Dirichlet Eta Function - Negative Integer Formula - Isaac Mor

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Many people are using the term “Assigned Value” or “Analytic Continuation” for divergent series
But this explantion is so lacking and can be replaced with a much easier and simpler term of explantion

For me (as I see it) when I am looking at the zeta function I dont see (or use) the term “Assigned Value” or “Analytic Continuation”
Instead I see “spirals” all around the grid

The simplest way is first to look at the part of $s > 1$ on the Complex plane $\zeta(x+iy) = a+ib$ and the behavior of convergent points
The spiral swirls around inwards to an unique point which the series Converges - Same goes for the other way around!

When I look at the the part of $s < 1$ on the Complex plane $\zeta(x+iy) = a+ib$ and the behavior of divergent points
The spiral swirls around outwards but if you look closely you will notice that the spiral has a “center point” or an “origin”
and that “origin” is the “Assigned Value” everyone is talking about

when I first started to read about the zeta function I didn’t know what are those “Assigned Values” or “Analytic Continuation” and how and why people are trying to give a value for divergent series? And why that specific value and not something else? I wanted an explantion that is more then “because the formula says so” and without going deeper into all the “Analytic Continuation” stuff.

Those “origin points” did the trick!

If you are assigning a value for a series that decreases to a specific value (case #1)
Then you can assigning a value for a series that increases from a specific value (case #2)

Other then those two cases there is one more

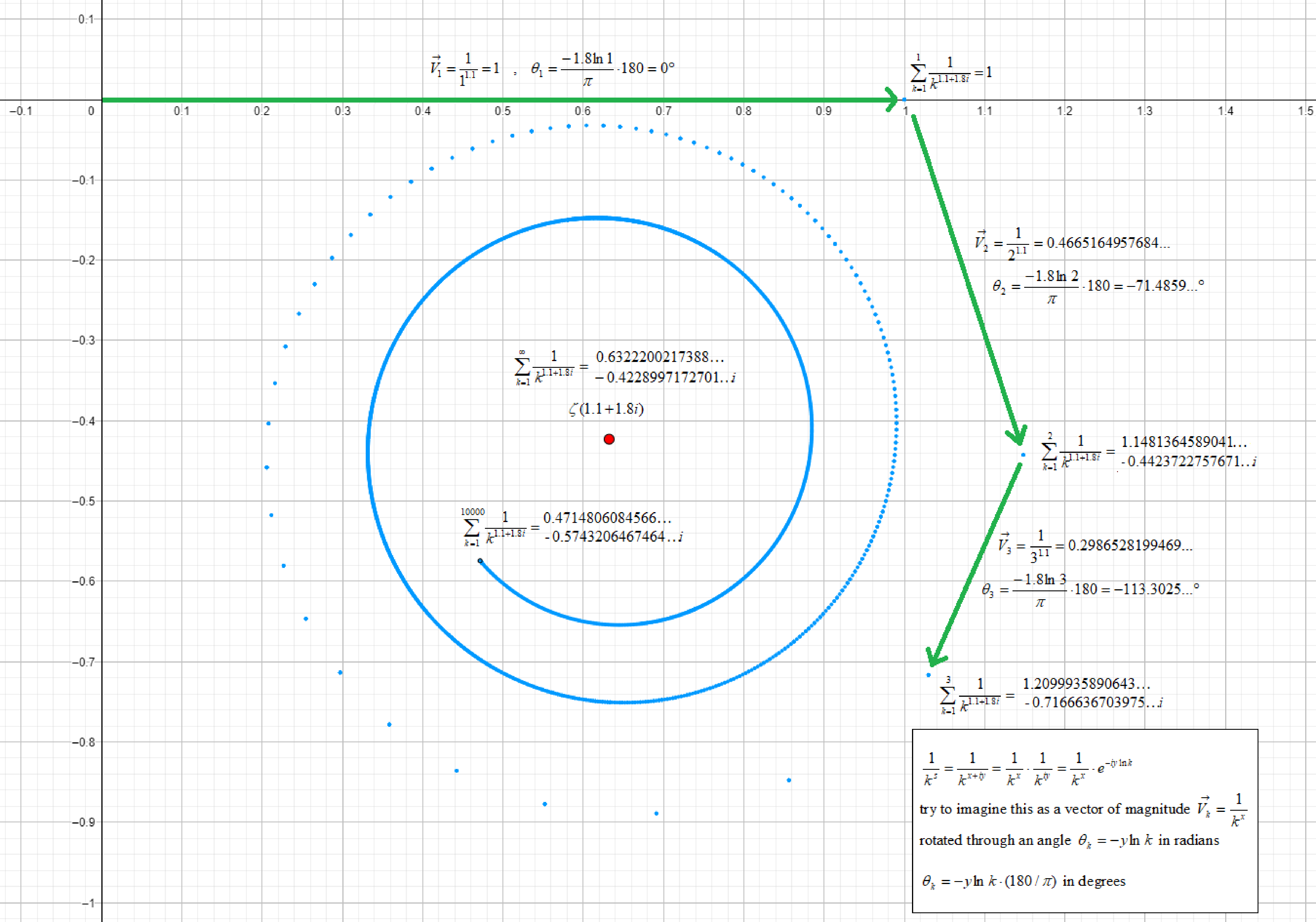
This is when the spiral at some point start to spin around a specific value with a “fixed radius”
those cases appears at the zeta function $\zeta(x+iy) = a+ib$ when $x = 1$ and the radius will be $1/y$
meaning that this is a divergent series with a “fixed radius”

I am not going to get into all the zeta function stuff!
This was a small intro for the eta function spirals

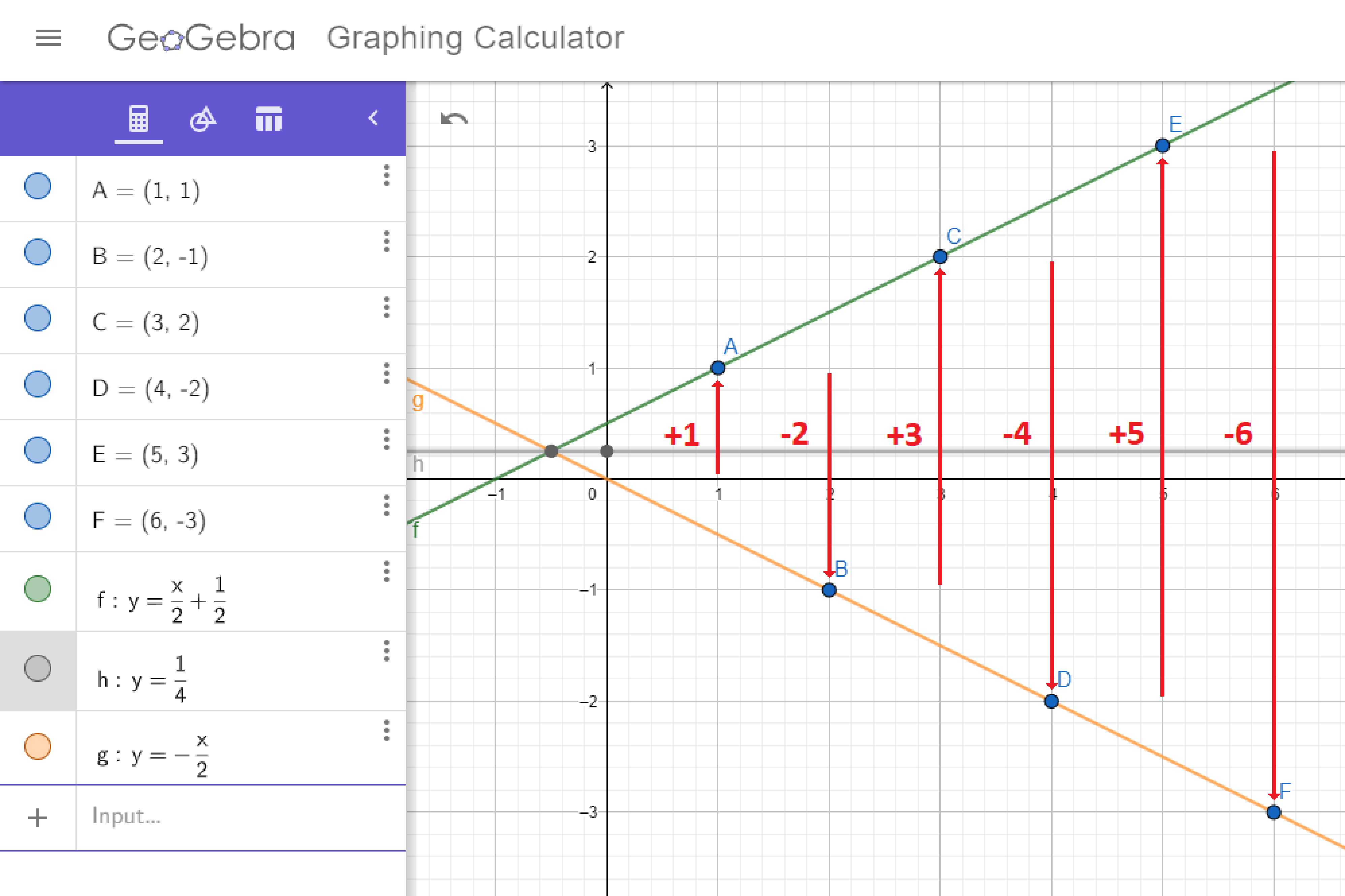
Its true that the zeta function spirals have 3 cases but they are all spirals with one arm
Now at the eta function the spirals have two arms (that is because of the +/- swapping) with the same 3 cases

Btw the “fixed radius” appears at the eta function $\eta(x+iy) = a+ib$ when $x = 0$

$$\eta(k) = 1^n - 2^n + 3^n - 4^n + \ldots \pm k^n$$



$\vec{V}_1 = \frac{1}{1^{1.1}} = 1$, $\theta_1 = \frac{-1.8 \ln 1}{\pi} \cdot 180 = 0°$
$\sum_{k=1}^{1} \frac{1}{k^{1.1+1.8i}} = 1$
$\vec{V}_2 = \frac{1}{2^{1.1}} = 0.4665164957684...$
$\theta_2 = \frac{-1.8 \ln 2}{\pi} \cdot 180 = -71.4859...°$
$\sum_{k=1}^{\infty} \frac{1}{k^{1.1+1.8i}} = 0.6322200217388... - 0.4228997172701...i$
$\zeta(1.1+1.8i)$
$\sum_{k=1}^{2} \frac{1}{k^{1.1+1.8i}} = 1.1481364589041... - 0.4423722757671...i$
$\sum_{k=1}^{10000} \frac{1}{k^{1.1+1.8i}} = 0.4714806084566... - 0.5743206467464...i$
$\vec{V}_3 = \frac{1}{3^{1.1}} = 0.2986528199469...$
$\theta_3 = \frac{-1.8 \ln 3}{\pi} \cdot 180 = -113.3025...°$
$\sum_{k=1}^{3} \frac{1}{k^{1.1+1.8i}} = 1.2099935890643... - 0.7166636703975...i$
$\frac{1}{k^s} = \frac{1}{k^{x+iy}} = \frac{1}{k^x} \cdot \frac{1}{k^{iy}} = \frac{1}{k^x} \cdot e^{-iy \ln k}$
try to imagine this as a vector of magnitude $\vec{V}_k = \frac{1}{k^x}$
rotated through an angle $\theta_k = -y \ln k$ in radians
$\theta_k = -y \ln k \cdot (180/\pi)$ in degrees


GeoGebra Graphing Calculator
A = (1, 1)
B = (2, -1)
C = (3, 2)
D = (4, -2)
E = (5, 3)
F = (6, -3)
f : y = x/2 + 1/2
h : y = 1/4
g : y = -x/2
Input...
+1
-2
+3
-4
+5
-6
A
B
C
D
E
F
f
g
h

Dirichlet Eta Function Negative Integer of order 0

$$\eta(k) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^0 - 10^0 + ... \pm k^0$$

Positive part of the eta function		Negative part of the eta function	
$f(x) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^0 - ... + x^0$		$g(x) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^0 - 10^0 + ... - x^0$	
$f(1) = 1^0$	1	$g(2) = 1^0 - 2^0$	0
$f(3) = 1^0 - 2^0 + 3^0$	1	$g(4) = 1^0 - 2^0 + 3^0 - 4^0$	0
$f(5) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0$	1	$g(6) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0$	0
$f(7) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0$	1	$g(8) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0$	0
$f(9) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^{00}$	1	$g(10) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^0 - 10^0$	0
$f(11) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^0 - 10^0 + 11^0$	1	$g(12) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^0 - 10^0 + 11^0 - 12^0$	0
$f(x) = 1$		$g(x) = 0$	

Dirichlet Eta Function Negative Integer of order 1

$$\eta(k) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1 - 6^1 + 7^1 - 8^1 + 9^1 - 10^1 + ... \pm k^1$$

Positive part of the eta function		Negative part of the eta function	
$f(x) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^0 - ... + x^0$		$g(x) = 1^0 - 2^0 + 3^0 - 4^0 + 5^0 - 6^0 + 7^0 - 8^0 + 9^0 - 10^0 + ... - x^0$	
$f(1) = 1^1$	1	$g(2) = 1^1 - 2^1$	-1
$f(3) = 1^1 - 2^1 + 3^1$	2	$g(4) = 1^1 - 2^1 + 3^1 - 4^1$	-2
$f(5) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1$	3	$g(6) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1 - 6^1$	-3
$f(7) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1 - 6^1 + 7^1$	4	$g(8) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1 - 6^1 + 7^1 - 8^1$	-4
$f(9) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1 - 6^1 + 7^1 - 8^1 + 9^1$	5	$g(10) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1 - 6^1 + 7^1 - 8^1 + 9^1 - 10^1$	-5
$f(11) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1 - 6^1 + 7^1 - 8^1 + 9^1 - 10^1 + 11^1$	6	$g(12) = 1^1 - 2^1 + 3^1 - 4^1 + 5^1 - 6^1 + 7^1 - 8^1 + 9^1 - 10^1 + 11^1 - 12^1$	-6
$f(x) = \frac{1}{2}x^1 + \frac{1}{2}$		$g(x) = -\frac{1}{2}x^1$	

Dirichlet Eta Function Negative Integer of order 2

$$\eta(k) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2 - 10^2 + \ldots \pm k^2$$

Positive part of the eta function		Negative part of the eta function	
$f(x) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2 - \ldots + x^2$		$g(x) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2 - 10^2 + \ldots - x^2$	
$f(1) = 1^2$	1	$g(2) = 1^2 - 2^2$	-3
$f(3) = 1^2 - 2^2 + 3^2$	6	$g(4) = 1^2 - 2^2 + 3^2 - 4^2$	-10
$f(5) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2$	15	$g(6) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2$	-21
$f(7) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2$	28	$g(8) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2$	-36
$f(9) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2$	45	$g(10) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2 - 10^2$	-55
$f(11) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2 - 10^2 + 11^2$	66	$g(12) = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2 - 10^2 + 11^2 - 12^2$	-78
$f(x) = \frac{1}{2}x^2 + \frac{1}{2}x^1$		$g(x) = -\frac{1}{2}x^2 - \frac{1}{2}x^1$	

Dirichlet Eta Function Negative Integer of order 3

$$\eta(k) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - 8^3 + 9^3 - 10^3 + \ldots \pm k^3$$

Positive part of the eta function		Negative part of the eta function	
$g(x) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - 8^3 + 9^3 - \ldots + x^3$		$g(x) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - 8^3 + 9^3 - 10^3 + \ldots - x^3$	
$f(1) = 1^3$	1	$g(2) = 1^3 - 2^3$	-7
$f(3) = 1^3 - 2^3 + 3^3$	20	$g(4) = 1^3 - 2^3 + 3^3 - 4^3$	-44
$f(5) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3$	81	$g(6) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3$	-135
$f(7) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3$	208	$g(8) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - 8^3$	-304
$f(9) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - 8^3 + 9^3$	425	$g(10) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - 8^3 + 9^3 - 10^3$	-575
$f(11) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - 8^3 + 9^3 - 10^3 + 11^3$	756	$g(12) = 1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - 8^3 + 9^3 - 10^3 + 11^3 - 12^3$	-972
$f(x) = \frac{1}{2}x^3 + \frac{3}{4}x^2 - \frac{1}{4}$		$g(x) = -\frac{1}{2}x^3 - \frac{3}{4}x^2$	

Dirichlet Eta Function Negative Integer of order 4

$$\eta(k) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4 - 8^4 + 9^4 - 10^4 + \ldots \pm k^4$$

Positive part of the eta function		Negative part of the eta function	
$f(x) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4 - 8^4 + 9^4 - \ldots + x^4$		$g(x) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4 - 8^4 + 9^4 - 10^4 + \ldots - x^4$	
$f(1) = 1^4$	1	$g(2) = 1^4 - 2^4$	-15
$f(3) = 1^4 - 2^4 + 3^4$	66	$g(4) = 1^4 - 2^4 + 3^4 - 4^4$	-190
$f(5) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4$	435	$g(6) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4$	-861
$f(7) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4$	1540	$g(8) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4 - 8^4$	-2556
$f(9) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4 - 8^4 + 9^4$	4005	$g(10) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4 - 8^4 + 9^4 - 10^4$	-5995
$f(11) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4 - 8^4 + 9^4 - 10^4 + 11^4$	8646	$g(12) = 1^4 - 2^4 + 3^4 - 4^4 + 5^4 - 6^4 + 7^4 - 8^4 + 9^4 - 10^4 + 11^4 - 12^4$	-12090
$f(x) = \frac{1}{2}x^4 + x^3 - \frac{1}{2}x^1$		$g(x) = -\frac{1}{2}x^4 - x^3 + \frac{1}{2}x^1$	

Dirichlet Eta Function Negative Integer of order 5

$$\eta(k) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5 - 8^5 + 9^5 - 10^5 + \ldots \pm k^5$$

Positive part of the eta function		Negative part of the eta function	
$g(x) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5 - 8^5 + 9^5 - \ldots + x^5$		$g(x) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5 - 8^5 + 9^5 - 10^5 + \ldots - x^5$	
$f(1) = 1^5$	1	$g(2) = 1^5 - 2^5$	-31
$f(3) = 1^5 - 2^5 + 3^5$	212	$g(4) = 1^5 - 2^5 + 3^5 - 4^5$	-812
$f(5) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5$	2313	$g(6) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5$	-5463
$f(7) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5$	11344	$g(8) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5 - 8^5$	-21424
$f(9) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5 - 8^5 + 9^5$	37625	$g(10) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5 - 8^5 + 9^5 - 10^5$	-62375
$f(11) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5 - 8^5 + 9^5 - 10^5 + 11^5$	98676	$g(12) = 1^5 - 2^5 + 3^5 - 4^5 + 5^5 - 6^5 + 7^5 - 8^5 + 9^5 - 10^5 + 11^5 - 12^5$	-150156
$f(x) = \frac{1}{2}x^5 + \frac{5}{4}x^4 - \frac{5}{4}x^2 + \frac{1}{2}$		$g(x) = -\frac{1}{2}x^5 - \frac{5}{4}x^4 + \frac{5}{4}x^2$	

Dirichlet Eta Function Negative Integer of order 6

$$\eta(k)=1^6-2^6+3^6-4^6+5^6-6^6+7^6-8^6+9^6-10^6+...\pm k^6$$

Positive part of the eta function		Negative part of the eta function	
$f(x)=1^6-2^6+3^6-4^6+5^6-6^6+7^6-8^6+9^6-...+x^6$		$g(x)=1^6-2^6+3^6-4^6+5^6-6^6+7^6-8^6+9^6-10^6+...-x^6$	
$f(1)=1^6$	1	$g(2)=1^6-2^6$	-63
$f(3)=1^6-2^6+3^6$	666	$g(4)=1^6-2^6+3^6-4^6$	-3430
$f(5)=1^6-2^6+3^6-4^6+5^6$	12195	$g(6)=1^6-2^6+3^6-4^6+5^6-6^6$	-34461
$f(7)=1^6-2^6+3^6-4^6+5^6-6^6+7^6$	83188	$g(8)=1^6-2^6+3^6-4^6+5^6-6^6+7^6-8^6$	-178956
$f(9)=1^6-2^6+3^6-4^6+5^6-6^6+7^6-8^6+9^6$	352485	$g(10)=1^6-2^6+3^6-4^6+5^6-6^6+7^6-8^6+9^6-10^6$	-647515
$f(11)=1^6-2^6+3^6-4^6+5^6-6^6+7^6-8^6+9^6-10^6+11^6$	1124046	$g(12)=1^6-2^6+3^6-4^6+5^6-6^6+7^6-8^6+9^6-10^6+11^6-12^6$	-1861938
$f(x)=\frac{1}{2}x^6+\frac{3}{2}x^5-\frac{5}{2}x^3+\frac{3}{2}x^1$		$g(x)=-\frac{1}{2}x^6-\frac{3}{2}x^5+\frac{5}{2}x^3-\frac{3}{2}x^1$	

Dirichlet Eta Function Negative Integer of order 7

$$\eta(k)=1^7-2^7+3^7-4^7+5^7-6^7+7^7-8^7+9^7-10^7+...\pm k^7$$

Positive part of the eta function		Negative part of the eta function	
$g(x)=1^7-2^7+3^7-4^7+5^7-6^7+7^7-8^7+9^7-...+x^7$		$g(x)=1^7-2^7+3^7-4^7+5^7-6^7+7^7-8^7+9^7-10^7+...-x^7$	
$f(1)=1^7$	1	$g(2)=1^7-2^7$	-127
$f(3)=1^7-2^7+3^7$	2060	$g(4)=1^7-2^7+3^7-4^7$	-14324
$f(5)=1^7-2^7+3^7-4^7+5^7$	63801	$g(6)=1^7-2^7+3^7-4^7+5^7-6^7$	-216135
$f(7)=1^7-2^7+3^7-4^7+5^7-6^7+7^7$	607408	$g(8)=1^7-2^7+3^7-4^7+5^7-6^7+7^7-8^7$	-1489744
$f(9)=1^7-2^7+3^7-4^7+5^7-6^7+7^7-8^7+9^7$	3293225	$g(10)=1^7-2^7+3^7-4^7+5^7-6^7+7^7-8^7+9^7-10^7$	-6706775
$f(11)=1^7-2^7+3^7-4^7+5^7-6^7+7^7-8^7+9^7-10^7+11^7$	12780396	$g(12)=1^7-2^7+3^7-4^7+5^7-6^7+7^7-8^7+9^7-10^7+11^7-12^7$	-23051412
$f(x)=\frac{1}{2}x^7+\frac{7}{4}x^6-\frac{35}{8}x^4+\frac{21}{4}x^2-\frac{17}{8}$		$g(x)=-\frac{1}{2}x^7-\frac{7}{4}x^6+\frac{35}{8}x^4-\frac{21}{4}x^2$	

Dirichlet Eta Function Negative Integer of order 8

$$\eta(k) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8 - 8^8 + 9^8 - 10^8 + ... \pm k^8$$

Positive part of the eta function		Negative part of the eta function	
$f(x) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8 - 8^8 + 9^8 - ... + x^8$		$g(x) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8 - 8^8 + 9^8 - 10^8 + ... - x^8$	
$f(1) = 1^8$	1	$g(2) = 1^8 - 2^8$	-255
$f(3) = 1^8 - 2^8 + 3^8$	6306	$g(4) = 1^8 - 2^8 + 3^8 - 4^8$	-59230
$f(5) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8$	331395	$g(6) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8$	-1348221
$f(7) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8$	4416580	$g(8) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8 - 8^8$	-12360636
$f(9) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8 - 8^8 + 9^8$	30686085	$g(10) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8 - 8^8 + 9^8 - 10^8$	-69313915
$f(11) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8 - 8^8 + 9^8 - 10^8 + 11^8$	145044966	$g(12) = 1^8 - 2^8 + 3^8 - 4^8 + 5^8 - 6^8 + 7^8 - 8^8 + 9^8 - 10^8 + 11^8 - 12^8$	-284936730
$f(x) = \frac{1}{2}x^8 + 2x^7 - 7x^5 + 14x^3 - \frac{17}{2}x^1$		$g(x) = -\frac{1}{2}x^8 - 2x^7 + 7x^5 - 14x^3 + \frac{17}{2}x^1$	

Dirichlet Eta Function Negative Integer of order 9

$$\eta(k) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9 - 8^9 + 9^9 - 10^9 + ... \pm k^9$$

Positive part of the eta function		Negative part of the eta function	
$g(x) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9 - 8^9 + 9^9 - ... + x^9$		$g(x) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9 - 8^9 + 9^9 - 10^9 + ... - x^9$	
$f(1) = 1^9$	1	$g(2) = 1^9 - 2^9$	-511
$f(3) = 1^9 - 2^9 + 3^9$	19172	$g(4) = 1^9 - 2^9 + 3^9 - 4^9$	-242972
$f(5) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9$	1710153	$g(6) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9$	-8367543
$f(7) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9$	31986064	$g(8) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9 - 8^9$	-102231664
$f(9) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9 - 8^9 + 9^9$	285188825	$g(10) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9 - 8^9 + 9^9 - 10^9$	-714811175
$f(11) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9 - 8^9 + 9^9 - 10^9 + 11^9$	1643136516	$g(12) = 1^9 - 2^9 + 3^9 - 4^9 + 5^9 - 6^9 + 7^9 - 8^9 + 9^9 - 10^9 + 11^9 - 12^9$	-3516643836
$f(x) = \frac{1}{2}x^9 + \frac{9}{4}x^8 - \frac{21}{2}x^6 + \frac{63}{2}x^4 - \frac{153}{4}x^2 + \frac{31}{2}$		$g(x) = -\frac{1}{2}x^9 - \frac{9}{4}x^8 + \frac{21}{2}x^6 - \frac{63}{2}x^4 + \frac{153}{4}x^2$	

Graphing Calculator - GeoGebra

geogebra.org/graphing

GeoGebra Graphing Calculator

SIGN IN

$A = (1, 1)$

$B = (2, 0)$

$C = (3, 1)$

$D = (4, 0)$

$E = (5, 1)$

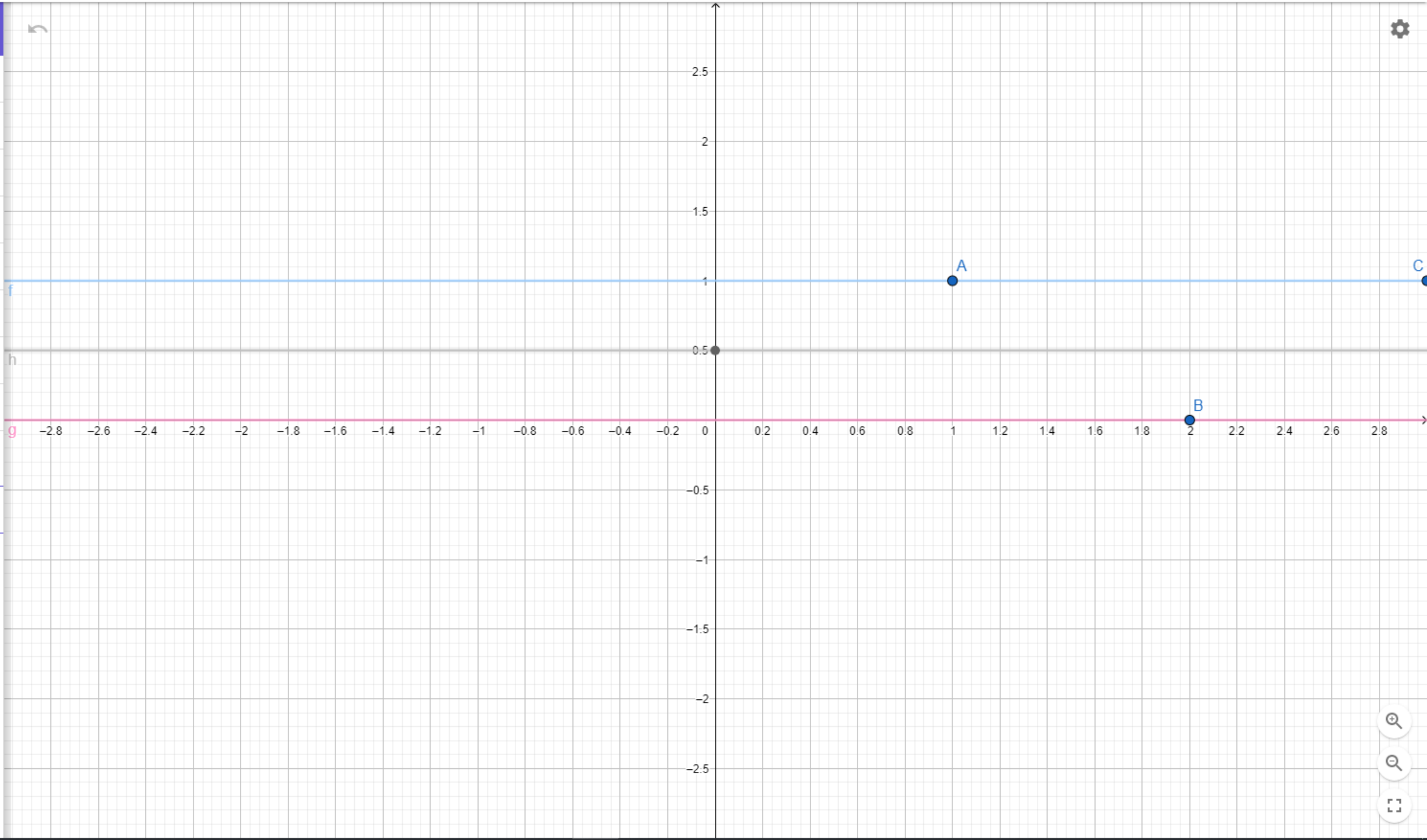
$F = (6, 0)$

$f : y = 1$

$g : y = 0$

$h : y = \frac{1}{2}$

Input...



Graphing Calculator - GeoGebra

geogebra.org/graphing

GeoGebra Graphing Calculator

SIGN IN

A = (1, 1)

B = (2, -1)

C = (3, 2)

D = (4, -2)

E = (5, 3)

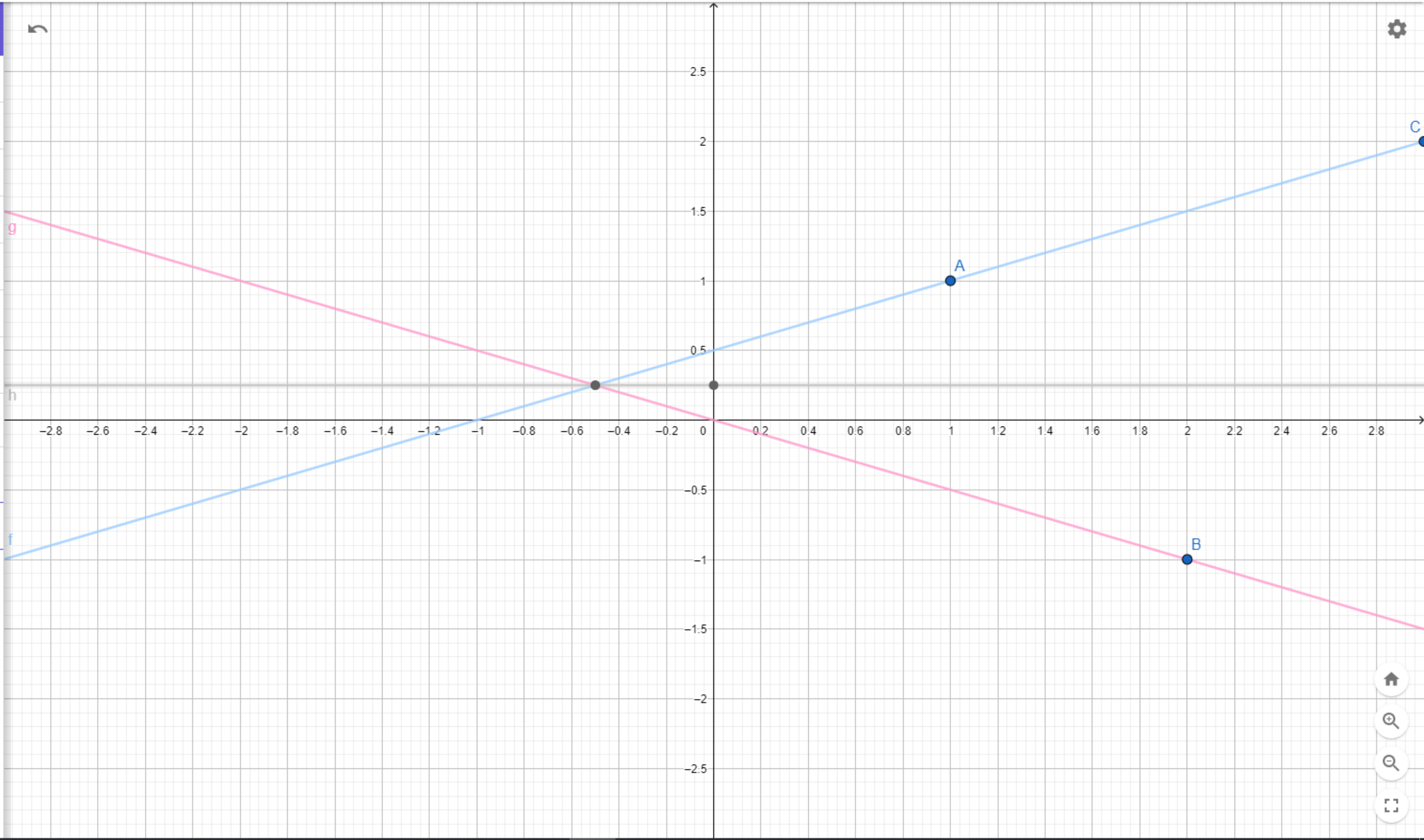
F = (6, -3)

$f : y = \frac{x}{2} + \frac{1}{2}$

$g : y = -\frac{x}{2}$

$h : y = \frac{1}{4}$

Input...



GeoGebra Graphing Calculator

A = (1, 1)

B = (2, -3)

C = (3, 6)

D = (4, -10)

E = (5, 15)

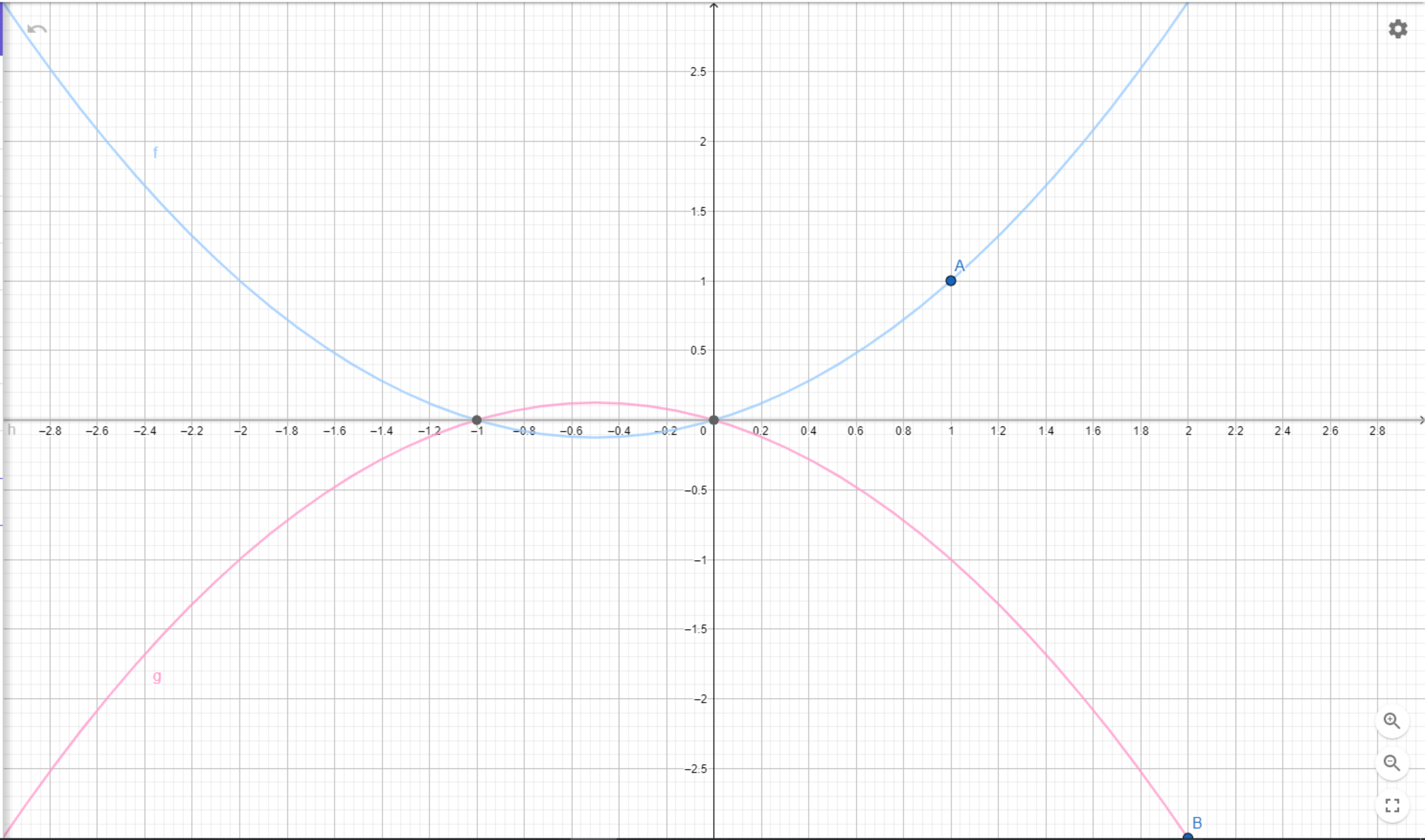
F = (6, -21)

f: y = 1 / 2 x² + 1 / 2 x

g: y = -1 / 2 x² - 1 / 2 x

h : y = 0

Input...



Graphing Calculator - GeoGebra

geogebra.org/graphing

GeoGebra Graphing Calculator

SIGN IN

A = (1, 1)

B = (2, -7)

C = (3, 20)

D = (4, -44)

E = (5, 81)

F = (6, -135)

G = (7, 208)

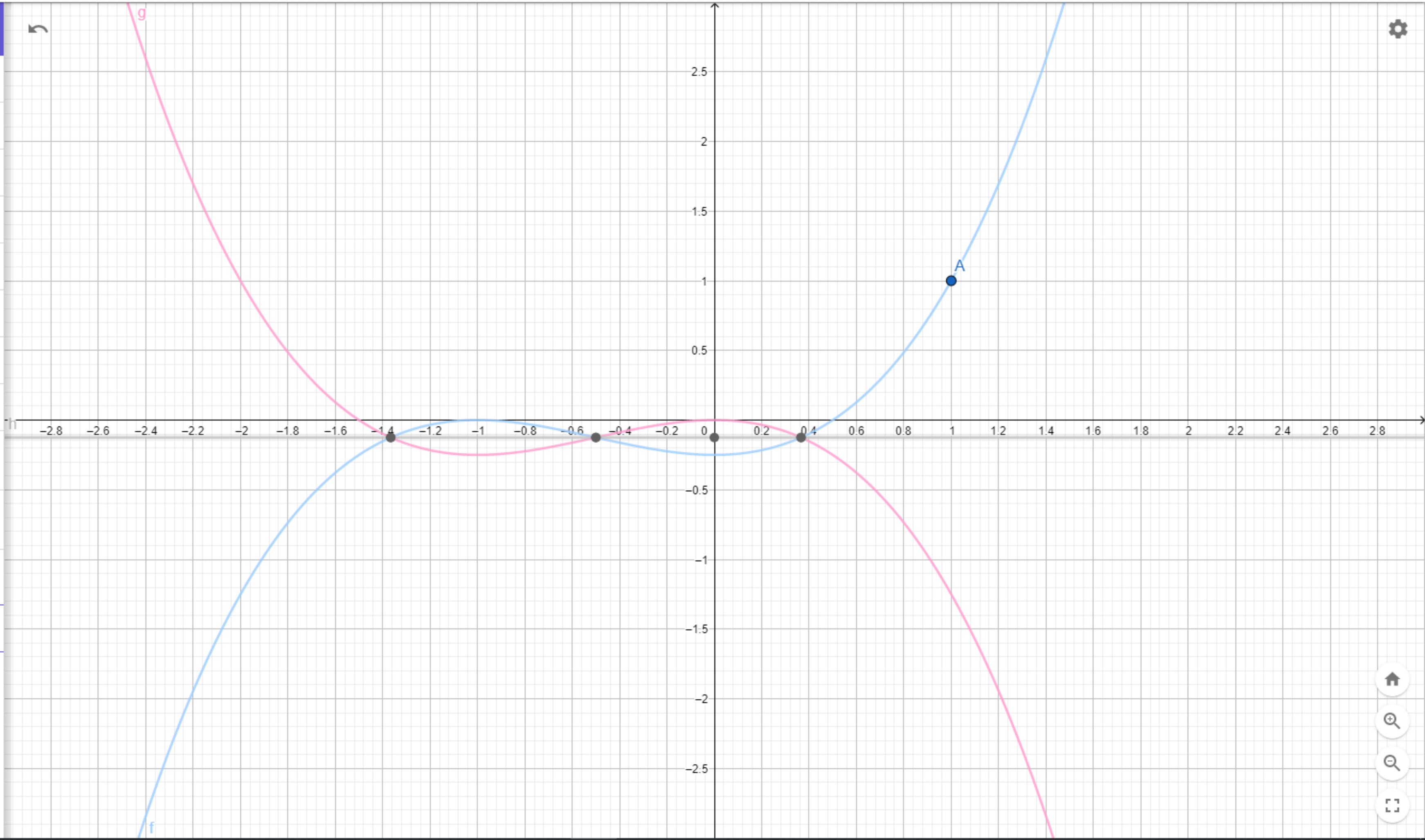
H = (8, -304)

$f : y = \frac{1}{2} x^3 + \frac{3 x^2}{4} - \frac{1}{4}$

$g : y = -\frac{1}{2} x^3 - \frac{3 x^2}{4}$

$h : y = -\frac{1}{8}$

Input...



Graphing Calculator - GeoGebra

geogebra.org/graphing

GeoGebra Graphing Calculator

SIGN IN

A = (1, 1)

B = (2, -15)

C = (3, 66)

D = (4, -190)

E = (5, 435)

F = (6, -861)

G = (7, 1540)

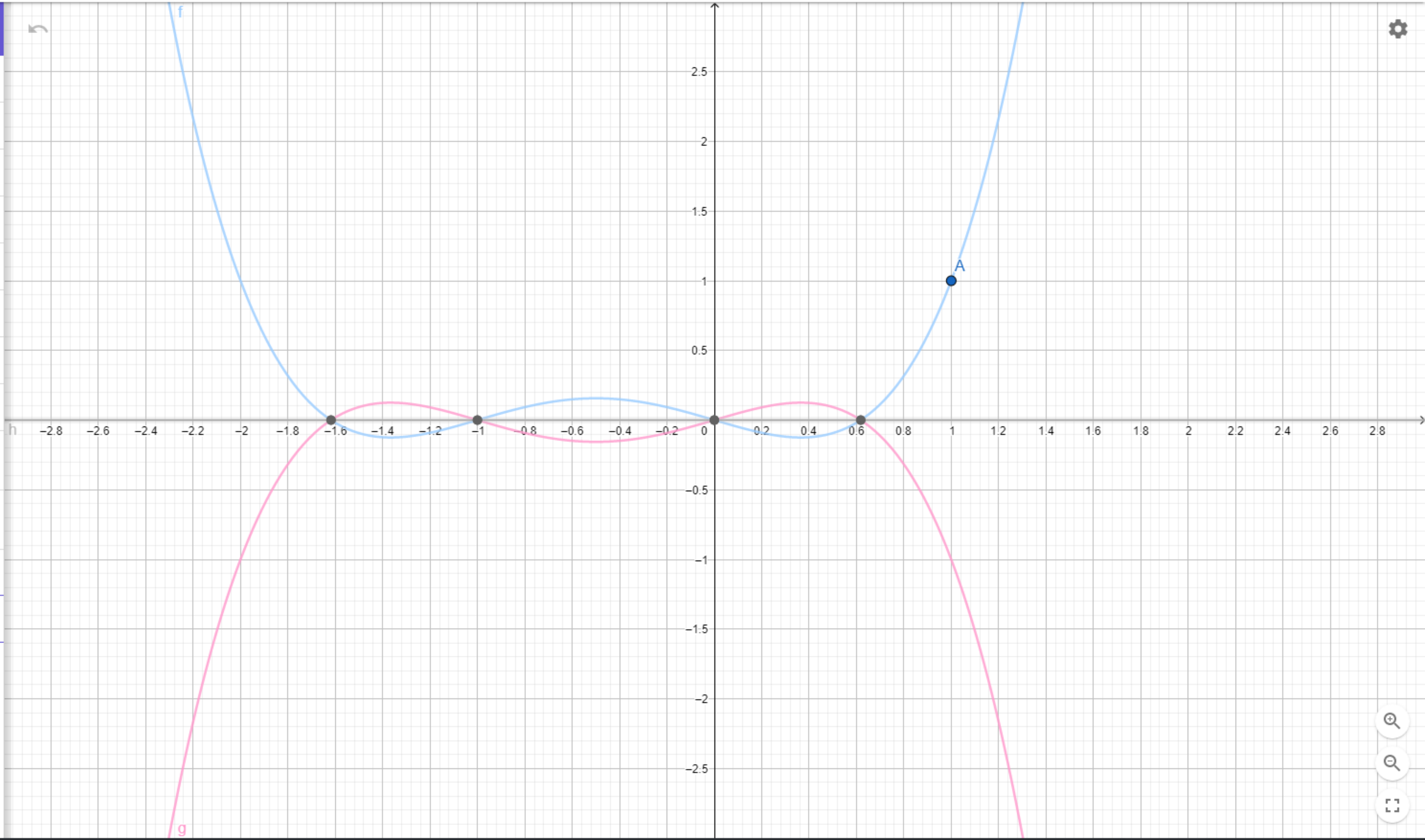
H = (8, -2556)

$f: y = \frac{x^4}{2} + x^3 - \frac{x}{2}$

$g: y = -\frac{x^4}{2} - x^3 + \frac{x}{2}$

$h: y = 0$

Input...



GeoGebra Graphing Calculator

SIGN IN

A = (1, 1)

B = (2, -31)

C = (3, 212)

D = (4, -812)

E = (5, 2313)

F = (6, -5463)

G = (7, 11344)

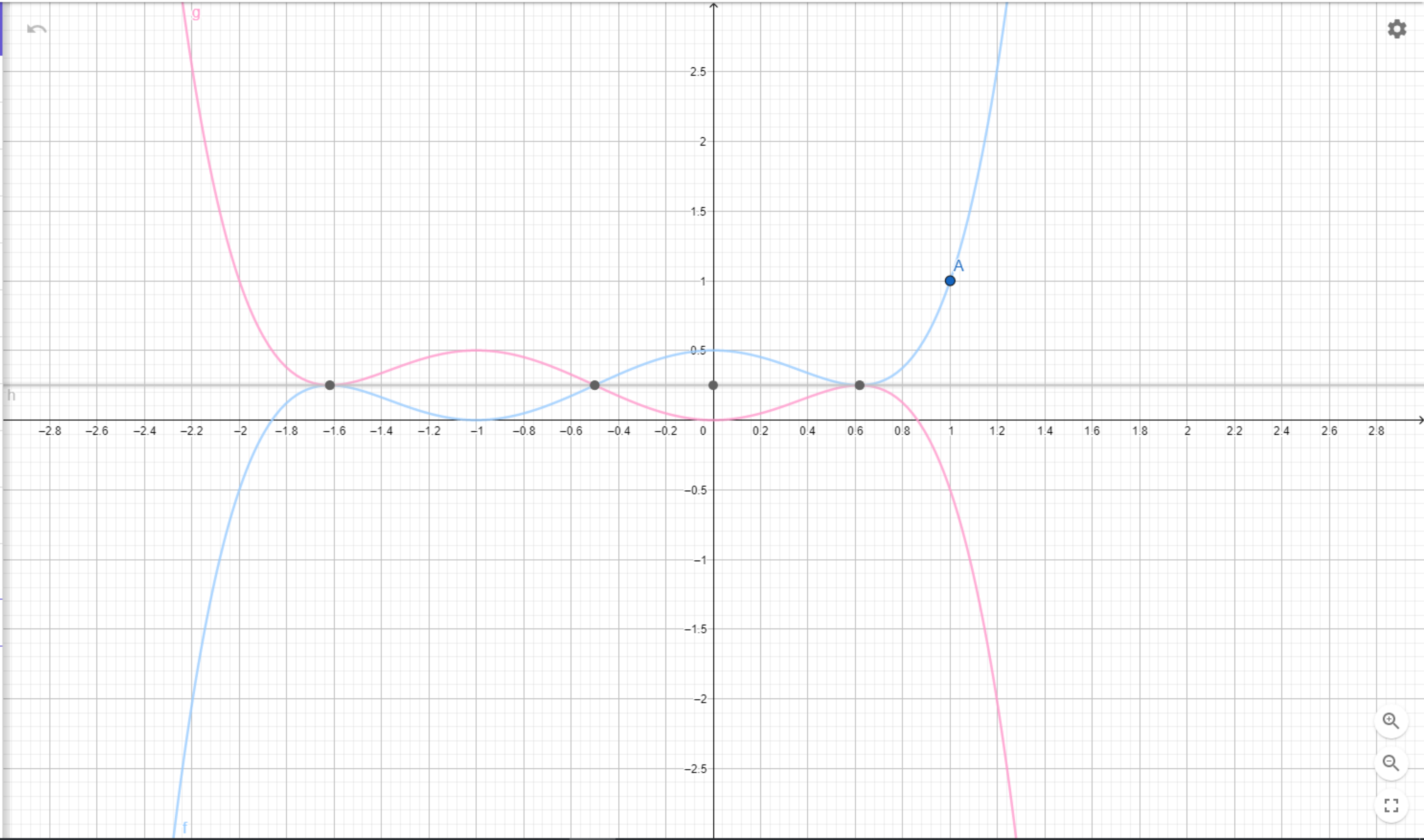
H = (8, -21424)

$f : y = \frac{2}{4} x^5 + \frac{5}{4} x^4 - \frac{5}{4} x^2 + \frac{2}{4}$

$g : y = -\frac{2}{4} x^5 - \frac{5}{4} x^4 + \frac{5}{4} x^2$

$h : y = \frac{1}{4}$

Input...



Graphing Calculator - GeoGebra

geogebra.org/graphing

GeoGebra Graphing Calculator

SIGN IN

A = (1, 1)

B = (2, -63)

C = (3, 666)

D = (4, -3430)

E = (5, 12195)

F = (6, -34461)

G = (7, 83188)

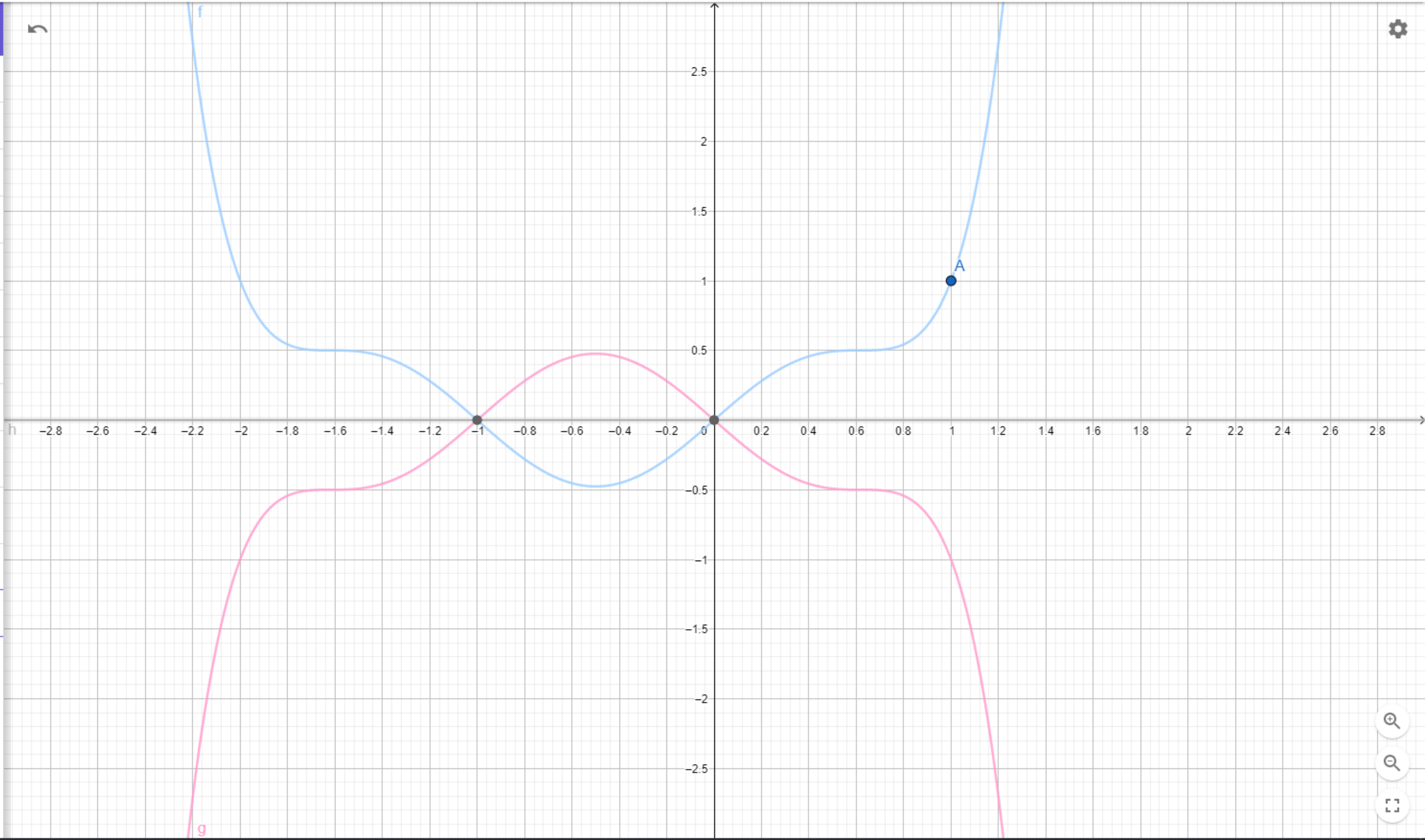
H = (8, -178956)

$f : y = \frac{1}{2} x^6 + \frac{6}{4} x^5 - \frac{20}{8} x^3 + \frac{6}{4} x$

$g : y = -\frac{1}{2} x^6 - \frac{6}{4} x^5 + \frac{20}{8} x^3 - \frac{6}{4} x$

$h : y = 0$

Input...



Graphing Calculator - GeoGebra

geogebra.org/graphing

GeoGebra Graphing Calculator

SIGN IN

$A = (1, 1)$

$B = (2, -127)$

$C = (3, 2060)$

$D = (4, -14324)$

$E = (5, 63801)$

$F = (6, -216135)$

$G = (7, 607408)$

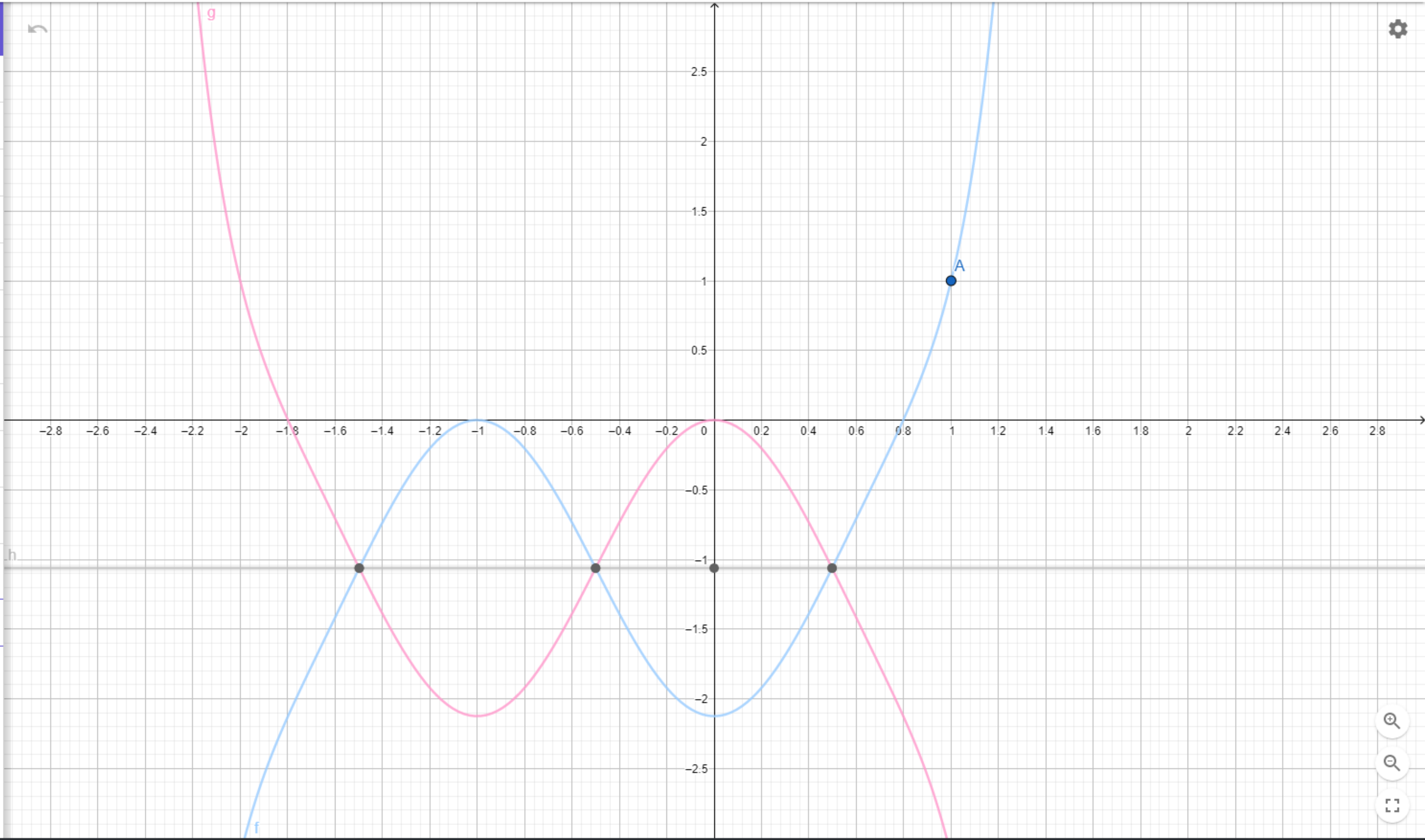
$H = (8, -1489744)$

$f: y = \frac{1}{2}x^7 + \frac{7}{4}x^6 - \frac{35}{8}x^4 + \frac{21}{4}x^2 - \frac{17}{8}$

$g: y = -\frac{1}{2}x^7 - \frac{7}{4}x^6 + \frac{35}{8}x^4 - \frac{21}{4}x^2$

$h: y = -\frac{17}{16}$

Input...



Graphing Calculator - GeoGebra

geogebra.org/graphing

GeoGebra Graphing Calculator

SIGN IN

A = (1, 1)

B = (2, -255)

C = (3, 6306)

D = (4, -59230)

E = (5, 331395)

F = (6, -1348221)

G = (7, 4416580)

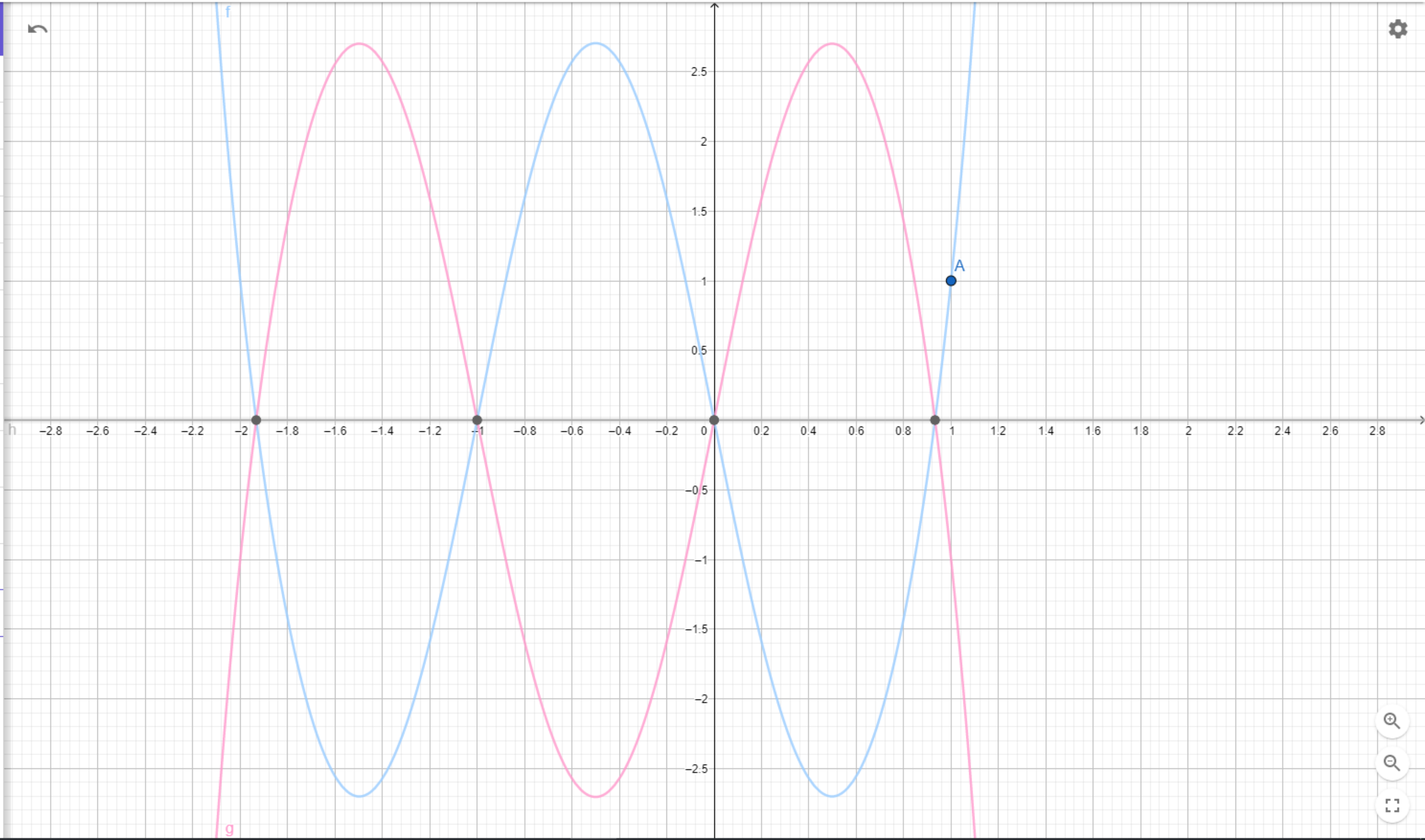
H = (8, -12360636)

$f : y = \frac{1}{2} x^8 + 2 x^7 - 7 x^5 + 14 x^3 - \frac{17}{2} x$

$g : y = -\frac{1}{2} x^8 - 2 x^7 + 7 x^5 - 14 x^3 + \frac{17}{2} x$

$h : y = 0$

Input...



Graphing Calculator - GeoGebra

geogebra.org/graphing

GeoGebra Graphing Calculator

SIGN IN

A = (1, 1)

B = (2, -511)

C = (3, 19172)

D = (4, -242972)

E = (5, 1710153)

F = (6, -8367543)

G = (7, 31986064)

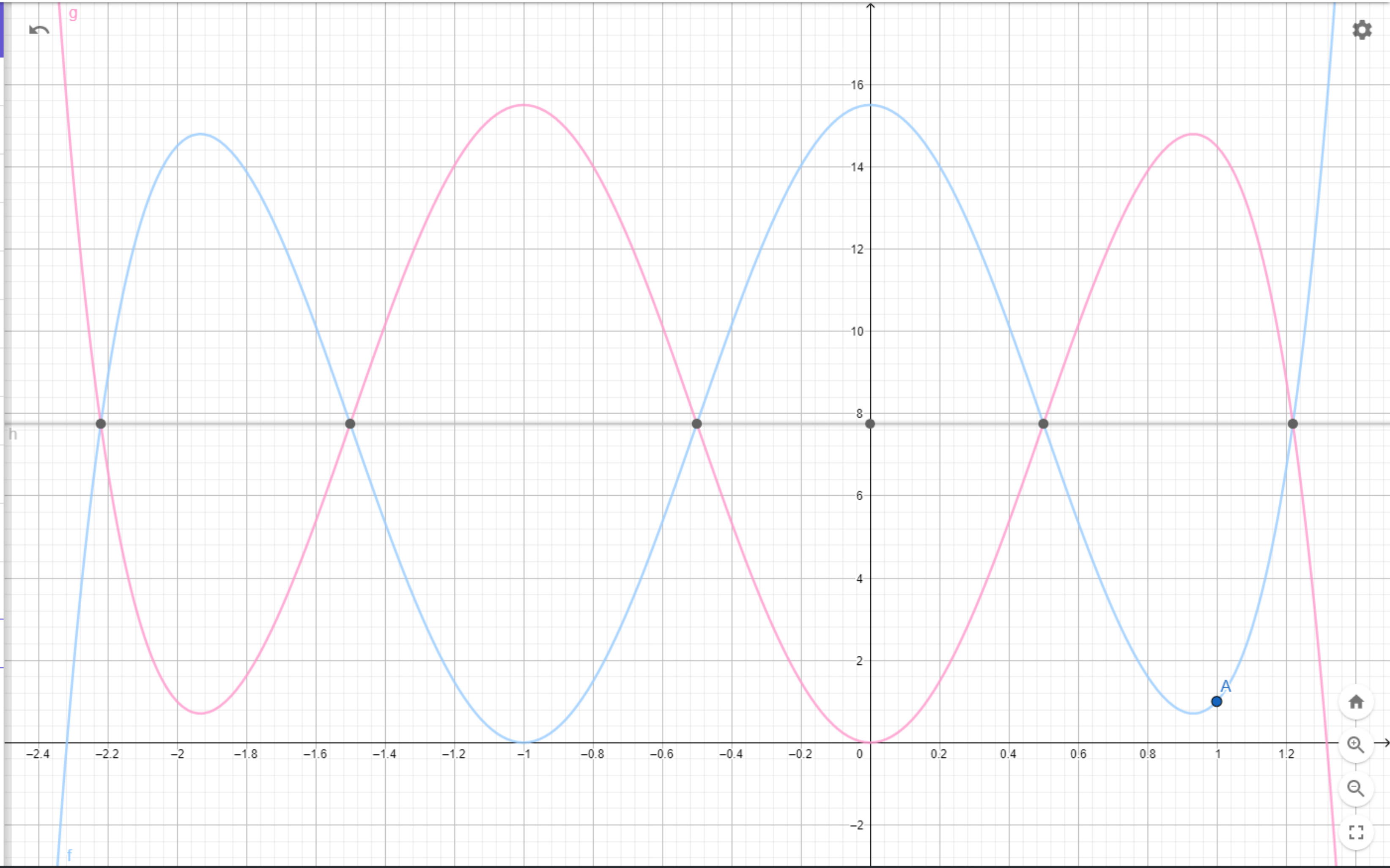
H = (8, -102231664)

$f : y = \frac{1}{2} x^9 + \frac{9}{4} x^8 - \frac{84}{8} x^6 + \frac{126}{4} x^4 - \frac{17}{16} \cdot 36 x^2 + \frac{31}{2}$

$g : y = -\frac{1}{2} x^9 - \frac{9}{4} x^8 + \frac{84}{8} x^6 - \frac{126}{4} x^4 + \frac{17}{16} \cdot 36 x^2$

$h : y = \frac{31}{4}$

Input...



Dirichlet Eta Function Negative Integer of orders 0 to 9

Order	$f(x)$ Positive part of the eta function	$g(x)$ Negative part of the eta function	Intersection $f(x)=g(x)$
0	1	0	$+\frac{1}{2}$
1	$\frac{1}{2}x^1+\frac{1}{2}$	$-\frac{1}{2}x^1$	$+\frac{1}{4}$
2	$\frac{1}{2}x^2+\frac{1}{2}x^1$	$-\frac{1}{2}x^2-\frac{1}{2}x^1$	0
3	$\frac{1}{2}x^3+\frac{3}{4}x^2-\frac{1}{4}$	$-\frac{1}{2}x^3-\frac{3}{4}x^2$	$-\frac{1}{8}$
4	$\frac{1}{2}x^4+x^3-\frac{1}{2}x^1$	$-\frac{1}{2}x^4-x^3+\frac{1}{2}x^1$	0
5	$\frac{1}{2}x^5+\frac{5}{4}x^4-\frac{5}{4}x^2+\frac{1}{2}$	$-\frac{1}{2}x^5-\frac{5}{4}x^4+\frac{5}{4}x^2$	$+\frac{1}{4}$
6	$\frac{1}{2}x^6+\frac{3}{2}x^5-\frac{5}{2}x^3+\frac{3}{2}x^1$	$-\frac{1}{2}x^6-\frac{3}{2}x^5+\frac{5}{2}x^3-\frac{3}{2}x^1$	0
7	$\frac{1}{2}x^7+\frac{7}{4}x^6-\frac{35}{8}x^4+\frac{21}{4}x^2-\frac{17}{8}$	$-\frac{1}{2}x^7-\frac{7}{4}x^6+\frac{35}{8}x^4-\frac{21}{4}x^2$	$-\frac{17}{16}$
8	$\frac{1}{2}x^8+2x^7-7x^5+14x^3-\frac{17}{2}x^1$	$-\frac{1}{2}x^8-2x^7+7x^5-14x^3+\frac{17}{2}x^1$	0
9	$\frac{1}{2}x^9+\frac{9}{4}x^8-\frac{21}{2}x^6+\frac{63}{2}x^4-\frac{153}{4}x^2+\frac{31}{2}$	$-\frac{1}{2}x^9-\frac{9}{4}x^8+\frac{21}{2}x^6-\frac{63}{2}x^4+\frac{153}{4}x^2$	$+\frac{31}{4}$

All the intersection points of x where $f(x)=g(x)$ will be on one line
because the two functions are a mirror image of the same function parallel to the yAxis
This Fixed value of y (the line of the intersection points) will be the “assigned value” of the eta function

let's just rearrange the coefficients

Order	$f(x)$ Positive part of the eta function	$g(x)$ Negative part of the eta function	
0	1	0	$+\frac{1}{2}$
1	$\frac{1}{2}x^1+\frac{1}{2}$	$-\left(\frac{1}{2}x^1\right)$	$+\frac{1}{4}$
2	$\frac{1}{2}x^2+\frac{1}{4}\cdot 2x^1$	$-\left(\frac{1}{2}x^2+\frac{1}{4}\cdot 2x^1\right)$	0
3	$\frac{1}{2}x^3+\frac{1}{4}\cdot 3x^2-\frac{1}{4}$	$-\left(\frac{1}{2}x^3+\frac{1}{4}\cdot 3x^2\right)$	$-\frac{1}{8}$
4	$\frac{1}{2}x^4+\frac{1}{4}\cdot 4x^3-\frac{1}{8}\cdot 4x^1$	$-\left(\frac{1}{2}x^4+\frac{1}{4}\cdot 4x^3-\frac{1}{8}\cdot 4x^1\right)$	0
5	$\frac{1}{2}x^5+\frac{1}{4}\cdot 5x^4-\frac{1}{8}\cdot 10x^2+\frac{1}{2}$	$-\left(\frac{1}{2}x^5+\frac{1}{4}\cdot 5x^4-\frac{1}{8}\cdot 10x^2\right)$	$+\frac{1}{4}$
6	$\frac{1}{2}x^6+\frac{1}{4}\cdot 6x^5-\frac{1}{8}\cdot 20x^3+\frac{1}{4}\cdot 6x^1$	$-\left(\frac{1}{2}x^6+\frac{1}{4}\cdot 6x^5-\frac{1}{8}\cdot 20x^3+\frac{1}{4}\cdot 6x^1\right)$	0
7	$\frac{1}{2}x^7+\frac{1}{4}\cdot 7x^6-\frac{1}{8}\cdot 35x^4+\frac{1}{4}\cdot 21x^2-\frac{17}{8}$	$-\left(\frac{1}{2}x^7+\frac{1}{4}\cdot 7x^6-\frac{1}{8}\cdot 35x^4+\frac{1}{4}\cdot 21x^2\right)$	$-\frac{17}{16}$
8	$\frac{1}{2}x^8+\frac{1}{4}\cdot 8x^7-\frac{1}{8}\cdot 56x^5+\frac{1}{4}\cdot 56x^3-\frac{17}{16}\cdot 8x^1$	$-\left(\frac{1}{2}x^8+\frac{1}{4}\cdot 8x^7-\frac{1}{8}\cdot 56x^5+\frac{1}{4}\cdot 56x^3-\frac{17}{16}\cdot 8x^1\right)$	0
9	$\frac{1}{2}x^9+\frac{1}{4}\cdot 9x^8-\frac{1}{8}\cdot 84x^6+\frac{1}{4}\cdot 126x^4-\frac{17}{16}\cdot 36x^2+\frac{31}{2}$	$-\left(\frac{1}{2}x^9+\frac{1}{4}\cdot 9x^8-\frac{1}{8}\cdot 84x^6+\frac{1}{4}\cdot 126x^4-\frac{17}{16}\cdot 36x^2\right)$	$+\frac{31}{4}$

let's just rearrange the coefficients even more …

Order	$f(x)$ Positive part of the eta function	$g(x)$ Negative part of the eta function	
0	$\frac{1}{2}x^0+\frac{1}{2}$	$-\left(\frac{1}{2}x^0-\frac{1}{2}\right)$	$+\frac{1}{2}$
1	$\frac{1}{2}x^1+\frac{1}{4}\cdot 1x^0+\frac{1}{4}$	$-\left(\frac{1}{2}x^1\right)$	$+\frac{1}{4}$
2	$\frac{1}{2}x^2+\frac{1}{4}\cdot 2x^1+0$	$-\left(\frac{1}{2}x^2+\frac{1}{4}\cdot 2x^1\right)$	0
3	$\frac{1}{2}x^3+\frac{1}{4}\cdot 3x^2-\frac{1}{8}\cdot 1x^0-\frac{1}{8}$	$-\left(\frac{1}{2}x^3+\frac{1}{4}\cdot 3x^2\right)$	$-\frac{1}{8}$
4	$\frac{1}{2}x^4+\frac{1}{4}\cdot 4x^3-\frac{1}{8}\cdot 4x^1+0$	$-\left(\frac{1}{2}x^4+\frac{1}{4}\cdot 4x^3-\frac{1}{8}\cdot 4x^1\right)$	0
5	$\frac{1}{2}x^5+\frac{1}{4}\cdot 5x^4-\frac{1}{8}\cdot 10x^2+\frac{1}{4}\cdot 1x^0+\frac{1}{4}$	$-\left(\frac{1}{2}x^5+\frac{1}{4}\cdot 5x^4-\frac{1}{8}\cdot 10x^2\right)$	$+\frac{1}{4}$
6	$\frac{1}{2}x^6+\frac{1}{4}\cdot 6x^5-\frac{1}{8}\cdot 20x^3+\frac{1}{4}\cdot 6x^1+0$	$-\left(\frac{1}{2}x^6+\frac{1}{4}\cdot 6x^5-\frac{1}{8}\cdot 20x^3+\frac{1}{4}\cdot 6x^1\right)$	0
7	$\frac{1}{2}x^7+\frac{1}{4}\cdot 7x^6-\frac{1}{8}\cdot 35x^4+\frac{1}{4}\cdot 21x^2-\frac{17}{16}\cdot 1x^0-\frac{17}{16}$	$-\left(\frac{1}{2}x^7+\frac{1}{4}\cdot 7x^6-\frac{1}{8}\cdot 35x^4+\frac{1}{4}\cdot 21x^2\right)$	$-\frac{17}{16}$
8	$\frac{1}{2}x^8+\frac{1}{4}\cdot 8x^7-\frac{1}{8}\cdot 56x^5+\frac{1}{4}\cdot 56x^3-\frac{17}{16}\cdot 8x^1+0$	$-\left(\frac{1}{2}x^8+\frac{1}{4}\cdot 8x^7-\frac{1}{8}\cdot 56x^5+\frac{1}{4}\cdot 56x^3-\frac{17}{16}\cdot 8x^1\right)$	0
9	$\frac{1}{2}x^9+\frac{1}{4}\cdot 9x^8-\frac{1}{8}\cdot 84x^6+\frac{1}{4}\cdot 126x^4-\frac{17}{16}\cdot 36x^2+\frac{31}{4}\cdot 1x^0+\frac{31}{4}$	$-\left(\frac{1}{2}x^9+\frac{1}{4}\cdot 9x^8-\frac{1}{8}\cdot 84x^6+\frac{1}{4}\cdot 126x^4-\frac{17}{16}\cdot 36x^2\right)$	$+\frac{31}{4}$

and even more …

	$f(x)$ Positive part of the eta function	$g(x)$ Negative part of the eta function	
0	$\frac{1}{2}x^0+\frac{1}{2}$	$-\left(\frac{1}{2}x^0\right)+\frac{1}{2}$	$+\frac{1}{2}$
1	$\frac{1}{2}x^1+\frac{1}{4}\cdot\binom{1}{1}x^0+\frac{1}{4}$	$-\left(\frac{1}{2}x^1+\frac{1}{4}\cdot\binom{1}{1}x^0\right)+\frac{1}{4}$	$+\frac{1}{4}$
2	$\frac{1}{2}x^2+\frac{1}{4}\cdot\binom{2}{1}x^1+0$	$-\left(\frac{1}{2}x^2+\frac{1}{4}\cdot\binom{2}{1}x^1\right)+0$	0
3	$\frac{1}{2}x^3+\frac{1}{4}\cdot\binom{3}{1}x^2-\frac{1}{8}\cdot\binom{3}{3}x^0-\frac{1}{8}$	$-\left(\frac{1}{2}x^3+\frac{1}{4}\cdot\binom{3}{1}x^2-\frac{1}{8}\cdot\binom{3}{3}x^0\right)-\frac{1}{8}$	$-\frac{1}{8}$
4	$\frac{1}{2}x^4+\frac{1}{4}\cdot\binom{4}{1}x^3-\frac{1}{8}\cdot\binom{4}{3}x^1+0$	$-\left(\frac{1}{2}x^4+\frac{1}{4}\cdot\binom{4}{1}x^3-\frac{1}{8}\cdot\binom{4}{3}x^1\right)+0$	0
5	$\frac{1}{2}x^5+\frac{1}{4}\cdot\binom{5}{1}x^4-\frac{1}{8}\cdot\binom{5}{3}x^2+\frac{1}{4}\cdot\binom{5}{5}x^0+\frac{1}{4}$	$-\left(\frac{1}{2}x^5+\frac{1}{4}\cdot\binom{5}{1}x^4-\frac{1}{8}\cdot\binom{5}{3}x^2+\frac{1}{4}\cdot\binom{5}{5}x^0\right)+\frac{1}{4}$	$+\frac{1}{4}$
6	$\frac{1}{2}x^6+\frac{1}{4}\cdot\binom{6}{1}x^5-\frac{1}{8}\cdot\binom{6}{3}x^3+\frac{1}{4}\cdot\binom{6}{5}x^1+0$	$-\left(\frac{1}{2}x^6+\frac{1}{4}\cdot\binom{6}{1}x^5-\frac{1}{8}\cdot\binom{6}{3}x^3+\frac{1}{4}\cdot\binom{6}{5}x^1\right)+0$	0
7	$\frac{1}{2}x^7+\frac{1}{4}\cdot\binom{7}{1}x^6-\frac{1}{8}\cdot\binom{7}{3}x^4+\frac{1}{4}\cdot\binom{7}{5}x^2-\frac{17}{16}\cdot\binom{7}{7}x^0-\frac{17}{16}$	$-\left(\frac{1}{2}x^7+\frac{1}{4}\cdot\binom{7}{1}x^6-\frac{1}{8}\cdot\binom{7}{3}x^4+\frac{1}{4}\cdot\binom{7}{5}x^2-\frac{17}{16}\cdot\binom{7}{7}x^0\right)-\frac{17}{16}$	$-\frac{17}{16}$
8	$\frac{1}{2}x^8+\frac{1}{4}\cdot\binom{8}{1}x^7-\frac{1}{8}\cdot\binom{8}{3}x^5+\frac{1}{4}\cdot\binom{8}{5}x^3-\frac{17}{16}\cdot\binom{8}{7}x^1+0$	$-\left(\frac{1}{2}x^8+\frac{1}{4}\cdot\binom{8}{1}x^7-\frac{1}{8}\cdot\binom{8}{3}x^5+\frac{1}{4}\cdot\binom{8}{5}x^3-\frac{17}{16}\cdot\binom{8}{7}x^1\right)+0$	0
9	$\frac{1}{2}x^9+\frac{1}{4}\cdot\binom{9}{1}x^8-\frac{1}{8}\cdot\binom{9}{3}x^6+\frac{1}{4}\cdot\binom{9}{5}x^4-\frac{17}{16}\cdot\binom{9}{7}x^2+\frac{31}{4}\cdot\binom{9}{9}x^0+\frac{31}{4}$	$-\left(\frac{1}{2}x^9+\frac{1}{4}\cdot\binom{9}{1}x^8-\frac{1}{8}\cdot\binom{9}{3}x^6+\frac{1}{4}\cdot\binom{9}{5}x^4-\frac{17}{16}\cdot\binom{9}{7}x^2+\frac{31}{4}\cdot\binom{9}{9}x^0\right)+\frac{31}{4}$	$+\frac{31}{4}$

Final form and I am not going to write g(x) so I can have more room to write f(x)

n	$f(x)$ Positive part of the eta function	$g(x)$ Negative part of the eta function	
0	$\frac{1}{2}\cdot\binom{0}{0}x^0+\frac{1}{2}$	$-f(x)+2\cdot\frac{1}{2}$	$+\frac{1}{2}$
1	$\frac{1}{2}\cdot\binom{1}{0}x^1+\frac{1}{4}\cdot\binom{1}{1}x^0+\frac{1}{4}$	$-f(x)+2\cdot\frac{1}{4}$	$+\frac{1}{4}$
2	$\frac{1}{2}\cdot\binom{2}{0}x^2+\frac{1}{4}\cdot\binom{2}{1}x^1+0\cdot\binom{2}{2}x^0+0$	$-f(x)+2\cdot 0$	0
3	$\frac{1}{2}\cdot\binom{3}{0}x^3+\frac{1}{4}\cdot\binom{3}{1}x^2+0\cdot\binom{3}{2}x^1-\frac{1}{8}\cdot\binom{3}{3}x^0-\frac{1}{8}$	$-f(x)-2\cdot\frac{1}{8}$	$-\frac{1}{8}$
4	$\frac{1}{2}\cdot\binom{4}{0}x^4+\frac{1}{4}\cdot\binom{4}{1}x^3+0\cdot\binom{4}{2}x^2-\frac{1}{8}\cdot\binom{4}{3}x^1+0\cdot\binom{4}{4}x^0+0$	$-f(x)+2\cdot 0$	0
5	$\frac{1}{2}\cdot\binom{5}{0}x^5+\frac{1}{4}\cdot\binom{5}{1}x^4+0\cdot\binom{5}{2}x^3-\frac{1}{8}\cdot\binom{5}{3}x^2+0\cdot\binom{5}{4}x^1+\frac{1}{4}\cdot\binom{5}{5}x^0+\frac{1}{4}$	$-f(x)+2\cdot\frac{1}{4}$	$+\frac{1}{4}$
6	$\frac{1}{2}\cdot\binom{6}{0}x^6+\frac{1}{4}\cdot\binom{6}{1}x^5+0\cdot\binom{6}{2}x^4-\frac{1}{8}\cdot\binom{6}{3}x^3+0\cdot\binom{6}{4}x^2+\frac{1}{4}\cdot\binom{6}{5}x^1+0\cdot\binom{6}{6}x^0+0$	$-f(x)+2\cdot 0$	0
7	$\frac{1}{2}\cdot\binom{7}{0}x^7+\frac{1}{4}\cdot\binom{7}{1}x^6+0\cdot\binom{7}{2}x^5-\frac{1}{8}\cdot\binom{7}{3}x^4+0\cdot\binom{7}{4}x^3+\frac{1}{4}\cdot\binom{7}{5}x^2+0\cdot\binom{7}{6}x^1-\frac{17}{16}\cdot\binom{7}{7}x^0-\frac{17}{16}$	$-f(x)-2\cdot\frac{17}{16}$	$-\frac{17}{16}$
8	$\frac{1}{2}\cdot\binom{8}{0}x^8+\frac{1}{4}\cdot\binom{8}{1}x^7+0\cdot\binom{8}{2}x^6-\frac{1}{8}\cdot\binom{8}{3}x^5+0\cdot\binom{8}{4}x^4+\frac{1}{4}\cdot\binom{8}{5}x^3+0\cdot\binom{8}{6}x^2-\frac{17}{16}\cdot\binom{8}{7}x^1+0\cdot\binom{8}{8}x^0+0$	$-f(x)+2\cdot 0$	0
9	$\frac{1}{2}\cdot\binom{9}{0}x^9+\frac{1}{4}\cdot\binom{9}{1}x^8+0\cdot\binom{9}{2}x^7-\frac{1}{8}\cdot\binom{9}{3}x^6+0\cdot\binom{9}{4}x^5+\frac{1}{4}\cdot\binom{9}{5}x^4+0\cdot\binom{9}{6}x^3-\frac{17}{16}\cdot\binom{9}{7}x^2+0\cdot\binom{9}{8}x^1+\frac{31}{4}\cdot\binom{9}{9}x^0+\frac{31}{4}$	$-f(x)+2\cdot\frac{31}{4}$	$+\frac{31}{4}$

$$f(x)=r_0\binom{n}{0}x^n+r_1\binom{n}{1}x^{n-1}+r_2\binom{n}{2}x^{n-2}+r_3\binom{n}{3}x^{n-3}+\ldots+r_{n-1}\binom{n}{n-1}x^{n-(n-1)}+r_n\binom{n}{n}x^{n-n}+r_n$$

r_n seems to be the assigned value of the eta function! Why? Because… first we will use $x=1$

$$f(1)=r_0\binom{n}{0}1^n+r_1\binom{n}{1}1^{n-1}+r_2\binom{n}{2}1^{n-2}+r_3\binom{n}{3}1^{n-3}+\ldots+r_{n-1}\binom{n}{n-1}1^1+r_n\binom{n}{n}1^0+r_n$$

$f(1) = 1^n = 1$ at any order :)

$$1 = r_0\binom{n}{0} + r_1\binom{n}{1} + r_2\binom{n}{2} + r_3\binom{n}{3} + \ldots + r_{n-1}\binom{n}{n-1} + r_n\binom{n}{n} + r_n$$

$$\boxed{r_n = 1 - r_0\binom{n}{0} - r_1\binom{n}{1} - r_2\binom{n}{2} - r_3\binom{n}{3} - r_4\binom{n}{4} - r_5\binom{n}{5} - r_6\binom{n}{6} - r_7\binom{n}{7} - r_8\binom{n}{8} - \ldots - r_{n-1}\binom{n}{n-1} - r_n\binom{n}{n}}$$

← if you want to include n=0

$$1 = r_0\binom{0}{0} + r_0$$

$$2r_0 = 1$$

$$2r_1 = 1 - r_0\binom{1}{0}$$

$$2r_2 = 1 - r_0\binom{2}{0} - r_1\binom{2}{1}$$

$$2r_3 = 1 - r_0\binom{3}{0} - r_1\binom{3}{1} - r_2\binom{3}{2}$$

$$2r_4 = 1 - r_0\binom{4}{0} - r_1\binom{4}{1} - r_2\binom{4}{2} - r_3\binom{4}{3}$$

$$2r_5 = 1 - r_0\binom{5}{0} - r_1\binom{5}{1} - r_2\binom{5}{2} - r_3\binom{5}{3} - r_4\binom{5}{4}$$

$$2r_6 = 1 - r_0\binom{6}{0} - r_1\binom{6}{1} - r_2\binom{6}{2} - r_3\binom{6}{3} - r_4\binom{6}{4} - r_5\binom{6}{5}$$

$$2r_7 = 1 - r_0\binom{7}{0} - r_1\binom{7}{1} - r_2\binom{7}{2} - r_3\binom{7}{3} - r_4\binom{7}{4} - r_5\binom{7}{5} - r_6\binom{7}{6}$$

$$2r_8 = 1 - r_0\binom{8}{0} - r_1\binom{8}{1} - r_2\binom{8}{2} - r_3\binom{8}{3} - r_4\binom{8}{4} - r_5\binom{8}{5} - r_6\binom{8}{6} - r_7\binom{8}{7}$$

$$2r_9 = 1 - r_0\binom{9}{0} - r_1\binom{9}{1} - r_2\binom{9}{2} - r_3\binom{9}{3} - r_4\binom{9}{4} - r_5\binom{9}{5} - r_6\binom{9}{6} - r_7\binom{9}{7} - r_8\binom{9}{8}$$

$$\boxed{2r_n = 1 - r_0\binom{n}{0} - r_1\binom{n}{1} - r_2\binom{n}{2} - r_3\binom{n}{3} - r_4\binom{n}{4} - r_5\binom{n}{5} - r_6\binom{n}{6} - r_7\binom{n}{7} - r_8\binom{n}{8} - \ldots - r_{n-1}\binom{n}{n-1}}$$

Self check :)

n		r_n	$\eta(x)\cdot\frac{1}{1-2^{1+x}}=\zeta(x)$
0	$2r_0=1$	$r_0=\frac{1}{2}$	$r_0\cdot\frac{1}{1-2^{1+0}}=\frac{1}{2}\cdot\frac{1}{1-2^1}=-\frac{1}{2}$
1	$2r_1=1-\left[\frac{1}{2}\binom{1}{0}\right]$	$r_1=\frac{1}{4}$	$r_1\cdot\frac{1}{1-2^{1+1}}=\frac{1}{4}\cdot\frac{1}{1-2^2}=-\frac{1}{12}$
2	$2r_2=1-\left[\frac{1}{2}\binom{2}{0}+\frac{1}{4}\binom{2}{1}\right]$	$r_2=0$	$r_2\cdot\frac{1}{1-2^{1+2}}=0\cdot\frac{1}{1-2^3}=0$
3	$2r_3=1-\left[\frac{1}{2}\binom{3}{0}+\frac{1}{4}\binom{3}{1}+0\binom{3}{2}\right]$	$r_3=-\frac{1}{8}$	$r_3\cdot\frac{1}{1-2^{1+3}}=-\frac{1}{8}\cdot\frac{1}{1-2^4}=\frac{1}{120}$
4	$2r_4=1-\left[\frac{1}{2}\binom{4}{0}+\frac{1}{4}\binom{4}{1}+0\binom{4}{2}-\frac{1}{8}\binom{4}{3}\right]$	$r_4=0$	$r_4\cdot\frac{1}{1-2^{1+4}}=0\cdot\frac{1}{1-2^5}=0$
5	$2r_5=1-\left[\frac{1}{2}\binom{5}{0}+\frac{1}{4}\binom{5}{1}+0\binom{5}{2}-\frac{1}{8}\binom{5}{3}+0\binom{5}{4}\right]$	$r_5=\frac{1}{4}$	$r_5\cdot\frac{1}{1-2^{1+5}}=\frac{1}{4}\cdot\frac{1}{1-2^6}=-\frac{1}{252}$
6	$2r_6=1-\left[\frac{1}{2}\binom{6}{0}+\frac{1}{4}\binom{6}{1}+0\binom{6}{2}-\frac{1}{8}\binom{6}{3}+0\binom{6}{4}+\frac{1}{4}\binom{6}{5}\right]$	$r_6=0$	$r_6\cdot\frac{1}{1-2^{1+6}}=0\cdot\frac{1}{1-2^7}=0$
7	$2r_7=1-\left[\frac{1}{2}\binom{7}{0}+\frac{1}{4}\binom{7}{1}+0\binom{7}{2}-\frac{1}{8}\binom{7}{3}+0\binom{7}{4}+\frac{1}{4}\binom{7}{5}+0\binom{7}{6}\right]$	$r_7=-\frac{17}{16}$	$r_7\cdot\frac{1}{1-2^{1+7}}=-\frac{17}{16}\cdot\frac{1}{1-2^8}=\frac{1}{240}$
8	$2r_8=1-\left[\frac{1}{2}\binom{8}{0}+\frac{1}{4}\binom{8}{1}+0\binom{8}{2}-\frac{1}{8}\binom{8}{3}+0\binom{8}{4}+\frac{1}{4}\binom{8}{5}+0\binom{8}{6}-\frac{17}{16}\binom{8}{7}\right]$	$r_8=0$	$r_8\cdot\frac{1}{1-2^{1+8}}=0\cdot\frac{1}{1-2^9}=0$
9	$2r_9=1-\left[\frac{1}{2}\binom{9}{0}+\frac{1}{4}\binom{9}{1}+0\binom{9}{2}-\frac{1}{8}\binom{9}{3}+0\binom{9}{4}+\frac{1}{4}\binom{9}{5}+0\binom{9}{6}-\frac{17}{16}\binom{9}{7}+0\binom{9}{8}\right]$	$r_9=\frac{31}{4}$	$r_9\cdot\frac{1}{1-2^{1+9}}=\frac{31}{4}\cdot\frac{1}{1-2^{10}}=\frac{1}{132}$

$f(x)$ is the Positive part of the eta function

$$f(x) = r_0\binom{n}{0}x^n + r_1\binom{n}{1}x^{n-1} + r_2\binom{n}{2}x^{n-2} + r_3\binom{n}{3}x^{n-3} + \ldots + r_{n-1}\binom{n}{n-1}x^{n-(n-1)} + r_n\binom{n}{n}x^{n-n} + r_n$$

$g(x)$ is the Negative part of the eta function

$$g(x) = -r_0\binom{n}{0}x^n - r_1\binom{n}{1}x^{n-1} - r_2\binom{n}{2}x^{n-2} - r_3\binom{n}{3}x^{n-3} - \ldots - r_{n-1}\binom{n}{n-1}x^{n-(n-1)} - r_n\binom{n}{n}x^{n-n} + r_n$$

or

$$g(x) = -r_0\binom{n}{0}x^n - r_1\binom{n}{1}x^{n-1} - r_2\binom{n}{2}x^{n-2} - r_3\binom{n}{3}x^{n-3} - \ldots - r_{n-1}\binom{n}{n-1}x^{n-(n-1)}$$

$$f(x) = -g(x) + 2r_n$$

center point $\eta(-x) = \dfrac{f(x)+g(x)}{2} = \dfrac{-g(x)+2r_n+g(x)}{2} = \dfrac{2r_n}{2} = r_n$

$$\eta(k) = (-1)^{k-1}\left[r_0\binom{n}{0}k^n + r_1\binom{n}{1}k^{n-1} + r_2\binom{n}{2}k^{n-2} + r_3\binom{n}{3}k^{n-3} + \ldots + r_{n-1}\binom{n}{n-1}k^{n-(n-1)} + r_n\binom{n}{n}k^{n-n}\right] + r_n$$

$$\eta(k) = 1^n - 2^n + 3^n - 4^n + \ldots \pm k^n$$

$$\eta(k) = (-1)^{k-1}r_0\binom{n}{0}k^n + (-1)^{k-1}r_1\binom{n}{1}k^{n-1} + (-1)^{k-1}r_2\binom{n}{2}k^{n-2} + (-1)^{k-1}r_3\binom{n}{3}k^{n-3} + \ldots + (-1)^{k-1}r_{n-1}\binom{n}{n-1}k^{n-(n-1)} + \left(1+(-1)^{k-1}\right)r_n$$

$$r_n = \frac{1}{2} - \frac{1}{2}\left[r_0\binom{n}{0} + r_1\binom{n}{1} + r_2\binom{n}{2} + r_3\binom{n}{3} + \ldots + r_{n-1}\binom{n}{n-1}\right]$$