2014 International Conference on Topology and its Applications,
July 3-7, 2014, Nafpaktos,
Greece

Selected papers of the 2014 International Conference on Topology and its Applications

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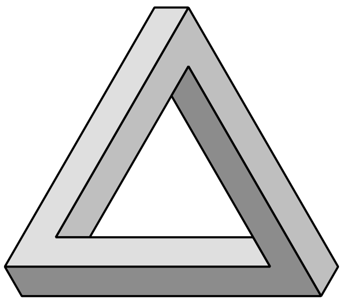

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Preface

The **2014 International Conference on Topology and its Applications** took place from July 3 to 7 in the **3rd High School of Nafpaktos, Greece**. It covered all areas of Topology and its Applications (especially General Topology, Set-Theoretic Topology, Geometric Topology, Topological Groups, Dimension Theory, Dynamical Systems and Continua Theory, Computational Topology, History of Topology). This conference was attended by 235 participants from 44 countries and the program consisted by 147 talks.

The Organizing Committee consisted of S.D. Iliadis (University of Patras), D.N. Georgiou (University of Patras), I.E. Kougias (Technological Educational Institute of Western Greece), A.C. Megaritis (Technological Educational Institute of Western Greece), and I. Boules (Mayor of the city of Nafpaktos).

The Organizing Committee is very much indebted to the City of Nafpaktos for its hospitality and for its excellent support during the conference.

The conference was sponsored by University of Patras, Technological Educational Institute of Western Greece, Municipality of Nafpaktos, New Media Soft – Internet Solutions, Loux Marlafekas A.B.E.E., TAXYTYPO – TAXYEKTYPOSEIS GRAVANIS EPE, Alpha Bank, and Wizard Solutions LTD.

This volume is a special volume under the title: "Selected papers of the 2014 International Conference on Topology and its Applications" which will be edited by the organizers (D.N. Georgiou, S.D. Iliadis, I.E. Kougias, and A.C. Megaritis) and published by the University of Patras. We thank the authors for their submissions.

Editors

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Contra $\widetilde{G}\alpha$-continuous functions

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Abstract

The concept of $\widetilde{g}\alpha$-closed sets in a topological space are introduced by R. Devi et. al. [4]. In this paper, we introduce the notion of contra $\widetilde{g}\alpha$-continuous functions utilizing $\widetilde{g}\alpha$-open sets and study some of its applications.

Key words: $\widetilde{g}\alpha$-closed sets, contra $\widetilde{g}\alpha$-continuous, $\widetilde{g}\alpha$-normal.
1991 MSC: 54A05, 54D05 54D10, 54D45.

1. Introduction and Preliminaries

In 1996, Dontchev [6] introduced the notions of contra continuity and strong S-closedness in topological spaces. He defined a function $f : X \to Y$ is contra continuous if the pre image of every open set of Y is closed in X. Also a new class of function called contra semi-continuous function is introduced and investigated by Dontchev and Noiri [7]. The notions of contra super continuous, contra pre continuous and contra α-continuous functions are introduced by Jafari and Noiri [11,12]. Nasef [16] has introduced and studied contra γ-continuous function. In this paper, we introduce the concept of contra $\widetilde{g}\alpha$-continuous functions via the notion of $\widetilde{g}\alpha$-open set and study some of the applications of this function.

All through this paper, (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of A and the interior of A will be denoted by $cl(A)$ and $int(A)$ respectively. A is regular open if $A = int(cl(A))$ and A is regular closed if its complement is regular open; equivalently A is regular closed if $A = cl(int(A))$, see [25].

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Definition 1.1. A subset A of a space (X,τ) is called a

1. semi-open set [13] if $A \subseteq cl(int(A))$ and a semi-closed set [13] if $int(cl(A)) \subseteq A$,
2. α-open set [17] if $A \subseteq int(cl(int(A)))$ and an α-closed set [17] if $cl(int(cl(A))) \subseteq A$,
3. pre open set [14] if $A \subseteq int(cl(A))$ and pre closed set [14] if $cl(int(A)) \subseteq A$,
4. γ-open set [9] if $A \subseteq cl(int(A)) \cup int(cl(A))$ and γ-closed set [9] if $int(cl(A)) \cup cl(int(A)) \subseteq A$,
5. β-open set [1] if $A \subseteq cl(int(cl(A)))$ and β-closed set [1] if $int(cl(int(A))) \subseteq A$,
6. δ-open set [24] if for each $x \in A$, there exists a regular open set G such that $x \in G \subset A$.

The semi-closure (resp. α-closure) of a subset A of a space (X,τ) is the intersection of all semi-closed (resp. α-closed) sets that contain A and is denoted by $scl(A)$ (resp. $\alpha cl(A)$).

Definition 1.2. A subset A of a space (X,τ) is called a

1. $\widehat{g}$-closed set [22,23] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ),
2. ${}^*g$-closed set [20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\widehat{g}$-open in (X,τ),
3. ${}^\sharp gs$-closed set [21] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is ${}^*g$-open in (X,τ).

Let (X,τ) be a space and let A be a subset of X. A is called $\widetilde{g}\alpha$-closed set [4] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ${}^\sharp gs$-open set of (X,τ). The complement of an $\widetilde{g}\alpha$-closed set is called $\widetilde{g}\alpha$-open. The union of two $\widetilde{g}\alpha$-closed sets is $\widetilde{g}\alpha$-closed set in (X,τ) [4, Let A and B be $\widetilde{g}\alpha$-closed sets. Let $A \cup B \subseteq U$, U is ${}^\sharp gs$-open. Since A and B are $\widetilde{g}\alpha$-closed sets, $\alpha cl(A) \subseteq U$, $\alpha cl(B) \subseteq U$. This implies that $\alpha cl(A \cup B) = \alpha cl(A) \cup \alpha cl(B) \subseteq U$, (since τ^α=α-open set forms a topology [13]) and so $\alpha cl(A \cup B) \subseteq U$. Therefore $A \cup B$ is $\widetilde{g}\alpha$-closed]. We set $\widetilde{g}\alpha O(X,x) = \{U : x \in U \text{ and } U \in \tau_{\widetilde{g}\alpha}\}$, where $\tau_{\widetilde{g}\alpha}$ denotes the family of all $\widetilde{g}\alpha$-open subsets of a space (X,τ). The collection of all closed subsets of X will be denoted by $C(X)$. We set $C(X,x) = \{V \in C(X) : x \in V\}$ for $x \in X$.

Definition 1.3. A function $f : (X,\tau) \to (Y,\sigma)$ is called a

1. contra continuous [6] if $f^{-1}(V)$ is closed in (X,τ) for every open set V of (Y,σ),
2. RC-continuous [7] if $f^{-1}(V)$ is regular-closed in (X,τ) for every open set V of (Y,σ),
3. contra super continuous [10] if $f^{-1}(V)$ is δ-closed in (X,τ) for every open set V of (Y,σ),
4. contra semi continuous [7] if $f^{-1}(V)$ is closed in (X,τ) for every open set

V of (Y, σ),

5. contra α-continuous [11] if $f^{-1}(V)$ is α-closed in (X, τ) for every open set V of (Y, σ),
6. contra pre continuous [11] if $f^{-1}(V)$ is pre-closed in (X, τ) for every open set V of (Y, σ),
7. contra γ-continuous [16] if $f^{-1}(V)$ is γ-closed in (X, τ) for every open set V of (Y, σ),
8. contra β-continuous [3] if $f^{-1}(V)$ is β-closed in (X, τ) for every open set V of (Y, σ),
9. $\widetilde{g}\alpha$-continuous [5] if $f^{-1}(V)$ is $\widetilde{g}\alpha$-closed in (X, τ) for every closed set V of (Y, σ),
10. $\widetilde{g}\alpha$-*irresolute* [5] if $f^{-1}(V)$ is $\widetilde{g}\alpha$-closed in (X, τ) for every $\widetilde{g}\alpha$-closed set V of (Y, σ).

2. Properties of contra $\widetilde{g}\alpha$-continuous functions

Definition 2.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called contra $\widetilde{g}\alpha$-continuous if $f^{-1}(U)$ is $\widetilde{g}\alpha$-closed in (X, τ) for each open set U in (Y, σ).

Theorem 2.2. Every contra α-continuous function is contra $\widetilde{g}\alpha$-continuous.

Proof. It follows from the fact that every α-closed set is $\widetilde{g}\alpha$-closed. ■

Corollary 2.3. Every contra-continuous function is contra $\widetilde{g}\alpha$-continuous.

Proof. It follows from the fact that every closed set is $\widetilde{g}\alpha$-closed. ■

The converse of the Theorem 2.2. and Corollary 2.3. need not be true by following example.

Example 2.4. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$ and

$$\sigma = \{Y, \phi, \{a\}, \{a, c\}\}.$$

Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Here $\{a\}$ is an open set of (Y, σ) but $f^{-1}(\{a\}) = \{b\}$ is not an α-closed and hence not closed set of (X, τ). Thus f is not contra-α-continuous and hence not contra-continuous, however f is contra- $\widetilde{g}\alpha$-continuous.

Definition 2.5. Let A be a subset of a space (X, τ).

(a) The set $\cap\{U \in \tau : A \subset U\}$ is called the kernel of A [13] and is denoted by $ker(A)$.

(b) The set $\cap\{F \subset X : A \subseteq F, F \text{ is } \widetilde{g}\alpha\text{-closed}\}$ is called the $\widetilde{g}\alpha$-closure of A and is denoted by $cl_{\widetilde{g}\alpha}(A)$.

(c) The set $\cup\{F \subset X : F \subseteq A, F$ is $\widetilde{g}\alpha$-open$\}$ is called $\widetilde{g}\alpha$-interior of A and is denoted by $int_{\widetilde{g}\alpha}(A)$.

Lemma 2.6. [10] The following properties hold for subsets A, B of a space X:

(a) $x \in ker(A)$ if and only if $A \cap F \neq \phi$ for any $F \in C(X, x)$.
(b) $A \subseteq ker(A)$ and $A = ker(A)$ if A is open in X.
(c) If $A \subseteq B$, then $ker(A) \subseteq ker(B)$.

Theorem 2.7. For a function $f : (X, \tau) \to (Y, \sigma)$ the following conditions are equivalent:

(1) f is contra $\widetilde{g}\alpha$-continuous;
(2) for every closed subset F of Y, $f^{-1}(F) \in \widetilde{g}\alpha O(X)$;
(3) for each $x \in X$ and each $F \in C(Y, f(x))$, there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq F$;
(4) $f(cl_{\widetilde{g}\alpha}(A)) \subseteq ker(f(A))$ for every subset A of X;
(5) $cl_{\widetilde{g}\alpha}(f^{-1}(B)) \subseteq f^{-1}(ker(B))$ for every subset B of Y.

Proof. The implications (1) $\Leftrightarrow$ (2) and (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (2) Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in \widetilde{g}\alpha O(X, x)$ such that $f(U_x) \subseteq F$. Therefore, we obtain $f^{-1}(F) = \cup\{U_x / x \in f^{-1}(F)\}$ and $f^{-1}(F)$ is $\widetilde{g}\alpha$-open, by [4, Theorem 3.15].

(2) $\Rightarrow$ (4) Let A be any subset of X. Suppose that $y \notin ker(f(A))$. Then by Lemma 2.6. there exists $F \in C(Y, f(x))$ such that $f(A) \cap F = \phi$. Thus, we have $A \cap f^{-1}(F) = \phi$ and since $f^{-1}(F)$ is $\widetilde{g}\alpha$-open then we have $cl_{\widetilde{g}\alpha}(A) \cap f^{-1}(F) = \phi$. Therefore, we obtain $f(cl_{\widetilde{g}\alpha}(A)) \cap F = \phi$ and $y \notin f(cl_{\widetilde{g}\alpha}(A))$. This implies that $f(cl_{\widetilde{g}\alpha}(A)) \subseteq ker(f(A))$.

(4) $\Rightarrow$ (5) Let B be any subset of Y. By (4) and Lemma 2.6., we have

$$f(cl_{\widetilde{g}\alpha}(f^{-1}(B))) \subseteq ker(f(f^{-1}(B))) \subseteq ker(B).$$

Thus $cl_{\widetilde{g}\alpha}(f^{-1}(B)) \subseteq f^{-1}(ker(B))$.

(5) $\Rightarrow$ (1) Let V be any open set of Y. Then, by Lemma 2.6., we have $cl_{\widetilde{g}\alpha}(f^{-1}(V)) \subseteq f^{-1}(ker(V)) = f^{-1}(V)$ and $cl_{\widetilde{g}\alpha}(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\widetilde{g}\alpha$-closed in X. ■

Theorem 2.8. Let $f : X \to Y$ be a function, then the following are equivalent.

(1) The function f is $\widetilde{g}\alpha$-continuous.
(2) For each point $x \in X$ and each open set V of Y with $f(x) \in V$, there exists a $\widetilde{g}\alpha$-open set U of X such that $x \in U$, $f(U) \subset U$.

Proof. (1) $\Rightarrow$ (2) Let $f(x) \in V$. Then $x \in f^{-1}(V) \in \widetilde{g}\alpha O(X)$, since f is $\widetilde{g}\alpha$-continuous. Let $U = f^{-1}(V)$. Then $x \in X$ and $f(U) \subset U$.

(2) $\Rightarrow$ (1) Let V be an open set of Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$. Then $f(x) \in V$ and thus there exists an $\widetilde{g}\alpha$-open set U_x of X such that $x \in U_x$ and $f(U) \subset V$. Now, $x \in U_x \subset f^{-1}(V)$ and $f^{-1}(V) = \cup U_x$. Then $f^{-1}(V)$ is $\widetilde{g}\alpha$-open in X. Therefore, f is $\widetilde{g}\alpha$-continuous. ■

Theorem 2.9. If a function $f : X \to Y$ is contra $\widetilde{g}\alpha$-continuous and Y is regular, then f is $\widetilde{g}\alpha$-continuous.

Proof. Let x be an arbitrary point of X and let V be an open set of Y containing $f(x)$; since Y is regular, there exists an open set W in Y containing $f(x)$ such that $cl(W) \subseteq V$. Since f is contra $\widetilde{g}\alpha$-continuous, so by Theorem 2.7.(3) there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq cl(W)$. Then $f(U) \subseteq cl(W) \subseteq V$. Hence, f is $\widetilde{g}\alpha$-continuous. ■

Corollary 2.10. If a function $f : X \to Y$ is contra $\widetilde{g}\alpha$-continuous and Y is regular, then f is continuous.

Proof. It suffices to observe that every continuous function is $\widetilde{g}\alpha$-continuous. ■

Remark 2.11. The converse of Corollary 2.10. is not true. The following example shows that continuity does not necessarily imply contra $\widetilde{g}\alpha$-continuity even if the range is regular.

Example 2.12. The identity function on the real line with the usual topology is continuous and hence $\widetilde{g}\alpha$-continuous. The inverse image of $(0, 1)$ is not $\widetilde{g}\alpha$-closed and consequently the function is not contra $\widetilde{g}\alpha$-continuous.

Definition 2.13. A space (X, τ) is said to be $\widetilde{g}\alpha$-space (resp. locally $\widetilde{g}\alpha$-indiscrete) if every $\widetilde{g}\alpha$-open set is open (resp. closed) in X.

Theorem 2.14. If a function $f : X \to Y$ is contra $\widetilde{g}\alpha$-continuous and X is $\widetilde{g}\alpha$-space, then f is contra-continuous.

Proof. Let V be a closed set in Y. Since f is contra-$\widetilde{g}\alpha$-continuous, $f^{-1}(V)$ is $\widetilde{g}\alpha$-open in X. Since X is $\widetilde{g}\alpha$-space, $f^{-1}(V)$ is open in X. Hence f is contra-continuous. ■

Corollary 2.15. If X is a $\widetilde{g}\alpha$-space, then for a $f : X \to Y$ function following statements are equivalent,

(1) f is contra-continuous.
(2) f is contra $\widetilde{g}\alpha$-continuous.

Theorem 2.16. Let X be locally $\widetilde{g}\alpha$-indiscrete. If a function $f : X \to Y$ is contra $\widetilde{g}\alpha$-continuous, then f is continuous.

Proof. Let V be a closed set in Y. Since f is contra-$\widetilde{g}\alpha$-continuous, $f^{-1}(V)$ is $\widetilde{g}\alpha$-open in X. Since X is locally $\widetilde{g}\alpha$-indiscrete, $f^{-1}(V)$ is closed in X. Hence

f is continuous. ■

Definition 2.17. A function $f : X \to Y$ is called almost $\widetilde{g}\alpha$-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq int_{\widetilde{g}\alpha}(cl(V))$.

Theorem 2.18. A function $f : X \to Y$ is almost $\widetilde{g}\alpha$-continuous if and only if for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq V$.

Proof. Let V be regular open set of Y containing $f(x)$ for each $x \in X$. This implies that V is an open set of Y containing $f(x)$ for each $x \in X$. Since f is almost $\widetilde{g}\alpha$-continuous, there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq int_{\widetilde{g}\alpha}(cl(V))$.
Conversely, if for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq V$. This implies V is an open set of Y containing $f(x)$, there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq V = int_{\widetilde{g}\alpha}(cl(V))$. Therefore f is almost $\widetilde{g}\alpha$-continuous. ■

Definition 2.19. A function $f : X \to Y$ is said to be pre $\widetilde{g}\alpha$-open if the image of each $\widetilde{g}\alpha$-open set is $\widetilde{g}\alpha$-open.

Theorem 2.20. If a function $f : X \to Y$ is a pre $\widetilde{g}\alpha$-open and contra $\widetilde{g}\alpha$-continuous, then f is almost $\widetilde{g}\alpha$-continuous.

Proof. Let x be any arbitrary point of X and V be an open set containing $f(x)$. Since f is contra $\widetilde{g}\alpha$-continuous, then by Theorem 2.7.(3) there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq cl(V)$. Since f is pre $\widetilde{g}\alpha$-open, $f(U)$ is $\widetilde{g}\alpha$-open in Y. Therefore, $f(U) = int_{\widetilde{g}\alpha}f(U) \subseteq int_{\widetilde{g}\alpha}(cl(f(U))) \subseteq int_{\widetilde{g}\alpha}(cl(V))$. This shows that f is almost $\widetilde{g}\alpha$-continuous. ■

Definition 2.21. The $\widetilde{g}\alpha$-frontier of A of a space (X, τ), denoted by $Fr_{\widetilde{g}\alpha}(A)$ is defined by $Fr_{\widetilde{g}\alpha}(A) = cl_{\widetilde{g}\alpha}(A) \cap cl_{\widetilde{g}\alpha}(X - A)$.

Theorem 2.22. Let $K = \{x \in X : V \cap U \neq \phi\}$ for every $\widetilde{g}\alpha$-open set V containing x, then $cl_{\widetilde{g}\alpha}(U) = K$.

Proof.

$$\begin{aligned}
\text{Let } x \in K &\Leftrightarrow V \cap U \neq \phi,\ x \in V,\ V \text{ is a } \widetilde{g}\alpha\text{-open set} \\
&\Leftrightarrow x \in U \text{ or every } \widetilde{g}\alpha\text{-open sets containing x contains} \\
&\quad\ \text{a point of } U \text{ other than } x \\
&\Leftrightarrow x \in cl_{\widetilde{g}\alpha}(U). \ ■
\end{aligned}$$

Theorem 2.23. The set of all points x of X at which $f : X \to Y$ is not contra $\widetilde{g}\alpha$-continuous is identical with the union of the $\widetilde{g}\alpha$-frontier of the inverse image of closed sets of Y containing $f(x)$.

Proof. Suppose f is not contra $\widetilde{g}\alpha$-continuous at $x \in X$. There exists $F \in C(Y, f(x))$ such that $f(U) \cap (Y - F) \neq \phi$ for every $U \in \widetilde{g}\alpha O(X, x)$. This implies that $U \cap f^{-1}(Y - F) \neq \phi$. Therefore, we have $x \in cl_{\widetilde{g}\alpha}(f^{-1}(Y - F)) = cl_{\widetilde{g}\alpha}(X - f^{-1}(F))$. However, since $x \in f^{-1}(F) \subseteq cl_{\widetilde{g}\alpha}(f^{-1}(F))$, thus $x \in cl_{\widetilde{g}\alpha}(f^{-1}(F)) \cap cl_{\widetilde{g}\alpha}(f^{-1}(Y - F))$. Therefore, we obtain $x \in Fr_{\widetilde{g}\alpha}(f^{-1}(F))$. Suppose that $x \in Fr_{\widetilde{g}\alpha}(f^{-1}(F))$ for some $F \in C(Y, f(x))$ and f is contra $\widetilde{g}\alpha$-continuous at x, then there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq F$. Therefore, we have $x \in U \subseteq f^{-1}(F)$ and hence $x \in int_{\widetilde{g}\alpha}(f^{-1}(F)) \subseteq X - Fr_{\widetilde{g}\alpha}(f^{-1}(F))$. This is a contradiction. This means that f is not contra $\widetilde{g}\alpha$-continuous. ■

Theorem 2.24. Let $(X_\lambda : \lambda \in \Lambda)$ be any family of topological spaces. If $f : X \to \prod X_\lambda$ is a contra $\widetilde{g}\alpha$-continuous function. Then $P_\lambda \circ f : X \to X_\lambda$ is contra $\widetilde{g}\alpha$-continuous for each $\lambda \in \Lambda$, where P_λ is the projection of $\prod X_\lambda$ onto X_λ.

Proof. We shall consider a fixed $\lambda \in \Lambda$. Suppose U_λ is an arbitrary open set in X_λ. Then $P_\lambda^{-1}(U_\lambda)$ is open in $\prod X_\lambda$. Since f is contra $\widetilde{g}\alpha$-continuous, we have by definition $f^{-1}(P_\lambda^{-1}(U_\lambda)) = (P_\lambda \circ f)^{-1}(U_\lambda)$ is $\widetilde{g}\alpha$-closed in X. Therefore $P_\lambda \circ f$ is contra $\widetilde{g}\alpha$-continuous. ■

Theorem 2.25. Let $f : X \to Y$ and $g : Y \to Z$ be two functions and $g \circ f : X \to Z$. Then

(i) $g \circ f$ is contra-$\widetilde{g}\alpha$-continuous, if g is continuous and f is contra-$\widetilde{g}\alpha$-continuous.

(ii) $g \circ f$ is contra-$\widetilde{g}\alpha$-continuous, if g is contra-continuous and f is $\widetilde{g}\alpha$-continuous.

(iii) $g \circ f$ is contra-$\widetilde{g}\alpha$-continuous, if f and g are $\widetilde{g}\alpha$-continuous and Y is locally $\widetilde{g}\alpha$-indiscrete.

Theorem 2.26. If $f : X \to Y$ be surjective $\widetilde{g}\alpha$-irresolute and pre-$\widetilde{g}\alpha$-open and $g : Y \to Z$ be any function. Then $g \circ f : X \to Z$ is contra $\widetilde{g}\alpha$-continuous if and only if g is contra $\widetilde{g}\alpha$-continuous.

Proof. The 'if' part is easy to prove. To prove the 'only if' part, let $g \circ f : X \to Z$ is contra $\widetilde{g}\alpha$-continuous and let F be a closed subset of Z. Then $(g \circ f)^{-1}(F)$ is a $\widetilde{g}\alpha$-open of X. That is $f^{-1}(g^{-1}(F))$ is an $\widetilde{g}\alpha$-open subset of X. Since f is pre-$\widetilde{g}\alpha$-open, $f(f^{-1}(g^{-1}(F)))$ is $\widetilde{g}\alpha$-open subset of Y. So, $g^{-1}(F)$ is an $\widetilde{g}\alpha$-open in Y. Hence g is contra $\widetilde{g}\alpha$-continuous. ■

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subseteq X \times Y$ is called the graph of f and is denoted by $Gr(f)$.

Definition 2.27. The graph $Gr(f)$ of a function $f : X \to Y$ is said to be contra $\widetilde{g}\alpha$-closed if for each $(x, y) \in (X \times Y) - Gr(f)$, there exists $U \in \widetilde{g}\alpha O(X, x)$ and $V \in C(Y, y)$ such that $(U \times V) \cap Gr(f) = \phi$ and it is denoted by $C\widetilde{g}\alpha$-closed.

Lemma 2.28. [8] Let $Gr(f)$ be the graph of f, for any subset $A \subseteq X$ and $B \subseteq Y$, we have $f(A) \cap B = \phi$ if and only if $(A \times B) \cap G(f) = \phi$.

Lemma 2.29. The graph $Gr(f)$ of a function $f : X \to Y$ is $C\widetilde{g}\alpha$-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - Gr(f)$, there exists $U \in \widetilde{g}\alpha O(X, x)$ and $V \in C(Y, y)$ such that $f(U) \cap V = \phi$.

Theorem 2.30. If $f : X \to Y$ is contra $\widetilde{g}\alpha$-continuous and Y is Urysohn, then f is $C\widetilde{g}\alpha$-closed in the product space $X \times Y$.

Proof. Let $(x, y) \in (X \times Y) - Gr(f)$. Then $y \neq f(x)$ and there exists open sets H_1, H_2 such that $f(x) \in H_1$, $y \in H_2$ and $cl(H_1) \cap cl(H_2) = \phi$. From hypothesis, there exists $V \in \widetilde{g}\alpha O(X, x)$ such that $f(V) \subseteq cl(H_1)$. Therefore, we obtain $f(V) \cap cl(H_2) = \phi$. This shows that f is $C\widetilde{g}\alpha$-closed. ■

Theorem 2.31. If $f : X \to Y$ and $g : X \to Y$ are contra $\widetilde{g}\alpha$-continuous and Y is Urysohn, then $K = \{x \in X : f(x) = g(x)\}$ is $\widetilde{g}\alpha$-closed in X.

Proof. Let $x \in X - K$. Then $f(x) \neq g(x)$. Since Y is Urysohn, there exist open sets U and V such that $f(x) \in U, g(x) \in V$ and $cl(U) \cap cl(V) = \phi$. Since f and g are contra $\widetilde{g}\alpha$-continuous, $f^{-1}(cl(U)) \in \widetilde{g}\alpha O(X)$ and $g^{-1}(cl(V)) \in \widetilde{g}\alpha O(X)$. Let $A = f^{-1}(cl(U))$ and $B = f^{-1}(cl(V))$, then A and B contains x. Set $C = A \cap B$. C is $\widetilde{g}\alpha$-open in X [4, Theorem 2.15]. Hence $f(C) \cap g(C) = \phi$ and $x \notin cl_{\widetilde{g}\alpha}(K)$. Thus, K is $\widetilde{g}\alpha$-closed in X. ■

Theorem 2.32. Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra $\widetilde{g}\alpha$-continuous, then f is contra $\widetilde{g}\alpha$-continuous.

Proof. Let U be an open set in Y, then $X \times U$ is an open set in $X \times Y$. Since g is contra $\widetilde{g}\alpha$-continuous, it follows that $f^{-1}(U) = g^{-1}(X \times U)$ is an $\widetilde{g}\alpha$-closed set in X. Thus, f is contra $\widetilde{g}\alpha$-continuous. ■

Theorem 2.33. If $f : X \to Y$ is $\widetilde{g}\alpha$-continuous and Y is T_1, then f is $C\widetilde{g}\alpha$-closed in $X \times Y$.

Proof. Let $(x, y) \in (X \times Y) - Gr(f)$. Then $f(x) \neq y$ and there exists an open set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is $\widetilde{g}\alpha$-continuous there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq V$. Therefore, we have $f(U) \cap (Y - V) = \phi$ and $Y - V \in C(Y, y)$. This shows that f is $C\widetilde{g}\alpha$-closed in $X \times Y$. ■

Definition 2.34.

(i) A space X is said to be $\widetilde{g}\alpha$-T_1 if for each pair of distinct points x and y in X, there exists $\widetilde{g}\alpha$-open sets U and V containing x and y respectively, such that $y \notin U$ and $x \notin V$.

(ii) A space X is said to be $\widetilde{g}\alpha$-T_2 if for each pair of distinct points x and y

in X, there exists $\widetilde{g}\alpha$-open sets U and V containing x and y respectively, such that $U \cap V = \phi$.

Theorem 2.35. Let X is a topological space and for each pair of distinct points x and y in X there exists a map f of X into a Urysohn topological space Y such that $f(x) \neq f(y)$ and f is contra $\widetilde{g}\alpha$-continuous at x and y, then X is $\widetilde{g}\alpha$-T_2.

Proof. Let x and y be any distinct points in X. Then, there exists a Urysohn space Y and a function $f : X \to Y$ such that $f(x) \neq f(y)$ and f is contra $\widetilde{g}\alpha$-continuous at x and y. Let $a = f(x)$ and $b = f(y)$. Then $a \neq b$. Since Y is Urysohn space, there exists open sets V and W containing a and b, respectively, such that $cl(V) \cap cl(W) = \phi$. Since f is contra $\widetilde{g}\alpha$-continuous at x and y, there exist $\widetilde{g}\alpha$-open sets A and B containing a and b, respectively, such that $f(A) \subseteq cl(V)$ and $f(B) \subseteq cl(W)$. Then $f(A) \cap f(B) = \phi$, so $A \cap B = \phi$. Hence, X is $\widetilde{g}\alpha$-T_2. ■

Corollary 2.36. Let $f : X \to Y$ be contra $\widetilde{g}\alpha$-continuous injection. If Y is an Urysohn space, then X is $\widetilde{g}\alpha$-T_2.

Definition 2.37. A space X is said to be weakly Hausdorff [18] if each element of X is an intersection of regular closed sets.

Theorem 2.38. If $f : X \to Y$ is a contra $\widetilde{g}\alpha$-continuous injection and Y is weakly Hausdorff, then X is $\widetilde{g}\alpha$-T_1.

Proof. Suppose that Y weakly Hausdorff. For any distinct points x_1 and x_2 in X, there exists regular closed sets U and V in Y such that $f(x_1) \in U, f(x_2) \notin U, f(x_1) \notin V$ and $f(x_2) \in V$. Since f is contra $\widetilde{g}\alpha$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $\widetilde{g}\alpha$-open subsets of X such that $x_1 \in f^{-1}(U), x_2 \notin f^{-1}(U), x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$. This shows that X is $\widetilde{g}\alpha$-T_1. ■

Theorem 2.39. Let $f : X \to Y$ have a $C\widetilde{g}\alpha$-graph. If f is injective, then X is $\widetilde{g}\alpha$-T_1.

Proof. Let x_1 and x_2 be any two distinct points of X. Then, we have

$$(x_1, f(x_2)) \in (X \times Y) - G(f).$$

Then, there exist a $\widetilde{g}\alpha$-open set U in X containing x_1 and $F \in C(Y, f(x_2))$ such that $f(U) \cap F = \phi$ hence $U \cap f^{-1}(F) = \phi$. Therefore we have $x_2 \notin U$. This implies that X is $\widetilde{g}\alpha$-T_1. ■

Definition 2.40. A topological space X is said to be ultra Hausdorff [19] if for each pair of distinct points x and y in X there exist clopen sets A and B containing x and y containing x and y, respectively such that $A \cap B = \phi$.

Theorem 2.41. Let $f : X \to Y$ be a contra $\widetilde{g}\alpha$-continuous injection. If Y is ultra Hausdorff space, then X is $\widetilde{g}\alpha$-T_2.

Proof. Let x_1 and x_2 be any distinct points in X, then $f(x_1) \neq f(x_2)$ and there exist clopen sets U and V containing $f(x_1)$ and $f(x_2)$ respectively such that $U \cap V = \phi$. Since f is contra $\widetilde{g}\alpha$-continuous, then $f^{-1}(U) \in \widetilde{g}\alpha O(X)$ and $f^{-1}(V) \in \widetilde{g}\alpha O(X)$ such that $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence, X is $\widetilde{g}\alpha$-T_2. ■

Definition 2.42. The graph $Gr(f)$ of $f : X \to Y$ is said to be strongly contra-$\widetilde{g}\alpha$-closed if for each $(x, y) \in (X, Y) - Gr(f)$, there exists $U \in \widetilde{g}\alpha O(X, x)$ and $V \in RC(Y, y)$ such that $(U \times V) \cap Gr(f) = \phi$.

Lemma 2.43. The graph $Gr(f)$ of $f : X \to Y$ is strongly contra-$\widetilde{g}\alpha$-closed graph in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - Gr(f)$, there exist $U \in \widetilde{g}\alpha O(X, x)$ and $V \in RC(Y, y)$ such that $f(U) \cap V = \phi$.

Theorem 2.44. If $f : X \to Y$ is almost $\widetilde{g}\alpha$-continuous and Y is Hausdorff, then $Gr(f)$ is strongly contra-$\widetilde{g}\alpha$-closed.

Proof. Suppose that $(x, y) \in (X \times Y) - Gr(f)$. Then $y \neq f(x)$. Since Y is Hausdorff, there exist open sets V and W in Y containing y and $f(x)$, respectively, such that $V \cap W = \phi$; hence, $cl(V) \cap int(cl(W)) = \phi$. Since f is almost $\widetilde{g}\alpha$-continuous and W is regular open by Theorem 2.18. there exists $U \in \widetilde{g}\alpha O(X, x)$ such that $f(U) \subseteq W \subseteq int(cl(W))$. This shows that $f(U) \cap cl(V) = \phi$ and hence by Lemma 2.43. we have $Gr(f)$ is strongly contra-$\widetilde{g}\alpha$-closed. ■

Remark 2.45. The following diagram shows the relationships established between contra $\widetilde{g}\alpha$-continuous functions and some other continuous functions. $A \to B$ represents A implies B but not conversely.

$$\begin{array}{cccccccccc}
A & & & & H & & & & \\
\downarrow & & & \nearrow & & \searrow & & & \\
B & & C & \to & D & \to & E & \to & I \\
\downarrow & \nearrow & \downarrow & & & & & & \\
F & \longrightarrow & G & & & & & &
\end{array}$$

Notation 2.46. A= RC-continuous, B= contra super continuous, C= contra α-continuous, D= contra semi-continuous, E= contra γ-continuous, F= contra continuous, G= contra $\widetilde{g}\alpha$-continuous, H= contra pre-continuous, I= contra β-continuous.

Remark 2.47. It should be mentioned that none of these implication is reversible as shown by the example stated below.

Example 2.48. [16] The digital line or the so-called Khalimsky line is the set of all integers Z, equipped with the topology k, generated by subbase $\tau_k = \left\{\{2n-1, 2n, 2n+1\} : n \in Z\right\}$. Let (Z, k) be the digital line and $f : (Z, k) \to (Z, k)$ be a function defined as follows: $f(x) = 0$, if x is odd; $f(x) = 1$, if x is even. It can be easily observed that f is contra super continuous but not RC-continuous.

Example 2.49. [16] Let $X = \{a, b\}$ be the Sierpinski space by setting $\tau = \{X, \phi, \{a\}\}$ and $\sigma = \{X, \phi, \{b\}\}$. The identity function $f : (X, \tau) \to (X, \sigma)$ is contra continuous but not contra super continuous.

Example 2.50. [11] Let $X = \{a, b, c\} = Y$, $\tau = \{X, \phi, \{a\}\}$ and $\sigma = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is contra α-continuous but not contra continuous.

Example 2.51. [16] Let $X = \{a, b\}$ with the indiscrete topology τ and $\sigma = \{X, \phi, \{a\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is contra γ-continuous but not contra semi continuous, since $A = \{a\} \in \sigma$ but A is not semi closed in (X, τ).

Example 2.52. [12] Let $X = \{a, b, c, d\}$ and

$$\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}\}.$$

Define a function $f : (X, \tau) \to (X, \tau)$ as follows : $f(a) = b$, $f(b) = a$ $f(c) = d$ and $f(d) = c$. Then f is contra semi-continuous. However, f is not contra α-continuous, since $\{c, d\}$ is closed set of (X, τ) and $f^{-1}(\{c, d\}) = \{c, d\}$ is not α-open.

Example 2.53. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $Y = \{1, 2\}$ be the Sierpinski space with the topology $\sigma = \{Y, \phi, \{1\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by: $f(a) = 1$ and $f(b) = f(c) = 2$. Then f is contra γ continuous but neither contra pre continuous nor contra $\widetilde{g}\alpha$-continuous.

Example 2.54. [7] A contra semi continuous function need not be contra pre continuous. Let $f : R \to R$ be the function $f(x) = [x]$, where $[x]$ is the Gaussion symbol. If V is a closed subset of the real line, its pre image $U = f^{-1}(V)$ is the union of the intervals of the form $[n, n+1]$, $n \in Z$; hence U is semi open being union of semi open sets. But f is not contra pre continuous, since $f^{-1}(0.5, 1.5) = [1, 2)$ is not pre closed in R.

Example 2.55. [7] A contra pre continuous function need not be contra semi continuous. Let $X = \{a, b\}$, $\tau = \{X, \phi\}$ and $\sigma = \{X, \phi, \{a\}\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is contra pre continuous as only the trivial subsets of X are open in (X, τ). However $f^{-1}(\{a\}) = \{a\}$ is not semi closed in (X, τ); hence f is not contra semi continuous.

Example 2.56. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $Y = \{p, q\}$, $\sigma = \{Y, \phi, \{p\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = p$ and $f(b) = f(c) = q$. Then f is contra β-continuous but neither contra pre continuous nor $\widetilde{g}\alpha$-continuous, since $f^{-1}(\{q\}) = \{b, c\}$ is β-open neither pre open nor $\widetilde{g}\alpha$-open.

Example 2.57. Let $X = \{a, b, c\} = Y$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \tau)$ be defined by: $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is contra semi continuous but not contra $\widetilde{g}\alpha$-continuous, since $f^{-1}(\{a, c\}) = \{a, c\}$ is not $\widetilde{g}\alpha$-open.

Example 2.58. Let $X = \{a, b, c\} = Y$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is contra pre continuous but not contra $\widetilde{g}\alpha$-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not $\widetilde{g}\alpha$-open.

3. Applications of contra-$\widetilde{g}\alpha$-Continuous Functions

Definition 3.1. A topological space X is said to be

(a) $\widetilde{g}\alpha$-normal if each pair of non-empty disjoint closed sets can be separated by disjoint $\widetilde{g}\alpha$-open sets,
(b) ultranormal [15] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

Theorem 3.2. If $f : X \to Y$ is a contra $\widetilde{g}\alpha$-continuous, closed injection and Y is ultranormal, then X is $\widetilde{g}\alpha$-normal.

Proof. Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is ultranormal, $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2, respectively. Hence $F_i \subset f^{-1}(V_i), f^{-1}(V_i) \in \widetilde{g}\alpha O(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus X is $\widetilde{g}\alpha$-normal. ■

Definition 3.3. A topological space X is said to be $\widetilde{g}\alpha$-connected if X is not the union of two disjoint non-empty $\widetilde{g}\alpha$-open subsets of X.

Theorem 3.4. A contra $\widetilde{g}\alpha$-continuous image of a $\widetilde{g}\alpha$-connected space is connected.

Proof. Let $f : X \to Y$ be a contra $\widetilde{g}\alpha$-continuous function of a $\widetilde{g}\alpha$-connected space X onto to a topological space Y. If possible, let Y is disconnected. Let A and B form a disconnected of Y. Then A and B are clopen and $Y = A \cup B$ where $A \cap B = \phi$. Since f is contra $\widetilde{g}\alpha$-continuous, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty $\widetilde{g}\alpha$-open sets in X. Also, $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence X is non-$\widetilde{g}\alpha$-connected which is a contradiction. Therefore Y is connected. ■

Theorem 3.5. Let X be $\widetilde{g}\alpha$-connected and Y be T_1. If $f : X \to Y$ is contra $\widetilde{g}\alpha$-continuous, then f is constant.

Proof. Since Y is T_1 space, $v = \{f^{-1}(y) : y \in Y\}$ is a disjoint $\widetilde{g}\alpha$-open partition of X. If $|v| \geq 2$, then X is the union of two non-empty $\widetilde{g}\alpha$-open sets. Since X is $\widetilde{g}\alpha$-connected, $|v| = 1$. Therefore, f is constant. ■

Theorem 3.6. If $f : X \to Y$ is a contra $\widetilde{g}\alpha$-continuous function from a $\widetilde{g}\alpha$-connected space X onto any space Y, then Y is not a discrete space.

Proof. Suppose that Y is discrete. Let A be a proper non-empty open and closed subset of Y. Then $f^{-1}(A)$ is a proper nonempty $\widetilde{g}\alpha$-clopen subset of X, which is a contradiction to the fact X is $\widetilde{g}\alpha$-connected. ■

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