

[A] Corollary [from a Theorem Proposed by the Author]

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Abstract

[From [a] theorem [proposed by the author], we can derive the generalized Dirichlet integral for any natural value when the whole integrand is raised to the n-th power.

Keywords: Dirichlet integral, [] theorem, Dirichlet generalization, Laplace transforms

1 Proof:

$$\begin{aligned}
 \int_0^{\infty} \frac{(\text{Sin}[t])^n}{t^n} dt &= \text{Lim}_{s \rightarrow 0} \int_s^{\infty} \frac{1}{\Pi_{(n-1)}} \mathcal{L}[D^{n-1}((\text{Sin}[t])^n)] d\sigma \\
 &= \text{Lim}_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L}[D^{n-1}\left(\left(\frac{e^{it} - e^{-it}}{2i}\right)^n\right)] d\sigma \\
 &= \text{Lim}_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L}[D^{n-1}\left(\sum_{k=0}^n \binom{n}{k} \frac{(e^{it})^{n-k} (-1)^k (e^{-it})^k}{(2i)^n}\right)] d\sigma \\
 &= \text{Lim}_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L}[D^{n-1}\left(\sum_{k=0}^n \binom{n}{k} \frac{(-1)^k e^{i(n-2k)t}}{(2i)^n}\right)] d\sigma
 \end{aligned}$$

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L} \left[\sum_{k=0}^n \binom{n}{k} \frac{(-1)^k (i(n-2k))^{n-1} e^{i(n-2k)t}}{(2i)^n} \right] d\sigma$$

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L} \left[\sum_{k=0}^n \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1} e^{i(n-2k)t}}{2^n i} \right] d\sigma$$

If n is even

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L} \left[\sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1} (e^{i(n-2k)t} - e^{-i(n-2k)t})}{2^{n-1} (2i)} \right] d\sigma$$

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L} \left[\sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1} \text{Sin}[(n-2k)t]}{2^{n-1}} \right] d\sigma$$

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^{n-1}} \frac{n-2k}{\sigma^2 + (n-2k)^2} d\sigma$$

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^{n-1}} \frac{1}{\left(\frac{\sigma}{n-2k}\right)^2 + 1} d\sigma$$

$$= \lim_{b \rightarrow \infty} \frac{1}{(n-1)!} \sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^{n-1}} \operatorname{Arctan}\left[\frac{b}{n-2k}\right]$$

$$= \frac{1}{(n-1)!} \sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^{n-1}} \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{(\operatorname{Sin}[t])^n}{t^n} dt = \frac{\pi}{(n-1)!} \sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^n}, n \text{ even}$$

If n is odd

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L} \left[\sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1} (e^{i(n-2k)t} - e^{-i(n-2k)t})}{2^{n-1} (2i)} \right] d\sigma$$

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \mathcal{L} \left[\sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1} \text{Sin}[(n-2k)t]}{2^{n-1}} \right] d\sigma$$

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^{n-1}} \frac{n-2k}{\sigma^2 + (n-2k)^2} d\sigma$$

$$= \lim_{s \rightarrow 0} \int_s^{\infty} \frac{1}{(n-1)!} \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^{n-1}} \frac{1}{\left(\frac{\sigma}{n-2k}\right)^2 + 1} d\sigma$$

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$$= \lim_{b \rightarrow \infty} \frac{1}{(n-1)!} \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^{n-1}} \text{Arctan} \left[\frac{b}{n-2k} \right]$$

$$= \frac{1}{(n-1)!} \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1} \pi}{2^{n-1}} \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{(\text{Sin}[t])^n}{t^n} dt = \frac{\pi}{(n-1)!} \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{(-1)^k (n-2k)^{n-1}}{2^n}, n \text{ odd}$$

2 Examples:

$$2.1) \int_0^{\infty} \frac{\text{Sin}[t]}{t} dt = \frac{\pi}{(1-1)!} \sum_{k=0}^{\frac{1-1}{2}} \binom{1}{k} \frac{(-1)^k (1-2k)^{1-1}}{2} = \frac{\pi}{2}$$

$$2.2) \int_0^{\infty} \frac{(\text{Sin}[t])^2}{t^2} dt = \frac{\pi}{(2-1)!} \sum_{k=0}^{\frac{2-1}{2}-1} \binom{2}{k} \frac{(-1)^k (2-2k)^{2-1}}{2^2} = \frac{\pi}{2}$$

$$2.3) \int_0^{\infty} \frac{(\text{Sin}[t])^3}{t^3} dt = \frac{\pi}{(3-1)!} \sum_{k=0}^{\frac{3-1}{2}} \binom{3}{k} \frac{(-1)^k (3-2k)^{3-1}}{2^3} = \frac{\pi}{16} (1 \cdot 3^2 - 3 \cdot 1^2) = \frac{3\pi}{8}$$

$$2.4) \int_0^{\infty} \frac{(\text{Sin}[t])^4}{t^4} dt = \frac{\pi}{(4-1)!} \sum_{k=0}^{\frac{4-1}{2}-1} \binom{4}{k} \frac{(-1)^k (4-2k)^{4-1}}{2^4} = \frac{\pi}{6} \left(\frac{1 \cdot 4^3 - 4 \cdot 2^3}{2^4} \right) = \frac{\pi}{3}$$