GENERAL SOLVING TRANSCEDENTAL AND POLYNOMIALS EQUATIONS- WITH THE GENERALIZED THEOREM OF LAGRANGE

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Abstract : While all the approximate methods mentioned or others that exist, give some specific solutions of the generalized transcendental equations or even polynomial, cannot resolve them completely.

"What we ask when we solve a generalized transcendental equation or polyonomial, is to find the total number of roots and not separate sets of roots in some random or specified this time. Mainly because this, too many categories transcendental equations have infinite number of solutions in the complex whole "

There are some particular equations or Logarithmic functions Trigonometric functions which solve particular problems in Physics, and mostly need the generalized solution. This is now the theory G.R $_L$E, to deal with the help of Super Simple geometric functions, or interlocking with very satisfactory answer to all this complex problem.

PART I. The Basic theorem G.R$_L$.E(N.Mantzakouras. 2007)

Types of functions

1. Definition

We define as **type of function** one of the 5 general forms of functions that are presented in the mathematics that is the following ones:

1. Exponential function.
2. Logarithmic function.
3. Trigonometric function.
4. Power function.
5. Power exponential function.

2. Definition

1. **Primary simple transcendental equation** is called each equation of the form $\sigma(z) = q(z) + t = 0$ with $t \in C$ that has roots in C
2. **Primary composite transcendental equation** is called each equation of the form $\sigma(z) = q(z) + m \cdot p(z) + t = 0$ which has roots in the total entire C Moreover the factors $m, t \neq 0$ which also take values from the set C and in general the functions $q(z), p(z)$ are of different type, with values above in the C

3. Theorem of existence and count of roots of primary composite Transcendental equation

Each **primary complex transcendental equation** of the form $\sigma(z) = q(z) + m \cdot p(z) + t = 0$ has as count of roots the union of individual fields of roots $L_1, L_2 \in C$ i.e.: $C.R(\text{Count Roots}) = \bigcup_{i=1}^{2} L_i$ **(1)** of the equations that follow:

$$\sigma_1(z) = m \cdot p(z) + t = 0 \quad \textbf{(σ1)}$$

$$\sigma_2(z) = q(z) + t = 0 \quad \textbf{(σ2)}$$

which come up with the **primary composite transcendental equation:** $q(z) + m \cdot p(z) + t = 0$ if also provided that if:

1. The functions $q(z), p(z)$ are functions of different type, or of a different form, or of a different power of the same type in general.
2. The factors $m, t \neq 0$ take values from the total entire C
3. The fields of L_1, L_2 roots of **(σ1, σ2)** equations are solved according to the theorem of Lagrange, and after the **primary simple transcendental functions** are solved per case, and belong in the total entire C

The Count of fields of the roots is 2, and consequently the set of the fields of the roots of

$$L = \{\exists z_i EC : \sigma_1(z) = 0\} \cup \{\exists z_j EC : \sigma_2(z) = 0\} \wedge q(z) + m \cdot p(z) + t = 0, \text{ or } \boxed{L = \bigcup_{i=1}^{2} L_i}$$

<u>**Case I**</u>

Let $q(z), p(z)$ and $f(z)$ and $\varphi(z)$ be functions of z analytic on and inside a contour C surrounding a point t and let m be such that the inequality $|-m \cdot p(z)| < |z - (-t)|$ is satisfied at all points z on the perimeter of C; letting $q(z) = \zeta$ and consequently $z = q^{-1}(\zeta)$; then doing inversion of the function I take $f(\zeta) = q^{-1}$ and then according to the **initial primary composite transcendental equation,** I have:

$$q(z)+m\cdot p(z)+t=0 \quad \textbf{(1)}$$

$$\zeta = q(z) = -m\cdot p(z)-t \quad \textbf{(2)}$$

If where $$\varphi(z) = p\left(q^{-1}(\zeta)\right) \quad \textbf{(3)}$$

Afterwards the equation **(2)** becomes: $\zeta = -m\cdot\varphi(z)-t$ regarded as an equation in ζ has one root in the interior of C; and further any function of ζ analytic on the inside C can be expanded as a power series with the use of a variable similar to ζ by the formula

$$f(\zeta) = f(w)_{w\to -t} + \sum_{n=1}^{\infty}\frac{(-m)^n}{n!}\frac{d^{n-1}}{dw^{n-1}}\left[f'(w)\{\varphi(w)\}^n\right]_{w\to -t} \quad \textbf{(4)}$$

And we come up with the root

$$\boxed{z_{i,1} = f(w)_{w\to -t} + \sum_{n=1}^{\infty}\frac{(-m)^n}{n!}\frac{d^{n-1}}{dw^{n-1}}\left[f'(w)\{\varphi(w)\}^n\right]_{w\to -t}} \quad \textbf{(5)}$$

that is also the final relation for the solution of the root of the **initial primary composite transcendental equation** $q(z)+m\cdot p(z)+t=0$ having count of the roots $z_{i,1}$ where the sum of the roots i is identified with the count of the roots of the **primary simple transcendental equation** $q(z)=-t$ which also determines the field of the roots of the equation $q(z)=-m\cdot p(z)-t$ that it is also L_1 and concerns only this form, that is to say **(σ1).**

Case II

As previously, we examine the second case

$p(z)=-\frac{1}{m}\cdot q(z)-\frac{t}{m}$ **(σ2)** which come up if we solve the initial equation $q(z)+m\cdot p(z)+t=0$ as for $p(z)$ Similarly the functions $q(z), p(z)$ and $f(z)$ are in effect which are regarded functions of analytic on and inside a contour C surrounding a point t and let m be such that the inequality

$$\left|-\frac{1}{m}\cdot q(z)\right| < \left|z-\left(-\frac{t}{m}\right)\right|$$

Then letting $p(z)=\zeta$ Doing inversion of the function I take $f(\zeta)=p^{-1}(\zeta)$ and from the initial primary complex transcendental equation $q(z)+m\cdot p(z)+t=0$ **(1*)** I take $\zeta=p(z)=-\frac{1}{m}\cdot q(z)-\frac{t}{m}$ **(2*)**

And if I take $\varphi(z)=q\left(p^{-1}(\zeta)\right)$ **(3*)** then the equation $\zeta=-\frac{1}{m}\cdot\varphi(\zeta)-\frac{t}{m}$ (more than one if the q (z) function exponential, logarithmic, trigonometric, Power function (n> 1), Power exponential function)regarded as an equation in ζ has one root in the interior of C; and further any function of ζ analytic on and inside C can be expanded as a power series with $w\to -t/m$ by the formula

$$f(\zeta)=f(w)_{w\to t/m}+\sum_{n=1}^{\infty}\frac{\left(-\frac{1}{m}\right)^n}{n!}\frac{d^{n-1}}{dw^{n-1}}\left[f'(w)\{\varphi(w)\}^n\right]_{w\to -t/m} \quad \textbf{(4*)}$$

And we come up with the root

$$\boxed{z_{i,2}=f(w)_{w\to -t/m}+\sum_{n=1}^{\infty}\frac{\left(-\frac{1}{m}\right)^n}{n!}\frac{d^{n-1}}{dw^{n-1}}\left[f'(w)\{\varphi(w)\}^n\right]_{w\to -t/m}} \quad \textbf{(5*)}$$

that is the relation for the solution of the root of the **initial primary composite transcendental equation** $q(z)+m\cdot p(z)+t=0$ having sum of roots $z_{i,2}$ so the count i is identified with the count of roots of the **primary simple transcendental equation**

$$p(z)=-\frac{t}{m}$$

which determines also the field of the roots of the equation $p(z)=-\frac{1}{m}\cdot q(z)-\frac{t}{m}$ that are L_2 and it also concerns only this form, that is to say **(σ2).** The total of roots, as simply comes up, will be the union of totals of solutions that is represented by the fields of the roots L_1, L_2 i.e. $\boxed{C.R(\text{Count Roots})=\bigcup_{i=1}^{2}L_i}$ and because the functions $q(z)$ and $p(z)$ are different between them, accordingly the totals of solutions L_1, L_2 will be different between them. In general, however, we accept the union of fields of the roots L_1, L_2 as the final field of roots of the equation $q(z)+m\cdot p(z)+t=0$ which we also symbolize as $L = L_1 \cup L_2$

1. **Generalised Theorem of existence and count of roots, of an accidental transcendental equation**

Each **accidental transcendental equation,** of the form

$$\sigma(z)=\sum_{i=1}^{n} m_i \cdot p_i(z)+t=0$$

has as count of roots the union of individual fields of the roots $L_1, L_2, \ldots L_n$, i.e.: the equations that follow:

$$m_1 \cdot p_1(z)+\sum_{i=2}^{n} m_i \cdot p_i(z)+t=0 \qquad (\sigma_1)$$

$$m_2 \cdot p_2(z)+\sum_{i=1, i\neq 2}^{n} m_i \cdot p_i(z)+t=0 \qquad (\sigma_2)$$

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$$m_\kappa \cdot p_\kappa(z)+\sum_{i=1, i\neq \kappa}^{n-1} m_i \cdot p_i(z)+t=0 \qquad (\sigma_k)$$

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$$m_n \cdot p_n(z)+\sum_{i=1}^{n-1} m_i \cdot p_i(z)+t=0 \qquad (\sigma_n)$$

which come up with the **generalised transcendental equation:**

$$\sigma(z)=\sum_{i=1}^{n} m_i \cdot p_i(z)+t=0$$

if also provided that if:

1. Functions $p_i(z)$ are analytical functions at all the point in the interior on total entire C simultaneously, they are functions of different type, or of different form, or of different power of the same type generally.

2. The factors $m_i, t \neq 0$ take values in the total entire C belong to the sequence of n (count of n factors) with at least 2 factors of m_i to be different of zero.
3. The fields of the roots $L_1, L_2, \ldots L_n$ of the $\sigma_1, \sigma_2, \ldots \sigma_n$ equations are solved according to the theorem of Lagrange and belong in the total entire C
4. The Count of fields of the roots is n, and consequently the set of the fields of th roots of $m_\kappa \cdot p_\kappa(z) + \sum_{i=1, i\neq\kappa}^{n-1} m_i \cdot p_i(z) + t = 0, \kappa = 1, 2, \ldots, n$ is $\boxed{L = \bigcup_{i=1}^{n} L_i}$

Case 1

Proof

Let $p_i(z)$ and $f(z)$ and $\varphi(z)$ be functions of z analytic on and inside a contour C surrounding a point t and let m_i be such that the inequality $\left|-\frac{1}{m_1}\sum_{i=2}^{n} m_i \cdot p_i(z)\right| < \left|z - (-t/m_1)\right|$ is satisfied at all points z on the perimeter of C; letting $p_i(z) = \zeta$ then doing inversion of the function I take $f(\zeta) = p_1^{-1}(\zeta)$ and from the generalised transcendental equation

$$\sigma(z) = \sum_{i=1}^{n} m_i \cdot p_i(z) + t = 0 \qquad (1)$$

I take
$$\zeta = p_1(z) = -\frac{1}{m_1}\sum_{i=2}^{n} m_i \cdot p_i(z) - t/m_1 \qquad (2)$$

If where
$$\boxed{\varphi(\zeta) = \sum_{i=2}^{n} m_i \cdot p_i\left(p_1^{-1}(\zeta)\right)} \qquad (3)$$

and also it is in effect $f(\zeta) = p_1^{-1}(\zeta)$ then the equation

$$\boxed{\zeta = -\frac{1}{m_1}\sum_{i=2}^{n} m_i \cdot p_i(z) - t/m_1}$$

regarded as an equation in ζ which has one root in the interior of C; (more than one if the q (z) function exponential, logarithmic, trigonometric, Power function (n> 1), Power exponential function)and further any function of ζ analytic on and inside C can be expanded as a power series with the use of a variable $w \to -t/m_1$ by the formula

$$f(\zeta) = f(w)_{w \to -t/m_1} + \sum_{n=1}^{\infty} \frac{\left(-\dfrac{m_r}{m_1}\right)^n}{n!} \frac{d^{n-1}}{dw^{n-1}} \left[f'(w) \{\varphi(w)\}^n \right]_{w \to -t/m_1} \quad \textbf{(4)}$$

where $$\varphi(w) = \sum_{i=2}^{n} (m_i/m_r) \cdot p_i\left(p_1^{-1}(w)\right) \quad \textbf{(5)}$$

with $|m_r| < |m_i|$ for any $i \geq 1,\ i \neq k,\ i \leq n, k \in Z$ or $m_r = \min\{|m_1|, |m_2|, \ldots, |m_n|\}$ so that $|m_r/m_1| < 1$

And we come up with the root

$$z_{i,1}^{full} = f(w)_{w \to -t/m_1} + \sum_{n=1}^{\infty} \frac{\left(-\dfrac{m_r}{m_1}\right)^n}{n!} \frac{d^{n-1}}{dw^{n-1}} \left[f'(w) \{\varphi(w)\}^n \right]_{w \to -t/m_1} < \quad \textbf{(6)}$$

that is the relation for the solution of the root of a **generalised transcendental equation** $\sigma(z) = \sum_{i=1}^{n} m_i \cdot p_i(z) + t = 0$ having count of roots $z_{i,1}^{full}$ so that the count i is identified with the count of the roots of the **primary simple transcendental equation** $p_1(z) = -\dfrac{t}{m_1}$

In this case now this determines also the field of the roots of the equation $m_1 \cdot p_1(z) + \sum_{i=2}^{n} m_i \cdot p_i(z) + t = 0$ that is L_1 and it also concerns only this form, that is to say the form **(σ1).**

Case 2

Proof in generally

The pairs of forms at line of the function

$$\sigma(z) = \sum_{i=1}^{n} m_i \cdot p_i(z) + t = 0 \qquad \textbf{(1**)}$$

in the formula $$m_k \cdot p_k(z) + \sum_{i=1, i \neq k}^{n} m_i \cdot p_i(z) + t = 0 \quad \textbf{(2**)}$$

belongs, as simply is proved, to the sequence of n (sum of n factors). **The proof is simple:** we suppose that we have a change κ with the attribute that follows:

If $m_1, m_2, ...m_k$ distinguished elements of the total $\{1,2,...n\}$ with the equivalence are existed, then π if $\alpha_1\pi = \alpha_2, \alpha_2\pi = \alpha_3, ..., \alpha_k\pi = \alpha_1$ is called a circle of length κ and it is written as follows. Consequently, we have line of changes κ Respectively, if $n = \kappa$, then we have the line of changes n According to the developed form **(2**)** we take the analysis in the forms that follow:

$$p_1(z) + \sum_{i=2}^{n} (m_i / m_1) \cdot p_i(z) + t / m_1 = 0 \qquad (\sigma_1)$$

$$p_2(z) + \sum_{i=1, i\neq 2}^{n} (m_i / m_2) \cdot p_i(z) + t / m_2 = 0 \qquad (\sigma_2)$$

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$$p_k(z) + \sum_{i=1, i\neq k}^{n} (m_i / m_k) \cdot p_i(z) + t / m_k = 0 \qquad (\sigma_k)$$

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$$p_n(z) + \sum_{i=1}^{n-1} (m_i / m_n) \cdot p_i(z) + t / m_n = 0 \qquad (\sigma_n)$$

Now let $p_i(z)$ and $f(z)$ and $\varphi(z)$ be functions of z analytic on and inside a contour C surrounding a point t and let m_i be such that the inequality $\left| -(1/m_\kappa) \sum_{i=2}^{n} m_i \cdot p_i(z) \right| < \left| z - (-t/m_\kappa) \right|$ is satisfied at all points z on the perimeter of C; letting $p_\kappa(z) = \zeta$ then doing inversion of the function I take $f(\zeta) = p_\kappa^{-1}(\zeta)$ and from the generalised transcendental equation $\sigma(z) = \sum_{i=1}^{n} m_i \cdot p_i(z) + t = 0$ **(1)**

I take with replacement

$$\boxed{\zeta = p_\kappa(z) = -\frac{1}{m_\kappa} \sum_{i=1, i\neq\kappa}^{n} m_i \cdot p_i(z) - t / m_\kappa} \qquad \textbf{(2)}$$

and $\varphi(\zeta)=\sum_{i=1,i\neq\kappa}^{n} m_i\cdot p_i\left(p_\kappa^{-1}(\zeta)\right)$ with $i\geq 1,\ i\neq\kappa,\ i\leq n, k\in Z$ after of course it is in effect $f(\zeta)=p_\kappa^{-1}(\zeta)$,then the equation $\zeta = -\frac{1}{m_k}\sum_{i=1,i\neq\kappa}^{n} m_i\cdot p_i(\zeta)-t/m_\kappa$ **(3)**

regarded as an equation in ζ which has one root in the interior of C; (more than one if the q (z) function exponential, logarithmic, trigonometric, Power function (n> 1)), and further any function of ζ analytic on and inside C can be expanded as a power series with a similar of ζ variable $w\rightarrow -t/m_\kappa$ by the formula

$$f(\zeta)=f(w)_{w\rightarrow -t/m_\kappa}+\sum_{n=1}^{\infty}\frac{\left(-\frac{m_r}{m_\kappa}\right)^n}{n!}\frac{d^{n-1}}{dw^{n-1}}\left[f'(w)\{\varphi(w)\}^n\right]_{w\rightarrow -t/m_\kappa} \quad \textbf{(4)}$$

where $$\varphi(w)=\sum_{i=1,i\neq\kappa}^{n}(m_i/m_r)\cdot p_i\left(p_\kappa^{-1}(w)\right) \quad \textbf{(5)}$$

with $|m_r|<|m_i|$ for any $i\geq 1,\ i\neq\kappa,\ i\leq n$ or $m_r=\min\{|m_1|,|m_2|,\ldots|m_n|\}$ so that $|m_r/m_\kappa|<1$ also for complex roots k>=1 and $k\in Z$ in general. Final we come up with the root

$$\boxed{z_{i,\kappa}^{full}=f(w)_{w\rightarrow -t/m_\kappa}+\sum_{n=1}^{\infty}\frac{\left(-\frac{m_r}{m_\kappa}\right)^n}{n!}\frac{d^{n-1}}{dw^{n-1}}\left[f'(w)\{\varphi(w)\}^n\right]_{w\rightarrow -t/m_\kappa}<} \quad \textbf{(6)}$$

that is the relation for the solution of the roots of the **generalised transcendental equation** $\sigma_\lambda(z)=\sum_{i=1}^{n}m_ip_i(z)+t=0$ with $1\leq\lambda\leq n$ and $1\leq i\leq n, y_\lambda\in Z$ having count of roots of $\sum_{i=1}^{n}m_ip_i(z)+t=0$ to is $\sum_{\lambda=1}^{n}y_\lambda$ and y_λ is the partial sets cardinality of roots and thus y_λ identified with the number of roots of the the **primary simple transcendental equations** $p_i=-\frac{t}{m_i}, i=\lambda$. Specifically, the case $p_i=-\frac{t}{m_i}, i=\lambda$ identified as field roots , with the field of the roots of the equation in form $m_kp_k(z)+\sum_{i=1,i\neq k}^{n}m_ip_i(z)+t=0$ that is the L_k and concerns only this form, that is to say **(σ_κ).** Now, for the generalisation of cases, because this κ takes values from 1 to n, consequently the count of fields of roots will also be n, and consequently the field of total of the roots of the equations

$\sigma_\kappa(z) = m_k p_k(z) + \sum_{i=1, i\neq k}^{n} m_i p_i(z) + t = 0, z \to -t/m_k, k \in \{1,2...,n\}$ will be $L = \bigcup_{i=1}^{n} L_i$

Of course, all these are in effect, provided that the functions of type $p_i(z)$ are of different type, or of different form, or of different power of the same type in general. In the case where we have powers of the same type, then we have the formula $\sigma_\lambda(z) = \sum_{i=1}^{n} m_i p_i^{r_i}(z) + t = 0$ with $r_i \in C$ and $i \in N$. More specifically, for the case of $r_i \in N$ and for the calculation of the roots, we will have the general relation that follows:

$$z_{i,\kappa}^{full} = f(w)_{w \to -t/m_\kappa} + \sum_{n=1}^{\infty} \frac{\left(-\frac{m_r}{m_\kappa}\right)^n}{n!} \frac{d^{n-1}}{dw^{n-1}} \left[f'(w) \{\varphi(w)\}^n \right]_{w \to -t/m_\kappa} <$$

with $f(w) = p_\kappa^{-1}\left(w^{1/r_j} \cdot e^{2\pi i/g}\right)$ and $\varphi(w) = \sum_{i=1, i\neq\kappa}^{n} m_i \cdot p_i\left(p_\kappa^{-1}\left(w^{1/r_j} \cdot e^{2\pi i/g}\right)\right)$ **(7)**

Within $g = 0,1,2...r_i - 1 \in N$

For the sum in general we have the following formula:

$$\sum_{n=1}^{\infty} \frac{\left(-\frac{m_r}{m_\kappa}\right)^n}{n!} \frac{d^{n-1}}{dw^{n-1}} \left[f'(w) \{\varphi(w)\}^n \right]_{w \to -t/m_\kappa} \quad \textbf{(8)}$$

That sum should converge in some limit of more generally complex number, and according to the conditions of the theorem of **Lagrange.**

This method gives very good results usually in cases where t <=1. For instances, however, where t> 1 is usually employs the inverse of the transformation F(1/ z) which facilitates the convergence of the sum of the formula (8).

The relevant literature on the original theorem of Lagrange and Burmann's derived from the classic book on the theorem "Course of modern Analysis of E.T Witteker and G.N.Watson "Press Campridge 2002 [0].

PART II . 7 FAMUS TRANSCENDENTAL EQUATIONS

1. SOLUTION OF THE EQUATION $z \cdot e^z = t$

The roots of the equation play a role in the iteration of the exponential function[2;3;11] and in the solution and application of certain difference - Equation [1;9;10;12]. For this reason, several authors [4; 5; 7; 8; 9; 12] have found various properties of some or all of the roots. There is a work by **E. M. Write,** communicated by **Richard Bellman**, December 15, 1958. Also must mention a very important offer of Wolfram in Mathematica program with the W--Function.

But now we will solve the with the method **($G.R_L.E$),** because it is the only method that throws ample light on general solve all equations. All the roots of our equation are given by $\log(z) + z = \log(t) + 2 \cdot k \cdot \pi \cdot i$ **(1)** where k takes all integral values as $k = 0, \pm 1, \pm 2, \pm 3, \ldots \pm \infty$. To solve the equation looking at three intervals, which in part are common and others differ in the method we choose.

A) Because we take the logarithm in both parties of the equation, the case $t < o \wedge t \in R$ leads only in complex roots. From the theory ($G.R_L.E$) we get two cases according to relation (I), because the relationship **(1)** has two functions, $p_1(z) = z = \zeta$ **(a)** and $p_2(z) = \log(z) = \zeta$ **(b).**

Thus the first case **(a)** the solution we are the roots of the equation

$z_k = \zeta + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (\zeta' \cdot \log^i(\zeta))$ **(i)** where ζ' is the first derivative of ζ with the type $\zeta = \log(t) + 2 \cdot k \cdot \pi \cdot i$ and k is integer, for a value of i. Also the case when t and is a complex number and especially when $|t| \geq e$, then the solution is represented by the same form **(i)**.

B) For interval $0 \leq t \leq \frac{1}{e} \wedge t \in R$ but also general where $0 \leq |t| \leq \frac{1}{e}$ in case that t is complex number and when $k \neq 0$, then the solutions illustrated from the equation :

$$z_k = \zeta + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (\zeta' \cdot \log^i(\zeta)) \quad \textbf{(i)}$$

and in case that $k = 0$ then using the form $p_2(z) = \log(z) = \zeta \Rightarrow z = Exp(\zeta)$ the Lagrange equation from ($G.R_L.E$) transformed to $z_k = Exp(\zeta) + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (Exp(\zeta)' \cdot Exp^i(\zeta))$ **(ii)** but this specific form translatable to $z_k = Exp(\zeta) + \sum_{i=1}^{\infty}(\frac{(-m)^i}{Gamma(i+1)}(i+1)^{i-1} \cdot Exp^{i+1}(\zeta))$ because we know the nth derivative of $Exp(m \cdot x) = m^n \cdot Exp(m \cdot x)$.

C) Specificity for the region $\frac{1}{e} \le t \le e \wedge t \in R$ but more generally $\frac{1}{e} \le |t| \le e$. Appears a small anomaly in the form **(i)** and as regards the complex or real value for k = 0 in $\zeta = \log(t) + 2 \cdot k \cdot \pi \cdot i$. The case for Complex roots we get as a solution of the equation by the form

$$z_k = \zeta + \sum_{i=1}^{\infty}(\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}}(\zeta' \cdot \log^i(\zeta)) \text{ except if } k \ne 0.$$

Eventually the case k = 0 is presented and the anomaly in the approach of the infinite sum in the form **(ii)**

$$z_s = Exp(\zeta) + \sum_{i=1}^{\infty}(\frac{(-m_*)^i}{Gamma(i+1)}(i+1)^{i-1} \cdot Exp^{i+1}(\zeta))$$

but $m_* = m / e^{s+1}$ with s>=1

Because the replay will be s times and $\zeta = z_{s-1}, s > 1$ we have to repeat. A very good approximation also in this special case is when we use the method approximate of Newton after obtaining an initial root z_s with s = 1 .

2. MAXIMUM THE SURFACE AREA AND VOLUME OF A HYPERSPHERE N DIM'S

In mathematics, an ***n*-sphere** is a generalization of the surface of an ordinary sphere to arbitrary dimension. For any natural number *n*, an *n*-sphere of radius *r* is defined as the set of points in (*n* + 1)-dimensional Euclidean space which are at distance *r* from a central point, where the radius *r* may be any positive real number. It is an *n*-dimensional manifold in Euclidean (*n* + 1)-space.

The n-hypersphere (often simply called the n-sphere) is a generalization of the circle (called by geometers the 2-sphere) and usual sphere (called by geometers the 3-sphere) to dimensions n>=4. The n-sphere is therefore defined (again, to a geometer; see below) as the set of n-tuples of points ($x_1, x_2, \ldots, x_n$) such that

$$x_1^2 + x_2^2 + \ldots. + x_n^2 = R^2 \quad (1)$$

where R is the radius of the hypersphere.

Let V_n denote the content of an n-hypersphere (in the geometer's sense) of radius R is given by

$V_n = \int_0^R S_n r^{n-1} dr = \frac{S_n \cdot R^n}{n}$ where S_n is the hyper-surface area of an n-sphere of unit radius. A unit hypersphere must satisfy

$$S_n \int_0^{\infty} e^{-r^2} r^{n-1} dr = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} e^{-(x_1^2+..+x_n^2)} dx_1 \ldots dx_n = (\int_{-\infty}^{\infty} e^{-x^2} dx)^n \Rightarrow \frac{1}{2} S_n \Gamma(n/2) = (\Gamma(1/2))^n$$

And to the end

$S_n = R^{n-1} 2(\Gamma(1/2))^n / \Gamma(n/2) = R^{n-1}(2\pi^{n/2}) / \Gamma(n/2)$ (1)

$V_n = R^n(\pi^{n/2}) / \Gamma(1+n/2)$ (2)But the gamma function can be defined by $\Gamma(m) = 2\int_0^{\infty} e^{-r^2} r^{2m-1} dr$

Strangely enough, the hyper-surface area reaches a maximum and then decreases towards 0 as n increases. The point of maximal hyper-surface area satisfies

$$\frac{dS_n}{dn} = R^{n-1}(2\pi^{n/2}) / \Gamma(n/2) = R^{n-1}\pi^{n/2} \cdot [\ln\pi - \psi_0(n/2)] / \Gamma(n/2) = 0 \ (3)$$

Where $\psi_0(x) = \Psi(x)$ is the digamma function . For maximum volume the same they be calculated

$$\frac{dV_n}{dn} = R^n(\pi^{n/2}) / \Gamma(1+n/2) = R^n \pi^{n/2} \cdot [\ln\pi - \psi_0(1+n/2)] / (2 \cdot \Gamma(1+n/2)) = 0 \ (4)$$

From Feng Qi and Bai –Ni-Guo exist theorem [arXiv:0902.2519v2 [math.CA] 19 Jan 2011]

For all $x \in (0,\infty)$, $\ln(x+\frac{1}{2}) - \frac{1}{x} < \psi(x) < \ln(x+e^{-\gamma}) - \frac{1}{x}$ the constant $e^{-\gamma} = 0.56$.
Taking advantage of the previous theorem solved in two levels ie...

From (3) we have 2 levels :

$\ln(\frac{1}{2}x + \frac{1}{2}) - \frac{1}{\frac{1}{2}x} = \ln\pi$ (a) and $\ln(\frac{1}{2}x + e^{-\gamma}) - \frac{1}{\frac{1}{2}x} = \ln\pi$ (b)

Both cases, if resolved in accordance with the theorem (G.R$_L$.E) from by the form..

$$z = 2 \cdot (e^{\zeta} - 1/2) \cdot + \sum_{i=1}^{\infty} (\frac{(-m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial \zeta^{i-1}} (2 \cdot e^{\zeta} \cdot (\frac{-2}{2(e^{\zeta} - 1/2)})^{\zeta}))$$

with $\zeta \to \log(\pi)$ but $m = 1/e^{s+1}$, with s>=1 as before in 1 case. The initial value for (a) case is 5.59464 and for (b) case is 5.48125. We use the method approximate of Newton after obtaining an initial root z_s with s =1 is 7.27218 and 7.18109 respectively, finally after a few iterations.This shows that ultimately we as integer result the integer 7, for maximum hyper-surface area.

Thereafter for the case of maximum volume, and before applying From Feng Qi and Bai –Ni-Guo *For all* $x\in(0,\infty)$, $\ln((\frac{1}{2}x+1+\frac{1}{2})-\frac{1}{(\frac{1}{2}x+1)}=\log(\pi)$ and $\ln(\frac{1}{2}x+1+e^{-\gamma})-\frac{1}{\frac{1}{2}x+1}=\ln\pi$. The results in both cases according to equation..

$$z=2\cdot(e^{\zeta}-3/2)\cdot+\sum_{i=1}^{\infty}(\frac{(-m)^{i}}{Gamma(i+1)}\frac{\partial^{i-1}}{\partial\zeta^{i-1}}(2\cdot e^{\zeta}\cdot(\frac{-2}{2(e^{\zeta}-3/2)+2})^{\zeta})$$ with $\zeta\to\log(\pi)$ but $m=1/e^{s+1}$ with s>=1 as before case. In two cases end up in the initial values 3.59464 and 3.48125. We use the method approximate of Newton arrive quickly in 5.27218 and 5.18109 respectively. Therefore the integer for the maximum volume hyper-surface is the 5.

3. THE KEPLER'S EQUATION

The kepler's equation allows determine the relation of the time angular displacement within an orbit. Kepler's equation is of fundamental importance in celestial mechanics, but cannot be directly inverted in terms of simple functions in order to determine where the planet will be at
a given time. Let M be the mean anomaly(a parameterization of time) and E the eccentric anomaly (a parameterization of polar angle) of a body orbiting on an ellipse with eccentricity e, then …

$$j=\frac{1}{2}a\cdot b\cdot(E-e\cdot SinE)\Rightarrow M=E-e\cdot SinE=(t-T)\cdot\sqrt{\frac{a^{3}}{\mu}}$$ and $h=\sqrt{p\cdot\mu}$

is angular momentum, $j=Area-angular$ .Eventually the equation of interest is in final form is $M=E-e\cdot SinE$ and calculate the E. The Kepler's equation [14]has a unique solution,but is a simple transcendental equation and so cannot be inverted and solved directly for E given an arbitrary M. However, many algorithms have been derived for solving the equation as a result of its importance in celestial mechanics. In essentially trying to solve the general equation $x-e\cdot Sinx=t$ where t,e are arbitrary in C more generally. According to the theory G.R$_L$.E we have two basic cases $p_1(z)=z=\zeta$ **(a)** and $p_2(z)=Sin(z)=\zeta$ **(b)** which if the solve separately,the total settlement will result from the union of the 2 fields of the individual solutions.The first case is this is of interest in relation to the equation Kepler, because $e<1$.From theory G.R$_L$.E we have the solution

$$z=\zeta+\sum_{i=1}^{\infty}(\frac{(e)^{i}}{Gamma(i+1)}\frac{\partial^{i-1}}{\partial\zeta^{i-1}}(\zeta'\cdot Sin^{i}(\zeta))=\zeta+\sum_{i=1}^{\infty}(\frac{(e)^{i}}{Gamma(i+1)}\frac{\partial^{i-1}}{\partial\zeta^{i-1}}(Sin^{i}(\zeta))\ (1)$$

for $\zeta\to t$. Since the exponents are changed from an odd to even we use two general expressions for the nth derivatives. If we have even exponent is

$$\frac{\partial^{2n-1}}{\partial x^{2n-1}} Sin^{2n}(x) = (1/2^{2*n-1}) * \sum_{k=0}^{n-1}(-1)^{n-k} * (2*n)!/(k! * (2n-k)!) * (2n-2k)^{2n-1} * \text{Sin}[(2n-2k)*t+(2n)\pi/2]$$

and for odd exponent is

$$\frac{\partial^{2n}}{\partial x^{2n}} Sin^{2n+1}(x) = \frac{1}{2^{2n}} * \sum_{k=0}^{n}(-1)^{n-k} * (2*n+1)!/(k! * (2n+1-k)!) * (2n-2k+1)^{2n} * \text{Sin}[(2n-2k+1)*x+(2n)\pi/2]$$

These formulas help greatly in finding the general solution of equation Kepler,because this is generalize the nth derivative of $Sin^i(\zeta)$ as sum of the two separate cases. So from (1) we can see the only solution for the equation Kepler's with the type (2)

$$z = t + \sum_{n=0}^{\infty}(1/2^{2n}) * \sum_{k=0}^{n}(-1)^{n-k} * ((m)^{2*n+1}/\text{Gamma}[2*n+2]) * (2*n+1)!/(k! * (2n+1-k)!) * (2n-2k+1)^{2n} * \text{Sin}[(2n-2k+1)*t+(2n)\pi/2] + \sum_{s=0}^{\infty}(1/2^{2*s-1}) * \sum_{k=0}^{s-1}((m)^{2*s}/\text{Gamma}[2*s+1]) * (-1)^{s-k} * (2*s)!/(k! * (2s-k)!) * (2s-2k)^{2s-1} * \text{Sin}[(2s-2k)*t+(2s)\pi/2]$$

The second case solution of the $x - e\cdot Sinx = t$ according to the theory G.R$_L$.E we can also from the $p_2(z) = Sin(z) = \zeta$ **(b)** that $z = ArcSin(z) + 2k\pi$ and also $z = -ArcSin(z) + (2k+1)\pi$ **.** So the full solution of the equation $x - e\cdot Sinx = t$ of the second field of roots is …

$$z_k = (ArcSin(\zeta) + 2k\pi) + \sum_{i=1}^{\infty}\left(\frac{(1/e)^i}{Gamma(i+1)}\frac{\partial^{i-1}}{\partial\zeta^{i-1}}((ArcSin(\zeta)'\cdot(ArcSin(\zeta)+2k\pi)^i)\right) \quad (3)$$

Or $$z_k = (-ArcSin(\zeta) + (2k+1)\pi) + \sum_{i=1}^{\infty}\left(\frac{(1/e)^i}{Gamma(i+1)}\frac{\partial^{i-1}}{\partial\zeta^{i-1}}((-ArcSin(\zeta)'\cdot(-ArcSin(\zeta)+(2k+1)\pi)^i)\right) \quad (4)$$

$ArcSin(\zeta)' = \dfrac{1}{\sqrt{1-\zeta^2}}$ (5) with $\zeta \to t/e$ and $k \in Z$. An example is the Jupiter, with data $M = 5\cdot 2\cdot\pi/11.8622$ with eccentricity e where $e = 0.04844$, then from equation (2) we find the value of (x or E) = 2.6704 radians.

4. THE NEUTRAL DIFFERENTIAL EQUATIONS (D.D.E)

In this part solve of transcendental equations we introduce another class of equations depending on past and present values but that involve derivatives with delays as well as function itself. Such equations historically have been referred as neutral differential difference equations[15].

The model non homogeneous equation is

$$\sum_{k=1}^{n} g_k \cdot \frac{\partial^k}{\partial x^k} x(t) + \sum_{r=1}^{m} c_r \cdot \frac{\partial^r}{\partial x^r} x(t-\tau_r) = a \cdot x(t) + \sum_{i=1}^{\sigma} w_i \cdot x(t-v_i) + f(t) \quad (1)$$

With g_k, c_r, a, w_i is constants and $w_i \neq 0$ and $f(t)$ is a continuous function on C. Of course any discussion of specific properties of the characteristic equation will be much more difficult since this equation transcendental, will be of the form :

$$h(\lambda) = a_0(\lambda) + \sum_{j=1}^{n1} a_j(\lambda) \cdot e^{-\lambda \tau_j} + \sum_{i=1}^{n2} b_i(\lambda) \cdot e^{-\lambda \cdot v_i} = 0 \quad (2)$$

Where $a_j(\lambda), b_i(\lambda), j > 0$ are polynomials of degree $\leq (m+\sigma)$ and $a_0(\lambda)$ is a polynomial of degree n also must $n_1 + n_2 \leq m+\sigma$. The equations (2) also resolved in accordance with the method G.R$_L$.E and the general solution is of as the form $x(t) = f_s(t) + \sum_j p_j(t) \cdot e^{\lambda_j t}$ where λ_j are the roots of the equation of characteristic and p_j are polynomials and also $f_s \neq f$ in generally. As an example we give the D.D.E differential equation $x'(t) - C \cdot x'(t-r) = a \cdot x(t) + w \cdot x(t-v) + f(t)$ (3) which is like an equation $h(\lambda)$ as of characteristic $h(\lambda) = \lambda(1 - C \cdot e^{-\lambda \cdot r}) - a - w \cdot e^{-\lambda \cdot v} = 0$ where $C \neq 0, r \geq 0, v \geq 0$ and a, w constants.

5. SOLUTION OF THE EQUATION $x^x - m \cdot x + t = 0$

The solution of the equation is based mostly on the solution of equation $x^x = z$ which has solution relying on the solution of $x \cdot e^x = v$ which solved before. Specifically because we know the function $W_k(z)$ is product log function $k \in Z$, and using it to solve the equation $x \cdot e^x = v$ is $z = W_k(v), v \neq 0$. Also $k \in Z$, all the solutions of the equation $x^x = z$ is for $z \neq 0$. According to this assumption we can solve the equation $x^x - m \cdot x + t = 0$ with the help of the method G.R$_L$.E. According to the theory G.R.E we have two basic cases $p_1(x) = x^x = \zeta$ **(a)** and $p_2(x) = x = \zeta$ **(b)** which if the solve separately, the total settlement will result from the union of the 2 fields of the individual solutions, $\zeta \in C$. The first case is of interest in relation to the equation has more options than the second, because it covers a large part of the real and the complex solutions. This situation leads to the solution for x such that it is in the form

$x = e^{ProductLog[(2k)\pi i + Log[\zeta]]}$ or taking and the other form

$$x = \frac{(\log(\zeta))}{(W_k(\log(\zeta)))}$$

From theory G.R$_L$.E we have the solution

$$x_k = e^{ProductLog[h,2\pi ik+\log[\zeta]]} + \sum_{v=1}^{\infty}\left(\frac{(-m)^v}{Gamma(i+1)}\frac{\partial^{v-1}}{\partial\zeta^{v-1}}((e^{ProductLog[h,2\pi ik+\log[\zeta]]})' \cdot e^{v\cdot ProductLog[h,2\pi ik+\log[\zeta]]}\right) \text{ with the } k \in Z$$

And $h = -1,0,1$ or more exactly

$$x_k = e^{ProductLog[h,2\pi ik+\log[\zeta]]} + \sum_{v=1}^{\infty}\left(\frac{(-m)^v}{Gamma(v+1)}\frac{\partial^{v-1}}{\partial\zeta^{v-1}}\left(\frac{1}{\zeta\cdot(1+ProductLog[h,2\pi ik+\log[\zeta]])}\right)\cdot e^{v\cdot ProductLog[h,2\pi ik+\log[\zeta]]}\right) \text{ with}$$

multiple roots in relation to k and $\zeta \to t$. Variations presented in case where, when we change the sign of m,t mainly in the sign of the complex roots. Even and in anomaly in the approach of the infinite sum we use the transformation but $m_* = m / e^{s+1}$ with s>=1, a very good approximation also in this special case is when we use the method approximate of Newton after obtaining an initial root z_s. The second group of solutions represents real mainly roots of equation where $p_2(x) = x = \zeta$

So we have

$$x = \zeta + \sum_{v=1}^{\infty}\left(\frac{(-1/m)^v}{Gamma(v+1)}\frac{\partial^{v-1}}{\partial\zeta^{v-1}}(\zeta'\cdot\zeta^{v\zeta}\right) = \zeta + \sum_{v=1}^{\infty}\left(\frac{(-1/m)^v}{Gamma(v+1)}\frac{\partial^{v-1}}{\partial\zeta^{v-1}}(\zeta^{v\cdot\zeta}\right) \text{ with } \zeta \to t/m \text{ ,for } m,t \in C$$

in generally.

6. SOLUTION OF THE EQUATION $x^q - m\cdot x^p + t = 0$

An equation seems simple but needs analysis primarily on the distinction of m, but also the powers specific p,q as to what look every time.

Distinguish two main cases:

$i)......p,q \in R$

The weight method would follow it takes m ,which regulates the method we will follow any time. But according to the method G.E.R we have two basic cases $p_1(x) = x^p = \zeta$ **(a)** and $p_2(x) = x^q = \zeta$ **(b)** whose solution gives the individual a comprehensive solution of the equation. For the case under consideration ie $m > 1, p > q$ transforms the original in two formats to assist us in connection with the logic employed by the general relation G.R$_L$.E .

The first transform given from the form $x^p - m \cdot x^q + t = 0 \Rightarrow x^q - (1/m) \cdot x^p - t/m = 0$ which is now in the normal form to solve equation. First we need to solve the relationship $x^p = \zeta$ in C. Following that we can get the form $x_k = e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/q}, k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[q/2]$ and the count of roots is maximum 2*IntegerPart[q/2] in generality.

Therefore so the first form of solution of the equation is..

$$x_k = e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/q} + \sum_{v=1}^{\infty} (\frac{(-1/m)^v}{Gamma(i+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} ((\frac{e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/q}}{q \cdot \zeta}) \cdot (e^{p \cdot (Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/q})^v)$$

Where $\partial_{\zeta} (e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/q}) = (e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/q}) / (q \cdot \zeta)$, with multiple roots in relation to k and $k \in Z, k = 0, \pm 1, \pm 2, \ldots IntegerPart[q/2]$ and $\zeta \to t/m$.

But for the complete solution of this case and find the other roots of the equation for this purpose i make the transformation $x = y^{-1}$ and we have $x^p - m \cdot x^q + t = 0 \Rightarrow y^{-q} - m \cdot y^{-p} + t = 0$ and then we transform in $1 - m \cdot y^{p-q} + t \cdot y^p = 0 \Rightarrow y^{p-q} - t/m \cdot y^p - 1/m = 0$ .In this way we find a whole other roots we have left from all the roots. The form of solution will be as above and assuming the that $g = p - q$ we have..

$y_k = e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/g} + \sum_{v=1}^{\infty} (\frac{(t/m)^v}{Gamma(i+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} ((\frac{e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/g}}{g\zeta}) \cdot (e^{p \cdot (Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/g})^v)$ and $x_k = 1/y_k$ which roots are in relation to $k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[g/2]$ with $\zeta \to -1/m$ .The second case related to m<1 has no procedure for dealing with the method. Starting from the original equation was originally found on the p and so the first transform given from the form $x^p - m \cdot x^q + t = 0$ to solve the relationship $x^p = \zeta$ in C, as helpful to the general equation G.R$_L$.E. So we have

$$x_k = e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/p} + \sum_{v=1}^{\infty} (\frac{(-m)^v}{Gamma(i+1)} \frac{\partial^{v-1}}{\partial \zeta^{v-1}} ((\frac{e^{(Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/p}}{p\zeta}) \cdot (e^{q \cdot (Log(\zeta)+2 \cdot k \cdot \pi \cdot i)/p})^v)$$

with $k \in Z, k = 0, \pm 1, \pm 2, \ldots \pm IntegerPart[p/2]$ with $\zeta \to t$.

To settle the issue of finding the roots, where roots arise other and with m<1 then i make the transformation $x = y^{-1}$ and we have $x^p - m \cdot x^q + t = 0 \Rightarrow y^{-q} - m \cdot y^{-p} + t = 0$ and then we transform in $y^q + 1/t \cdot y^{q-p} - m/t = 0$ with the pre case $p < q$. This transformation is relevant to the case remains as a final case before us. The solution in this case has form and assuming the that $g = p - q$ we have..

$y_k = e^{(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q} + \sum_{v=1}^{\infty}\left(\frac{(-1/t)^v}{Gamma(i+1)}\frac{\partial^{v-1}}{\partial\zeta^{v-1}}\left(\left(\frac{e^{(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q}}{q\cdot\zeta}\right)\cdot\left(e^{g\cdot(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q}\right)^v\right)\right)$ and $x_k = 1/y_k$ which roots are in relation to $k \in Z, k = 0,\pm1,\pm2,....\pm IntegerPart[q/2]$ with $\zeta \to m/t$.

$ii)......p,q \in C$

In this case should first solve the equation , $z^q - m\cdot z^p + t = 0, z \in C$. The solution for z variable, after several operations in concordance with the type De Moivre, we get the relation connecting the real and imaginary parts the general case of complex numbers.. $z^{a+bi} = x + yi$

and the solution is

$$z_k = e^{\frac{b(2k\pi+Arg(x+yi))}{a^2+b^2}}(x^2+y^2)^{\frac{a}{2(a^2+b^2)}}Cos[\frac{a(2k\pi+Arg(x+yi))}{a^2+b^2}-\frac{bLog[x^2+y^2]}{2(a^2+b^2)}]+$$
$$ie^{\frac{b(2k\pi+Arg(x+yi))}{a^2+b^2}}(x^2+y^2)^{\frac{a}{2(a^2+b^2)}}Sin[\frac{a(2k\pi+Arg(x+yi))}{a^2+b^2}-\frac{bLog[x^2+y^2]}{2(a^2+b^2)}]$$

we see that the number of solutions, resulting from the denominator of the fraction that the full line equals with the $c = (a^2+b^2)/a$ if prices of $k \in Z, k = 0,\pm1,\pm2,....\pm IntegerPart[c/2]$. For the case under consideration ie $m > 1, p > q$ transforms the original in two formats to assist us in connection with the logic employed by the general relation G.R$_L$.E.

The first transform given from the form $x^p - m\cdot x^q + t = 0 \Rightarrow x^q - (1/m)\cdot x^p - t/m = 0$ which is now in the normal form to solve equation. First we need to solve the relationship $x^p = \zeta$ in C. Following that we can get the form $k \in Z, k = 0,\pm1,\pm2,....\pm IntegerPart[q/2]$ and the count of roots is maximum 2*IntegerPart[q/2] in generality. The solution is when we analyze the power as

$y_k = e^{(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q} + \sum_{v=1}^{\infty}\left(\frac{(-1/m)^v}{Gamma(v+1)}\frac{\partial^{v-1}}{\partial\zeta^{v-1}}\left(\left(\frac{e^{(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q}}{q\cdot\zeta}\right)\cdot\left(e^{p\cdot(Log(\zeta)+2\cdot k\cdot\pi\cdot i)/q}\right)^v\right)\right)$ and $x_k = y_k$ which roots are in relation to $k \in Z, k = 0,\pm1,\pm2,....\pm IntegerPart[q/2]$ with $\zeta \to t/m$. The remaining cases are similar to previous with $p,q \in R$. The sole change is in relation to the number of cases is $Integer((a^2+b^2)/a)$ for (+/- x axes) and $z^{a+bi} = x + y\cdot i = w$ for any $w,z \in C$.

7. 2 FAMOUS EQUATIONS OF PHYSICS

i)The ***difraction phenomena*** due to "capacity" of the waves bypass obstacles in their way, so to be observed in regions of space behind the barriers, which could be described as ***geometric shadow*** areas . In essence the phenomena of diffraction phenomena [17] is contribution, that is due to superposition of waves of the same frequency that coexist at the same point in space.

If virus is where the intensity at a distance ro from the slot at $\theta = 0$, ie opposite to the slit. So finally we write the relationship in the form

$$I(\theta) = I_o \frac{\sin^2 w}{w^2}$$
$$w = \frac{1}{2} kD \sin\theta$$

The maximum intensity appears to correspond to the extremefunction sin w / w.Derivative of and equating to zero will take the trigonometric equation **$w = \tan w$** a solution which provides the values of w corresponding to maximum intensity. With the assist of a second of the relations We can then, for a given problem is know the wave number k (or wavelength λ) and width D the slit, to calculate the addresses corresponding to θ are the greatest.

Consider many tears as a crowd of $2N+1$ parallel between the cracks width D, the distance from center to center is a and which we have numbered from $-N$ to N. Such a device called a ***diffraction grating*** slits.We accept that sufficiently met the criterion for Fraunhofer diffraction and find the equation for the volume.

$$I(\theta) = I_o \frac{\sin^2 w}{w^2} \frac{\sin^2 Mu}{\sin^2 u}$$

where

$$w = \frac{\pi D \sin\theta}{\lambda}$$
$$u = \frac{\pi a \sin\theta}{\lambda}$$

There fringes addresses for which zero quantity sin u, and therefore the intensity of which is determined by the factor

So we must solve the relation **$w = \tan w$**.

Where $k=\alpha/D$ and $u=kw$, $m=M$.Trying solving the general form of the equation **$w = m*\tan w$** *with* $m \in C$, consider 2 general forms of solution, arising from the form $Cos(w)=\zeta$ and

$$Cos(w) = \zeta \Rightarrow w = \pm ArcCos(w) + 2k\pi$$

$k \in Z, k = 0, \pm1, \pm2, ...$ so we have..

$$w_p = (ArcCos(\zeta) + 2k\pi) + \sum_{i=1}^{\infty} \left(\frac{(m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial\zeta^{i-1}} ((ArcCos(\zeta)' \cdot (Sin[ArcCos(\zeta) + 2k\pi)] / (ArcCos(\zeta) + 2\kappa\pi))^i \right)$$

and the form

$$w_q = (-ArcCos(\zeta) + 2k\pi) + \sum_{i=1}^{\infty} \left(\frac{(m)^i}{Gamma(i+1)} \frac{\partial^{i-1}}{\partial\zeta^{i-1}} ((-ArcCos(\zeta)' \cdot (Sin[-ArcCos(\zeta) + 2k\pi)] / (-ArcCos(\zeta) + 2\kappa\pi))^i \right.$$

Then the general solution is $w_q \cup w_p$.

ii) The spectral density of black body is given by the equation

$$u(\nu) = \bar{E}\rho(\nu) = \frac{h\nu}{e^{bh\nu}-1}\frac{8\pi\nu^2}{c^3} = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}$$

according to the relationship of Plank.

The correlated u (λ)

$$u(\lambda) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/kT\lambda}-1}$$

By $c = \lambda / \mathrm{T} = \lambda\nu$ which is extreme if the derivative zero. Thus we have the relationship

$$\frac{d}{d\lambda}u(\lambda) = 8\pi hc\frac{-5\lambda^4\left(e^{hc/kT\lambda}-1\right)-\lambda^5 e^{hc/kT\lambda}\left(-\frac{1}{\lambda^2}\frac{hc}{kT}\right)}{\lambda^{10}\left(e^{hc/kT\lambda}-1\right)^2}$$

Zeroing the derivative will have the relationship

$$-5\left(e^{hc/kT\lambda}-1\right)+e^{hc/kT\lambda}\left(\frac{1}{\lambda}\frac{hc}{kT}\right)=0$$

And if $x = hc / kT\lambda$ then we get the equation

$$5-5e^{-x}-x=0$$

Finding the solution of x we find the relationship

$$\lambda_{max}T = b$$

By $b = hc / 4.965 \cdot k$ Is called constant Bin, called displacement law. **T**hen we need to calculate the general solution of the equation by the method G,R_LE

The first group of solutions represents real mainly roots of equation where $p_1(x) = x = \zeta$

So we have

$$x = \zeta + \sum_{v=1}^{\infty}\left(\frac{(m)^v}{Gamma(v+1)}\frac{\partial^{v-1}}{\partial\zeta^{v-1}}(\zeta' \cdot Exp[-\zeta])^{v}\right) = \zeta + \sum_{v=1}^{\infty}\left(\frac{(m)^v}{Gamma(v+1)}(-v^{v-1}e^{-\zeta v}\right)$$ with $\zeta \to t$, for $m,t \in C$.

In this case t=5 and m=-5, we calculate the **x=4.9651142317442763037** the nearest 20 ignored.

Because apply relation

$$\frac{\partial^r}{\partial x^r} e^{-xw} = (-w)^r e^{-xw}$$

$$\frac{\partial^r}{\partial x^r} e^{xw} = (w)^r e^{xw}$$

The second group of solutions represents complex roots of equation where

$p_2(x) = e^x = \zeta \Rightarrow x = \log(\zeta) + 2\kappa\pi i$

But this does not refer to real solutions and not the physical Evol equations for this and omitted.

References

[0]. Course of modern analysis: An introduction to the general theory of modern analysis, E.T. Witteker and G.N. Watson ,press Campridge 2002

[1]. R. Bellman, Ann. of Math. vol. 50 (1949) pp. 347-355.

[2]. G. Eisenstein, J. Reine Angew. Math. vol. 28 (1844) pp. 49-52.

[3]. L. Euler, *Opera Omnia* (i) vol. 15, Leipzig and Berne, 1927, pp. 268-297.

[4]. N. D. Hayes, J. London Math. Soc. vol. 25 (1950) pp. 226-232.

[5]. , Quart. J. Math. Oxford Ser. (2) vol. 3 (1952) pp. 81-90.

[6]. A. Hurwitzand R. Courant, *Funktionentheorie,* 3d éd., Berlin, 1929, pp. 141-142.

[7]. E. M. Lémeray, Nouv. Ann. de Math. (3) vol. 15 (1896) pp. 548-556 and vol. 16 (1896) pp. 54-61.

[8]. , Proc. Edinburgh Math. Soc. vol. 16 (1897) pp. 13-35.

[9]. O. Polossuchin, Thesis, Zurich, 1910.

[10]. F. Schürer, Ber. Verh. Sachs. Akad. Wiss. Leipzig Math.-phys. Kl. vol. 64(1912) pp. 167-236 and vol. 65 (1913) pp. 239-246.

[11]. E. M. Wright, Quart. J. Math. Oxford Ser. (2) vol. 18 (1947) pp. 228-235.

[12]. , J. Reine Angew. Math. vol. 194 (1955) pp. 66-87, esp. pp. 72-74.

[13]. http://mathworld.wolfram.com/Hypersphere.html

[14]. http://mathworld.wolfram.com/KeplersEquation.html

[15]. Delay Differential Equations: Recent Advances and New DirectionsBy Balakumar Balachandran, Tamás Kalmár-Nagy, David E. Gilsinn

[16]. yako.physics.upatras.gr/waves/12.pdf

[17]. Εισαγωγή στην Κβαντομηχανική ,Αντώνης Στρέκλας