{\frac{180074375}{729}}} + \frac{5}{3})$$

i: imaginary value.

There are 8 values for x, let s = 1, can find out matching and unsuitable values among them, the matching values satisfy the equation $x^5 - 5x^2 + 2 = 0$.

References

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