A Theoretical Approach to Complex Systems Analysis: Simple Non-Directed Graphs as Homogenous, Morphological Models

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Abstract

Recent advances have begun to blur the lines between theoretical mathematics and applied mathematics. Oftentimes, in a variety of fields, concepts from not only applied mathematics but theoretical mathematics have been employed to great effect. As more and more researchers come to utilize, deploy, and develop both abstract and concrete mathematical models (both theoretical and applied), the demand for highly generalizable, accessible, and versatile mathematical models has increased drastically (Rosen, 2011). Specifically in the case of Complex Systems and the accompanying field of Complex Systems Analysis, this phenomenon has had profound effects. As researchers, academics, and scholars from these fields turn to mathematical models to assist in their scientific inquiries (specifically, concepts and ideas taken from various subsets of graph theory), the limitations of our current mathematical frameworks becomes increasingly apparent. To remedy this, we present the Chang Graph, a simple graph defined by an n-sided regular polygon surrounding a 2n-sided regular polygon. Various properties and applications of this graph are discussed, and further research is proposed for the study of this mathematical model.

Keywords: Complex Systems, Complex Systems Analysis, Novel Methods in Mathematical Research, Applications of Theoretical Mathematics, Theoretical Mathematics, Applied Mathematics, Mathematics, Philosophy of Mathematics, Discrete Mathematics

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1. Introduction

For any academic (or professional, for that matter) working within the field of Complex Systems, finds it prerequisite to understand the associated field of Complex Systems Analysis. Instead of delving into this field directly, as many before have (Bar-Yam, 2002), we instead seek to present a valuable theoretical framework with which to approach the main concepts present in the study of Complex Systems Analysis.

For us, this theoretical framework takes the form of, generally, the many concepts and insights of the mathematical field of Graph Theory, and, specifically, our proposition of the Chang Graph as a morphological model for comprehensive utilization in the study of, and development within, the field of Complex Systems Analysis. As will be shown, this graph (and its varied insights into Graph Theory) can be a powerful theoretical tool for those seeking to harvest homogenous, morphological, mathematical models for implementation in Complex Systems Analysis.

2. Definition of the Chang Graph

For ease of use, we will be using the abbreviated notation of the Chang Graph as "CG". The definition of the CG is as follows:

1. A simple, non-directed graph formed by the union of two regular polygons:
   (a) A regular, n-sided convex polygon. Most commonly this takes the form of a square (n=4) but can also take the form of a triangle (n=3), pentagon (n=5), hexagon (n=6), and so on.
   (b) A regular, 2n-sided star polygon, inscribed into the first shape such that the vertices of both shapes overlap (as is illustrated in Fig. 1 below).

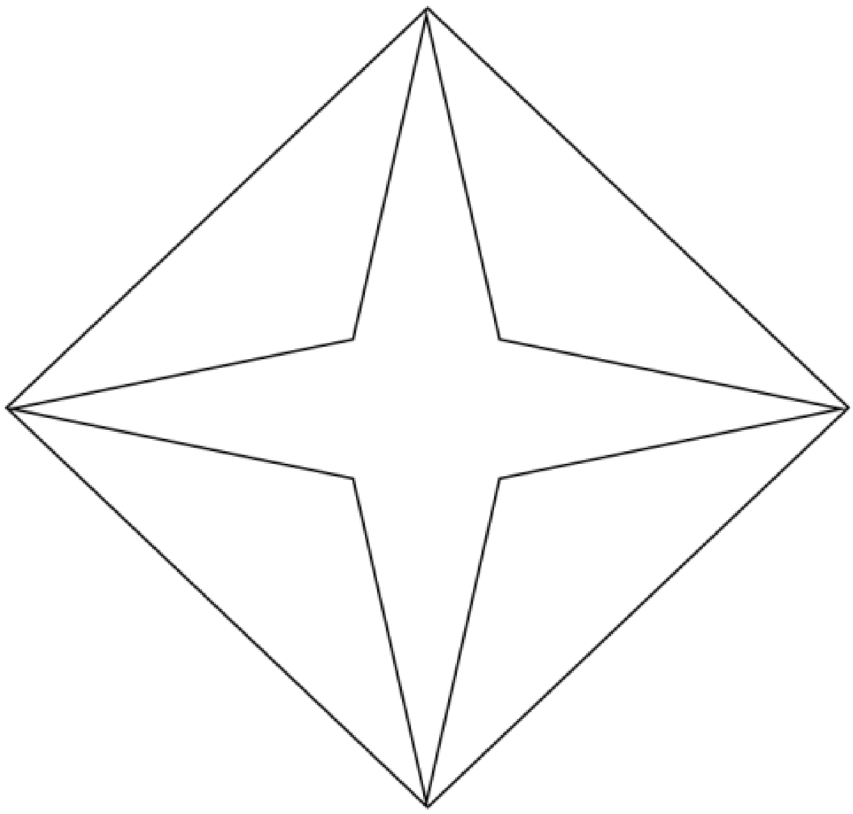

Figure 1: The common Chang Graph, comprised of an outer 4-sided convex polygon (square) and an inner 8-sided star polygon.

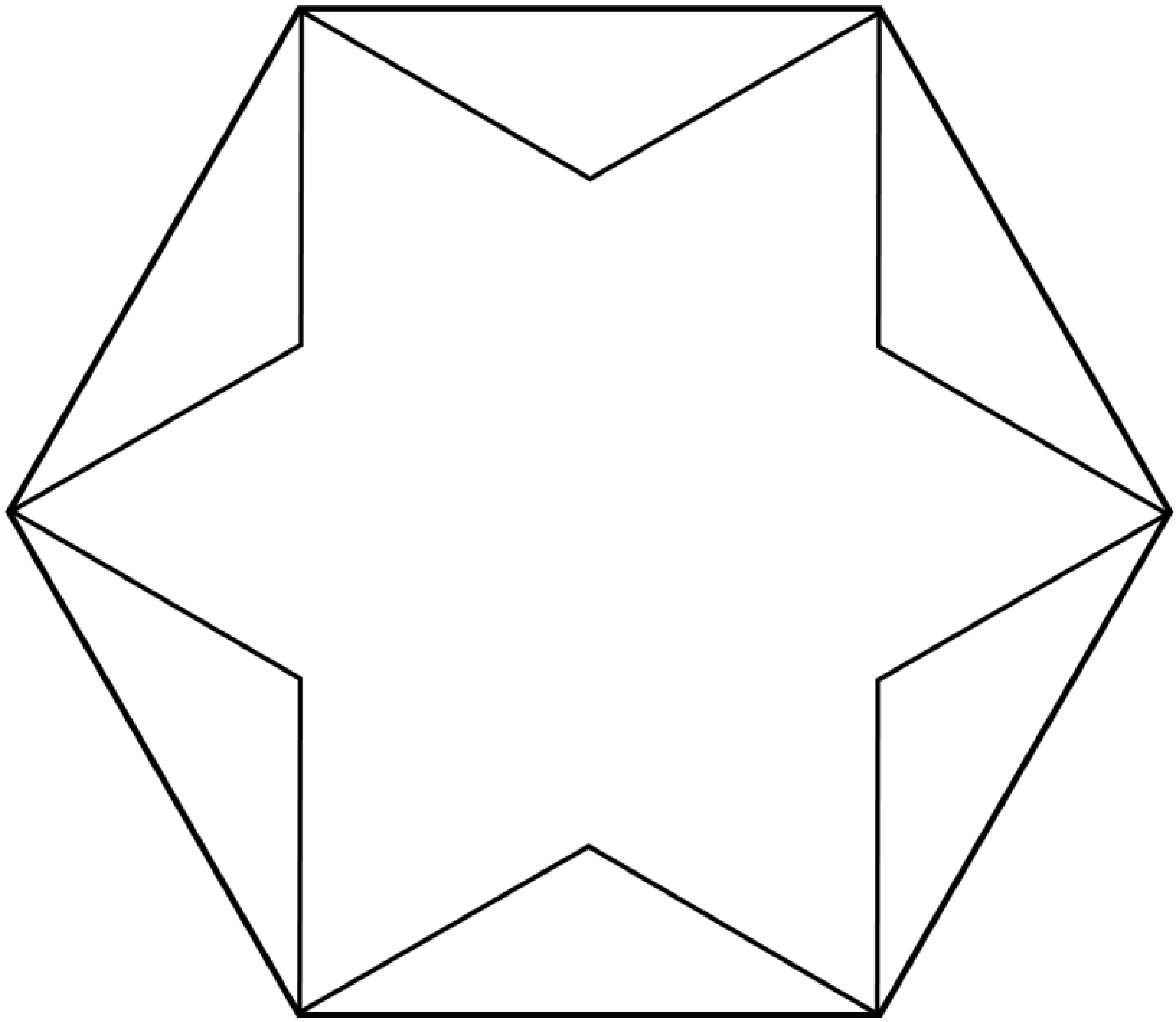

Figure 2: An uncommon Chang Graph, featuring an outer polygon with 6 sides (hexagon) and an inner polygon resembling a slightly-tilted Star of David.

3. Properties of the Chang Graph

The Chang Graph contains a variety of interesting features (some trivial, some puzzlingly counter-intuitive) and presents an array of insights into the complex concepts unpinning much of modern Graph Theory. However, the present paper does not intend to delve fully into all of its features, but rather opts for a cursory outline of its more intriguing aspects.

For the Common Chang Graph (CCG), we can develop the following list of properties inherent in its structure by definition:

For prove of the CCG's Chromatic Index, we can refer to Fig. 3 below:

Table 1: Various Structural Features of the CCG

Vertices	8
Edges	12
Chromatic Number	3
Chromatic Index	4

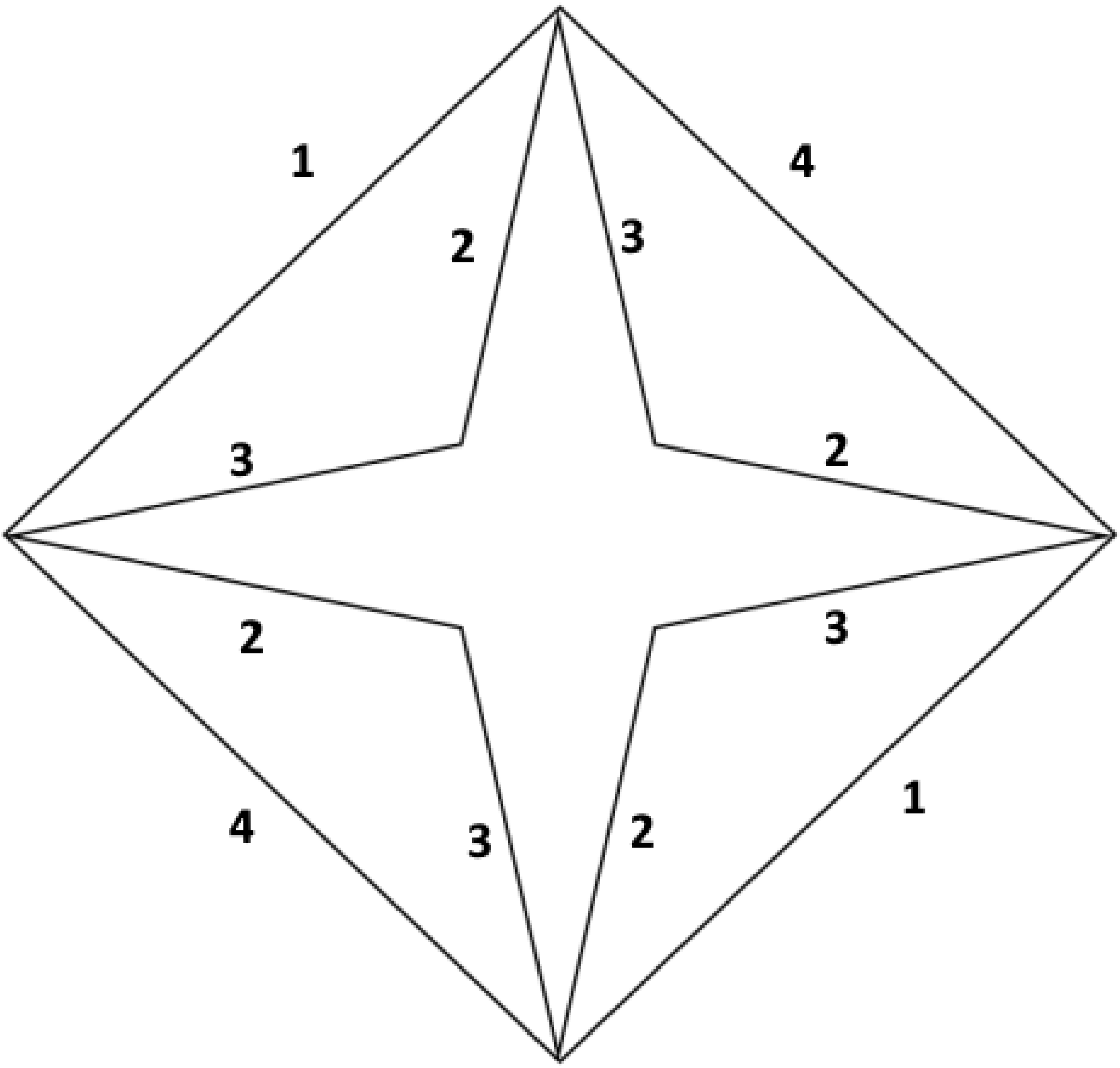

Figure 3: A comprehensive coloring of the 12 vertices comprising the CCG. Note how only four colors are used in this coloring.

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4. Applications of the Chang Graph

The specific applications of the Chang Graph in Complex Systems Analysis are varied. They can be used to introduce laymen to the concept of mathematical graphs, as well as the concepts of vertices and edges. They can also be implemented specifically within Complex Systems Analysis as discrete, yet nonetheless advanced, homogeneous morphological models.

The Chang Graph also presents several interesting parallels to the legendary Petersen Graph, of which the revered Donald Knuth once described as:

"a remarkable configuration that serves as a counterexample to many optimistic predictions about what might be true for graphs in general."

Of the many similarities between just the CCG and the Petersen Graph: both possess a Chromatic Index of 4 while simultaneously retaining a Chromatic Number of 3.

Acknowledgements

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Appendix A. References

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