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Title :THE PROBLEM P=NP (version 2)

ABSTRACT:

In this article, we are going to solve the problem P=NP for a particular kind of problems called *basic problems of numerical determination*. Nonetheless, this solution can be generalized to all problems belonging to class P or NP. We are going to propose 3 fundamental Axioms permitting to solve the problem P=NP, but those Axioms can also be considered as pure logical assertions, intuitively evident and never contradicted, permitting to understand the solution of the problem P=NP. Indeed, we will see that the conclusion of this article solves the considered problem.

I)INTRODUCTION

In this article, we are going to give a solution to the problem P=NP. We know that the conjecture P=NP (Any problem belonging to class P belongs to class NP and conversely) has never been proved nor its negation P≠NP. In this article we are going to propose 3 assertions of pure logic, intuitively evident and never contradicted, called *Axioms* for this reason, permitting to give a solution to the problem P=NP. Indeed, we know that in a mathematical theory, we can use and introduce Axioms, propositions that are evident or own an intuitively evident justification and that have never been contradicted. Moreover, the fact that none fundamental results linked to the problem P=NP have ever been obtained using classical mathematical theories suggests that obtaining the solution of the problem P=NP needs to introduce and to use new Axioms, and cannot be obtained using only classical mathematical theories. It seems to be evident that the Axioms that we are going to admit cannot be proved using classical mathematical theories. To begin with, we will only consider a kind of problems belonging to class P or to class NP, called *basic problems of numerical determination*. But we will see that we can immediately generalize our Axioms and give the solution of the problem P=NP in the general case.

We will define in this article a *basic problem of numerical determination*. This definition is important because it contains a very general kind of problems that are potentially of class P or of class NP, and consequently because it constitutes a very concrete basis permitting to justify intuitively the Axioms that we are going to introduce, and also to test their validity.

The theory presented on this article is not a purely mathematical theory but a logical mathematical theory. It is quasi-certain that the solution of the problem P=NP obtained by this theory could not be obtained by a purely mathematical theory. We can consider the theory exposed in this article as purely logical because it does not introduce any new equation. Nonetheless, it is in agreement with all (established) equations relative to the problem P=NP, its implications concern all those equations and in order to prove its invalidity, it should be necessary to use such equations.

In this article, the approach of the problem P=NP is completely new and does not use any anterior published article concerning this problem. But we will see that its conclusion is in agreement with all articles previously published about this problem. We are going to justify that it is not possible to prove P=NP nor P≠NP. Our article can be consider either as a mathematical logical proof (using intuitive Axioms) either as a logical justification (Considering our Axioms as logical assertions with intuitive justification). In both cases the conclusion is a fundamental mathematical result that a priori cannot be obtained without using logical assertions analogous to those introduced in this article.

II)SOLUTION OF THE PROBLEM P=NP

A)BASIC PROBLEMS OF NUMERICAL DETERMINATION (DEFINITION).

By definition, *a basic problem of numerical determination* contains the following data:

-A natural n different from 0.

-A finite set A(n) defined as a function of n.

-A function k(n) belonging to $F(\mathbf{N},\mathbf{N})$ (That is possibly constant).

-In some cases r (r being a given number) finite sets $B_1,..,B_r$ with for i belonging to {1,..,r} B_i verifying a proposition of the kind $P_{Bi}(B_i,A(n),n)$.

By definition solving this problem signifies to find k(n) distinct elements of A(n) $a_1,..,a_{k(n)}$ verifying a proposition of the kind $P(a_1,..,a_{k(n)},A(n),n, k(n), B_1,..,B_r)$. (This last proposition is contained by the definition of the considered basic problem of numerical determination).

(We could have generalized the preceding definition in including in this definition analogous problems in which k(n) is not included in the data, but in which we want to obtain a sequence $(a_1,.,a_s)$, the length of this sequence being defined in the proposition $P(a_1,..,a_{k(n)},A(n),n, B_1,..,B_r)$, or in which the a_i are not necessarily distinct).

We remind that we will say that such a problem *belongs to class* p (or *is of class* p) if it exists a polynomial algorithm permitting to obtain for any natural n at least a sequence $(a_1,..,a_{k(n)})$ if it exists. This polynomial algorithm can use n,A(n), $B_1,..,B_r$. Then we will say that such algorithm is *a polynomial algorithm permitting to solve* the considered problem *(or solving it).*

We remind that we will say that such a problem *belongs to class* np (or *is of class* np) if it exists a polynomial algorithm permitting, for any natural n and any distinct elements of A(n) $a_{10},..,a_{k(n)0}$, to determine if $P(a_{10},..,a_{k(n)0},n,A(n),B_1,..,B_r)$ is true. This polynomial algorithm can use $a_{10},..,a_{k(n)0}$, n, $B_1,..,B_r$,k(n) and A(n). Then we will say that such an algorithm is *a polynomial algorithm permitting to verify* the considered problem.

It exists some problems that are of class P or of class NP that are not basic problems of numerical determination but this latter kind of problems constitute most of the interesting and classical of problems of class P or class NP.

For instance we can consider the classical basic problem of numerical determination consisting in finding if they exist 2 distinct divisors a_1 and a_2 of a natural n.

Then we have for this basic problem of numerical determination A(n)={1,..,n},k(n)=2 and $P(a_1,a_2,n)$: « a_1 et a_2 are 2 distinct naturals different from n and $a_1 \times a_2 = n$ ».

For the example of the Clay's institute (in which n=400), A(n)={1,..,n}, k(n)=Int(n/4) (or 100) and B_1 is a set verifying $P_{B1}(B_1,A(n),n)$: "B_1 is a set with Card(B_1)=Int(n/4) (or 100) and for any x element of B_1, it exists b_1 and b_2 distinct elements of A(n) such that $x=\{b_1,b_2\}$.".

And $P(a_1,.. ,a_{k(n)}, k(n),B_1)$: "For any i,j distinct elements of {1,..,k(n)}, $\{a_i,a_j\}$ is not element of B_1 ".

It is clear that this example is not interesting for the considered values of k(n) and Card(B_1). But they can be modified.

P=NP, for the basic problems of numerical determination, signifies that any basic problem of numerical determination belonging to class p belongs to class np and conversely. We are going to prove, using assertions of pure logic intuitively evident and never contradicted that we called "Axioms", that this problem has not classical solution, and that we have:

-If P=NP, it is impossible to prove it.

-If P≠NP, it is impossible to prove it.

In what follows we will consider only basic problems of numerical determination.

B)IMPOSSIBILITY TO PROVE P≠NP.

In order to prove P≠NP, we must prove either than P is not included in NP either that NP is not included in P. Consequently we must find a problem of class P that is not of class NP or a problem of class NP that is not of class P.

But we admit the following Axiom:

AXIOM 1:

It is impossible to prove that a basic problem of numerical determination is not of class P or is not of class NP.

Indeed, it does not exist general Axioms permitting to prove that it does not exist any polynomial algorithm permitting to solve or to verify a given basic problem of numerical determination P_{DN}. Consequently in order to prove that none polynomial algorithm permits to solve (resp. to verify) P_{DN}, we should consider each existing polynomial algorithm and verify that it does not solve (resp. verify) P_{DN}, which is evidently impossible. Moreover, this Axiom 1 has never been contradicted, it has never been proved that a basic problem of numerical determination is not of class P or is not of class NP, which would be necessary in order to prove the invalidity of this Axiom 1.The part of the Axiom 1 relative to basic problems of numerical determination can be justified the same way.

So because of this Axiom 1, it is impossible to prove that a given basic problem of numerical determination is not of class P or is not of class NP, and consequently to prove that P is not included in NP or that NP is not included in P, and consequently to prove P≠NP.

C)IMPOSSIBILITY TO PROVE P=NP.

In order to prove P=NP, we must prove that any problem of class p is of class np and conversely.

But we admit the following Axiom:

AXIOM 2:

In order to prove that a basic problem of numerical determination is of class P (resp. of class NP), we must necessarily give a polynomial algorithm permitting to solve it (resp.to verify it).

This Axiom 2 is the consequence of the fact that it does not exist general Axioms permitting to justify that it exists a polynomial algorithm permitting to solve or to verify a given basic problem of numerical determination. This is confirmed also by the fact that it has never been proved that a given basic problem of numerical determination was of class P or of class NP by another way than giving explicitly a polynomial algorithm permitting to solve or to verify it. In order to prove the invalidity of this Axiom, it should be necessary to prove that a basic problem of numerical determination belongs to class P without giving the polynomial algorithm solving it (or a way to obtain this polynomial algorithm).

A consequence of this Axiom 2 is the following logical assertion, that can also be considered as its 2nd part:

ASSERTION 1:

In order to prove that NP is included in P (For the basic problems of numerical determination), it will be necessary to provide a (general) polynomial algorithm permitting to solve all the problems of numerical determination of class NP.

We can find an analogous assertion in order to prove that P is included in NP.

But it is evident that it will be impossible to find such a polynomial algorithm which we admit in the 3rd following Axiom:

AXIOM 3:

It does not exist a (general) polynomial algorithm permitting to solve all the basic problems of numerical determination of class NP.

Proving the invalidity of this Axiom would imply to find such a general algorithm, which seems to be completely impossible. Indeed it is evident that if we consider a basic problem of numerical determination of class NP corresponding to the general definition that we gave, we do not have enough elements to build a polynomial algorithm solving this problem. We have not a single line of such an algorithm. It is true that we can obtain a general algorithm permitting to solve it (see the following section), but this algorithm being not always polynomial is not the needed algorithm.

So we justified using the preceding Axioms that it is impossible to prove P=NP (Even if it is true), because in order to prove P=NP we must prove that P is included in NP and NP is included in P.

D)REMARK

So we justified using the introduced Axioms that if P=NP is true, we cannot prove it and if P≠NP, we also cannot prove it.

We remark that if a problem of numerical determination is of class NP, we can easily find an algorithm permitting to solve it: We just need to apply the polynomial algorithm permitting to verify the considered problem to each element of $A(n)^{k(n)}$. But the obtained algorithm is not compulsory polynomial. (We can also replace $A(n)^{k(n)}$ by the set whose the elements are the elements of $A(n)^{k(n)}$ with distinct terms).

We remind that we can also consider the Axioms 1,2,3 as assertions of pure logic, never contradicted, that permit to solve the problem P=NP.

III)CONCLUSION

So we did not prove that neither P=NP nor P≠NP was true, but we justified that in both cases it will be impossible to prove it. This means that if it exists a basic problem of numerical determination of class NP such that whatever be Ag_p polynomial algorithm Ag_p does not solve this problem, then P≠NP, but this will be impossible to prove according to the Axiom 1 and its intuitive justification. On the contrary, if for every basic problem of numerical determination of class NP (resp.P) it exist a polynomial algorithm permitting to solve it (resp.to verify it), then P=NP but this will be impossible to prove according to Axiom 2 and Axiom 3 and their intuitive justifications.

We considered only problems of class P and class NP that were basic problems of numerical determination, but the solution that we gave to the problem P=NP can be easily generalized to all problems of class P and class NP. (We generalize the Axioms 1,2,3 replacing "problems of numerical determination " by "problems"). We remark nonetheless that the fact that we cannot prove P=NP for the basic problems of numerical determination implies that we cannot also prove this in the general case.

The theory that we presented is not a purely mathematical theory but is a logical mathematical theory. It seems quasi-certain that it is not possible to obtain the solution that we gave of the problem P=NP with a purely mathematical theory. We remind that we can consider the Axioms 1,2,3 as assertions of pure logic admitted because they have an intuitive evident justification and have never been contradicted. Moreover, the fact that we have never obtained fundamental result concerning the problem P=NP suggests that the solution of this problem needs compulsory to introduce new Axioms, and cannot be obtained using only classical mathematical theories. It is very possible that any theory solving the problem P=NP must admit Axioms analogous to Axioms we introduced in this article. We can consider the theory exposed in this article as purely logical because it does not introduce any new equation. Nonetheless, it is in agreement with all the (established) equations relative to the problem P=NP , its implications concern all those equations and it would be necessary to use those equations in order to prove its invalidity (and the invalidity of its axioms). We remind that our theory can be considered either as a mathematical logical proof (using intuitive Axioms) either as a logical justification (Considering our Axioms as logical assertions with intuitive justification). In both cases the conclusion is a fundamental mathematical result that a priori cannot be obtained without using logical assertions analogous to those introduced in this article.

We remind that the definition of a *basic problem of numerical determination* is important because it contains a very general kind of problems that are potentially of class P or of class NP, and consequently because it constitutes a very concrete basis permitting to justify intuitively the Axioms that we introduced, and also to test their validity.

Our proof can be generalized immediately to the case of other kind of algorithms, for instance polynomial algorithms whose the polynomial degree is inferior to a given natural.

So we did not prove P=NP nor P≠NP but we solved the problem P=NP the same way the proof that it did not exist any algorithm permitting to obtain the trisection of the angle or the quadrature of the circle with a compass solved those problems. The conclusion of this article is therefore in agreement with all articles previously published about the problem P=NP. In order to prove P=NP or its contradiction, it should be necessary at first to prove that the theory exposed in this article and its Axioms are wrong, which should be clearly much easier than proving P=NP or its negation.