A runner cheating thanks to Riemann:

Akram Louiz
Independent researcher, Settat, Morocco.
louiz.akram@gmail.com

Abstract: Pure mathematics should be used very carefully when applying it to many fields that have special considerations and special axioms.
I used a simple example of runners waiting for the start of a race. I concluded thanks to Riemann's definition of integrals that a runner can cheat if he has time in order to win.
The demonstration in this paper is very simple but the analogy of the proposed example with many fields can make the researcher be careful when using the definition of Riemann for the integrals.

Keywords: Riemann's definition, integrals, applied mathematics, analogy.

1. Introduction:

Let's consider that a runner wants to run a race on a running track of a field. The shape of the race field is continuous and thus the track of this field can be considered as a curve representing a continuous integrable function f of real numbers for each runner.
The chronometer starts if and only if the runner overrides the starting line A.
The chronometer stops if and only if the runner reaches the finishing line B.
Let's consider that A and B are two points of the line of real numbers. Consequently, for all the runners, the distance between A and B is the distance where the chronometer isn't stopped, and thus the distance is the length l of the interval]A,B].
We conclude that l= B-A and consequently the length l_1 of]A,B[is l_1= B-A-ε and the length l_2 of [A,B] is l_2= B-A+ε (1)

For the runners and in many scientific fields, ε exists otherwise the three intervals above will be considered the same during the study, with ε as follows:
$\forall A, B\in R$ with B>A we have: $\exists M\in R\backslash Q$with M= Max {x/x ϵ [A,B[} (2)
Hence: $\exists \varepsilon\in R\backslash Q$ with ε>0 and M+ ε=B. (3)

For the runners, M exists even if we can't determine it exactly . It is the real number that is sticking to the point B at the left of this point in the real number line.
Let's consider that $\frac{C}{n}$ is the length of any part of the real number line, then we have:
$\forall n\in N\backslash 0$and $\forall C\epsilon$]0,+∞ [$\quad B-\frac{C}{n}\leqslant B-\epsilon<B$ (4)
So we can consider that $\varepsilon=0^+$.

Remark:

In many scientific fields where applied mathematics are required, any part of the line of the real numbers (an interval) has a length L>0 otherwise it doesn't exist. In this case, the infinitesimal length of a singleton is l=ε>0 and it exists because the infinite sum of singletons' lengths is the only cause that makes the length L of an interval.

2. Investigation:

The referee of the race is a mathematician who should know the field of the race. Hence the referee makes an integration of the function f since the field of the race represents the integrable function f.

The referee makes a definition to the integral: $\int_A^B f(x).dx$ **by Riemann's Definition [1] on time scales [2]:**

The correct subdivision of the interval [A,B] should be made with intervals which are: $[a_{i-1},a_i[$ since their length is exactly $(a_i - a_{i-1})$ even in the considerations made above and since they are separated (they make separate surfaces). **This is very important because the sum of infinite infinitesimal common surfaces caused by the not separated intervals $[a_{i-1},a_i[$ can make a surface that won't be neglected.**

Consequently, we consider that: $a_i > a_{i-1}$ $\forall n \in N\backslash 0$and a_0=A and a_n=B

Hence: $U_{i=1}^{n}[a_{i-1},a_i[$ <=> [A,B[(5)

And: $\forall i \in N\backslash 0$: $A_i = A + i\frac{B-A}{n}$ and $a_i - a_{i-1} = h = \frac{B-A}{n}$ (6)

We define also: $x_i = A + \frac{(i-1+\gamma)\times(B-A)}{n} = a_{i-1} + \gamma\times h$ with: $\gamma \in [0,1[$. (7)

Finally we have: $S_f = \sum_{i=1}^{n}(f(x_i)\times(a_i - a_{i-1})) = \frac{B-A}{n}\times\sum_{i=1}^{n}(f(A + \frac{(i-1+\gamma)\times(B-A)}{n}))$ (8)

S_f becomes mathematically the Riemann sum of the function f as the partitions get finer. However, in order to respect even the considerations of the runners, we should also have:

[A,B] <=>$U_{i=1}^{n}[a_{i-1},a_i[U\{B\}$ (9)

And by considering that: M= Max $\{x/x \in [a_{n-1},B[\}$ we conclude that: {B}<=>]M,B] (10)

And thus: [A,B] <=>$U_{i=1}^{n}[a_{i-1},a_i[$ *U]M,B]* *(11)*

And we considered that: B-M= ε so we conclude that:

$$S_f + (B - M)\times f(B) = S_f + \varepsilon\times f(B) = F_A^B \quad (12)$$

Important:

We considered that: $\frac{B-A}{n} \geqslant \epsilon > 0$ from the beginning with F_A^B the corrected **Riemann sum** applied for $\int_A^B f(x).dx$ by respecting even the considerations of the runners.

We get finally: $$\lim_{n\to+\infty} F_A^B = \int_A^B f(x).dx \quad (13)$$

which means that:

$$\int_A^B f(x).dx - \varepsilon\times f(B) = \lim_{n\to+\infty}\frac{B-A}{n}\times\sum_{i=1}^{+\infty}(f(A + (i - 1 + \gamma))\times\frac{B-A}{n}) \quad (14)$$

Now let's use a subdivision with the intervals: $]a_{i-1},a_i]$ and $\gamma \in]0,1]$. In this case, the considered ε above stays the same for all the real numbers since the positions of all real numbers are distributed in the same manner in the line of the real numbers.
S_f stays also the same but we get finally:

$$\int_A^B f(x).dx - \varepsilon\times f(A) = \lim_{n\to+\infty} \frac{B-A}{n} \times \sum_{i=1}^{+\infty} (f(A + (i - 1 + \gamma))\times\frac{B-A}{n}) \quad (15)$$

We conclude that: $\varepsilon\times(f(A) - f(B)) = 0.$ (16)

And since f(A) and f(B) depending on the studied field represented by the function f, then $\varepsilon = 0$.

Consequently:

After using riemann's definition of integrals and despite the considerations of the runners, the mathematician referee made these conclusions:
M= Max $\{x/x \in [a_{n-1},B[\}$= Sup $\{x/x \in [a_{n-1},B[\}$=B (17)
And M'= Min $\{x/x \in]A,a_1]\}$= Inf $\{x/x \in]A,a_1]\}$=A (18)

3. Conclusion:

Now let's return back to our example:
The mathematician referee has all the proof that the small distance "epsilon" equals zero $\varepsilon = 0$ since he studied the field of the race. However, "epsilon" is real and strictly positive $\varepsilon>0$ for all the runners who are not mathematicians.
We can conclude that if a runner comes to the field of the race a long period of time before that the race starts, then that period of time can be sufficient for the not mathematician runner to cheat many times with the small distance "epsilon" without that the referee notices because $\varepsilon = 0$ for mathematicians but $\varepsilon=0^+$ for the runner. In this case, the runner can use his positive real short distance "epsilon" in order to move ahead many times and cross the starting line by cheating without that the mathematician notices. The runner can even reach the finishing line B and win the race without any competition if the referee postpones the start of the race for an infinite Time since the runner will use his "epsilon" to move ahead infinite times.
The analogy exists between this example of race and many scientific and engineering fields where applied mathematics can be used.

NB: I advise you also to read one of my previous works in order to remark the difficulties that happen when applying mathematics in other scientific fields, especially in Physics [3].

References:

[1] Bohner, M., & Guseinov, G. (2003). Riemann and Lebesgue integration. In *Advances in dynamic equations on time scales* (pp. 117-163). Birkhäuser, Boston, MA.
[2] Guseinov, G. S. (2003). Integration on time scales. *Journal of Mathematical Analysis and Applications*, *285*(1), 107-127.
[3] Louiz, A. (2020). A thesis about Newtonian mechanics rotations and about differential operators. Maghrebian Journal of Pure and Applied Science, 6(1), 26-50.