The Absolute Smallest Possible Money Unit! When Money Crashes into the Laws of Physics*

Espen Gaarder Haug
Norwegian University of Life Sciences
e-mail espenhaug@mac.com

November 7, 2019

Abstract

In this paper, we demonstrate that there is an absolute physical limit on how small the smallest money unit can be, no matter how much we are able to improve our technology. The smallest money unit seems to be directly linked to the smallest possible energy unit needed to store one bit. If the smallest money unit is smaller than the cost of energy of storing one bit then there seems to be an arbitrage, which will also constrain money producers such as central banks from issuing money with a smaller denomination than this minimum money unit.

Keywords: Money units, money creation, arbitrage, information theory, Launder limit, the Planck constant.

1 Introduction

Historically, the smallest unit of money has been limited by the smallest coin unit. For example, if the smallest coin unit is one cent, then one cannot use a money unit below one cent. This can be enforced by law because only legal tender can be used. Of course, the money authority, the King, the Royal Mint, or the central bank could decide to introduce a smaller money unit.

In older times, in some countries the coins were made of gold and silver. However, in those days, coins were not perfectly round so it was hard to notice if a very small piece was cut off; this is known as "coin clipping." Obviously, in one regard, this was a type of theft, as coin clipping was not legal. Yet this activity could have been used to create smaller money units when they were needed., e.g., by cutting a gold coin into two equal parts one suddenly had a coin of half the value of the complete coin.

Today most money exists in an electronic form. There is also a rapid development in different types of electronic money. There are cryptocurrencies such as Bitcoins, Ethereum, and many others. The smallest Bitcoin unit is a Satoshi, which is 0.00000001 of a Bitcoin, that is 10^{-8}, and is also known as 1 SAT.

At the national level, China has recently introduced a government-backed digital money; the central bank of Sweden and several other central banks are planning to do the same. As a computer can easily handle many decimals, from a technical point of view there is almost no limit on how small the currency units might be, at least at first glance. Could we, for example, change the existing USD to have units as small as 10^{-5} \$ or even a 10^{-100} \$ unit? Why not? As we will see, a 10^{-5} \$ unit is practical and physically possible. On the other hand, such a move could prove to be quite destructive: 10^{-100} USD units might lead to a collapse of the USD, based on some assumptions that we will describe shortly.

2 Minimum Money Unit When Using the 100% Gold Standard

Before we return to electronic money, let's go back to an older monetary standard and see what the smallest money unit could be from a technical standpoint, hypothetically. Throughout several eras in the past, certain regions in the world had a 100% gold, or 100% silver, or gold and silver standard. The simplest implementation of a 100% gold standard is to have gold coins only. The gold coins will not have any notional value (which is imprinted on them as modern coins do) ? they are simply a standardized weight of gold, so merchants and other business or trades people can easily know how much gold they are receiving or paying. Such gold coins even exist today. The South African Krugerand, for example, comes in a unit of 1 Troy Ounce (31.103 476 8 gram) of gold and follows the price of gold. So, in this case and others like it, gold and money are the same. This is

*Thanks to Victoria Terces for helping me edit this manuscript.

a system of 100% gold money, where gold is money and money is gold. If one wants to make a smaller money unit, as we have seen before, one can simply cut such a coin into exactly two pieces. Now we have two half-ounce coins. By extension, the smallest possible money unit under a 100% gold standard will then be limited to one gold atom. A gold atom cannot be divided without turning gold into something else. And to turn other atoms into gold, even if it has been accomplished in particle accelerators, would be far too expensive energy-wise. One gold atom is about 3.27×10^{-25} kg. Assume that the price of gold is about \$1,500 per Troy ounce, that is for $\frac{31.1034768}{1000} = 0.0311034768$ kg. The dollar value of one gold atom must then be $\frac{3.27\times 10^{-25}}{0.0311034768} \times 1500 \approx 1.58 \times 10^{-20}$ USD.

Naturally, we are working under idealized theoretical assumptions, such that a piece of gold could easily be split into all of its atoms without using a tremendous amount of energy in the process. Still, the point is that if the government introduced a fixed price for gold again (that is, it linked the USD to gold and backed it with a 100% gold standard, not a fractional gold standard, for example), then one could not have a smaller money unit than one gold atom. We will assume the owner of the money can, upon his or her own choosing, decide to get the money delivered in separated gold atoms.

3 Minimum Money Unit for "Electronic" Money

When we consider electronic money, we see that the smallest unit of money must be represented by at least one bit. In 1961, Landauer [1] made a connection between the minimum amount of energy needed to store one bit (or to erase one bit, which is two sides of the same coin). The so-called Landauer's limit is given by

$$K_b T \ln(2) \text{ Joule} \tag{1}$$

where K_b is the Boltzmann constant and T is the temperature in Kelvin (of the computer). According to Landauer's limit, it takes a minimum of 2.8×10^{-21} Joule to store one bit of energy if we assume a temperature of 20 Celcius (293.15 Kelvin, 68 Fahrenheit). Additional work has been done on this limit; for example, [2, 3] have explained that the Launder limit has already been reached in laboratory work. In the broader computing world, it is expected that we will reach this limit by the year 2050 for on-the-shelf computers. One Kilowatt hour (KwH) is 3.6×10^6 Joule. Over the past few years, the wholesale price of one KwH in the New York metropolitan area has been around 5-6 cents for industrial customers, although the price of electricity can change significantly over time. This means the Landauer's limit for storing one bit costs a minimum of 3.89×10^{-29} USD (based on the 5 cent KwH price). Assume we have optimized computers storing information at the Landauer's limit. Then assume that the central bank opens the way for money units smaller than 3.89×10^{-29} USD. Someone could deposit a thousand dollars, for example, and require the ability to withdraw it in its smallest possible units. The bank will have to deliver the stored money (perhaps on some type of modern hard drive), where all of these small money units are stored as separate bits. We can easily imagine a case where the energy to store the bits (money units) would be more expensive than the notional value of the units themselves.

This may sound strange, but it is similar to a situation that unfolded with physical money several years ago. Around 2005 and 2006, the "melt" value of 10 cent coins in the United States (dimes) was higher than the notional imprinted on them (10 cents). Arbitrageurs could withdraw \$1,000 in 10 cent coins and melt them down. Some activity must have been taking place because the US Mint issued a statement on December 14, 2006 that made US coin melting illegal. The regulation also limited the export of nickels and pennies; now a person can take a maximum of \$5 of these coins in face value out of the country. The new regulations authorize a fine of not more than \$10,000, or imprisonment of not more than five years, or both, against a person who knowingly violates the regulations. See Appendix A.

Viewed from a different standpoint, if it not illegal to melt physical money, then the holder of the money has an embedded call option to melt the currency if the melt value goes above the face value, as described by [4]. For electronic money, there is a similar option: if the energy value of the bits used to store the smallest money units is above the notional value of that money unit, then there is an arbitrage. Get the money, take it out as energy, and sell the energy.

Returning to the Landauer limit, which again is the minimum amount of energy needed store (or erase) one bit. Recently, [5] has claimed to have beaten the Landauer limit. It is reasonable to ask if the Landauer limit is the ultimate limit for storing one bit; at least it is not certain. As one instance of a possible limit breaker, there is much discussion on quantum computers these days, which are based on the principles of quantum mechanics. Yet, although quantum mechanics has been very successful at predictions and confirmed by hundreds of experiments, there are still many unanswered questions, especially regarding the limits of quantum mechanics and the completeness and interpretation of the theory. Further, modern physics has not been able to unify quantum mechanics with gravity, nor is quantum mechanics likely to be fully compatible with Minkiowski space-time, as discussed by [6], for example. This shows how perspectives change over time, as new information becomes available and frontiers in science are crossed.

Another interesting and more fundamental absolute limit on the minimum energy needed to store or erase one bit is linked to the quantization of energy and the theories of Max Planck. Planck [7] figured out that energy exists in discrete units, something that has been observed in many experiments. Specifically, Planck showed that energy took the form of quanta, which are linked to Planck's constant h. Energy is a frequency of action, where each action unit is directly linked to Planck's constant. The energy of pure energy (light) is given by

$$E = fh \tag{2}$$

where f is the frequency over a given time period, and h is Planck's constant. The minimum frequency we can have inside a given time window is one. So, the minimum energy unit must, in our view, be h. The frequency is given by $f = \frac{c}{\lambda}$, where lambda is the wavelength. In order to explain this in depth would require a long debate and philosophical reasoning. However, for the moment let's assume that the minimum energy is $f = 1 \times h = h$. The Planck constant is 6.6×10^{-34} Joule seconds. Planck's constant represents the smallest energy unit, which moves at the speed of light; it is a photon. Clearly, this is quite complex from deeper philosophical and physical points of view, but let's assume that the minimum energy unit is indeed h. The question then is how much KwH there are in the smallest energy unit. Again, one KwH is 3.6×10^6 J, so the smallest energy unit (we will claim) is $\frac{6.6\times10^{-34}}{3.6\times10^{6}} \approx 1.84 \times 10^{-40}$ KwH. Assuming the rate of 5 cents per KwH, then the cost of the smallest energy unit is 9.20×10^{-42} USD. We can say this is the smallest money unit that can exist in space and time, no matter where we are in the universe (although it would be adjusted for different energy prices in different places). So, following this line of reasoning, the ultimate limit on the smallest money is directly linked to Planck's constant.

The energy limit we have derived from Planck's constant is much smaller than the Landauer's limit. However, we have to keep in mind that that Landauer's limit is dependent on the temperature of the heat sink (the "computer"). The lower the temperature, the lower the Landauer limit. From standard quantum mechanics, it is clear one can never reach absolute zero (Kelvin) in temperature. Zero Kelvin would implicate that it would take zero energy to erase one bit, which would be absurd. However, [8] has recently suggested that the absolute minimum temperature is given by the Planck temperature linked to the Planck second, which is a temperature of 7.6×10^{-12} Kelvin (per second). This gives a Landauer limit of $h\ln(2) \approx 0.69h$, where h is Planck's constant. This is 30% lower than our minimum energy limit given from the earlier reasoning around the Planck constant and has a frequency of one. We think the reason for this difference is simply that the ln(2) part in Landauer's formula is a very good approximation for large particle systems, which, in fact, likely gives an error at the very depth of reality. This error is not so important, because both energy limits are in the same range. The conclusion remains that the minimum money unit is directly linked to Planck's constant.

Table 1 summarizes our suggested minimum money units

Money	Minimum Unit				USD
100% Gold Standard, 1 Troy Oz $1500	Gold atom				1.58×10^{-20}
		Minimum energy one bit	**Joule 1 bit**	**KwH 1 bit**	
Digital at room temperature	1 bit	Landauer's limit	2.8×10^{-21}	7.79×10^{-28}	3.89×10^{-29}
Digital at minimum temperature	1 bit	Landauer's limit[a]	6.63×10^{-34}	1.84×10^{-40}	9.20×10^{-42}
Digital	1 bit	Planck limit	6.63×10^{-34}	1.84×10^{-40}	9.20×10^{-42}

Table 1: Minimum Money Units. This is based on an energy price of 5 cents per KwH.

[a]This is adjusted by a factor of ln(2) compared to formula given before.

4 Conclusion

We can conclude that the smallest money unit in a system of a 100% gold standard is a gold atom (at least until one discovers cold fusion). For electronic money, the smallest possible money unit is linked to the smallest unit of energy needed to store one bit. We have discussed both the Landauer limit and what we can call the Planck limit. If the smallest money unit is smaller in value than the cost of energy for the amount of energy used to store one bit, then there is an arbitrage possibility (under ideal conditions) and such a discrepancy will not last long. The absolute smallest money unit is directly linked to the Planck scale and the Planck constant. We have demonstrated that in the future, money will likely crash into the fundamental limits of physics, since energy comes in discrete units (quanta) and it is not continuous, as believed by many physicists before Max Planck's pioneering research in the 1890s.

References

[1] R. Landauer. Irreversibility and heat generation in the computing process. *IBM J. Res. Dev.*, 5, 1961.

[2] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz. Beating landauer?s bound by memory erasure using time multiplexed potentials. *Nature*, 483, 2012.

[3] J. P. S. Peterson, R. S. Sarthour, A. M. Souza, I. S. Oliveira, J. Goold, K. Modi, D. O. Soares-Pinto, and L. C. Céleri. Experimental demonstration of information to energy conversion in a quantum system at the landauer limit. *Proceedings of The Royal Society A*, 5, 2016.

[4] E. G. Haug and J. Stevenson. Options embedded in physical money. *Wilmott Magazine*, 2009.

[5] S. Talukdar, S. Bhaban, and M. Salapaka. Beating landauer?s bound by memory erasure using time multiplexed potentials. *IFAC-PapersOnLine*, 50.

[6] W. G. Unruh. *Chapter: Minkowski Space-Time and Quantum Mechanics, in the book Minkowski Spacetime: A Hundred Years Later, Edited by V. Petkov.* Springer, 2009.

[7] Max Planck. Ueber das gesetz der energieverteilung im normalspectrum. *Annalen der Physik*, 4, 1901.

[8] E. G. Haug. A new absolute minimum temperature from Haug's new unified quantum gravity theory. *Working paper*, 2019.

Appendix

This appendix is a reprint of the statement given by the United States Mint to limit melting and exporting of certain circulating coins.

December 14, 2006 United States Mint Moves to Limit Exportation & Melting of Coins

Interim Rule Goes Into Effect Immediately

WASHINGTON – The United States Mint has implemented regulations to limit the exportation, melting, or treatment of one-cent (penny) and 5-cent (nickel) United States coins, to safeguard against a potential shortage of these coins in circulation. The United States Mint is soliciting public comment on the interim rule, which is being published in the Federal Register.

Prevailing prices of copper, nickel and zinc have caused the production costs of pennies and nickels to significantly exceed their respective face values. The United States Mint also has received a steady flow of inquiries from the public over the past several months concerning the metal value of these coins and whether it is legal to melt them. ?We are taking this action because the Nation needs its coinage for commerce,? said Director Ed Moy. ?We don't want to see our pennies and nickels melted down so a few individuals can take advantage of the American taxpayer. Replacing these coins would be an enormous cost to taxpayers.?

Specifically, the new regulations prohibit, with certain exceptions, the melting or treatment of all one-cent and 5-cent coins. The regulations also prohibit the unlicensed exportation of these coins, except that travelers may take up to $ 5 in these coins out of the country, and individuals may ship up to $ 100 in these coins out of the country in any one shipment for legitimate coinage and numismatic purposes. In all essential respects, these regulations are patterned after the Department of the Treasury's regulations prohibiting the exportation, melting, or treatment of silver coins between 1967 and 1969, and the regulations prohibiting the exportation, melting, or treatment of one-cent coins between 1974 and 1978.

The new regulations authorize a fine of not more than $ 10,000, or imprisonment of not more than five years, or both, against a person who knowingly violates the regulations. In addition, by law, any coins exported, melted, or treated in violation of the regulation shall be forfeited to the United States Government.

The regulations are being issued in the form of an interim rule, to be effective for a period of 120 days from the time of publication. The interim rule states that during a 30-day period from the date of publication, the public can submit written comments to the United States Mint on the regulations. Upon consideration of such comments, the Director of the United States Mint would then issue the final rule.

Those interested in providing comments to the United States Mint regarding this interim rule must submit them in writing to the Office of Chief Counsel, United States Mint, 801 9th Street, N.W., Washington D.C. 20220, by January 14, 2007. The interim rule appears on the United States Mint website at www.usmint.gov. The United States Mint will make public all comments it receives regarding this interim rule, and may not consider confidential any information contained in comments.