SEMIOPEN SETS IN IDEAL BITOPOLOGICAL SPACES

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ABSTRACT. The aim of this paper is to introduced and characterized the concepts of semiopen sets and their related notions in ideal bitopological spaces.

1. INTRODUCTION

The important and useful notion of semiopen sets introduced and to some extent studied by N. Levine [19]. Since then several papers investigated this notions in not only in topological spaces but also in different contexts. The concept of ideals in topological spaces has been introduced and studied by Kuratowski [18] and Vaidyanathasamy [23]. Hamlett and Janković (see [12], [13], [16] and [17]) utilized topological ideals to generalize many notions and properties in general topology. Since then many contributed to this field of research such as M. E. Abd El-Monsef, A. Al-Omari, F. G. Arenas, M. Caldas, J. Dontchev, M. Ganster, D. N. Georgiou, T. R. Hamlett, E. Hatir, S. D. Iliadis, S. Jafari, D. Jankovic, E. F. Lashien, M. Maheswari, , H. Maki, A. C. Megaritis, F. I. Michael, A. A. Nasef, T. Noiri, B. K. Papadopoulos, M. Parimala, G. A. Prinos, M. L. Puertas, M. Rajamani, N. Rajesh, D. Rose, A. Selvakumar, Jun-Iti Umehara and many others (see [1], [2], [5], [7], [8], [9], [10], [11], [14], [15], [17], [22], [20], [21]).An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a bitopological space (X, τ_1, τ_2) with an ideal $\mathcal{I}$ on X and if $\mathcal{P}(X)$ is the set of all subsets of X, a set operator $(.)_i^*$: $\mathcal{P}(X) \to \mathcal{P}(X)$, called the local function [23] of A with respect to τ_i and $\mathcal{I}$, is defined as follows: for $A \subset X$, $A_i^*(\tau_i, \mathcal{I}) = \{x \in X | U \cap A \notin \mathcal{I}$ for every $U \in \tau_i(x)\}$, where $\tau_i(x) = \{U \in \tau_i | x \in U\}$. A Kuratowski closure operator $\mathrm{Cl}_i^*(.)$ is defined by $\mathrm{Cl}_i^*(A) = A \cup A_i^*(\tau_i, \mathcal{I})$ when there is no chance of confusion, $A_i^*(\mathcal{I})$ is denoted by A_i^*. The aim of this paper is to introduce and characterize the concepts of semiopen sets and their related notions in ideal bitopological spaces.

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2. Preiliminaries

Let A be a subset of a bitopological space (X, τ_1, τ_2). We denote the closure of A and the interior of A with respect to τ_i by τ_i-Cl(A) and τ_i-Int(A), respectively.

Definition 2.1. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i, j)-semiopen* [3] *if $A \subset \tau_i\text{-Int}(\tau_j\text{-Cl}(A))$, where $i, j = 1, 2$ and $i \neq j$.*

Definition 2.2. *A subset S of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be semi-$\mathcal{I}$-open* [24] *if $S \subset \text{Int}(\text{Cl}^*(S))$. The family of all semi-$\mathcal{I}$-open sets of $(X, \tau, \mathcal{I})$ is denoted by $S\mathcal{I}O(X, \tau)$.*

Definition 2.3. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-semicontinuous* [3] *if the inverse image of every σ_j-open set in (Y, σ_1, σ_2) is (i, j)-semiopen in $(X, \tau_1, \tau_2, \mathcal{I})$, where $i \neq j$, i, j=1, 2.*

Definition 2.4. *A subset A of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is said to be*

(i) *(i, j)-α-$\mathcal{I}$-open* [6] *if $A \subset \tau_i\text{-Int}(\tau_j\text{-Cl}^*(\tau_i\text{-Int}(A)))$.*
(ii) *(i, j)-pre-$\mathcal{I}$-open* [4] *if $A \subset \tau_i\text{-Int}(\tau_j\text{-Cl}^*(A))$.*

Definition 2.5. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is said to be*

(i) *pairwise pre-$\mathcal{I}$-continuous* [4] *if the inverse image of every σ_i-open set of Y is (i, j)-pre-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*
(i) *pairwise α-$\mathcal{I}$-continuous* [6] *if the inverse image of every σ_i-open set of Y is (i, j)-α-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*

3. (i, j)-semi-$\mathcal{I}$-open sets

Definition 3.1. *A subset A of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is said to be (i, j)-semi-$\mathcal{I}$-open if and only if $A \subset \tau_j\text{-Cl}^*(\tau_i\text{-Int}(A))$, where $i, j = 1, 2$ and $i \neq j$.*
The family of all (i, j)-semi-$\mathcal{I}$-open sets of $(X, \tau_1, \tau_2, \mathcal{I})$ is denoted by $S\mathcal{I}O(X, \tau_1, \tau_2)$ or (i, j)-$S\mathcal{I}O(X)$. Also, the family of all (i, j)-semi-$\mathcal{I}$-open sets of $(X, \tau_1, \tau_2, \mathcal{I})$ containing x is denoted by (i, j)-$S\mathcal{I}O(X, x)$.

Remark 3.2. *Let $\mathcal{I}$ and $\mathcal{J}$ be two ideals on (X, τ_1, τ_2). If $\mathcal{I} \subset \mathcal{J}$, then $S\mathcal{I}O(X, \tau_1, \tau_2) \subset S\mathcal{J}O(X, \tau_1, \tau_2)$.*

Remark 3.3. *It is clear that $S\mathcal{I}O(X, \tau_1, \tau_2) \neq S\mathcal{I}O(X, \tau_1) \cup S\mathcal{I}O(X, \tau_2)$.*

Example 3.4. *Let $X = \{a, b, c\}$ $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $S\mathcal{I}O(X, \tau_1) = \{\varnothing, \{a\}, X\}$, $S\mathcal{I}O(X, \tau_2) = \{\varnothing, \{a\}, \{a, b\}, X\}$. But $S\mathcal{I}O(X, \tau_1, \tau_2) = \{\varnothing, \{a\}, X\}$.*

Proposition 3.5. (i) *Every (i, j)-α-$\mathcal{I}$-open set is (i, j)-semi-$\mathcal{I}$-open.*
(ii) *Every (i, j)-semi-$\mathcal{I}$-open set is (i, j)-semiopen.*

Proof. The proof follows from the definitions. □

Remark 3.6. *Let* $X = \{a, b, c\}$ $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then the set* $\{b, c\}$ *is* (i, j)*-semi-*$\mathcal{I}$*-open but not* (i, j)*-*α*-*$\mathcal{I}$*-open. Also notice that it is clear that* (i, j)*-semi-*$\mathcal{I}$*-openness and* (i, j)*-pre-*$\mathcal{I}$*-openness are independent notions. For example, the set* $\{a, c\}$ *is* (i, j)*-pre-*$\mathcal{I}$*-open in* X *but not* (i, j)*-semi-*$\mathcal{I}$*-open in* X *and the set* $\{b, c\}$ *is* (i, j)*-semi-*$\mathcal{I}$*-open in* X *but not* (i, j)*-pre-*$\mathcal{I}$*-open in* X.

Proposition 3.7. *For an ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *and* $A \subset X$ *we have:*

(i) *If* $\mathcal{I} = \{\emptyset\}$, *then* A *is* (i, j)*-semi-*$\mathcal{I}$*-open if and only if* A *is* (i, j)*-semiopen.*
(ii) *If* $\mathcal{I} = \mathcal{P}(X)$, *then* A *is* (i, j)*-semi-*$\mathcal{I}$*-open if and only if* A *is* τ_i*-open.*

Proof. The proof follows from the fact that

(i) If $\mathcal{I} = \{\emptyset\}$, then $A^* = \mathrm{Cl}(A)$.
(ii) If $\mathcal{I} = \mathcal{P}(X)$, then $A^* = \emptyset$ for every subset A of X.

□

Proposition 3.8. *In an ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$, A *is* (i, j)*-semi-*$\mathcal{I}$*-open if and only if there exists* $U \in \tau_i$ *such that* $U \subset A \subset \tau_j\text{-}\mathrm{Cl}^*(U)$.

Proof. Let $A \in \mathcal{SIO}(X, \tau_1, \tau_2)$. Then we have $A \subset \tau_j\text{-}\mathrm{Cl}^*(\tau_i\text{-}\mathrm{Int}(A))$. Take $\tau_i\text{-}\mathrm{Int}(A) = U$. Then $U \subset A \subset \tau_j\text{-}\mathrm{Cl}^*(U)$. Conversely, let U be a τ_i-open set such that $U \subset A \subset \tau_j\text{-}\mathrm{Cl}^*(U)$. Since $U \subset A$, $U \subset \tau_i\text{-}\mathrm{Int}(A)$ and hence $\tau_j\text{-}\mathrm{Cl}^*(U) \subset \tau_j\text{-}\mathrm{Cl}^*(\tau_i\text{-}\mathrm{Int}(A))$. Thus, we obtain $A \subset \tau_j\text{-}\mathrm{Cl}^*(\tau_i\text{-}\mathrm{Int}(A))$. □

Proposition 3.9. *A subset* A *of an ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)*-semi-*$\mathcal{I}$*-open if and only if* $\tau_j\text{-}\mathrm{Cl}^*(A) = \tau_j\text{-}\mathrm{Cl}^*(\tau_i\text{-}\mathrm{Int}(A))$.

Proof. Let $A \in \mathcal{SIO}(X, \tau_1, \tau_2)$. Then we have $A \subset \tau_j\text{-}\mathrm{Cl}^*(\tau_i\text{-}\mathrm{Int}(A))$. Then $\tau_j\text{-}\mathrm{Cl}^*(A) \subset \tau_j\text{-}\mathrm{Cl}^*(\tau_i\text{-}\mathrm{Int}(A))$ and hence $\tau_j\text{-}\mathrm{Cl}^*(A) = \tau_j\text{-}\mathrm{Cl}^*(\tau_i\text{-}\mathrm{Int}(A))$. The converse is obvious. □

Proposition 3.10. *If* A *is an* (i, j)*-semi-*$\mathcal{I}$*-open set in an ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *and* $A \subset B \subset \tau_j\text{-}\mathrm{Cl}^*(A)$, *then* B *is an* (i, j)*-semi-*$\mathcal{I}$*-open set in* $(X, \tau_1, \tau_2, \mathcal{I})$.

Proof. Since A is (i, j)-semi-$\mathcal{I}$-open, there exists a τ_i-open set U such that $U \subset A \subset \tau_j\text{-}\mathrm{Cl}^*(U)$. Then we have $U \subset A \subset B \subset \tau_j\text{-}\mathrm{Cl}^*(A) \subset \tau_j\text{-}\mathrm{Cl}^*(\tau_j\text{-}\mathrm{Cl}^*(U)) = \tau_j\text{-}\mathrm{Cl}^*(U)$ and hence $U \subset B \subset \tau_j\text{-}\mathrm{Cl}^*(U)$. By Proposition 3.8, we obtain $B \in \mathcal{SIO}(X, \tau_1, \tau_2)$. □

Theorem 3.11. *If* $\{A_\alpha\}_{\alpha \in \Omega}$ *is a family of* (i, j)*-semi-*$\mathcal{I}$*-open sets in* $(X, \tau_1, \tau_2, \mathcal{I})$, *then* $\bigcup_{\alpha \in \Omega} A_\alpha$ *is* (i, j)*-semi-*$\mathcal{I}$*-open in* $(X, \tau_1, \tau_2, \mathcal{I})$.

Proof. Since $\{A_\alpha : \alpha \in \Omega\} \subset (i,j)$-$S\mathcal{I}O(X)$, then $A_\alpha \subset \tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A_\alpha))$ for every $\alpha \in \Omega$. Thus, $\underset{\alpha\in\Omega}{\cup} A_\alpha \subset \underset{\alpha\in\Omega}{\cup} \tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A_\alpha)) \subset \tau_j$-$\mathrm{Cl}^*(\underset{\alpha\in\Omega}{\cup} \tau_i$-$\mathrm{Int}(A_\alpha)) = \tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(\underset{\alpha\in\Omega}{\cup} A_\alpha))$. Therefore, we obtain $\underset{\alpha\in\Omega}{\cup} A_\alpha \subset \tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(\underset{\alpha\in\Omega}{\cup} A_\alpha))$. Hence any union of (i,j)-semi-$\mathcal{I}$-open sets is (i,j)-semi-$\mathcal{I}$-open. □

Theorem 3.12. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space. If U is biopen (that is, τ_i-open and τ_j-open) and $V \in S\mathcal{I}O(X,\tau_1,\tau_2,)$, then $U \cap V \in SO(X,\tau_1,\tau_2,\mathcal{I})$.*

Proof. By definition, we have $U\cap V \subset U\cap\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(V)) \subset U\cap(\tau_i$-$\mathrm{Int}(V)\cup(\tau_i$-$\mathrm{Int}(V)^*_j)) = (U\cap\tau_i$-$\mathrm{Int}(V))\cup(U\cap(\tau_i$-$\mathrm{Int}(V))^*_j) = (U\cap\tau_i$-$\mathrm{Int}(V))\cup(U\cap(\tau_i$-$\mathrm{Int}(V)))^*_j = \tau_i$-$\mathrm{Int}(U\cap V)\cup(\tau_i$-$\mathrm{Int}(U\cap V))^*_j = \tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(U\cap V))$. Therefore, $U\cap V \in SO(X,\tau_1,\tau_2,\mathcal{I})$. □

Lemma 3.13. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space with $A \subset B \subset X$, then $A^*_i(\mathcal{I}_{|B},\tau_{|B}) = A^*_i(\mathcal{I},\tau_i)\cap B$ for $i = 1,2$.*

Theorem 3.14. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space. If U is biopen and $W \in (i,j)$-$S\mathcal{I}O(X)$, then $U\cap W \in (i,j)$-$SO(U,\tau_{1|U},\tau_{2|U},\mathcal{I}|U)$.*

Proof. Since U is biopen, we have τ_i-$\mathrm{Int}_U(A) = \tau_i$-$\mathrm{Int}(A)$ for any subset A of U. By using this fact and Lemma 3.13, the result follows immediately. □

Definition 3.15. *In an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$, $A \subset X$ is said to be (i,j)-semi-$\mathcal{I}$-closed if $X\backslash A$ is (i,j)-semi-$\mathcal{I}$-open in X, $i,j = 1,2$ and $i \neq j$.*

Theorem 3.16. *If A is an (i,j)-semi-$\mathcal{I}$-closed set in an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$, then τ_j-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}^*(A)) \subset A$.*

Proof. Since $A \in S\mathcal{I}C(X)$, $X\backslash A \in S\mathcal{I}O(X)$. Hence, $X\backslash A \subset \tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(X\backslash A)) \subset \tau_j$-$\mathrm{Cl}(\tau_i$-$\mathrm{Int}(X\backslash A)) = X\backslash(\tau_j$-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}(A))) \subset X\backslash(\tau_j$-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}^*(A)))$. Therefore, we obtain τ_i-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}^*(A)) \subset A$. □

Proposition 3.17. *A subset A of an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-semi-$\mathcal{I}$-closed if and only if τ_j-$\mathrm{Int}(\tau_i$-$\mathrm{Cl}^*(A)) = \tau_j$-$\mathrm{Int}(A)$.*

Proof. The proof is clear. □

Theorem 3.18. *Arbitrary intersection of (i,j)-semi-$\mathcal{I}$-closed sets is always (i,j)-semi-$\mathcal{I}$-closed.*

Proof. Follows from Theorems 3.11 and 3.16. □

Definition 3.19. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space, S a subset of X and x be a point of X. Then*

(i) *x is called an (i,j)-semi-$\mathcal{I}$-interior point of S if there exists $V \in (i,j)$-$S\mathcal{I}O(X,\tau_1,\tau_2)$ such that $x \in V \subset S$.*

ii) *the set of all* (i, j)*-semi-*$\mathcal{I}$*-interior points of* S *is called* (i, j)*-semi-*$\mathcal{I}$*-interior of* S *and is denoted by* (i, j)-$s\mathcal{I}\operatorname{Int}(S)$.

Theorem 3.20. *Let* A *and* B *be subsets of* $(X, \tau_1, \tau_2, \mathcal{I})$*. Then the following properties hold:*

(i) (i, j)-$s\mathcal{I}\operatorname{Int}(A) = \cup\{T : T \subset A$ *and* $A \in (i, j)$-$S\mathcal{I}O(X)\}$.
(ii) (i, j)-$s\mathcal{I}\operatorname{Int}(A)$ *is the largest* (i, j)*-semi-*$\mathcal{I}$*-open subset of* X *contained in* A.
(iii) A *is* (i, j)*-semi-*$\mathcal{I}$*-open if and only if* $A = (i, j)$-$s\mathcal{I}\operatorname{Int}(A)$.
(iv) (i, j)-$s\mathcal{I}\operatorname{Int}((i, j)$-$s\mathcal{I}\operatorname{Int}(A)) = (i, j)$-$s\mathcal{I}\operatorname{Int}(A)$.
(v) *If* $A \subset B$*, then* (i, j)-$s\mathcal{I}\operatorname{Int}(A) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(B)$.
(vi) (i, j)-$s\mathcal{I}\operatorname{Int}(A \cap B) = (i, j)$-$s\mathcal{I}\operatorname{Int}(A) \cap (i, j)$-$s\mathcal{I}\operatorname{Int}(B)$.
(vii) (i, j)-$s\mathcal{I}\operatorname{Int}(A \cup B) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(A) \cup (i, j)$-$s\mathcal{I}\operatorname{Int}(B)$.

Proof. (i). Let $x \in \cup\{T : T \subset A$ and $A \in (i, j)$-$S\mathcal{I}O(X)\}$. Then, there exists $T \in (i, j)$-$S\mathcal{I}O(X, x)$ such that $x \in T \subset A$ and hence $x \in (i, j)$-$s\mathcal{I}\operatorname{Int}(A)$. This shows that $\cup\{T : T \subset A$ and $A \in (i, j)$-$S\mathcal{I}O(X)\}$ $\subset (i, j)$-$s\mathcal{I}\operatorname{Int}(A)$. For the reverse inclusion, let $x \in (i, j)$-$s\mathcal{I}\operatorname{Int}(A)$. Then there exists $T \in (i, j)$-$S\mathcal{I}O(X, x)$ such that $x \in T \subset A$. we obtain $x \in \cup\{T : T \subset A$ and $A \in (i, j)$-$S\mathcal{I}O(X)\}$. This shows that (i, j)-$s\mathcal{I}\operatorname{Int}(A) \subset \cup\{T : T \subset A$ and $A \in (i, j)$-$S\mathcal{I}O(X)\}$. Therefore, we obtain (i, j)-$s\mathcal{I}\operatorname{Int}(A) = \cup\{T : T \subset A$ and $A \in (i, j)$-$S\mathcal{I}O(X)\}$.
The proof of (ii)-(v) are obvious.
(vi). By (v), we have (i, j)-$s\mathcal{I}\operatorname{Int}(A) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(A \cup B)$ and (i, j)-$s\mathcal{I}\operatorname{Int}(B) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(A \cup B)$. Then we obtain (i, j)-$s\mathcal{I}\operatorname{Int}(A) \cup (i, j)$-$s\mathcal{I}\operatorname{Int}(B) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(A \cup B)$ Since (i, j)-$s\mathcal{I}\operatorname{Int}(A) \subset A$ and (i, j)-$s\mathcal{I}\operatorname{Int}(B) \subset B$, we obtain (i, j)-$s\mathcal{I}\operatorname{Int}(A \cup B) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(A)$ $\cup$ (i, j)-$s\mathcal{I}\operatorname{Int}(B)$. It follows that (i, j)-$s\mathcal{I}\operatorname{Int}(A \cap B) = (i, j)$-$s\mathcal{I}\operatorname{Int}(A)$ $\cap$ (i, j)-$s\mathcal{I}\operatorname{Int}(B)$.
(vii). Since $A \cap B \subset A$ and $A \cap B \subset B$, by (v), we have (i, j)-$s\mathcal{I}\operatorname{Int}(A \cap B) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(A)$ and (i, j)-$s\mathcal{I}\operatorname{Int}(A \cap B) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(B)$. Therefore, (i, j)-$s\mathcal{I}\operatorname{Int}(A) \cup (i, j)$-$s\mathcal{I}\operatorname{Int}(B) \subset (i, j)$-$s\mathcal{I}\operatorname{Int}(A \cap B)$. □

Definition 3.21. *Let* $(X, \tau_1, \tau_2, \mathcal{I})$ *be an ideal bitopological space,* S *a subset of* X *and* x *be a point of* X*. Then*

(i) x *is called an* (i, j)*-semi-*$\mathcal{I}$*-cluster point of* S *if* $V \cap S \neq \emptyset$ *for every* $V \in (i, j)$-$S\mathcal{I}O(X, x)$.
(ii) *the set of all* (i, j)*-semi-*$\mathcal{I}$*-cluster points of* S *is called* (i, j)*-semi-*$\mathcal{I}$*-closure of* S *and is denoted by* (i, j)-$s\mathcal{I}\operatorname{Cl}(S)$.

Theorem 3.22. *Let* A *and* B *be subsets of* $(X, \tau_1, \tau_2, \mathcal{I})$*. Then the following properties hold:*

(i) (i, j)-$s\mathcal{I}\operatorname{Cl}(A) = \cap\{F : A \subset F$ *and* $F \in (i, j)$-$S\mathcal{I}C(X)\}$.
(ii) (i, j)-$s\mathcal{I}\operatorname{Cl}(A)$ *is the smallest* (i, j)*-semi-*$\mathcal{I}$*-closed subset of* X *containing* A.
(iii) A *is* (i, j)*-semi-*$\mathcal{I}$*-closed if and only if* $A = (i, j)$-$s\mathcal{I}\operatorname{Cl}(A)$.
(iv) (i, j)-$s\mathcal{I}\operatorname{Cl}((i, j)$-$s\mathcal{I}\operatorname{Cl}(A) = (i, j)$-$s\mathcal{I}\operatorname{Cl}(A)$.

(v) *If* $A \subset B$*, then* (i,j)-$s\mathcal{I}\,\mathrm{Cl}(A) \subset (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(B)$.
(vi) (i,j)-$s\mathcal{I}\,\mathrm{Cl}(A \cup B) = (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A) \cup (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(B)$.
(vii) (i,j)-$s\mathcal{I}\,\mathrm{Cl}(A \cap B) \subset (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A) \cap (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(B)$.

Proof. (i). Suppose that $x \notin (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$. Then there exists $F \in (i,j)$-$S\mathcal{I}O(X)\}$ such that $V \cap S \neq \emptyset$. Since $X \backslash V$ is (i,j)-semi-$\mathcal{I}$-closed set containing A and $x \notin X \backslash V$, we obtain $x \notin \cap\{F : A \subset F$ and $F \in (i,j)$-$S\mathcal{I}C(X)\}$. Then there exists $F \in (i,j)$-$S\mathcal{I}C(X)$ such that $A \subset F$ and $x \notin F$. Since $X \backslash V$ is (i,j)-semi-$\mathcal{I}$-closed set containing x, we obtain $(X \backslash F) \cap A = \emptyset$. This shows that $x \notin (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$. Therefore, we obtain (i,j)-$s\mathcal{I}\,\mathrm{Cl}(A) = \cap\{F : A \subset F$ and $F \in (i,j)$-$S\mathcal{I}C(X)\}$.
The other proofs are obvious. □

Theorem 3.23. *Let* $(X, \tau_1, \tau_2, \mathcal{I})$ *be an ideal bitopological space and* $A \subset X$*. A point* $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$ *if and only if* $U \cap A \neq \emptyset$ *for every* $U \in (i,j)$-$S\mathcal{I}O(X,x)$.

Proof. Suppose that $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$. We shall show that $U \cap A \neq \emptyset$ for every $U \in (i,j)$-$S\mathcal{I}O(X,x)$. Suppose that there exists $U \in (i,j)$-$S\mathcal{I}O(X,x)$ such that $U \cap A = \emptyset$. Then $A \subset X \backslash U$ and $X \backslash U$ is (i,j)-semi-$\mathcal{I}$-closed. since $A \subset X \backslash U$, (i,j)-$s\mathcal{I}\,\mathrm{Cl}(A) \subset (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(X \backslash U)$. Since $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$, we have $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(X \backslash U)$. Since $X \backslash U$ is (i,j)-semi-$\mathcal{I}$-closed, we have $x \in X \backslash U$; hence $x \notin U$, which is a contradicition that $x \in U$. Therefore, $U \cap A \neq \emptyset$. Conversely, suppose that $U \cap A \neq \emptyset$ for every $U \in (i,j)$-$S\mathcal{I}O(X,x)$. We shall show that $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$. Suppose that $x \notin (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$. Then there exists $U \in (i,j)$-$S\mathcal{I}O(X,x)$ such that $U \cap A = \emptyset$. This is a contradicition to $U \cap A \neq \emptyset$; hence $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$. □

Theorem 3.24. *Let* $(X, \tau_1, \tau_2, \mathcal{I})$ *be an ideal bitopological space and* $A \subset X$*. Then the following propeties hold:*
(i) (i,j)-$s\mathcal{I}\,\mathrm{Int}(X \backslash A) = X \backslash (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$;
(i) (i,j)-$s\mathcal{I}\,\mathrm{Cl}(X \backslash A) = X \backslash (i,j)$-$s\mathcal{I}\,\mathrm{Int}(A)$.

Proof. (i). Let $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$. Since $x \notin (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$, there exists $V \in (i,j)$-$s\mathcal{I}O(X,x)$ such that $V \cap A \neq \emptyset$; hence we obtain $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Int}(X \backslash A)$. This shows that $X \backslash (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A) \subset (i,j)$-$s\mathcal{I}\,\mathrm{Int}(X \backslash A)$. Let $x \in (i,j)$-$s\mathcal{I}\,\mathrm{Int}(X \backslash A)$. Since (i,j)-$s\mathcal{I}\,\mathrm{Int}(X \backslash A) \cap A = \emptyset$, we obtain $x \notin (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$; hence $x \in X \backslash (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$. Therefore, we obtain (i,j)-$s\mathcal{I}\,\mathrm{Int}(X \backslash A) = X \backslash (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(A)$.
(ii). Follows from (i). □

Definition 3.25. *A subset* B_x *of an ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *is said to be an* (i,j)*-semi-*$\mathcal{I}$*-neighbourhood of a point* $x \in X$ *if there exists an* (i,j)*-semi-*$\mathcal{I}$*-open set* U *such that* $x \in U \subset B_x$.

Theorem 3.26. *A subset of an ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i,j)*-semi-*$\mathcal{I}$*-open if and only if it is an* (i,j)*-semi-*$\mathcal{I}$*-neighbourhood of each of its points.*

Proof. Let G be an (i,j)-semi-$\mathcal{I}$-open set of X. Then by definition, it is clear that G is an (i,j)-semi-$\mathcal{I}$-neighbourhood of each of its points, since for every $x \in G$, $x \in G \subset G$ and G is (i,j)-semi-$\mathcal{I}$-open. Conversely, suppose G is an (i,j)-semi-$\mathcal{I}$-neighbourhood of each of its points. Then for each $x \in G$, there exists $S_x \in (i,j)$-$S\mathcal{I}O(X)$ such that $S_x \subset G$. Then $G = \bigcup\{S_x : x \in G\}$. Since each S_x is (i,j)-semi-$\mathcal{I}$-open, G is (i,j)-semi-$\mathcal{I}$-open in $(X, \tau_1, \tau_2, \mathcal{I})$. □

Proposition 3.27. *The product of two (i,j)-semi-$\mathcal{I}$-open sets is (i,j)-semi-$\mathcal{I}$-open.*

Proof. The proof follows from Lemma 3.3 of [24]. □

4. PAIRWISE SEMI-$\mathcal{I}$-CONTINUOUS FUNCTIONS

Definition 4.1. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-semi-$\mathcal{I}$-continuous if the inverse image of every σ_i-open set of Y is (i,j)-semi-$\mathcal{I}$-open in X, where $i \neq j$, i,j=1, 2.*

Proposition 4.2.
- (i) *Every (i,j)-α-$\mathcal{I}$-continuous function is (i,j)-semi-$\mathcal{I}$-continuous but not conversely.*
- (ii) *Every (i,j)-semi-$\mathcal{I}$-continuous function is (i,j)-continuous but not conversely.*
- (iii) *(i,j)-semi-$\mathcal{I}$-continuity and (i,j)-pre-$\mathcal{I}$-continuity are independent.*

Proof. The proof follows from Proposition 3.5 and Remark 3.6. □

Theorem 4.3. *For a function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:*
- (i) *f is pairwise semi-$\mathcal{I}$-continuous;*
- (ii) *For each point x in X and each σ_i-open set F in Y such that $f(x) \in F$, there is a (i,j)-semi-$\mathcal{I}$-open set A in X such that $x \in A$, $f(A) \subset F$;*
- (iii) *The inverse image of each σ_i-closed set in Y is (i,j)-semi-$\mathcal{I}$-closed in X;*
- (iv) *For each subset A of X, $f((i,j)\text{-}s\mathcal{I}\,\mathrm{Cl}(A)) \subset \sigma_i\text{-}\mathrm{Cl}(f(A))$;*
- (v) *For each subset B of Y, $(i,j)\text{-}s\mathcal{I}\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-}\mathrm{Cl}(B))$;*
- (vi) *For each subset C of Y, $f^{-1}(\sigma_i\text{-}\mathrm{Int}(C)) \subset (i,j)\text{-}s\mathcal{I}\,\mathrm{Int}(f^{-1}(C))$.*

Proof. (i)⇒(ii): Let $x \in X$ and F be a σ_j-open set of Y containing $f(x)$. By (i), $f^{-1}(F)$ is (i,j)-semi-$\mathcal{I}$-open in X. Let $A = f^{-1}(F)$. Then $x \in A$ and $f(A) \subset F$.
(ii)⇒(i): Let F be σ_j-open in Y and let $x \in f^{-1}(F)$. Then $f(x) \in F$. By (ii), there is an (i,j)-semi-$\mathcal{I}$-open set U_x in X such that $x \in U_x$ and $f(U_x) \subset F$. Then $x \in U_x \subset f^{-1}(F)$. Hence $f^{-1}(F)$ is (i,j)-semi-$\mathcal{I}$-open in X.
(i)⇔(iii): This follows due to the fact that for any subset B of Y, $f^{-1}(Y \backslash B) = X \backslash f^{-1}(B)$.

(iii)⇒(iv): Let A be a subset of X. Since $A \subset f^{-1}(f(A))$ we have $A \subset f^{-1}(\sigma_j\text{-Cl}(f(A)))$. Now, $(i,j)\text{-}s\mathcal{I}\,\text{Cl}(f(A))$ is σ_j-closed in Y and hence $f^{-1}(\sigma_j\text{-Cl}(A)) \subset f^{-1}(\sigma_j\text{-Cl}(f(A)))$, for $(i,j)\text{-}s\mathcal{I}\,\text{Cl}(A)$ is the smallest (i,j)-semi-$\mathcal{I}$-closed set containing A. Then $f((i,j)\text{-}s\mathcal{I}\,\text{Cl}(A)) \subset \sigma_j\text{-Cl}(f(A))$.
(iv)⇒(iii): Let F be any (i,j)-semi-$\mathcal{I}$-closed subset of Y. Then $f((i,j)\text{-}s\mathcal{I}\,\text{Cl}(f^{-1}(F))) \subset (i,j)\text{-}\sigma_i\text{-Cl}(f(f^{-1}(F))) = (i,j)\text{-}\sigma_i\text{-Cl}(F) = F$. Therefore, $(i,j)\text{-}s\mathcal{I}\,\text{Cl}(f^{-1}(F)) \subset f^{-1}(F)$. Consequently, $f^{-1}(F)$ is (i,j)-semi-$\mathcal{I}$-closed in X.
(iv)⇒(v): Let B be any subset of Y. Now, $f((i,j)\text{-}s\mathcal{I}\,\text{Cl}(f^{-1}(B))) \subset (i,j)\text{-}\sigma_i\text{-Cl}(f(f^{-1}(B))) \subset \sigma_i\text{-Cl}(B)$. Consequently, $(i,j)\text{-}s\mathcal{I}\,\text{Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-Cl}(B))$.
(v)⇒(iv): Let $B = f(A)$ where A is a subset of X. Then, $(i,j)\text{-}s\mathcal{I}\,\text{Cl}(A) \subset (i,j)\text{-}s\mathcal{I}\,\text{Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-Cl}(B)) = f^{-1}(\sigma_i\text{-Cl}(f(A)))$. This shows that $f((i,j)\text{-}s\mathcal{I}\,\text{Cl}(A)) \subset \sigma_i\text{-Cl}(f(A))$.
(i)⇒(vi): Let B be a σ_j-open set in Y. Clearly, $f^{-1}(\sigma_i\text{-Int}(B)$ is (i,j)-semi-$\mathcal{I}$-open and we have $f^{-1}(\sigma_i\text{-Int}(B)) \subset (i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}\sigma_i\text{-Int}(B)) \subset (i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}B)$.
(vi)⇒(i): Let B be a σ_j-open set in Y. Then $\sigma_i\text{-Int}(B) = B$ and $f^{-1}(B) \backslash f^{-1}(\sigma_i\text{-Int}(B)) \subset (i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(B))$. Hence we have $f^{-1}(B) = (i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(B))$. This shows that $f^{-1}(B)$ is (i,j)-semi-$\mathcal{I}$-open in X. □

Theorem 4.4. *If $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is pairwise semi-$\mathcal{I}$-continuous and A is biopen, then $f_{|A} : (A, \tau_{1|A}, \tau_{2|A}, \mathcal{I}_|A) \to (Y, \sigma_1, \sigma_2)$ is pairwise semi-$\mathcal{I}_{|A}$-continuous.*

Proof. Follows from Theorem 3.14. □

Theorem 4.5. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a pairwise semi-$\mathcal{I}$-continuous function. Then for each subset V of Y, $f^{-1}(\sigma_i\text{-Int}(V)) \subset \tau_j\text{-Cl}^*(\tau_i\text{-Int}(f^{-1}(V)))$.*

Proof. Let V be any subset of Y. Then $\sigma_i\text{-Int}(V)$ is σ_i-open in Y and so $f^{-1}(\sigma_i\text{-Int}(V))$ is (i,j)-semi-$\mathcal{I}$-open in X. Hence $f^{-1}(\sigma_i\text{-Int}(V)) \subset \tau_j\text{-Cl}^*(\tau_i\text{-Int}(f^{-1}(\sigma_i\text{-Int}(V)))) \subset \tau_j\text{-Cl}^*(\tau_i\text{-Int}(f^{-1}(V)))$. □

Corollary 4.6. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a pairwise semi-$\mathcal{I}$-continuous function. Then for each subset V of Y, $\tau_j\text{-Int}(\tau_i\text{-Cl}^*(f^{-1}(V))) \subset f^{-1}(\sigma_i\text{-Cl}(V))$.*

Theorem 4.7. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a bijection. Then f is pairwise semi-$\mathcal{I}$-continuous if and only if $\sigma_i\text{-Int}(f(U)) \subset f((i,j)\text{-}s\mathcal{I}\,\text{Int}(U))$ for each subset U of X.*

Proof. Let U be any subset of X. Then by Theorem 4.3, $f^{-1}(\sigma_i\text{-Int}(f(U))) \subset (i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(f(U)))$. Since f is bijection, $\sigma_i\text{-Int}(f(U)) = f(f^{-1}(\sigma_i\text{-Int}(f(U))) \subset f((i,j)\text{-}s\mathcal{I}\,\text{Int}(U))$. Conversely, let V be any subset of Y. Then $\sigma_i\text{-Int}(f(f^{-1}(V))) \subset f((i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(V)))$. Since

f is bijection, σ_i-Int$(V) = \sigma_i$-Int$(f(f^{-1}(V))) \subset f((i,j)$-$s\mathcal{I}\,\text{Int}(f^{-1}(V)))$; hence $f^{-1}(\sigma_i$-Int$(V) \subset (i,j)$-$s\mathcal{I}\,\text{Int}(f^{-1}(V))$. Therefore, by Theorem 4.3, f is pairwise semi-$\mathcal{I}$-continuous. □

Theorem 4.8. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a function. If $g : (X, \tau_1, \tau_2, \mathcal{I}) \to (X \times Y, \sigma_1 \times \sigma_2)$ defined by $g(x) = (x, f(x))$ is a pairwise semi-$\mathcal{I}$-continuous function, then f is pairwise semi-$\mathcal{I}$-continuous.*

Proof. Let V be a σ_i-open set of Y. Then $f^{-1}(V) = X \cap f^{-1}(V) = g^{-1}(X \times V)$. Since g is a pairwise semi-$\mathcal{I}$-continuous function and $X \times V$ is a $\tau_i \times \sigma_i$-open set of $X \times Y$, $f^{-1}(V)$ is a (i,j)-semi-$\mathcal{I}$-open set of X. Hence f is pairwise semi-$\mathcal{I}$-continuous. □

Definition 4.9. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ is said to be:*

(i) *pairwise semi-$\mathcal{I}$-open if $f(U)$ is a (i,j)-semi-$\mathcal{I}$-open set of Y for every τ_i-open set U of X.*
(ii) *pairwise semi-$\mathcal{I}$-closed if $f(U)$ is a (i,j)-semi-$\mathcal{I}$-closed set of Y for every τ_i-closed set U of X.*

Theorem 4.10. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$, the following statements are equivalent:*

(i) *f is pairwise semi-$\mathcal{I}$-open;*
(ii) *$f(\tau_i$-Int$(U)) \subset (i,j)$-$s\mathcal{I}\,\text{Int}(f(U))$ for each subset U of X;*
(iii) *τ_i-Int$(f^{-1}(V)) \subset f^{-1}((i,j)$-$s\mathcal{I}\,\text{Int}(V))$ for each subset V of Y.*

Proof. *(i)* ⇒ *(ii)*: Let U be any subset of X. Then τ_i-Int(U) is a τ_i-open set of X. Then $f(\tau_i$-Int$(U))$ is a (i,j)-semi-$\mathcal{I}$-open set of Y. Since $f(\tau_i$-Int$(U)) \subset f(U)$, $f(\tau_i$-Int$(U)) = (i,j)$-$s\mathcal{I}\,\text{Int}(f(\tau_i$-Int$(U))) \subset (i,j)$-$s\mathcal{I}\,\text{Int}(f(U))$.
(ii) ⇒ *(iii)*: Let V be any subset of Y. Then $f^{-1}(V)$ is a subset of X. Hence $f(\tau_i$-Int$(f^{-1}(V))) \subset (i,j)$-$s\mathcal{I}\,\text{Int}(f(f^{-1}(V))) \subset (i,j)$-$s\mathcal{I}\,\text{Int}(V))$. Then τ_i-Int$(f^{-1}(V)) \subset f^{-1}(f(\tau_i$-Int$(f^{-1}(V)))) \subset f^{-1}((i,j)$-$s\mathcal{I}\,\text{Int}(V))$.
(iii) ⇒ *(i)*: Let U be any τ_i-open set of X. Then τ_i-Int$(U) = U$ and $f(U)$ is a subset of Y. Now, $V = \tau_i$-Int$(V) \subset \tau_i$-Int$(f^{-1}(f(V))) \subset f^{-1}((i,j)$-$s\mathcal{I}\,\text{Int}(f(V)))$. Then $f(V) \subset f(f^{-1}((i,j)$-$s\mathcal{I}\,\text{Int}(f(V)))) \subset (i,j)$-$s\mathcal{I}\,\text{Int}(f(V))$ and (i,j)-$s\mathcal{I}\,\text{Int}(f(V)) \subset f(V)$. Hence $f(V)$ is a (i,j)-semi-$\mathcal{I}$-open set of Y; hence f is pairwise semi-$\mathcal{I}$-open. □

Theorem 4.11. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a function. Then f is a pairwise semi-$\mathcal{I}$-closed function if and only if for each subset V of X, (i,j)-$s\mathcal{I}\,\text{Cl}(f(V)) \subset f(\text{Cl}(V))$.*

Proof. Let f be a pairwise semi-$\mathcal{I}$-closed function and V any subset of X. Then $f(V) \subset f(\tau_i$-Cl$(V))$ and $f(\tau_i$-Cl$(V))$ is a (i,j)-semi-$\mathcal{I}$-closed set of Y. We have (i,j)-$s\mathcal{I}\,\mathrm{Cl}(f(V)) \subset (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(f(\tau_i\text{-}\mathrm{Cl}(V))) = f(\tau_i$-$\mathrm{Cl}(V))$. Conversely, let V be a τ_i-open set of X. Then $f(V) \subset (i,j)$-$s\mathcal{I}\,\mathrm{Cl}(f(V)) \subset f(\tau_i\text{-}\mathrm{Cl}(V)) = f(V)$; hence $f(V)$ is a (i,j)-semi-$\mathcal{J}$-closed subset of Y. Therefore, f is a pairwise semi-$\mathcal{I}$-closed function. □

Theorem 4.12. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a function. Then f is a pairwise semi-$\mathcal{I}$-closed function if and only if for each subset V of Y, $f^{-1}((i,j)\text{-}s\mathcal{I}\,\mathrm{Cl}(V)) \subset \tau_i\text{-}\mathrm{Cl}(f^{-1}(V))$.*

Proof. Let V be any subset of Y. Then by Theorem 4.11, (i,j)-$s\mathcal{I}\,\mathrm{Cl}(V) \subset f(\tau_i\text{-}\mathrm{Cl}(f^{-1}(V)))$. Since f is bijection, $f^{-1}((i,j)\text{-}s\mathcal{I}\,\mathrm{Cl}(V)) = f^{-1}((i,j)\text{-}s\mathcal{I}\,\mathrm{Cl}(f(f^{-1}(V)))) \subset f^{-1}(f(\tau_i\text{-}\mathrm{Cl}(f^{-1}(V)))) = \tau_i\text{-}\mathrm{Cl}(f^{-1}(V))$. Conversely, let U be any subset of X. Since f is bijection, (i,j)-$s\mathcal{I}\,\mathrm{Cl}(f(U)) = f(f^{-1}((i,j)\text{-}s\mathcal{I}\,\mathrm{Cl}(f(U))) \subset f(\tau_i\text{-}\mathrm{Cl}(f^{-1}(f(U)))) = f(\tau_i$-$\mathrm{Cl}(U))$. Therefore, by Theorem 4.11, f is a pairwise semi-$\mathcal{I}$-closed function. □

Theorem 4.13. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a pairwise semi-$\mathcal{I}$-open function. If V is a subset of Y and U is a τ_i-closed subset of X containing $f^{-1}(V)$, then there exists a (i,j)-semi-$\mathcal{I}$-closed set F of Y containing V such that $f^{-1}(F) \subset U$.*

Proof. Let V be any subset of Y and U a τ_i-closed subset of X containing $f^{-1}(V)$, and let $F = Y\backslash(f(X\backslash V))$. Then $f(X\backslash V) \subset f(f^{-1}(X\backslash V)) \subset X\backslash V$ and $X\backslash U$ is a τ_i-open set of X. Since f is pairwise semi-$\mathcal{I}$-open, $f(X\backslash U)$ is a (i,j)-semi-$\mathcal{I}$-open set of Y. Hence F is an (i,j)-semi-$\mathcal{I}$-closed set of Y and $f^{-1}(F) = f^{-1}(Y\backslash(f(X\backslash U)) \subset U$. □

Theorem 4.14. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a pairwise semi-$\mathcal{I}$-closed function. If V is a subset of Y and U is a open subset of X containing $f^{-1}(V)$, then there exists (i,j)-semi-$\mathcal{I}$-open set F of Y containing V such that $f^{-1}(F) \subset U$.*

Proof. The proof is similar to the Theorem 4.13. □

Theorem 4.15. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a pairwise semi-$\mathcal{I}$-open function. Then for each subset V of Y, $f^{-1}(\tau_j\text{-}\mathrm{Int}(\tau_i\text{-}\mathrm{Cl}^*(V)) \subset \tau_i\text{-}\mathrm{Cl}(f^{-1}(V)$.*

Proof. Let V be any subset of Y. Then $\tau_i\text{-}\mathrm{Cl}(f^{-1}(V)$ is a τ_i-closed set of X containing $f^{-1}(V)$. Since f is pairwise semi-$\mathcal{I}$-open, by Theorem 4.13, there is a (i,j)-semi-$\mathcal{I}$-open set F of Y containing V such that $f^{-1}(\tau_j\text{-}\mathrm{Int}(\tau_i\text{-}\mathrm{Cl}^*(V)) \subset \tau_j\text{-}\mathrm{Int}(\tau_i\text{-}\mathrm{Cl}^*(F)) \subset f^{-1}(F) \subset \tau_i\text{-}\mathrm{Cl}(f^{-1}(V))$. □

Theorem 4.16. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a bijection such that for each subset V of Y, $f^{-1}(\tau_j\text{-}\mathrm{Int}(\tau_i\text{-}\mathrm{Cl}^*(V)) \subset \tau_i\text{-}\mathrm{Cl}(f^{-1}(V))$. Then f is a (i,j)-semi-$\mathcal{I}$-open function.*

Proof. Let U be a τ_i-open subset of X. Then $f(X\backslash U)$ is a subset of Y and $f^{-1}(\tau_j\text{-Int}(\tau_i\text{-Cl}^*(f(X\backslash U)))) \subset \tau_i\text{-Cl}(f^{-1}(f(X\backslash U))) = \tau_i\text{-Cl}(X\backslash U) = X\backslash U$, and so $\tau_j\text{-Int}(\tau_i\text{-Cl}^*(f(X\backslash U))) \subset f(X\backslash U)$. Hence $f(X\backslash U)$ is a (i,j)-semi-$\mathcal{I}$-closed set of Y and $f(U) = X\backslash(f(X\backslash U))$ is a (i,j)-semi-$\mathcal{I}$-open set of Y. Therefore, f is a (i,j)-semi-$\mathcal{I}$-open function. □

Definition 4.17. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ is said to be:*

(i) *pairwise semi-$(\mathcal{I}, \mathcal{J})$-open if $f(U)$ is a (i,j)-semi-$\mathcal{J}$-open set of Y for every (i,j)-semi-$\mathcal{I}$-open set U of X.*
(ii) *pairwise semi-$(\mathcal{I}, \mathcal{J})$-closed if $f(U)$ is a (i,j)-semi-$\mathcal{J}$-closed set of Y for every (i,j)-semi-$\mathcal{I}$-closed set U of X.*

It is clear that every pairwise semi-$(\mathcal{I}, \mathcal{J})$-open (resp. pairwise semi-$(\mathcal{I}, \mathcal{J})$-closed) function is pairwise semi-$\mathcal{J}$-open (resp. pairwise semi-$\mathcal{J}$-closed) function. But the converse is not true in general.

Example 4.18. *Let $X = \{a, b, c\}$ $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$ and $\mathcal{I}) = \{\emptyset, \{a\}\}$. Then the function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (X, \tau_1, \tau_2, \mathcal{I})$ defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$ is pairwise semi-$\mathcal{J}$-open but not pairwise semi-$(\mathcal{I}, \mathcal{I})$-open.*

Theorem 4.19. *For a function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$, the following statements are equivalent:*

(i) *f is pairwise semi-$(\mathcal{I}, \mathcal{J})$-open;*
(ii) *$f((i,j)\text{-}s\mathcal{I}\,\text{Int}(U)) \subset (i,j)\text{-}s\mathcal{J}\,\text{Int}(f(U))$ for each subset U of X;*
(iii) *$(i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(V)) \subset f^{-1}((i,j)\text{-}s\mathcal{J}\,\text{Int}(V))$ for each subset V of Y.*

Proof. *(i)* ⇒ *(ii)*: Let U be any subset of X. Then $(i,j)\text{-}s\mathcal{I}\,\text{Int}(U)$ is a (i,j)-semi-$\mathcal{I}$-open set of X. Then $f((i,j)\text{-}s\mathcal{I}\,\text{Int}(U))$ is a (i,j)-semi-$\mathcal{I}$-open set of Y. Since $f((i,j)\text{-}s\mathcal{I}\,\text{Int}(U)) \subset f(U)$, $f((i,j)\text{-}s\mathcal{I}\,\text{Int}(U)) = (i,j)\text{-}s\mathcal{I}\,\text{Int}(f((i,j)\text{-}s\mathcal{I}\,\text{Int}(U))) \subset (i,j)\text{-}s\mathcal{J}\,\text{Int}(f(U))$.
(ii) ⇒ *(iii)*: Let V be any subset of Y. Then $f^{-1}(V)$ is a subset of X. Hence $f((i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(V))) \subset (i,j)\text{-}s\mathcal{J}\,\text{Int}(f(f^{-1}(V))) \subset (i,j)\text{-}s\mathcal{I}\,\text{Int}(V))$. Then $(i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(V)) \subset f^{-1}(f((i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(V)))) \subset f^{-1}((i,j)\text{-}s\mathcal{I}\,\text{Int}(V))$.
(iii) ⇒ *(i)*: Let U be any (i,j)-semi-$\mathcal{I}$-open set of X. Then $(i,j)\text{-}s\mathcal{I}\,\text{Int}(U) = U$ and $f(U)$ is a subset of Y. Now, $U = (i,j)\text{-}s\mathcal{I}\,\text{Int}(U) \subset (i,j)\text{-}s\mathcal{I}\,\text{Int}(f^{-1}(f(U))) \subset f^{-1}((i,j)\text{-}s\mathcal{J}\,\text{Int}(f(U)))$. Then $f(U) \subset f(f^{-1}((i,j)\text{-}s\mathcal{J}\,\text{Int}(f(U))) \subset (i,j)\text{-}s\mathcal{J}\,\text{Int}(f(U))$ and $(i,j)\text{-}s\mathcal{J}\,\text{Int}(f(U)) \subset f(U)$. Hence $f(U)$ is a (i,j)-semi-$\mathcal{J}$-closed set of Y; hence f is pairwise semi-$(\mathcal{I}, \mathcal{J})$-open. □

Theorem 4.20. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function. Then f is a pairwise semi-$(\mathcal{I}, \mathcal{J})$-closed function if and only if for each subset U of X, $(i,j)\text{-}s\mathcal{J}\,\text{Cl}(f(U)) \subset f((i,j)\text{-}s\mathcal{I}\,\text{Cl}(U))$.*

Proof. Let f be a pairwise semi-$(\mathcal{I},\mathcal{J})$-closed function and U any subset of X. Then $f(U) \subset f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(U))$ and $f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(U))$ is a (i,j)-semi-$\mathcal{J}$-closed set of Y. We have $(i,j)\text{-}s\mathcal{J}\operatorname{Cl}(f(U)) \subset (i,j)\text{-}s\mathcal{J}\operatorname{Cl}(f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(U))) = f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(U))$. Conversely, let U be a (i,j)-semi-$\mathcal{I}$-open set of X. Then $f(U) \subset (i,j)\text{-}s\mathcal{J}\operatorname{Cl}(f(U)) \subset f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(U)) = f(U)$; hence $f(U)$ semi-$\mathcal{J}$-closed subset of Y. Therefore, f is a pairwise semi-$(\mathcal{I},\mathcal{J})$-closed function. □

Theorem 4.21. *Let $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$ be a function. Then f is a pairwise semi-$(\mathcal{I},\mathcal{J})$-closed function if and only if for each subset V of Y, $f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Cl}(V)) \subset (i,j)\text{-}s\mathcal{I}\operatorname{Cl}(f^{-1}(V))$.*

Proof. Let V be any subset of Y. Then by Theorem 4.20, $(i,j)\text{-}s\mathcal{J}\operatorname{Cl}(f(f^{-1}(V))) \subset f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(f^{-1}(V)))$. Since f is bijection, $f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Cl}(V)) \subset (i,j)\text{-}s\mathcal{I}\operatorname{Cl}(f^{-1}(V))$. Conversely, let U be any subset of X. Then $f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Cl}(f(U))) \subset (i,j)\text{-}s\mathcal{I}\operatorname{Cl}(f^{-1}(f(U)))$. Hence $(i,j)\text{-}s\mathcal{J}\operatorname{Cl}(f(U)) \subset f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(f^{-1}(f(U))))$. Therefore, by Theorem 4.20 f is a pairwise semi-$(\mathcal{I},\mathcal{J})$-closed function. □

Theorem 4.22. *Let $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$ be a pairwise semi-$(\mathcal{I},\mathcal{J})$-open function. If V is a subset of Y and U is a (i,j)-semi-$\mathcal{I}$-closed subset of X containing $f^{-1}(V)$, then there exists (i,j)-semi-$\mathcal{I}$-closed set F of Y containing V such that $f^{-1}(F) \subset V$.*

Proof. The proof is similar to the Theorem 4.13. □

Theorem 4.23. *Let $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$ be a pairwise semi-$(\mathcal{I},\mathcal{J})$-closed function. If V is a subset of Y and U is a (i,j)-semi-$\mathcal{I}$-open subset of X containing $f^{-1}(V)$, then there exists (i,j)-semi-$\mathcal{J}$-open set F of Y containing V such that $f^{-1}(F) \subset V$.*

Proof. The proof is similar to the Theorem 4.13. □

Theorem 4.24. *For a bijective function $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$, the following statements are equivalent:*

(i) *f is pairwise semi-$(\mathcal{I},\mathcal{J})$-closed;*
(ii) *f is pairwise semi-$(\mathcal{I},\mathcal{J})$-open.*

Proof. The proof is clear. □

5. Pairwise semi-$\mathcal{I}$-irresolute functions

Definition 5.1. *A function $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$ is said to be (i,j)-semi-$\mathcal{I}$-irresolute if the inverse image of every (i,j)-semi-$\mathcal{J}$-open set of Y is (i,j)-semi-$\mathcal{I}$-open in X, where $i \neq j$, i,j=1, 2.*

Proposition 5.2. *Every pairwise semi-$\mathcal{I}$-irresolute function is pairwise semi-$\mathcal{I}$-continuous.*

Proof. Straigtforward. □

The function defined as in Example 4.18 is pairwise semi-$\mathcal{I}$-continuous but not pairwise semi-$\mathcal{I}$-irresolute.

Theorem 5.3. *If $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ is pairwise semi-$\mathcal{I}$-continuous and $f^{-1}(\tau_j\text{-Cl}^*(V)) \subset \tau_j\text{-Cl}^*(f^{-1}(V))$ for each $V \in \sigma_i$, then f is pairwise semi-$\mathcal{I}$-irresolute.*

Proof. Let B be any (i,j)-semi-$\mathcal{J}$-open subset of Y. By Proposition 3.8, there exists $V \in \sigma_i$ such that $V \subset B \subset \tau_j\text{-Cl}^*(V)$. Therefore, we have $f^{-1}(V) \subset f^{-1}(B) \subset f^{-1}(\tau_j\text{-Cl}^*(V)) \subset \tau_j\text{-Cl}^*(f^{-1}(V))$. Since f is pairwise semi-$\mathcal{I}$-continuous and $V \in \sigma_i$, $f^{-1}(V)$ is (i,j)-semi-$\mathcal{I}$-open set of X; hence $f^{-1}(B)$ is (i,j)-semi-$\mathcal{I}$-open set of X. This shows that f is pairwise semi-$\mathcal{I}$-irresolute. □

Theorem 5.4. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function, then*

(1) *f is pairwise semi-$\mathcal{I}$-irresolute;*
(2) *the inverse image of each (i,j)-semi-$\mathcal{J}$-closed subset of Y is (i,j)-semi-$\mathcal{I}$-closed in X;*
(3) *for each $x \in X$ and each $V \in S\mathcal{J}O(Y)$ containing $f(x)$, there exists $U \in S\mathcal{I}O(X)$ containing x such that $f(U) \subset V$.*

Proof. The proof is obvious from that fact that the arbitrary union of (i,j)-semi-$\mathcal{I}$-open subsets is (i,j)-semi-$\mathcal{I}$-open. □

Theorem 5.5. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function, then*

(1) *f is pairwise semi-$\mathcal{I}$-irresolute;*
(2) *(i,j)-$s\mathcal{I}\,\text{Cl}(f^{-1}(V)) \subset f^{-1}((i,j)\text{-}s\mathcal{J}\,\text{Cl}(V))$ for each subset V of Y;*
(3) *$f((i,j)\text{-}s\mathcal{I}\,\text{Cl}(U) \subset (i,j)\text{-}s\mathcal{J}\,\text{Cl}(f(U))$ for each subset U of X.*

Proof. (1) ⇒ (2): Let V be any subset of Y. Then $V \subset (i,j)\text{-}s\mathcal{J}\,\text{Cl}(V)$ and $f^{-1}(V) \subset f^{-1}((i,j) - s\mathcal{I}\,\text{Cl}(V))$. Since f is pairwisr semi-$\mathcal{I}$-irresolute, $f^{-1}((i,j) - s\mathcal{J}\,\text{Cl}(V))$ is a (i,j)-semi-$\mathcal{I}$-closed subset of X. Hence $(i,j) - s\mathcal{I}\,\text{Cl}(f^{-1}(V)) \subset (i,j) - s\mathcal{I}\,\text{Cl}(f^{-1}((i,j) - s\mathcal{J}\,\text{Cl}(V))) = f^{-1}((i,j) - s\mathcal{J}\,\text{Cl}(V))$.
(2) ⇒ (3): Let U be any subset of X. Then $f(U) \subset (i,j) - s\mathcal{J}\,\text{Cl}(f(U))$ and $(i,j) - s\mathcal{I}\,\text{Cl}(U) \subset (i,j) - s\mathcal{I}\,\text{Cl}(f^{-1}(f(U))) \subset f^{-1}((i,j) - s\mathcal{J}\,\text{Cl}(f(U)))$. This implies that $f((i,j) - s\mathcal{I}\,\text{Cl}(U)) \subset f(f^{-1}((i,j) - s\mathcal{J}\,\text{Cl}(f(U)))) \subset (i,j) - s\mathcal{J}\,\text{Cl}(f(U))$.
(3) ⇒ (1): Let V be a (i,j)-semi-$\mathcal{J}$-closed subset of Y. Then $f((i,j) - s\mathcal{I}\,\text{Cl}(f^{-1}(V)) \subset (i,j) - s\mathcal{I}\,\text{Cl}(f^{-1}(f(V))) \subset (i,j) - s\mathcal{I}\,\text{Cl}(V) = V$. This implies that $(i,j) - s\mathcal{I}\,\text{Cl}(f^{-1}(V)) \subset f^{-1}(f((i,j) - s\mathcal{I}\,\text{Cl}(f^{-1}(V)))) \subset f^{-1}(V)$. Therefore, $f^{-1}(V)$ is a (i,j)-semi-$\mathcal{I}$-closed subset of X and consequently f is a pairwise semi-$\mathcal{I}$-irresolute function. □

Theorem 5.6. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ is a pairwise semi-$\mathcal{I}$-irresolute if and only if $f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Int}(V)) \subset (i,j)\text{-}s\mathcal{I}\operatorname{Int}(f^{-1}(V))$ for each subset V of Y.*

Proof. Let V be any subset of Y. Then $(i,j)\text{-}s\mathcal{J}\operatorname{Int}(V) \subset V$. Since f is pairwise semi-$\mathcal{I}$-irresolute, $f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Int}(V))$ is a (i,j)-semi-$\mathcal{I}$-open subset of X. Hence $f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Int}(V)) = (i,j)\text{-}s\mathcal{I}\operatorname{Int}(f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Int}(V))) \subset (i,j)\text{-}\mathcal{I}s\operatorname{Int}(f^{-1}(V))$. Conversely, let V be a (i,j)-semi-$\mathcal{J}$-open subset of Y. Then $f^{-1}(V) = f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Int}(V)) \subset (i,j)\text{-}s\mathcal{I}\operatorname{Int}(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is a (i,j)-semi-$\mathcal{I}$-open subset of X and consequently f is a pairwisr semi-$\mathcal{I}$-irresolute function. □

Corollary 5.7. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function. Then f is pairwise semi-$\mathcal{I}$-closed and pairwise semi-$\mathcal{I}$-irresolute if and only if $f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(V)) = (i,j)\text{-}s\mathcal{J}\operatorname{Cl}(f(V)))$ for evey subset V of X.*

Corollary 5.8. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function. Then f is pairwise semi-$\mathcal{I}$-open and pairwise semi-$\mathcal{I}$-irresolute if and only if $f^{-1}(((i,j)\text{-}s\mathcal{J}\operatorname{Cl}(V))) = (i,j)\text{-}s\mathcal{I}\operatorname{Cl}(f^{-1}((V)))$ for evey subset V of Y.*

Definition 5.9. *An ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is called pairwise semi-$\mathcal{I}$-Hausdorff if for each two distinct points $x \neq y$, there exist disjoint (i,j)-semi-$\mathcal{I}$-open sets U and V containing x and y, respectively.*

Theorem 5.10. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a pairwise semi-$\mathcal{I}$-irresolute function. If Y is pairwise semi-$\mathcal{J}$-Hausdorff, then X is pairwise semi-$\mathcal{I}$-Hausdorff.*

Proof. The proof is clear. □

Definition 5.11. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ is said to be pairwise semi-$\mathcal{I}$-homeomorphism if f and f^{-1} are pairwise semi-$\mathcal{I}$-irresolute.*

Theorem 5.12. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a bijection. Then the following statements are equivalent:*

(1) *f is pairwise semi-$\mathcal{I}$-homeomorphism;*
(2) *f^{-1} is pairwise semi-$\mathcal{I}$-homeomorphism;*
(3) *f and f^{-1} are pairwise semi-$(\mathcal{I},\mathcal{J})$-open (pairwise semi-$(\mathcal{J},\mathcal{I})$-closed);*
(4) *f is pairwise semi-$\mathcal{I}$-irresolute and pairwise semi-$(\mathcal{I},\mathcal{J})$-open (pairwise semi-$(\mathcal{J},\mathcal{I})$-closed);*
(5) *$f((i,j)\text{-}s\mathcal{I}\operatorname{Cl}(V)) = (i,j)\text{-}s\mathcal{J}\operatorname{Cl}(f(V))$ for each subset V of X;*
(6) *$f((i,j)\text{-}s\mathcal{I}\operatorname{Int}(V)) = (i,j)\text{-}s\mathcal{J}\operatorname{Int}(f(V))$ for each subset V of X;*
(7) *$f^{-1}((i,j)\text{-}s\mathcal{J}\operatorname{Int}(V)) = (i,j)\text{-}s\mathcal{I}\operatorname{Int}(f^{-1}(V))$ for each subset V of Y;*

(8) (i,j)-$s\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)) = f^{-1}((i,j)$-$s\mathcal{J}\,\mathrm{Cl}(V))$ *for each subset* V *of* Y*;*

Proof. (1) ⇒ (2): It follows immediately from the definition of a pairwise semi-$\mathcal{I}$-homeomorphism.
(2) ⇒ (3) ⇒ (4): It follows from Theorem 4.24.
(4) ⇒ (5): It follows from Theorem 4.21 and Corollary 5.7.
(5) ⇒ (6): Let U be a subset of X. Then by Theorem 3.24, $f((i,j)$-$s\mathcal{I}\,\mathrm{Int}(U)) = X\backslash f((i,j)$-$s\mathcal{I}\,\mathrm{Cl}(X\backslash U)) = X\backslash(i,j)$-$s\mathcal{I}\,\mathrm{Cl}(f(X\backslash U)) = (i,j)$-$s\mathcal{I}\,\mathrm{Int}(f(U))$.
(6) ⇒ (7): Let V be a subset of Y. Then $f((i,j)$-$s\mathcal{I}\,\mathrm{Int}(f^{-1}(V))) = (i,j)$-$s\mathcal{I}\,\mathrm{Int}(f(f^{-1}(V))) = (i,j)$-$s\mathcal{I}\,\mathrm{Int}(f(V))$. Hence $f^{-1}(f((i,j)$-$s\mathcal{I}\,\mathrm{Int}(f^{-1}(V)))) = f^{-1}((i,j)$-$s\mathcal{I}\,\mathrm{Int}(V))$. Therefore, $f^{-1}((i,j)$-$s\mathcal{J}\,\mathrm{Int}(V)) = (i,j)$-$s\mathcal{I}\,\mathrm{Int}(f^{-1}(V))$.
(7) ⇒ (8): Let V be a subset of Y. Then by Theorem 3.24, (i,j)-$s\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)) = X\backslash(f^{-1}((i,j)$-$s\mathcal{J}\,\mathrm{Int}(Y\backslash V))) = X\backslash((i,j)$-$s\mathcal{I}\,\mathrm{Int}(f^{-1}((X\backslash V)))) = f^{-1}((i,j)$-$s\mathcal{I}\,\mathrm{Cl}(V))$.
(8) ⇒ (1): It follows from Theorem 4.21 and Corollary 5.8. □

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