The *y-Increment Puzzle* And The *Law-Of-Large-Numbers*.

Michael C. Nwogugu
Enugu 400007, Enugu State, Nigeria
Emails: Mcn2225@aol.com; or mcn2225@gmail.com
Phone: 234-814-906-2100; 234-909-606-8162

Abstract.

The *y-Increment Puzzle* is a new phenomenon that is introduced herein, and occurs in exponential equations of the type $x = r^s - q^s$; in positive integers where $r = (n+y_1)$, $q=(n-y_2)$, and y_1 and y_2 may or may not be equal. This class of functions has wide applications in computer science, applied math, physics and finance/economics. Given the *y-Increment Puzzle*, the Law-of-Large-Numbers (LLN) isn't valid for all time series.

Keywords: *Nonlinearity*; Dynamical Systems; Ill-Posed Problems; Law-of-Large-numbers; Homomorphisms; Prime Numbers; Mathematical Cryptography.

1. Introduction.

A *y-Increment Function* is a type of function that: i) contains only polynomials, and ii) has only two variable, iii) has two polynomial terms that have either "*Equi-Distant Opposite Increments*" or "*Unequal Opposite Increments*". The *y-Increment Puzzle* or *Asymmetrical y-Increment Puzzle* occurs when a *y-Increment function* has a different value than that produced by an expansion of its component variables (its "Equivalent Formula"). On equations that are tangentially related to the *y-Increment Puzzle*, see the comments in: Lu (1959), Miyazaki & Togbé (2012), Terai & Hibino (2015), Ibarra & Dang (2006), Rahmawati, Sugandha, et. al. (2019), and Wang & Deng (1996).

Chu (2008) and Lu & Wu (2016) studied dynamical systems pertaining to Diophantine equations. Luca, Moree & Weger (2011) discussed *Group Theory*. On Homomorphisms, see: Wang & Chin (2012). Elia (2005), Jones, Sato, et. al. (1976) and Matijasevič (1981) noted that primes can also be represented as Diophantine equations or as polynomials. On uses of Diophantine Equations in Cryptography, see: Ding, Kudo, et. al. (2018), Okumura (2015), and Ogura (2012) (ie. the equations $x = r^s - q^s$, and $x=(n+y)^s-(n-y)^s$ and $a^x+b^x=c^x$ can be used to create public-keys and in cryptoanalysis). Zadeh (2019) notes that Diophantine equations have been used in analytic functions.

Chen, Wu & Li (2013) and Seneta (2013) analyzed the LLN (Law Of Large Numbers); and Lan & Zhang (2017) analyzed limit theorems.

2. A *y-Increment Function* With *Equi-Distant Increments*.

The function $x = (n+y)^s - (n-y)^s$ is defined as having "*Equi-Distant Opposite Increments*" because it has only two terms, and in the first term ($(n+y)^2$), *y* is added to *n*, whereas in the second term ($(n-y)^2$), *y* is subtracted from *n*. The function is a special case of the function ($x = r^s-q^s$). Here, $r = (n+y)$; $q=(n-y)$; and $y=2$.

Theorem-1: The "*y-Increment Puzzle*" Exists In Integers.

Proof: The function $x = (n+2)^2 - (n-2)^2$ is a special case of the function ($x = r^s-q^s$). Here, $r = (n+y)$; $q=(n-y)$; and $y=2$.

Where *n* is any real number (and y=2):

1.1) $x_2 = (n+2)^2 - (n-2)^2 = n^2 + 4n + 4 - n^2 +4n -4 = 8n$

1.2) $\partial(n^2)/\partial n >1$, as $(0<n) \to +\infty$.

1.3) $\partial(n^2)^2/\partial^2 n > 0$, as $(0<n) \to +\infty$.

Thus as $(n,y) \to +\infty$, and *n* is very large (eg. $n> 7E+10^{14}$); $dx/dn > d(4yn)/dn$

Where *n* is any real number; and *y* is a real odd number:
1.4) $x_2 = (n+y)^2 - (n-y)^2 = n^2 + 2yn + y^2 - n2 + 2yn - y^2 = 4yn$

As shown in Tables 1-4, x = 8n (where y=2) and x=4yn (where y is a real odd number) are valid for some positive values of *n*, but not for very large positive *n*. That is henceforth referred to as the "*y-increment puzzle*" because as shown in Tables 1-5: i) when *n* exceeds a specific positive-number threshold, the function $x = (n+2)^2 - (n-2)^2$ looses its homomorphism; and ii) when *n* exceeds a specific positive-number threshold, the values of *x* change significantly from positive to negative and back; and iii) when n exceeds a specific positive-number threshold, $x = (n+2)^2 - (n-2)^2$ should be equal to, but differs substantially from x=4yn (which is its "**Equivalent Formula**").

In the case of $x_3 = [(n+y)^3 - (n-y)^3]$:
1.5) $(n+y)^3 = (n^2 + 2yn + y2)(n+y) = n^3 + 2yn^2 + ny^2 + yn^2 + 2y^2n + y^3 = n^3 + 3yn^2 + 3y^2n + y^3$
1.6) $(n-y)^3 = (n2 - 2yn + y2)(n-y) = (n^3 - 2yn^2 + ny^2 - yn^2 + 2y^2n - y^3) = n^3 - 3yn^2 + 3y^2n - y^3$
1.7) Thus, $x_3 = (n+y)^3 - (n-y)^3 = n^3 + 3yn^2 + 3y^2n + y^3 - n^3 + 3yn^2 - 3y^2n + y^3 = 6yn^2 + 2y^3$.
But under the *y-Increment Puzzle*, when n exceeds a specific positive-number threshold, equation $x_3 = [(n+y)^3 - (n-y)^3]$ should be equal to, but behaves differently from $x_3 = 6yn^2 + 2y^3$ (which is its "**Equivalent Formula**") and both equations' Homomorphisms change drastically over $-\infty < n,y < +\infty$.
As above:

$\partial(n^3)/\partial n > 1$, as $(0<n) \rightarrow +\infty$.
$\partial(n^3)^2/\partial^2 n > 0$, as $(0<n) \rightarrow +\infty$.
Thus as $(n,y) \rightarrow +\infty$, and *n* is very large (eg. n> 7E+10[14]); $dx/dn > d(6yn^2+2y^3)/dn$

In the case of $x_6 = (n+y)^6 - (n-y)^6$:
1.8) $(n+y)^6 = (n^3+3yn^2+3y^2n+y^3)(n^3+3yn^2+3y^2n+y^3) = n^6+3yn^5+3y^2n^3+n^3y^3+3yn^5+9y^2n^4+9y^3n^3+3y^4n^2+3y^2n^4+9y^3n^3+9y^4n^2+3y^5n+n^3y^3+3y^4n^2+3y^5n+y^6$
$= n^6+6yn^5+3y^2n^3+20n^3y^3+12y^2n^4+15y^4n^2+6y^5n+y^6$

1.9) $(n-y)^6 = (n^3-3yn^2+3y^2n-y^3)(n^3-3yn^2+3y^2n-y^3) = n^6-3yn^5+3y^2n^4-y^3n^3-3yn^5+9y^2n^4-9y^3n^3+3y^4n^2+3y^2n^4-9y^3n^3+9y^4n^2-3y^5n-y^3n^3+3y^3n^2-3y^5n+y^6$
$= n^6-6yn^5+6y^2n^4+9y^2n^4-20y^3n^3+12y^4n^2-6y^5n+3y^3n^2+y^6$

1.10) Thus $x_6 = n^6+6yn^5+3y^2n^3+20n^3y^3+12y^2n^4+15y^4n^2+6y^5n+y^6 - [n^6-6yn^5+15y^2n^4-20y^3n^3+12y^4n^2-6y^5n+3y^3n^2+y^6]$
$= n^6+6yn^5+3y^2n^3+2n^3y^3+12y^2n^4+18y^3n^3+15y^4n^2+6y^5n+y^6 - n^6+6yn^5-15y^2n^4+20y^3n^3-12y^4n^2+6y^5n-3y^3n^2-y^6$
$x_6 = 12yn^5+3y^2n^3+40n^3y^3-3y^2n^4+3y^4n^2+12y^5n-3y^3n^2$

But under the *y-Increment Puzzle*, when *n* exceeds a specific positive-number threshold, equation $x_6=[(n+y)^6 - (n-y)^6]$ should be equal to, but behaves differently from $x_6 = [12yn^5+3y^2n^3+40n^3y^3-3y^2n^4+3y^4n^2+12y^5n-3y^3n^2]$ (which is its "**Equivalent Formula**") and both equations' Homomorphisms change drastically over real numbers $-\infty < n,y < +\infty$.

Where *n, y and s* are real numbers and positive integers:
1.11) $x = (n+y)^s - (n-y)^s = n^s + (s)yn^s + 2yn + y^2 - n2 + 2yn \ldots - y^s = (s^2)yn$

■

2. A *y-Increment Function* With Un-equal Increments (The "*Asymmetrical y-Increment Puzzle*").
The function $x = (n+y_1)^2 - (n-y_2)^2$ (where $y_1 \neq y_2$ are positive integers) is a special case of the function ($x = r^2 - q^2$) and both have numerous applications in computer science, applied math, physic and finance/economics. Here, $r = (n+y_1)$ and $q=(n-y_2)$.

Theorem-2: The "*Asymmetrical y-Increment Puzzle*" Exists In Integers.

Proof:

Where n is any real number (and where $y1 \neq y2$ are positive integers):

2.1) $x_2 = (n+y_1)^2 - (n-y_2)^2 = n^2 + 2ny_1 + y_1^2 - n^2 + 2ny_2 - y_2^2 = 2n(y_1 + y_2) + y_1^2 - y_2^2$

As shown in Tables 6-11, equations $x = (n+y_1)^2 - (n-y_2)^2$ =, and $x = 2n(y_1 + y_2) + y_1^2 - y_2^2$, (where y_1 and y_2 are unequal positive integers) are equal only for some positive values of n, but not for very large positive n.

That is henceforth referred to as the "*Asymmetrical y-Increment Puzzle*" because: i) when n exceeds a specific positive-number threshold, the functions $x = (n+y_1)^2 - (n-y_2)^2$, and $x = 2n(y_1 + y_2) + y_1^2 - y_2^2$ alternately lose and re-gain their homomorphisms (both equations' Homomorphisms change drastically); and ii) when n exceeds a specific positive-number threshold, the values of $x = (n+y_1)^2 - (n-y_2)^2$ and $x = 2n(y_1 + y_2) + y_1^2 - y_2^2$ (which is its "**Equivalent Formula**") change significantly from positive to negative and back.

Separately,

2.2) if $x_4 = (n+y_1)^4 - (n-y_2)^4 = (n^2 + 2ny_1 + y_1^2)^2 - (n^2 - 2ny_2 - y_2^2)^2$

Then:

2.3) $(n^2 + 2ny_1 + y_1^2)(n^2 + 2ny_1 + y_1^2) = n^4 + 2n^3y_1 + n^2y_1^2 + 2n^3y_1 + 4n^2y_1^2 + 2ny_1^3 + n^2y_1^2 + 2ny_1^3 + y_1^4$

$= n^4 + 4n^3y_1 + 6n^2y_1^2 + 4ny_1^3 + y_1^4$

2.4) $(n^2 - 2ny_2 - y_2^2)(n^2 - 2ny_2 - y_2^2) = n^4 - 2n^3y_2 - n^2y_2^2 - 2n^3y_2 + 4n^2y_2^2 + 2ny_2^3 - n^2y_2^2 + 2ny_2^3 + y_2^4$

$n^4 - 4n^3y_2 + 2n^2y_2^2 + 4ny_2^3 + y_2^4$

Thus,

2.5) $x_4 = (n+y_1)^4 - (n-y_2)^4 = (n^2 + 2ny_1 + y_1^2)^2 - (n^2 - 2ny_2 - y_2^2)^2 =$

$= n^4 + 4n^3y_1 + 6n^2y_1^2 + 4ny_1^3 + y_1^4 - (n^4 - 4n^3y_2 + 2n^2y_2^2 + 4ny_2^3 + y_2^4)$

$= n^4 + 4n^3y_1 + 6n^2y_1^2 + 4ny_1^3 + y_1^4 - n^4 + 4n^3y_2 - 2n^2y_2^2 - 4ny_2^3 - y_2^4$

$= 4n^3y_1 + 6n^2y_1^2 + 4ny_1^3 + y_1^4 + 4n^3y_2 - 2n^2y_2^2 - 4ny_2^3 - y_2^4$

2.6) $\mathbf{x_4 = 4n^3(y_1+y_2) + 2n^2(3y_1^2 - y_2^2) + 4n(y_1^3 - y_2^3) + y_1^4 - y_2^4}$

As above, and under the ***Asymmetrical y-Increment Puzzle***, when n exceeds a specific positive-number threshold, equation $x_4 = [(n+y_1)^4 - (n-y_2)^4]$ should be equal to (but isn't), and behaves differently from $x_4 = [\mathbf{4n^3(y_1+y_2) + 2n^2(3y_1^2 - y_2^2) + 4n(y_1^3 - y_2^3) + y_1^4 - y_2^4}]$ (which is its "**Equivalent Formula**") and both equations' Homomorphisms change drastically over real numbers $-\infty < n, y < +\infty$.

Separately,

2.7) if $x_8 = (n+y_1)^8 - (n-y_2)^8 = (n^4 + 4n^3y_1 + 6n^2y_1^2 + 4ny_1^3 + y_1^4)^2 - (n^4 - 4n^3y_2 + 2n^2y_2^2 + 4ny_2^3 + y_2^4)^2$

Then:

2.8) $(n^4 + 4n^3y_1 + 6n^2y_1^2 + 4ny_1^3 + y_1^4)^2 = (n^4 + 4n^3y_1 + 6n^2y_1^2 + 4ny_1^3 + y_1^4)(n^4 + 4n^3y_1 + 6n^2y_1^2 + 4ny_1^3 + y_1^4)$

$= n^8 + 4n^7y_1 + 6n^6y_1^2 + 4n^5y_1^3 + n^4y_1^4 + 4n^7y_1 + 16n^6y_1^2 + 24n^5y_1^3 + 16n^4y_1^4 + 4n^3y_1^5 + 6n^6y_1^2 + 24n^5y_1^3 + 36n^4y_1^4 + 24n^3y_1^5 + 6n^2y_1^6 + 4n^5y_1^3 + 16n^4y_1^4 + 24n^3y_1^5 + 16n^2y_1^6 + 4ny_1^7 + n^4y_1^4 + 4n^3y_1^5 + 6n2y_1^6 + 4ny_1^7 + y_1^8$

$= n^8 + 8n^7y_1 + 28n^6y_1^2 + 56n^5y_1^3 + 56n^3y_1^5 + 70n^4y_1^4 + 28n^2y_1^6 + 8ny_1^7 + y_1^8$

and:

$(n^4 - 4n^3y_2 + 2n^2y_2^2 + 4ny_2^3 + y_2^4)^2 = (n^4 - 4n^3y_2 + 2n^2y_2^2 + 4ny_2^3 + y_2^4)(n^4 - 4n^3y_2 + 2n^2y_2^2 + 4ny_2^3 + y_2^4)$

$= n^8 - 4n^7y_2 + 2n^6y_22 + 4n^5y_2^3 + n^4y_2^4 - 4n^7y_2^2 - 8n^5y_2^3 - 16n^4y_2^4 - 4n^3y_2^5 + 2n^6y_2^2 - 8n^5y_2^3 + 4n^4y_2^4 + 8n^3y_2^5 + 2n^2y_2^6 + 4n^5y^{23} - 16n^4y^4 + 8n^3y_2^5 + 16n^2y_2^6 + 4ny_2^7 + n^4y_2^4 - 4n^3y_2^5 + 2n^2y_2^6 + 4ny_2^7 + y_2^8$

$= n^8 - 8n^7y_2 + 4n^6y_2^2 - 8n^5y_2^3 - 26n^4y_2^4 + 8n^3y_2^5 + 20n^2y_2^6 - 8ny_2^7 + y_2^8$

2.9) Thus: $x_8 = (n+y_1)^8 - (n-y_2)^8 = n^8 + 8n^7y_1 + 28n^6y_1^2 + 56n^5y_1^3 + 56n^3y_1^5 + 70n^4y_1^4 + 28n^2y_1^6 + 8ny_1^7 + y_1^8 - [n^8 - 8n^7y_2 + 4n^6y_2^2 - 8n^5y_2^3 - 26n^4y_2^4 + 8n^3y_2^5 + 20n^2y_2^6 - 8ny_2^7 + y_2^8]$

$= n^8 + 8n^7y_1 + 28n^6y_1^2 + 56n^5y_1^3 + 56n^3y_1^5 + 70n^4y_1^4 + 28n^2y_1^6 + 8ny_1^7 + y_1^8 - n^8 + 8n^7y_2 - 4n^6y_2^2 + 8n^5y_2^3 + 26n^4y_2^4 - 8n^3y_2^5 - 20n^2y_2^6 + 8ny_2^7 - y_2^8$

$= 8n^7y_1 + 28n^6y_1^2 + 56n^5y_1^3 + 56n^3y_1^5 + 70n^4y_1^4 + 28n^2y_1^6 + 8ny_1^7 + y_1^8 + 8n^7y_2 - 4n^6y_2^2 + 8n^5y_2^3 + 26n^4y_2^4 - 8n^3y_2^5 - 20n^2y_2^6 + 8ny_2^7 - y_2^8$

$= 8n^7(y_1+y_2) + 8n(y_1^7 + y_2^7) + 8n^3(n^2y_2^3 - y_2^5) + 28(n^6y_1^2 + n^2y_1^6) + 56(n^5y_1^3 + n^3y_1^5) + n^4(70y_1^4 + 26y_2^4) - 4n^6y_2^2 - 20n^2y_2^6 + y_1^8 - y_2^8$

As above, and under the ***Asymmetrical y-Increment Puzzle***, when *n* exceeds a specific positive-number threshold, equation $x_8=[(n+y_1)^8 – (n-y_2)^8]$ should be equal to (but isn't), and behaves differently from $x_8 = [8n^7(y_1+y_2)+8n(y_1^7+y_2^7)+8n^3(n^2y_2^3-y_2^5)+28(n^6y_1^2+n^2y_1^6)+56(n^5y_1^3+n^3y_1^5)+n^4(70y_1^4 +26y_2^4) -4n^6y_2^2-20n^2y_2^6+y_1^8-y_2^8]$ (which is its "**Equivalent Formula**") and both equations' Homomorphisms change drastically over integers $-\infty<n,y<+\infty$. ■

Theorem-3: **Both The Weak And Strong Law-Of-Large-Numbers (LLN) Are Wrong Or Aren't Valid For Some Series.**

Proof:

The existence of the *y-Increment Puzzle* and the *Asymmetrical y-Increment Puzzle* implies that LLN is or can be invalid. If a series is defined as $x=[(n+y_1)^a–(n-y_2)^a]$, then given **Theorems 1 & 2** herein and above, and Tables 1-11 below, the average of *x* wont converge to its mean over many "trials", because of the changing Homomorphisms implicit in $x=[(n+y_1)^a–(n-y_2)^a]$ and the differences between $[(n+y_1)^a–(n-y_2)^a]$ and its "*Equivalent Formula*"; and thus the LLN is wrong or isn't valid for some time series. ■

Corrollary-1: **Invalidity Of Big-Data Studies That Are Based On Real Numbers.**

The existence of the *y-Increment Puzzle* and the *Asymmetrical y-Increment Puzzle* implies that Variance is or can be invalid. In many circumstances, Variance is calculated as:

i) Variance = $EX-E(X)^2$, or

ii) Variance = Expected/Desired Value – Current Value.

That can be expressed as Variance = $\sum_{i=0}$[Current Value - Expected/Desired Value]/n, where n is the number of data points or occurrences.

Such Variance can also be expressed as Variance = $\sum_{i=0}[(c+y_1)^a–(c-y_2)^a]/n$; and in such cases, and given **Theorems 1 & 2** herein and above, and Tables 1-11 below, the Variance cannot be accurate, primarily because of the changing (positive and negative) Homomorphisms implicit in $[(c+y_1)^a–(c-y_2)^a]$ and the differences between $[(c+y_1)^a–(c-y_2)^a]$ and its "*Equivalent Formula*".

Similarly, where the mean of a series can be expressed as Mean = $\sum_{i=0}[(c+y_1)^a–(c-y_2)^a]/n$, such Mean may be wrong as a measure of the average. ■

3. Conclusion.

The *y-Increment Puzzle* and the *Asymmetrical y-Increment Puzzle* have significant implications in data analysis and functional analysis (and for the Law-Of-Large-Numbers which isn't valid for all time series).

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Tables.

Table-1 (equal y-Increments): y=9; s=2.

n	$x = (n+y)^2 - (n-y)^2$	4yn	Difference (4yn-x)	% Difference	$f = r^2-q^2$	Difference (f-x)	Difference (f-8n)
4	144	144	0	0.0000000%	144	0	0
20	720	720	0	0.0000000%	720	0	0
2,000	72,000	72,000	0	0.0000000%	72,000	0	0
500,000	18,000,000	18,000,000	0	0.0000000%	18,000,000	0	0
5,000,000	180,000,000	180,000,000	0	0.0000000%	180,000,000	0	0
80,000,000	2,880,000,000	2,880,000,000	0	0.0000000%	2,880,000,000	0	0
600,000,000	21,599,999,983	21,600,000,000	17	0.0000001%	21,600,000,000	17	0
10,000,000,000	359,999,997,871	360,000,000,000	2,129	0.0000006%	359,999,995,904	(1,967)	(4,096)
100,000,000,000	3,600,000,241,583	3,600,000,000,000	(241,583)	-0.0000067%	3,600,000,483,328	241,745	483,328
3.00000E+12	1.08000E+14	1.08000E+14	449,929,297	0.0004166%	1.0800E+14	623,812,689	173,883,392
1.7000E+14	6.1210E+15	6.1200E+15	-1.0404E+12	-0.0169995%	6.1177E+15	-3.358E+12	-2.317E+12
2.3000E+15	8.3058E+16	8.2800E+16	-2.5830E+14	-0.3119524%	8.2191E+16	-8.6760E+14	-6.0931E+14
7.8000E+16	2.5569E+18	2.8080E+18	2.5108E+17	8.9415419%	0.0000E+00	-2.5569E+18	-2.8080E+18
6.4000E+17	1.1520E+19	2.3040E+19	1.1520E+19	50.0000000%	0.0000E+00	-1.1520E+19	-2.3040E+19
3.4500E+18	6.2100E+19	1.2420E+20	6.2100E+19	50.0000000%	0.0000E+00	-6.2100E+19	-1.2420E+20
9.2000E+19	1.6560E+21	3.3120E+21	1.6560E+21	50.0000000%	0.0000E+00	-1.6560E+21	-3.3120E+21

Table-2 (equal y-Increments): y = 175; s=2

n	$x = (n+y)^2 - (n-y)^2$	4yn	Difference (4yn-x)	% Difference	$f = r^2-q^2$	Difference (f-x)	Difference (f-8n)
4	2,800	2,800	0	0.0000000%	2,800	0	0
20	14,000	14,000	0	0.0000000%	14,000	0	0
2,000	1,400,000	1,400,000	0	0.0000000%	1,400,000	0	0
500,000	350,000,000	350,000,000	0	0.0000000%	350,000,000	0	0
5,000,000	3,500,000,000	3,500,000,000	0	0.0000000%	3,500,000,000	0	0
80,000,000	56,000,000,000	56,000,000,000	0	0.0000000%	56,000,000,000	0	0
600,000,000	420,000,000,031	420,000,000,000	(31)	0.0000000%	420,000,000,000	(31)	0
10,000,000,000	7,000,000,004,191	7,000,000,000,000	(4,191)	-0.0000001%	7,000,000,004,096	(95)	4,096
100,000,000,000	70,000,000,940,127	70,000,000,000,000	(940,127)	-0.0000013%	70,000,001,941,504	1,001,377	1,941,504
3.00000E+12	2.10000E+15	2.10000E+15	(318,498,911)	-0.0000152%	2.1000E+15	(755,181,663)	(436,682,752)
1.70000E+14	1.19001E+17	1.19000E+17	-1.1712E+12	-0.0009842%	1.1900E+17	-3.227E+12	-2.056E+12
2.3000E+15	1.6100E+18	1.6100E+18	-1.8433E+13	-0.0011449%	1.6100E+18	1.8433E+13	3.6867E+13
7.8000E+16	5.4970E+19	5.4600E+19	-3.7012E+17	-0.6778683%	5.5340E+19	3.7012E+17	7.4023E+17
6.4000E+17	4.4536E+20	4.4800E+20	2.6391E+18	0.5890784%	0.0000E+00	-4.4536E+20	-4.4800E+20
3.4500E+18	3.5687E+21	2.4150E+21	-1.1537E+21	-47.7715628%	0.0000E+00	-3.5687E+21	-2.4150E+21
9.2000E+19	3.2200E+22	6.4400E+22	3.2200E+22	50.0000000%	0.0000E+00	-3.2200E+22	-6.4400E+22

Table-3 (equal y-Increments): y=3,300; s=2

n	$x = (n+y)^2 - (n-y)^2$	4yn	Difference (4yn-x)	% Difference	$f = r^2-q^2$	Difference (f-x)	Difference (f-8n)
4	52,800	52,800	0	0.0000000%	52,800	0	0
20	264,000	264,000	0	0.0000000%	264,000	0	0
2,000	26,400,000	26,400,000	0	0.0000000%	26,400,000	0	0
500,000	6,600,000,000	6,600,000,000	0	0.0000000%	6,600,000,000	0	0
5,000,000	66,000,000,000	66,000,000,000	0	0.0000000%	66,000,000,000	0	0
80,000,000	1,056,000,000,000	1,056,000,000,000	0	0.0000000%	1,056,000,000,000	0	0
600,000,000	7,919,999,999,984	7,920,000,000,000	16	0.0000000%	7,920,000,000,000	16	0
10,000,000,000	131,999,999,997,168	132,000,000,000,000	2,832	0.0000000%	132,000,000,000,000	2,832	0
100,000,000,000	1.32000E+15	1.32000E+15	1,272,592	0.0000001%	1.3200E+15	1,633,040	360,448
3.0000E+12	3.9600E+16	3.9600E+16	(13,391,088)	0.0000000%	3.9600E+16	35,171,088	48,562,176
1.70000E+14	2.24400E+18	2.24400E+18	1.16042E+12	0.0000517%	2.2440E+18	3.238E+12	2.077E+12
2.30000E+15	3.03605E+19	3.03600E+19	-5.08444E+14	-0.0016747%	3.0360E+19	-6.1746E+14	-1.0901E+14
7.80000E+16	1.03016E+21	1.02960E+21	-5.55913E+17	-0.0539931%	1.0284E+21	-1.7499E+18	-1.1940E+18
6.40000E+17	8.42986E+21	8.44800E+21	1.81424E+19	0.2147532%	8.4855E+21	5.5645E+19	3.7502E+19
3.45000E+18	4.63818E+22	4.55400E+22	-8.41832E+20	-1.8485560%	4.2501E+22	-3.8805E+21	-3.0387E+21
9.20000E+19	1.81613E+24	1.21440E+24	-6.01726E+23	-49.5492276%	0.0000E+00	-1.8161E+24	-1.2144E+24

Table-4 (equal y-Increments): y=11;
s=3

n	$x = (n+y)^3 - (n-y)^3$	$6yn^2+2y^3$	Difference ($[6yn^2+2y^3]-x$)	% Difference	$f = r^3-q^3$	Difference (f-x)	Difference (f-$[6yn^2+2y^3]$)
4	3,718	3,718	0	0.0000000%	3,718	0	0
20	29,062	29,062	0	0.0000000%	29,062	0	0
2,000	264,002,662	2.640E+08	0	0.0000000%	264,002,662	0	0
500,000	1.650E+13	1.650E+13	6.000E+00	0.0000000%	1.650E+13	0.000E+00	-6.000E+00
5,000,000	1.650E+15	1.650E+15	-5.530E+03	0.0000000%	1.650E+15	0.000E+00	5.530E+03
80,000,000	4.224E+17	4.224E+17	1.888E+07	0.0000000%	4.224E+17	0.000E+00	-1.888E+07
600,000,000	2.376E+19	2.376E+19	2.240E+10	0.0000001%	2.376E+19	0.000E+00	-2.240E+10
10,000,000,000	6.600E+21	6.600E+21	7.884E+13	0.0000012%	6.600E+21	0.000E+00	-7.884E+13
100,000,000,000	6.600E+23	6.600E+23	-1.475E+17	-0.0000223%	6.600E+23	0.000E+00	1.475E+17
3.0000E+12	5.940E+26	5.940E+26	-2.868E+21	-0.0004828%	5.940E+26	0.000E+00	2.868E+21
1.70000E+14	1.908E+30	1.907E+30	-2.656E+26	-0.0139247%	1.908E+30	0.000E+00	2.656E+26
2.30000E+15	3.473E+32	3.491E+32	1.804E+30	0.5166224%	3.473E+32	0.000E+00	-1.804E+30
7.80000E+16	0.000E+00	4.015E+35	4.015E+35	100.0000000%	0.000E+00	0.000E+00	-4.015E+35
6.40000E+17	0.000E+00	2.703E+37	2.703E+37	100.0000000%	0.000E+00	0.000E+00	-2.703E+37
3.45000E+18	0.000E+00	7.856E+38	7.856E+38	100.0000000%	0.000E+00	0.000E+00	-7.856E+38
9.20000E+19	0.000E+00	5.586E+41	5.586E+41	100.0000000%	0.000E+00	0.000E+00	-5.586E+41

Table-5 (equal y-Increments):
y=3300; s=3

n	$x = (n+y)^3 - (n-y)^3$	$6yn^2+2y^3$	Difference ($[6yn^2+2y^3]-x$)	% Difference	$f = r^3-q^3$	Difference (f-x)	Difference (f-$[6yn^2+2y^3]$)
4	71,874,316,800	71,874,316,800	0	0.0000000%	71,874,316,800	0	0
20	21,791	0	(21,791)	#DIV/0!	71,881,920,000	71,881,898,209	71,881,920,000
2,000	132,727,331	0	(132,727,331)	#DIV/0!	151,074,000,000	150,941,272,669	151,074,000,000
500,000	8.250E+12	0	-8.250E+12	#DIV/0!	4.95E+15	4.942E+15	4.950E+15
5,000,000	8.250E+14	0.000E+00	-8.250E+14	#DIV/0!	4.95E+17	4.942E+17	4.950E+17
80,000,000	2.112E+17	0.000E+00	-2.112E+17	#DIV/0!	1.27E+20	1.265E+20	1.267E+20
600,000,000	1.188E+19	0.000E+00	-1.188E+19	#DIV/0!	7.13E+21	7.116E+21	7.128E+21
10,000,000,000	3.300E+21	0.000E+00	-3.300E+21	#DIV/0!	1.98E+24	1.977E+24	1.980E+24
100,000,000,000	3.300E+23	0.000E+00	-3.300E+23	#DIV/0!	1.98E+26	1.977E+26	1.980E+26
3.0000E+12	2.970E+26	0.000E+00	-2.970E+26	#DIV/0!	1.78E+29	1.779E+29	1.782E+29
1.70000E+14	9.538E+29	0.000E+00	-9.538E+29	#DIV/0!	5.72E+32	5.713E+32	5.722E+32
2.30000E+15	1.749E+32	0.000E+00	-1.749E+32	#DIV/0!	1.05E+35	1.046E+35	1.047E+35
7.80000E+16	0	0	0	#DIV/0!	1.20E+38	1.203E+38	1.203E+38
6.40000E+17	0	0	0	#DIV/0!	8.12E+39	8.124E+39	8.124E+39
3.45000E+18	0	0	0	#DIV/0!	2.18E+41	2.178E+41	2.178E+41
9.20000E+19	0	0	0	#DIV/0!	0.00E+00	0.000E+00	0.000E+00

Table-6 (equal y-Increments): y=11;
s=6

n	$x = (n+y)^6 - (n-y)^6$	$[12yn^5+3y^2n^3+40n^3y^3-3y^2n^4+3y^4n^2+12y^5n-3y^3n^2]$	Difference ($[12yn^5+3y^2n^3+40n^3y^3-3y^2n^4+3y^4n^2+12y^5n-3y^3n^2]-x$)	% Difference	$f = r^6-q^6$	Difference (f-x)	Difference (f-$[12yn^5+3y^2n^3+40n^3y^3-3y^2n^4+3y^4n^2+12y^5n-3y^3n^2]$)
4	11,272,976	11,842,160	569,184	4.8064205%	11,272,976	0	(569,184)
20	886,972,240	847,768,240	(39,204,000)	-4.6243771%	886,972,240	0	39,204,000
2,000	4.2244E+18	4.2186E+18	-5.8049E+15	-0.1376027%	4.2244E+18	0.0000E+00	5.8049E+15
500,000	4.1250E+30	4.1250E+30	-2.2687E+25	-0.0005500%	4.1250E+30	0.0000E+00	2.2687E+25
5,000,000	4.1250E+35	4.1250E+35	-2.2687E+29	-0.0000550%	4.1250E+35	4.8357E+24	2.2688E+29
80,000,000	4.3254E+41	4.3254E+41	-1.4881E+34	-0.0000034%	4.3254E+41	0.0000E+00	1.4881E+34
600,000,000	1.0264E+46	1.0264E+46	-5.2334E+37	-0.0000005%	1.0264E+46	-5.3169E+36	4.7017E+37
10,000,000,000	1.3200E+52	1.3200E+52	3.3826E+44	0.0000026%	1.3200E+52	5.3522E+44	1.9696E+44
100,000,000,000	1.3200E+57	1.3200E+57	-2.1804E+50	-0.0000165%	1.3200E+57	-1.8707E+50	3.0970E+49
3.0000E+12	3.2076E+64	3.2076E+64	-1.9074E+59	-0.0005946%	3.2076E+64	-1.0043E+59	9.0301E+58
1.70000E+14	1.8745E+73	1.8742E+73	-3.0307E+69	-0.0161704%	1.8749E+73	3.4509E+69	6.4816E+69
2.30000E+15	8.4239E+78	8.4960E+78	7.2098E+76	0.8486170%	8.4818E+78	5.7896E+76	-1.4202E+76
7.80000E+16	4.9732E+86	3.8111E+86	-1.1622E+86	-30.4943795%	5.5949E+86	6.2165E+85	1.7838E+86
6.40000E+17	0.0000E+00	1.4173E+91	1.4173E+91	100.0000000%	0.0000E+00	0.0000E+00	-1.4173E+91
3.45000E+18	0.0000E+00	6.4516E+94	6.4516E+94	100.0000000%	0.0000E+00	0.0000E+00	-6.4516E+94
9.20000E+19	0.0000E+00	8.6999E+101	8.6999E+101	100.0000000%	0.0000E+00	0.0000E+00	-8.6999E+101

Table-7 (equal y-Increments):

y=3300; s=6

n	x = (n+y)^6 - (n-y)^6	$[12yn^5+3y^2n^3+40n^3y^3-3y^2n^4+3y^4n^2+12y^5n-3y^3n^2]$	Difference $([12yn^5+3y^2n^3+40n^3y^3-3y^2n^4+3y^4n^2+12y^5n-3y^3n^2]-x)$	% Difference	f = r6-q6	Difference (f-x)	Difference $(f-[12yn^5+3y^2n^3+40n^3y^3-3y^2n^4+3y^4n^2+12y^5n-3y^3n^2])$
4	1.8785E+19	1.8791E+19	5.6907E+15	0.0302845%	1.8785E+19	0.0000E+00	-5.6907E+15
20	9.3936E+19	9.4079E+19	1.4226E+17	0.1512164%	9.3936E+19	0.0000E+00	-1.4226E+17
2,000	2.2160E+22	2.3060E+22	9.0022E+20	3.9038382%	2.2160E+22	0.0000E+00	-9.0022E+20
500,000	1.2377E+33	1.2356E+33	-2.0418E+30	-0.1652411%	1.2377E+33	0.0000E+00	2.0418E+30
5,000,000	1.2375E+38	1.2373E+38	-2.0419E+34	-0.0165027%	1.2375E+38	2.4179E+24	2.0419E+34
80,000,000	1.2976E+44	1.2976E+44	-1.3382E+39	-0.0010313%	1.2976E+44	4.0565E+31	1.3382E+39
600,000,000	3.0793E+48	3.0793E+48	-4.2340E+42	-0.0001375%	3.0793E+48	5.3169E+36	4.2340E+42
10,000,000,000	3.9600E+54	3.9600E+54	-3.2672E+47	-0.0000083%	3.9600E+54	0.0000E+00	3.2672E+47
100,000,000,000	3.9600E+59	3.9600E+59	-3.3805E+51	-0.0000009%	3.9600E+59	-1.8707E+50	3.1935E+51
3.0000E+12	9.6228E+66	9.6228E+66	-7.6437E+58	-0.0000008%	9.6228E+66	-1.0043E+59	-2.3997E+58
1.70000E+14	5.6226E+75	5.6226E+75	-1.6237E+69	-0.0000289%	5.6226E+75	0.0000E+00	1.6237E+69
2.30000E+15	2.5488E+81	2.5488E+81	-2.3640E+76	-0.0009275%	2.5488E+81	-2.8948E+76	-5.3077E+75
7.80000E+16	1.1420E+89	1.1433E+89	1.3426E+86	0.1174270%	1.1417E+89	-3.1083E+85	-1.6534E+86
6.40000E+17	4.2696E+93	4.2520E+93	-1.7610E+91	-0.4141512%	4.2696E+93	0.0000E+00	1.7610E+91
3.45000E+18	1.7889E+97	1.9355E+97	1.4660E+96	7.5742961%	1.8156E+97	2.6700E+95	-1.1990E+96
9.20000E+19	0.0000E+00	2.6100E+104	2.6100E+104	100.0000000%	0.0000E+00	0.0000E+00	-2.6100E+104

Table-8 (un-equal y-Increments): y1=11 and y2=7; s=2

n	$x = (n+y_1)^2 - (n-y_2)^2$	$[2n(y_1+y_2) + y_1^2 - y_2^2]$	Difference $([2n(y_1+y_2) + y_1^2 - y_2^2]-x)$	% Difference	$f = r^2-q^2$	Difference (f-x)	Difference $(f-[2n(y_1+y_2) + y_1^2 - y_2^2])$
4	2.1600E+02	1.6200E+02	-5.4000E+01	-33.3333333%	2.1600E+02	0.0000E+00	5.4000E+01
20	7.9200E+02	7.3800E+02	-5.4000E+01	-7.3170732%	7.9200E+02	0.0000E+00	5.4000E+01
2,000	7.2072E+04	7.2018E+04	-5.4000E+01	-0.0749813%	7.2072E+04	0.0000E+00	5.4000E+01
500,000	1.8000E+07	1.8000E+07	-5.4000E+01	-0.0003000%	1.8000E+07	0.0000E+00	5.4000E+01
5,000,000	1.8000E+08	1.8000E+08	-5.4000E+01	-0.0000300%	1.8000E+08	0.0000E+00	5.4000E+01
80,000,000	2.8800E+09	2.8800E+09	-5.4000E+01	-0.0000019%	2.8800E+09	0.0000E+00	5.4000E+01
600,000,000	2.1600E+10	2.1600E+10	-4.6000E+01	-0.0000002%	2.1600E+10	0.0000E+00	4.6000E+01
10,000,000,000	3.6000E+11	3.6000E+11	4.1140E+03	0.0000011%	3.6000E+11	0.0000E+00	-4.1140E+03
100,000,000,000	3.6000E+12	3.6000E+12	-4.8331E+05	-0.0000134%	3.6000E+12	0.0000E+00	4.8331E+05
3.0000E+12	1.0800E+14	1.0800E+14	-1.7388E+08	-0.0001610%	1.0800E+14	0.0000E+00	1.7388E+08
1.70000E+14	6.1221E+15	6.1200E+15	-2.0807E+12	-0.0339991%	6.1221E+15	0.0000E+00	2.0807E+12
2.30000E+15	8.2191E+16	8.2800E+16	6.0931E+14	0.7358778%	8.2191E+16	0.0000E+00	-6.0931E+14
7.80000E+16	0.0000E+00	2.8080E+18	2.8080E+18	100.0000000%	0.0000E+00	0.0000E+00	-2.8080E+18
6.40000E+17	0.0000E+00	2.3040E+19	2.3040E+19	100.0000000%	0.0000E+00	0.0000E+00	-2.3040E+19
3.45000E+18	0.0000E+00	1.2420E+20	1.2420E+20	100.0000000%	0.0000E+00	0.0000E+00	-1.2420E+20
9.20000E+19	0.0000E+00	3.3120E+21	3.3120E+21	100.0000000%	0.0000E+00	0.0000E+00	-3.3120E+21

Table-9 (un-equal y-Increments): y1=3300 and y2=2240; s=2

n	$x = (n+y_1)^2 - (n-y_2)^2$	$[2n(y_1+y_2) + y_1^2 - y_2^2]$	Difference $([2n(y_1+y_2) + y_1^2 - y_2^2]-x)$	% Difference	$f = r^2-q^2$	Difference (f-x)	Difference $(f-[2n(y_1+y_2) + y_1^2 - y_2^2])$
4	5.9167E+06	4.9860E+04	-5.8669E+06	-11766.6666667%	5.9167E+06	0.0000E+00	5.8669E+06
20	6.0940E+06	2.2714E+05	-5.8669E+06	-2582.9268293%	6.0940E+06	0.0000E+00	5.8669E+06
2,000	2.8032E+07	2.2166E+07	-5.8669E+06	-26.4683829%	2.8032E+07	0.0000E+00	5.8669E+06
500,000	5.5459E+09	5.5400E+09	-5.8669E+06	-0.1058999%	5.5459E+09	0.0000E+00	5.8669E+06
5,000,000	5.5406E+10	5.5400E+10	-5.8669E+06	-0.0105900%	5.5406E+10	0.0000E+00	5.8669E+06
80,000,000	8.8641E+11	8.8640E+11	-5.8669E+06	-0.0006619%	8.8641E+11	0.0000E+00	5.8669E+06
600,000,000	6.6480E+12	6.6480E+12	-5.8668E+06	-0.0000882%	6.6480E+12	0.0000E+00	5.8668E+06
10,000,000,000	1.1080E+14	1.1080E+14	-5.8681E+06	-0.0000053%	1.1080E+14	0.0000E+00	5.8681E+06
100,000,000,000	1.1080E+15	1.1080E+15	-5.6797E+06	-0.0000005%	1.1080E+15	0.0000E+00	5.6797E+06
3.0000E+12	3.3240E+16	3.3240E+16	-6.6548E+08	-0.0000020%	3.3240E+16	0.0000E+00	6.6548E+08
1.70000E+14	1.8836E+18	1.8836E+18	2.4218E+11	0.0000129%	1.8836E+18	0.0000E+00	-2.4218E+11
2.30000E+15	2.5484E+19	2.5484E+19	3.8151E+14	0.0014971%	2.5484E+19	0.0000E+00	-3.8151E+14
7.80000E+16	8.6354E+20	8.6424E+20	7.0179E+17	0.0812035%	8.6354E+20	0.0000E+00	-7.0179E+17
6.40000E+17	7.1573E+21	7.0912E+21	-6.6137E+19	-0.9326588%	7.1573E+21	0.0000E+00	6.6137E+19
3.45000E+18	3.5418E+22	3.8226E+22	2.8083E+21	7.3464432%	3.5418E+22	0.0000E+00	-2.8083E+21
9.20000E+19	0.0000E+00	1.0194E+24	1.0194E+24	100.0000000%	0.0000E+00	0.0000E+00	-1.0194E+24

Table-10 (un-equal y-Increments): y1=11 and y2=7; s=4

n	$x = (n+y1)^4 - (n-y2)^4$	$[4n^3(y_1+y_2) +2n^2(3y_1^2-y_2^2) +4n(y_1^3-y_2^3) +y_1^4-y_2^4]$	Difference $([4n^3(y_1+y_2) +2n^2(3y_1^2-y_2^2) +4n(y_1^3-y_2^3) +y_1^4-y_2^4]-x)$	% Difference	f = r4-q4	Difference (f-x)	Difference $(f-[4n^3(y_1+y_2) +2n^2(3y_1^2-y_2^2) +4n(y_1^3-y_2^3) +y_1^4-y_2^4])$
4	5.0544E+04	4.2704E+04	-7.8400E+03	-18.3589359%	5.0544E+04	0.0000E+00	7.8400E+03
20	8.9496E+05	9.1848E+05	2.3520E+04	2.5607525%	8.9496E+05	0.0000E+00	-2.3520E+04
2,000	5.7774E+11	5.7852E+11	7.7851E+08	0.1345696%	5.7774E+11	0.0000E+00	-7.7851E+08
500,000	9.0001E+18	9.0002E+18	4.8999E+13	0.0005444%	9.0001E+18	-8.3886E+06	-4.8999E+13
5,000,000	9.0000E+21	9.0000E+21	4.9000E+15	0.0000544%	9.0000E+21	0.0000E+00	-4.9000E+15
80,000,000	3.6864E+25	3.6864E+25	1.2538E+18	0.0000034%	3.6864E+25	0.0000E+00	-1.2538E+18
600,000,000	1.5552E+28	1.5552E+28	6.6049E+19	0.0000004%	1.5552E+28	-1.8447E+19	-8.4496E+19
10,000,000,000	7.2000E+31	7.2000E+31	1.8831E+23	0.0000003%	7.2000E+31	1.2089E+24	1.0206E+24
100,000,000,000	7.2000E+34	7.2000E+34	1.0216E+27	0.0000014%	7.2000E+34	0.0000E+00	-1.0216E+27
3.0000E+12	1.9440E+39	1.9440E+39	-6.3284E+33	-0.0003255%	1.9440E+39	0.0000E+00	6.3284E+33
1.70000E+14	3.5385E+44	3.5374E+44	-1.1411E+41	-0.0322572%	3.5385E+44	0.0000E+00	1.1411E+41
2.30000E+15	8.6777E+47	8.7602E+47	8.2574E+45	0.9426001%	8.7348E+47	5.7090E+45	-2.5484E+45
7.80000E+16	0.0000E+00	3.4168E+52	3.4168E+52	100.0000000%	0.0000E+00	0.0000E+00	-3.4168E+52
6.40000E+17	0.0000E+00	1.8874E+55	1.8874E+55	100.0000000%	0.0000E+00	0.0000E+00	-1.8874E+55
3.45000E+18	0.0000E+00	2.9566E+57	2.9566E+57	100.0000000%	0.0000E+00	0.0000E+00	-2.9566E+57
9.20000E+19	0.0000E+00	5.6066E+61	5.6066E+61	100.0000000%	0.0000E+00	0.0000E+00	-5.6066E+61

Table-11 (un-equal y-Increments): y1=3,300 and y2=2,240; s=4

n	$x = (n+y1)^4 - (n-y2)^4$	$[4n^3(y_1+y_2) +2n^2(3_y1^2-y_2^2) +4n(y_1^3-y_2^3) +y_1^4-y_2^4]$	Difference $([4n^3(y_1+y_2) +2n^2(3_y1^2-y_2^2) +4n(y_1^3-y_2^3) +y_1^4-y_2^4]-x)$	% Difference	f = r4-q4	Difference (f-x)	Difference $(f-[4n^3(y_1+y_2) +2n^2(3_y1^2-y_2^2) +4n(y_1^3-y_2^3) +y_1^4-y_2^4])$
4	9.4171E+13	9.3812E+13	-3.5934E+11	-0.3830438%	9.4171E+13	0.0000E+00	3.5934E+11
20	9.7204E+13	9.5414E+13	-1.7903E+12	-1.8763301%	9.7204E+13	0.0000E+00	1.7903E+12
2,000	7.8904E+14	6.8950E+14	-9.9549E+13	-14.4379724%	7.8904E+14	0.0000E+00	9.9549E+13
500,000	2.7789E+21	2.7839E+21	4.9726E+18	0.1786230%	2.7789E+21	0.0000E+00	-4.9726E+18
5,000,000	2.7709E+24	2.7714E+24	5.0131E+20	0.0180888%	2.7709E+24	0.0000E+00	-5.0131E+20
80,000,000	1.1346E+28	1.1346E+28	1.2844E+23	0.0011320%	1.1346E+28	0.0000E+00	-1.2844E+23
600,000,000	4.7866E+30	4.7866E+30	7.2253E+24	0.0001509%	4.7866E+30	1.8447E+19	-7.2253E+24
10,000,000,000	2.2160E+34	2.2160E+34	2.0067E+27	0.0000091%	2.2160E+34	0.0000E+00	-2.0067E+27
100,000,000,000	2.2160E+37	2.2160E+37	2.0089E+29	0.0000009%	2.2160E+37	0.0000E+00	-2.0089E+29
3.0000E+12	5.9832E+41	5.9832E+41	-1.8025E+34	-0.0000030%	5.9832E+41	-1.0385E+34	7.6408E+33
1.70000E+14	1.0887E+47	1.0887E+47	1.6167E+40	0.0000148%	1.0887E+47	0.0000E+00	-1.6167E+40
2.30000E+15	2.6962E+50	2.6962E+50	2.2129E+45	0.0008207%	2.6962E+50	0.0000E+00	-2.2129E+45
7.80000E+16	1.0506E+55	1.0516E+55	1.0097E+52	0.0960152%	1.0506E+55	0.0000E+00	-1.0097E+52
6.40000E+17	5.8603E+57	5.8091E+57	-5.1152E+55	-0.8805462%	5.8848E+57	2.4520E+55	7.5672E+55
3.45000E+18	8.2858E+59	9.0997E+59	8.1393E+58	8.9445264%	8.5369E+59	2.5108E+58	-5.6284E+58
9.20000E+19	0.0000E+00	1.7256E+64	1.7256E+64	100.0000000%	0.0000E+00	0.0000E+00	-1.7256E+64