SOLVING THE PROBLEM OF THE 8 QUEENS USING HARBOUR LANGUAGE

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ABSTRACT

In this article we present the solution to the classic 8 queens problem which aims to look for states on a chessboard where 8 queens are positioned without one attack to another, that is, they should not share columns, rows or diagonals.

Keywords: eight queens, combinatorial analysis.

I. INTRODUCTION

In this work we present the solution to the problem of the 8 queens. In item II we present some mathematical data about the combinatorics involved in this problem. Next we show how we eliminate most of the research universe through an interchangeable mapping. In item IV we show a diagonal mapping that allows us to quickly check if one house is diagonal to another. Finally in item V we present the solution algorithm. Annex I presents 4 element vectors formed by the combinations of the numbers [12345678], Annex II presents the 92 possible solutions for the 8 queens problem and Annex III contains the source code of the solution in Harbour language.

II. UNDERSTANDING THE PROBLEM

The problem of the 8 queens was proposed in Schachzeitung magazine by chess player Max Bezzel in 1848[1] and since then has attracted the curiosity of many. It is actually a combinatorial analysis problem where we have 64 positions on the board, 8 of them to be occupied by the queens and an unknown a priori number of solutions that meet the requirements imposed by the problem. Mathematically we have that the number of possible positions for the 8 queens is of:

$$\frac{64*63*62*61*60*59*58*57}{8!} = \frac{1{,}784629876 \times 10^{14}}{40320}$$

number of combinations

This gives exactly 4.426.165.368 possibilities to place the 8 queens on the board. However, this number can be reduced to 8! (40320 possibilities) by employing a permutation in a vector of 8 elements.

$$[1,2,3,4,5,6,7,8]$$

Diagonal representation vector

Let us imagine that each element of the vector represents the line that each queen is in and that each position of the vector represents the column in which the queen is. This way we eliminate all row and column conflicts between queens, reducing the number of possibilities to 8! (40320 possibilities).

III. ORGANIZING DATA

The first step in finding the solution to the problem is to map these 40320 combinations which correspond to a group permutation [12345678]. A quick strategy to obtain these permutations is to do all the permutations of 4 elements **[abcd]** by replacing the elements **a**, **b**, **c** and **d** with all the elements of the group [12345678]. This way we will obtain 1680 permutations[2] like the ones below:

1234, 1235, 1236, 1237, 1238, 1243, 1245, 1246, 1247,
1248, 1253, 1254, 1256, 1257, 1258, 1263, 1264, 1265,
1267, 1268, 1273, 1274, 1275, 1276, 1278, 1283, 1284,
1285, 1286, 1287, 1324, 1325, 1326, 1327, 1328, 1342

The complete table with all 1680 permutations can be seen in **Annex I**. However we have to create the permutations with groups of 8 queens and not only with 4 queens. To solve this impasse we will make a combination of each of these combinations with themselves, one at a time, discarding those that obtain repeated numbers in their composition. The result of this work is a vector with 40320 combinations of 8 unrepeated elements. Exactly what we need to start processing our algorithm. The final vector starts as follows:

1 https://pt.wikipedia.org/wiki/Problema_das_oito_damas

2 The data here has been sorted. In the original algorithm we didn't do it to gain performance.

12345678, 12345687, 12345768, 12345786, 12345867,
12345876, 12346578, 12346587, 12346758, 12346785,
12346857, 12346875, 12347568, 12347586, 12347658,
12347685, 12347856, 12347865, 12348567, 12348576

It remains to be noted that these two processes to generate the final vector with 40320 positions are very fast. This procedure does not compromise the efficiency of the algorithm.

IV. MAPPING THE DIAGONALS

For the purpose of checking the queens' positions, it is useful to know from a given box on the board which boxes are diagonal to it. This way, through a simple algorithm, we map the 64 houses of the board and all the diagonal houses in relation to each one of these houses. Next we will list this diagonal mapping. The first number of each reference corresponds to a row of the board and the second to a column.

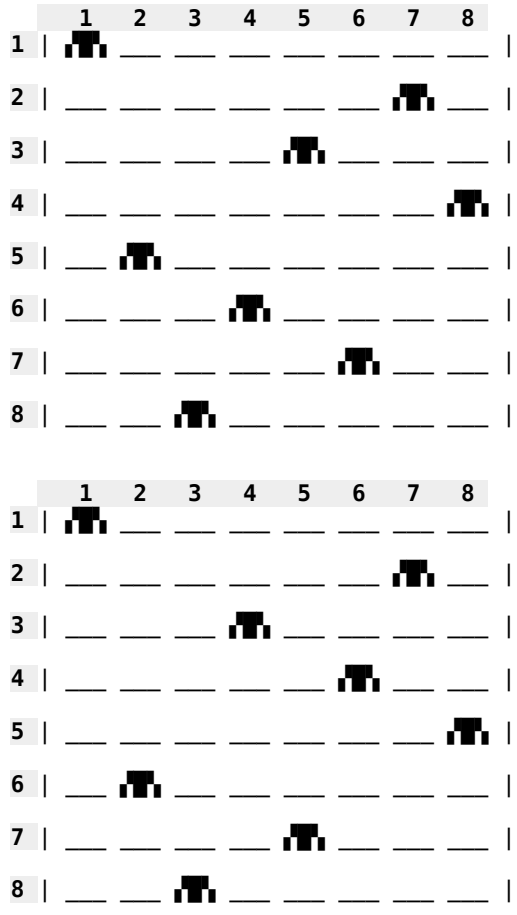
11 : 22-33-44-55-66-77-88
12 : 23-34-45-56-67-78-21
13 : 24-35-46-57-68-22-31
14 : 25-36-47-58-23-32-41
15 : 26-37-48-24-33-42-51
16 : 27-38-25-34-43-52-61
17 : 28-26-35-44-53-62-71
18 : 27-36-45-54-63-72-81
21 : 32-43-54-65-76-87-12
22 : 11-33-44-55-66-77-88-13-31
23 : 12-34-45-56-67-78-14-32-41
24 : 13-35-46-57-68-15-33-42-51
25 : 14-36-47-58-16-34-43-52-61
26 : 15-37-48-17-35-44-53-62-71
27 : 16-38-18-36-45-54-63-72-81
28 : 17-37-46-55-64-73-82
31 : 42-53-64-75-86-22-13
32 : 21-43-54-65-76-87-23-14-41
33 : 11-22-44-55-66-77-88-24-15-42-51
34 : 12-23-45-56-67-78-25-16-43-52-61
35 : 13-24-46-57-68-26-17-44-53-62-71
36 : 14-25-47-58-27-18-45-54-63-72-81
37 : 15-26-48-28-46-55-64-73-82
38 : 16-27-47-56-65-74-83
41 : 52-63-74-85-32-23-14
42 : 31-53-64-75-86-33-24-15-51
43 : 21-32-54-65-76-87-34-25-16-52-61
44 : 11-22-33-55-66-77-88-35-26-17-53-62-71
45 : 12-23-34-56-67-78-36-27-18-54-63-72-81
46 : 13-24-35-57-68-37-28-55-64-73-82
47 : 14-25-36-58-38-56-65-74-83
48 : 15-26-37-57-66-75-84
51 : 62-73-84-42-33-24-15
52 : 41-63-74-85-43-34-25-16-61
53 : 31-42-64-75-86-44-35-26-17-62-71
54 : 21-32-43-65-76-87-45-36-27-18-63-72-81
55 : 11-22-33-44-66-77-88-46-37-28-64-73-82
56 : 12-23-34-45-67-78-47-38-65-74-83
57 : 13-24-35-46-68-48-66-75-84
58 : 14-25-36-47-67-76-85
61 : 72-83-52-43-34-25-16
62 : 51-73-84-53-44-35-26-17-71
63 : 41-52-74-85-54-45-36-27-18-72-81
64 : 31-42-53-75-86-55-46-37-28-73-82
65 : 21-32-43-54-76-87-56-47-38-74-83
66 : 11-22-33-44-55-77-88-57-48-75-84
67 : 12-23-34-45-56-78-58-76-85
68 : 13-24-35-46-57-77-86
71 : 82-62-53-44-35-26-17
72 : 61-83-63-54-45-36-27-18-81
73 : 51-62-84-64-55-46-37-28-82
74 : 41-52-63-85-65-56-47-38-83
75 : 31-42-53-64-86-66-57-48-84
76 : 21-32-43-54-65-87-67-58-85
77 : 11-22-33-44-55-66-88-68-86
78 : 12-23-34-45-56-67-87
81 : 72-63-54-45-36-27-18
82 : 71-73-64-55-46-37-28
83 : 61-72-74-65-56-47-38
84 : 51-62-73-75-66-57-48
85 : 41-52-63-74-76-67-58
86 : 31-42-53-64-75-77-68
87 : 21-32-43-54-65-76-78
88 : 11-22-33-44-55-66-77

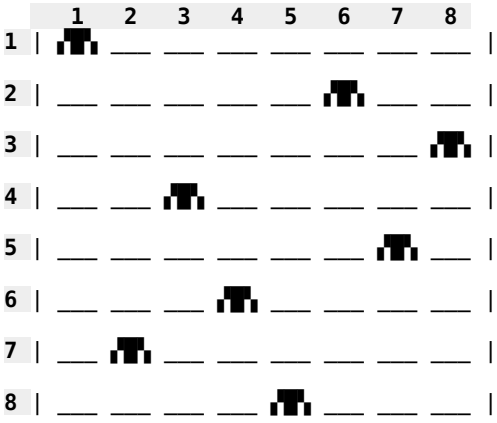
V. THE SOLUTION ALGORITHM

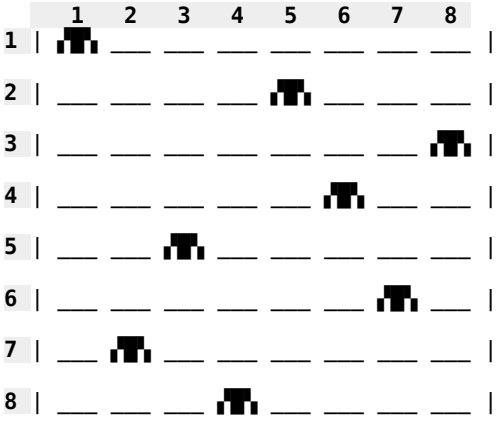
Having prepared the search terrain and the map of diagonals we left analyzing the 40320 possibilities and easily mapped all 92 that meet the requirement of the problem: no queen attacks the other.

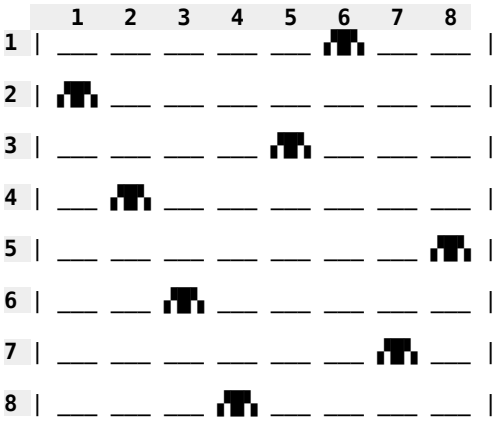
Basically we have to check the position of a queen and, through the diagonal map, check if in any of the houses presented there is another queen. We will repeat this process for the 7 remaining queens, unless we find a diagonal house occupied. In this case we interrupt the processing by discarding the combination under analysis and move on to the next permutation.

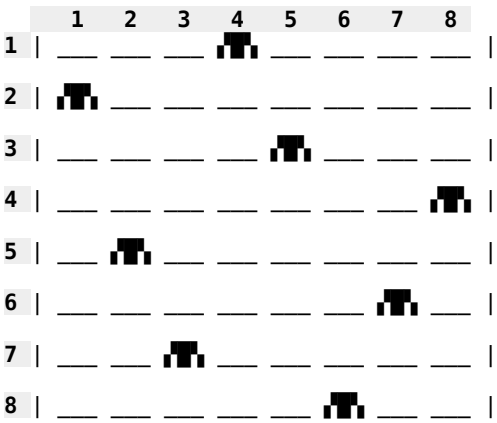
It is important to say that going through this vector of 40320 elements is something very simple for any current computer. We will then arrive at solutions like these:

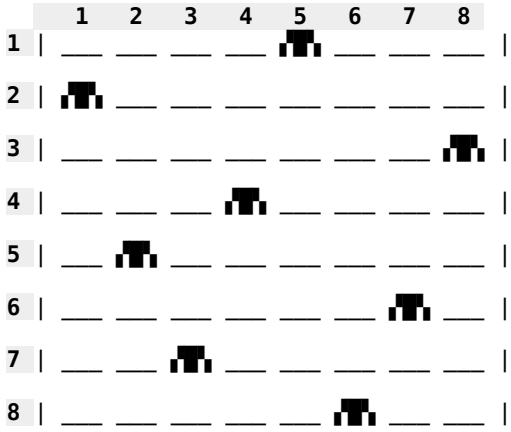


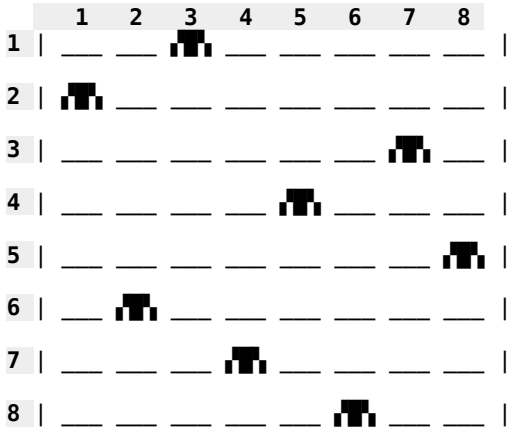












VI. CONCLUSION

This work presented an algorithm written in harbour language (https://github.com/vszakats/harbour-core/) to solve the problem of the 8 queens.

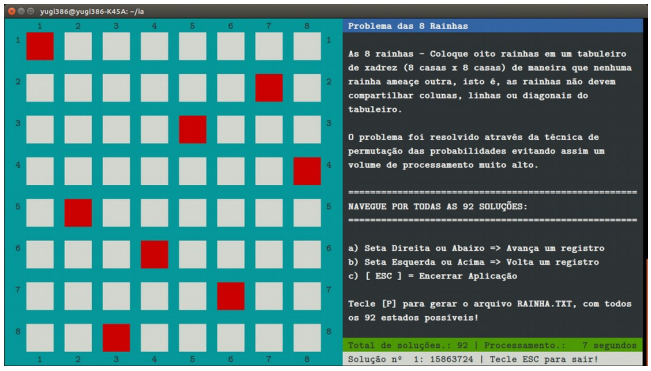


Figure 1: Program interface

The algorithm finds all 92 possible solutions (see Annex II) in just 7 seconds using an Intel® processor Core™ i5-3210M CPU @ 2.50GHz × 4 64-bit, with 6 GB of RAM, running the Ubuntu 16.04 LTS operating system. Although this solution to the problem of the 8 queens uses a conventional algorithm, and not a heuristic or genetic algorithm, its response is accurate and fast serving our purpose. The source code of the algorithm is available in Annex III.

ANNEX I

This annex contains all combinations of four items containing the numbers [12345678].

1234, 1235, 1236, 1237, 1238, 1243, 1245, 1246, 1247, 1248, 1253, 1254, 1256, 1257, 1258, 1263, 1264, 1265, 1267, 1268, 1273,
1274, 1275, 1276, 1278, 1283, 1284, 1285, 1286, 1287, 1324, 1325, 1326, 1327, 1328, 1342, 1345, 1346, 1347, 1348, 1352, 1354,
1356, 1357, 1358, 1362, 1364, 1365, 1367, 1368, 1372, 1374, 1375, 1376, 1378, 1382, 1384, 1385, 1386, 1387, 1423, 1425, 1426,
1427, 1428, 1432, 1435, 1436, 1437, 1438, 1452, 1453, 1456, 1457, 1458, 1462, 1463, 1465, 1467, 1468, 1472, 1473, 1475, 1476,
1478, 1482, 1483, 1485, 1486, 1487, 1523, 1524, 1526, 1527, 1528, 1532, 1534, 1536, 1537, 1538, 1542, 1543, 1546, 1547, 1548,
1562, 1563, 1564, 1567, 1568, 1572, 1573, 1574, 1576, 1578, 1582, 1583, 1584, 1586, 1587, 1623, 1624, 1625, 1627, 1628, 1632,
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1756, 1758, 1762, 1763, 1764, 1765, 1768, 1782, 1783, 1784, 1785, 1786, 1823, 1824, 1825, 1826, 1827, 1832, 1834, 1835, 1836,
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2684, 2685, 2687, 2713, 2714, 2715, 2716, 2718, 2731, 2734, 2735, 2736, 2738, 2741, 2743, 2745, 2746, 2748, 2751, 2753, 2754,
2756, 2758, 2761, 2763, 2764, 2765, 2768, 2781, 2783, 2784, 2785, 2786, 2813, 2814, 2815, 2816, 2817, 2831, 2834, 2835, 2836,
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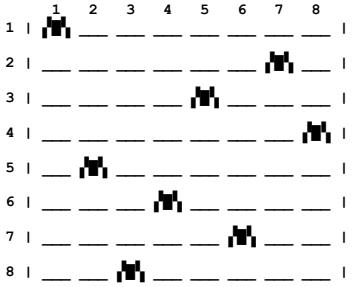
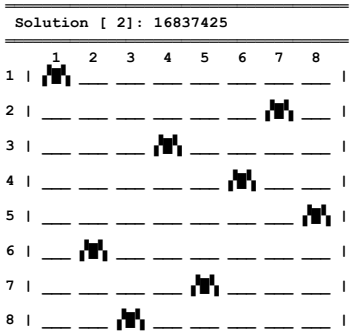
ANNEX II
All 92 possible solutions

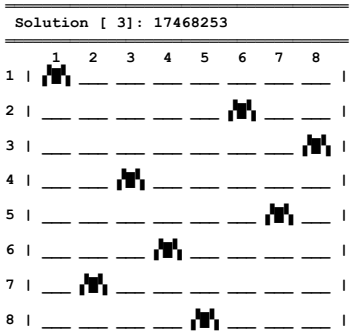
Solution [1]: 15863724



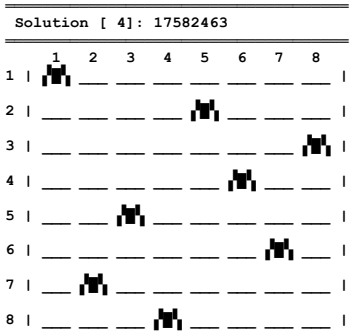
Solution [2]: 16837425



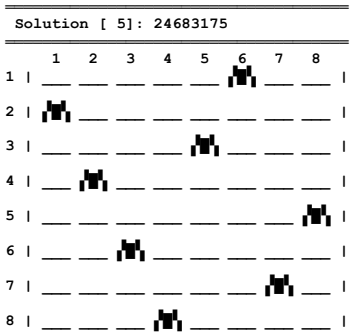
Solution [3]: 17468253



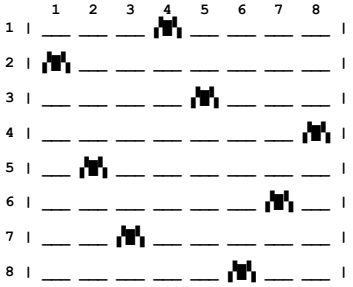
Solution [4]: 17582463



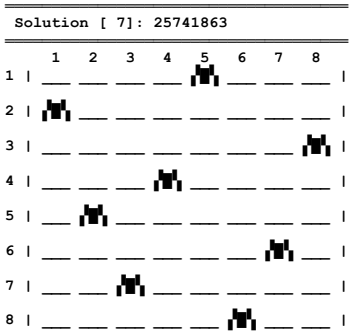
Solution [5]: 24683175



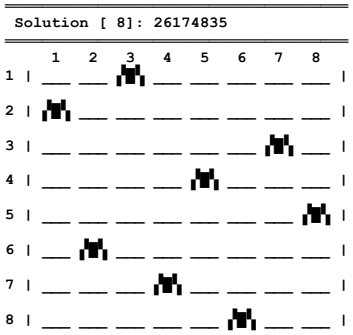
Solution [6]: 25713864



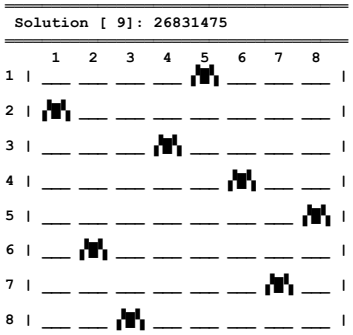
Solution [7]: 25741863



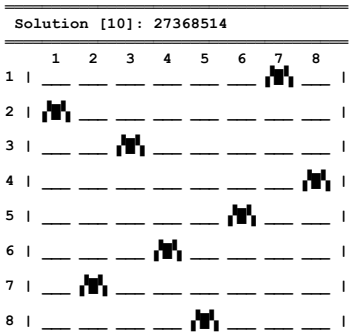
Solution [8]: 26174835



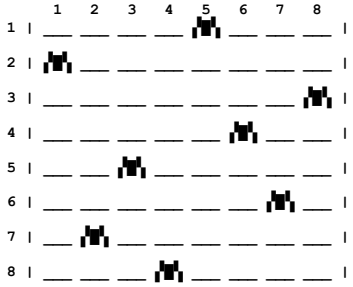
Solution [9]: 26831475



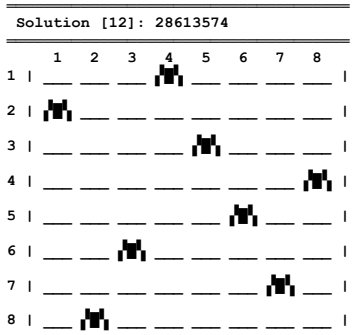
Solution [10]: 27368514



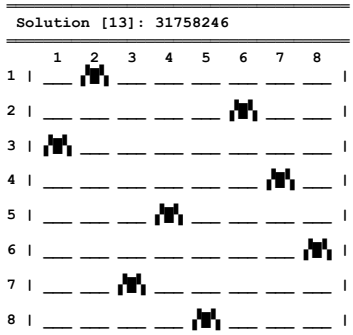
Solution [11]: 27581463



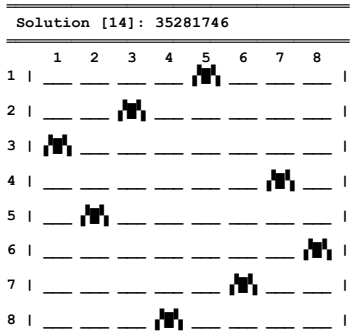
Solution [12]: 28613574



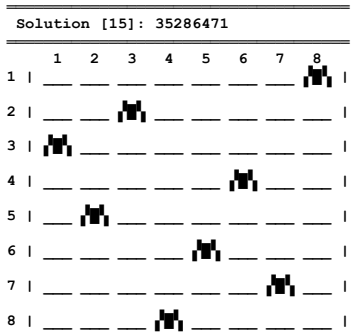
Solution [13]: 31758246



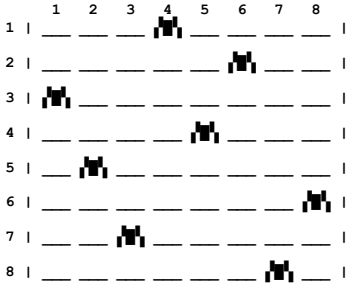
Solution [14]: 35281746



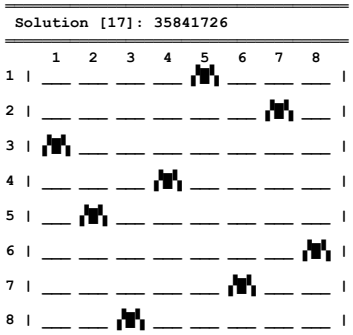
Solution [15]: 35286471



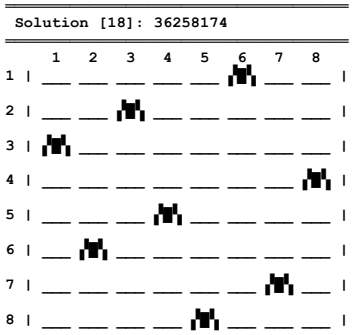
Solution [16]: 35714286



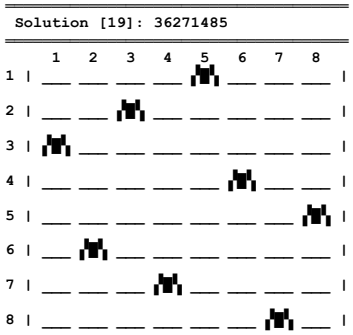
Solution [17]: 35841726



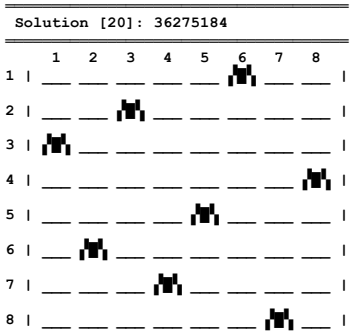
Solution [18]: 36258174



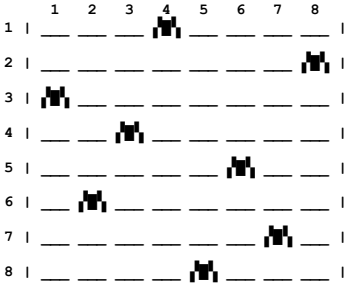
Solution [19]: 36271485



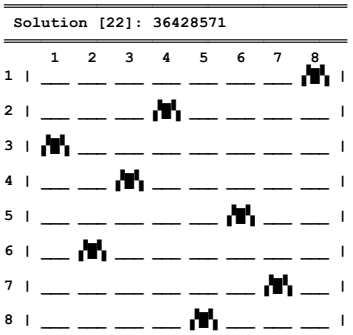
Solution [20]: 36275184



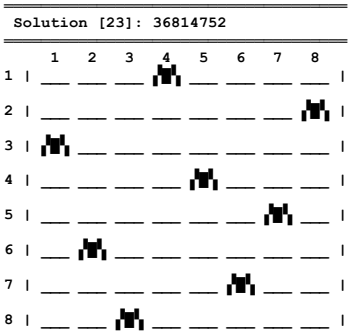
Solution [21]: 36418572



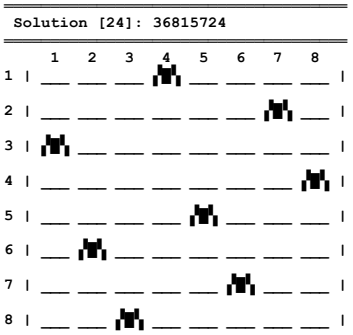
Solution [22]: 36428571



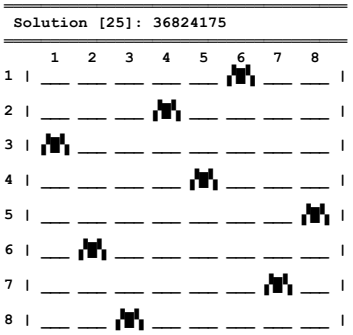
Solution [23]: 36814752



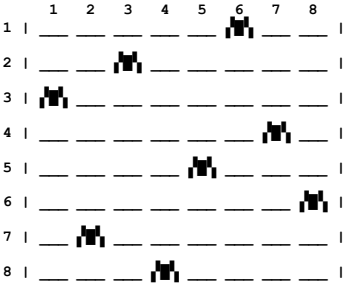
Solution [24]: 36815724



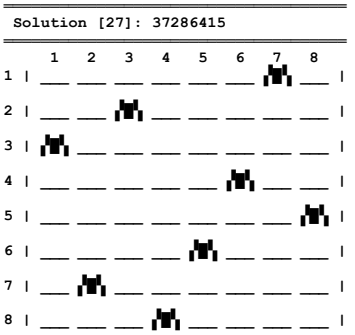
Solution [25]: 36824175



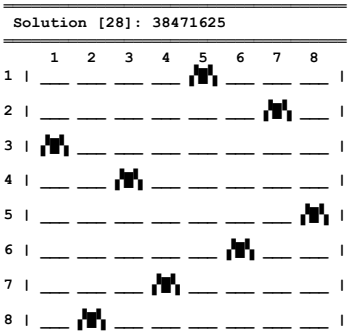
Solution [26]: 37285146



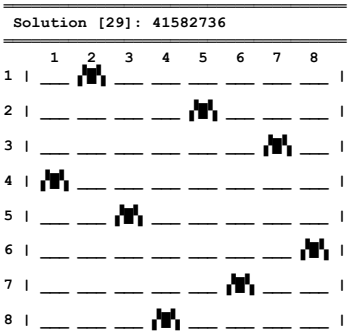
Solution [27]: 37286415



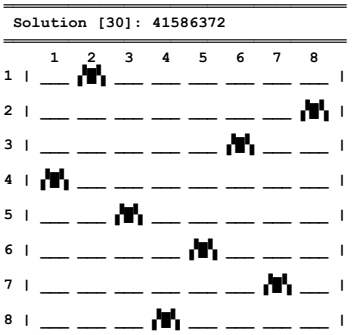
Solution [28]: 38471625



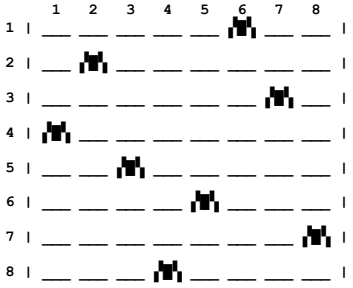
Solution [29]: 41582736



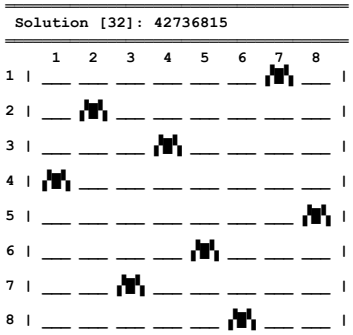
Solution [30]: 41586372



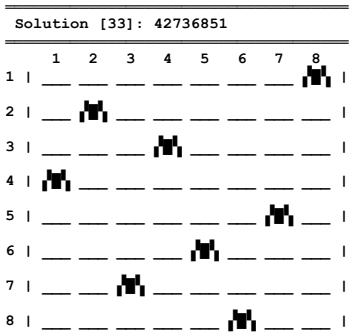
Solution [31]: 42586137



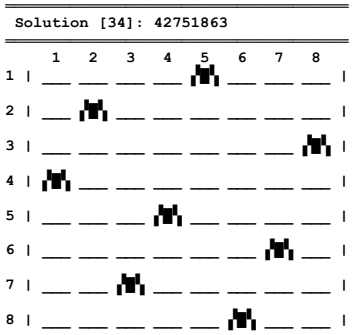
Solution [32]: 42736815



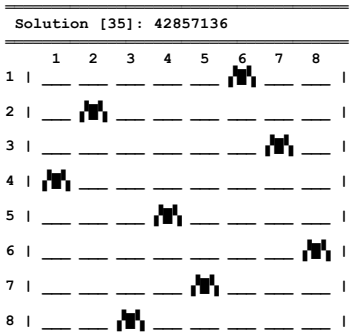
Solution [33]: 42736851



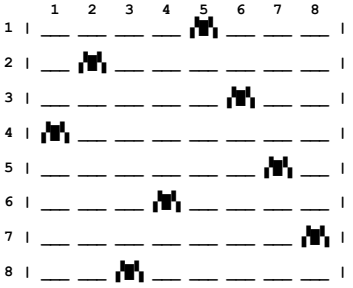
Solution [34]: 42751863



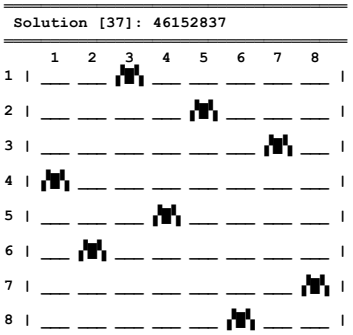
Solution [35]: 42857136



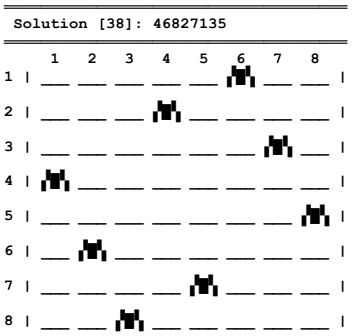
Solution [36]: 42861357



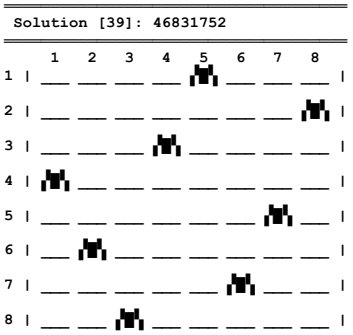
Solution [37]: 46152837



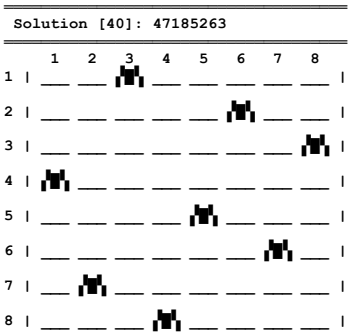
Solution [38]: 46827135



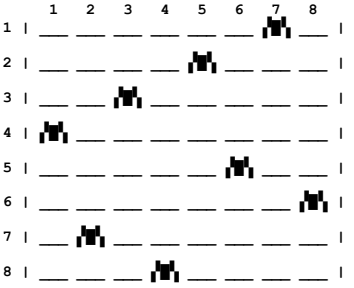
Solution [39]: 46831752



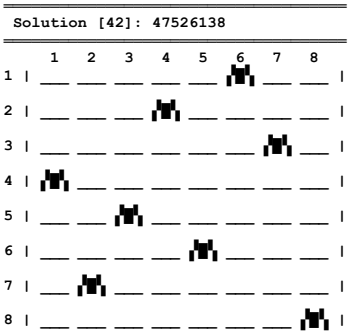
Solution [40]: 47185263



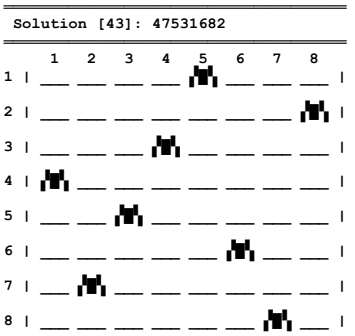
Solution [41]: 47382516



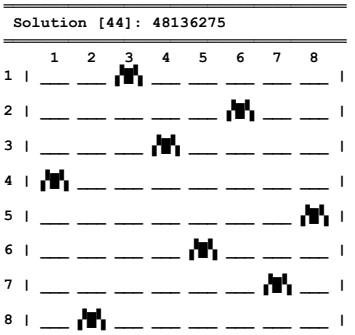
Solution [42]: 47526138



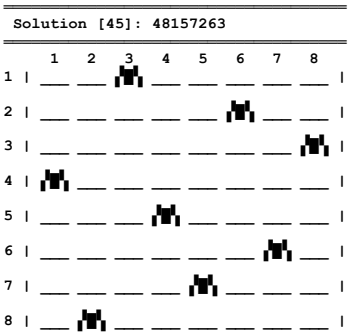
Solution [43]: 47531682



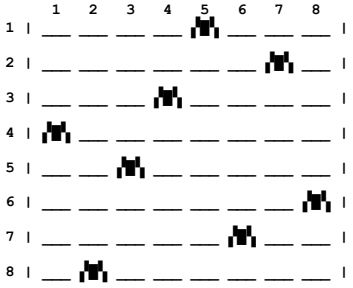
Solution [44]: 48136275



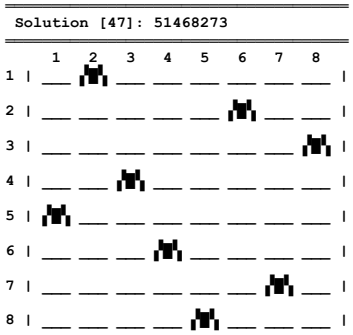
Solution [45]: 48157263



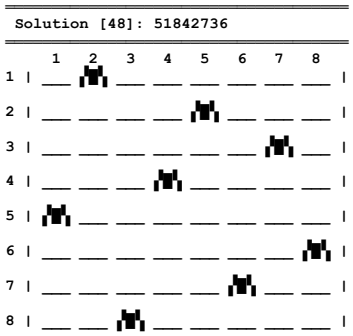
Solution [46]: 48531726



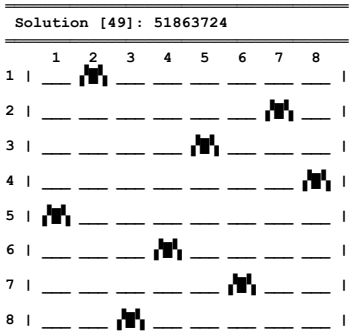
Solution [47]: 51468273



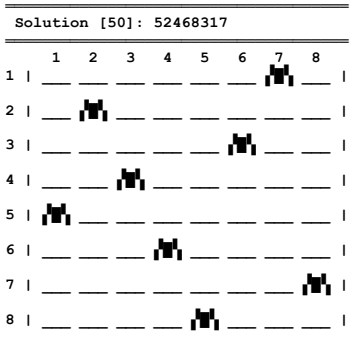
Solution [48]: 51842736



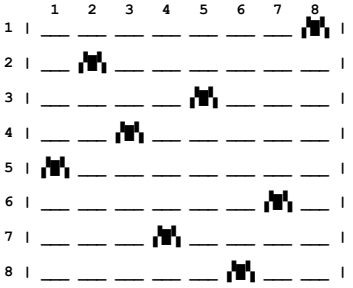
Solution [49]: 51863724



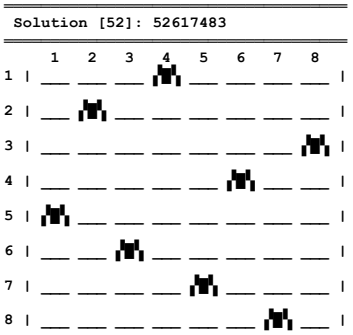
Solution [50]: 52468317



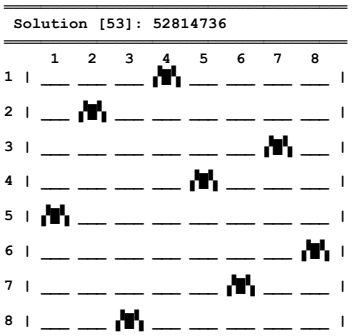
Solution [51]: 52473861



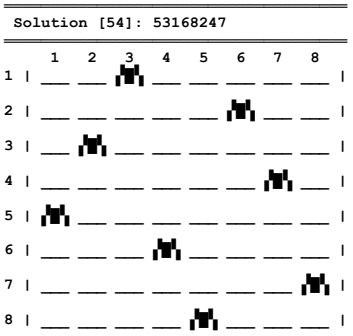
Solution [52]: 52617483



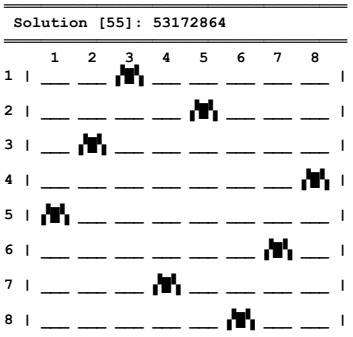
Solution [53]: 52814736



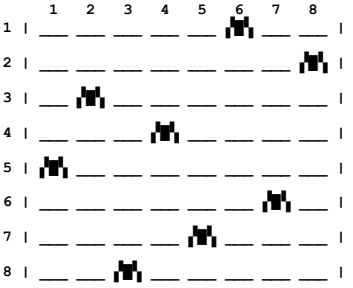
Solution [54]: 53168247



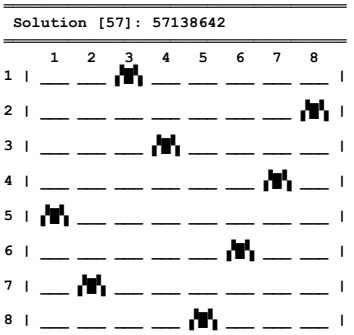
Solution [55]: 53172864



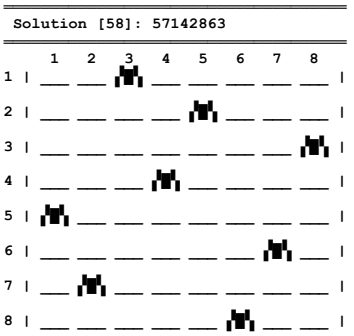
Solution [56]: 53847162



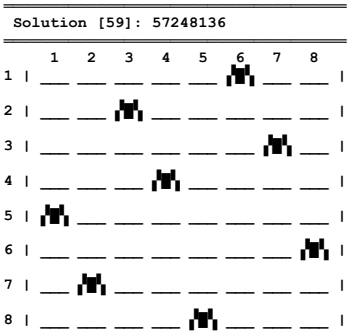
Solution [57]: 57138642



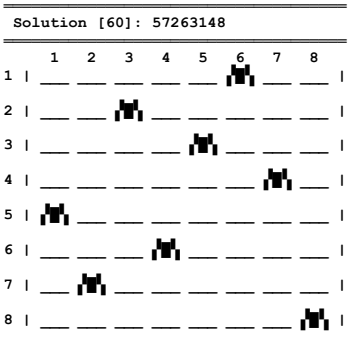
Solution [58]: 57142863



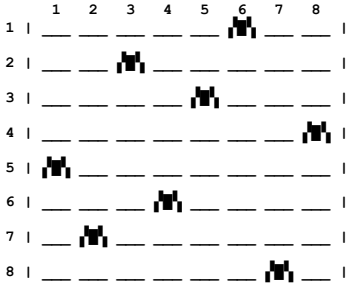
Solution [59]: 57248136



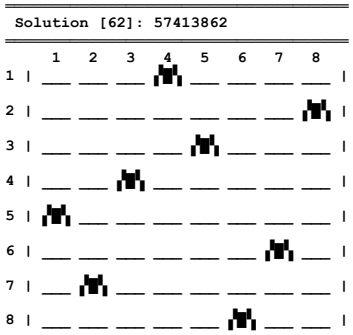
Solution [60]: 57263148



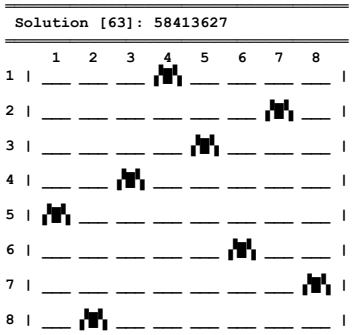
Solution [61]: 57263184



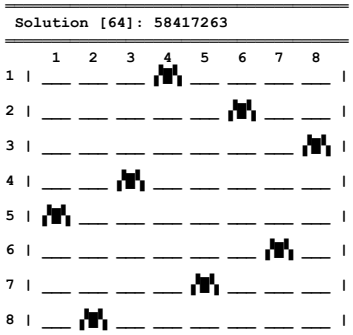
Solution [62]: 57413862



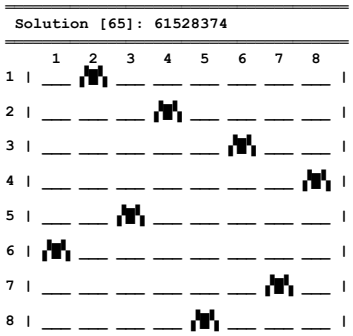
Solution [63]: 58413627



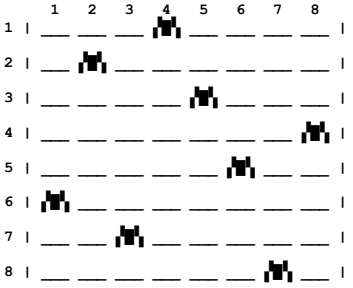
Solution [64]: 58417263



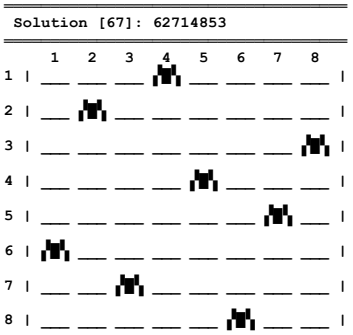
Solution [65]: 61528374



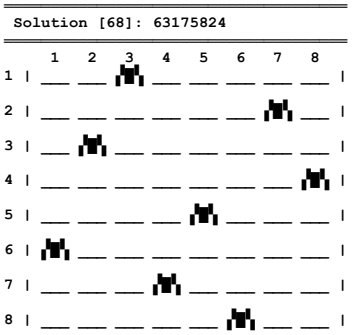
Solution [66]: 62713584



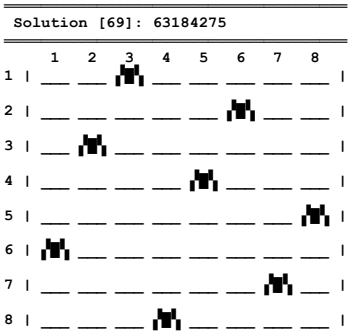
Solution [67]: 62714853



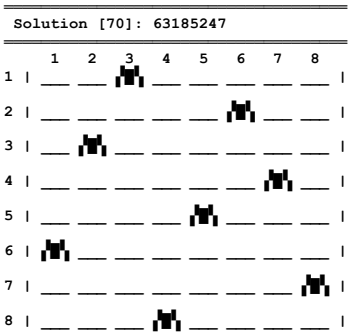
Solution [68]: 63175824



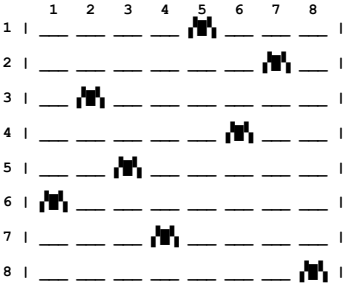
Solution [69]: 63184275



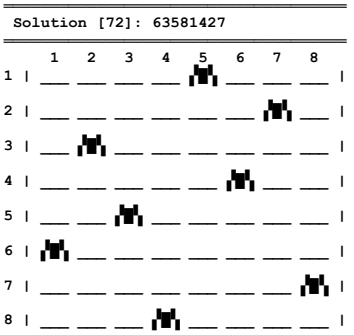
Solution [70]: 63185247



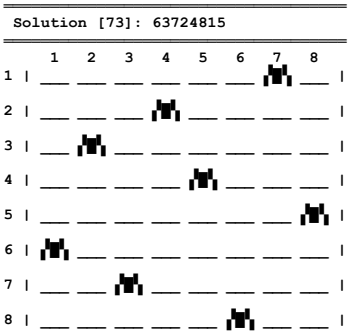
Solution [71]: 63571428



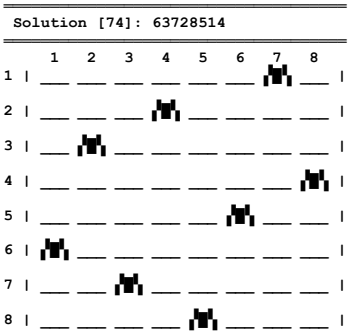
Solution [72]: 63581427



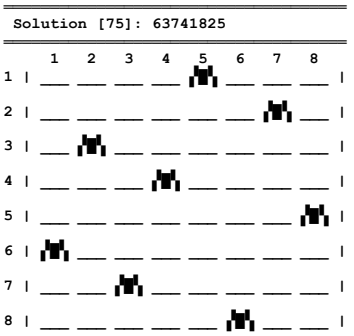
Solution [73]: 63724815



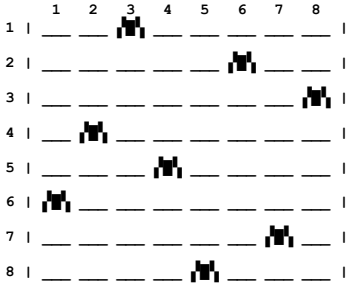
Solution [74]: 63728514



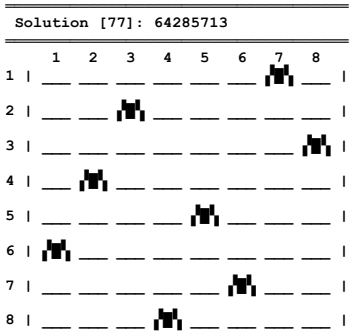
Solution [75]: 63741825



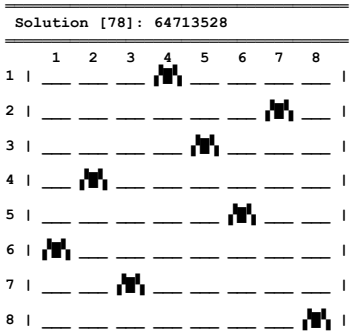
Solution [76]: 64158273



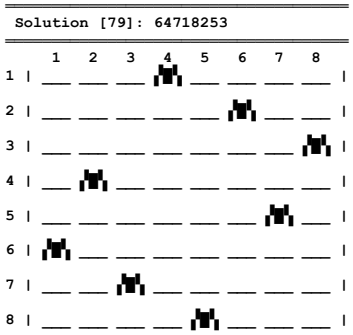
Solution [77]: 64285713



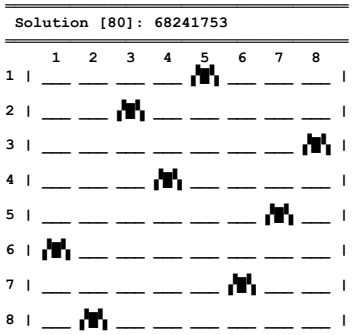
Solution [78]: 64713528



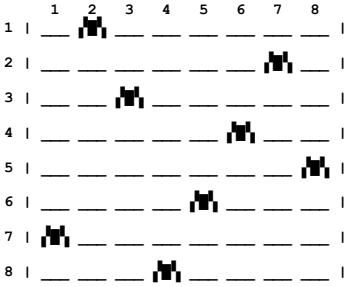
Solution [79]: 64718253



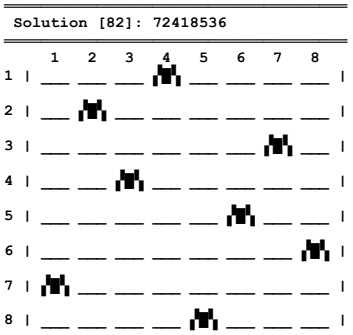
Solution [80]: 68241753



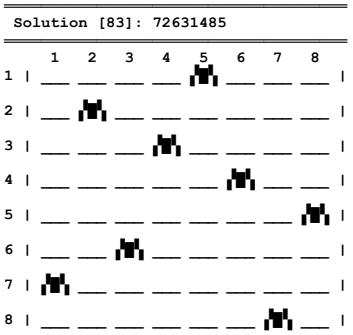
Solution [81]: 71386425



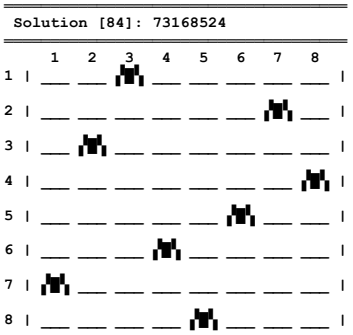
Solution [82]: 72418536



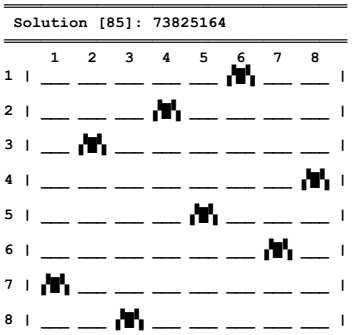
Solution [83]: 72631485



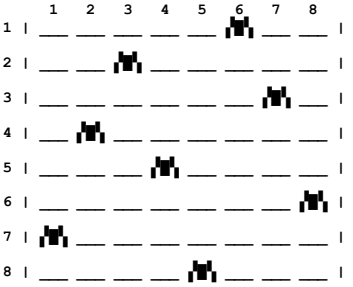
Solution [84]: 73168524



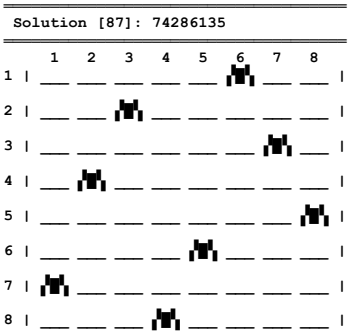
Solution [85]: 73825164



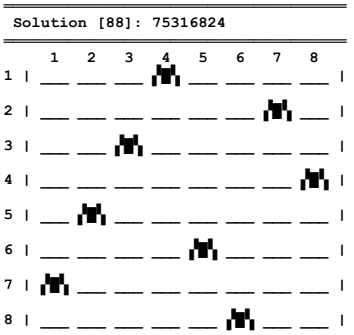
Solution [86]: 74258136



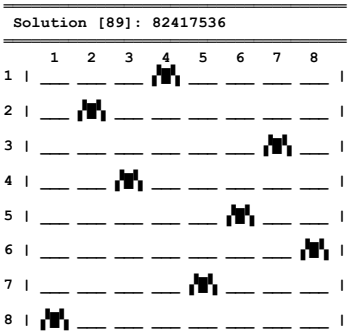
Solution [87]: 74286135



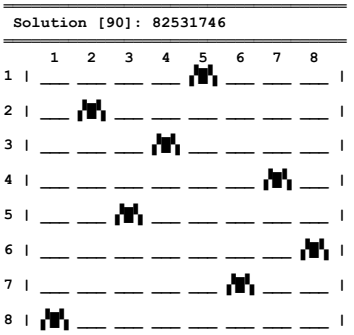
Solution [88]: 75316824



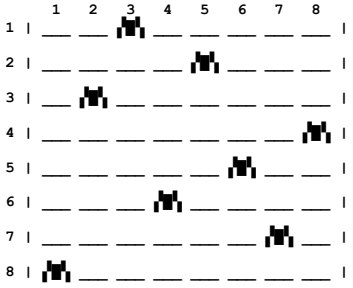
Solution [89]: 82417536



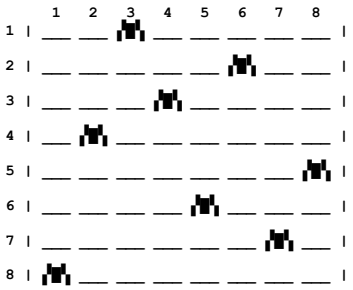
Solution [90]: 82531746



Solution [91]: 83162574



Solution [92]: 84136275



ANNEX III - SOURCE CODE OF THE SOLUTION IN HARBOUR LANGUAGE

To compile the program you must download the Harbour 3.4 compiler (https://github.com/vszakats/harbour-core/). The compilation command is **hbmk2 -fullstatic -gc3 source_name** replacing the **source_name** with the file. The file name must have the extension .prg.

```
/*
_____________________________________________________________________________
[1] PROBLEM OF THE 8 QUEENS

The 8 queens - Place eight queens on a chessboard (8 squares x 8 squares) so that no queen threatens another,
i.e. queens must not share columns, rows or diagonals of the board.
_____________________________________________________________________________
*/

/*
Módulo principal do programa
*/
FUNCTION MAIN()
LOCAL ct:=0, verifica:=.t., inicio:=0, fim:=0
PUBLIC posicao:= array(8), permutacao:={}, tabuleiro:=array(8,8), solucao:={}, total:=0,;
               mapaDiagonal:=array(64,2)

        set decimals to 0
        configuracoes(25,132,24,1)

        /* Preparação: Eliminando estados inválidos do tabuleiro e
                       criando um mapa com as casas diagonais em relação
                                  a uma casa fixa
        */
        cls
        inicio = seconds()      // Marca tempo
        interfaceGeral()        // Mostra interface inicial da aplicação

        gerandoPermutacao()     // Gerando Universo que contém as soluções que se quer buscar

        // Gera um mapa que a partir de qualquer casa do tabuleiro indica as casas
        // que lhe são diagonais
        criarMapaDiagonal()

        // Eliminando os estados inválidos

        asort(permutacao)
        total:=len(permutacao)
        for ct:= 1 to total
                limpaTabuleiro(permutacao[ct])
                verifica = verificarPosicionamento(permutacao[ct])

                if verifica == .t.
                        aadd(solucao,permutacao[ct])
                endif

        next

        // Apresentação do resultado:
        setcolor("N/G")
        @ 23,62 clear to 23,131
        @ 23,63 say "Total de soluções.: " + alltrim(str(len(solucao))) + " | Processamento.: " +;
                 str(seconds()-inicio,3) + " segundos"
        setcolor("N/W")
        @ 24,62 clear to 24,131

        ct = 1
        do while.t.
                setcolor("N/W")
                @ 24,62 say " Solution nº " + str(ct,2) + ": " + alltrim(solucao[ct]) + " | Tecle ESC para sair!"
                limpaTabuleiro(solucao[ct])
                mostraTabuleiro()
                inkey(0)

                if lastkey() == 27
                        exit
                endif

                if lastkey() == 4 .or. lastkey() = 24
                        ++ct
                        if ct > len(solucao)
                                ct = len(solucao)
```

```
                endif
        endif

        if lastkey() == 19 .or. lastkey() = 5
                --ct
                if ct < 1
                        ct = 1
                endif
        endif

        if chr(lastkey()) == "P" .or. chr(lastkey()) = "p"
                imprimeArquivo(solucao)
        endif

    enddo

    clear all
    set color to
    cls

RETURN NIL

//_____________________________________________________________________________
/*
Gerando o mapa de permutações que contém as soluções possíveis
*/
FUNCTION gerandoPermutacao()
LOCAL ct:=0, ct2:=0, ct3:=0, numeros:={"1","2","3","4","5","6","7","8"}, valorPrimo:={2,3,5,7,11,13,17,19},;
      posicao:="", p1:=1, p2:=1, p3:=1, p4:=1, matrizPermutacao4:={}, permutacaoValida:=.t., valor:=0, total:=0

    // Calculando as posições possíveis para 4 rainhas
    do while.t.
        posicao = numeros[p1] + numeros[p2] + numeros[p3] + numeros[p4]
        valor = valorPrimo[p1] * valorPrimo[p2] * valorPrimo[p3] * valorPrimo[p4]

        // verificando permutação válida:
        permutacaoValida:=.t.
        for ct:= 1 to 8
                if valor % (valorPrimo[ct]^2) == 0
                        permutacaoValida:=.f.
                        exit
                endif
        next

        if (permutacaoValida == .t.)
                aadd(matrizPermutacao4,posicao)
        endif

        // Avançando os ponteiros contadores:
        ++p1
        if p1 > 8
                p1 = 1
                ++p2
        endif

        if p2 > 8
                p2 = 1
                ++p3
        endif

        if p3 > 8
                p3 = 1
                ++p4
        endif

        if p4 > 8
                exit
        endif

    enddo

    // Lendo o total de permutações para 4 Rainhas e juntando para formar 8
    total := len(matrizPermutacao4)

    for ct:= 1 to total
        for ct2:= 1 to total
                valor = valorPrimo[ val(substr(matrizPermutacao4[ct],1,1)) ] * ;
                        valorPrimo[ val(substr(matrizPermutacao4[ct],2,1)) ] * ;
                        valorPrimo[ val(substr(matrizPermutacao4[ct],3,1)) ] * ;
                        valorPrimo[ val(substr(matrizPermutacao4[ct],4,1)) ] * ;
```

```
                                valorPrimo[ val(substr(matrizPermutacao4[ct2],1,1)) ] * ;
                                valorPrimo[ val(substr(matrizPermutacao4[ct2],2,1)) ] * ;
                                valorPrimo[ val(substr(matrizPermutacao4[ct2],3,1)) ] * ;
                                valorPrimo[ val(substr(matrizPermutacao4[ct2],4,1)) ]

                        // verificando permutação válida:
                        permutacaoValida:=.t.
                        for ct3:= 1 to 8
                                if valor % (valorPrimo[ct3]^2) == 0
                                        permutacaoValida:=.f.
                                        exit
                                endif
                        next

                        if (permutacaoValida == .t.)
                                aadd(permutacao,matrizpermutacao4[ct] + matrizpermutacao4[ct2])
                        endif
                next
        next

RETURN NIL

//_____________________________________________________________________________
/*
Função para limpar o tabuleiro e preencher com um posicionamento
*/
FUNCTION limpaTabuleiro(rposicao)
LOCAL ct:=0, ct2:=0

        // Limpando o tabuleiro
        for ct:= 1 to 8
                for ct2:=1 to 8
                        tabuleiro[ct,ct2] = 0
                next
        next

        // Posicionando as Rainhas nas linhas e colunas
        for ct:= 1 to 8
                tabuleiro[val(substr(rposicao,ct,1)),ct] = ct
        next

RETURN NIL

//_____________________________________________________________________________
/*
Função para mostrar a interface inicial da aplicação
*/
FUNCTION interfaceGeral()
LOCAL ct:=0, linha:=0, coluna:=0

        setcolor("N/N+")
        @ 00,00 clear to 24,131
        setcolor("W+/B")
        @ 00,62 clear to 00,131
        @ 00,63 say "Problema das 8 Rainhas"
        setcolor("N/W")
        @ 24,62 clear to 24,131
        @ 24,63 say "Aguarde... calculando soluções!"
        setcolor("N/BG")
        @ 00,00 clear to 24,61

        linha = 1
        coluna = 4

        setcolor("N/W")
        do while.t.
                coluna = 4
                for ct:= 1 to 8
                        @ linha,coluna clear to linha+1,coluna+4
                        coluna = coluna + 7
                next

                linha = linha + 3

                if linha > 24
                        exit
                endif
        enddo

        setcolor("N*/BG")
```

```
        coluna = 6
        for ct:= 1 to 8
                @ 00,coluna say alltrim(str(ct))
                @ 24,coluna say alltrim(str(ct))
                coluna = coluna + 7
        next

        setcolor("N*/BG")
        linha = 1
        for ct:= 1 to 8
                @ linha,2 say alltrim(str(ct))
                @ linha,59 say alltrim(str(ct))
                linha = linha + 3
        next

        setcolor("W+/N")
        @ 02,63 say "As 8 rainhas - Coloque oito rainhas em um tabuleiro"
        @ 03,63 say "de xadrez (8 casas x 8 casas) de maneira que nenhuma"
        @ 04,63 say "rainha ameaçe outra, isto é, as rainhas não devem"
        @ 05,63 say "compartilhar colunas, linhas ou diagonais do "
        @ 06,63 say "tabuleiro."

        @ 08,63 say "O problema foi resolvido através da técnica de"
        @ 09,63 say "permutação das probabilidades evitando assim um"
        @ 10,63 say "volume de processamento muito alto."

        @ 20,63 say "Tecle [P] para gerar o arquivo RAINHA.TXT, com todos"
        @ 21,63 say "os 92 estados possíveis!"

        @ 12,63 say replicate("=",53)
        @ 13,63 say "NAVEGUE POR TODAS AS 92 SOLUÇÕES:"
        @ 14,63 say replicate("=",53)
        @ 16,63 say "a) Seta Direita ou Abaixo => Avança um registro"
        @ 17,63 say "b) Seta Esquerda ou Acima => Volta um registro"
        @ 18,63 say "c) [ ESC ] = Encerrar Aplicação"

RETURN NIL

//_____________________________________________________________________________
/*
Função para mostrar o tabuleiro
*/
FUNCTION mostraTabuleiro()
LOCAL ct:=0, ct2:=0, coluna, linha, marca_linha

        linha = 1
        coluna = 4
        marca_linha = 1

        setcolor("N/W")
        do while.t.
                coluna = 4
                for ct:= 1 to 8
                        setcolor("N/W")
                        @ linha,coluna clear to linha+1,coluna+4
                        if tabuleiro[marca_linha,ct] <> 0
                                setcolor("N/R+")
                                @ linha,coluna clear to linha+1,coluna+4
                        endif

                        coluna = coluna + 7
                next

                ++ marca_linha
                linha = linha + 3

                if linha > 24
                        exit
                endif
        enddo

RETURN NIL

//_____________________________________________________________________________
/*
Função para imprimir um arquivo com todas os estados possíveis
*/
FUNCTION imprimeArquivo(solucao)
local tam:= len(solucao), ct:=0, contador:=0
```

```
        set console off
        set device to printer
        set printer on
        set printer to "RAINHA.TXT"

        for contador:= 1 to tam
                limpaTabuleiro(solucao[contador])

                ?  "═══════════════════════════════════════"
                ?  " Solution [" + str(contador,2) + "]: " + solucao[contador]
                ?  "═══════════════════════════════════════"
                ?  "     1   2   3   4   5   6   7   8 "

                for ct:= 1 to 8
                        ? alltrim(str(ct)) + space(1) + "| "
                        for ct2:= 1 to 8
                                if ct2 < 8
                                        if tabuleiro[ct,ct2] <> 0
                                                ?? "▟▙ "
                                        else
                                                ?? "___ "
                                        endif
                                else
                                        if tabuleiro[ct,ct2] <> 0
                                                ?? "▟▙"
                                        else
                                                ?? "___"
                                        endif
                                endif
                        next
                        ?? " |"
                        ?
                next
                ?
        next

        set console on
        set device to screen
        set printer off
        set printer to

RETURN NIL
//_____________________________________________________________________________
/*
Função para mapear as casas que formam as diagonais do tabuleiro
*/
FUNCTION criarMapaDiagonal()
local ct:=0, ct2:=0, coluna:=0, linha:=0, diagonais:=""

        contador = 1
        for ct:= 1 to 8
                for ct2:= 1 to 8
                        coluna = ct
                        linha = ct2
                        diagonais = ""

                        // ****************************************************
                        // Voltando a primeira linha ou coluna
                        coluna = ct
                        linha = ct2

                        // diminui coluna e linha
                        do while.t.
                                if linha == 1 .or. coluna == 1
                                        exit
                                else
                                        --linha
                                        --coluna
                                endif
                        enddo
                        // ****************************************************

                        // [1] Mapeando as diagonais da esquerda para a direita
                        do while.t.
                                diagonais = diagonais + alltrim(str(coluna)) + alltrim(str(linha)) + " "
                                ++linha
                                ++coluna

                                if coluna > 8 .or. linha > 8
                                        exit
```

```
                        endif
                enddo

                // *****************************************************
                // Voltando a primeira posicao inicial
                coluna = ct
                linha = ct2

                // *****************************************************


                // [2] Mapeando as diagonais da direita para a esquerda
                do while.t.
                        diagonais = diagonais + alltrim(str(coluna)) + alltrim(str(linha)) + " "
                        ++linha
                        --coluna

                        if coluna < 1 .or. linha > 8
                                exit
                        endif
                enddo


                // *****************************************************
                // Voltando a posição inicial
                coluna = ct
                linha = ct2

                // *****************************************************

                // [3] Mapeando as diagonais da direita para a esquerda
                do while.t.
                        diagonais = diagonais + alltrim(str(coluna)) + alltrim(str(linha)) + " "
                        --linha
                        ++coluna

                        if coluna > 8 .or. linha < 1
                                exit
                        endif
                enddo

                // Preenchendo
                mapaDiagonal[contador,1] = alltrim(str(ct)) + alltrim(str(ct2))
                mapaDiagonal[contador,2] = strtran(diagonais,(alltrim(str(ct)) +;
                 alltrim(str(ct2))+" ")," ")
                mapaDiagonal[contador,2] = strtran(mapaDiagonal[contador,2]," ","")
                ++contador
        next
next

RETURN NIL

//_____________________________________________________________________________
/*
Função para verificar posicionamento onde as Rainhas não se atacam
*/
FUNCTION verificarPosicionamento(rposicao)
LOCAL ct:=0, ct2:=0, coluna:=0, linha:=0, posicao:=0, mapa:="",tam:=0

        // Limpando o tabuleiro
        for ct:= 1 to 8

                // Lendo a posição da Rainha e preparando para verificar a diagonal
                coluna = ct
                linha = val(substr(rposicao,ct,1))

                for ct2:= 1 to 64
                        if (substr(rposicao,ct,1)) + alltrim(str(coluna)) == mapaDiagonal[ct2,1]
                                exit
                        endif
                next

                // Verificando se existem outras Rainhas nas diagonais
                tam = len(mapaDiagonal[ct2,2])
                mapa = mapaDiagonal[ct2,2]

                for ct2:= 1 to tam step 2
                        posicao = substr(mapa,ct2,2)
                        if ( tabuleiro[val(substr(posicao,1,1)), val(substr(posicao,2,1))] <> 0 )
                                return .f.
```

```
                        endif
                next

        next

RETURN .t.

//_____________________________________________________________________________
/*
Configurações básicas de inicialização do HARBOUR
*/
FUNCTION CONFIGURACOES(linhas,colunas,lstatus,modo)

        CLS
        REQUEST HB_LANG_PT
        REQUEST HB_CODEPAGE_UTF8EX
        HB_LANGSELECT( 'PT' )
        HB_CDPSELECT( "UTF8EX" )

        set date french
        set century on
        set scoreboard off
        set cursor off
        setblink(.f.)
        set confirm on
        setmode(linhas,colunas)
    if modo == 1
        set message to lstatus center
    else
        set message to lstatus
    endif
        set wrap On
    set exact on

RETURN NIL
```