Pre-print

On the Origin of 1/f Noise due to Generated Entropy - Version 4

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Abstract

Noise measurements analysis has been associated with degradation. In particular one such type is called 1/f noise and is the most likely measure of subtle degradation occurring in materials. This measurement is done using random vibration methods. It is important to determine if this is the likely region of the spectrum related to degradation occurring in materials to aid in noise type of reliability tests. The literature on 1/f noise appears to have a broad commonality in explanations that can be associated with entropy in materials. It is reasonable in this regard to look at 1/f noise aspects in terms of disorder and the associated spectral content. This lends itself to a thermodynamic entropy frame work for analysis. We review some of the key aspects of 1/f noise in the literature and discuss how observations relate to entropy. Once describe, we suggest two thermodynamics models that might be used to model 1/f noise. Results help to provide a broader understanding of 1/f noise, help to identify the region of the spectrum related to degradation, and use it to do prognostics. Such interpretation suggests that 1/f noise is a good tool for measuring certain aspects of disorder in materials. Experiments are suggested to demonstrate the importance of 1/f noise as a prognostic tool for reliability testing to identify and predict degradation in materials over time.

1.0 Introduction – Resistor flicker Noise

In this article we slowly build a case that *1/f* noise (also called flicker noise), is a sensitive measure of irreversibility. As well we suggest experiments to investigate the possibility of using flicker noise as a prognostic tool for making reliability predictions of degradation in materials. We will initially look at the literature and illustrate why it likely makes sense to explain flicker noise in terms of generated entropy. But first, consider a fundamental situation of a current flowing through a resistor to illustrate how noise in general appears to be a fundamental measure of entropy production. The typical argument of entropy change occurring due to current flow is

$$\Delta s = \Delta s_{resistor} + \Delta s_{Environment} \geq 0 \tag{1}$$

The entropy change to the environment is the heat dissipated

$$\Delta s_{Environemnt} = \frac{\Delta Q}{T} = \frac{i^2 R \Delta t}{T} \tag{2}$$

The entropy change to the resistor is often considered to be negligible since the average current is considered constant ($\Delta i=0$) as is the average temperature. Therefore, no net change to ΔQ and $\Delta s_{resistor} = 0$

However, in noise measurements, a much more sensitive measure, we observe voltage fluctuations across a resistor. Therefore, current is not constant, a temperature gradient exist in order to dissipate heat and current fluctuation must generate complex entropy at the microscopic level. Clearly, this is not a reversible process. We have the possibility that the resistor entropy could increase $\Delta s_R \neq 0$, and the current itself becomes disorganized ($\Delta s_{Current} \neq 0$) this observed when sensitive 1/f noise measurements are made. We suggest a possible model for the complex entropy generated

$$\Delta s = \Delta s_{resistor}(W, s_{resistor}) + \Delta s_{current}(s_{resistor}) + \Delta s_{Environment}(i^2 R) \geq 0 \tag{3}$$

The $\Delta s_{Environment}$ in general represent entropy flow of heat to the environment and is not of immediate interest. Here we will focus on the resistor and current generate entropy change. Often in thermodynamics entropy flow (heat for example) is distinguished from generated entropy which causes damage or disorder to the material and in this case to the current flow.

Many of the features of flicker noise in resistors are illustrated by the phenomenological equation due to Hooge [1]

$$S(f) = \gamma \frac{V_{DC}^2}{f} \tag{4}$$

Here capital S, is the noise spectral density, γ is the Hogg constant, and V_{DC} is the applied voltage. We see that noise power $S(f) \sim <V^2> = <(IR)^2>$, where I is the driving current and R is the sample resistance. In terms of an entropy interpretation, R is a direct measure of internal friction, which when interacting with current flow, generates entropy.

The concept that 1/f noise related to internal friction is not new as it has been described in metals by Kogan and Nagaev (1982) [2], Their argument was that 1/f noise low frequency fluctuations could occur in mechanical strain and then electrical resistance would depends on the strain displacements. Their detailed model is a type of mechanical approach. Here we are providing an interpretation from an energy approach.

1.1 Current Fluctuation Entropy

If we consider the random current fluctuations observable in sensitive *1/f* noise measurements, we can consider this as a type of disorganization occurring in the current; therefore quantifiable by generated entropy current change. Above, we have assumed this disorganization is a function of the entropy state of the resistor. This is theoretically supported by the fact that if current instead were to flow through a material with zero resistance; the process would be reversible with no generated entropy.

1.2 Resistance Entropy Change

Theoretically, any thermodynamic process creates entropy due to work. Thermodynamic stress in the material creates strain and this work is denoted as *W in Eq. 3,* likely created by current interactions in the material resulting in current fluctuations, other neighboring thermodynamic process in the material may also occur. For example, the resistor may not be in complete thermodynamic equilibrium, even in the absence of current flow! This is likely due to manufacturing stresses built into any fabricated material. The best likely way to observe Δs_R according to Eq. 3, is to turn off the current in a 1/f noise measurement. Note that in the argument this term also depends on the state of a system's entropy. This argument comes about since system entropy itself can create stress due to lack of internal structural internal integrity. This might suggest that the entropy change in the material (the first term $\Delta s(W,s_R)$ on the RHS of Eq. 3) goes as the entropy in the material. In the absence of current in Eq. 3

$$\frac{ds}{dt} = \lambda s(W)_R \tag{5}$$

Entropy changes are easier to measure than entropy. Clarke and Voss [12] found that 1/f noise was present if there was no driving current at equilibrium but they could not guarantee true thermal equilibrium. In this view, non thermal equilibrium would likely be due to fabrication internal stresses. Recall that the thermodynamic work *W in Eq. 3* is defined to be due to the measurement current or any other neighboring thermodynamic work process.

At this point, we will need to look further into modeling to determine how an entropy approach leads to the 1/f noise dependence.

1.3 Wire Wound vs. Carbon Resistor Entropy Comparison

From our above discussion, we assumed current noise entropy is a function of the entropy state of the resistor depending on the resistance (Eq. 4). It also depends on the resistor type. For example, 1/f noise observations indicate that wire wound (*w*) resistors have less noise than carbon (*c*) resistors [3]. In terms of entropy, a comparison of entropy created in the current *i* has

$$\Delta s_{i-c}(s_c) - \Delta s_{i-w}(s_w) \geq 0 \tag{6}$$

so that $s_c > s_w$. Furthermore, a comparison of any damage entropy contribution in the two materials indicates

$$\Delta s_{R-c}(W_i) - \Delta s_{R-w}(W_i) \geq 0 \tag{7}$$

so damage entropy in a carbon resistor being higher than a wire wound resistor, will occur for the same amount of work, or alternately, wire would resistors are likely manufactured with higher stability.

1.4 Oscillator Phase Noise Entropy

The phase noise of an oscillator is perhaps one of the most important parameters. Here 1/f noise is known to dominate. Phase noise affects the purity of the carrier frequency in transmission. It is known that the unloaded Q in flicker noise goes as the inverse of Q to the forth power observed [4,5,6]. Here again, damping (an inverse function of Q), is characteristic of another form of internal friction and strongly associated with entropy generation. A higher Q also indicates a more stable material and less susceptible to generated entropy damage.

1.5 MOS Observation and Entropy

Flicker models vary widely for MOSFET devices. One basic theory results due to fluctuations in bulk mobility based on Hooge's [1] empirical relation is

$$S(f) = \gamma \frac{I^2}{N\ f} \tag{8}$$

Capital S is the noise spectral density, γ is the Hooge constant, I is the current, N is the number of charge carriers, A simply entropy interpretation indicates that charge carriers reduce internal friction or alternately resistance in the bulk varies inversely with N. The current flow I increases the amplitude of the flicker noise is perturbed in the channel [7].

1.6 Effect of Temperature

We note the spectral density in the Hooge's Equation 8 is independent of temperature. This reinforces the fact of generated entropy damage compared with entropy flow (heat added) is the issue in 1/f noise. However, 1/f noise shows some atypical temperature dependent characteristic (Eberhard and Horn, 1978 [8]) where they noted an adhoc function of γ(T). We would view this due to damage created by heat as part of the thermodynamic entropy damage process.

1.7 Noise Measurement as a Tool to Observe Entropy Damage

Noise in operating systems has been linked to degradation [9,10]. An interesting example is congestive heart patients compared with healthy hearts had a distinctly different noise spectrum [11]. We see that no process is truly reversible, a common thermodynamic argument. If a system process is in thermal equilibrium, then the process is reversible, but in thermal equilibrium there is no measurement process!

2.0 Overview of Entropy Models

Here we suggest two possible models to illustrate how an entropy approach can be modeled resulting in a 1/f noise dependence. The first is a time domain model that is transformed to the frequency domain, leading to the *1/f* dependence. The second is based on Schottky's original model [13] that is compared with Eq. 5.

2.1 Time Domain Entropy Model

The definitions of entropy, s, for discrete and continuous variable X are:

Discrete *X, p(x):*

$$s(X) = -\sum p(x) \log_2 P(x) \tag{9}$$

and Continuous *X, f(x), Differential Entropy [14,15]:*

$$s(X) = -\int f(x) \log\ (f(x))\ dx \tag{10}$$

Here we are concerned with the continuous variations in time t distributed by *f(t).* Consider a Gaussian spectral density due to a process that generates entropy current fluctuations with distribution

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(t-\mu)^2}{2\sigma^2} \right) \tag{11}$$

Gaussian spectra density for 1/f noise time domain processes is often described as logical in the literature [such as 16]. Milotti [17] summarized the question noting:

> Voss [18] produced experimental plots of the quantity <*V*(*t*)|*V*0>/ *V*0 in several conductors and was able to show that the noise processes observed were reasonably Gaussian. Further, it was noted that the superposition of many non Gaussian microscopic processes can results as Gaussian at the macroscopic level (demonstrated via the central limit theorem). J. B. Johnson in his 1925 experiment [19] in vacuum tubes asserted that the spectral density characterizes a noise process completely only if the process is stationary, ergodic and Gaussian: does the observed *1/f* noise satisfy all these constraints.

When Eq. 11 is inserted into the differential entropy Eq. 10, the result for a temporal process is [15]

$$s(t) = \frac{1}{2} \log(2\pi e \sigma(t)^2) \tag{12}$$

Note that the entropy does not depend on the mean only on the variance, a more sensitive measure. Solving in terms of the variance and expanding terms by assuming small change to the entropy and looking at the temporal part

$$\sigma(t)^2 = \frac{1}{2\pi e} Exp\{4.606\, s(t)\} \approx \frac{1}{2\pi e}(1 + 4.6 s(t) \approx C + 4.6\{s(0) + s'(0)t + ...\} \tag{13}$$

A common noise measure related to the variance in the time domain is the Allan Variance

$$\text{Allan Variance} : \sigma^2(\tau) = \frac{1}{2(n-1)} \sum_i (\bar{y}(\tau)_{i+1} - \bar{y}(\tau)_i)^2 \tag{14}$$

The Allan variance transforms to the frequency domain are well established and the frequency domain spectral density of Equation 13 is [20]

$$S(f)=\frac{2.3}{2\pi\ln 2}\frac{1}{f}s(0)=\frac{1}{f}\frac{2.3}{2\pi\ln 2}\frac{1}{k}\frac{ds(0)}{dt} \tag{15}$$

Here the noise shows a *1/f* dependence is a function of the entropy and the RHS we have included the concept of Eq. 5 where entropy changes are easier measured than entropy itself which brings in the constant *k*. Note the second term in the Taylor expansion in Eq. 13 would lead to Brownian motion (see discussion below).

A second alternate approach perhaps more insightful is to look at the rate of change of the entropy in Equation 12

$$\frac{ds(t)}{dt}=\frac{1}{2}\frac{1}{\sigma(t)}\frac{\sigma(t)}{dt} \tag{16}$$

We write this as

$$\sigma^2(t)=\frac{1}{4}\left(\frac{\sigma(t)}{dt}\Big/\frac{ds(t)}{dt}\right)^2=b(s)t^u \tag{17}$$

Here we assume some possible time dependence power with entropy dependent constant *b(s).* In this model we assume the variance rate of change likely goes as the entropy rate in Eq. 17, so

$$u=0 \tag{18}$$

When this is the case, we get a *stationary process* and the equivalent frequency domain spectrum *S* is transformed for $\sigma^2(t)\propto t^0$ to [20] to the frequency domain, so Eq. 17 becomes

$$S(f)=\frac{b(s)}{2\ln 2}\frac{1}{f} \tag{19}$$

This illustrates the *1/f* dependence. Note if we had some temporal dependence (non stochastic process) in the ratio $\sigma^2(t)\propto t$ in Eq. 17 where

$$u=1 \tag{20}$$

we would get Brownian noise as it transform to [17,20]

$$S(f)\propto 1/f^2 \tag{21}$$

We note that the temporal model, *u=0*, indicates that the variance and entropy rates change together. Therefore, we anticipate 1/f noise provides more fundamental significance to generated entropy damage sensitivity then say brown noise (u=1).

2.2 Schottky-Entropy Flicker Model

Equation 5 is actually similar to Schottky's (1926) [13] original premise. In his model, contribution to the vacuum tube current where he assumed surface trapping sites that released electrons according to a simple exponential relaxation.

$$N(t)=N_o\exp(-\lambda t) \tag{22}$$

In comparison, the solution to Equation 5 has similar form in entropy terms with solution

$$s(W)=s_o Exp\{\lambda t\} \tag{22}$$

The results lead to the Schottky's [13,17] spectrum model which can be put in terms of entropy

$$S(\omega)=\frac{N_0^2 n}{(\lambda^2+f^2)}=\frac{s_0^2 n}{(\lambda^2+f^2)} \tag{24}$$

In the Schottky model n is an average pulse rate. Here it is the interactive stress such as the current. To be consistent with Eq. 15 we might let $N_o^2 \sim s_o$.

In terms of the Schottky model, Bernamont [16] pointed out that only a superposition of processes with a variety of relaxation rates λ would yield *1/f* noise for a reasonable range of frequencies. He showed that if λ is uniformly distributed between λ_1 and λ_2, and the amplitudes remain constant, the spectrum can be interpreted in the pink *1/f* noise region

$$S(\omega)=\frac{N_0^2 n\pi}{2\omega(\lambda_2-\lambda_1)},\qquad \lambda_1<<\omega<<\lambda_2 \tag{25}$$

and Brown noise for example

$$S(\omega)=\frac{N_0^2 n}{\omega^2}, \qquad \lambda_1 << \lambda_2 << \omega \tag{26}$$

3.0 Suggested Experiments to Illustrate the Usefulness of the Entropy Approach

Accelerated testing of materials and products is often done in industry. Since entropy increase with aging time, and we have illustrated how flicker noise is a likely a sensitive measure of entropy change, it is important to work with standardize test so degradation can be quantified through noise. Below are some suggested calibrated experiments.

3.1 Suggested Flicker Aging Experiments

Using Eq. 15, the entropy flicker model with the aid of Equation 3 can be written

$$S_{Material-stress}(f)=\frac{1}{f}\chi\Delta s_{Material-stress}=\frac{1}{f}\chi\{\Delta s_{Material}(W,s_{Material})+\Delta s_{stress}(s_{Material})\} \tag{27}$$

Here χ is a calibration constant (similar to the Hooge constant) discussed in Sec 3.2.

Now we suggest in this section some aging experiments below. In this case the noise level will be dependent on aging test time

$$S_{Material-stress}(f, aging\ test\ time)=\frac{1}{f}\chi\Delta s_{Material-stress}(aging\ test\ time) \tag{28}$$

Note in the absence of stress current, we can observe the material flicker noise entropy as indicated in Eq. 27. Flicker reliability noise measurements could be done at room temperature after periodic stress exposure. For example, in an aging oven so the measurement itself is stationary even though reliability aging is a non stationary process.

The spectral *1/f* characteristic of the material and its material-stress interaction spectrum likely provide a unique spectral characteristic and can be used to build a *1/f* library similar to how FTIR spectroscopy libraries are used to identify organic material.

3.1.1 Thin Film Resistors

Thin film resistors are known to age as a power law in aging time t over temperature (for example $\Delta s_R=kt^n$). Since it is known that thin film resistance increases with temperature over time, then flicker noise S-level and entropy will also increase. However, now the option is available to look at aging rates at lower temperatures to observe the flicker aging law and if needed transfer it to the time domain and compare it to gross measurements (i.e. higher temperatures and longer macroscopic gross measurements).

3.1.2 Biological Aging Experiment in living systems

The human heart is known to have different noise characteristics for Congestive Heart Failure (CHF) compared to healthy heart [11]. However, now the option in this view is available to study aging in normal healthy hearts using flicker noise measurement over a person's lifetime. Here we might suggest both long term tracking of a group of people and also looking at different aging groups. All measurements should be first done with a calibration standard as suggested below.

3.2 Calibration Standard Measurement Process

Prior to any *1/f* measurement, a calibration standard of material like a 1K, 10K, and 100K wire wound resistor of a specified reasonably stable material. Each measurement made at room temperature conditions at a given voltage (10 Volt). We see in the literature that it can be hard to compare measurements from different researcher when calibration is not reported. A calibration standard is suggested so measurements can be compared more easily between researchers. The noise level should be provided in a reasonable frequency region such as 1 Hz to 1000 Hz.

$$S_{Cal}(f)=\frac{1}{f}\chi\Delta s_{Cal}=\frac{1}{f}\chi\{\Delta s_{resistor}(W,s_{resistor})+\Delta s_{current}(s_{resistor})\} \tag{29}$$

From Eq. 27, χ is the calibration factor (similar to Hooge's constant) and Δs_{Cal} is the entropy of the calibration sample. To separate out χ from Δs would likely require two or three standards (the 1K, 10K, 100K) at a specified voltage. Here we assume χ is independent of stress. Note one might turn off the resistor current $\Delta s_{current}$ to further separate values for the RHS of Eq. 29.

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Appendix: Particle Analogy "PhoDons" and Measurement Uncertainty

We briefly can be a bit creative and assign a word "Pho*D*on" for a fundamental *D*amage particle created with energy change $\Delta\phi_i$. The "phodon" associated energy of creation would then be somewhat analogous to phonon energy, but unlike phonons with modes of vibration in say a crystal structure, phodon wave nature becomes associated with random current fluctuation in the measurement process.

Degradation can then be thought of as creating phodon damage particles created with energies that depend on the material properties and interaction with the neighboring environment so that according to the second law and Eq. 3

$$\delta W = T ds_{Damage} - d\phi$$

Looking at just the work in creating phodons we see the phodon creation causes a change to the free energy via the current work (consistent with Eq 3).

In Eq. 3, when the current flow is zero (i.e. $\Delta s_{current}=0$), it is apparent that phodons can be created. This is because we described *W* typically due to measurement current of any other thermodynamic process such as fabrication stresses.

Finally, we should be mindful of the difficulty of taking very low frequencies measurement (near 0), the observation time must be long enough to be certain of the frequency value $\Delta t \geq 1/\Delta\nu$ due to the uncertainty principle for measurement accuracy.