ON Λ_b-SETS AND THE ASSOCIATED TOPOLOGY τ^{Λ_b}*

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Dedicated to Professor Alexander Arhangelskii
on the occasion of his 65th birthday§

Abstract

In this paper we define the concept of Λ_b-sets (resp. V_b-sets) of a topological space, i.e., the intersection of b-open (resp. the union of b-closed) sets. We study the fundamental property of Λ_b-sets (resp. V_b-sets) and investigate the topologies defined by these families of sets.

1 Introduction

In 1996, Andrijević [2] introduced a new class of generalized open sets called b-open sets into the field of topology. This class is a subset of the class of semi-preopen sets [3], i.e. a subset of a topological space which is contained in the closure of the interior of its closure. Also the class of b-open sets is a superset of the class of semi-open sets [7], i.e. a set which is contained in the closure of its interior, and the class of locally dense sets [5] or preopen sets

*1991 Math. Subject Classification –Primary: 54D30, 54A05; Secondary: 54H05, 54G99.
Keywords and phrases: b-open sets, Λ_b-sets, V_b-sets, topology τ^{Λ_b}.

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[11], i.e. a set which is contained in the interior of its closure. Andrijević studied several fundamental and interesting properties of b-open sets. Among others, he showed that a rare b-open set is preopen [[2], Proposition 2.2]. Recall that a rare set [4] is a set with no interior points. It is well-known that for a topological space X, every rare b-open set is semi-open if and only if the interior of a dense subset is dense.

Throughout the present paper, the space (X, τ) always means a topological space on which no separation axioms are assumed unless explicitly stated. Let $A \subseteq X$, then A is said to be *b-open* [2] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$, where $Cl(A)$ and $Int(A)$ denotes the closure and the interior of A in (X, τ), respectively. The complement A^c of a b-open set A is called *b-closed* and the *b-closure* of a set A, denoted by $Cl_b(A)$, is the intersection of all b-closed sets containing A. The *b-interior* of a set A denoted by $Int_b(A)$, is the union of all b-open sets contained in A.

The family of all b-open (resp. b-closed) sets in (X, τ) will be denoted by $BO(X, \tau)$ (resp. $BC(X, \tau)$).

PROPOSITION 1.1 *(Andrijević [2])* (a) The union of any family of b-open sets is b-open.
(b) The intersection of an open and a b-open set is a b-open set.

LEMMA 1.2 The *b-closure* $Cl_b(A)$, is the set of all $x \in X$ such that $O \cap A \neq \emptyset$ for every $O \in BO(X, x)$, *where* $BO(X, x) = \{U \mid x \in U, U \in BO(X, \tau)\}$.

It is the aim of this paper to introduce the concept of Λ_b-sets (resp. V_b-sets) which is the intersection of b-open (resp. the union of b-closed) sets. We also investigate the notions of generalized Λ_b-sets and generalized V_b-sets in a topological space (X, τ). Moreover, we present a new topology τ^{Λ_b} on (X, τ) by utilizing the notions of Λ_b-sets and V_b-sets. In this connection, we examine some of the properties of this new topology.

2 Λ_b-sets and V_b-sets

DEFINITION 1 Let B be a subset of a topological space (X, τ). We define the subsets B^{Λ_b} and B^{V_b} as follows:

$B^{\Lambda_b} = \bigcap\{O/O \supseteq B,\ O \in BO(X,\tau)\}$ and $B^{V_b} = \bigcup\{F/F \subseteq B,\ F^c \in BO(X,\tau)\}$.

PROPOSITION 2.1 Let A, B and $\{B_\lambda : \lambda \in \Omega\}$ be subsets of a topological space (X, τ). Then the following properties are valid:

(a) $B \subseteq B^{\Lambda_b}$;

(b) If $A \subseteq B$, then $A^{\Lambda_b} \subseteq B^{\Lambda_b}$;

(c) $(B^{\Lambda_b})^{\Lambda_b} = B^{\Lambda_b}$;

(d) $[\bigcup_{\lambda\in\Omega} B_\lambda]^{\Lambda_b} = \bigcup_{\lambda\in\Omega} B_\lambda^{\Lambda_b}$;

(e) If $A \in BO(X,\tau)$, then $A = A^{\Lambda_b}$;

(f) $(B^c)^{\Lambda_b} = (B^{V_b})^c$;

(g) $B^{V_b} \subseteq B$;

(h) If $B \in BC(X,\tau)$, then $B = B^{V_b}$;

(i) $[\bigcap_{\lambda\in\Omega} B_\lambda]^{\Lambda_b} \subseteq \bigcap_{\lambda\in\Omega} B_\lambda^{\Lambda_b}$;

(j) $[\bigcup_{\lambda\in\Omega} B_\lambda]^{V_b} \supseteq \bigcup_{\lambda\in\Omega} B_\lambda^{V_b}$.

PROOF. (a) Clear by Definition 1.

(b) Suppose that $x \notin B^{\Lambda_b}$. Then there exists a subset $O \in BO(X,\tau)$ such that $O \supseteq B$ with $x \notin O$. Since $B \supseteq A$, then $x \notin A^{\Lambda_b}$ and thus $A^{\Lambda_b} \subseteq B^{\Lambda_b}$.

(c) Follows from (a) and Definition 1.

(d) Suppose that there exists a point x such that $x \notin [\bigcup_{\lambda\in\Omega} B_\lambda]^{\Lambda_b}$. Then, there exists a subset $O \in BO(X,\tau)$ such that $\bigcup_{\lambda\in\Omega} B_\lambda \subseteq O$ and $x \notin O$. Thus, for each $\lambda \in \Omega$ we have $x \notin B_\lambda^{\Lambda_b}$. This implies that $x \notin \bigcup_{\lambda\in\Omega} B_\lambda^{\Lambda_b}$. Conversely, suppose that there exists a point $x \in X$ such that $x \notin \bigcup_{\lambda\in\Omega} B_\lambda^{\Lambda_b}$. Then by Definition 1, there exist subsets $O_\lambda \in BO(X,\tau)$ (for each $\lambda \in \Omega$) such that $x \notin O_\lambda$, $B_\lambda \subseteq O_\lambda$. Let $O = \bigcup_{\lambda\in\Omega} O_\lambda$. Then we have that $x \notin \bigcup_{\lambda\in\Omega} O_\lambda$, $\bigcup_{\lambda\in\Omega} B_\lambda \subseteq O$ and $O \in BO(X,\tau)$. This implies that $x \notin [\bigcup_{\lambda\in\Omega} B_\lambda]^{\Lambda_b}$. Thus, the proof of (d) is completed.

(e) By Definition 1 and since $A \in BO(X, \tau)$, we have $A^{\Lambda_b} \subseteq A$. By (a) we have that $A^{\Lambda_b} = A$.

(f) $(B^{V_b})^c = \bigcap \{F^c / F^c \supseteq B^c, F^c \in BO(X, \tau)\} = (B^c)^{\Lambda_b}$.

(g) Clear by Definition 1.

(h) If $B \in BC(X, \tau)$, then $B^c \in BO(X, \tau)$. By (e) and (f): $B^c = (B^c)^{\Lambda_b} = (B^{V_b})^c$. Hence $B = B^{V_b}$.

(i) Suppose that there exists a point x such that $x \notin \bigcap\limits_{\lambda \in \Omega} B_\lambda^{\Lambda_b}$. Then, there exists $\lambda \in \Omega$ such that $x \notin B_\lambda^{\Lambda_b}$. Hence there exists $O \in BO(X, \tau)$ such that $O \supseteq B_\lambda$ and $x \notin O$. Thus $x \notin [\bigcap\limits_{\lambda \in \Omega} B_\lambda]^{\Lambda_b}$.

(j) $[\bigcup\limits_{\lambda \in \Omega} B_\lambda]^{V_b} = [((\bigcup\limits_{\lambda \in \Omega} B_\lambda)^c)^{\Lambda_b}]^c = [(\bigcap\limits_{\lambda \in \Omega} B_\lambda^c)^{\Lambda_b}]^c \supseteq [\bigcap\limits_{\lambda \in \Omega} (B_\lambda^c)^{\Lambda_b}]^c = [\bigcap\limits_{\lambda \in \Omega} (B_\lambda^{V_b})^c]^c = \bigcup\limits_{\lambda \in \Omega} B_\lambda^{V_b}$ (by (f) and (i)). □

REMARK 2.2 In general $(B_1 \bigcap B_2)^{\Lambda_b} \neq B_1^{\Lambda_b} \bigcap B_2^{\Lambda_b}$, as the following example shows.

EXAMPLE 2.3 *Let* $X = \{a, b, c\}$ *and* $\tau = \{\emptyset, \{a\}, X\}$. *Let* $B_1 = \{b\}$ *and* $B_2 = \{c\}$. *Then we have* $(B_1 \bigcap B_2)^{\Lambda_b} = \emptyset$ *but* $B_1^{\Lambda_b} \bigcap B_2^{\Lambda_b} = \{a\}$.

DEFINITION 2 In a topological space (X, τ), a subset B is a Λ_b-*set* (resp. V_b-*set*) of (X, τ) if $B = B^{\Lambda_b}$ (resp. $B = B^{V_b}$). *By* Λ_b *(resp.* V_b*), we denote the family of all* Λ_b*-sets (resp.* V_b*-sets) of* (X, τ).

REMARK 2.4 By Proposition 2.1 (e) and (h) we have that:

(a) If $B \in BO(X, \tau)$, then B is a Λ_b-set.

(b) If $B \in BC(X, \tau)$, then B is a V_b-set.

THEOREM 2.5 (a) The subsets $\emptyset$ and X are Λ_b-sets and V_b-sets.

(b) Every union of Λ_b-sets (resp. V_b-sets) is a Λ_b-set (resp. V_b-set).

(c) Every intersection of Λ_b-sets (resp. V_b-sets) is a Λ_b-set (resp. V_b-set).

(d) A subset B is a Λ_b-set if and only if B^c is a V_b-set.

PROOF. (a) and (d) are obvious.

(b) Let $\{B_\lambda \colon \lambda \in \Omega\}$ be a family of Λ_b-sets in a topological space (X, τ). Then by Definition 2 and Proposition 2.1 (d), $\bigcup\limits_{\lambda\in\Omega} B_\lambda = \bigcup\limits_{\lambda\in\Omega} B_\lambda^{\Lambda_b} = [\bigcup\limits_{\lambda\in\Omega} B_\lambda]^{\Lambda_b}$.

(c) Let $\{B_\lambda : \lambda \in \Omega\}$ be a family of Λ_b-sets in (X, τ). Then by Proposition 2.1 (h) and Definition 2 $[\bigcap\limits_{\lambda\in\Omega} B_\lambda]^{\Lambda_b} \subseteq \bigcap\limits_{\lambda\in\Omega} B_\lambda^{\Lambda_b} = \bigcap\limits_{\lambda\in\Omega} B_\lambda$. Hence by Proposition 2.1 (a) $\bigcap\limits_{\lambda\in\Omega} B_\lambda = [\bigcap\limits_{\lambda\in\Omega} B_\lambda]^{\Lambda_b}$. □

REMARK 2.6 *By Theorem 2.5, Λ_b (resp. V_b) is a topology on X containing all b-open (resp. b-closed) sets. Clearly (X, Λ_b) and (X, V_b) are Alexandroff spaces [1], i.e. arbitrary intersections of open sets are open.*

A topological space (X, τ) is said to be b-T_1 if for each pair of distinct points x and y of X, there exist a b-open set U_x containing x but not y and a b-open set U_y containing y but not x. It is obvious that (X, τ) is b-T_1 if and only if for each $x \in X$, the singleton $\{x\}$ is b-closed.

THEOREM 2.7 *For a topological space (X, τ), the following properties are equivalent:*
(a) (X, τ) is b-T_1;
(b)Every subset of X is a Λ_b-set;
(c) Every subset of X is a V_b-set.

PROOF. It is obvious that (b) $\Leftrightarrow$ (c).
(a) $\Rightarrow$ (c): Let A be any subset of X. Since $A = \cup\{\{x\} \mid x \in A\}$, A is the union of b-closed sets, hence a V_b-set.
(c) $\Rightarrow$ (a): Since by (c),we have that every singleton is an union of b-closed sets, i.e. it is b-closed, then (X, τ) is an b-T_1 space. □

Recall that a subset A of a topological space (X, τ) is said to be generalized closed (briefly g-closed) [8] if $Cl(A) \subset U$ whenever $A \subset U$ and $U \in \tau$. A topological space (X, τ) is said to be $T_{\frac{1}{2}}$ if every g-closed subset of X is closed. Dunham [6] pointed out that (X, τ) is $T_{\frac{1}{2}}$ if and only if for each $x \in X$ the singleton $\{x\}$ is open or closed.

THEOREM 2.8 *For a topological space (X, τ), the following properties hold:*
(a) (X, Λ_b) and (X, V_b) are $T_{\frac{1}{2}}$,
(b) If (X, τ) is b-T_1, then both (X, Λ_b) and (X, V_b) are discrete spaces.

PROOF. (a) Let $x \in X$. Then $\{x\}$ is either preclosed or open and hence $\{x\}$ is either b-open or b-closed. If $\{x\}$ is b-open, $\{x\} \in \Lambda_b$. If $\{x\}$ is b-closed in (X, τ), then $X \setminus \{x\}$ is b-open and hence $X \setminus \{x\} \in \Lambda_b$. Therefore $\{x\}$ is closed in (X, Λ_b). Hence (X, Λ_b) and (X, V_b) are $T_{\frac{1}{2}}$ spaces.
(b) This follows from Theorem 2.7. □

3 G.Λ_b-sets and g.V_b-sets

In this section, by using the Λ_b-operator and V_b-operator, we introduce the classes of generalized Λ_b-sets ($= g.\Lambda_b$-sets) and generalized V_b-sets ($= g.V_b$-sets) as an analogy of the sets introduced by Maki [9].

DEFINITION 3 In a topological space (X, τ), a subset B is called a *$g.\Lambda_b$-set* of (X, τ) if $B^{\Lambda_b} \subseteq F$ whenever $B \subseteq F$ and F is b-closed.

DEFINITION 4 In a topological space (X, τ), a subset B is called a *$g.V_b$-set* of (X, τ) if B^c is a $g.\Lambda_b$-set of (X, τ).

REMARK 3.1 *We shall see, however, that we obtain nothing new according to the following results.*

PROPOSITION 3.2 For a subset B of a topological space (X, τ), the following properties hold:
(a) B is a $g.\Lambda_b$-set if and only if B is a Λ_b-set,
(b) B is a $g.V_b$-set if and only if B is a V_b-set.

PROOF. (a) Every Λ_b-set is a $g.\Lambda_b$-set. Now, let B be a $g.\Lambda_b$-set. Suppose that $x \in \Lambda_b(B) \setminus B$. It follows from theorems 2.24 and 2.27 of [10] that for each $x \in X$, the singleton $\{x\}$ is preopen or preclosed. If $\{x\}$ is preopen, then $\{x\}$ is b-open and hence $X \setminus \{x\}$ is b-closed. Since $B \subset X \setminus \{x\}$, we have $B^{\Lambda_b} \subset X \setminus \{x\}$ which is a contradiction. If $\{x\}$ is preclosed, $X \setminus \{x\}$ is b-open and $B \subset X \setminus \{x\}$. Therefore, we have $B^{\Lambda_b} \subset X \setminus \{x\}$. This is a contradiction. Hence $B^{\Lambda_b} = B$ and B is a Λ_b-set.
(b) This is proved in a similar way. □

4 The associated topology τ^{Λ_b}

In this section , we define a closure operator C^{Λ_b} and the associated topology τ^{Λ_b} on the topological spaces (X, τ) by using the family of Λ_b-sets .

DEFINITION 5 For any subset B of a topological space (X, τ), define
$C^{\Lambda_b}(B) = \bigcap\{U : B \subseteq U, U \epsilon \Lambda_b\}$ and $Int^{V_b}(B) = \bigcup\{F : B \supseteq F, F \epsilon V_b\}$.

PROPOSITION 4.1 For any subset B of a topological space (X, τ),
(a) $B \subseteq C^{\Lambda_b}(B)$.
(b) $C^{\Lambda_b}(B^c) = (Int^{V_b}(B))^c$.
(c) $C^{\Lambda_b}(\emptyset) = \emptyset$.
(d) Let $\{B_\lambda : \lambda \epsilon \Omega\}$ be a family of (X, τ). Then $\bigcup\limits_{\lambda \epsilon \Omega} C^{\Lambda_b}(B_\lambda) = C^{\Lambda_b}(\bigcup\limits_{\lambda \epsilon \Omega} B_\lambda)$.
(e) $C^{\Lambda_b}(C^{\Lambda_b}(B)) = C^{\Lambda_b}(B)$.
(f) If $A \subseteq B$ then $C^{\Lambda_b}(A) \subseteq C^{\Lambda_b}(B)$.
(g) If B is a Λ_b-set then $C^{\Lambda_b}(B) = B$.
(h) If B is a V_b-set then $Int^{V_b}(B) = B$.

Proof. (a), (b) and (c): Clear.
(d) Suppose that there exists a point x such that $x \notin C^{\Lambda_b}(\bigcup\limits_{\lambda \epsilon \Omega} B_\lambda)$. Then, there exists a subset $U \epsilon \Lambda_b$ such that $\bigcup\limits_{\lambda \epsilon \Omega} B_\lambda \subseteq U$ and $x \notin U$. Thus, for each $\lambda \epsilon \Omega$ we have $x \notin C^{\Lambda_b}(B_\lambda)$. This implies that $x \notin \bigcup\limits_{\lambda \epsilon \Omega} C^{\Lambda_b}(B_\lambda)$.

Conversely we suppose that there exists a point $x\epsilon X$ such that $x \notin \bigcup_{\lambda\epsilon\Omega} C^{\Lambda_b}(B_\lambda)$. Then, there exist subsets $U_\lambda\epsilon\Lambda_b$ for all $\lambda\epsilon\Omega$, such that $x \notin U_\lambda$, $B_\lambda \subseteq U_\lambda$. Let $U = \bigcup_{\lambda\epsilon\Omega} U_\lambda$. From this and Proposition 2.1(c) we have that $x \notin U$, $\bigcup_{\lambda\epsilon\Omega} B_\lambda \subseteq U$ and $U\epsilon\Lambda_b$. Thus, $x \notin C^{\Lambda_b}(\bigcup_{\lambda\epsilon\Omega} B_\lambda)$.
(e) Suppose that there exists a point $x\epsilon X$ such that $x \notin C^{\Lambda_b}(B)$. Then there exists a subset $U\epsilon\Lambda_b$ such that $x \notin U$ and $U \supseteq B$. Since $U\epsilon\Lambda_b$ we have $C^{\Lambda_b}(B) \subseteq U$. Thus we have $x \notin C^{\Lambda_b}(C^{\Lambda_b}(B))$. Therefore $C^{\Lambda_b}(C^{\Lambda_b}(B)) \subseteq C^{\Lambda_b}(B)$. The converse containment relation is clear by (a).
(f) Clear.
(g) By (a) and Definition 5, the proof is clear.
(h) By Definition 5, by (g) and (b). □

Then we have the following :

THEOREM 4.2 C^{Λ_b} is a Kuratowski closure operator on X.

DEFINITION 6 Let τ^{Λ_b} be the topology on X generated by C^{Λ_b} in the usual manner, i.e., $\tau^{\Lambda_b} = \{B : B \subseteq X, C^{\Lambda_b}(B^c) = B^c\}$.
We define a family ρ^{Λ_b}, by $\rho^{\Lambda_b} = \{B : B \subseteq X, C^{\Lambda_b}(B) = B\}$
By Definition 6, $\rho^{\Lambda_b} = \{B : B \subseteq X, B^c\epsilon\tau^{\Lambda_b}\}$.

PROPOSITION 4.3 Let (X, τ) be a topological space. Then ,
(a) $\tau^{\Lambda b} = \{B : B \subseteq X, Int^{V_b}(B) = B\}$.
(b) $\Lambda_b = \rho^{\Lambda_b}$.
(c) $V_b = \tau^{\Lambda_b}$.
(d) If $BC(X, \tau) = \tau^{\Lambda_b}$ then every Λ_b-set of (X, τ) is b-open (i.e., $BO(X, \tau) = \Lambda_b$).
(e) If every Λ_b-set of (X, τ) is b-open (i.e., $\Lambda_b \subseteq BO(X, \tau)$), then $\tau^{\Lambda_b} = \{B : B \subseteq X, B = B^{V_b}\}$.
(f) If every Λ_b-set of (X, τ) is b-closed (i.e., $\Lambda_b \subseteq BC(X, \tau)$), then $BO(X, \tau) = \tau^{\Lambda_b}$.

Proof. (a) By Definition 6 and Proposition 4.1(b) we have,
if $A \subset X$ then $A\epsilon\tau^{\Lambda_b}$ if and only if $C^{\Lambda_b}(A^c) = A^c$, if and only if $(Int^{V_b}(A))^c = A^c$, if and only if $Int^{V_b}(A) = A$ if and only if, $A\epsilon\{B : B \subset X, Int^{V_b}(B) = B\}$.

(b) Let B be a subset of X. By Proposition 2.1(e) $BO(X,\tau) \subset \Lambda_b$ and $C^{\Lambda_b}(B) = \bigcap\{U \mid B \subset U, U \in \Lambda_b\} \subset \bigcap\{U \mid B \subset U, U \in BO(X,\tau)\} = B^{\Lambda_b}$. Therefore, we have $C^{\Lambda_b}(B) \subset B^{\Lambda_b}$. Now suppose that $x \notin C^{\Lambda_b}(B)$. There exists $U \in \Lambda_b$ such that $B \subset U$ and $x \notin U$. Since $U \in \Lambda_b$, $U = U^{\Lambda_b} = \{V \mid U \subset V \in BO(X,\tau)\}$ and hence there exists $V \in BO(X,\tau)$ such that $U \subset V$ and $x \notin V$. Thus, $x \notin V$ and $B \subset V \in BO(X,\tau)$. This shows that $x \notin B^{\Lambda_b}$. Therefore, $B^{\Lambda_b} \subset C^{\Lambda_b}(B)$ and hence $B^{\Lambda_b} = C^{\Lambda_b}(B)$ for any subset B of X. By the definitions of Λ_b and ρ^{Λ_b}, we obtain $\Lambda_b = \rho^{\Lambda_b}$.
(c) Let $B \in \tau^{\Lambda_b}$. Then $C^{\Lambda_b}(B^c) = B^c$ and $B^c \in \rho^{\Lambda_b}$. By (b), $B^c \in \Lambda_b$ and $B^c = (B^c)^{\Lambda_b}$. Therefore, by Proposition 2.1(f) $B^c = (B^{V_b})^c$ and $B = B^{V_b}$. This shows that $B \in V_b$. Consequently, we obtain $\tau^{\Lambda_b} \subset V_b$. Quite similarly, we obtain $\tau^{\Lambda_b} \supset V_b$ and hence $V_b = \tau^{\Lambda_b}$.
(d) Let B be any Λ_b-set i.e., $B \epsilon \Lambda_b$. By (b), $B \epsilon \rho^{\Lambda_b}$ thus, $B^c \epsilon \tau^{\Lambda_b}$. From the assumption we have $B^c \epsilon BC(X,\tau)$ and hence $B \epsilon BO(X,\tau)$.
(e) Let $A \subseteq X$ and $A \epsilon \tau^{\Lambda_b}$. Then by Definitions 5 and 6
$A^c = C^{\Lambda_b}(A^c) = \bigcap\{U : U \supseteq A^c, U \epsilon \Lambda_b\} = \bigcap\{U : U \supseteq A^c, U \epsilon BO(X,\tau)\} = (A^c)^{\Lambda_b}$.
Using Proposition 2.1(f) we have $A = A^{V_b}$, i.e., $A \epsilon \{B : B \subseteq X, B = B^{V_b}\}$.
Conversely ,if $A \epsilon \{B : B \supseteq X, B = B^{V_b}\}$ then by Proposition 3.2(b)) A is a $g.V_b$-set. Thus $A \epsilon V_b$. By using (c) $A \epsilon \tau^{\Lambda_b}$.
(f) Let $A \subseteq X$ and $A \epsilon \tau^{\Lambda_b}$. Then
$A = (C^{\Lambda_b}(A^c))^c = (\bigcap\{U : A^c \subseteq U, U \epsilon \Lambda_b\})^c = \bigcup\{U^c : U^c \subseteq A, U \in \Lambda_b\}$.
Conversely, if $A \epsilon BO(X,\tau)$ then by (b) $A \epsilon \Lambda_b$. By assumption $A \epsilon BC(X,\tau)$. By using (c) $A \epsilon \tau^{\Lambda_b}$. □

PROPOSITION 4.4 If $BO(X,\tau) = \tau^{\Lambda_b}$, then (X, τ^{Λ_b}) is a discrete space.

Proof. Suppose that $\{x\}$ is not b-open in (X,τ). Then $\{x\}$ is b-closed in (X,τ). Thus $\{x\} \epsilon \tau^{\Lambda_b}$ by Proposition 4.3 (c). Suppose that $\{x\}$ is b-open in (X,τ), then $\{x\} \epsilon BO(X,\tau) = \tau^{\Lambda_b}$. Therefore, every singleton $\{x\}$ is τ^{Λ_b}-open and hence every subset of X is τ^{Λ_b}-open. □

Acknowledgement. The authors are very grateful to the referee for his observations which improved the value of this paper.

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