Weak Signal Detection in Geophysical Data using Vector-FFT

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Abstract: The gravimeter in CB measures at 50.65 µHz a hitherto unknown, amplitude-modulated and very weakly damped natural vibration of the earth, the cause of which is not recognizable. It could be a member of the famous "Slichter Triplet".

Introduction

A few years ago, I started to work out finer and finer ways of measuring to see how far the predictions of the theory are consistent with measurable signals in geophysical data series. In the end, a riddle remained: can one measure the oscillations of the Earth's core here on the surface? 58 years ago, their existence was predicted by Slichter[1], but so far no convincing proof has been obtained.

The theory gives a meager hint: the resonance frequency of the 1S1 oscillation should be somewhere near 60 µHz and – as the earth rotates – three frequencies should be measurable, the legendary Slichter triple. That's all.

It is not particularly difficult to identify spectral lines, ie special frequencies in data series, as long as the signal-to-noise ratio exceeds 20 dB. The usual method of measurement is based on the fast Fourier transformation (FFT), which is framed by further processing operations, which are not without problems, especially if the signal-to-noise ratio of the sought signal is so bad that there is doubt, if there is any signal is present or whether it is simple noise.

All processing steps were examined in detail[2], not with simulated data, but with real geophysical records. Here are the results:

- If the SNR is very low, "windowing" of the data series, for example with a Hamming function, should be avoided, as this will increase the already very high background noise.
- Stacking, ie the addition of time-shifted spectra, does not yield any gain when the magnitudes of the complex Fourier components are summed up. Stacking is only useful if the *complex* components are added directly. However, this requires an individual phase correction of each individual frequency to compensate for the consequences of the frame shift.
- Stacking the complex FFT results has two advantages: it provides the temporal evolution of amplitude and phase. Extremely weak spectral lines can be detected more easily by tracking the phase than by looking for a small amplitude change in the spectrum.
- Vector stacking reduces the half-width of true spectral lines but not of random noise maxima.

Data preparation

The published CORMIN data from selected SG stations were concatenated into long time series spanning several years. The main criterion was that the measured data had already attracted attention in previous examinations due to the particularly low level of noise. The following studies are based on the data recorded in Canberra / Australia in the years 1997 to 2010.

Two parameters were iteratively optimized: the influence of the air mass over the gravimeter

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(atmospheric admittance) and the optimal length for the median smoothing of the data series. The admittance α is always in the range 3.5 to 3.9; $\alpha_{CB} = 3.72$. The median length can be chosen without noticeable deterioration between five and seven minutes. 19 minutes are way too long because then many unwanted mixed products are generated.

In the subsequent data reduction, it must always be borne in mind that every process will generate additional noise. The first filter step is always a comb filter with sufficiently high notch frequency to attenuate the extremely strong tide signals. With the value of 1800 µHz you also achieve a pleasantly flat course of the noise level. However, the choice of such a high notch frequency produces very strong phase shifts in the range around 60 µHz. But that does not bother because we are looking for isolated frequencies.

The output of the comb filter can be used to optimize the admittance α. Before that you have to decide what exactly should be minimized. With regard to the planned search, it was defined: Take a record that is 500,000 minutes long and in which there are neither strong earthquakes nor noticeable data gaps. After FFT, the mean value of the amplitudes for all frequencies between 70 Hz and 77 µHz is formed because there are no noticeable spectral peaks in this area. This is done for some values of α to determine the minimum.

The next step is a sinc-highpass, which suppresses all frequencies below 45 µHz. Earlier research has shown that FIR or even IIR filters significantly increase the noise level. The dynamic range of the time series generated in this way is sufficiently low that the following filters do not have to meet any special requirements. Finally, before storing, the length of the records is shortened by factor 60 by decimation. The following figure shows a short excerpt from the spectrum. Remarkable are the surprisingly strong spectral lines near 68 µHz, which are nonlinear mixed products of tidal frequencies[B].

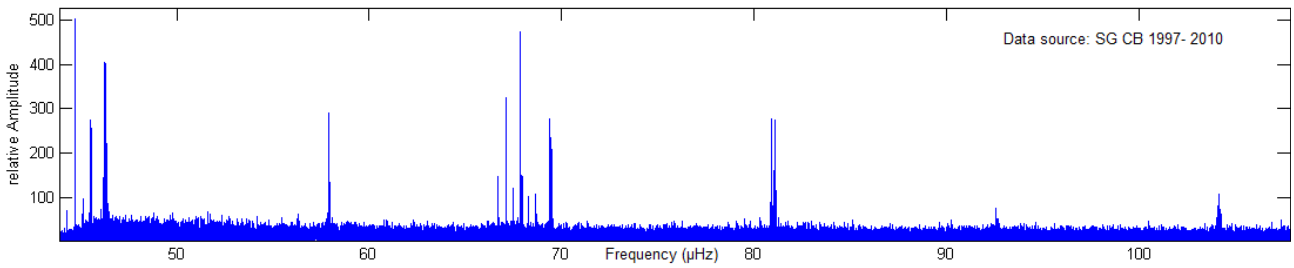


Fig 1: Low-noise spectrum from multi-year data measured by the CB gravimeter

How it started

The frequency range shown above was examined for structures that are unlikely to be random constellations and deserve more detailed analysis. At 50.65 µHz, a strange group of four closely spaced spectral lines is noticeable, all with a very moderate signal-to-noise ratio (SNR ≈ 6 dB). It should be noted that the spectrum in figure 2 was calculated with straight FFT, ie without "windowing". If the data is modulated with a Hamming window, the FFT gives a completely different result: instead of four peaks at 50.65 µHz, only two broad peaks of reduced amplitude are observed, which do not differ from the surrounding noise. There would have been no reason to study the environment of this frequency in more detail.

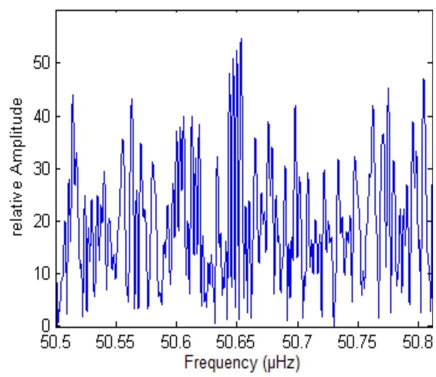


Fig 2: The detail enlargement of Figure 1 shows a structure that is probably no noise.

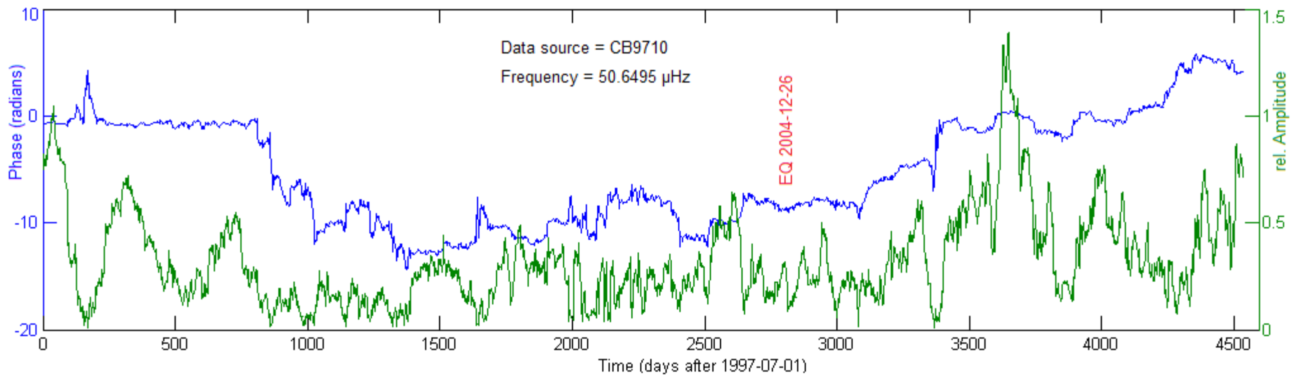
Windowing destroys fine structures!

Now it had to be clarified, if this strange quartet is real, reproducible and constantly measurable. Or

(B) Severine Rosat, ondes_non_lineaires, personal communication can it only be detected during short periods of time? When and how often? Is it initiated by events like earthquake? These questions can be resolved by a procedure such as Vector FFT[1].

Before the measurement, the frequency is reduced by 50 µHz (offset) to increase the relative frequency spacing. The procedure is common in the telecommunications industry and there are several proven methods. Since the examination frequency now has dropped to about 0.65 µHz, the file can be decimated by the factor 10, which corresponds to a sampling interval of 10 hours.

Finally, you decide which frequencies (from the total FFT spectrum) are of particular interest and should be analyzed. Then the vector FFT starts, calculates 2700 spectra in succession and stores the amplitude and phase that FFT delivers step by step. The next figure shows the amazing result for f = 0.6496 µHz (+offset!).



In the first 808 days after commissioning, the gravimeter in CB has measured something very special: Except for a short-term disturbance caused by too low an amplitude, the phase of the completely inconspicuous frequency 50.6496 µHz did not change its phase during an extended period. That's no coincidence, that's a signal. The strong earthquake in 2004 obviously did not stimulate this vibration.

Tracking the phase is obviously very well suited to find weak signals of constant frequency in noisy data streams. Which high-precision clock was triggered sometime before 1997-07-01 and stopped oscillating two years later? In the following years 2000 till 2010, the phase is quite chaotic, also the amplitude. The strong earthquake in December 2004 obviously has no stimulating effect on this oscillation. Did we really find a damped vibration in the first 1000 days, which is additionally amplitude modulated? To find out, the investigation is limited to this period.

Here are the parameters used by the Vector-FFT:

NFFT = 2^{17} (=131072);
frame-length = 300 data points (=3000 hours), extended by zero-padding;
advance of the frame = 4 data points (=40 hours);
total number of frames = 2721;

The method "Vector-FFT" is very robust, because even larger changes of some or all parameters (up to ± 50%) only marginally influence the results.

Properties of the long-term oscillation

To determine further characteristics of this enigmatic vibration, the file length is limited and the calculation aborted as soon as the phase of the mysterious oscillation becomes too restless. The detail spectrum (Figure 3) of this ominous oscillation now shows symmetric sidebands around the carrier frequency (50.6496 µHz), typical of an amplitude modulated oscillation. The distance of the sideband frequencies from the carrier frequency makes one sit up.

The carrier is modulated with (at least) two different frequencies:

$$f_{m1}=\tfrac{1}{2}\cdot(50.6845-50.6161)\,\mu Hz=33.226\,nHz \rightarrow T_1=338\,days$$

$$f_{m2}=\tfrac{1}{2}\cdot(50.7021-50.5993)\,\mu Hz=51.392\,nHz \rightarrow T_2=225\,days$$

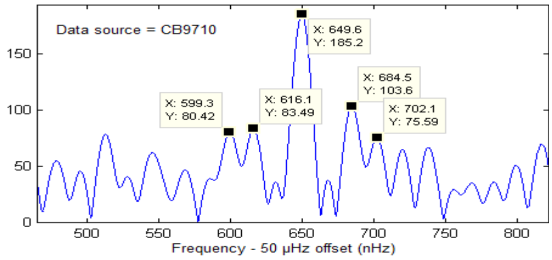


Fig 3: Spectrum of the data from the station CB; The recording period is 1997 - 1999

Remarkably, T_1 and T_2 are not independent, the relation $T_2=\tfrac{2}{3}\cdot T_1$ applies. Perhaps this connection can help to elucidate the generation mechanism of the 50.65 µHz oscillation.

The two oscillation periods T_1 and T_2 determine the rhythm of the total amplitude during the first 1000 days, as can be clearly seen in Fig 4. The initial amplitude is halved after about 650 days. Assuming that the mean amplitude decreases exponentially, we use the formula α=πf/Q and estimate the quality factor Q ≈ 13000.

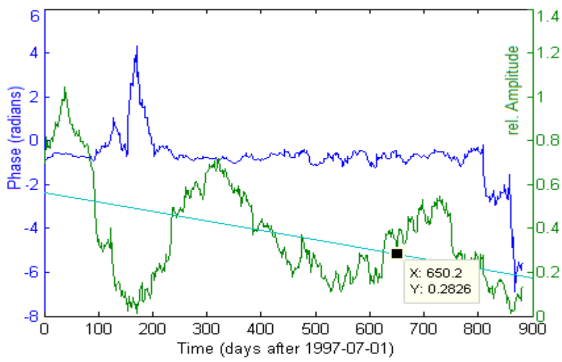
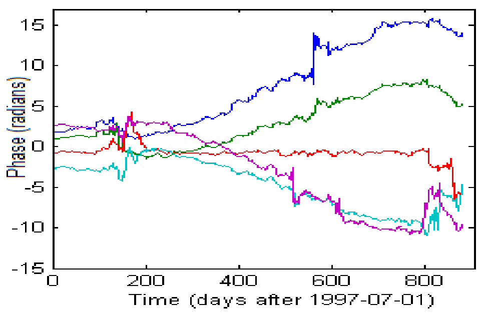


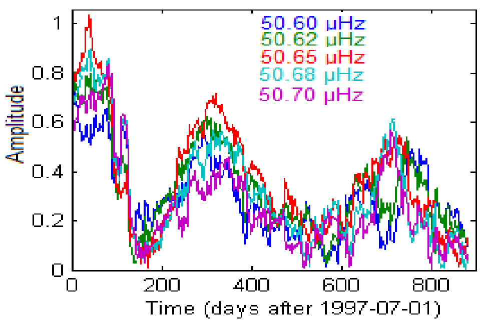
Fig 4: The mean amplitude loss is approximated by the turquoise line.

Further checks

Vector FFT makes it possible to prove that the five marked spectral lines in Figure 3 behave in the same way, ie have a common cause. The program "Vector FFT" is extended by a few lines of code so that the calculated amplitude and phase of each of the five frequencies is recorded step by step in separate memories. Then one can check by a simple comparison whether the spectral lines are related or not.



The time dependencies in both images are convincing proofs that neither the phases nor the amplitudes of all five tested frequencies are independent or even random. Given the very poor signal-to-noise ratio of less than 6 dB, it is amazing how exactly the phase curves can be determined. Tracking the phase is obviously a robust way to identify weak signals with SNR <10 dB.



The procedure Vector-FFT makes it possible to simultaneously calculate and store the temporal course of amplitude and phase of several frequencies. This makes it easier to check whether different frequencies behave similarly and therefore may have a common cause.

Summary

All results and especially the smooth phase during a remarkably long period of time allow only one conclusion: The gravimeter in CB measures at 50.65 µHz a hitherto unknown, very weakly damped natural vibration of the earth, whose cause is not recognizable.

[1] L. Slichter, The fundamental free mode of the Earth's inner core, Proc. Natl. Acad. Sci., 47, 186-19, (1961)
[2] H. Weidner, Frequency Measurements in Noisy Records using Vector-FFT, 2019