Preprint

1/f Phase Noise in Oscillators Modeled with Q and Its Entropy Significance

A. Feinberg

DfRSoft Research, dfrsoft@gmail.com

Abstract

Noise in operating systems has been strongly linked to degradation [1,2,3]. One such type of noise of interest in this paper is phase noise, which we model and describe its significance in thermodynamic degradation science [3]. Phase noise of an oscillator is perhaps one of the most important oscillator parameters and the source of the noise is not well understood. Phase noise is important as it degrades the purity of the carrier frequency when used in transmission which is only one of the many applications in oscillator usage. It is known that the unloaded Q in phase noise goes as the inverse of Q to the forth power observed [4-5] in the low frequency area (i.e. near the carrier frequency) as noted in oscillator power noise spectral density. In this paper we provide a model that leads to this observed unloaded Q dependence noted in the power spectral density. We will then provide specific comparison to an LRC oscillator circuit to establish a parametric useful analogy. A second model links Q to entropy which we show produces this type of noise. Although this noise's origin, is not well understood in terms of reliability, we have previously found [1-3] that noise is typically attributed to entropy (disorder). This is because temporal coherence of a signal from an operating device can be correlated to disorder in the spatial coherence in the device [3]. Once understood, the phase noise has an entropy explanation that yields the inverse frequency dependence observed. Therefore, this type of noise measurement is of importance in thermodynamic degradation process as it has applications to understanding noise in other areas besides oscillators. Results show consistency with damage entropy principles in terms of purity of materials and measurement methods observed in the literature. Because entropy is an expression of the disorder, or randomness of a system, we anticipate that such results can be applied in assessing stability issues in many fabricated electronic devices.

1.0 Introduction

Phase noise is actually frequency instability. In particular, we are concerned with short term stability of the carrier frequency mainly at low frequency related to what is often termed the flicker noise region in the frequency domain. This is illustrate in Figure 1 where phase variations $\theta(t)$ and the amplitude variation $A(t)$ also occurring are shown.

In this paper we using a standard unloaded oscillator equation we will demonstrate the Q dependence goes as $1/Q^4$ [4,5] in the spectral power density (frequency domain associated with the random vibration occurring in the time domain). We will further define the phase noise frequency region. Then we will compare the standard oscillator equation with an LRC resonance circuit to provide design guidelines for phase noise related to Q and the anticipated phase noise region. Lastly we will provide an entropy physics description of phase noise to better understand its origin.

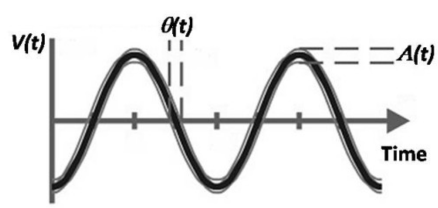

Figure 1 Illustrating instability of oscillator frequency and amplitude in the time domain represented random phase variations $\theta(t)$ and amplitude variation $A(t)$

2.0 Standard Harmonic Oscillator and Q dependence of Phase Noise

Since we are concerned with the unloaded Q of a harmonica oscillators frequency stability. This means that the oscillator is not driven by an external frequency or is driving any load. Therefore, the RHS is set equal to zero for the standard oscillator equation

$$\ddot{X} + \Gamma_0 \dot{X} + \omega_0^2 X = 0 \tag{1}$$

The damping term Γ can be written in terms of Q which we are interested in for frequency stability

$$\Gamma_0 = \frac{\omega_0}{Q} \tag{2}$$

The differential equation solution to the standard harmonica oscillator Eq. 1 is

$$X = X_0 e^{-i\zeta t} \tag{3}$$

Here ζ is the complex frequency of the unloaded oscillator that is shifted from the natual frequency ω_o due to the damping. That is, if we insert the assumed solution into the differential equation we have

$$\zeta^2 + i\zeta \frac{\omega_0}{Q} - \omega_0^2 = 0 \tag{4}$$

and the solution for ζ is perturbed from the natural frequency given by

$$\zeta_{+/-} = i\frac{\omega_0}{2Q} \pm \omega_0 \sqrt{1 - \frac{1}{4Q^2}} = i\frac{\omega_0}{2Q} \pm \omega \tag{5}$$

Where we have let real part of the frequency set to

$$\omega = \omega_0 \sqrt{1 - \frac{1}{4Q^2}} \approx \omega_0 (1 - \frac{1}{8Q^2}) \tag{6}$$

Since Q is assumed large we have expanded the square root terms above. Then the shift from the natural frequency is

$$\omega - \omega_0 = \Delta\omega = \frac{\omega_0}{8Q^2} \quad \text{or} \quad 8\frac{\Delta\omega}{\omega_0} = \frac{1}{Q^2} \tag{7}$$

This shift is assumed here to be the maximum variation of the oscillator's carrier frequency. Ideally this shift would be stable. One way to capture this variation is by using phase in the frequency argument of the carrier frequency. For example, an oscillator with carrier frequency ω, the voltage output is

$$V(t) = (V_o + A(t))\sin(\omega_o t + \theta(t)) \tag{8}$$

This is shown in Figure 1. Here V_o is the nominal voltage amplitude, $A(t)$ is random amplitude changes, $\theta(t)$ is the random phase changes or the oscillator's instability often termed jitter in the time domain. Then $V(t)$ is related to phase and amplitude fluctuations.

$$\frac{d\theta(t)}{dt} = \omega - \omega_0(t) = \Delta\omega(t) = \frac{\omega_0(t)}{8Q^2} \tag{9}$$

Therefore the frequency instability is captured by the phase variation in time. For small instability in phase variation, the oscillator voltage amplitude (Eq. 8) is proportional to the phase variations [XYZ]. Letting $\theta(f)$ be the phase noise spectrum of $\theta(t)$, the RMS voltage is proportional to the phase as

$$\Delta V_{RMS}(f) = K_\theta \Delta\theta_{RMS}(f) \qquad \langle \Delta\omega(t)\Delta\omega(0) \rangle \sim \theta(f)^2 \sim \frac{1}{Q^4} \tag{10}$$

Hhere K_θ is the proportionality constant. The noise spectral density $S_\theta(f)$ for phase $\theta(t)$ is defined as

$$S_\theta(f) = \frac{\theta(f)^2{}_{RMS}}{\Gamma_0} \sim \frac{1}{Q^4} \tag{11}$$

This is one of the desired results. Here we have obtained the experimentally observed Q dependence on phase noise [xyz]. It is shown to be highly sensitive function of Q to the forth power. The larger the Q, the smaller is the phase noise by Q^4. This is close to the carrier frequency as oscillators tend to have high Q so the frequency region from the carrier is small and from Eq. 9 and 11 the region (shown in Fig. 2) is

$$0 < f_{Flicker} < \left(\frac{2\pi f_0}{8Q^2}\right) \tag{12}$$

Note we have not found other derivations with this identified region. This does not really explain why flicker noise exists. This explanation is discussed further in Section 4.

3.0 LRC Circuit Parameter Related to Phase Noise

What we would like to do is understand the phase noise in terms of a circuit model which can be helpful in terms of engineering.

$$-V_C = L\frac{di}{dt} + iR = 0 \tag{13}$$

Letting $dV_C/dt=1/C\ i(t)$ we obtain

$$\frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0 \tag{14}$$

By comparison with Equation 1 we have

$$\omega_0 = \frac{1}{\sqrt{LC}} \ \ and \ \ \Gamma_0 = \frac{R}{L} = \frac{\omega_0}{Q} \tag{15}$$

Then with substitution the Q is

$$Q = \omega_0 \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{16}$$

The flicker region provided by Eq. 12 can now be assessed in terms of the electrical circuit equivalence. This helps to establish design guides for oscillator so that phase noise can be estimated based on the design and anticipated Q.

4.0 Entropy Model for Phase Noise

We have described flicker noise in reference [6]. The key model in that reference 4 indicated the noise spectral density is given in terms of entropy in the material of the device, in this case, a resonator, and the associated disorder occurring to the current given by

$$S_{Material-current}(f) = \frac{1}{f}\chi\,\Delta s_{Material-current} = \frac{1}{f}\chi\{\Delta s_{Material}(W, s_{Material}) + \Delta s_{current}(s_{Material})\} \tag{17}$$

Here capital S is the noise spectral density in the frequency domain associated with the actual random vibration occurring in time domain. Here χ is a constant, small s is the entropy, W is the work to create phase noise in the material. Note the equation is written in terms of entropy variation which is easier to measure than entropy itself. If we turn off the current in Eq. 17, we still have flicker noise and this has been observed as well

$$S_{Material}(f) = \frac{1}{f}\chi\,\Delta s_{Material-current} = \frac{1}{f}\chi\{\Delta s_{Material}(W, s_{Material})\} \tag{18}$$

From Eq. 11, the entropy change of the disorder in the oscillator is quantified by its Q and the flicker noise without current is also a function of Q and internal existing fabrication stresses. These stresses are in-turn dependent on the internal order which is related to material stability. That is, the change in the oscillator's entropy goes as the entropy in the material, which has the Q^4 dependence (Eq. 11) [6]

$$\frac{ds}{dt} = \lambda s(W)_Q = \frac{F(W)}{Q^4} \tag{19}$$

Here F(W) is a function of thermodynamic work that is create entropy change such as built in fabrication stresses. Material stability changes are minute but without current flow, flicker noise still has been observed. From Equations 18 and 19 then noise due to the material is

$$S_{Oscillator}(f) = \frac{\chi F(W)}{f\,Q^4} \tag{20}$$

When current flows in the oscillator, the current itself interacts with the internal frictional resistance both creating more entropy in the material, but also amplifying the noise in the material (Eq. 20). Figure 2 summarizes some of the concepts for the noise spectral density frequency dependence. The dotted line represents an oscillator that has degraded over time with lower Q and thus increase entropy.

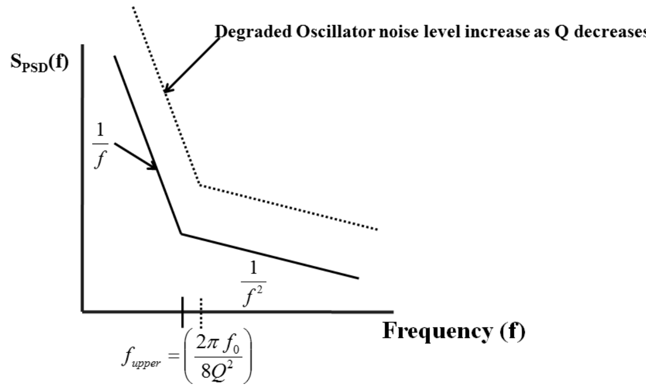

Figure 2 Frequency domain noise spectral density of the time domain jitter shown in Fig. 1 with the predicted upper flicker frequency. Dashed area is a degraded oscillator with lower Q

Conclusion

The observed dependence of oscillator phase noise has been derived. In the frequency domain we have identified it flicker noise region. LRC oscillator mode parameters have been included to aid in electrical design. We have combined prior work and modeled the frequency domain spectral power density for oscillator in the absence of an oscillator load using an entropy model. Results indicate that the phase noise is highly dependent on the structural integrity of the oscillator material which is quantified by its Q^4 dependence.

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