Stefan-Boltzmann constant incorrect by a factor 2π

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Abstract- Due to a wrong conception of Planck's mathematical presentation of the power density spectrum of a black body radiator, a wrong Stefan-Boltzmann constant has been created. This article explains what that misconception is and how this factor 2π arose from it.

1 Planck's description of his black body power density spectrum

These descriptions are found in [1]. The relevant parameters are:

h	Planck's constant	$6.6 \cdot 10^{-34}$	Js
c	speed of light	$3.0 \cdot 10^{8}$	m/s
k	Boltzmann's constant	$1.4 \cdot 10^{-23}$	J/K

> "*Moreover the specific intensity* K_ν *of a monochromatic plane polarized ray of frequency* ν *is, according to equation..*
>
> $$K_\nu = h\nu^3 c^{-2}/(\exp(h\nu/kT) - 1) \qquad (274)"$$
>
> *This is the specific intensity of a monochromatic plane polarized ray of the frequency* ν *which is emitted from a black body at the temperature T into vacuum in a direction perpendicular to the surface.*"

Attention has to be paid to the condition: ".......emitted .... in a direction *perpendicular* to the surface" and to the restriction "in vacuum". The index ν in K_ν is from the author, not from Planck.

The characteristic property of a black body thus is that for all frequencies, from 0 to ∞, each frequency obeys the relation as shown by K_ν. For that reason Planck uses the word 'monochromatic'. The dimension of K_ν is $W/m^2/Hz$, thus is the dimension W/m^2 of $\int_0^\infty K_\nu \, d\nu$.

The just mentioned integral plays the main role in the investigation of the subject under consideration, because in practical situations, meaning the measurement of the power density of a radiator in order to determine its temperature, all the frequencies from 0 to ∞ are meant to be received simultaneously.

The related integral can be written as $C \cdot \int_0^\infty x^3/(e^x - 1)dx$,

with $x = h\nu/kT$, so $\nu = xkT/h$, thus $d\nu = dx \cdot kT/h$. As a result: $C = (h/c^2)\cdot(kT/h)^4$.

According to [2] the integral $\int_0^\infty x^3/(e^x - 1)dx$ is: quote 'a particular case of a Bose–Einstein integral, the polylogarithm, or the Riemann zeta function ζ(s). The value of the integral is $6 \cdot \zeta(4) = \pi^4/15$' end quote.

The final result is: $\int_0^\infty h\nu^3 c^{-2}/(\exp(h\nu/kT) - 1)\, d\nu = \pi^4/15\cdot(h/c^2)\cdot(kT/h)^4 = \pi^4/15\cdot h^{-3}c^{-2}k^4\cdot T^4 = \sigma_U T^4$ W/m^2.

This σ_U thus is a factor 2π lower than the generally accepted σ: $2\pi \cdot \pi^4/15\, h^{-3}c^{-2}k^4$ $W/m^2/K^4$.

2 Explanation of the wrong Stefan–Boltzmann constant

Reference [2] writes:

"The Stefan–Boltzmann law describes the power radiated from a black body in terms of its temperature."

Comment:

There is no Stefan–Boltzmann law, *anymore!* It has just been shown that the integration of Planck's spectrum over the frequency results in the expression σT^4. The constant σ can be called the Stefan–Boltzmann constant, but why? The answer to this question is also found in [2] under "History":

"*In 1864, John Tyndall presented measurements of the infrared emission by a platinum filament and the corresponding color of the filament. The proportionality to the fourth power of the absolute temperature was deduced by Josef Stefan in 1879 on the basis of Tyndall's experimental measurements,.........*"

Reference [1] shows that Planck wrote his book on thermal radiation in 1913. So Planck wrote this stuff more than 20 years after Stefan experimentally discovered the relationship "power density = σT4". Reference [2] refers to the article of Stefan: "Über die Beziehung zwischen der Wärmestrahlung und der Temperatur". It is a 40 pages long report showing a large number of measurements and calculations. In the last 4 pages the temperature of the Sun is considered. The results vary from 1700 to 10300 K, depending on the scientist carrying out the measurement. Most likely, the experiments were not performed in accordance with the conditions attached to Planck's theory, especially with regard to the emission and reception in vacuum circumstances. That might explain the enormous different results in the measured temperature of Sun's surface. But still the question remains why in reference [2] a theory has been presented, adapted to the results of Stefan's experiments, instead of accepting the inevitable theoretical result, shown in section 1. It presents a theoretical approach that lies in between the pure theoretical approach and a practical approach, by showing the application of Planck's spectrum to a small flat black body. Parts of the text from [2] have been copied and presented in Italics, followed by comments.

"The law (meant is Stefan–Boltzmann law) *can be derived by considering a small flat black body surface radiating out into a half-sphere. This derivation uses spherical coordinates, with θ as the zenith angle and φ as the azimuthal angle; and the small flat blackbody surface lies on the xy-plane, where* $\theta = \pi/2$."

Comment:

The root of the problem is ultimately found in the words: "The law can be derived by....", because Planck has already presented the law theoretically and in the words "radiating out into a half-sphere", as shown hereafter.

" *The intensity of the light emitted from the blackbody surface is given by Planck's law:*

$$I(\nu,T) = 2\cdot h\nu^3 c^{-2}/(\exp(h\nu/kT) - 1)$$

$I(\nu,T)$ *is the amount of power per unit surface area per unit solid angle per unit frequency emitted at a frequency* ν *by a black body at temperature T.* "

Comment:

Planck's spectrum is violated here in two ways: by the addition of the factor 2 and by the addition of: "per unit solid angle". Planck's original text does not contain any reference to fixed angles, nor is a numerical constant used that could lead to the introduction of a factor 2.

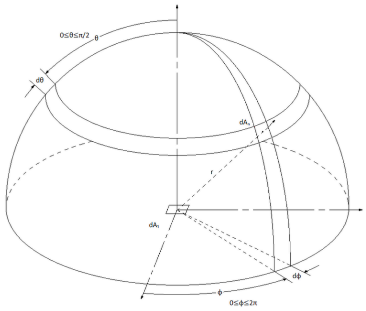
" *The quantity I(ν,T) A dν dΩ is the power radiated by a surface of area A through a solid angle dΩ in the frequency range between* ν *and* ν + *dν* "

Comment:

The addition "through a solid angle dΩ" is fundamentally wrong, because the quantity I(ν,T) A dν dΩ has no physical meaning. The text has to be: The quantity I(ν,T) A dν is the power P radiated by a surface A in a direction perpendicular to A in the frequency range ν to ν + dν. The resulting power density is P/A.

"*Note that the cosine appears* (in $P/A = \int_0^\infty I(\nu,T)\, d\nu \int \cos\theta\, d\Omega$) *because black bodies are Lambertian (i.e. they obey Lambert's cosine law), meaning that the intensity observed along the sphere will be the actual intensity times the cosine of the zenith angle. To derive the Stefan–Boltzmann law, we must integrate* $d\Omega = \sin(\theta)\, d\theta\, d\varphi$ *over the half-sphere and integrate* ν *from 0 to ∞.*"

Comment:
The figure below from [2] shows the meant flat black body A at the centre and the bottom of the drawn half-sphere. The only relevant intensity is found along the perpendicular axis on A, excluding all other possible radiations. The Lambertian character of the source, nor the half-sphere do play any role in the situation under consideration.



Remark:
At the end of 2019, criticism of [2] was published at viXra in an article which has also been sent to 7500 physicists around the world. It showed the wrong definition of the angles θ and φ as a minor error. Before the end of 2020 the corresponding drawing has been replaced and corrected, but the other, much more important, criticism has been ignored.

The miscalculation with the factor π, out of 2π, is thus caused by a misconception of the area related to the radiation. In the example in [2], the surface $2\pi r^2$ of a hemisphere is initially assumed as most relevant, corrected for the Lambertian character of the flat black body to πr^2. However, such a surface has nothing to do with a representative part of the surface at the black body.

Summarized: In order to measure the temperature of a *black body*, a representative small area S of that source has to be observed *perpendicularly* by the beam of a measuring device, *in vacuum*. The thus received power P_r has to be divided by that area S. Only in such a configuration it is allowed to use the relation $P_r/S = \sigma_U T^4$, in order to calculate the temperature as $(P_r/S/\sigma_U)^{1/4}$.

Conclusions

1 The Stefan-Boltzmann constant $\sigma=2\pi\cdot\pi^4k^4/15h^3c^2$ (W/m²/K⁴) has to be replaced by $\sigma_U = \sigma/2\pi$.

2 One of the consequences is that the alleged temperature of Sun's surface: 5777 K, assumed that the related power density has been measured correctly, has to be corrected to 9146 K.

References

[1] http://www.gutenberg.org/files/40030/40030-pdf.pdf

[2] https://en.wikipedia.org/wiki/Stefan-Boltzmann_law