

Continued fractions and real numbers

A.Balan

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Abstract

We define the real numbers by mean of the continued fractions.

1 The real numbers

It is usual to define the real numbers by a set of axioms among others the axiom of Archimède. The real numbers are moreover defined by the completion of the rational numbers or the decomposition over a basis n or lastly by the cuts of Dedekind.

2 The continued fractions

We propose here a fourth definition of the real numbers by mean of the continued fractions. The continued fractions are defined by example by Khinchin.

3 The definition

We consider the sequences of integers and we define the addition and multiplication by truncation of the sequences and the associated rational numbers.

$$(x + y)_n = x_n + y_n$$

$$(x.y)_n = x_n.y_n$$

As we can associate a continued fraction to a real number, we can prove that the set of real numbers is well-defined by the sequences of integers by mean of the continued fractions.

4 The bibliography

A.Ya.Khinchin,"Continued Fractions", Dover, Chicago, 1997.