2014 International Conference on Topology and its Applications,
July 3-7, 2014, Nafpaktos,
Greece

Selected papers of the 2014 International Conference on Topology and its Applications

Editors

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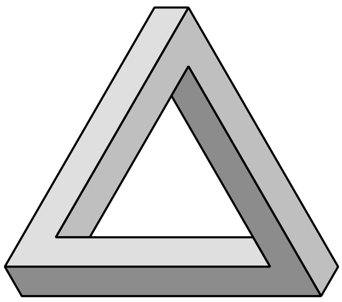

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ISBN: 978-618-80609-1-3

Preface

The **2014 International Conference on Topology and its Applications** took place from July 3 to 7 in the **3rd High School of Nafpaktos, Greece**. It covered all areas of Topology and its Applications (especially General Topology, Set-Theoretic Topology, Geometric Topology, Topological Groups, Dimension Theory, Dynamical Systems and Continua Theory, Computational Topology, History of Topology). This conference was attended by 235 participants from 44 countries and the program consisted by 147 talks.

The Organizing Committee consisted of S.D. Iliadis (University of Patras), D.N. Georgiou (University of Patras), I.E. Kougias (Technological Educational Institute of Western Greece), A.C. Megaritis (Technological Educational Institute of Western Greece), and I. Boules (Mayor of the city of Nafpaktos).

The Organizing Committee is very much indebted to the City of Nafpaktos for its hospitality and for its excellent support during the conference.

The conference was sponsored by University of Patras, Technological Educational Institute of Western Greece, Municipality of Nafpaktos, New Media Soft – Internet Solutions, Loux Marlafekas A.B.E.E., TAXYTYPO – TAXYEKTYPOSEIS GRAVANIS EPE, Alpha Bank, and Wizard Solutions LTD.

This volume is a special volume under the title: "Selected papers of the 2014 International Conference on Topology and its Applications" which will be edited by the organizers (D.N. Georgiou, S.D. Iliadis, I.E. Kougias, and A.C. Megaritis) and published by the University of Patras. We thank the authors for their submissions.

Editors

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Selected papers of the 2014 International Conference on Topology and its Applications
(2015) Pages 159–167

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Some properties of $\widetilde{G}\alpha$-closed graphs

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Abstract

R.Devi et al. [4] introduced the concept of $\widetilde{g}\alpha$-open sets. In this paper, we introduce and study some properties of functions with ultra $\widetilde{g}\alpha$-closed graphs and strongly $\widetilde{g}\alpha$-closed graphs by utilizing $\widetilde{g}\alpha$-open sets and the $\widetilde{g}\alpha$-closure operator.

Key words: $\widetilde{g}\alpha$-open set, ultra $\widetilde{g}\alpha$-closed graphs, strongly $\widetilde{g}\alpha$-closed graph, $\widetilde{g}\alpha$-Urysohn space.
1991 MSC: 54A05, 54D05 54D10, 54D45.

1. Introduction and Preliminaries

Quite recently, R.Devi et al. [4] introduced the notion of $\widetilde{g}\alpha$-open sets in topological spaces and introduced the concept of $\widetilde{g}\alpha$-closure of a set by utilizing the notion of $\widetilde{g}\alpha$-open sets defined in [4]. In 2009, the concept of functions with strongly λ-closed graphs was introduced and studied by M.Caldas et al. [1]. In this paper, we introduce and study some properties of functions with ultra $\widetilde{g}\alpha$-closed graphs and strongly $\widetilde{g}\alpha$-closed graphs by utilizing $\widetilde{g}\alpha$-open sets and the $\widetilde{g}\alpha$-closure operator.

Throughout this paper, by (X, τ) and (Y, σ) (or X and Y) we always mean topological spaces. For a subset A of a space (X, τ), $cl(A)$ and $int(A)$ denote the closure of A and the interior of A respectively.

We recall the following definitions, which are useful in the sequel.

Definition 1.1. A subset A of a space (X, τ) is called

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1. a semi-open set [6] if $A \subseteq cl(int(A))$ and a semi-closed set [6] if $int(cl(A)) \subseteq A$ and
2. an α-open set [7] if $A \subseteq int(cl(int(A)))$ and an α-closed set [7] if $cl(int(cl(A))) \subseteq A$.

The semi-closure (resp. α-closure) of a subset A of a space (X, τ) is the intersection of all semi-closed (resp. α-closed) sets that contain A and is denoted by $scl(A)$ (resp. $\alpha cl(A)$).

Definition 1.2. A subset A of a space (X, τ) is called

1. a $\widehat{g}$-closed set [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ); the complement of a $\widehat{g}$-closed set is called a $\widehat{g}$-open set,
2. a $^*g$-closed set [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\widehat{g}$-open in (X, τ); the complement of a $^*g$-closed set is called a $^*g$-open set,
3. a $^\sharp gs$-closed set [10] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^*g$-open in (X, τ); the complement of a $^\sharp gs$-closed set is called a $^\sharp gs$-open set and
4. a $\widetilde{g}\alpha$-closed set [4] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^\sharp gs$-open in (X, τ); the complement of a $\widetilde{g}\alpha$-closed set is called a $\widetilde{g}\alpha$-open set.

Notation 1.3. For a topological space (X, τ), $\widetilde{G}\alpha C(X, \tau)$ (resp. $\widetilde{G}\alpha O(X, \tau)$) denotes the class of all $\widetilde{g}\alpha$-closed (resp. $\widetilde{g}\alpha$-open) subsets of (X, τ). We set $\widetilde{G}\alpha O(X, x) = \{U : x \in U \text{ and } U \in \widetilde{G}\alpha O(X, \tau)\}$.

Definition 1.4. A function $f : (X, \tau) \to (Y, \sigma)$ is called a

1. $\widetilde{g}\alpha$-continuous [3] if $f^{-1}(V)$ is $\widetilde{g}\alpha$-closed in (X, τ) for every closed set V of (Y, σ) and
2. $\widetilde{g}\alpha$-irresolute [3] if $f^{-1}(V)$ is $\widetilde{g}\alpha$-closed in (X, τ) for every $\widetilde{g}\alpha$-closed set V of (Y, σ).

Definition 1.5. [2]

(i) A space X is said to be $\widetilde{g}\alpha$-T_1 if for each pair of distinct points x and y in X, there exists $\widetilde{g}\alpha$-open sets U and V containing x and y respectively, such that $y \notin U$ and $x \notin V$.

(ii) A space X is said to be $\widetilde{g}\alpha$-T_2 if for each pair of distinct points x and y in X, there exists $\widetilde{g}\alpha$-open sets U and V containing x and y respectively, such that $U \cap V = \phi$.

2. Ultra $\widetilde{g}\alpha$-Closed Graphs

If $f : (X, \tau) \to (Y, \sigma)$ is any function, then the subset

$$G(f) = \{(x, f(x)) : x \in X\}$$

of the product space $(X \times Y, \tau \times \sigma)$ is called graph of f ([5]).

Definition 2.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to have a ultra $\widetilde{g}\alpha$-closed graph if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in \widetilde{G}\alpha O(X, x)$ and $V \in \widetilde{G}\alpha O(Y, y)$ such that $f(U) \cap \widetilde{g}\alpha cl(V) = \phi$.

Theorem 2.2. If $f : (X, \tau) \to (Y, \sigma)$ is a function with a ultra $\widetilde{g}\alpha$-closed graph, then for each $x \in X$, $f(x) = \cap\{\widetilde{g}\alpha cl(f(U))|U \in \widetilde{G}\alpha O(X, x)\}$.

Proof. Suppose the theorem is false. Then there exists a $y \neq f(x)$ such that $y \in \cap\{\widetilde{g}\alpha cl(f(U))|U \in \widetilde{G}\alpha O(X, x)\}$. This implies that $y \in \widetilde{g}\alpha cl(f(U))$, for every $U \in \widetilde{G}\alpha O(X, x)$. So $V \cap f(U) \neq \phi$ for every $V \in \widetilde{G}\alpha O(Y, y)$. This indicates that $\widetilde{g}\alpha cl(V) \cap f(U) \supset V \cap f(U) \neq \phi$ which contradicts the hypothesis that f is a function with a ultra $\widetilde{g}\alpha$-closed graph. Hence the theorem holds. ■

Theorem 2.3. If $f : (X, \tau) \to (Y, \sigma)$ is $\widetilde{g}\alpha$-irresolute and Y is $\widetilde{g}\alpha$-T_2, then $G(f)$ is ultra $\widetilde{g}\alpha$-closed.

Proof. Let $(x, y) \in (X \times Y) - G(f)$ and $V \in \widetilde{G}\alpha O(Y, y)$ such that $f(x) \notin \widetilde{g}\alpha cl(V)$. It follows that there is $U \in \widetilde{G}\alpha O(X, x)$ such that $f(U) \subset Y - \widetilde{g}\alpha cl(V)$. Hence, $f(U) \cap \widetilde{g}\alpha cl(V) = \phi$. ■

The converse need not be true by the following example.

Example 2.4. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and define the identity map $f : (X, \tau) \to (X, \tau)$. Then f is clearly $\widetilde{g}\alpha$-irresolute and X is not $\widetilde{g}\alpha$-T_2 space. Hence we obtain $G(f)$ is not ultra $\widetilde{g}\alpha$-closed.

Theorem 2.5. If $f : (X, \tau) \to (Y, \sigma)$ is surjective and has a ultra $\widetilde{g}\alpha$-closed graph $G(f)$, then Y is both $\widetilde{g}\alpha$-T_2 and $\widetilde{g}\alpha$-T_1.

Proof. Let $y_1, y_2 (y_1 \neq y_2) \in Y$. The surjectivity of f gives a $x_1 \in X$ such that $f(x_1) = y_1$. Now $(x_1, y_2) \in (X \times Y) - G(f)$. The ultra $\widetilde{g}\alpha$-closed graph $G(f)$ gives $U \in \widetilde{G}\alpha O(X, x_1)$ and $V \in \widetilde{G}\alpha O(Y, y_2)$ such that $f(U) \cap \widetilde{g}\alpha cl(V) = \phi$, since $y_1 \notin \widetilde{g}\alpha cl(V)$. This means that there exists $W \in \widetilde{G}\alpha O(Y, y_1)$ such that $W \cap V = \phi$. So, Y is $\widetilde{g}\alpha$-T_2 and hence is $\widetilde{g}\alpha$-T_1. ■

Theorem 2.6. If $f : (X, \tau) \to (Y, \sigma)$ is an injection and $G(f)$ is ultra $\widetilde{g}\alpha$-closed, then X is $\widetilde{g}\alpha$-T_1.

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. Then $(x_1, f(x_2)) \in (X \times Y) - G(f)$. Since $G(f)$ is ultra $\widetilde{g}\alpha$-closed, there exist $U \in \widetilde{G}\alpha O(X, x_1)$ and $V \in \widetilde{G}\alpha O(Y, f(x_2))$ such that $f(U) \cap \widetilde{g}\alpha cl(V) = \phi$. Therefore, $x_2 \notin U$. We obtain a set $W \in \widetilde{G}\alpha O(X, x_2)$ such that $x_1 \notin W$. Hence, X is $\widetilde{g}\alpha$-T_1. ■

Theorem 2.7. If $f : (X, \tau) \to (Y, \sigma)$ is bijective function with ultra $\widetilde{g}\alpha$-closed graph $G(f)$, then (X, τ) and (Y, σ) are $\widetilde{g}\alpha$-T_1 space.

Proof. The proof is an immediate consequence of Theorem 2.5. and Theorem 2.6. ■

Theorem 2.8. A space X is $\widetilde{g}\alpha$-T_2 if and only if the identity function $f : (X, \tau) \to (X, \tau)$ has a ultra $\widetilde{g}\alpha$-closed graph $G(f)$.

Proof. Necessity. Let X be a $\widetilde{g}\alpha$-T_2 space. Since the identity function $f : (X, \tau) \to (X, \tau)$ is $\widetilde{g}\alpha$-irresolute, it follows from Theorem 2.3, that $G(f)$ is ultra $\widetilde{g}\alpha$-closed.
Sufficiency. Let $G(f)$ be a ultra $\widetilde{g}\alpha$-closed graph. Then the surjectivity of f and ultra $\widetilde{g}\alpha$-closed graph of $G(f)$ implies, by Theorem 2.5, that X is $\widetilde{g}\alpha$-T_2. ■

Definition 2.9. A function $f : (X, \tau) \to (Y, \sigma)$ is called quasi $\widetilde{g}\alpha$-irresolute if for each $x \in X$ and each $V \in \widetilde{G}\alpha O(Y, f(x))$, there exist $U \in \widetilde{G}\alpha O(X, x)$ such that $f(U) \subset \widetilde{g}\alpha cl(V)$.

Theorem 2.10. If a function $f : (X, \tau) \to (Y, \sigma)$ is a quasi $\widetilde{g}\alpha$-irresolute injection with a ultra $\widetilde{g}\alpha$-closed graph $G(f)$, then X is $\widetilde{g}\alpha$-T_2.

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. Therefore $(x_1, f(x_2)) \in (X \times Y) - G(f)$. Since $G(f)$ is ultra $\widetilde{g}\alpha$-closed, there exist $U \in \widetilde{G}\alpha O(X, x_1)$ and $V \in \widetilde{G}\alpha O(Y, f(x_2))$ such that $f(U) \cap \widetilde{g}\alpha cl(V) = \phi$, hence we obtain $U \cap f^{-1}(\widetilde{g}\alpha cl(V)) = \phi$. Consequently,

$$f^{-1}(\widetilde{g}\alpha cl(V)) \subset X - U.$$

Since f is quasi $\widetilde{g}\alpha$-irresolute, there exists $W \in \widetilde{G}\alpha O(X, x_2)$ such that $f(W) \subset \widetilde{g}\alpha cl(V)$. It follows that $W \subset f^{-1}(\widetilde{g}\alpha cl(V)) \subset X - U$, hence $W \cap U = \phi$. Thus for the pair of distinct points $x_1, x_2 \in X$, there exist $U \in \widetilde{G}\alpha O(X, x_1)$ and $W \in \widetilde{G}\alpha O(X, x_2)$ such that $W \cap U = \phi$. Hence, X is $\widetilde{g}\alpha$-T_2. ■

Corollary 2.11. If a function $f : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}\alpha$-irresolute injection with a ultra $\widetilde{g}\alpha$-closed graph $G(f)$, then X is $\widetilde{g}\alpha$-T_2.

Proof. The proof follows from Theorem 2.10, and the fact that every $\widetilde{g}\alpha$-irresolute function is quasi $\widetilde{g}\alpha$-irresolute. ■

Theorem 2.12. If a function $f : (X, \tau) \to (Y, \sigma)$ is a quasi $\widetilde{g}\alpha$-irresolute bijection with a ultra $\widetilde{g}\alpha$-closed graph $G(f)$, then X and Y are $\widetilde{g}\alpha$-T_2.

Proof. The proof follows from Theorem 2.10 and Theorem 2.5. ■

We recall that the union of any two $\widetilde{g}\alpha$-closed sets are $\widetilde{g}\alpha$-closed.

Definition 2.13. A topological space X is called,

(i) $\widetilde{g}\alpha$-extremely disconnected if the $\widetilde{g}\alpha$-closure of every $\widetilde{g}\alpha$-open set is $\widetilde{g}\alpha$-open.

(ii) X is called nearly $\widetilde{g}\alpha$-compact (resp. a subset A of X is said to be nearly $\widetilde{g}\alpha$-compact relative to X), if every $\widetilde{g}\alpha$-open cover of X (resp. if every cover of A by $\widetilde{g}\alpha$-open sets of X) has a finite subfamily such that the union of their $\widetilde{g}\alpha$-closures covers X (resp. has a finite subfamily such that the union of their $\widetilde{g}\alpha$-closures covers A).

Lemma 2.14. Every open subset of a nearly $\widetilde{g}\alpha$-compact space X is nearly $\widetilde{g}\alpha$-compact relative to X.

Proof. Let B be any open (hence $\widetilde{g}\alpha$-clopen) subset of a nearly $\widetilde{g}\alpha$-compact space X. Let $\{O_\alpha|\alpha \in \Omega\}$ be any cover of B by $\widetilde{g}\alpha$-open sets in X. Then the family $F = \{O_\alpha|\alpha \in \Omega\} \cup \{X - B\}$ is a cover of X by $\widetilde{g}\alpha$-open sets in X. Because of near $\widetilde{g}\alpha$-compactness of X, there exists a finite subfamily $F^* = \{O_{\alpha_i}|1 \leq i \leq n\} \cup \{X - B\}$ of F such that the union of $\widetilde{g}\alpha$-closure covers X. So, because of $\widetilde{g}\alpha$-clopenness of B we have the family $\{\widetilde{g}\alpha cl(O_{\alpha_i})|1 \leq i \leq n\}$ which covers B. Therefore B is nearly $\widetilde{g}\alpha$-compact relative to X. ■

Theorem 2.15. Let (X, τ) be a $\widetilde{g}\alpha$-space. If Y is a nearly $\widetilde{g}\alpha$-compact and $\widetilde{g}\alpha$-extremely disconnected space, then a function $f : (X, \tau) \to (Y, \sigma)$ with a ultra $\widetilde{g}\alpha$-closed graph is quasi $\widetilde{g}\alpha$-irresolute.

Proof. Let $x \in X$ and $V \in \widetilde{G}\alpha O(Y, f(x))$. Take any $y \in Y - \widetilde{g}\alpha cl(V)$. Then $(x, y) \in (X \times Y) - G(f)$. Now the ultra $\widetilde{g}\alpha$-closedness of $G(f)$ induces the existence of $U_y(x) \in \widetilde{G}\alpha O(X, x)$ and $V_y(x) \in \widetilde{G}\alpha O(Y, y)$ such that

$$f(U_y(x)) \cap \widetilde{g}\alpha cl(V_y) = \phi. \tag{1}$$

Now $\widetilde{g}\alpha$-extremal disconnectedness of Y induces the $\widetilde{g}\alpha$-clopenness of $\widetilde{g}\alpha cl(V)$ and hence $Y - \widetilde{g}\alpha cl(V)$ is also $\widetilde{g}\alpha$-clopen. Now $\{V_y : y \in Y|\widetilde{g}\alpha cl(V)\}$ is a cover of $Y - \widetilde{g}\alpha cl(V)$ by $\widetilde{g}\alpha$-open sets in Y. By Lemma 2.14, there exists a finite subfamily $\{V_{y_i} : 1 \leq i \leq n\}$ such that $Y - \widetilde{g}\alpha cl(V) \subset \cup_{i=1}^{n} \widetilde{g}\alpha cl(V_{y_i})$. Let $W = \cap_{i=1}^{n} U_{y_i}(x)$, where $U_{y_i}(x)$ are $\widetilde{g}\alpha$-open sets in X satisfying (1). Also $W \in \widetilde{G}\alpha O(X, x)$. Now $f(W) \cap (Y - \widetilde{g}\alpha cl(V)) \subset f[\cap_{i=1}^{n} U_{y_i}(x)] \cap (\cup_{i=1}^{n} \widetilde{g}\alpha cl(V_{y_i})) \subset \cup_{i=1}^{n} (f[U_{y_i}(x)] \cap \widetilde{g}\alpha cl(V_{y_i})) = \phi$, by (1). Therefore, $f(W) \subset \widetilde{g}\alpha cl(V)$ and this indicates that f is quasi $\widetilde{g}\alpha$-irresolute. ■

Corollary 2.16. Let (X, τ) be a $\widetilde{g}\alpha$-space. If Y is a nearly $\widetilde{g}\alpha$-compact and $\widetilde{g}\alpha$-extremely disconnected space, then the surjection $f : (X, \tau) \to (Y, \sigma)$ with a ultra $\widetilde{g}\alpha$-closed graph is quasi $\widetilde{g}\alpha$-irresolute.

Proof. The proof follows from Theorem 2.5 and Theorem 2.15. ■

3. Strongly $\widetilde{G}\alpha$-Closed Graphs

Definition 3.1. A graph $G(f)$ of a function $f : (X, \tau) \to (Y, \sigma)$ is strongly $\widetilde{g}\alpha$-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in \widetilde{G}\alpha O(X, x)$ and an open set V of Y containing y such that $f(U) \cap V = \phi$.

Theorem 3.2. Every ultra $\widetilde{g}\alpha$-closed graph is strongly $\widetilde{g}\alpha$-closed graph.

Proof. It follows from the definitions. $\blacksquare$

Theorem 3.3. If $f : (X, \tau) \to (Y, \sigma)$ is $\widetilde{g}\alpha$-continuous and Y is Hausdroff, then $G(f)$ is strongly $\widetilde{g}\alpha$-closed in $X \times Y$.

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $f(x) \neq y$. Since Y is Hausdorff, there exist open sets V and W in Y containing $f(x)$ and y respectively such that $V \cap W = \phi$. Since f is $\widetilde{g}\alpha$-continuous, there exists $U \in \widetilde{G}\alpha O(X, x)$ such that $f(U) \subset V$. Therefore, $f(U) \cap W = \phi$, $G(f)$ is strongly $\widetilde{g}\alpha$-closed. $\blacksquare$

Theorem 3.4. If $f : (X, \tau) \to (Y, \sigma)$ is surjective and has a strongly $\widetilde{g}\alpha$-closed graph $G(f)$, then Y is T_1.

Proof. Let $y_1, y_2 (y_1 \neq y_2) \in Y$. The surjectivity of f gives a $x \in X$ such that $f(x) = y_2$. Hence $(x, y_1) \notin G(f)$. Then by definition, there exists $\widetilde{g}\alpha$-open set U and an open set V containing x and y_1 respectively, such that $f(U) \cap V = \phi$. Hence $y_2 \notin V$. This means that Y is T_1. $\blacksquare$

Theorem 3.5. If $f : (X, \tau) \to (Y, \sigma)$ is a function with a strongly $\widetilde{g}\alpha$-closed graph, then for each $x \in X$, $f(x) = \cap\{\widetilde{g}\alpha cl(f(U)) | U \in \widetilde{G}\alpha O(X, x)\}$.

Proof. It follows from the Theorem 2.2 and Theorem 3.2. $\blacksquare$

Theorem 3.6. If $f : (X, \tau) \to (Y, \sigma)$ is surjective and has a strongly $\widetilde{g}\alpha$-closed graph $G(f)$, then Y is both $\widetilde{g}\alpha$-T_2 and $\widetilde{g}\alpha$-T_1.

Proof. The proof follows from Theorem 2.5 and Theorem 3.2. $\blacksquare$

Theorem 3.7. If $f : (X, \tau) \to (Y, \sigma)$ is an injection and $G(f)$ is strongly $\widetilde{g}\alpha$-closed, then X is $\widetilde{g}\alpha$-T_1.

Proof. It follows from the Theorem 2.6 and Theorem 3.2. $\blacksquare$

Theorem 3.8. If $f : (X, \tau) \to (Y, \sigma)$ is bijective function with strongly $\widetilde{g}\alpha$-closed graph $G(f)$, then (X, τ) and (Y, σ) are $\widetilde{g}\alpha$-T_1 space.

Proof. The proof is an immediate consequence of Theorem 2.7 and Theorem 3.2. $\blacksquare$

Theorem 3.9. If $f : (X, \tau) \to (Y, \sigma)$ is $\widetilde{g}\alpha$-irresolute and Y is $\widetilde{g}\alpha$-T_2, then $G(f)$ is strongly $\widetilde{g}\alpha$-closed.

Proof. It follows from the Theorem 2.3 and Theorem 3.2. $\blacksquare$

The converse need not be true by the following example.

Example 3.10. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and define the identity map $f : (X, \tau) \to (X, \tau)$. Then f is clearly $\widetilde{g}\alpha$-irresolute and X is not $\widetilde{g}\alpha$-T_2

space. Hence we obtain $G(f)$ is not strongly $\widetilde{g}\alpha$-closed.

Theorem 3.11. A space X is $\widetilde{g}\alpha$-T_2 if and only if the identity function $f : (X,\tau) \to (X,\tau)$ has a strongly $\widetilde{g}\alpha$-closed graph $G(f)$.

Proof. It follows from the Theorem 2.8 and Theorem 3.2. ■

Theorem 3.12. If a function $f : (X,\tau) \to (Y,\sigma)$ is a quasi $\widetilde{g}\alpha$-irresolute injection with a strongly $\widetilde{g}\alpha$-closed graph $G(f)$, then X is $\widetilde{g}\alpha$-T_2.

Proof. It follows from the Theorem 2.10 and Theorem 3.2. ■

Corollary 3.13. If a function $f : (X,\tau) \to (Y,\sigma)$ is a $\widetilde{g}\alpha$-irresolute injection with a strongly $\widetilde{g}\alpha$-closed graph $G(f)$, then X is $\widetilde{g}\alpha$-T_2.

Proof. The proof follows from Theorem 2.11 and the fact that every $\widetilde{g}\alpha$-irresolute function is quasi $\widetilde{g}\alpha$-irresolute. ■

Theorem 3.14. If a function $f : (X,\tau) \to (Y,\sigma)$ is a quasi $\widetilde{g}\alpha$-irresolute bijection with a strongly $\widetilde{g}\alpha$-closed graph $G(f)$, then X and Y are $\widetilde{g}\alpha$-T_2.

Proof. The proof follows from Theorem 2.12 and Theorem 3.2. ■

Theorem 3.15. Let (X,τ) be a $\widetilde{g}\alpha$-space. If Y is a nearly $\widetilde{g}\alpha$-compact and $\widetilde{g}\alpha$-extremely disconnected space, then a function $f : (X,\tau) \to (Y,\sigma)$ with a strongly $\widetilde{g}\alpha$-closed graph is quasi $\widetilde{g}\alpha$-irresolute.

Proof. It follows from the Theorem 2.15 and Theorem 3.2. ■

Corollary 3.16. Let (X,τ) be a $\widetilde{g}\alpha$-space. If Y is a nearly $\widetilde{g}\alpha$-compact and $\widetilde{g}\alpha$-extremely disconnected space, then the surjective $f : (X,\tau) \to (Y,\sigma)$ with a strongly $\widetilde{g}\alpha$-closed graph is quasi $\widetilde{g}\alpha$-irresolute.

Proof. The proof follows from Theorem 2.16 and Theorem 3.2. ■

4. Additional Properties

Definition 4.1. A topological space X is called $\widetilde{g}\alpha$-Urysohn if every pair of distinct points $x, y \in X$, there exists $U \in \widetilde{G}\alpha O(X,x)$ and $V \in \widetilde{G}\alpha O(X,y)$ such that $\widetilde{g}\alpha cl(U) \cap \widetilde{g}\alpha cl(V) = \phi$.

Theorem 4.2. A $\widetilde{g}\alpha$-Urysohn space is $\widetilde{g}\alpha$-T_2.

Proof. Let x and y be two distinct points of X. Since X is $\widetilde{g}\alpha$-Urysohn, there exist $U \in \widetilde{G}\alpha O(X,x)$ and $V \in \widetilde{G}\alpha O(X,y)$ such that $\widetilde{g}\alpha cl(U) \cap \widetilde{g}\alpha cl(V) = \phi$, hence $U \cup V = \phi$. Therefore, X is $\widetilde{g}\alpha$-T_2. ■

Theorem 4.3. If Y is $\widetilde{g}\alpha$-Urysohn and $f : (X,\tau) \to (Y,\sigma)$ is quasi $\widetilde{g}\alpha$-

irresolute injection, then X is $\tilde{g}\alpha$-T_2.

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. The $\tilde{g}\alpha$-Urysohn property of Y indicates that there exist

$$V_i \in \widetilde{G}\alpha O(Y, f(x_i)),\ i = 1, 2$$

such that $\tilde{g}\alpha cl(V_1) \cap \tilde{g}\alpha cl(V_2) = \phi$. Hence $f^{-1}(\tilde{g}\alpha cl(V_1)) \cap f^{-1}(\tilde{g}\alpha cl(V_2)) = \phi$. Since f is quasi $\tilde{g}\alpha$-irresolute, there exists $U_i \in \widetilde{G}\alpha O(X, x_i), i = 1, 2$ such that $f(U_i) \subset \tilde{g}\alpha cl(V_i), i = 1, 2$. It follows that $U_i \subset f^{-1}(\tilde{g}\alpha cl(V_i)), i = 1, 2$. Hence $U_1 \cap U_2 \subset f^{-1}(\tilde{g}\alpha cl(V_1)) \cap f^{-1}(\tilde{g}\alpha cl(V_2)) = \phi$. Therefore, X is $\tilde{g}\alpha$-T_2. ■

Definition 4.4. [2] A function $f : (X, \tau) \to (Y, \sigma)$ is pre $\tilde{g}\alpha$-open if $f(A) \in \widetilde{G}\alpha O(Y)$ for all $A \in \widetilde{G}\alpha O(X)$.

Lemma 4.5. Let a bijection $f : (X, \tau) \to (Y, \sigma)$ be pre $\tilde{g}\alpha$-open. Then for any $B \in \widetilde{G}\alpha c(X)$, $f(B) \in \widetilde{G}\alpha c(Y)$.

Theorem 4.6. If a bijection $f : (X, \tau) \to (Y, \sigma)$ is pre $\tilde{g}\alpha$-open and X is $\tilde{g}\alpha$-Urysohn, then Y is $\tilde{g}\alpha$-Urysohn.

Proof. Let $y_1, y_2 \in Y$ and $y_1 \neq y_2$. Since f is bijective, $f^{-1}(y_1), f^{-1}(y_2) \in X$ and $f^{-1}(y_1) \neq f^{-1}(y_2)$. The $\tilde{g}\alpha$-Urysohn property of X gives the existence of sets $U \in \widetilde{G}\alpha O(X, f^{-1}(y_1))$ and $V \in \widetilde{G}\alpha O(X, f^{-1}(y_2))$ such that $\tilde{g}\alpha cl(U) \cap \tilde{g}\alpha cl(V) = \phi$. As $\tilde{g}\alpha cl(U)$ is a $\tilde{g}\alpha$-closed set in X, the bijectivity and $\tilde{g}\alpha$-openness of f together indicate by Lemma 4.5, that $f(\tilde{g}\alpha cl(U)) \in \widetilde{G}\alpha c(Y)$. Again from $U \subset \tilde{g}\alpha cl(U)$ it follows that $f(U) \subset f(\tilde{g}\alpha cl(U))$ and hence

$$\tilde{g}\alpha cl(f(U)) \subset \tilde{g}\alpha cl(f(\tilde{g}\alpha cl(U))) = f(\tilde{g}\alpha cl(U)).$$

Simillarly we have $\tilde{g}\alpha cl(f(V)) \subset f(\tilde{g}\alpha cl(V))$. Therefore, by the injectivity of f, $\tilde{g}\alpha cl(f(U)) \cap \tilde{g}\alpha cl(f(V)) \subset f(\tilde{g}\alpha cl(U)) \cap f(\tilde{g}\alpha cl(V)) = f(\tilde{g}\alpha cl(U) \cap \tilde{g}\alpha cl(V)) = \phi$. Thus $\tilde{g}\alpha$-openness of f gives the existence of two sets $f(U) \in \widetilde{G}\alpha O(Y, y_1)$ and $f(V) \in \widetilde{G}\alpha O(Y, y_2)$ such that $\tilde{g}\alpha cl(f(U)) \cap \tilde{g}\alpha cl(f(V)) = \phi$, which shows that Y is $\tilde{g}\alpha$-urysohn. ■

Theorem 4.7. If a bijection $f : (X, \tau) \to (Y, \sigma)$ is pre $\tilde{g}\alpha$-open and X is $\tilde{g}\alpha$-T_2, then $G(f)$ is ultra $\tilde{g}\alpha$-closed.

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$. Since f is bijective, $x \neq f^{-1}(y)$. Since X is $\tilde{g}\alpha$-T_2, there exist $U_x, U_y \in \widetilde{G}\alpha O(X)$ such that $x \in U_x$, $f^{-1}(y) \in U_y$ and $U_x \cap U_y = \phi$. Moreover f is pre $\tilde{g}\alpha$-open and bijective, therefore $f(x) \in f(U_x) \in \widetilde{G}\alpha O(Y)$, $y \in f(U_y) \in \widetilde{G}\alpha O(Y)$ and $f(U_x) \cap f(U_y) = \phi$. Hence $f(U_x) \cap \tilde{g}\alpha cl(f(U_y)) = \phi$. This shows that $G(f)$ is ultra $\tilde{g}\alpha$-closed. ■

Theorem 4.8. If $f : (X, \tau) \to (Y, \sigma)$ is quasi $\tilde{g}\alpha$-irresolute and Y is $\tilde{g}\alpha$-urysohn, then $G(f)$ is ultra $\tilde{g}\alpha$-closed.

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$. since Y is $\widetilde{g}\alpha$-urysohn, there exist $V \in \widetilde{g}\alpha O(Y, y)$ and $W \in \widetilde{G}\alpha O(Y, f(x))$ such that $\widetilde{g}\alpha cl(V) \cap \widetilde{g}\alpha cl(W) = \phi$. Since f is quasi $\widetilde{g}\alpha$-irresolute, there exists $U \in \widetilde{G}\alpha O(X, x)$ such that $f(U) \subset \widetilde{g}\alpha cl(W)$. This, implies that $f(U) \cap \widetilde{g}\alpha cl(V) = \phi$. By definition, $G(f)$ is ultra $\widetilde{g}\alpha$-closed. ■

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ISBN: 978-618-80609-1-3