$\widetilde{G}\alpha$-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract

In this paper, we introduce the notion of $\widetilde{g}\alpha$-*closed* sets in topological spaces and investigate some of their basic properties.

Keywords. $\widetilde{g}\alpha$-*closed* set and $^\sharp gs$-*closed* set.

AMS(2000) Subject classification : 54A05,54D10,54F65,54G05.

1. Introduction and Preliminaries

Levine [6,7] introduced the concept of *generalized closed* sets and *semi-closed* sets in topological spaces. Maki et al. introduced *generalized* α-*closed* sets (briefly $g\alpha$-*closed* sets) [9] and α-*generalized closed* sets (briefly αg-closed sets) [8]. The concept of $\widehat{g}$-*closed* sets [16,17], $^*g$-*closed* sets [14] and $^\sharp gs$-*closed* sets [15] are introduced by M.K.R.S. Veera Kumar. In this paper, we introduce a new class of sets, namely, $\widetilde{g}\alpha$-*closed* sets and present some of its properties.

Throughout this paper (X,τ), (Y,σ) and (Z,η) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$ and $int(A)$ denote the closure of A and the interior of A, respectively. $P(X)$ denotes the power set of X.

We recall the following definitions which are useful in the sequel.

Definition 1.1. A subset A of a space (X,τ) is called

1. a *pre-open* set [10] if $A \subseteq int(cl(A))$ and a *pre-closed* set if $cl(int(A)) \subseteq A$,

2. a *semi-open* set [7] if $A \subseteq cl(int(A))$ and a *semi-closed* set [7] if $int(cl(A)) \subseteq A$,

3. an α-*open* set [11] if $A \subseteq int(cl(int(A)))$ and an α-*closed* set [11] if $cl(int(cl(A))) \subseteq A$,

4. a *semi-preopen* set [1] if $A \subseteq cl(int(cl(A)))$ and a *semi-preclosed* set [1] if $int(cl(int(A))) \subseteq A$ and

5. a *regular open* set if $A = int(cl(A))$ and a *regular closed* set if $cl(int(A)) = A$.

The pre-closure (resp. semi-closure, α-closure, semi-preclosure) of a subset A of a space (X, τ) is the intersection of all *pre-closed* (resp. *semi-closed*, α-*closed*, *semi-preclosed*) sets that contain A and is denoted by $pcl(A)$ (resp. $scl(A)$, $\alpha cl(A)$, $spcl(A)$).

Definition 1.2. A subset A of a space (X, τ) is called a

1. a *generalized closed* (briefly *g-closed*) set [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *open* in (X, τ); the complement of a *g-closed* set is called a *g-open* set,

2. a *semi-generalized closed* (briefly *sg-closed*) set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is *semi-open* in (X, τ),

3. a *generalized semi-closed* (briefly *gs-closed*) set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is *open* in (X, τ),

4. an α-*generalized closed* (briefly αg-*closed*) set [8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *open* in (X, τ),

5. a *generalized* α-*closed* (briefly $g\alpha$-*closed*) set [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-*open* in (X, τ),

6. a $g\alpha^*$-*closed* set [9] if $\alpha cl(A) \subseteq int(U)$ whenever $A \subseteq U$ and U is α-*open* in (X, τ),

7. a *generalized semi-preclosed* (briefly *gsp-closed*) set [4] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is *open* in (X, τ),

8. a *generalized preregular-closed* (briefly *gpr-closed*) set [5] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is *regular open* in (X, τ),

9. a g^*-*closed* set [13] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *g-open* in (X, τ),

10. a $\widehat{g}$-*closed* set [16,17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *semi-open* in (X, τ); the complement of a $\widehat{g}$-*closed* set is called a $\widehat{g}$-*open* set,

11. a $^*g$-*closed* set [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\widehat{g}$-*open* in (X, τ); the complement of a $^*g$-*closed* set is called a $^*g$-*open* set,

12. a $^\sharp gs$-*closed* set [15] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^*g$-*open* in (X, τ); the complement of a $^\sharp gs$-*closed* set is called a $^\sharp gs$-*open* set and

13. a $\widetilde{g}s$-*closed* set [12] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^\sharp gs$-*open* in (X, τ).

Notation 1.3. For a topological space (X, τ), $C(X, \tau)$ (resp. $\alpha C(X, \tau)$, $GC(X, \tau)$, $SGC(X, \tau)$, $GSC(X, \tau)$, $\alpha GC(X, \tau)$, $G\alpha C(X, \tau)$, $G\alpha^* C(X, \tau)$, $GSPC(X, \tau)$, $GPRC(X, \tau)$, $G^*C(X, \tau)$, $^*GC(X, \tau)$, $^\sharp GSC(X, \tau)$, $\widetilde{G}SC(X, \tau)$) denotes the class of all *closed* (resp. *α-closed*, *g-closed*, *sg-closed*, *gs-closed*, *αg-closed*, *gα-closed*, $g\alpha^*$-*closed*, *gsp-closed*, *gpr-closed*, g^*-*closed*, $^*g$-*closed*, $^\sharp gs$-*closed*, $\widetilde{g}s$-*closed*) subsets of (X, τ).

2. $\widetilde{g}\alpha$-*closed* sets

We introduce the following definition.

Definition 2.1. A subset A of (X, τ) is called a $\widetilde{g}\alpha$-*closed* set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^\sharp gs$-*open* in (X, τ).

Theorem 2.2. *Every α-closed set is a $\widetilde{g}\alpha$-closed set and thus every closed set is $\widetilde{g}\alpha$-closed.*

Proof. Let A be an *α-closed* set in (X, τ), then $A = \alpha cl(A)$. Let $A \subseteq U$ such that U is $^\sharp gs$-*open* in (X, τ). Since A is *α-closed*, $A = \alpha cl(A) \subseteq U$. This shows that A is $\widetilde{g}\alpha$-*closed* set. The second part of the theorem follows from the fact that every *closed* set is *α-closed*.

The converse of Theorem 2.2 is not true as it can be seen by the following example.

Example 2.3. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. Here $\alpha C(X, \tau) =$

$\{X, \phi, \{c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{c\}, \{b, c\}, \{a, c\}\}$ and let $A = \{b, c\}$. Then A is not an α-*closed* and thus it is not *closed*. However A is a $\widetilde{g}\alpha$-*closed* set.

Thus the class of $\widetilde{g}\alpha$-*closed* sets properly contains the classes of α-*closed* sets and *closed* sets.

Theorem 2.4.

(a) *Every $\widetilde{g}\alpha$-closed set is a gs-closed set and thus gsp-closed and gpr-closed.*

(b) *Every $\widetilde{g}\alpha$-closed set is a $g\alpha$-closed set and thus αg-closed.*

(c) *Every $\widetilde{g}\alpha$-closed set is a sg-closed set and thus semi-preclosed.*

Proof. It follows from the definitions.

The following examples show that these implications are not reversible.

Example 2.5. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Here $GSC(X, \tau) = P(X)$, $GSPC(X, \tau) = P(X)$, $GPRC(X, \tau) = P(X)$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$ and let $A = \{b\}$. Then A is *gs-closed*, *gsp-closed* and *gpr-closed*. However A is not a $\widetilde{g}\alpha$-*closed* set.

Example 2.6. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{b\}, \{b, c\}\}$. Here $G\alpha C(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$, $\alpha GC(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$ and let $A = \{a, b\}$. Then A is $g\alpha$-*closed* and αg-*closed*. However A is not a $\widetilde{g}\alpha$-*closed* set.

Example 2.7. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Here $SGC(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$, $SPC(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{c\}, \{b, c\}, \{a, c\}\}$ and let $A = \{a\}$. Then A is *sg-closed* and *semi-preclosed*. However A is not a $\widetilde{g}\alpha$-*closed* set.

Theorem 2.8. *Every $\widetilde{g}\alpha$-closed set is $\widetilde{g}s$-closed set.*
Proof. It follows from the definitions.

The converse of Theorem 2.8 need not be true by the following example.

Example 2.9. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. Here $\widetilde{G}SC(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$, $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Let $A = \{a\}$. Then A is $\widetilde{g}s$-*closed* but not a $\widetilde{g}\alpha$-*closed* set.

Theorem 2.10.

(a) $\widetilde{g}\alpha$*-closedness is independent of g-closedness,* g^**-closedness and* $^*g$*-closedness.*

(b) $\widetilde{g}\alpha$*-closedness is independent of* $\widehat{g}$*-closedness.*

(c) $\widetilde{g}\alpha$*-closedness is independent of* $g\alpha^*$*-closedness.*

Proof. It follows from the following examples.

Example 2.11. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, c\}\}$. Here $GC(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$, $G^*C(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$, $^*GC(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{b\}.\{c\}, \{b, c\}\}$. Then $\{a, b\}$ is *g-closed*, g^**-closed* and $^*g$*-closed*, but not $\widetilde{g}\alpha$*-closed* set and also $\{c\}$ is $\widetilde{g}\alpha$*-closed*, but not even a *g-closed*, g^**-closed* and $^*g$*-closed*.

Example 2.12. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, c\}\}$. Here $\widehat{G}C(X, \tau) = \{X, \phi, \{b\}, \{b, c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Then $\{c\}$ is $\widetilde{g}\alpha$*-closed*, but not a $\widehat{g}$*-closed* set.

Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Here $\widehat{G}C(X, \tau) = P(X)$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$. Then $\{b\}$ is $\widehat{g}$*-closed*, but not a $\widetilde{g}\alpha$*-closed* set.

Example 2.13. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Here $G\alpha^*C(X, \tau) = P(X)$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$. Then $\{b\}$ is $g\alpha^*$*-closed*, but not a $\widetilde{g}\alpha$*-closed* set.

Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. Here $G\alpha^*C(X, \tau) = \{X, \phi, \{b, c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Then $\{b\}$ is $\widetilde{g}\alpha$*-closed*, but not a $g\alpha^*$*-closed* set.

Theorem 2.14. *Let A be a subset of* (X, τ)*.*

(a) *If A is* $\widetilde{g}\alpha$*-closed, then* $\alpha cl(A) - A$ *does not contain any non-empty* $^\sharp gs$*-closed set.*

(b) *If A is* $\widetilde{g}\alpha$*-closed and* $A \subseteq B \subseteq \alpha cl(A)$*, then B is* $\widetilde{g}\alpha$*-closed.*

Proof.

(a) Suppose that A is $\widetilde{g}\alpha$*-closed* and let F be a non-empty $^\sharp gs$*-closed* set with $F \subseteq \alpha cl(A) - A$. Then $A \subseteq X - F$ and so $\alpha cl(A) \subseteq X - F$. Hence $F \subseteq X - \alpha cl(A)$, a contradiction.

(b) Let U be a $^\sharp gs$-*open* set of (X,τ) such that $B \subseteq U$. Then $A \subseteq U$. Since A is $\widetilde{g}\alpha$-*closed*, $\alpha cl(A) \subseteq U$. Now $\alpha cl(B) \subseteq \alpha cl(\alpha cl(A)) \subseteq U$. Therefore B is also a $\widetilde{g}\alpha$-*closed* set of (X,τ).

Theorem 2.15. *Let A and B be subsets of a topological space (X,τ). Then the union of two $\widetilde{g}\alpha$-closed set is $\widetilde{g}\alpha$-closed set in (X,τ).*
Proof. Let A and B be $\widetilde{g}\alpha$-*closed* sets. Let $A \cup B \subseteq U$ such that U is $^\sharp gs$-*open*. Since A and B are $\widetilde{g}\alpha$-*closed* sets, $\alpha cl(A) \subseteq U$ and $\alpha cl(B) \subseteq U$. This implies that $\alpha cl(A \cup B) = \alpha cl(A) \cup \alpha cl(B) \subseteq U$, (since τ^α=α-*open* set forms a topology [9]) and so $\alpha cl(A \cup B) \subseteq U$. Therefore $A \cup B$ is $\widetilde{g}\alpha$-*closed*.

We need the following notations:
For a subset E of a space (X,τ), we define the following subsets of E.
$E_\tau = \{x \in E/\{x\} \in \tau\}$;
$E_\mathcal{F} = \{x \in E/\{x\}$ is *closed* in $(X,\tau)\}$;
$E_{\widetilde{g}\alpha o} = \{x \in E/\{x\}$ is $\widetilde{g}\alpha$-*open* in $(X,\tau)\}$;
$E_{^\sharp gsc} = \{x \in E/\{x\}$ is $^\sharp gs$-*closed* in $(X,\tau)\}$.

Lemma 2.16. *For any space (X,τ), $X = X_{^\sharp gsc} \cup X_{\widetilde{g}\alpha o}$ holds.*
Proof. Let $x \in X$. Suppose that $\{x\}$ is not $^\sharp gs$-*closed* set in (X,τ). Then X is a unique $^\sharp gs$-*open* set containing $X - \{x\}$. Thus $X - \{x\}$ is $\widetilde{g}\alpha$-*closed* in (X,τ) and so $\{x\}$ is $\widetilde{g}\alpha$-*open*. Therefore $x \in X_{^\sharp gsc} \cup X_{\widetilde{g}\alpha o}$ holds.

We need more notations:

For a subset A of (X,τ), $ker(A) = \cap\{U/U \in \tau$ and $A \subseteq U\}$;
$^\sharp GSO$-$ker(A) = \cap\{U/U \in {}^\sharp GSO(X,\tau)$ and $A \subseteq U\}$.

Theorem 2.17. *For a subset A of (X,τ), the following conditions are equivalent.*

(1) *A is $\widetilde{g}\alpha$-closed in (X,τ).*

(2) *$\alpha cl(A) \subseteq {}^\sharp GSO$-$ker(A)$ holds.*

(3) (i) *$\alpha cl(A) \cap X_{^\sharp gsc} \subseteq A$ and (ii) $\alpha cl(A) \cap X_{^\sharp gso} \subseteq {}^\sharp GSO$-$ker(A)$ holds.*

Proof.
(1) ⇒ (2) Let $x \notin {}^\sharp GSO$-$ker(A)$. Then there exists a set $U \in {}^\sharp GSO(X,\tau)$ such

that $x \notin U$ and $A \subseteq U$. Since A is $\widetilde{g}\alpha$-*closed*, $\alpha cl(A) \subseteq U$ and so $x \notin \alpha cl(A)$. This shows that $\alpha cl(A) \subseteq {}^{\sharp}GSO\text{-}ker(A)$.

(2) $\Rightarrow$ (1) Let $U \in {}^{\sharp}GSO(X, \tau)$ such that $A \subseteq U$. Then we have that ${}^{\sharp}GSO\text{-}ker(A) \subseteq U$ and so by (2) $\alpha cl(A) \subseteq U$. Therefore A is $\widetilde{g}\alpha$-*closed.*

(2) $\Rightarrow$ (3) (i) First we claim that ${}^{\sharp}GSO\text{-}ker(A) \cap X_{\sharp gsc} \subseteq A$. Indeed, let $x \in {}^{\sharp}GSO\text{-}ker(A) \cap X_{\sharp gsc}$ and assume that $x \notin A$. Since the set $X - \{x\} \in {}^{\sharp}GSO(X, \tau)$ and $A \subseteq X - \{x\}$, ${}^{\sharp}GSO\text{-}ker(A) \subseteq X - \{x\}$. Then we have that $x \in X - \{x\}$ and so this is a contradiction. Thus we show that ${}^{\sharp}GSO\text{-}ker(A) \cap X_{\sharp gsc} \subseteq A$. By using (2), $\alpha cl(A) \cap X_{\sharp gsc} \subseteq {}^{\sharp}GSO\text{-}ker(A) \cap X_{\sharp gsc} \subseteq A$.

(ii) It is obtained by (2).

(3) $\Rightarrow$ (2) By lemma 2.16 and (3),

$$\begin{aligned} \alpha cl(A) &= \alpha cl(A) \cap X = \alpha cl(A) \cap (X_{\sharp gsc} \cup X_{\widetilde{g}\alpha o}) \\ &= (\alpha cl(A) \cap X_{\sharp gsc}) \cup (\alpha cl(A) \cap X_{\widetilde{g}\alpha o}) \\ &= A \cup {}^{\sharp}GSO\text{-}ker(A) \\ &= {}^{\sharp}GSO\text{-}ker(A) \text{ holds.} \end{aligned}$$

Theorem 2.18. Let (X, τ) be a space and A and B are subsets.

(i) If A is ${}^{\sharp}gs$-*open* and $\widetilde{g}\alpha$-*closed*, then A is α-*closed* in (X, τ).

(ii) Suppose that (X, τ) is an α-space. A $\widetilde{g}\alpha$-*closed* set A is α-*closed* in (X, τ) if and only if $\alpha cl(A) - A$ is α-*closed* in (X, τ).

(iii) For each $x \in X$, $\{x\}$ is ${}^{\sharp}gs$-*closed* or $X - \{x\}$ is $\widetilde{g}\alpha$-*closed* in (X, τ).

(iv) Every subset is $\widetilde{g}\alpha$-*closed* in (X, τ) if and only if ${}^{\sharp}gs$-*open* set is α-*closed.*

Proof.

(ii) (Necessity) If A is α-*closed*, then $\alpha cl(A) - A = \phi$.
(Sufficiency) Suppose that A is $\widetilde{g}\alpha$-*closed* and $\alpha cl(A) - A$ is α-*closed.* It follows from assumptions that $\tau = \tau^{\alpha}$. Then, $\alpha cl(A) - A$ is ${}^{\sharp}gs$-*closed* in (X, τ) and by Theorem 2.14., $\alpha cl(A) - A = \phi$. Therefore A is α-*closed* in (X, τ).

(iii) If $\{x\}$ is not ${}^{\sharp}gs$-*closed*, then $X - \{x\}$ is not ${}^{\sharp}gs$-*open.* Therefore $X - \{x\}$ is $\widetilde{g}\alpha$-*closed* in (X, τ).

(iv) (Necessity) Let U be a ${}^{\sharp}gs$-*open* set. Then we have that $\alpha cl(U) \subseteq U$ and hence U is α-*closed.*

(Sufficiency) Let A be a subset and U is a $^\sharp gs$-*open* set such that $A \subseteq U$. Then $\alpha cl(A) \subseteq \alpha cl(U) = U$ and hence A is $\widetilde{g}\alpha$-*closed*.

Remark 2.19. The following diagram shows the relationships established between $\widetilde{g}\alpha$-*closed* sets and some other sets. $A \to B$ represents A implies B but not conversely.

$$
\begin{array}{ccccc}
\alpha-closed & & gs-closed & & g\alpha-closed \\
& \searrow & \uparrow & \nearrow & \\
closed & \longrightarrow & \widetilde{g}\alpha-closed & \longrightarrow & gsp-closed \\
& \swarrow & \downarrow & \searrow & \\
semi-preclosed & & sg-closed & & gpr-closed
\end{array}
$$

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