Something is rotten in the state of QED

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Quantum electrodynamics (QED) is considered the most accurate theory in the history of science. However, this precision is based on a single experimental value: the anomalous magnetic moment of the electron (g-factor). An examination of QED history reveals that this value was obtained using illegitimate mathematical traps, manipulations and tricks. These traps included the fraud of Kroll & Karplus, who acknowledged that they lied in their presentation of the most relevant calculation in QED history. As we will demonstrate in this paper, the Kroll & Karplus scandal was not a unique event. Instead, the scandal represented the fraudulent manner in which physics has been conducted from the creation of QED through today.

1 Introduction

After the end of World War II, American physicists organized a series of three transcendent conferences for the development of modern physics: Shelter Island (1947), Pocono (1948) and Oldstone (1949). These conferences were intended to be a continuation of the mythical Solvay conferences. But, after World War II, the world had changed.

The launch of the atomic bombs in Hiroshima and Nagasaki (1945), followed by the immediate surrender of Japan, made the Manhattan Project scientists true war heroes. Physicists were no longer a group of harmless intellectuals; they had become the powerful holders of the secrets of the atomic bomb. The members of the Manhattan Project had been militarized, and their knowledge had become a state secret. There was a positive aspect to this development: the US government created the Atomic Energy Commission (AEC) and appointed Oppenheimer as its chief advisor. The former members of the Manhattan Project took control of universities, and American research centers received generous government grants. With these grants, the research centers were able to invest in expensive experimental resources, such as atomic explosion tests, particle accelerators and supercomputers.

Although this situation provided the American scientists with unlimited resources to conduct their research, no one had considered the great risk it posed to the future of science. The former members of the Manhattan Project now enjoyed unlimited credibility. Their hypotheses were automatically accepted, and no one could refute their theories. Their calculations and experimental data were subject to military secrecy, and the cost of the equipment necessary to perform the experiments was prohibitive for the rest of the international scientific community. Consequently, calculations and experiments could no longer be reproduced. Modern physics had become “not falsifiable,” according to the criteria of the philosopher Popper. It was no longer possible to differentiate scientific theories from dogmas. Those who accepted as truth the hypotheses of the former members of the Manhattan Project were rewarded with positions of responsibility in research centers, while those who criticized their work were separated and ostracized. The devil’s seed had been planted in the scientific community, and its inevitable consequences would soon grow and flourish.

2 Shelter Island (1947)

2.1 The problem of infinities

After the success of the Dirac equation in 1928, quantum mechanics theorists attempted to quantify the electromagnetic field by creating the quantum field theory (QFT). Unfortunately, QFT was a complete failure since any attempted calculation under this theory resulted in an infinite number.

The only solution the proponents could devise was to simply ignore these infinities. Many methods can be used to ignore infinities, but the primary ones are

- Substitution: replacing a divergent series with a specific finite value that has been arbitrarily chosen (for example, the energy of an electron).
- Separation: separating an infinite series into two components, one that diverges to infinity and another that converges to a finite value. Eventually, the infinite component is ignored and only the finite part remains.
- Cut-off: focusing on an arbitrary term in the evolution of a series that diverges to infinity and ignoring the rest of the terms of the series.

All these techniques are illegitimate from a mathematical perspective, as demonstrated by Dirac: ***“I must say that I am very dissatisfied with the situation because this so-called ’good theory’ does involve neglecting infinities which appear in its equations, ignoring them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves disregarding a quantity when it is small – not neglecting it just because it is infinitely great and you do not want it!.” [33]***

This technique of ignoring infinities is called renormalization. Feynman also recognized that this technique was not mathematically legitimate: ***"The shell game that we play is technically called 'renormalization'. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate." [1]***

The extent of the magic used by the QFT mathematicians is exemplified by the sum of the whole numbers. According to an illegitimate mathematical demonstration conducted by the Indian mathematician Ramanujan, the result of the sum of all positive integers is not infinite, but -1/12 [2]:

$$\sum_{n=1}^{\infty} n = 1 + 2 + 3 + 4 + 5 + ... = -\frac{1}{12} \tag{1}$$

Despite the absurdity of these renormalization techniques, they have been accepted in modern physics, and they are constantly used in current theories.

2.2 Nature is absurd

The acceptance of quantum mechanics meant the acceptance of seemingly illogical physical explanations, such as the wave-corpuscle duality, the uncertainty principle and the collapse of the wave function. With the quantization of the electromagnetic field, these physical explanations became even more confusing. The long list of meaningless explanations includes the polarization of the quantum vacuum, electrons and photons interacting with their own electromagnetic fields, particles traveling back in time, the emission and reception of virtual photons, and the continuous creation and destruction of electron-positron pairs in a quantum vacuum.

Feynman summarized this quite clearly: ***"The theory of quantum electrodynamics describes nature as absurd from the point of view of common sense. And it fully agrees with experiment. So I hope you can accept nature as She is — Absurd." [1]***

Accepting QED means giving up on understanding Nature. After all, ***"Nobody understands quantum mechanics."***

2.3 The Shelter Island conference

From June 2 to 4, 1947, the first international physics conference after World War II was held at Shelter Island. The conference brought together 24 physicists from the Manhattan Project, including Bethe, Bohm, Breit, Feynman, Kramers, Lamb, von Neumann, Pauling, Rabi, Schwinger, Teller, Uhlenbeck, Weisskopf and Wheeler. Oppenheimer acted as the master of ceremonies for the congress. The participants were received as celebrities, and the conference made a significant impact in the press. Despite high expectations, the conference ended in disappointment.

Fig. 1: Shelter Island Conference participants

Two important experimental measures were presented at the Shelter Island conference: the Lamb shift and the anomalous magnetic moment of the electron. Lamb [3] presented an experiment that showed that the 2S1/2 and the 2P1/2 energy levels of the hydrogen atom were not identical; instead they differed by about 1000 MHz. Rabi's team [4] presented a 0.1% anomaly in the hyperfine structure of hydrogen. Later, Breit [5] interpreted this anomaly as the anomalous magnetic moment of the electron (g-factor). These two measurements contradicted the Dirac equation, published in 1928, that had worked correctly for 20 years without exception.

Questioning the validity of the Dirac equation meant questioning the validity of quantum mechanics, which had generated many controversies in its beginning stages. Questioning quantum mechanics also meant questioning the legitimacy and status of the Manhattan Project heroes.

The meeting's participants discussed how to handle this crisis. However, all the options that were considered involved accepting errors in their theories without offering alternative solutions. They devised a compromise solution to manage this dilemma by defining QED as the renormalized perturbation theory of the electromagnetic quantum vacuum. Under this hypothesis, it was assumed that the Dirac equation of the electron was absolutely correct and that the small measurement discrepancies were due to disturbances caused by the polarization of the quantum vacuum. It was also assumed that these perturbations could be calculated using the QFT and that the infinities of this theory could be corrected using renormalization techniques.

For this compromise solution to work, it was necessary to use the QED equations to calculate the experimental values of the Lamb shift and the g-factor—something that no one knew how to do.

3 Pocono (1948)

3.1 Bethe's fudge factor

On the train trip home after the conference ended, Bethe starred in one of the most epic moments in the history of theoretical physics. Using only a pencil and paper, he solved the Lamb shift electron equation, obtaining a result of 1040 MHz.

This is the story, as recalled by Bethe: ***"The combination of these two talks that by Kramers and that by Lamb, stimulated me greatly and I said to myself, well, let's try to calculate that Lamb shift. And indeed, once the conference was over, I traveled by train to the General Electric research lab. And on the train I figured out how much that difference might be. I had to remember the interaction of electromagnetic quanta with electrons. And I wasn't sure about the factor of two. So if I remembered correctly, I seemed to get just about the right energy separation of 1000MHz. But I might be wrong by a factor of two. So the first thing I did when I came to the library at General Electric was to look up Heitler's book on Radiation Theory, and I found that, indeed, I had remembered the number correctly and that, indeed, I'd got a 1000MHz." [40]***

The article published by Bethe in 1947 [6] was a short three pages. In the article, Bethe proposed this equation for the Lamb shift.

$$W_{ns'} = \frac{8}{3\pi}\left(\frac{e^2}{\hbar c}\right)^3 Ry \left(\frac{Z^4}{n^3}\right) ln \, \frac{K}{< E_n - E_m >_{Av}} \qquad (2)$$

In this equation, K is a series that diverges to infinity. Bethe decided to apply renormalization by substituting this infinite value for the finite value of the electron's energy ($K = mc^2$).

$$W_{ns'} = 136 \, ln \left(\frac{K}{kp}\right) = 136 \, ln \left(\frac{m_e c^2}{17.8 \, Ry}\right) = 1040 \text{ Mhz} \qquad (3)$$

All the values in Bethe's equation were known physical constants, except for the value of 17.8 Ry. The origin of this value is unknown, but it is essential to obtain the desired result. According to the document, ***"The average excitation energy $kp =< E_n - E_m >_{Av}$ for the 2s state of hydrogen has been calculated numerically by Dr. Stehn and Miss Steward and found to be 17.8 Ry, an amazingly high value."*** That is, Bethe's fantastic calculation is based on data that was calculated later, data that Bethe could not have known on his train journey. It is a value that was entered ad hoc to match the theoretical value with the experimental value. In the field of physics, this illegitimate trick is known as a fudge factor.

The value of 1 Ry is defined as the ionization energy of the hydrogen atom (13.6 eV). Ionization energy is the maximum energy that an electron can receive in an atom, so the electrons' excitation values must be much lower than this amount of energy. By definition, the average excitation energy must be less than the maximum excitation value, that is, less than 1 Ry. However, in this case, Bethe proposed 17.8 Ry (242 eV), an absurd value. Bethe, himself, considered this fudge factor ***"an amazingly high value."***

3.2 Schwinger's numerology

A few months after Bethe calculated the value of the Lamb shift, Schwinger devised an even more epic calculation for the g-factor of the electron. This value was known as the Schwinger factor [7].

$$g.factor = 1 + \frac{\alpha}{2\pi} = 1.001162 \qquad (4)$$

Kush and Foley [8] [9] had obtained an experimental g-factor value of 1.00119. Schwinger published his result in February 1948 in a short article, comprising just one sheet, that said, ***"the detailed application of the theory shows that the radiative correction corresponds to an additional magnetic moment associated with the electron spin of magnitude $\delta\mu/\mu = \alpha/2\pi$ = 0.001162." [7]*** At no time was it explained how that value had been obtained. Instead, it was said that ***"a paper dealing with the details of this theory and its applications is in course of preparation."***

The Schwinger factor had a significant impact on the scientific community due to its simplicity and accuracy. Everyone waited expectantly for the fabulous new theory he had used to calculate this factor. Schwinger's theory must signify a revolution in modern physics. But, the days passed, and Schwinger did not publish his theory. Why did he not publish this long-awaited theory?

We suspect that Schwinger did not publish the theory because he had no theory. How did he obtain such a spectacular result without a theory? We suspect that he used a technique known as numerology. Schwinger assumed that the g-factor should be directly related to the fine structure constant (α), which has an approximate value of 0.7%. Dividing this value by 6 provides an approximate value of 0.1%, which is the value obtained by Rabi [4]. And 2π is about 6.

3.3 The Pocono conference

The Pocono conference took place from March 30 to April 2, 1948. This conference was attended by the same participants as the Shelter Island conference, as well as three of the greatest physicists of the time: Bohr, Dirac and Fermi. As with the Shelter Island conference, the expectations were high due to the recent progress of Bethe and Schwinger. As in the Shelter Island conference, the results were again disappointing.

The conference expectations were focused on Schwinger's presentation. Everyone hoped that he would finally explain the elegant way in which the Schwinger factor had been calculated. Schwinger's presentation lasted for five unbearable hours and comprised a series of complex, totally incomprehensible equations. Oppenheimer expressed his displeasure: ***"others gave talks to show others how to do the calculation, while Schwinger gave talks to show that only he could do it. [41]"*** Gradually, the attendees left the presentation until only Bethe and Fermi remained. The overall feeling was one of disappointment, as it was clear that Schwinger's theory was not based on an elegant solution.

The next day, Feynman presented his theory, explaining for the first time his famous Feynman diagrams. However, the attendees did not respond positively to this presentation. Feynman was convinced of the accuracy of his calculations simply because they produced the correct results. He did not bother to defend the mathematical basis for the equations or to present any physical hypotheses. For Feynman, the result was everything.

Bethe remembers the conference like this: ***"At Pocono, Schwinger and Feynman, respectively presented their theories. (...) Their theories seemed to be totally different. (...) Schwinger's was closely connected to the known quantum electrodynamics, so Niels Bohr, who was in the audience, immediately was convinced this was correct. And then Feynman came with his completely new ideas, which among other things involved positrons going backwards in time. And Niels Bohr was shocked, that couldn't possibly be true, and gave Feynman a very hard time. [39]"***

Feynman's recollection of the conference is also enlightening: ***"This meeting at Pocono was very exciting, because Schwinger was going to tell how he did things and I was to explain mine. (...). We could talk back and forth, without going into details, but nobody there understood either of us. (...) When he tried to explain his theory, he encountered great difficulty. (...) As soon as he would try to explain the ideas physically, the wolves would descend on him, he had great difficulty. Also, people were getting more and more tired (...) I didn't have a mathematical scheme to talk about. Actually I had discovered one mathematical expression, from which all my diagrams, rules and formulas would come out. The only way I knew that one of my formulas worked was when I got the right result from it. (...) I said in my talk: "This is my mathematical formula, and I'll show you that it produces all the results of quantum electrodynamics." immediately I was asked: "Where does the formula come from?' I said, "It doesn't matter where it comes from; it works, it's the right formula!" "How do you know it's the right formula?" "Because it works, it gives the right results!" "How do you know it gives the right answers?" ' (...) They got bored when I tried to go into the details. (...) Then I tried to go into the physical ideas. I got deeper and deeper into difficulties, everything chaotic. I tried to explain the tricks I had employed. (...) I had discovered from empirical rules that if you don't pay attention to it, you get the right answers anyway, and if you do pay attention to it then you have to worry about this and that. [42]"***

After the disappointing explanations of Schwinger and Feynman, the scientists returned home, aware of the need for a new unified QED theory that could elegantly explain Bethe's Lamb shift results and the Schwinger factor for the anomalous magnetic moment of the electron. Upon his return to Princeton, Oppenheimer received a third QED theory from a Japanese physicist named Tomonaga. Now, there were three QED theories, and all of them were inconsistent and incompatible with one other.

4 Oldstone (1949)

4.1 Dyson's series

After the Pocono meeting, the physics community searched for a unified, covariant QED theory. The person in charge of addressing this problem was a young 26-year-old English scientist named Dyson, who managed to reconcile the three QED theories in his article "The Radiation Theories of Tomonaga, Schwinger, and Feynman." [11] The article abstract indicated that ***"The chief results obtained are (a) a demonstration of the equivalence of the Feynman and Schwinger theories, and (b) a considerable simplification of the procedure involved in applying the Schwinger theory."*** In reality, Dyson had created his own QED theory based on the ideas of Tomonaga, Schwinger and Feynman. Dyson's theory was subsequently published in an article titled "The S-Matrix in Quantum Electrodynamics." [12] In this article, Dyson proposed that the Heisenberg S-matrix could be used to calculate the electron's g-factor, transforming it into a series called the Dyson series. The Dyson series was an infinite series of powers of alpha, where the first coefficient was precisely the Schwinger factor, and where each coefficient could be calculated by solving a certain number of Feynman diagrams.

$$a = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 \ldots \quad (5)$$

Dyson's theory, based on Feynman's diagrams, appeared to provide the definitive solution for which his peers had been waiting. Enthusiasm returned to the American scientific community.

4.2 Internal criticism

However, not all scientists were excited about Feynman's and Dyson's results. The primary critic of this new QED theory was Dirac: ***"How then do they manage with these incorrect equations? These equations lead to infinities when one tries to solve them; these infinities ought not to be there. They remove them artificially. (...) Just because the results happen to be in agreement with observations does not prove that one's theory is correct." [32].***

Another critic was Oppenheimer, as Dyson relates: ***"When after some weeks I had a chance to talk to Oppenheimer, I was astonished to discover that his reasons for being uninterested in my work were quite the opposite of what I had imagined. I had expected that he would disparage my program as merely unoriginal, a minor adumbration of Schwinger and Feynman. On the contrary, he considered it to be fundamentally on the wrong track. He thought adumbrating Schwinger and Feynman to be a wasted effort, because he did not believe that the ideas of Schwinger and Feynman had much to do with reality. I had known that he***

had never appreciated Feynman, but it came as a shock to hear him now violently opposing Schwinger, his own student, whose work he had acclaimed so enthusiastically six months earlier. He had somehow become convinced during his stay in Europe that physics was in need of radically new ideas, that this quantum electrodynamics of Schwinger and Feynman was just another misguided attempt to patch up old ideas with fancy mathematics. [35]"

According to Dyson, Fermi also did not agree with this new way of conducting science: ***"When Dyson met Fermi, he quickly put aside the graphs he was being shown indicating agreement between theory and experiment. His verdict, as Dyson remembered, was "There are two ways of doing calculations in theoretical physics. One way, and this is the way I prefer, is to have a clear physical picture of the process you are calculating. The other way is to have a precise and self-consistent mathematical formalism. You have neither." When a stunned Dyson tried to counter by emphasizing the agreement between experiment and the calculations, Fermi asked him how many free parameters he had used to obtain the fit. Smiling after being told "Four," Fermi remarked, "I remember my old friend Johnny von Neumann used to say, with four parameters I can fit an elephant, and with five I can make him wiggle his trunk." There was little to add." [37]***

Feynman's response to these criticisms is well known: ***"Shut up and Calculate!" [1]***

4.3 The Oldstone conference

From April 11 to 14, 1949, a third conference was held at Oldstone, with the same participants as the Shelter Island and Pocono conferences. As on the previous occasions, the Oldstone conference began with great expectations, this time based on Dyson's advances. As with the previous conferences, the results were disappointing.

The star of the Oldstone conference was Feynman, who used his immense charisma to present Dyson's theory as the definitive formalism of the QED theory. From that moment on, Feynman's diagrams became a popular tool among American physicists, and Feynman became the leader of this new generation of scientists.

In parallel to the QED consolidation, the conference presented important experimental results on subatomic particles that were called pi-mesons or pions. These particles had been discovered thanks to the new synchrocyclotron particle accelerator at the University of Berkeley. Interest in QED rapidly declined due to its extreme complexity and lack of practical utility, while the pions became the primary focus. As a result, Oppenheimer decided not to convene any further QED conferences; instead, he created the International Conference of High Energy Physics (ICHEP).

New research in high energy physics resulted in quantum chromodynamics (QCD), the electroweak theory and the standard theory of particle physics. All these developments relied heavily on the use of Feynman diagrams. However, the Feynman diagrams are only valid when the coupling constant has a very low value. In the case of electromagnetism, the coupling constant alpha is much smaller than one. But, in the case of fermions, the coupling constant is greater than one, so it is not mathematically legitimate to use the Feynman diagrams for these calculations. In 1951, Feynman himself warned Fermi of this problem: ***"Don't believe any calculation in meson theory that uses a Feynman diagram." [43]***

4.4 More fudge factors for Bethe

In 1950, Bethe [15] published a new calculation of the Lamb shift that adjusted the fudge factor value from 17.8 Ry to 16.646 Ry. This value has not been modified since this change. Other researchers, such as Kroll, Feynman, French and Weisskopf, have expanded Bethe's original equation with new fudge factors, resulting in a value of 1052 Mhz with the following equation:

$$W_{ns'} = \frac{\alpha^3}{3\pi\, Ry}\left(ln\, \frac{mc^2}{16.646\, Ry} - ln\, 2 + \frac{5}{6} - \frac{1}{5} \right) \tag{6}$$

This strategy of adding to existing equations new factors of diverse origin with the objective of matching the theoretical and experimental values has been widely used in QED. This strategy is known as perturbation theory, and it is often used recursively. That is, each of these factors is, in turn, formed by another series of factors. For example, it was assumed that the difference between the theoretical and experimental values of the Lamb shift was due to relativistic corrections. These corrections were calculated by Baranger in 1951 [16], who obtained a theoretical Lamb shift value of 1058.3 MHz, while the experimental value was $1061 \pm 2Mhz$. As expected, this new factor was, in turn, composed of three other factors of diverse origin.

$$\Delta W = \alpha^4 Ry \left(1 + \frac{11}{128} - \frac{ln\, 2}{2} \right) = 6.894 \text{ Mhz} \tag{7}$$

5 The Kroll & Karplus Scandal (1950-1957)

5.1 Fourth Order Correction

In 1949, Gardner and Purcell obtained a new experimental result for the g-factor of 1.001,146 [13]. With this new experimental value, Schwinger's factor was no longer considered accurate. Feynman used this new crisis as an opportunity to demonstrate the validity of his theory. Using the QED reformulation with Dyson's S-matrix, the renormalization of infinities could be performed in a consistent manner. According to this theory, Schwinger's factor was only the first coefficient of the Dyson series. The calculation of each coefficient in the series required the resolution of an exponential number of extremely complex equations. The calculation of the next factor in the series (the fourth order correction) required seven Feynman diagrams. Kroll and Karplus, two of Feynman's assistants, performed these calculations. In 1950, they published their results [14].

The second coefficient of the Dyson series, as calculated by Kroll and Karplus, was -2.973. Consequently, the new theoretical value of the g-factor was 1.001,147, which was almost the same as the experimental result that had been reported by Gardner and Purcell.

$$g.factor = 1 + \frac{\alpha}{2\pi} - 2.973\left(\frac{\alpha}{2\pi}\right)^2 = 1.001,147 \quad (8)$$

As indicated in the paper, ***"The details of two independent calculations which were performed so as to provide some check of the final result are available from the authors." [14]*** That is, the calculations had been performed independently by two teams of mathematicians who had obtained the same result; therefore, it was impossible that there were any errors in the calculations. Nor was it possible to imagine that a theoretical result that was identical to the experimental result could have been achieved by chance. This was the definitive test. QED had triumphed. Along the way, logic had been renounced, and rigorous mathematics had been dispensed. These faults did not matter; the theoretical calculations coincided with the experimental data with great precision. There was nothing more to discuss. Feynman's prestige dramatically increased, and he began to be mentioned as a candidate for the Nobel Prize.

5.2 Dyson's betrayal

In 1952, two years after this great success, Dyson published an article entitled "Divergence of Perturbation Theory in Quantum Electrodynamics," [17] which said that ***"An argument is presented which leads tentatively to the conclusion that all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge."*** The creator of the QED theory had questioned its theoretical validity by stating that his Dyson series was divergent.

Dyson had hinted at this fact in his previous works. The abstract of the article "The Radiation Theories of Tomonaga, Schwinger, and Feynman" [11] indicated that ***"the theory of these higher order processes is a program rather than a definitive theory, since no general proof of the convergence of these effects is attempted."*** The abstract of the article "The S-Matrix in Quantum Electrodynamics" [12] also indicated that ***"Not considered in this paper to prove the convergence of the theory as the order of perturbation itself tends to infinity."*** After this difficult confession, Dyson moved to England, abandoned this line of research and dedicated the rest of his career to other areas of physics. Perhaps this confession is the reason he never received the Nobel Prize.

5.3 The scandal

Surprisingly, Dyson's claim that the series was divergent did not diminish QED's credibility. However, in 1956, Franken and Liebes [18] published new, more precise experimental data that provided a very different g-factor value (1.001,165). This value was higher than the Schwinger factor, so the value of the second coefficient that had been calculated by Kroll and Karplus not only did not improve the Schwinger factor; it made the calculation worse.

With the new experimental data from Franken and Liebes, the value of the second coefficient of the series should have been +0.7 instead of -2.973. The difference between these values is huge and unjustifiable. The probative force of QED was upended. QED must necessarily be an incorrect theory. In addition, there was no explanation for why Kroll and Karplus's calculation provided the exact expected experimental value when that value was incorrect. It was evident that the QED calculations had matched the experimental data because they were manipulated. It was a fraud, a scandal.

But the creators of QED refused to accept defeat. QED could not be an incorrect theory because that placed them in an indefensible situation. All the developments in the field of theoretical physics that had occurred in the last decade were based on this theory. All the privileges they had obtained after the success of the Manhattan Project were at stake. Even worse, now the scientists' own lives were in jeopardy. In 1949, the USSR had obtained the atomic bomb thanks to information provided by Fucks, a Manhattan Project researcher with communist sympathies. From then on, espionage accusations became widespread among the American scientific community. Senator McCarthy began a witch hunt in which Oppenheimer was accused of treason and had to submit to trial. The witch hunt ended in 1957, when the Russians sent the Sputnik satellite into space and the US government realized that it needed scientists to create NASA and win the space race.

In response to this scandal, surprising facts were revealed. First, Kroll and Karplus confessed that they had not independently reached the same result; instead, they had reached a consensus result. Therefore, it was possible that there were errors in the calculation. Next, Petermann [19] detected an error in the Kroll and Karplus calculations (one that no one had detected in the seven years since the article was published). Petermann made the correct calculation and obtained a result of -0.328, which was almost 10 times lower than the previous calculation of -2.973. This correct calculation resulted in a new theoretical value of the g-factor (1.001,159,6) that was within the margin of error of the new experimental value (1,001,165). The same error was independently detected by Sommerfield [21]. Once again, two independent calculations provided the same theoretical value. Miraculously, QED had been saved.

For the third time in 10 years, experimental data had contradicted theoretical calculations (Kursh, Gardner and Franken), and, for the third time in 10 years, a theoretical correction had allowed the reconciliation of the theoretical data with the experimental data (Schwinger, Kroll and Karplus, and Petermann). This was all very suspicious. After the Kroll and Karplus scandal, two facts had become clear: no one was reviewing the theoretical calculations that were being published, and the researchers had lied to indicate that the calculations had been performed independently by two different groups.

According to Kroll: ***"Karplus and I carried out the first major application of that program, to calculate the fourth order magnetic moment, which calculation subsequently turned out to have some errors in it, which has been a perpetual source of embarrassment to me, but nevertheless the paper I believe was quite influential. (...) The errors were arithmetic (...) We had some internal checks but not nearly enough. (...) it was refereed and published and was a famous paper and now it's an infamous paper." [38]***

5.4 The analysis

At this point, we have doubts about everything that was reported, so we reviewed the article published by Kroll and Karplus in 1950 [14], as well as the corrections of Petermann [19] and Sommerfield [21] that were published in 1957.

Kroll and Karplus's article consists of 14 pages and is full of complex mathematical calculations. The document indicates that to obtain the coefficient, it is necessary to calculate the 18 Feynman diagrams that are presented in Fig. 2. These diagrams are grouped into five groups (I, II, III, IV and V).

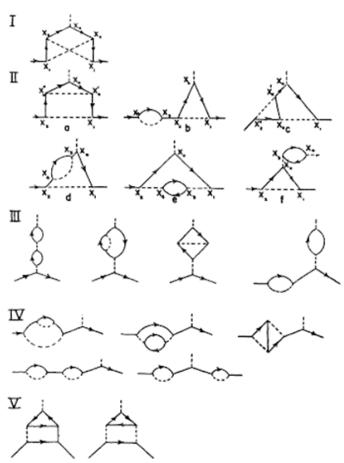

Fig. 2: Feynman diagrams for the fourth-order corrections

Subsequently, it is argued that groups III, IV and V are not necessary, so only seven Feynman diagrams (I, IIa, IIb, IIc, IId, IIe and IIf) need to be calculated. A large number of calculations are then performed to demonstrate that diagrams IIb and IIf are also not necessary. This occurs on page 11 of the article. The values for diagrams IId and IIe, which appear the simplest, are quickly calculated, while indicating that ***"The expressions for I, IIa and IIc become successively more complicated and very much more tedious to evaluate and cannot be given in detail here."*** Consequently, the calculations for three of the five diagrams were never published.

Finally, the results summary shows the results of each of the five Feynman diagrams. The sum of the five diagrams provides a result of -2.973.

Petermann's paper consists of only two pages, and the calculations are not shown; only the results are presented. Petermann indicated that he only found relevant errors in the calculations for diagram IIc. The calculations for diagrams IIa, IId and IIe were correct, while diagram I had a very small error. The following table demonstrates the contribution of each diagram to the final coefficient:

	I	IIa	IIc	IId	IIe	Total
K&K	-0.499	0.78	-3.178	-0.09	0.02	-2.973
Pet.	-0.467	0.78	-0.564	-0.09	0.02	-0.328
Diff.	6%	0%	82%	0%	0%	89%

Table 1: Feynman diagrams values.

The error in diagram I is small (6%), but the error in diagram IIc is huge (82%). Petermann focused on diagram IIc which is the dominant diagram. Suspiciously, this is the diagram that included the main errors. The results of the other four diagrams are not relevant and practically cancel each other out.

In the article summary, Petermann showed the result of the original calculation of diagram IIc, the result of the corrected calculation and the difference between the calculations:

[Karplus & Kroll]

$$II_c = -\frac{323}{24} + \frac{31}{9}\pi^2 - \frac{49}{6}\pi^2 \, ln(2) + \frac{107}{4}\zeta(3) \tag{9}$$

[Petermann]

$$II_c = -\frac{67}{24} + \frac{1}{18}\pi^2 + \frac{1}{3}\pi^2 \, ln(2) - \frac{1}{2}\zeta(3) \tag{10}$$

[Difference]

$$II_c = \frac{32}{3} - \frac{61}{18}\pi^2 + \frac{17}{2}\pi^2 \, ln(2) - \frac{109}{4}\zeta(3) \tag{11}$$

If we analyze the calculations for diagram IIc (Table 2), we observe that the four components of IIc have abnormally high values (-13, 34, -55 and 32). When added together, surprisingly result in -3.18, a figure that is an order of magnitude lower. In contrast, Petermann's corrections were enormous, the size of one or two orders of magnitude for each component of diagram IIc. It is difficult to believe that Kroll and Karplus made so many large mistakes. These circumstances cast doubt on Kroll's assertion that the discrepancies were due to ***"simple arithmetic errors."***

	Const.	π^2	$\pi^2\ ln(2)$	$\zeta(3)$	Total
K&K	-13,46	34,00	-55,87	32,15	-3,18
Pet.	-2,79	0,55	2,28	-0,60	-0,56
Diff.	10,67	-33,45	58,15	-32,75	2,61

Table 2: Components of IIc Feynman diagram.

We analyzed another paper by Petermann that was published a few months earlier in the journal of Nuclear Physics [20]. In this paper, Petermann indicated that ***"some of the big contributions have been evaluated analytically, the others estimated by analytic upper and lower bounds. The numerical value for this term has been found to satisfy IIc = -1.02 +/- 0.53".*** The types of calculations and the obtained results clearly indicate that the problem was not due to simple arithmetic errors; the issue was related to discrepancies about how and where renormalization techniques should be applied to eliminate infinities.

The Sommerfeld paper [21] is a press release that confirms Petermann's results without providing any further information. The press release merely indicates that: ***"The present calculation has been checked several times and all of the auxiliary integrals have been done in at least two different ways,"*** without offering any substantive proof.

Since the original calculations for diagrams I and IIc were not published, Petermann and Sommerfeld must have had access to private data to find the errors. Since they also did not publish their corrective calculations, we cannot know what the errors were or if the corrections were accurate.

The version of the facts conveyed by Feynman does not correlate with reality and completely ignores the seriousness of what occurred: ***"It took two 'independent' groups of physicists two years to calculate this next term, and then another year to find out there was a mistake — experimenters had measured the value to be slightly different, and it looked for a while that the theory didn't agree with experiment for the first time, but no: it was a mistake in arithmetic. How could two groups make the same mistake? It turns out that near the end of the calculation the two groups compared notes and ironed out the differences between their calculations, so they were not really independent". [1]***

6 The Nobel Prize (1965)

6.1 Direct g-factor measurement

In 1953, a research team from the University of Michigan [22] proposed a new experiment to calculate the magnetic moment of the electron directly from the precession of the free electron spin. This new technique provided more precise experimental values than the previous techniques that were based on atomic levels. The Michigan experiment only presented a proof of concept, demonstrating that the idea was viable while obtaining irrelevant results.

A few years later, the Michigan team obtained the necessary resources to conduct the real experiment. In 1961, Schupp, Pidd and Crane [23] published their results with an experimental value of 1.0011609 (24). The experiment was revolutionary because of the measured precision, however, the authors were cautious with their results, presenting large margins of error. The explanation for this strange decision is found in the paper: ***"In deciding upon a single value for a to give as the result of the experiment, our judgement is that we should recognize the trend of the points (...). The value a=0.0011609, obtained in this way, may be compared with a simple weighted average of the data of Table IV, which is 0.0011627. We adopt the value 0.0011609 but assign a standard error which is great enough to include the weighted average of Table IV, namely ±0.0000020. Finally, we combine with this the estimated systematic standard errors (...). This results in a final value of* 0.0011609 ± 0.0000024"**.

B (gauss)	Energy (kev)	ω_D/ω_0	Standard error
82.074	50	1.16448×10^{-3}	1.24×10^{-6}
88.674	58	1.16255×10^{-3}	1.23×10^{-6}
91.698	60	1.16308×10^{-3}	0.74×10^{-6}
98.764	70	1.16346×10^{-3}	0.62×10^{-6}
105.830	81	1.16224×10^{-3}	1.21×10^{-6}
115.892	98	1.16168×10^{-3}	1.20×10^{-6}
117.319	100	1.16232×10^{-3}	0.45×10^{-6}

Fig. 3: Table IV, the g-factor anomaly calculated for the various electron energies

According to this explanation, ***"the estimated systematic standard error"*** was 0.0000004. If this error had been published, the result would have been 0.0011609 ± 0.0000004, leaving Petermann's theoretical value outside the margin of error and creating a new crisis in the development of QED. The authors proposed another possible approach: they averaged the measurements in Table IV, generating a result of 0.0011627 ± 0.0000024. But this alternative result also left out of the margin of error the Petermann's theoretical value.

Finally, the authors published a meaningless result. Although they published what they considered to be the correct result (0.0011609), they added a margin of error of +0.0000024 to include the average of the actual results. They also added a negative symmetrical margin of error of -0.0000024, without any logical basis; this was the only way to keep Petermann's theoretical value within the margin of error.

6.2 The experimenter's bias

At that time the situation was dramatic again. Predictably, subsequent experiments would discredit the g-factor theoretical value. And after the Kroll and Karplus scandal, the theoretical calculations could not be modified again to adapt them to the experimental data without completely distorting the QED.

And the moment come in 1963, Wilkinson and Crane [24] published a third improved version of the experiment. In the report of the results of this third experiment, all the previous cautionary language disappeared. The accuracy of this result was presented as 100 times higher than that of the previous experiment, and the tone of the paper was blunt: ***"mainly for experimental reasons, we here conclude the 10-year effort of the laboratory on the g factor of the free negative electron."***

Just when QED seemed doomed to disaster, the miracle happened again. This time, the new experimental value was $1,001,159,622 \pm 0.000,000,027$, nearly the same as Petermann's theoretical value (1,001,159,615).

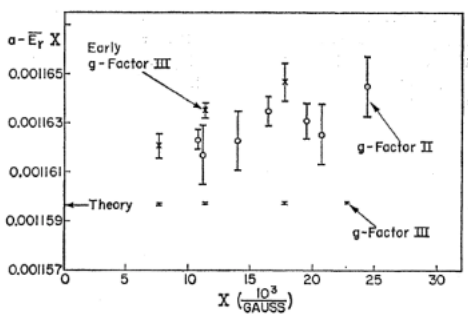

Fig. 4: Experimental values

This experimental result is incredibly suspicious. It was obtained after a simple improvement of the previous experiment, and it was conducted at the same University, with the same teams, only two years later. The margin of error could not have improved so much from one experiment to another, and it is extremely strange that all the measurements from the previous experiment were outside the range of the new experimental value. Even stranger, the theoretical value fit perfectly within the experimental value. Most disturbing, this value is not correct, as was demonstrated in later experiments. It is evident that the measuring devices were calibrated to obtain the expected theoretical value. This type of error is known as experimenter bias.

In this case, the error does not seem to be involuntary; instead, it appears to be a conscious manipulation of the experimental data with the sole objective of, once again, saving QED. The trap worked perfectly. After this experiment, all doubts about QED were cleared, and, in 1965, Feynman, Schwinger and Tomonaga were awarded the Nobel Prize in physics.

This experimental manipulation is the most serious aspect of this story. We suspect that some corrections to systemic errors that were fraudulently added to this experiment, are still maintained in current experiments. As an indirect consequence of this manipulation, no alternative theory to QED can offer better results, since the theoretical results must be compared with manipulated experimental data.

7 To Infinity and Beyond

7.1 The Penning trap

The story did not end here, as the cycle was repeated a fifth time. Between 1977 and 1987, Van Dyck and Dehmelt of the University of Washington published experimental results using a new technique known as free electron spin resonance. These measurements were based on a device called a Penning trap, which allowed measurements to be obtained from individual electrons. These experiments improved the previous results by three orders of magnitude, and, again, the new result excluded Petermann's theoretical value (0.001,159,615).

- [1977] : $0.001,159,652,410(200)$ [25]
- [1981] : $0.001,159,652,222(50)$ [26]
- [1984] : $0.001,159,652,193(4)$ [27]
- [1987] : $0.001,159,652,188,4(43)$ [28]

To resolve this discrepancy, the theoretical physicists decided to calculate another coefficient of the Dyson series (sixth-order correction). This new coefficient required solving 72 Feynman diagrams. In 1965, Drell and Pagels [29] had published a first approximate theoretical value of 0.15 for this coefficient, the precision of which was now insufficient. Increasingly precise numerical calculations were presented for 30 years until Laporta and Remiddi [45] published their final analytical calculation in 1996. Their result was 1.181241 (10 times higher than the initial calculation), leaving the theoretical value of the g-factor as 0.001,159,652,201.2(271), within the range of experimental error. Once again, QED had been miraculously saved.

What happened to Dyson's controversial statement about the divergence of his series? Dyson's argument is correct, as his series does diverge. How is it possible that 72 Feynman diagrams resulted in a value close to 1? For some unknown reason, the results of the Feynman diagrams tend to cancel each other out, leading to a total result on the order of 1, regardless of the number of diagrams that are calculated. Just another Christmas miracle.

7.2 Harvard experiment

After 20 years of tranquility, history repeated for a sixth time. A team from Harvard University led by Gabrielse improved the experimental results of Van Dyck and Dehmelt by two orders of magnitude. The Harvard University data were not compatible with previous experimental data provided by the

University of Washington. These new data also excluded the theoretical value of the g-factor.

- [2006] : 1.001, 159, 652, 180, 85(76) [30]
- [2008] : 1.001, 159, 652, 180, 73(28) [31]

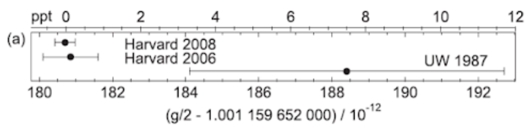

Fig. 5: Harvard vs Washington errors

To resolve this new discrepancy, the theoretical physicists decided to calculate two new coefficients of the Dyson series (eighth-order and tenth-order corrections). These new coefficients involved the calculation of 891 and 12,672 Feynman diagrams respectively.

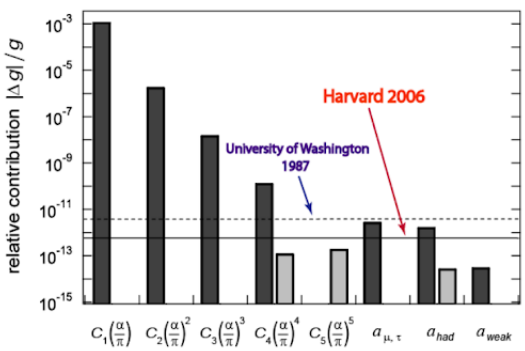

Fig. 6: electron g-factor errors

Unfortunately these two new coefficients were not enough to fit the experimental value. As in the case of Bethe's fudge factors, three new factors were added to adjust the result.

$$a = a(QED) + a(\mu, \tau) + a(weak) + a(hadron) \quad (12)$$

The first coefficient was derived from the interaction of the electron with leptons, the second coefficient was derived from the electroweak interaction and the third coefficient was derived from the electron's interaction with hadrons.

Despite these difficulties, the QED theorists returned to work the miracle. A team led by the Japanese physicist Kinoshita managed to perform the necessary calculations, thanks to the use of supercomputers, and obtained a g-factor of 1,001,159,652,182,032(720), within the margins of experimental error [48] .

7.3 Supercomputer calculations

In 2017, after 36 years of calculation refinements, Laporta [46] published a final calculation of the fourth coefficient of the Dyson series, which required solving 841 Feynman diagrams. As Kinoshita [48] indicates, ***"It took 36 years since the preliminary value A8 = -0.8 was reported. For the purpose of this article it is sufficient to list the first ten digits of Laporta's result: A8 = -1.912 245 764."***

This implies a final result more than double the initial estimation. Furthermore, Laporta's published value had an accuracy of 1100 digits (sic), about 100 times the necessary precision. Presenting a theoretical value with this unnecessary level of precision leads us to suspect the results even more ("Excusatio non petita, accusatio manifesta.").

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-1.91224576492644557415264716743983005406087339065872534517132984800 60
384439806517061427608927000036315837558415331473270056378514912854539 1
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Fig. 7: First 1100 digits of A8

In 2018, the Kinoshita team published [48] a new theoretical value for the fifth coefficient of the series. As usual, this most recent calculation included a review of the previous calculation that was published in 2014. The paper indicated, ***"During this work, we found that one of the integrals, called X024, was given a wrong value in the previous calculation due to an incorrect assignment of integration variables. The correction of this error causes a shift of -1.25 to the Set V contribution, and hence to the tenth-order universal term."*** The value of the fifth coefficient that was calculated in 2014 was 7,795; while the value that was calculated in 2018 was 6,675, which means admitting an error of 15%.

Given the serious errors committed by Karplus and Kroll in 1950 in the calculation of a single Feynman diagram (IIc), it seems ridiculous to propose that 12,672 Feynman diagrams could be calculated without errors

The lack of critical review of the theoretical results that have been published is evident. The theoretical results are only scrutinized when they do not match the experimental values. That errors continuously appear in theoretical calculations is no longer a surprise. Recall that the calculation of each Feynman diagram implies the resolution of multiple factors, and that each of these factors diverges to infinity. Therefore, renormalization techniques must be arbitrarily applied to eliminate these infinities and to obtain finite results. Moreover, these calculations are extremely complex and are not published in their entirety, so it is impossible to independently validate them. None of this matters, so long as you can affirm without blushing that ***"QED is the most accurate theory man has produced"***.

7.4 The muon anomaly

QED was also used to calculate the anomalous magnetic moment of the muon. To date, the most accurate experimental value was obtained in 2004 in the E821 experiment that was conducted at the Brookhaven National Laboratory (BNL). The experimental result was 0.001,165,920.9(6) [47]. Unfortunately, the theoretical value did not match the experimental value and had an error that was greater than 3 sigmas. Despite enormous effort in recent decades, this discrepancy could not be eliminated. Currently, the theoretical value of the muon g-factor is 0.001,165,918.04(51).

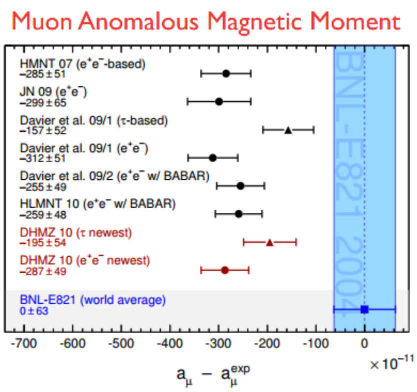

Fig. 8: Muon anomaly

Theoretical physicists are concerned about this discrepancy, as it is perhaps the most palpable evidence that the standard model is incomplete.

In 2011, the E989 experiment was devised to improve the accuracy of the E821 experiment. This extremely complex experiment will be performed at the Fermilab's Tevatron. Before the experiment could be conducted, a gigantic magnet (15 meters in diameter and 600 tons in weight) had to be moved 1300 km, from BNL to Fermilab. This delicate operation was successfully performed in June 2013. The magnet transfer lasted 35 days and cost 3 million dollars.

Fig. 9: Transportation of the 600 ton magnet to Fermilab

In addition, the Fermilab particle accelerator had to be enlarged. The related investment plan, the PIP-II Reference Design Report, had an estimated cost of 600 million dollars and was approved in July 2018. It is expected that the E989 experiment can be concluded in 2020 and that a new experimental value will be presented for the muon g-factor.

Given the scientific precedents, we are convinced that, one way or the other, the discrepancy will be resolved, and the myth of QED's precision will be preserved.

8 Summary

According Feynman ***"We have found nothing wrong with the theory of quantum electrodynamics. It is, therefore, I would say, the jewel of physics — our proudest possession." [1]*** But this statement is nothing more than a false myth.

The reality of the QED is better reflected by Dyson's description in a letter to Gabrielse in 2006: ***"As one of the inventors of QED, I remember that we thought of QED in 1949 as a temporary and jerry-built structure, with mathematical inconsistencies and renormalized infinities swept under the rug. We did not expect it to last more than 10 years before some more solidly built theory would replace it. Now, 57 years have gone by and that ramshackle structure still stands." [44]***

QED should be the quantized version of Maxwell's laws, but it is not that at all. QED is a simple addition to quantum mechanics that attempts to justify two experimental discrepancies in the Dirac equation: the Lamb shift and the anomalous magnetic moment of the electron.

The reality is that QED is a bunch of fudge factors, numerology, ignored infinities, hocus-pocus, manipulated calculations, illegitimate mathematics, incomprehensible theories, hidden data, biased experiments, miscalculations, suspicious coincidences, lies, arbitrary substitutions of infinite values and budgets of 600 million dollars to continue the game.

Maybe it is time to consider alternative proposals. Winter is coming.

1 February 2020

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