Essential Spaces

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Abstract

We introduce the idea of an Essential Spaces for 2-dimensional compact manifolds. We raise the question whether essential spaces do exist also for 3-dimensional manifolds.

Key Words: compact manifold, manifold decomposition.

1 Introduction

There are many ways to decompose compact n-dimensional manifolds in simpler elements. A possible way is to decompose them as a connected sum of prime[1] compact manifolds. For example, for 2-dimensional manifolds, any compact manifold can be obtained by connected sum of 2-real projective planes (cross-caps) and 2-tori (handles) to a 2-sphere (see [1]).

However, from the example in the parafgraph below, we will see that projective planes are somehow more fundamental then tori and, we will call such kind of fundamental spaces **Essential Spaces**.

2 Essential Spaces

It is possible to obtain any compact 2-dimensional manifold by adding and removing projective planes from a 2-sphere without using handles. For example you can obtain a tours by the following procedure:

1. Add three cross-caps to a sphere by means of connected sums.
2. Combine two of the three cross-caps on the sphere in order to obtain a Klein Bottle attached to the sphere.
3. Readjust the Klein bottle so that it can be seen as an handle with one end attached to one side of the sphere and the other end attached to the opposite side of the sphere (see Fig. 1a).

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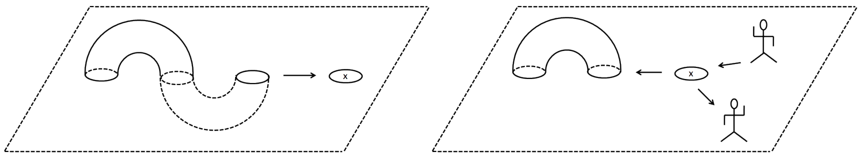
[1]A prime manifold is an n-manifold that cannot be expressed as a non-trivial connected sum of two n-manifolds. Non-trivial means that neither of the two is an n-sphere

4. Drag one end of the handle (Klein Bottle) inside the third cross-cap (see Fig 1a), and move it in order to make it emerge from the cross-cap attached to the opposite side of the sphere (see Fig. 1b). Now you have a proper handle (2-torus).

5. Remove the third cross-cap. Now you have eventually a 2-torus.



(a) (b)

Figure 1: From Klein Bottle to Torus

From the example above we see that in two dimensions the set of prime spaces (which contains only 2 element) has a proper subset of spaces which are more fundamental. This fundamental spaces are the ones that we have called Essential Spaces in the Introduction.

3 Further Studies

The set of 3-dimensional compact manifold is infinite. We raise the question whether this set has a proper subset of Essential Spaces, which are prime spaces that can be used, by means of a procedure similar to the one described in the paragraph above, to build other prime spaces. If these spaces exist we would like to find some example.

If not yet addressed in literature, we believe this is something that deserve further mathematical investigation.

References

[1] J. Gallier, D. Xu. *A Guide to the Classification Theorem for Compact Surfaces.* Springer (2013)