Bengali language, Romanisation and Onsager Core

Anindya Kumar Biswas, Department of Physics;
North-Eastern Hill University, Mawkynroh-Umshing, Shillong-793022.
email:anindya@nehu.ac.in

March 26, 2020

Abstract

We continue to study Chalantika, a bengali to bengali dictionary. We romanise the bengali alphabet via wikipedia scheme. We reduce the romanised alphabet to English alphabet i.e. the alphabet which appears in an English dictionary. In the reduced alphabet scheme, we draw the natural logarithm of the number of words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised(unnormalised). We observe that behind the words of the dictionary of the bengali language, in the reduced alphabet scheme, the magnetisation curve is BP(4,$\beta H = 0.08$), in the Bethe-Peierls approximation of Ising model with four nearest neighbours, in presence of liitle external magnetic fields, $\beta H = 0.08$. Moreover, words of the bengali language in the reduced alphabet scheme, nearly go over to Onsager solution, on few successive normalisations. β is $\frac{1}{k_B T}$ where, T is temperature, H is external magnetic field and k_B is tiny Boltzmann constant.

I Introduction

How come a bengali boy in the author's locality in Kolkata looks like a Mising boy from upper Assam, in India? Does it mean that there is another coincidence, if not many? Do the bengali language and the Mising language coincide in one level? In the previous paper, [1], we have found that the words of the Mising language written in Roman alphabet, [2], i.e. with diacritical marks, when subjected to reduction to English alphabet, exhibits Onsager Core. The graph of $\frac{lnf}{lnf_{nnnn-max}}$ vs $\frac{lnk}{lnk_{lim}}$ looks like the Onsager solution. $\frac{lnf}{lnf_{nnnn-max}}$ corresponds to normalisation once we decide not to use the words beginning with the first four letters in terms of ranking,[3]. k is a positive integer called rank. k_{lim} or, k_d is the limiting or, maximum rank where, f_{lim} is one. f is a number of words once set of words is arranged in strictly descending order.

The two dimensional Ising model,[4], in absence of external magnetic field, is prototype of an Ising model. In case of square lattice of planar spins, one spin

interacts with four other nearest neighbour spins i.e. on an average to another one spin. Below a certain ambient temperature, denoted as T_c, the two dimensional array of spins reduces to a planar magnet with magnetic moment per site varying as a function of $\frac{T}{T_c}$. This function was inferred, [5], by Lars Onsager way back in 1948, [6] and thoroughly deduced thereafter by C.N.Yang[7]. This function we are referring to as Onsager solution. Moreover, systems, [8], showing behaviour like Onsager solution is rare to come across.
Studying a bengali to bengali dictionary, [9] we have found that a a graphical law underlies that dictionary of the bengali language, in the article, [10]. The letters of the dictionary is of the bengali alphabet. What happens if we follow the same steps as in Mising? This paper proceeds along that. We add a new step. We write the bengali alphabet in the Roman alphabet, [11],[12] before reducing the Roman alphabet to the English alphabet i.e. to the alphabet that appears in an English dictionary. Then we do the rank analysis. We find that the graph of $\frac{lnf}{lnf_{nnnnnn-max}}$ vs $\frac{lnk}{lnk_{lim}}$ looks almost like the Onsager solution.
We were led to the present paper as follows. We studied natural languages, [13] and have found, in the preliminary study, existence of a curve magnetisation related to two approximations of Ising model, under each language. We termed this phenomenon as graphical law. Then we looked into, [14], dictionaries of five discipline of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of graphical law behind bengali, [10], Basque, [15], Romanian, [16], five more disciplines of knowledge, [17] and Onsager core of Abor-Miri, Mising languages,[1].
The planning of this paper is as follows. We review counting of Chalantika, [9],[10]; wiki romanisation, [12], reduced alphabet scheme in the section II, the standard curves of magnetisation of Ising model in the section III. In the section IV, we describe analysis of bengali words in the reduced alphabet scheme,[1] and the Onsager solution at the core. Section V is discussion. The appendix section VI describes how to draw the curves of magnetisations used in this paper. Next section VII is acknowledgement section. The last section is bibliography.

II Bengali

"Jato mot, tato path."
——a bengali proverb.
We have studied Chalantika, a bengali to bengali dictionary, [9] and found that a a graphical law underlies that dictionary of the bengali language, in the article, [10]. The number of words counted by us in [10], for each letter is tabulated in the table, 1.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
2595	1397	177	35	1034	30	25	237	28	113	30	2314	599	1157	316	988	350	895	235	236	137	191	134	1078	102	1392	515	1463	3196	392	3170	791	1773	356	737	434	955	47	2530	629

Table 1: Bengali words: the first row represents letters of the Bengali alphabet,[9] in the serial order.

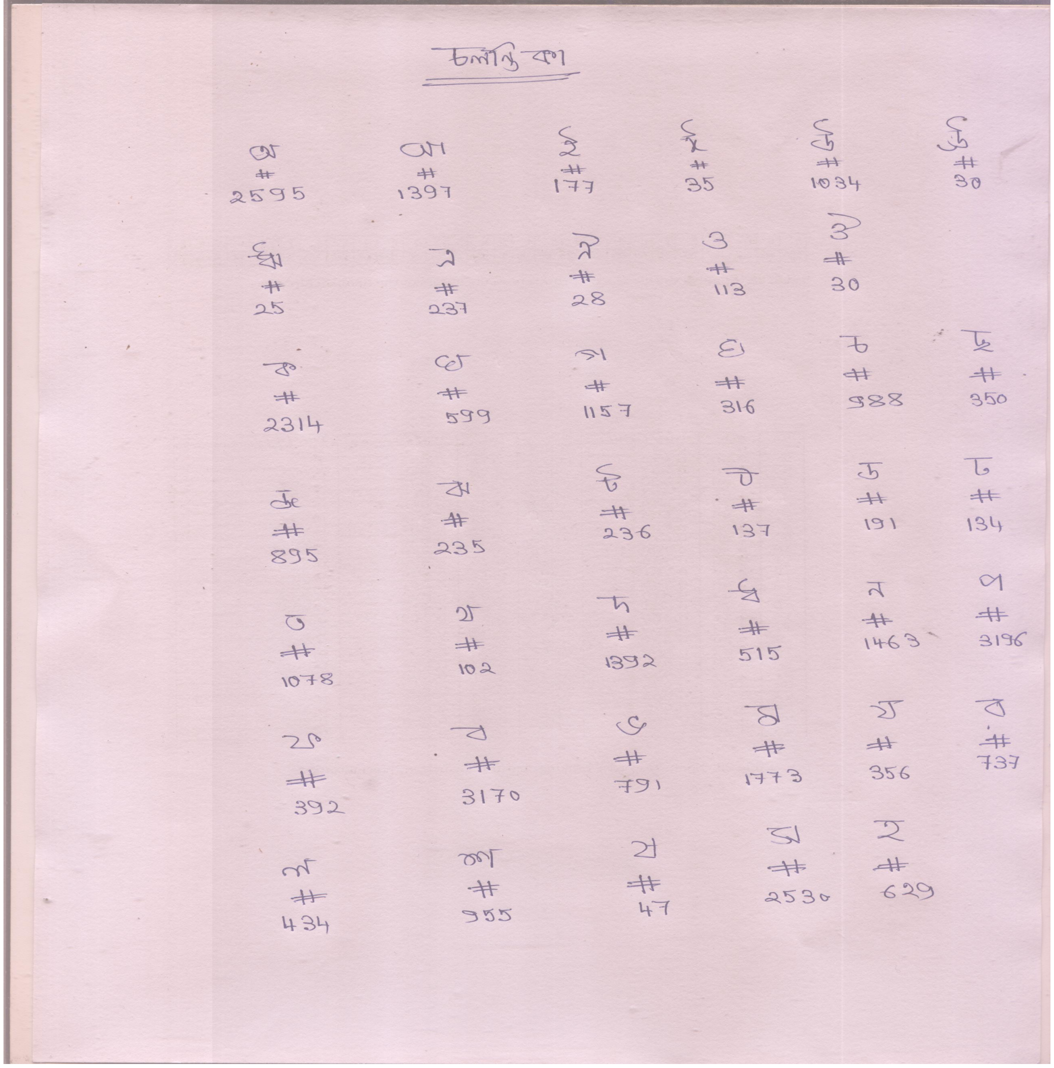

Figure 1: Number of words starting with various letters of the bengali alphabet

$\hat{O}$	A	I	Ī	U	Ū	Ṛ	E	OI	O	OU	K$\hat{O}$	KH$\hat{O}$	G$\hat{O}$	GH$\hat{O}$	CH$\hat{O}$	CHH$\hat{O}$	J̌$\hat{O}$	J̌H$\hat{O}$	Ṭ$\hat{O}$
2595	1397	177	35	1034	30	25	237	28	113	30	2314	599	1157	316	988	350	895	235	236
ṬH$\hat{O}$	Ḍ$\hat{O}$	ḌH$\hat{O}$	T$\hat{O}$	TH$\hat{O}$	D$\hat{O}$	DH$\hat{O}$	N$\hat{O}$	P$\hat{O}$	PH$\hat{O}$	B$\hat{O}$	BH$\hat{O}$	M$\hat{O}$	J$\hat{O}$	R$\hat{O}$	L$\hat{O}$	SH$\hat{O}$	Ṣ$\hat{O}$	S$\hat{O}$	H$\hat{O}$
137	191	134	1078	102	1392	515	1463	3196	392	3170	791	1773	356	737	434	955	47	2530	629

Table 2: Bengali words: the odd rows represent letters of the Bengali alphabet,[9] in the serial order, once romanised, [12]

A	B	C	D	E	G	H	I	J	K	L	M	N	O	P	R	S	T	U
1397	3961	1338	2232	237	1473	629	212	1486	2913	434	1773	1463	2766	3588	762	3532	1553	1064

Table 3: Bengali words: the first row represents letters of the English alphabet in the serial order, in the reduced alphabet scheme, for the words of the bengali language([9]).

As we write explicitly in terms of letters of the bengali alphabet, the table, 1, takes the form of the figure,1. Next we take resort to Romanisation of the bengali alphabet, [11]. There are many ways of doing so. We do not know which is the royal road to Romanisation for the bengali alphabet. We take a practical approach. We choose the wikipedia romanisation,[12]. We obtain the table, 2. In the next step, we reduce the romanised alphabet in the table 2, down to the English alphabet ala [1] as follows. We combine B$\hat{O}$ and BH$\hat{O}$ into B to get the resulting number of words starting with B as three thousand nine hundred sixty one, combine CH$\hat{O}$ and CHH$\hat{O}$ into C to get the resulting number of words starting with C as one thousand three hundred thirty eight, put together Ḍ$\hat{O}$, Ḍ$H\hat{O}$, D$\hat{O}$, DH$\hat{O}$ into D to obtain the resulting number of words starting with D as two thousand two hundred thirty two; G$\hat{O}$ and GH$\hat{O}$ into G to obtain the resulting number of words starting with G as one thousand four hundred seventy three; I and $\bar{I}$ into I to obtain the resulting number of words starting with I as two hundred; J̌$\hat{O}$, J̌H$\hat{O}$ and J$\hat{O}$ into J to obtain the resulting number of words starting with J as one thousand four hundred eighty six; K$\hat{O}$ and KH$\hat{O}$ into K to obtain the resulting number of words starting with K as two thousand nine hundred thirteen; $\hat{O}$, Oi,O and OU into O to obtain the resulting number of words starting with O as two thousand seven hundred sixty six; P$\hat{O}$ and PH$\hat{O}$ into P to obtain the resulting number of words starting with P as three thousand three hundred ninty two; Ṛ and R$\hat{O}$ into R to to obtain the resulting number of words starting with R as seven hundred sixty two; SH$\hat{O}$, Ṣ$\hat{O}$ and S$\hat{O}$ into S to obtain the resulting number of words starting with S as three thousand five hundred thirty two;Ṭ$\hat{O}$,Ṭ$H\hat{O}$, T$\hat{O}$ and TH$\hat{O}$ into T to obtain the resulting number of words starting with T as one thousand five hundred fifty three; U and $\bar{U}$ into U to obtain the resulting number of words starting with U as one thousand sixty four. The result is the table, 3. To visualise the content of the table,3, we plot the number of words against respective letters the figure fig.2. In the next to next section, we carry on analysis looking for onsager core

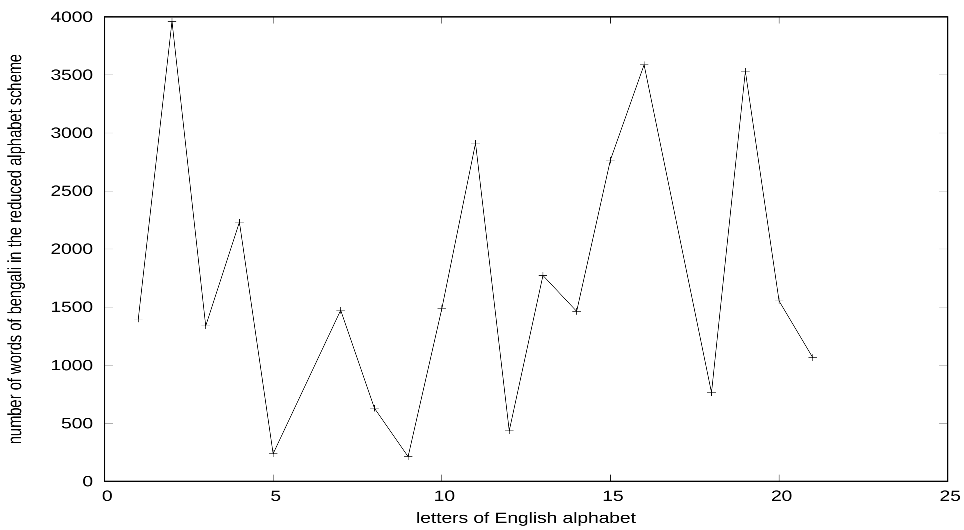

Figure 2: Vertical axis is number of words, in the table 3, of the bengali language in the reduced alphabet scheme and horizontal axis is the respective letters of the English alphabet. Letters are represented by the number in the English alphabet sequence.

as well as graphical law, basing on the table, 3. As a preparation to that, in the next section, we give an overview of the Onsager solution of the two dimensional Ising model as well as curves of magnetisation in Bethe-Peierls approximation of Ising model in presence of external magnetic field.

III Curves of Magnetisation

The Ising Hamiltonian,[4],[18],for a lattice of spins is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs, σ_i is i-th spin, H is external magnetic field and ϵ is coupling between two nearest neighbour spins. σ_i is binary i.e. can take values ± 1. At a temperature T, below a certain temperature called phase transition temperature, T_c, for the two dimensional Ising model in absence of external magnetic field i.e. for H equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [7], [18],

$$\frac{M}{M_{max}} = [1 - (sinh\frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

Graphically, the Onsager solution appears as in fig.3. In the Bragg-Williams and

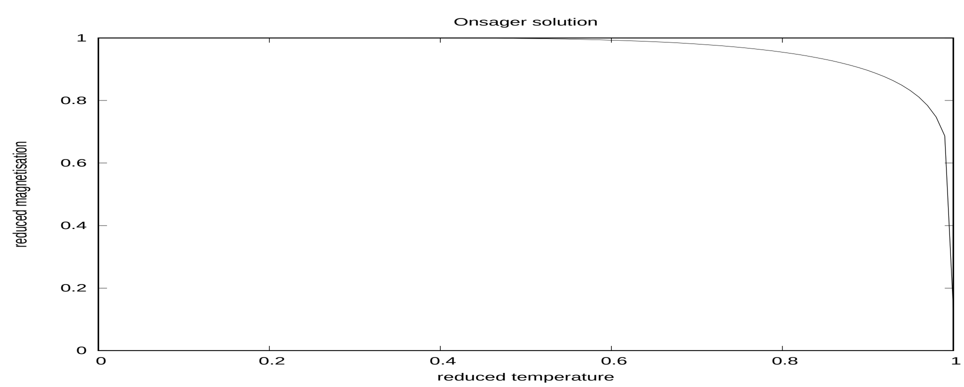

Figure 3: Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in absence of external magnetic field

Bethe-Peierls approximations for an Ising model in any dimension, in presence of external magnetic fields, reduced magnetisation as a function of reduced temperature, below the phase transition temperature, T_c, vary as in the figures 4-5. Related details are in the appendix.

The graphs in the figures,3-5, are used in the sections to follow as refernce curves.

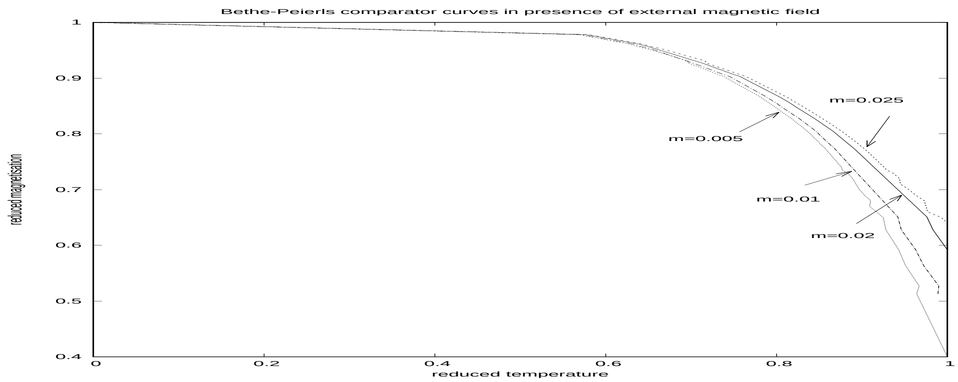

Figure 4: Reduced magnetisation vs reduced temperature curves, BP(4,βH), for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

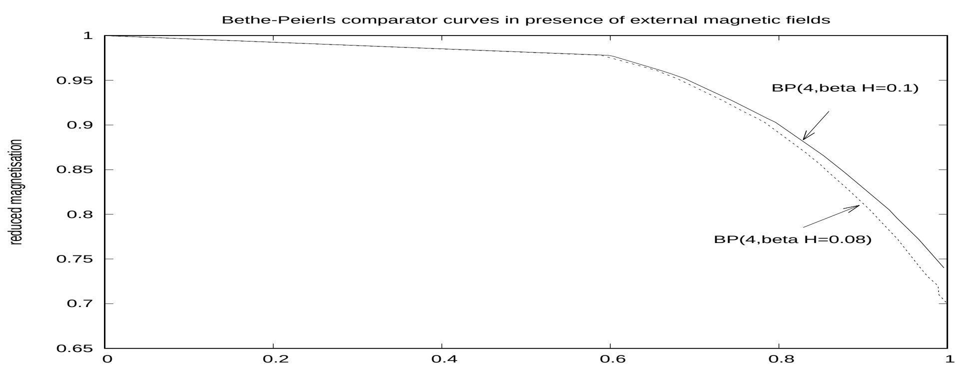

Figure 5: Reduced magnetisation vs reduced temperature curves, BP(4,βH=0.1) and BP(4,βH=0.08) in the Bethe-Peierls approximation in presence of little external magnetic field, for four nearest neighbours.

IV Analysis in the reduced alphabet scheme

IV.1 Method of study

For the purpose of searching for Onsager core as well as exploring graphical law, we assort the letters according to the number of words of the table, 3, in the descending order, denoted by f and the respective rank, denoted by k. k is a positive integer starting from one. Moreover, we attach a limiting rank, k_{lim}, and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty and the limiting number of words is one. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, 4 and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.6.
We then ignore the letter with the highest of words, tabulate in the adjoining table, 4 and redo the plot, normalising the lnfs with next-to-maximum $lnf_{nextmax}$, and starting from $k = 2$ in the figure fig.7. Normalising the lnfs with next-to-next-to-maximum $lnf_{nextnextmax}$, we tabulate in the adjoining table, 4 and starting from $k = 3$ we draw in the figure fig.8. Normalising the lnfs with next-to-next-to-next-to-maximum $lnf_{nextnextnextmax}$ we record in the adjoining table, 4 and plot starting from $k = 4$ in the figure fig.9. Normalising the lnfs with next-to-next-to-next-to-next-to-maximum $lnf_{nnnnmax}$ we record in the adjoining table, 4 and plot starting from $k = 5$ in the figure fig.10. Normalising the lnfs with nextnextnextnextnext-maximum $lnf_{nnnnnmax}$ we record in the adjoining table, 4 and plot starting from $k = 6$ in the figure fig.11. Normalising the lnfs with nextnextnextnextnextnext-maximum $lnf_{nnnnnnmax}$ we record in the adjoining table, 4 and plot starting from $k = 7$ in the figure fig.12. Normalising the lnfs with nnnnnnn-max $lnf_{nnnnnnnmax}$ we record in the adjoining table, 4 and plot starting from $k = 8$ in the figure fig.13.

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{nmax}	lnf/lnf_{nnmax}	lnf/lnf_{nnnmax}	lnf/$lnf_{nnnnmax}$	lnf/$lnf_{nnnnnmax}$	lnf/$lnf_{nnnnnnmax}$	lnf/$lnf_{nnnnnnnnmax}$
1	0	0	3961	8.284	1	Blank	Blank	Blank	Blank	Blank	Blank	Blank
2	0.69	0.230	3588	8.185	0.988	1	Blank	Blank	Blank	Blank	Blank	Blank
3	1.10	0.367	3532	8.170	0.986	0.998	1	Blank	Blank	Blank	Blank	Blank
4	1.39	0.463	2913	7.977	0.963	0.975	0.976	1	Blank	Blank	Blank	Blank
5	1.61	0.537	2766	7.925	0.957	0.968	0.970	0.993	1	Blank	Blank	Blank
6	1.79	0.597	2232	7.711	0.931	0.942	0.944	0.967	0.973	1	Blank	Blank
7	1.95	0.650	1773	7.480	0.903	0.914	0.916	0.938	0.944	0.970	1	Blank
8	2.08	0.693	1553	7.348	0.887	0.898	0.899	0.921	0.927	0.953	0.982	1
9	2.20	0.733	1486	7.304	0.882	0.892	0.894	0.916	0.922	0.947	0.976	0.994
10	2.30	0.767	1473	7.295	0.881	0.891	0.893	0.915	0.921	0.946	0.975	0.993
11	2.40	0.800	1463	7.288	0.880	0.890	0.892	0.914	0.920	0.945	0.974	0.992
12	2.48	0.827	1397	7.242	0.874	0.885	0.886	0.908	0.914	0.939	0.968	0.986
13	2.56	0.853	1338	7.199	0.869	0.880	0.881	0.902	0.908	0.934	0.962	0.980
14	2.64	0.880	1064	6.970	0.841	0.852	0.853	0.874	0.879	0.904	0.932	0.949
15	2.71	0.903	762	6.636	0.801	0.811	0.812	0.832	0.837	0.861	0.887	0.903
16	2.77	0.923	629	6.444	0.778	0.787	0.789	0.808	0.813	0.836	0.861	0.877
17	2.83	0.943	434	6.073	0.733	0.742	0.743	0.761	0.766	0.788	0.812	0.826
18	2.89	0.963	237	5.468	0.660	0.668	0.669	0.685	0.690	0.709	0.731	0.744
19	2.94	0.980	212	5.357	0.647	0.654	0.656	0.672	0.676	0.695	0.716	0.729
20	3.00	0	0	0	0	0	0	0	0	0	0	0

Table 4: Bengali words in the reduced alphabet scheme: ranking,natural logarithm, normalisations

IV.2 Results

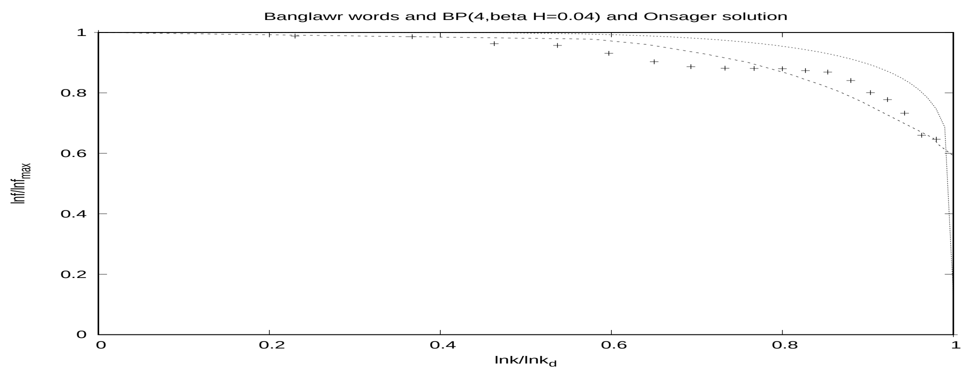

Figure 6: Vertical axis is $\frac{lnf}{lnf_{max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the bengali language ([9]) , in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.02$ or, $\beta H = 0.04$. The uppermost curve is the Onsager solution.

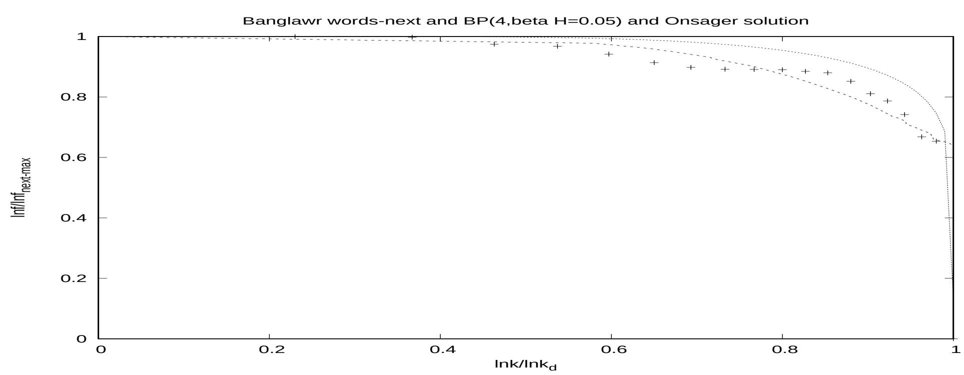

Figure 7: Vertical axis is $\frac{lnf}{lnf_{next-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.025$ or, $\beta H = 0.05$. The uppermost curve is the Onsager solution.

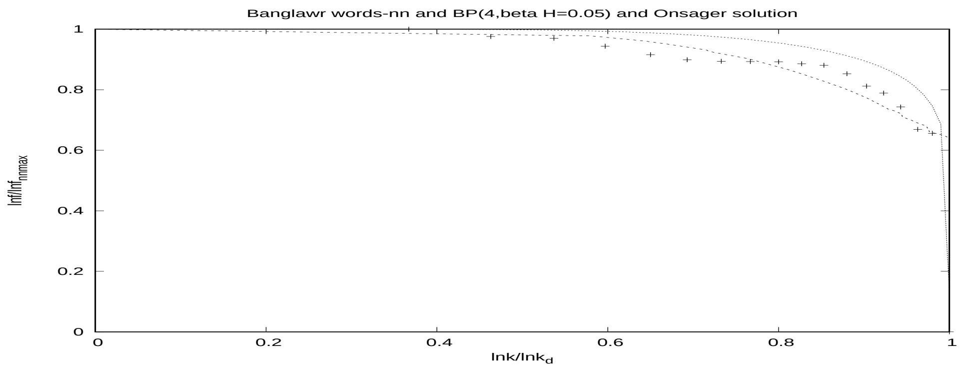

Figure 8: Vertical axis is $\frac{lnf}{lnf_{nextnext-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.025$ or, $\beta H = 0.05$.The uppermost curve is the Onsager solution

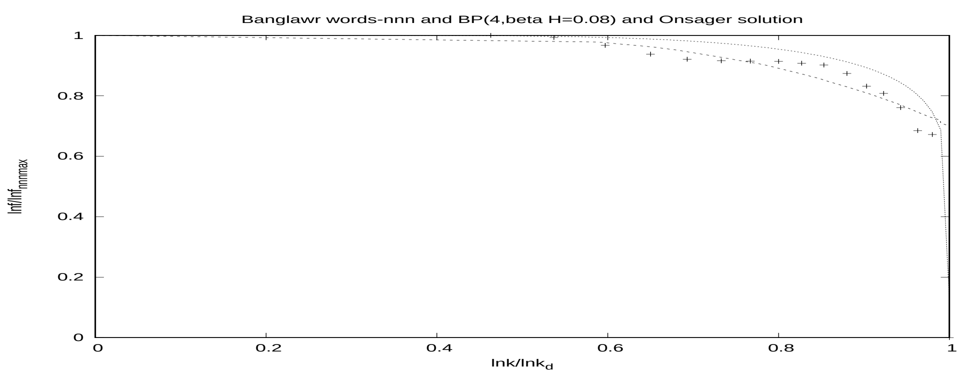

Figure 9: Vertical axis is $\frac{lnf}{lnf_{nextnextnext-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.04$ or, $\beta H = 0.08$. The uppermost curve is the Onsager solution.

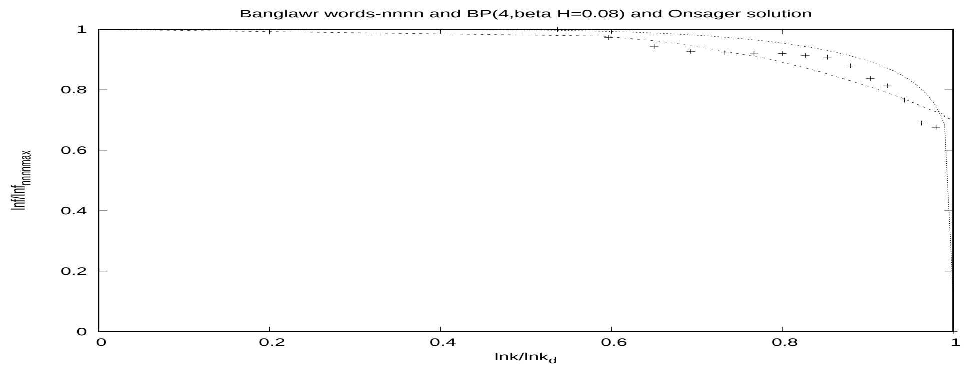

Figure 10: Vertical axis is $\frac{lnf}{lnf_{nextnextnextnext-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four neighbours and little magnetic field, $m = 0.04$ or, $\beta H = 0.08$. The uppermost curve is the Onsager solution.

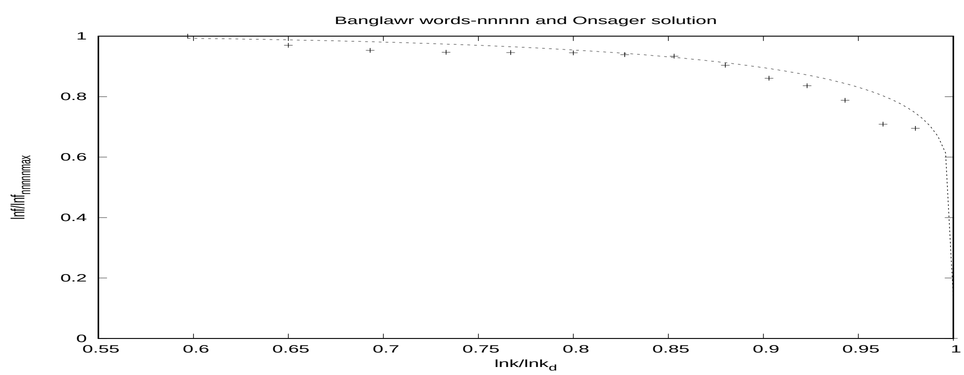

Figure 11: Vertical axis is $\frac{lnf}{lnf_{nnnnn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being the Onsager solution.

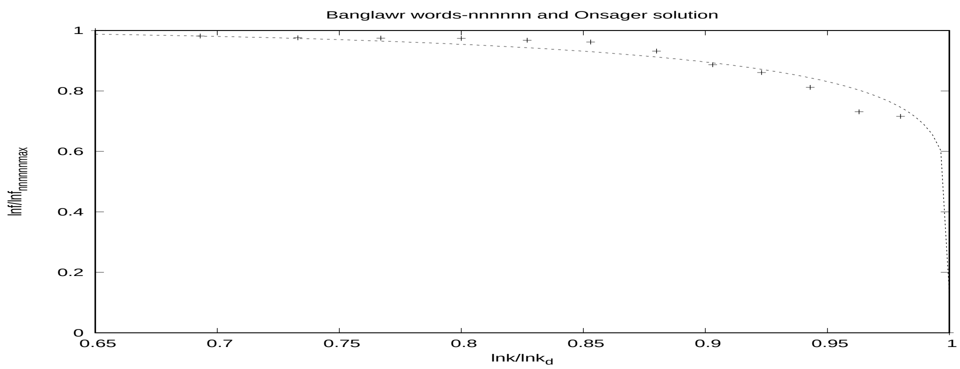

Figure 12: Vertical axis is $\frac{lnf}{lnf_{nnnnnn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being the Onsager solution.

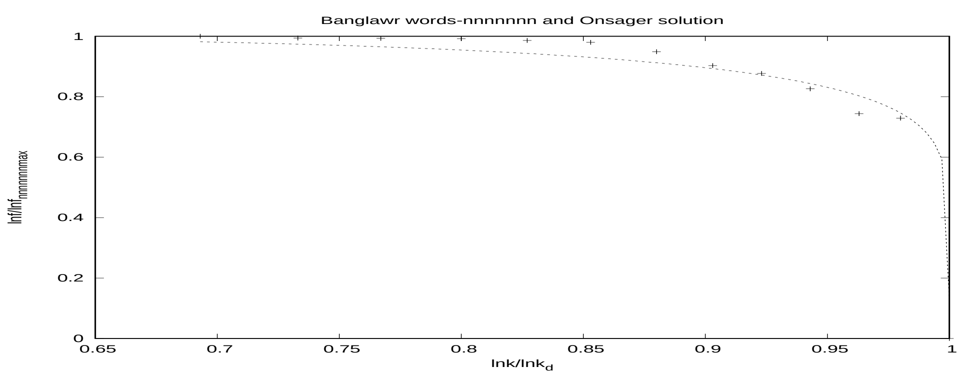

Figure 13: Vertical axis is $\frac{lnf}{lnf_{nnnnnn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the bengali language ([9]), in the reduced alphabet scheme, with the fit curve being the the Onsager solution.

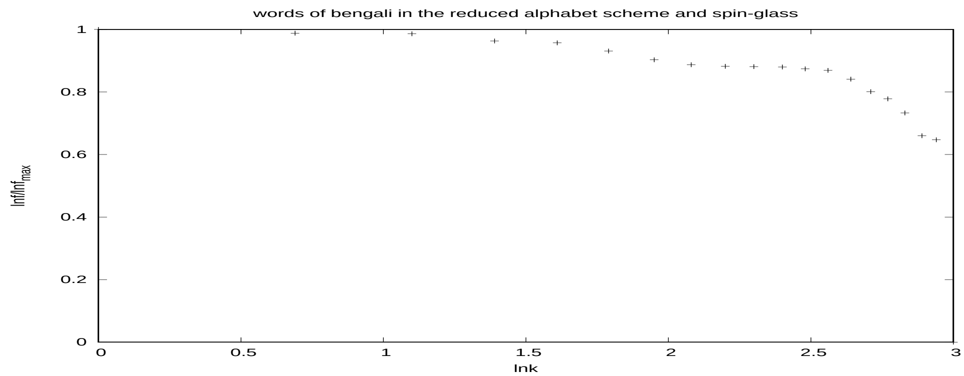

Figure 14: Vertical axis is $\frac{lnf}{lnf_{max}}$ and horizontal axis is lnk. The + points represent the words of the bengali language in the reduced alphabet scheme.

IV.3 Conclusion

From the figures (fig.6-fig.13), we observe that there is a curve of magnetisation, behind words of bengali language ([9]), in the reduced alphabet scheme. This is magnetisation curve, BP(4,βH = 0.08), in the Bethe-Peierls approximation with four nearest neighbours, in presence of liitle magnetic field, $\beta H = 0.08$. Moreover, the associated correspondance is,

$$\frac{lnf}{lnf_{next-to-next-to-next-to-maximum}} \longleftrightarrow \frac{M}{M_{max}},$$

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [19].
On the top of it, on successive higher normalisations, words of the bengali language, in the reduced alphabet scheme, almost go over to Onsager solution in the $\frac{lnf}{lnf_{nnnnnn-max}}$ vs $\frac{lnk}{lnk_{lim}}$ graph.
For the shake of completeness, we draw $\frac{lnf}{lnf_{max}}$ and $\frac{lnf}{lnf_{next-max}}$ against lnk in the figures fig.(14,15) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying Bengali language words, in the reduced alphabet scheme.

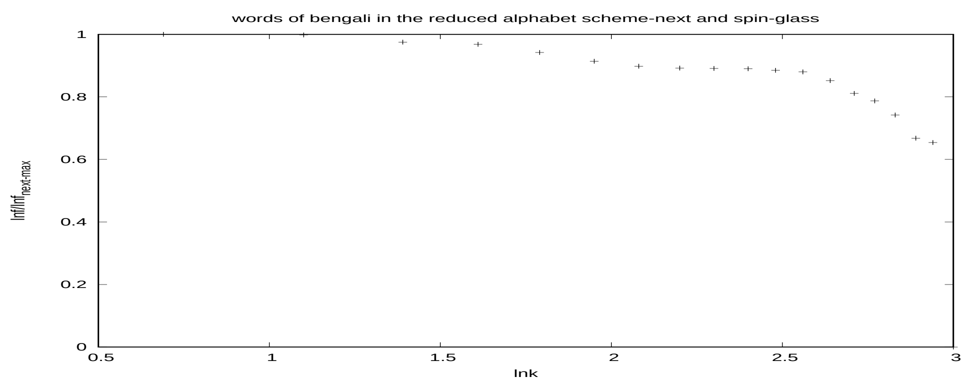

Figure 15: Vertical axis is $\frac{lnf}{lnf_{next-max}}$ and horizontal axis is lnk. The + points represent the words of the bengali language in the reduced alphabet scheme.

In the figures 14 and 15, the points has a transition, above transition the line is almost horizontal. Hence, the words of the bengali language in the reduced alphabet scheme, can be described to underlie a Spin-Glass magnetisation curve, [20], in the presence of magnetic field.

V Discussion

We have observed that there is a curve of magnetisation, behind words of the bengali language,[9], in the reduced alphabet scheme. This is magnetisation curve, BP(4,$\beta H = 0.08$), in the Bethe-Peierls approximation with four nearest neighbours, in presence of liitle magnetic field, $\beta H = 0.08$. Moreover, words of the bengali language, [9], go over under successive normalisations nearly to the Onsager solution. i.e has Onsager core. Incidentally, this is the same conclusion we reached in [1].
It will be of interest to do the analysis counting all the words of [9], i.e. without ignoring the dialectical variation. Moreover, there is a spectrum of bengali dictionaries available. It will be more interesting to do the analysis for as many of those as possible, from the standpoint of uniqueness, as pointed out in [17].

VI appendix:Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [18], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{1}$$

Derivation of this formula ala [18] is given in the appendix of [17].
$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{2}$$

In the following, we describe datas in the table, 5, generated from the equation(2) and curves of magnetisation plotted on the basis of those datas. BP(4, $\beta H = 0.06$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(2). BP(4, $\beta H = 0.05$)

stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.05$. calculated from the equation(2). BP(4, $\beta H = 0.04$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(2). BP(4, $\beta H = 0.02$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(2). BP(4, $\beta H = 0.01$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(2). The data set is used to plot fig.4 and fig.5. Empty spaces in the table, 5, mean corresponding point pairs were not used for plotting a line.

BP(4,$\beta H=0.1$)	BP(4,$\beta H=0.08$)	BP(4,$\beta H=0.06$)	BP(4,$\beta H=0.05$)	BP(4,$\beta H=0.04$)	BP(4,$\beta H=0.02$)	BP(4,$\beta H=0.01$)	reduced magnetisation
0	0	0	0	0	0	0	1
0.597	0.589	0.583	0.580	0.577	0.572	0.569	0.978
0.603	0.593	0.587	0.584	0.581	0.575	0.572	0.977
0.660	0.655	0.647	0.643	0.639	0.632	0.628	0.961
0.673	0.665	0.657	0.653	0.649	0.641	0.637	0.957
0.688	0.679	0.671	0.667		0.654	0.650	0.952
			0.716			0.696	0.931
0.745	0.734	0.723	0.718	0.713	0.702	0.697	0.927
0.766	0.754	0.743	0.737	0.731	0.720	0.714	0.917
0.787	0.775	0.762	0.756	0.749	0.737	0.731	0.907
0.796	0.783	0.770	0.764	0.757	0.745	0.738	0.903
0.848	0.832	0.816	0.808	0.800	0.785	0.778	0.869
0.854	0.837	0.821	0.813	0.805	0.789	0.782	0.865
0.866	0.849	0.832	0.823	0.815	0.799	0.791	0.856
0.878	0.859	0.841	0.833	0.824	0.807	0.799	0.847
0.902	0.882	0.863	0.853	0.844	0.826	0.817	0.828
0.931	0.908	0.887	0.876	0.866	0.846	0.836	0.805
0.940	0.917	0.895	0.884	0.873	0.852	0.842	0.796
0.966	0.941	0.916	0.904	0.892	0.869	0.858	0.772
0.996	0.968	0.940	0.926	0.914	0.888	0.876	0.740
1			0.929			0.877	0.735
	0.977		0.936			0.883	0.730
	0.989		0.944			0.889	0.720
	0.990		0.945				0.710
	1.00		0.955			0.897	0.700
			0.963			0.903	0.690
			0.973			0.910	0.680
						0.909	0.670
			0.993			0.925	0.650
				0.976	0.942		0.651
			1.00				0.640
				0.983	0.946	0.928	0.628
				1.00	0.963	0.943	0.592
					0.972	0.951	0.564
					0.990	0.967	0.527
						0.964	0.513
					1.00		0.500
						1.00	0.400
							0.300
							0.200
							0.100
							0

Table 5: Reduced magentisation vs reduced temperature datas in Bethe-Peierls approx. of Ising model in presence of little external magnetic fields, from right to left along a row.

VI.1 Spin-Glass

In the case coupling between(among) the spins, not necessarily n.n, for the Ising model is(are) random, we get Spin-Glass, [20, 21, 22, 23, 24, 25, 26]. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like $\frac{1}{T-T_c}$ upto the the phase transition temperature, followed by very little increase,[20, 26], in magnetisation, as the ambient temperature continues to drop. This happens at least in the replica approach of the Spin-Glass theory, [23, 24].

VII Acknowledgement

We have used gnuplot for plotting the figures in this paper.

References

[1] Anindya Kumar Biswas, ”Onsager Core of Abor-Miri and Mising Languages”, viXra: 2003.0343[Condensed Matter].

[2] Tabu Ram Taid, ”A Dictionary of the Mising Language”, Anundoram Borooah Institute of Language, Art and Culture, March,2010, Assam, India.

[3] A. M. Gun, M. K. Gupta and B. Dasgupta, Fundamentals of Statistics Vol 1, Chapter 12, eighth edition, 2012, The World Press Private Limited, Kolkata.

[4] E. Ising, Z.Physik 31,253(1925).

[5] S. M. Bhattacharjee and A. Khare, ”Fifty Years of the Exact solution of the Two-dimensional Ising Model by Onsager”, arXiv:cond-mat/9511003v2.

[6] L. Onsager, Nuovo Cim. Supp.6(1949)261.

[7] C. N. Yang, Phys. Rev. 85, 809(1952).

[8] K. I. Keda and K. Hirakawa, Solid Stat. Comm. 14 (1974) 529.

[9] Rajsekhar Basu Sankalita(edited), ”Chalantika, Adhunik Bangabhashar Abhidhan”, thirteenth edition(teroso unonobboi bongabda (1982)), M. C. Sarkar and Sons private limited, 14, Bankim Chatterjee Street, Kolkata, © Dipankar Basu.

[10] Anindya Kumar Biswas, ”Bengali language and Graphical law ”, viXra: 1908:0090[Linguistics].

[11] `http://en.wikipedia.org/wiki/Romanisation_of_Bengali`

[12] `http://en.wikipedia.org/wiki/Bengali_alphabet`

[13] Anindya Kumar Biswas, "Graphical Law beneath each written natural language", arXiv:1307.6235v3[physics.gen-ph]. A preliminary study of words of dictionaries of twenty six languages, more accurate study of words of dictionary of Chinese usage and all parts of speech of dictionary of Lakher(Mara) language and of verbs, adverbs and adjectives of dictionaries of six languages are included.

[14] Anindya Kumar Biswas, "A discipline of knowledge and the graphical law", IJARPS Volume 1(4), p 21, 2014; viXra: 1908:0090[Linguistics].

[15] Anindya Kumar Biswas, "Basque language and the Graphical Law", viXra: 1908:0414[Linguistics].

[16] Anindya Kumar Biswas, "Romanian language, the Graphical Law and More ", viXra: 1909:0071[Linguistics].

[17] Anindya Kumar Biswas, "Discipline of knowledge and the graphical law, part II", viXra:1912.0243 [Condensed Matter],International Journal of Arts Humanities and Social Sciences Studies Volume 5 Issue 2 February 2020.

[18] K. Huang, Statistical Mechanics, second edition, John Wiley and Sons(Asia) Pte Ltd.

[19] Sonntag, Borgnakke and Van Wylen, Fundamentals of Thermodynamics, p206-207, fifth edition, John Wiley and Sons Inc.

[20] `http://en.wikipedia.org/wiki/Spin_glass`

[21] S. F Edwards and P. W. Anderson, J. Phys.F: Metal Phys. 5, 965-74, 1975.

[22] P. W. Anderson, "Spin-Glass III, Theory raises Its Head", Physics Today June(1988).

[23] S. Guchhait and R. L. Orbach, "Magnetic Field Dependence of Spin Glass Free Energy Barriers", PRL 118, 157203 (2017).

[24] T. Jorg, H. G. Katzgraber, F. Krzakala, "Behavior of Ising Spin Glasses in a Magnetic Field", PRL 100, 197202(2008).

[25] J. R. L. de Almeida and D. J. Thouless, "Stability of the Sherrington-Kirkpatrick solution of a spin glass model", J. Phys. A: Math.Gen.,Vol. 11, No. 5,1978.

[26] D. Sherrington and S. Kirkpatrick, PRL 35, 1792-6, 1975.