Difference cordial labeling of the graphs related to duplication of an edge or vertex of a cycle and total graph

[1]A. Sugumaran, [2]V. Mohan

[1, 2]Department of mathematics, Government Arts College, Thiruvannamalai-606603,

Tamilnadu, India.

[1]e-mail: sugumaranaruna@gmail.com

[2]e-mail: vmb5685@gmail.com

Abstract. A *difference cordial labeling* of a graph G is a bijective function f from V(G) onto {1, 2, 3, ⋯ , |V(G)|} such that each edge uv is assigned the label 1 if $|f(u) - f(v)| = 1$, and the label 0 otherwise, satisfying the condition that the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1. A graph with difference cordial labeling is called a *difference cordial graph*. In this paper we proved that the umbrella graph U(m, n), duplication of a vertex by an edge in a cycle C_n, duplication of an edge by a vertex in a cycle C_n and the total graph of a path P_n are difference cordial graphs.

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Keywords. Difference cordial labeling, duplication of an arbitrary edge by a new vertex, duplication of an arbitrary vertex by a new edge, total graph and umbrella graph.

1. Introduction.

We deal here with finite, simple and undirected graphs. Graph labeling connects many branches of mathematics and is consider one of the important blocks of graph theory. Labeled graphs plays an important role in communication network addressing and models for constraint programming over finite domains [2]. Cordial labeling concept was first introduced in 1987 by Cahit [1], and there has been a major effort in this area made this topic growing steadily and widely [2]. Initially the difference cordial labeling was stated in [4] and some of the standard graphs were proved for the same in [5, 6, 7, 8]. Seoud and Salman [9, 10] studied some characterization of ladder graphs for difference cordial labeling. Sugumaran and Mohan [11, 12, 13] have proved some of the graphs such as plus graph Pl_n, path union of plus graph $P(r. Pl_n)$, cycle union of plus graph $C(r. Pl_n)$, barycentric subdivision of Pl_n, hanging pyramid HPy_n graph, path union of hanging pyramid graph $P(r. HPy_n)$, the graphs obtained by switching of a pendant vertex in P_n, switching of an apex vertex in CH_n, the graph obtained by duplication of each vertex of path by an edge, the graph obtained by barycentric subdivision of crown graph $C_n \Theta K_1$, path union of r copies of fan $P(r. F_n)$, cycle union of r copies of fan $C(r. F_n)$ and open star of r copies of fan $S(r. F_n)$ graph and some graphs related to square graphs are difference cordial graphs. In this paper, we proved that the umbrella graph U(m, n), duplication of an arbitrary vertex by a new edge in a cycle C_n, duplication of an arbitrary edge by a new vertex in a cycle C_n and the total graph $T(P_n)$ of path P_n are difference cordial graphs.

2. Basic definitions

Definition 2.1. A *difference cordial labeling* of a graph G is a bijective function f from V(G) to {1, 2, 3, ⋯ , |V(G)|} such that if each edge uv is assigned the label 1 if |f(u) – f(v)| = 1 and 0 otherwise, satisfying the condition that the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1. A graph with difference cordial labeling is called a *difference cordial graph*. We denote $e_f(0)$ and $e_f(1)$ are the number of edges labeled with 0 and the number of edges labeled with 1 respectively.

Definition 2.2. An *umbrella graph* U(m, n) is the graph obtained by joining a path P_n with the apex vertex of a fan F_m.

Definition 2.3. *Duplication of an edge* e = v_iv_{i+1} *by a new vertex v* in a graph G produces a new graph G' such that the neighborhood of v, that is N(v) = { v_i, v_{i+1}}.

Definition 2.4. *Duplication of a vertex v by a new edge* e = $v'v''$ in a graph G produces a new graph G' such that the neighbourhood of v' and v'' are respectively N(v') = { v, v''} and N(v'') = { v, v'}.

Definition 2.5. The *total graph* of a graph G is the graph whose vertex set is V(G)∪E(G) and two vertices in this graph are adjacent whenever they are either adjacent or incident in G. The total graph of G is denoted by T(G).

3. Main results

Theorem 3.1. Every umbrella graph U(m, n) is a difference cordial graph, where m > 1 and n >1.

Proof. Let V(U(m, n)) = {$x_1, x_2, x_3, \dots , x_m, y_1, y_2, y_3, \dots , y_n$} and E(U(m, n)) = {($x_i$, x_{i+1}) ; i = 1, 2, 3, … , (m-1), (y_i, y_{i+1}) ; i = 1, 2, 3, … , (n-1), (y_n, x_i) ; i = 1, 2, 3, … , m}, be the vertex set and edge set of an umbrella graph. Let y_n be the apex vertex of fan F_m. Note that |V(U(m, n))| = m + n and |E(U(m, n))| = 2(m-1) + n. We define a bijective map f : V(U(m, n)) → {1, 2, 3, … , (m + n)} as follows:

We distinguish the vertex labeling into six cases.

Case 1. When n = 2 and m > 1.

$f(x_i)$ = i + 1 for i = 1, 2, 3, … , m,

$f(y_1)$ = m + n,

$f(y_2)$ = 1.

In view of the above labeling pattern, we get $e_f(0) = e_f(1)$ = m. Hence, | $e_f(0) - e_f(1)$| = 0.

Case 2. When n = 3 and m > 1.

$f(x_i)$ = i + 1 for i = 1, 2, 3, … , m,

$f(y_1)$ = m + n,

$f(y_2)$ = m + n – 1,

$f(y_3) = 1$.

In view of the above labeling pattern, we get $e_f(0) = m$ and $e_f(1) = m+1$. Hence, $| e_f(0) - e_f(1)| = 1$.

Case 3. When $n \equiv 0 \pmod 4$ and $m > 1$.

Subcase 3.1. When $n = 4$.

$f(x_i) = 4 + i$ for $1 \leq i \leq m$,

$f(y_1) = 1$, $f(y_2) = 2$, $f(y_3) = 4$, $f(y_4) = 3$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = m+1$. Hence, $| e_f(0) - e_f(1)| = 0$.

Subcase 3.2. When $n = 8$.

$f(x_i) = 8 + i$ for $1 \leq i \leq m$,

$f(y_1) = 1$, $f(y_2) = 2$, $f(y_3) = 3$, $f(y_4) = 4$, $f(y_5) = 6$, $f(y_6) = 7$, $f(y_7) = 5$, $f(y_8) = 8$ and we interchange the labels of the vertices y_1 and y_2.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = m+1$. Hence, $| e_f(0) - e_f(1)| = 0$.

Subcase 3.3. When $n \geq 12$.

$f(x_i) = n + i$ for $1 \leq i \leq m$,

$f(y_i) = i$ for $1 \leq i \leq \frac{n+2}{2}$,

$f(y_{\frac{n+2}{2}+i}) = \frac{n+2}{2} + 2i$ for $1 \leq i \leq \frac{n-4}{4}$,

$f(y_{\frac{3n}{4}+i}) = \frac{n+2}{2} + 2i - 1$ for $1 \leq i \leq \frac{n}{4}$ and then we interchange the labels of the vertices y_1 and y_2.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = m-1+\frac{n}{2}$. Hence, $| e_f(0) - e_f(1)| = 0$.

Case 4. When $n \equiv 1 \pmod 4$ and $m > 1$.

$f(x_i) = n + i$ for $1 \leq i \leq m$,

$f(y_i) = i$ for $1 \leq i \leq \frac{n+1}{2}$,

$f(y_{\frac{n+1}{2}+i}) = \frac{n+1}{2} + 2i$ for $1 \leq i \leq \frac{n-1}{4}$,

$f(y_{\frac{3n+1}{4}+i}) = \frac{n+1}{2} + 2i - 1$ for $1 \leq i \leq \frac{n-1}{4}$.

In view of the above labeling pattern, we get $e_f(0) = 3$ and $e_f(1) = 4$ when $n=5$. For $n \geq 9$, we get $e_f(0) = \frac{2m+n-1}{2}$ and $e_f(1) = \frac{2m+n-3}{2}$. Hence, $| e_f(0) - e_f(1)|=1$.

Case 5. When $n \equiv 2 \pmod 4$ and $m > 1$.

Subcase 5.1. When $n = 6$.

$f(x_i) = 6 + i$ for $1 \leq i \leq m$,

$f(y_1) = 1$, $f(y_2) = 2$, $f(y_3) = 3$, $f(y_4) = 5$, $f(y_5) = 6$, $f(y_6) = 4$.

Subcase 5.2. When $n \geq 10$.

$f(x_i) = n + i$ for $1 \leq i \leq m$,

$f(y_i) = i$ for $1 \leq i \leq \frac{n}{2}$,

$f(y_{\frac{n}{2}+i}) = \frac{n}{2} + 2i$ for $1 \leq i \leq \frac{n-2}{4}$,

$f(y_{\frac{3n-2}{4}+i}) = \frac{n}{2} + 2i - 1$ for $1 \leq i \leq \frac{n+2}{4}$.

In view of the above labeling pattern in both subcases 5.1 and 5.2, we get $e_f(0) = e_f(1) = \frac{2(m-1)+n}{2}$. Hence, $| e_f(0) - e_f(1)| = 0$.

Case 6. When $n \equiv 3 \pmod 4$ and $m > 1$.

Subcase 6.1. When $n = 7$.

$f(x_i) = 7 + i$ for $1 \leq i \leq m$,

$f(y_1) = 1$, $f(y_2) = 2$, $f(y_3) = 3$, $f(y_4) = 6$, $f(y_5) = 4$, $f(y_6) = 5$, $f(y_7) = 7$.

In view of the above labeling pattern, we get $e_f(0) = 4$ and $e_f(1) = .5$ Hence, $| e_f(0) - e_f(1)| = 1$.

Subcase 6.2. When $n \geq 11$.

$f(x_i) = n + i$ for $1 \leq i \leq m$,

$f(y_i) = i$ for $1 \leq i \leq \frac{n+1}{2}$,

$f(y_{\frac{n+1}{2}+i}) = \frac{n+1}{2} + 2i$ for $1 \leq i \leq \frac{n-3}{4}$,

$f(y_{\frac{3n-1}{4}+i}) = \frac{n+1}{2} + 2i - 1$ for $1 \leq i \leq \frac{n+1}{4}$.

In view of the above labeling pattern, we get $e_f(0) = \frac{2m+n-3}{2}$ and $e_f(1) = \frac{2m+n-1}{2}$. Hence, $| e_f(0) - e_f(1)| = 1$. Therefore, the umbrella graph $U(m, n)$ is a difference cordial graph.

Example 1. The difference cordial labeling of the umbrella graph $U(5, 4)$ is shown in Figure 1.

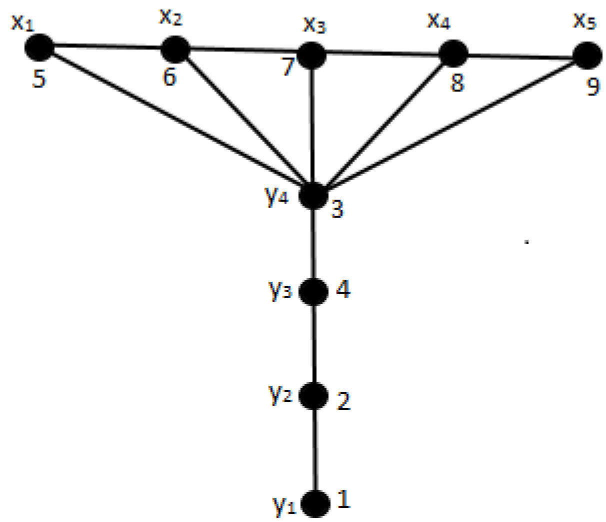

Figure 1. The difference cordial labeling of the umbrella graph $U(5,4)$

Theorem 3.2. The graph obtained by duplication of an arbitrary edge by a vertex in a cycle admits difference cordial labeling.

Proof. Consider the cycle C_n. Let $v_1, v_2, v_3, \ldots, v_n$ be the consecutive vertices of C_n and let G be the graph obtained by duplication of an edge $e = v_j v_{j+1}$ by a new vertex w. Let us assume that j = 1, without loss of generality. i.e., $e = v_1v_2$. Note that $|V(G)| = n + 1$ and $|E(G)| = n + 2$. We define a vertex labeling function $f : V(G) \rightarrow \{1, 2, 3, \ldots, (n + 1)\}$ as follows:

We consider the following cases.

Case 1. When $n \equiv 0 \pmod 4$.

Subcase 1.1. When n = 4.

$f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 5$, $f(v_4) = 4$, $f(w) = 2$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = 3$. Hence, $|e_f(0) - e_f(1)|=0$.

Subcase 1.2. When n = 8.

$f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 4$, $f(v_4) = 5$, $f(v_5) = 8$, $f(v_6) = 6$, $f(v_7) = 7$, $f(v_8) = 9$, $f(w) = 2$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = 5$. Hence, $|e_f(0) - e_f(1)|=0$.

Subcase 1.3. When $n \geq 12$.

$f(v_1) = 1$, $f(v_2) = 3$, $f(w) = 2$,

$f(v_i) = i + 1$ for $3 \leq i \leq \frac{n+2}{2}$,

$f(v_{\frac{n+2}{2}+i}) = \frac{n+4}{2} + 2i$ for $1 \leq i \leq \frac{n-4}{4}$,

$f(v_{\frac{3n}{4}+i}) = \frac{n+4}{2} + 2i - 1$ for $1 \leq i \leq \frac{n}{4}$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = \frac{n+2}{2}$. Hence, $|e_f(0) - e_f(1)| = 0$.

Case 2. When $n \equiv 1 \pmod 4$.

Subcase 2.1. When $n = 5$.

$f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 5$, $f(v_4) = 6$, $f(v_5) = 4$, $f(w) = 2$.

In view of the above labeling pattern, we get $e_f(0) = 4$ and $e_f(1) = 3$. Hence, $|e_f(0) - e_f(1)| = 1$.

Subcase 2.2. When $n \geq 9$.

$f(v_1) = 1$, $f(v_2) = 3$, $f(w) = 2$,

$f(v_i) = i + 1$ for $3 \leq i \leq \frac{n+1}{2}$,

$f(v_{\frac{n+1}{2}+i}) = \frac{n+3}{2} + 2i$ for $1 \leq i \leq \frac{n-1}{4}$,

$f(v_{\frac{3n+1}{4}+i}) = \frac{n+3}{2} + 2i - 1$ for $1 \leq i \leq \frac{n-1}{4}$.

In view of the above labeling pattern, we get $e_f(0) = \frac{n+3}{2}$ and $e_f(1) = \frac{n+1}{2}$. Hence, $|e_f(0) - e_f(1)| = 1$.

Case 3. When $n \equiv 2 \pmod 4$.

Subcase 3.1. When $n = 6$.

$f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 4$, $f(v_4) = 6$, $f(v_5) = 5$, $f(v_6) = 7$, $f(w) = 2$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = 4$. Hence, $|e_f(0) - e_f(1)| = 0$.

Subcase 3.2. When $n \geq 10$.

$f(v_1) = 1$, $f(v_2) = 3$, $f(w) = 2$,

$f(v_i) = i + 1$ for $3 \leq i \leq \frac{n+2}{2}$,

$f(v_{\frac{n+2}{2}+i}) = \frac{n+4}{2} + 2i$ for $1 \leq i \leq \frac{n-2}{4}$,

$f(v_{\frac{3n+2}{4}+i}) = \frac{n+4}{2} + 2i - 1$ for $1 \leq i \leq \frac{n-2}{4}$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = \frac{n+2}{2}$. Hence, $|e_f(0) - e_f(1)| = 0$.

Case 4. When $n \equiv 3 \pmod 4$.

Subcase 4.1. When $n = 3$.

$f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 4$, $f(w) = 2$.

In view of the above labeling pattern, we get $e_f(0) = 4$ and $e_f(1) = 5$. Hence, $|e_f(0) - e_f(1)| = 1$.

Subcase 4.2. When $n = 7$.

$f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 4$, $f(v_4) = 5$, $f(v_5) = 7$, $f(v_6) = 6$, $f(v_7) = 8$, $f(w) = 2$.

In view of the above labeling pattern, we get $e_f(0) = 2$ and $e_f(1) = 3$. Hence, $|e_f(0) - e_f(1)|=1$.

Subcase 4.3. When $n \geq 11$.

$f(v_1) = 1$, $f(v_2) = 3$, $f(w) = 2$,

$f(v_i) = i + 1$ for $3 \leq i \leq \frac{n+1}{2}$,

$f(v_{\frac{n+1}{2}+i}) = \frac{n+3}{2} + 2i$ for $1 \leq i \leq \frac{n-3}{4}$,

$f(v_{\frac{3n-1}{4}+i}) = \frac{n+3}{2} + 2i - 1$ for $1 \leq i \leq \frac{n+1}{4}$.

In view of the above labeling pattern, we get $e_f(0) = \frac{n+3}{2}$. and $e_f(1) = \frac{n+1}{2}$.. Hence, $|e_f(0) - e_f(1)| = 1$.

Therefore, in all the above cases, we have $|e_f(0) - e_f(1)| \leq 1$. Hence G is a difference cordial graph.

Example 2. The difference cordial labeling of duplication of an edge by a vertex in a cycle $C_{16}$is shown in Figure 2.

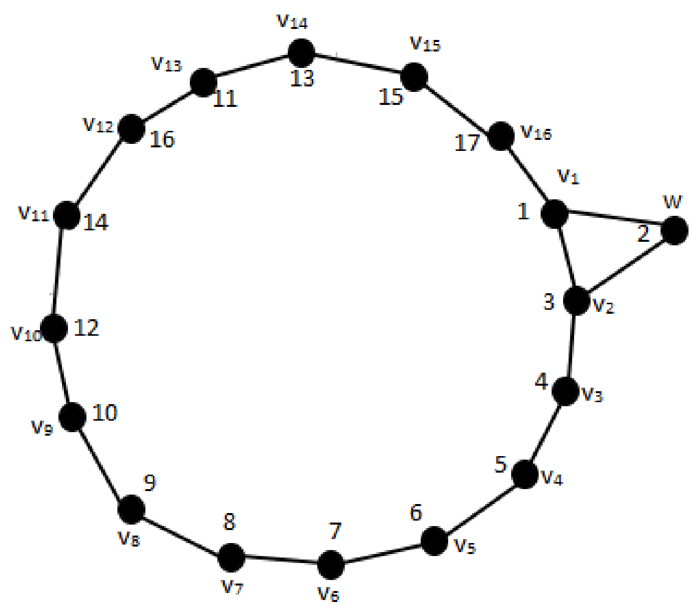

Figure 2. The difference cordial labeling of duplication of an edge by a vertex in a cycle C_{16}

Theorem 3.3. The graph obtained by duplication of an arbitrary vertex by an edge in a cycle admits difference cordial labeling.

Proof. Consider a cycle C_n. Let $v_1, v_2, v_3, \dots, v_n$ be the consecutive vertices of C_n and let G be the graph obtained by duplication of an arbitrary vertex by a new edge. Without loss of generality we assume that the arbitrary vertex is the vertex v_1 and the new edge is the $e = v_1' v_1''$. Note that $|V(G)| = n + 2$ and $|E(G)| = n + 3$. We define a vertex labeling function $f : V(G) \to \{1, 2, 3, \dots, (n + 2)\}$ as follows:

We consider the following cases.

Case 1. When $n \equiv 0 \pmod 4$.

Subcase 1.1. When n = 4.

$f(v_1) = 1$, $f(v_2) = 2$, $f(v_3) = 3$, $f(v_4) = 4$, $f(v_1') = 5$, $f(v_1'') = 6$.

In view of the above labeling pattern, we get $e_f(0) = 3$ and $e_f(1) = 4$. Hence, $|e_f(0) - e_f(1)| = 1$.

Subcase 1.2. When n = 8.

$f(v_i) = i$ for $1 \leq i \leq 5$

$f(v_6) = 7$, $f(v_7) = 6$, $f(v_8) = 8$, $f(v_1') = 9$, $f(v_1'') = 10$.

In view of the above labeling pattern, we get $e_f(0) = 5$ and $e_f(1) = 6$. Hence, $|e_f(0) - e_f(1)| = 1$.

Subcase 1.3. When $n \geq 12$.

$f(v_1') = n + 1$, $f(v_1'') = n + 2$,

$f(v_i) = i$ for $1 \leq i \leq \frac{n+2}{2}$,

$f(v_{\frac{n+2}{2}+i}) = \frac{n+2}{2} + 2i$ for $1 \leq i \leq \frac{n-4}{4}$,

$f(v_{\frac{3n}{4}+i}) = \frac{n+2}{2} + 2i - 1$ for $1 \leq i \leq \frac{n}{4}$.

In view of the above labeling pattern, we get $e_f(0) = \frac{n+4}{2}$ and $e_f(1) = \frac{n+2}{2}$. Hence, $|e_f(0) - e_f(1)| = 1$.

Case 2. When $n \equiv 1 \pmod 4$.

Subcase 2.1. When n = 5.

$f(v_1) = 1$, $f(v_2) = 2$, $f(v_3) = 3$, $f(v_4) = 5$, $f(v_5) = 4$, $f(v_1') = 6$, $f(v_1'') = 7$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = 4$. Hence, $|e_f(0) - e_f(1)| = 0$.

Subcase 2.2. When n = 9.

$f(v_1) = 1$, $f(v_2) = 2$, $f(v_3) = 3$, $f(v_4) = 4$, $f(v_5) = 5$, $f(v_6) = 8$, $f(v_7) = 6$, $f(v_8) = 7$, $f(v_9) = 9$, $f(v_1') = 10$, $f(v_1'') = 11$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = 6$. Hence, $|e_f(0) - e_f(1)| = 0$.

Subcase 2.3. When $n \geq 13$.

$f(v_1') = n + 1$, $f(v_1'') = n + 2$,

$f(v_i) = i$ for $1 \leq i \leq \frac{n+3}{2}$,

$f(v_{\frac{n+3}{2}+i}) = \frac{n+3}{2} + 2i$ for $1 \leq i \leq \frac{n-5}{4}$,

$f(v_{\frac{3n+1}{4}+i}) = \frac{n+3}{2} + 2i - 1$ for $1 \le i \le \frac{n-1}{4}$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = \frac{n+3}{2}$. Hence, $|e_f(0) - e_f(1)| = 0$.

Case 3. When $n \equiv 2 \pmod 4$.

Subcase 3.1. When n = 6.

$f(v_1) = 1$, $f(v_2) = 2$, $f(v_3) = 3$, $f(v_4) = 5$, $f(v_5) = 4$, $f(v_6) = 6$, $f(v_1') = 7$, $f(v_1'') = 8$.

In view of the above labeling pattern, we get $e_f(0) = 5$ and $e_f(1) = 4$. Hence, $|e_f(0) - e_f(1)| = 1$.

Subcase 3.2. When $n \geq 10$.

$f(v_1') = n + 1$, $f(v_1'') = n + 2$,

$f(v_i) = i$ for $1 \le i \le \frac{n+2}{2}$,

$f(v_{\frac{n+2}{2}+i}) = \frac{n+2}{2} + 2i$ for $1 \le i \le \frac{n-2}{4}$,

$f(v_{\frac{3n+2}{4}+i}) = \frac{n+2}{2} + 2i - 1$ for $1 \le i \le \frac{n-2}{4}$.

In view of the above labeling pattern, we get $e_f(0) = \frac{n+4}{2}$ and $e_f(1) = \frac{n+2}{2}$. Hence, $|e_f(0) - e_f(1)| = 1$.

Case 4. When $n \equiv 3 \pmod 4$.

Subcase 4.1. When n = 3.

$f(v_1) = 1$, $f(v_2) = 2$, $f(v_3) = 3$, $f(v_1') = 4$, $f(v_1'') = 5$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = 3$. Hence, $|e_f(0) - e_f(1)| = 0$.

Subcase 4.2. When n = 7.

$f(v_1) = 1$, $f(v_2) = 2$, $f(v_3) = 3$, $f(v_4) = 4$, $f(v_5) = 6$, $f(v_6) = 5$, $f(v_7) = 7$, $f(v_1') = 8$, $f(v_1'') = 9$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = 5$. Hence, $|e_f(0) - e_f(1)| = 0$.

Subcase 4.3. When $n \geq 11$.

$f(v_1') = n + 1$, $f(v_1'') = n + 2$,

$f(v_i) = i$ for $1 \le i \le \frac{n+3}{2}$,

$f(v_{\frac{n+3}{2}+i}) = \frac{n+3}{2} + 2i$ for $1 \le i \le \frac{n-3}{4}$,

$f(v_{\frac{3n+3}{4}+i}) = \frac{n+3}{2} + 2i - 1$ for $1 \le i \le \frac{n-3}{4}$.

In view of the above labeling pattern, we get $e_f(0) = e_f(1) = \frac{n+3}{2}$. Hence, $|e_f(0) - e_f(1)| = 0$.

In all the above cases, we have $|e_f(0) - e_f(1)| \leq 1$. Hence G is a difference cordial graph.

Example 3. The difference cordial labeling of duplication of vertex by an edge in a cycle C_{11} is shown in Figure 3.

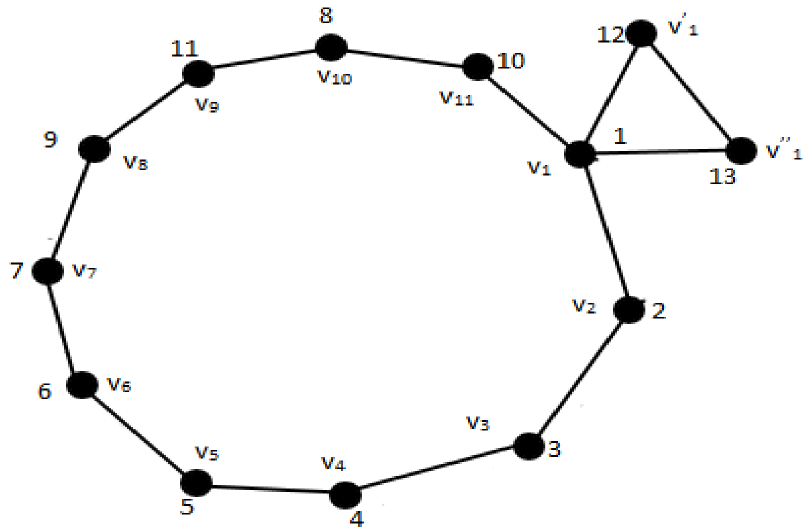

Figure 3. The difference cordial labeling of duplication of vertex by an edge in a cycle C_{11}

Theorem 3.4. The total graph of path P_n is a difference cordial graph.

Proof. Let G be the total graph of path P_n. Let us denote $u_1, u_2, u_3, \ldots, u_n$ be the vertices of path P_n and let $v_1, v_2, v_3, \ldots, v_{n-1}$ be the vertices corresponding to the edges $v_1v_2, v_2v_3, v_3v_4, \ldots, v_{n-1}v_n$ respectively. Note that $|V(G)| = 2n-1$ and $|E(G)| = 4n-5$. We define a vertex labeling function $f : V(G) \rightarrow \{1, 2, 3, \ldots, (2n-1)\}$ as follows:

$f(u_i) = i$ for $1 \leq i \leq n$,

$f(v_i) = n + i$ for $1 \leq i \leq n-1$.

In view of the above labeling pattern, we have $e_f(0) = 2n-2$ and $e_f(1) = 2n-3$. Hence, $|e_f(0) - e_f(1)| = 1$. Therefore G is a difference cordial graph.

Example 4. The difference cordial labeling of the total graph $T(P_5)$ is shown in figure 4.

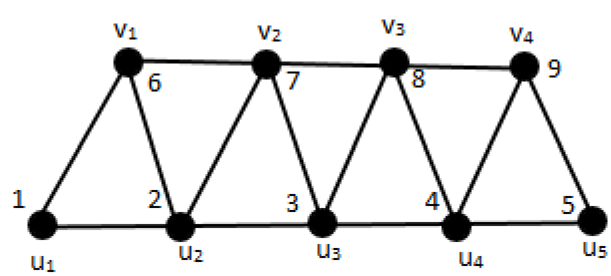

Figure 4. The difference cordial labeling of the total graph $T(P_5)$

Conclusion.

Here we investigated the difference cordial labeling concept to the graphs such as umbrella graph U(m, n), duplication of an arbitrary vertex by a new edge in a cycle C_n, duplication of an arbitrary edge by a new vertex in a cycle C_n and the total graph $T(P_n)$ of path P_n.

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