ON SOME PROPERTIES OF WEAKLY LC-CONTINUOUS FUNCTIONS

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Abstract

M. Ganster and I.L. Reilly [2] introduced a new decomposition of continuity called LC-continuity. In this paper, we introduce and investigate a generalization LC-continuity called weakly LC-continuity.

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1 Introduction and Preliminaries

M. Ganster and I.L. Reilly in [2] introduced three types of continuity, that is, LC-irresoluteness, LC-continuity and sub-LC- continuity based on a notion, namely locally closed sets, implicitly introduced in Kuratowski and Sierpinski's work [4]. They have further investigated LC-continuity in [3]. In this paper, we introduce and investigate the class of LC-continuous functions.
In what follows (X,τ) and (Y,σ) (or X and Y) denote topological spaces. Let A be a subset of X. We denote the interior, the closure and the complement of a set A by $Int(A)$, $Cl(A)$ and $X\backslash A$, respectively.

Definition 1 *A subset A of a topological space X is said to be locally closed [1] in X if it is the intersection of an open subset of X and a closed subset of X. The complement of a locally closed set is said to be locally open.*

The family of all locally closed sets of X containing a point $x \in X$ is denoted by $LC(X,x)$. The family of all locally closed (resp. locally open) sets of X is denoted by $LC(X)$ (resp. $LO(X)$). Similarly, we denoted by $O(X,x)$ (resp. $C(X,x)$) the family of all open (resp. closed) sets of X containing a point $x \in X$.

Remark 1.1 *The following properties are well-known.*
(i) A subset A of X is locally closed if and only if its complement $X \backslash A$ is locally open, it is the union of an open set and a closed set.
(ii) Every open (resp. closed) subset of X is locally closed.
(iii) The complement of a locally closed set need not be locally closed.

Definition 2 *[2] A function $f : (X,\tau) \to (Y,\sigma)$ is said to be*
(1) LC-continuous if $f^{-1}(V) \in LC(X,\tau)$ for each $V \in \sigma$.
(2) LC-irresolute if $f^{-1}(F) \in LC(X,\tau)$ for each $F \in LC(Y,\sigma)$.

2 Some fundamental properties

We introduce the following notions.

Definition 3 *A point $x \in X$ is called a LC-cluster point of a subset A of X if $U \cap A \neq \emptyset$ for every $U \in LC(X,x)$. The set of all LC-cluster points of A is called the LC-closure of A and is denoted by $[A]_{LC}$. A subset A is said to be LC-closed if $A = [A]_{LC}$.*
The complement of a LC-closed set A is said to be LC-open.

Remark 2.1 *For a subset A of a space X, $[A]_{LC} = \bigcap\{V : A \subset V, V \in LO(X)\}$.*

Observe that as an example for Definition 3, take $X = \{a,b,c\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{a,b\}\}$. Then $\{a,c\}$ is LC-closed but not locally closed.

Definition 4 *A function $f : X \to Y$ is said to be weakly LC-continuous at $x \in X$ if for each open set V of Y containing $f(x)$, there exists a locally closed set U in X containing x such that $f(U) \subset V$. If f is weakly LC-continuous at every point of X, then it is called weakly LC-continuous on X.*

It should be noticed that:
continuity $\Rightarrow$ LC-irresolute $\Rightarrow$ LC-continuity $\Rightarrow$ weak LC-continuity by ([2], p. 421) and Example 3 of [2]. Example 3 is an example of a weakly LC-continuous function which is not LC-continuous.

Theorem 2.2 *For a function $f : X \to Y$, the following are equivalent:*
(1) f is weakly LC-continuous ;
(2) $f([A]_{LC}) \subset Cl(f(A))$ for every subset A of X;
(3) $[f^{-1}(B)]_{LC} \subset f^{-1}(Cl(B))$ for every subset B of Y;
(4) $f^{-1}(F)$ is LC-closed for every closed set F of Y;
(5) $f^{-1}(V)$ is LC-open for every open set V of Y.

Proof. (1) $\Rightarrow$ (2): Let $y \in f([A]_{LC})$ and let V be any open set of Y containing y. Then, there exists a point $x \in [A]_{LC}$ such that $f(x) = y \in V$. Since f is weakly LC-continuous, there exists $U \in LC(X, x)$ such that $f(U) \subset V$. Since $x \in [A]_{LC}$, $U \cap A \neq \emptyset$ holds and hence $f(A) \cap V \neq \emptyset$. Therefore we have $y = f(x) \in Cl(f(A))$.
(2) $\Rightarrow$ (3): Let B be an arbitrary set containing of Y and let $A = f^{-1}(B)$. Then by (2), we have $f([A]_{LC}) \subset Cl(f(A)) \subset Cl(B)$. This implies that $[A]_{LC} \subset f^{-1}(Cl(B))$. That is $[f^{-1}(B)]_{LC} \subset f^{-1}(Cl(B))$.
(3) $\Rightarrow$ (4): Let F be any closed set of Y. By (3), we have $[f^{-1}(F)]_{LC} \subset f^{-1}(Cl(F)) = f^{-1}(F)$. By Remark 2.1, $[f^{-1}(F)]_{LC} \supset f^{-1}(F)$ and hence $[f^{-1}(F)]_{LC} = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is LC-closed.
(4) $\Rightarrow$ (5): Let V be any open set of Y. We have $f^{-1}(X \backslash V) = X \backslash f^{-1}(V)$ and by (4), $f^{-1}(V)$ is LC-open.
(5) $\Rightarrow$ (1): Let $x \in X$ and $V \in O(Y, f(x))$. By (5), $x \in f^{-1}(V)$ and $f^{-1}(V)$ LC-open. Therefore, $X \backslash f^{-1}(V)$ is LC-closed and $x \notin [X \backslash f^{-1}(V)]$. Hence there exists $U \in LC(X, x)$ such that $U \cap (X \backslash f^{-1}(V)) = \emptyset$; hence $U \subset f^{-1}(V)$. Therefore, we obtain $f(U) \subset V$. This shows that f is weakly LC-continuous.

Definition 5 *Let (X, τ) be a topological space. Since $LC(X)$ is closed under a finite intersection, $LC(X)$ is a base of some topology for X. We denote it by τ_{LC}.*

Theorem 2.3 *A function $f : (X, \tau) \to (Y, \sigma)$ is weakly LC-continuous if and only if $f : (X, \tau_{LC}) \to (Y, \sigma)$ is continuous.*

Proof. Necessity. Let $V \in \sigma$ and $x \in f^{-1}(V)$. Then there exists $U_x \in LC(X, x)$ such that $f(U_x) \subset V$. Hence we obtain $\bigcup\{U_x : x \in f^{-1}(V)\} = f^{-1}(V) \in \tau_{LC}$. Therefore, $f : (X, \tau_{LC}) \to (Y, \sigma)$ is continuous.
Sufficiency. Let $x \in X$ and $V \in O(Y, f(x))$. Then $x \in f^{-1}(V) \in \tau_{LC}$ and there exists $U \in LC(X, x)$ such that $x \in U \subset f^{-1}(V)$; hence $f(U) \subset V$. This shows that f is weakly LC-continuous.

Definition 6 *Let A be a subset of X. A mapping $r : X \to A$ is called a weakly LC-continuous retraction if r is weakly LC-continuous and the restriction $r \mid_A$ is the identity mapping on A.*

Theorem 2.4 *Let A be a subset of X and $r : X \to A$ be a weakly LC-continuous retraction. If X is Hausdorff, then A is a LC-closed set of X.*

Proof. Suppose that A is not LC-closed. Then, there exists a point x in X such that $x \in [A]_{LC}$ but $x \notin A$. It follows that $r(x) \neq x$ because r is weakly LC-continuous retraction. Since X is Hausdorff there exists disjoint open sets U and V in X such that $x \in U$ and $r(x) \in V$. Now let W be an arbitrary locally closed set containing x. Then $W \cap U$ is a locally closed set containing x. Since $x \in [A]_{LC}$, we have $(W \cap U) \cap A \neq \phi$. Therefore, there exists a point y in $W \cap U \cap A$. Since $y \in A$, we have $r(y) = y \in U$ and hence $r(y) \notin V$. This implies that $r(W) \not\subset V$ because $y \in W$. This is contrary to the weakly LC-continuity of r. Consequently, A is a LC-closed set of X.

Definition 7 *the LC-frontier of a subset A of a space X denoted by LC-$fr(A)$, is given by LC-$fr(A) = [A]_{LC} \cap [X \backslash A]_{LC}$.*

Theorem 2.5 *The set of all points $x \in X$ at which $f : (X, \tau) \to (Y, \sigma)$ is not weakly LC-continuous is identical with the union of the LC-frontiers of the inverse images of open subsets of Y containing $f(x)$.*

Proof. Necessity. Suppose that f is not weakly LC-continuous at a point x of X. Then, there exists an open set $V \subset Y$ containing $f(x)$ such that $f(U)$ is not a subset of V for every $U \in LC(X,x)$. Hence we have $U \cap (X \setminus f^{-1}(V)) \neq \emptyset$ for every $U \in LC(X,x)$. It follows that $x \in [X \setminus f^{-1}(V)]_{LC}$. We also have $x \in f^{-1}(V) \subset [f^{-1}(V)]_{LC}$. This means that $x \in LC\text{-}fr(f^{-1}(V))$.

Sufficiency. Suppose that $x \in LC\text{-}fr(f^{-1}(V))$ for some $V \in O(Y, f(x))$ Now, we assume that f is weakly LC-continuous at $x \in X$. Then there exists $U \in LC(X,x)$ such that $f(U) \subset V$. Therefore, we have $x \in U \subset f^{-1}(V)$. Thus $x \notin [X \setminus f^{-1}(V)]_{LC}$. This is a contradiction. This means that f is not weakly LC-continuous at x.

Definition 8 *A filter base B is said to be LC-convergent to a point $x \in X$ if for any locally closed set A containing x, there exists $B_1 \in B$ such that $B_1 \subset A$.*

Theorem 2.6 *A function $f : X \to Y$ is weakly LC-continuous if and only if for each point $x \in X$ and each filter base B on X LC-converging to x, the filter base $f(B)$ is convergent to $f(x)$.*

Proof. Suppose that f is weakly LC-continuous. Let $x \in X$ and B be any filter base LC-converging to x. Since f is weakly LC-continuous, for each open set $V \subset Y$ containing $f(x)$, there exists a locally closed set U in X containing x such that $f(U) \subset V$. Since B is LC-converging to x, then there exists $B_1 \in B$ such that $B_1 \in U$. This implies that $f(B_1) \subset V$. It follows that $f(B_1)$ is convergent to $f(x)$.

Conversely, let $x \in X$ and V be any open set containing $f(x)$. Suppose that $B = LC(X,x)$. Then it follows that B is a filter base LC-converging to x. Hence there exists U in B such that $f(U) \subset V$, as we wished to prove.

Definition 9 *A space X is said to be LC-separate if for every pair of distinct points x and y in X, there exist locally closed sets B_1 and B_2 containing x and y, respectively, such that $B_1 \cap B_2 = \emptyset$.*

Let $X = \{a, b\}$ with $\tau = \{X, \emptyset, \{a\}\}$. (X, τ) is LC-separate but not separate.

Theorem 2.7 *If $f : X \to Y$ is a weakly LC-continuous injection and Y is Hausdorff, then X is LC-separate.*

Proof. Let x and y be distinct points of X. Then $f(x) \neq f(y)$. Since Y is Hausdorff, there exist disjoint open sets V and W in Y containing $f(x)$ and $f(y)$, respectively. Since f is weakly LC-continuous, there exist locally closed sets U_1 and U_2 containing x and y, respectively, such that $f(U_1) \subset V$ and $f(U_2) \subset W$. It follows that $U_1 \cap U_2 = \emptyset$. This shows clearly that X is LC-separate.

Theorem 2.8 *If $f, g : X \to Y$ are weakly LC-continuous functions and Y is Hausdorff, then $A = \{x \in X : f(x) = g(x)\}$ is LC-closed in X.*

Proof. Suppose that $x \notin A$. Then $f(x) \neq g(x)$. Since Y is Hausdorff, there exist $V \in O(Y, f(x))$ and $W \in O(Y, g(x))$ such that $V \cap W = \emptyset$. Since f and g are weakly LC-continuous, there exist $U \in LC(X, x)$ and $G \in LC(X, x)$ such that $f(U) \subset V$ and $f(G) \subset W$. Set $D = U \cap G$, so $D \in LC(X, x)$. Hence we have $f(D) \cap g(D) \subset V \cap W = \emptyset$. This shows clearly that $x \notin [A]_{LC}$. It follows that $[A]_{LC} \subset A$, that is A is LC-closed in X.

Definition 10 *For a function $f : X \to Y$, the graph $G(f) = \{(x, f(x)) : x \in X\}$ is said to be LC-closed if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in LC(X, x)$ and $V \in O(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 2.9 *A function $f : X \to Y$ has a LC-closed graph $G(f)$ if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in LC(X, x)$ and $V \in O(Y, y)$ such that $f(U) \cap V = \emptyset$.*

Proof. It is an immediate consequence of Definition 10 and the fact that for any subsets $U \subset X$ and $V \subset Y$, $(U \times V) \cap G(f) = \emptyset$ if and only if $f(U) \cap V = \emptyset$.

Theorem 2.10 *If $f : X \to Y$ is weakly LC-continuous and Y is Hausdorff, then $G(f)$ is LC-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) \backslash G(f)$. It follows that $f(x) \neq y$. Since Y is Hausdorff, there exist disjoint open sets V and W in Y containing $f(x)$ and y, respectively. Since f is weakly LC-continuous, there exists $U \in LC(X, x)$ such that $f(U) \subset V$. Therefore $f(U) \cap W = \emptyset$ and $G(f)$ is LC-closed in $X \times Y$.

Definition 11 *Let A be a subset of X, then we say that A is LC-compact relative to X if every cover of A by locally closed sets of X has a finite subcover. A space X is said to be LC-compact if X is LC-compact in X.*

Theorem 2.11 *If $f : X \to Y$ is a weakly LC-continuous function and A is LC-compact relative to X, then $f(A)$ is compact relative to Y.*

Proof. Suppose that $f : X \to Y$ is weakly LC-continuous and let A be LC-compact relative to X. Let $\{V_\alpha : \alpha \in \nabla\}$ be an open cover of $f(A)$. For each point $x \in A$, there exists $\alpha(x) \in \nabla$ such that $f(x) \in V_{\alpha(x)}$. Since f is weakly LC-continuous, there exists $U_x \in LC(X, x)$ such that $f(U_x) \subset V_{\alpha(x)}$. The family $\{U_x : x \in A\}$ is a cover of A by locally closed sets of X and hence there exists a finite set A_0 of A such that $A \subset \cup_{x \in A_0} U_x$. Therefore, we obtain $f(A) \subset \cup_{x \in A_0} V_{\alpha(x)}$. This shows that $f(A)$ is compact in Y.

Definition 12 *A space X is said to be LC-connected if X can not be expressed as the union of two nonempty LC-open sets.*

Observe that the Sierpinski space is connected but it is not LC-connected.

Theorem 2.12 *If $f : X \to Y$ is a weakly LC-continuous function and X is LC-connected, then Y is connected.*

Proof. Suppose that Y is not connected. Then there exist nonempty open sets V and W such that $V \cap W = \emptyset$ and $V \cup W = Y$. It follows that $f^{-1}(V) \cap f^{-1}(W) = \emptyset$ and $f^{-1}(V) \cup f^{-1}(W) = X$. By weakly LC-continuity of f, it follows from Theorem 2.1 that $f^{-1}(V)$ and $f^{-1}(W)$ are nonempty LC-open sets in X. This shows that X is not LC-connected. But this is a contradiction. Hence Y is connected.

Definition 13 *The intersection of all locally closed sets containing a set A is called the LC^*-closure of A and is denoted by $[A]^*_{LC}$. This is, for any $A \subset X$, $[A]^*_{LC} = \cap\{F \in LC(X) : A \subset F\}$.*

Remark 2.13 *If B is a locally closed set in a space X, then $[B]^*_{LC} = B$. The converse is false. If X denote the real line with the cofinite topology and if $B = \{\frac{1}{n} : n \in N\}$. Then $[B]^*_{LC} = B$. But B is not locally closed. However, the converse is true if the space X is an Alexandorff space. A space is said to be Alexandorff if the intersection of any open sets of X is open in X.*

Definition 14 *Let p be a point of X and N be a subset of X. N is called a LC-neighborhood of p in X if there exists a locally open set O of X such that $p \in O \subset N$.*

Lemma 2.14 *Let A be a subset of X. Then, $p \in [A]^*_{LC}$ if and only if for any LC-neighborhood N_p of p in X, $A \cap N_p \neq \phi$.*

Proof. Necessity. Suppose that $p \in [A]^*_{LC}$. If there exists a LC- neighborhood N of the point p in X such that $N \cap A = \phi$, then by definition, there exists a locally open set O_p such that $p \in O_p \subset N$. Therefore, we have $O_p \cap A = \phi$, so that $A \subset X \backslash O_p$. Since $X \backslash O_p$ is locally closed, then $[A]^*_{LC} \subset X \backslash O_p$. As $p \notin [A]^*_{LC}$ which is contrary to the hypothesis.
Sufficiency. If $p \notin [A]^*_{LC}$, then by definition of $[A]^*_{LC}$, there exists a locally closed set F of X such that $A \subset F$ and $p \notin F$. Therefore, we have $p \in X \backslash F$ such that $X \backslash F$ is a locally open set. Hence $X \backslash F$ is a LC-neighborhood of p in X, but $(X \backslash F) \cap A = \phi$. This is contrary to the hypothesis.

Definition 15 *A function $f : X \to Y$ is said to be LC^*-continuous if the inverse image of every closed in Y is locally closed in X.*

Theorem 2.15 *Let $f : X \to Y$ be a function.*
(i) The following statements are equivalent:
(a) f is LC^*-continuous.
(b) The inverse image of each open set of Y is locally open in X.

(ii) If f is LC^*-continuous, then $f([A]^*_{LC}) \subset Cl(f(A))$ for every $A \subset X$.

(iii) The following statements are equivalent:

(a) For each point $x \in X$ and each open set V of Y containing $f(x)$, there exists a locally open set U in X containing x such that $f(U) \subset V$.

(b) $f([A]^*_{LC}) \subset Cl(f(A))$ for every $A \subset X$.

(iv) For the following statements $(a) \Rightarrow (b) \Rightarrow (c)$, and they are equivalent if X is Alexandorff.

(a) f is LC^*-continuous.

(b) $f([A]^*_{LC}) \subset Cl(f(A))$ for every $A \subset X$.

(c) $[f^{-1}(B)]^*_{LC}) \subset f^{-1}(Cl(B))$ for every $B \subset Y$.

Proof. (i) The equivalence is proved by definitions.

(ii) Since $A \subset f^{-1}(Cl(f(A)))$, it is obtained that $f([A]^*_{LC}) \subset Cl(f(A))$ by using assumptions.

(iii) $(a) \Rightarrow (b)$: Let $y \in f([A]^*_{LC})$ and let V any open neighborhood of y. Then, there exists a point $x \in X$ and a locally open set U such that $f(x) = y$, $x \in U$, $x \in [A]^*_{LC}$ and $f(U) \subset V$. Since $x \in [A]^*_{LC}$, $U \cap A \neq \emptyset$ holds and hence $f(A) \cap V \neq \emptyset$. Therefore we have $y = f(x) \in Cl(f(A))$.

$(b) \Rightarrow (a)$: Let $x \in X$ and V be any open set containing $f(x)$. Let $A = f^{-1}(Y \backslash V)$, then $x \notin A$. Since $f([A]^*_{LC}) \subset Cl(f(A)) \subset (Y \backslash V)$, it is shown that $[A]^*_{LC} = A$. Then, since $x \notin [A]^*_{LC}$, there exists a locally open set U containing x such that $U \cap A = \emptyset$ and hence $f(U) \subset f(X \backslash A) \subset V$.

(iv) $(a) \Rightarrow (b)$: Let A be any subset of X. Let $y \notin Cl(f(A))$. Then there exist $V \in O(Y, y)$ such that $V \cap f(A) = \emptyset$; hence $A \cap f^{-1}(V) = \emptyset$. By (i), $f^{-1}(V) \in LO(X)$ and $A \subset X \backslash f^{-1}(V) \in LC(X)$. Therefore, we have $[A]^*_{LC} \subset X \backslash f^{-1}(V)$ and hence $[A]^*_{LC} \cap f^{-1}(V) = \emptyset$. We obtain $f([A]^*_{LC}) \cap V = \emptyset$ and $y \notin f([A]^*_{LC})$. Hence $f([A]^*_{LC}) \subset Cl(f(A))$.

$(b) \Rightarrow (c)$: Let B be any subset of Y. By (b) $f([f^{-1}(B)]^*_{LC}) \subset Cl(B)$ and $[f^{-1}(B)]^*_{LC}) \subset f^{-1}(Cl(B))$.

Let X be Alexandorff and we prove that $(c) \Rightarrow (a)$. Let F be any closed set of Y. By (c), $[f^{-1}(B)]^*_{LC}) \subset f^{-1}(Cl(F)) = f^{-1}(F)$ and hence $[f^{-1}(B)]^*_{LC}) = f^{-1}(F)$. Since X

is Alexandorff, $[f^{-1}(B)]^*_{LC}) \in LC(X)$ and $f^{-1}(F)$ is locally closed. Therefore, f is LC^*-continuous.

Theorem 2.16 *If $f : X \to Y$ be a function, and let $g : X \to X \times Y$ be the graph function of f, defined by $g(x) = \{(x, f(x))\}$ for every $x \in X$. If g is LC^*-continuous, then f is LC^*-continuous.*

Proof. Let U be an open set in Y, Then $X \times U$ is an open set in $X \times Y$. Since g is LC^*-continuous, it follows of Theorem 2.13(i) that $f^{-1}(U) = g^{-1}(X \times U)$ is a locally open set in X. Thus f is LC^*-continuous.

Theorem 2.17 *Let $\{X_i : i \in I\}$ be any family of topological spaces. If $f : X \to \prod X_i$ is a LC^*-continuous function, then $Pr_i \circ f : X \to X_i$ is LC^*-continuous for each $i \in I$, where Pr_i is the projection of $\prod X_j$ onto X_i.*

Proof. We shall consider a fixed $i \in I$. Suppose U_i is an arbitrary open set in X_i . Then $Pr_i^{-1}(U_i)$ is open in $\prod X_i$. Since f is LC^*-continuous, $f^{-1}(Pr_i^{-1}(U_i)) = (Pr_i \circ f)^{-1}(U_i)$ is locally open in X. Therefore $Pr_i \circ f$ is LC^*-continuous.

Definition 16 *A space X is said to be:*
(i) L-connected if X can not be expressed as the union of two disjoint nonempty locally open sets.
(ii) L-normal if each pair of non-empty disjoint closed sets can be separated by disjoint locally open sets.

Theorem 2.18 *If $f : X \to Y$ is a LC^*-continuous surjection and X is L-connected, then Y is connected.*

Proof. Suppose that Y is not connected. Then there exist nonempty open sets V and W such that $V \cap W = \emptyset$ and $V \cup W = Y$. It follows that $f^{-1}(V) \cap f^{-1}(W) = \emptyset$ and $f^{-1}(V) \cup f^{-1}(W) = X$. By LC^*-continuity of f, it follows that $f^{-1}(V)$ and $f^{-1}(W)$ are nonempty locally open sets in X. This shows that X is not L-connected. But this is a contradiction. Hence Y is connected.

Theorem 2.19 *If $f : X \to Y$ is a LC^*-continuous, closed injection and Y is normal, then X is L-normal.*

Proof. Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint open sets V_1 and V_2 respectively. Hence $F_i \subset f^{-1}(V_i)$, $f^{-1}(V_i) \in LO(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$ and thus X is L-normal.

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