A short remark on the result of József Sándor

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Abstract : It is pointed out that, one of the results in the recently published article, *'On the Iyengar-Madhava Rao-Nanjundiah inequality and it's hyperbolic version'* [3] by József Sándor is logically incorrect and new corrected result with it's proof is presented.

Keywords : Iyengar-Madhava Rao-Nanjundiah inequality; logically incorrect; mathematical mistake; corrected version.

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1 Introduction

The well known inequality [1, pp. 236], [2]

$$\cos x < \frac{\sin x}{x};\ x \in (0, \pi/2) \tag{1.1}$$

has many applications in Mathematics. The inequality (1.1) has been studied and used extensively by many researchers in the recent past. Its refinements and generalizations are given by many others. Recently in [3] József Sándor proved the following statement:

Statement 1. *([3, Corollary 2.2]): The best constants c, d such that*

$$\cos(x + c) < \frac{\sin x}{x} < \cos(x + d) \tag{1.2}$$

for $x \in (0, \pi/2)$ *are* $c = 0$ *and* $d = \arccos \frac{2}{\pi} - \frac{\pi}{2} \approx -0.690107$.

The left inequality of (1.2) is undoubtedly valid; but right inequality in (1.2) is not valid in some part of the interval $(0, \pi/2)$. This can be seen as follows:

In the proof of Statement 1, lastly József Sándor arrived at the conclusion that

$$x + d < \arccos\left(\frac{\sin x}{x}\right) < x \tag{1.3}$$

in $(0, \pi/2)$. Then he applied on (1.3) the function cosine considering it as decreasing in the intervals of values of functions in (1.3), which is logically incorrect for left inequality of (1.3), since $x + d$ is negative in the interval $(0, -d)$, whereas $\arccos(\sin x/x)$ is positive in the same interval. For applying cosine function on left inequality of (1.3), $x + d$ and $\arccos(\sin x/x)$ should lie in the same interval where cosine is increasing or decreasing. This thing seems to be not considered by author of [3], so the result in Statement 1 is partially incorrect due to this simple mathematical mistake. However we must emphasize that the technique of applying cosine function to obtain other inequalities in the same paper is interesting.

2 Main Result

We present a corrected version of Statement 1 in this section.

Theorem 1. *The best constants c, d such that*

$$\cos(x + c) < \frac{\sin x}{x};\ x \in (0, \pi/2) \tag{2.1}$$

and

$$\frac{\sin x}{x} < \cos(x + d);\ x \in (\lambda, \pi/2) \tag{2.2}$$

where $\lambda \approx 0.43715$ *are* $c = 0$ *and* $d = \arccos\frac{2}{\pi} - \frac{\pi}{2} \approx -0.690107$.

Proof. The inequality (2.1) is already proved in [3]. So we prove only (2.2). As in the proof of [3, Corollary 2.2], consider the left inequality of (1.3) as

$$x + d < \arccos\left(\frac{\sin x}{x}\right)$$

where $d = \arccos\frac{2}{\pi} - \frac{\pi}{2} \approx -0.690107$. Clearly, in the interval $[-d, \pi/2)$ both the functions $x+d$ and $\arccos(\sin x/x)$ are non-negative. Therefore, applying strictly decreasing function cosine we get

$$\frac{\sin x}{x} < \cos(x+d);\ x \in [-d, \pi/2) \tag{2.3}$$

where $d \approx -0.690107$. Now it remains to consider the validity of (2.3) in $(0, -d)$. It is not difficult to check that $\cos(x+d)$ is strictly increasing and $\frac{\sin x}{x}$ is strictly decreasing in $(0, -d)$. Again the solution of equation $\cos(x+d) - \frac{\sin x}{x} = 0$ can be found by Numerical methods techniques to be $x \approx 0.43715$. This shows that (2.3) is valid in $(\lambda, -d)$ also. The proof is complete. □

The following graphical comparison of functions in (2.2) supports the proof.

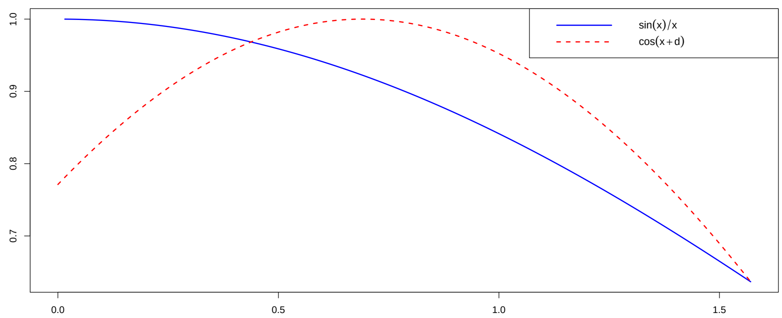

Figure 1: Graphs of the functions in (2.2) for $x \in (0, \pi/2)$.

References

[1] D. S. Mitrinovic, *Analytic Inequalities*, Springer-Verlag, Berlin **1970**.

[2] K. S. K. Iyengar, B. S. Madhava Rao and T. S. Nanjundiah, *Some trigonometrical inequalities*, Half-Yearly J. Mysore Univ., **B**(N.S.)6, 1-12(1945).

[3] J. Sándor, *On the Iyengar-Madhava Rao-Nanjundiah inequality and its hyperbolic version*, Notes on Number Theory and Discrete Mathematics, Vol. **24**, No. 2, 2018, pp. 134-139.