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Compact Open Topology and Evaluation Map via Neutrosophic Sets

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Abstract: The concept of neutrosophic locally compact and neutrosophic compact open topology are introduced and some interesting propositions are discussed.

Keywords: neutrosophic locally Compact Hausdorff space; neutrosophic product topology; neutrosophic compact open topology; neutrosophic homeomorphism; neutrosophic evaluation map; Exponential map.

1 Introduction and Preliminaries

In 1965, Zadeh [19] introduced the useful notion of a fuzzy set and Chang [6] three years later offered the notion of fuzzy topological space. Since then, several authors have generalized numerous concepts of general topology to the fuzzy setting. The concept of intuitionistic fuzzy set was introduced and studied by Atanassov [1] and subsequently some important research papers published by him and his colleagues [2,3,4]. The concept of fuzzy compact open topology was introduced by S.Dang and A . Behera[9]. The concepts of intuitionistic evaluation maps by R.Dhavaseelan et al[9]. After the introduction of the concepts of neutrosophy and neutrosophic set by F. Smarandache [[11], [12]], the concepts of neutrosophic crisp set and neutrosophic crisp topological spaces were introduced by A. A. Salama and S. A. Alblowi[10].

In this paper the notion of neutrosophic compact open topology is introduced. Some interesting properties are discussed. Moreover, neutrosophic local compactness and neutrosophic product topology are developed. We have also utilized the notion of fuzzy locally compactness due to Wong[17], Christoph [8] and fuzzy product topology due to Wong [18].

Throughout this paper neutrosophic topological spaces (X,T),(Y,S) and (Z,R) will be replaced by X,Y and Z respectively.

Definition 1.1. Let T,I,F be real standard or non standard subsets of $]0^-,1^+[$, with $sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n-sup = t_{sup} + i_{sup} + f_{sup}$
$n-inf = t_{inf}+i_{inf}+f_{inf}$. T,I,F are neutrosophic components.

Definition 1.2. Let X be a nonempty fixed set. A neutrosophic set [briefly NS] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

Remark 1.1. (1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^-,1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

We introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 1.3. $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 1.4. [8] A neutrosophic topology (NT) on a nonempty set X consists of a family T of neutrosophic sets in X which satisfies the following:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (NTS) and each neutrosophic set in T is called a neutrosophic open set (NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (NCS) in X.

Definition 1.5. [8] Let A be a neutrosophic subset of a neutrosophic topological space X. The neutrosophic interior and neutrosophic closure of A are denoted and defined by

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and

$G \subseteq A\}$;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$.

2 Neutrosophic Locally Compact and Neutrosophic Compact Open Topology

Definition 2 .1. Let X be a nonempty set and $x \in X$ a fixed element in X. If $r, t \in I_0 = (0, 1]$ and $s \in I_1 = [0, 1)$ are fixed real numbers such that $0 < r + t + s < 3$, then $x_{r,t,s} = \langle x, r, t, s\rangle$ is called a neutrosophic point (in short NP) in X, where r denotes the degree of membership of $x_{r,t,s}$, t denotes the degree of indeterminacy and s denotes the degree of nonmembership of $x_{r,t,s}$ and $x \in X$ the support of $x_{r,t,s}$.

The neutrosophic point $x_{r,t,s}$ is contained in the neutrosophic $A(x_{r,t,s} \in A)$ if and only if $r < \mu_A(x), t < \sigma_A(x), s > \gamma_A(x)$.

Definition 2.2. A neutrosophic set $A = \langle x, \mu_A, \sigma_A, \gamma_A\rangle$ in a neutrosophic topological space (X, T) is said to be a neutrosophic neighbourhood of a neotrosophic point $x_{r,t,s}, x \in X$, if there exists a neutrosophic open set $B = \langle x, \mu_B, \sigma_B, \gamma_B\rangle$ with $x_{r,t,s} \subseteq B \subseteq A$.

Definition 2.3. *Let X and Y be neutrosophic topological spaces.A mapping $f : X \to Y$ is said to be a neutrosophic homeomorphism if f is bijective, neutrosophic continuous and neutrosophic open.*

Definition 2.4. *An neutrosophic topological space (X, T) is called a neutrosophic Hausdorff space or T_2-space if for any pair of distinct neutrosophic points(i.e., neutrosophic points with distinct supports) $x_{r,t,s}$ and $y_{u,v,w}$,there exist neutrosophic open sets U and V such that $x_{r,t,s} \in U$,$y_{u,v,w} \in V$ and $U \wedge V = 0_N$*

Definition 2.5. *An neutrosophic topological space (X, T) is said to be neutrosophic locally compact if and only if for every neutrosophic point $x_{r,t,s}$ in X, there exists a neutrosophic open set $U \in T$ such that $x_{r,t,s} \in U$ and U is neutrosophic compact,i.e., each neutrosophic open cover of U has a finite subcover.*

Definition 2.6. *Let $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ and $B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y)\rangle$ be neutrosophic sets of X and Y respectively.The product of two neutrosophic sets A and B in a neutrosophic topological space X is defined as*
$(A\times B)(x, y) = \langle (x, y), min(\mu_A(x), \mu_B(y)), min(\sigma_A(x), \sigma_B(y)), max(\gamma_A(x), \gamma_B(y))\rangle$ *for all* $(x, y) \in X \times Y$.

Definition 2.7. *Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined by: $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))\ \forall(x_1, x_2) \in X_1 \times X_2$.*

Lemma 2.1. *Let $f_i : X_i \to Y_i\ (i = 1, 2)$ be functions and U, V are neutrosophic sets of Y_1, Y_2, respectively, then $(f_1 \times f_2)^{-1}(U \times V) = f_1^{-1}(U) \times f_2^{-1}(V)\ \forall\, U \times V \in Y_1 \times Y_2$*

Definition 2.8. *A mapping $f : X \to Y$ is neutrosophic continuous iff for each neutrosophic point $x_{r,t,s}$ in X and each neutrosophic neighbourhood B of $f(x_{r,t,s})$ in Y,there is a neutrosophic neighbourhood A of $x_{r,t,s}$ in X such that $f(A) \subseteq B$.*

Definition 2.9. *A mapping $f : X \to Y$ is said to be neutrosophic homeomorphism if f is bijective ,neutrosophic continuous and neutrosophic open.*

Definition 2.10. *A neutrosophic topological space X is called a neutrosophic Hausdorff space or T_2 space if for any distinct neutrosophic points $x_{r,t,s}$ and $y_{u,v,w}$,there exists neutrosophic open sets G_1 and G_2, such that $x_{r,t,s} \in G_1$,$y_{u,v,w} \in G_2$ and $G_1 \cap G_2 = 0_\sim$*

Definition 2.11. *A neutrosophic topological space X is said to be a neutrosophic locally compact iff for any neutrosophic point $x_{r,t,s}$ in X, there exists a neutrosophic open set $U \in T$ such that $x_{r,t,s} \in U$ and U is neutrosophic compact that is, each neutrosophic open cover of U has a finite subcover.*

Proposition 2.1. In a neutrosophic Hausdorff topological space X, the following conditions are equivalent.

(a) X is a neutrosophic locally compact

(b) for each neutrosophic point $x_{r,t,s}$ in X, there exists a neutrosophic open set G in X such that $x_{r,t,s} \in G$ and $Ncl(G)$ is neutrosophic compact

Proof. $(a) \Rightarrow (b)$ By hypothesis for each neutrosophic point $x_{r,t,s}$ in X, there exists a neutrosophic open set G which is neutrosophic compact.Since X is neutrosophic Hausdorff (neutrosophic compact subspace of neutrosophic Hausdorff space is neutrosophic closed), G is neutrosophic closed, thus $G = Ncl(G)$. Hence $x_{r,t,s} \in G$ and $Ncl(G)$ is neutrosophic compact.
$(b) \Rightarrow (a)$ Proof is simple. □

Proposition 2.2. Let X be a neutrosophic Hausdorff topological space.Then X is neutrosophic locally compact at a neutrosophic point $x_{r,t,s}$ in X iff for every neutrosophic open set G containing $x_{r,t,s}$ there exists a neutrosophic open set V such that $x_{r,t,s} \in V$, $Ncl(V)$ is neutrosophic compact and $Ncl(V) \subseteq G$.

Proof. Suppose that X is neutrosophic locally compact at a neutrosophic point $x_{r,t,s}$. By Definition 2.11, there exists a neutrosophic open set G such that $x_{r,t,s} \in G$ and G is neutrosophic compact. Since X is a neutrosophic Hausdorff space,(neutrosophic compact subspace of neutrosophic Hausdorff space is neutrosophic closed), G is neutrosophic closed. Thus $G = Ncl(G)$. Consider a neutrosophic point $x_{r,t,s} \in \overline{G}$. Since X is neutrosophic Hausdorff space, by Definition 2.10, there exist neutrosophic open sets C and D such that $x_{r,t,s} \in C$, $y_{u,v,w} \in D$ and $C \cap D = 0_\sim$. Let $V = C \cap G$. Hence $V \subseteq G$ implies $Ncl(V) \subseteq Ncl(G) = G$. Since $Ncl(V)$ is neutrosophic closed and G is neutrosophic compact, (every neutrosophic closed subset of a neutrosophic compact space is neutrosophic compact) it follows that $Ncl(V)$ is neutrosophic compact. Thus $x_{r,t,s} \in Ncl(V) \subseteq G$ and $Ncl(G)$ is neutrosophic compact.

The converse follows from Proposition 2.1(b). □

Definition 2.12. *Let X and Y be two neutrosophic topological spaces.The function $T : X \times Y \to Y \times X$ defined by $T(x, y) = (y, x)$ for each $(x, y) \in X \times Y$ is called a switching map.*

Proposition 2.3. The switching map $T : X \times Y \to Y \times X$ defined as above is neutrosophic continuous.

We now introduce the concept of a neutrosophic compact open topology in the set of all neutrosophic continuous functions from a neutrosophic topological space X to a neutrosophic topological space Y.

Definition 2.13. *Let X and Y be two neutrosophic topological spaces and let $Y^X = \{f : X \to Y$ such that f is neutrosophic continuous$\}$. We give this class Y^X a topology called the neutrosophic compact open topology as follows: Let $\mathcal{K} = \{K \in I^X :$ K is neutrosophic compact on $X\}$ and $\mathcal{V} = \{V \in I^Y : V$ is neutrosophic open in $Y\}$.For any $K \in \mathcal{K}$ and $V \in \mathcal{V}$,let $S_{K,V} = \{f \in Y^X : f(K) \subseteq V\}$.*

The collection of all such $\{S_{K,V} : K \in \mathcal{K}, V \in \mathcal{V}\}$ is a neutrosophic subbase to generate a neutrosophic topology on the class Y^X. The class Y^X with this topology is called a neutrosophic compact open topological space.

3 Neutrosophic Evaluation Map and Exponential Map

We now consider the neutrosophic product topological space $Y^X \times X$ and define a neutrosophic continuous map from $Y^X \times X$ into Y.

Definition 3.1. *The mapping $e : Y^X \times X \to Y$ defined by $e(f, x_{r,t,s}) = f(x_{r,t,s})$ for each neutrosophic point $x_{r,t,s} \in X$ and $f \in Y^X$ is called the neutrosophic evaluation map.*

Definition 3.2. *Let X,Y,Z be neutrosophic topological spaces and $f : Z \times X \to Y$ be any function. Then the induced map $\widehat{f} : X \to Y^Z$ is defined by $(\widehat{f}(x_{r,t,s}))(z_{t,u,v}) = f(z_{t,u,v}, x_{r,t,s})$ for neutrosophic point $x_{r,t,s} \in X$ and $z_{t,u,v} \in Z$.*
Conversely, given a function $\widehat{f} : X \to Y^Z$, a corresponding function f can also be defined by the same rule.

Proposition 3.1. Let X be a neutrosophic locally compact Hausdorff space. Then the neutrosophic evaluation map $e : Y^X \times X \to Y$ is neutrosophic continuous.

Proof. Consider $(f, x_{r,t,s}) \in Y^X \times X$,where $f \in Y^X$ and $x_{r,t,s} \in X$.Let V be a neutrosophic open set containing $f(x_{r,t,s}) = e(f, x_{r,t,s})$ in Y. Since X is neutrosophic locally compact and f is neutrosophic continuous, by Proposition 2.2, there exists a neutrosophic open set U in X such that $x_{r,t,s} \in Ncl(U)$ is neutrosophic compact and $f(Ncl(U)) \subseteq V$.

Consider the neutrosophic open set $S_{Ncl(U),V} \times U$ in $Y^X \times X$. Clearly $(f, x_{r,t,s}) \in S_{Ncl(U),V} \times U$.Let $(g, x_{t,u}) \in S_{Ncl(U),V} \times U$ be arbitrary. Thus $g(Ncl(U)) \subseteq V$. Since $x_{t,u} \in U$,we have $g(x_{t,u}) \in V$ and $e(g, x_{t,u}) = g(x_{t,u}) \in V$.Thus $e(S_{Ncl(U),V} \times U) \subseteq V$.Hence e is neutrosophic continuous. □

Proposition 3.2. Let X and Y be two neutrosophic topological spaces with Y being neutrosophic compact. Let $x_{r,t,s}$ be any neutrosophic point in X and N be a neutrosophic open set in the neutrosophic product space $X \times Y$ containing $\{x_{r,t,s}\} \times Y$. Then there exists some neutrosophic neighbourhood W of $x_{r,t,s}$ in X such that $\{x_{r,t,s}\} \times Y \subseteq W \times Y \subseteq N$.

Proposition 3.3. Let Z be a neutrosophic locally compact Hausdorff space and X, Y be arbitrary neutrosophic topological spaces. Then a map $f : Z \times X \to Y$ is neutrosophic continuous iff $\widehat{f} : X \to Y^Z$ is neutrosophic continuous,where $\widehat{f}$ is defined by the rule $(\widehat{f}(x_{r,t,s}))(z_{t,u,v}) = f(z_{t,u,v}, x_{r,t,s})$.

Proposition 3.4. Let X and Z be a neutrosophic locally compact Hausdorff spaces. Then for any neutrosophic topological space Y,the function $E : Y^{Z\times X} \to (Y^Z)^X$ defined by $E(f) = \widehat{f}$(that is $E(f)(x_{r,t,s})(z_{t,u,v}) = f(z_{t,u,v}, x_{r,t,s}) = (\widehat{f}(x_{r,t,s})(z_{t,u,v})))$ for all $f : Z \times X \to Y$ is a neutrosophic homeomorphism.

Proof. (a) Clearly E is onto.

(b) For E to be injective, let $E(f) = E(g)$ for $f, g : Z \times X \to Y$. Thus $\widehat{f} = \widehat{g}$, where $\widehat{f}$ and $\widehat{g}$ are the induced map of f and g, respectively. Now for any neutrosophic point $x_{r,t,s}$ in X and any neutrosophic point $z_{t,u,v}$ in Z, $f(z_{t,u,v}, x_{r,t,s}) = (\widehat{f}(x_{r,t,s})(z_{t,u,v})) = (\widehat{g}(x_{r,t,s})(z_{t,u,v})) = g(z_{t,u,v}, x_{r,t,s})$. Thus $f = g$.

(c) For proving the neutrosophic continuity of E, consider any neutrosophic subbasis neighbourhood V of $\widehat{f}$ in $(Y^Z)^X$, i.e V is of the form $S_{K,W}$ where K is a neutrosophic compact subset of X and W is neutrosophic open in Y^Z. Without loss of generality, we may assume that $W = S_{L,U}$, where L is a neutrosophic compact subset of Z and U is a neutrosophic open set in Y. Then $\widehat{f}(K) \subseteq S_{L,U} = W$ and this implies that $\widehat{f}(K)(L) \subseteq U$. Thus for any neutrosophic point $x_{r,t,s}$ in K and for every neutrosophic point $z_{t,u,v}$ in L, we have $(\widehat{f}(x_{r,t,s}))(z_{t,u,v}) \in U$, that is $f(z_{t,u,v}, x_{r,t,s}) \in U$ and therefore $f(L \times K) \subseteq U$. Now since L is a neutrosophic compact in Z and K is a neutrosophic compact in X, $L \times K$ is also a neutrosophic compact in $Z \times X$[7] and since U is a neutrosophic open set in Y, we conclude that $f \in S_{L\times K,U} \subseteq Y^{Z\times X}$. We assert that $E(S_{L\times K,U}) \subseteq S_{K,W}$. Let $g \in S_{L\times K,U}$ be arbitrary. Thus $g(L \times K) \subseteq U$, i.e $g(z_{t,u,v}, x_{r,t,s}) = (\widehat{g}(x_{r,t,s}))(z_{t,u,v}) \in U$ for all neutrosophic points $z_{t,u,v} \in L \subseteq Z$ and for every neutrosophic point $x_{r,t,s} \in L \subseteq X$. So $(\widehat{g}(x_{r,t,s}))(L) \subseteq U$ for every neutrosophic point $x_{r,t,s} \in K \subseteq X$, that is $(\widehat{g}(x_{r,t,s})) \in S_{L,U} = W$ for every neutrosophic points $x_{r,t,s} \in K \subseteq X$, that is $\widehat{g}(x_{r,t,s}) \in S_{L,U} = W$ for every neutrosophic point $x_{r,t,s} \in K \subseteq U$. Hence we have $\widehat{g}(K) \subseteq W$, that is $\widehat{g} = E(g) \in S_{K,W}$ for any $g \in S_{L\times K,U}$.

Thus $E(S_{L\times K,U}) \subseteq S_{K,W}$. This proves that E is a neutrosophic continuous.

(d) For proving the neutrosophic continuity of E^{-1},we consider the following neutrosophic evaluation maps: $e_1 : (Y^Z)^X \times X \to Y^Z$ defined by $e_1(\widehat{f}, x_{r,t,s}) = \widehat{f}(x_{r,t,s})$ where $\widehat{f} \in (Y^Z)^X$ and $x_{r,t,s}$ is any neutrosophic point in X and $e_2 : Y^Z \times Z \to Y$ defined by $e_2(g, z_{t,u,v}) = g(z_{t,u,v})$, where $g \in Y^Z$ and $z_{t,u,v}$ is a neutrosophic point in Z. Let ψ denote the composition of the following neutrosophic continuous functions $\psi : (Z \times X) \times (Y^Z)^X \xrightarrow{T} (Y^Z)^X \times (Z \times X) \xrightarrow{i\times t} (Y^Z)^X \times (X \times Z) \xrightarrow{=} ((Y^Z)^X \times X) \times Z \xrightarrow{e_1 \times i_Z} (Y^Z) \times Z \xrightarrow{e_2} Y$, where i, i_Z denote the neutrosophic identity maps on $(Y^Z)^X$ and Z, respectively and T, t denote the switching maps. Thus $\psi : (Z \times X) \times (Y^Z)^X \to Y$, that is $\psi \in Y^{(Z\times X)\times (Y^Z)^X}$. We consider the map $\widetilde{E} : Y^{(Z\times X)\times (Y^Z)^X} \to (Y^{(Z\times X)})^{(Y^Z)^X}$ (as defined in the statement of the Proposition 3.4 in fact it is E). So $\widetilde{E}(\psi) : (Y^Z)^X \to Y^{(Z\times X)}$. Now for any neutrosophic points $z_{t,u,v} \in Z, x_{r,t,s} \in X$ and $f \in Y^{(Z\times X)}$, again we have that $(\widetilde{E}(\psi) \circ E)(f)(z_{t,u,v}, x_{r,t,s}) = f(z_{t,u,v}, x_{r,t,s})$;hence $\widetilde{E}(\psi) \circ E$=identity. Similarly for any $\widehat{g} \in (Y^Z)^X$ and neutrosophic points $x_{r,t,s} \in X, z_{t,u,v} \in Z$, so we have that $(E \circ \widetilde{E}(\psi))(\widehat{g})(x_{r,t,s}, z_{t,u,v}) = (\widehat{g}(x_{r,t,s}))(z_{t,u,v})$;hence $E \circ \widetilde{E}(\psi)$=identity. Thus E is a neutrosophic homeomorphism.

□

Definition 3.3. *The map E in Proposition 3.4 is called the exponential map.*

As easy consequence of Proposition 3.4 is as follows.

Proposition 3.5. Let X, Y, Z be neutrosophic locally compact Hausdorff spaces. Then the map $N : Y^X \times Z^Y \to Z^X$ defined by $N(f, g) = g \circ f$ is neutrosophic continuous.

Proof. Consider the following compositions: $X \times Y^X \times Z^Y \xrightarrow{T} Y^X \times Z^Y \times X \xrightarrow{t\times i_X} Z^Y \times Y^X \times X \xrightarrow{=} Z^Y \times (Y^X \times X) \xrightarrow{i\times e_2} Z^Y \times Y \xrightarrow{e_2} Z$, where T, t denote the switching maps, i_X, i denote the neutrosophic identity functions on X and Z^Y, respectively and e_2 denotes the neutrosophic evaluation maps. Let $\varphi = e_2 \circ (i \times e_2) \circ (t \times i_X) \circ T$. By proposition 3.4, we have an exponential map $E : Z^{X\times Y^X \times Z^Y} \to (Z^X)^{Y^X \times Z^Y}$. Since $\varphi \in Z^{X\times Y^X\times Z^Y}$, $E(\varphi) \in (Z^X)^{Y^X\times Z^Y}$. Let $N = E(\varphi)$ that is, $N : Y^X \times Z^Y \to Z^X$ is neutrosophic continuous. For $f \in Y^X, g \in Z^Y$ and for any neutrosophic point $x_{r,t,s} \in X$,it easy to see that $N(f, g)(x_{r,t,s}) = g(f(x_{r,t,s}))$. □

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