TOWARDS M-MATRIX

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Preface

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. Zero energy ontology and the interpretation of parton orbits as light-like "wormhole throats" suggests that virtual particle do not differ from on mass shell particles only in that the four- and three- momenta of wormhole throats fail to be parallel. The two throats of the wormhole contact defining virtual particle would contact carry on mass shell quantum numbers but for virtual particles the four-momenta need not be parallel and can also have opposite signs of energy.

The localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

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Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 32 years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss about my work. I have had also stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Pekka Rapinoja has offered his help in this respect and I am especially grateful for him for my Python skills. Also Matti Vallinkoski has helped me in computer related problems.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation to CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. In particualr, I am grateful for Mark McWilliams and Ulla Matfolk for providing links to possibly interesting web sites and articles. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the publicity through the iron wall of the academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as individual. Homepage and blog are however not enough since only the formally published

result is a result in recent day science. Publishing is however impossible without a direct support from power holders- even in archives like arXiv.org.

Situation changed for five years ago as Andrew Adamatsky proposed the writing of a book about TGD when I had already got used to the thought that my work would not be published during my life time. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loop holes. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy. And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his sixty year birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from the society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During last few years when the right wing has held the political power this trend has been steadily strengthening. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Matti Pitkänen

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Contents

IV MISCELLANEOUS TOPICS 719

List of Figures

Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. The CMAP files at my homepage provide an overview about ideas and evolution of TGD and make easier to understand what TGD and its applications are about (`http://www.tgdtheory.fi/cmaphtml.html` [L12]).

1.1.1 Basic vision very briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections ofCP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and

lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. Imbedding space H has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogenuity in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last thirty seven years for the realization of this dream and this has resulted in eight online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

1.1.2 Two manners to see TGD and their fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A91, A69, A80, A66].

The identification of the space-time as a sub-manifold [A59, A89] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

1.1.3 Basic objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 p-Adic variants of space-time surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. Also the hierarchy of Planck constants forces a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

1.1.5 The threads in the development of quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name 'TGD as a generalized number theory'. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K75, K56, K48, K93, K64, K92, K91, K62] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K68, K6, K52, K5, K30, K36, K38, K61, K86] are warmly recommended to the interested reader.

Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH ("world of classical worlds",WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A76, A95, A97]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B+C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory [1]. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian density matrix and unitary S-matrix.

   M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the complex square roots of density matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense: its own symmetries would define the symmetries of the theory. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian

[1] There are four kinds of Dirac operators in TGD. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of M^4 Dirac operator appearing in propagators

square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible.

4. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The modified gamma matrices define as anti-commutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.

5. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak

form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worldss", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

1. p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that

clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

The notion of p-adic manifold [K97] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheets as chart maps provides a possible solution of the basic challenge. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, "thought bubbles"). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. The quantum dynamics should be such that it allows quantum transitions transforming space-time sheets belonging to different number fields to each other. The space-time sheets in the intersection of real and p-adic worlds are of special interest and the hypothesis is that living matter resides in this intersection. This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book).

The basic principle is number theoretic universality stating roughly that the physics in various number fields can be obtained as completion of rational number based physics to various number fields. Rational number based physics would in turn describe physics in finite measurement resolution and cognitive resolution. The notion of finite measurement resolution has become one of the basic principles of quantum TGD and leads to the notions of braids as representatives of 3-surfaces and inclusions of hyper-finite factors as a representation for finite measurement resolution. The braids actually co-emerge with string world sheets implied by the condition that em charge is well-defined for spinor modes.

2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces X^4 be regarded as associative or co-associative ("quaternionic" is equivalent with "associative") surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of X^4 [K67]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.

2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K67]? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained

by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4. This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.

3. M^8-H duality can be generalized to a duality $H \to H$ if the images of the associative surface in M^8 is associative surface in H. One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.

4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.

5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K67] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4 \subset M^8$. One can also speak about M^8-H duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the "free" gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$.

2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic) sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.

3. For gamma matrix option induced rather than modified gamma matrices must be in question since modified gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially interesting is that p-adic and real regions of the space-time surface might aso emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

1.1.6 Hierarchy of Planck constants and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phases

D. Da Rocha and Laurent Nottale [E2] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K59] .

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Hierarchy of Planck constants from the anomalies of neuroscience and biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K53, K54, K84]) support the view that dark matter might be a key player in living matter.

Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple $\hbar = n\hbar_0$ of the ordinary Planck constant $\hbar_0$ is assigned with a multiple singular covering of the imbedding space [K21]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invariance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer n is highly suggestive notion and would imply that n sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book). n would naturally correspond the value of h_{eff} and its factors negentropic entanglement with unit density matrix would be between the n sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of n.

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of biochemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.2 Bird's eye of view about the topics of the book

This book is devoted to a detailed representation of what quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory. The most important guiding principle is quantum classical correspondence whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory. A further mathematical guideline is the mathematics associated with hyper-finite factors of type II_1 about which the spinors of the world of classical worlds represent a canonical example.

1. *Quantum classical correspondence*

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

1. Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

   The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales.

2. Also long ranged classical color and electro-weak fields are an unavoidable prediction It however took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.

3. The new view about energy and time finding a justification in the framework of zero energy ontology means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.

4. The general properties of Kähler action, in particular its vacuum degeneracy and the failure of the classical determinism in the conventional sense, have also strong implications. Space-time surface as a generalization of Bohr orbit provides not only a representation of quantum states but also of sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.

5. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights. CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

   Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and Z^0 fields are ideal for communication purposes and charged W MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy W MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

2. Physics as infinite-dimensional geometry in the "world of classical worlds"

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. Configuration space must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyper-finite factors of type II_1 having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

3. Physics as a generalized number theory

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I have christened hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space. Field equations would state that space-time surfaces are Kähler calibrations with Kähler action density reducing to a closed 4-form at space-time surfaces. Hence TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

4. The organization of the book

The first part of the book describes basic quantum TGD in its recent form.

1. The properties of the preferred extremals of Kähler action are crucial for the construction and the discussion of known extremals is therefore included.

2. General coordinate invariance and generalized super-conformal symmetries - the latter present only for 4-dimensional space-time surfaces and for 4-D Minkowski space - define the basic symmetries of quantum TGD.

3. In zero energy ontology S-matrix is replaced with M-matrix and identified as time-like entanglement coefficients between positive and negative energy parts of zero energy states assignable to the past and future boundaries of 4-surfaces inside causal diamond defined as intersection of future and past directed light-cones. M-matrix is a product of diagonal density matrix and unitary S-matrix and there are reasons to believe that S-matrix is universal. Generalized Feynman rules based on the generalization of Feynman diagrams obtained by replacing lines with light-like 3-surfaces and vertices with 2-D surfaces at which the lines meet.

4. A category theoretical formulation of quantum TGD is considered. Finite measurement resolution realized in terms of a fractal hierarchy of causal diamonds inside causal diamonds leads to a stringy formulation of quantum TGD involving effective replacement of the 3-D light-like surface with a collection of braid strands representing the ends of strings. A formulation in terms of category theoretic concepts is proposed and leads to a hierarchy of algebras forming what is known as operads.

5. Twistors emerge naturally in TGD framework and could allow the formulation of low energy limit of the theory in the approximation that particles are massless. The replacement of massless plane waves with states for which amplitudes are localized are light-rays is suggestive in twistor theoretic framework. Twistors could allow also a dual representation of space-time surfaces in terms of surfaces of $X \times CP_2$, where X is 8-D twistor space or its 6-D projective variant. These surfaces would have dimension higher than four in non-perturbative phases meaning an analogy with branes. In full theory a massive particles must be included but represent a problem in approach based on standard twistors. The interpretation of massive particles in 4-D sense as massless particles in 8-D sense would resolve the problem and requires

a generalization of twistor concept involving in essential manner the triality of vector and spinor representations of $SO(7,1)$.

6. In TGD Universe bosons are in well-defined sense bound states of fermion and anti-fermion. This leads to the notion of bosonic emergence meaning that the fundamental action is just Dirac action coupled to gauge potentials and bosonic action emerges as part of effective action as one functionally integrates over the spinor fields. This kind of approach predicts the evolution of all coupling constants if one is able to fix the necessary UV cutoffs of mass and hyperbolic angle in loop integrations. The guess for the hyperbolic cutoff motivated by the geometric view about finite measurement resolution predicts coupling constant evolution which is consistent with that predicted by standard model. The condition that all N-vertices defined by fermiomic loops vanish for $N > 3$ when incoming particles are massless gives hopes of fixing completely the hyperbolic cutoff from fundamental principles.

1.3 Sources

The eight online books about TGD [K75, K56, K93, K64, K48, K92, K91, K62] and nine online books about TGD inspired theory of consciousness and quantum biology [K68, K6, K52, K5, K30, K36, K38, K61, K86] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://www.tgdtheory.com/curri.html`) contains a lot of material about TGD. In particular, there is summary about TGD and its applications using CMAP representation serving also as a TGD glossary [L12, L13] (see `http://www.tgdtheory.fi/cmaphtml.html` and `http://www.tgdtheory.fi/tgdglossary.pdf`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://journals.sfu.ca/jnonlocality/index.php/jnonlocality` founded by Lian Sidorov and in *Prespacetime Journal* (`http://prespacetime.com`), *Journal of Consciousness Research and Exploration* (`https://www.createspace.com/4185546`), and *DNA Decipher Journal* (`http://dnadecipher.com`), all of them founded by Huping Hu. One can find the list about the articles published at `http://www.tgdtheory.com/curri.html`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

1.4.1 Part I: The recent view about field equations

Basic extremals of the Kähler action

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at fundamental level. The strategy is simple: try to guess the general physical consequences of the configuration space geometry and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The 4-surface associated with given 3-surface defined by Kähler function K as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to hyper-quaternionic and co-hyper-quaternionic regions. The reduction of the classical theory to the level of the modified Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [?] It is not clear whether criticality and hyper-quaternionicity are consistent with each other.

   Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.

3. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $exp(K)$ is analogous to the exponent $exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy.

4. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with modified Dirac action defines classical theory: this is in complete accordance with the proposed definition of the configuration space spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2, gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

2. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extemals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of CP_2 must be replaced with that for $S^2 \times CP_2$ in order to obtain a configuration space metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariace from $SO(3,1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

The recent vision about preferred extremals and solutions of the modified Dirac equation

During years several approaches to what preferred extremals of Kähler action and solutions of the modified Dirac equation could be have been proposed and the challenge is to see whether at least some of these approaches are consistent with each other. It is good to list various approaches first.

1. For preferred extremals generalization of conformal invariance to 4-D situation is very attractive approach and leads to concrete conditions formally similar to those encountered in string model. The approach based on basic heuristics for massless equations, on effective 3-dimensionality, and weak form of electric magnetic duality is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic imbedding space.

2. There are also several approaches for solving the modified Dirac equation. The most promising approach is assumes that other than right-handed neutrino modes are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of number theoretic vision. The conditions stating that electric charge is conserved for preferred extremals is an alternative very promising approach.

In this chapter the question whether these various approaches are mutually consistent is discussed. It indeed turns out that the approach based on the conservation of electric charge leads under rather general assumptions to the proposal that solutions of the modified Dirac equation are localized on 2-dimensional string world sheets and/or partonic 2-surfaces. Einstein's equations are satisfied for the preferred extremals and this implies that the earlier proposal for the realization of Equivalence Principle is not needed. This leads to a considerable progress in the understanding of super Virasoro representations for super-symplectic and super-Kac-Moody algebra. In particular, the proposal is that super-Kac-Moody currents assignable to string world sheets define duals of gauge potentials and their generalization for gravitons: in the approximation that gauge group is Abelian - motivated by the notion of finite measurement resolution - the exponents for the sum of KM charges would define non-integrable phase factors. One can also identify Yangian as the algebra generated by these charges. The approach allows also to understand the special role of the right handed neutrino in SUSY according to TGD.

1.4.2 Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

This chapter represents the updated version of the construction providing a solution to the problems of the previous construction. The basic formulas remain as such but the expressions for WCW super-Hamiltonians defining WCW Hamiltonians (and matrix elements of WCW metric) as their anticommutator are replaced with those following from the dynamics of the modified Dirac action.

1.4.3 Unified Number Theoretical Vision

An updated view about M^8-H duality is discussed. M^8-H duality allows to deduce $M^4 \times CP_2$ via number theoretical compactification. One important correction is that octonionic spinor structure makes sense only for M^8 whereas for $M^4 \times CP_2$ complefixied quaternions characterized the spinor structure.

Octonions, quaternions, quaternionic space-time surfaces, octonionic spinors and twistors and twistor spaces are highly relevant for quantum TGD. In the following some general observations distilled during years are summarized.

There is a beautiful pattern present suggesting that $H = M^4 \times CP_2$ is completely unique on number theoretical grounds. Consider only the following facts. M^4 and CP_2 are the unique 4-D spaces allowing twistor space with Kähler structure. Octonionic projective space OP_2 appears as octonionic twistor space (there are no higher-dimensional octonionic projective spaces). Octotwistors generalise the twistorial construction from M^4 to M^8 and octonionic gamma matrices make sense also for H with quaternionicity condition reducing OP_2 to to 12-D $G_2/U(1) \times U(1)$ having same dimension as the the twistor space $CP_3 \times SU(3)/U(1) \times U(1)$ of H assignable to complexified quaternionic representation of gamma matrices.

A further fascinating structure related to octo-twistors is the non-associated analog of Lie group defined by automorphisms by octonionic imaginary units: this group is topologically six-sphere. Also the analogy of quaternionicity of preferred extremals in TGD with the Majorana condition central in super string models is very thought provoking. All this suggests that associativity indeed could define basic dynamical principle of TGD.

Number theoretical vision about quantum TGD involves both p-adic number fields and classical number fields and the challenge is to unify these approaches. The challenge is non-trivial since the p-adic variants of quaternions and octonions are not number fields without additional conditions. The key idea is that TGD reduces to the representations of Galois group of algebraic numbers realized in the spaces of octonionic and quaternionic adeles generalizing the ordinary adeles as Cartesian products of all number fields: this picture relates closely to Langlands program. Associativity would force sub-algebras of the octonionic adeles defining 4-D surfaces in the space of octonionic adeles so that 4-D space-time would emerge naturally. $M^8 - H$ correspondence in turn would map the space-time surface in M^8 to $M^4 \times CP_2$.

1.4.4 Various General Ideas Related to Quantum TGD

I have gathered to this chapter those ideas related to quantum TGD which are not absolutely central and whose status is not clear in the long run. I have represented earlier these ideas in chapters and the outcome was a total chaos and reader did not have a slightest idea what is they real message. I hope that this organization of material makes it easier for the reader to grasp the topology of TGD correctly.

1.4.5 Part II: General Theory

Construction of Quantum Theory: Symmetries

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

1. Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the configuration space forms what I have used to call super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

3. Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

4. Modified Dirac equation gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the modified Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

2. p-adic physics and p-adic variants of basic symmetries

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

3. Hierarchy of Planck constants and dark matter hierarchy

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzles leads to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of CP_2 or of a causal diamond (CD) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups.

4. Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the modified gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane M^2 of M^4, one ends up with $M^8 - H$ duality stating that space-time surfaces can be equivalently regarded as surfaces in M^8 or $M^4 \times CP_2$. One can actually generalize M^2 to a two-dimensional Minkowskian sub-manifold of M^4. One ends up with quantum TGD by considering associative sub-algebras of the local octonionic Clifford algebra of M^8 or H. so that TGD could be seen as a generalized number theory.

Construction of Quantum Theory: M-matrix

The construction of M-matrix has remained the key challenge of quantum TGD from the very beginning when it had become clear that path integral approach and canonical quantization make no sense in TGD framework. My intuitive feeling that the problems are not merely technical has turned out to be correct.

The rapid evolution of a bundle of new ideas has taken place during last five years (zero energy ontology, the notion of finite measurement resolution, the role of hyper-finite factors of type II_1, the hierarchy of Planck constants, the construction of configuration space geometry in terms of second quantized induced spinor fields, number theoretic compactification,...). These ideas are now converging to an overall view in which various approaches to quantum TGD (physics as infinite dimensional geometry, physics as generalized number theory, physics from number theoretical universality, physics from finite measurement resolution implying effective discretization, TGD as almost topological QFT) neatly fuse together to single coherent overall view. Many ideas have been of course thrown away because they have not produced anything useful.

In this chapter the overall view about the construction of the TGD counterpart of S-matrix - M-matrix -is discussed. It is perhaps wise to summarize briefly the vision about M-matrix.

1. Zero energy ontology and interpretation of light-like 3-surfaces as generalized Feynman diagrams

1. Zero energy ontology is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface X_l^3 connecting the light-like boundaries of a causal diamond CD identified as intersection of future and past directed light-cones. There is entire hierarchy of CDs, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to CDs corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.

2. Light-like 3-surfaces are the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

2. Identification of the counterpart of S-matrix as time-like entanglement coefficients

1. The TGD counterpart of S-matrix -call it M-matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of CD. One can also assign to quantum jump between zero energy states a matrix- call it U-matrix - which is unitary and assumed to be expressible in terms of M-matrices. M-matrix need not be unitary unlike the U-matrix characterizing the unitary process forming part of quantum jump. There are several good arguments suggesting that that M-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In fact, M-matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.

2. In the general case only thermal M-matrix defines a normalizable zero energy state so that thermodynamics becomes part of quantum theory. One can assign to M-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

3. Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as

ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

4. Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

5. Input from the construction of configuration space spinor structure

The construction of configuration space spinor structure in terms of second quantized induced spinor fields is certainly the most important step made hitherto towards explicit formulas for M-matrix elements.

1. Number theoretical compactification ($M^8 - H$ duality) states that space-time surfaces can be equivalently regarded as 4-dimensional surfaces of either $H = M^4 \times CP_2$ or of 8-D Minkowski space M^8, and consisting of hyper-quaternionic and co-hyper-quaternionic regions identified as regions with Minkowskian and Euclidian signatures of induced metric. Duality preserves induced metric and Kähler form. This duality poses very strong constraints on the geometry of the preferred extremals of Kähler action implying dual slicings of the space-time surface by string worlds sheets and partonic 2-surfaces as also by light-like 1-surfaces and light-like 3-surfaces. These predictions are consistent what is known about the extremals of Kähler action. The predictions of number theoretical compactification lead to dramatic progress in

the construction of configurations space spinor structure and geometry. One consequence is dimensional reduction of space-time surface to string world sheet allowing to understand how the space-time correlate for Equivalence Principle is realized in TGD framework (its quantum counterpart emerges from coset construction for super-symplectic and super Kac-Moody algebras).

2. The construction of configuration space geometry and spinor structure in terms of induced spinor fields leads to the conclusion that finite measurement resolution is an intrinsic property of quantum states basically due to the vacuum degeneracy of Kähler action. This gives a justification for the notion of number theoretic braid effectively replacing light-like 3-surfaces. Hence the infinite-dimensional configuration space is replaced with a finite-dimensional space $(\delta M^4_\pm \times CP_2)^n/S_n$. A possible interpretation is that the finite fermionic oscillator algebra for given partonic 2-surface X^2 represents the factor space $\mathcal{M}/\mathcal{N}$ identifiable as quantum variant of Clifford algebra. $(\delta M^4_\pm \times CP_2)^n/S_n$ would represent its bosonic analog.

3. The isometries of the configuration space corresponds to X^2 local symplectic transformations $\delta M^4_\pm \times CP_2$ depending only on the value of the invariant $\epsilon^{\mu\nu} J_{\mu\nu}$, where $J_{\mu\nu}$ can correspond to the Kähler form induced from $\delta M^4_\pm$ or CP_2. This group parameterizes quantum fluctuating degrees of freedom. Zero modes correspond to coordinates which cannot be made complex, in particular to the values of the induced symplectic form which thus behaves as a classical field so that configuration space allows a slicing by the classical field patterns $J_{\mu\nu}(x)$ representing zero modes.

4. By the effective 2-dimensionality of light-like 3-surfaces X^3_l (holography) the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. Quantum classical correspondence suggests that gauge fixing in Kac-Moody degrees of freedom takes place and implies correlation between the quantum numbers of the physical state and X^3_l. There would be no path integral over X^3_l and only functional integral defined by configuration space geometry over partonic 2-surfaces.

5. The condition that the Noether currents assignable to the modified Dirac equation are conserved requires that space-time surfaces correspond to extremals for which second variation of Kähler action vanishes. A milder condition is that the rank of the matrix defined by the second variation of Kähler action is less than maximal. Preferred extremals of Kähler action can be identified as this kind of 4-surface and the interpretation is in terms of quantum criticality.

6. Conformal symmetries and stringy diagrammatics

The modified Dirac equation has rich super-conformal symmetries helping to achieve concrete vision about the structure of M-matrix in terms of generalized Feynman diagrammatics

1. Both super-conformal symmetries, the slicing of space-time surface by string worlds sheets, and the reduction of space-time sheet to string world sheet as a consequence of finite measurement resolution suggest that the generalized Feynman diagrams have as vertices N-point functions of a conformal field theory assignable to the partonic 2-surfaces at which the lines of Feynman diagram meet. Finite measurement resolution means that this conformal theory is defined in the discrete set defined by the number theoretic braid. The presence of symplectic invariants in turn suggest a symplectic variant of conformal field theory leading to a concrete construction of symplectic fusion rules relying in crucial manner to discretization.

2. The effective 3-dimensionality implied by the modified Dirac operator associated with Kähler action plays crucial role in the construction of both configuration space geometry (Kähler function is identified as Dirac determinant assignable to the modified Dirac operator) and of M-matrix. By effective 3-dimensionality the propagators reduce to the propagators assignable the light-like 3-surfaces. This does not give stringy propagators and massive stringy excitations would not appear at all in propagators. This does not conform with what p-adic mass calculations and conformal symmetries suggest.

3. The solution of the problem is provided by the addition of measurement interaction term to the modified Dirac action and assignable to wormhole throats or equivalently any light-like 3-surface parallel to them int the slicing of space-time sheet: this condition defines additional symmetry. Measurement interaction term implies that the preferred extremals of Kähler action depend on quantum numbers of the states of super-conformal representations as quantum classical correspondence requires. The coupling constants appearing in the measurement interaction term are fixed by the condition that Kähler function transforms only by a real part of a holomorphic function of complex coordinates of WCW depending also on zero modes so that Kähler metric of WCW remains unchanged. This realizes also the effective 2-dimensionality of space-like 3-surfaces but only in finite regions where the slicing by light-like 3-surfaces makes sense.

7. TGD as almost topological QFT

The idea that TGD could be regarded as almost topological QFT has been very fruitful although the hypothesis that Chern-Simons term for induced Kähler gauge potential assignable to light-like 3-surfaces identified as regions of space-time where the Euclidian signature of induced metric assignable to the interior or generalized Feynman diagram changes to Minkowskian one turned out to be too strong. The reduction of configuration space and its Clifford algebra to finite dimensional structures due to finite measurement resolution however realizes this idea but in different manner.

1. There is functional integral over the small deformations of Feynman cobordisms corresponding to the maxima of Kähler function which is finite-dimensional if finite measurement resolution is taken into account. Almost topological QFT property of quantum suggests the identification of M-matrix as a functor from the category of generalized Feynman cobordisms (generalized Feynman diagrams) to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces.

2. The limit at which momenta vanish is well-defined for M-matrix since the modified Dirac action contains measurement interaction term and at this limit one indeed obtains topological QFT.

3. Almost TQFT property suggests that braiding S-matrices should have important role in the construction. It is indeed possible to assign the with the lines of the generalized Feynman diagram. The reduction of quantum TGD to topological QFT should occur at quantum criticality with respect to the change of Planck constant since in this situation the M-matrix should not depend at all on Planck constant. Factoring QFTs in 1+1 dimensions give examples of this kind of theories.

8. Bosonic emergence

The construction of QFT limit of quantum TGD based on the notion of bosonic emergence led to the most concrete picture about M-matrix achieved hitherto.

1. An "almost stringy" fermion propagator arises as one adds to the modified Dirac action a term coupling the charges in a Cartan algebra of the isometry group of $H = M^4 \times CP_2$ to conserved fermionic currents (there are several of them). Also more general observables allow this kind of coupling and the interpretation in terms of measurement interaction. This term also realizes quantum classical correspondence by feeding information about quantum numbers of partons to the geometry of space-time sheet so that quantum numbers entangle with the geometry of space-time sheet as holography requires. This measurement interaction was the last piece in the puzzle "What are the basic equations of quantum TGD" and unified several visions about the physics predicted by quantum TGD. "Almost stringy" means that the on mass shell fermions obey stringy mass formulas dictated by super-conformal symmetry but that propagator itself -although it depends on four-momentum- is not the inverse of super-Virasoro generator G_0 as it would be in string models.

2. The identification of bosons as wormhole contacts means that bosonic propagation reduces to a propagation of fermion and antifermion at opposite throats of the wormhole throat. In this framework bosonic n-vertex would correspond to the decay of bosons to fermion-antifermion pairs in the loop. Purely bosonic gauge boson couplings would be generated radiatively from triangle and box diagrams involving only fermion-boson couplings. In particular, bosonic propagator would be generated as a self-energy loop: bosons would propagate by decaying to fermion-antifermion pair and then fusing back to the boson. TGD counterpart for gauge theory dynamics would be emergent and bosonic couplings would have form factors with IR and UV behaviors allowing finiteness of the loops constructed from them since the constraint that virtual fermion pair corresponds to wormhole contact poses strong constraint on virtual momenta of fermion and antifermion.

This picture leads to generalized Feynman rules for M-matrix. The QFT limit based on this picture is able to reproduce the p-adic length scale evolution of various gauge coupling strengths with simple cutoffs on mass squared and hyperbolic angle characterizing the state of fermion in the rest system of virtual boson. The presence of these cutoffs is dictated by geometric picture about loops provided by zero energy ontology. The condition that the bosonic $N > 3$-vertices vanish when incoming states are on mass shell gives an infinite number of conditions which could fix the cutoffs uniquely.

More about Matrices

This chapter is a second part of chapter representing material related to the construction of U-, M, and S-matrices. The general philosophy is discussed in the first part of the chapter and I will not repeat the discussion.

The views about M-matrix as a characterizer of time-like entanglement and M-matrix as a functor are analyzed. The role of hyper-finite factors in the construction of M-matrix is considered. One section is devoted to the possibility that Connes tensor product could define fundamental vertices. The last section is devoted to the construction of unitary U-matrix characterizing the unitary process forming part of quantum jump.

The last section is about the anatomy of quantum jump. The first part of the chapter began with a similar piece of text. This reflects the fact that the ideas are developing all the time so that the vision about the matrices is by no means top-down view beginning from precisely state assumption and proceeding to conclusions.

Category Theory and Quantum TGD

Possible applications of category theory to quantum TGD are discussed. The so called 2-plectic structure generalizing the ordinary symplectic structure by replacing symplectic 2-form with 3-form and Hamiltonians with Hamiltonian 1-forms has a natural place in TGD since the dynamics of the light-like 3-surfaces is characterized by Chern-Simons type action. The notion of planar operad was developed for the classification of hyper-finite factors of type II_1 and its mild generalization allows to understand the combinatorics of the generalized Feynman diagrams obtained by gluing 3-D light-like surfaces representing the lines of Feynman diagrams along their 2-D ends representing the vertices.

The fusion rules for the symplectic variant of conformal field theory, whose existence is strongly suggested by quantum TGD, allow rather precise description using the basic notions of category theory and one can identify a series of finite-dimensional nilpotent algebras as discretized versions of field algebras defined by the fusion rules. These primitive fusion algebras can be used to construct more complex algebras by replacing any algebra element by a primitive fusion algebra. Trees with arbitrary numbers of branches in any node characterize the resulting collection of fusion algebras forming an operad. One can say that an exact solution of symplectic scalar field theory is obtained.

Conformal fields and symplectic scalar field can be combined to form symplecto-formal fields. The combination of symplectic operad and Feynman graph operad leads to a construction of Feynman diagrams in terms of n-point functions of conformal field theory. M-matrix elements with a finite measurement resolution are expressed in terms of a hierarchy of symplecto-conformal n-point functions such that the improvement of measurement resolution corresponds to an algebra homomorphism mapping conformal fields in given resolution to composite conformal fields in improved resolution. This expresses the idea that composites behave as independent conformal fields. Also other applications are briefly discussed.

1.4.6 Part III: Twistors and TGD

Yangian Symmetry,Twistors, and TGD

There has been impressive steps in the understanding of $\mathcal{N} = 4$ maximally sypersymmetric YM theory possessing 4-D super-conformal symmetry. This theory is related by AdS/CFT duality to certain string theory in $AdS_5 \times S^5$ background. Second stringy representation was discovered by Witten and is based on 6-D Calabi-Yau manifold defined by twistors. The unifying proposal is that so called Yangian symmetry is behind the mathematical miracles involved.

In the following I will discuss briefly the notion of Yangian symmetry and suggest its generalization in TGD framework by replacing conformal algebra with appropriate super-conformal algebras. Also a possible realization of twistor approach and the construction of scattering amplitudes in terms of Yangian invariants defined by Grassmannian integrals is considered in TGD framework and based on the idea that in zero energy ontology one can represent massive states as bound states of massless particles. There is also a proposal for a physical interpretation of the Cartan algebra of Yangian algebra allowing to understand at the fundamental level how the mass spectrum of n-particle bound states could be understood in terms of the n-local charges of the Yangian algebra.

Twistors were originally introduced by Penrose to characterize the solutions of Maxwell's equations. Kähler action is Maxwell action for the induced Kähler form of CP_2. The preferred extremals allow a very concrete interpretation in terms of modes of massless non-linear field. Both conformally compactified Minkowski space identifiable as so called causal diamond and CP_2 allow a description in terms of twistors. These observations inspire the proposal that a generalization of Witten's twistor string theory relying on the identification of twistor string world sheets with certain holomorphic surfaces assigned with Feynman diagrams could allow a formulation of quantum TGD in terms of 3-dimensional holomorphic surfaces of $CP_3 \times CP_3$ mapped to 6-surfaces dual $CP_3 \times CP_3$, which are sphere bundles so that they are projected in a natural manner to 4-D space-time surfaces. Very general physical and mathematical arguments lead to a highly unique proposal for the holomorphic differential equations defining the complex 3-surfaces conjectured to correspond to the preferred extremals of Kähler action.

Some Fresh Ideas about Twistorialization of TGD

The reading of the article of Tim Adamo and the recent work of Nima Arkani Hamed and Jaroslav Trnka has inspired a fresh look on twistors and a possible answer to several questions (I have written two chapters about twistors and TGD giving a view about development of ideas).

Both M^4 and CP_2 are highly unique in that they allow twistor structure and in TGD one can overcome the fundamental "googly" problem of the standard twistor program preventing twistorialization in general space-time metric by lifting twistorialization to the level of the imbedding space containg M^4 as a Cartesian factor. Also CP_2 allows twistor space identifiable as flag manifold $SU(3)/U(1) \times U(1)$ as the self-duality of Weyl tensor indeed suggests. This provides an additional "must" in favor of sub-manifold gravity in $M^4 \times CP_2$. Both octonionic interpretation of M^8 and triality possible in dimension 8 play a crucial role in the proposed twistorialization of $H = M^4 \times CP_2$. It also turns out that $M^4 \times CP_2$ allows a natural twistorialization respecting Cartesian product: this is far from obvious since it means that one considers space-like geodesics of H with light-like M^4 projection as basic objects. p-Adic mass calculations however require tachyonic ground states and in generalized Feynman diagrams fermions propagate as massless particles in M^4 sense. Furthermore, light-like H-geodesics lead to non-compact candidates for the twistor space of H. Hence the twistor space would be 12-dimensional manifold $CP_3 \times SU(3)/U(1) \times U(1)$.

Generalisation of 2-D conformal invariance extending to infinite-D variant of Yangian symmetry; light-like 3-surfaces as basic objects of TGD Universe and as generalised light-like geodesics; light-likeness condition for momentum generalized to the infinite-dimensional context via super-conformal algebras. These are the facts inspiring the question whether also the "world of classical worlds" (WCW) could allow twistorialization. It turns out that center of mass degrees of freedom (imbedding space) allow natural twistorialization: twistor space for $M^4 \times CP_2$ serves as moduli space for choice of quantization axes in Super Virasoro conditions. Contrary to the original optimistic expectations it turns out that although the analog of incidence relations holds true for Kac-Moody algebra, twistorialization in vibrational degrees of freedom does not look like a good idea since incidence relations force an effective reduction of vibrational degrees of freedom to four.

The Grassmannian formalism for scattering amplitudes is expected to generalize for generalized Feynman diagrams: the basic modification is due to the possible presence of CP_2 twistorialization and the fact that 4-fermion vertex -rather than 3-boson vertex- and its super counterparts define now the fundamental vertices. Both QFT type BFCW and stringy BFCW can be considered.

1. For QFT type BFCW BFF and BBB vertices would be an outcome of bosonic emergence (bosons idealized as wormhole contacts) and 4-fermion vertex is proportional to factor with dimensions of inverse mass squared and naturally identifiable as proportional to the factor $1/p^2$ assignable to each boson line. This predicts a correct form for the bosonic propagators for which mass squared is in general non-vanishing unlike for fermion lines. The usual BFCW construction would emerge naturally in this picture. There is however a problem: the emergent bosonic propagator diverges or vanishes depending on whether one assumes SUSY at the level of single wormhole throat or not. By the special properties of SUSY generated by right handed neutrino the SUSY cannot be applied to single wormhole throat but only to a pair of wormhole throats.

2. This as also the fact that physical particles are necessarily pairs of wormhole contacts connected by fermionic strings forces stringy variant of BFCW avoiding the problems caused by non-planar diagrams. Now boson line BFCW cuts are replaced with stringy cuts and loops with stringy loops. By generalizing the earlier QFT twistor Grassmannian rules one ends up with their stringy variants in which super Virasoro generators $G, G^\dagger$ and L bringing in CP_2 scale appear in propagator lines: most importantly, the fact that G and $G^\dagger$ carry fermion number in TGD framework ceases to be a problem since a string world sheet carrying fermion number has $1/G$ and $1/G^\dagger$ at its ends. Twistorialization applies because all fermion lines are light-like.

3. A more detailed analysis of the properties of right-handed neutrino demonstrates that modified gamma matrices in the modified Dirac action mix right and left handed neutrinos but that this happens markedly only in very short length scales comparable to CP_2 scale. This makes neutrino massive and also strongly suggests that SUSY generated by right-handed neutrino emerges as a symmetry at very short length scales so that spartners would be very massive and effectively absent at low energies. Accepting CP_2 scale as cutoff in order to avoid divergent gauge boson propagators QFT type BFCW makes sense. The outcome is consistent with conservative expectations about how QFT emerges from string model type description.

Perhaps it is not exaggeration to say that the architecture of generalized Feynman diagrams and their connection to twistor approach is now reasonably well-understood. There are of course several problems to be solved. On must feed in p-adic thermodynamics for external particles (here zero energy ontology might be highly relevant). Also the description of elementary particle families in terms of elementary particle functionals in the space of conformal equivalence classes of partonic 2-surface must be achieved.

1.4.7 Part IV: Miscellaneous topics

Quantum Field Theory Limit of TGD from Bosonic Emergence

This chapter summarizes the basic mathematical realization of the modified Feynman rules hoped to give rise to a unitary M-matrix (recall that M-matrix is product of a positive square root of density matrix and unitary S-matrix in TGD framework and need not be unitary in the general case). The basic idea is that bosonic propagators emerge as fermionic loops. The approach is bottom up and leads to a precise general formulation for how the counterpart of YM action emerges from Dirac action coupled to gauge bosons and to modified Feynman rules. An essential element of the approach is a physical formulation for UV cutoff. Actually cutoff in both mass squared and hyperbolic angle is needed since Wick rotation does not make sense in TGD framework. This approach predicts all gauge couplings and assuming a geometrically very natural hyperbolic UV cutoff motivated by zero energy ontology one can understand the evolution of standard model gauge couplings and reproduce correctly the values of fine structure constant at electron and intermediate boson length scales. Also asymptotic freedom follows as a basic prediction. The UV cutoff for the hyperbolic angle as a function of p-adic length scale is somewhat ad hoc element of the model and a quantitative model for how this function could follow from the requirement of quantum criticality is formulated and discussed.

These considerations and numerical calculations lead to a general vision about how real and p-adic variants of TGD relate to each other and how p-adic fractalization takes place. As in case of twistorialization Cutkosky rules allowing unitarization of the tree amplitudes in terms of $TT^\dagger$ contribution involving only light-like momenta seems to be the only working option and requires that $TT^\dagger$ makes sense p-adically. The vanishing of the fermionic loops defining bosonic vertices for the incoming massless momenta emerges as a consistency condition suggested also by quantum criticality and by the fact that only BFF vertex is fundamental vertex if bosonic emergence is accepted. The vanishing of on mass shell N-vertices gives an infinite number of conditions on the hyperbolic cutoff as function of the integer k labeling p-adic length scale at the limit when bosons are massless and IR cutoff for the loop mass scale is taken to zero. It is not yet clear whether dynamical symmetries, in particular super-conformal symmetries, are involved with the realization of the vanishing conditions or whether hyperbolic cutoff is all that is needed.

Does the QFT Limit of TGD Have Space-Time Super-Symmetry?

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the modified Dirac action. The introduction of a measurement interaction term to the action allows to understand how stringy propagator results and provides profound insights about physics predicted by TGD.

The appearance of the momentum and color quantum numbers in the measurement interaction couples space-time degrees of freedom to quantum numbers and allows also to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation a finite-dimensional SUSY algebra or the fermionic part of super-conformal algebra with an infinite number of oscillator operators results. The addition of a fermion in particular mode would define particular super-symmetry. Zero energy ontology implies that fermions as wormhole throats correspond to chiral super-fields assignable to positive or negative energy SUSY algebra whereas bosons as wormhole contacts with two throats correspond to the direct sum of positive and negative energy algebra and fields which are chiral or antichiral with respect to both positive and negative energy theta parameters. This super-symmetry is badly broken due to the dynamics of the modified Dirac operator which also mixes M^4 chiralities inducing massivation. Since righthanded neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.

2. The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The the general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in D=10 and D=11 as the only possible candidates for TOE after it turned out that chiral anomalies cancel.

3. The massivation of particles is basic problem of both SUSYs and twistor approach. The fact that particles which are massive in M^4 sense can be interpreted as massless particles in $M^4 \times CP_2$ suggests a manner to understand super-symmetry breaking and massivation in TGD framework. The octonionic realization of twistors is a very attractive possibility in this framework and quaternionicity condition guaranteing associativity leads to twistors which are almost equivalent with ordinary 4-D twistors.

4. The first approach is based on an approximation assuming only the super-multiplets generated by right-handed neutrino or both right-handed neutrino and its antineutrino. The assumption that right-handed neutrino has fermion number opposite to that of the fermion associated with the wormhole throat implies that bosons correspond to $\mathcal{N} = (1,1)$ SUSY and fermions to $\mathcal{N} = 1$ SUSY identifiable also as a short representation of $\mathcal{N} = (1,1)$ SUSY algebra trivial with respect to positive or negative energy algebra. This means a deviation from the standard view but the standard SUSY gauge theory formalism seems to apply in this case.

5. A more ambitious approach would put the modes of induced spinor fields up to some cutoff into super-multiplets. At the level next to the one described above the lowest modes of the induced spinor fields would be included. The very large value of $\mathcal{N}$ means that $\mathcal{N} \leq \ni\in$ SUSY cannot define the QFT limit of TGD for higher cutoffs. One must generalize SUSYs gauge theories to arbitrary value of $\mathcal{N}$ but there are reasons to expect that the formalism becomes rather complex. More ambitious approach working at TGD however suggest a more general manner to avoid this problem.

   (a) One of the key predictions of TGD is that gauge bosons and Higgs can be regarded as bound states of fermion and antifermion located at opposite throats of a wormhole contact. This implies bosonic emergence meaning that it QFT limit can be defined in terms of Dirac action. The resulting theory was discussed in detail in [**?**]nd it was shown that bosonic propagators and vertices can be constructed as fermionic loops so that all coupling constant follow as predictions. One must however pose cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops in order to obtain finite theory and to avoid massivation of bosons. The resulting coupling constant evolution is consistent with low energy phenomenology if the cutoffs in hyperbolic angle as a function of p-adic length scale is chosen suitably.

   (b) The generalization of bosonic emergence that the TGD counterpart of SUSY is obtained by the replacement of Dirac action with action for chiral super-field coupled to vector field as the action defining the theory so that the propagators of bosons and all their super-counterparts would emerge as fermionic loops.

   (c) The huge super-symmetries give excellent hopes about the cancelation of infinities so that this approach would work even without the cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops. Cutoffs have a physical motivation in zero energy ontology but it could be an excellent approximation to take them to infinity. Alternatively, super-symmetric dynamics provides cutoffs dynamically.

6. The condition that $\mathcal{N} = \infty$ variants for chiral and vector superfields exist fixes completely the identification of these fields in zero energy ontology.

   (a) In this framework chiral fields are generalizations of induced spinor fields and vector fields those of gauge potentials obtained by replacing them with their super-space counterparts. Chiral condition reduces to analyticity in theta parameters thanks to the different definition of hermitian conjugation in zero energy ontology (θ is mapped to a derivative with respect to theta rather than to $\overline{\theta}$) and conjugated super-field acts on the product of all theta parameters.

   (b) Chiral action is a straightforward generalization of the Dirac action coupled to gauge potentials. The counterpart of YM action can emerge only radiatively as an effective action so that the notion emergence is now unavoidable and indeed basic prediction of TGD.

(c) The propagators associated with the monomials of n theta parameters behave as $1/p^n$ so that only $J = 0, 1/2, 1$ states propagate in normal manner and correspond to normal particles. The presence of monomials with number of thetas higher than 2 is necessary for the propagation of bosons since by the standard argument fermion and scalar loops cancel each other by super-symmetry. This picture conforms with the identification of graviton as a bound state of wormhole throats at opposite ends of string like object.

(d) This formulation allows also to use modified gamma matrices in the measurement interaction defining the counterpart of super variant of Dirac operator. Poincare invariance is not lost since momenta and color charges act on the tip of CD rather than the coordinates of the space-time sheet. Hence what is usually regarded as a quantum theory in the background defined by classical fields follows as exact theory. This feeds all data about space-time sheet associated with the maximum of Kähler function. In this approach WCW as a Kähler manifold is replaced by a cartesian power of CP_2, which is indeed quaternionic Kähler manifold. The replacement of light-like 3-surfaces with number theoretic braids when finite measurement resolution is introduced, leads to a similar replacement.

(e) Quantum TGD as a "complex square root" of thermodynamics approach suggests that one should take a superposition of the amplitudes defined by the points of a coherence region (identified in terms of the slicing associated with a given wormhole throat) by weighting the points with the Kähler action density. The situation would be highly analogous to a spin glass system since the modified gamma matrices defining the propagators would be analogous to the parameters of spin glass Hamiltonian allowed to have a spatial dependence. This would predict the proportionality of the coupling strengths to Kähler coupling strength and bring in the dependence on the size of CD coming as a power of 2 and give rise to p-adic coupling constant evolution. Since TGD Universe is analogous to 4-D spin glass, also a sum over different preferred extremals assignable to a given coherence regions and weighted by $exp(K)$ is probably needed.

(f) In TGD Universe graviton is necessarily a bi-local object and the emission and absorption of graviton are bi-local processes involving two wormhole contacts: a pair of particles rather than single particle emits graviton. This is definitely something new and defies a description in terms of QFT limit using point like particles. Graviton like states would be entangled states of vector bosons at both ends of stringy curve so that gravitation could be regarded as a square of YM interactions in rather concrete sense. The notion of emergence would suggest that graviton propagator is defined by a bosonic loop. Since bosonic loop is dimensionless, IR cutoff defined by the largest CD present must be actively involved. At QFT limit one can hope a description as a bi-local process using a bi-local generalization of the QFT limit. It turns out that surprisingly simple candidate for the bi-local action exists.

Chapter 2

Coupling Constant Evolution in Quantum TGD

This chapter summarizes the recent views about p-adic coupling constant evolution.

1. The most recent view about coupling constant evolution

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-canonical and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination makes possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules.

2. Equivalence Principle and evolution of gravitational constant

Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

1. The coset construction for super-symplectic and super Kac-Moody algebras implies Equivalence Principle in the sense that four-momenta assignable to the Super Virasoro generators of the two algebras are identical. The challenge is to understand this result in more concrete terms.

2. The progress made in the understanding of number theoretical compactification led to a dramatic progress in the construction of configuration space geometry and spinor structure in terms of the modified Dirac operator associated with light-like 3-surfaces appearing in the slicing of the preferred extremal $X^(X^3_l)$ of Kähler action to light-like 3-surfaces Y^3_l "parallel" to X^3_l. Even more the M^4 projection is predicted to have a slicing into 2-dimensional stringy worldsheets having $M^2(x) \subset M^4$ as a tangent space at point x.

3. By dimensional reduction one can assign to any stringy slice Y^2 a stringy action obtained by integrating Kähler action over the transversal degrees of freedom labeling the copies of Y^2. One can assign length scale evolution to the string tension $T(x)$, which in principle can depend on the point of the string world sheet and thus evolves. $T(x)$ is not identifiable as inverse of gravitational constant but by general arguments proportional to $1/L_p^2$, where L_p is p-adic length scale.

4. Gravitational constant can be understood as a product of L_p^2 with the exponential of the Kähler action for the two pieces of CP_2 type vacuum extremals representing wormhole contacts assignable to graviton connected by the string world sheets. The volume of the typical CP_2 type extremal associated with the graviton increases with L_p so that the exponential

factor decreases reducing the growth due to the increase of L_p. Hence G could be RG invariant in p-adic coupling constant evolution. It does not make sense to formulate evolution of gravitational constant at space-time level and gravitational constant characterizes given CD.

5. Gravitational mass is assigned to the stringy world sheet and should be identical with the inertial mass identified as Noether charge assignable to the preferred extremal. By construction there are good hopes that for a proper choice of G gravitational and inertial masses are identical.

3. The RG invariance of gauge couplings inside causal diamond

Quantum classical correspondence suggests that the notion of p-adic coupling constant evolution should have space-time correlate. Zero energy ontology suggests that this counterpart is realized in terms of CDs in the sense that coupling constant evolution has formulation at space-time level inside CD of given size scale and that RG invariance holds true for this evolution. Number theoretic compactification forces to conclude that space-time surfaces has slicing into light-like 3-surfaces Y_l^3: this prediction is consistent with that is known about the extremals. General Coordinate Invariance requires that basic theory can be formulated by replacing the light-like 3-surface X_l^3 associated with wormhole throats with any surface Y_l^3 appearing in the associated slicing.

The natural identification for the renormalization group parameter is as the light-like coordinate labeling different light-like slices. The light-likeness of the RG parameter suggests RG invariance. Quantum classical correspondence requires that the classical gauge fluxes to X_l^3 selected by stationary phase approximation correspond to the expectation values of gQ_g, where g is coupling constant and Q_g the expectation (eigen) value of corresponding charge matrix in the state in question. If the gauge currents are light-like and in direction of Y_l^3 as they are for known extremals under proper selection of X_l^3, RG invariance follows because Abelian gauge fluxes are conserved due to the absence of the component of vacuum current in the direction of slicing.

In principle TGD predicts the values of all coupling constants including also the value of Kähler coupling strength which follows from the identification of Kähler action of the preferred extremal $X^4(X_l^3)$ of Kähler action as Dirac determinant associated with modified Dirac action. Hence Kähler coupling strength could have several values. Quantum criticality in the strongest form however motivates the hypothesis that g_K^2 is invariant under p-adic coupling constant evolution and evolution under evolution associated with the hierarchy of Planck constants.

4. Quantitative predictions for the values of coupling constants

The latest progress in the understanding of p-adic coupling constant evolution comes from a formula for Kähler coupling strength α_K in terms of Dirac determinant of the modified Dirac operator associated with $C-S$ action. The progress came from the realization about how that data about preferred extremal of Kähler action is feeded into the eigenvalue spectrum, which - due to the almost topological character of $C-S$ action - is otherwise far from fixed.

The formula for α_K fixes its number theoretic anatomy and also that of other coupling strengths. The assumption that simple rationals (p-adicization) are involved can be combined with the input from p-adic mass calculations and with an old conjecture for the formula of gravitational constant allowing to express it in terms of CP_2 length scale and Kähler action of topologically condensed CP_2 type vacuum extremal. The prediction is that α_K is renormalization group invariant and equals to the value of fine structure constant at electron length scale characterized by M_{127}. Newton's constant is proportional to p-adic length scale squared and ordinary gravitons correspond to M_{127}. The number theoretic anatomy of R^2/G allows to consider two options. For the first one only M_{127} gravitons are possible number theoretically. For the second option gravitons corresponding to $p \simeq 2^k$ are possible.

A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

5. p-Adic length scale evolution of gauge couplings

Understanding the dependence of gauge couplings constants on p-adic prime is one of the basic challenges of quantum TGD. The problem has been poorly understood even at the conceptual level to say nothing about concrete calculations. The generalization of the motion of S-matrix to that of M-matrix changed however the situation. M-matrix is always defined with respect to measurement resolution characterized in terms of an inclusion of von Neumann algebra. Coupling constant evolution reduces to a discrete evolution involving only octaves of $T(k) = 2^k T_0$ of the fundamental time scale $T_0 = R$, where R CP_2 scale. p-Adic length scale $L(k)$ is related to $T(k)$ by $L^2(k) = T(k)T_0$. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, is automatic prediction of the theory. There is also a close connection with the description of coupling constant evolution in terms of radiative corrections.

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level and why certain p-adic primes are favored.

1. Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges invariant: this seems to be the case for known extremals which form scaling invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

2. Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

Part I

THE RECENT VIEW ABOUT FIELD EQUATIONS

Chapter 3

Basic Extremals of the Kähler Action

3.1 Introduction

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable. The progress made during next five years led to a detailed understanding of quantum TGD at the fundamental parton level and this provides considerable additional insights concerning the interpretation of field equations.

3.1.1 About the notion of preferred extremal

The notion of preferred extremal has been central in classical TGD although the known solutions could be preferred or not: the main challenge has been to understand what "preferred" could mean.

In zero energy ontology (ZEO) one can also consider the releaving possibility that all extremals are preferred ones! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. The non-determinism of Kähler action suggests that "preferred" could be obsolete in given length scale resolution.

Actually all the discussions of this chapter are about known extremals in general so that the attribute "preferred" is not relevant for them.

3.1.2 Beltrami fields and extremals

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that preferred extremals satisfy the condition. The vanishing of the Lorentz 4-force in turn implies a local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. If Kähler action is defined by CP_2 Kähler form alone, the condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteeing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

An important outcome is the notion of Hamilton-Jacobi structure meaning dual slicings of M^4 projection of preferred extremals to string world sheets and partonic 2-surfaces. The necessity of this slicing was discovered years later from number theoretic compactification and is now a key element of quantum TGD allowing to deduce Equivalence Principle in its stringy form from quantum TGD and formulate and understand quantum TGD in terms of modified Dirac action assignable to Kähler action. The conservation of Noether charges associated with modified Dirac action requires the vanishing of the second second variation of Kähler action for preferred extremals. Preferred extremals would thus define space-time representation for quantum criticality. Infinite-dimensional variant for the hierarchy of criticalities analogous to the hierarchy assigned to the extrema of potential function with levels labeled by the rank of the matrix defined by the second derivatives of the potential function in catastrophe theory would suggest itself.

A natural interpretation for deformations would be as conformal gauge symmetries due to the non-determinism of Kähler action. They would transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They would preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

3.1.3 In what sense field equations could mimic dissipative dynamics?

By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. The nontrivial question is what this means in TGD framework.

1. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extremals of Kähler action correspond to space-time sheets which at least asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final (rather than initial!) 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property of Kähler action abstracted to purely algebraic generalized Beltrami conditions would make sense also in the p-adic context. The general solution ansatz discussed in the last section of the chapter assumes that all conserved isometry currents are proportional to instanton current so that various charges are conserved separately for all flow lines: this means essentially the integrability of the theory. This ansatz is forced by the hypothesis that TGD reduces to almost topological QFT and this idea. The basic consequence is that dissipation is impossible classically.

2. A more radical view inspired by zero energy ontology is that the light-like 3-surfaces and corresponding space-time regions with Euclidian signature defining generalized Feynman diagrams provide a space-time representation of dissipative dynamics just as they provide this

representation in quantum field theory. Minkowskian regions would represent empty space so that the vanishing of Lorentz 4-force and absence of dissipation would be natural. This would mean very precise particle field duality and the topological pattern associated with the generalized Feynman diagram would represent dissipation. One could also interprete dissipation as transfer of energy between sheets of the many-sheeted space time and thus as an essentially topological phenomenon. This option seems to be the only viable one.

3.1.4 The dimension of CP_2 projection as classifier for the fundamental phases of matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. The natural guess would be that this phase is chaotic and analogous to de-magnetized phase. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. It seems however that preferred extremals can correspond only to small perturbations of these extremals resulting by topological condensation of CP_2 type vacuum extremals and through topological condensation to larger space-time sheets. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase could be seen as the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

The original proposal was that $D(CP_2) = 4$ phase is completely chaotic. This is not true if the reduction to almost topological QFT takes place. This phase must correspond to Maxwellian phase with a vanishing Kähler current as concluded already earlier. Various isometry currents are however proportional to the instanton current and conserved along the flow lines of the instanton current whose flow parameter extends to a global coordinate. Hence a completely chaotic phase is not in question even in this case.

3.1.5 Specific extremals of Kähler action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynman diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

3. There are also various non-vacuum extremals.

   (a) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.

(b) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

(c) In the so called Maxwell phase, ordinary Maxwell equations for the induced Kähler field would be satisfied in an excellent approximation. It is however far from clear whether this kind of extremals exist. Their non-existence would actually simplify the theory enormously since all extremals would have quantal character. The recent view indeed is that Maxwell phase makes sense only as as genuinely many-sheeted structure and solutions of Maxwell's equation appear only at the level of effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Gauge potentials in effective space-time are determined in the same manner. Since the gauge potentials sum up, it is possible to understand how field configurations of Maxwell's theory emerge at this limit.

3.1.6 The weak form of electric-magnetic duality and modification of Kähler action

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

Generalized Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector and there are reasons to hope that TGD is completely integrable theory.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- Classical TGD [L16]
- Topological field quantization [L46]
- Identification of preferred extremals of Kaehler action [L25]
- TGD and GRT [L42]
- Holography [L23]
- 4-D spin glass degeneracy [L14]

3.2 General considerations

The solution families of field equations studied in this chapter were found already during eighties. The physical interpretation turned out to be the the really tough problem. What is the principle selecting preferred extremals of Kähler action as analogs of Bohr orbits assigning to 3-surface X^3 a unique space-time surface $X^4(X^3)$? Does Equivalence Principle hold true and if so, in what sense? These have been the key questions. The realization that light-like 3-surfaces X_l^3 associated with the light-like wormhole throats at which the signature of the induced metric changes from Minkowskian to Euclidian led to the formulation of quantum TGD in terms of second quantized induced spinor fields at these surfaces. Together with the notion of number theoretical compactification this approach allowed to identify the conditions characterizing the preferred extremals. What is remarkable that these conditions are consistent with what is known about extremals.

Also a connection with string models emerges and partial understanding of the space-time realization of Equivalence Principle suggests itself. However, much more general argument allows to understand how GRT space-time appears from the many-sheeted space-time of TGD (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book) as effective concept [K72]: this more general view is not in conflict with the much earlier proposal discussed below.

In this section the theoretical background behind field equations is briefly summarized. I will not repeat the discussion of previous two chapters [K26, K27] summarizing the general vision about many-sheeted space-time, and consideration will be restricted to those aspects of vision leading to direct predictions about the properties of preferred extremals of Kähler action.

3.2.1 Number theoretical compactification and $M^8 - H$ duality

The notion of hyper-quaternionic and octonionic manifold makes sense but it not plausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Situation changes if H is replaced with hyper-octonionic M^8. Suppose that $X^4 \subset M^8$ consists of hyper-quaternionic and co-hyper-quaternionic regions. The basic observation is that the hyper-quaternionic sub-spaces of M^8 with a fixed hyper-complex structure (containing in their tangent space a fixed hyper-complex subspace M^2 or at least one of the light-like lines of M^2) are labeled by points of CP_2. Hence each hyper-quaternionic and co-hyper-quaternionic four-surface of M^8 defines a 4-surface of $M^4 \times CP_2$. One can loosely say that the number-theoretic analog of spontaneous compactification occurs: this of course has nothing to do with dynamics.

This picture was still too naive and it became clear that not all known extremals of Kähler action contain fixed $M^2 \subset M^4$ or light-like line of M^2 in their tangent space.

1. The first option represents the minimal form of number theoretical compactification. M^8 is interpreted as the tangent space of H. Only the 4-D tangent spaces of light-like 3-surfaces X_l^3 (wormhole throats or boundaries) are assumed to be hyper-quaternionic or co-hyper-quaternionic and contain fixed M^2 or its light-like line in their tangent space. Hyper-quaternionic regions would naturally correspond to space-time regions with Minkowskian signature of the induced metric and their co-counterparts to the regions for which the signature is Euclidian. What is of special importance is that this assumption solves the problem of identifying the boundary conditions fixing the preferred extremals of Kähler action since in the generic case the intersection of M^2 with the 3-D tangent space of X_l^3 is 1-dimensional. The surfaces $X^4(X_l^3) \subset M^8$ would be hyper-quaternionic or co-hyper-quaternionic but would not allow a local mapping between the 4-surfaces of M^8 and H.

2. One can also consider a more local map of $X^4(X_l^3) \subset H$ to $X^4(X_l^3) \subset M^8$. The idea is to allow $M^2 \subset M^4 \subset M^8$ to vary from point to point so that $S^2 = SO(3)/SO(2)$ characterizes the local choice of M^2 in the interior of X^4. This leads to a quite nice view about strong geometric form of $M^8 - H$ duality in which M^8 is interpreted as tangent space of H and $X^4(X_l^3) \subset M^8$ has interpretation as tangent for a curve defined by light-like 3-surfaces at X_l^3 and represented by $X^4(X_l^3) \subset H$. Space-time surfaces $X^4(X_l^3) \subset M^8$ consisting of hyper-quaternionic and co-hyper-quaternionic regions would naturally represent a preferred extremal of E^4 Kähler action. The value of the action would be same as CP_2 Kähler action. $M^8 - H$ duality would apply also at the induced spinor field and at the level of WCW. The possibility to assign $M^2(x) \subset M^4$ to each point of M^4 projection $P_{M^4}(X^4(X_l^3))$ is consistent with what is known about extremals of Kähler action with only one exception: CP_2 type vacuum extremals. In this case M^2 can be assigned to the normal space.

3. Strong form of $M^8 - H$ duality satisfies all the needed constraints if it represents Kähler isometry between $X^4(X_l^3) \subset M^8$ and $X^4(X_l^3) \subset H$. This implies that light-like 3-surface is mapped to light-like 3-surface and induced metrics and Kähler forms are identical so that also Kähler action and field equations are identical. The only differences appear at the level

of induced spinor fields at the light-like boundaries since due to the fact that gauge potentials are not identical.

4. The map of $X_l^3 \subset H \rightarrow X_l^3 \subset M^8$ would be crucial for the realization of the number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as those preferred coordinates in which the points of imbedding space are rational/algebraic. Thus the point of $X^4 \subset H$ is algebraic if it is mapped to algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactification could thus be motivated by the number theoretical universality.

5. The possibility to use either M^8 or H picture might be extremely useful for calculational purposes. In particular, M^8 picture based on $SO(4)$ gluons rather than $SU(3)$ gluons could perturbative description of low energy hadron physics. The strong $SO(4)$ symmetry of low energy hadron physics can be indeed seen direct experimental support for the $M^8 - H$ duality.

Number theoretical compactification has quite deep implications for quantum TGD and is actually responsible for most of the progress in the understanding of the mathematical structure of quantum TGD. A very powerful prediction is that preferred extremals should allow slicings to either stringy world sheets or dual partonic 2-surfaces as well as slicing by light-like 3-surfaces. Both predictions are consistent with what is known about extremals.

1. If the distribution of planes $M^2(x)$ is integrable, it is possible to slice $X^4(X^3)$ to a union of 2-dimensional surfaces having interpretation as string world sheets and dual 2-dimensional copies of partonic surfaces X^2. This decomposition defining 2+2 Kaluza-Klein type structure could realize quantum gravitational holography and might allow to understand Equivalence Principle at space-time level in the sense that dimensional reduction defined by the integral of Kähler action over the 2-dimensional space labeling stringy world sheets gives rise to the analog of stringy action and one obtains string model like description of quantum TGD as dual for a description based on light-like partonic 3-surfaces. String tension is not however equal to the inverse of gravitational constant as one might naively expect but the connection is more delicate. As already mentioned, TGD-GRT connection and EP can be understood at general level only from very general arguments [K72].

2. Second implication is the slicing of $X^4(X_l^3)$ to light-like 3-surfaces Y_l^3 "parallel" to X_l^3. Also this slicing realizes quantum gravitational holography if one requires General Coordinate Invariance in the sense that the Dirac determinant differs for two 3-surfaces Y_l^3 in the slicing only by an exponent of a real part of a holomorphic function of WCW complex coordinates giving no contribution to the Kähler metric.

3. The square of the Dirac determinant would be equal to the modulus squared for the exponent of vacuum functional and would be formally defined as the product of conformal weights assignable to the modes of the Dirac operator at string world sheets at the ends of strings at partonic 2-surfaces defining the ends of Y_l^3. The detailed definition requires to specify what one means with the conformal weights assignable with the modes of the Kähler-Dirac operator.

4. The localization of the modes of Kähler-Dirac operator to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) [K87] following from the condition that electromagnetic charges of the modes is well-defined is very strong restriction and reduces Dirac determinant to a product of Dirac determinants assignable with these 2-surfaces.

3.2.2 Dirac determinant as exponent of Kähler action for preferred extremal

An attractive hypothesis is that Dirac determinant reduces to the vacuum functional identifiable as exponent of Kähler action S_K for a preferred extremal came first. The contribution from Euclidian regions corresponds to Kähler function and that from Minkowskian regions serves as analog of Minkowskian action defining Morse function at the level of WCW.

There is no hope of reducing Kähler action to Dirac action since Kähler action and Kähler-Dirac action are in completely democratic position since Kähler-Dirac gamma matrices are defined in terms of the canonical momentumd densities for Kähler action. The value of Kähler coupling strength is however expected to follow from the condition that Dirac determinant equals to vacuum functional.

1. The realization that well-definedness of em charge requires the localization of the modes of induced spinor field to string world sheets or partonic 2-surfaces was an important step in process trying to make the notion of Dirac determinant more concrete [K87]. Dirac determinants reduce to those assignable to string world sheets and possibly also partonic 2-surfaces and would naturally correspond to square roots of determinants defined by the products of the eigenvalues of the mass squared operator for incoming on mass shell states and given by stringy mass formula. Zeta function regularization should allow to defined these determinants and one can hope that it reduces to the exponent of Kähler action for preferred extremal. Thus coupling constant evolution might allow a reduction to string model type description.

2. If weak form of electric magnetic duality and by $j \cdot A = 0$ condition for Kähler current and gauge potential in the interior of space-time sheets are satisfied, Kähler action reduces to Chern-Simons terms at light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface. Induced metric would apparently disappear from the action in accordance with the idea about TGD as almost topological QFT. In order to obtain perturbation theory one must add Chern-Simons term to partonic orbits such that it compensates the contribution of Kähler action. This term has also fermionic counterparts and means that the Kähler-Dirac action reduces to Chern-Simons Dirac action. If spinor modes are generalized eigen modes of C-S-D Dirac operator with eigenvalues of $p^k\gamma_k$. where p^k is the virtual momentum at fermion line identified as boundary string world sheet, one obtains ordinary massless Dirac propagator.

Measurement interaction terms would be completely analogous to those fixing the values of observables in thermodynamics and thus Lagrange multiplier terms fixing the values of certain classical conserved charges in Cartan algebra to their quantal counterparts. By supersymmetry this would give rise to measurement interaction term in Kähler-Dirac action at the space-like ends of the space-time surface and this term would give additional term to the the boundary conditions of Kähler Dirac equation. Typically the massless incoming states would generate mass due to the Lagrange multiplier terms.

In absence of measurement interaction terms one would have $\Gamma^n\Psi = p^k\gamma_k\Psi = 0$ where Γ^n is the normal component for Kähler-Dirac gamma matrix vector and depends on Kähler action and p^k is four-momentum assignable to the fundamental fermion associated with fermionic string. Lagrange multiplier terms imply that Γ^n is replaced with its sum with the 3-D modified Dirac operator defined by the constraint terms.

$$(\Gamma^n + \sum_i \lambda_i \Gamma^\alpha_{Q_i} D_\alpha)\Psi = 0 \ ,$$

where Q_i refers to i:th conserved charge.

3.2.3 Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries. The natural identification would be as conformal symmetries. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K/\partial h^k_\alpha \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K/\partial h^k_\alpha \delta h^k$

over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally.

The vanishing of the second variation in interior of $X^4(X_l^3)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

For instance, the natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \to X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

The basic question is whether number theoretic view about preferred extremals imply absolute minimization or something analogous to it.

1. The number theoretic conditions defining preferred extremals are purely algebraic and make sense also p-adically and this is enough since p-adic variants of field equations make sense although the notion of Kähler action does not make sense as integral. Despite this the identification of the vacuum functional as exponent of Kähler function as Dirac determinant allows to define the exponent of Kähler function as a p-adic number [K10] .

2. The general objection against all extremization principles is that they do not make sense p-adically since p-adic numbers are not well-ordered.

3. These observations do not encourage the idea about equivalence of the two approaches. On the other hand, real and p-adic sectors are related by algebraic continuation and it could be quite enough if the equivalence were true in real context alone.

The finite-dimensional analogy allows to compare absolute minimization and criticality with each other.

1. Absolute minimization would select the branch of Thom's catastrophe surface with the smallest value of potential function for given values of control variables. In general this value would not correspond to criticality since absolute minimization says nothing about the values of control variables (zero modes).

2. Criticality forces the space-time surface to belong to the bifurcation set and thus fixes the values of control variables, that is the interior of 3-surface assignable to the partonic 2-surface, and realized holography. If the catastrophe has more than $N = 3$ sheets, several preferred extremals are possible for given values of control variables fixing $X^3(X^2)$ unless one assumes that absolute minimization or some other criterion is applied in the bifurcation set. In this sense absolute minimization might make sense in the real context and if the selection is between finite number of alternatives is in question, it should be possible carry out the selection in number theoretically universal manner.

It must be emphasized that there are several proposals for what preferred extremal property could mean. For instance, one can consider the identification of space-time surface as quaternionic sub-manifold meaning that tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of imbedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of imbedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K4] defining also this kind of slicing and the approaches could be equivalent. A further approach is based on the identification of preferred extremal property as quantum criticality [K4].

3.2.4 Can one determine experimentally the shape of the space-time surface?

The question 'Can one determine experimentally the shape of the space-time surface?' does not relate directly to the topic of this chapter in technical sense, and the only excuse for its inclusion is the title of this section plus the fact that the general conceptual framework behind quantum TGD assumes an affirmative answer to this question. If physics were purely classical physics, operationalism in the strong sense of the word would require that one can experimentally determine the shape of the space-time as a surface of the imbedding space with arbitrary accuracy by measuring suitable classical observables. In quantum physics situation is considerably more complex and quantum effects are both a blessing and a curse.

Measuring classically the shape of the space-time surface

Consider first the purely classical situation to see what is involved.

1. All classical gauge fields are expressible in terms of CP_2 coordinates and their space-time gradients so that the measurement of four field quantities with some finite resolution in some space-time volume could in principle give enough information to deduce the remaining field quantities. The requirement that space-time surface corresponds to an extremal of Kähler action gives a further strong consistency constraint and one can in principle test whether this constraint is satisfied. A highly over-determined system is in question.

2. The freedom to choose the space-time coordinates freely causes complications and it seems that one must be able to determine also the distances between the points at which the field quantities are determined. At purely classical Riemannian level this boils down to the measurement of the induced metric defining classical gravitational field. In macroscopic length scales one could base the approach to iterative procedure in which one starts from the assumption that the coordinates used are Minkowski coordinates and gravitational corrections are very weak.

3. The measurement of induced Kähler form in some space-time volume determines space-time surface only modulo canonical transformations of CP_2 and isometries of the imbedding space. If one measures classical electromagnetic field, which is not canonical invariant in general case, with some precision, one can determine to what kind of surface space-time region corresponds apart from the action of the isometries of H.

Quantum measurement of the shape of the space-time surface

In practice the measurement of the shape of the space-time surface is necessarily a bootstrap procedure based on the model for space-time region and on the requirement of internal consistency. Many-sheeted space-time and quantum phenomena produce considerable complications but also provide universal measurement standards.

Consider first how quantum effects could help to measure classical fields and distances.

1. The measurement of distances by measuring first induced metric at each point of space-time sheet is rather unpractical procedure. Many-sheeted space-time however comes in rescue here. p-Adic length scale hypothesis provides a hierarchy of natural length scales and one can use p-adic length and time scales as natural units of length and time: space-time sheets serve as meter sticks. For instance, length measurement reduces in principle to a finite number of operations using various space-time sheets with standardized lengths given by p-adic length scales. Also various transition frequencies and corresponding wavelengths provide universal time and length units. Atomic clock provides a standard example of this kind of time unit. A highly nontrivial implication is the possibility to deduce the composition of distant star from its spectral lines. Without p-adic length scale hypothesis the scales for the mass spectra of the elementary particles would be variable and atomic spectra would vary from point to point in TGD universe.

   Do the p-adic length scales correspond to the length units of the induced metric or of M^4_+ metric? If the topological condensation a meter stick space-time sheet at a larger space-time sheet does not stretch the meter stick but only bends it, the length topologically condensed meter stick in the induced metric equals to its original length measured using M^4_+ metric.

2. If superconducting order parameters are expressible in terms of the CP_2 coordinates (there is evidence for this, see the chapter "Macroscopic quantum phenomena and CP_2 geometry"), one might determine directly the CP_2 coordinates as functions of Minkowski coordinates and this would allow to estimate all classical fields directly and thus to deduce strong consistency constraints.

3. At quantum level only the fluxes of the classical fields through surface areas with some minimum size determined by the length scale resolution can be measured. In case of magnetic fields the quantization of the magnetic flux simplifies the situation dramatically. Topological field quantization quite generally modifies the measurement of continuous field variables to the measurement of fluxes. Interestingly, the construction of WCW geometry uses as WCW coordinates various electric and magnetic fluxes over 2-dimensional cross sections of 3-surface.

Quantum effects introduce also difficulties and restrictions.

1. Canonical transformations localized with respect to the boundary of the light cone or more general light like surfaces act as isometries of WCW and one can determine the space-time surface only modulo these isometries. Even more, only the values of the non-quantum fluctuating zero modes characterizing the shape and size of the space-time surface are measurable with arbitrary precision in quantum theory. At the level of conscious experience quantum

fluctuating degrees of freedom correspond to sensory qualia like color having no classical geometric content.

2. Space-time surface is replaced by a new one in each quantum jump (or rather the superposition of perceptively equivalent space-time surfaces). Only in the approximation that the change of the space-time region in single quantum jump is negligible, the measurement of the shape of space-time surface makes sense. The physical criterion for this is that dissipation is negligible. The change of the space-time region in single quantum jump can indeed be negligible if the measurement is performed with a finite resolution.

3. Conscious experience of self is an average over quantum jumps defining moments of consciousness. In particular, only the average increment of the zero modes is experienced and this means that one cannot fix the space-time surface apart from canonical transformation affecting the zero modes. Again the notion of measurement resolution comes in rescue.

4. The possibility of coherent states of photons and gravitons brings in a further quantum complication since the effective classical em and gravitational fields are superpositions of classical field and the order parameter describing the coherent state. In principle the extremely strong constraints between the classical field quantities allow to measure both the order parameters of the coherent phases and classical fields.

Quantum holography and the shape of the space-time surface

If the Dirac determinant asssognable to the mass squared eigenvalue spectrum of the modified Dirac operator $D_K(X^2)$ equals to the exponent of Kähler action of a preferred extremal, it is fair to say that a lot of information about the shape of the space-time surface is coded to physical observables, which eigenvalues indeed represent. Quantum gravitational holography due to the Bohr orbit like character of space-time surface reduces the amount of information needed. Only a finite number of eigenvalues is involved and the eigen modes are associated with the 3-D light-like wormhole throats rather than with the space-time surface itself. If the eigenvalues were known or could be measured with infinite accuracy, one could in principle fix the boundary conditions at X_l^3 and solve field equations determining the preferred extremal of Kähler action.

What is of course needed is the complete knowledge of the light-like 3-surfaces X_l^3. Needless to say, in practice a complete knowledge of X_l^3 is impossible since measurement resolution is finite. The notion number theoretic braid provides a precise realization for the finite measurement accuracy at space-time level. At the level of WCW spinors fields (world of classical worlds) just the fact that the number of eigenvalues is finite is correlate for the finite measurement accuracy. Furthermore, quantum states are actually quantum superpositions of 3-surfaces, which means that one can only speak about quantum average space-time surface for which the phase factors coding for the quantum numbers of elementary particles assigned to the strands of number theoretic braids are stationary so that correlation of classical gauge charges with quantum gauge charges is obtained.

3.3 General view about field equations

In this section field equations are deduced and discussed in general level. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general, in principle makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the selection of preferred extremal of Kähler action. This condition implies that covariant divergence of energy momentum tensor vanishes and in General Relativity context this leads to Einstein's equations. If preferred extremals correspond to absolute minima this principle would be essentially equivalent with the second law of thermodynamics. There are however could reasons to keep the identification of preferred extremely property open.

3.3.1 Field equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$\begin{aligned} D_\beta(T^{\alpha\beta}h^k_\alpha) &- j^\alpha J^k{}_l\partial_\alpha h^l = 0 \ , \\ T^{\alpha\beta} &= J^{\nu\alpha}J_\nu{}^\beta - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \end{aligned} \tag{3.3.1}$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$\begin{aligned} T^{\alpha\beta}H^k_{\alpha\beta} &- j^\alpha(J_\alpha{}^\beta h^k{}_\beta + J^k{}_l\partial_\alpha h^l) = 0 \ . \\ H^k_{\alpha\beta} &= D_\beta\partial_\alpha h^k \ . \end{aligned} \tag{3.3.2}$$

$H^k_{\alpha\beta}$ denotes the components of the
second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 and at wormhole throats the field equations are given by the expression

$$\frac{\partial L_K}{\partial_n h^k} = T^{n\beta}\partial_\beta h^k - J^{n\alpha}(J_\alpha{}^\beta\partial_\beta h^k + J^k_l)\partial_\alpha h^k) = 0 \ . \tag{3.3.3}$$

At wormhole throats problems are caused by the vanishing of metric determinant implying that contravariant metric is singular.

For M^4 coordinates boundary conditions are satisfied if one assumes

$$T^{n\beta} = 0 \tag{3.3.4}$$

stating that there is no flow of four-momentum through the boundary component or wormhole throat. This means that there is no energy exchange between Euclidian and Minkowskian regions so that Euclidian regions provide representations for particles as autonomous units. This is in accordance with the general picture [K27] . Note that momentum transfer with external world necessarily involves generalized Feynman diagrams also at classical level.

For CP_2 coordinates the boundary conditions are more delicate. The construction of WCW spinor structure [K10] led to the conditions

$$g_{ni} = 0 \ , \ J_{ni} = 0 \ . \tag{3.3.5}$$

$J^{ni} = 0$ does not and should not follow from this condition since contravariant metric is singular. It seems that limiting procedure is necessary in order to see what comes out.

The condition that Kähler electric charge defined as a gauge flux is non-vanishing would require that the quantity $J^{nr}\sqrt{g}$ is finite (here r refers to the light-like coordinate of X^3_l). Also $g^{nr}\sqrt{g_4}$ which is analogous to gravitational flux if n is interpreted as time coordinate could be non-vanishing. These conditions are consistent with the above condition if one has

$$\begin{array}{llll} J_{ni} = 0 \ , & g_{ni} = 0 \ , & J_{ir} = 0 \ , & g_{ir} = 0 \ , \\ J^{nk} = 0 \ \ k \neq r \ , & g^{nk} = 0 \ \ k \neq r \ , & J^{nr}\sqrt{g_4} \neq 0 \ , & g^{nr}\sqrt{g_4} \neq 0 \ . \end{array} \tag{3.3.6}$$

The interpretation of this conditions is rather transparent.

1. The first two conditions state that covariant form of the induced Kähler electric field is in direction normal to X^3_l and metric separate into direct sum of normal and tangential contributions. Fifth and sixth condition state the same in contravariant form for $k \neq n$.

2. Third and fourth condition state that the induced Kähler field at X_l^3 is purely magnetic and that the metric of x_l^3 reduces to a block diagonal form. The reduction to purely magnetic field is of obvious importance as far as the understanding of the generalized eigen modes of the modified Dirac operator is considered [K10] .

3. The last two conditions must be understood as a limit and $\neq$ means only the possibility of non-vanishing Kähler gauge flux or analog of gravitational flux through X_l^3.

4. The vision inspired by number theoretical compactification allows to identify r and n in terms of the light-like coordinates assignable to an integrable distribution of planes $M^2(x)$ assumed to be assignable to M^4 projection of $X^4(X_l^3)$. Later it will be found that Hamilton-Jacobi structure assignable to the extremals indeed means the existence of this kind of distribution meaning slicing of $X^4(X_l^3)$ both by string world sheets and dual partonic 2-surfaces as well as by light-like 3-surfaces Y_l^3.

5. The physical analogy for the situation is the surface of an ideal conductor. It would not be surprising that these conditions are satisfied by all induced gauge fields.

3.3.2 Topologization and light-likeness of the Kähler current as alternative manners to guarantee vanishing of Lorentz 4-force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

Topologization of the Kähler current for $D_{CP_2} = 3$: covariant formulation

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2} < 4$. For $D_{CP_2} = 2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

If the simplest vision about light-like 3-surfaces as basic dynamical objects is accepted $D_{CP_2} = 2$, corresponds to a non-physical situation and only the deformations of these surfaces - most naturally resulting by gluing of CP_2 type vacuum extremals on them - can represent preferred extremals of Kähler action. One can however speak about $D_{CP_2} = 2$ phase if 4-surfaces are obtained are obtained in this manner.

$$j^\alpha \equiv D_\beta J^{\alpha\beta} = \psi \times j_I^\alpha = \psi \times \epsilon^{\alpha\beta\gamma\delta} J_{\beta\gamma} A_\delta \ . \tag{3.3.7}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2} < 4$. Also the contraction of $\nabla\psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2} < 4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^\alpha J_{\alpha\beta} &= \psi \times j_I^\alpha J_{\alpha\beta} \\ &= \psi \times \epsilon^{\alpha\mu\nu\delta} J_{\mu\nu} A_\delta J_{\alpha\beta} \ . \end{aligned} \tag{3.3.8}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2} < 4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} \;=\; 0 \ . \tag{3.3.9}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection.

Topologization of the Kähler current for $D_{CP_2} = 3$: non-covariant formulation

In order to gain a concrete understanding about what is involved it is useful to repeat these arguments using the 3-dimensional notation. The components of the instanton 4-current read in three-dimensional notation as

$$\overline{j}_I = \overline{E}\times\overline{A} + \phi\overline{B} \ \ , \quad \rho_I = \overline{B}\cdot\overline{A} \ . \tag{3.3.10}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation and read

$$\begin{aligned} \nabla\times\overline{B} - \partial_t\overline{E} &= \overline{j} = \psi\overline{j}_I = \psi\left(\phi\overline{B} + \overline{E}\times\overline{A}\right) \ , \\ \nabla\cdot E &= \rho = \psi\rho_I \ . \end{aligned} \tag{3.3.11}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami condition

$$\nabla\times\overline{B} = \alpha\overline{B} \ \ , \quad \alpha = \psi\phi \ . \tag{3.3.12}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami condition with $\alpha = \phi = constant$, which allows an explicit solution [B44] .

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz 3-force can be written as

$$\rho_I\overline{E} + \overline{j}\times\overline{B} = \psi\overline{B}\cdot\overline{AE} + \psi\left(\overline{E}\times\overline{A} + \phi\overline{B}\right)\times\overline{B} = 0 \ . \tag{3.3.13}$$

The fourth component of the Lorentz force reads as

$$\overline{j}\cdot\overline{E} = \psi\overline{B}\cdot\overline{E} + \psi\left(\overline{E}\times\overline{A} + \phi\overline{B}\right)\cdot\overline{E} = 0 \ . \tag{3.3.14}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity density of the so called helicity charge $\rho = \psi\rho_I = \psi B\cdot A$. This charge is topological charge in the sense that it does not depend on the induced metric at all. Note the presence of arbitrary function ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 independent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality condition for the Kähler current allows to understand the general features of the solution ansatz to very high degree without any detailed knowledge about the detailed solution. The question whether field equations allow solutions consistent with the self duality conditions of the current will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler current relate to each other very intimately.

Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for $D_{CP_2} = 2$

For $D_{CP_2} = 2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla \times \overline{B} = \alpha \overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2} = 2$.

$D_{CP_2} = 2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2} = 2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

Under what conditions topologization of Kähler current yields Beltrami conditions?

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

1. The $\overline{E} \times \overline{A}$ term contributing besides $\phi \overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} = \nabla \Phi - \partial_t \overline{A} = \beta \overline{A} \tag{3.3.15}$$

This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinates and and $\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

2. The vector $\overline{E} \times \overline{A}$ is parallel to $\overline{B}$.

$$\overline{E} \times \overline{A} = \beta \overline{B} \tag{3.3.16}$$

The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B \cdot A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B} \cdot \overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A}, \overline{B}) \rightarrow_{\nabla \times} (\overline{B}, \overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

Hydrodynamic analogy

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the imbedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [B26] .

In hydrodynamics the role of the magnetic field is taken by the velocity field. This raises the idea that the incompressible flow could occur along the field lines of some natural vector field. The considerations of the last section show that the instanton current defines a universal candidate as far as the general solution of the field equations is considered. All conserved currents defined by the isometry charges would be parallel to the instanton current: one can say each flow line of instanton current is a carrier of conserved quantum numbers. Perhaps even the flow lines of an incompressible hydrodynamic flow could in reasonable approximation correspond to those of instanton current.

The conservation laws are satisfied for each flow line separately and therefore it seems that one cannot have the analog of viscous hydrodynamic flow in this framework. One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Does something go badly wrong?

The following argument suggests a way out of the problem. Dissipation is certaily due to the quantum jumps at scales below that associated with causal diamond (CD) associated with the observer and is thus assignable to sub-CDs. The quantum jumps for sub-CDs would eventually lead to a thermal ensemble of sub-CDs.

The usual description of dissipation in terms of viscocity and similar parameters emerges at the GRT-QFT limit of TGD replacing in long length scales the many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book) with a piece of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. This lumping of space-time sheets means that induced gauge fields and gravitational fields from various space-time sheet sum up and become random (by central limit theorems). Thus locally the dynamics is dissipation free for individual space-time sheets and dissipation emerges at the level of GRT space-time carrying effective metric and effective gauge fields.

The stability of generalized Beltrami fields

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds contact . Contact form is a one-form A (that is covariant vector field A_α) with the property $A\wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta}A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta}J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

1. The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi)\wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X\wedge dX = 0$. In the three-dimensional case this gives $\overline{X}\cdot(\nabla\times\overline{X}) = 0$.

2. This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B}\cdot\overline{A}\neq 0$ states that the charge density for the topologized Kähler

current is non-vanishing. The condition that the field lines of the magnetic field allow a global coordinate requires $\overline{B} \cdot \nabla \times \overline{B} = 0$. The condition is not satisfied by Beltrami fields with $\alpha \neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi) = 0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X) = 1$ and $dA(X;) = 0$ hold true. The latter condition states the vanishing of the cross product $X \times B$ so that X is parallel to the Kähler magnetic field B^α and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. Stability of the Beltrami flow and contact structures

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [B34] , and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [B34]!

Beltrami flows associated with Euler equations are known to be unstable [B34] . Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2} = 4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

3.3.3 How to satisfy field equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 \ . \tag{3.3.17}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The following approach utilizes the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

String model as a starting point

String model serves as a starting point.

1. In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u, v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.

2. For Euclidian minimal surfaces (u,v) is replaced by complex coordinates $(w,\overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{w}\overline{w}}$. The mechanism should generalize to the recent case.

The general form of energy momentum tensor as a guideline for the choice of coordinates

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$T = \begin{pmatrix} \frac{E^2+B^2}{2} & 0 & 0 & EB \\ 0 & \frac{E^2+B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{-E^2+B^2}{2} & 0 \\ EB & 0 & 0 & \frac{E^2-B^2}{2} \end{pmatrix} \tag{3.3.18}$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E}\times\overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla\times X\cdot X = 0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E}\times\overline{B}$ such that the coordinates $u = t-z$ and $v = t+z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

Hamilton Jacobi structures in M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2\oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition $(S^+ \equiv u = t+z, S^- \equiv v = t-z, w = x+iy, \overline{w} = x-iy)$.

2. In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$(\nabla S_\pm)^2 = 0 \quad .$$

The gradients of $S_\pm$ are obviously analogous to local light like velocity vectors $v = (1,\overline{v})$ and $\tilde{v} = (1,-\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with the interpretation of massless extremals as Bohr orbits of em field. $S_\pm = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_\pm$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t - z \rightarrow v = t + z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \rightarrow S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t-z, t+z, x, y) \rightarrow S^-(t-z, t+z, x, y) = S^+(t+z, t-z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

3. The coordinates $(S_\pm, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$\begin{aligned} ds^2 &= g_{+-}dS^+dS^- + g_{w\overline{w}}dwd\overline{w} \\ &+ g_{+w}dS^+dw + g_{+\overline{w}}dS^+d\overline{w} \\ &+ g_{-w}dS^-dw + g_{-\overline{w}}dS^-d\overline{w} \ . \end{aligned} \tag{3.3.19}$$

Conformal transformations of M^4_+ leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \rightarrow f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \rightarrow f(S^+), S^- \rightarrow f(S^-)$ in longitudinal degrees of freedom preserve this structure.

4. The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$\begin{aligned} g_{w\overline{w}} &= \partial_w\partial_{\overline{w}}K \ , \quad g_{+-} = \partial_{S^+}\partial_{S^-}K \ , \\ g_{w\pm} &= \partial_w\partial_{S^\pm}K \ , \quad g_{\overline{w}\pm} = \partial_{\overline{w}}\partial_{S^\pm}K \ . \end{aligned} \tag{3.3.20}$$

for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard light-cone coordinates the Kähler function is given by

$$K = w_0\overline{w}_0 + uv \ , \quad w_0 = x + iy \ , \quad u = t - z \ , \quad v = t + z \ . \tag{3.3.21}$$

The Christoffel symbols satisfy the conditions

$$\left\{ {k \atop w\ \overline{w}} \right\} = 0 \ , \quad \left\{ {k \atop +-} \right\} = 0 \ . \tag{3.3.22}$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M^4_+ degrees of freedom.

5. The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M^4_+. Obviously there should be quite large number of them.

Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +\ or\ = -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

Hamilton Jacobi structure means the existence of foliations of the M^4 projection of X^4 by 2-D surfaces analogous to string word sheets labeled by w and the dual of this foliation defined by partonic 2-surfaces labeled by the values of S_i. Also the foliation by light-like 3-surfaces Y_l^3 labeled by $S_\pm$ with $S_\mp$ serving as light-like coordinate for Y_l^3 is implied. This is what number theoretic compactification and $M^8 - H$ duality predict when space-time surface corresponds to hyper-quaternionic surface of M^8 [K27, K67] .

Contact structure and generalized Kähler structure of CP_2 projection

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi, \overline{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2 projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M_+^4 case.

$$s_{w\overline{w}} = \partial_w \partial_{\overline{w}} K \quad , \quad s_{ws} = \partial_w \partial_s K \quad , \quad s_{\overline{w}s} = \partial_{\overline{w}} \partial_s K \ . \tag{3.3.23}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{{}_{\xi\ \overline{\xi}}^{\ k}\}$.

$$\{{}_{\xi\ \overline{\xi}}^{\ k}\} \ = \ 0 \ . \tag{3.3.24}$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s, \xi, \overline{\xi}, S^-)$ as coordinates for the space-time surface defined by the ansatz $(w = w(\xi, s), S^+ = S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^- = constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi, \overline{\xi}, s)$ and some coordinate of M^4_+, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta(J^{\alpha\beta}\sqrt{g}) &= 0 \text{ for } \alpha \in \{\xi, \overline{\xi}, s\} \ , \\ g^{4i} \neq 0 \ . \end{aligned} \tag{3.3.25}$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

A solution ansatz yielding light-like current in $D_{CP_2} = 3$ case

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M^4_+ respecting this structure and guaranteeing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\overline{\xi}}$ and T^{s-} in the coordinates $(\xi, \overline{\xi}, s, S^-)$.

1. The coordinates (w, S^+) are assumed to holomorphic functions of the CP_2 coordinates (s, ξ)

$$S^+ = S^+(s) \ , \quad w = w(\xi, s) \ . \tag{3.3.26}$$

   Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s, w) and (S^+, ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

2. Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\overline{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M^4_+ Christoffel symbols are vanishing for these lower index pairs.

3. By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi, \overline{\xi}, s, S^-)$ has as non-vanishing components only $g_{\xi\overline{\xi}}$ and g_{s-}

$$g_{ss} = 0 \ , \quad g_{\xi s} = 0 \ , \quad g_{\overline{\xi} s} = 0 \ . \tag{3.3.27}$$

   Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

4. The condition guaranteeing the product structure of the metric is

$$\begin{aligned} s_{ss} &= m_{+w}\partial_s w(\xi, s)\partial_s S^+(s) + m_{+\overline{w}}\partial_s \overline{w}(\xi, s)\partial_s S^+(s) \ , \\ s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\ s_{s\overline{\xi}} &= m_{+w}\partial_{\overline{\xi}} w(\overline{\xi})\partial_s S^+(s) \ . \end{aligned} \tag{3.3.28}$$

   Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

5. The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$\begin{aligned} & j^{\xi}\sqrt{g} = \partial_\beta(J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\overline{\xi}}\sqrt{g} = \partial_\beta(J^{\overline{\xi}\beta}\sqrt{g}) = 0 \ , \\ & j^{+}\sqrt{g} = \partial_\beta(J^{+\beta}\sqrt{g}) = 0 \ . \end{aligned} \quad (3.3.29)$$

Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g}j^{+} = \partial_\beta(J^{+\beta}\sqrt{g}) \quad = \quad 0 \quad (3.3.30)$$

is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

1. The light-like character of the Kähler current brings in mind CP_2 extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\overline{\xi}}, J^{\xi -}, J^{\overline{\xi}-})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.

2. Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If preferred extremals correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.

3. The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \overline{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.

4. Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

Are solutions with time-like or space-like Kähler current possible in $D_{CP_2} = 3$ case?

As noticed in the section about number theoretical compactification, the flow of gauge currents along slices Y_l^3 of $X^4(X_l^3)$ "parallel" to X_l^3 requires only that gauge currents are parallel to Y_l^3 and can thus space-like. The following ansatz gives good hopes for obtaining solutions with space-like and perhaps also time-like Kähler currents.

1. Assign to light-like coordinates coordinates (T, Z) by the formula $T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \overline{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.

2. The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z, s) \ , \quad w = w(\xi, s) \ . \tag{3.3.31}$$

   If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.

3. The components of the induced metric are

$$\begin{aligned} g_{ZZ} &= m_{ZZ} + m_{TT}\partial_Z T \partial_s T \ , & g_{Zs} &= m_{TT}\partial_Z T \partial_s T \ , \\ g_{ss} &= s_{ss} + m_{TT}\partial_s T \partial_s T \ , & g_{w\overline{w}} &= s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w \partial_{\overline{\xi}} \overline{w} \ , \\ g_{s\xi} &= s_{s\xi} \ , & g_{s\overline{\xi}} &= s_{s\overline{\xi}} \ . \end{aligned} \tag{3.3.32}$$

   Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} \ = \ 0 \tag{3.3.33}$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\begin{aligned} \partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) &= 0 \ , \\ \partial_{\xi}(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi} Z}\sqrt{g}) &= 0 \ . \end{aligned} \tag{3.3.34}$$

In the special case that the induced metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M_+^4 Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\begin{aligned} \left\{ {}^{\ k}_{T\ w} \right\} &= 0 \ , & \left\{ {}^{\ k}_{T\ \overline{w}} \right\} &= 0 \ , \\ \left\{ {}^{\ k}_{Z\ w} \right\} &= 0 \ , & \left\{ {}^{\ k}_{Z\ \overline{w}} \right\} &= 0 \end{aligned} \tag{3.3.35}$$

hold true. The conditions are equivalent with the conditions

$$\{^{\ k}_{\pm\ w}\} = 0 \ , \quad \{^{\ k}_{\pm\ \overline{w}}\} = 0 \ . \tag{3.3.36}$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s, Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

$D_{CP_2} = 4$ **case**

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

1. Hamilton Jacobi structure for M^4_+ is expected to be crucial also now.

2. One might hope that for $D_{CP_2} = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A \wedge dA = A \wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi) \wedge X$ and implies $X \wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl}A_l$. By a brief calculation one finds that $X \wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations would be indeed satisfied, provided this phase exists at all. It however seems that Maxwell phase is probably realized differently.

1. Solution ansatz with a 3-dimensional M^4_+ projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1, ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1, \xi^2) \ , \quad w = w(\xi^1, \xi^2) \ , \quad S^- = constant \ . \tag{3.3.37}$$

The induced metric does possesses only components of type $g_{i\bar{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \tag{3.3.38}$$

This guarantees that energy momentum tensor has only components of type $T^{i\bar{j}}$ in coordinates (ξ^1, ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M^4_+ degrees of freedom one must pose the conditions

$$\{^{\ k}_{w+}\} = 0 \ , \quad \{^{\ k}_{\overline{w}+}\} = 0 \ , \quad \{^{\ k}_{++}\} = 0 \ . \tag{3.3.39}$$

on Christoffel symbols. These conditions are satisfied if the the M^4_+ metric does not depend on S^+:

$$\partial_+ m_{kl} \ = \ 0 \ . \tag{3.3.40}$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ = a_k\xi^k + \overline{a}_k\overline{\xi}^k \ . \quad (3.3.41)$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M^4_+ coordinates (u, w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j(J^{j\bar{i}}\sqrt{g}) = 0 \ , \quad \partial_{\bar{j}}(J^{\bar{j}i}\sqrt{g}) = 0 \ , \quad (3.3.42)$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M^4_+ projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M^4_+ projection is a light like curve correspond to a special case of this solution ansatz: transversal M^4_+ coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M^4_+ projection is 1-dimensional.

2. Are solutions with a 4-dimensional M^4_+ projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M^4_+ projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i}S^+\partial_{\xi^j}S^- + m_{+-}\partial_{\xi^i}S^-\partial_{\xi^j}S^+)$ are non-vanishing.

1. The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.

2. The solution ansatz has the same general form as in $D_{CP_2} = 3$ case and must be symmetric with respect to the exchange of M^4_+ and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s, r), S^-(s, r)) \ , \quad w = w(\xi) \ . \quad (3.3.43)$$

This ansatz would describe ordinary Maxwell field in M^4_+ since the roles of M^4_+ coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M^4_+ projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

The recent view conforms with this intuition. The Maxwell phase is certainly physical notion but would correspond effective fields experience by particle in many-sheeted space-time. Test particle topological condenses to all the space-time sheets with projection to a given region of Minkowski space and experiences essentially the sum of the effects caused by the induced gauge fields at different sheets. This applies also to gravitational fields interpreted as deviations from Minkowski metric.

The transition to GRT and QFT picture means the replacement of many-sheeted space-time with piece of Minkowski space with effective metric defined as the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Effective gauge potentials are sums of the induced gauge potentials. Hence the rather simple topologically quantized induced gauge fields associated with space-time sheets become the classical fields in the sense of Maxwell's theory and gauge theories.

$D_{CP_2} = 2$ **case**

Hamilton Jacobi structure for M^4_+ is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D_{CP_2} = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

1. String like objects, which are products $X^2 \times Y^2 \subset M^4_+ \times CP_2$ of minimal surfaces Y^2 of M^4_+ with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M^4_+. $w = constant$ surfaces define minimal surfaces X^2 of M^4_+. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the imbedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves moving along string with light velocity, and for more general solutions they become TGD counterparts of Alfven waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfven waves are a phenomenological notion not really justified by the properties of Maxwell's equations.

2. Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi, \overline{\xi}, u, v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s, r) = (s(u, v), r(u, v)) \quad , \quad \xi = constant \quad , \tag{3.3.44}$$

and corresponds to a vanishing Kähler current.

3. Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B} \cdot \overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2} = 2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s, r) = (s, r)(u, v, w, \overline{w}) \quad , \quad \xi = constant \quad . \tag{3.3.45}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

1. Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e = f(w, \overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.

2. The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfven waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \ , \ w = w(\xi) \ , \ S^- = s^- \ , \ S^+ = s^+ + f(\xi, \overline{\xi}) \ .$$

   Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi \partial_+ h^k$ and $\partial_{\overline{\xi}} \partial_+ h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

Could $D_{CP_2} = 2 \rightarrow 3$ transition occur in rotating magnetic systems?

I have studied the imbeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [K69] .

Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates $(U \equiv cos(\theta), \Phi)$ and let (t, ρ, ϕ, z) denote cylindrical coordinates for a cylindrical space-time sheet. The simplest possible space-time surfaces $X^4 \subset M^4_+ \times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$\begin{array}{ll} U = U(\rho) \ , & \Phi = n\phi + kz \ , \\ J_{\rho\phi} = n\partial_\rho U \ , & J_{\rho z} = k\partial_\rho U \ . \end{array} \qquad (3.3.46)$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$\Phi = \omega t + kz + n\phi$$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \rightarrow 3$. This could help to understand various strange effects related to the rotating magnetic systems [K69] . For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

3.3.4 $D_{CP_2} = 3$ phase allows infinite number of topological charges characterizing the linking of magnetic field lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D_{CP_2} = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i \int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D_{CP_2} = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [B46] . Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times} B(r) = \int dV' \frac{(r - r')}{|r - r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r - r')}{|r - r'|^3} \times B(r') \right) \ ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D_{CP_2} = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D_{CP_2} = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to WCW Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

3.3.5 Preferred extremal property and the topologization/light-likeness of Kähler current?

The basic question is under what conditions the Kähler current is either topologized or light-like so that the Lorentz force vanishes. Does this hold for all preferred extremals of Kähler action? Or only asymptotically as suggested by the fact that generalized Beltrami fields can be interpreted as asymptotic self-organization patterns, when dissipation has become insignificant. Or does topologization take place in regions of space-time surface having Minkowskian signature of the induced metric? And what asymptotia actually means? Do absolute minima of Kähler action correspond to preferred extremals?

One can challenge the interpretation in terms of asymptotic self organization patterns assigned to the Minkowskian regions of space-time surface.

1. Zero energy ontology challenges the notion of approach to asymptotia in Minkowskian sense since the dynamics of light-like 3-surfaces is restricted inside finite volume $CD \subset M^4$ since the partonic 2-surfaces representing their ends are at the light-like boundaries of causal diamond in a given p-adic time scale.

2. One can argue that generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$. The weak point of this argument is that it is 3-D light-like 3-surfaces rather than space-time surfaces which are the basic dynamical objects so that the generic and only possible case corresponds to $D_{CP_2} = 3$ for X_l^3. It is quite possible that preferred extremal property implies that $D_{CP_2} = 3$ holds true in the Minkowskian regions since these regions indeed represent empty space. Geometrically this would mean that the CP_2 projection does not change as the light-like coordinate labeling Y_l^3 varies. This conforms nicely with the notion of quantum gravitational holography.

3. The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field for which also Lorentz force vanishes since $\overline{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields would represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles. This would seem to support the view that $D_{CP_2} = 4$ Minkowskian regions are present. The weak point of this argument is that there is fractal hierarchy of length scales represented by the hierarchy of causal diamonds (CDs) and that the resulting hierarchy of generalized Feynman graphs might be enough to represent dissipation classically.

4. One objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all space-like 3-surfaces approaching same asymptotic state have the same value of Kähler function assuming that the Kähler function assignable to space-like 3-surface is same for all space-like sections of $X^4(X_l^3)$ (assuming that one can realize general coordinate invariance also in this sense). This need not be the case. In any case, this need not be a problem since it would mean an additional symmetry extending general

coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature.

It seems that asymptotic self-organization pattern need not be correct interpretation for non-dissipating regions, and the identification of light-like 3-surfaces as generalized Feynman diagrams encourages an alternative interpretation.

1. $M^8 - H$ duality states that also the H counterparts of co-hyper-hyperquaternionic surfaces of M^8 are preferred extremals of Kähler action. CP_2 type vacuum extremals represent the basic example of these and a plausible conjecture is that the regions of space-time with Euclidian signature of the induced metric represent this kind of regions. If this conjecture is correct, dissipation could be assigned with regions having Euclidian signature of the induced metric. This makes sense since dissipation has quantum description in terms of Feynman graphs and regions of Euclidian signature indeed correspond to generalized Feynman graphs. This argument would suggest that generalized Beltrami conditions or light-likeness hold true inside Minkowskian regions rather than only asymptotically.

2. One could of course play language games and argue that asymptotia is with respect to the Euclidian time coordinate inside generalized Feynman graps and is achieved exactly when the signature of the induced metric becomes Minkowskian. This is somewhat artificial attempt to save the notion of asymptotic self-organization pattern since the regions outside Feynman diagrams represent empty space providing a holographic representations for the matter at X_l^3 so that the vanishing of $j^\alpha F_{\alpha\beta}$ is very natural.

3. What is then the correct identification of asymptotic self-organization pattern. Could correspond to the negative energy part of the zero energy state at the upper light-like boundary δM^4_- of CD? Or in the case of phase conjugate state to the positive energy part of the state at δM^4_+? An identification consistent with the fractal structure of zero energy ontology and TGD inspired theory of consciousness is that the entire zero energy state reached by a sequence of quantum jumps represents asymptotic self-organization pattern represented by the asymptotic generalized Feynman diagram or their superposition. Biological systems represent basic examples about self-organization, and one cannot avoid the questions relating to the relationship between experience and geometric time. A detailed discussion of these points can be found in [K3] .

Absolute minimization of Kähler action was the first guess for the criterion selecting preferred extremals. Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number. The generalized Beltrami conditions and the boundary conditions defining the preferred extremals are however local and purely algebraic and make sense also p-adically. If absolute minimization reduces to these algebraic conditions, it would make sense.

3.3.6 Generalized Beltrami fields and biological systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2} = 2$ corresponds to ordered phase, $D_{CP_2} = 3$ to spin glass phase and $D_{CP_2} = 4$ to chaos, with $D_{CP_2} = 3$ defining life as a phenomenon at the boundary between order and chaos. If the criteria suggested by the number theoretic compactification are accepted, it is not clear whether D_{CP_2} extremals can define preferred extremals of Kähler action. For instance, cosmic strings are not preferred extremals and the Y_l^3 associated with MEs allow only covariantly constant right handed neutrino eigenmode of $D_K(X^2)$. The topological condensation of CP_2 type vacuum extremals around $D_{CP_2} = 2$ type extremals is however expected to give preferred extremals and if the density of the condensate is low enough one can still speak about $D_{CP_2} = 2$ phase. A natural guess is also that the deformation of $D_{CP_2} = 2$ extremals transforms light-like gauge currents to space-like topological currents allowed by $D_{CP_2} = 3$ phase.

Why generalized Beltrami fields are important for living systems?

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

1. Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.

2. Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [B26] .

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example. One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

$D_{CP_2} = 3$ systems as boundary between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. $D_{CP_2} = 4$ phase, dead matter, and chaos

$D_{CP_2} = 4$ corresponds to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary "dead matter". If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality.

2. $D_{CP_2} = 2$ phase as ordered phase

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D_{CP_2} = 2$ and the vanishing of em field requires that that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. $D_{CP_2} = 3$ corresponds to living matter

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A}\cdot\overline{B} \neq 0$ and Kähler current $\overline{E}\times\overline{A}+\phi\overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [K24] . For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D_{CP_2} = 3$ phase to $D_{CP_2} = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. It is impossible to assign slicing of 4-surface by 3-D surfaces labeled by a coordinate t varying along the flow lines. This means that one cannot speak about a continuous evolution of Schrödinger amplitude with t playing the role of time coordinate. One could perhaps say that the entire space-time sheet represents single quantum event which cannot be decomposed to evolution. This would conform with the assignment of macroscopic and macro-temporal quantum coherence with living matter.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$ phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

Can one assign a continuous Schrödinger time evolution to light-like 3-surfaces?

Alain Connes wrote [A54] about factors of various types using as an example Schrödinger equation for various kinds of foliations of space-time to time=constant slices. If this kind of foliation does not exist, one cannot speak about time evolution of Schrödinger equation at all. Depending on the character of the foliation one can have factor of type I, II, or III. For instance, torus with slicing $dx = ady$ in flat coordinates, gives a factor of type I for rational values of a and factor of type II for irrational values of a.

1. 3-D foliations and type III factors

Connes mentioned 3-D foliations V which give rise to type III factors. Foliation property requires a slicing of V by a one-form v to which slices are orthogonal (this requires metric).

1. The foliation property requires that v multiplied by suitable scalar is gradient. This gives the integrability conditions $dv = w \wedge v$, $w = -d\psi/\psi = -dlog(\psi)$. Something proportional to $log(\psi)$ can be taken as a third coordinate varying along flow lines of v: the flow defines a continuous sequence of maps of 2-dimensional slice to itself.

2. If the so called Godbillon-Vey invariant defined as the integral of $dw \wedge w$ over V is non-vanishing, factor of type III is obtained using Schrödinger amplitudes for which the flow lines of foliation define the time evolution. The operators of the algebra in question are transversal operators acting on Schrödinger amplitudes at each slice. Essentially Schrödinger equation in 3-D space-time would be in question with factor of type III resulting from the exotic choice of the time coordinate defining the slicing.

2. What happens in case of light-like 3-surfaces?

In TGD light-like 3-surfaces are natural candidates for V and it is interesting to look what happens in this case. Light-likeness is of course a disturbing complication since orthogonality condition and thus contravariant metric is involved with the definition of the slicing. Light-likeness is not however involved with the basic conditions.

1. The one-form v defined by the induced Kähler gauge potential A defining also a braiding is a unique identification for v. If foliation exists, the braiding flow defines a continuous sequence of maps of partonic 2-surface to itself.

2. Physically this means the possibility of a super-conducting phase with order parameter satisfying covariant constancy equation $D\psi = (d/dt - ieA)\psi = 0$. This would describe a supra current flowing along flow lines of A.

3. If the integrability fails to be true, one *cannot* assign Schrödinger time evolution with the flow lines of v. One might perhaps say that 3-surface behaves like single quantum event not allowing slicing by a continuous Schrödinger time evolution.

4. The condition that the modes of the induced spinor field have well-defined em charge implies that CP_2 projection for the region of space-time in which induced spinor field is non-vanishing is 2-dimensional. In the generic case a localization to 2-surfaces - string world sheets and possibly partonic 2-surface. At light-like 3-surfaces this implies that modes are localized at 1-D curves so that the hydrodynamic picture is realized [K87].

3. Extremals of Kähler action

Some comments relating to the interpretation of the classification of the extremals of Kähler action by the dimension of their CP_2 projection are in order. It has been already found that the extremals can be classified according to the dimension D of the CP_2 projection of space-time sheet in the case that $A_a = 0$ holds true.

1. For $D_{CP_2} = 2$ integrability conditions for the vector potential can be satisfied for $A_a = 0$ so that one has generalized Beltrami flow and one can speak about Schrödinger time evolution associated with the flow lines of vector potential defined by covariant constancy condition $D\psi = 0$ makes sense. Kähler current is vanishing or light-like. This phase is analogous to a super-conductor or a ferromagnetic phase. For non-vanishing A_a the Beltrami flow property is lost but the analogy with ferromagnetism makes sense still.

2. For $D_{CP_2} = 3$ foliations are lost. The phase is dominated by helical structures. This phase is analogous to spin glass phase around phase transition point from ferromagnetic to non-magnetized phase and expected to be important in living matter systems.

3. $D_{CP_2} = 4$ is analogous to a chaotic phase with vanishing Kähler current and to a phase without magnetization. The interpretation in terms of non-quantum coherent "dead" matter is suggestive.

An interesting question is whether the ordinary 8-D imbedding space which defines one sector of the generalized imbedding space could correspond to $A_a = 0$ phase. If so, then all states for this sector would be vacua with respect to M^4 quantum numbers. M^4-trivial zero energy states in this sector could be transformed to non-trivial zero energy states by a leakage to other sectors.

3.3.7 About small perturbations of field equations

The study of small perturbations of the known solutions of field equations is a standard manner to get information about the properties of the solutions, their stability in particular. Fourier expansion is the standard manner to do the perturbation theory. In the recent case an appropriate modification of this ansatz might make sense if the solution in question is representable as a map $M^4_+ \to CP_2$, and the perturbations are rapidly varying when compared to the components of the induced metric and Kähler form so that one can make adiabatic approximation and approximate them as being effectively constant. Presumably also restrictions on directions of wave 4-vectors $k_\mu = (\omega, \overline{k}))$ are necessary so that the direction of wave vector adapts to the slowly varying background as in ray optics. Also Hamilton Jacobi structure is expected to modify the most straightforward approach. The four CP_2 coordinates are the dynamical variables so that the situation is relatively simple.

A completely different approach is inspired by the physical picture. In this approach one glues CP_2 type vacuum extremal to a known extremal and tries to deduce the behavior of the deformed extremal in the vicinity of wormhole throat by posing the general conditions on the slicing by light-like 3-surfaces Y^3_l. This approach is not followed now.

Generalized plane waves

Individual plane waves are geometrically very special since they represent a deformation of the space-time surface depending on single coordinate only. Despite this one might hope that plane waves or their appropriate modifications allowing to algebraize the treatment of small perturbations could give useful information also now.

1. Lorentz invariance plus the translational invariance due to the assumption that the induced metric and Kähler form are approximately constant encourage to think that the coordinates reduce Minkowski coordinates locally with the orientation of the local Minkowski frame depending slowly on space-time position. Hamilton Jacobi $(S^+, S^-, w, \overline{w})$ are a good candidate for this kind of coordinates. The properties of the Hamilton Jacobi structure and of the solution ansatz suggest that excitations are generalized plane waves in longitudinal degrees of freedom only so that four-momentum would be replaced by the longitudinal momentum. In transverse degrees of freedom one might expect that holomorphic plane-waves $exp(ik_T w)$, where k_T is transverse momentum, make algebraization possible.

   For time-like longitudinal momenta one can choose the local M^4 coordinates in such a manner that longitudinal momentum reduces to $(\omega_0, 0)$, where ω_0 plays the role of rest mass and is analogous to the plasma frequency serving as an infrared cutoff for plasma waves. In these coordinates the simplest candidates for excitations with time-like momentum would be of form $\Delta s^k = \epsilon a^k exp(i\omega_0 u)$, where s^k are some real coordinates for CP_2, a^k are Fourier coefficients, and time-like coordinate is defined as $u = S^+ + S^-$. The excitations moving with light velocity correspond to $\omega_0 = 0$, and one must treat this case separately using plane wave $exp(i\omega S^\pm)$, where ω has continuum of values.

2. It is possible that only some preferred CP_2 coordinates are excited in longitudinal degrees of freedom. For $D_{CP_2} = 3$ ansatz the simplest option is that the complex CP_2 coordinate ξ depends analytically on w and the longitudinal CP_2 coordinate s obeys the plane wave ansatz. $\xi(w) = a \times exp(ik_T w)$, where k_T is transverse momentum allows the algebraization of the solution ansatz also in the transversal degrees of freedom so that a dispersion relation results. For imaginary values of k_T and ω the equations are real.

2. General form for the second variation of the field equations

For time-like four-momentum the second variation of field equations contains three kinds of terms. There are terms quadratic in ω_0 and coming from the second derivatives of the deformation, terms proportional to $i\omega_0$ coming from the variation with respect to the derivatives of CP_2 coordinates, and terms which do not depend on ω_0 and come from the variations of metric and Kähler form with respect to the CP_2 coordinates.

In standard perturbation theory the terms proportional to $i\omega_0$ would have interpretation as analogs of dissipative terms. This forces to assume that ω_0 is complex: note that in purely imaginary ω_0 the equations are real. The basic assumption is that Kähler action is able to mimic dissipation despite the fact that energy and momentum are conserved quantities. The vanishing of the Lorentz force has an interpretation as the vanishing of the dissipative effects. This would suggest that the terms proportional to $i\omega_0$ vanish for the perturbations of the solution preserving the non-dissipative character of the asymptotic solutions. This might quite well result from the vanishing of the contractions with the deformation of the energy momentum tensor with the second fundamental form and of energy momentum tensor with the deformation of the second fundamental form coming from first derivatives.

Physical intuition would suggest that dissipation-less propagation is possible only along special directions. Thus the vanishing of the linear terms should occur only for special directions of the longitudinal momentum vector, say for light-like four-momenta in the direction of coordinate lines of S^+ or S^-. Quite generally, the sub-space of allowed four-momenta is expected to depend on position since the components of metric and Kähler form are slowly varying. This dependence is completely analogous with that appearing in the Hamilton Jacobi (ray-optics) approach to the approximate treatment of wave equations and makes sense if the phase of the plane wave varies rapidly as compared to the variation of CP_2 coordinates for the unperturbed solution.

Complex values of ω_0 are also possible, and would allow to deduce important information about the rate at which small deviations from asymptotia vanish as well as about instabilities of the asymptotic solutions. In particular, for imaginary values of ω_0 one obtains completely well-defined solution ansatz representing exponentially decaying or increasing perturbation.

High energy limit

One can gain valuable information by studying the perturbations at the limit of very large four-momentum. At this limit the terms which are quadratic in the components of momentum dominate and come from the second derivatives of the CP_2 coordinates appearing in the second fundamental form. The resulting equations reduce for all CP_2 coordinates to the same condition

$$T^{\alpha\beta}k_\alpha k_\beta = 0 \ .$$

This condition is generalization of masslessness condition with metric replaced by the energy momentum tensor, which means that light velocity is replaced by an effective light velocity. In fact, energy momentum tensor effectively replaces metric also in the modified Dirac equation whose form is dictated by super symmetry. Light-like four momentum is a rather general solution to the condition and corresponds to $\omega_0 = 0$ case.

Reduction of the dispersion relation to the graph of swallowtail catastrophe

Also the general structure of the equations for small perturbations allows to deduce highly non-trivial conclusions about the character of perturbations.

1. The equations for four CP_2 coordinates are simultaneously satisfied if the determinant associated with the equations vanishes. This condition defines a 3-dimensional surface in the 4-dimensional space defined by ω_0 and coordinates of 3-space playing the role of slowly varying control parameters. 4×4 determinant results and corresponds to a polynomial which is of order $d = 8$ in ω_0. If the determinant is real, the polynomial can depend on ω_0^2 only so that a fourth order polynomial in $w = \omega_0^2$ results.

2. Only complex roots are possible in the case that the terms linear in $i\omega_0$ are non-vanishing. One might hope that the linear term vanishes for certain choices of the direction of slowly varying four-momentum vector $k^\mu(x)$ at least. For purely imaginary values of ω_0 the equations determinant are real always. Hence catastrophe theoretic description applies in this case at least, and the so called swallow tail [A102] with three control parameters applies to the situation.

3. The general form of the vanishing determinant is

$$D(w,a,b,c) = w^4 - ew^3 - cw^2 - bw - a \ .$$

The transition from the oscillatory to purely dissipative case changes only the sign of w. By the shift $w = \hat{w} + e/4$ the determinant reduces to the canonical form

$$D(\hat{w},a,b,c) = \hat{w}^4 - c\hat{w}^2 - b\hat{w} - a$$

of the swallowtail catastrophe. This catastrophe has three control variables, which basically correspond to the spatial 3-coordinates on which the induced metric and Kähler form depend. The variation of these coefficients at the space-time sheet of course covers only a finite region of the parameter space of the swallowtail catastrophe. The number of real roots for $w = \omega_0^2$ is four, two, or none since complex roots appear in complex conjugate pairs for a real polynomial. The general shape of the region of 3-space is that for a portion of swallow tail catastrophe.

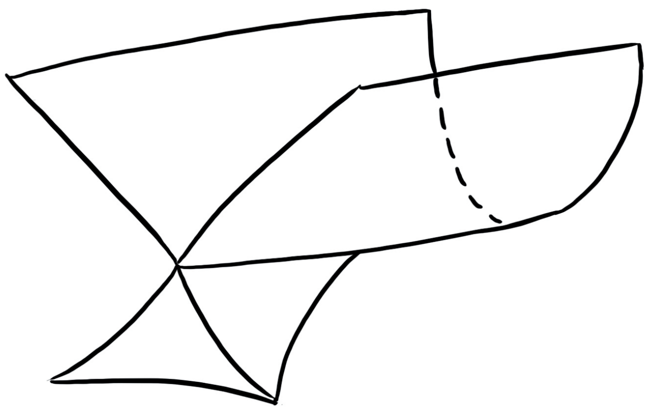

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Figure 3.1: The projection of the bifurcation set of the swallowtail catastrophe to the 3-dimensional space of control variables. The potential function has four extrema in the interior of the swallowtail bounded by the triangles, no extrema in the valley above the swallowtail, and 2 extrema elsewhere.

4. The dispersion relation for the "rest mass" ω_0 (decay rate for the imaginary value of ω_0) has at most four real branches, which conforms with the fact that there are four dynamical variables. In real case ω_0 is analogous to plasma frequency acting as an infrared cutoff for the frequencies of plasma excitations. To get some grasp on the situation notice that for $a = 0$ the swallowtail reduces to $\hat{w} = 0$ and

$$\hat{w}^3 - c\hat{w} - b = 0 \ ,$$

which represents the cusp catastrophe easy to illustrate in 3-dimensional space. Cusp in turn reduces for $b = 0$ to $\hat{w} = 0$ and fold catastrophe $\hat{w} = \pm\sqrt{c}$. Thus the catastrophe surface becomes 4-sheeted for $c \geq 0$ for sufficiently small values of the parameters a and b. The possibility of negative values of $\hat{w}$ in principle allows $\omega^2 = \hat{w} + e/4 < 0$ solutions identifiable as exponentially decaying or amplified perturbations. At the high frequency limit the 4 branches degenerate to a single branch $T^{\alpha\beta}k_\alpha k_\beta = 0$, which as a special case gives light-like four-momenta corresponding to $\omega_0 = 0$ and the origin of the swallowtail catastrophe.

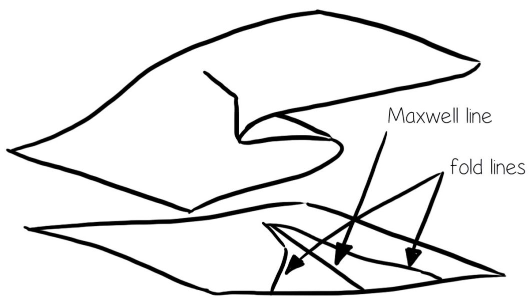

Figure 3.2: Cusp catastrophe. Vertical direction corresponds to the behavior variable and orthogonal directions to control variables.

5. It is quite possible that the imaginary terms proportional to $i\omega_0$ cannot be neglected in the time-like case. The interpretation would be as dissipative effects. If these effects are not too large, an approximate description in terms of butterfly catastrophe makes still sense. Note however that the second variation contains besides gravitational terms potentially large dissipative terms coming from the variation of the induced Kähler form and from the variation of CP_2 Christoffel symbols.

6. Additional complications are encountered at the points, where the induced Kähler field vanishes since the second variation vanishes identically at these points. By the arguments represented earlier, these points quite generally represent instabilities.

3.4 Vacuum extremals

Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance in TGD universe. Vacuum extremals are not gravitational vacua and they are indeed fundamental in TGD inspired cosmology.

3.4.1 CP_2 type extremals

CP_2 type vacuum extremals

These extremals correspond to various isometric imbeddings of CP_2 to $M^4_+ \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the imbedding is given by the formula

$$\begin{aligned} m^k &= m^k(u) \ , \\ m_{kl}\dot{m}^k\dot{m}^l &= 0 \ , \end{aligned} \tag{3.4.1}$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1,2,3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of

CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-sub-manifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the p-adic QFT limit of TGD. Furthermore, it has turned out that Super Virasoro invariance is a general symmetry of WCW geometry and quantum TGD and appears both at the level of imbedding space and space-time surfaces.

The action for all extremals is same and given by the Kähler action for the imbedding of CP_2. The value of the action is given by

$$S = -\frac{\pi}{8\alpha_K} . \tag{3.4.2}$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action.

The absolute minimization of Kähler action was the original suggestion for what preferred extremal property could mean, and suggested that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. The same conclusion however follows also from the mere vacuum degeneracy of Käbler action. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. This identification seems reasonable: the 4-D lines of generalized Feynman graphs [K29] would be regions with Euclidian signature of induced metric and identifiable as deformations of CP_2 type vacuum extremals, and even TGD counterparts of blackholes would be analogous to lines of Feynman diagrams. Their M^4 projection would be of course arbitrarily of macroscopic size. The nice generalization of the area law for the entropy of black hole [K25] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere zitterbewegung so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [A66] . A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of "colorons": states with zero momentum but non-vanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

Are CP_2 type non-vacuum extremals possible?

The isometric imbeddings of CP_2 are all vacuum extremals so that these extremals as such cannot correspond to physical particles. One obtains however non-vacuum extremals as deformations of these solutions. There are several types of deformations leading to non-vacuum solutions. In order to describe some of them, recall the expressions of metric and Kähler form of CP_2 in the coordinates (r,Θ,Ψ,Φ) [A91] are given by

$$\begin{aligned}
\frac{ds^2}{R^2} &= \frac{dr^2}{(1+r^2)^2}+\frac{r}{2(1+r^2))^2}(d\Psi+cos(\Theta)d\Phi)^2\\
&+ \frac{r^2}{(4(1+r^2)}(d\Theta^2+sin^2\Theta d\Phi^2)\ ,\\
J &= \frac{r}{(1+r^2)}dr\wedge(d\Psi+cos(\Theta)d\Phi)\\
&- \frac{r^2}{(2(1+r^2)}sin(\Theta)d\Theta\wedge d\Phi\ .
\end{aligned} \tag{3.4.3}$$

The scaling of the line element is defined so that πR is the length of the CP_2 geodesic line. Note that Φ and Ψ appear as "cyclic" coordinates in metric and Kähler form: this feature plays important role in the solution ansatze to be described.

Let $M^4=M^2\times E^2$ denote the decomposition of M^4 to a product of 2-dimensional Minkowski space and 2-dimensional Euclidian plane. This decomposition corresponds physically to the decomposition of momentum degrees of freedom for massless particle: E^2 corresponds to polarization degrees of freedom.

There are several types of non-vacuum extremals.

1. "Virtual particle" extremals: the mass spectrum is continuous (also Euclidian momenta are allowed) but these extremals reduce to vacuum extremals in the massless limit.

2. Massless extremals.

Consider first an example of virtual particle extremal. The simplest extremal of this type is obtained in the following form

$$m^k = a^k\Psi+b^k\Phi\ . \tag{3.4.4}$$

Here a^k and b^k are some constant quantities. Field equations are equivalent to the conditions expressing four-momentum conservation and are identically satisfied the reason being that induced metric and Kähler form do not depend on the coordinates Ψ and Φ.

Extremal describes 3-surface, which moves with constant velocity in M^4. Four-momentum of the solution can be both space and time like. In the massless limit solution however reduces to a vacuum extremal. Therefore the interpretation as an off mass shell massless particle seems appropriate.

Massless extremals are obtained from the following solution ansatz.

$$\begin{aligned}
m^0 &= m^3 = a\Psi+b\Phi\ ,\\
(m^1,m^2) &= (m^1(r,\Theta),m^2(r,\Theta))\ .
\end{aligned} \tag{3.4.5}$$

Only E^2 degrees of freedom contribute to the induced metric and the line element is obtained from

$$ds^2 = ds^2_{CP_2}-(dm^1)^2-(dm^2)^2\ . \tag{3.4.6}$$

Field equations reduce to conservation condition for the componenents of four-momentum in E^2 plane. By their cyclicity the coordinates Ψ and Φ disappear from field equations and one obtains essentially current conservation condition for two-dimensional field theory defined in space spanned by the coordinates r and Θ.

$$\begin{aligned} (J_a^i)_{,i} &= 0 \ , \\ J_a^i &= T^{ij} f^a_{,j} \sqrt{g} \ . \end{aligned} \qquad (3.4.7)$$

Here the index i and a refer to r and Θ and to E^2 coordinates m^1 and m^2 respectively. T^{ij} denotes the canonical energy momentum tensor associated with Kähler action. One can express the components of T^{ij} in terms of induced metric and CP_2 metric in the following form

$$T^{ij} = (-g^{ik}g^{jl} + g^{ij}g^{kl}/2)s_{kl} \ . \qquad (3.4.8)$$

This expression holds true for all components of the energy momentum tensor.

Since field equations are essentially two-dimensional conservation conditions they imply that components of momentum currents can be regarded as vector fields of some canonical transformations

$$J_a^i = \varepsilon^{ij} H^a_{,j} \ , \qquad (3.4.9)$$

where ε^{ij} denotes two-dimensional constant symplectic form. An open problem is whether one could solve field equations exactly and whether there exists some nonlinear superposition principle for the solutions of these equations. Solutions are massless since transversal momentum densities vanish identically.

Consider as a special case the solution obtained by assuming that one E^2 coordinate is constant and second coordinate is function $f(r)$ of the variable r only. Field equations reduce to the following form

$$f_{,r} = \pm \frac{k}{(1+r^2)^{1/3}} \sqrt{r^2 - k^2(1+r^2)^{4/3}} \ . \qquad (3.4.10)$$

The solution is well defined only for sufficiently small values of the parameter k appearing as integration constant and becomes ill defined at two singular values of the variable r. Boundary conditions are identically satisfied at the singular values of r since the radial component of induced metric diverges at these values of r. The result leads to suspect that the generation of boundary components dynamically is a general phenomenon so that all non-vacuum solutions have boundary components in accordance with basic ideas of TGD.

$CP_2\#CP_2\#...\#CP_2$:s as generalized Feynman graphs

There are reasons to believe that point like particles might be identified as CP_2 type extremals in TGD approach. Also the geometric counterparts of the massless on mass shell particles and virtual particles have been identified. It is natural to extend this idea to the level of particle interactions: the lines of Feynman diagrams of quantum field theory are thickened to four-manifolds, which are in a good approximation CP_2 type vacuum extremals. This would mean that generalized Feynman graphs are essentially connected sums of CP_2:s (see Fig. 3.4.1): $X^4 = CP_2\#CP_2....\#CP_2$).

Unfortunately, this picture seems to be oversimplified. First, it is questionable whether the cross sections for the scattering of CP_2 type extremals have anything to do with the cross sections associated with the standard gauge interactions. A naive geometric argument suggests that the cross section should reflect the geometric size of the scattered objects and therefore be of the order of CP_2 radius for topologically non-condensed CP_2 type extremals. The observed cross sections would result at the first level of condensation, where particles are effectively replaced by surfaces with size of order Compton length. Secondly, the $h_{vac} = -D$ rule, considered in the previous chapter, suggests that only real particles correspond to the CP_2 type extremals whereas virtual particles in general correspond to the vacuum extremals with a vanishing Kähler action. The reason is that the negative exponent of the Kähler action reduces the contribution of the CP_2 type extremals to the functional integral very effectively. Therefore the exchanges of CP_2 type extremals are suppressed by the negative exponent of the Kähler action very effectively so that geometric scattering cross section is obtained.

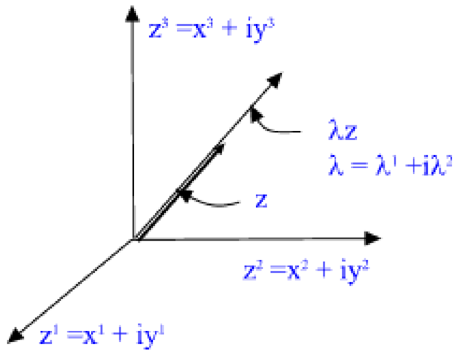

Figure 3.3: Topological sum of CP_2:s as Feynman graph with lines thickened to four-manifolds

3.4.2 Vacuum extremals with vanishing Kähler field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Legendre manifold. The condition expressing Legendre manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A = \sum_k P_k dQ^k \ . \tag{3.4.11}$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) \ , \tag{3.4.12}$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M^4_+ diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the WCW in the proposed construction of the configuration space metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D_{CP_2} = 3$ for CP_2 type extremals, $D_{CP_2} = 2$ for string like objects, $D_{CP_2} = 1$ for membranes and $D_{CP_2} = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynman diagram is not suppressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M^4_+ \rightarrow CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenuities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

The reason would be "Yin-Yang principle".

1. Consider first the option for which Kähler function corresponds to an absolute minimum of Kähler action. Vacuum functional as an exponent of Kähler function is expected to concentrate on those 3-surfaces for which the Kähler action is non-negative. On the other hand, the requirement that Kähler action is absolute minimum for the space-time associated with a given 3-surface, tends to make the action negative. Therefore the vacuum functional is expected to differ considerably from zero only for 3-surfaces with a vanishing Kähler action per volume. It could also occur that the degeneracy of 3-surfaces with same large negative action compensates the exponent of Kähler function.

2. If preferred extrema correspond to Kähler calibrations or their duals [K67] , Yin-Yang principle is modified to a more local principle. For Kähler calibrations (their duals) the absolute value of action in given region is minimized (maximized). A given region with positive (negative sign) of action density favors Kähler electric (magnetic) fields. In long length scales the average density of Kähler action per four-volume tends to vanish so that Kähler function of the entire universe is expected to be very nearly zero. This regularizes the theory automatically and implies that average Kähler action per volume vanishes. Positive and finite values of Kähler function are of course favored.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M^4_+ \to D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

3.5 Non-vacuum extremals

3.5.1 Cosmic strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M^4_+ (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2. The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the absolute minimization of the Kähler action is global rather than a local principle. A more general approach gives up absolute minimization as definition of preferred extremal property and there are indeed several proposals for what preferred extremal property could mean. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T = \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} \ , \tag{3.5.1}$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

3.5.2 Massless extremals

Massless extremals (or topological light rays) are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$\begin{aligned} &s^k = f^k(u,v) \ , \\ &u = p\cdot m \ , \qquad v = \varepsilon \cdot m \ , \\ &p\cdot \varepsilon = 0 \ , \qquad p^2 = 0 \ . \end{aligned}$$

CP_2 coordinates are arbitrary functions of $p\cdot m$ and $\varepsilon \cdot m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2+m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta} \propto p^\alpha p^\beta$ the conditions $T^{n\beta} = 0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$\begin{aligned} &H(p\cdot m, \varepsilon\cdot m, \varepsilon_1\cdot m) = 0 \ , \\ &\qquad \varepsilon \cdot p = 0 \ , \qquad \varepsilon_1 \cdot p = 0 \ , \quad \varepsilon\cdot\varepsilon_1 = 0 \ . \end{aligned} \tag{3.5.2}$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many manners to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta} J^k_{\ l} \partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a manner to satisfy all boundary conditions but it is not clear whether there are any other manners to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamcis this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are bounded this is in accordance with the general ideas about topological condensation. The fact that massless plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

3.5.3 Generalization of the solution ansatz defining massless extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

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Local light cone coordinates

The solution involves a decomposition of M^4_+ tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2 \oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct *orthogonal* sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray.

2. With these assumptions the coordinates $(S_\pm, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 = 2g_{+-}dS^+dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{3.5.3}$$

   If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.

3. This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

A conformally invariant family of local light cone coordinates

The simplest solutions to the equations defining local light cone coordinates are of form $S_\pm = k \cdot m$ giving as a special case $S_\pm = m^0 \pm m^3$. For more general solutions of from

$$S_\pm = m^0 \pm f(m^1, m^2, m^3) \ , \ (\nabla_3 f)^2 = 1 \ ,$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \ .$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many manners.

Closer inspection of the conditions defining local light cone coordinates

Whether the conformal transforms of the local light cone coordinates $\{S_\pm = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

1. Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.

2. The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement

$$\epsilon_i \rightarrow \hat{\epsilon}_i = \epsilon_i + k\nabla S_+ \ ,$$

where k is any function. With the choice

$$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-}$$

one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.

3. The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition

$$k = k(S^+)$$

is satisfiedin this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.

4. The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions

$$(\nabla S^+)^2 = (\nabla S^-)^2 = 0 \ , \quad \nabla S^+ \cdot \nabla S^- \neq 0 \ ,$$

$$\nabla S^+ \cdot \nabla E^i = 0 \ , \qquad \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \ .$$

The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \rightarrow -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M^4_+ must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

General solution ansatz for MEs for given choice of local light cone coordinates

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M^4_+ tangent space has been found.

1. Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.

2. Quite a general family of MEs corresponds to the solution family of the field equations having the general form

$$s^k = f^k(S^+, E) \ ,$$

where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

3. The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then 'free' solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.

4. For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.

5. One can try to generalize the solution ansatz further by allowing the metric of M^4_+ to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm} = 0$ holds true for the induced metric. For $s^k = s^k(S^+, E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the imbedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 \ = \ 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{3.5.4}$$

$g_{1+} = 0$ can be achieved by an additional condition

$$m_{1+} \ = \ s_{kl}\partial_1 s^k \partial_+ s^k \ . \tag{3.5.5}$$

The diagonalization of the metric seems to be a general aspect of preferred extremals. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

1. The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M^4_+ can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k \cdot m$ and $\tilde{k} \cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.

2. Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d = 2$ and is characterized by an infinite-dimensional super-symplectic and super-conformal symmetries just like the boundary of the imbedding space $M^4_+ \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).

3. These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the absolute minimization of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d = 2$. This does not yet imply that light like surfaces of imbedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

3.5.4 Maxwell phase

"Maxwell phase" corresponds to small deformations of the M^4 type vacuum extremals. Since energy momentum tensor is quadratic in Kähler field the term proportional to the contraction of the energy momentum tensor with second fundamental form drops from field equations and one obtains in lowest order the following field equations

$$j^\alpha J^k_{\ l} s^l_{,\alpha} = 0 \ . \tag{3.5.6}$$

These equations are satisfied if Maxwell's equations

$$j^\alpha = 0 \tag{3.5.7}$$

hold true. Massless extremals and Maxwell phase clearly exclude each other and it seems that they must corresponds to different space-time sheets.

The explicit construction of these extremals reduces to the task of finding an imbedding for an arbitrary free Maxwell field to H. One can also allow source terms corresponding to the presence of the point like charges: these should correspond to the regions of the space-time, where the flat space-time approximation of the space-time fails. The regions where the approximation defining the Maxwell phase fails might correspond to a topologically condensed CP_2 type extremals, for example. As a consequence, Kähler field is superposition of radiation type Kähler field and of Coulomb term. A second possibility is the generation of "hole" with similar Coulombic Kähler field.

An important property of the Maxwell phase (also of massless extremals) is its approximate canonical invariance. Canonical transformations do not spoil the extremal property of the four-surface in the approximation used, since it corresponds to a mere $U(1)$ gauge transformation. This implies the counterpart of the vacuum degeneracy, that is, the existence of an enormous number of four-surfaces with very nearly the same action. Also there is an approximate $Diff(M^4_+)$ invariance.

The canonical degeneracy has some very interesting consequences concerning the understanding of the electro-weak symmetry breaking and color confinement. Kähler field is canonical invariant and satisfies Maxwells equations. This is in accordance with the identification of Kähler field as $U(1)$ part of the electro-weak gauge field. Electromagnetic gauge field is a superposition of Kähler field and Z^0 field$\gamma = 3J - sin^2(\theta_W)Z^0/2$ so that also electromagnetic gauge field is long ranged assuming that Z^0 and W^+ fields are short ranged. These fields are not canonical invariants and their behavior seems to be essentially random, which implies short range correlations and the consequent massivation.

There is an objection against this argument. For the known $D < 4$ solutions of field equations weak fields are not random at all. These situations could represent asymptotic configurations assignable to space-time sheets. This conforms with the interpretation that weak gauge fields are essentially massless within the asymptotic space-time sheets representing weak bosons. Gauge fields are however transferred between space-time sheets through # contacts modellable as pieces of CP_2 type extremals having $D_{CP_2} = 4$. In contrast to Kähler and color gauge fluxes, weak gauge fluxes are not conserved in the Euclidian time evolution between the 3-D causal horizons separating the Euclidian # contact from space-time sheets with Minkowskian signature. This non-conservation implying the loss of coherence in the transfer of fields between space-time sheets is a plausible mechanism for the loss of correlations and massivation of the weak gauge fields.

Classical gluon fields are proportional to Kähler field and to the Hamiltonians associated with the color isometry generators.

$$g^A_{\alpha\beta} = kH^A J_{\alpha\beta} \ . \tag{3.5.8}$$

This implies that the direction of gluon fields in color algebra is random. One can always perform a canonical transformation, which reduces to a global color rotation in some arbitrary small region of space-time and reduces to identity outside this region. The proportionality of a gluon field to Kähler form implies that there is a classical long range correlation in X^4 degrees of freedom: in this sense classical gluon fields differ from massive electro-weak fields in Maxwell phase.

3.5.5 Stationary, spherically symmetric extremals

The stationary, spherically symmetric extremals of the Kähler action imbeddable in $M^4 \times S^2$, where S^2 is geodesic sphere, are the simplest extremals, which one can study as models for the space-time surrounding a topologically condensed particle, say CP_2 type vacuum extremal. In the region near the particle the spherical symmetry is an unrealistic assumption since it excludes the presence of magnetic fields needed to cancel the total Kähler action. The stationarity is also unrealistic assumption since zitterbewegung seems to provide a necessary mechanism for generating Kähler magnetic field and for satisfying boundary conditions. Also the imbeddability to $M^4 \times S^2$ implies unrealistic relationship between Z^0 and photon charges.

According to the general wisdom, the generation of a Kähler electric field must take place in order to minimize the action and it indeed turns out that the extremal is characterized by essentially $1/r^2$ Kähler electric field. The necessary presence of a hole or of a topologically condensed object is also demonstrated: it is impossible to find extremals well defined in the region surrounding the origin. It is impossible to satisfy boundary conditions at a hole: this is in accordance with the idea that Euclidian region corresponding to a CP_2 type extremal performing zitterbewegung is generated. In case of CP_2 extremal radius is of the order of the Compton length of the particle and in case of a "hole" of the order of Planck length. The value of the vacuum frequency ω is of order of particle mass whereas for macroscopic vacuum extremals it must be of the order of $1/R$. This does not lead to a contradiction if the concept of a many-sheeted space-time is accepted.

The Poincare energy of the exterior region is considerably smaller than the gravitational mass; this conforms with the interpretation that gravitational mass is sum of absolute values of positive and negative inertial masses associated with matter and negative energy antimatter. It is quite possible that classical considerations cannot provide much understanding concerning the inertial masses of topologically condensed particles. Electro-weak gauge forces are considerably weaker than the gravitational force at large distances, when the value of the frequency parameter ω is of order $1/R$. Both these desirable properties fail to be true if CP_2 radius is of order Planck length as believed earlier.

In light of the general ideas about topological condensation it is clear that in planetary length scales these kind of extremals cannot provide a realistic description of space-time. Indeed, spherically symmetric extremals predict a wrong rate for the precession of the perihelion of Mercury. Scwhartschild and Reissner-Nordström metric do this and indeed allow imbedding as vacuum extremals for which the inertial masses of positive energy matter and negative energy antimatter sum up to zero.

This does not yet resolve the interpretational challenge due to the unavoidable long range color and weak gauge fields. A dark matter hierarchy giving rise to a hierarchy of color and electro-weak physics characterized by increasing values of weak and confinement scales explains these fields. # contacts involve a pair of causal horizons at which the Euclidian metric signature of # contact transforms to Minkowskian one. These causal horizons have interpretation as partons so that # contact can be regarded as a bound state of partons bound together by a gravitational instanton (CP_2 type extremal). # contacts provide basic example of dark matter creating long ranged weak fields.

An important result is the correlation between the sign of the vacuum frequency ω and that of the Kähler charge, which is of opposite sign for fermions and anti-fermions. This suggests an explanation for matter-antimatter asymmetry. Matter and antimatter condense stably on disjoint regions of the space-time surface at different space-time sheets. Stable antimatter could correspond to negative time orientation and negative energy. This leads to a model for the primordial generation of matter as spontaneous generation of zero energy # contacts between space-time sheets of opposite time orientations. If CP conjugation is not exact symmetry, # contacts and their CP conjugates are created with slightly different rates and this gives rise to CP asymmetry at each of the two space-time sheets involved. After the splitting of # contacts and subsequent annihilation of particles and antiparticles at each space-time sheet, the two space-time sheets contain only positive energy matter and negative energy antimatter.

General solution ansatz

The general form of the solution ansatz is obtained by assuming that the space-time surface in question is a sub-manifold of $M^4 \times S^2$, where S^2 is the homologically non-trivial geodesic sphere of CP_2. S^2 is most conveniently realized as $r = \infty$ surface of CP_2, for which all values of the coordinate Ψ correspond to same point of CP_2 so that one can use Θ and Φ as the coordinates of S^2.

The solution ansatz is given by the expression

$$\begin{aligned} cos(\Theta) &= u(r) \ , \\ \Phi &= \omega t \ , \\ m^0 &= \lambda t \ , \\ r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ . \end{aligned} \tag{3.5.9}$$

The induced metric is given by the expression

$$ds^2 = \left[\lambda^2 - \frac{R^2}{4}\omega^2(1-u^2)\right]dt^2 - (1+\frac{R^2}{4}\theta_{,r}^2)dr^2 - r^2 d\Omega^2 \ . \tag{3.5.10}$$

The value of the parameter λ is fixed by the condition $g_{tt}(\infty) = 1$:

$$\lambda^2 - \frac{R^2}{4}\omega^2(1-u(\infty)^2) = 1 \ . \tag{3.5.11}$$

From the condition $e^0 \wedge e^3 = 0$ the non-vanishing components of the induced Kähler field are given by the expression

$$J_{tr} = \frac{\omega}{4}u_{,r} \ . \tag{3.5.12}$$

Geodesic sphere property implies that Z^0 and photon fields are proportional to Kähler field:

$$\begin{aligned} \gamma &= (3-p/2)J \ , \\ Z^0 &= J \ . \end{aligned} \tag{3.5.13}$$

From this formula one obtains the expressions

$$\begin{aligned} Q_{em} &= \frac{(3-p/2)}{4\pi\alpha_{em}} Q_K \ , \quad Q_Z = \frac{1}{4\pi\alpha_Z} Q \ , \\ Q &\equiv \frac{J_{tr} 4\pi r^2}{\sqrt{-g_{rr}g_{tt}}} \ . \end{aligned} \tag{3.5.14}$$

for the electromagnetic and Z^0 charges of the solution using e and g_Z as unit.

Field equations can be written as conditions for energy momentum conservation (two equations is in principle all what is needed in the case of geodesic sphere). Energy conservation holds identically true and conservation of momentum, say, in z-direction gives the equation

$$(T^{rr} z_{,r})_{,r} + (T^{\theta\theta} z_{,\theta})_{,\theta} = 0 \ . \tag{3.5.15}$$

Using the explicit expressions for the components of the energy momentum tensor

$$\begin{aligned} T^{rr} &= g^{rr} L/2 \ , \\ T^{\theta\theta} &= -g^{\theta\theta} L/2 \ , \\ L &= g^{tt} g^{rr} (J_{tr})^2 \sqrt{g}/2 \ , \end{aligned} \tag{3.5.16}$$

and the following notations

$$\begin{aligned} A &= g^{tt} g^{rr} r^2 \sqrt{-g_{tt}g_{rr}} \ , \\ X &\equiv (J_{tr})^2 \ , \end{aligned} \tag{3.5.17}$$

the field equations reduce to the following form

$$(g^{rr} AX)_{,r} - \frac{2AX}{r} = 0 \ . \tag{3.5.18}$$

In the approximation $g^{rr} = 1$ this equation can be readily integrated to give $AX = C/r^2$. Integrating Eq. (7.2.7), one obtains integral equation for X

$$J_{tr} = \frac{q}{r_c} (|g_{rr}|^3 g_{tt})^{1/4} exp(\int_{r_c}^{r} dr \frac{g_{rr}}{r}) \frac{1}{r} \ , \tag{3.5.19}$$

where q is integration constant, which is related to the charge parameter of the long range Kähler electric field associated with the solution. r_c denotes the critical radius at which the solution ceases to be well defined.

The inspection of this formula shows that J_{tr} behaves essentially as $1/r^2$ Coulomb field. This behavior doesn't depend on the detailed properties of the solution ansatz (for example the imbeddability to $M^4 \times S^2$): stationarity and spherical symmetry is what matters only. The compactness of CP_2 means that stationary, spherically symmetric solution is not possible in the region containing origin. This is in concordance with the idea that either a hole surrounds the origin or there is a topologically condensed CP_2 extremal performing zitterbewegung near the origin and making the solution non-stationary and breaking spherical symmetry.

Second integration gives the following integral equation for CP_2 coordinate $u = cos(\Theta)$

$$u(r) \quad = \quad u_0 + \frac{4q}{\omega}\int_{r_c}^{r}(-g_{rr}^3 g_{tt})^{1/4}\frac{1}{r}exp(\int_{r_c}^{r} dr\frac{g_{rr}}{r}) \quad . \tag{3.5.20}$$

Here u_0 denotes the value of the coordinate u at $r = r_0$.

The form of the field equation suggests a natural iterative procedure for the numerical construction of the solution for large values of r.

$$u_n(r) \quad = \quad T_{n-1} \quad , \tag{3.5.21}$$

where T_{n-1} is evaluated using the induced metric associated with u_{n-1}. The physical content of the approximation procedure is clear: estimate the gravitational effects using lower order solution since these are expected to be small.

A more convenient manner to solve u is based on Taylor expansion around the point $V \equiv 1/r = 0$. The coefficients appearing in the power series expansion $u = \sum_n u_n A^n V^n \ : \ A = q/\omega$ can be solved by calculating successive derivatives of the integral equation for u.

The lowest order solution is simply

$$u_0 \quad = \quad u_\infty \quad , \tag{3.5.22}$$

and the corresponding metric is flat metric. In the first order one obtains for $u(r)$ the expression

$$u \quad = \quad u_\infty - \frac{4q}{\omega r} \quad , \tag{3.5.23}$$

which expresses the fact that Kähler field behaves essentially as $1/r^2$ Coulomb field. The behavior of u as a function of r is identical with that obtained for the imbedding of the Reissner-Nordström solution.

To study the properties of the solution we fix the signs of the parameters in the following manner:

$$u_\infty < 0 \ , \quad q < 0 \ , \quad \omega > 0 \tag{3.5.24}$$

(reasons become clear later).

Concerning the behavior of the solution one can consider two different cases.

1. The condition $g_{tt} > 0$ hold true for all values of Θ. In this case u decreases and the rate of decrease gets faster for small values of r. This means that in the lowest order the solution becomes certainly ill defined at a critical radius $r = r_c$ given by the the condition $u = 1$: the reason is that u cannot get values large than one. The expression of the critical radius is given by

$$\begin{aligned} r_c \quad &\geq \quad \frac{4q}{(|u_\infty| + 1)\omega} \\ &= \quad \frac{4\alpha Q_{em}}{(3 - p/2)}\frac{1}{(|u_\infty| + 1)\omega} \quad . \end{aligned} \tag{3.5.25}$$

   The presence of the critical radius for the actual solution is also a necessity as the inspection of the expression for J_{tr} shows: $\partial_r\theta$ grows near the origin without bound and $u = 1$ is reached at some finite value of r. Boundary conditions require that the quantity $X = T^{rr}\sqrt{g}$ vanishes at critical radius (no momentum flows through the boundary). Substituting the expression of J_{tr} from the field equation to T^{rr} the expression for X reduces to a form, from which it is clear that X cannot vanish. The cautious conclusion is that boundary conditions cannot be satisfied and the underlying reason is probably the stationarity and spherical symmetry of the solution. Physical intuition suggests that that CP_2 type extremal performing zitterbewegung is needed to satisfy the boundary conditions.

2. g_{tt} vanishes for some value of Θ. In this case the radial derivative of u together with g_{tt} can become zero for some value of $r = r_c$. Boundary conditions can be satisfied only provided $r_c = 0$. Thus it seems that for the values of ω satisfying the condition $\omega^2 = \frac{4\lambda^2}{R^2 sin^2(\Theta_0)}$ it might be possible to find a globally defined solution. The study of differential equation for u however shows that the ansatz doesn't work. The conclusion is that although the boundary is generated it is not possible to satisfy boundary conditions.

A direct calculation of the coefficients u_n from power series expansion gives the following third order polynomial approximation for u ($V = 1/r$)

$$\begin{aligned} u &= \sum_n u_n A^n V^n \ , \\ u_0 &= u_\infty(<0) \ , \quad u_1 = 1 \ , \\ u_2 &= K|u_\infty| \ , \quad u_3 = K(1+4K|u_\infty|) \ , \\ A &\equiv \frac{4q}{\omega} \ , \quad K \equiv \omega^2 \frac{R^2}{4} \ . \end{aligned} \tag{3.5.26}$$

The coefficients u_2 and u_3 are indeed positive which means that the value of the critical radius gets larger at least in these orders.

Solution contains three parameters: Kähler electric flux $Q = 4\pi q$, parameter ωR and parameter u_∞. The latter parameters can be regarded as parameters describing the properties of a flat vacuum extremal (lowest order solution) to which particle like solution is glued and are analogous to the parameters describing symmetry broken vacuum in gauge theories.

Solution is not a realistic model for topological condensation

The solution does not provide realistic model for topological condensation although it gives indirect support for some essential assumptions of TGD based description of Higgs mechanism.

1. When the value of ω is of the order of CP_2 mass the solution could be interpreted as the "exterior metric" of a "hole".
   i) The radius of the hole is of the order of CP_2 length and its mass is of the order of CP_2 mass.
   ii) Kähler electric field is generated and charge renormalization takes place classically at CP_2 length scales as is clear from the expression of $Q(r)$: $Q(r) \propto (\frac{-g_{rr}}{g_{tt}})^{1/4}$ and charge increases at short distances.
   iii) The existence of the critical radius is unavoidable but boundary conditions cannot be satisfied. The failure to satisfy boundary conditions might be related to stationarity or to the absence of magnetic field. The motion of the boundary component with velocity of light might be the only manner to satisfy boundary conditions. Second possibility is the breaking of spherical symmetry by the generation of a static magnetic field.
   iv) The absence of the Kähler magnetic field implies that the Kähler action has an infinite magnitude and the probability of the configuration is zero. A more realistic solution ansatz would break spherical symmetry containing dipole type magnetic field in the nearby region of the hole. The motion of the boundary with a velocity of light could serves as an alternative mechanism for the generation of magnetic field. The third possibility, supported by physical intuition, is that one must give up "hole" type extremal totally.

2. For sufficiently large values of r and for small values of ω (of the order of elementary particle mass scale), the solution might provide an approximate description for the region surrounding elementary particle. Although it is not possible to satisfy boundary conditions the order of magnitude estimate for the size of critical radius ($r_c \simeq \alpha/\omega$) should hold true for more realistic solutions, too. The order of magnitude for the critical radius is smaller than Compton length or larger if the vacuum parameter ω is larger than the mass of the particle. In macroscopic length scales the value of ω is of order $1/R$. This does not lead to a contradiction if the many-sheeted space-time concept is accepted so that $\omega < m$ corresponds to elementary particle

space-time sheet. An unrealistic feature of the solution is that the relationship between Z^0 and em charges is not correct: Z^0 charge should be very small in these length scales.

Exterior solution cannot be identified as a counterpart of Schwartshild solution

The first thing, which comes into mind is to ask whether one might identify exterior solution as the TGD counterpart of the Schwartshild solution. The identification of gravitational mass as absolute value of inertial mass which is negative for antimatter implies that vacuum extremals are vacua only with respect to the inertial four-momentum and have a non-vanishing gravitational four-momentum. Hence, in the approximation that the net density of inertial mass vanishes, vacuum extremals provide the proper manner to model matter, and the identification of the ansatz for a spherically symmetric extremal as the counterpart of Scwhartschild metric is certainly not possible. It is however useful to show explicitly that the identification is indeed unrealistic. The solution is consistent with Equivalence Principle but the electro-weak gauge forces are considerably weaker than gravitational forces. A wrong perihelion shift is also predicted so that the identification as an exterior metric of macroscopic objects is out of question.

1. Is Equivalence Principle respected?

The following calculation demonstrates that Equivalence Principle might not be satisfied for the solution ansatz (which need not actually define a preferred extremal!).

The gravitational mass of the solution is determined from the asymptotic behavior of g_{tt} and is given by

$$M_{gr} = \frac{R^2}{G}\omega q u_{\infty} \ , \tag{3.5.27}$$

and is proportional to the Kähler charge q of the solution.

One can estimate the gravitational mass density also by applying Newtonian approximation to the time component of the metric $g_{tt} = 1 - 2\Phi_{gr}$. One obtains Φ_{gr} corresponds in the lowest order approximation to a solution of Einstein's equations with the source consisting of a mass point at origin and the energy density of the Kähler electric field. The effective value of gravitational constant is however $G_{eg} = 8R^2\alpha_K$. Thus the only sensible interpretation is that the density of Kähler (inertial) energy is only a fraction $G/G_{eq} \equiv \epsilon \simeq .22 \times 10^{-6}$ of the density of gravitational mass. Hence the densities of positive energy matter and negative energy antimatter cancel each other in a good approximation.

The work with cosmic strings lead to a possible interpretation of the solution as a space-time sheet containing topologically condensed magnetic flux tube idealizable as a point. The negative Kähler electric action must cancel the positive Kähler magnetic action. The resulting structure in turn can condense to a vacuum extremal and Schwartshild metric is a good approximation for the metric.

One can estimate the contribution of the exterior region ($r > r_c$) to the inertial mass of the system and Equivalence principle requires this to be a fraction of order ϵ about the gravitational mass unless the region $r < r_c$ contains negative inertial mass density, which is of course quite possible. Approximating the metric with a flat metric and using first order approximation for $u(r)$ the energy reduces just to the standard Coulomb energy of charged sphere with radius r_c

$$\begin{aligned} M_I(ext) &= \frac{1}{32\pi\alpha_K}\int_{r>r_c} E^2\sqrt{g}d^3x \\ &\simeq \frac{\lambda q^2}{2\alpha_K r_c} \ , \\ \lambda &= \sqrt{1+\frac{R^2}{4}\omega^2(1-u_{\infty}^2)} \ (>1) \ . \end{aligned} \tag{3.5.28}$$

Approximating the metric with flat metric the contribution of the region $r > r_c$ to the energy of the solution is given by

$$M_I(ext) = \frac{1}{8\alpha_K}\lambda q\omega(1+|u_\infty|) \ . \tag{3.5.29}$$

The contribution is proportional to Kähler charge as expected. The ratio of external inertial and gravitational masses is given by the expression

$$\begin{aligned} \frac{M_I(ext)}{M_{gr}} &= \frac{G}{4R^2\alpha_K}x \ , \\ x &= \frac{(1+|u_\infty|)}{|u_\infty|} > 1 \ . \end{aligned} \tag{3.5.30}$$

In the approximation used the the ratio of external inertial and gravitational masses is of order 10^{-6} for $R \sim 10^4\sqrt{G}$ implied by the p-adic length scale hypothesis and for $x \sim 1$. The result conforms with the above discussed interpretation.

The result forces to challenge the underlying implicit assumptions behind the calculation.

1. Many-sheeted space-time means that single space-time sheet need not be a good approximation for astrophysical systems. The GRT limit of TGD can be interpreted as obtained by lumping many-sheeted space-time time to Minkowski space with effective metric defined as sum M^4 metric and sum of deviations from M^4 metric for various space-time sheets involved [K72]. This effective metric should correspond to that of General Relativity and Einstein's equations reflect the underlying Poincare invariance. Gravitational and cosmological constants follow as predictions and EP is satisfied.

2. The systems considered need not be preferred extremals of Kähler action so that one cannot take the results obtained too seriously. For vacuum extremals one does not encounter this problem at all and it could be that vacuum extremals with induced metric identified as GRT metric are a good approximation in astrophysical systems. This requires that single-sheetedness is a good approximation. TGD based single-sheeted models for astrophysical and cosmological systems rely on this assumption.

2. Z^0 and electromagnetic forces are much weaker than gravitational force

The extremal in question carries Kähler charge and therefore also Z^0 and electromagnetic charge. This implies long range gauge interactions, which ought to be weaker than gravitational interaction in the astrophysical scales. This is indeed the case as the following argument shows.

Expressing the Kähler charge using Planck mass as unit and using the relationships between gauge fields one obtains a direct measure for the strength of the Z^0 force as compared with the strength of gravitational force.

$$Q_Z \equiv \varepsilon_Z M_{gr}\sqrt{G} \ . \tag{3.5.31}$$

The value of the parameter ε_Z should be smaller than one. A transparent form for this condition is obtained, when one writes $\Phi = \omega t = \Omega m^0 : \Omega = \lambda\omega$:

$$\varepsilon_Z = \frac{\alpha_K}{\alpha_Z}\frac{1}{\pi(1+|u_\infty|)\Omega R}\sqrt{\frac{G}{R}} \ . \tag{3.5.32}$$

The order of magnitude is determined by the values of the parameters $\sqrt{\frac{G}{R^2}} \sim 10^{-4}$ and ΩR. Global Minkowskian signature of the induced metric implies the condition $\Omega R < 2$ for the allowed values of the parameter ΩR. In macroscopic length scales one has $\Omega R \sim 1$ so that Z^0 force is by a factor of order 10^{-4} weaker than gravitational force. In elementary particle length scales with $\omega \sim m$ situation is completely different as expected.

3. The shift of the perihelion is predicted incorrectly

The g_{rr} component of Reissner-Nordström and TGD metrics are given by the expressions

$$g_{rr} = -\frac{1}{(1-\frac{2GM}{r})} \ , \tag{3.5.33}$$

and

$$g_{rr} \simeq 1 - \frac{\frac{Rq}{\omega^2}}{\left[1-(u_\infty - \frac{4q}{\omega r})^2\right] r^4} \ , \tag{3.5.34}$$

respectively. For reasonable values of q, ω and u_∞ the this terms is extremely small as compared with $1/r$ term so that these expressions differ by $1/r$ term.

The absence of the $1/r$ term from g_{rr}-component of the metric predicts that the shift of the perihelion for elliptic plane orbits is about $2/3$ times that predicted by GRT so that the identification as a metric associated with objects of a planetary scale leads to an experimental contradiction. Reissner-Nordström solutions are obtained as vacuum extremals so that standard predictions of GRT are obtained for the planetary motion.

One might hope that the generalization of the form of the spherically symmetric ansatz by introducing the same modification as needed for the imbedding of Reissner-Nordstrm̈ metric might help. The modification would read as

$$\begin{aligned} cos(\Theta) &= u(r) \ , \\ \Phi &= \omega t + f(r) \ , \\ m^0 &= \lambda t + h(r) \ , \\ r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ . \end{aligned} \tag{3.5.35}$$

The vanishing of the g_{tr} component of the metric gives the condition

$$\lambda\partial_r h - \frac{R^2}{4}sin^2(\Theta)\omega\partial_r f = 0 \ . \tag{3.5.36}$$

The expression for the radial component of the metric transforms to

$$g_{rr} \simeq \partial_r h^2 - 1 - \frac{R^2}{4}(\partial_r\Theta)^2 - \frac{R^2}{4}sin^2(\Theta)\partial_r f^2 \ , \tag{3.5.37}$$

Essentially the same perihelion shift as for Schwartschild metric is obtained if g_{rr} approaches asymptotically to its expression for Schwartschild metric. This is guaranteed if the following conditions hold true:

$$f(r)_{r\rightarrow\infty} \rightarrow \omega r \ , \ \Lambda^2 - 1 = \frac{R^2\omega^2}{4}sin^2(\Theta_\infty) \ll \frac{2GM}{\langle r\rangle} \ . \tag{3.5.38}$$

In the second equation $\langle r\rangle$ corresponds to the average radius of the planetary orbit.

The field equations for this ansatz can be written as conditions for energy momentum and color charge conservation. Two equations are enough to determine the functions $\Theta(r)$ and $f(r)$. The equation for momentum conservation is same as before. Second field equation corresponds to the conserved isometry current associated with the color isometry $\Phi \rightarrow \Phi + \epsilon$ and gives equation for f.

$$[T^{rr}f_{,r}s_{\Phi\Phi}\sqrt{g}]_{,r} = 0 \ . \tag{3.5.39}$$

The conservation laws associated with other infinitesimal $SU(2)$ rotations of S^2_I should be satisfied identically. This equation can be readily integrated to give

$$T^{rr}f_{,r}s_{\Phi\Phi}\sqrt{g_{tt}g_{rr}} = \frac{C}{r^2} \ . \tag{3.5.40}$$

Unfortunately, the result is inconsistent with the $1/r^4$ behavior of T^{rr} and $f \rightarrow \omega r$ implies by correct red shift.

It seems that the only possible way out of the difficulty is to replace spherical symmetry with a symmetry with respect to the rotations around z-axis. The simplest modification of the solution ansatz is as follows:

$$m^0 = \lambda t + h(\rho) \ , \quad \Phi = \omega t + k\rho \quad .$$

Thanks to the linear dependence of Φ on ρ, the conservation laws for momentum and color isospin reduce to the same condition. The ansatz induces a small breaking of spherical symmetry by adding to $g_{\rho\rho}$ the term

$$(\partial_\rho h)^2 - \frac{R^2}{4}sin^2(\Theta)k^2 \ .$$

One might hope that in the plane $\theta = \pi/2$, where $r = \rho$ holds true, the ansatz could behave like Schwartschild metric if the conditions discussed above are posed (including the condition $k = \omega$). The breaking of the spherical symmetry in the planetary system would be coded already to the gravitational field of Sun.

Also the study of the imbeddings of Reissner-Nordström metric as vacuum extremals and the investigation of spherically symmetric (inertial) vacuum extremals for which gravitational four-momentum is conserved [K72] leads to the conclusion that the loss of spherical symmetry due to rotation is inevitable characteristic of realistic solutions.

3.5.6 Maxwell hydrodynamics as a toy model for TGD

The field equations of TGD are extremely non-linear and all known solutions have been discovered by symmetry arguments. Chern-Simons term plays essential role also in the construction of solutions of field equations and at partonic level defines braiding for light-like partonic 3-surfaces expected to play key role in the construction of S-matrix. The inspiration for this section came from Terence Tao's blog posting *2006 ICM: Etienne Ghys, "Knots and dynamics"* [A93] giving an elegant summary about amazing mathematical results related to knots, links, braids and hydrodynamical flows in dimension $D = 3$. Posting tells about really amazing mathematical results related to knots.

Chern-Simons term as helicity invariant

Tao mentions helicity as an invariant of fluid flow. Chern-Simons action defined by the induced Kähler gauge potential for light-like 3-surfaces has interpretation as helicity when Kähler gauge potential is identified as fluid velocity. This flow can be continued to the interior of space-time sheet. Also the dual of the induced Kähler form defines a flow at the light-like partonic surfaces but not in the interior of space-time sheet. The lines of this flow can be interpreted as magnetic field lines. This flow is incompressible and represents a conserved charge (Kähler magnetic flux).

The question is which of these flows should define number theoretical braids. Perhaps both of them can appear in the definition of S-matrix and correspond to different kinds of partonic matter (electric/magnetic charges, quarks/leptons?,...). Second kind of matter could not flow in the interior of space-time sheet. Or could interpretation in terms of electric magnetic duality make sense?

Helicity is not gauge invariant and this is as it must be in TGD framework since CP_2 symplectic transformations induce $U(1)$ gauge transformation, which deforms space-time surface an modifies induced metric as well as classical electroweak fields defined by induced spinor connection. Gauge degeneracy is transformed to spin glass degeneracy.

Maxwell hydrodynamics

In TGD Maxwell's equations are replaced with field equations which express conservation laws and are thus hydrodynamical in character. With this background the idea that the analogy between gauge theory and hydrodynamics might be applied also in the reverse direction is natural. Hence one might ask what kind of relativistic hydrodynamics results if assumes that the action principle is Maxwell action for the four-velocity u^α with the constraint term saying that light velocity is maximal signal velocity.

1. For massive particles the length of four-velocity equals to 1: $u^\alpha u_\alpha = 1$. In massless case one has $u^\alpha u_\alpha = 0$. Geometrically this means that one has sigma model with target space which is 3-D Lobatschevski space or at light-cone boundary. This condition means the addition of constraint term

$$\lambda(u^\alpha u_\alpha - \epsilon) \tag{3.5.41}$$

to the Maxwell action. $\epsilon = 1/0$ holds for massive/massless flow. In the following the notation of electrodynamics is used to make easier the comparison with electrodynamics.

2. The constraint term destroys gauge invariance by allowing to express A^0 in terms of A^i but in general the constraint is not equivalent to a choice of gauge in electrodynamics since the solutions to the field equations with constraint term are not solutions of field equations without it. One obtains field equations for an effectively massive em field with Lagrange multiplier λ having interpretation as photon mass depending on space-time point:

$$\begin{aligned} j^\alpha &= \partial_\beta F^{\alpha\beta} = \lambda A^\alpha \ , \\ A^\alpha &\equiv u^\alpha \ , \ F^{\alpha\beta} = \partial^\beta A^\alpha - \partial^\alpha A^\beta \ . \end{aligned} \tag{3.5.42}$$

3. In electrodynamic context the natural interpretation would be in terms of spontaneous massivation of photon and seems to occur for both values of ϵ. The analog of em current given by λA^α is in general non-vanishing and conserved. This conservation law is quite strong additional constraint on the hydrodynamics. What is interesting is that breaking of gauge invariance does not lead to a loss of charge conservation.

4. One can solve λ by contracting the equations with A_α to obtain

$$\lambda = j^\alpha A_\alpha$$

for $\epsilon = 1$. For $\epsilon = 0$ one obtains

$$j^\alpha A_\alpha = 0$$

stating that the field does not dissipate energy: λ can be however non-vanishing unless field equations imply $j^\alpha = 0$. One can say that for $\epsilon = 0$ spontaneous massivation can occur. For $\epsilon = 1$ massivation is present from the beginning and dissipation rate determines photon mass: a natural interpretation for $\epsilon = 1$ would be in terms of thermal massivation of photon. Non-tachyonicity fixes the sign of the dissipation term so that the thermodynamical arrow of time is fixed by causality.

5. For $\epsilon = 0$ massless plane wave solutions are possible and one has

$$\partial_\alpha \partial_\beta A^\beta = \lambda A_\alpha \ .$$

$\lambda = 0$ is obtained in Lorentz gauge which is consistent with the condition $\epsilon = 0$. Also superpositions of plane waves with same polarization and direction of propagation are solutions of field equations: these solutions represent dispersionless precisely targeted pulses. For superpositions of plane waves λ with 4-momenta, which are not all parallel λ is non-vanishing so that non-linear self interactions due to the constraint can be said to induce massivation. In asymptotic states for which gauge symmetry is not broken one expects a decomposition of solutions to regions of space-time carrying this kind of pulses, which brings in mind final states of particle reactions containing free photons with fixed polarizations.

6. Gradient flows satisfying the conditions

$$A_\alpha = \partial_\alpha \Phi \ , \ A^\alpha A_\alpha = \epsilon \tag{3.5.43}$$

give rise to identically vanishing hydrodynamical gauge fields and $\lambda = 0$ holds true. These solutions are vacua since energy momentum tensor vanishes identically. There is huge number of this kind of solutions and spin glass degeneracy suggests itself. Small deformations of these vacuum flows are expected to give rise to non-vacuum flows.

7. The counterparts of charged solutions are of special interest. For $\epsilon = 0$ the solution $(u^0, u^r) = (Q/r)(1,1)$ is a solution of field equations outside origin and corresponds to electric field of a point charge Q. In fact, for $\epsilon = 0$ any ansatz $(u^0, u^r) = f(r)(1,1)$ satisfies field equations for a suitable choice of $\lambda(r)$ since the ratio of equations associate with j^0 and j^r gives an equation which is trivially satisfied. For $\epsilon = 1$ the ansatz $(u^0, u^r) = (cosh(u), sinh(u))$ expressing solution in terms of hyperbolic angle linearizes the field equation obtained by dividing the equations for j^0 and j^r to eliminate λ. The resulting equation is

$$\partial_r^2 u + \frac{2\partial_r u}{r} = 0$$

for ordinary Coulomb potential and one obtains $(u^0, u^r) = (cosh(u_0 + k/r), sinh(u_0 + k/r))$. The charge of the solution at the limit $r \to \infty$ approaches to the value $Q = sinh(u_0)k$ and diverges at the limit $r \to 0$. The charge increases exponentially as a function of $1/r$ near origin rather than logarithmically as in QED and the interpretation in terms of thermal screening suggests itself. Hyperbolic ansatz might simplify considerably the field equations also in the general case.

Similarities with TGD

There are strong similarities with TGD which suggests that the proposed model might provide a toy model for the dynamics defined by Kähler action.

1. Also in TGD field equations are essentially hydrodynamical equations stating the conservation of various isometry charges. Gauge invariance is broken for the induced Kähler field although Kähler charge is conserved. There is huge vacuum degeneracy corresponding to vanishing of induced Kähler field and the interpretation is in terms of spin glass degeneracy.

2. Also in TGD dissipation rate vanishes for the known solutions of field equations and a possible interpretation is as space-time correlates for asymptotic non-dissipating self organization patterns.

3. In TGD framework massless extremals represent the analogs for superpositions of plane waves with fixed polarization and propagation direction and representing targeted and dispersionless propagation of signal. Gauge currents are light-like and non-vanishing for these solutions. The decomposition of space-time surface to space-time sheets representing particles is much more general counterpart for the asymptotic solutions of Maxwell hydrodynamics with vanishing λ.

4. In TGD framework one can consider the possibility that the four-velocity assignable to a macroscopic quantum phase is proportional to the induced Kähler gauge potential. In this kind of situation one could speak of a quantal variant of Maxwell hydrodynamics, at least for light-like partonic 3-surfaces. For instance, the condition

$$D^{\alpha}D_{\alpha}\Psi = 0 \ , \ D_{\alpha}\Psi = (\partial_{\alpha} - iq_K A_{\alpha})\Psi$$

for the order parameter of the quantum phase corresponds at classical level to the condition $p^{\alpha} = q_K Q^{\alpha} + l^{\alpha}$, where q_K is Kähler charge of fermion and l^{α} is a light-like vector field naturally assignable to the partonic boundary component. This gives $u^{\alpha} = (q_K Q^{\alpha} + l^{\alpha})/m$, $m^2 = p^{\alpha}p_{\alpha}$, which is somewhat more general condition. The expressibility of u^{α} in terms of the vector fields provided by the induced geometry is very natural.

   The value ϵ depends on space-time region and it would seem that also $\epsilon = -1$ is possible meaning tachyonicity and breaking of causality. Kähler gauge potential could however have a time-like pure gauge component in M^4 possibly saving the situation. The construction of quantum TGD at parton level indeed forces to assume that Kähler gauge potential has Lorentz invariant M^4 component $A_a = constant$ in the direction of the light-cone proper time coordinate axis a. Note that the decomposition of WCW to sectors consisting of space-time sheets inside future or past light-cone of M^4 is an essential element of the construction of WCW geometry and does not imply breaking of Poincare invariance. Without this component $u_{\alpha}u^{\alpha}$ could certainly be negative. The contribution of M^4 component could prevent this for preferred extremals.

If TGD is taken seriously, these similarities force to ask whether Maxwell hydrodynamics might be interpreted as a nonlinear variant of electrodynamics. Probably not: in TGD em field is proportional to the induced Kähler form only in special cases and is in general non-vanishing also for vacuum extremals.

3.6 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B8] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K12] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural. Also Chern-Simons Dirac equation implies the localization of solutions to flow lines, and this is consistent with the localization solutions of Kähler-Dirac equation to string world sheets.

3.6.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the

identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} \quad = \quad KJ_{12} \ . \tag{3.6.1}$$

A more general form of this duality is suggested by the considerations of [K31] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B2] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} \quad = \quad K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{3.6.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m \quad = \quad (1+K)J_{12} \ , \tag{3.6.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

   n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \qquad (3.6.4)$$

   Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \qquad (3.6.5)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \quad p = sin^2(\theta_W) \ . \end{aligned} \qquad (3.6.6)$$

   Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

   The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \qquad (3.6.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K51] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \to \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{3.6.8}$$

   This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{3.6.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \to 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{3.6.10}$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K55]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K72]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

3.6.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D3] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K23] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_{\pm}$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K39] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K40] .

3.6.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta}A_\beta\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_\beta\sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \to r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K4]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha(J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\ gamma})\sqrt{g_4}d^3x \ . \tag{3.6.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta}K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j^\alpha_K \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{3.6.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j^\alpha_K$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} \ = \ 0 \ . \tag{3.6.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B44] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K4] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j^\alpha_I \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j^\alpha_K \phi$ and $j^\alpha_I \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j^\alpha_K \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha (j^\alpha \phi) \ = \ 0 \ . \tag{3.6.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_\phi = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q^m_\phi = \sum\int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to modified Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

   One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

3.6.4 About the notion of measurement interaction

The notion of measurement has been central notion in quantum TGD but the precise definition of this notion is far from clear. In the following two possibly equivalent formulations are considered. The first formulation relies on the gauge transformations leaving Coulomb term of Kähler action unchanged and the second one to the interpretation of TGD as a square root of thermodynamics allowing to fix the values of conserved classical charges for zero energy energy state using Lagrange multipliers analogous to chemical potentials.

1. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j^\alpha_K \partial_\alpha\phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha\phi) = 0 . \tag{3.6.15}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_\phi = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q^m_\phi = \sum\int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

2. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action.

   The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant of Chern-Simons Dirac operator (after many turns and twists) and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to instanton term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

3. A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

In zero energy ontology (ZEO) TGD can be seen as square root of thermodynamics and this suggests an alternative manner to define what measurement interaction term means.

1. The condition that the space-time sheets appearing in superposition of space-time surfaces with given quantum numbers in Cartan algebra have same classical quantum numbers associated with Kähler action can be realized in terms of Lagrange multipliers in standard manner. These kind of terms would be analogous to various chemical potential terms in the partition function. One could call them measurement interaction terms. Measurement interaction terms would code the values of quantum charges to the space-time geometry.

   Kähler action contains also Chern-Simons term at partonic orbits compensating the Chern-Simons terms coming from Kähler action when weak form of electric-magnetic duality is assumed. This guarantees that Kähler action for preferred extremals reduces to Chern-Simons terms at the space-like ends of the spacetime surface and one obtains almost topological QFT.

2. If Kähler-Dirac action is constructed from Kähler action in super-symmetric manner by defining the modified gamma matrices in terms of canonical momentum densities one obtains also the fermionic counterparts of the Lagrange multiplier terms at partonic orbits and could call also them measurement interaction terms. Besides this one has also the Chern-Simons Dirac terms associated with the partonic orbits giving ordinary massless Dirac propagator.

In presence of measurement interaction terms at the space-like ends of the space-time surface the boundary conditions $\Gamma^n\Psi = 0$ at the ends would be modified by the addition of term coming from the modified gamma matrix associated with the Lagrange multiplier terms. The original generalized massless generalized eigenvalue spectrum $p^k\gamma_k$ of Γ^n would be modified to massive spectrum given by the condition

$$(\Gamma^n + \sum_i \lambda_i \Gamma^{\alpha}_{Q_i} D_{\alpha})\Psi = 0 \ ,$$

where Q_i refers to i:th conserved charge.

An interesting question is whether these two manners to introduce measurement interaction terms are actually equivalent.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

3.6.5 Kähler action for Euclidian regions as Kähler function and Kähler action for Minkowskian regions as Morse function?

One of the nasty questions about the interpretation of Kähler action relates to the square root of the metric determinant. If one proceeds completely straightforwardly, the only reason conclusion is that the square root is imaginary in Minkowskian space-time regions so that Kähler action would be complex. The Euclidian contribution would have a natural interpretation as positive definite Kähler function but how should one interpret the imaginary Minkowskian contribution? Certainly the path integral approach to quantum field theories supports its presence. For some mysterious reason I was able to forget this nasty question and serious consideration of the obvious answer to it. Only when I worked between possibile connections between TGD and Floer homology [K82] I realized that the Minkowskian contribution is an excellent candidate for Morse function whose critical points give information about WCW homology. This would fit nicely with the vision about TGD as almost topological QFT.

Euclidian regions would guarantee the convergence of the functional integral and one would have a mathematically well-defined theory. Minkowskian contribution would give the quantal interference effects and stationary phase approximation. The analog of Floer homology would represent quantum superpositions of critical points identifiable as ground states defined by the extrema of Kähler action for Minkowskian regions. Perturbative approach to quantum TGD would rely on functional integrals around the extrema of Kähler function. One would have maxima also for the Kähler function but only in the zero modes not contributing to the WCW metric.

There is a further question related to almost topological QFT character of TGD. Should one assume that the reduction to Chern-Simons terms occurs for the preferred extremals in *both* Minkowskian and Euclidian regions or only in Minkowskian regions?

1. All arguments for this have been represented for Minkowskian regions [K22] involve local light-like momentum direction which does not make sense in the Euclidian regions. This does not however kill the argument: one can have non-trivial solutions of Laplacian equation in the region of CP_2 bounded by wormhole throats: for CP_2 itself only covariantly constant right-handed neutrino represents this kind of solution and at the same time supersymmetry. In the general case solutions of Laplacian represent broken super-symmetries and should be in one-one correspondences with the solutions of the modified Dirac equation. The interpretation for

the counterparts of momentum and polarization would be in terms of classical representation of color quantum numbers.

2. If the reduction occurs in Euclidian regions, it gives in the case of CP_2 two 3-D terms corresponding to two 3-D gluing regions for three coordinate patches needed to define coordinates and spinor connection for CP_2 so that one would have two Chern-Simons terms. I have earlier claimed that without any other contributions the first term would be identical with that from Minkowskian region apart from imaginary unit and different coefficient. This statement is wrong since the space-like parts of the corresponding 3-surfaces are discjoint for Euclidian and Minkowskian regions.

3. There is also an argument stating that Dirac determinant for Chern-Simons Dirac action equals to Kähler function, which would be lost if Euclidian regions would not obey holography. The argument obviously generalizes and applies to both Morse and Kähler function which are definitely not proportional to each other.

CP breaking and ground state degeneracy

The Minkowskian contribution of Kähler action is imaginary due to the negativity of the metric determinant and gives a phase factor to vacuum functional reducing to Chern-Simons terms at wormhole throats. Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

1. In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

2. A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.

3. There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and shortlived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral measons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

3.6.6 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. This requires that the flow parameters associated with the flow lines of isometry currents and

Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

Basic field equations

Consider first the equations at general level.

1. The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T\times SO(3)\times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

2. Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha\left[D_\beta(J^{\alpha\beta}H_A) - j^\alpha_K H^A + T^{\alpha\beta} j^l_A h_{kl}\partial_\beta h^l\right] \quad = \quad 0 \ . \tag{3.6.16}$$

   The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha\left[j^\alpha_K H^A - T^{\alpha\beta} j^k_A h_{kl}\partial_\beta h^l\right] \quad = \quad 0 \ . \tag{3.6.17}$$

   For energy one has $H_A=1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

3. One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j^\alpha_K J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j^\alpha_K D_\alpha H^A \quad = \quad j^\alpha_K J_\alpha^{\ \beta} j^A_\beta + T^{\alpha\beta} H^k_{\alpha\beta} j^A_k \ . \tag{3.6.18}$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A\wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J=\Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

1. The most general assumption is that the conserved isometry currents

$$J_A^{\alpha} = j_K^{\alpha} H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \quad (3.6.19)$$

and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

2. The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \quad (3.6.20)$$

The conservation of J_A gives

$$d*(\Psi_A d\Phi_A) = 0 \ . \quad (3.6.21)$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

3. The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d*d\Phi_A = 0 \ , \ d\Psi_A \cdot d\Phi_A = 0 \ . \quad (3.6.22)$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p \cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

1. The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \tag{3.6.23}$$

   In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d * d\Phi = 0 \ , \ d\Psi_A \cdot d\Phi = 0. \tag{3.6.24}$$

   This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

2. Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A \Psi_A d\Phi_{G(A)} \ , \ d * (d\Phi_G(A)) = 0 \ , \ d\Psi_A \cdot d\Phi_G(A)) = 0 \ . \tag{3.6.25}$$

   where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [K31] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

3.6.7 Hydrodynamic picture in fermionic sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the modified Dirac equation. This would mean that the solutions of Dirac equation can be localized to lower-dimensional surface or even flow lines.

Basic objection

The obvious objection against the localization to sub-manifolds is that it is not consistent with uncertainty principle in transversal degrees of freedom. More concretely, the assumption that the mode is localized to a lower-dimensional surface of X^4 implies that the action of the transversal part of Dirac operator in question acts on delta function and gives something singular.

The situation changes if the Dirac operator in question has vanishing transversal part at the lower-dimensional surface. This is not possible for the Dirac operator defined by the induced metric but is quite possible in the case of Kähler-Dirac operator. For instance, in the case of massless extremals Kähler-Dirac gamma matrices are non-vanishing in single direction only and the solution modes could be one-dimensional. For more general preferred extremals such as cosmic strings this is not the case.

In fact, there is a strong physical argument in favor of the localization of spinor modes at 2-D string world sheets so that hydrodynamical picture would result but with flow lines replaced with fermionic string world sheets.

1. Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

2. The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action and requires that the part of the Kähler-Dirac operator transversal to 2-surface vanishes.

3. This localization does not hold for cosmic string solutions which however have 2-D CP_2 projection which should have vanishing weak fields so that 4-D spinor modes with well-defined em charge are possible.

4. A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

4-dimensional modified Dirac equation and hydrodynamical picture

In following consideration is restricted to preferred extremals for which one has decomposition to regions characterized by local light-like vector and polarization direction. In this case one has good hopes that the modes can be restricted to 1-D light-like geodesics.

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

1. The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)}\Gamma_k \ . \end{aligned} \tag{3.6.26}$$

   The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^\alpha_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \tag{3.6.27}$$

   The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D modified Dirac equation would reduce everything to initial values at partonic 2-surfaces.

2. One might hope that the conservation of these super currents for all modes is equivalent with the modified Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

3. The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

1. The modified gamma matrices are of form $T^\alpha_k\Gamma^k$, $T^\alpha_k = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^α_k can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h^k_l = j^{Ak}j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k}j^A_k j^k_A = T^{\alpha A}j^k_A$. One can defined gamma matrices Γ_A as $\Gamma_k j^k_A$ to get $T^\alpha_k\Gamma^k = T^{\alpha A}\Gamma_A$.

2. This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the modified Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA} j_A D_t \Psi \quad = \quad 0 \ . \tag{3.6.28}$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

3.6.8 Possible role of Beltrami flows and symplectic invariance in the description of gauge and gravitational interactions

One of the most recent observations made by people working with twistors is the finding of Monteiro and O'Connell described in the preprint The Kinematic Algebra From the Self-Dual Sector [B48]. The claim is that one can obtain supergravity amplitudes by replacing the color factors with kinematic factors which obey formally 2-D symplectic algebra defined by the plane defined by light-like momentum direction and complexified variable in the plane defined by polarizations. One could say that momentum and polarization dependent kinematic factors are in exactly the same role as the factors coming from Yang-Mills couplings. Unfortunately, the symplectic algebra looks rather formal object since the first coordinate is light-like coordinate and second coordinate complex transverse coordinate. It could make sense only in the complexification of Minkowski space.

In any case, this would suggest that the gravitational gauge group (to be distinguished from diffeomorphisms) is symplectic group of some kind having enormous representative power as we know from the fact that the symmetries of practically any physical system are realized in terms of symplectic transformations. According to the authors of [B48] one can identify the Lie algebra of symplectic group of sphere with that of $SU(N)$ at large N limit in suitable basis. What makes this interesting is that at large N limit non-planar diagrams which are the problem of twistor Grassmann approach vanish: this is old result of t'Hooft, which initiated the developments leading to AdS/CFT correspondence.

The symplectic group of $\delta M^4_\pm \times CP_2$ is the isometry algebra of WCW and I have proposed that the effective replacement of gauge group with this group implies the vanishing of non-planar diagrams [K80]. The extension of SYM to a theory of also gravitation in TGD framework could make Yangian symmetry exact, resolve the infrared divergences, and the problems caused by non-planar diagrams. It would also imply stringy picture in finite measurement resolution. Also the the construction of the non-commutative homology and cohomology in TGD framework led to the lifting of Galois group algebras to their braided variants realized as symplectic flows [K82] and to the conjecture that in finite measurement resolution the cohomology obtained in this manner represents WCW ("world of classical worlds") spinor fields (or at least something very essential about them).

It is however difficult to understand how one could generalize the symplectic structure so that also symplectic transformations involving light-like coordinate and complex coordinate of the partonic 2-surface would make sense in some sense. In fact, a more natural interpretation for the kinematic algebra would in terms of volume preserving flows which are also Beltrami flows [B44, B47]. This gives a connection with quantum TGD since Beltrami flows define a basic dynamical symmetry for the preferred extremals of Kähler action which might be called Maxwellian phase.

1. Classical TGD is defined by Kähler action which is the analog of Maxwell action with Maxwell field expressed as the projection of CP_2 Kähler form. The field equations are extremely non-linear and only the second topological half of Maxwell equations is satisfied. The remaining equations state conservation laws for various isometry currents. Actually much more general conservation laws are obtained.

2. As a special case one obtains solutions analogous to those for Maxwell equations but there are also other objects such as CP_2 type vacuum extremals providing correlates for elementary particles and string like objects: for these solutions it does not make sense to speak about QFT in Minkowski space-time. For the Maxwell like solutions linear superposition is lost but a superposition holds true for solutions with the same local direction of polarization

and massless four-momentum. This is a very quantal outcome (in accordance with quantum classical correspondence) since also in quantum measurement one obtains final state with fixed polarization and momentum. So called massless extremals (topological light rays) analogous to wave guides containing laser beam and its phase conjugate are solutions of this kind. The solutions are very interesting since no dispersion occurs so that wave packet preserves its form and the radiation is precisely targeted.

3. Maxwellian preferred extremals decompose in Minkowskian space-time regions to regions that can be regarded as classical space-time correlates for massless particles. Massless particles are characterized by polarization direction and light-like momentum direction. Now these directions can depend on position and are characterized by gradients of two scalar functions Φ and Ψ. Φ defines light-like momentum direction and the square of the gradient of Φ in Minkowski metric must vanish. Ψ defines polarization direction and its gradient is orthogonal to the gradient of Φ since polarization is orthogonal to momentum.

4. The flow has the additional property that the coordinate associated with the flow lines integrates to a global coordinate. Beltrami flow is the term used by mathematicians. Beltrami property means that the condition $j \wedge dj = 0$ is satisfied. In other words, tjhe current is in the plane defined by its exterior derivative. The above representation obviously guarantees this. Beltrami property allows to assign order parameter to the flow depending only the parameter varying along flow line.

   This is essential for the hydrodynamical interpretation of the preferred extremals which relies on the idea that varies conservation laws hold along flow lines. For instance, super-conducting phase requires this kind of flow and velocity along flow line is gradient of the order parameter. The breakdown of super-conductivity would mean topologically the loss of the Beltrami flow property. One might say that the space-time sheets in TGD Universe represent analogs of supra flow and this property is spoiled only by the finite size of the sheets. This strongly suggests that the space-time sheets correspond to perfect fluid flows with very low viscosity to entropy ratio and one application is to the observed perfect flow behavior of quark gluon plasma.

5. The current $J = \Phi\nabla\Psi$ has vanishing divergence if besides the orthogonality of the gradients the functions Ψ and Φ satisfy massless d'Alembert equation. This is natural for massless field modes and when these functions represent constant wave vector and polarization also d'Alembert equations are satisfied. One can actually add to $\nabla\Psi$ a gradient of an arbitrary function of Φ this corresponds to U(1) gauge invariance and the addition to the polarization vector a vector parallel to light-like four-momentum. One can replace Φ by any function of Φ so that one has Abelian Lie algebra analogous to $U(1)$ gauge algebra restricted to functions depending on Φ only.

The general Beltrami flow gives as a special case the kinetic flow associated by Monteiro and O'Connell with plane waves. For ordinary plane wave with constant direction of momentum vector and polarization vector one could take $\Phi = cos(\phi)$, $\phi = k \cdot m$ and $\Psi = \epsilon \cdot m$. This would give a real flow. The kinematical factor in SYM diagrams corresponds to a complexified flow $\Phi = exp(i\phi)$ and $\Psi = \phi + w$, where w is complex coordinate for polarization plane or more naturally, complexificaton of the coordinate in polarization direction. The flow is not unique since gauge invariance allows to modify ϕ term. The complexified flow is volume preserving only in the formal algebraic sense and satisfies the analog of Beltrami condition only in Dolbeault cohomology where d is identified as complex exterior derivative ($df = df/dz\ dz$ for holomorphic functions). In ordinary cohomology it fails. This formal complex flow of course does not define a real diffeomorphism at space-time level: one should replace Minkowski space with its complexification to get a genuine flow.

The finding of Monteiro and O'Connel encourages to think that the proposed more general Abelian algebra pops up also in non-Abelian YM theories. Discretization by braids would actually select single polarization and momentum direction. If the volume preserving Beltrami flows characterize the basic building bricks of radiation solutions of both general relativity and YM theories, it would not be surprising if the kinematic Lie algebra generators would appear in the vertices of YM theory and replace color factors in the transition from YM theory to general relativity. In

TGD framework the construction of vertices at partonic two-surfaces would define local kinematic factors as effectively constant ones.

3.7 An attempt to understand preferred extremals of Kähler action

Preferred extremal of Kähler action is one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what "preferred" really means. For instance, the conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K80]. The problem is however how to assign a complex coordinate with the string world sheet having Minkowskian signature of metric. One can hope that the understanding of preferred extremals could allow to identify two preferred complex coordinates whose existence is also suggested by number theoretical vision giving preferred role for the rational points of partonic 2-surfaces in preferred coordinates. The best one could hope is a general solution of field equations in accordance with the hints that TGD is integrable quantum theory.

3.7.1 What "preferred" could mean?

The first question is what preferred extremal could mean.

1. In positive energy ontology preferred extremal would be a space-time surface assignable to given 3-surface and unique in the ideal situation: since one cannot pose conditions to the normal derivatives of imbedding space coordinates at 3-surface, there is infinity of extremals. Some additional conditions are required and space-time surface would be analogous to Bohr orbit : hence the attribute "preferred". The problem would be to understand what "preferred" could mean. The non-determinism of Kähler action however destroyed this dream in its original form and led to zero energy ontology (ZEO).

2. In ZEO one considers extremals as space-time surfaces connecting two space-like 3-surfaces at the boundaries. One might hope that these 4-surfaces are unique. The non-determinism of Kähler action suggests that this is not the case. At least there is conformal invariance respecting the light-likeness of the 3-D parton orbits at which the signature of the induced metric changes: the conformal transformations would leave the space-like 3-D ends or at least partonic 2-surfaces invariant. This non-determinism would correspond to quantum criticality.

3. Effective 2-dimensionality follows from strong form of general coordinate invariance (GCI) stating that light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface are equivalent physically: partonic 2-surfaces and their 4-D tangent space data would determine everything. One can however worry about how effective 2-dimensionality relates to the the fact that the modes of the induced spinor field are localized at string world sheets and partonic 2-surface. Are the tangent space data equivalent with the data characterizing string world sheets as surfaces carrying vanishing electroweak fields?

   There is however a problem: the hierarchy of Planck constants (dark matter) requires that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom so that either space-like or light-like surfaces do not seem to be quite enough.

   Should one then include also the light-like partonic orbits to the what one calls 3-surface? The resulting connected 3-surfaces would define analogs of Wilson loops. Could the conformal equivalence class of the preferred extremal be unique without any additional conditions? If so, one could get rid of the attribute "preferred". The fractal character of the many-sheeted space-time however suggests that one can have this kind of uniqueness only in given length scale resolution and that "radiative corrections" due to the non-determinism are always present.

These considerations show that the notion of preferred extremal is still far from being precisely defined and it is not even clear whether the attribute "preferred" is needed. If not then the question is what are the extremals of Kähler action.

3.7.2 What is known about extremals?

A lot is is known about properties of extremals and just by trying to integrate all this understanding, one might gain new visions. The problem is that all these arguments are heuristic and rely heavily on physical intuition. The following considerations relate to the space-time regions having Minkowskian signature of the induced metric. The attempt to generalize the construction also to Euclidian regions could be very rewarding. Only a humble attempt to combine various ideas to a more coherent picture is in question.

The core observations and visions are following.

1. Hamilton-Jacobi coordinates for M^4 (discussed in this chapter) define natural preferred coordinates for Minkowskian space-time sheet and might allow to identify string world sheets for X^4 as those for M^4. Hamilton-Jacobi coordinates consist of light-like coordinate m and its dual defining local 2-plane $M^2 \subset M^4$ and complex transversal complex coordinates $(w, \overline{w})$ for a plane E^2_x orthogonal to M^2_x at each point of M^4. Clearly, hyper-complex analyticity and complex analyticity are in question.

2. Space-time sheets allow a slicing by string world sheets (partonic 2-surfaces) labelled by partonic 2-surfaces (string world sheets).

3. The quaternionic planes of octonion space containing preferred hyper-complex plane are labelled by CP_2, which might be called CP_2^{mod} [K67]. The identification $CP_2 = CP_2^{mod}$ motivates the notion of $M^8 - -M^4 \times CP_2$ duality [K15]. It also inspires a concrete solution ansatz assuming the equivalence of two different identifications of the quaternionic tangent space of the space-time sheet and implying that string world sheets can be regarded as strings in the 6-D coset space $G_2/SU(3)$. The group G_2 of octonion automorphisms has already earlier appeared in TGD framework.

4. The duality between partonic 2-surfaces and string world sheets in turn suggests that the $CP_2 = CP_2^{mod}$ conditions reduce to string model for partonic 2-surfaces in $CP_2 = SU(3)/U(2)$. String model in both cases could mean just hypercomplex/complex analyticity for the coordinates of the coset space as functions of hyper-complex/complex coordinate of string world sheet/partonic 2-surface.

The considerations of this section lead to a revival of an old very ambitious and very romantic number theoretic idea.

1. To begin with express octonions in the form $o = q_1 + Iq_2$, where q_i is quaternion and I is an octonionic imaginary unit in the complement of fixed a quaternionic sub-space of octonions. Map preferred coordinates of $H = M^4 \times CP_2$ to octonionic coordinate, form an arbitrary octonion analytic function having expansion with real Taylor or Laurent coefficients to avoid problems due to non-commutativity and non-associativity. Map the outcome to a point of H to get a map $H \rightarrow H$. This procedure is nothing but a generalization of Wick rotation to get an 8-D generalization of analytic map.

2. Identify the preferred extremals of Kähler action as surfaces obtained by requiring the vanishing of the imaginary part of an octonion analytic function. Partonic 2-surfaces and string world sheets would correspond to commutative sub-manifolds of the space-time surface and of imbedding space and would emerge naturally. The ends of braid strands at partonic 2-surface would naturally correspond to the poles of the octonion analytic functions. This would mean a huge generalization of conformal invariance of string models to octonionic conformal invariance and an exact solution of the field equations of TGD and presumably of quantum TGD itself.

3.7.3 Basic ideas about preferred extremals

The slicing of the space-time sheet by partonic 2-surfaces and string world sheets

The basic vision is that space-time sheets are sliced by partonic 2-surfaces and string world sheets. The challenge is to formulate this more precisely at the level of the preferred extremals of Kähler action.

1. Almost topological QFT property means that the Kähler action reduces to Chern-Simons terms assignable to 3-surfaces. This is guaranteed by the vanishing of the Coulomb term in the action density implied automatically if conserved Kähler current is proportional to the instanton current with proportionality coefficient some scalar function.

2. The field equations reduce to the conservation of isometry currents. An attractive ansatz is that the flow lines of these currents define global coordinates. This means that these currents are Beltrami flows [B44] so that corresponding 1-forms J satisfy the condition $J \wedge dJ = 0$. These conditions are satisfied if

$$J = \Phi \nabla \Psi$$

hold true for conserved currents. From this one obtains that Ψ defines global coordinate varying along flow lines of J.

3. A possible interpretation is in terms of local polarization and momentum directions defined by the scalar functions involved and natural additional conditions are that the gradients of Ψ and Φ are orthogonal:

$$\nabla \Phi \cdot \nabla \Psi = 0 \ ,$$

and that the Ψ satisfies massless d'Alembert equation

$$\nabla^2 \Psi = 0$$

as a consequence of current conservation. If Ψ defines a light-like vector field - in other words

$$\nabla \Psi \cdot \nabla \Psi = 0 \ ,$$

the light-like dual of Φ -call it Φ_c- defines a light-like like coordinate and Φ and Φ_c defines a light-like plane at each point of space-time sheet.

If also Φ satisfies d'Alembert equation

$$\nabla^2 \Phi = 0 \ ,$$

also the current

$$K = \Psi \nabla \Phi$$

is conserved and its flow lines define a global coordinate in the polarization plane orthogonal to time-lik plane defined by local light-like momentum direction.

If Φ allows a continuation to an analytic function of the transversal complex coordinate, one obtains a coordinatization of space-time surface by Ψ and its dual (defining hyper-complex coordinate) and $w, \overline{w}$. Complex analyticity and its hyper-complex variant would allow to provide space-time surface with four coordinates very much analogous with Hamilton-Jacobi coordinates of M^4.

This would mean a decomposition of the tangent space of space-time surface to orthogonal planes defined by light-like momentum and plane orthogonal to it. If the flow lines of J defined Beltrami flow it seems that the distribution of momentum planes is integrable.

4. General arguments suggest that the space-time sheets allow a slicing by string world sheets parametrized by partonic 2-surfaces or vice versa. This would mean a intimate connection with the mathematics of string models. The two complex coordinates assignable to the Yangian of affine algebra would naturally relate to string world sheets and partonic 2-surfaces and the highly non-trivial challenge is to identify them appropriately.

Hamilton-Jacobi coordinates for M^4

The earlier attempts to construct preferred extremals [K4] led to the realization that so called Hamilton-Jacobi coordinates (m, w) for M^4 define its slicing by string world sheets parametrized by partonic 2-surfaces. m would be pair of light-like conjugate coordinates associated with an integrable distribution of planes M^2 and w would define a complex coordinate for the integrable distribution of 2-planes E^2 orthogonal to M^2. There is a great temptation to assume that these coordinates define preferred coordinates for M^4.

1. The slicing is very much analogous to that for space-time sheets and the natural question is how these slicings relate. What is of special interest is that the momentum plane M^2 can be defined by massless momentum. The scaling of this vector does not matter so that these planes are labelled by points z of sphere S^2 telling the direction of the line $M^2 \cap E^3$, when one assigns rest frame and therefore S^2 with the preferred time coordinate defined by the line connecting the tips of CD. This direction vector can be mapped to a twistor consisting of a spinor and its conjugate. The complex scalings of the twistor $(u, \overline{u}) \rightarrow \lambda u, \overline{u}/\lambda)$ define the same plane. Projective twistor like entities defining CP_1 having only one complex component instead of three are in question. This complex number defines with certain prerequisites a local coordinate for space-time sheet and together with the complex coordinate of E^2 could serve as a pair of complex coordinates (z, w) for space-time sheet. This brings strongly in mind the two complex coordinates appearing in the expansion of the generators of quantum Yangian of quantum affine algebra [K80].

2. The coordinate Ψ appearing in Beltrami flow defines the light-like vector field defining M^2 distribution. Its hyper-complex conjugate would define Ψ_c and conjugate light-like direction. An attractive possibility is that Φ allows analytic continuation to a holomorphic function of w. In this manner one would have four coordinates for M^4 also for space-time sheet.

3. The general vision is that at each point of space-time surface one can decompose the tangent space to $M^2(x) \subset M^4 = M^2_x \times E^2_x$ representing momentum plane and polarization plane $E^2 \subset E^2_x \times T(CP_2)$. The moduli space of planes $E^2 \subset E^6$ is 8-dimensional and parametrized by $SO(6)/SO(2) \times SO(4)$ for a given E^2_x. How can one achieve this selection and what conditions it must satisfy? Certainly the choice must be integrable but this is not the only condition.

Space-time surfaces as associative/co-associative surfaces

The idea that number theory determines classical dynamics in terms of associativity condition means that space-time surfaces are in some sense quaternionic surfaces of an octonionic space-time. It took several trials before the recent form of this hypothesis was achieved.

1. Octonionic structure is defined in terms of the octonionic representaton of gamma matrices of the imbedding space existing only in dimension $D = 8$ since octonion units are in one-one correspondence with tangent vectors of the tangent space. Octonionic real unit corresponds to a preferred time axes (and rest frame) identified naturally as that connecting the tips of CD. What modified gamma matrices mean depends on variational principle for space-time surface. For volume action one would obtain induced gamma matrices. For Kähler action one obtains something different. In particular, the modified gamma matrices do not define vector basis identical with tangent vector basis of space-time surface.

2. Quaternionicity means that the modified gamma matrices defined as contractions of gamma matrices of H with canonical momentum densities for Kähler action span quaternionic sub-space of the octonionic tangent space [K22]. A further condition is that each quaternionic space defined in this manner contains a preferred hyper-complex subspace of octonions.

3. The sub-space defined by the modified gamma matrices does not co-incide with the tangent space of space-time surface in general so that the interpretation of this condition is far from obvious. The canonical momentum densities need not define four independent vectors at given point. For instance, for massless extremals these densities are proportional to light-like vector so that the situation is degenerate and the space in question reduces to 2-D hyper-complex sub-space since light-like vector defines plane M^2.

The obvious questions are following.

1. Does the analog of tangent space defined by the octonionic modified gammas contain the local tangent space $M^2 \subset M^4$ for preferred extremals? For massless extremals [K4] this condition would be true. The orthogonal decomposition $T(X^4) = M^2 \oplus_\perp E^2$ can be defined at each point if this is true. For massless extremals also the functions Ψ and Φ can be identified.

2. One should answer also the following delicate question. Can M^2 really depend on point x of space-time? CP_2 as a moduli space of quaternionic planes emerges naturally if M^2 is *same* everywhere. It however seems that one should allow an integrable distribution of M^2_x such that M^2_x is same for all points of a given partonic 2-surface.

   How could one speak about fixed CP_2 (the imbedding space) at the entire space-time sheet even when M^2_x varies?

   (a) Note first that G_2 defines the Lie group of octonionic automorphisms and G_2 action is needed to change the preferred hyper-octonionic sub-space. Various $SU(3)$ subgroups of G_2 are related by G_2 automorphism. Clearly, one must assign to each point of a string world sheet in the slicing parameterizing the partonic 2-surfaces an element of G_2. One would have Minkowskian string model with G_2 as a target space. As a matter fact, this string model is defined in the target space $G_2/SU(3)$ having dimension $D = 6$ since $SU(3)$ automorphisms leave given $SU(3)$ invariant.

   (b) This would allow to identify at each point of the string world sheet standard quaternionic basis - say in terms of complexified basis vectors consisting of two hyper-complex units and octonionic unit q_1 with "color isospin" $I_3 = 1/2$ and "color hypercharge" $Y = -1/3$ and its conjugate $\overline{q}_1$ with opposite color isospin and hypercharge.

   (c) The CP_2 point assigned with the quaternionic basis would correspond to the $SU(3)$ rotation needed to rotate the standard basis to this basis and would actually correspond to the first row of $SU(3)$ rotation matrix. Hyper-complex analyticity is the basic property of the solutions of the field equations representing Minkowskian string world sheets. Also now the same assumption is highly natural. In the case of string models in Minkowski space, the reduction of the induced metric to standard form implies Virasoro conditions and similar conditions are expected also now. There is no need to introduce action principle -just the hyper-complex analycitity is enough-since Kähler action already defines it.

3. The WZW model inspired approach to the situation would be following. The parameterization corresponds to a map $g : X^2 \to G_2$ for which g defines a flat G_2 connection at string world sheet. WZW type action would give rise to this kind of situation. The transition $G_2 \to G_2/SU(3)$ would require that one gauges $SU(3)$ degrees of freedom by bringing in $SU(3)$ connection. Similar procedure for $CP_2 = SU(3)/U(2)$ would bring in $SU(3)$ valued chiral field and $U(2)$ gauge field. Instead of introducing these connections one can simply introduce $G_2/SU(3)$ and $SU(3)/U(2)$ valued chiral fields. What this observation suggests that this ansatz indeed predicts gluons and electroweak gauge bosons assignable to string like objects so that the mathematical picture would be consistent with physical intuition.

The two interpretations of CP_2

An old observation very relevant for what I have called $M^8 - H$ duality [K15] is that the moduli space of quaternionic sub-spaces of octonionic space (identifiable as M^8) containing preferred hyper-complex plane is CP_2. Or equivalently, the space of two planes whose addition extends hyper-complex plane to some quaternionic subspace can be parametrized by CP_2. This CP_2 can be called it CP_2^{mod} to avoid confusion. In the recent case this would mean that the space $E^2(x) \subset E^2_x \times T(CP_2)$ is represented by a point of CP_2^{mod}. On the other hand, the imbedding of space-time surface to H defines a point of "real" CP_2. This gives two different CP_2s.

1. The highly suggestive idea is that the identification $CP_2^{mod} = CP_2$ (apart from isometry) is crucial for the construction of preferred extremals. Indeed, the projection of the space-time point to CP_2 would fix the local polarization plane completely. This condition for $E^2(x)$ would be purely local and depend on the values of CP_2 coordinates only. Second condition for $E^2(x)$ would involve the gradients of imbedding space coordinates including those of CP_2 coordinates.

2. The conditions that the planes M^2_x form an integrable distribution at space-like level and that M^2_x is determined by the modified gamma matrices. The integrability of this distribution for M^4 could imply the integrability for X^2. X^4 would differ from M^4 only by a deformation in degrees of freedom transversal to the string world sheets defined by the distribution of M^2s.

   Does this mean that one can begin from vacuum extremal with constant values of CP_2 coordinates and makes them non-constant but allows to depend only on transversal degrees of freedom? This condition is too strong even for simplest massless extremals for which CP_2 coordinates depend on transversal coordinates defined by $\epsilon \cdot m$ and $\epsilon \cdot k$. One could however allow dependence of CP_2 coordinates on light-like M^4 coordinate since the modification of the induced metric is light-like so that light-like coordinate remains light-like coordinate in this modification of the metric.

   Therefore, if one generalizes directly what is known about massless extremals, the most general dependence of CP_2 points on the light-like coordinates assignable to the distribution of M^2_x would be dependence on either of the light-like coordinates of Hamilton-Jacobi coordinates but not both.

3.7.4 What could be the construction recipe for the preferred extremals assuming $CP_2 = CP_2^{mod}$ identification?

The crucial condition is that the planes $E^2(x)$ determined by the point of $CP_2 = CP_2^{mod}$ identification and by the tangent space of $E^2_x \times CP_2$ are same. The challenge is to transform this condition to an explicit form. $CP_2 = CP_2^{mod}$ identification should be general coordinate invariant. This requires that also the representation of E^2 as (e^2, e^3) plane is general coordinate invariant suggesting that the use of preferred CP_2 coordinates - presumably complex Eguchi-Hanson coordinates - could make life easy. Preferred coordinates are also suggested by number theoretical vision. A careful consideration of the situation would be required.

The modified gamma matrices define a quaternionic sub-space analogous to tangent space of X^4 but not in general identical with the tangent space: this would be the case only if the action were 4-volume. I will use the notation $T^m_x(X^4)$ about the modified tangent space and call the vectors of $T^m_x(X^4)$ modified tangent vectors. I hope that this would not cause confusion.

$CP_2 = CP_2^{mod}$ condition

Quaternionic property of the counterpart of $T^m_x(X^4)$ allows an explicit formulation using the tangent vectors of $T^m_x(X^4)$.

1. The unit vector pair (e_2, e_3) should correspond to a unique tangent vector of H defined by the coordinate differentials dh^k in some natural coordinates used. Complex Eguchi-Hanson coordinates [L1] are a natural candidate for CP_2 and require complexified octonionic imaginary units. If octonionic units correspond to the tangent vector basis of H uniquely, this is possible.

2. The pair (e_2, e_3) as also its complexification $(q_1 = e_2 + ie_3, \overline{q}_1 = e_2 - ie_3)$ is expressible as a linear combination of octonionic units $I_2, ...I_7$ should be mapped to a point of $CP_2^{mod} = CP_2$ in canonical manner. This mapping is what should be expressed explicitly. One should

express given (e_2, e_3) in terms of $SU(3)$ rotation applied to a standard vector. After that one should define the corresponding CP_2 point by the bundle projection $SU(3) \rightarrow CP_2$.

3. The tangent vector pair

$$(\partial_w h^k, \partial_{\overline{w}} h^k)$$

defines second representation of the tangent space of $E^2(x)$. This pair should be equivalent with the pair $(q_1, \overline{q}_1)$. Here one must be however very cautious with the choice of coordinates. If the choice of w is unique apart from constant the gradients should be unique. One can use also real coordinates (x, y) instead of $(w = x + iy, \overline{w} = x - iy)$ and the pair (e_2, e_3). One can project the tangent vector pair to the standard vielbein basis which must correspond to the octonionic basis

$$(\partial_x h^k, \partial_y h^k) \rightarrow (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A) e_A) \leftrightarrow (e_2, e_3) \ ,$$

where the e_A denote the octonion units in 1-1 correspondence with vielbein vectors. This expression can be compared to the expression of (e_2, e_3) derived from the knowledge of CP_2 projection.

Formulation of quaternionicity condition in terms of octonionic structure constants

One can consider also a formulation of the quaternionic tangent planes in terms of (e_2, e_3) expressed in terms of octonionic units deducible from the condition that unit vectors obey quaternionic algebra. The expressions for octonionic *resp.* quaternionic structure constants can be found at [A17] *resp.* [A24].

1. The ansatz is

$$\begin{aligned}
\{E_k\} &= \{1, I_1, E_2, E_3\} \ , \\
E_2 &= E_{2k} e^k \equiv \sum_{k=2}^{7} E_{2k} e^k \ \ , \ \ E_3 = E_{3k} e^k \equiv \sum_{k=2}^{7} E_{3k} e^k \ , \\
|E_2| &= 1 \ , \ |E_3| = 1 \ .
\end{aligned} \tag{3.7.1}$$

2. The multiplication table for octonionic units expressible in terms of octonionic triangle [A17] gives

$$f^{1kl} E_{2k} = E_{3l} \ \ , \ \ f^{1kl} E_{3k} = -E_{2l} \ \ , \ \ f^{klr} E_{2k} E_{3l} = \delta_1^r \ . \tag{3.7.2}$$

Here the indices are raised by unit metric so that there is no difference between lower and upper indices. Summation convention is assumed. Also the contribution of the real unit is present in the structure constants of third equation but this contribution must vanish.

3. The conditions are linear and quadratic in the coefficients E_{2k} and E_{3k} and are expected to allow an explicit solution. The first two conditions define homogenous equations which must allow solution. The coefficient matrix acting on (E_2, E_3) is of the form

$$\begin{pmatrix} f_1 & 1 \\ -1 & f_1 \end{pmatrix} \ ,$$

where 1 denotes unit matrix. The vanishing of the determinant of this matrix should be due to the highly symmetric properties of the structure constants. In fact the equations can be written as eigen conditions

$$f_1 \circ (E_2 \pm iE_3) = \mp i(E_2 \pm iE_3) \ ,$$

and one can say that the structure constants are eigenstates of the hermitian operator defined by I_1 analogous to color hyper charge. Both values of color hyper charged are obtained.

Explicit expression for the $CP_2 = CP_2^{mod}$ conditions

The symmetry under $SU(3)$ allows to construct the solutions of the above equations directly.

1. One can introduce complexified basis of octonion units transforming like $(1,1,3,\overline{3})$ under SU(3). Note the analogy of triplet with color triplet of quarks. One can write complexified basis as $(1, e_1, (q_1, q_2, q_3), (\overline{q}_1 \overline{q}_2, \overline{q}_3))$. The expressions for complexified basis elements are

$$(q_1, q_2, q_3) = \frac{1}{\sqrt{2}}(e_2 + ie_3, e_4 + ie_5, e_6 + ie_7) \ .$$

   These options can be seen to be possible by studying octonionic triangle in which all lines containing 3 units defined associative triple: any pair of octonion units at this kind of line can be used to form pair of complexified unit and its conjugate. In the tangent space of $M^4 \times CP_2$ the basis vectors q_1, and q_2 are mixtures of E^2_x and CP_2 tangent vectors. q_3 involves only CP_2 tangent vectors and there is a temptation to interpret it as the analog of the quark having no color isospin.

2. The quaternionic basis is real and must transform like $(1, 1, q_1, \overline{q}_1)$, where q_1 is any quark in the triplet and $\overline{q}_1$ its conjugate in antitriplet. Having fixed some basis one can perform SU(3) rotations to get a new basis. The action of the rotation is by 3×3 special unitary matrix. The over all phases of its rows do not matter since they induce only a rotation in (e_2, e_3) plane not affecting the plane itself. The action of $SU(3)$ on q_1 is simply the action of its first row on (q_1, q_2, q_3) triplet:

$$\begin{aligned} q_1 &\rightarrow (Uq)_1 = U_{11}q_1 + U_{12}q_2 + U_{13}q_3 \equiv z_1 q_1 + z_2 q_2 + z_3 q_3 \\ &= z_1(e_2 + ie_3) + z_2(e_4 + ie_5) + z_3(e_6 + ie_7) \ . \end{aligned} \tag{3.7.3}$$

   The triplets (z_1, z_2, z_3) defining a complex unit vector and point of S^5. Since overall phase does not matter a point of CP_2 is in question. The new real octonion units are given by the formulas

$$\begin{aligned} e_2 &\rightarrow Re(z_1)e_2 + Re(z_2)e_4 + Re(z_3)e_6 - Im(z_1)e_3 - Im(z_2)e_5 - Im(z_3)e_7 \ , \\ e_3 &\rightarrow Im(z_1)e_2 + Im(z_2)e_4 + Im(z_3)e_6 + Re(z_1)e_3 + Re(z_2)e_5 + Re(z_3)e_7 \ . \end{aligned} \tag{3.7.4}$$

   For instance the CP_2 coordinates corresponding to the coordinate patch (z_1, z_2, z_3) with $z_3 \neq 0$ are obtained as $(\xi_1, \xi_2) = (z_1/z_3, z_2/z_3)$.

Using these expressions the equations expressing the conjecture $CP_2 = CP_2^{mod}$ equivalence can be expressed explicitly as first order differential equations. The conditions state the equivalence

$$(e_2, e_3) \leftrightarrow (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A e_A) \ , \tag{3.7.5}$$

where e_A denote octonion units. The comparison of two pairs of vectors requires normalization of the tangent vectors on the right hand side to unit vectors so that one takes unit vector in the direction of the tangent vector. After this the vectors can be equated. This allows to expresses the contractions of the partial derivatives with vielbein vectors with the 6 components of e_2 and e_3. Each condition gives 6+6 first order partial differential equations which are non-linear by the presence of the overal normalization factor for the right hand side. The equations are invariant under scalings of (x, y). The very special form of these equations suggests that some symmetry is involved.

It must be emphasized that these equations make sense only in preferred coordinates: ordinary Minkowski coordinates and Hamilton-Jacobi coordinates for M^4 and Eguchi-Hanson complex coordinates in which $SU(2) \times U(1)$ is represented linearly for CP_2. These coordinates are preferred because they carry deep physical meaning.

Does TGD boil down to two string models?

It is good to look what have we obtained. Besides Hamilton-Jacobi conditions, and $CP_2 = CP_2^{mod}$ conditions one has what one might call string model with 6-dimensional $G_2/SU(3)$ as targent space. The orbit of string in $G_2/SU(3)$ allows to deduce the G_2 rotation identifiable as a point of $G_2/SU(3)$ defining what one means with standard quaternionic plane at given point of string world sheet. The hypothesis is that hyper-complex analyticity solves these equations.

The conjectured electric-magnetic duality implies duality between string world sheet and partonic 2-surfaces central for the proposed mathematical applications of TGD [K32, K33, K65, K82]. This duality suggests that the solutions to the $CP_2 = CP_2^{mod}$ conditions could reduce to holomorphy with respect to the coordinate w for partonic 2-surface plus the analogs of Virasoro conditions. The dependence on light-like coordinate would appear as a parametric dependence.

If this were the case, TGD would reduce at least partially to what might be regarded as dual string models in $G_2/SU(3)$ and $SU(3)/U(2)$ and also to string model in M^4 and X^4! In the previous arguments one ends up to string models in moduli spaces of string world sheets and partonic 2-surfaces. TGD seems to yield an inflation of string models! This not actually surprising since the slicing of space-time sheets by string world sheets and partonic 2-surfaces implies automatically various kinds of maps having interpretation in terms of string orbits.

3.8 In what sense TGD could be an integrable theory?

During years evidence supporting the idea that TGD could be an integrable theory in some sense has accumulated. The challenge is to show that various ideas about what integrability means form pieces of a bigger coherent picture. Of course, some of the ideas are doomed to be only partially correct or simply wrong. Since it is not possible to know beforehand what ideas are wrong and what are right the situation is very much like in experimental physics and it is easy to claim (and has been and will be claimed) that all this argumentation is useless speculation. This is the price that must be paid for real thinking.

Integrable theories allow to solve nonlinear classical dynamics in terms of scattering data for a linear system. In TGD framework this translates to quantum classical correspondence. The solutions of modified Dirac equation define the scattering data. This data should define a real analytic function whose octonionic extension defines the space-time surface as a surface for which its imaginary part in the representation as bi-quaternion vanishes. There are excellent hopes about this thanks to the reduction of the modified Dirac equation to geometric optics.

In the following I will first discuss briefly what integrability means in (quantum) field theories, list some bits of evidence for integrability in TGD framework, discuss once again the question whether the different pieces of evidence are consistent with other and what one really means with various notions. An an outcome I represent what I regard as a more coherent view about integrability of TGD. The notion of octonion analyticity developed in the previous section is essential for the for what follows.

3.8.1 What integrable theories are?

The following is an attempt to get some bird's eye of view about the landscape of integrable theories.

Examples of integrable theories

Integrable theories are typically non-linear 1+1-dimensional (quantum) field theories. Solitons and various other particle like structures are the characteristic phenomenon in these theories. Scattering matrix is trivial in the sense that the particles go through each other in the scattering and suffer only a phase change. In particular, momenta are conserved. Korteveg- de Vries equation [B5]was motivated by the attempt to explain the experimentally discovered shallow water wave preserving its shape and moving with a constant velocity. Sine-Gordon equation [B14] describes geometrically constant curvature surfaces and defines a Lorentz invariant non-linear field theory in 1+1-dimensional space-time, which can be applied to Josephson junctions (in TGD inspired quantum biology it is encountered in the model of nerve pulse [K55]). Non- linear Schrödinger equation [B11] having applications to optics and water waves represents a further example. All these equations have various variants.

From TGD point of view conformal field theories represent an especially interesting example of integrable theories. (Super-)conformal invariance is the basic underlying symmetry and by its infinite-dimensional character implies infinite number of conserved quantities. The construction of the theory reduces to the construction of the representations of (super-)conformal algebra. One can solve 2-point functions exactly and characterize them in terms of (possibly anomalous) scaling dimensions of conformal fields involved and the coefficients appearing in 3-point functions can be solved in terms of fusion rules leading to an associative algebra for conformal fields. The basic applications are to 2-dimensional critical thermodynamical systems whose scaling invariance generalizes to conformal invariance. String models represent second application in which a collection of super-conformal field theories associated with various genera of 2-surface is needed to describe loop corrections to the scattering amplitudes. Also moduli spaces of conformal equivalence classes become important.

Topological quantum field theories are also examples of integrable theories. Because of its independence on the metric Chern-Simons action is in 3-D case the unique action defining a topological quantum field theory. The calculations of knot invariants (for TGD approach see [K32]), topological invariants of 3-manifolds and 4-manifolds, and topological quantum computation (for a model of DNA as topological quantum computer see [K20]) represent applications of this approach. TGD as almost topological QFT means that the Kähler action for preferred extremals reduces to a surface term by the vanishing of Coulomb term in action and by the weak form of electric-magnetic duality reduces to Chern-Simons action. Both Euclidian and Minkowskian regions give this kind of contribution.

$\mathcal{N} = 4$ SYM is the a four-dimensional and very nearly realistic candidate for an integral quantum field theory. The observation that twistor amplitudes allow also a dual of the 4-D conformal symmetry motivates the extension of this symmetry to its infinite-dimensional Yangian variant [A36]. Also the enormous progress in the construction of scattering amplitudes suggests integrability. In TGD framework Yangian symmetry would emerge naturally by extending the symplectic variant of Kac-Moody algebra from light-cone boundary to the interior of causal diamond and the Kac-Moody algebra from light-like 3-surface representing wormhole throats at which the signature of the induced metric changes to the space-time interior [K80].

About mathematical methods

The mathematical methods used in integrable theories are rather refined and have contributed to the development of the modern mathematical physics. Mention only quantum groups, conformal algebras, and Yangian algebras.

The basic element of integrability is the possibility to transform the non-linear classical problem for which the interaction is characterized by a potential function or its analog to a linear scattering problem depending on time. For instance, for the ordinary Schrödinger function one can solve potential once single solution of the equation is known. This does not work in practice. One can however gather information about the asymptotic states in scattering to deduce the potential. One cannot do without information about bound state energies too.

In TGD framework asymptotic states correspond to partonic 2-surfaces at the two light-like boundaries of CD (more precisely: the largest CD involved and defining the IR resolution for momenta). From the scattering data coding information about scattering for various values of energy of the incoming particle one deduced the potential function or its analog.

1. The basic tool is inverse scattering transform known as Gelfand-Marchenko-Levitan (GML) transform described in simple terms in [B17].

   (a) In 1+1 dimensional case the S-matrix characterizing scattering is very simple since the only thing that can take place in scattering is reflection or transmission. Therefore the S-matrix elements describe either of these processes and by unitarity the sum of corresponding probabilities equals to 1. The particle can arrive to the potential either from left or right and is characterized by a momentum. The transmission coefficient can have a pole meaning complex (imaginary in the simplest case) wave vector serving as a signal for the formation of a bound state or resonance. The scattering data are represented by the reflection and transmission coefficients as function of time.

   (b) One can deduce an integral equation for a propagator like function $K(t,x)$ describing how delta pulse moving with light velocity is scattered from the potential and is expressible in terms of time integral over scattering data with contributions from both scattering states and bound states. The derivation of GML transform [B17] uses time reversal and time translational invariance and causality defined in terms of light velocity. After some tricks one obtains the integral equation as well as an expression for the time independent potential as $V(x) = K(x,x)$. The argument can be generalized to more complex problems to deduce the GML transform.

2. The so called Lax pair is one manner to describe integrable systems [B6]. Lax pair consists of two operators L and M. One studies what might be identified as "energy" eigenstates satisfying $L(x,t)\Psi = \lambda\Psi$. λ does not depend on time and one can say that the dynamics is associated with x coordinate whereas as t is time coordinate parametrizing different variants of eigenvalue problem with the same spectrum for L. The operator $M(t)$ does not depend on x at all and the independence of λ on time implies the condition

   $$\partial_t L = [L,M] \ .$$

   This equation is analogous to a quantum mechanical evolution equation for an operator induced by time dependent "Hamiltonian" M and gives the non-linear classical evolution equation when the commutator on the right hand side is a multiplicative operator (so that it does not involve differential operators acting on the coordinate x). Non-linear classical dynamics for the time dependent potential emerges as an integrability condition.

   One could say that $M(t)$ introduces the time evolution of $L(t,x)$ as an automorphism which depends on time and therefore does not affect the spectrum. One has $L(t,x) = U(t)L(0,x)U^{-1}(t)$ with $dU(t)/dt = M(t)U(t)$. The time evolution of the analog of the quantum state is given by a similar equation.

3. A more refined view about Lax pair is based on the observation that the above equation can be generalized so that M depends also on x. The generalization of the basic equation for $M(x,t)$ reads as

   $$\partial_t L - \partial_x M - [L,M] = 0 \ .$$

   The condition has interpretation as a vanishing of the curvature of a gauge potential having components $A_x = L, A_t = M$. This generalization allows a beautiful geometric formulation of the integrability conditions and extends the applicability of the inverse scattering transform. The monodromy of the flat connection becomes important in this approach. Flat connections in moduli spaces are indeed important in topological quantum field theories and in conformal field theories.

4. There is also a connection with the so called Riemann- Hilbert problem [A27]. The monodromies of the flat connection define monodromy group and Riemann-Hilbert problem concerns the existence of linear differential equations having a given monodromy group. Monodromy group emerges in the analytic continuation of an analytic function and the action

of the element of the monodromy group tells what happens for the resulting many-valued analytic function as one turns around a singularity once ('mono-'). The linear equations obviously relate to the linear scattering problem. The flat connection (M, L) in turn defines the monodromy group. What is needed is that the functions involved are analytic functions of (t, x) replaced with a complex or hyper-complex variable. Again Wick rotation is involved. Similar approach generalizes also to higher dimensional moduli spaces with complex structures.

In TGD framework the effective 2-dimensionality raises the hope that this kind of mathematical apparatus could be used. An interesting possibility is that finite measurement resolution could be realized in terms of a gauge group or Kac-Moody type group represented by trivial gauge potential defining a monodromy group for n-point functions. Monodromy invariance would hold for the full n-point functions constructed in terms of analytic n-point functions and their conjugates. The ends of braid strands are natural candidates for the singularities around which monodromies are defined.

3.8.2 Why TGD could be integrable theory in some sense?

There are many indications that TGD could be an integrable theory in some sense. The challenge is to see which ideas are consistent with each other and to build a coherent picture where everything finds its own place.

1. 2-dimensionality or at least effective 2-dimensionality seems to be a prerequisite for integrability. Effective 2-dimensionality is suggested by the strong form of General Coordinate Invariance implying also holography and generalized conformal invariance predicting infinite number of conservation laws. The dual roles of partonic 2-surfaces and string world sheets supports a four-dimensional generalization of conformal invariance. Twistor considerations [K78] indeed suggest that Yangian invariance and Kac-Moody invariances combine to a 4-D analog of conformal invariance induced by 2-dimensional one by algebraic continuation.

2. Octonionic representation of imbedding space Clifford algebra and the identification of the space-time surfaces as quaternionic space-time surfaces would define a number theoretically natural generalization of conformal invariance. The reason for using gamma matrix representation is that vector field representation for octonionic units does not exist. The problem concerns the precise meaning of the octonionic representation of gamma matrices.

   Space-time surfaces could be quaternionic also in the sense that conformal invariance is analytically continued from string curve to 8-D space by octonion real-analyticity. The question is whether the Clifford algebra based notion of tangent space quaternionicity is equivalent with octonionic real-analyticity based notion of quaternionicity.

   The notions of co-associativity and co-quaternionicity make also sense and one must consider seriously the possibility that associativity-co-associativity dichotomy corresponds to Minkowskian-Euclidian dichotomy.

3. Field equations define hydrodynamic Beltrami flows satisfying integrability conditions of form $J \wedge dJ = 0$.

   (a) One can assign local momentum and polarization directions to the preferred extremals and this gives a decomposition of Minkowskian space-time regions to massless quanta analogous to the 1+1-dimensional decomposition to solitons. The linear superposition of modes with 4-momenta with different directions possible for free Maxwell action does not look plausible for the preferred extremals of Kähler action. This rather quantal and solitonic character is in accordance with the quantum classical correspondence giving very concrete connection between quantal and classical particle pictures. For 4-D volume action one does not obtain this kind of decomposition. In 2-D case volume action gives superposition of solutions with different polarization directions so that the situation is nearer to that for free Maxwell action and is not like soliton decomposition.

   (b) Beltrami property in strong sense allows to identify 4 preferred coordinates for the space-time surface in terms of corresponding Beltrami flows. This is possible also in

Euclidian regions using two complex coordinates instead of hyper-complex coordinate and complex coordinate. The assumption that isometry currents are parallel to the same light-like Beltrami flow implies hydrodynamic character of the field equations in the sense that one can say that each flow line is analogous to particle carrying some quantum numbers. This property is not true for all extremals (say cosmic strings).

(c) The tangent bundle theoretic view about integrability is that one can find a Lie algebra of vector fields in some manifold spanning the tangent space of a lower-dimensional manifolds and is expressed in terms of Frobenius theorem [A7]). The gradients of scalar functions defining Beltrami flows appearing in the ansatz for preferred exremals would define these vector fields and the slicing. Partonic 2-surfaces would correspond to two complex conjugate vector fields (local polarization direction) and string world sheets to light-like vector field and its dual (light-like momentum directions). This slicing generalizes to the Euclidian regions.

4. Infinite number of conservation laws is the signature of integrability. Classical field equations follow from the condition that the vector field defined by modified gamma matrices has vanishing divergence and can be identified an integrability condition for the modified Dirac equation guaranteeing also the conservation of super currents so that one obtains an infinite number of conserved charges.

5. Quantum criticality is a further signal of integrability. 2-D conformal field theories describe critical systems so that the natural guess is that quantum criticality in TGD framework relates to the generalization of conformal invariance and to integrability. Quantum criticality implies that Kähler coupling strength is analogous to critical temperature. This condition does affects classical field equations only via boundary conditions expressed as weak form of electric magnetic duality at the wormhole throats at which the signature of the metric changes.

   For finite-dimensional systems the vanishing of the determinant of the matrix defined by the second derivatives of potential is similar signature and applies in catastrophe theory. Therefore the existence of vanishing second variations of Kähler action should characterize criticality and define a property of preferred extremals. The vanishing of second variations indeed leads to an infinite number of conserved currents [K22, K4] following the conditions that the deformation of modified gamma matrix is also divergenceless and that the modified Dirac equation associated with it is satisfied.

3.8.3 Questions

There are several questions which are not completely settled yet. Even the question what preferred extremals are is still partially open. In the following I try to de-learn what I have possibly learned during these years and start from scratch to see which assumptions might be un-necessarily strong or even wrong.

3.8.4 Could TGD be an integrable theory?

Consider first the abstraction of integrability in TGD framework. Quantum classical correspondence could be seen as a correspondence between linear quantum dynamics and non-linear classical dynamics. Integrability would realize this correspondence. In integrable models such as Sine-Gordon equation particle interactions are described by potential in 1+1 dimensions. This too primitive for the purposes of TGD. The vertices of generalized Feynman diagrams take care of this. At lines one has free particle dynamics so that the situation could be much simpler than in integrable models if one restricts the considerations to the lines or Minkowskian space-time regions surrounding them.

The non-linear dynamics for the space-time sheets representing incoming lines of generalized Feynman diagram should be obtainable from the linear dynamics for the induced spinor fields defined by modified Dirac operator. There are two options.

1. Strong form of the quantum classical correspondence states that each solution for the linear dynamics of spinor fields corresponds to space-time sheet. This is analogous to solving the

potential function in terms of a single solution of Schrödinger equation. Coupling of space-time geometry to quantum numbers via measurement interaction term is a proposal for realizing this option. It is however the quantum numbers of positive/negative energy parts of zero energy state which would be visible in the classical dynamics rather than those of induced spinor field modes.

2. Only overall dynamics characterized by scattering data- the counterpart of S-matrix for the modified Dirac operator- is mapped to the geometry of the space-time sheet. This is much more abstract realization of quantum classical correspondence.

3. Can these two approaches be equivalent? This might be the case since quantum numbers of the state are not those of the modes of induced spinor fields.

What the scattering data could be for the induced spinor field satisfying modified Dirac equation?

1. If the solution of field equation has hydrodynamic character, the solutions of the modified Dirac equation can be localized to light-like Beltrami flow lines of hydrodynamic flow. These correspond to basic solutions and the general solution is a superposition of these. There is no dispersion and the dynamics is that of geometric optics at the basic level. This means geometric optics like character of the spinor dynamics.

   Solutions of the modified Dirac equation are completely analogous to the pulse solutions defining the fundamental solution for the wave equation in the argument leading from wave equation with external time independent potential to Marchenko-Gelfand-Levitan equation allowing to identify potential in terms of scattering data. There is however no potential present now since the interactions are described by the vertices of Feynman diagram where the particle lines meet. Note that particle like regions are Euclidian and that this picture applies only to the Minkowskian exteriors of particles.

2. Partonic 2-surfaces at the ends of the line of generalized Feynman diagram are connected by flow lines. Partonic 2-surfaces at which the signature of the induced metric changes are in a special position. Only the imaginary part of the bi-quaternionic value of the octonion valued map is non-vanishing at these surfaces which can be said to be co-complex 2-surfaces. By geometric optics behavior the scattering data correspond to a diffeomorphism mapping initial partonic 2-surface to the final one in some preferred complex coordinates common to both ends of the line.

3. What could be these preferred coordinates? Complex coordinates for S^2 at light-cone boundary define natural complex coordinates for the partonic 2-surface. With these coordinates the diffeomorphism defining scattering data is diffeomorphism of S^2. Suppose that this map is real analytic so that maps "real axis" of S^2 to itself. This map would be same as the map defining the octonionic real analyticity as algebraic extension of the complex real analytic map. By octonionic analyticity one can make large number of alternative choices for the coordinates of partonic 2-surface.

4. There can be non-uniqueness due to the possibility of $G_2/SU(3)$ valued map characterizing the local octonionic units. The proposal is that the choice of octonionic imaginary units can depend on the point of string like orbit: this would give string model in $G_2/SU(3)$. Conformal invariance for this string model would imply analyticity and helps considerably but would not probably fix the situation completely since the element of the coset space would constant at the partonic 2-surfaces at the ends of CD. One can of course ask whether the $G_2/SU(3)$ element could be constant for each propagator line and would change only at the 2-D vertices?

This would be the inverse scattering problem formulated in the spirit of TGD. There could be also dependence of space-time surface on quantum numbers of quantum states but not on individual solution for the induced spinor field since the scattering data of this solution would be purely geometric.

3.9 About deformations of known extremals of Kähler action

I have done a considerable amount of speculative guesswork to identify what I have used to call preferred extremals of Kähler action. The difficulty is that the mathematical problem at hand is extremely non-linear and that I do not know about existing mathematical literature relevant to the situation. One must proceed by trying to guess the general constraints on the preferred extremals which look physically and mathematically plausible. The hope is that this net of constraints could eventually chrystallize to Eureka! Certainly the recent speculative picture involves also wrong guesses. The need to find explicit ansatz for the deformations of known extremals based on some common principles has become pressing. The following considerations represent an attempt to combine the existing information to achieve this.

3.9.1 What might be the common features of the deformations of known extremals

The dream is to discover the deformations of all known extremals by guessing what is common to all of them. One might hope that the following list summarizes at least some common features.

Effective three-dimensionality at the level of action

1. Holography realized as effective 3-dimensionality also at the level of action requires that it reduces to 3-dimensional effective boundary terms. This is achieved if the contraction $j^\alpha A_\alpha$ vanishes. This is true if j^α vanishes or is light-like, or if it is proportional to instanton current in which case current conservation requires that CP_2 projection of the space-time surface is 3-dimensional. The first two options for j have a realization for known extremals. The status of the third option - proportionality to instanton current - has remained unclear.

2. As I started to work again with the problem, I realized that instanton current could be replaced with a more general current $j = *B \wedge J$ or concretely: $j^\alpha = \epsilon^{\alpha\beta\gamma\delta} B_\beta J_{\gamma\delta}$, where B is vector field and CP_2 projection is 3-dimensional, which it must be in any case. The contractions of j appearing in field equations vanish automatically with this ansatz.

3. Almost topological QFT property in turn requires the reduction of effective boundary terms to Chern-Simons terms: this is achieved by boundary conditions expressing weak form of electric magnetic duality. If one generalizes the weak form of electric-magnetic duality to $J = \Phi * J$ one has $B = d\Phi$ and j has a vanishing divergence for 3-D CP_2 projection. This is clearly a more general solution ansatz than the one based on proportionality of j with instanton current and would reduce the field equations in concise notation to $Tr(TH^k) = 0$.

4. Any of the alternative properties of the Kähler current implies that the field equations reduce to $Tr(TH^k) = 0$, where T and H^k are shorthands for Maxwellian energy momentum tensor and second fundamental form and the product of tensors is obvious generalization of matrix product involving index contraction.

Could Einstein's equations emerge dynamically?

For j^α satisfying one of the three conditions, the field equations have the same form as the equations for minimal surfaces except that the metric g is replaced with Maxwell energy momentum tensor T.

1. This raises the question about dynamical generation of small cosmological constant Λ: $T = \Lambda g$ would reduce equations to those for minimal surfaces. For $T = \Lambda g$ modified gamma matrices would reduce to induced gamma matrices and the modified Dirac operator would be proportional to ordinary Dirac operator defined by the induced gamma matrices. One can also consider weak form for $T = \Lambda g$ obtained by restricting the consideration to a sub-space of tangent space so that space-time surface is only "partially" minimal surface but this option is not so elegant although necessary for other than CP_2 type vacuum extremals.

2. What is remarkable is that $T = \Lambda g$ implies that the divergence of T which in the general case equals to $j^\beta J^\alpha_\beta$ vanishes. This is guaranteed by one of the conditions for the Kähler current. Since also Einstein tensor has a vanishing divergence, one can ask whether the condition to $T = \kappa G + \Lambda g$ could the general condition. This would give Einstein's equations with cosmological term besides the generalization of the minimal surface equations. GRT would emerge dynamically from the non-linear Maxwell's theory although in slightly different sense as conjectured [K72]! Note that the expression for G involves also second derivatives of the imbedding space coordinates so that actually a partial differential equation is in question. If field equations reduce to purely algebraic ones, as the basic conjecture states, it is possible to have $Tr(GH^k) = 0$ and $Tr(gH^k) = 0$ separately so that also minimal surface equations would hold true.

   What is amusing that the first guess for the action of TGD was curvature scalar. It gave analogs of Einstein's equations as a definition of conserved four-momentum currents. The recent proposal would give the analog of ordinary Einstein equations as a dynamical constraint relating Maxwellian energy momentum tensor to Einstein tensor and metric.

3. Minimal surface property is physically extremely nice since field equations can be interpreted as a non-linear generalization of massless wave equation: something very natural for non-linear variant of Maxwell action. The theory would be also very "stringy" although the fundamental action would not be space-time volume. This can however hold true only for Euclidian signature. Note that for CP_2 type vacuum extremals Einstein tensor is proportional to metric so that for them the two options are equivalent. For their small deformations situation changes and it might happen that the presence of G is necessary. The GRT limit of TGD discussed in [K72] [L10] indeed suggests that CP_2 type solutions satisfy Einstein's equations with large cosmological constant and that the small observed value of the cosmological constant is due to averaging and small volume fraction of regions of Euclidian signature (lines of generalized Feynman diagrams).

4. For massless extremals and their deformations $T = \Lambda g$ cannot hold true. The reason is that for massless extremals energy momentum tensor has component T^{vv} which actually quite essential for field equations since one has $H^k_{vv} = 0$. Hence for massless extremals and their deformations $T = \Lambda g$ cannot hold true if the induced metric has Hamilton-Jacobi structure meaning that g^{uu} and g^{vv} vanish. A more general relationship of form $T = \kappa G + \Lambda G$ can however be consistent with non-vanishing T^{vv} but require that deformation has at most 3-D CP_2 projection (CP_2 coordinates do not depend on v).

5. The non-determinism of vacuum extremals suggest for their non-vacuum deformations a conflict with the conservation laws. In, also massless extremals are characterized by a non-determinism with respect to the light-like coordinate but like-likeness saves the situation. This suggests that the transformation of a properly chosen time coordinate of vacuum extremal to a light-like coordinate in the induced metric combined with Einstein's equations in the induced metric of the deformation could allow to handle the non-determinism.

Are complex structure of CP_2 and Hamilton-Jacobi structure of M^4 respected by the deformations?

The complex structure of CP_2 and Hamilton-Jacobi structure of M^4 could be central for the understanding of the preferred extremal property algebraically.

1. There are reasons to believe that the Hermitian structure of the induced metric ((1,1) structure in complex coordinates) for the deformations of CP_2 type vacuum extremals could be crucial property of the preferred extremals. Also the presence of light-like direction is also an essential elements and 3-dimensionality of M^4 projection could be essential. Hence a good guess is that allowed deformations of CP_2 type vacuum extremals are such that (2,0) and (0,2) components the induced metric and/or of the energy momentum tensor vanish. This gives rise to the conditions implying Virasoro conditions in string models in quantization:

$$g_{\xi^i\xi^j} = 0 \ , \quad g_{\overline{\xi}^i\overline{\xi}^j} = 0 \ , \quad i,j = 1,2 \ . \tag{3.9.1}$$

Holomorphisms of CP_2 preserve the complex structure and Virasoro conditions are expected to generalize to 4-dimensional conditions involving two complex coordinates. This means that the generators have two integer valued indices but otherwise obey an algebra very similar to the Virasoro algebra. Also the super-conformal variant of this algebra is expected to make sense.

These Virasoro conditions apply in the coordinate space for CP_2 type vacuum extremals. One expects similar conditions hold true also in field space, that is for M^4 coordinates.

2. The integrable decomposition $M^4(m) = M^2(m)+E^2(m)$ of M^4 tangent space to longitudinal and transversal parts (non-physical and physical polarizations) - Hamilton-Jacobi structure- could be a very general property of preferred extremals and very natural since non-linear Maxwellian electrodynamics is in question. This decomposition led rather early to the introduction of the analog of complex structure in terms of what I called Hamilton-Jacobi coordinates $(u, v, w, \overline{w})$ for M^4. (u, v) defines a pair of light-like coordinates for the local longitudinal space $M^2(m)$ and $(w, \overline{w})$ complex coordinates for $E^2(m)$. The metric would not contain any cross terms between $M^2(m)$ and $E^2(m)$: $g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$.

   A good guess is that the deformations of massless extremals respect this structure. This condition gives rise to the analog of the constraints leading to Virasoro conditions stating the vanishing of the non-allowed components of the induced metric. $g_{uu} = g_{vv} = g_{ww} = g_{\overline{w}\overline{w}} = g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$. Again the generators of the algebra would involve two integers and the structure is that of Virasoro algebra and also generalization to super algebra is expected to make sense. The moduli space of Hamilton-Jacobi structures would be part of the moduli space of the preferred extremals and analogous to the space of all possible choices of complex coordinates. The analogs of infinitesimal holomorphic transformations would preserve the modular parameters and give rise to a 4-dimensional Minkowskian analog of Virasoro algebra. The conformal algebra acting on CP_2 coordinates acts in field degrees of freedom for Minkowskian signature.

Field equations as purely algebraic conditions

If the proposed picture is correct, field equations would reduce basically to purely algebraically conditions stating that the Maxwellian energy momentum tensor has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1,1) and second fundamental form H^k a tensor of type (2,0) and (0,2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

3.9.2 What small deformations of CP_2 type vacuum extremals could be?

I was led to these arguments when I tried find preferred extremals of Kähler action, which would have 4-D CP_2 and M^4 projections - the Maxwell phase analogous to the solutions of Maxwell's equations that I conjectured long time ago. It however turned out that the dimensions of the projections can be $(D_{M^4} \leq 3, D_{CP_2} = 4)$ or $(D_{M^4} = 4, D_{CP_2} \leq 3)$. What happens is essentially breakdown of linear superposition so that locally one can have superposition of modes which have 4-D wave vectors in the same direction. This is actually very much like quantization of radiation field to photons now represented as separate space-time sheets and one can say that Maxwellian superposition corresponds to union of separate photonic space-time sheets in TGD.

Approximate linear superposition of fields is fundamental in standard physics framework and is replaced in TGD with a linear superposition of effects of classical fields on a test particle topologically condensed simultaneously to several space-time sheets. One can say that linear superposition is replaced with a disjoint union of space-time sheets. In the following I shall restrict the consideration to the deformations of CP_2 type vacuum extremals (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

Solution ansatz

I proceed by the following arguments to the ansatz.

1. Effective 3-dimensionality for action (holography) requires that action decomposes to vanishing $j^\alpha A_\alpha$ term + total divergence giving 3-D "boundary" terms. The first term certainly vanishes (giving effective 3-dimensionality) for

$$D_\beta J^{\alpha\beta} = j^\alpha = 0 \ .$$

   Empty space Maxwell equations, something extremely natural. Also for the proposed GRT limit these equations are true.

2. How to obtain empty space Maxwell equations $j^\alpha = 0$? The answer is simple: assume self duality or its slight modification:

$$J = *J$$

   holding for CP_2 type vacuum extremals or a more general condition

$$J = k * J \ ,$$

   In the simplest situation k is some constant not far from unity. * is Hodge dual involving 4-D permutation symbol. $k = constant$ requires that the determinant of the induced metric is apart from constant equal to that of CP_2 metric. It does not require that the induced metric is proportional to the CP_2 metric, which is not possible since M^4 contribution to metric has Minkowskian signature and cannot be therefore proportional to CP_2 metric.

   One can consider also a more general situation in which k is scalar function as a generalization of the weak electric-magnetic duality. In this case the Kähler current is non-vanishing but divergenceless. This also guarantees the reduction to $Tr(TH^k) = 0$. In this case however the proportionality of the metric determinant to that for CP_2 metric is not needed. This solution ansatz becomes therefore more general.

3. Field equations reduce with these assumptions to equations differing from minimal surfaces equations only in that metric g is replaced by Maxwellian energy momentum tensor T. Schematically:

$$Tr(TH^k) = 0 \ ,$$

   where T is the Maxwellian energy momentum tensor and H^k is the second fundamental form - asymmetric 2-tensor defined by covariant derivative of gradients of imbedding space coordinates.

How to satisfy the condition $Tr(TH^k) = 0$?

It would be nice to have minimal surface equations since they are the non-linear generalization of massless wave equations. It would be also nice to have the vanishing of the terms involving Kähler current in field equations as a consequence of this condition. Indeed, $T = \kappa G + \Lambda g$ implies this. In the case of CP_2 vacuum extremals one cannot distinguish between these options since CP_2 itself is constant curvature space with $G \propto g$. Furthermore, if G and g have similar tensor structure the algebraic field equations for G and g are satisfied separately so that one obtains minimal surface property also now. In the following minimal surface option is considered.

1. The first opton is achieved if one has

$$T = \Lambda g \ .$$

Maxwell energy momentum tensor would be proportional to the metric! One would have dynamically generated cosmological constant! This begins to look really interesting since it appeared also at the proposed GRT limit of TGD [L10]. Note that here also non-constant value of Λ can be considered and would correspond to a situation in which k is scalar function: in this case the the determinant condition can be dropped and one obtains just the minimal surface equations.

2. Very schematically and forgetting indices and being sloppy with signs, the expression for T reads as

$$T = JJ - g/4Tr(JJ) \ .$$

Note that the product of tensors is obtained by generalizing matrix product. This should be proportional to metric.

Self duality implies that $Tr(JJ)$ is just the instanton density and does not depend on metric and is constant.

For CP_2 type vacuum extremals one obtains

$$T = -g + g = 0 \ .$$

Cosmological constant would vanish in this case.

3. Could it happen that for deformations a small value of cosmological constant is generated? The condition would reduce to

$$JJ = (\Lambda - 1)g \ .$$

Λ must relate to the value of parameter k appearing in the generalized self-duality condition. For the most general ansatz Λ would not be constant anymore.

This would generalize the defining condition for Kähler form

$$JJ = -g \ \ (i^2 = -1 \ \ geometrically)$$

stating that the square of Kähler form is the negative of metric. The only modification would be that index raising is carried out by using the induced metric containing also M^4 contribution rather than CP_2 metric.

4. Explicitly:

$$J_{\alpha\mu}J^{\mu}_{\ \beta} = (\Lambda - 1)g_{\alpha\beta} \ .$$

Cosmological constant would measure the breaking of Kähler structure. By writing $g = s+m$ and defining index raising of tensors using CP_2 metric and their product accordingly, this condition can be also written as

$$Jm = (\Lambda - 1)mJ \ .$$

If the parameter k is constant, the determinant of the induced metric must be proportional to the CP_2 metric. If k is scalar function, this condition can be dropped. Cosmological constant would not be constant anymore but the dependence on k would drop out from the field equations and one would hope of obtaining minimal surface equations also now. It however seems that the dimension of M^4 projection cannot be four. For 4-D M^4 projection the contribution of the M^2 part of the M^4 metric gives a non-holomorphic contribution to CP_2 metric and this spoils the field equations.

For $T = \kappa G + \Lambda g$ option the value of the cosmological constant is large - just as it is for the proposed GRT limit of TGD [K72] [L10]. The interpretation in this case is that the average value of cosmological constant is small since the portion of space-time volume containing generalized Feynman diagrams is very small.

More detailed ansatz for the deformations of CP_2 type vacuum extremals

One can develop the ansatz to a more detailed form. The most obvious guess is that the induced metric is apart from constant conformal factor the metric of CP_2. This would guarantee self-duality apart from constant factor and $j^\alpha = 0$. Metric would be in complex CP_2 coordinates tensor of type (1,1) whereas CP_2 Riemann connection would have only purely holomorphic or anti-holomorphic indices. Therefore CP_2 contributions in $Tr(TH^k)$ would vanish identically. M^4 degrees of freedom however bring in difficulty. The M^4 contribution to the induced metric should be proportional to CP_2 metric and this is impossible due to the different signatures. The M^4 contribution to the induced metric breaks its Kähler property but would preserve Hermitian structure.

A more realistic guess based on the attempt to construct deformations of CP_2 type vacuum extremals is following.

1. Physical intuition suggests that M^4 coordinates can be chosen so that one has integrable decomposition to longitudinal degrees of freedom parametrized by two light-like coordinates u and v and to transversal polarization degrees of freedom parametrized by complex coordinate w and its conjugate. M^4 metric would reduce in these coordinates to a direct sum of longitudinal and transverse parts. I have called these coordinates Hamilton-Jacobi coordinates.

2. w would be holomorphic function of CP_2 coordinates and therefore satisfy the analog of massless wave equation. This would give hopes about rather general solution ansatz. u and v cannot be holomorphic functions of CP_2 coordinates. Unless wither u or v is constant, the induced metric would receive contributions of type (2,0) and (0,2) coming from u and v which would break Kähler structure and complex structure. These contributions would give no-vanishing contribution to all minimal surface equations. Therefore either u or v is constant: the coordinate line for non-constant coordinate -say u- would be analogous to the M^4 projection of CP_2 type vacuum extremal.

3. With these assumptions the induced metric would remain $(1,1)$ tensor and one might hope that $Tr(TH^k)$ contractions vanishes for all variables except u because the there are no common index pairs (this if non-vanishing Christoffel symbols for H involve only holomorphic or anti-holomorphic indices in CP_2 coordinates). For u one would obtain massless wave equation expressing the minimal surface property.

4. If the value of k is constant the determinant of the induced metric must be proportional to the determinant of CP_2 metric. The induced metric would contain only the contribution from the transversal degrees of freedom besides CP_2 contribution. Minkowski contribution has however rank 2 as CP_2 tensor and cannot be proportional to CP_2 metric. It is however enough that its determinant is proportional to the determinant of CP_2 metric with constant proportionality coefficient. This condition gives an additional non-linear condition to the solution. One would have wave equation for u (also w and its conjugate satisfy massless wave equation) and determinant condition as an additional condition.

   The determinant condition reduces by the linearity of determinant with respect to its rows to sum of conditions involved 0,1,2 rows replaced by the transversal M^4 contribution to metric given if M^4 metric decomposes to direct sum of longitudinal and transversal parts. Derivatives with respect to derivative with respect to particular CP_2 complex coordinate appear linearly in this expression they can depend on u via the dependence of transversal metric components on u. The challenge is to show that this equation has (or does not have) non-trivial solutions.

5. If the value of k is scalar function the situation changes and one has only the minimal surface equations and Virasoro conditions.

What makes the ansatz attractive is that special solutions of Maxwell empty space equations are in question, equations reduces to non-linear generalizations of Euclidian massless wave equations, and possibly space-time dependent cosmological constant pops up dynamically. These properties are true also for the GRT limit of TGD [L10].

3.9.3 Hamilton-Jacobi conditions in Minkowskian signature

The maximally optimistic guess is that the basic properties of the deformations of CP_2 type vacuum extremals generalize to the deformations of other known extremals such as massless extremals, vacuum extremals with 2-D CP_2 projection which is Lagrangian manifold, and cosmic strings characterized by Minkowskian signature of the induced metric. These properties would be following.

1. The recomposition of M^4 tangent space to longitudinal and transversal parts giving Hamilton-Jacobi structure. The longitudinal part has hypercomplex structure but the second light-like coordinate is constant: this plays a crucial role in guaranteeing the vanishing of contractions in $Tr(TH^k)$. It is the algebraic properties of g and T which are crucial. T can however have light-like component T^{vv}. For the deformations of CP_2 type vacuum extremals $(1,1)$ structure is enough and is guaranteed if second light-like coordinate of M^4 is constant whereas w is holomorphic function of CP_2 coordinates.

2. What could happen in the case of massless extremals? Now one has 2-D CP_2 projection in the initial situation and CP_2 coordinates depend on light-like coordinate u and single real transversal coordinate. The generalization would be obvious: dependence on single light-like coordinate u and holomorphic dependence on w for complex CP_2 coordinates. The constraint is $T = \Lambda g$ cannot hold true since T^{vv} is non-vanishing (and light-like). This property restricted to transversal degrees of freedom could reduce the field equations to minimal surface equations in transversal degrees of freedom. The transversal part of energy momentum tensor would be proportional to metric and hence covariantly constant. Gauge current would remain light-like but would not be given by $j = *d\phi \wedge J$. $T = \kappa G + \Lambda g$ seems to define the attractive option.

It therefore seems that the essential ingredient could be the condition

$$T = \kappa G + \lambda g \ ,$$

which has structure (1,1) in both $M^2(m)$ and $E^2(m)$ degrees of freedom apart from the presence of T^{vv} component with deformations having no dependence on v. If the second fundamental form has (2,0)+(0,2) structure, the minimal surface equations are satisfied provided Kähler current satisfies on of the proposed three conditions and if G and g have similar tensor structure.

One can actually pose the conditions of metric as complete analogs of stringy constraints leading to Virasoro conditions in quantization to give

$$g_{uu} = 0 \ , \quad g_{vv} = 0 \ , \quad g_{ww} = 0 \ , \quad g_{\overline{w}\overline{w}} = 0 \ . \tag{3.9.2}$$

This brings in mind the generalization of Virasoro algebra to four-dimensional algebra for which an identification in terms of non-local Yangian symmetry has been proposed [K80]. The number of conditions is four and the same as the number of independent field equations. One can consider similar conditions also for the energy momentum tensor T but allowing non-vanishing component T^{vv} if deformations has no v-dependence. This would solve the field equations if the gauge current vanishes or is light-like. On this case the number of equations is 8. First order differential equations are in question and they can be also interpreted as conditions fixing the coordinates used since there is infinite number of manners to choose the Hamilton-Jacobi coordinates.

One can can try to apply the physical intuition about general solutions of field equations in the linear case by writing the solution as a superposition of left and right propagating solutions:

$$\xi^k = f_+^k(u,w) + f_+^k(v,w) \ . \tag{3.9.3}$$

This could guarantee that second fundamental form is of form (2,0)+(0,2) in both M^2 and E^2 part of the tangent space and these terms if $Tr(TH^k)$ vanish identically. The remaining terms involve contractions of T^{uw}, $T^{u\overline{w}}$ and T^{vw}, $T^{v\overline{w}}$ with second fundamental form. Also these terms should sum up to zero or vanish separately. Second fundamental form has components coming from f_+^k and f_-^k

Second fundamental form H^k has as basic building bricks terms $\hat{H}^k$ given by

$$\hat{H}^k_{\alpha\beta} = \partial_\alpha\partial_\beta h^k + \left\{ {k \atop l\ m} \right\} \partial_\alpha h^l \partial_\beta h^m \ . \tag{3.9.4}$$

For the proposed ansatz the first terms give vanishing contribution to H^k_{uv}. The terms containing Christoffel symbols however give a non-vanishing contribution and one can allow only f^k_+ or f^k_- as in the case of massless extremals. This reduces the dimension of CP_2 projection to $D = 3$.

What about the condition for Kähler current? Kähler form has components of type $J_{w\overline{w}}$ whose contravariant counterpart gives rise to space-like current component. J_{uw} and $J_{u\overline{w}}$ give rise to light-like currents components. The condition would state that the $J^{w\overline{w}}$ is covariantly constant. Solutions would be characterized by a constant Kähler magnetic field. Also electric field is represent. The interpretation both radiation and magnetic flux tube makes sense.

3.9.4 Deformations of cosmic strings

In the physical applications it has been assumed that the thickening of cosmic strings to Kähler magnetic flux tubes takes place. One indeed expects that the proposed construction generalizes also to the case of cosmic strings having the decomposition $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 a complex homologically non-trivial sub-manifold of CP_2. Now the starting point structure is Hamilton-Jacobi structure for $M^2_m \times Y^2$ defining the coordinate space.

1. The deformation should increase the dimension of either CP_2 or M^4 projection or both. How this thickening could take place? What comes in mind that the string orbits X^2 can be interpreted as a distribution of longitudinal spaces $M^2(x)$ so that for the deformation w coordinate becomes a holomorphic function of the natural Y^2 complex coordinate so that M^4 projection becomes 4-D but CP_2 projection remains 2-D. The new contribution to the X^2 part of the induced metric is vanishing and the contribution to the Y^2 part is of type $(1,1)$ and the the ansatz $T = \kappa G + \Lambda g$ might be needed as a generalization of the minimal surface equations The ratio of κ and G would be determined from the form of the Maxwellian energy momentum tensor and be fixed at the limit of undeformed cosmic strong to $T = (ag(Y^2) - bg(Y^2)$. The value of cosmological constant is now large, and overall consistency suggests that $T = \kappa G + \Lambda g$ is the correct option also for the CP_2 type vacuum extremals.

2. One could also imagine that remaining CP_2 coordinates could depend on the complex coordinate of Y^2 so that also CP_2 projection would become 4-dimensional. The induced metric would receive holomorphic contributions in Y^2 part. As a matter fact, this option is already implied by the assumption that Y^2 is a complex surface of CP_2.

3.9.5 Deformations of vacuum extremals?

What about the deformations of vacuum extremals representable as maps from M^4 to CP_2?

1. The basic challenge is the non-determinism of the vacuum extremals. One should perform the deformation so that conservation laws are satisfied. For massless extremals there is also non-determinism but it is associated with the light-like coordinate so that there are no problems with the conservation laws. This would suggest that a properly chosen time coordinate consistent with Hamilton-Jacobi decomposition becomes light-like coordinate in the induced metric. This poses a conditions on the induced metric.

2. Physical intuition suggests that one cannot require $T = \Lambda g$ since this would mean that the rank of T is maximal whereas the original situation corresponds to the vanishing of T. For small deformations rank two for T looks more natural and one could think that T is proportional to a projection of metric to a 2-D subspace. The vision about the long length scale limit of TGD is that Einstein's equations are satisfied and this would suggest $T = kG$ or $T = \kappa G + \Lambda g$. The rank of T could be smaller than four for this ansatz and this conditions binds together the values of κ and G.

3. These extremals have CP_2 projection which in the generic case is 2-D Lagrangian sub-manifold Y^2. Again one could assume Hamilton-Jacobi coordinates for X^4. For CP_2 one could assume Darboux coordinates (P_i, Q_i), $i = 1, 2$, in which one has $A = P_i dQ^i$, and that $Y^2 \subset CP_2$ corresponds to $Q_i = constant$. In principle P_i would depend on arbitrary manner on M^4 coordinates. It might be more convenient to use as coordinates (u, v) for M^2 and (P_1, P_2) for Y^2. This covers also the situation when M^4 projection is not 4-D. By its 2-dimensionality Y^2 allows always a complex structure defined by its induced metric: this complex structure is not consistent with the complex structure of CP_2 (Y^2 is not complex sub-manifold).

   Using Hamilton-Jacobi coordinates the pre-image of a given point of Y^2 is a 2-dimensional sub-manifold X^2 of X^4 and defines also 2-D sub-manifold of M^4. The following picture suggests itself. The projection of X^2 to M^4 can be seen for a suitable choice of Hamilton-Jacobi coordinates as an analog of Lagrangian sub-manifold in M^4 that is as surface for which v and $Im(w)$ vary and u and $Re(w)$ are constant. X^2 would be obtained by allowing u and $Re(w)$ to vary: as a matter fact, (P_1, P_2) and $(u, Re(w))$ would be related to each other. The induced metric should be consistent with this picture. This would requires $g_{uRe(w)} = 0$.

   For the deformations Q_1 and Q_2 would become non-constant and they should depend on the second light-like coordinate v only so that only g_{uu} and g_{uw} and $gu\overline{w}$ $g_{w,w}$ and $g_{\overline{w},\overline{w}}$ receive contributions which vanish. This would give rise to the analogs of Virasoro conditions guaranteeing that T is a tensor of form $(1, 1)$ in both M^2 and E^2 indices and that there are no cross components in the induced metric. A more general formulation states that energy momentum tensor satisfies these conditions. The conditions on T might be equivalent with the conditions for g and G separately.

4. Einstein's equations provide an attractive manner to achieve the vanishing of effective 3-dimensionality of the action. Einstein equations would be second order differential equations and the idea that a deformation of vacuum extremal is in question suggests that the dynamics associated with them is in directions transversal to Y^2 so that only the deformation is dictated partially by Einstein's equations.

5. Lagrangian manifolds do not involve complex structure in any obvious manner. One could however ask whether the deformations could involve complex structure in a natural manner in CP_2 degrees of freedom so that the vanishing of g_{ww} would be guaranteed by holomorphy of CP_2 complex coordinate as function of w.

   One should get the complex structure in some natural manner: in other words, the complex structure should relate to the geometry of CP_2 somehow. The complex coordinate defined by say $z = P_1 + iQ^1$ for the deformation suggests itself. This would suggest that at the limit when one puts $Q_1 = 0$ one obtains $P_1 = P_1(Re(w))$ for the vacuum extremals and the deformation could be seen as an analytic continuation of real function to region of complex plane. This is in spirit with the algebraic approach. The vanishing of Kähler current requires that the Kähler magnetic field is covariantly constant: $D_z J^{z\overline{z}} = 0$ and $D_{\overline{z}} J^{z\overline{z}} = 0$.

6. One could consider the possibility that the resulting 3-D sub-manifold of CP_2 can be regarded as contact manifold with induced Kähler form non-vanishing in 2-D section with natural complex coordinates. The third coordinate variable- call it s- of the contact manifold and second coordinate of its transversal section would depend on time space-time coordinates for vacuum extremals. The coordinate associated with the transversal section would be continued to a complex coordinate which is holomorphic function of w and u.

7. The resulting thickened magnetic flux tubes could be seen as another representation of Kähler magnetic flux tubes: at this time as deformations of vacuum flux tubes rather than cosmic strings. For this ansatz it is however difficult to imagine deformations carrying Kähler electric field.

3.9.6 About the interpretation of the generalized conformal algebras

The long-standing challenge has been finding of the direct connection between the super-conformal symmetries assumed in the construction of the geometry of the "world of classical worlds" (WCW) and possible conformal symmetries of field equations. 4-dimensionality and Minkowskian signature have been the basic problems. The recent construction provides new insights to this problem.

1. In the case of string models the quantization of the Fourier coefficients of coordinate variables of the target space gives rise to Kac-Moody type algebra and Virasoro algebra generators are quadratic in these. Also now Kac-Moody type algebra is expected. If one were to perform a quantization of the coefficients in Laurents series for complex CP_2 coordinates, one would obtain interpretation in terms of $su(3) = u(2)+t$ decomposition, where t corresponds to CP_3: the oscillator operators would correspond to generators in t and their commutator would give generators in $u(2)$. SU(3)/SU(2) coset representation for Kac-Moody algebra would be in question. Kac-Moody algebra would be associated with the generators in both M^4 and CP_2 degrees of freedom. This kind of Kac-Moody algebra appears in quantum TGD.

2. The constraints on induced metric imply a very close resemblance with string models and a generalization of Virasoro algebra emerges. An interesting question is how the two algebras acting on coordinate and field degrees of freedom relate to the super-conformal algebras defined by the symplectic group of $\delta M^4_+ \times CP_2$ acting on space-like 3-surfaces at boundaries of CD and to the Kac-Moody algebras acting on light-like 3-surfaces. It has been conjectured that these algebras allow a continuation to the interior of space-time surface made possible by its slicing by 2-surfaces parametrized by 2-surfaces. The proposed construction indeed provides this kind of slicings in both M^4 and CP_2 factor.

3. In the recent case, the algebras defined by the Fourier coefficients of field variables would be Kac-Moody algebras. Virasoro algebra acting on preferred coordinates would be expressed in terms of the Kac-Moody algebra in the standard Sugawara construction applied in string models. The algebra acting on field space would be analogous to the conformal algebra assignable to the symplectic algebra so that also symplectic algebra is present. Stringy pragmatist could imagine quantization of symplectic algebra by replacing CP_2 coordinates in the expressions of Hamiltonians with oscillator operators. This description would be counterpart for the construction of spinor harmonics in WCW and might provide some useful insights.

4. For given type of space-time surface either CP_2 or M^4 corresponds to Kac-Moody algebra but not both. From the point of view of quantum TGD it looks as that something were missing. An analogous problem was encountered at GRT limit of TGD [L10]. When Euclidian space-time regions are allowed Einstein-Maxwell action is able to mimic standard model with a surprising accuracy but there is a problem: one obtains either color charges or M^4 charges but not both. Perhaps it is not enough to consider either CP_2 type vacuum extremal or its exterior but both to describe particle: this would give the direct product of the Minkowskian and Euclidian algebras acting on tensor product. This does not however seem to be consistent with the idea that the two descriptions are duality related (the analog of T-duality).

3.10 Do geometric invariants of preferred extremals define topological invariants of space-time surface and code for quantum physics?

The recent progress in the understanding of preferred extremals [K4] led to a reduction of the field equations to conditions stating for Euclidian signature the existence of Kähler metric. The resulting conditions are a direct generalization of corresponding conditions emerging for the string world sheet and stating that the 2-metric has only non-diagonal components in complex/hypercomplex coordinates. Also energy momentum of Kähler action and has this characteristic (1,1) tensor structure. In Minkowskian signature one obtains the analog of 4-D complex structure combining hyper-complex structure and 2-D complex structure.

The construction lead also to the understanding of how Einstein's equations with cosmological term follow as a consistency condition guaranteeing that the covariant divergence of the Maxwell's energy momentum tensor assignable to Kähler action vanishes. This gives $T = kG+\Lambda g$. By taking trace a further condition follows from the vanishing trace of T:

$$R = \frac{4\Lambda}{k} \ . \tag{3.10.1}$$

That any preferred extremal should have a constant Ricci scalar proportional to cosmological constant is very strong prediction. Note that the accelerating expansion of the Universe would support positive value of Λ. Note however that both Λ and $k \propto 1/G$ are both parameters characterizing one particular preferred extremal. One could of course argue that the dynamics allowing only constant curvature space-times is too simple. The point is however that particle can topologically condense on several space-time sheets meaning effective superposition of various classical fields defined by induced metric and spinor connection.

The following considerations demonstrate that preferred extremals can be seen as canonical representatives for the constant curvature manifolds playing central role in Thurston's geometrization theorem [A33] known also as hyperbolization theorem implying that geometric invariants of space-time surfaces transform to topological invariants. The generalization of the notion of Ricci flow to Maxwell flow in the space of metrics and further to Kähler flow for preferred extremals in turn gives a rather detailed vision about how preferred extremals organize to one-parameter orbits. It is quite possible that Kähler flow is actually discrete. The natural interpretation is in terms of dissipation and self organization.

Quantum classical correspondence suggests that this line of thought could be continued even further: could the geometric invariants of the preferred extremals could code not only for space-time topology but also for quantum physics? How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge of quantum TGD. Could the correlation functions be reduced to statistical geometric invariants of preferred extemals? The latest (means the end of 2012) and perhaps the most powerful idea hitherto about coupling constant evolution is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

3.10.1 Preferred extremals of Kähler action as manifolds with constant Ricci scalar whose geometric invariants are topological invariants

An old conjecture inspired by the preferred extremal property is that the geometric invariants of space-time surface serve as topological invariants. The reduction of Kähler action to 3-D Chern-Simons terms [K4] gives support for this conjecture as a classical counterpart for the view about TGD as almost topological QFT. The following arguments give a more precise content to this conjecture in terms of existing mathematics.

1. It is not possible to represent the scaling of the induced metric as a deformation of the space-time surface preserving the preferred extremal property since the scale of CP_2 breaks scale invariance. Therefore the curvature scalar cannot be chosen to be equal to one numerically. Therefore also the parameter $R = 4\Lambda/k$ and also Λ and k separately characterize the equivalence class of preferred extremals as is also physically clear.

   Also the volume of the space-time sheet closed inside causal diamond CD remains constant along the orbits of the flow and thus characterizes the space-time surface. Λ and even $k \propto 1/G$ can indeed depend on space-time sheet and p-adic length scale hypothesis suggests a discrete spectrum for Λ/k expressible in terms of p-adic length scales: $\Lambda/k \propto 1/L_p^2$ with $p \simeq 2^k$ favored by p-adic length scale hypothesis. During cosmic evolution the p-adic length scale would increase gradually. This would resolve the problem posed by cosmological constant in GRT based theories.

2. One could also see the preferred extremals as 4-D counterparts of constant curvature 3-manifolds in the topology of 3-manifolds. An interesting possibility raised by the observed

negative value of Λ is that most 4-surfaces are constant negative curvature 4-manifolds. By a general theorem coset spaces H^4/Γ, where $H^4 = SO(1,4)/SO(4)$ is hyperboloid of M^5 and Γ a torsion free discrete subgroup of $SO(1,4)$ [A10]. It is not clear to me, whether the constant value of Ricci scalar implies constant sectional curvatures and therefore hyperbolic space property. It could happen that the space of spaces with constant Ricci curvature contain a hyperbolic manifold as an especially symmetric representative. In any case, the geometric invariants of hyperbolic metric are topological invariants.

By Mostow rigidity theorem [A15] finite-volume hyperbolic manifold is unique for $D > 2$ and determined by the fundamental group of the manifold. Since the orbits under the Kähler flow preserve the curvature scalar the manifolds at the orbit must represent different imbeddings of one and hyperbolic 4-manifold. In 2-D case the moduli space for hyperbolic metric for a given genus $g > 0$ is defined by Teichmueller parameters and has dimension $6(g-1)$. Obviously the exceptional character of $D = 2$ case relates to conformal invariance. Note that the moduli space in question plays a key role in p-adic mass calculations [K13].

In the recent case Mostow rigidity theorem could hold true for the Euclidian regions and maybe generalize also to Minkowskian regions. If so then both "topological" and "geometro" in "Topological GeometroDynamics" would be fully justified. The fact that geometric invariants become topological invariants also conforms with "TGD as almost topological QFT" and allows the notion of scale to find its place in topology. Also the dream about exact solvability of the theory would be realized in rather convincing manner.

These conjectures are the main result independent of whether the generalization of the Ricci flow discussed in the sequel exists as a continuous flow or possibly discrete sequence of iterates in the space of preferred extremals of Kähler action. My sincere hope is that the reader could grasp how far reaching these result really are.

3.10.2 Is there a connection between preferred extremals and AdS$_4$/CFT correspondence?

The preferred extremals satisfy Einstein Maxwell equations with a cosmological constant and have negative scalar curvature for negative value of Λ. 4-D space-times with hyperbolic metric provide canonical representation for a large class of four-manifolds and an interesting question is whether these spaces are obtained as preferred extremals and/or vacuum extremals.

4-D hyperbolic space with Minkowski signature is locally isometric with AdS$_4$. This suggests at connection with AdS$_4$/CFT correspondence of M-theory. The boundary of AdS would be now replaced with 3-D light-like orbit of partonic 2-surface at which the signature of the induced metric changes. The metric 2-dimensionality of the light-like surface makes possible generalization of 2-D conformal invariance with the light-like coordinate taking the role of complex coordinate at light-like boundary. AdS could represent a special case of a more general family of space-time surfaces with constant Ricci scalar satistying Einstein-Maxwell equations and generalizing the AdS$_4$/CFT correspondence. There is however a strong objection from cosmology: the accelerated expansion of the Universe requires positive value of Λ and favors De Sitter Space dS_4 instead of AdS_4.

These observations provide motivations for finding whether AdS$_4$ and/or dS_4 allows an imbedding as a vacuum extremal to $M^4 \times S^2 \subset M^4 \times CP_2$, where S^2 is a homologically trivial geodesic sphere of CP_2. It is easy to guess the general form of the imbedding by writing the line elements of, M^4, S^2, and AdS$_4$.

1. The line element of M^4 in spherical Minkowski coordinates (m, r_M, θ, ϕ) reads as

$$ds^2 = dm^2 - dr_M^2 - r_M^2 d\Omega^2 \ . \tag{3.10.2}$$

2. Also the line element of S^2 is familiar:

$$ds^2 = -R^2(d\Theta^2 + sin^2(\theta)d\Phi^2) \ . \tag{3.10.3}$$

3. By visiting in Wikipedia one learns that in spherical coordinate the line element of AdS_4/dS_4 is given by

$$\begin{aligned} ds^2 &= A(r)dt^2 - \frac{1}{A(r)}dr^2 - r^2 d\Omega^2 \ , \\ A(r) &= 1+\epsilon y^2 \ , \ y = \frac{r}{r_0} \ , \\ \epsilon &= 1 \text{ for } AdS_4 \ , \ \epsilon = -1 \text{ for } dS_4 \ . \end{aligned} \tag{3.10.4}$$

4. From these formulas it is easy to see that the ansatz is of the same general form as for the imbedding of Schwartschild-Nordstöm metric:

$$\begin{array}{ll} m = \Lambda t + h(y) \ , & r_M = r \ , \\ \Theta = s(y) \ , & \Phi = \omega(t+f(y)) \ . \end{array} \tag{3.10.5}$$

The non-trivial conditions on the components of the induced metric are given by

$$\begin{aligned} g_{tt} &= \Lambda^2 - x^2 sin^2(\Theta) = A(r) \ , \\ g_{tr} &= \frac{1}{r_0}\left[\Lambda\frac{dh}{dy} - x^2 sin^2(\theta)\frac{df}{dr}\right] = 0 \ , \\ g_{rr} &= \frac{1}{r_0^2}\left[(\frac{dh}{dy})^2 - 1 - x^2 sin^2(\theta)(\frac{df}{dy})^2 - R^2(\frac{d\Theta}{dy})^2\right] = -\frac{1}{A(r)} \ , \\ x &= R\omega \ . \end{aligned} \tag{3.10.6}$$

By some simple algebraic manipulations one can derive expressions for $sin(\Theta)$, df/dr and dh/dr.

1. For $\Theta(r)$ the equation for g_{tt} gives the expression

$$\begin{aligned} sin(\Theta) &= \pm\frac{P^{1/2}}{x} \ , \\ P &= \Lambda^2 - A = \Lambda^2 - 1 - \epsilon y^2 \ . \end{aligned} \tag{3.10.7}$$

The condition $0 \leq sin^2(\Theta) \leq 1$ gives the conditions

$$\begin{array}{rcll} (\Lambda^2 - x^2 - 1)^{1/2} \leq & y & \leq (\Lambda^2-1)^{1/2} & \text{for } \epsilon = 1 \ (AdS_4) \ , \\ (-\Lambda^2+1)^{1/2} \leq & y & \leq (x^2+1-\Lambda^2)^{1/2} & \text{for } \epsilon = -1 \ (dS_4) \ . \end{array} \tag{3.10.8}$$

Only a spherical shell is possible in both cases. The model for the final state of star considered in [K72] predicted similar layer layer like structure and inspired the proposal that stars quite generally have an onion-like structure with radii of various shells characterize by p-adic length scale hypothesis and thus coming in some powers of $\sqrt{2}$. This brings in mind also Titius-Bode law.

2. From the vanishing of g_{tr} one obtains

$$\frac{dh}{dy} = \frac{P}{\Lambda}\frac{df}{dy} \ . \tag{3.10.9}$$

3. The condition for g_{rr} gives

$$(\frac{df}{dy})^2 = \frac{r_0^2}{AP}[A^{-1} - R^2(\frac{d\Theta}{dy})^2] \ . \quad (3.10.10)$$

Clearly, the right-hand side is positive if $P \geq 0$ holds true and $Rd\Theta/dy$ is small. One can express $d\Theta/dy$ using chain rule as

$$(\frac{d\Theta}{dy})^2 = \frac{x^2y^2}{P(P-x^2)} \ . \quad (3.10.11)$$

One obtains

$$(\frac{df}{dy})^2 = \Lambda r_0^2 \frac{y^2}{AP}\left[\frac{1}{1+y^2} - x^2(\frac{R}{r_0})^2\frac{1}{P(P-x^2)}\right] \ . \quad (3.10.12)$$

The right hand side of this equation is non-negative for certain range of parameters and variable y. Note that for $r_0 \gg R$ the second term on the right hand side can be neglected. In this case it is easy to integrate $f(y)$.

The conclusion is that both AdS_4 and dS^4 allow a local imbedding as a vacuum extremal. Whether also an imbedding as a non-vacuum preferred extremal to $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere is possible, is an interesting question.

3.10.3 Generalizing Ricci flow to Maxwell flow for 4-geometries and Kähler flow for space-time surfaces

The notion of Ricci flow has played a key part in the geometrization of topological invariants of Riemann manifolds. I certainly did not have this in mind when I choose to call my unification attempt "Topological Geometrodynamics" but this title strongly suggests that a suitable generalization of Ricci flow could play a key role in the understanding of also TGD.

Ricci flow and Maxwell flow for 4-geometries

The observation about constancy of 4-D curvature scalar for preferred extremals inspires a generalization of the well-known volume preserving Ricci flow [A26] introduced by Richard Hamilton. Ricci flow is defined in the space of Riemann metrics as

$$\frac{dg_{\alpha\beta}}{dt} = -2R_{\alpha\beta} + 2\frac{R_{avg}}{D}g_{\alpha\beta} \ . \quad (3.10.13)$$

Here R_{avg} denotes the average of the scalar curvature, and D is the dimension of the Riemann manifold. The flow is volume preserving in average sense as one easily checks ($\langle g^{\alpha\beta}dg_{\alpha\beta}/dt\rangle = 0$). The volume preserving property of this flow allows to intuitively understand that the volume of a 3-manifold in the asymptotic metric defined by the Ricci flow is topological invariant. The fixed points of the flow serve as canonical representatives for the topological equivalence classes of 3-manifolds. These 3-manifolds (for instance hyperbolic 3-manifolds with constant sectional curvatures) are highly symmetric. This is easy to understand since the flow is dissipative and destroys all details from the metric.

What happens in the recent case? The first thing to do is to consider what might be called Maxwell flow in the space of all 4-D Riemann manifolds allowing Maxwell field.

1. First of all, the vanishing of the trace of Maxwell's energy momentum tensor codes for the volume preserving character of the flow defined as

$$\frac{dg_{\alpha\beta}}{dt} = T_{\alpha\beta} \ . \tag{3.10.14}$$

   Taking covariant divergence on both sides and assuming that d/dt and D_α commute, one obtains that $T^{\alpha\beta}$ is divergenceless.

   This is true if one assumes Einstein's equations with cosmological term. This gives

$$\frac{dg_{\alpha\beta}}{dt} = kG_{\alpha\beta} + \Lambda g_{\alpha\beta} = kR_{\alpha\beta} + (-\frac{kR}{2} + \Lambda)g_{\alpha\beta} \ . \tag{3.10.15}$$

   The trace of this equation gives that the curvature scalar is constant. Note that the value of the Kähler coupling strength plays a highly non-trivial role in these equations and it is quite possible that solutions exist only for some critical values of α_K. Quantum criticality should fix the allow value triplets (G, Λ, α_K) apart from overall scaling

$$(G, \Lambda, \alpha_K) \rightarrow (xG, \Lambda/x, x\alpha_K \ .$$

   Fixing the value of G fixes the values remaining parameters at critical points. The rescaling of the parameter t induces a scaling by x.

2. By taking trace one obtains the already mentioned condition fixing the curvature to be constant, and one can write

$$\frac{dg_{\alpha\beta}}{dt} = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \ . \tag{3.10.16}$$

   Note that in the recent case $R_{avg} = R$ holds true since curvature scalar is constant. The fixed points of the flow would be Einstein manifolds [A5, A46] satisfying

$$R_{\alpha\beta} = \frac{\Lambda}{k} g_{\alpha\beta} \tag{3.10.17}$$

   .

3. It is by no means obvious that continuous flow is possible. The condition that Einstein-Maxwell equations are satisfied might pick up from a completely general Maxwell flow a discrete subset as solutions of Einstein-Maxwell equations with a cosmological term. If so, one could assign to this subset a sequence of values t_n of the flow parameter t.

4. I do not know whether 3-dimensionality is somehow absolutely essential for getting the topological classification of closed 3-manifolds using Ricci flow. This ignorance allows me to pose some innocent questions. Could one have a canonical representation of 4-geometries as spaces with constant Ricci scalar? Could one select one particular Einstein space in the class four-metrics and could the ratio Λ/k represent topological invariant if one normalizes metric or curvature scalar suitably. In the 3-dimensional case curvature scalar is normalized to unity. In the recent case this normalization would give $k = 4\Lambda$ in turn giving $R_{\alpha\beta} = g_{\alpha\beta}/4$. Does this mean that there is only single fixed point in local sense, analogous to black hole toward which all geometries are driven by the Maxwell flow? Does this imply that only the 4-volume of the original space would serve as a topological invariant?

Maxwell flow for space-time surfaces

One can consider Maxwell flow for space-time surfaces too. In this case Kähler flow would be the appropriate term and provides families of preferred extremals. Since space-time surfaces inside CD are the basic physical objects are in TGD framework, a possible interpretation of these families would be as flows describing physical dissipation as a four-dimensional phenomenon polishing details from the space-time surface interpreted as an analog of Bohr orbit.

1. The flow is now induced by a vector field $j^k(x,t)$ of the space-time surface having values in the tangent bundle of imbedding space $M^4 \times CP_2$. In the most general case one has Kähler flow without the Einstein equations. This flow would be defined in the space of all space-time surfaces or possibly in the space of all extremals. The flow equations reduce to

$$h_{kl} D_\alpha j^k(x,t) D_\beta h^l = \frac{1}{2} T_{\alpha\beta} . \tag{3.10.18}$$

   The left hand side is the projection of the covariant gradient $D_\alpha j^k(x,t)$ of the flow vector field $j^k(x,t)$ to the tangent space of the space-time surface. D_{alpha} is covariant derivative taking into account that j^k is imbedding space vector field. For a fixed point space-time surface this projection must vanish assuming that this space-time surface reachable. A good guess for the asymptotia is that the divergence of Maxwell energy momentum tensor vanishes and that Einstein's equations with cosmological constant are well-defined.

   Asymptotes corresponds to vacuum extremals. In Euclidian regions CP_2 type vacuum extremals and in Minkowskian regions to any space-time surface in any 6-D sub-manifold $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähler form. Symplectic transformations of CP_2 combined with diffeomorphisms of M^4 give new Lagrangian manifolds. One would expect that vacuum extremals are approached but never reached at second extreme for the flow.

   If one assumes Einstein's equations with a cosmological term, allowed vacuum extremals must be Einstein manifolds. For CP_2 type vacuum extremals this is the case. It is quite possible that these fixed points do not actually exist in Minkowskian sector, and could be replaced with more complex asymptotic behavior such as limit, chaos, or strange attractor.

2. The flow could be also restricted to the space of preferred extremals. Assuming that Einstein Maxwell equations indeed hold true, the flow equations reduce to

$$h_{kl} D_\alpha j^k(x,t) \partial_\beta h^l = \frac{1}{2}(kR_{\alpha\beta} - \Lambda g_{\alpha\beta}) . \tag{3.10.19}$$

   Preferred extremals would correspond to a fixed sub-manifold of the general flow in the space of all 4-surfaces.

3. One can also consider a situation in which $j^k(x,t)$ is replaced with $j^k(h,t)$ defining a flow in the entire imbedding space. This assumption is probably too restrictive. In this case the equations reduce to

$$(D_r j_l(x,t) + D_l j_r)\partial_\alpha h^r \partial_\beta h^l = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} . \tag{3.10.20}$$

   Here D_r denotes covariant derivative. Asymptotia is achieved if the tensor $D_k j_l + D_k j_l$ becomes orthogonal to the space-time surface. Note for that Killing vector fields of H the left hand side vanishes identically. Killing vector fields are indeed symmetries of also asymptotic states.

It must be made clear that the existence of a continuous flow in the space of preferred extremals might be too strong a condition. Already the restriction of the general Maxwell flow in the space of metrics to solutions of Einstein-Maxwell equations with cosmological term might lead to discretization, and the assumption about reprentability as 4-surface in $M^4 \times CP_2$ would give a further condition reducing the number of solutions. On the other hand, one might consiser a possibility of a continuous flow in the space of constant Ricci scalar metrics with a fixed 4-volume and having hyperbolic spaces as the most symmetric representative.

Dissipation, self organization, transition to chaos, and coupling constant evolution

A beautiful connection with concepts like dissipation, self-organization, transition to chaos, and coupling constant evolution suggests itself.

1. It is not at all clear whether the vacuum extremal limits of the preferred extremals can correspond to Einstein spaces except in special cases such as CP_2 type vacuum extremals isometric with CP_2. The imbeddability condition however defines a constraint force which might well force asymptotically more complex situations such as limit cycles and strange attractors. In ordinary dissipative dynamics an external energy feed is essential prerequisite for this kind of non-trivial self-organization patterns.

   In the recent case the external energy feed could be replaced by the constraint forces due to the imbeddability condition. It is not too difficult to imagine that the flow (if it exists!) could define something analogous to a transition to chaos taking place in a stepwise manner for critical values of the parameter t. Alternatively, these discrete values could correspond to those values of t for which the preferred extremal property holds true for a general Maxwell flow in the space of 4-metrics. Therefore the preferred extremals of Kähler action could emerge as one-parameter (possibly discrete) families describing dissipation and self-organization at the level of space-time dynamics.

2. For instance, one can consider the possibility that in some situations Einstein's equations split into two mutually consistent equations of which only the first one is independent

$$\begin{aligned} xJ^{\alpha}{}_{\nu}J^{\nu\beta} &= R^{\alpha\beta} \ , \\ L_K &= xJ^{\alpha}{}_{\nu}J^{\nu\beta} = 4\Lambda \ , \\ x &= \frac{1}{16\pi\alpha_K} \ . \end{aligned} \tag{3.10.21}$$

   Note that the first equation indeed gives the second one by tracing. This happens for CP_2 type vacuum extremals.

   Kähler action density would reduce to cosmological constant which should have a continuous spectrum if this happens always. A more plausible alternative is that this holds true only asymptotically. In this case the flow equation could not lead arbitrary near to vacuum extremal, and one can think of situation in which $L_K = 4\Lambda$ defines an analog of limiting cycle or perhaps even strange attractor. In any case, the assumption would allow to deduce the asymptotic value of the action density which is of utmost importance from calculational point of view: action would be simply $S_K = 4\Lambda V_4$ and one could also say that one has minimal surface with Λ taking the role of string tension.

3. One of the key ideas of TGD is quantum criticality implying that Kähler coupling strength is analogous to critical temperature. Second key idea is that p-adic coupling constant evolution represents discretized version of continuous coupling constant evolution so that each p-adic prime would correspond a fixed point of ordinary coupling constant evolution in the sense that the 4-volume characterized by the p-adic length scale remains constant. The invariance of the geometric and thus geometric parameters of hyperbolic 4-manifold under the Kähler flow would conform with the interpretation as a flow preserving scale assignable to a given p-adic prime. The continuous evolution in question (if possible at all!) might correspond to

a fixed p-adic prime. Also the hierarchy of Planck constants relates to this picture naturally. Planck constant $\hbar_{eff} = n\hbar$ corresponds to a multi-furcation generating n-sheeted structure and certainly affecting the fundamental group.

4. One can of course question the assumption that a continuous flow exists. The property of being a solution of Einstein-Maxwell equations, imbeddability property, and preferred extremal property might allow allow only discrete sequences of space-time surfaces perhaps interpretable as orbit of an iterated map leading gradually to a fractal limit. This kind of discrete sequence might be also be selected as preferred extremals from the orbit of Maxwell flow without assuming Einstein-Maxwell equations. Perhaps the discrete p-adic coupling constant evolution could be seen in this manner and be regarded as an iteration so that the connection with fractality would become obvious too.

Does a 4-D counterpart of thermodynamics make sense?

The interpretation of the Kähler flow in terms of dissipation, the constancy of R, and almost constancy of L_K suggest an interpretation in terms of 4-D variant of thermodynamics natural in zero energy ontology (ZEO), where physical states are analogs for pairs of initial and final states of quantum event are quantum superpositions of classical time evolutions. Quantum theory becomes a "square root" of thermodynamics so that 4-D analog of thermodynamics might even replace ordinary thermodynamics as a fundamental description. If so this 4-D thermodynamics should be qualitatively consistent with the ordinary 3-D thermodynamics.

1. The first naive guess would be the interpretation of the action density L_K as an analog of energy density $e = E/V_3$ and that of R as the analog to entropy density $s = S/V_3$. The asymptotic states would be analogs of thermodynamical equilibria having constant values of L_K and R.

2. Apart from an overall sign factor ϵ to be discussed, the analog of the first law $de = Tds - pdV/V$ would be

$$dL_K = kdR + \Lambda\frac{dV_4}{V_4} \quad .$$

   One would have the correspondences $S \rightarrow \epsilon RV_4$, $e \rightarrow \epsilon L_K$ and $k \rightarrow T$, $p \rightarrow -\Lambda$. $k \propto 1/G$ indeed appears formally in the role of temperature in Einstein's action defining a formal partition function via its exponent. The analog of second law would state the increase of the magnitude of ϵRV_4 during the Kähler flow.

3. One must be very careful with the signs and discuss Euclidian and Minkowskian regions separately. Concerning purely thermodynamic aspects at the level of vacuum functional Euclidian regions are those which matter.

   (a) For CP_2 type vacuum extremals $L_K \propto E^2 + B^2$, $R = \Lambda/k$, and Λ are positive. In thermodynamical analogy for $\epsilon = 1$ this would mean that pressure is negative.

   (b) In Minkowskian regions the value of $R = \Lambda/k$ is negative for $\Lambda < 0$ suggested by the large abundance of 4-manifolds allowing hyperbolic metric and also by cosmological considerations. The asymptotic formula $L_K = 4\Lambda$ considered above suggests that also Kähler action is negative in Minkowskian regions for magnetic flux tubes dominating in TGD inspired cosmology: the reason is that the magnetic contribution to the action density $L_K \propto E^2 - B^2$ dominates.

Consider now in more detail the 4-D thermodynamics interpretation in Euclidian and Minkowskian regions assuming that the the evolution by quantum jumps has Kähler flow as a space-time correlate.

1. In Euclidian regions the choice $\epsilon = 1$ seems to be more reasonable one. In Euclidian regions $-\Lambda$ as the analog of pressure would be negative, and asymptotically (that is for CP_2 type

vacuum extremals) its value would be proportional to $\Lambda \propto 1/GR^2$, where R denotes CP_2 radius defined by the length of its geodesic circle.

A possible interpretation for negative pressure is in terms of string tension effectively inducing negative pressure (note that the solutions of the modified Dirac equation indeed assign a string to the wormhole contact). The analog of the second law would require the increase of RV_4 in quantum jumps. The magnitudes of L_K, R, V_4 and Λ would be reduced and approach their asymptotic values. In particular, V_4 would approach asymptotically the volume of CP_2.

2. In Minkowskian regions Kähler action contributes to the vacuum functional a phase factor analogous to an imaginary exponent of action serving in the role of Morse function so that thermodynamics interpretation can be questioned. Despite this one can check whether thermodynamic interpretation can be considered. The choice $\epsilon = -1$ seems to be the correct choice now. $-\Lambda$ would be analogous to a negative pressure whose gradually decreases. In 3-D thermodynamics it is natural to assign negative pressure to the magnetic flux tube like structures as their effective string tension defined by the density of magnetic energy per unit length. $-R \geq 0$ would entropy and $-L_K \geq 0$ would be the analog of energy density.

   $R = \Lambda/k$ and the reduction of Λ during cosmic evolution by quantum jumps suggests that the larger the volume of CD and thus of (at least) Minkowskian space-time sheet the smaller the negative value of Λ.

   Assume the recent view about state function reduction explaining how the arrow of geometric time is induced by the quantum jump sequence defining experienced time [K3]. According to this view zero energy states are quantum superpositions over CDs of various size scales but with common tip, which can correspond to either the upper or lower light-like boundary of CD. The sequence of quantum jumps the gradual increase of the average size of CD in the quantum superposition and therefore that of average value of V_4. On the other hand, a gradual decrease of both $-L_K$ and $-R$ looks physically very natural. If Kähler flow describes the effect of dissipation by quantum jumps in ZEO then the space-time surfaces would gradually approach nearly vacuum extremals with constant value of entropy density $-R$ but gradually increasing 4-volume so that the analog of second law stating the increase of $-RV_4$ would hold true.

3. The interpretation of $-R > 0$ as negentropy density assignable to entanglement is also possible and is consistent with the interpretation in terms of second law. This interpretation would only change the sign factor ϵ in the proposed formula. Otherwise the above arguments would remain as such.

3.10.4 Could correlation functions, S-matrix, and coupling constant evolution be coded the statistical properties of preferred extremals?

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

1. The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

2. The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

3. The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

   Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root " of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

4. Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix having M-matrices as its orthonormal rows.

5. In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

1. General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

   Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

2. The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \to \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

3. What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

4. What about Higgs field? TGD in principle allows scalar and pseudo-scalars which could be called Higgs like states. These states are however not necessary for particle massivation although they can represent particle massivation and must do so if one assumes that QFT limit exist. p-Adic thermodynamics however describes particle massivation microscopically.

   The problem is that Higgs like field does not seem to have any obvious space-time correlate. The trace of the second fundamental form is the obvious candidate but vanishes for preferred extremals which are both minimal surfaces and solutions of Einstein Maxwell equations with cosmological constant. If the string world sheets at which all spinor components except right handed neutrino are localized for the general solution ansatz of the modified Dirac equation, the corresponding second fundamental form at the level of imbedding space defines a candidate for classical Higgs field. A natural expectation is that string world sheets are minimal surfaces of space-time surface. In general they are however not minimal surfaces of the imbedding space so that one might achieve a microscopic definition of classical Higgs field and its vacuum expectation value as an average of one point correlation function over the string world sheet.

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

3.11 Does thermodynamics have a representation at the level of space-time geometry?

R. Kiehn has proposed what he calls Topological Thermodynamics (TTD) [B42] as a new formulation of thermodynamics. The basic vision is that thermodynamical equations could be translated to differential geometric statements using the notions of differential forms and Pfaffian system [A20]. That TTD differs from TGD by a single letter is not enough to ask whether some relationship between them might exist. Quantum TGD can however in a well-defined sense be regarded as a square root of thermodynamics in zero energy ontology (ZEO) and this leads leads to ask seriously whether TTD might help to understand TGD at deeper level. The thermodynamical interpretation of space-time dynamics would obviously generalize black hole thermodynamics to TGD framework and already earlier some concrete proposals have been made in this direction.

One can raise several questions. Could the preferred extremals of Kähler action code for the square root of thermodynamics? Could induced Kähler gauge potential and Kähler form (essentially Maxwell field) have formal thermodynamic interpretation? The vacuum degeneracy of Kähler action implies 4-D spin glass degeneracy and strongly suggests the failure of strict determinism for the dynamics of Kähler action for non-vacuum extremals too. Could thermodynamical irreversibility and preferred arrow of time allow to characterize the notion of preferred extremal more sharply?

It indeed turns out that one can translate Kiehn's notions to TGD framework rather straightforwardly.

1. Kiehn's work 1- form corresponds to induced Kähler gauge potential implying that the vanishing of instanton density for Kähler form becomes a criterion of reversibility and irreversibility is localized on the (4-D) "lines" of generalized Feyman diagrams, which correspond to space-like signature of the induced metric. The localization of heat production to generalized Feynman diagrams conforms nicely with the kinetic equations of thermodynamics based on reaction rates deduced from quantum mechanics. It also conforms with Kiehn's vision that dissipation involves topology change.

2. Heat produced in a given generalized Feynman diagram is just the integral of instanton density and the condition that the arrow of geometric time has definite sign classically fixes the sign of produced heat to be positive. In this picture the preferred extremals of Kähler action would allow a trinity of interpretations as non-linear Maxwellian dynamics, thermodynamics, and integrable hydrodynamics.

3. The 4-D spin glass degeneracy of TGD breaking of ergodicity suggests that the notion of global thermal equilibrium is too naive. The hierarchies of Planck constants and of p-adic length scales suggests a hierarchical structure based on CDs withing CDs at imbedding space level and space-time sheets topologically condensed at larger space-time sheets at space-time level. The arrow of geometric time for quantum states could vary for sub-CDs and would have thermodynamical space-time correlates realized in terms of distributions of arrows of geometric time for sub-CDs, sub-sub-CDs, etc...

The hydrodynamical character of classical field equations of TGD means that field equations reduce to local conservation laws for isometry currents and Kähler gauge current. This requires the extension of Kiehn's formalism to include besides forms and exterior derivative also induced metric, index raising operation transforming 1-forms to vector fields, duality operation transforming k- forms to n-k forms, and divergence which vanishes for conserved currents.

3.11.1 Motivations and background

It is good to begin by discussing the motivations for the geometrization of thermodynamics and by introducing the existing mathematical framework identifying space-time surfaces as preferred extremals of Kähler action.

ZEO and the need for the space-time correlates for square root of thermodynamics

Quantum classical correspondence is basic guiding principle of quantum TGD. In ZEO TGD can be regarded as a complex square root of thermodynamics so that the thermodynamics should have correlates at the level of the geometry of space-time.

1. Zero energy states consist of pairs of positive and negative energy states assignable to opposite boundaries of a causal diamond (CD). There is entire hierarchy of CDs characterized by their scale coming as an integer multiple of a basic scale (also their Poincare transforms are allowed).

2. In ZEO zero energy states are automatically time-irreversible in the sense that either end of the causal diamond (CD) corresponds to a state consisting of single particle states with well-defined quantum numbers. In other words, this end of CD carries a prepared state. The other end corresponds to a superposition of states which can have even different particle numbers: this is the case in particle physics experiment typically. State function reduction reduces the second end of CD to a prepared state. This process repeats itself. This suggests that the arrow of time or rather, its geometric counterpart which we experience, alternates. This need not however be the case if quantum classical correspondence holds true.

3. To illustrate what I have in mind consider a path towel, which has been been folded forth and back. Assume that the direction in which folding is carried is time direction. Suppose that the inhabitant of bath towel Universe is like the habitant of the famous Flatland and therefore not able to detect the folding of the towel. If the classical dynamics of towel is time irreversible (time corresponds to the direction in which the folding takes place), the inhabitant sees ever lasting irreversible time evolution with single arrow of geometric time identified as time coordinate for the towel: no changes in the arrow of geometric time. If the inhabitant is able to make measurements about 3-D space the situation he or she might be able to see that his time evolution takes place forth and back with respect to the time coordinate of higher-dimensional imbedding space.

4. One might understand the arrow of time - albeit differently as in normal view about the situation - if classical time evolution for the preferred extremals of Kähler action defines a geometric correlate for quantum irreversibility of zero energy states. There are of course other space-time sheets and other CDs present an it might be possible to detect the alternation of the arrow of geometric time at imbedding space level by making measurements giving information about their geometric arrows of time [K3].

By quantum classical correspondence one expects that the geometric arrow of time - irreversibility - for zero energy states should have classical counterparts at the level of the dynamics of preferred extremals of Kähler action. What could be this counterpart? Thermodynamical evolution by quantum jumps does not obey ordinary variational principle that would make it deterministic: Negentropy Maximization Principle (NMP) [K39] for state function reductions of system is analogous to Second Law for an ensemble of copies of system and actually implies it. Could one mimic irreversibility by single classical evolution defined by a preferred extremal? Note that the dynamics of preferred extremals is not actually strictly deterministic in the ordinary sense of the word: the reason is the enormous vacuum degeneracy implying 4-D spin glass degeneracy. This makes it possible to mimic not only quantum states but also sequences of quantum jumps by piece-wise deterministic evolution.

Preferred extremals of Kähler action

In Quantum TGD the basic arena of quantum dynamics is "world of classical worlds" (WCW) [K56]. Purely classical spinor fields in this infinite-dimensional space define quantum states of the Universe. General Coordinate Invariance (GCI) implies that classical worlds can be regarded as either 3-surfaces or 4-D space-time surfaces analogous to Bohr orbits. Strong form of GCI implies in ZEO strong form of holography in the sense that the points of WCW effectively correspond to collections of partonic 2-surfaces belonging to both ends of causal diamonds (CDs) plus their 4-D tangent space-time data.

Kähler geometry reduces to the notion of Kähler function [K31] and by quantum classical correspondence a good guess is that Kähler function corresponds to so called Kähler action for Euclidian space-time regions. Minkowskian space-time regions give a purely imaginary to Kähler action (square root of metric determinant is imaginary) and this contribution plays the role of Morse function for WCW. Stationary phase approximation imples that in first the approximation the extremals of the Kähler *function* (to be distinguished from preferred extremals of Kähler *action*!) select one particular 3-surface and corresponding classical space-time surface (Bohr orbit) as that defining "classical physics".

GCI implies holography and holography suggests that action reduces to 3-D terms. This is true if one has $j^\mu A_\mu = 0$ in the interior of space-time. If one assumes so called weak form of electric-magnetic duality [K22] at the real and effective boundaries of space-time surface (3-D surfaces at the ends of CDs and the light-like 3-surfaces at which the signature of induced 4-metric changes so that 4-metric is degenerate), one obtains a reduction of Kähler action to Chern-Simons terms at the boundaries. TGD reduces to almost topological QFT. "Almost" means that the induced metric does not disappear completely from the theory since it appears in the conditions expressing weak form of electric magnetic duality and in the condition $j^\mu A_\mu = 0$.

The strong form of holography implies effective 2-dimensionality and this suggests the reduction of Chern-Simons terms to 2-dimensional areas of string world sheets and possible of partonic 2-surfaces. This would mean almost reduction to string theory like theory with string tension becoming a dynamic quantity.

Under additional rather general conditions the contributions from Minkowskian and Euclidian regions of space-time surface are apart from the value of coefficient identical at light-like 3-surfaces. At space-like 3-surfaces at the ends of space-time surface they need not be identical.

Quantum classical correspondence suggests that space-time surfaces provide a representation for the square root of thermodynamics and therefore also for thermodynamics. In general relativity black hole thermodynamics suggests the same. This idea is not new in TGD framework. For instance, Hawking- Bekenstein formula for blackbody entropy [B1] allows a p-adic generalization in terms of area of partonic 2-surfaces [K47]. The challenge is to deduce precise form of this correspondence and here Kiehn's topological thermodynamics might help in this task.

3.11.2 Kiehn's topological thermodynamics (TTD)

The basic in the work of Kiehn is that thermodynamics allows a topological formulation in terms of differential geometry.

1. Kiehn introduces also the notions of hrefhttp://www22.pair.com/csdc/pdf/irevtors.pdfPfaff system and Pfaff dimension as the number of non-vanishing forms in the sequence for given 1-form such as W or Q: W, dW, $W \wedge dW$, $dW \wedge dW$. Pfaff dimension $D \leq 4$ tells that one can describe W as sum $W = \sum W_k dx^k$ of gradients of D variables. $D = 4$ corresponds to open system, $D = 3$ to a closed system and $W \wedge dW \neq 0$ defines what can be regarded as a chirality. For $D = 2$ chirality vanishes no spontaneous parity breaking.

2. Kiehn's king idea that Pfaffian systems provide a universal description of thermodynamical reversibility. Kiehn introduces heat 1- form Q. System is thermodynamically reversible if Q is integrable. In other words, the condition $Q \wedge dQ = 0$ holds true which implies that one can write $Q = TdS$: Q allows an integrable factor T and is expressible in terms of the gradient of entropy. $Q = TdS$ condition implies that Q correspond to a global flow defined by the coordinate lines of S. This in turn implies that it is possible define phase factors depending on S along the flow line: this relates to macroscopic quantum coherence for macroscopic quantum phases.

3. The first law expressing the work 1-form W as $W = Q - dU = TdS - dU$ for reversible processes. This gives $dW \wedge dW = 0$. The condition $dW \wedge dW \neq 0$ therefore characterizes irreversible processes.

4. Symplectic transformations are natural in Kiehn's framework but not absolutely essential.

Reader is encouraged to get familiar with Kiehn's examples [B42] about the description of various simple thermodynamical systems in this conceptual framework. Kiehn has also worked with the differential topology of electrodynamics and discussed concepts like integrable flows known as Beltrami flows. These flows generalized to TGD framework and are in key role in the construction of proposals for preferred extremals of Kähler action: the basic idea would be that various conserved isometry currents define Beltrami flows so that theit flow lines can be associated with coordinate lines.

3.11.3 Attempt to identify TTD in TGD framework

Let us now try to identify TTD or its complex square root in TGD framework.

The role of symplectic transformations

Symplectic transformations are important in Kiehn's approach although they are not a necessary ingredient of it and actually impossible to realize in Minkowski space-time.

1. Symplectic symmetries of WCW induced by synmplectic symmetries of CP_2 and light-like boundary of CD are important also in TGD framework [K12] and define the isometries of WCW. As a matter fact, symplectic group parameterizes the quantum fluctuating degrees of freedom and zero modes defining classical variables are symplectic invariants. One cannot assign to entire space-time surfaces symplectic structure although this is possible for partonic 2-surfaces.

2. The symplectic transformations of CP_2 act on the Kähler gauge potential as $U(1)$ gauge transformations formally but modify the shape of the space-time surface. These symplectic transformations are symmetries of Kähler action only in the vacuum sector which as such does not belong to WCW whereas small deformations of vacua belong. Therefore genuine gauge symmetries are not in question. One can of course formally assign to Kähler gauge potential a separate $U(1)$ gauge invariance.

3. Vacuum extremals with at most 2-D CP_2 projection (Lagrangian sub-manifold) form an infinite-dimensional space. Both M^4 diffeomorphisms and symplectic transformations of CP_2 produce new vacuum extremals, whose small deformations are expected to correspond preferred extremals. This gives rise to 4-D spin glass degeneracy [K47] to be distinguished from 4-D gauge degeneracy.

Identification of basic 1-forms of TTD in TGD framework

Consider nex the identification of the basic variables which are forms of various degrees in TTD.

1. Kähler gauge potential is analogous to work 1-form W. In classical electrodynamics vector potential indeed has this interpretation. $dW \wedge dW$ is replaced with $J \wedge J$ defining instanton density ($E_K \cdot B_K$ in physicist's notation) for Kähler form and its non-vanishing - or equivalently 4-dimensionality of CP_2 projection of space-time surface - would be the signature of irreversibility. $dJ = 0$ holds true only locally and one can have magnetic monopoles since CP_2 has non-trivial homology. Therefore the non-trivial topology of CP_2 implying that the counterpart of W is not globablly defined, brings in non-trivial new element to Kiehn's theory.

2. Chirality $C - S = A \wedge J$ is essentially Chern-Simons 3-form and in ordinary QFT non-vanishing of $C - S$- if present in action - means parity breaking in ordinary quantum field theories. Now one must be very cautious since parity is a symmetry of the imbedding space rather than that of space-time sheet.

3. Pfaff dimension equals to the dimension of CP_2 projection and has been used to classify existing preferred extremals. I have called the extremals with 4-D CP_2 projection chaotic and so called CP_2 vacuum extremals with 4-D CP_2 projection correspond to such extremals. Massless extremals or topological light rays correspond to $D = 2$ as do also cosmic strings. In Euclidian regions preferred extremals with $D = 4$ are are possible but not in Minkowskian regions if one accepts effective 3-dimensionality. Here one must keep mind open.

Irreversibility identified as a non-vanishing of the instanton density $J \wedge J$ has a purely geometrical and topological description in TGD Universe if one accepts effective 3-dimensionality.

1. The effective 3-dimensionality for space-time sheets (holography implied by general coordinate invariance) implies that Kähler action reduces to Chern-Simons terms so that the Pfaff dimension is at most $D = 3$ for Minkowskian regions of space-time surface so that they are are thermodynamically reversible.

2. For Euclidian regions (say deformations of CP_2 type vacuum extremals) representing orbits of elementary particles and lines of generalized Feynman diagrams $D = 4$ is possible. Therefore Euclidian space-like regions of space-time would be solely responsible for the irreversibility. This is quite strong conclusion but conforms with the standard quantum view about thermodynamics according to which various particle reaction rates deduced from quantum theory appear in kinetic equations giving rise to irreversible dynamics at the level of ensembles. The presence of Morse function coming from Minkowskian regions is natural since square root of thermodynamics is in question. Morse function is analogous to the action in QFTs whereas Kähler function is analogous to Hamiltonian in thermodynamics. Also this conforms with the square root of TTD interpretation.

Instanton current, instanton density, and irreversibility

Classical TGD has the structure of hydrodynamics in the sense that field equations are conservation laws for isometry currents and Kähler current. These are vector fields although induced metric allows to transform them to forms. This aspect should be visible also in thermodynamic interpretation and forces to add to the Kiehn's formulation involving only forms and exterior derivative also induced metric transforming 1-forms to vector fields, the duality mapping 4-k forms and k-forms to each other, and divergence operation.

It was already found that irreversibility and dissipation corresponds locally to a non-vanishing instanton density $J \wedge J$. This form can be regarded as exterior derivative of Chern-Simons 3-form or equivalently as divergence of instanton current.

1. The dual of C-S 3-form given by $*(A \wedge J)$ defines what I have called instanton current. This current is not conserved in general and the interpretation as a heat current would be natural. The exterior derivative of C-S gives instanton density $J \wedge J$. Equivalently, the divergence of instanton currengives the dual of $J \wedge J$ and the integral of instanton density gives the analog of instanton number analogous to the heat generated in a given space-time volume. Note that in Minkowskian regions one can multiply instanton current with a function of CP_2 coordinates without losing closedness property so that infinite number similar conserved currents is possible.

   The heat 3-form is expressible in terms of Chern-Simons 3-form and for preferred extremals it would be proportional to the weight sum of Kähler actions from Minkowskian and Euclidian regions (coeffecients are purely imaginary and real in these two regions). Instead of single real quantity one would have complex quantity characterizing irreversibility. Complexity would conform with the idea that quantum TGD is complex square root of thermodynamics.

2. The integral of heat 3-form over effective boundaries associated with a given space-time region define the net heat flow from that region. Only the regions defining the lines of generalized Feynman diagrams give rise to non-vanishing heat fluxes. Second law states that one has $\Delta Q \geq 0$. Generalized second law means at the level of quantum classical correspondence would mean that depending on the arrow of geometric time for zero energy state ΔQ is defined as difference between upper and lower or lower and upper boundaries of CD. This condition applied to CD and sub-CD:s would generalize the conditions familiar from hydrodynamics (stating for instance that for shock waves the branch of bifurcation for which the entropy increases is selected). Note that the field equations of TGD are hydrodynamical in the sense that they express conservation of various isometry currents. The naive picture about irreversibility is that classical dynamics generates CP_2 type vacuum extremals so that the number of outgoing lines of generalized Feynman diagram is higher than that of

incoming ones. Therefore that the number of space-like 3-surfaces giving rise to Chern-Simons contribution is larger at the end of CD corresponding to the final (negative energy) state.

3. A more precise characterization of the irreversible states involves several non-trivial questions.

   (a) By the failure of strict classical determinism the condition that for a given CD the number outgoing lines is not smaller than incoming lines need not provide a unique manner to fix the preferred extremal when partonic 2-surfaces at the ends are fixed. Could the arrow of geometric time depend on sub-CD as the model for living matter suggests (recall also phase conjugate light rays)?

   In ordinary quantum mechanical approach to kinetic equations also the reactions, which decrease entropy are allowed but their weight is smaller in thermal equilibrium. Could this fact be described as a probability distribution for the arrow of time associated for the sub-CDs, sub-sub-CDs, etc... ? Space-time correlates for quantal thermodynamics would be probability distributions for space-time sheets and hierarchy of sub-CDs.

   (b) 4-D spin glass degeneracy suggests breaking of ergodic hypothesis: could this mean that one does not have thermodynamical equilibrium but very large number of spin glass states caused by the frustation for which induced Kähler form provides a representation? Could these states correspond to a varying arrow of geometric time for sub-CDs? Or could different deformed vacuum extremals correspond to different space-time sheets in thermal equilibrium with different thermal parameters.

Also Kähler current and isometry currents are needed

The conservation Kähler current and of isometry currents imply the hydrodynamical character of TGD.

1. The conserved Kähler current j_K is defined as 3-form $j_K = *(d * J)$, where $d * J$ is closed 3-form and defines the counterpart of d^*dW. Field equations for preferred extremals require $*j_K \wedge A = 0$ satsfied if one Kähler current is proportional to instanton current: $*j_K \propto A \wedge J$. As a consequence Kähler action reduces to 3-dimensional Chern-Simons terms (classical holography) and Minkowskian space-time regions have at most 3-D CP_2 projection (Pfaff dimension $D \leq 3$) so that one has $J \wedge J = 0$ and reversibility. This condition holds true for preferred extremals representing macroscopically the propagation of massless quanta but not Euclidian regions representing quanta themselves and identifiable as basic building bricks of wormhole contacts between Minkowskian space-time sheets.

2. A more general proposal is that all conserved currents transformed to 1-forms using the induced metric (classical gravitation comes into play!) are integrable: in other words, on has $j \wedge dj = 0$ for both isometry currents and Kähler current. This would mean that they are analogous to heat 1-forms in the reversible case and therefore have a representation analogous to $Q = TdS$, $W = PdV$, μdN and the coordinate along flowline defines the analog of S, V, or N (note however that dS,dV, dN would more naturally correspond to 3-forms than 1-forms, see below) A stronger form corresponds to the analog of hydrodynamics for one particle species: all one-forms are proportional (by scalar function) to single 1-form which is $A \wedge J$ (all quantum number flows are parallel to each other).

Questions

There are several questions to be answered.

1. In Darboux coordinates in which one has $A = P_1 dQ^1 + P_2 dQ_2$. The identification of A as counterpart for $W = PdV - \mu dN$ comes first in mind. For thermodynamical equilibria one would have $TdS = dU + W$ translating to $TdS = dU + A$ so that Q for reversible processes would be apart from $U(1)$ gauge transformation equal to the Kähler gauge potential. Symplectic transformations of CP_2 generate $U(1)$ gauge transformations and dU might have interpretation in terms of energy flow induced by this kind of transformation. Recall however

that symplectic transformations are not symmetries of space-time surfaces but only of the WCW metric and act on partonic 2-surfaces and their tangent space data as such.

2. Does the conserved Kähler current j_K have any thermodynamical interpretation? Clearly the counterparts of conserved (and also non-conserved quantities) in Kiehn's formulation would be 3-forms with vanishing curl $d(*j_K) = 0$ in conserved case. Therefore it seems impossible to reduce them to 1-forms unless one introduces divergence besides exterior derivative as a basic differential operation.

   The hypothesis that the flow lines of these 1-forms associated with j_K vector field are integrable implies that they are gradients apart from the presence of integrating factor. Reduction to a gradient ($j = dU$) means that U satisfies massless d'Alembert equation $d * dU = 0$. Note that local polarization and light-like momentum are gradients of scalar functions which satisfy massless d'Alembert equation for the Minkowskian space-time regions representing propagating of massless quanta.

3. In genuinely 3-dimensional context S, V, N are integrals of 3-forms over 3-surfaces for some current defining 3-form. This is in conflict with Kiehn's description where they are 0-forms. One can imagine three cures and first two ones look

   (a) The integrability of the flows allows to see them as superposition of independent 1-dimensional flows. This picture would make it natural to regard the TGD counterparts of S, V, N as 0-forms rather than 2-forms. This would also allow to deduce $J \wedge J = 0$ as a reversibility condition using Kiehn's argument.

   (b) Unless one requires integrable flows, one must consider the replacement of $Q = TdS\ resp.\ W = PdV\ resp.\ \mu dN\ Q = TdS\ resp.\ W = PdV\ resp.\ \mu dN$ where W, Q, dS,dV, anddN with 3-forms. So that S, V, N would be 2-forms and the 3-integrals of dS, dV, dN over 3-surfaces would reduce to integrals over partonic 2-surfaces, which is of course highly non-trivial but physically natural implication of the effective 2-dimensionality. First law should now read as $*W = T*dS - *dU$ and would give $d*W = dT \wedge *dS + Td*dS + d*dU$. If S and U as 2-forms satisfy massless d'Alembert equation, one obtains $d*W = dT \wedge *dS$ giving $d * W \wedge d * W = 0$ as the reversibility condition. If one replaces $W \leftrightarrow A$ correspondence with $*W \leftrightarrow A$ correspondence, one obtains the vanishing of instanton density as a condition for reversibility. For the preferred extremals having interpretation as massless modes the massless d'Alembert equations are satisfied and it might that this option makes sense and be equivalent with the first option.

   (c) In accordance with the idea that finite measurement resolution is realized at the level of modified Dirac equation, its solutions at light-like 3-surfaces reduces to solutions restricted to lines connecting partonic 2-surfaces. Could one regard W, Q, dS, dV, and dN as singular one-forms restricted to these lines? The vanishing of instanton density would be obtained as a condition for reversibility only at the braid strands, and one could keep the original view of Kiehn. Note however that the instanton density could be non-vanishing elsewhere unless one develops a separate argument for its vanishing. For instance, the condition that isometries of imbedding space say translations produce braid ends points for which instanton density also vanishes for the reversible situation might be enough.

To sum up, it seems that TTD allows to develop considerable insights about how classical space-time surfaces could code for classical thermodynamics. An essential ingredient seems to be the reduction of the hydrodynamical flows for isometry currents to what might be called perfect flows decomposing to 1-dimensional flows with conservation laws holding true for individual flow lines. An interesting challenge is to find expressions for total heat in terms of temperature and entropy. Blackhole-elementary particle analogy suggest the reduction as well as effective 2-dimensionality suggest the reduction of the integrals of Chern-Simons terms defining total heat flux to two 2-D volume integrals over string world sheets and/or partonic 2-surfaces and this would be quite near to Hawking-Bekenstein formula.

3.12 Robert Kiehn's ideas about Falaco solitons and generation of turbulent wake from TGD perspective

I have been reading two highly interesting articles by Robert Kiehn. The first article has the title "Hydrodynamics wakes and minimal surfaces with fractal boundaries" [B40]. Second article is titled "Instability patterns, wakes and topological limit sets" [B41]. There are very many contacts on TGD inspired vision and its open interpretational problems.

The notion of Falaco soliton has surprisingly close resemblance with Kähler magnetic flux tubes defining fundamental structures in TGD Universe. Fermionic strings are also fundamental structures of TGD accompanying magnetic flux tubes and this supports the vision that these string like objects could allow reduction of various condensed matter phenomena such as sound waves -usually regarded as emergent phenomena allowing only highly phenomenological description - to the fundamental microscopic level in TGD framework. This can be seen as the basic outcome of this article.

Kiehn proposed a new description for the generation of various instability patterns of hydrodynamics flows (Kelvin-Helmholtz and Rayleigh-Taylor instabilities) in terms of hyperbolic dynamics so that a connection with wave phenomena like interference and diffraction would emerge. The role of characteristic surfaces as surfaces of tangential and also normal discontinuities is central for the approach. In TGD framework the characteristic surfaces have as analogs light-like wormhole throats at which the signature of the induced 4-metric changes and these surfaces indeed define boundaries of two phases and of material objects in general. This inspires a more detailed comparison of Kiehn's approach with TGD.

3.12.1 Falaco solitons and TGD

In the first article [B40] Kiehn tells about his basic motivations. The first motivating observations were related to so called Falaco solitons. Second observation was related to the so called mushroom pattern associated with Rayleigh Taylor instability or fingering instability [B12], which appears in very many contexts, the most familiar being perhaps the mushroom shaped cloud created by a nuclear explosion. The idea was that both structures whose stability is not easy to understand in standard hydrodynamics, could have topological description.

Falaco solitons are very fascinating objects. Kiehn describes in detail the formation and properties in [B40]: anyone possessing swimming pool can repeat these elegant and simple experiments. The vortex string connecting the end singularities - dimpled indentations at the surface of water - is the basic notion. Kiehn asks whether there migh be a deeper connection with a model of mesons in which strings connecting quark and antiquark appear. The formation of spiral structures around the end gaps in the initial formative states of Falaco soliton is emphasized and compared to the structure of spiral galaxies. The suggestion is that galaxies could appear as pairs connected by strings.

Kähler magnetic tubes carrying monopole flux are central in TGD and have several interesting resemblances with Falaco solutions.

1. In TGD framework so called cosmic strings fundamental primordial objects. They have 2-D Minkowski space projection and 2-D CP_2 projection so that one can say that there is no space-time in ordinary sense present during the primordial phase. During cosmic evolution their time= constant M^4 projection gradually thickens from ideal string to a magnetic flux tube. Among other things this explains the presence of magnetic fields in all cosmic scale not easy to understand in standard view. The decay of cosmic strings generates visible and dark matter much in the same manner as the decay of inflaton field does in inflationary scenario. One however avoids the many problems of inflationary scenario.

   Cosmic strings would contain ordinary matter and dark matter around them like necklace contains pearls along it. Cosmic strings carry Kähler magnetic monopole flux which stabilizes them. The magnetic field energy explains dark energy. Magnetic tension explains the negative "pressure" explaining accelerated expansion. The linear distribution of field energy along cosmic strings gives rise to logarithmic gravitational potential, which explains the constant velocity spectrum of distant stars around galaxy and therefore galactic dark matter.

2. Magnetic flux tubes form a fractal structure and the notion of Falaco soliton has also an analogy in TGD based description of elementary particles. In TGD framework the ends caps of vortices correspond to pairs of wormhole throats connected by short wormhole contact and there is a magnetic flux tube carrying monopole flux at both space-time sheets.

   So called modified Dirac equation assigns with this flux tube 1-D closed string and to it string world sheets, which might be 2-D minimal surface of space-time surface [K87]. Rather surprisingly, string model in 4-D space-time emerges naturally in TGD framework and has also very special properties due to the knotting of strings as 1-knots and knotting of string world sheets as 2-knots. Braiding and linking of strings is also involved and make dimension D=4 for space-time completely unique.

   Both elementary particles and hadron like state are describable in terms of these string like objects. Wormhole throats are the basic building brick of particles which are in the simplest situation two-sheeted structure with wormhole contact structures connecting the sheets and giving rise to one or more closed flux tubes accompanied by closed strings.

3.12.2 Stringy description of condensed matter physics and chemistry?

What is important that magnetic flux tubes and associated string world sheets can also connect wormhole throats associated with different elementary particles in the sense that their boundaries are along light-like wormhole throats assignable to different elementary particles. These string worlds sheets therefore mediate interactions between elementary particles.

1. What these interactions are? Could *string world sheets could provide a microscopic first principle description of condensed matter phenomena* - in particular of sound waves and various waves analogs of sound waves usually regarded as emergent phenomena requiring phenomenological models of condensed matter?

   The hypothesis that this is the case would allow to test basic assumptions of quantum TGD at the level of condensed matter physics. String model in 4-D space-time could describe concrete experimental everyday reality rather than esoteric Planck length scale physics! The phenomena of condensed matter physics often thought to be high level emergent phenomena would have first principle microscopic description at the level of space-time geometry.

2. The idea about stringy reductionism extends also to chemistry. One of the poorly understanding basic notions of molecular chemistry is the formation of valence bond as pairing of two valence electrons belonging to different atoms. Could this pairing correspond to a formation of a closed Kähler magnetic flux tube with two wormhole contacts carrying quantum numbers of electron? Could also Cooper pairs be regarded as this kind of structure with long connecting pair of flux tubes between electron carrying wormhole contacts as has been suggested already earlier?

3. The proposal indeed is that TGD inspired biochemistry and neuroscience indeed has magnetic flux tubes and flux sheets as a key element. For instance, the notion of magnetic body plays a key role in TGD inspired view about EEG and magnetic flux tubes represent braid strands in the model for DNA-cell membrane system as topological quantum computer [K20].

One can argue that this is not a totally new idea: basically one particular variant of holography[1] is in question and follows in TGD framework from general coordinate invariance alone: the geometry of world of classical worlds must assign to a given 3-surface a unique space-time surface.

1. The fashionable manner to realize holography is by replacing 4-D space-time with 10-D one. String world sheets in 10-D space-time $AdS_5 - S_5$ connecting the points of 4+5-D boundary of $AdS_5 - S_5$ are hoped to provide a dual description of even condensed matter phenomena in the case that the system is described by a theory enjoying conformal invariance in 4-D sense.

[1]The equivalent of holography emerged from the construction of the Kähler geometry of "world of classical worlds" as an implication of general coordinate invariance around 1990, about five years before it was introduced by t'Hooft and Susskind.

2. In TGD framework holography is much more concrete: 3-D light-like 3-surfaces (giving rise to generalized conformal invariance by their metric 2-dimensionality) are enough. One has actually a strong form of holography stating that 2-D partonic 2-surfaces plus their 4-D tangent space data are enough. Partonic 2-surfaces define the ends of light-like 3-surfaces at the ends of space-time surface at the light-like 7-D boundaries of causal diamonds. 10-D space is replaced with the familiar 4-D space-time and 4+5-D boundary with end 2-D ends of 3-D light-like wormhole orbits (plus 4-D tangent space data). These partonic 2-surfaces are highly analogous to the 2-D sections of your characteristic surfaces.

Consider now how sound waves as and various oscillations of this kind could be understood in terms of string word sheets. String world sheets have both geometric and fermionic degrees of freedom.

1. A good first guess is that string world sheet is minimal surface in space-time - this does not mean minimal surface property in imbedding space and the non-vanishing second fundamental form- in particular its CP_2 part should have physical meaning - maybe the parameter that would be called Higgs vacuum expectation in QFT limit of TGD could relate to it.

2. Another possibility that I have proposed is that a minimal surface of imbedding space (not the minimal surface is geometric analog for a solution of massless wave equation) but in the effective metric defined by the anti-commutators of modified gamma matrices defined by the canonical momentum densities of Kähler action is in question: in this case one might even dream about the possibility that the analog of light-velocity defined by the effective metric has interpretation as sound velocity.

For string world sheets as minimal surfaces of X^4 (the first option) oscillations would propagate with light-velocity but as one adds massive particle momenta at wormhole throats defining their ends the situation changes due to the additional inertia making impossible propagation with light-velocity. Consideration of the situation for ordinary non-relativistic condensed matter string with masses at ends as a simple example, the velocity of propagation is in the first naive estimate just square root of the ratio of the magnetic energy of string portion to its total energy which also concludes the mass at its ends. Kähler magnetic energy is given by string tension which has a spectrum determined by p-adic length scale hypothesis so that one ends up with a rough quantitative picture and coil understand the dependence of the sound velocity on temperature.

In TGD framework massless quanta moving in different directions correspond to different space-time sheets: linear superposition for fields is replaced with a set theoretic union and effects superpose instead of fields. This would hold true also for sound waves which would always be restricted at stringy world sheets: superposition can make sense only for wave moving in exactly the same direction. This of course conforms with the properties of phonons so that Bohr orbitology would be realized for sound waves and ordinary description of sound waves would be only an approximation. The fundamental difference between light and sound defining fundamental qualia would be the dimension of the quanta as geometric structures.

3.12.3 New manner to understand the generation of turbulent wake

Kiehn proposes a new manner to understand the generation of turbulent wake [B41]. The dynamics generating it would be that of hyperbolic wave equation rather than diffusive parabolic or elliptic dynamics. The decay of the turbulence would however obey the diffusive parabolic dynamics. Therefore sound velocity and supersonic velocities would be involved with the generation of the turbulence.

Kiehn considers Landau's nonlinear model for a scalar potential of velocity in the case of 2-D compressible isentropic fluid as an example. The wave equation is given by

$$(c^2 - \Phi_x^2)\Phi_{xx} + (c^2 - \Phi_y^2)\Phi_{yy} - 2\Phi_x\Phi_u\Phi_{xy} = 0 . \quad (3.12.1)$$

Here c denotes sound velocity and velocity is given by $v = \nabla\Phi$. 3-D generalization is obvious. This partial differential equation for the velocity potential is quasi-linear equation of the form

$$A\Phi_{\eta\eta} + 2B\Phi_{\eta\xi} + C\Phi_{\xi\xi} = 0 \ . \tag{3.12.2}$$

The characteristic surfaces contain imbedded curves which are given by solutions to ordinary differentia equations

$$\frac{d\eta}{d\xi} = \frac{B \pm (B^2AC)^{1/2}}{C} \ . \tag{3.12.3}$$

Real solutions are possible when the argument of the square root is positive. This is true when the local velocity exceeds the local characteristic speed c. These characteristic lines combine to form characteristic surfaces.

Velocity field would be compressible ($\nabla \cdot v \neq 0$) but irrotational ($\nabla \times v = 0$) in this approach whereas in standard approach velocity field would be incompressible ($\nabla \cdot v = 0$) but irrotational ($\nabla \times v \neq 0$). There would be two phases in which these two different options would be realized and at the boundary the dynamics would be both in-compressible and irrotational and these boundaries would correspond to characteristic surfaces which are minimal surfaces which evolve with time somehow. The presence of scalar function satisfying Laplace equation ($\nabla^2\Phi = 0$) would serve as a signature of this.

The emergence of this hyperbolic dynamics would explain the sharpness and long-lived character of the singular structures. Kiehn also proposes that the formation of wake could have analogies with diffraction and interference - basic aspects of wave motion. This picture does not conform with standard view which assumes diffusive parabolic or elliptic dynamics as the origin of the wake turbulence.

Characteristic surfaces and light-like wormhole throat orbits

Characteristic surface is key notion in Kiehn's approach and he suggests that the creation of wakes relies on hyperbolic dynamics in restricted regions [B41]. If I have understood correctly, the boundaries of vortices created in the process could be seen as this kind of characteristic surfaces: some physical quantities would have tangential discontinuities at them since a boundary between different phases (fluid and air) would be in question.

Another situation corresponds to a shock wave in which case there is a flow of matter through the characteristic surface. Also boundary patterns associated with Kelvin-Helmholtz instability (formation of waves due to wind and their breaking) and Rayleigh -Taylor instability (the formation of mushroom like fingers of heavier substance resting above lighter one).

The proposal of Kiehn is that the characteristic minimal surfaces have the following general form:

$$\begin{array}{ll} u = \frac{d\eta}{ds} = A(\rho) \times sin(Q(s)) \ , & v = \frac{d\eta}{ds} = -A(\rho) \times cos(Q(s)) \ , \\ w = F(u,v) = Q(u/v = s) \ per, & Q(s) = arctan(s) \ . \end{array} \tag{3.12.4}$$

If $F(u,v)$ satisfies the equation

$$(1 + F_v^2)F_{uu} + (1 + F_u^2)F_{vv} - 2F_uF_vF_{uv} = 0 \ . \tag{3.12.5}$$

This expresses the vanishing of the trace of the second fundamental form, actually the component corresponding to the coordinate w. The minimal surface in question is known as right helicoid.

In TGD framework light-like 3-surfaces defined by wormhole throats are the counterparts of characteristic surfaces.

1. By their light-likeness the light-like wormhole throats are analogous to characteristic surfaces (In TGD context light-velocity of course replace local sound velocity). Since the signature of the metric changes at wormhole throats, the 4-D tangent space reduces to 3-D in metric sense at them so that they indeed are singular in a unique sense. Gravitational effects imply that they need not look expanding in Minkowski coordinates. The light-velocity in the induced metric is in general smaller than maximal signal velocity in Minkowski space and can be arbitrarily small.

2. In TGD framework light-like 3-surfaces would be naturally associated with phase boundaries defining boundaries of physical objects. They would be light-like metrically degenerate 3-surfaces in space-time along which the space-time sheet assignable to fluid flow meets the space-time sheet assignable to say air. The generation of wake turbulence would in TGD framework mean the decay of a large 3-surface representing a laminar flow to sheet of separate cylindrical 3-surfaces representing vortex sheet. Also the amalgamation of vortices can be considered as a reverse process.

3. Interesting question related to the time evolution of these 2-D boundaries. In TGD framework it should give rise to 3-D light-like surface. The simulations for the evolution of Kelvin-Helmholtz instability and Rayleigh-Taylor mushroom pattern in Wikipedia and its seems that at the initial stages there is period of growth bringing in mind expanding light-front: the velocity of expansion is not its value in Minkowski space but corresponds to that assignable to the induced metric and can be much smaller. Recall also that in TGD framework gravitational effects are large near the singularity so that growth is not with the light-velocity in vacuum.

The proposal of Kiehn that very special minimal surfaces (right helicoids) are in question would in TGD framework correspond to a light-like 3-surfaces representing light-like orbits of these minimal surfaces presumably expanding at least in the beginning of the time evolution.

Minkowskian hydrodynamics/Maxwellian dynamics as hyperbolic dynamics and Euclidian hydrodynamics as elliptic dynamics

In Kiehn's proposal both the hyperbolic wave dynamics (about which Maxwell's equations provide a simple linear example) and diffusive elliptic or parabolic dynamics are present. In TGD framework both aspects are present at the level of field equations and correspond to the hyperbolic dynamics in Minkowskian space-time regions and elliptic dynamics in Euclidian space-time regions.

The dynamics of preferred extremals can be seen in two manners. Either as hydrodynamics or as Maxwellian dynamics with Bohr rules expressing the decomposition of the field to quanta-magnetic flux quanta or massless radiation quanta.

1. Maxwellian hydrodynamics involves a considerable restriction: superposition of modes moving in different directions is not allowed: one has just left-movers or right-movers in given direction, not both. Preferred extremals are "Bohr orbit like" and resemble outcomes of state function reduction measuring polarization and wave vector. The linear superposition of fields is replaced with the superposition of effects. The test particle topologically condenses to several space-time sheets simultaneously and experiences the sum of the forces of classical fields associated with the space-time sheets. Therefore one avoids the worst objection against TGD that I have been able to invent. Only four primary field like variables would replace the multitude of primary fields encountered in a typical unification. Besides this one has second quantized induced spinor fields.

2. Field equations are hydrodynamical in the sense that the field equations state classical conservation laws of four-momentum and color charges. In fermionic sector conservation of electromagnetic charge (in quantum sense so that different charge states for spinor mode do not mix) requires the localization of solutions to 2-D string world sheets for all states except right-handed neutrino. This leads to 2-D conformal invariance. A possible identification of string world sheet is as 2-D minimal surface of space-time (rather than that of imbedding space).

   What is remarkable that in Minkowskian space-time regions most preferred extremals (magnetic flux tube structures define an exception to this) are locally analogous to the modes of massless field with polarization direction and light-like momentum direction which in the general case can depend on position so that one has curvilinear light-like curve as analog of light-ray. The curvilinear light-like orbits results when two parallel preferred extremals with constant light-like direction form bound states via the formation of magnetically charged wormhole contact structures identifiable as elementary particles. Total momentum is conserved and is time-like for this kind of states, and the hypothesis is that the values of mass squared are given by p-adic thermodynamics. The conservation of Kähler current holds true as also its integrability in the sense of Frobenius giving $j = \Psi\nabla\Phi$. Besides this massless wave equations hold true for both Ψ and Φ. This looks like 4-D generalization of your equations at the characteristic defined by phase boundary.

3. In Euclidian regions one has naturally elliptic "hydrodynamics". Euclidian regions correspond for 4-D CP_2 projection to the 4-D "lines" of generalized Feynman diagrams. Their M^4 projections can be arbitrary large and the proposal is that the space-time sheet characterizing the macroscopic objects is actually Euclidian. In $AdS_5 - S^5$correspondence the corresponding idea is that macroscopic object is described as a blackhole in 10-D space. Now blackhole interiors have Euclidian signature as lines of generalized Feynman diagrams and blackhole interior does not differ from the interior of any system in any dramatical manner. Whether the Euclidian and Minkowskian dynamics are dual of each other or whether both are necessary is an open question.

Chapter 4

The Recent Vision about Preferred Extremals and Solutions of the Modified Dirac Equation

4.1 Introduction

During years several approaches to what preferred extremals of Kähler action and solutions of the modified Dirac equation could be have been proposed and the challenge is to see whether at least some of these approaches are consistent with each other.

The notion of preferred extremal emerged when I still lived in positive energy ontology. In zero energy ontology (ZEO) situation changes since 3-surfaces are now unions of space-like 3-surfaces at the opposite boundaries of causal diamond (CD). If Kähler action were deterministic, the attribute "preferred" would become obsolete. One of the most important outcomes of non-determinism is quantum criticality realized as a conformal invariance acting as gauge symmetries. The transformations in question are Kac-Moody type symmetries respecting the light-likeness of partonic orbits identified as surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. The orbits can be grouped to conformal equivalence classes and their number n would define in a natural manner the value of the effective Planck constant $h_{eff} = n \times h$.

One might hope that in finite measurement resolution the attribute "preferred" would not be needed. Bohr orbitology in ZEO would mean that one has Bohr orbits connecting 3-surfaces at boundaries of CD and this would give strong correlations between these 3-surfaces. Not all of them could be connected. Despite these uncertainties, I will talk in the following about preferred extremals. This means no loss since what is known recently is known for extremals.

It is good to list various approaches first.

4.1.1 Construction of preferred extremals

There has been considerable progress in the understanding of both preferred extremals and Kähler-Dirac equation.

1. For preferred extremals the generalization of conformal invariance to 4-D situation is very attractive idea and leads to concrete conditions formally similar to those encountered in string model [K4]. In particular, Einstein's equations with cosmological constant would solve consistency conditions and field equations would reduce to a purely algebraic statements analogous to those appearing in equations for minimal surfaces if one assumes that space-time surface has Hermitian structure or its Minkowskian variant Hamilton-Jacobi structure (Appendix). The older approach based on basic heuristics for massless equations, on effective 3-dimensionality, weak form of electric magnetic duality, and Beltrami flows is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic imbedding space [K67].

The basic step of progress was the realization that the known extremals of Kähler action - certainly limiting cases of more general extremals - can be deformed to more general extremals having interpretation as preferred extremals.

(a) The generalization boils down to the condition that field equations reduce to the condition that the traces $Tr(TH^k)$ for the product of energy momentum tensor and second fundamental form vanish. In string models energy momentum tensor corresponds to metric and one obtains minimal surface equations. The equations reduce to purely algebraic conditions stating that T and H^k have no common components. Complex structure of string world sheet makes this possible.

Stringy conditions for metric stating $g_{zz} = g_{\overline{z}\overline{z}} = 0$ generalize. The condition that field equations reduce to $Tr(TH^k) = 0$ requires that the terms involving Kähler gauge current in field equations vanish. This is achieved if Einstein's equations hold true (one can consider also more general manners to satisfy the conditions). The conditions guaranteeing the vanishing of the trace in turn boil down to the existence of Hermitian structure in the case of Euclidian signature and to the existence of its analog - Hamilton-Jacobi structure - for Minkowskian signature (Appendix). These conditions state that certain components of the induced metric vanish in complex coordinates or Hamilton-Jacobi coordinates.

In string model the replacement of the imbedding space coordinate variables with quantized ones allows to interpret the conditions on metric as Virasoro conditions. In the recent case a generalization of classical Virasoro conditions to four-dimensional ones would be in question. An interesting question is whether quantization of these conditions could make sense also in TGD framework at least as a useful trick to deduce information about quantum states in WCW degrees of freedom.

The interpretation of the extended algebra as Yangian [A36] [B43] suggested previously [K80] to act as a generalization of conformal algebra in TGD Universe is attractive. There is also the conjecture that preferred extremals could be interpreted as quaternionic of co-quaternionic 4-surface of the octonionic imbedding space with octonionic representation of the gamma matrices defining the notion of tangent space quanternionicity.

4.1.2 Understanding Kähler-Dirac equation

There are several approaches for solving the modified Dirac (or Kähler-Dirac) equation.

(a) The most promising approach is discussed in this chapter. It assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

(b) One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the modified Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the modified Dirac operator generate badly broken super-symmetries.

(c) Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry.This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

4.1.3 Measurement interaction term and boundary conditions

Quantum classical correspondence (QCC) requires a coupling between quantum and classical and this coupling should also give rise to a generalization of quantum measurement theory. The big question is how to realize this coupling.

(a) The proposal discussed in previous chapter was that the addition of a measurement interaction term to the modified Dirac action could do the job and solve a handful of problems of quantum TGD and unify various visions about the physics predicted by quantum TGD. This proposal implies QCC at the level of modified Dirac action and Kähler action. The simplest form of this term is completely analogous to algebraic form of Dirac action in M^4 but with integration measure $det(g_4)^{1/2}d^3x$ restricted to the 3-D surface in question.

(b) Another possibility consistent with the considerations of this chapter is that QCC is realized at the level of WCW Dirac operator and modified Dirac operator contains only interior term. I have indeed proposed that WCW spinor fields with given quantum charges in Cartan algebra are superpositions of space-time surfaces with same classical charges. A stronger form of QCC at the level of WCW would be that classical correlation functions for various geometric observables are identical with quantal correlation functions.

The boundary conditions for modified Dirac equation at space-like 3-surfaces are determined by the sum the analog of algebraic massless Dirac operator $p^k\gamma_k$ in M^4 coupled to the formal analog of Higgs field defined by the normal component Γ^n of the Kähler-Dirac gamma matrix. Higgs field is not in question. Rather the equation allows to formulate space-time correlate for stringy mass formula and also to understand how the ground state conformal weight can be negative half-integer as required by the p-adic mass calculations. At lightlike 3-surfaces Γ^n must vanish and the measurement interaction involving $p^k\gamma_k$ vanishes identically.

4.1.4 Progress in the understanding of super-conformal symmetries

The considerations in the sequel lead to a considerable progress in the understanding of super Virasoro representations for super-symplectic and super-Kac-Moody algebra. In particular, the proposal is that super-Kac-Moody currents assignable to string world sheets define duals of gauge potentials and their generalization for gravitons: in the approximation that gauge group is Abelian - motivated by the notion of finite measurement resolution - the exponents for the sum of KM charges would define non-integrable phase factors. One can also identify Yangian as the algebra generated by these charges. The approach allows also to understand the special role of the right handed neutrino in SUSY according to TGD. It must be however emphasized that also a weaker form of Einstein's equations can be considered solving the condition that the energy momentum tensor for Kähler action has vanishing divergence [K96] implying Einstein's equations with cosmological constant in general relativity. The weaker form involves several non-constant parameters analogous to cosmological constant.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http:`

//www.tgdtheory.fi/cmaphtml.html [L12]. Pdf representation of same files serving as a kind of glossary can be found at http://www.tgdtheory.fi/tgdglossary.pdf [L13]. The topics relevant to this chapter are given by the following list.

- TGD as infinite-dimensional geometry [L44]
- WCW spinor fields [L50]
- KD equation [L27]
- Kaehler-Dirac action [L26]

4.2 About deformations of known extremals of Kähler action

I have done a considerable amount of speculative guesswork to identify what I have used to call preferred extremals of Kähler action. The difficulty is that the mathematical problem at hand is extremely non-linear and that I do not know about existing mathematical literature relevant to the situation. One must proceed by trying to guess the general constraints on the preferred extremals which look physically and mathematically plausible. The hope is that this net of constraints could eventually chrystallize to Eureka! Certainly the recent speculative picture involves also wrong guesses. The need to find explicit ansatz for the deformations of known extremals based on some common principles has become pressing. The following considerations represent an attempt to combine the existing information to achieve this.

4.2.1 What might be the common features of the deformations of known extremals

The dream is to discover the deformations of all known extremals by guessing what is common to all of them. One might hope that the following list summarizes at least some common features.

Effective three-dimensionality at the level of action

(a) Holography realized as effective 3-dimensionality also at the level of action requires that it reduces to 3-dimensional effective boundary terms. This is achieved if the contraction $j^\alpha A_\alpha$ vanishes. This is true if j^α vanishes or is light-like, or if it is proportional to instanton current in which case current conservation requires that CP_2 projection of the space-time surface is 3-dimensional. The first two options for j have a realization for known extremals. The status of the third option - proportionality to instanton current - has remained unclear.

(b) As I started to work again with the problem, I realized that instanton current could be replaced with a more general current $j = *B \wedge J$ or concretely: $j^\alpha = \epsilon^{\alpha\beta\gamma\delta} B_\beta J_{\gamma\delta}$, where B is vector field and CP_2 projection is 3-dimensional, which it must be in any case. The contractions of j appearing in field equations vanish automatically with this ansatz.

(c) Almost topological QFT property in turn requires the reduction of effective boundary terms to Chern-Simons terms: this is achieved by boundary conditions expressing weak form of electric magnetic duality. If one generalizes the weak form of electric-magnetic duality to $J = \Phi * J$ one has $B = d\Phi$ and j has a vanishing divergence for 3-D CP_2 projection. This is clearly a more general solution ansatz than the one based on proportionality of j with instanton current and would reduce the field equations in concise notation to $Tr(TH^k) = 0$.

(d) Any of the alternative properties of the Kähler current implies that the field equations reduce to $Tr(TH^k) = 0$, where T and H^k are shorthands for Maxwellian energy momentum tensor and second fundamental form and the product of tensors is obvious generalization of matrix product involving index contraction.

Could Einstein's equations emerge dynamically?

For j^α satisfying one of the three conditions, the field equations have the same form as the equations for minimal surfaces except that the metric g is replaced with Maxwell energy momentum tensor T.

(a) This raises the question about dynamical generation of small cosmological constant Λ: $T = \Lambda g$ would reduce equations to those for minimal surfaces. For $T = \Lambda g$ modified gamma matrices would reduce to induced gamma matrices and the modified Dirac operator would be proportional to ordinary Dirac operator defined by the induced gamma matrices. One can also consider weak form for $T = \Lambda g$ obtained by restricting the consideration to a sub-space of tangent space so that space-time surface is only "partially" minimal surface but this option is not so elegant although necessary for other than CP_2 type vacuum extremals.

(b) What is remarkable is that $T = \Lambda g$ implies that the divergence of T which in the general case equals to $j^\beta J^\alpha_\beta$ vanishes. This is guaranteed by one of the conditions for the Kähler current. Since also Einstein tensor has a vanishing divergence, one can ask whether the condition to $T = \kappa G + \Lambda g$ could the general condition. This would give Einstein's equations with cosmological term besides the generalization of the minimal surface equations. GRT would emerge dynamically from the non-linear Maxwell's theory although in slightly different sense as conjectured [K72]! Note that the expression for G involves also second derivatives of the imbedding space coordinates so that actually a partial differential equation is in question. If field equations reduce to purely algebraic ones, as the basic conjecture states, it is possible to have $Tr(GH^k) = 0$ and $Tr(gH^k) = 0$ separately so that also minimal surface equations would hold true.

What is amusing that the first guess for the action of TGD was curvature scalar. It gave analogs of Einstein's equations as a definition of conserved four-momentum currents. The recent proposal would give the analog of ordinary Einstein equations as a dynamical constraint relating Maxwellian energy momentum tensor to Einstein tensor and metric.

(c) Minimal surface property is physically extremely nice since field equations can be interpreted as a non-linear generalization of massless wave equation: something very natural for non-linear variant of Maxwell action. The theory would be also very "stringy" although the fundamental action would not be space-time volume. This can however hold true only for Euclidian signature. Note that for CP_2 type vacuum extremals Einstein tensor is proportional to metric so that for them the two options are equivalent. For their small deformations situation changes and it might happen that the presence of G is necessary. The GRT limit of TGD discussed in [K72] [L10] indeed suggests that CP_2 type solutions satisfy Einstein's equations with large cosmological constant and that the small observed value of the cosmological constant is due to averaging and small volume fraction of regions of Euclidian signature (lines of generalized Feynman diagrams).

(d) For massless extremals and their deformations $T = \Lambda g$ cannot hold true. The reason is that for massless extremals energy momentum tensor has component T^{vv} which actually quite essential for field equations since one has $H^k_{vv} = 0$. Hence for massless extremals and their deformations $T = \Lambda g$ cannot hold true if the induced metric has Hamilton-Jacobi structure meaning that g^{uu} and g^{vv} vanish. A more general relationship of form $T = \kappa G + \Lambda G$ can however be consistent with non-vanishing T^{vv} but require that deformation has at most 3-D CP_2 projection (CP_2 coordinates do not depend on v).

(e) The non-determinism of vacuum extremals suggest for their non-vacuum deformations a conflict with the conservation laws. In, also massless extremals are characterized by a non-determinism with respect to the light-like coordinate but like-likeness saves the situation. This suggests that the transformation of a properly chosen time coordinate of vacuum extremal to a light-like coordinate in the induced metric combined with Einstein's equations in the induced metric of the deformation could allow to handle the non-determinism.

Are complex structure of CP_2 and Hamilton-Jacobi structure of M^4 respected by the deformations?

The complex structure of CP_2 and Hamilton-Jacobi structure of M^4 could be central for the understanding of the preferred extremal property algebraically.

(a) There are reasons to believe that the Hermitian structure of the induced metric ((1,1) structure in complex coordinates) for the deformations of CP_2 type vacuum extremals could be crucial property of the preferred extremals. Also the presence of light-like direction is also an essential elements and 3-dimensionality of M^4 projection could be essential. Hence a good guess is that allowed deformations of CP_2 type vacuum extremals are such that (2,0) and (0,2) components the induced metric and/or of the energy momentum tensor vanish. This gives rise to the conditions implying Virasoro conditions in string models in quantization:

$$g_{\xi^i\xi^j} = 0 \ , \quad g_{\overline{\xi}^i\overline{\xi}^j} = 0 \ , \quad i,j = 1,2 \ . \tag{4.2.1}$$

Holomorphisms of CP_2 preserve the complex structure and Virasoro conditions are expected to generalize to 4-dimensional conditions involving two complex coordinates. This means that the generators have two integer valued indices but otherwise obey an algebra very similar to the Virasoro algebra. Also the super-conformal variant of this algebra is expected to make sense.

These Virasoro conditions apply in the coordinate space for CP_2 type vacuum extremals. One expects similar conditions hold true also in field space, that is for M^4 coordinates.

(b) The integrable decomposition $M^4(m) = M^2(m)+E^2(m)$ of M^4 tangent space to longitudinal and transversal parts (non-physical and physical polarizations) - Hamilton-Jacobi structure- could be a very general property of preferred extremals and very natural since non-linear Maxwellian electrodynamics is in question. This decomposition led rather early to the introduction of the analog of complex structure in terms of what I called Hamilton-Jacobi coordinates $(u,v,w,\overline{w})$ for M^4. (u,v) defines a pair of light-like coordinates for the local longitudinal space $M^2(m)$ and $(w,\overline{w})$ complex coordinates for $E^2(m)$. The metric would not contain any cross terms between $M^2(m)$ and $E^2(m)$: $g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$.

A good guess is that the deformations of massless extremals respect this structure. This condition gives rise to the analog of the constraints leading to Virasoro conditions stating the vanishing of the non-allowed components of the induced metric. $g_{uu} = g_{vv} = g_{ww} = g_{\overline{w}\overline{w}} = g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$. Again the generators of the algebra would involve two integers and the structure is that of Virasoro algebra and also generalization to super algebra is expected to make sense. The moduli space of Hamilton-Jacobi structures would be part of the moduli space of the preferred extremals and analogous to the space of all possible choices of complex coordinates. The analogs of infinitesimal holomorphic transformations would preserve the modular parameters and give rise to a 4-dimensional Minkowskian analog of Virasoro algebra. The conformal algebra acting on CP_2 coordinates acts in field degrees of freedom for Minkowskian signature.

Field equations as purely algebraic conditions

If the proposed picture is correct, field equations would reduce basically to purely algebraically conditions stating that the Maxwellian energy momentum tensor has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1,1) and second fundamental form H^k a tensor of type (2,0) and (0,2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

4.2.2 What small deformations of CP_2 type vacuum extremals could be?

I was led to these arguments when I tried find preferred extremals of Kähler action, which would have 4-D CP_2 and M^4 projections - the Maxwell phase analogous to the solutions of Maxwell's equations that I conjectured long time ago. It however turned out that the dimensions of the projections can be ($D_{M^4} \leq 3, D_{CP_2} = 4$) or ($D_{M^4} = 4, D_{CP_2} \leq 3$). What happens is essentially breakdown of linear superposition so that locally one can have superposition of modes which have 4-D wave vectors in the same direction. This is actually very much like quantization of radiation field to photons now represented as separate space-time sheets and one can say that Maxwellian superposition corresponds to union of separate photonic space-time sheets in TGD.

Approximate linear superposition of fields is fundamental in standard physics framework and is replaced in TGD with a linear superposition of effects of classical fields on a test particle topologically condensed simultaneously to several space-time sheets. One can say that linear superposition is replaced with a disjoint union of space-time sheets. In the following I shall restrict the consideration to the deformations of CP_2 type vacuum extremals.

Solution ansatz

I proceed by the following arguments to the ansatz.

(a) Effective 3-dimensionality for action (holography) requires that action decomposes to vanishing $j^\alpha A_\alpha$ term + total divergence giving 3-D "boundary" terms. The first term certainly vanishes (giving effective 3-dimensionality) for

$$D_\beta J^{\alpha\beta} = j^\alpha = 0 \ .$$

Empty space Maxwell equations, something extremely natural. Also for the proposed GRT limit these equations are true.

(b) How to obtain empty space Maxwell equations $j^\alpha = 0$? The answer is simple: assume self duality or its slight modification:

$$J = *J$$

holding for CP_2 type vacuum extremals or a more general condition

$$J = k * J \ ,$$

In the simplest situation k is some constant not far from unity. * is Hodge dual involving 4-D permutation symbol. $k = constant$ requires that the determinant of the induced metric is apart from constant equal to that of CP_2 metric. It does not require that the induced metric is proportional to the CP_2 metric, which is not possible since M^4 contribution to metric has Minkowskian signature and cannot be therefore proportional to CP_2 metric.

One can consider also a more general situation in which k is scalar function as a generalization of the weak electric-magnetic duality. In this case the Kähler current is non-vanishing but divergenceless. This also guarantees the reduction to $Tr(TH^k) = 0$.

In this case however the proportionality of the metric determinant to that for CP_2 metric is not needed. This solution ansatz becomes therefore more general.

(c) Field equations reduce with these assumptions to equations differing from minimal surfaces equations only in that metric g is replaced by Maxwellian energy momentum tensor T. Schematically:

$$Tr(TH^k) = 0 \ ,$$

where T is the Maxwellian energy momentum tensor and H^k is the second fundamental form - asymmetric 2-tensor defined by covariant derivative of gradients of imbedding space coordinates.

How to satisfy the condition $Tr(TH^k) = 0$?

It would be nice to have minimal surface equations since they are the non-linear generalization of massless wave equations. It would be also nice to have the vanishing of the terms involving Kähler current in field equations as a consequence of this condition. Indeed, $T = \kappa G + \Lambda g$ implies this. In the case of CP_2 vacuum extremals one cannot distinguish between these options since CP_2 itself is constant curvature space with $G \propto g$. Furthermore, if G and g have similar tensor structure the algebraic field equations for G and g are satisfied separately so that one obtains minimal surface property also now. In the following minimal surface option is considered.

(a) The first opton is achieved if one has

$$T = \Lambda g \ .$$

Maxwell energy momentum tensor would be proportional to the metric! One would have dynamically generated cosmological constant! This begins to look really interesting since it appeared also at the proposed GRT limit of TGD [L10]. Note that here also non-constant value of Λ can be considered and would correspond to a situation in which k is scalar function: in this case the the determinant condition can be dropped and one obtains just the minimal surface equations.

(b) Very schematically and forgetting indices and being sloppy with signs, the expression for T reads as

$$T = JJ - g/4Tr(JJ) \ .$$

Note that the product of tensors is obtained by generalizing matrix product. This should be proportional to metric.

Self duality implies that $Tr(JJ)$ is just the instanton density and does not depend on metric and is constant.

For CP_2 type vacuum extremals one obtains

$$T = -g + g = 0 \ .$$

Cosmological constant would vanish in this case.

(c) Could it happen that for deformations a small value of cosmological constant is generated?

The condition would reduce to

$$JJ = (\Lambda - 1)g \ .$$

Λ must relate to the value of parameter k appearing in the generalized self-duality condition. For the most general ansatz Λ would not be constant anymore.

This would generalize the defining condition for Kähler form

$$JJ = -g \ \ (i^2 = -1 \ \ geometrically)$$

stating that the square of Kähler form is the negative of metric. The only modification would be that index raising is carried out by using the induced metric containing also M^4 contribution rather than CP_2 metric.

(d) Explicitly:

$$J_{\alpha\mu}J^{\mu}_{\ \beta} = (\Lambda - 1)g_{\alpha\beta} \ \ .$$

Cosmological constant would measure the breaking of Kähler structure. By writing $g = s + m$ and defining index raising of tensors using CP_2 metric and their product accordingly, this condition can be also written as

$$Jm = (\Lambda - 1)mJ \ \ .$$

If the parameter k is constant, the determinant of the induced metric must be proportional to the CP_2 metric. If k is scalar function, this condition can be dropped. Cosmological constant would not be constant anymore but the dependence on k would drop out from the field equations and one would hope of obtaining minimal surface equations also now. It however seems that the dimension of M^4 projection cannot be four. For 4-D M^4 projection the contribution of the M^2 part of the M^4 metric gives a non-holomorphic contribution to CP_2 metric and this spoils the field equations.

For $T = \kappa G + \Lambda g$ option the value of the cosmological constant is large - just as it is for the proposed GRT limit of TGD [K72] [L10]. The interpretation in this case is that the average value of cosmological constant is small since the portion of space-time volume containing generalized Feynman diagrams is very small.

More detailed ansatz for the deformations of CP_2 type vacuum extremals

One can develop the ansatz to a more detailed form. The most obvious guess is that the induced metric is apart from constant conformal factor the metric of CP_2. This would guarantee self-duality apart from constant factor and $j^\alpha = 0$. Metric would be in complex CP_2 coordinates tensor of type (1,1) whereas CP_2 Riemann connection would have only purely holomorphic or anti-holomorphic indices. Therefore CP_2 contributions in $Tr(TH^k)$ would vanish identically. M^4 degrees of freedom however bring in difficulty. The M^4 contribution to the induced metric should be proportional to CP_2 metric and this is impossible due to the different signatures. The M^4 contribution to the induced metric breaks its Kähler property but would preserve Hermitian structure.

A more realistic guess based on the attempt to construct deformations of CP_2 type vacuum extremals is following.

(a) Physical intuition suggests that M^4 coordinates can be chosen so that one has integrable decomposition to longitudinal degrees of freedom parametrized by two light-like coordinates u and v and to transversal polarization degrees of freedom parametrized by complex coordinate w and its conjugate. M^4 metric would reduce in these coordinates to a direct sum of longitudinal and transverse parts. I have called these coordinates Hamilton-Jacobi coordinates.

(b) w would be holomorphic function of CP_2 coordinates and therefore satisfy the analog of massless wave equation. This would give hopes about rather general solution ansatz. u and v cannot be holomorphic functions of CP_2 coordinates. Unless wither u or v is constant, the induced metric would receive contributions of type (2,0) and (0,2) coming from u and v which would break Kähler structure and complex structure. These contributions would give no-vanishing contribution to all minimal surface equations. Therefore either u or v is constant: the coordinate line for non-constant coordinate -say u- would be analogous to the M^4 projection of CP_2 type vacuum extremal.

(c) With these assumptions the induced metric would remain $(1,1)$ tensor and one might hope that $Tr(TH^k)$ contractions vanishes for all variables except u because the there are no common index pairs (this if non-vanishing Christoffel symbols for H involve only holomorphic or anti-holomorphic indices in CP_2 coordinates). For u one would obtain massless wave equation expressing the minimal surface property.

(d) If the value of k is constant the determinant of the induced metric must be proportional to the determinant of CP_2 metric. The induced metric would contain only the contribution from the transversal degrees of freedom besides CP_2 contribution. Minkowski contribution has however rank 2 as CP_2 tensor and cannot be proportional to CP_2 metric. It is however enough that its determinant is proportional to the determinant of CP_2 metric with constant proportionality coefficient. This condition gives an additional non-linear condition to the solution. One would have wave equation for u (also w and its conjugate satisfy massless wave equation) and determinant condition as an additional condition.

The determinant condition reduces by the linearity of determinant with respect to its rows to sum of conditions involved 0,1,2 rows replaced by the transversal M^4 contribution to metric given if M^4 metric decomposes to direct sum of longitudinal and transversal parts. Derivatives with respect to derivative with respect to particular CP_2 complex coordinate appear linearly in this expression they can depend on u via the dependence of transversal metric components on u. The challenge is to show that this equation has (or does not have) non-trivial solutions.

(e) If the value of k is scalar function the situation changes and one has only the minimal surface equations and Virasoro conditions.

What makes the ansatz attractive is that special solutions of Maxwell empty space equations are in question, equations reduces to non-linear generalizations of Euclidian massless wave equations, and possibly space-time dependent cosmological constant pops up dynamically. These properties are true also for the GRT limit of TGD [L10].

4.2.3 Hamilton-Jacobi conditions in Minkowskian signature

The maximally optimistic guess is that the basic properties of the deformations of CP_2 type vacuum extremals generalize to the deformations of other known extremals such as massless extremals, vacuum extremals with 2-D CP_2 projection which is Lagrangian manifold, and cosmic strings characterized by Minkowskian signature of the induced metric. These properties would be following.

(a) The recomposition of M^4 tangent space to longitudinal and transversal parts giving Hamilton-Jacobi structure. The longitudinal part has hypercomplex structure but the second light-like coordinate is constant: this plays a crucial role in guaranteeing the vanishing of contractions in $Tr(TH^k)$. It is the algebraic properties of g and T which are crucial. T can however have light-like component T^{vv}. For the deformations of CP_2 type vacuum extremals $(1,1)$ structure is enough and is guaranteed if second light-like coordinate of M^4 is constant whereas w is holomorphic function of CP_2 coordinates.

(b) What could happen in the case of massless extremals? Now one has 2-D CP_2 projection in the initial situation and CP_2 coordinates depend on light-like coordinate u and single real transversal coordinate. The generalization would be obvious: dependence on single light-like coordinate u and holomorphic dependence on w for complex CP_2 coordinates. The constraint is $T = \Lambda g$ cannot hold true since T^{vv} is non-vanishing (and light-like). This property restricted to transversal degrees of freedom could reduce the field equations to minimal surface equations in transversal degrees of freedom. The transversal part of energy momentum tensor would be proportional to metric and hence covariantly constant. Gauge current would remain light-like but would not be given by $j = *d\phi \wedge J$. $T = \kappa G + \Lambda g$ seems to define the attractive option.

It therefore seems that the essential ingredient could be the condition

$$T = \kappa G + \lambda g \ ,$$

which has structure (1,1) in both $M^2(m)$ and $E^2(m)$ degrees of freedom apart from the presence of T^{vv} component with deformations having no dependence on v. If the second fundamental form has (2,0)+(0,2) structure, the minimal surface equations are satisfied provided Kähler current satisfies on of the proposed three conditions and if G and g have similar tensor structure.

One can actually pose the conditions of metric as complete analogs of stringy constraints leading to Virasoro conditions in quantization to give

$$g_{uu} = 0 \ , \quad g_{vv} = 0 \ , \quad g_{ww} = 0 \ , \quad g_{\overline{w}\overline{w}} = 0 \ . \tag{4.2.2}$$

This brings in mind the generalization of Virasoro algebra to four-dimensional algebra for which an identification in terms of non-local Yangian symmetry has been proposed [K80]. The number of conditions is four and the same as the number of independent field equations. One can consider similar conditions also for the energy momentum tensor T but allowing non-vanishing component T^{vv} if deformations has no v-dependence. This would solve the field equations if the gauge current vanishes or is light-like. On this case the number of equations is 8. First order differential equations are in question and they can be also interpreted as conditions fixing the coordinates used since there is infinite number of manners to choose the Hamilton-Jacobi coordinates.

One can can try to apply the physical intuition about general solutions of field equations in the linear case by writing the solution as a superposition of left and right propagating solutions:

$$\xi^k = f_+^k(u,w) + f_+^k(v,w) \ . \tag{4.2.3}$$

This could guarantee that second fundamental form is of form (2,0)+(0,2) in both M^2 and E^2 part of the tangent space and these terms if $Tr(TH^k)$ vanish identically. The remaining terms involve contractions of T^{uw}, $T^{u\overline{w}}$ and T^{vw}, $T^{v\overline{w}}$ with second fundamental form. Also these terms should sum up to zero or vanish separately. Second fundamental form has components coming from f_+^k and f_-^k

Second fundamental form H^k has as basic building bricks terms $\hat{H}^k$ given by

$$\hat{H}^k_{\alpha\beta} \ = \ \partial_\alpha\partial_\beta h^k + \left({}_{l}{}^{k}{}_{m}\right)\partial_\alpha h^l \partial_\beta h^m \ . \tag{4.2.4}$$

For the proposed ansatz the first terms give vanishing contribution to H^k_{uv}. The terms containing Christoffel symbols however give a non-vanishing contribution and one can allow only f_+^k or f_-^k as in the case of massless extremals. This reduces the dimension of CP_2 projection to $D = 3$.

What about the condition for Kähler current? Kähler form has components of type $J_{w\overline{w}}$ whose contravariant counterpart gives rise to space-like current component. J_{uw} and $J_{u\overline{w}}$ give rise to light-like currents components. The condition would state that the $J^{w\overline{w}}$ is covariantly constant. Solutions would be characterized by a constant Kähler magnetic field. Also electric field is represent. The interpretation both radiation and magnetic flux tube makes sense.

4.2.4 Deformations of cosmic strings

In the physical applications it has been assumed that the thickening of cosmic strings to Kähler magnetic flux tubes takes place. One indeed expects that the proposed construction generalizes also to the case of cosmic strings having the decomposition $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 a complex homologically non-trivial sub-manifold of CP_2. Now the starting point structure is Hamilton-Jacobi structure for $M^2_m \times Y^2$ defining the coordinate space.

(a) The deformation should increase the dimension of either CP_2 or M^4 projection or both. How this thickening could take place? What comes in mind that the string orbits X^2 can be interpreted as a distribution of longitudinal spaces $M^2(x)$ so that for the deformation w coordinate becomes a holomorphic function of the natural Y^2 complex coordinate so that M^4 projection becomes 4-D but CP_2 projection remains 2-D. The new contribution to the X^2 part of the induced metric is vanishing and the contribution to the Y^2 part is of type $(1,1)$ and the the ansatz $T = \kappa G + \Lambda g$ might be needed as a generalization of the minimal surface equations The ratio of κ and G would be determined from the form of the Maxwellian energy momentum tensor and be fixed at the limit of undeformed cosmic strong to $T = (ag(Y^2) - bg(Y^2)$. The value of cosmological constant is now large, and overall consistency suggests that $T = \kappa G + \Lambda g$ is the correct option also for the CP_2 type vacuum extremals.

(b) One could also imagine that remaining CP_2 coordinates could depend on the complex coordinate of Y^2 so that also CP_2 projection would become 4-dimensional. The induced metric would receive holomorphic contributions in Y^2 part. As a matter fact, this option is already implied by the assumption that Y^2 is a complex surface of CP_2.

4.2.5 Deformations of vacuum extremals?

What about the deformations of vacuum extremals representable as maps from M^4 to CP_2?

(a) The basic challenge is the non-determinism of the vacuum extremals. One should perform the deformation so that conservation laws are satisfied. For massless extremals there is also non-determinism but it is associated with the light-like coordinate so that there are no problems with the conservation laws. This would suggest that a properly chosen time coordinate consistent with Hamilton-Jacobi decomposition becomes light-like coordinate in the induced metric. This poses a conditions on the induced metric.

(b) Physical intuition suggests that one cannot require $T = \Lambda g$ since this would mean that the rank of T is maximal whereas the original situation corresponds to the vanishing of T. For small deformations rank two for T looks more natural and one could think that T is proportional to a projection of metric to a 2-D subspace. The vision about the long length scale limit of TGD is that Einstein's equations are satisfied and this would suggest $T = kG$ or $T = \kappa G + \Lambda g$. The rank of T could be smaller than four for this ansatz and this conditions binds together the values of κ and G.

(c) These extremals have CP_2 projection which in the generic case is 2-D Lagrangian sub-manifold Y^2. Again one could assume Hamilton-Jacobi coordinates for X^4. For CP_2 one could assume Darboux coordinates (P_i, Q_i), $i = 1,2$, in which one has $A = P_i dQ^i$, and that $Y^2 \subset CP_2$ corresponds to $Q_i = constant$. In principle P_i would depend on arbitrary manner on M^4 coordinates. It might be more convenient to use as coordinates (u,v) for M^2 and (P_1,P_2) for Y^2. This covers also the situation when M^4 projection is not 4-D. By its 2-dimensionality Y^2 allows always a complex structure defined by its induced metric: this complex structure is not consistent with the complex structure of CP_2 (Y^2 is not complex sub-manifold).

Using Hamilton-Jacobi coordinates the pre-image of a given point of Y^2 is a 2-dimensional sub-manifold X^2 of X^4 and defines also 2-D sub-manifold of M^4. The following picture suggests itself. The projection of X^2 to M^4 can be seen for a suitable choice of Hamilton-Jacobi coordinates as an analog of Lagrangian sub-manifold in M^4 that is as surface for which v and $Im(w)$ vary and u and $Re(w)$ are constant. X^2 would be obtained by allowing u and $Re(w)$ to vary: as a matter fact, (P_1, P_2) and $(u, Re(w))$ would be related to each other. The induced metric should be consistent with this picture. This would requires $g_{uRe(w)} = 0$.

For the deformations Q_1 and Q_2 would become non-constant and they should depend on the second light-like coordinate v only so that only g_{uu} and g_{uw} and $gu\overline{w}$ $g_{w,w}$ and $g_{\overline{w},\overline{w}}$ receive contributions which vanish. This would give rise to the analogs of Virasoro conditions guaranteeing that T is a tensor of form $(1,1)$ in both M^2 and E^2 indices and that there are no cross components in the induced metric. A more general formulation states that energy momentum tensor satisfies these conditions. The conditions on T might be equivalent with the conditions for g and G separately.

(d) Einstein's equations provide an attractive manner to achieve the vanishing of effective 3-dimensionality of the action. Einstein equations would be second order differential equations and the idea that a deformation of vacuum extremal is in question suggests that the dynamics associated with them is in directions transversal to Y^2 so that only the deformation is dictated partially by Einstein's equations.

(e) Lagrangian manifolds do not involve complex structure in any obvious manner. One could however ask whether the deformations could involve complex structure in a natural manner in CP_2 degrees of freedom so that the vanishing of g_{ww} would be guaranteed by holomorphy of CP_2 complex coordinate as function of w.

One should get the complex structure in some natural manner: in other words, the complex structure should relate to the geometry of CP_2 somehow. The complex coordinate defined by say $z = P_1 + iQ^1$ for the deformation suggests itself. This would suggest that at the limit when one puts $Q_1 = 0$ one obtains $P_1 = P_1(Re(w))$ for the vacuum extremals and the deformation could be seen as an analytic continuation of real function to region of complex plane. This is in spirit with the algebraic approach. The vanishing of Kähler current requires that the Kähler magnetic field is covariantly constant: $D_z J^{z\overline{z}} = 0$ and $D_{\overline{z}} J^{z\overline{z}} = 0$.

(f) One could consider the possibility that the resulting 3-D sub-manifold of CP_2 can be regarded as contact manifold with induced Kähler form non-vanishing in 2-D section with natural complex coordinates. The third coordinate variable- call it s- of the contact manifold and second coordinate of its transversal section would depend on time spacetime coordinates for vacuum extremals. The coordinate associated with the transversal section would be continued to a complex coordinate which is holomorphic function of w and u.

(g) The resulting thickened magnetic flux tubes could be seen as another representation of Kähler magnetic flux tubes: at this time as deformations of vacuum flux tubes rather than cosmic strings. For this ansatz it is however difficult to imagine deformations carrying Kähler electric field.

4.2.6 About the interpretation of the generalized conformal algebras

The long-standing challenge has been finding of the direct connection between the superconformal symmetries assumed in the construction of the geometry of the "world of classical worlds" (WCW) and possible conformal symmetries of field equations. 4-dimensionality and Minkowskian signature have been the basic problems. The recent construction provides new insights to this problem.

(a) In the case of string models the quantization of the Fourier coefficients of coordinate variables of the target space gives rise to Kac-Moody type algebra and Virasoro algebra generators are quadratic in these. Also now Kac-Moody type algebra is expected. If one were to perform a quantization of the coefficients in Laurents series for complex CP_2 coordinates, one would obtain interpretation in terms of $su(3) = u(2)+t$ decomposition, where t corresponds to CP_3: the oscillator operators would correspond to generators in t

and their commutator would give generators in $u(2)$. SU(3)/SU(2) coset representation for Kac-Moody algebra would be in question. Kac-Moody algebra would be associated with the generators in both M^4 and CP_2 degrees of freedom. This kind of Kac-Moody algebra appears in quantum TGD.

(b) The constraints on induced metric imply a very close resemblance with string models and a generalization of Virasoro algebra emerges. An interesting question is how the two algebras acting on coordinate and field degrees of freedom relate to the super-conformal algebras defined by the symplectic group of $\delta M^4_+ \times CP_2$ acting on space-like 3-surfaces at boundaries of CD and to the Kac-Moody algebras acting on light-like 3-surfaces. It has been conjectured that these algebras allow a continuation to the interior of space-time surface made possible by its slicing by 2-surfaces parametrized by 2-surfaces. The proposed construction indeed provides this kind of slicings in both M^4 and CP_2 factor.

(c) In the recent case, the algebras defined by the Fourier coefficients of field variables would be Kac-Moody algebras. Virasoro algebra acting on preferred coordinates would be expressed in terms of the Kac-Moody algebra in the standard Sugawara construction applied in string models. The algebra acting on field space would be analogous to the conformal algebra assignable to the symplectic algebra so that also symplectic algebra is present. Stringy pragmatist could imagine quantization of symplectic algebra by replacing CP_2 coordinates in the expressions of Hamiltonians with oscillator operators. This description would be counterpart for the construction of spinor harmonics in WCW and might provide some useful insights.

(d) For given type of space-time surface either CP_2 or M^4 corresponds to Kac-Moody algebra but not both. From the point of view of quantum TGD it looks as that something were missing. An analogous problem was encountered at GRT limit of TGD [L10]. When Euclidian space-time regions are allowed Einstein-Maxwell action is able to mimic standard model with a surprising accuracy but there is a problem: one obtains either color charges or M^4 charges but not both. Perhaps it is not enough to consider either CP_2 type vacuum extremal or its exterior but both to describe particle: this would give the direct product of the Minkowskian and Euclidian algebras acting on tensor product. This does not however seem to be consistent with the idea that the two descriptions are duality related (the analog of T-duality).

4.3 Under what conditions electric charge is conserved for the modified Dirac equation?

One might think that talking about the conservation of electric charge at 21st century is a waste of time. In TGD framework this is certainly not the case.

(a) In quantum field theories there are two manners to define em charge: as electric flux over 2-D surface sufficiently far from the source region or in the case of spinor field quantum mechanically as combination of fermion number and vectorial isospin. The latter definition is quantum mechanically more appropriate.

(b) There is however a problem. In standard approach to gauge theory Dirac equation in presence of charged classical gauge fields does not conserve electric charge as quantum number: electron is transformed to neutrino and vice versa. Quantization solves the problem since the non-conservation can be interpreted in terms of emission of gauge bosons. In TGD framework this does not work since one does not have path integral quantization anymore. Preferred extremals carry classical gauge fields and the question whether em charge is conserved arises. Heuristic picture suggests that em charge must be conserved.

It seems that one should pose the well-definedness of spinorial em charge as an additional condition. Well-definedness of em charge is not the only problem. How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? Does this require that classical weak fields vanish in the regions where the modes of induced spinor fields are non-vanishing?

This condition might be one of the conditions defining what it is to be a preferred extremal/solution of Kähler Dirac equation. It is not however trivial whether this kind of additional condition can be posed unless it follows automatically from the recent formulation for Kähler action and Kähler Dirac action. The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string/parton picture part of TGD. The vanishing of classical weak fields has also number theoretic interpretation: space-time surfaces would have quaternionic (hyper-complex) tangent space and the 2-surfaces carrying spinor fields complex (hyper-complex) tangent space.

4.3.1 Conservation of em charge for Kähler Dirac equation

What does the conservation of em charge imply in the case of the modified Dirac equation? The obvious guess that the em charged part of the modified Dirac operator must annihilate the solutions, turns out to be correct as the following argument demonstrates.

(a) Em charge as coupling matrix can be defined as a linear combination $Q = aI + bI_3$, $I_3 = J_{kl}\Sigma^{kl}$, where I is unit matrix and I_3 vectorial isospin matrix, J_{kl} is the Kähler form of CP_2, Σ^{kl} denotes sigma matrices, and a and b are numerical constants different for quarks and leptons. Q is covariantly constant in $M^4 \times CP_2$ and its covariant derivatives at space-time surface are also well-defined and vanish.

(b) The modes of the modified Dirac equation should be eigen modes of Q. This is the case if the modified Dirac operator D commutes with Q. The covariant constancy of Q can be used to derive the condition

$$\begin{aligned} [D,Q]\,\Psi &= D_1\Psi = 0 \ , \\ D &= \hat{\Gamma}^\mu D_\mu \ , \ D_1 = [D,Q] = \hat{\Gamma}_1^\mu D_\mu \ , \ \hat{\Gamma}_1^\mu = \left[\hat{\Gamma}^\mu, Q\right] \ . \end{aligned} \tag{4.3.1}$$

Covariant constancy of J is absolutely essential: without it the resulting conditions would not be so simple.
It is easy to find that also $[D_1, Q]\Psi = 0$ and its higher iterates $[D_n, Q]\Psi = 0$, $D_n = [D_{n-1}, Q]$ must be true. The solutions of the modified Dirac equation would have an additional symmetry.

(c) The commutator $D_1 = [D, Q]$ reduces to a sum of terms involving the commutators of the vectorial isospin $I_3 = J_{kl}\Sigma^{kl}$ with the CP_2 part of the gamma matrices:

$$D_1 = [Q,D] = [I_3,\Gamma_r]\partial_\mu s^r T^{\alpha\mu} D_\alpha \ . \tag{4.3.2}$$

In standard complex coordinates in which $U(2)$ acts linearly the complexified gamma matrices can be chosen to be eigenstates of vectorial isospin. Only the charged flat space complexified gamma matrices Γ^A denoted by Γ^+ and Γ^- possessing charges +1 and -1 contribute to the right hand side. Therefore the additional Dirac equation $D_1\Psi = 0$ states

$$\begin{aligned} D_1\Psi &= [Q,D]\Psi = I_3(A)e_{Ar}\Gamma^A\partial_\mu s^r T^{\alpha\mu} D_\alpha\Psi \\ &= (e_{+r}\Gamma^+ - e_{-r}\Gamma^-)\partial_\mu s^r T^{\alpha\mu} D_\alpha\Psi = 0 \ . \end{aligned} \tag{4.3.3}$$

The next condition is

$$D_2\Psi = [Q,D]\Psi = (e_{+r}\Gamma^+ + e_{-r}\Gamma^-)\partial_\mu s^r T^{\alpha\mu} D_\alpha\Psi = 0 \ . \tag{4.3.4}$$

Only the relative sign of the two terms has changed. The remaining conditions give nothing new.

(d) These equations imply two separate equations for the two charged gamma matrices

$$\begin{aligned} D_+\Psi &= T^\alpha_+\Gamma^+ D_\alpha\Psi = 0 \ , \\ D_-\Psi &= T^\alpha_-\Gamma^- D_\alpha\Psi = 0 \ , \\ T^\alpha_\pm &= e_{\pm r}\partial_\mu s^r T^{\alpha\mu} \ . \end{aligned} \tag{4.3.5}$$

These conditions state what one might have expected: the charged part of the modified Dirac operator annihilates separately the solutions. The reason is that the classical W fields are proportional to $e_{r\pm}$.

The above equations can be generalized to define a decomposition of the energy momentum tensor to charged and neutral components in terms of vierbein projections. The equations state that the analogs of the modified Dirac equation defined by charged components of the energy momentum tensor are satisfied separately.

(e) In complex coordinates one expects that the two equations are complex conjugates of each other for Euclidian signature. For the Minkowskian signature an analogous condition should hold true. The dynamics enters the game in an essential manner: whether the equations can be satisfied depends on the coefficients a and b in the expression $T = aG + bg$ implied by Einstein's equations in turn guaranteeing that the solution ansatz generalizing minimal surface solutions holds true [K4].

(f) As a result one obtains three separate Dirac equations corresponding to the the neutral part $D_0\Psi = 0$ and charged parts $D_\pm\Psi = 0$ of the modified Dirac equation. By acting on the equations with these Dirac operators one obtains also that the commutators $[D_+, D_-]$, $[D_0, D_\pm]$ and also higher commutators obtained from these annihilate the induced spinor field model. Therefore entire -possibly- infinite-dimensional algebra would annihilate the induced spinor fields. In string model the counterpart of Dirac equation when quantized gives rise to Super-Virasoro conditions. This analogy would suggest that modified Dirac equation gives rise to the analog of Super-Virasoro conditions in 4-D case. But what the higher conditions mean? Could they relate to the proposed generalization to Yangian algebra? Obviously these conditions resemble structurally Virasoro conditions $L_n|phys\rangle = 0$ and their supersymmetric generalizations, and might indeed correspond to a generalization of these conditions just as the field equations for preferred extremals could correspond to the Virasoro conditions if one takes seriously the analogy with the quantized string.

What could this additional symmetry mean from the point of view of the solutions of the modified Dirac equation? The field equations for the preferred extremals of Kähler action reduce to purely algebraic conditions in the same manner as the field equations for the minimal surfaces in string model. Could this happen also for the modified Dirac equation and could the condition on charged part of the Dirac operator help to achieve this?

This argument was very general and one can ask for simple manners to realize these conditions. Obviously the vanishing of classical W fields in the region where the spinor mode is non-vanishing defines this kind of condition.

4.3.2 About the solutions of Kähler Dirac equation for known extremals

To gain perpective consider first Dirac equation in in H. Quite generally, one can construct the solutions of the ordinary Dirac equation in H from covariantly constant right-handed neutrino spinor playing the role of fermionic vacuum annihilated by the second half of complexified gamma matrices. Dirac equation reduces to Laplace equation for a scalar function and solution can be constructed from this "vacuum" by multiplying with the spherical harmonics of CP_2 and applying Dirac operator [K37]. Similar construction works quite generally thanks to the existence of covariantly constant right handed neutrino spinor. Spinor harmonics of CP_2 are only replaced with those of space-time surface possessing either hermitian structure of Hamilton-Jacobi structure (corresponding to Euclidian and Minkowskian signatures of the induced metric [K4, K87]). What is remarkable is that these solutions possess well-defined em charge although classical W boson fields are present.

This in sense that H d'Alembertian commutes with em charge matrix defined as a linear combination of unit matrix and the covariantly constant matrix $J^{kl}\Sigma_{kl}$ since the commutators of the covariant derivatives give constant Ricci scalar and $J^{kl}\Sigma_{kl}$ term to the d'Alembertian besides scalar d'Alembertian commuting with em charge. Dirac operator itself does not commute with em charge matrix since gamma matrices not commute with em charge matrix.

Consider now Kähler Dirac operator. The square of Kähler Dirac operator contains commutator of covariant derivatives which contains contraction $[\Gamma^\mu, \Gamma^\nu] F^{weak}_{\mu\nu}$ which is quadratic in sigma matrices of $M^4 \times CP_2$ and does not reduce to a constant term commuting which em charge matrix. Therefore additional condition is required even if one is satisfies with the commutativity of d'Alembertian with em charge. Stronger condition would be commutativity with the Kähler Dirac operator and this will be considered in the following.

To see what happens one must consider space-time regions with Minkowskian and Euclidian signature. What will be assumed is the existence of Hamilton-Jacobi structure [K4] meaning complex structure in Euclidian signature and hyper-complex plus complex structure in Minkowskian signature. The goal is to get insights about what the condition that spinor modes have a well-defined em charge eigenvalue requires. Or more concretely: is the localization at string world sheets guaranteeing well-defined value of em charge predicted by Kähler Dirac operator or must one introduce this condition separately? One can also ask whether this condition reduces to commutativity/co-commutativity in number theoretic vision.

(a) CP_2 type vacuum extremals serve as a convenient test case for the Euclidian signature. In this case the modified Dirac equation reduces to the massless ordinary Dirac equation in CP_2 allowing only covariantly constant right-handed neutrino as solution. Only part of CP_2 so that one give up the constraint that the solution is defined in the entire CP_2. In this case holomorphic solution ansatz obtained by assuming that solutions depend on the coordinates ξ^i, $i = 1,2$ but not on their conjugates and that the gamma matrices $\Gamma^{\bar{i}}$, $i = 1,2$, annihilate the solutions, works. The solutions ansatz and its conjugate are of exactly the same form as in case string models where one considers string world sheets instead of CP_2 region.

The solutions are not restricted to 2-D string world sheets and it is not clear whether one can assign to them a well-defined em charge in any sense. Note that for massless Dirac equation in H one obtains all CP_2 harmonics as solutions, and it is possible to talk about em charge of the solution although solution itself is not restricted to 2-D surface of CP_2.

(b) For massless extremals and a very wide class of solutions produced by Hamilton-Jacobi structure - perhaps all solutions representable locally as graphs for map $M^4 \rightarrow CP_2$ - canonical momentum densities are light-like and solutions are hyper-holomorphic in the coordinates associated with with string world sheet and annihilated by the conjugate gamma and arbitrary functions in transversal coordinates. This allows localization to string world sheets. The localization is now strictly dynamical and implied by the properties of Kähler Dirac operator.

(c) For string like objects one obtains massless Dirac equation in $X^2 \times Y^2 \subset M^4 \times Y^2$, Y^2 a complex 2-surface in CP_2. Homologically trivial geodesic sphere corresponds to the simplest choice for Y^2. Modified Dirac operator reduces to a sum of massless Dirac operators associated with X^2 and Y^2. The most general solutions would have Y^2 mass. Holomorphic solutions reduces to product of hyper-holomorphic and holomorphic solutions and massless 2-D Dirac equation is satisfied in both factors.

For instance, for S^2 a geodesic sphere and $X^2 = M^2$ one obtains M^2 massivation with mass squared spectrum given by Laplace operator for S^2. Conformal and hyper-conformal symmetries are lost, and one might argue that this is quite not what one wants. One must be however resist the temptation to make too hasty conclusions since the massivation of string like objects is expected to take place. The question is whether it takes place already at the level of fundamental spinor fields or only at the level of elementary particles constructed as many-fermion states of them as twistor Grassmann approach assuming massless M^4 propagators for the fundamental fermions strongly suggests [K58].

(d) For vacuum extremals the Kähler Dirac operator vanishes identically so that it does not make sense to speak about solutions.

What can one conclude from these observations?

(a) The localization of solutions to 2-D string world sheets follows from Kähler Dirac equation only for the Minkowskian regions representable as graphs of map $M^4 \rightarrow CP_2$ locally. For string like objects and deformations of CP_2 type vacuum extremals this is not expected to take place.

(b) It is not clear whether one can speak about well-defined em charge for the holomorphic spinors annihilated by the conjugate gamma matrices or their conjugates. As noticed, for imbedding space spinor harmonics this is however possible.

(c) Strong form of conformal symmetry and the condition that em charge is well-defined for the nodes suggests that the localization at 2-D surfaces at which the charged parts of induced electroweak gauge fields vanish must be assumed as an additional condition. Number theoretic vision would suggest that these surfaces correspond to 2-D commutative or co-commutative surfaces. The string world sheets inside space-time surfaces would not emerge from theory but would be defined as basic geometric objects.

This kind of condition would also allow duals of string worlds sheets as partonic 2-surfaces identified number theoretically as co-commutative surfaces. Commutativity and co-commutativity would become essential elemenents of the number theoretical vision.

(d) The localization of solutions of the modified Dirac action at string world sheets and partonic 2-surfaces as a constraint would mean induction procedure for Kähler-Dirac matrices from SX^4 to X^2 - that is projection. The resulting em neutral gamma matrices would correspond to tangent vectors of the string world sheet. The vanishing of the projections of charged parts of energy momentum currents would define these surfaces. The conditions would apply both in Minkowskian and Euclidian regions. An alternative interpretation would be number theoretical: these surface would be commutative or co-commutative.

4.3.3 Concrete realization of the conditions guaranteeing well-defined em charge

Well-definedness of the em charge is the fundamental condiiton on spinor modes. Physical intuition suggests that also classical Z^0 field should vanish - at least in scales longer than weak scale. Above the condition guaranteeing vanishing of em charge has been discussed at very general level. It has however turned out that one can understand situation by simply posing the simplest condition that one can imagine: the vanishing of classical W and possibly also Z^0 fields inducing mixing of different charge states.

(a) Induced W fields mean that the modes of Kähler-Dirac equation do not in general have well-defined em charge. The problem disappears if the induced W gauge fields vanish. This does not yet guarantee that couplings to classical gauge fields are physical in long scales. Also classical Z^0 field should vanish so that the couplings would be purely vectorial. Vectoriality might be true in long enough scales only. If W and Z^0 fields vanish in all scales then electroweak forces are due to the exchanges of corresponding gauge bosons described as string like objects in TGD and represent non-trivial space-time geometry and topology at microscopic scale.

(b) The conditions solve also another long-standing interpretational problem. Color rotations induce rotations in electroweak-holonomy group so that the vanishing of all induced weak fields also guarantees that color rotations do not spoil the property of spinor modes to be eigenstates of em charge.

One can study the conditions quite concretely by using the formulas for the components of spinor curvature [L1] (`http://www.tgdtheory.fi/public_html/pdfpool/append.pdf`).

(a) The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \tag{4.3.6}$$

$R_{01}=R_{23}$ and $R_{03}=-R_{31}$ combine to form purely left handed classical W boson fields and Z^0 field corresponds to $Z^0=2R_{03}$.
Kähler form is given by

$$J = 2(e^0\wedge e^3 + e^1\wedge e^2) \ . \tag{4.3.7}$$

(b) The vanishing of classical weak fields is guaranteed by the conditions

$$\begin{array}{c} e^0\wedge e^1 - e^2\wedge e^3 = 0 \ , \\ e^0\wedge e^2 - e^3\wedge e^1 \ , \\ 4e^0\wedge e^3 + 2e^1\wedge e^2 \ . \end{array} \tag{4.3.8}$$

(c) There are many manners to satisfy these conditions. For instance, the condition $e^1 = a\times e^0$ and $e^2=-a\times e^3$ with arbitrary a which can depend on position guarantees the vanishing of classical W fields. The CP_2 projection of the tangent space of the region carrying the spinor mode must be 2-D.
Also classical Z^0 vanishes if $a^2=2$ holds true. This guarantees that the couplings of induced gauge potential are purely vectorial. One can consider other alternaties. For instance, one could require that only classical Z^0 field or induced Kähler form is non-vanishing and deduce similar condition.

(d) The vanishing of the weak part of induced gauge field implies that the CP_2 projection of the region carrying spinor mode is 2-D. Therefore the condition that the modes of induced spinor field are restricted to 2-surfaces carrying no weak fields sheets guarantees well-definedness of em charge and vanishing of classical weak couplings. This condition does not imply string world sheets in the general case since the CP_2 projection of the space-time sheet can be 2-D.

How string world sheets could emerge?

(a) Additional consistency condition to neutrality of string world sheets is that Kähler-Dirac gamma matrices have no components orthogonal to the 2-surface in question. Hence various fermionic would flow along string world sheet.

(b) If the Kähler-Dirac gamma matrices at string world sheet are expressible in terms of two non-vanishing gamma matrices parallel to string world sheet and sheet and thus define an integrable distribution of tangent vectors, this is achieved. What is important that modified gamma matrices can indeed span lower than 4-D space and often do so as already described. Induced gamma matrices defined always 4-D space so that the restriction of the modes to string world sheets is not possible.

(c) String models suggest that string world sheets are minimal surfaces of space-time surface or of imbedding space but it might not be necessary to pose this condition separately.

In the proposed scenario string world sheets emerge rather than being postulated from beginning.

(a) The vanishing conditions for induced weak fields allow also 4-D spinor modes if they are true for entire spatime surface. This is true if the space-time surface has 2-D projection. One can expect that the space-time surface has foliation by string world sheets and the general solution of K-D equation is continuous superposition of the 2-D modes in this case and discrete one in the generic case.

(b) If the CP_2 projection of space-time surface is homologically non-trivial geodesic sphere S^2, the field equations reduce to those in $M^4 \times S^2$ since the second fundamental form for S^2 is vanishing. It is possible to have geodesic sphere for which induced gauge field has only em component?

(c) If the CP_2 projection is complex manifold as it is for string like objects, the vanishing of weak fields might be also achieved.

(d) Does the phase of cosmic strings assumed to dominate primordial cosmology correspond to this phase with no classical weak fields? During radiation dominated phase 4-D string like objects would transform to string world sheets.Kind of dimensional transmutation would occur.

Right-handed neutrino has exceptional role in K-D action.

(a) Electroweak gauge potentials do not couple to ν_R at all. Therefore the vanishing of W fields is un-necessary if the induced gamma matrices do not mix right handed neutrino with left-handed one. This is guaranteed if M^4 and CP_2 parts of Kähler-Dirac operator annihilate separately right-handed neutrino spinor mode. Also ν_R modes can be interpreted as continuous superpositions of 2-D modes and this allows to define overlap integrals for them and induced spinor fields needed to define WCW gamma matrices and super-generators.

(b) For covariantly constant right-handed neutrino mode defining a generator of super-symmetries is certainly a solution of K-D. Whether more general solutions of K-D exist remains to be checked out.

4.3.4 Connection with number theoretic vision?

The interesting potential connection of the Hamilton-Jacobi vision to the number theoretic vision about field equations has been already mentioned.

(a) The vision that associativity/co-associativity defines the dynamics of space-time surfaces boils down to $M^8 - H$ duality stating that space-time surfaces can be regarded as associative/co-associative surfaces either in M^8 or H [K95]. Associativity reduces to hyper-quaternionicity implying that that the tangent/normal space of space-time surface at each point contains preferred sub-space $M^2(x) \subset M^8$ and these sub-spaces forma an integrable distribution. An analogous condition is involved with the definition of Hamilton-Jacobi structure.

(b) The octonionic representation of the tangent space of M^8 and H effectively replaces $SO(7,1)$ as tangent space group with its octonionic analog obtained by the replacement of sigma matrices with their octonionic counterparts defined by anti-commutators of gamma matrices. By non-associativity the resulting algebra is not ordinary Lie-algebra and exponentiates to a non-associative analog of Lie group. The original wrong belief was that the reduction takes place to the group G_2 of octonionic automorphisms acting as a subgroup of $SO(7)$. One can ask whether the conditions on the charged part of energy momentum tensor could relate to the reduction of $SO(7,1)$

(c) What puts bells ringing is that the modified Dirac equation for the octonionic representation of gamma matrices allows the conservation of electromagnetic charge in the proposed sense. The reason is that the left handed sigma matrices (W charges are left-handed) in the octonionic representation of gamma matrices vanish identically! What remains are vectorial=right-handed em and Z^0 charge which becomes proportional to em charge since its left-handed part vanishes. All spinor modes have a well-defined em charge in the octonionic sense defined by replacing imbedding space spinor locally by its octonionic variant? Maybe this could explain why H spinor modes can have well-defined em charge contrary to the naive expectations.

(d) The non-associativity of the octonionic spinors is however a problem. Even non-commutativity poses problems - also at space-time level if one assumes quaternion-real analyticity for the spinor modes. Could one assume commutativity or co-commutativity for the induced spinor modes? This would mean restriction to associative or co-associative 2-surfaces and (hyper-)holomorphic depends on its (hyper-)complex coordinate. The outcome would be a localization to a hyper-commutative of commutative 2-surface, string world sheet or partonic 2-surface.

(e) These conditions could also be interpreted by saying that for the Kähler Dirac operator the octonionic induced spinors assumed to be commutative/co-commutative are equivalent with ordinary induced spinors. The well-definedness of em charge for ordinary spinors would correspond to commutativity/co-commutativity for octonionic spinors. Even the Dirac equations based on induced and modified gamma matrices could be equivalent since it is essentially holomorphy which matters.

To sum up, these considerations inspire to ask whether the associativity/co-associativity of the space-time surface is equivalent with the reduction of the field equations to stringy field equations stating that certain components of the induced metric in complex/Hamilton-Jacobi coordinates vanish in turn guaranteeing that field equations reduce to algebraic identifies following from the fact that energy momentum tensor and second fundamental form have no common components? Commutativity/co-commutativity would characterize fermionic dynamics and would have physical representation as possibility to have em charge eigenspinors. This should be the case if one requires that the two solution ansätze are equivalent.

4.4 Preferred extremals and solutions of the modified Dirac equation and super-conformal symmetries

The previous considerations concerning super-conformal symmetries and space-time SUSY have been based on general arguments. The new vision about preferred extremals and modified Dirac equation [K87] however leads to a detailed understanding of super-conformal symmetries at the level of field equations and is bound to modify the existing vision about super-conformal symmetries. One important discovery is that Einstein's equations imply the vanishing of terms proportional to Kähler current in field equations for preferred extremals and Equivalence Principle at the classical level could be realized automatically in all scales in contrast to the earlier belief. This obviously must have implications to the general vision about Super-Virasoro representations and one must be ready to modify the existing picture based on the assumption that quantum version of Equivalence Principle is realized in terms coset representations.

The very special role of right handed neutrino is also bound to have profound implications. A further important outcome is the identification of gauge potentials as duals of Kac-Moody currents at the boundaries of string world sheets: quantum gauge potentials are defined only where they are needed that is the curves defining the non-integrable phase factors. This gives also rise to the realization of the conjecture Yangian in terms of the Kac-Moody charges and commutators in accordance with the earlier conjecture.

4.4.1 Super-conformal symmetries

It is good to summarize first the basic ideas about Super-Virasoro representations. TGD allows two kinds of super-conformal symmetries.

(a) The first super-conformal symmetry is associated with $\delta M^4_\pm \times CP_2$ and corresponds to symplectic symmetries of $\delta M^4_\pm \times CP_2$. The reason for extension of conformal symmetries is metric 2-dimensionality of the light-like boundary $\delta M^4_\pm$ defining upper/lower boundary of causal diamond (CD). This super-conformal symmetry is something new and corresponds to replacing finite-dimensional Lie-group G for Kac-Moody symmetry with infinite-dimensional symplectic group. The light-like radial coordinate of $\delta M^4_\pm$ takes the role of the real part of complex coordinate z for ordinary conformal symmetry. Together with complex coordinate of S^2 it defines 3-D restriction of Hamilton-Jacobi variant of 4-D super-conformal symmetries. One can continue the conformal symmetries from light-cone boundary to CD by forming a slicing by parallel copies of $\delta M^4_\pm$. There are two possible slicings corresponding to the choices δM^4_+ and δM^4_- assignable to the upper and lower boundaries of CD. These two choices correspond to two arrows of geometric time for the basis of zero energy states in ZEO.

(b) Super-symplectic degrees of freedom determine the electroweak and color quantum numbers of elementary particles. Bosonic emergence implies that ground states assignable to partonic 2-surfaces correspond to partial waves in $\delta M^4_\pm$ and one obtains color partial waves in particular. These partial waves correspond to the solutions for the Dirac equation in imbedding space and the correlation between color and electroweak quantum numbers is not quite correct. Super-Kac-Moody generators give the compensating color for massless states obtained from tachyonic ground states guaranteeing that standard correlation is obtained. Super-symplectic degrees are therefore directly visible in particle spectrum. One can say that at the point-like limit the WCW spinors reduce to tensor products of imbedding space spinors assignable to the center of mass degrees of freedom for the partonic 2-surfaces defining wormhole throats.

    I have proposed a physical interpretation of super-symplectic vibrational degrees of freedom in terms of degrees of freedom assignable to non-perturbative QCD. These degrees of freedom would be responsible for most of the baryon masses but their theoretical understanding is lacking in QCD framework.

(c) The second super-conformal symmetry is assigned light-like 3-surfaces and to the isometries and holonomies of the imbedding space and is analogous to the super-Kac-Moody symmetry of string models. Kac-Moody symmetries could be assigned to the light-like deformations of light-like 3-surfaces. Isometries give tensor factor $E^2 \times SU(3)$ and holonomies factor $SU(2)_L \times U(1)$. Altogether one has 5 tensor factors to super-conformal algebra. That the number is just five is essential for the success p-adic mass calculations [K43, K37].

    The construction of solutions of the modified Dirac equation suggests strongly that the fermionic representation of the Super-Kac-Moody algebra can be assigned as conserved charges associated with the space-like braid strands at both the 3-D space-like ends of space-time surfaces and with the light-like (or space-like with a small deformation) associated with the light-like 3-surfaces. The extension to Yangian algebra involving higher multi-linears of super-Kac Moody generators is also highly suggestive. These charges would be non-local and assignable to several wormhole contacts simultaneously. The ends of braids would correspond points of partonic 2-surfaces defining a discretization of the partonic 2-surface having interpretation in terms of finite measurement resolution.

    These symmetries would correspond to electroweak and strong gauge fields and to gravitation. The duals of the currents giving rise to Kac-Moody charges would define the counterparts of gauge potentials and the conserved Kac-Moody charges would define the counterparts of non-integrable phase factors in gauge theories. The higher Yangian charges would define generalization of non-integrable phase factors. This would suggest a rather direct connection with the twistorial program for calculating the scattering amplitudes implies also by zero energy ontology.

Quantization recipes have worked in the case of super-string models and one can ask whether the application of quantization to the coefficients of powers of complex coordinates or Hamilton-Jacobi coordinates could lead to the understanding of the 4-D variants of the conformal symmetries and give detailed information about the representations of the Kac-Moody algebra too.

4.4.2 What is the role of the right-handed neutrino?

A highly interesting aspect of Super-Kac-Moody symmetry is the special role of right handed neutrino.

(a) Only right handed neutrino allows besides the modes restricted to 2-D surfaces also the 4D modes de-localized to the entire space-time surface. The first ones are holomorphic functions of single coordinate and the latter ones holomorphic functions of two complex/Hamilton-Jacobi coordinates. Only ν_R has the full $D = 4$ counterpart of the conformal symmetry and the localization to 2-surfaces has interpretation as super-conformal symmetry breaking halving the number of super-conformal generators.

(b) This forces to ask for the meaning of super-partners. Are super-partners obtained by adding ν_R neutrino localized at partonic 2-surface or de-localized to entire space-time surface or its Euclidian or Minkowskian region accompanying particle identified as wormhole throat? Only the Euclidian option allows to assign right handed neutrino to a unique partonic 2-surface. For the Minkowskian regions the assignment is to many particle state defined by the partonic 2-surfaces associated with the 3-surface. Hence for spartners the 4-D right-handed neutrino must be associated with the 4-D Euclidian line of the generalized Feynman diagram.

(c) The orthogonality of the localized and de-localized right handed neutrino modes requires that 2-D modes correspond to higher color partial waves at the level of imbedding space. If color octet is in question, the 2-D right handed neutrino as the candidate for the generator of standard SUSY would combine with the left handed neutrino to form a massive neutrino. If 2-D massive neutrino acts as a generator of super-symmetries, it is is in the same role as badly broken super-symmeries generated by other 2-D modes of the induced spinor field (SUSY with rather large value of $\mathcal{N}$) and one can argue that the counterpart of standard SUSY cannot correspond to this kind of super-symmetries. The right-handed neutrinos de-localized inside the lines of generalized Feynman diagrams, could generate $\mathcal{N} = 2$ variant of the standard SUSY.

How particle and right handed neutrino are bound together?

Ordinary SUSY means that apart from kinematical spin factors sparticles and particles behave identically with respect to standard model interactions. These spin factors would allow to distinguish between particles and sparticles. But is this the case now?

(a) One can argue that 2-D particle and 4-D right-handed neutrino behave like independent entities, and because ν_R has no standard model couplings this entire structure behaves like a particle rather than sparticle with respect to standard model interactions: the kinematical spin dependent factors would be absent.

(b) The question is also about the internal structure of the sparticle. How the four-momentum is divided between the ν_R and and 2-D fermion. If ν_R carries a negligible portion of four-momentum, the four-momentum carried by the particle part of sparticle is same as that carried by particle for given four-momentum so that the distinctions are only kinematical for the ordinary view about sparticle and trivial for the view suggested by the 4-D character of ν_R.

Could sparticle character become manifest in the ordinary scattering of sparticle?

(a) If ν_R behaves as an independent unit not bound to the particle, it would continue in the original direction as particle scatters: sparticle would decay to particle and right-handed neutrino. If ν_R carries a non-negligible energy the scattering could be detected via a missing energy. If not, then the decay could be detected by the interactions revealing the presence of ν_R. ν_R can have only gravitational interactions. What these gravitational interactions are is not however quite clear since the proposed identification of gravitational gauge potentials is as duals of Kac-Moody currents analogous to gauge potentials located at the boundaries of string world sheets. Does this mean that 4-D right-handed neutrino has no quantal gravitational interactions? Does internal consistency require ν_R to have a vanishing gravitational and inertial masses and does this mean that this particle carries only spin?

(b) The cautious conclusion would be following: if de-localized ν_R and parton are uncorrelated particle and sparticle cannot be distinguished experimentally and one might perhaps understand the failure to detect standard SUSY at LHC. Note however that the 2-D fermionic oscillator algebra defines badly broken large $\mathcal{N}$ SUSY containing also massive (longitudinal momentum square is non-vanishing) neutrino modes as generators.

Taking a closer look on sparticles

It is good to take a closer look at the de-localized right handed neutrino modes.

(a) At imbedding space level that is in cm mass degrees of freedom they correspond to covariantly constant CP_2 spinors carrying light-like momentum which for causal diamond could be discretized. For non-vanishing momentum one can speak about helicity having opposite sign for ν_R and $\overline{\nu}_R$. For vanishing four-momentum the situation is delicate since only spin remains and Majorana like behavior is suggestive. Unless one has momentum continuum, this mode might be important and generate additional SUSY resembling standard $\mathcal{N} = 1$ SUSY.

(b) At space-time level the solutions of modified Dirac equation are holomorphic or anti-holomorphic.

   i. For non-constant holomorphic modes these characteristics correlate naturally with fermion number and helicity of ν_R . One can assign creation/annihilation operator to these two kinds of modes and the sign of fermion number correlates with the sign of helicity.

   ii. The covariantly constant mode is naturally assignable to the covariantly constant neutrino spinor of imbedding space. To the two helicities one can assign also oscillator operators $\{a_\pm, a_\pm^\dagger\}$. The effective Majorana property is expressed in terms of non-orthogonality of ν_R and and $\overline{\nu}_R$ translated to the the non-vanishing of the anti-commutator $\{a_+^\dagger, a_-\} = \{a_-^\dagger, a_+\} = 1$. The reduction of the rank of the 4×4 matrix defined by anti-commutators to two expresses the fact that the number of degrees of freedom has halved. $a_+^\dagger = a_-$ realizes the conditions and implies that one has only $\mathcal{N} = 1$ SUSY multiplet since the state containing both ν_R and $\overline{\nu}_R$ is same as that containing no right handed neutrinos.

   iii. One can wonder whether this SUSY is masked totally by the fact that sparticles with all possible conformal weights n for induced spinor field are possible and the branching ratio to $n = 0$ channel is small. If momentum continuum is present, the zero momentum mode might be equivalent to nothing.

What can happen in spin degrees of freedom in super-symmetric interaction vertices if one accepts this interpretation? As already noticed, this depends solely on what one assumes about the correlation of the four-momenta of particle and ν_R.

(a) For SUSY generated by covariantly constant ν_R and $\overline{\nu}_R$ there is no neutrino four-momentum involved so that only spin matters. One cannot speak about the change of direction for ν_R. In the scattering of sparticle the direction of particle changes and introduces different spin quantization axes. ν_R retains its spin and in new system it is superposition of two spin projections. The presence of both helicities requires that the transformation $\nu_R \rightarrow \overline{\nu}_R$ happens with an amplitude determined purely kinematically by spin rotation matrices. This is consistent with fermion number conservation modulo 2. $\mathcal{N}=1$ SUSY based on Majorana spinors is highly suggestive.

(b) For SUSY generated by non-constant holomorphic and anti-holomorphic modes carrying fermion number the behavior in the scattering is different. Suppose that the sparticle does not split to particle moving in the new direction and ν_R moving in the original direction so that also ν_R or $\overline{\nu}_R$ carrying some massless fraction of four-momentum changes its direction of motion. One can form the spin projections with respect to the new spin axis but must drop the projection which does not conserve fermion number. Therefore the kinematics at the vertices is different. Hence $\mathcal{N}=2$ SUSY with fermion number conservation is suggestive when the momentum directions of particle and ν_R are completely correlated.

(c) Since right-handed neutrino has no standard model couplings, p-adic thermodynamics for 4-D right-handed neutrino must correspond to a very low p-adic temperature $T=1/n$. This implies that the excitations with non-vanishing conformal weights are effectively absent and one would have $\mathcal{N}=1$ SUSY effectively.

The simplest assumption is that particle and sparticle correspond to the same p-adic mass scale and have degenerate masses: it is difficult to imagine any good reason for why the p-adic mass scales should differ. This should have been observed -say in decay widths of weak bosons - unless the spartners correspond to large $\hbar$ phase and therefore to dark matter. Note that for the badly broken 2-D N=2 SUSY in fermionic sector this kind of almost degeneracy cannot be excluded and I have considered an explanation for the mysterious X and Y mesons in terms of this degeneracy [K40].

Why space-time SUSY is not possible in TGD framework?

LHC suggests that one does not have $\mathcal{N}=1$ SUSY in standard sense. Why one cannot have standard space-time SUSY in TGD framework. Let us begin by listing all arguments popping in mind.

(a) Could covariantly constant ν_R represents a gauge degree of freedom? This is plausible since the corresponding fermion current is non-vanishing.

(b) The original argument for absence of space-time SUSY years ago was indirect: $M^4 \times CP_2$ does not allow Majorana spinors so that $\mathcal{N}=1$ SUSY is excluded.

(c) One can however consider $\mathcal{N}=2$ SUSY by including both helicities possible for covariantly constant ν_R. For ν_R the four-momentum vanishes so that one cannot distinguish the modes assigned to the creation operator and its conjugate via complex conjugation of the spinor. Rather, one oscillator operator and its conjugate correspond to the two different helicities of right-handed neutrino with respect to the direction determined by the momentum of the particle. The spinors can be chosen to be real in this basis. This indeed gives rise to an irreducible representation of spin 1/2 SUSY algebra with right-handed neutrino creation operator acting as a ladder operator. This is however $\mathcal{N}=1$ algebra and right-handed neutrino in this particular basis behaves effectively like Majorana spinor. One can argue that the system is mathematically inconsistent. By choosing the spin projection axis differently the spinor basis becomes complex. In the new basis one would have $\mathcal{N}=2$, which however reduces to $\mathcal{N}=1$ in the real basis.

(d) Or could it be that fermion and sfermion do exist but cannot be related by SUSY? In standard SUSY fermions and sfermions forming irreducible representations of super Poincare algebra are combined to components of superfield very much like finite-dimensional representations of Lorentz group are combined to those of Poincare. In TGD framework ν_R generates in space-time interior generalization of 2-D super-conformal symmetry but covarianlty constant ν_R cannot give rise to space-time SUSY.

This would be very natural since right-handed neutrinos do not have any electroweak interactions and are are de-localized into the interior of the space-time surface unlike other particles localized at 2-surfaces. It is difficult to imagine how fermion and ν_R could behave as a single coherent unit reflecting itself in the characteristic spin and momentum dependence of vertices implied by SUSY. Rather, it would seem that fermion and sfermion should behave identically with respect to electroweak interactions.

The third argument looks rather convincing and can be developed to a precise argument.

(a) If sfermion is to represent elementary bosons, the products of fermionic oscillator operators with the oscillator operators assignable to the covariantly constant right handed neutrinos must define might-be bosonic oscillator operators as $b_n = a_n a$ and $b_n^\dagger = a_n^\dagger a^\dagger$ One can calculate the commutator for the product of operators. If fermionic oscillator operators commute, so do the corresponding bosonic operators. The commutator $[b_n, b_n^\dagger]$ is however proportional to occupation number for ν_R in $\mathcal{N} = 1$ SUSY representation and vanishes for the second state of the representation. Therefore $\mathcal{N} = 1$ SUSY is a pure gauge symmetry.

(b) One can however have both irreducible representations of SUSY: for them either fermion or sfermion has a non-vanishing norm. One would have both fermions and sfermions but they would not belong to the same SUSY multiplet, and one cannot expect SUSY symmetries of 3-particle vertices.

(c) For instance, γFF vertex is closely related to $\gamma \tilde{F}\tilde{\overline{F}}$ in standard SUSY. Now one expects this vertex to decompose to a product of $\gamma F\overline{F}$ vertex and amplitude for the creation of $\nu_R \tilde{\nu}_R$ from vacuum so that the characteristic momentum and spin dependent factors distinguishing between the couplings of photon to scalar and and fermion are absent. Both states behave like fermions. The amplitude for the creation of $\nu_R \tilde{\nu}_R$ from vacuum is naturally equal to unity as an occupation number operator by crossing symmetry. The presence of right-handed neutrinos would be invisible if this picture is correct. Whether this invisible label can have some consequences is not quite clear: one could argue that the decay rates of weak bosons to fermion pairs are doubled unless one introduces $1/\sqrt{2}$ factors to couplings.

Where the sfermions might make themselves visible are loops. What loops are? Consider boson line first. Boson line is replaced with a sum of two contributions corresponding to ordinary contribution with fermion and anti-fermion at opposite throats and second contribution with fermion and anti-fermion accompanied by right-handed neutrino ν_R and its antiparticle which now has opposite helicity to ν_R. The loop for ν_R decomposes to four pieces since also the propagation from wormhole throat to the opposite wormhole throat must be taken into account. Each of the four propagators equals to $a^\dagger_{1/2} a^\dagger_{-1/2}$ or its hermitian conjugate. The product of these is slashed between vacuum states and anti-commutations give imaginary unit per propagator giving $i^4 = 1$. The two contributions are therefore identical and the scaling $g \rightarrow g/\sqrt{2}$ for coupling constants guarantees that sfermions do not affect the scattering amplitudes at all. The argument is identical for the internal fermion lines.

4.4.3 WCW geometry and super-conformal symmetries

The vision about the geometry of WCW has been roughly the following and the recent steps of progress induce to it only small modifications if any.

(a) Kähler geometry is forced by the condition that hermitian conjugation allows geometrization. Kähler function is given by the Kähler action coming from space-time regions with Euclidian signature of the induced metric identifiable as lines of generalized Feynman diagrams. Minkowskian regions give imaginary contribution identifiable as the analog of Morse function and implying interference effects and stationary phase approximation. The vision about quantum TGD as almost topological QFT inspires the proposal that Kähler action reduces to 3-D terms reducing to Chern-Simons terms by the weak form of electric-magnetic duality. The recent proposal for preferred extremals is consistent with this property realizing also holography implied by general coordinate invariance.

Strong form of general coordinate invariance implying effective 2-dimensionality in turn suggests that Kähler action is expressible in terms of areas of partonic 2-surfaces and string world sheets.

(b) The complexified gamma matrices of WCW come as hermitian conjugate pairs and anti-commute to the Kähler metric of WCW. Also bosonic generators of symplectic transformations of $\delta M^4_\pm \times CP_2$ a assumed to act as isometries of WCW geometry can be complexified and appear as similar pairs. The action of isometry generators coincides with that of symplectic generators at partonic 2-surfaces and string world sheets but elsewhere inside the space-time surface it is expected to be deformed from the symplectic action. The super-conformal transformations of $\delta M^4_\pm \times CP_2$ acting on the light-like radial coordinate of $\delta M^4_\pm$ act as gauge symmetries of the geometry meaning that the corresponding WCW vector fields have zero norm.

(c) WCW geometry has also zero modes which by definition do not contribute to WCW metric expect possibly by the dependence of the elements of WCW metric on zero modes through a conformal factor. In particular, induced CP_2 Kähler form and its analog for sphere $r_M = constant$ of light cone boundary are symplectic invariants, and one can define an infinite number of zero modes as invariants defined by Kähler fluxes over partonic 2-surfaces and string world sheets. This requires however the slicing of CD parallel copies of δM^4_+ or δM^4_-. The physical interpretation of these non-quantum fluctuating degrees of freedom is as classical variables necessary for the interpretation of quantum measurement theory. Classical variable would metaphorically correspond the position of the pointer of the measurement instrument.

(d) The construction receives a strong philosophical inspiration from the geometry of loop spaces. Loop spaces allow a unique Kähler geometry with maximal isometry group identifiable as Kac-Moody group. The reason is that otherwise Riemann connection does not exist. The only problem is that curvature scalar diverges since the Riemann tensor is by constant curvature property proportional to the metric. In 3-D case one would have union of constant curvature spaces labelled by zero modes and the situation is expected to be even more restrictive. The conjecture indeed is that WCW geometry exists only for $H = M^4 \times CP_2$: infinite-D Kähler geometric existence and therefore physics would be unique. One can also hope that Ricci scalar is finite and therefore zero by the constant curvature property so that Einstein's equations are satisfied.

(e) WCW Hamiltonians determined the isometry currents and WCW metric is given in terms of the anti-commutators of the Killing vector fields associated with symplectic isometry currents. The WCW Hamiltonians generating symplectic isometries correspond to the Hamiltonians spanning the symplectic group of $\delta M^4_\pm \times CP_2$. One can say that the space of quantum fluctuating degrees of freedom is this symplectic group of $\delta M^4_\pm \times CP_2$ or its subgroup or coset space: this must have very deep implications for the structure of the quantum TGD.

(f) Zero energy ontology brings in additional delicacies. Basic objects are now unions of partonic 2-surfaces at the ends of CD. Also string world sheets would naturally contribute. One can generalize the expressions for the isometry generators in a straightforward manner by requiring that given isometry restricts to a symplectic transformation at partonic 2-surfaces and string world sheets.

(g) One could criticize the effective metric 2-dimensionality forced by general consistency arguments as something non-physical. The Hamiltonians are expressed using only the data at partonic 2-surfaces: this includes also 4-D tangent space data via the weak form of electric-magnetic duality so that one has only effective 2-dimensionality. Obviously WCW geometry must have large gauge symmetries besides zero modes. The super-conformal symmetries indeed represent gauge symmetries of this kind. Effective 2-dimensionality realizing strong form of holography in turn is induced by the strong form of general coordinate invariance. Light-like 3-surfaces at which the signature of the induced metric changes must be equivalent with the 3-D space-like ends of space-time surfaces at the light-boundaries of space-time surfaces as far as WCW geometry is considered. This requires that the data from their 2-D intersections defining partonic

2-surfaces should dictate the WCW geometry. Note however that Super-Kac-Moody charges giving information about the interiors of 3-surfaces appear in the construction of the physical states.

What is the role of the right handed neutrino in this construction?

(a) In the construction of components of WCW metric as anti-commutators of super-generators only the covariantly constant right-handed neutrino appears in the super-generators analogous to super-Kac-Moody generators. All holomorphic modes of right handed neutrino characterized by two integers could in principle contribute to the WCW gamma matrices identified as fermionic super-symplectic generators anti-commuting to the metric. At the space-like ends of space-time surface the holomorphic generators would restrict to symplectic generators since the radial light-like coordinate r_M identified and complex coordinate of CP_2 allowing identification as restrictions of two complex coordinates or Hamilton-Jacobi coordinates to light-like boundary.

(b) The non-covariantly constant modes could also correspond to purely super-conformal gauge degrees of freedom. Originally the restriction to right-handed neutrino looked somewhat un-satisfactory but the recent view about Super-Kac-Moody symmetries makes its special role rather natural. One could say that WCW geometry possesses the maximal $D = 4$ supersymmetry.

(c) One can of course ask whether the Super-Kac-Moody generators assignable to the isometries of H and expressible as conserved charges associated with the boundaries of string world sheets could contribute to the WCW geometry via the anti-commutators. This option cannot be excluded but in this case the interpretation in terms of Hamiltonians is not obvious.

4.4.4 The relationship between inertial gravitational masses

The relationship between inertial and gravitational masses and Equivalence Principle have been on of the longstanding problems in TGD. Not surprisingly, the realization how GRT space-time relates to the many-sheeted space-time of TGD finally allowed to solve the problem.

ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invarance of TGD is in sharpt contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4. If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

Equivalence Principle at quantum level

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

(a) At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.

(b) The most recent view (2014) about understanding how EP emerges in TGD is described in [K72] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets associated with Käbler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).

(c) The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the modified Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

Equivalence Principle at classical level

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD spacetime and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K96].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

4.4.5 Constraints from p-adic mass calculations and ZEO

A further important physical input comes from p-adic thermodynamics forming a core element of p-adic mass calculations.

(a) The first thing that one can get worried about relates to the extension of conformal symmetries. If the conformal symmetries generalize to $D = 4$, how can one take seriously the results of p-adic mass calculations based on 2-D conformal invariance? There is no reason to worry. The reduction of the conformal invariance to 2-D one for the preferred extremals takes care of this problem. This however requires that the fermionic contributions assignable to string world sheets and/or partonic 2-surfaces - Super- Kac-Moody contributions - should dictate the elementary particle masses. For hadrons also symplectic contributions should be present. This is a valuable hint in attempts to identify the mathematical structure in more detail.

(b) ZEO suggests that all particles, even virtual ones correspond to massless wormhole throats carrying fermions. As a consequence, twistor approach would work and the kinematical constraints to vertices would allow the cancellation of divergences. This would suggest that the p-adic thermal expectation value is for the longitudinal M^2 momentum squared (the definition of CD selects $M^1 \subset M^2 \subset M^4$ as also does number theoretic vision). Also propagator would be determined by M^2 momentum. Lorentz invariance would be obtained by integration of the moduli for CD including also Lorentz boosts of CD.

(c) In the original approach one allows states with arbitrary large values of L_0 as physical states. Usually one would require that L_0 annihilates the states. In the calculations however mass squared was assumed to be proportional L_0 apart from vacuum contribution. This is a questionable assumption. ZEO suggests that total mass squared vanishes and that one can decompose mass squared to a sum of longitudinal and transversal parts. If one can do the same decomposition to longitudinal and transverse parts also for the Super Virasoro algebra then one can calculate longitudinal mass squared as a p-adic thermal expectation in the transversal super-Virasoro algebra and only states with $L_0 = 0$ would contribute and one would have conformal invariance in the standard sense.

(d) In the original approach the assumption motivated by Lorentz invariance has been that mass squared is replaced with conformal weight in thermodynamics, and that one first calculates the thermal average of the conformal weight and then equates it with mass squared. This assumption is somewhat ad hoc. ZEO however suggests an alternative interpretation in which one has zero energy states for which longitudinal mass squared of positive energy state derive from p-adic thermodynamics. Thermodynamics - or rather, its square root - would become part of quantum theory in ZEO. M-matrix is indeed product of hermitian square root of density matrix multiplied by unitary S-matrix and defines the entanglement coefficients between positive and negative energy parts of zero energy state.

(e) The crucial constraint is that the number of super-conformal tensor factors is $N = 5$: this suggests that thermodynamics applied in Super-Kac-Moody degrees of freedom assignable to string world sheets is enough, when one is interested in the masses of fermions and gauge bosons. Super-symplectic degrees of freedom can also contribute and determine the dominant contribution to baryon masses. Should also this contribution obey p-adic thermodynamics in the case when it is present? Or does the very fact that this contribution need not be present mean that it is not thermal? The symplectic contribution should correspond to hadronic p-adic length prime rather the one assignable to (say) u quark. Hadronic p-adic mass squared and partonic p-adic mass squared cannot be summed since primes are different. If one accepts the basic rules [K46], longitudinal energy and momentum are additive as indeed assumed in perturbative QCD.

(f) Calculations work if the vacuum expectation value of the mass squared must be assumed to be tachyonic. There are two options depending on whether one whether p-adic thermodynamics gives total mass squared or longitudinal mass squared.

   i. One could argue that the total mass squared has naturally tachyonic ground state expectation since for massless extremals longitudinal momentum is light-like and transversal momentum squared is necessary present and non-vanishing by the localization to topological light ray of finite thickness of order p-adic length scale. Transversal degrees of freedom would be modeled with a particle in a box.

   ii. If longitudinal mass squared is what is calculated, the condition would require that transversal momentum squared is negative so that instead of plane wave like behavior exponential damping would be required. This would conform with the localization in transversal degrees of freedom.

4.4.6 The emergence of Yangian symmetry and gauge potentials as duals of Kac-Moody currents

Yangian symmetry plays a key role in $\mathcal{N} = 4$ super-symmetric gauge theories. What is special in Yangian symmetry is that the algebra contains also multi-local generators. In TGD framework multi-locality would naturally correspond to that with respect to partonic 2-surfaces and string world sheets and the proposal has been that the Super-Kac-Moody algebras assignable to string worlds sheets could generalize to Yangian.

Witten has written a beautiful exposition of Yangian algebras [B43]. Yangian is generated by two kinds of generators J^A and Q^A by a repeated formation of commutators. The number of commutations tells the integer characterizing the multi-locality and provides the Yangian algebra with grading by natural numbers. Witten describes a 2-dimensional QFT like situation in which one has 2-D situation and Kac-Moody currents assignable to real axis define the Kac-Moody charges as integrals in the usual manner. It is also assumed that the gauge potentials defined by the 1-form associated with the Kac-Moody current define a flat connection:

$$\partial_\mu j^A_\nu - \partial_\nu j^A_\nu + [j^A_\mu, j^A_\nu] = 0 \quad . \tag{4.4.1}$$

This condition guarantees that the generators of Yangian are conserved charges. One can however consider alternative manners to obtain the conservation.

(a) The generators of first kind - call them J^A - are just the conserved Kac-Moody charges. The formula is given by

$$J_A = \int_{-\infty}^{\infty} dx j^{A0}(x,t) \ . \tag{4.4.2}$$

(b) The generators of second kind contain bi-local part. They are convolutions of generators of first kind associated with different points of string described as real axis. In the basic formula one has integration over the point of real axis.

$$Q^A = f^A_{BC}\int_{-\infty}^{\infty} dx \int_x^{\infty} dy j^{B0}(x,t)j^{C0}(y,t) - 2\int_{-\infty}^{\infty} j^A_x dx \ . \tag{4.4.3}$$

These charges are indeed conserved if the curvature form is vanishing as a little calculation shows.

How to generalize this to the recent context?

(a) The Kac-Moody charges would be associated with the braid strands connecting two partonic 2-surfaces - Strands would be located either at the space-like 3-surfaces at the ends of the space-time surface or at light-like 3-surfaces connecting the ends. Modified Dirac equation would define Super-Kac-Moody charges as standard Noether charges. Super charges would be obtained by replacing the second quantized spinor field or its conjugate in the fermionic bilinear by particular mode of the spinor field. By replacing both spinor field and its conjugate by its mode one would obtain a conserved c-number charge corresponding to an anti-commutator of two fermionic super-charges. The convolution involving double integral is however not number theoretically attractive whereas single 1-D integrals might make sense.

(b) An encouraging observation is that the Hodge dual of the Kac-Moody current defines the analog of gauge potential and exponents of the conserved Kac-Moody charges could be identified as analogs for the non-integrable phase factors for the components of this gauge potential. This identification is precise only in the approximation that generators commute since only in this case the ordered integral $P(exp(i\int Adx))$ reduces to $P(exp(i\int Adx))$.Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization implying that Abelian approximation works. This conforms with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

This would make possible a direct identification of Kac-Moody symmetries in terms of gauge symmetries. For isometries one would obtain color gauge potentials and the analogs of gauge potentials for graviton field (in TGD framework the contraction with M^4 vierbein would transform tensor field to 4 vector fields). For Kac-Moody generators corresponding to holonomies one would obtain electroweak gauge potentials. Note that super-charges would give rise to a collection of spartners of gauge potentials automatically. One would obtain a badly broken SUSY with very large value of $\mathcal{N}$ defined by the number of spinor modes as indeed speculated earlier [K23].

(c) The condition that the gauge field defined by 1-forms associated with the Kac-Moody currents are trivial looks unphysical since it would give rise to the analog of topological QFT with gauge potentials defined by the Kac-Moody charges. For the duals of Kac-Moody currents defining gauge potentials only covariant divergence vanishes implying that curvature form is

$$F_{\alpha\beta} = \epsilon_{\alpha\beta}[j_\mu, j^\mu] \ , \tag{4.4.4}$$

so that the situation does not reduce to topological QFT unless the induced metric is diagonal. This is not the case in general for string world sheets.

(d) It seems however that there is no need to assume that j_μ defines a flat connection. Witten mentions that although the discretization in the definition of J^A does not seem to be possible, it makes sense for Q^A in the case of $G = SU(N)$ for any representation of G. For general G and its general representation there exists no satisfactory definition of Q. For certain representations, such as the fundamental representation of $SU(N)$, the definition of Q^A is especially simple. One just takes the bi-local part of the previous formula:

$$Q^A = f^A_{BC}\sum_{i<j} J_i^B J_j^C \quad . \tag{4.4.5}$$

What is remarkable that in this formula the summation need not refer to a discretized point of braid but to braid strands ordered by the label i by requiring that they form a connected polygon. Therefore the definition of J^A could be just as above.

(e) This brings strongly in mind the interpretation in terms of twistor diagrams. Yangian would be identified as the algebra generated by the logarithms of non-integrable phase factors in Abelian approximation assigned with pairs of partonic 2-surfaces defined in terms of Kac-Moody currents assigned with the modified Dirac action. Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization. This would fit nicely with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

The resulting algebra satisfies the basic commutation relations

$$\left[J^A, J^B\right] = f_C^{AB} J^C \quad , \quad \left[J^A, Q^B\right] = f_C^{AB} Q^C \quad . \tag{4.4.6}$$

plus the rather complex Serre relations described in [B43].

4.4.7 Quantum criticality and electroweak symmetries

In the following quantum criticali and electroweak symmetries are discussed for Kähler-Dirac action.

What does one mean with quantum criticality?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

(a) What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the imbedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

(b) At more technical level one would expect criticality to corresponds to deformations of a given preferred extremal defining a vanishing second variation of Kähler Khler function or Kähler action.

i. For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

ii. Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A102]. Cusp catastrophe [A3] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

(c) Quantum criticality makes sense also for Kähler action.

i. Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K21] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

ii. Also now one expects a hierarchy of criticalities and and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

iii. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

(d) I have discussed what criticality could mean for modified Dirac action [K22] .

i. I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the modified Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

ii. The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since imbedding space coordinates appear as parameters in modified Dirac action. The existence of conserved currents does not actually require the vanishing of the second variation of Kähler action as claimed earlier. It is enough that the first variation of the canonical momentum densities contracted with the imbedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation:it does not imply the vanishing of four-momentum, only the vanishing of mass. Hence conserved currents are obtained also outside the quantum criticality.

iii. It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generaic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for currents associated with the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the modified gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor. This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied.

In the following these arguments are formulated more precisely. The unexpected result is that critical deformations induce conformal scalings of the modified metric and electro-weak gauge transformations of the induced spinor connection at X^2. Therefore holomorphy brings in the Kac-Moody symmetries associated with isometries of H (gravitation and color gauge group) and quantum criticality those associated with the holonomies of H (electro-weak-gauge group) as additional symmetries.

The variation of modes of the induced spinor field in a variation of space-time surface respecting the preferred extremal property

Consider first the variation of the induced spinor field in a variation of space-time surface respecting the preferred extremal property. The deformation must be such that the deformed modified Dirac operator D annihilates the modified mode. By writing explicitly the variation of the modified Dirac action (the action vanishes by modified Dirac equation) one obtains deformations and requiring its vanishing one obtains

$$\delta\Psi = D^{-1}(\delta D)\Psi \ . \tag{4.4.7}$$

D^{-1} is the inverse of the modified Dirac operator defining the analog of Dirac propagator and δD defines vertex completely analogous to $\gamma^k\delta A_k$ in gauge theory context. The functional integral over preferred extremals can be carried out perturbatively by expressing δD in terms of δh^k and one obtains stringy perturbation theory around X^2 associated with the preferred extremal defining maximum of Kähler function in Euclidian region and extremum of Kähler action in Minkowskian region (stationary phase approximation).

What one obtains is stringy perturbation theory for calculating n-points functions for fermions at the ends of braid strands located at partonic 2-surfaces and representing intersections of string world sheets and partonic 2-surfaces at the light-like boundaries of CDs. δD- or more precisely, its partial derivatives with respect to functional integration variables - appear atthe vertices located anywhere in the interior of X^2 with outgoing fermions at braid ends. Bosonic propagators are replaced with correlation functions for δh^k. Fermionic propagator is defined by D^{-1}.

After 35 years or hard work this provides for the first time a reasonably explicit formula for the N-point functions of fermions. This is enough since by bosonic emergence [K50] these N-point functions define the basic building blocks of the scattering amplitudes. Note that bosonic emergence states that bosons corresponds to wormhole contacts with fermion and anti-fermion at the opposite wormhole throats.

What critical modes could mean for the induced spinor fields?

What critical modes could mean for the induced spinor fields at string world sheets and partonic 2-surfaces. The problematic part seems to be the variation of the modified Dirac operator since it involves gradient. One cannot require that covariant derivative remains invariant since this would require that the components of the induced spinor connection remain invariant and this is quite too restrictive condition. Right handed neutrino solutions de-localized into entire X^2 are however an exception since they have no electro-weak gauge couplings and in this case the condition is obvious: modified gamma matrices suffer a local scaling for critical deformations:

$$\delta\Gamma^{\mu} = \Lambda(x)\Gamma^{\mu} \ . \tag{4.4.8}$$

This guarantees that the modified Dirac operator D is mapped to ΛD and still annihilates the modes of ν_R labelled by conformal weight, which thus remain unchanged.

What is the situation for the 2-D modes located at string world sheets? The condition is obvious. Ψ suffers an electro-weak gauge transformation as does also the induced spinor connection so that D_{μ} is not affected at all. Criticality condition states that the deformation of the space-time surfaces induces a conformal scaling of Γ^{μ} at X^2. It might be possible to continue this conformal scaling of the entire space-time sheet but this might be not necessary and this would mean that all critical deformations induced conformal transformations of the effective metric of the space-time surface defined by $\{\Gamma^{\mu},\Gamma^{\nu}\} = 2G^{\mu\nu}$. Thus it seems that effective metric is indeed central concept (recall that if the conjectured quaternionic structure is associated with the effective metric, it might be possible to avoid problem related to the Minkowskian signature in an elegant manner).

In fact, one can consider even more general action of critical deformation: the modes of the induced spinor field would be mixed together in the infinitesimal deformation besides infinitesimal electroweak gauge transformation, which is same for all modes. This would extend electroweak gauge symmetry. Modified Dirac equation holds true also for these deformations. One might wonder whether the conjectured dynamically generated gauge symmetries assignable to finite measurement resolution could be generated in this manner.

The infinitesimal generator of a critical deformation J_M can be expressed as tensor product of matrix A_M acting in the space of zero modes and of a generator of infinitesimal electro-weak gauge transformation $T_M(x)$ acting in the same manner on all modes: $J_M = A_M \otimes T_M(x)$. A_M is a spatially constant matrix and $T_M(x)$ decomposes to a direct sum of left- and right-handed $SU(2) \times U(1)$ Lie-algebra generators. Left-handed Lie-algebra generator can be regarded as a quaternion and right handed as a complex number. One can speak of a direct sum of left-handed local quaternion $q_{M,L}$ and right-handed local complex number $c_{M,R}$. The commutator $[J_M, J_N]$ is given by $[J_M, J_N] = [A_M, A_N] \otimes \{T_M(x), T_N(x)\} + \{A_M, A_N\} \otimes [T_M(x), T_N(x)]$. One has $\{T_M(x), T_N(x)\} = \{q_{M,L}(x), q_{N,L}(x)\} \oplus \{c_{M,R}(x), c_{N,R}(x)\}$ and $[T_M(x), T_N(x)] = [q_{M,L}(x), q_{N,L}(x)]$. The commutators make sense also for more general gauge group but quaternion/complex number property might have some deeper role.

Thus the critical deformations would induce conformal scalings of the effective metric and dynamical electro-weak gauge transformations. Electro-weak gauge symmetry would be a dynamical symmetry restricted to string world sheets and partonic 2-surfaces rather than acting at the entire space-time surface. For 4-D de-localized right-handed neutrino modes the conformal scalings of the effective metric are analogous to the conformal transformations of M^4 for $\mathcal{N} = 4$ SYMs. Also ordinary conformal symmetries of M^4 could be present for string world sheets and could act as symmetries of generalized Feynman graphs since even virtual wormhole throats are massless. An interesting question is whether the conformal invariance associated with the effective metric is the analog of dual conformal invariance in $\mathcal{N} = 4$ theories.

Critical deformations of space-time surface are accompanied by conserved fermionic currents. By using standard Noetherian formulas one can write

$$J_i^\mu = \overline{\Psi}\Gamma^\mu \delta_i \Psi + \delta_i \overline{\Psi}\Gamma^\mu \Psi \ . \qquad (4.4.9)$$

Here $\delta\Psi_i$ denotes derivative of the variation with respect to a group parameter labeled by i. Since $\delta\Psi_i$ reduces to an infinitesimal gauge transformation of Ψ induced by deformation, these currents are the analogs of gauge currents. The integrals of these currents along the braid strands at the ends of string world sheets define the analogs of gauge charges. The interpretation as Kac-Moody charges is also very attractive and I have proposed that the 2-D Hodge duals of gauge potentials could be identified as Kac-Moody currents. If so, the 2-D Hodge duals of J would define the quantum analogs of dynamical electro-weak gauge fields and Kac-Moody charge could be also seen as non-integral phase factor associated with the braid strand in Abelian approximation (the interpretation in terms of finite measurement resolution is discussed earlier).

One can also define super currents by replacing $\overline{\Psi}$ or Ψ by a particular mode of the induced spinor field as well as c-number valued currents by performing the replacement for both $\overline{\Psi}$ or Ψ. As expected, one obtains a super-conformal algebra with all modes of induced spinor fields acting as generators of super-symmetries restricted to 2-D surfaces. The number of the charges which do not annihilate physical states as also the effective number of fermionic modes could be finite and this would suggest that the integer $\mathcal{N}$ for the supersymmetry in question is finite. This would conform with the earlier proposal inspired by the notion of finite measurement resolution implying the replacement of the partonic 2-surfaces with collections of braid ends.

Note that Kac-Moody charges might be associated with "long" braid strands connecting different wormhole throats as well as short braid strands connecting opposite throats of wormhole contacts. Both kinds of charges would appear in the theory.

What is the interpretation of the critical deformations?

Critical deformations bring in an additional gauge symmetry. Certainly not all possible gauge transformations are induced by the deformations of preferred extremals and a good guess is that they correspond to holomorphic gauge group elements as in theories with Kac-Moody symmetry. What is the physical character of this dynamical gauge symmetry?

(a) Do the gauge charges vanish? Do they annihilate the physical states? Do only their positive energy parts annihilate the states so that one has a situation characteristic for the representation of Kac-Moody algebras. Or could some of these charges be analogous to the gauge charges associated with the constant gauge transformations in gauge theories and be therefore non-vanishing in the absence of confinement. Now one has electro-weak gauge charges and these should be non-vanishing. Can one assign them to deformations with a vanishing conformal weight and the remaining deformations to those with non-vanishing conformal weight and acting like Kac-Moody generators on the physical states?

(b) The simplest option is that the critical Kac-Moody charges/gauge charges with non-vanishing positive conformal weight annihilate the physical states. Critical degrees of freedom would not disappear but make their presence known via the states labelled by different gauge charges assignable to critical deformations with vanishing conformal weight. Note that constant gauge transformations can be said to break the gauge symmetry also in the ordinary gauge theories unless one has confinement.

(c) The hierarchy of quantum criticalities suggests however entire hierarchy of electro-weak Kac-Moody algebras. Does this mean a hierarchy of electro-weak symmetries breakings in which the number of Kac-Moody generators not annihilating the physical states gradually increases as also modes with a higher value of positive conformal weight fail to annihilate the physical state?

The only manner to have a hierarchy of algebras is by assuming that only the generators satisfying $n\ mod\ N = 0$ define the sub-Kac-Moody algebra annihilating the physical states so that the generators with $n\ mod\ N \neq 0$ would define the analogs of gauge charges. I have suggested for long time ago the relevance of kind of fractal hierarchy of Kac-Moody and Super-Virasoro algebras for TGD but failed to imagine any concrete realization.

A stronger condition would be that the algebra reduces to a finite dimensional algebra in the sense that the actions of generators Q_n and Q_{n+kN} are identical. This would correspond to periodic boundary conditions in the space of conformal weights. The notion of finite measurement resolution suggests that the number of independent fermionic oscillator operators is proportional to the number of braid ends so that an effective reduction to a finite algebra is expected.

Whatever the correct interpretation is, this would obviously refine the usual view about electro-weak symmetry breaking.

These arguments suggests the following overall view. The holomorphy of spinor modes gives rise to Kac-Moody algebra defined by isometries and includes besides Minkowskian generators associated with gravitation also SU(3) generators associated with color symmetries. Vanishing second variations in turn define electro-weak Kac-Moody type algebra.

Note that criticality suggests that one must perform functional integral over WCW by decomposing it to an integral over zero modes for which deformations of X^4 induce only an electro-weak gauge transformation of the induced spinor field and to an integral over moduli corresponding to the remaining degrees of freedom.

4.4.8 The importance of being light-like

The singular geometric objects associated with the space-time surface have become increasingly important in TGD framework. In particular, the recent progress has made clear that these objects might be crucial for the understanding of quantum TGD. The singular objects are associated not only with the induced metric but also with the effective metric defined by the anti-commutators of the modified gamma matrices appearing in the modified Dirac equation and determined by the Kähler action.

The singular objects associated with the induced metric

Consider first the singular objects associated with the induced metric.

(a) At light-like 3-surfaces defined by wormhole throats the signature of the induced metric changes from Euclidian to Minkowskian so that 4-metric is degenerate. These surfaces are carriers of elementary particle quantum numbers and the 4-D induced metric degenerates locally to 3-D one at these surfaces.

(b) Braid strands at light-like 3-surfaces are most naturally light-like curves: this correspond to the boundary condition for open strings. One can assign fermion number to the braid strands. Braid strands allow an identification as curves along which the Euclidian signature of the string world sheet in Euclidian region transforms to Minkowskian one. Number theoretic interpretation would be as a transformation of complex regions to hyper-complex regions meaning that imaginary unit i satisfying $i^2 = -1$ becomes hyper-complex unit e satisfying $e^2 = 1$. The complex coordinates $(z, \overline{z})$ become hyper-complex coordinates $(u = t + ex, v = t - ex)$ giving the standard light-like coordinates when one puts $e = 1$.

The singular objects associated with the effective metric

There are also singular objects assignable to the effective metric. According to the simple arguments already developed, string world sheets and possibly also partonic 2-surfaces are singular objects with respect to the effective metric defined by the anti-commutators of the modified gamma matrices rather than induced gamma matrices. Therefore the effective metric seems to be much more than a mere formal structure.

(a) For instance, quaternionicity of the space-time surface could allow an elegant formulation in terms of the effective metric avoiding the problems due to the Minkowski signature. This is achieved if the effective metric has Euclidian signature $\epsilon \times (1,1,1,1)$, $\epsilon = \pm 1$ or a complex counterpart of the Minkowskian signature $\epsilon(1,1,-1,-1)$.

(b) String word sheets and perhaps also partonic 2-surfaces could be understood as singularities of the effective metric. What happens that the effective metric with Euclidian signature $\epsilon \times (1,1,1,1)$ transforms to the signature $\epsilon(1,1,-1,-1)$ (say) at string world sheet so that one would have the degenerate signature $\epsilon \times (1,1,0,0)$ at the string world sheet.

   What is amazing is that this works also number theoretically. It came as a total surprise to me that the notion of hyper-quaternions as a closed algebraic structure indeed exists. The hyper-quaternionic units would be given by $(1, I, iJ, iK)$, where i is a commuting imaginary unit satisfying $i^2 = -1$. Hyper-quaternionic numbers defined as combinations of these units with real coefficients do form a closed algebraic structure which however fails to be a number field just like hyper-complex numbers do. Note that the hyper-quaternions obtained with real coefficients from the basis $(1, iI, iJ, iK)$ fail to form an algebra since the product is not hyper-quaternion in this sense but belongs to the algebra of complexified quaternions. The same problem is encountered in the case of hyper-octonions defined in this manner. This has been a stone in my shoe since I feel strong disrelish towards Wick rotation as a trick for moving between different signatures.

(c) Could also partonic 2-surfaces correspond to this kind of singular 2-surfaces? In principle, 2-D surfaces of 4-D space intersect at discrete points just as string world sheets and partonic 2-surfaces do so that this might make sense. By complex structure the situation is algebraically equivalent to the analog of plane with non-flat metric allowing all possible signatures (ϵ_1, ϵ_2) in various regions. At light-like curve either ϵ_1 or ϵ_2 changes sign and light-like curves for these two kinds of changes can intersect as one can easily verify by drawing what happens. At the intersection point the metric is completely degenerate and simply vanishes.

(d) Replacing real 2-dimensionality with complex 2-dimensionality, one obtains by the universality of algebraic dimension the same result for partonic 2-surfaces and string world sheets. The braid ends at partonic 2-surfaces representing the intersection points of 2-surfaces of this kind would have completely degenerate effective metric so that the modified gamma matrices would vanish implying that energy momentum tensor vanishes as does also the induced Kähler field.

(e) The effective metric suffers a local conformal scaling in the critical deformations identified in the proposed manner. Since ordinary conformal group acts on Minkowski space and leaves the boundary of light-cone invariant, one has two conformal groups. It is not however clear whether the M^4 conformal transformations can act as symmetries in TGD, where the presence of the induced metric in Kähler action breaks M^4 conformal symmetry. As found, also in TGD framework the Kac-Moody currents assigned to the braid strands generate Yangian: this is expected to be true also for the Kac-Moody counterparts of the conformal algebra associated with quantum criticality. On the other hand, in twistor program one encounters also two conformal groups and the space in which the second conformal group acts remains somewhat mysterious object. The Lie algebras for the two conformal groups generate the conformal Yangian and the integrands of the scattering amplitudes are Yangian invariants. Twistor approach should apply in TGD if zero energy ontology is right. Does this mean a deep connection?

   What is also intriguing that twistor approach in principle works in strict mathematical sense only at signatures $\epsilon \times (1,1,-1-1)$ and the scattering amplitudes in Minkowski signature are obtained by analytic continuation. Could the effective metric give rise to the desired signature? Note that the notion of massless particle does not make sense in the signature $\epsilon \times (1,1,1,1)$.

These arguments provide genuine a support for the notion of quaternionicity and suggest a connection with the twistor approach.

4.4.9 Realization of large $\mathcal{N}$ SUSY in TGD

The generators large $\mathcal{N}$ SUSY algebras are obtained by taking fermionic currents for second quantized fermions and replacing either fermion field or its conjugate with its particular mode. The resulting super currents are conserved and define super charges. By replacing both fermion and its conjugate with modes one obtains c number valued currents. Therefore $\mathcal{N} = \infty$ SUSY - presumably equivalent with super-conformal invariance - or its finite $\mathcal{N}$ cutoff is realized in TGD framework and the challenge is to understand the realization in more detail.

Super-space viz. Grassmann algebra valued fields

Standard SUSY induces super-space extending space-time by adding anti-commuting coordinates as a formal tool. Many mathematicians are not enthusiastic about this approach because of the purely formal nature of anti-commuting coordinates. Also I regard them as a non-sense geometrically and there is actually no need to introduce them as the following little argument shows.

Grassmann parameters (anti-commuting theta parameters) are generators of Grassmann algebra and the natural object replacing super-space is this Grassmann algebra with coefficients of Grassmann algebra basis appearing as ordinary real or complex coordinates. This is just an ordinary space with additional algebraic structure: the mysterious anti-commuting coordinates are not needed. To me this notion is one of the conceptual monsters created by the over-pragmatic thinking of theoreticians.

This allows allows to replace field space with super field space, which is completely well-defined object mathematically, and leave space-time untouched. Linear field space is simply replaced with its Grassmann algebra. For non-linear field space this replacement does not work. This allows to formulate the notion of linear super-field just in the same manner as it is done usually.

The generators of super-symmetries in super-space formulation reduce to super translations , which anti-commute to translations. The super generators Q_α and $\overline{Q}_{\dot{\beta}}$ of super Poincare algebra are Weyl spinors commuting with momenta and anti-commuting to momenta:

$$\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\ \dot{beta}} P_\mu \quad . \tag{4.4.10}$$

One particular representation of super generators acting on super fields is given by

$$\begin{aligned} D_\alpha &= i\frac{\partial}{\partial_{\theta_\alpha}} \ , \\ D_{\dot{\alpha}} &= i\frac{\partial}{\partial_{\overline{\theta}}{}_{\dot{alpha}}} + \theta^\beta \sigma^\mu_{\beta\dot{\alpha}} \partial_\mu \end{aligned} \tag{4.4.11}$$

Here the index raising for 2-spinors is carried out using antisymmetric 2-tensor $\epsilon^{\alpha\beta}$. Super-space interpretation is not necessary since one can interpret this action as an action on Grassmann algebra valued field mixing components with different fermion numbers.

Chiral superfields are defined as fields annihilated by $D_{\dot{\alpha}}$. Chiral fields are of form $\Psi(x^\mu + i\overline{\theta}\sigma^\mu\theta, \theta)$. The dependence on $\overline{\theta}_{\dot{\alpha}}$ comes only from its presence in the translated Minkowski coordinate annihilated by $D_{\dot{\alpha}}$. Super-space enthusiast would say that by a translation of M^4 coordinates chiral fields reduce to fields, which depend on θ only.

The space of fermionic Fock states at partonic 2-surface as TGD counterpart of chiral super field

As already noticed, another manner to realize SUSY in terms of representations the super algebra of conserved super-charges. In TGD framework these super charges are naturally associated with the modified Dirac equation, and anti-commuting coordinates and super-fields do not appear anywhere. One can however ask whether one could identify a mathematical structure replacing the notion of chiral super field.

In [K23] it was proposed that generalized chiral super-fields could effectively replace induced spinor fields and that second quantized fermionic oscillator operators define the analog of SUSY algebra. One would have $\mathcal{N} = \infty$ if all the conformal excitations of the induced spinor field restricted on 2-surface are present. For right-handed neutrino the modes are labeled by two integers and de-localized to the interior of Euclidian or Minkowskian regions of space-time sheet.

The obvious guess is that chiral super-field generalizes to the field having as its components many-fermions states at partonic 2-surfaces with theta parameters and their conjugates in one-one correspondence with fermionic creation operators and their hermitian conjugates.

(a) Fermionic creation operators - in classical theory corresponding anti-commuting Grassmann parameters - replace theta parameters. Theta parameters and their conjugates are not in one-one correspondence with spinor components but with the fermionic creation operators and their hermitian conjugates. One can say that the super-field in question is defined in the "world of classical worlds" (WCW) rather than in space-time. Fermionic Fock state at the partonic 2-surface is the value of the chiral super field at particular point of WCW.

(b) The matrix defined by the $\sigma^\mu\partial_\mu$ is replaced with a matrix defined by the modified Dirac operator D between spinor modes acting in the solution space of the modified Dirac equation. Since modified Dirac operator annihilates the modes of the induced spinor field, super covariant derivatives reduce to ordinary derivatives with respect the theta parameters labeling the modes. Hence the chiral super field is a field that depends on θ_m or conjugates $\overline{\theta}_m$ only. In second quantization the modes of the chiral super-field are many-fermion states assigned to partonic 2-surfaces and string world sheets. Note that this is the only possibility since the notion of super-coordinate does not make sense now.

(c) It would seem that the notion of super-field does not bring anything new. This is not the case. First of all, the spinor fields are restricted to 2-surfaces. Second point is that one cannot assign to the fermions of the many-fermion states separate non-parallel or even parallel four-momenta. The many-fermion state behaves like elementary particle. This has non-trivial implications for propagators and a simple argument [K23] leads to the proposal that propagator for N-fermion partonic state is proportional to $1/p^N$. This would mean that only the states with fermion number equal to 1 or 2 behave like ordinary elementary particles.

How the fermionic anti-commutation relations are determined?

Understanding the fermionic anti-commutation relations is not trivial since all fermion fields except right-handed neutrino are assumed to be localized at 2-surfaces. Since fermionic conserved currents must give rise to well-defined charges as 3-D integrals the spinor modes must be proportional to a square root of delta function in normal directions. Furthermore, the modified Dirac operator must act only in the directions tangential to the 2-surface in order that the modified Dirac equation can be satisfied.

The square root of delta function can be formally defined by starting from the expansion of delta function in discrete basis for a particle in 1-D box. The product of two functions in x-space is convolution of Fourier transforms and the coefficients of Fourier transform of delta function are apart from a constant multiplier equal to 1: $\delta(x) = K\sum_n exp(inx/2\pi L)$.

Therefore the Fourier transform of square root of delta function is obtained by normalizing the Fourier transform of delta function by $1/\sqrt{N}$, where $N \to \infty$ is the number of plane waves. In other words: $\sqrt{\delta(x)} = \sqrt{\frac{K}{N}} \sum_n \sum exp(inx/2\pi L)$.

Canonical quantization defines the standard approach to the second quantization of the Dirac equation.

(a) One restricts the consideration to time=constant slices of space-time surface. Now the 3-surfaces at the ends of CD are natural slices. The intersection of string world sheet with these surfaces is 1-D whereas partonic 2-surfaces have 2-D Euclidian intersection with them.

(b) The canonical momentum density is defined by

$$\begin{aligned} \Pi_\alpha &= \frac{\partial L}{\partial_t \overline{\Psi}_\alpha(x)} = \Gamma^t \Psi \ , \\ \Gamma^t &= \frac{\partial L_K}{\partial(\partial_t h^k)} \ . \end{aligned} \tag{4.4.12}$$

L_K denotes Kähler action density: consistency requires $D_\mu \Gamma^\mu = 0$, and this is guaranteed only by using the modified gamma matrices defined by Kähler action. Note that Γ^t contains also the $\sqrt{g_4}$ factor. Induced gamma matrices would require action defined by four-volume. t is time coordinate varying in direction tangential to 2-surface.

(c) The standard equal time canonical anti-commutation relations state

$$\{\Pi_\alpha, \overline{\Psi}_\beta\} = \delta^3(x,y)\delta_{\alpha\beta} \ . \tag{4.4.13}$$

Can these conditions be applied both at string world sheets and partonic 2-surfaces.

(a) Sttring world sheets do not pose problems. The restriction of the modes to string world sheets means that the square root of delta function in the normal direction of string world sheet takes care of the normal dimensions and the dynamical part of anti-commutation relations is 1-dimensional just as in the case of strings.

(b) Partonic 2-surfaces are problematic. The $\sqrt{g_4}$ factor in Γ^t implies that Γ^t approaches zero at partonic 2-surfaces since they belong to light-like wormhole throats at which the signature of the induced metric changes. Energy momentum tensor appearing in Γ^t involves to index raisins by induced metric so that it can grow without limit as one approaches partonic two-surface. Therefore it is quite possible that the limit is finite and the boundary conditions defined by the weak form of electric magnetic duality might imply that the limit is finite. The open question is whether one can apply canonical quantization at partonic 2-surfaces. One can also ask whether one can define induced spinor fields at wormhole throats only at the ends of string world sheets so that partonic 2-surface would be effectively discretized. This cautious conclusion emerged in the earlier study of the modified Dirac equation [K22].

(c) Suppose that one can assume spinor modes at partonic 2-surfaces. 2-D conformal invariance suggests that the situation reduces to effectively one-dimensional also at the partonic two-surfaces. If so, one should pose the anti-commutation relations at some 1-D curves of the partonic 2-surface only. This is the only sensical option. The point is that the action of the modified Dirac operator is tangential so that also the canonical momentum current must be tangential and one can fix anti-commutations only at some set of curves of the partonic 2-surface.

One can of course worry what happens at the limit of vacuum extremals. The problem is that Γ^t vanishes for space-time surfaces reducing to vacuum extremals at the 2-surfaces carrying fermions so that the anti-commutations are inconsistent. Should one require - as done earlier- that the anti-commutation relations make sense at this limit and cannot therefore have the standard form but involve the scalar magnetic flux formed from the induced Kähler form by permuting it with the 2-D permutations symbol? The restriction to preferred extremals, which are always non-vacuum extremals, might allow to avoid this kind of problems automatically.

In the case of right-handed neutrino the situation is genuinely 3-dimensional and in this case non-vacuum extremal property must hold true in the regions where the modes of ν_R are non-vanishing. The same mechanism would save from problems also at the partonic 2-surfaces. The dynamics of induced spinor fields must avoid classical vacuum. Could this relate to color confinement? Could hadrons be surrounded by an insulating layer of Kähler vacuum?

4.4.10 Comparison of TGD and stringy views about super-conformal symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

(a) In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K12] . This kind of representation applies also in Kac-Moody sector since the local transversal isometries localized in X^3_l and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

(b) A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K14] .

(c) The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an 1/2 disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

(d) If Kähler action defines the modified Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

(e) What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D\ mod\ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator $S_0 = S + S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n > 0$ creates fermions whereas G_n, $n < 0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{aligned} \{G_m, G_n^\dagger\} &= 2L_{m+n} + \tfrac{c}{3}(m^2 - \tfrac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} &= 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} &= 0 \ . \end{aligned} \tag{4.4.14}$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n < 0$ and $G_n^\dagger$, $n > 0$.

What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

(a) Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K12] . Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

(b) The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em charge must be localized at string world sheets makes the connection with strings even more explicit [K87].

One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

(c) The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X_l^3 (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M_\pm^4 \times CP_2$. Clearly, wormhole throats X_l^3 would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

(d) An alternative identification for TGD parts of conformal fields is inspired by $M^8 - H$ duality. Conformal fields would be fields in WCW. The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

4.5 Appendix: Hamilton-Jacobi structure

In the following the definition of Hamilton-Jacobi structure is discussed in detail.

4.5.1 Hermitian and hyper-Hermitian structures

The starting point is the observation that besides the complex numbers forming a number field there are hyper-complex numbers. Imaginary unit i is replaced with e satisfying $e^2 = 1$. One obtains an algebra but not a number field since the norm is Minkowskian norm $x^2 - y^2$, which vanishes at light-cone $x = y$ so that light-like hypercomplex numbers $x \pm e$) do not have inverse. One has "almost" number field.

Hyper-complex numbers appear naturally in 2-D Minkowski space since the solutions of a massless field equation can be written as $f = g(u = t-ex)+h(v = t+ex)$ whith $e^2 = 1$ realized by putting $e = 1$. Therefore Wick rotation relates sums of holomorphic and antiholomorphic functions to sums of hyper-holomorphic and anti-hyper-holomorphic functions. Note that u and v are hyper-complex conjugates of each other.

Complex n-dimensional spaces allow Hermitian structure. This means that the metric has in complex coordinates $(z_1, ..., z_n)$ the form in which the matrix elements of metric are non-vanishing only between z_i and complex conjugate of z_j. In 2-D case one obtains just $ds^2 = g_{z\overline{z}}dzd\overline{z}$. Note that in this case metric is conformally flat since line element is proportional to the line element $ds^2 = dzd\overline{z}$ of plane. This form is always possible locally. For complex n-D case one obtains $ds^2 = g_{i\overline{j}}dz^id\overline{z}^j$. $g_{i\overline{j}} = \overline{g_{j\overline{i}}}$ guaranteeing the reality of ds^2. In 2-D case this condition gives $g_{z\overline{z}} = \overline{g_{z\overline{z}}}$.

How could one generalize this line element to hyper-complex n-dimensional case. In 2-D case Minkowski space M^2 one has $ds^2 = g_{uv}dudv$, $g_{uv} = 1$. The obvious generalization would be the replacement $ds^2 = g_{u_iv_j}du^idv^j$. Also now the analogs of reality conditions must hold with respect to $u_i \leftrightarrow v_i$.

4.5.2 Hamilton-Jacobi structure

Consider next the path leading to Hamilton-Jacobi structure.

4-D Minkowski space $M^4 = M^2 \times E^2$ is Cartesian product of hyper-complex M^2 with complex plane E^2, and one has $ds^2 = dudv + dzd\overline{z}$ in standard Minkowski coordinates. One can also consider more general integrable decompositions of M^4 for which the tangent space $TM^4 = M^4$ at each point is decomposed to $M^2(x) \times E^2(x)$. The physical analogy would be a position dependent decomposition of the degrees of freedom of massless particle to longitudinal ones ($M^2(x)$: light-like momentum is in this plane) and transversal ones ($E^2(x)$: polarization vector is in this plane). Cylindrical and spherical variants of Minkowski coordinates define two examples of this kind of coordinates (it is perhaps a good exercise to think what kind of decomposition of tangent space is in question in these examples). An interesting mathematical problem highly relevant for TGD is to identify all possible decompositions of this kind for empty Minkowski space.

The integrability of the decomposition means that the planes $M^2(x)$ are tangent planes for 2-D surfaces of M^4 analogous to Euclidian string world sheet. This gives slicing of M^4 to Minkowskian string world sheets parametrized by euclidian string world sheets. The question is whether the sheets are stringy in a strong sense: that is minimal surfaces. This is not the case: for spherical coordinates the Euclidian string world sheets would be spheres which are not minimal surfaces. For cylindrical and spherical coordinates howeverr $M^2(x)$ integrate to plane M^2, which is minimal surface.

Integrability means in the case of $M^2(x)$ the existence of light-like vector field J whose flow lines define a global coordinate. Its existence implies also the existence of its conjugate and together these vector fields give rise to $M^2(x)$ at each point. This means that one has $J = \Psi\nabla\Phi$: Φ indeed defines the global coordinate along flow lines. In the case of M^2 either the coordinate u or v would be the coordinate in question. This kind of flows are called Beltrami flows. Obviously the same holds for the transversal planes E^2.

One can generalize this metric to the case of general 4-D space with Minkowski signature of metric. At least the elements g_{uv} and $g_{z\overline{z}}$ are non-vanishing and can depend on both u, v

and $z, \overline{z}$. They must satisfy the reality conditions $g_{z\overline{z}} = \overline{g_{z\overline{z}}}$ and $g_{uv} = \overline{g_{vu}}$ where complex conjugation in the argument involves also $u \leftrightarrow v$ besides $z \leftrightarrow \overline{z}$.

The question is whether the components g_{uz}, g_{vz}, and their complex conjugates are non-vanishing if they satisfy some conditions. They can. The direct generalization from complex 2-D space would be that one treats u and v as complex conjugates and therefore requires a direct generalization of the hermiticity condition

$$g_{uz} = \overline{g_{v\overline{z}}} \ \ , \ \ g_{vz} = \overline{g_{u\overline{z}}} \ \ .$$

This would give complete symmetry with the complex 2-D (4-D in real sense) spaces. This would allow the algebraic continuation of hermitian structures to Hamilton-Jacobi structures by just replacing i with e for some complex coordinates.

Chapter 5

Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

5.1 Introduction

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry [A63]. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

The basic idea is that WCW is union of symmetric spaces G/H labelled by zero modes which do not contribute to the WCW metric. There have been many open questions and it seems the details of the ealier approach [**?**]ust be modified at the level of detailed identifications and interpretations.

(a) A longstanding question has been whether one could assign Equivalence Principle (EP) with the coset representation formed by the super-Virasoro representation assigned to G and H in such a manner that the four-momenta associated with the representations and identified as inertial and gravitational four-momenta would be identical. This does not seem to be the case. The recent view will be that EP reduces to the view that the classical four-momentum associated with Kähler action is equivalent with that assignable to modified Dirac action supersymmetrically related to Kähler action: quantum classical correspondence (QCC) would be in question. Also strong form of general coordinate invariance implying strong form of holography in turn implying that the super-symplectic representations assignable to space-like and light-like 3-surfaces are equivalent could imply EP with gravitational and inertial four-momenta assigned to these two representations.

   At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least.

(b) The detailed identification of groups G and H and corresponding algebras has been a longstanding problem. Symplectic algebra associated with$\delta M^4_\pm \times CP2$ ($\delta M^4_\pm$ is light-cone boundary - or more precisely, with the boundary of causal diamond (CD) defined as Cartesian product of CP_2 with intersection of future and past direct light cones of M^4 has Kac-Moody type structure with light-like radial coordinate replacing complex coordinate z. Virasoro algebra would correspond to radial diffeomorphisms. I have also introduced Kac-Moody algebra assigned to the isometries and localized with respect to internal coordinates of the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and which serve as natural correlates for elementary particles (in very general sense!). This kind of localization by force could be however argued to be rather ad hoc as opposed to the inherent localization of the symplectic algebra containing the symplectic algebra of isometries as sub-algebra. It turns out that one obtains direct sum of representations of symplectic algebra and Kac-Moody algebra of isometries naturally as required by the success of p-adic mass calculations.

(c) The dynamics of Kähler action is not visible in the earlier construction. The construction also expressed WCW Hamiltonians as 2-D integrals over partonic 2-surfaces. Although strong form of general coordinate invariance (GCI) implies strong form of holography meaning that partonic 2-surfaces and their 4-D tangent space data should code for quantum physics, this kind of outcome seems too strong. The progress in the understanding of the solutions of modified Dirac equation led however to the conclusion that spinor modes other than right-handed neutrino are localized at string world sheets with strings connecting different partonic 2-surfaces. This leads to a modification of earlier construction in which WCW super-Hamiltonians are essentially integrals with integrand identified as a Noether super current for the deformations in G Each spinor mode gives rise to super current and the modes of right-handed neutrino and other fermions differ in an essential manner. Right-handed neutrino would correspond to symplectic algebra and other modes to the Kac-Moody algebra and one obtains the crucial 5 tensor factors of Super Virasoro required by p-adic mass calculations.

The matrix elements of WCW metric between Killing vectors are expressible as anti-commutators of super-Hamiltonians identifiable as contractions of WCW gamma matrices with these vectors and give Poisson brackets of corresponding Hamiltonians. The anti-commutation relates of induced spinor fields are dictated by this condition. Everything is 3-dimensional although one expects that symplectic transformations localized within interior of X^3 act as gauge symmetries so that in this sense effective 2-dimensionality is achieved. The components of WCW metric are labelled by standard model quantum numbers so that the connection with physics is extremely intimate.

(d) An open question in the earlier visions was whether finite measurement resolution is realized as discretization at the level of fundamental dynamics. This would mean that only certain string world sheets from the slicing by string world sheets and partonic 2-surfaces are possible. The requirement that anti-commutations are consistent suggests that string world sheets correspond to surfaces for which Kähler magnetic field is constant along string in well defined sense ($J_{\mu\nu}\epsilon^{\mu\nu}g^{1/2}$ remains constant along string). It however turns that by a suitable choice of coordinates of 3-surface one can guarantee that this quantity is constant so that no additional constraint results.

(e) Quantum criticality is one of the basic notions of quantum TGD and its relationship to coset construction has remained unclear. In this chapter the concrete realization of criticality in terms of symmetry breaking hierarchy of Super Virasoro algebra acting on symplectic and Kac-Moody algebras. Also a connection with finite measurement resolution - second key notion of TGD - emerges naturally.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- Hierarchy of Planck constants [L22]

- Hyperfinite factors and TGD [L24]
- Structure of WCW [L38]
- TGD as infinite-dimensional geometry [L44]
- WCW gamma matrices [L49]
- WCW spinor fields [L50]
- Weak form of electric-magnetic duality [L51]
- Zero Energy Ontology (ZEO) [L52]
- 4-D spin glass degeneracy [L14]
- Equivalence Principle [L18]
- Holography [L23]
- Quantum Classical Correspondence [L32]
- Quantum criticality [L33]
- Symmetries of WCW [L40]
- TGD as ATQFT [L43]
- Vacuum functional in TGD [L48]
- KD equation [L27]
- Kaehler-Dirac action [L26]

5.2 WCW as a union of homogenous or symmetric spaces

In the following the vision about WCW as union of coset spaces is discussed in more detail.

5.2.1 Basic vision

The basic view about coset space construction for WCW has not changed.

(a) The idea about WCW as a union of coset spaces G/H labelled by zero modes is extremely attractive. The structure of homogenous space [A9] (`http://en.wikipedia.org/wiki/Homogenous_space`) means at Lie algebra level the decomposition $g = h \oplus t$ to sub-Lie-algebra h and its complement t such that $[h,t] \subset t$ holds true. Homogeneous spaces have G as its isometries. For symmetric space the additional condition $[t,t] \subset h$ holds true and implies the existence of involution changing at the Lie algebra level the sign of elements of t and leaving the elements of h invariant. The assumption about the structure of symmetric space [A30] (`http://en.wikipedia.org/wiki/Symmetric_space`) implying covariantly constant curvature tensor is attractive in infinite-dimensional case since it gives hopes about calculability.

An important source of intuition is the analogy with the construction of CP_2, which is symmetric space A particular choice of h corresponds to Lie-algebra elements realized as Killing vector fields which vanish at particular point of WCW and thus leave 3-surface invariant. A preferred choice for this point is as maximum or minimum of Kähler function. For this 3-surface the Hamiltonians of h should be stationary. If symmetric space property holds true then commutators of $[t,t]$ also vanish at the minimum/maximum. Note that Euclidian signature for the metric of WCW requires that Kähler function can have only maximum or minimum for given zero modes.

(b) The basic objection against TGD is that one cannot use the powerful canonical quantization using the phase space associated with configuration space - now WCW. The reason is the extreme non-linearity of the Kähler action and its huge vacuum degeneracy, which do not allow the construction of Hamiltonian formalism. Symplectic and Kähler structure must be realized at the level of WCW. In particular, Hamiltonians must be represented in completely new manner. The key idea is to construct WCW

Hamiltonians as anti-commutators of super-Hamiltonians defining the contractions of WCW gamma matrices with corresponding Killing vector fields and therefore defining the matrix elements of WCW metric in the tangent vector basis defined by Killing vector fields. Super-symmetry therefre gives hopes about constructing quantum theory in which only induced spinor fields are second quantized and imbedding space coordinates are treated purely classically.

(c) It is important to understand the difference between symmetries and isometries assigned to the Kähler function. Symmetries of Kähler function do not affect it. The symmetries of Kähler action are also symmetries of Kähler action because Kähler function is Kähler action for a preferred extremal (here there have been a lot of confusion). Isometries leave invariant only the quadratic form defined by Kähler metric $g_{M\overline{N}} = \partial_M \partial_{\overline{L}} K$ but not Kähler function in general. For G/H decomposition G represents isometries and H both isometries and symmetries of Kähler function.

CP_2 is familiar example: $SU(3)$ represents isometries and $U(2)$ leaves also Kähler function invariant since it depends on the $U(2)$ invariant radial coordinate r of CP_2. The origin $r = 0$ is left invariant by $U(2)$ but for $r > 0$ $U(2)$ performs a rotation at $r = constant$ 3-sphere. This simple picture helps to understand what happens at the level of WCW.

How to then distinguish between symmetries and isometries? A natural guess is that one obtains also for the isometries Noether charges but the vanishing of boundary terms at spatial infinity crucial in the argument leading to Noether theorem as $\Delta S = \Delta Q = 0$ does not hold true anymore and one obtains charges which are not conserved anymore. The symmetry breaking contributions would now come from effective boundaries defined by wormhole throats at which the induce metric changes its signature from Minkowskian to Euclidian. A more delicate situation is in which first order contribution to ΔS vanishes and therefore also ΔQ and the contribution to ΔS comes from second variation allowing also to define Noether charge which is not conserved.

(d) The simple picture about CP_2 as symmetric space helps to understand the general vision if one assumes that WCW has the structure of symmetric space. The decomposition $g = h+t$ corresponds to decomposition of symplectic deformations to those which vanish at 3-surface (h) and those which do not (t).

For the symmetric space option, the Poisson brackets for super generators associated with t give Hamiltonians of h identifiable as the matrix elements of WCW metric. They would not vanish although they are stationary at 3-surface meaning that Riemann connection vanishes at 3-surface. The Hamiltonians which vanish at 3-surface X^3 would correspond to t and the Hamiltonians for which Killing vectors vanish and which therefore are stationary at X^3 would correspond to h. Outside X^3 the situation would of course be different. The metric would be obtained by parallel translating the metric from the preferred point of WCW to elsewhere and symplectic transformations would make this parallel translation.

For the homogenous space option the Poisson brackets for super generators of t would still give Hamiltonians identifiable as matrix elements of WCW metric but now they would be necessary those of h. In particular, the Hamiltonians of t do not in general vanish at X^3.

5.2.2 Equivalence Principle and WCW

5.2.3 EP at quantum and classical level

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

(a) The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

(b) Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K72].

A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the modified Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

(c) A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K96].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

5.2.4 Criticism of the earlier construction

The earlier detailed realization of super-Hamiltonians and Hamiltonians can be criticized.

(a) Even after these more than twenty years it looks strange that the Hamiltonians should reduce to flux integrals over partonic 2-surfaces. The interpretation has been in terms of effective 2-dimensionality suggested strongly by strong form of general coordinate invariance stating that the descriptions based on light-like orbits of partonic 2-surfaces and space-like three surfaces at the ends of causal diamonds are dual so that only partonic 2-surfaces and 4-D tangent space data at them would matter. Strong form of holography implies effective 2-dimensionality but this should correspond gauge character for the action of symplectic generators in the interior the space-like 3-surfaces at the ends of CDs, which is something much milder.

    One expects that the strings connecting partonic 2-surfaces could bring something new to the earlier simplistic picture. The guess is that imbedding space Hamiltonian assignable to a point of partonic 2-surface should be replaced with that defined as integral over string attached to the point. Therefore the earlier picture would suffer no modification at the level of general formulas.

(b) The fact that the dynamics of Kähler action and modified Dirac action are not directly involved with the earlier construction raises suspicions. I have proposed that Kähler function could allow identification as Dirac determinant [K22] but one would expect more intimate connection. Here the natural question is whether super-Hamiltonians for the modified Dirac action could correspond to Kähler charges constructible using Noether's theorem for corresponding deformations of the space-time surface and would also be identifiable as WCW gamma matrices.

5.2.5 Is WCW homogenous or symmetric space?

A key question is whether WCW can be symmetric space [A30] (`http://en.wikipedia.org/wiki/Riemannian_symmetric_space`) or whether only homogenous structure is needed. The lack of covariant constancy of curvature tensor might produce problems in infinite-dimensional context.

The algebraic conditions for symmetric space are $g = h + t$, $[h,t] \subset t$, $[t,t] \subset h$. The latter condition is the difficult one.

(a) δCD Hamiltonians should induce diffeomorphisms of X^3 indeed leaving it invariant. The symplectic vector fields would be parallel to X^3. A stronger condition is that they induce symplectic transformations for which all points of X^3 remain invariant. Now symplectic vector fields vanish at preferred 3-surface (note that the symplectic transformations are r_M local symplectic transformations of $S^2 \times CP_2$).

(b) For Kac-Moody algebra inclusion $H \subset G$ for the finite-dimensional Lie-algebra induces the structure of symmetric space. If entire algebra is involved this does not look physically very attractive idea unless one believes on symmetry breaking for both $SU(3)$, $U(2)_{ew}$, and $SO(3)$ and E_2 (here complex conjugation corresponds to the involution). If one assumes only Kac-Moody algebra as critical symmetries, the number of tensor factors is 4 instead of five, and it is not clear whether one can obtain consistency with p-adic mass calculations.

Examples of 3-surfaces remaining invariant under $U(2)$ are 3-spheres of CP_2. They could correspond to intersections of deformations of CP_2 type vacuum extremals with the boundary of CD. Also geodesic spheres S^2 of CP_2 are invariant under $U(2)$ subgroup and would relate naturally to cosmic strings. The corresponding 3-surface would be $L \times S^2$, where L is a piece of light-like radial geodesic.

(c) In the case of symplectic algebra one can construct the imbedding space Hamiltonians inducing WCW Hamiltonians as products of elements of the isometry algebra of $S^2 \times CP_2$ for with parity under involution is well-defined. This would give a decomposition of the symplectic algebra satisfying the symmetric space property at the level imbedding space. This decomposition does not however look natural at the level of WCW since the only single point of CP_2 and light-like geodesic of δM^4_+ can be fixed by $SO(2) \times U(2)$ so that the 3-surfaces would reduce to pieces of light rays.

(d) A more promising involution is the inversion $r_M \rightarrow 1/r_M$ of the radial coordinate mapping the radial conformal weights to their negatives. This corresponds to the inversion in Super Virasoro algebra. t would correspond to functions which are odd functions of $u \equiv log(r_M/r_0)$ and h to even function of u. Stationary 3-surfaces would correspond to $u = 1$ surfaces for which $log(u) = 0$ holds true. This would assign criticality with Virasoro algebra as one expects on general grounds.

$r_M = constant$ surface would most naturally correspond to a maximum of Kähler function which could indeed be highly symmetric. The elements with even u-parity should define Hamiltonians, which are stationary at the maximum of Kähler function. For other 3-surfaces obtained by $/r_M$-local) symplectic transformations the situation is different: now H is replaced with its symplectic conjugate hHg^{-1} of H is acceptable and if the conjecture is true one would obtained 3-surfaces assignable to perturbation theory around given maximum as symplectic conjugates of the maximum. The condition that H leaves X^3 invariant in poin-twise manner is certainly too strong and imply that the 3-surface has single point as CP_2 projection.

(e) One can also consider the possibility that critical deformations correspond to h and non-critical ones to t for the preferred 3-surface. Criticality for given h would hold only for a preferred 3-surface so that this picture would be very similar that above. Symplectic conjugates of h would define criticality for other 3-surfaces. WCW would decompose to a union corresponding to different criticalities perhaps assignable to the hierarchy of sub-algebras of conformal algebra labelled by integer whose multiples give the allowed conformal weights. Hierarchy of breakings of conformal symmetries would characterize this hierarchy of sectors of WCW.

For sub-algebras of the conformal algebras (Kac-Moody and symplectic algebra) the condition $[t, t] \subset h$ cannot hold true so that one would obtain only the structure of homogenous space.

5.2.6 Symplectic and Kac-Moody algebras as basic building bricks

The basic building bricks are symplectic algebra of δCD (this includes CP_2 besides light-cone boundary) and Kac-Moody algebra assignable to the isometries of δCD [K12]. It seems however that the longheld view about the role of Kac-Moody algebra must be modified. Also the earlier realization of super-Hamiltonians and Hamiltonians seems too naive.

(a) I have been accustomed to think that Kac-Moody algebra could be regarded as a sub-algebra of symplectic algebra. p-Adic mass calculations however requires five tensor factors for the coset representation of Super Virasoro algebra naturally assigned to the coset structure G/H of a sector of WCW with fixed zero modes. Therefore Kac-Moody algebra cannot be regarded as a sub-algebra of symplectic algebra giving only single tensor factor and thus inconsistent with interpretation of p-adic mass calculations.

(b) The localization of Kac-Moody algebra generators with respect to the internal coordinates of light-like 3-surface taking the role of complex coordinate z in conformal field theory is also questionable: the most economical option relies on localization with respect to light-like radial coordinate of light-cone boundary as in the case of symplectic algebra. Kac-Moody algebra cannot be however sub-algebra of the symplectic algebra assigned with covariantly constant right-handed neutrino in the earlier approach.

(c) Right-handed covariantly constant neutrino as a generator of super symmetries plays a key role in the earlier construction of symplectic super-Hamiltonians. What raises doubts is that other spinor modes - both those of right-handed neutrino and electroweakly charged spinor modes - are absent. All spinor modes should be present and thus provide direct mapping from WCW geometry to WCW spinor fields in accordance with super-symmetry and the general idea that WCW geometry is coded by WCW spinor fields.

    Hence it seems that Kac-Moody algebra must be assigned with the modes of the induced spinor field which carry electroweak quantum numbers. If would be natural that the modes of right-handed neutrino having no weak and color interactions would generate the huge symplectic algebra of symmetries and that the modes of fermions with electroweak charges generate much smaller Kac-Moody algebra.

(d) The dynamics of Kähler action and modified Dirac action action are invisible in the earlier construction. This suggests that the definition of WCW Hamiltonians is too simplistic. The proposal is that the conserved super charges derivable as Noether charges and identifiable as super-Hamiltonians define WCW metric and Hamiltonians as their anti-commutators. Spinor modes would become labels of Hamiltonians and WCW geometry relates directly to the dynamics of elementary particles.

(e) Note that light-cone boundary $\delta M^4_+ = S^2 \times R_+$ allows infinite-dimensional group of isometries consisting of conformal transformation of the sphere S^2 with conformal scaling compensated by an S^2 local scaling or the light-like radial coordinate of R_+. These isometries contain as a subgroup symplectic isometries and could act as gauge symmetries of the theory.

5.3 Preferred extremals of Kähler action, solutions of the modified Dirac operator, and quantum criticality

Perhaps due to my natural laziness I have not bothered to go through the basic construction [K12, K10] although several new ideas have emerged during last years [K87].

(a) The new view about preferred extremals of Kähler action involves the slicing of space-time surface to string world sheets labelled by points of any partonic two-surface or vice versa. I have called this structure Hamilton-Jacobi structure [K4]. A number theoretic interpretation based on the octonionic representation of imbedding space gamma matrices. A gauge theoretic interpretation in terms of two orthogonal 2-D spaces assignable to polarization and momentum of massless field mode is also possible. The slicing suggests duality between string world sheets and conformal field theory at partonic 2-surfaces analogous to AdS/CFT. Strong form of holography implied by strong form of GCI would be behind the duality.

(b) The new view about the solutions of modified Dirac equation involves localization of the modes at string world sheets: this emerges from the condition that electric charge is well defined quantum number for the modes. The effective 2-dimensionality of the space of the modified gamma matrices is crucial for the localization. This leads to a concrete model of elementary particles as string like objects involving two space-time sheets and flux tubes carrying Kähler magnetic monopole flux. Holomorphy and complexification of modified gamma matrices are absolutely essential consequences of the localization and is expected to be crucial also in the construction of WCW geometry. The weakest interpretation is that the general solution of modified Dirac is superposition of these localized modes parametrized by the points of partonic 2-surface and integer labelling the modes themselves as in string theory. One has the same general picture as in ordinary quantum theory.

One can wonder whether finite measurement resolution is realized dynamically in the sense that a discrete set of stringy world sheets are possible. It will be found that quantization of induced spinor fields leads to a concrete proposal realizing this: strings would be identified as curves along which Kähler magnetic field has constant value.

(c) Quantum criticality is central notion in TGD framework: Kähler coupling strength is the only coupling parameter appearing in Kähler action and is analogous to temperature. The idea of quantum criticality is that TGD Universe is quantum critical so that Kähler coupling strength is analogous to critical temperature. The hope is that this could make the theory unique. I have not however been able to really understand it and relate it to the coset space construction of WCW and to coset representations of Super Virasoro.

5.3.1 What criticality is?

The basic technical problem has been characterization of it quantitatively [K22]. Here there is still a lot of fuzzy thinking and unanswered questions. What is the precise definition of criticality and what is its relation to G/H decomposition of WCW? Could H correspond to critical deformations so that it would have purely group theoretical characterization, and one would have nice unification of two approaches to quantum TGD?

1. Does criticality correspond to the failure of classical determinism?

The intuitive guess is that quantum criticality corresponds classically to the criticality of Kähler action implying non-determinism. The preferred extremal associated with given 3-surface at the boundary of CD is not unique. There are several deformations of space-time surface *vanishing* at X^3 and leaving the Kähler action and thus Kähler function invariant.

Some nitpicking before continuing is in order.

(a) The key word is *"vanishing"* in the above definition of criticality relying on classical non-determinism. Could one allow also non-vanishing deformations of X^3 with the property that Kähler function and Kähler action are not changed? This would correspond to the idea that critical directions correspond to flat directions for the potential in quadratic approximation: now it would be Kähler function in quadratic approximation. The flat direction would not contribute to Kähler metric $G_{K\overline{L}} = \partial_K \partial_{\overline{L}}$.

Clearly, the subalgebra h associated with H would satisfy criticality in this sense for all 3-surfaces except the one for which it acts as isotropy group: in this case one would have criticality in the strong sense.

This identification of criticality is consistent with that based on non-determinism *only if* the deformations in H leaving X^3 fixed do not leave $X^4(X^3)$ fixed. This would apply also to h. One would have bundle like structure: 3-surface would represent base point of the bundle and space-time surfaces associated with it would correspond to the points in the fiber permuted by h.

(b) What about zero modes, which appear only in the conformal scaling factor of WCW metric but not in the differentials appearing the line element? Are the critical modes zero modes but only up to second order in functional Taylor expansion?

Returning to the definition of criticality relying on classical non-determinism. One can try to fix $X^4(X^3)$ uniquely by fixing 3-surface at the second end of CD but even this need not be enough? One expects non-uniqueness in smaller scales in accordance with approximate scaling invariance and fractality assignable to criticality.

A possible interpretation would be in terms of dynamical symmetry analogous to gauge symmetry assignable to H and having interpretation in terms of measurement resolution. Increasing the resolution would mean fixing X^3 at upper and lower boundaries in shorter scale. Finite measurement resolution would give rise to dynamical gauge symmetry. This conforms with the idea that TGD Universe is analogous to a Turing machine able to mimic any gauge dynamics. The hierarchy of inclusions for hyper-finite factors of type II_1 supports this view too [K79].

Criticality would be a space-time correlate for quantum non-determinism. I have assigned this nondeterminism to multi-furcations of space-time sheets giving rise to the hierarchy of Planck constants. This involves however something new: namely the idea that several alternative paths are selected in the multi-furcation simultaneously [K21, K81].

2. Further aspects of criticality

(a) Mathematically the situation at criticality of Kähler action for $X^4(X^3)$ (as distinguished from Kähler function for X^3) is analogous to that at the extremum of potential when the Hessian defined by second derivatives has vanishing determinant and there are zero modes. Now one would have an infinite number of deformations leaving Kähler action invariant in second order. What is important that critical deformations leave X^3 invariant so that they cannot correspond to the sub-algebra h except possibly at point for which H acts as an isotropy group.

(b) Criticality would suggest that conserved charges linear in deformation vanish: this because deformation vanishes at X^3. Second variation would give rise to charges to and invariance of the Kähler action in this action would mean that $\Delta S_2 = \Delta Q_2 = 0$ holds true unless effective boundary terms spoil the situation. Second order charges would be quadratic in the variation and it is not at all clear whether there is any hope about having a non-linear analog of Lie-algebra or super algebra structure. I do not know whether mathematicians have considered this kind of possibility. Yangian algebra represent involving besides Lie algebra generators also generators coming as their multilinears have some formal resemblance with this kind of non-linear structure.

(c) Supersymmetry would suggest that criticality for the Kähler action implies criticality for the modified Dirac action. The first order charges for Dirac action involve the partial derivatives of the canonical momentum currents T^α_k with respect to partial derivatives $\partial_\beta h^l$ of imbedding space coordinates just as the second order charges for Kähler action do. First order Noether charges vanish if criticality means that variation vanishes at X^3 but not at $X^4(X^3)$ since they involve linearly δh^k vanishing at X^3. Second order charges for modified Dirac action get second contribution from the modification of the induced spinor field by a term involving spin rotation and from the second variation of the modified gamma matrices. Here it is essential that derivatives of $\partial_k \delta h^l$, which need not vanish, are involved.

    Note: I use the notation ∂_α for space-time partial derivatives and ∂_k for imbedding space partial derivatives).

5.3.2 Do critical deformations correspond to Super Virasoro algebra?

One can try to formulate criticality a in terms of super-conformal algebras and their subalgebras $h_{c,m}$ for which conformal weights are integer multiples of integer m. Now I mean with super-conformal algebra also symplectic and super Kac-Moody algebras. These decompositions - call them just $g_c = t_c \oplus h_c$ need *not* correspond to $g + h$ associated with G/H although it could do so. For instance, if g_c corresponds to Super Virasoro algebra then the decomposition $g_c = t_c \oplus h_c$ does not correspond to $g = t \oplus h$.

(a) There would be a hierarchy of included sub-algebras $h_{c,m}$, which corresponds to hierarchy of conformal algebras assignable to the light-like radial coordinate of the boundary of light-cone and criticalities could form hierarchy in this sense. The algebras form inclusion hierarchies $h_{m_1} \supset h_{m_2} \supset ...$ labelled by sequences consisting of integers such that given integer is divisible by the previous integer in the sequence: $m_n \ mod \ m_{n-1} = 0$.

Critical deformations assignable to $h_{c,m}$ would vanish at preferred X^3 for which H is isotropy group and leave Kähler action invariant and would not therefore contribute to Kähler metric at X^3. They could however affect $X^4(X^3)$.

Non-critical deformation would correspond to the complement of this sub-algebra affecting both $X^4(X^3)$ and X^3. This hierarchy would correspond to an infinite hierarchy of conformal symmetry breakings and would be manifested at the level of WCW geometry. Also a connection with the inclusion hierarchy for hyper-finite factors of type II_1 [K79] having interpretation in terms of finite measurement resolution is suggested by this hierarchy. Super Virasoro generators with conformal weight coming as a multiple of m would annihilate physical states so that effectively the criticality correspond to finite-D Hilbert space. This is something new as compared to the ordinary view about criticality for which all Super Virasoro generators annihilate the states.

(b) A priori $g = t + h$ decomposition need not have anything to do with the decomposition of deformations to non-critical and critical ones. Critical deformations could indeed appear as sub-algebra of $g = t + h$ and be present for both t and h in the same manner: that is as sub-algebras of super- Virasoro algebras: Super Virasoro would represent the non-determinism and criticality and in 2-D conformal theories describing criticality this is indeed the case. In this case the actions of G and H identified as super-symplectic and super Kac-Moody algebras could be unique and non-deterministic aspect would not be present. This corresponds to the physical intuition.

If criticality corresponds to G/H structure, symmetric space property $[t, t] \subset h$ would not hold true as is clear from the additivity of super-conformal weights in the commutators of conformal algebras. The reduction of G/H structure to criticality would be very nice but personally I would give up covariant constancy of curvature tensor in infinite-dimensional context only with heavy heart.

(c) The super-symmetric relation between Kähler action and corresponding modified Dirac action suggests that the criticality of Kähler action implies vanishing conserved charges also for the modified Dirac action (both ordinary and super charges so that one has super-symmetry). The reason is that conserved charge is linear in deformation. Conservation in turn means that Kähler action is not changed: $\Delta S = \Delta Q = 0$. For non-critical deformations the boundary terms at the orbits wormhole throats imply non-conservation so that ΔQ (the difference of charges at space-like ends of space-time surface) is non-vanishing although local conservation law holds strue. This in terms implies that the contribution to the Kähler metric is non-trivial.

At criticality both bosonic and fermionic conserved currents can be assigned to the second variation and are thus quadratic in deformation just like that associated with Kähler action. If effective boundary terms vanish the criticality for Kähler action implies the conservation of second order charges by $\Delta_2 S = \Delta_2 Q = 0$.

5.3.3 Connection with the vanishing of second variation for Kähler action

There are three general conjectures related to modified Dirac equation and the conserved currents associated with the vanishing second variation of Kähler action at critical points analogous to extrema of potential function at which flat directions appear and the determinant defined by second derivatives of the potential function does not have maximal rank.

(a) Quantum criticality has as a correlate the vanishing of the second variation of Kähler action for critical deformations. The conjecture is that the number of these directions is infinite and corresponds to sub-algebras of Super Virasorol algebra corresponding to conformal weights coming as integer multiples of integer. Super Virasoro hypothesis implies that preferred extremals have same algebra of critical deformations at all points.

Noether theorem applied to critical variations gives rise to conserved currents and charges which are quadratic in deformation. For non-critical deformations one obtains linearity in deformation and this charges define the super- conformal algebras.

Super Virasoro algebra indeed has a standard representation in which generators are indeed quadratic in Kac-Moody (and symplectic generators in the recent case). This quadratic character would have interpretation in terms of criticality not allowing linear representation.

(b) Modified Dirac operator is assumed to have a solution spectrum for which both non-critical and critical deformations act as symmetries. The critical currents vanish in the first order. Second variation involving first variation for the modified gamma matrices and first variation for spinors (spinor rotation term) gives and second variation for canonical momentum currents gives conserved current. The general form of the current is very similar to the corresponding current associated with Kähler action.

(c) The currents associated with the modified Dirac action and Kähler action have same origin. In other words: the conservation of Kähler currents implies the conservation of the currents associated with the modes of the modified Dirac operator. A question inspired by quantum classical correspondence is whether the eigen values of the fermionic charges correspond to the values of corresponding classical conserved charges for Kähler action in the Cartan algebra. This would imply that all space-time surfaces in superposition representing momentum eigen state have the same value of classical four-momentum. A stronger statement of QCC would be that classical correlation functions are same as the quantal ones.

5.4 Quantization of the modified Dirac action

The quantization of the modified Dirac action follows standard rules.

(a) The general solution is written as a superposition of modes, which are for other fermions than ν_R localized to string world sheets and parametrized by a point of partonic 2-surface which can be chosen to be the intersection of light-like 3-surface at which induced metric changes signature with the boundary of CD.

(b) The anti-commutations for the induced spinor fields are dictated from the condition that the anti-commutators of the super-Hamiltonians identified as WCW gamma matrices give WCW Hamiltonians as matrix elements of WCW metric. Super Hamiltonians are identified as Noether charges for the modified Dirac action assignable to the symplectic algebra of δCD being labelled also by the quantum numbers labelling the modes of the induced spinor field.

(c) Consistency conditions for the modified Dirac operator require that the modified gamma matrices have vanishing divergence: this is true for the extremals of Kähler action.

(d) The guess for the critical algebra is as sub-algebra of Super Virasoro algebra affecting on the radial light-like coordinate of δCD as diffeomorphisms. The deformations of the modified Dirac operator should annihilate spinor modes. This requires that the deformation corresponds to a gauge transformation for the induced gauge fields. Furthermore, the deformation for the modified gamma matrices determined by the deformation of the canonical momentum densities contracted covariant derivatives should annihilate the spinor modes. The situation is analogous to that for massless Dirac operator: Dirac equation for momentum eigenstate does not imply vanishing of the momentum but only that of mass. The condition that the divergence for the deformation of the modified gamma matrices vanishes as does also the divergence of the modified gamma matrices implies that the second variation of Kähler action vanishes. One obtains classical Kähler charges and Dirac charges: the latter act as operators. The equivalence of the two definitions of of four-momenta would corresponds to EP and QCC.

(e) An interesting question of principle is what the almost topological QFT property meaning that Kähler action reduces to Chern-Simons form integrated over boundary of space-time and over the light-like 3-surfaces means. Could one write the currents in terms of Chern-Simons form alone? Could one use also Chern-Simons analog of modified Dirac action. What looks like problem at the first glance is that only the charges associated with the symplectic group of CP_2 would be non-vanishing. Here the weak form of electric-magnetic duality [K22, K87] however introduce constraint terms to the action implying that all charges can be non-vanishing.

The challenge is to construct explicit representations of super charges and demonstrate that suitably defined anti-commutations for spinor fields reproduce the anti-commutations of the super-symplectic algebra.

5.4.1 Integration measure in the superposition over modes

One can express Ψ as a superposition over modes as usually. Except for ν_R, the modes are localized at string world sheets and can be labelled by a point of X^2, integer characterizing the mode and analogous to conformal weight, and quantum numbers characterizing spin, electroweak quantum numbers, and M^4 handedness. The de-localization of the modes of ν_R decouple from left-handed neutrino if the modified gamma matrices involved only M^4 or CP_2 gamma matrices. It might be possible to choose the string coordinate to be light-like radial coordinate of δCD but this is by no means necessary.

The integration measure $d\mu$ in the superposition of modes has nothing to do with the metric determinants assignable to 3-surface X^3 or with the corresponding space-time surface at X^3. $d\mu$ at partonic 1-surface X^2 must be taken to be such that its square multiplied by transversal delta function resulting in anti-commutation of two modes gives a measure defined by the Kähler form $J_{\mu\nu}$ and given by $d\mu = J_{\mu\nu}dx^\mu dx^\nu = J\sqrt{g_2}dx^1 \wedge dx^2$, $J = J_{\mu\nu}\epsilon^{\mu\nu}$ (note that permutation tensor is inversely proportional $\sqrt{g_2}$). This measure appears in the earlier definition of WCW Hamiltonian as the analog of flux integral $\oint H_A J dx^1 \wedge dx^2$, where H_A is Hamiltonian to be replaced with its integral over string.

There are two manners to get J to the measure for Hamiltonian flux.

- Option I: One uses for super charges has "half integration measure" given by $d\mu_{1/2} = \sqrt{J\sqrt{g_2}}dx^1 \times dx^2$. Note that $\sqrt{J}$ is imaginary for $J < 0$ and also the unique choice of sign of the square root might produce problems.
- Option II: The integration measure is $d\mu = J(x, end)\sqrt{g_2}dx^1 \wedge dx^2$ for the super charge and anti-commutations of Ψ at string are proportional to $1/J(x, end)\sqrt{g_2}$ so that anti-commutator of supercharges would be proportional to $J(x, end)\sqrt{g_2}$ and metric determinant disappears from the integration measure. Note that the vanishing of $J(x, end)$ does not produce any problems in anti-commutators.

  $J(x, end)$ means a non-locality in the anti-commutator. If the string is interpreted as beginning from the partonic surface at its second end, one obtains two different anti-commutation relations unless strings are $J(x, y)\sqrt{g_2} = constant$ curves. This could make sense for flux tubes which are indeed assumed to carry the Kähler flux. Note also that partonic 2-surface decomposes naturally into regions with fixed sign of J forming flux tubes.

  $J(x, y)\sqrt{g_2} = constant$ condition seems actually trivial. The reason is that by a suitable coordinate transformations $(x, y) \rightarrow (f(x, y)$ leaving string coordinate invariant the $\sqrt{g_2}$ gains a factor equal to the Jacobian of the transformation which reduces to 2-D Jacobian for the transformation for the coordinates of partonic 2-surface. By a suitable choice of this transformation $J(x, y)\sqrt{g_2} = constant$ condition is satisfied along string world sheets. This transformation is determined only modulo an area preserving - thus symplectic - transformation for each partonic 2-surface in the slicing. One obtains space-time analog of symplectic invariance as an additional symmetry having identification as a remnant of 3-D GCI. Since also string parameterizations $t \rightarrow f(t)$ are allowed so that 3-D GCI reduces to 1-D Diff and 2-D Sympl. Natural 4-D extension of string reparameterizations would be to the analogs of conformal transformations associated with the effective metric defined by modified gamma matrices so that 4-D Diff would reduce to a product of 2-D conformal and symplectic groups.

The physical state is specified by a finite number of fermion number carrying string world sheets (one can of course have a superposition of these states with different locations of string world sheets). One can ask whether QCC forces the space-time surface to code this state in its geometry in the sense that only these string world sheets are possible. $J(x,y)\sqrt{g_2} = constant$ condition does not force this.

- Option III: If one assumes slicing by partonic 2-surfaces with common coordinates $x = (x^1, x^2)$ and that $J(x,y)\sqrt{g}$ is included to current density at the point of string and that $1/J(x,y)\sqrt{g_2}$ in the anti-commutations is evaluated at the point x of the partonic surface intersecting the string at y, the flux is replaced with the superposition of local fluxes from all points in the slicing by partonic 2-surfaces and $J(x,y)$. For $J\sqrt{g_2}$= constant along strings Options II an III are equivalent.

On basis of physical picture Option II with $J\sqrt{g_2}$= constant achieved by a proper choice of partonic coordinates for the slicing looks very attractive.

5.4.2 Fermionic supra currents as Noether currents

Fermionic supra currents can be taken as Noether currents assignable to the modified Dirac action. Charges are obtained by integrating over string. Here possible technical problems relate to the correct identification of the integration measure. In the normal situation the integration measure is $\sqrt{g_4}$ but now 2-D delta function restricts the charge density for a given mode to the string world sheet and might produce additional factors.

The general form of the super current at given string world sheet corresponding to a given string world sheet is given by

$$\begin{aligned} J^\alpha &= \left[\overline{\Psi_n} O^\alpha_{\beta,k}\delta h^k D_\alpha \Psi + \overline{\Psi_n}\Gamma^\alpha \delta\Psi\right]\sqrt{g_4} \ , \\ O^\alpha_{\beta,k} &= \frac{\partial \Gamma^\alpha}{\partial(\partial_\beta h^k)} \ . \end{aligned} \tag{5.4.1}$$

The covariant divergence of J^α vanishes. Modified gamma matrices appearing in the equation are defined as contractions of the canonical momentum densities T^α_k of Kähler action with imbedding space gamma matrices Γ^k as

$$\begin{aligned} \Gamma^\alpha &= T^\alpha_k \Gamma^k \ , \\ T^\alpha_k &= \frac{L_K}{\partial(\partial_\beta h^k)} \ , \end{aligned} \tag{5.4.2}$$

Ψ_n is the mode of induced spinor field considered. $\delta\Psi$ is the change of Ψ in spin rotation given by

$$\delta\Psi = \partial_l j_k \Sigma^{kl} \ . \tag{5.4.3}$$

The corresponding current is obtained by replacing Ψ_n with Ψ and integrating over the modes.

The current could quite well vanish. The reason is that holography means that one half of modified gamma matrices whose number is effectively 2 annihilates the spinor modes. Also the covariant derivative D_z or $D_{\overline{z}}$ annihilates it. One obtains vanishing result if the quantity $O^z_{\beta,k}$ is proportional to Γ^z. This can be circumvented if it is superposition of gamma matrices which are not parallel to the string world sheet or if is superposition of Γ^z and $\Gamma^{\overline{z}}$: this could have interpretation as breaking of conformal invariance.

For critical deformations vanishing at X^3 δh^k appearing in the formula of current vanishes so that one obtains non-vanishing charge only for second variation.

Note that the quantity $O^\alpha_{\beta,k}$ involves terms $J^{\alpha k}J^l_\beta$ and can be non-vanishing even when J vanishes. The replacement of ordinary γ^0 in fermionic anti-commutation relations with the modified gamma matrix Γ^0 helps here since modified gamma matrices vanish when J vanishes.

Note that for option II favoured by the existing physical picture J is constant along the strings and anti-commutation relations are non-singular for $J \neq 0$.

5.4.3 Anti-commutators of super-charges

The anti-commutators for fermionic fields- or more generally, quantities related to them - should be such that the anti-commutator of fermionic super-Hamiltonians defines WCW Hamiltonian with correct group theoretical properties. To obtain the correct anti-commutator requires that one obtains Poisson bracket of δCD Hamiltonians appearing in the super-Hamiltonians. This is the case if the anti-commutator involved is proportional to iJ_{kl} since this gives the desired Poisson bracket

$$J_{kl}j^k_A j^l_B = \{H_A, H_B\} \ . \tag{5.4.4}$$

This is achieved if one replaces the anti-commutators of Ψ and $\overline{\Psi}$ with anti-commutator of $A_k \equiv O^0_k\Psi$ and $\overline{A_l} \equiv \overline{\Psi}O^0_l$ (O^α_k was defined in Eq. 5.4.1) and assumes

$$\{A_k, \overline{A}_l\} = iJ_{kl}\Gamma^0\delta_2(x_2, y_2)\delta_1(y_1, y_2)\frac{X}{g_4^{1/2}} \ . \tag{5.4.5}$$

Here Γ^0 is modified gamma matrix and δ_2 is delta function assignable to the partonic 2-surface and δ_1 is delta function assignable with the string. Depending on whether one assumes option I, II, or III one has $X = 1$, $X = 1/J_{x,end}$ or $1/J(x_1, x_2, y)$.

The modified anti-commutation relations do not make sense in higher imbedding space dimensions since the number of spinor components exceeds imbedding space dimension. For $D = 8$ the dimension of H and the number of independent spinor components with given H-chirality are indeed same (leptons and quarks have opposite H-chirality). This makes the dimension $D = 8$ unique in TGD framework.

5.4.4 Strong form of General Coordinate Invariance and strong form of holography

Strong form of general coordinate invariance (GCI) suggests a duality between descriptions using light-like 3-surfaces X^3_l at which the signature of the induced metric changes and space-like 3-surface X^3 at the ends of the space-time surface. Also the translates of these surfaces along slicing might define the theory but with a Kähler function to which real part of a holomorphic function defined in WCW is added.

In order to define the formalism for light-like 3-surfaces, one should be able to define the symplectic algebra. This requires the translation of the boundaries of the light-cone along the line connecting the tips of the CD so that the Hamiltonians of δM^4_+ or δM^4_- make sense at X^3_l. Depending on whether the the state function reduction has occurred on upper or lower boundary of CD one must use translates of δM^4_+ or δM^4_-: this would be one particular manifestation for the arrow of time.

5.4.5 Radon, Penrose ja TGD

The construction of the induced spinor field as a superposition of modes restricted to string world sheets to have well-defined em charge (except in the case of right-handed neutrino) brings in mind Radon transform [A25] (`http://en.wikipedia.org/wiki/Radon_transform`) and Penrose transform [A19] (`http://en.wikipedia.org/wiki/Penrose_transform`). In Radon transform the function defined in Euclidian space E^n is coded by its integrals over $n-1$ dimensional hyper-planes. All planes are allowed and are characterized by their normal whose direction corresponds to a point of $n-1$-dimensional sphere S^{n-1} and by the orthogonal distance of the plane from the origin. Note that the space of hyper-planes is n-dimensional as it should be if it is to carry same information as the function itself. One can easily demonstrate that n-dimensional Fourier transform is composite of 1-dimensional Fourier transform in the direction normal vector parallel to wave vector obtained integrating over the distance parameter associated with $n-$dimensional Radon transform defined by function multplied by the plane wave.

In the case of Penrose transform [A19] (`http://en.wikipedia.org/wiki/Penrose_transform`) one has 6-dimensional twistor space CP_3 and the space of complex two - planes- topologically spheres in CP_3 - one for each point of in CP_3 - defines 4-D compactified Minkowski space. A massless field in M^4 has a representation in CP_3 with field value at given point of M^4 represented as an integral over S^3 of holomorphic field in CP_3.

In the recent case the situation resembles very much that for Penrose transform. In the case of space-like 3-surface CP_3 is replaced with the space of strings emanating from the partonic 2-surface and its points are labelled by points of partonic 2-surface and points of string so that dimension is still $D = 3$. The transform describes second quantize spinor field as a collection of "Fourier components" along stringy curves. In 4-D case one has 4-D space-time surface and collection of "Fourier components" along string world sheets. One could say that charge densities assignable to partonic 2-surfaces replace the massless fields in M^4. Now however the decomposition into strings and string world sheets takes place at the level of physics rather than only mathematically.

5.5 About the notion of four-momentum in TGD framework

The starting point of TGD was the energy problem of General Relativity [K72]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K72, K96]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of zero energy ontology (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K92] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K80, K58]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [B20]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

5.5.1 Scale dependent notion of four-momentum in zero energy ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K60], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of zero energy ontology (ZEO) [K3, K88], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K88]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K87].

5.5.2 Are the classical and quantal four-momenta identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

5.5.3 What Equivalence Principle (EP) means in quantum TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

(a) Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

(b) QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say

$P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. A more general solution ansatz involves several functions analogous to cosmological constant corresponding to the decomposition of energy momentum tensor to terms proportional to Einstein tensor and several lower-dimensional projection operators [K96]. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

(c) Does EP at quantum level reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K92] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors is extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep. It is of course possible that the two realizations of EP are equivalent and the natural conjecture is that this is the case.

For QCC option the GRT inspired interpretation of Equivalence Principle at space-time level remains to be understood. Is it needed at all? The condition that the energy momentum tensor of Kähler action has a vanishing divergence leads in General Relativity to Einstein equations with cosmological term. In TGD framework preferred extremals satisfying the analogs of Einstein's equations with several cosmological constant like parameters can be considered.

Should one give up this idea, which indeed might be wrong? Could the divergence of of energy momentum tensor vanish only asymptotically as was the original proposal? Or should one try to generalize the interpretation? QCC states that quantum physics has classical correlate at space-time level and implies EP. Could also quantum classical correspondence itself have a correlate at space-time level. If so, space-time surface would able to represent abstractions as statements about statements about.... as the many-sheeted structure and the vision about TGD physics as analog of Turing machine able to mimic any other Turing machine suggest.

5.5.4 TGD-GRT correspondence and Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K96].

5.5.5 How translations are represented at the level of WCW?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n - - > T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

The action of translations at space-time sheets

The action of imbedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred extremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

5.5.6 Yangian and four-momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [B20]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K80], where also references to the work of pioneers can be found.

Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K80] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B43] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

(a) The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A11] and Virasoro algebras [A29] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

(b) The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

(c) This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

(a) At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

(b) Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

(c) The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

(d) Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

Chapter 6

Unified Number Theoretical Vision

6.1 Introduction

Octonions, quaternions, quaternionic space-time surfaces, octonionic spinors and twistors and twistor spaces are highly relevant for quantum TGD. In the following some general observations distilled during years are summarized. This summary involves several corrections to the picture which has been developing for a decade or so.

A brief updated view about $M^8 - H$ duality and twistorialization is in order. There is a beautiful pattern present suggesting that $M^8 - H$ duality makes sense and that $H = M^4 \times CP_2$ is completely unique on number theoretical grounds.

(a) $M^8 - H$ duality allows to deduce $M^4 \times CP_2$ via number theoretical compactification. For the option with minimal number of conjectures the associativity/co-associativity of the space-time surfaces in M^8 guarantees that the space-time surfaces in M^8 define space-time surfaces in H. The tangent/normal spaces of quaternionic/hyper-quaternionic surfaces in M^8 contain also an integrable distribution of hyper-complex tangent planes $M^2(x)$.

An important correction is that associativity/co-associativity does not make sense at the level of H since the spinor structure of H is already complex quaternionic and reducible to the ordinary one by using matrix representations for quaternions. The associativity condition should however have some counterpart at level of H. One could require that the induced gamma matrices at each point could span a *real*-quaternionic sub-space of complexified quaternions for quaternionicity and a purely imaginary quaternionic sub-space for co-quaternionicity. One might hope that it is consistent with - or even better, implies - preferred extremal property. I have not however found a viable definition of quaternionic "reality". On the other hand, it is possible assigne the tangent space M^8 of H with octonion structure and define associativity as in case of M^8.

$M^8 - H$ duality could generalize to $H - H$ duality in the sense that also the image of the space-time surface under duality map is not only preferred extremal but also associative (co-associative) surface. The duality map $H \rightarrow H$ could be iterated and would define the arrow for the category formed by preferred extremals.

(b) M^4 and CP_2 are the unique 4-D spaces allowing twistor space with Kähler structure. M^8 allows twistor space for octonionic spinor structure obtained by direct generalization of the standard construction for M^4. $M^4 \times CP_2$ spinors can be regarded as tensor products of quaternionic spinors associated with M^4 and CP_2: this trivial observation forces to challenge the earlier rough vision, which however seems to stand up the challenge.

(c) Octotwistors generalise the twistorial construction from M^4 to M^8 and octonionic gamma matrices make sense also for H with quaternionicity condition reducing 12-D

$T(M^8) = G_2/U(1) \times U(1)$ to the 12-D twistor space $T(H) = CP_3 \times SU^3/U(1) \times U(1)$. The interpretation of the twistor space in the case of M^8 is as the space of choices of quantization axes for the 2-D Cartan algebra of G_2 acting as octonionic automorphisms. For CP_2 one has space for the chocies of quantization axes for the 2-D $SU(3)$ Cartan algebra.

(d) It is also possible that the dualities extend to a sequence $M^8 \to H \to H$... by mapping the associative/co-associative tangent space to CP_2 and M^4 point to M^4 point at each step. One has good reasons to expect that this iteration generates fractal as the limiting space-time surface.

(e) A fascinating structure related to octo-twistors is the non-associated analog of Lie group defined by automorphisms by octonionic imaginary units: this group is topologically 7-sphere. Second analogous structure is the 7-D Lie algebra like structure defined by octonionic analogs of sigma matrices.

The analogy of quaternionicity of M^8 pre-images of preferred extremals and quaternionicity of the tangent space of space-time surfaces in H with the Majorana condition central in super string models is very thought provoking. All this suggests that associativity at the level of M^8 indeed could define basic dynamical principle of TGD.

Number theoretical vision about quantum TGD involves both p-adic number fields and classical number fields and the challenge is to unify these approaches. The challenge is non-trivial since the p-adic variants of quaternions and octonions are not number fields without additional conditions. The key idea is that TGD reduces to the representations of Galois group of algebraic numbers realized in the spaces of octonionic and quaternionic adeles generalizing the ordinary adeles as Cartesian products of all number fields: this picture relates closely to Langlands program. Associativity would force sub-algebras of the octonionic adeles defining 4-D surfaces in the space of octonionic adeles so that 4-D space-time would emerge naturally. $M^8 - H$ correspondence in turn would map the space-time surface in M^8 to $M^4 \times CP_2$.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- Physics as generalized number theory [L31]
- Quantum physics as generalized number theory [L34]
- TGD and classical number fields [L41]
- $M^8 - H$ duality [L28]
- Basic notions behind $M^8 - H$ duality [L15]
- Quaternionic planes of octonions [L37]

6.2 Number theoretic compactification and $M^8 - H$ duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (actually its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely. However, if quaternionicity can be realized in terms of M_c^8 using O_c-real analytic functions and if quaternionicity is equivalent with preferred extremal property, a huge simplification results and one can say that field equations are exactly solvable.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 or even M^8_c composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

Remark: The original assumption was that space-times could be regarded as surfaces in M^8 rather than in its complexification M^8_c identifiable as complexified octonions. This assumption is un-necessarily strong and if one assumes that octonion-real analytic functions characterize these surfaces M^8_c must be assumed.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere K̈hler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3) \times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick. $M^8_c = O_c$ provides the proper formulation.

(a) The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j. If complexified quaternions are used for H, Minkowskian signature requires the introduction of two commuting imaginary units j and i meaning double complexification.

(b) Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the sub-space of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

(c) Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$. Tangent space vectors of H correspond hyper-quaternions $q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

(a) Kähler form for M^8 non-trivial only in $E^4 \subset M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ needed to define $M^8 - H$ duality uniquely. This applies also to M^8_c. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the WCWs associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?

(b) One can formulate associativity in M^8 (or M^8_c) by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

   The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest. Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *imbedding space level* and *space-time level*. One must have imbedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.

(c) Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

(d) What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \to H \to H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. This brings in mind the functional composition of O_c-real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of modified gamma matrices defined by Kähler action (certainly not M^8).

(a) All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the modified gamma matrices need not span the entire tangent space. The space spanned by the modified gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the modified gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices.

   If the space spanned by modified gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.

(b) For modified gamma matrices the notion of co-associativity can produce problems since modified gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by modified gamma matrices? Co-associativity option must be considered for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the modified gamma matrices reduce to sums of ordinary CP_2 gamma matrices and ligt-like M^4 contribution. The orthogonal complement for the modified gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

6.2.1 Basic idea behind $M^8 - M^4 \times CP_2$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

(a) One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This decomposition generalizes also to the case of M^8_c. This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.

(b) The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.

(c) Quaternionic sub-algebras of M^8 (and M^8_c) are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

(d) The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1} e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

(a) The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \rightarrow S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.

(b) Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \rightarrow M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.

(c) One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

(d) Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.

(e) The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

(f) Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

(a) This definition generalizes to the case of M_c^8: all that matters is that tangent space-is is complexified quaternionic and there is a unique identification $M^4 \subset M_c^8$: this allows to assign the point of 4-surfaces a point of $M^4 \times CP_2$. The generalization is needed if one wants to formulate the hypothesis about O_c real-analyticity as a manner to build quaternionic space-time surfaces properly.

(b) This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.

(c) The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K4]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.

(d) Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K22] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of modified Dirac operator.

For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

(e) Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in H. One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.

(f) One can also consider a variant of associativity based on modified gamma matrices - but only in H. This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

6.2.2 Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [K78]).

(a) According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

(b) Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \to H \to H....$ This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and modified gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

6.2.3 Are Kähler and spinor structures necessary in M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

(a) By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.

(b) One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

(c) Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naive replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.

(d) The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

(a) The complexified octonions representing H spinors decompose to $1 + 1 + 3 + \overline{3}$ under SU(3) representing color automorphisms but the interpretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

(b) One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1 + kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

(c) One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.

(d) The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

    If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1 + ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.

(e) In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of imbedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

6.2.4 How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \to H \to H...$ iteration generating new solutions from existing ones.

Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1+io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1, -1, -, 1-, 1)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M^8_c but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1, -1, -1, -1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M^8_c (not M^8!) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assignes to them a 4-surface in $M^4 \times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

(a) Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a, b, c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

(b) If one is able to choose the coordinates in such a manner that one of the tangent vectors corresponds to real unit (in the imbedding map imbedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the imbedding space vectors.

    One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

(c) Field equations would reduce to tri-linear equations in in the gradients of imbedding space coordinates (rather than involving imbedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

(d) Written explicitly field equations give in terms of vielbein projections e_α^A, vielbein vectors e_k^A, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e_\alpha^A e_\beta^B e_\gamma^C A_{ABC}^E &= 0 \ , \\ A_{ABC}^E &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e_\alpha^A &= \partial_\alpha h^k e_k^A \ , \\ \Gamma_k &= e_k^A \gamma_A \ . \end{aligned} \tag{6.2.1}$$

The very naive idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F_{\alpha\beta}^A = D_\alpha e_\beta^A - D_\beta e_\alpha^A = 0 \ . \tag{6.2.2}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associatitivity conditions.

(e) The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://en.wikipedia.org/wiki/Hyperdeterminant`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A_{BCD}^E x^B y^C z^D = 0$ of trilinear forms defined by the associators. The conditions say somethig only about the octonioni structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A43] expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product e_1e_2 (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "time-like" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

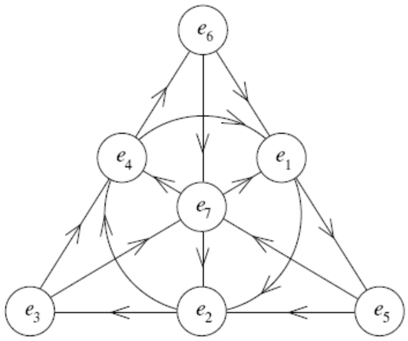

Figure 6.1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

6.2.5 Quaternionicity at the level of imbedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A43] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2:s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

6.2.6 Questions

In following some questions related to $M^8 - H$ duality are represented.

Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

(a) For Kähler action the modified gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k \gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space.

The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.

(b) The space spanned by the modified gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection modified gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence modified gamma matrices are flexible enough to realize associativity in H.

(c) Modified gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the modified gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \to k$ duality.

Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

(a) If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

(b) $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

(c) $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

    This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

$M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

(a) At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

(b) The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

(c) The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

(d) The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

(e) Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K46] .

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

6.2.7 Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \to H \to H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

6.3 Octo-twistors and twistor space

The basic problem of the twistor approach is that one cannot represent massive momenta in terms of twistors in an elegant manner. One can also consider generalization of the notion of spinor and twistor. I have proposed a possible representation of massive states based on the existence of preferred plane of M^2 in the basic definition of theory allowing to express four-momentum as one of two light-like momenta allowing twistor description. One could however ask whether some more elegant representation of massive M^4 momenta might be possible by generalizing the notion of twistor -perhaps by starting from the number theoretic vision.

The basic idea is obvious: in quantum TGD massive states in M^4 can be regarded as massless states in M^8 and $M^4 \times CP_2$ (recall $M^8 - H$ duality). One can therefore map any massive M^4 momentum to a light-like M^8 momentum and hope that this association could be made in a unique manner. One should assign to a massless 8-momentum an 8-dimensional spinor of fixed chirality. The spinor assigned with the light-like four-momentum is not unique without additional conditions. The existence of covariantly constant right-handed neutrino in CP_2 degrees generating the super-conformal symmetries could allow to eliminate the non-uniqueness. 8-dimensional twistor in M^8 would be a pair of this kind of spinors fixing the momentum of massless particle and the point through which the corresponding light-geodesic goes through: the set of these points forms 8-D light-cone and one can assign to each point a spinor. In $M^4 \times CP_2$ definitions makes also in the case of $M^4 \times CP_2$ and twistor space would also now be a lifting of the space of light-like geodesics.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the modified gamma matrices both in M^8 and H.

The basic challenge is to achieve twistorial description of four-momenta or even $M^4 \times CP_2$ quantum numbers: this applies both to the momenta of fundamental fermions at the lines of generalized Feynman diagrams and to the massive incoming and outcoming states identified as their composites.

(a) A rather attractive way to overcome the problem at the level of fermions propagating along the braid strands at the light-like orbits of partonic 2-surfaces relies on the assumption that generalized Feynman diagrammatics effectively reduces to a form in which all fermions in the propagator lines are massless although they can have non-physical helicity [K58]. One can use ordinary M^4 twistors. This is consistent with the idea that space-time surfaces are quaternionic sub-manifolds of octonionic imbedding space.

(b) Incoming and outgoing states are composites of massless fermions and not massless. They are however massless in 8-D sense. This suggests that they could be described using generalization of twistor formalism from M^4 to M^8 and even betterm to $M^4 \times CP_2$.

In the following two possible twistorializations are considered.

6.3.1 Two manners to twistorialize imbedding space

In the following the generalization of twistor formalism for M^8 or $M^4 \times CP_2$ will be considered in more detail. There are two options to consider.

(a) For the first option one assigns to $M^4 \times CP_2$ twistor space as a product of corresponding twistor spaces $T(M_4) = CP_3$ and the flag-manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ parameterizing the choices of quantization axes for $SU(3)$: $T_H = T(M^4) \times T(CP_2)$. Quite remarkably, M^4 and CP_2 are the only 4-D manifolds allowing twistor space with Kähler structure. The twistor space is 12-dimensional. The choice of quantization axis is certainly a physically well-define operation so that $T(CP_2)$ has physical interpretation. If all observable physical states are color singlets situation becomes more complex. If one assumes QCC for color quantum numbers Y and I_3, then also the choice of color quantization axis is fixed at the level of Kähler action from the condition that Y and I_3 have classically their quantal values.

(b) For the second option one generalizes the usual construction for M^8 regarded as tangent space of $M^4 \times CP_2$ (unless one takes $M^8 - H$ duality seriously).

The tangent space option looks like follows.

(a) One can map the points of M^8 to octonions. One can consider 2-component spinors with octonionic components and map points of M^8 light-cone to linear combinations of 2×2 Pauli sigma matrices but with octonionic components. By the same arguments as in the deduction of ordinary twistor space one finds that 7-D light-cone boundary is mapped to 7+8 D space since the octonionic 2-spinor/its conjugate can be multiplied/divided by arbitrary octonion without changing the light-like point. By standard argument this space extends to 8+8-D space. The points of M^8 can be identified as 8-D octonionic planes (analogs of complex sphere CP_1 in this space. An attractive identification is as octonionic projective space OP_2. Remarkably, octonions do not allow higher dimensional projective spaces.

(b) If one assumes that the spinors are quaternionic the twistor space should have dimension 7+4+1=12. This dimension is same as for $M^4 \times CP_2$. Does this mean that quaternionicity assumption reduces $T(M^8) = OP_2$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$? Or does it yield 12-D space $G_2/U(1) \times U(1)$, which is also natural since G_2 has 2-D Cartan algebra? Number theoretical compactification would transform $T(M^8) = G_2/U(1) \times U(1)$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$. This would not be surprising since in $M^8 - H$-duality CP_2 parametrizes (hyper)quaternionic planes containing preferred plane M^2.

Quaternionicity is certainly very natural in TGD framework. Quaternionicity for 8-momenta does not in general imply that they reduce to the observed M^4-momenta unless one identifies M^4 as one particular subspace of M^8. In $M^8 - H$ duality one in principle allows all choices of M^4: it is of course unclear whether this makes any physical difference. Color confinement could be interpreted as a reduction of M^8 momenta to M^4 momenta and would also allow the interpretational problems caused by the fact that CP_2 momenta are not possible.

(c) Since octonions can be regarded as complexified quaternions with non-commuting imaginary unit, one can say that quaternionic spinors in M^8 are "real" and thus analogous to Majorana spinors. Similar interpretation applies at the level of H. Could one can interpret the quaternionicity condition for space-time surfaces and imbedding space spinors as TGD analog of Majorana condition crucial in super string models? This would also be crucial for understanding supersymmetry in TGD sense.

6.3.2 Octotwistorialization of M^8

Consider first the twistorialization in 4-D case. In M^4 one can map light-like momoment to spinors satisfying massless Dirac equation. General point m of M^4 can be mapped to a pair of massless spinors related by incidence relation defining the point m. The essential element of this association is that mass squared can be defined as determinant of the 2×2 matrix resulting in the assignment. Light-likeness is coded to the vanishing of the determinant implying that the spinors defining its rows are linearly independent. The reduction of M^4 inner product to determinant occurs because the 2×2 matrix can be regarded as a matrix representation of complexified quaternion. Massless means that the norm of a complexified quaternion defined as the product of q and its conjugate vanishes. Incidence relation $s_1 = xs_2$ relating point of M^4 and pair of spinors defining the corresponding twistor, can be interpreted in terms of product for complexified quaternions.

The generalization to the 8-D situation is straightforward: replace quaternions with octonions.

(a) The transition to M^8 means the replacement of quaternions with octonions. Masslessness corresponds to the vanishing norm for complexified octonion (hyper-octonion).

(b) One should assign to a massless 8-momentum an 8-dimensional spinor identifiable as octonion - or more precisely as hyper-octonion obtained by multiplying the imaginary part of ordinary octonion with commuting imaginary unit j and defining conjugation as a change of sign of j or that of octonionic imaginar units.

(c) This leads to a generalization of the notion of twistor consisting of pair of massless octonion valued spinors (octonions) related by the incidence relation fixing the point of

M^8. The incidence relation for Euclidian octonions says $s_1 = xs_2$ and can be interpreted in terms of triality for $SO(8)$ relating conjugate spinor octet to the product of vector octed and spinor octet. For Minkowskian subspace of complexified octonions light-like vectors and s_1 and s_2 can be taken light-like as octonions. Light like x can annihilate s_2.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the modified gamma matrices both in M^8 and H.

6.3.3 Octonionicity, $SO(1,7)$, G_2, and non-associative Malcev group

The symmetries assignable with octonions are rather intricate. First of all, octonions (their hyper-variants defining M^8) have SO(8) (SO(1,7)) as isometries. $G_2 \subset SO(7)$ acts as automorphisms of octonions and $SO(1,7) \rightarrow G_2$ clearly means breaking of Lorentz invariance.

John Baez has described in a lucid manner G_2 geometrically (`http://math.ucr.edu/home/baez/octonions/node14.html`). The basic observation is that that quaternionic sub-space is generated by two linearly independent imaginary units and by their product. By adding a fourth linearly independent imaginary unit, one can generated all octonions. From this and the fact that G_2 represents subgroup of $SO(7)$, one easily deduces that G_2 is 14-dimensional. The Lie algebra of G_2 corresponds to derivations of octonionic algebra as follows infinitesimally from the condition that the image of product is the product of images. The entire algebra $SO(8)$ is direct sum of G_2 and linear transformations generated by right and left multiplication by imaginary octonion: this gives $14 + 14 = 28 = D(SO(8))$. The subgroup $SO(7)$ acting on imaginary octonsions corresponds to the direct sum of derivations and adjoint transformations defined by commutation with imaginary octonions, and has indeed dimension $14 + 7 = 21$.

One can identify also a non-associative group-like structure.

(a) In the case of octonionic spinors this group like structure is defined by the analog of phase multiplication of spinor generalizing to a multiplication with octonionic unit expressible as linear combinations of 8 octonionic imaginary units and defining 7-sphere plays appear as analog of automorphisms $o \rightarrow uou^{-1} = uou^*$.
One can associate with these transformations a non-associative Lie group and Lie algebra like structures by defining the commutators just as in the case of matrices that is as $[a, b] = ab - ba$. One 7-D non-associative Lie group like structure with topology of 7-sphere S^7 whereas G_2 is 14-dimensional exceptional Lie group (having S^6 as coset space $S^6 = G_2/SU(3)$). This group like object might be useful in the treatment of octonionic twistors. In the case of quaternions one has genuine group acting as SO(3) rotations.

(b) Octonionic gamma matrices allow to define as their commutators octonionic sigma matrices:

$$\Sigma_{kl} = \frac{i}{2}\left[\gamma_k, \gamma_l\right] \quad . \tag{6.3.1}$$

This algebra is 14-dimensional thanks to the fact that octonionic gamma matrices are of form $\gamma_0 = \sigma_1 \otimes 1$, $\gamma_i = \sigma_2 \otimes e_i$. Due to the non-associativity of octonions this algebra does not satisfy Jacobi identity - as is easy to verify using Fano triangle - and is therefore not a genuine Lie-algebra. Therefore these sigma matrices do not define a representation of G_2 as I thought first.
This algebra has decomposition $g = h + t$, $[h, t] \subset t$, $[t, t] \subset h$ characterizing for symmetric spaces. h is the 7-D algebra generated by Σ_{ij} and identical with the non-associative Malcev algebra generated by the commutators of octonionic units. The complement t corresponds to the generators Σ_{0i}. The algebra is clearly an octonionic non-associative analog fo $SO(1,7)$.

6.3.4 Octonionic spinors in M^8 and real complexified-quaternionic spinors in H?

This above observations about the octonionic sigma matrices raise the problem about the octonionic representation of spinor connection. In $M^8 = M^4 \times E^4$ the spinor connection is trivial but for $M^4 \times CP_2$ not. There are two options.

(a) Assume that octonionic spinor structure makes sense for M^8 only and spinor connection is trivial.

(b) An alternative option is to identify M^8 as tangent space of $M^4 \times CP_2$ possessing quaternionic structure defined in terms of octonionic variants of gamma matrices. Should one replace sigma matrices appearing in spinor connection with their octonionic analogs to get a sigma matrix algebra which is pseudo Lie algebra. Or should one map the holonomy algebra of CP_2 spinor connection to a sub-algebra of $G_2 \subset SO(7)$ and define the action of the sigma matrices as ordinary matrix multiplication of octonions rather than octonionic multiplication? This seems to be possible formally.

   The replacement of sigma matrices with their octonionic counterparts seems to lead to weird looking results. Octonionic multiplication table implies that the electroweak sigma matrices associated with CP_2 tangent space reduce to M^4 sigma matrices so that the spinor connection is quaternionic. Furthermore, left-handed sigma matrices are mapped to zero so that only the neutral part of spinor connection is non-vanishing. This supports the view that only M^8 gamma matrices make sense and that Dirac equation in M^8 is just free massless Dirac equation leading naturally also to the octonionic twistorialization.

   One might think that distinction between different H-chiralities is difficult to make but it turns out that quarks and leptons can be identified as different components of 2-component complexified octonionic spinors.

The natural question is what associativization of octonions gives. This amounts to a condition putting the associator $a(bc)-(ab)c$ to zero. It is enough to consider octonionic imaginary units which are different. By using the decomposition of the octonionic algebra to quaternionic sub-algebra and its complement and general structure of structure constants, one finds that quaternionic sub-algebra remains as such but the products of all imaginary units in the complement with different imaginary units vanish. This means that the complement behaves effectively as 4-D flat space-gamma matrix algebra annihilated by the quaternionic sub-algebra whose imaginary part acts like Lie algebra of $SO(3)$.

6.3.5 What the replacement of $SO(7,1)$ sigma matrices with octonionic sigma matrices could mean?

The basic implication of octonionization is the replacement of $SO(7,1)$ sigma matrices with octonionic sigma matrices. For M^8 this has no consequences since since spinor connection is trivial.

For $M^4 \times CP_2$ situation would be different since CP_2 spinor connection would be replaced with its octonionic variant. This has some rather unexpected consequences and suggests that one should not try to octonionize at the level of $M^4 \times CP_2$ but interepret gamma matrices as tensor products of quaternionic gamma matrices, which can be replaced with their matrix representations. There are however some rather intriguing observations which force to keep mind open.

Octonionic representation of 8-D gamma matrices

Consider first the representation of 8-D gamma matrices in terms of tensor products of 7-D gamma matrices and 2-D Pauli sigma matrices.

(a) The gamma matrices are given by

$$\gamma^0 = 1 \times \sigma_1 \ , \ \gamma^i = \gamma^i \otimes \sigma_2 \ , \ i = 1,..,7 \ . \quad (6.3.2)$$

7-D gamma matrices in turn can be expressed in terms of 6-D gamma matrices by expressing γ^7 as

$$\gamma^{7)}_{i+1} = \gamma^{6)}_i , i = 1, ..., 6 \ , \ \gamma^{7)}_1 = \gamma^{6)}_7 = \prod_{i=1}^{6} \gamma^{6)}_i \ . \quad (6.3.3)$$

(b) The octonionic representation is obtained as

$$\gamma_0 = 1 \otimes \sigma_1 \ , \ \gamma_i = e_i \otimes \sigma_2 \ . \quad (6.3.4)$$

where e_i are the octonionic units. $e_i^2 = -1$ guarantees that the M^4 signature of the metric comes out correctly. Note that $\gamma_7 = \prod \gamma_i$ is the counterpart for choosing the preferred octonionic unit and plane M^2.

(c) The octonionic sigma matrices are obtained as commutators of gamma matrices:

$$\Sigma_{0i} = je_i \times \sigma_3 \ , \ \Sigma_{ij} = jf_{ij}{}^{k} e_k \otimes 1 \ . \quad (6.3.5)$$

Here j is commuting imaginary unit. These matrices span G_2 algebra having dimension 14 and rank 2 and having imaginary octonion units and their conjugates as the fundamental representation and its conjugate. The Cartan algebra for the sigma matrices can be chosen to be Σ_{01} and Σ_{23} and belong to a quaternionic sub-algebra.

(d) The lower dimension $D = 14$ of the non-associative version of sigma matrix algebra algebra means that some combinations of sigma matrices vanish. All left or right handed generators of the algebra are mapped to zero: this explains why the dimension is halved from 28 to 14. From the octonionic triangle expressing the multiplication rules for octonion units [A17] one finds $e_4e_5 = e_1$ and $e_6e_7 = -e_1$ and analogous expressions for the cyclic permutations of e_4, e_5, e_6, e_7. From the expression of the left handed sigma matrix $I^3_L = \sigma_{23} + \sigma^{30}$ representing left handed weak isospin (see the Appendix about the geometry of CP_2 [L1]) one can conclude that this particular sigma matrix and left handed sigma matrices in general are mapped to zero. The quaternionic sub-algebra $SU(2)_L \times SU(2)_R$ is mapped to that for the rotation group $SO(3)$ since in the case of Lorentz group one cannot speak of a decomposition to left and right handed subgroups. The elements of the complement of the quaternionic sub-algebra are expressible in terms of Σ_{ij} in the quaternionic sub-algebra.

Some physical implications of the reduction of $SO(7,1)$ to its octonionic counterpart

The octonization of spinor connection of CP_2 has some weird physical implications forcing to keep mind to the possibility that the octonionic description even at the level of H might have something to do with reality.

(a) If $SU(2)_L$ is mapped to zero only the right-handed parts of electro-weak gauge field survive octonionization. The right handed part is neutral containing only photon and Z^0 so that the gauge field becomes Abelian. Z^0 and photon fields become proportional to each other ($Z^0 \rightarrow sin^2(\theta_W)\gamma$) so that classical Z^0 field disappears from the dynamics, and one would obtain just electrodynamics.

(b) The gauge potentials and gauge fields defined by CP_2 spinor connection are mapped to fields in $SO(2) \subset SU(2) \times U(1)$ in quaternionic sub-algebra which in a well-defined sense corresponds to M^4 degrees of freedom and gauge group becomes $SO(2)$ subgroup of rotation group of $E^3 \subset M^4$. This looks like catastrophe. One might say that electroweak interactions are transformed to gravimagnetic interactions.

(c) In very optimistic frame of mind one might ask whether this might be a deeper reason for why electrodynamics is an excellent description of low energy physics and of classical physics. This is consistent with the fact that CP_2 coordinates define 4 field degrees of freedom so that single Abelian gauge field should be enough to describe classical physics. This would remove also the interpretational problems caused by the transitions changing the charge state of fermion induced by the classical W boson fields.

(d) Interestingly, the condition that electromagnetic charge is well-defined quantum number for the modes of the induced spinor field for $X^4 \subset H$ leads to the proposal that the solutions of the modified Dirac equation are localized to string world sheets in Minkowskian regions of space-time surface at least. For CP_2 type vacuum extremals one has massless Dirac and this allows only covariantly constant right-handed neutrino as solution. One has however only a piece of CP_2 (wormhole contact) so that holomorphic solutions annihilated by two complexified gamma matrices are possible in accordance with the conformal symmetries.

Can one assume non-trivial spinor connection in M^8

(a) The simplest option encouraged by the requirement of maximal symmetries is that it is absent. Massless 8-momenta would characterize spinor modes in M^8 and this would give physical justification for the octotwistors.

(b) If spinor connection is present at all, it reduces essentially to Kähler connection having different couplings to quarks and leptons identifiable as components of octonionic 2-spinors. It should be $SO(4)$ symmetric and since CP_2 is instant one might argue that now one has also instanton that is self-dual U(1) gauge field in $E^4 \subset M^4 \times E^4$ defining Kähler form. One can loosely say that that one has of constant electric and magnetic fields which are parallel to each other. The rotational symmetry in E^4 would break down to $SO(2)$.

(c) Without spinor connection quarks and leptons are in completely symmetric position at the level of M^8: this is somewhat disturbing. The difference between quarks and leptons in H is made possible by the fact that CP_2 does not allow standard spinor structure. Now this problem is absent. I have also consider the possibility that only leptonic spinor chirality is allowed and quarks result via a kind of anyonization process allowing them to have fractional em charges (see `http://www.tgdtheory.fi/public_html/articles/genesis.pdf`).

(d) If the solutions of the Kähler Dirac equation in Minkowskian regions are localized to two surfaces identifiable as integrable distributions of planes $M^2(x)$ and characterized by a local light-like direction defining the direction of massless momentum, they are holomorphic (in the sense of hyper-complex numbers) such that the second complexified modified gamma matrix annihilates the solution. Same condition makes sense also at the level of M^8 for solutions restricted to string world sheets and the presence or absence of spinor connection does not affect the situation.

Does this mean that the difference between quarks and leptons becomes visible only at the imbedding space level where ground states of super-conformal representations correspond to to imbedding space spinor harmonics which in CP_2 cm degrees are different for quarks and leptons?

Octo-spinors and their relation to ordinary imbedding space spinors

Octo-spinors are identified as octonion valued 2-spinors with basis

$$
\begin{aligned}
\Psi_{L,i} &= e_i \left(\begin{array}{c} 1 \\ 0 \end{array} \right) \quad , \\
\Psi_{q,i} &= e_i \left(\begin{array}{c} 0 \\ 1 \end{array} \right) \quad .
\end{aligned} \tag{6.3.6}
$$

One obtains quark and lepton spinors and conjugation for the spinors transforms quarks to leptons. Note that octospinors can be seen as 2-dimensional spinors with components which have values in the space of complexified octonions.

The leptonic spinor corresponding to real unit and preferred imaginary unit e_1 corresponds naturally to the two spin states of the right handed neutrino. In quark sector this would mean that right handed U quark corresponds to the real unit. The octonions decompose as $1+1+3+\overline{3}$ as representations of $SU(3) \subset G_2$. The concrete representations are given by

$$
\begin{array}{ll}
\{1 \pm ie_1\} \ , & e_R \text{ and } \nu_R \text{ with spin } 1/2 \ , \\
\{e_2 \pm ie_3\} \ , & e_R \text{ and } \nu_L \text{ with spin -}1/2 \ , \\
\{e_4 \pm ie_5\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ , \\
\{e_6 \pm ie_7\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ .
\end{array} \tag{6.3.7}
$$

Instead of spin one could consider helicity. All these spinors are eigenstates of e_1 (and thus of the corresponding sigma matrix) with opposite values for the sign factor $\epsilon = \pm$. The interpretation is in terms of vectorial isospin. States with $\epsilon = 1$ can be interpreted as charged leptons and D type quarks and those with $\epsilon = -1$ as neutrinos and U type quarks. The interpretation would be that the states with vanishing color isospin correspond to right handed fermions and the states with non-vanishing SU(3) isospin (to be not confused with QCD color isospin) and those with non-vanishing SU(3) isospin to left handed fermions.

The importance of this identification is that it allows a unique map of the candidates for the solutions of the octonionic modified Dirac equation to those of ordinary one. There are some delicacies involved due to the possibility to chose the preferred unit e_1 so that the preferred subspace M^2 can corresponds to a sub-manifold $M^2 \subset M^4$.

6.4 Abelian class field theory and TGD

The context leading to the discovery of adeles (`http://en.wikipedia.org/wiki/Adele_ring`) was so called Abelian class field theory. Typically the extension of rationals means that the ordinary primes decompose to the primes of the extension just like ordinary integers decompose to ordinary primes. Some primes can appear several times in the decomposition of ordinary non-square-free integers and similar phenomenon takes place for the integers of extension. If this takes place one says that the original prime is ramified. The simplest example is provided Gaussian integers $Q(i)$. All odd primes are unramified and primes $p\ mod\ 4 = 1$ they decompose as $p = (a + ib)(a - ib)$ whereas primes $p\ mos\ 4 = 3$ do not decompose at all. For $p = 2$ the decomposition is $2 = (1 + i)(1 - i) = -i(1 + i)^2 = i(1 - i)^2$ and is not unique $\{\pm 1, \pm i\}$ are the units of the extension. Hence $p = 2$ is ramified.

There goal of Abelian class field theory (`http://en.wikipedia.org/wiki/Class_field_theory`) is to understand the complexities related to the factorization of primes of the original field. The existence of the isomorphism between ideles modulo rationals - briefly ideles - and maximal Abelian Galois Group of rationals (MAGG) is one of the great discoveries of Abelian class field theory. Also the maximal - necessarily Abelian - extension of finite field G_p has Galois group isomorphic to the ideles. The Galois group of $G_p(n)$ with p^n elements is actually the cyclic group Z_n. The isomorphism opens up the way to study the representations of Abelian Galois group and also those of the AGG. One can indeed see these representations as special kind of representations for which the commutator group of AGG is represented trivially playing a role analogous to that of gauge group.

This framework is extremely general. One can replace rationals with any algebraic extension of rationals and study the maximal Abelian extension or algebraic numbers as its extension. One can consider the maximal algebraic extension of finite fields consisting of union of all all finite fields associated with given prime and corresponding adele. One can study function fields defined by the rational functions on algebraic curve defined in finite field and its maximal extension to include Taylor series. The isomorphisms applies in al these cases. One ends up with the idea that one can represent maximal Abelian Galois group in function space of complex valued functions in $GL_e(A)$ right invariant under the action of $GL_e(Q)$. A denotes here adeles.

In the following I will introduce basic facts about adeles and ideles and then consider a possible realization of the number theoretical vision about quantum TGD as a Galois theory for the algebraic extensions of classical number fields with associativity defining the dynamics. This picture leads automatically to the adele defined by p-adic variants of quaternions and octonions, which can be defined by posing a suitable restriction consistent with the basic physical picture provide by TGD.

6.4.1 Adeles and ideles

Adeles and ideles are structures obtained as products of real and p-adic number fields. The formula expressing the real norm of rational numbers as the product of inverses of its p-adic norms inspires the idea about a structure defined as produc of reals and various p-adic number fields.

Class field theory (`http://en.wikipedia.org/wiki/Class_field_theory`) studies Abelian extensions of global fields (classical number fields or functions on curves over finite fields), which by definition have Abelian Galois group acting as automorphisms. The basic result of class field theory is one-one correspondence between Abelian extensions and appropriate classes of ideals of the global field or open subgroups of the ideal class group of the field. For instance, Hilbert class field, which is maximal unramied extension of global field corresponds to a unique class of ideals of the number field. More precisely, reciprocity homomorphism generalizes the quadratic resiprocity for quadratic extensions of rationals. It maps the idele class group of the global field defined as the quotient of the ideles by the multiplicative group of the field - to the Galois group of the maximal Abelian extension of the global field. Each open subgroup of the idele class group of a global field is the image with respect to the norm map from the corresponding class field extension down to the global field.

The idea of number theoretic Langlands correspondence, [A13, A65, A64]. is that n-dimensional representations of Absolute Galois group correspond to infinite-D unitary representations of group $Gl_n(A)$. Obviously this correspondence is extremely general but might be highly relevant for TGD, where imbedding space is replaced with Cartesian product of real imbedding space and its p-adic variants - something which might be related to octonionic and quaternionic variants of adeles. It seems however that the TGD analogs for finite-D matrix groups are analogs of local gauge groups or Kac-Moody groups (in particular symplectic group of $\delta M^4_+ \times CP_2$) so that quite heavy generalization of already extremely abstract formalism is expected.

The following gives some more precise definitions for the basic notions.

(a) Prime ideals of global field, say that of rationals, are defined as ideals which do not decompose to a product of ideals: this notion generalizes the notion of prime. For instance, for p-adic numbers integers vanishing mod p^n define an ideal and ideals can be multiplied. For Abelian extensions of a global field the prime ideals in general decompose to prime ideals of the extension, and the decompostion need not be unique: one speaks of ramification. One of the challenges of tjhe class field theory is to provide information about the ramification. Hilbert class field is define as the maximal unramified extension of global field.

(b) The ring of integral adeles (see http://en.wikipedia.org/wiki/Adele_ring) is defined as $A_Z = R \times \hat{Z}$, where $\hat{Z} = \prod_p Z_p$ is Cartesian product of rings of p-adic integers for all primes (prime ideals) p of assignable to the global field. Multiplication of element of A_Z by integer means multiplication in all factors so that the structure is like direct sum from the point of view of physicist.

(c) The ring of rational adeles can be defined as the tensor product $A_Q = Q \otimes_Z A_Z$. $_Z$ means that in the multiplication by element of Z the factors of the integer can be distributed freely among the factors $\hat{Z}$. Using quantum physics language, the tensor product makes possible entanglement between Q and A_Z.

(d) Another definition for rational adeles is as $R \times \prod_p' Q_p$: the rationals in tensor factor Q have been absorbed to p-adic number fields: given prime power in Q has been absorbed to corresponding Q_p. Here all but finite number of Q_p elements ar p-adic integers. Note that one can take out negative powers of p_i and if their number is not finite the resulting number vanishes.The multiplication by integer makes sense but the multiplication by a rational does not smake sense since all factors Q_p would be multiplied.

(e) Ideles are defined as invertible adeles (http://en.wikipedia.org/wiki/Idele_class_groupIdele class group). The basic result of the class field theory is that the quotient of the multiplicative group of ideles by number field is homomorphic to the maximal Abelian Galois group!

6.4.2 Questions about adeles, ideles and quantum TGD

The intriguing general result of class field theory (http://en.wikipedia.org/wiki/Class_field_theory) is that the the maximal Abelian extension for rationals is homomorphic with the multiplicative group of ideles. This correspondence plays a key role in Langlands correspondence.

Does this mean that it is not absolutely necessary to introduce p-adic numbers? This is actually not so. The Galois group of the maximal abelian extension is rather complex objects (absolute Galois group, AGG, defines as the Galois group of algebraic numbers is even more complex!). The ring $\hat{Z}$ of adeles defining the group of ideles as its invertible elements homeomorphic to the Galois group of maximal Abelian extension is profinite group (http://en.wikipedia.org/wiki/Profinite_group). This means that it is totally disconnected space as also p-adic integers and numbers are. What is intriguing that p-dic integers are however a continuous structure in the sense that differential calculus is possible. A concrete example is provided by 2-adic units consisting of bit sequences which can have literally infinite non-vanishing bits. This space is formally discrete but one can construct differential calculus since the situation is not democratic. The higher the pinary digit in the expansion is, the less significant it is, and p-adic norm approaching to zero expresses the reduction of the insignificance.

1. Could TGD based physics reduce to a representation theory for the Galois groups of quaternions and octonions?

Number theoretical vision about TGD raises questions about whether adeles and ideles could be helpful in the formulation of TGD. I have already earlier considered the idea that quantum TGD could reduce to a representation theory of appropriate Galois groups. I proceed to make questions.

(a) Could real physics and various p-adic physics on one hand, and number theoretic physics based on maximal Abelian extension of rational octonions and quaternions on one hand, define equivalent formulations of physics?

(b) Besides various p-adic physics all classical number fields (reals, complex numbers, quaternions, and octonions) are central in the number theoretical vision about TGD. The technical problem is that p-adic quaternions and octonions exist only as a ring unless one poses some additional conditions. Is it possible to pose such conditions so that one could define what might be called quaternionic and octonionic adeles and ideles?

It will be found that this is the case: p-adic quaternions/octonions would be products of rational quaternions/octonions with a p-adic unit. This definition applies also to algebraic extensions of rationals and makes it possible to define the notion of derivative for corresponding adeles. Furthermore, the rational quaternions define non-commutative automorphisms of quaternions and rational octonions at least formally define a non-associative analog of group of octonionic automorphisms [K95].

(c) I have already earlier considered the idea about Galois group as the ultimate symmetry group of physics. The representations of Galois group of maximal Abelian extension (or even that for algebraic numbers) would define the quantum states. The representation space could be group algebra of the Galois group and in Abelian case equivalently the group algebra of ideles or adeles. One would have wave functions in the space of ideles.

The Galois group of maximal Abelian extension would be the Cartan subgroup of the absolute Galois group of algebraic numbers associated with given extension of rationals and it would be natural to classify the quantum states by the corresponding quantum numbers (number theoretic observables).

If octonionic and quaternionic (associative) adeles make sense, the associativity condition would reduce the analogs of wave functions to those at 4-dimensional associative sub-manifolds of octonionic adeles identifiable as space-time surfaces so that also space-time physics in various number fields would result as representations of Galois group in the maximal Abelian Galois group of rational octonions/quaternions. TGD would reduce to classical number theory! One can hope that WCW spinor fields assignable to the associative and co-associative space-time surfaces provide the adelic representations for super-conformal algebras replacing symmetries for point like objects.

This of course involves huge challenges: one should find an adelic formulation for WCWin terms octonionic and quaternionic adeles, similar formulation for WCW spinor fields in terms of adelic induced spinor fields or their octonionic variants is needed. Also zero energy ontology, causal diamonds, light-like 3-surfaces at which the signature of the induced metric changes, space-like 3-surfaces and partonic 2-surfaces at the boundaries of CDs, M^8-H duality, possible representation of space-time surfaces in terms of of O_c-real analytic functions (O_c denotes for complexified octonions), etc. should be generalized to adelic framework.

(d) Absolute Galois group is the Galois group of the maximal algebraic extension and as such a poorly defined concept. One can however consider the hierarchy of all finite-dimensional algebraic extensions (including non-Abelian ones) and maximal Abelian extensions associated with these and obtain in this manner a hierarchy of physics defined as representations of these Galois groups homomorphic with the corresponding idele groups.

(e) In this approach the symmetries of the theory would have automatically adelic representations and one might hope about connection with Langlands program [K33], [A13, A65, A64].

2. Adelic variant of space-time dynamics and spinorial dynamics?

As an innocent novice I can continue to pose stupid questions. Now about adelic variant of the space-time dynamics based on the generalization of Kähler action discussed already earlier but without mentioning adeles ([K97]).

(a) Could one think that adeles or ideles could extend reals in the formulation of the theory: note that reals are included as Cartesian factor to adeles. Could one speak about adelic space-time surfaces endowed with adelic coordinates? Could one formulate variational principle in terms of adeles so that exponent of action would be product of actions exponents associated with various factors with Neper number replaced by p for Z_p. The minimal interpretation would be that in adelic picture one collects under the same umbrella real physics and various p-adic physics.

(b) Number theoretic vision suggests that 4:th/8:th Cartesian powers of adeles have interpretation as adelic variants of quaternions/ octonions. If so, one can ask whether adelic quaternions and octonions could have some number theoretical meaning. Adelic quaternions and octonions are not number fields without additional assumptions since the moduli squared for a p-adic analog of quaternion and octonion can vanish so that the inverse fails to exist at the light-cone boundary which is 17-dimensional for complexified octonions and 7-dimensional for complexified quaternions. The reason is that norm squared is difference $N(o_1) - N(o_2)$ for $o_1 \oplus io_2$. This allows to define differential calculus for Taylor series and one can consider even rational functions. Hence the restriction is not fatal.

If one can pose a condition guaranteeing the existence of inverse for octonionic adel, one could define the multiplicative group of ideles for quaternions. For octonions one would obtain non-associative analog of the multiplicative group. If this kind of structures exist then four-dimensional associative/co-associative sub-manifolds in the space of non-associative ideles define associative/co-associative adeles in which ideles act. It is easy to find that octonionic ideles form 1-dimensional objects so that one must accept octonions with arbitrary real or p-adic components.

(c) What about equations for space-time surfaces. Do field equations reduce to separate field equations for each factor? Can one pose as an additional condition the constraint that p-adic surfaces provide in some sense cognitive representations of real space-time surfaces: this idea is formulated more precisely in terms of p-adic manifold concept [K97] (see the appendix of the book). Or is this correspondence an outcome of evolution?

Physical intuition would suggest that in most p-adic factors space-time surface corresponds to a point, or at least to a vacuum extremal. One can consider also the possibility that same algebraic equation describes the surface in various factors of the adele. Could this hold true in the intersection of real and p-adic worlds for which rationals appear in the polynomials defining the preferred extremals.

(d) To define field equations one must have the notion of derivative. Derivative is an operation involving division and can be tricky since adeles are not number field. The above argument suggests this is not actually a problem. Of course, if one can guarantee that the p-adic variants of octonions and quaternions are number fields, there are good hopes about well-defined derivative. Derivative as limiting value $df/dx = \lim(f(x+dx) - f(x))/dx$ for a function decomposing to Cartesian product of real function $f(x)$ and p-adic valued functions $f_p(x_p)$ would require that $f_p(x)$ is non-constant only for a finite number of primes: this is in accordance with the physical picture that only finite number of p-adic primes are active and define "cognitive representations" of real space-time surface. The second condition is that dx is proportional to product $dx \times \prod dx_p$ of differentials dx and dx_p, which are rational numbers. dx goes to xero as a real number but not p-adically for any of the primes involved. dx_p in turn goes to zero p-adically only for Q_p.

(e) The idea about rationals as points common to all number fields is central in number theoretical vision. This vision is realized for adeles in the minimal sense that the action of rationals is well-defined in all Cartesian factors of the adeles. Number theoretical vision allows also to talk about common rational points of real and various p-adic space-time surfaces in preferred coordinate choices made possible by symmetries of the imbedding space, and one ends up to the vision about life as something residing in the intersection of real and p-adic number fields. It is not clear whether and how adeles could allow to formulate this idea.

(f) For adelic variants of imbedding space spinors Cartesian product of real and p-adc variants of imbedding spaces is mapped to their tensor product. This gives justification for the physical vision that various p-adic physics appear as tensor factors. Does this mean that the generalized induced spinors are infinite tensor products of real and various p-adic spinors and Clifford algebra generated by induced gamma matrices is obtained by tensor product construction? Does the generalization of massless Dirac equation reduce to a sum of d'Alembertians for the factors? Does each of them annihilate the appropriate spinor? If only finite number of Cartesian factors corresponds to a space-time surface which is not vacuum extremal vanishing induced Kähler form, Kähler Dirac equation is non-trivial only in finite number of adelic factors.

3. Objections leading to the identification of octonionic adeles and ideles

The basic idea is that appropriately defined invertible quaternionic/octonionic adeles can be regarded as elements of Galois group assignable to quaternions/octonions. The best manner to proceed is to invent objections against this idea.

(a) The first objection is that p-adic quaternions and octonions do not make sense since p-adic variants of quaternions and octonions do not exist in general. The reason is that the p-adic norm squared $\sum x_i^2$ for p-adic variant of quaternion, octonion, or even complex number can vanish so that its inverse does not exist.

(b) Second objection is that automorphisms of the ring of quaternions (octonions) in the maximal Abelian extension are products of transformations of the subgroup of $SO(3)$ (G_2) represented by matrices with elements in the extension and in the Galois group of the extension itself. Ideles separate out as 1-dimensional Cartesian factor from this group so that one does not obtain 4-field (8-fold) Cartesian power of this Galois group.

One can define quaternionic/octonionic ideles in terms of rational quaternions/octonions multiplied by p-adic number. For adeles this condition produces non-sensical results.

(a) This condition indeed allows to construct the inverse of p-adic quaternion/octonion as a product of inverses for rational quaternion/octonion and p-adic number. The reason is that the solutions to $\sum x_i^2 = 0$ involve always p-adic numbers with an infinite number of pinary digits - at least one and the identification excludes this possibility. The ideles form also a group as required.

(b) One can interpret also the quaternionicity/octonionicity in terms of Galois group. The 7-dimensional non-associative counterparts for octonionic automorphisms act as transformations $x \rightarrow gxg^{-1}$. Therefore octonions represent this group like structure and the p-adic octonions would have interpretation as combination of octonionic automorphisms with those of rationals.

(c) One cannot assign to ideles 4-D idelic surfaces. The reason is that the non-constant part of all 8-coordinates is proportional to the same p-adic valued function of space-time point so that space-time surface would be a disjoint union of effectively 1-dimensional structures labelled by a subset of rational points of M^8. Induced metric would be 1-dimensional and induced Kähler and spinor curvature would vanish identically.

(d) One must allow p-adic octonions to have arbitrary p-adic components. The action of ideles representing Galois group on these surfaces is well-defined. Number field property is lost but this feature comes in play as poles only when one considers rational functions. Already the Minkowskian signature forces to consider complexified octonions and quaternions leading to the loss of field property. It would not be surprising if p-adic poles would be associated with the light-like orbits of partonic 2-surfaces. Both p-adic and Minkowskian poles might therefore be highly relevant physically and analogous to the poles of ordinary analytic functions. For instance, n-point functions could have poles at the light-like boundaries of causal diamonds and at light-like partonic orbits and explain their special physical role.

    The action of ideles in the quaternionic tangent space of space-time surface would be analogous to the action of of adelic linear group $Gl_n(A)$ in n-dimensional space.

(e) Adelic variants of octonions would be Cartesian products of ordinary and various p-adic octonions and would define a ring. Quaternionic 4-surfaces would define associative local sub-rings of octonion-adelic ring.

Chapter 7

Ideas Emerging from TGD

7.1 Introduction

I have gathered to this chapter those ideas related to quantum TGD which are not absolutely central and whose status is not clear in the long run. I have represented earlier these ideas in chapters and the outcome was a total chaos and reader did not have a slightest idea what is they real message. I hope that this organization of material makes it easier for the reader to grasp the topology of TGD correctly. The representation includes various ideas and notions such as weak form of electric magnetic duality, $M^8 - H$ duality, hierarchy of Planck constants, and the notion of number theoretic braid. Sections about twistor approach and octonionic spinors are included as well as considerations related to WCW integration and about possible topological invariances defined by geometric invariants for preferred extremals of Kähler action.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- Quantum theory [L36]
- Emergent ideas and notions [L17]
- Weak form of electric-magnetic duality [L51]
- $M^8 - H$ duality [L28]
- Hierarchy of Planck constants [L22]
- Hyperfinite factors and TGD [L24]
- The unique role of twistors in TGD [L45]
- Twistors and TGD [L47]

7.2 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B8] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K12] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

(a) The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

(b) This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

(c) The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

(d) The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

(e) One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

    The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural. Also Chern-Simons Dirac equation implies the localization of solutions to flow lines, and this is consistent with the localization solutions of Kähler-Dirac equation to string world sheets.

7.2.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

(a) The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

(b) Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

(c) A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

(d) To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{7.2.1}$$

A more general form of this duality is suggested by the considerations of [K31] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B2] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}\sqrt{g_4} \ . \tag{7.2.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

(e) Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} \ , \tag{7.2.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

(a) The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

(b) The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{7.2.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{7.2.5}$$

(c) The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \quad p = sin^2(\theta_W) \ . \end{aligned} \tag{7.2.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned}\alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ .\end{aligned} \quad (7.2.7)$$

(d) There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

(a) The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

(b) The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K51] supports this interpretation.

(c) The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

(d) The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n\times\frac{g_K^2}{\hbar}, n\in Z \ . \quad (7.2.8)$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{7.2.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

(a) Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{7.2.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

(b) For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

(c) The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K55]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

(a) The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.
(b) GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K72]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.
(c) Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.
(d) GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

7.2.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

(a) In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

(b) One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D3] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K23] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

(a) Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

(b) The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

(c) How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K39] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

(d) What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K40] .

7.2.3 Could Quantum TGD reduce to almost topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

(a) Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^{\alpha} A_{\alpha}$ plus and integral of the boundary term $J^{n\beta} A_{\beta} \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_{\beta} \sqrt{g_4}$ over the ends of the 3-surface.

(b) If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

(a) For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K4]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

(b) The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \ . \tag{7.2.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

(c) This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

(d) The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{7.2.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} = 0 \ . \tag{7.2.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B44] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K4] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

(e) There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 \ . \tag{7.2.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_\phi = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q^m_\phi = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

(f) The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

(g) A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

(h) Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to modified Dirac action as boundary term.

Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

7.2.4 About the notion of measurement interaction

The notion of measurement has been central notion in quantum TGD but the precise definition of this notion is far from clear. In the following two possibly equivalent formulations are considered. The first formulation relies on the gauge transformations leaving Coulomb term of Kähler action unchanged and the second one to the interpretation of TGD as a square root of thermodynamics allowing to fix the values of conserved classical charges for zero energy energy state using Lagrange multipliers analogous to chemical potentials.

(a) There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \to A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 \ . \tag{7.2.15}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J \phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

(b) The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action.

The gauge transformed Kähler potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function. Kähler function is identified as a Dirac determinant of Chern-Simons Dirac operator (after many turns and twists) and the spectrum of this operator should not be invariant under these gauge transformations if this picture is correct. This is is achieved if the gauge transformation is carried only in the Dirac action corresponding to instanton term: this assumption is motivated by the breaking of time reversal invariance induced by quantum measurements. The modification of Kähler action can be guessed to correspond just to the Chern-Simons contribution from the instanton term.

(c) A reasonable looking guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

In zero energy ontology (ZEO) TGD can be seen as square root of thermodynamics and this suggests an alternative manner to define what measurement interaction term means.

(a) The condition that the space-time sheets appearing in superposition of space-time surfaces with given quantum numbers in Cartan algebra have same classical quantum numbers associated with Kähler action can be realized in terms of Lagrange multipliers in standard manner. These kind of terms would be analogous to various chemical potential terms in the partition function. One could call them measurement interaction terms. Measurement interaction terms would code the values of quantum charges to the space-time geometry.

Kähler action contains also Chern-Simons term at partonic orbits compensating the Chern-Simons terms coming from Kähler action when weak form of electric-magnetic duality is assumed. This guarantees that Kähler action for preferred extremals reduces to Chern-Simons terms at the space-like ends of the spacetime surface and one obtains almost topological QFT.

(b) If Kähler-Dirac action is constructed from Kähler action in super-symmetric manner by defining the modified gamma matrices in terms of canonical momentum densities one obtains also the fermionic counterparts of the Lagrange multiplier terms at partonic orbits and could call also them measurement interaction terms. Besides this one has also the Chern-Simons Dirac terms associated with the partonic orbits giving ordinary massless Dirac propagator. In presence of measurement interaction terms at the space-like ends of the space-time surface the boundary conditions $\Gamma^n\Psi = 0$ at the ends would be modified by the addition of term coming from the modified gamma matrix associated with the Lagrange multiplier terms. The original generalized massless generalized eigenvalue spectrum $p^k\gamma_k$ of Γ^n would be modified to massive spectrum given by the condition

$$(\Gamma^n + \sum_i \lambda_i \Gamma^\alpha_{Q_i} D_\alpha)\Psi = 0 \ ,$$

where Q_i refers to i:th conserved charge.

An interesting question is whether these two manners to introduce measurement interaction terms are actually equivalent.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

7.2.5 Kähler action for Euclidian regions as Kähler function and Kähler action for Minkowskian regions as Morse function?

One of the nasty questions about the interpretation of Kähler action relates to the square root of the metric determinant. If one proceeds completely straightforwardly, the only reason conclusion is that the square root is imaginary in Minkowskian space-time regions so that Kähler action would be complex. The Euclidian contribution would have a natural interpretation as positive definite Kähler function but how should one interpret the imaginary Minkowskian contribution? Certainly the path integral approach to quantum field theories supports its presence. For some mysterious reason I was able to forget this nasty question and serious consideration of the obvious answer to it. Only when I worked between possibile connections between TGD and Floer homology [K82] I realized that the Minkowskian contribution is an excellent candidate for Morse function whose critical points give information about WCW homology. This would fit nicely with the vision about TGD as almost topological QFT.

Euclidian regions would guarantee the convergence of the functional integral and one would have a mathematically well-defined theory. Minkowskian contribution would give the quantal interference effects and stationary phase approximation. The analog of Floer homology would represent quantum superpositions of critical points identifiable as ground states defined by the extrema of Kähler action for Minkowskian regions. Perturbative approach to quantum TGD would rely on functional integrals around the extrema of Kähler function. One would have maxima also for the Kähler function but only in the zero modes not contributing to the WCW metric.

There is a further question related to almost topological QFT character of TGD. Should one assume that the reduction to Chern-Simons terms occurs for the preferred extremals in *both* Minkowskian and Euclidian regions or only in Minkowskian regions?

(a) All arguments for this have been represented for Minkowskian regions [K22] involve local light-like momentum direction which does not make sense in the Euclidian regions. This does not however kill the argument: one can have non-trivial solutions of Laplacian equation in the region of CP_2 bounded by wormhole throats: for CP_2 itself only covariantly constant right-handed neutrino represents this kind of solution and at the same time supersymmetry. In the general case solutions of Laplacian represent broken super-symmetries and should be in one-one correspondences with the solutions of the modified Dirac equation. The interpretation for the counterparts of momentum and polarization would be in terms of classical representation of color quantum numbers.

(b) If the reduction occurs in Euclidian regions, it gives in the case of CP_2 two 3-D terms corresponding to two 3-D gluing regions for three coordinate patches needed to define coordinates and spinor connection for CP_2 so that one would have two Chern-Simons terms. I have earlier claimed that without any other contributions the first term would be identical with that from Minkowskian region apart from imaginary unit and different coefficient. This statement is wrong since the space-like parts of the corresponding 3-surfaces are discjoint for Euclidian and Minkowskian regions.

(c) There is also an argument stating that Dirac determinant for Chern-Simons Dirac action equals to Kähler function, which would be lost if Euclidian regions would not obey holography. The argument obviously generalizes and applies to both Morse and Kähler function which are definitely not proportional to each other.

CP breaking and ground state degeneracy

The Minkowskian contribution of Kähler action is imaginary due to the negativity of the metric determinant and gives a phase factor to vacuum functional reducing to Chern-Simons terms at wormhole throats. Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

(a) In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

(b) A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.

(c) There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and shortlived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral measons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

7.2.6 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

Basic field equations

Consider first the equations at general level.

(a) The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T \times SO(3) \times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

(b) Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha \left[D_\beta (J^{\alpha\beta} H_A) - j_K^\alpha H^A + T^{\alpha\beta} j_A^l h_{kl} \partial_\beta h^l \right] = 0 \ . \tag{7.2.16}$$

The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha \left[j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \right] = 0 \ . \tag{7.2.17}$$

For energy one has $H_A = 1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

(c) One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j_K^\alpha J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j_K^\alpha D_\alpha H^A = j_K^\alpha J_\alpha{}^\beta j_\beta^A + T^{\alpha\beta} H_{\alpha\beta}^k j_k^A \ . \tag{7.2.18}$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

(a) The most general assumption is that the conserved isometry currents

$$J_A^\alpha = j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \tag{7.2.19}$$

and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

(b) The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \tag{7.2.20}$$

The conservation of J_A gives

$$d*(\Psi_A d\Phi_A) = 0 . \tag{7.2.21}$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

(c) The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d*d\Phi_A = 0 \ , \ d\Psi_A \cdot d\Phi_A = 0 . \tag{7.2.22}$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p \cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

(a) The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi . \tag{7.2.23}$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d*d\Phi = 0 \ , \ d\Psi_A \cdot d\Phi = 0. \tag{7.2.24}$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

(b) Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A\Psi_A d\Phi_{G(A)} \ , \ d*(d\Phi_G(A)) = 0 \ , \ d\Psi_A \cdot d\Phi_G(A)) = 0 . \tag{7.2.25}$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [K31] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

7.2.7 Hydrodynamic picture in fermionic sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the modified Dirac equation. This would mean that the solutions of Dirac equation can be localized to lower-dimensional surface or even flow lines.

Basic objection

The obvious objection against the localization to sub-manifolds is that it is not consistent with uncertainty principle in transversal degrees of freedom. More concretely, the assumption that the mode is localized to a lower-dimensional surface of X^4 implies that the action of the transversal part of Dirac operator in question acts on delta function and gives something singular.

The situation changes if the Dirac operator in question has vanishing transversal part at the lower-dimensional surface. This is not possible for the Dirac operator defined by the induced metric but is quite possible in the case of Kähler-Dirac operator. For instance, in the case of massless extremals Kähler-Dirac gamma matrices are non-vanishing in single direction only and the solution modes could be one-dimensional. For more general preferred extremals such as cosmic strings this is not the case.

In fact, there is a strong physical argument in favor of the localization of spinor modes at 2-D string world sheets so that hydrodynamical picture would result but with flow lines replaced with fermionic string world sheets.

(a) Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

(b) The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action and requires that the part of the Kähler-Dirac operator transversal to 2-surface vanishes.

(c) This localization does not hold for cosmic string solutions which however have 2-D CP_2 projection which should have vanishing weak fields so that 4-D spinor modes with well-defined em charge are possible.

(d) A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

4-dimensional modified Dirac equation and hydrodynamical picture

In following consideration is restricted to preferred extremals for which one has decomposition to regions characterized by local light-like vector and polarization direction. In this case one has good hopes that the modes can be restricted to 1-D light-like geodesics.

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

(a) The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)} \Gamma_k \ . \end{aligned} \tag{7.2.26}$$

The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^\alpha_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \tag{7.2.27}$$

The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D modified Dirac equation would reduce everything to initial values at partonic 2-surfaces.

(b) One might hope that the conservation of these super currents for all modes is equivalent with the modified Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

(c) The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

(a) The modified gamma matrices are of form $T^{\alpha}_k\Gamma^k$, $T^{\alpha}_k = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^{α}_k can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h^k_l = j^{Ak}j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k}j^A_k j^k_A = T^{\alpha A}j^k_A$. One can defined gamma matrices Γ_A as $\Gamma_k j^k_A$ to get $T^{\alpha}_k\Gamma^k = T^{\alpha A}\Gamma_A$.

(b) This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the modified Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA}j_A D_t\Psi = 0 . \tag{7.2.28}$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

7.3 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

7.3.1 The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

(a) The starting point was the proposal of Nottale [E2] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K59, K49] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

(b) Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K60] . TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

(c) The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

(d) Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K70] .

(e) The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E2] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

(f) Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L2, K70] , [L2] .

7.3.2 The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

(a) The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

(b) CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is unacceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   i. The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

   ii. The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

(c) For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

(d) The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

7.3.3 About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

(a) How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

(b) One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

(c) A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

7.3.4 How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

(a) One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

(b) If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

(c) The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

7.3.5 Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s}+1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} seem to be especially favored as values of n_a in living matter [K18] .

7.3.6 How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

7.3.7 Could the dynamics of Kähler action predict the hierarchy of Planck constants?

The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. The formula for the Planck constant involves heuristic guess work and physical plausibility arguments. There are good arguments in favor of the hypothesis that only coverings are possible. Only a finite number of pages of the Big Book correspond to a given value of Planck constant, biological evolution corresponds to a gradual dispersion to the pages of the Big Book with larger Planck constant, and a connection with the hierarchy of infinite primes and p-adicization program based on the mathematical realization of finite measurement resolution emerges.

One can however ask whether this hierarchy could emerge directly from the basic quantum TGD rather than as a separate hypothesis. The following arguments suggest that this might be possible. One finds also a precise geometric interpretation of preferred extremal property interpreted as criticality in zero energy ontology.

1-1 correspondence between canonical momentum densities and time derivatives fails for Kähler action

The basic motivation for the geometrization program was the observation that canonical quantization for TGD fails. To see what is involved let us try to perform a canonical quantization in zero energy ontology at the 3-D surfaces located at the light-like boundaries of $CD \times CP_2$.

(a) In canonical quantization canonical momentum densities $\pi_k^0 \equiv \pi_k = \partial L_K/\partial(\partial_0 h^k)$, where $\partial_0 h^k$ denotes the time derivative of imbedding space coordinate, are the physically natural quantities in terms of which to fix the initial values: once their value distribution is fixed also conserved charges are fixed. Also the weak form of electric-magnetic duality given by $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and a mild generalization of this condition to be discussed below can be interpreted as a manner to fix the values of conserved gauge charges (not Noether charges) to their quantized values since Kähler magnetic flux equals to the integer giving the homology class of the (wormhole) throat. This condition alone need not characterize criticality, which requires an infinite number of deformations of X^4 for which the second variation of the Kähler action vanishes and implies infinite number conserved charges. This in fact gives hopes of replacing π_k with these conserved Noether charges.

(b) Canonical quantization requires that $\partial_0 h^k$ in the energy is expressed in terms of π_k. The equation defining π_k in terms of $\partial_0 h^k$ is however highly non-linear although algebraic. By taking squares the equations reduces to equations for rational functions of $\partial_0 h^k$. $\partial_0 h^k$ appears in contravariant and covariant metric at most quadratically and in the induced Kähler electric field linearly and by multiplying the equations by $det(g_4)^3$ one can transform the equations to a polynomial form so that in principle $\partial_0 h^k$ can obtained as a solution of polynomial equations.

(c) One can always eliminate one half of the coordinates by choosing 4 imbedding space coordinates as the coordinates of the space-time surface so that the initial value conditions reduce to those for the canonical momentum densities associated with the remaining four coordinates. For instance, for space-time surfaces representable as map $M^4 \to CP_2$ M^4 coordinates are natural and the time derivatives $\partial_0 s^k$ of CP_2 coordinates are multi-valued. One would obtain four polynomial equations with $\partial_0 s^k$ as unknowns. In regions where CP_2 projection is 4-dimensional -in particular for the deformations of CP_2 vacuum extremals the natural coordinates are CP_2 coordinates and one can regard $\partial_0 m^k$ as unknowns. For the deformations of cosmic strings, which are of form $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, one can use coordinates of $M^2 \times S^2$, where S^2 is geodesic sphere as natural coordinates and regard as unknowns E^2 coordinates and remaining CP_2 coordinates.

(d) One can imagine solving one of the four polynomials equations for time derivaties in terms of other obtaining N roots. Then one would substitute these roots to the remaining 3 conditions to obtain algebraic equations from which one solves then second variable. Obviously situation is very complex without additional symmetries. The criticality of the preferred extremals might however give additional conditions allowing simplifications. The reasons for giving up the canonical quantization program was following. For the vacuum extremals of Kähler action π_k are however identically vanishing and this means that there is an infinite number of value distributions for $\partial_0 h^k$. For small deformations of vacuum extremals one might however hope a finite number of solutions to the conditions and thus finite number of space-time surfaces carrying same conserved charges.

If one assumes that physics is characterized by the values of the conserved charges one must treat the the many-valuedness of $\partial_0 h^k$. The most obvious guess is that one should replace the space of space-like 4-surfaces corresponding to different roots $\partial_0 h^k = F^k(\pi_l)$ with four-surfaces in the covering space of $CD \times CP_2$ corresponding to different branches of the many-valued function $\partial_0 h^k = F(\pi_l)$ co-inciding at the ends of CD.

Do the coverings forces by the many-valuedness of $\partial_0 h^k$ correspond to the coverings associated with the hierarchy of Planck constants?

The obvious question is whether this covering space actually corresponds to the covering spaces associated with the hierarchy of Planck constants. This would conform with quantum classical correspondence. The hierarchy of Planck constants and hierarchy of covering spaces was introduced to cure the failure of the perturbation theory at quantum level. At classical level the multi-valuedness of $\partial_0 h^k$ means a failure of perturbative canonical quantization and forces the introduction of the covering spaces. The interpretation would be that when the density of matter becomes critical the space-time surface splits to several branches so that the density at each branches is sub-critical. It is of course not at all obvious whether the proposed structure of the Big Book is really consistent with this hypothesis and one also consider modifications of this structure if necessary. The manner to proceed is by making questions.

(a) The proposed picture would give only single integer characterizing the covering. Two integers assignable to CD and CP_2 degrees of freedom are however needed. How these two coverings could emerge?

   i. One should fix also the values of $\pi^n_k = \partial L_K/\partial h^k_n$, where n refers to space-like normal coordinate at the wormhole throats. If one requires that charges do not flow between regions with different signatures of the metric the natural condition is $\pi^n_k = 0$ and allows also multi-valued solution. Since wormhole throats carry magnetic charge and since weak form of electric-magnetic duality is assumed, one can assume that CP_2 projection is four-dimensional so that one can use CP_2 coordinates and regard $\partial_0 m^k$ as un-knows. The basic idea about topological condensation in turn suggests that M^4 projection can be assumed to be 4-D inside space-like 3-surfaces so that here $\partial_0 s^k$ are the unknowns. At partonic 2-surfaces one would have conditions for both π^0_k and π^n_k. One might hope that the numbers of solutions are finite for preferred extremals because of their symmetries and given by n_a for $\partial_0 m^k$ and by n_b for $\partial_0 s^k$. The optimistic guess is that n_a and n_b corresponds to the numbers of sheets for singular coverings of CD and CP_2. The covering could be visualized as replacement of space-time surfaces with space-time surfaces which have $n_a n_b$ branches. n_b branches would degenerate to single branch at the ends of diagrams of the generaled Feynman graph and n_a branches would degenerate to single one at wormhole throats.

   ii. This picture is not quite correct yet. The fixing of π^0_k and π^n_k should relate closely to the effective 2-dimensionality as an additional condition perhaps crucial for criticality. One could argue that both π^0_k and π^n_k must be fixed at X^3 and X^3_l in order to effectively bring in dynamics in two directions so that X^3 could be interpreted as a an orbit of partonic 2-surface in space-like direction and X^3_l as its orbit in light-like direction. The additional conditions could be seen as gauge conditions made possible by symplectic and Kac-Moody type conformal symmetries. The conditions for π^k_0 would give n_b branches in CP_2 degrees of freedom and the conditions for π^n_k would split each of these branches to n_a branches.

   iii. The existence of these two kinds of conserved charges (possibly vanishing for π^n_k) could relate also very closely to the slicing of the space-time sheets by string world sheets and partonic 2-surfaces.

(b) Should one then treat these branches as separate space-time surfaces or as a single space-time surface? The treatment as a single surface seems to be the correct thing to do. Classically the conserved changes would be $n_a n_b$ times larger than for single branch. Kähler action need not (but could!) be same for different branches but the total action is $n_a n_b$ times the average action and this effectively corresponds to the replacement of the $\hbar_0/g_K^2$ factor of the action with $\hbar/g_K^2$, $r \equiv \hbar/\hbar_0 = n_a n_b$. Since the conserved quantum charges are proportional to $\hbar$ one could argue that $r = n_a n_b$ tells only that the charge conserved charge is $n_a n_b$ times larger than without multi-valuedness. $\hbar$ would be only effectively $n_a n_b$ fold. This is of course poor man's argument but might catch something essential about the situation.

(c) How could one interpret the condition $J^{03}\sqrt{g_4} = 4\pi\alpha_K J_{12}$ and its generalization to be discussed below in this framework? The first observation is that the total Kähler electric charge is by $\alpha_K \propto 1/(n_a n_b)$ same always. The interpretation would be in terms of charge fractionization meaning that each branch would carry Kähler electric charge $Q_K = ng_K/n_a n_b$. I have indeed suggested explanation of charge fractionization and quantum Hall effect based on this picture.

(d) The vision about the hierarchy of Planck constants involves also assumptions about imbedding space metric. The assumption that the M^4 covariant metric is proportional to $\hbar^2$ follows from the physical idea about $\hbar$ scaling of quantum lengths as what Compton length is. One can always introduce scaled M^4 coordinates bringing M^4 metric into the standard form by scaling up the M^4 size of CD. It is not clear whether the scaling up of CD size follows automatically from the proposed scenario. The basic question is why the M^4 size scale of the critical extremals must scale like $n_a n_b$? This should somehow relate to the weak self-duality conditions implying that Kähler field at each branch is reduced by a factor $1/r$ at each branch. Field equations should posses a dynamical symmetry involving the scaling of CD by integer k and $J^{0\beta}\sqrt{g_4}$ and $J^{n\beta}\sqrt{g_4}$ by $1/k$. The scaling of CD should be due to the scaling up of the M^4 time interval during which the branched light-like 3-surface returns back to a non-branched one.

(e) The proposed view about hierarchy of Planck constants is that the singular coverings reduce to single-sheeted coverings at $M^2 \subset M^4$ for CD and to $S^2 \subset CP_2$ for CP_2. Here S^2 is any homologically trivial geodesic sphere of CP_2 and has vanishing Kähler form. Weak self-duality condition is indeed consistent with any value of $\hbar$ and impies that the vacuum property for the partonic 2-surface implies vacuum property for the entire space-time sheet as holography indeed requires. This condition however generalizes. In weak self-duality conditions the value of $\hbar$ is free for any 2-D Lagrangian sub-manifold of CP_2.

The branching along M^2 would mean that the branches of preferred extremals always collapse to single branch when their M^4 projection belongs to M^2. Magnetically charged light-light-like throats cannot have M^4 projection in M^2 so that self-duality conditions for different values of $\hbar$ do not lead to inconsistencies. For space-like 3-surfaces at the boundaries of CD the condition would mean that the M^4 projection becomes light-like geodesic. Straight cosmic strings would have M^2 as M^4 projection. Also CP_2 type vacuum extremals for which the random light-like projection in M^4 belongs to M^2 would represent this of situation. One can ask whether the degeneration of branches actually takes place along any string like object $X^2 \times Y^2$, where X^2 defines a minimal surface in M^4. For these the weak self-duality condition would imply $\hbar = \infty$ at the ends of the string. It is very plausible that string like objects feed their magnetic fluxes to larger space-times sheets through wormhole contacts so that these conditions are not encountered.

Connection with the criticality of preferred extremals

Also a connection with quantum criticality and the criticality of the preferred extremals suggests itself. Criticality for the preferred extremals must be a property of space-like 3-surfaces and light-like 3-surfaces with degenerate 4-metric and the degeneration of the $n_a n_b$ branches of the space-time surface at the its ends and at wormhole throats is exactly what happens at criticality. For instance, in catastrophe theory roots of the polynomial equation giving extrema of a potential as function of control parameters co-incide at criticality. If this picture is correct the hierarchy of Planck constants would be an outcome of criticality and of preferred extremal property and preferred extremals would be just those multi-branched space-time surfaces for which branches co-incide at the the boundaries of $CD \times CP_2$ and at the throats.

7.3.8 Updated view about the hierarchy of Planck constants

The original hypothesis was that the hierarchy of Planck constants is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. In this formulation the singular covering of the imbedding space became only a convenient auxiliary tool. It is no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

The formulation based on multi-furcations of space-time surfaces to N branches. For some reason I assumed that they are simultaneously present. This is too restrictive an assumption. The N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural. As a matter fact, this interpretation was the original one, and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

(a) The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K71].

(b) Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K51] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

(c) In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [E2] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the modified Dirac equation. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices [K87] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

(a) The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

(b) The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

(c) In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

(d) For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elemenetary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

   This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar_m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

(a) The scaling $\hbar \rightarrow k\hbar$ in the formula $E_n = (n + 1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.

(b) It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

(a) The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement automatically.

(b) If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.

(c) Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

(a) In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

(b) The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionalization and fractionalization of spin.

(c) The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionalization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N + 1$:th branch corresponds to the original one. This suggests that angular momentum fractionalization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N - 1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N - 1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionalization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionalization however leads to problems with fractionalization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

(a) Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g_{eff}^{\alpha\beta}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the modified Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

(b) At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g^{kl}_{eff} = K^2 m^{kl}$.

One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g^{\alpha\beta}_{eff} p_\alpha p_\beta = g^{\alpha\beta}_{eff} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g^{kl}_{eff} p_k p_l = n^2 \frac{\hbar^2}{L^2} \quad . \tag{7.3.1}$$

Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g^{kl}_{eff} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2 \hbar^2}{K^2 L^2} \quad .$$

$\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.

(c) Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0} \frac{1}{L} \quad . \tag{7.3.2}$$

For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.

(d) Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

(e) One can consider similar equation for gravitational angular momentum:

$$g^{\alpha\beta}_{eff} L_\alpha L_\beta = g^{kl}_{eff} L_k L_l = l(l+1)\hbar^2 \quad . \tag{7.3.3}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{7.3.4}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m^{kl}_{eff} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m^{kl}_{eff}.

Could $h_{gr} = h_{eff}$ hold true?

The obvious question is whether the gravitational Planck constant deduced from the Nottale's considerations and the effective Planck constant $h_{eff} = nh$ deduced from ELF effects on vertebrate brain and explained in terms of non-determinism of Kähler action could be identical. At first this seems to be non-sensical idea since $h_{gr} = GMm/v_0$ has gigantic value.

It is however essential to realize that by Equivalence Principle one describe gravitational interaction by reducing it to elementary particle level. For instance, gravitational Compton lengths do not depend at all on the masses of particles. Also the radii of the planetary orbits are independent of the mass of particle mass in accordance with Equivalence Principle. For elementary particles the values of h_{gr} are in the same range as in quantum biological applications. Typically 10 Hz ELF radiation should correspond to energy $E = h_{eff}f$ of UV photon if one assumes that dark ELF photons have energies of biophotons and transform to them. The order of magnitude for n would be therefore $n \simeq 10^{14}$.

The experiments of M. Tajmar et al [E1, E3] discussed in [K94] provide a support for this picture. The value of gravimagnetic field needed to explain the findings is 28 orders of magnitude higher than theoretical value if one extrapolates the model of Meissner effect to gravimagnetic context. The amazing finding is that if one replaces Planck constant in the formula of gravimagnetic field with h_{gr} associated with Earth-Cooper pair system and assumes that the velocity parameter v_0 appearing in it corresponds to the Earth's rotation velocity around its axis, one obtains correct order of magnitude for the effect requiring $r \simeq 3.6 \times 10^{14}$.

The most important implications are in quantum biology and Penrose's vision about importance of quantum gravitation in biology might be correct.

(a) This result allows by Equivalence Principle the identification $h_{gr} = h_{eff}$ at elementary particle level at least so that the two views about hierarchy of Planck constants would be equivalent. If the identification holds true for larger units it requires that space-time sheet identifiable as quantum correlates for physical systems are macroscopically quantum coherent and gravitation causes this. If the values of Planck constant are really additive, the number of parallel space-time sheets corresponding to non-determinism evolution for the flux tube connecting systems with masses M and m is proportional to the masses M and m using Planck mass as unit. Information theoretic interpretation is suggestive since hierarchy of Planck constants is assumed to relate to negentropic entanglement very closely in turn providing physical correlate for the notions of rule and concept.

(b) That gravity would be fundamental for macroscopic quantum coherence would not be surprising since by EP all particles experience same acceleration in constant gravitational field, which therefore has tendency to create coherence unlike other basic interactions. This in principle allows to consider hierarchy in which the integers $h_{gr,i}$ are additive but give rise to the same universal dark Compton length.

(c) The model for quantum biology relying on the notions of magnetic body and dark matter as hierarchy of phases with $h_{eff} = nh$, and biophotons [K90, K89] identified as decay produces of dark photons. The assumption $h_{gr} \propto m$ becomes highly predictable since cyclotron frequencies would be independent of the mass of the ion.

   i. If dark photons with cyclotron frequencies decay to biophotons, one can conclude that biophoton spectrum reflects the spectrum of endogenous magnetic field strengths. In the model of EEG [K18] it has been indeed assumed that this kind spectrum is there: the inspiration came from music metaphors suggesting that musical scales are realized in terms of values of magnetic field strength. The new quantum physics associated with gravitation would also become key part of quantum biophysics in TGD Universe.

   ii. For the proposed value of h_{gr} 1 Hz cyclotron frequency associated to DNA sequences would correspond to ordinary photon frequency $f = 3.6 \times 10^{14}$ Hz and energy 1.2 eV just at the lower limit of visible frequencies. For 10 Hz alpha band the energy would be 12 eV in UV. This plus the fact that molecular energies are in eV range suggests very simple realization of biochemical control by magnetic body. Each ion has its own cyclotron frequency but same energy for the corresponding biophoton.

iii. Biophoton with a given energy would activate transitions in specific bio-molecules or atoms: ionization energies for atoms except hydrogen have lower bound about 5 eV (http://en.wikipedia.org/wiki/Ionization_energy). The energies of molecular bonds are in the range 2-10 eV (http://en.wikipedia.org/wiki/Bond-dissociation_energy). If one replaces v_0 with $2v_0$ in the estimate, DNA corresponds to .62 eV photon with energy of order metabolic energy currency and alpha band corresponds to 6 eV energy in the molecular region and also in the region of ionization energies. Each ion at its specific magnetic flux tubes with characteristic palette of magnetic field strengths would resonantly excite some set of biomolecules.This conforms with the earlier vision about dark photon frequencies as passwords.
It could be also that biologically important ions take care of their ionization self. This would be achieved if the magnetic field strength associated with their flux tubes is such that dark cyclotron energy equals to ionization energy. EEG bands labelled by magnetic field strengths could reflect ionization energies for these ions.

iv. The hypothesis means that the scale of energy spectrum of biophotons depends on the ratio M/v_0 of the planet and on the strength of the endogenous magnetic field, which is .2 Gauss for Earth (2/5 of the nominal value of the Earth's magnetic field). Therefore the astrophysical characteristics of planets should be tuned for molecular life. Taking v_0 to be rotational velocity one obtains for the ratio $M(planet)/v_0(planet)$ using the ratio for Earth as unit the following numbers for the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptune): $M/v_0 = (8.5, 209, 1, .214223, 1613, 6149, 9359)$. If the energy scale of biophotons is required to be the same, the scale of endogenous magnetic field should be divided by this ratio in order to obtain the same situation as in Earth. For instance, in Mars the magnetic field should be roughly 5 times stronger: in reality the magnetic field of Mars is much weaker. Just for fun one can notice that for Sun the ratio is 1.4×10^6 so that magnetic field should be by the inverse of this factor weaker.

(d) An interesting question is how large systems can behave as coherent units with $h_{gr} = GMm/v_0$. In living matter one might consider the possibility that entire organism might be this kind of system. Interestingly, for larger masses the gravitational quantum coherence would be easier. For particle with mass m $h_{gr}/h > 1$ requires larger mass to satisfy $M > M_P^2/m_e$. The first guess that life has evolved from long to shorter scales and reached elementary particle last. Planck mass is the critical mass corresponds to the mass of water blog with volume of size scale of 10^{-4} m (big neuron) is the limit.

(e) The Universal gravitational Compton wave length of $GM/v_0 \simeq 864$ meters gives an idea about largest possible living matter system if Earth is the second body. Of course, also other large bodies are possible.In the case of solar system this length is 3×10^3 km. The radius of Earth is 6.37×10^3 km - roughly twice the Compton length. The radii of Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptunus are (.38,.99, .533, 1, 10.6, 8.6, 4.0, 3.9) using Earth radius as unit the value of h_{gr} is by factor 5 larger than for three inner planets so that the values are reasonably near to gravitational Compton length or twice it. Does this mean that dark matter associated with Earth and maybe also other planets is in macroscopic quantum state at some level of the hierarchy of space-time sheets? Does this mean that Mother Gaia as conscious entity might make sense. One can of course make same question in the case of Sun. The universal gravitational Compton length in Sun would be 18 per cent of the radius of Sun if v_0 is taken to be the rotational velocity at the surface of Sun. The radius of solar core, where fusion takes place, is 20-25 per cent of solar radius.

(f) There are further interesting numerical co-incidences. One can for a moment forget the standard hostility of scientist towards horoscopes and ask whether Sun and Moon could have somehow affect our life via astrocopic quantum coherence. The gravitational Compton length for particle-Moon or particle-Sun system multiplied by the natural value of magnetic field is the relevant parameter. For Sun the parameters in question are mass of Sun, and rotational velocity of Earth with respect to Sun, plus magnetic fields of Sun at flux tubes associated with solar magnetic field measured to be about 5 nT at the position of Earth and 100 times stronger than expected from dipole field behavior. This gives that the range of biophoton energies is scaled down with factor of 1/4 in good approximation so that Father Sun might affect terrestrial biology! If one uses for the rotational velocity of particle at surface of Moon as parameter v_0 (particle would be at Moon), biophoton energy scaled scaled up by factor 1.2.

The general proposal discussed above is testable. In particular, a detailed study of molecular energies with those associated with resonances of EEG could be highly rewarding and reveal the speculated spectroscopy of consciousness.

Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and the hierarchy is only effective. Anyonic physics and effective charge fractionalization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [K87]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level , to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [K80] so that conformal dynamics represents conformal evolution) [K67].

7.4 Number theoretic braids and global view about anti-commutations of induced spinor fields

The anti-commutations of the induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-D partonic surface and the effective 2-dimensionality means that partonic 2-surfaces plus there 4-D tangent space take the role of fundamental dynamical objects. This is expressed concretely by the condition that the ends of the space-time surface and wormhole throats are extremals of Chern-Simons action. Conformal invariance would in turn make the 2-D partons 1-D objects (analogous to Euclidian strings) and braids, which can be regarded as the ends of string world sheets with Minkowskian signature, in turn would discretize these Euclidian strings. It must be however noticed that the status of Euclidian strings is uncertain.

Somehow these views should be unifiable into a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales.

(a) The notions of measurement resolution and braid concept indeed provides the needed physical insights in this respect. The precise definition of the notion of braid and its number theoretic counterpart has however remained open and I have considered several alternatives. The topological character of braid indeed allows flexibility in its definition but it would be nice to have some canonical definition with a clear physical meaning.

(b) It turned out that the braid concept emerges automatically from the localization of the modes of Kähler-Dirac action to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - with vanishing induced W fields and above weak scale also induced Z^0 fields. The boundaries of string world sheets can be identified as braids and string world sheets as 2-braids. Hence the identification of braids is unique although their topological character does not necessitate this. The attribute "number theoretic" would mean that the intersections of braids with partonic 2-surfaces corresponds to points with preferred imbedding space coordinates having values which are algebraic numbers in some extension of rational numbers. This selects preferred extremals among all extremals and they could perhaps be said to belong to the intersection of real and p-adic space-time sheets.

7.4.1 Quantization of the modified Dirac action and configuration space geometry

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. The naive approach would be based on standard canonical quantization of induced spinor fields by posing anti-commutation relations between Ψ and canonical momentum density $\partial L/\partial(\partial_t\Psi)$.

One can imagine two alternative forms of the anti-commutation relations.

(a) The standard canonical anti-commutation relations for the induced the spinor fields would be given by

$$\{\overline{\Psi}\hat{\Gamma}^0(x),\Psi(y)\} = \delta^2_{x,y} . \quad (7.4.1)$$

The factor that $\hat{\Gamma}^0(x)$ corresponds to the canonical momentum density associated with Kähler action. The discrete variant of the anti-commutation relations applying in the case of non-stringy space-time sheets is

$$\{\overline{\Psi}\hat{\Gamma}^0(x_i),\Psi(x_j)\} = \delta_{i,j} . \quad (7.4.2)$$

where x_i and x_j label the points of the number theoretic braid. These anti-commutations are are inconsistent at the limit of vacuum extremal and also extremely non-linear in the imbedding space coordinates.

(b) The construction of WCW gamma matrices leads to a nonsingular form of anti-commutation relations given by

$$\{\overline{\Psi}(x)\gamma^0,\Psi(x)\} = (1+K)J\delta_{x,y} . \quad (7.4.3)$$

Here J denotes the Kähler magnetic flux J_m and Kähler electric flux relates to via the formula $J_e = KJ_m$, where K is symplectic invariant. What is nice that at the limit of vacuum extremals the right hand side vanishes so that spinor fields become non-dynamical. Therefore this option- actually the original one - seems to be the only reasonable choice.

For the latter option the super counterparts of local flux Hamiltonians can be written in the form

$$\begin{aligned}
H_{A,+,n} &= H_{A,+,q,n} + H_{A,+,L,n} \ , \ H_{A,-,n} = H_{A,-,q,n} + H_{A,-,L,n} \ , \\
H_{A,+,q,n} &= \oint \overline{\Psi} J^A_+ q_n d^2x \ , \\
H_{A,-,q,n} &= \oint \overline{q}_n J^A_- \Psi d^2x \ , \\
H_{A,-,L,n} &= \oint \overline{\Psi} J^A_+ L_n d^2x \ , \\
H_{A,+,L,n} &= \oint \overline{L}_n J^A_- \Psi d^2x \ , \\
J^A_+ &= j^{Ak}\Gamma_k \ , \ J^A_- = j^{A\overline{k}}\Gamma_{\overline{k}} \ .
\end{aligned} \tag{7.4.4}$$

Suppose that there is a one-one correspondence between quark modes and leptonic modes is satisfied and the label n decomposes as $n = (m, i)$, where n labels a scalar function basis and i labels spinor components. This would give

$$\begin{aligned}
q_n = q_{m,i} &= \Phi_m q_i \ , \\
L_n = L_{m,i} &= \Phi_m L_i \ , \\
\overline{q}_i \gamma^0 q_j &= \overline{L}_i \gamma^0 L_j = g_{ij} \ .
\end{aligned} \tag{7.4.5}$$

Suppose that the inner products g_{ij} are constant. The simplest possibility is $g_{ij} = \delta_{ij}$ Under these assumptions the anti-commutators of the super-symmetric flux Hamiltonians give flux Hamiltonians.

$$\{H_{A,+,n}, H_{A,-,n}\} = g_{ij} \oint (1+K)\overline{\Phi}_m \Phi_n H_A J d^2x \ . \tag{7.4.6}$$

The product of scalar functions can be expressed as

$$\overline{\Phi}_m \Phi_n = c_{mn}^{\ \ k} \Phi_k \ . \tag{7.4.7}$$

Note that the notion of symplectic QFT led to a scalar function algebra of similar kind consisting of phase factors and there excellent reasons to consider the possibility that there is a deep connection with this approach.

One expects that the symplectic algebra is restricted to a direct sum of symplectic algebras localized to the regions where the induced Kähler form is non-vanishing implying that the algebras associated with different region form to a direct sum. Also the contributions to WCW metric are direct sums. The symplectic algebras associated with different region can be truncated to finite-dimensional spaces of symplectic algebras $S^2 \times S$ associated with the regions in question. As far as coordinatization of the reduced configuration space is considered, these symplectic sub-spaces are enough. These truncated algebras naturally correspond to the hyper-finite factor property of the Clifford algebra of WCW.

7.4.2 Expressions for WCW super-symplectic generators in finite measurement resolution

The expressions of WCW Hamiltonians and their super counterparts just discussed were based on 2-dimensional integrals. This is problematic for several reasons.

(a) In p-adic context integrals do not makes sense so that this representation fails in p-adic context. Sums would be more appropriate if one wants number theoretic universality at the level of basic formulas.

(b) The use of sums would also conform with the notion of finite measurement resolution having discretization in terms of intersections of X^2 with number theoretic braids as a space-time correlate.

(c) Number theoretic duality suggests a unique realization of the discretization in the sense that only the points of partonic 2-surface X^2 whose $\delta M^4_\pm$ projections commute in hyper-octonionic sense and thus belong to the intersections of the projection $P_{M^4}(X^2)$ with radial light-like geodesics $M_\pm$ representing intersections of $M^2 \subset M^4 \subset M^8$ with $\delta M^4_\pm \times CP_2$ contribute to WCW Hamiltonians and super Hamiltonians and therefore to the WCW metric.

Clearly, finite measurement resolution seems to be an unavoidable aspect of the geometrization of WCW as one can expect on basis of the fact that WCW Clifford algebra provides representation for hyper-finite factors of type II_1 whose inclusions provide a representation for the finite measurement resolution. This means that the infinite-dimensional WCW can be represented as a finite-dimensional space in arbitrary precise approximation so that also also configuration Clifford algebra and WCW spinor fields becomes finite-dimensional.

The modification of anti-commutation relations to this case is

$$\{\overline{\Psi}(x_m)\gamma^0, \Psi(x_n)\} = (1+K)J\delta_{x_m,x_n} . \tag{7.4.8}$$

Note that the constancy of γ^0 implies a complete symmetry between the two points. The number of points must be the maximal one consistent with the Kronecker delta type anti-commutation relations so that information is not lost.

The question arises about the choice of the points x_m. This choice should ge general coordinate invariant. As already described, the localization of the modes of the Kähler-Dirac action to 2-D surfaces resolves this problem: the points x_m correspond to points of imbedding space which in preferred imbedding space coordinates have values in some algebraic extension of rationals.

7.4.3 QFT description of particle reactions at the level of braids

The overall view conforms with zero energy ontology in which hierarchy of causal diamonds (CDs) within CDs gives rise to a hierarchy of generalized Feynman diagrams and geometric description of the radiative corrections. Each sub-CD gives also rise to to zero energy states and thus particle reactions in its own time scale so that improvement of the time resolution brings in also new physics as it does also in reality.

The natural question is what happens to the braids at vertices.

(a) The vision based on infinite primes led to the conclusion that the selection rules of arithmetic quantum field theory based on the conservation of the total number theoretic momentum $P = \sum n_i log(p_i)$ dictate the selection rules at the vertices. For given p_i the momentum $n_i log(p_i)$ can be shared between the outgoing lines and this allows several combinations of infinite primes in outgoing lines having interpretations in terms of singular coverings of CD and CP_2.

(b) What happens then to the braid strands? If the bosons and fermions with given p_i are shared between several outgoing particles, does this require that the braid strands replicate? Or is their number preserved if one regards each braid strand as having n_a *resp.* n_b copies at the sheets of the corresponding coverings? This is required by the conservation of number theoretic momentum if one accepts the connection between the hierarchy of Planck constants and infinite primes.

(c) The question raised already earlier is whether DNA replication could have a counterpart at the level of fundamental physics. The interpretation of the incoming lines of generalized Feynman diagram as representations of topological quantum computations and the virtual particle lines as representations of quantum communications would support this picture. The no-cloning theorem [B10] would hold true since exact copies of quantum states would not be possible by the conservation of the number theoretical momentum. One could however say that the bosonic occupation number n_i means the presence of n_i-fold copy of same piece of information so that the sharing of information by sharing the pages of the singular covering associated with n_i would be possible in the limits posed by the values of n_i. Note again that the identification $n_i = n_a$ or $n_i = n_b$ (two infinite primes characterize the quantum state) makes sense only if only one of the p-adic primes associated with the 3-surface is realized as a physical state since the identification forces the selection of the covering. The quantum model for DNA based on hierarchy of Planck constants [K73] inspires the question whether DNA replication could be actually accompanied by its proposed counterpart at the fundamental level defining the fundamental information transfer process.

(d) The localization of the quantum numbers to braid strands suggests that braid ends of a given braid continue to one particular line or more generally, are shared between several lines. This condition is quite strong since without additional quantization conditions the ends of the braids of outgoing particles do not co-incide with the ends of the incoming braid. These kind of quantization conditions would conform with the generalized Bohr orbit property of light-like 3-surfaces.

(e) Without these quantization conditions one meets the challenge of calculating the anti-commutators of fermionic oscillator operators associated with non-co-inciding points of the incoming and outgoing braids. This raises the question whether one should regard the quantizations of induced spinor fields based on the L_{min} as one possible gauge only and allow the variation of L_{min} in some limits. If these quantizations are equivalent, the fermionic oscillator operators would be unitarily related. How to deduce this unitary transformation would be the non-trivial problem and it seems that the simpler picture is much more attractive.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

7.4.4 How do generalized braid diagrams relate to the perturbation theory?

The association of generalized braid diagrams characterized by infinite primes to the incoming and outgoing partonic legs and internal lines of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The basic question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams.

(a) If one believes in perturbation theoretic approach, the basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancelation could occur only for critical values of Kähler coupling strength α_K: for general values of α_K the cancellation would require separate vanishing of each term in the sum and does not occur.

In perturbative approach the expression of Kähler function as Chern-Simons action could be used and propagator would correspond to the inverse of the 1-1 part of the second variation of the Chern-Simons action with respect to complex WCW coordinates evaluated allowing only the extrema of Chern-Simons action for the ends of space-time surface and for wormhole throats. One would have perturbation theory for a sum over maxima of Kähler function. From the expression of the Kähler function as Dirac determinant the maxima would correspond to the local minima of $L_p = \sqrt{p}L_{min}$ for a given infinite prime. The connection between Chern-Simons representation and Dirac determinant representation of Kähler function would be obviously highly desirable.

(b) The possibility to define WCW functional integral in terms of harmonic analysis for infinite-dimensional spaces leads to a non-perturbative approach to functional integration allowing also a generalization the p-adic context [K67] . In this approach there is no need to make additional assumptions.

For both cases the assignment of the collection of braids characterized by pairs of infinite primes allows to organize the generalized Feynman diagrams into a sum of generalized Feynman diagrams and for each diagram type the exponent of Kähler function - if given by the Dirac determinant- would be simply the product $\prod_i L_{p_i}^{-1}$, $L_p = \sqrt{p}L_{min}$. One should perform a sum over different infinite primes in the internal lines subject to the conservation of the total number theoretic momenta. The conservation of the incoming number theoretic momentum would allow only a finite number of configurations for the intermediate lines. For the approach based on harmonic analysis the expression of the Kähler function in terms of the Dirac determinant would be optimal since it is manifestly algebraic function.

Both approaches involve a perturbative summation in the sense of introducing sub-CDs with time scales coming as 2^{-n} powers of the time scale of CD defining the infrared cutoff.

(a) The addition of zero energy insertions corresponding to sub-CDs as radiative corrections allows to improve measurement resolution. Hence a connection with QFT type Feyman diagram expansion would be obtained and Connes tensor product would have a practical computational realization.

(b) The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

7.4.5 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

The condition $T_n = 2^nT_0$ would assign to the hierarchy of CDs as hierarchy of time scales coming as octaves. A weaker condition would be $T_p = pT_0$, p prime, and would assign all secondary p-adic time scales to the size scale hierarchy of CDs.

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution. Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^nT_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks like an attractive idea but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

(a) The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

(b) p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

(c) In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

7.5 Twistor revolution and TGD

Lubos Motl wrote a nice summary about the talk of Nima Arkani Hamed about twistor revolution in Strings 2012 and gave also a link to the talk [B21]. It seems that Nima and collaborators are ending to a picture about scattering amplitudes which strongly resembles that provided by generalized Feynman diagrammatics in TGD framework

TGD framework is much more general than $\mathcal{N} = 4$ SYM and is to it same as general relativity for special relativity whereas the latter is completely explicit. Of course, I cannot hope that TGD view could be taken seriously - at least publicly. One might hope that these approaches could be combined some day: both have a lot to give for each other. Below I compare these approaches.

The recent approach below emerges from the study of preferred extremals of Kähler and solutions of the modified Dirac equations so that it begins directly from basic TGD whereas the approaches hitherto have been based on general arguments and the precise role of right-handed neutrino has remained enigmatic. Chapters "Construction of quantum TGD: Symmetries" [K15] and "The recent vision about preferred extremals and solutions of the modified Dirac equation" [K87] contain section explaining how super-conformal and Yangian algebras crucial for the Grassmannian approach emerge from the basic TGD.

7.5.1 The origin of twistor diagrammatics

In TGD framework zero energy ontology forces to replace the idea about continuous unitary evolution in Minkowski space with something more general assignable to causal diamonds (CDs), and S-matrix is replaced with a square root of density matrix equal to a hermitian l square root of density matrix multiplied by unitary S-matrix. Also in twistor approach unitarity has ceased to be a star actor. In p-Adic context continuous unitary time evolution fails to make sense also mathematically.

Twistor diagrammatics involves only massless on mass shell particles on both external and internal lines. Zero energy ontology (ZEO) requires same in TGD: wormhole lines carry parallelly moving massless fermions and anti-fermions. The mass shell conditions at vertices are enormously powerful and imply UV finiteness. Also IR finiteness follows if external particles are massive.

What one means with mass is however a delicate matter. What does one mean with mass? I have pondered 35 years this question and the recent view is inspired by p-adic mass calculations and ZEO, and states that observed mass is in a well-defined sense expectation value of longitudinal mass squared for all possible choices of $M^2 \subset M^4$ characterizing the choices of quantization axis for energy and spin at the level of "world of classical worlds" (WCW) assignable with given causal diamond CD.

The choice of quantization axis thus becomes part of the geometry of WCW. All wormhole throats are massless but develop non-vanishing longitudinal mass squared. Gauge bosons correspond to wormhole contacts and thus consist of pairs of massless wormhole throats. Gauge bosons could develop 4-D mass squared but also remain massless in 4-D sense if the throats have parallel massless momenta. Longitudinal mass squared is however non-vanishing and p-adic thermodynamics predicts it.

7.5.2 The emergence of 2-D sub-dynamics at space-time level

Nima et al introduce ordering of the vertices in 4-D case. Ordering and related braiding are however essentially 2-D notions. Somehow 2-D theory must be a part of the 4-D theory also at space-time level, and I understood that understanding this is the challenge of the twistor approach at this moment.

The twistor amplitude can be represented as sum over the permutations of n external gluons and all diagrams corresponding to the same permutation are equivalent. Permutations are more like braidings since they carry information about how the permutation proceeded as a homotopy. Yang-Baxter equation emerges and states associativity of the braid group. The allowed braidings are minimal braidings in the sense that the repetitions of permutations of two adjacent vertices are not considered to be separate. Minimal braidings reduce to ordinary permutations. Nima also talks about affine braidings which I interpret as analogs of Kac-Moody algebras meaning that one uses projective representations which for Kac-Moody algebra mean non-trivial central extension. Perhaps the condition is that the square of a permutation permuting only two vertices which each other gives only a non-trivial phase factor. Lubos suggests an alternative interpretation which would select only special permutations and cannot be therefore correct.

There are rules of identifying the permutation associated with a given diagram involving only basic 3-gluon vertex with white circle and its conjugate. Lubos explains this "Mickey Mouse in maze" rule in his posting in detail: to determine the image $p(n)$ of vertex n in the permutation put a mouse in the maze defined by the diagram and let it run around obeying single rule: if the vertex is black turn to the right and if the vertex is white turn to the left. The mouse cannot remain in a loop: if it would do so, the rule would force it to run back to n after single full loop and one would have a fixed point: $p(n) = n$. The reduction in the number of diagrams is enormous: the infinity of different diagrams reduces to $n!$ diagrams!

What happens in TGD framework?

(a) In TGD framework string world sheets and partonic 2-surfaces (or either or these if they are dual notions as conjectured) at space-time surface would define the sought for 2-D theory, and one obtains indeed perturbative expansion with fermionic propagator defined by the inverse of the modified Dirac operator and bosonic propagator defined by the correlation function for small deformations of the string world sheet. The vertices of twistor diagrams emerge as braid ends defining the intersections of string world sheets and partonic 2-surfaces.

String model like description becomes part of TGD and the role of string world sheets in X^4 is highly analogous to that of string world sheets connecting branes in $AdS^5 \times S^5$ of $\mathcal{N} = 4$ SYM. In TGD framework 10-D $AdS^5 \times S^5$ is replaced with 4-D space-time surface in $M^4 \times CP_2$. The meaning of the analog of AdS^5 duality in TGD framework should be understood. In particular, it could it be that the descriptions involving string world sheets on one hand and partonic 2-surfaces - or 3-D orbits of wormhole throats defining the generalized Feynman diagram- on the other hand are dual to each other. I have conjectured something like this earlier but it takes some time for this kind of issues to find their natural answer.

(b) As described in the article, string world sheets and partonic 2-surfaces emerge directly from the construction of the solutions of the modified Dirac equation by requiring conservation of em charge. This result has been conjectured already earlier but using other less direct arguments. 2-D "string world sheets" as sub-manifolds of the space-time surface make the ordering possible, and guarantee the finiteness of the perturbation theory involving n-point functions of a conformal QFT for fermions at wormhole throats and n-point functions for the deformations of the space-time surface. Conformal invariance should dictate these n-point functions to a high degree. In TGD framework the fundamental 3-vertex corresponds to joining of light-like orbits of three wormhole contacts along their 2-D ends (partonic 2-surfaces).

7.5.3 The emergence of Yangian symmetry

Yangian symmetry associated with the conformal transformations of M^4 is a key symmetry of Grassmannian approach. Is it possible to derive it in TGD framework?

(a) TGD indeed leads to a concrete representation of Yangian algebra as generalization of color and electroweak gauge Kac-Moody algebra using general formula discussed in Witten's article about Yangian algebras (see the article).

(b) Article discusses also a conjecture about 2-D Hodge duality of quantized YM gauge potentials assignable to string world sheets with Kac-Moody currents. Quantum gauge potentials are defined only where they are needed - at string world sheets rather than entire 4-D space-time.

(c) Conformal scalings of the effective metric defined by the anti-commutators of the modified gamma matrices emerge as realization of quantum criticality. They are induced by critical deformations (second variations not changing Kähler action) of the space-time surface. This algebra can be generalized to Yangian using the formulas in Witten's article (see the article).

(d) Critical deformations induce also electroweak gauge transformations and even more general symmetries for which infinitesimal generators are products of U(n) generators permuting n modes of the modified Dirac operator and infinitesimal generators of local electro-weak gauge transformations. These symmetries would relate in a natural manner to finite measurement resolution realized in terms of inclusions of hyperfinite factors with included algebra taking the role of gauge group transforming to each other states not distinguishable from each other.

(e) How to end up with Grassmannian picture in TGD framework? This has inspired some speculations in the past. From Nima's lecture one however learns that Grassmannian picture emerges as a convenient parameterization. One starts from the basic 3-gluon vertex or its conjugate expressed in terms of twistors. Momentum conservation implies that with the three twistors λ_i or their conjugates are proportional to each other (depending on which is the case one assigns white or black dot with the vertex). This constraint can be expressed as a delta function constraint by introducing additional integration variables and these integration variables lead to the emergence of the Grassmannian $G_{n,k}$ where n is the number of gluons, and k the number of positive helicity gluons.

Since only momentum conservation is involved, and since twistorial description works because only massless on mass shell virtual particles are involved, one is bound to end up with the Grassmannian description also in TGD.

7.5.4 The analog of AdS^5 duality in TGD framework

The generalization of AdS^5 duality of $\mathcal{N}=4$ SYMs to TGD framework is highly suggestive and states that string world sheets and partonic 2-surfaces play a dual role in the construction of M-matrices. Some terminology first.

(a) Let us agree that string world sheets and partonic 2-surfaces refer to 2-surfaces in the slicing of space-time region defined by Hermitian structure or Hamilton-Jacobi structure.

(b) Let us also agree that *singular* string world sheets and partonic 2-surfaces are surfaces at which the *effective* metric defined by the anti-commutators of the modified gamma matrices degenerates to effectively 2-D one.

(c) Braid strands at wormhole throats in turn would be loci at which the *induced* metric of the string world sheet transforms from Euclidian to Minkowskian as the signature of induced metric changes from Euclidian to Minkowskian.

AdS^5 duality suggest that string world sheets are in the same role as string world sheets of 10-D space connecting branes in AdS^5 duality for $\mathcal{N}=4$ SYM. What is important is that there should exist a duality meaning two manners to calculate the amplitudes. What the duality could mean now?

(a) Also in TGD framework the first manner would be string model like description using string world sheets. The second one would be a generalization of conformal QFT at light-like 3-surfaces (allowing generalized conformal symmetry) defining the lines of generalized Feynman diagram. The correlation functions to be calculated would have points at the intersections of partonic 2-surfaces and string world sheets and would represent braid ends.

(b) General Coordinate Invariance (GCI) implies that physics should be codable by 3-surfaces. Light-like 3-surfaces define 3-surfaces of this kind and same applies to space-like 3-surfaces. There are also preferred 3-surfaces of this kind. The orbits of 2-D wormhole throats at which 4-metric degenerates to 3-dimensional one define preferred light-like 3-surfaces. Also the space-like 3-surfaces at the ends of space-time surface at light-like boundaries of causal diamonds (CDs) define preferred space-like 3-surfaces. Both light-like and space-like 3-surfaces should code for the same physics and therefore their intersections defining partonic 2-surfaces plus the 4-D tangent space data at them should be enough to code for physics. This is strong form of GCI implying effective 2-dimensionality. As a special case one obtains singular string world sheets at which the effective metric reduces to 2-dimensional and singular partonic 2-surfaces defining the wormhole throats. For these 2-surfaces situation could be especially simple mathematically.

(c) The guess inspired by strong GCI is that string world sheet -partonic 2-surface duality holds true. The functional integrals over the deformations of 2 kinds of 2-surfaces should give the same result so that functional integration over either kinds of 2-surfaces should be enough. Note that the members of a given pair in the slicing intersect at discrete set of points and these points define braid ends carrying fermion number. Discretization and braid picture follow automatically.

(d) Scattering amplitudes in the twistorial approach could be thus calculated by using *any* pair in the slicing - or only either member of the pair if the analog of AdS^5 duality holds true as argued. The possibility to choose any pair in the slicing means general coordinate invariance as a symmetry of the Kähler metric of WCW and of the entire theory suggested already early: Kähler functions for difference choices in the slicing would differ by a real part of holomorphic function and give rise to same Kähler metric of "world of classical worlds" (WCW). For a general pair one obtains functional integral over deformations of space-time surface inducing deformations of 2-surfaces with only other kind 2-surface contributing to amplitude. This means the analog of stringy QFT: Minkowskian or Euclidian string theory depending on choice.

(e) For singular string world sheets and partonic 2-surfaces an enormous simplification results. The propagators for fermions and correlation functions for deformations reduce to 1-D instead of being 2-D: the propagation takes place only along the light-like lines at which the string world sheets with Euclidian signature (inside CP_2 like regions) change to those with Minkowskian signature of induced metric. The local reduction of space-time dimension would be very real for particles moving along sub-manifolds at which higher dimensional space-time has reduced metric dimension: they cannot get out from lower-D sub-manifold. This is like ending down to 1-D black hole interior and one would obtain the analog of ordinary Feynman diagrammatics. This kind of Feynman diagrammatics involving only braid strands is what I have indeed ended up earlier so that it seems that I can trust good intuition combined with a sloppy mathematics sometimes works;-).

These singular lines represent orbits of point like particles carrying fermion number at the orbits of wormhole throats. Furthermore, in this representation the expansions coming from string world sheets and partonic 2-surfaces are identical automatically. This follows from the fact that only the light-like lines connecting points common to singular string world sheets and singular partonic 2-surfaces appear as propagator lines!

(f) The TGD analog of AdS^5 duality of $\mathcal{N} = 4$ SUSYs would be trivially true as an identity in this special case, and the good guess is that it is true also generally. One could indeed use integral over either string world sheets or partonic 2-sheets to deduce the amplitudes.

What is important to notice that singularities of Feynman diagrams crucial for the Grassmannian approach of Nima and others would correspond at space-time level 2-D singularities of the effective metric defined by the modified gamma matrices defined as contractions of canonical momentum currents for Kähler action with ordinary gamma matrices of the imbedding space and therefore directly reflecting classical dynamics.

7.5.5 Problems of the twistor approach from TGD point of view

Twistor approach has also its problems and here TGD suggests how to proceed. Signature problem is the first problem.

(a) Twistor diagrammatics works in a strict mathematical sense only for $M^{2,2}$ with metric signature (1,1,-1,-1) rather than M^4 with metric signature (1,-1,-1,-1). Metric signature is wrong in the physical case. This is a real problem which must be solved eventually.

(b) Effective metric defined by anti-commutators of the modified gamma matrices (to be distinguished from the induced gamma matrices) could solve that problem since it would have the correct signature in TGD framework (see the article). String world sheets and partonic 2-surfaces would correspond to the 2-D singularities of this effective metric at which the even-even signature (1,1,1,1) changes to even-even signature (1,1,-1,-1). Space-time at string world sheet would become locally 2-D with respect to effective metric just as space-time becomes locally 3-D with respect to the induced metric at the light-like orbits of wormhole throats. String world sheets become also locally 1-D at light-like curves at which Euclidian signature of world sheet in induced metric transforms to Minkowskian.

(c) Twistor amplitudes are indeed singularities and string world sheets implied in TGD framework by conservation of em charge would represent these singularities at space-time level. At the end of the talk Nima conjectured about lower-dimensional manifolds of space-time as representation of space-time singularities. Note that string world sheets and partonic 2-surfaces have been part of TGD for years. TGD is of course to $\mathcal{N} = 4$

SYM what general relativity is for the special relativity. Space-time surface is dynamical and possesses induced and effective metrics rather than being flat.

Second limitation is that twistor diagrammatics works only for planar diagrams. This is a problem which must be also fixed sooner or later.

(a) This perhaps dangerous and blasphemous statement that I will regret it some day but I will make it;-). Nima and others have not yet discovered that $M^2 \subset M^4$ must be there but will discover it when they begin to generalize the results to non-planar diagrams and realize that Feynman diagrams are analogous to knot diagrams in 2-D plane (with crossings allowed) and that this 2-D plane must correspond to $M^2 \subset M^4$. The different choices of causal diamond CD correspond to different choices of M^2 representing choice of quantization axes 4-momentum and spin. The integral over these choices guarantees Lorentz invariance. Gauge conditions are modified: longitudinal M^2 projection of massless four-momentum is orthogonal to polarization so that three polarizations are possible: states are massive in longitudinal sense.

(b) In TGD framework one replaces the lines of Feynman diagrams with the light-like 3-surfaces defining orbits of wormhole throats. These lines carry many fermion states defining braid strands at light-like 3-surfaces. There is internal braiding associated with these braid strands. String world sheets connect fermions at different wormhole throats with space-like braid strands. The M^2 projections of generalized Feynman diagrams with 4-D "lines" replaced with genuine lines define the ordinary Feynman diagram as the analog of braid diagram. The conjecture is that one can reduce non-planar diagrams to planar diagrams using a procedure analogous to the construction of knot invariants by un-knotting the knot in Alexandrian manner by allowing it to be cut temporarily.

(c) The permutations of string vertices emerge naturally as one constructs diagrams by adding to the interior of polygon sub-polygons connected to the external vertices. This corresponds to the addition of internal partonic two-surfaces. There are very many equivalent diagrams of this kind. Only permutations matter and the permutation associated with a given diagram of this kind can be deduced by the Mickey-Mouse rule described explicitly by Lubos. A connection with planar operads is highly suggestive and also conjecture already earlier in TGD framework.

7.5.6 Could $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM be a part of TGD after all?

Whether right-handed neutrinos generate a supersymmetry in TGD has been a long standing open question. $\mathcal{N} = 1$ SUSY is certainly excluded by fermion number conservation but already $\mathcal{N} = 2$ defining a "complexification" of $\mathcal{N} = 1$ SUSY is possible and could generate right-handed neutrino and its antiparticle. These states should however possess a non-vanishing light-like momentum since the fully covariantly constant right-handed neutrino generates zero norm states. So called massless extremals (MEs) allow massless solutions of the modified Dirac equation for right-handed neutrino in the interior of space-time surface, and this seems to be case quite generally in Minkowskian signature for preferred extremals. This suggests that particle represented as magnetic flux tube structure with two wormhole contacts sliced between two MEs could serve as a starting point in attempts to understand the role of right handed neutrinos and how $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM emerges at the level of space-time geometry. The following arguments inspired by the article of Nima Arkani-Hamed et al [B32] about twistorial scattering amplitudes suggest a more detailed physical interpretation of the possible SUSY associated with the right-handed neutrinos.

The fact that right handed neutrinos have only gravitational interaction suggests a radical re-interpretation of SUSY: no SUSY breaking is needed since it is very difficult to distinguish between mass degenerate spartners of ordinary particles. In order to distinguish between different spartners one must be able to compare the gravitomagnetic energies of spartners in slowly varying external gravimagnetic field: this effect is extremely small.

Scattering amplitudes and the positive Grassmannian

The work of Nima Arkani-Hamed and others represents something which makes me very optimistic and I would be happy if I could understand the horrible technicalities of their work. The article Scattering Amplitudes and the Positive Grassmannian by Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, and Trnka [B32] summarizes the recent situation in a form, which should be accessible to ordinary physicist. Lubos has already discussed the article. The following considerations do not relate much to the main message of the article (positive Grassmannians) but more to the question how this approach could be applied in TGD framework.

1. All scattering amplitudes have on shell amplitudes for massless particles as building bricks

The key idea is that all planar amplitudes can be constructed from on shell amplitudes: all virtual particles are actually real. In zero energy ontology I ended up with the representation of TGD analogs of Feynman diagrams using only mass shell massless states with both positive and negative energies. The enormous number of kinematic constraints eliminates UV and IR divergences and also the description of massive particles as bound states of massless ones becomes possible.

In TGD framework quantum classical correspondence requires a space-time correlate for the on mass shell property and it indeed exists. The mathematically ill-defined path integral over all 4-surfaces is replaced with a superposition of preferred extremals of Kähler action analogous to Bohr orbits, and one has only a functional integral over the 3-D ends at the light-like boundaries of causal diamond (Euclidian/Minkowskian space-time regions give real/imaginary Chern-Simons exponent to the vacuum functional). This would be obviously the deeper principle behind on mass shell representation of scattering amplitudes that Nima and others are certainly trying to identify. This principle in turn reduces to general coordinate invariance at the level of the world of classical worlds.

Quantum classical correspondence and quantum ergodicity would imply even stronger condition: the quantal correlation functions should be identical with classical correlation functions for any preferred extremal in the superposition: all preferred extremals in the superposition would be statistically equivalent [K87]. 4-D spin glass degeneracy of Kähler action however suggests that this is is probably too strong a condition applying only to building bricks of the superposition.

Minimal surface property is the geometric counterpart for masslessness and the preferred extremals are also minimal surfaces: this property reduces to the generalization of complex structure at space-time surfaces, which I call Hamilton-Jacobi structure for the Minkowskian signature of the induced metric. Einstein Maxwell equations with cosmological term are also satisfied.

2. Massless extremals and twistor approach

The decomposition $M^4 = M^2 \times E^2$ is fundamental in the formulation of quantum TGD, in the number theoretical vision about TGD, in the construction of preferred extremals, and for the vision about generalized Feynman diagrams. It is also fundamental in the decomposition of the degrees of string to longitudinal and transversal ones. An additional item to the list is that also the states appearing in thermodynamical ensemble in p-adic thermodynamics correspond to four-momenta in M^2 fixed by the direction of the Lorentz boost. In twistor approach to TGD the possibility to decompose also internal lines to massless states at parallel space-time sheets is crucial.

Can one find a concrete identification for $M^2 \times E^2$ decomposition at the level of preferred extremals? Could these preferred extremals be interpreted as the internal lines of generalized Feynman diagrams carrying massless momenta? Could one identify the mass of particle predicted by p-adic thermodynamics with the sum of massless classical momenta assignable to two preferred extremals of this kind connected by wormhole contacts defining the elementary particle?

Candidates for this kind of preferred extremals indeed exist. Local $M^2 \times E^2$ decomposition and light-like longitudinal massless momentum assignable to M^2 characterizes "massless extremals" (MEs, "topological light rays"). The simplest MEs correspond to single space-time sheet carrying a conserved light-like M^2 momentum. For several MEs connected by wormhole contacts the longitudinal massless momenta are not conserved anymore but their sum defines a time-like conserved four-momentum: one has a bound states of massless MEs. The stable wormhole contacts binding MEs together possess Kähler magnetic charge and serve as building bricks of elementary particles. Particles are necessary closed magnetic flux tubes having two wormhole contacts at their ends and connecting the two MEs.

The sum of the classical massless momenta assignable to the pair of MEs is conserved even when they exchange momentum. Quantum classical correspondence requires that the conserved classical rest energy of the particle equals to the prediction of p-adic mass calculations. The massless momenta assignable to MEs would naturally correspond to the massless momenta propagating along the internal lines of generalized Feynman diagrams assumed in zero energy ontology. Masslessness of virtual particles makes also possible twistor approach. This supports the view that MEs are fundamental for the twistor approach in TGD framework.

3. Scattering amplitudes as representations for braids whose threads can fuse at 3-vertices

Just a little comment about the content of the article. The main message of the article is that non-equivalent contributions to a given scattering amplitude in $\mathcal{N} = 4$ SYM represent elements of the group of permutations of external lines - or to be more precise - decorated permutations which replace permutation group S_n with $n!$ elements with its decorated version containing $2^n n!$ elements. Besides 3-vertex the basic dynamical process is permutation having the exchange of neighboring lines as a generating permutation completely analogous to fundamental braiding. BFCW bridge has interpretation as a representations for the basic braiding operation.

This supports the TGD inspired proposal (TGD as almost topological QFT) that generalized Feynman diagrams are in some sense also knot or braid diagrams allowing besides braiding operation also two 3-vertices [K32]. The first 3-vertex generalizes the standard stringy 3-vertex but with totally different interpretation having nothing to do with particle decay: rather particle travels along two paths simultaneously after $1 \to 2$ decay. Second 3-vertex generalizes the 3-vertex of ordinary Feynman diagram (three 4-D lines of generalized Feynman diagram identified as Euclidian space-time regions meet at this vertex). The main idea is that in TGD framework knotting and braiding emerges at two levels.

(a) At the level of space-time surface string world sheets at which the induced spinor fields (except right-handed neutrino [K87]) are localized due to the conservation of electric charge can form 2-knots and can intersect at discrete points in the generic case. The boundaries of strings world sheets at light-like wormhole throat orbits and at space-like 3-surfaces defining the ends of the space-time at light-like boundaries of causal diamonds can form ordinary 1-knots, and get linked and braided. Elementary particles themselves correspond to closed loops at the ends of space-time surface and can also get knotted (possible effects are discussed in [K32]).

(b) One can assign to the lines of generalized Feynman diagrams lines in M^2 characterizing given causal diamond. Therefore the 2-D representation of Feynman diagrams has concrete physical interpretation in TGD. These lines can intersect and what suggests itself is a description of non-planar diagrams (having this kind of intersections) in terms of an algebraic knot theory. A natural guess is that it is this knot theoretic operation which allows to describe also non-planar diagrams by reducing them to planar ones as one does when one constructs knot invariant by reducing the knot to a trivial one. Scattering amplitudes would be basically knot invariants.

"Almost topological" has also a meaning usually not assigned with it. Thurston's geometrization conjecture stating that geometric invariants of canonical representation of manifold as Riemann geometry, defined topological invariants, could generalize somehow. For instance, the geometric invariants of preferred extremals could be seen as topological or more refined invariants (symplectic, conformal in the sense of 4-D generalization of conformal structure). If quantum ergodicity holds true, the statistical geometric invariants defined by the classical correlation functions of various induced classical gauge fields for preferred extremals could be regarded as this kind of invariants for sub-manifolds. What would distinguish TGD from standard topological QFT would be that the invariants in question would involve length scale and thus have a physical content in the usual sense of the word!

Could $\mathcal{N}$ =2 or $\mathcal{N}=4$ SUSY have something to do with TGD?

$\mathcal{N}=4$ SYM has been the theoretical laboratory of Nima and others. $\mathcal{N}=4$ SYM is definitely a completely exceptional theory, and one cannot avoid the question whether it could in some sense be part of fundamental physics. In TGD framework right handed neutrinos have remained a mystery: whether one should assign space-time SUSY to them or not. Could they give rise to something resembling $\mathcal{N}=2$ or $\mathcal{N}=4$ SUSY with fermion number conservation?

1. Earlier results

My latest view is that *fully* covariantly constant right-handed neutrinos decouple from the dynamics completely. I will repeat first the earlier arguments which consider only fully covariantly constant right-handed neutrinos.

(a) $\mathcal{N}=1$ SUSY is certainly excluded since it would require Majorana property not possible in TGD framework since it would require superposition of left and right handed neutrinos and lead to a breaking of lepton number conservation. Could one imagine SUSY in which both MEs between which particle wormhole contacts reside have $\mathcal{N}=2$ SUSY which combine to form an $\mathcal{N}=4$ SUSY?

(b) Right-handed neutrinos which are covariantly constant right-handed neutrinos in both M^4 degrees of freedom cannot define a non-trivial theory as shown already earlier. They have no electroweak nor gravitational couplings and carry no momentum, only spin.

    The fully covariantly constant right-handed neutrinos with two possible helicities at given ME would define representation of SUSY at the limit of vanishing light-like momentum. At this limit the creation and annihilation operators creating the states would have vanishing anti-commutator so that the oscillator operators would generate Grassmann algebra. Since creation and annihilation operators are hermitian conjugates, the states would have zero norm and the states generated by oscillator operators would be pure gauge and decouple from physics. This is the core of the earlier argument demonstrating that $\mathcal{N}=1$ SUSY is not possible in TGD framework: LHC has given convincing experimental support for this belief.

2. Could massless right-handed neutrinos covariantly constant in CP_2 degrees of freedom define $\mathcal{N}=2$ or $\mathcal{N}=4$ SUSY?

Consider next right-handed neutrinos, which are covariantly constant in CP_2 degrees of freedom but have a light-like four-momentum. In this case fermion number is conserved but this is consistent with $\mathcal{N}=2$ SUSY at both MEs with fermion number conservation. $\mathcal{N}=2$ SUSYs could emerge from $\mathcal{N}=4$ SUSY when one half of SUSY generators annihilate the states, which is a basic phenomenon in supersymmetric theories.

(a) At space-time level right-handed neutrinos couple to the space-time geometry - gravitation - although weak and color interactions are absent. One can say that this coupling forces them to move with light-like momentum parallel to that of ME. At the level of space-time surface right-handed neutrinos have a spectrum of excitations of four-dimensional analogs of conformal spinors at string world sheet (Hamilton-Jacobi structure).

    For MEs one indeed obtains massless solutions depending on longitudinal M^2 coordinates only since the induced metric in M^2 differs from the light-like metric only by a contribution which is light-like and contracts to zero with light-like momentum in the same direction. These solutions are analogs of (say) left movers of string theory. The dependence on E^2 degrees of freedom is holomorphic. That left movers are only possible would suggest that one has only single helicity and conservation of fermion number at given space-time sheet rather than 2 helicities and non-conserved fermion number: two real Majorana spinors combine to single complex Weyl spinor.

(b) At imbedding space level one obtains a tensor product of ordinary representations of $\mathcal{N}=2$ SUSY consisting of Weyl spinors with opposite helicities assigned with the ME. The state content is same as for a reduced $\mathcal{N}=4$ SUSY with four $\mathcal{N}=1$ Majorana spinors replaced by two complex $\mathcal{N}=2$ spinors with fermion number conservation. This gives 4 states at both space-time sheets constructed from ν_R and its antiparticle. Altogether the two MEs give 8 states, which is one half of the 16 states of $\mathcal{N}=4$ SUSY so that a degeneration of this symmetry forced by non-Majorana property is in question.

3. Is the dynamics of $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM possible in right-handed neutrino sector?

Could $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM be a part of quantum TGD? Could TGD be seen a fusion of a degenerate $\mathcal{N}=4$ SYM describing the right-handed neutrino sector and string theory like theory describing the contribution of string world sheets carrying other leptonic and quark spinors? Or could one imagine even something simpler?

What is interesting that the net momenta assigned to the right handed neutrinos associated with a pair of MEs would correspond to the momenta assignable to the particles and obtained by p-adic mass calculations. It would seem that right-handed neutrinos provide a representation of the momenta of the elementary particles represented by wormhole contact structures. Does this mimicry generalize to a full duality so that all quantum numbers and even microscopic dynamics of defined by generalized Feynman diagrams (Euclidian space-time regions) would be represented by right-handed neutrinos and MEs? Could a generalization of $\mathcal{N}=4$ SYM with non-trivial gauge group with proper choices of the ground states helicities allow to represent the entire microscopic dynamics?

Irrespective of the answer to this question one can compare the TGD based view about supersymmetric dynamics with what I have understood about $\mathcal{N}=4$ SYM.

(a) In the scattering of MEs induced by the dynamics of Kähler action the right-handed neutrinos play a passive role. Modified Dirac equation forces them to adopt the same direction of four-momentum as the MEs so that the scattering reduces to the geometric scattering for MEs as one indeed expects on basic of quantum classical correspondence. In ν_R sector the basic scattering vertex involves four MEs and could be a re-sharing of the right-handed neutrino content of the incoming two MEs between outgoing two MEs respecting fermion number conservation. Therefore $\mathcal{N}=4$ SYM with fermion number conservation would represent the scattering of MEs at quantum level.

(b) $\mathcal{N}=4$ SUSY would suggest that also in the degenerate case one obtains the full scattering amplitude as a sum of permutations of external particles followed by projections to the directions of light-like momenta and that BCFW bridge represents the analog of fundamental braiding operation. The decoration of permutations means that each external line is effectively doubled. Could the scattering of MEs can be interpreted in terms of these decorated permutations? Could the doubling of permutations by decoration relate to the occurrence of pairs of MEs?

One can also revert these questions. Could one construct massive states in $\mathcal{N}=4$ SYM using pairs of momenta associated with particle with integer label k and its decorated copy with label $k+n$? Massive external particles obtained in this manner as bound states of massless ones could solve the IR divergence problem of $\mathcal{N}=4$ SYM.

(c) The description of amplitudes in terms of leading singularities means picking up of the singular contribution by putting the fermionic propagators on mass shell. In the recent case it would give the inverse of massless Dirac propagator acting on the spinor at the end of the internal line annihilating it if it is a solution of Dirac equation.

The only way out is a kind of cohomology theory in which solutions of Dirac equation represent exact forms. Dirac operator defines the exterior derivative d and virtual lines correspond to non-physical helicities with $d\Psi \neq 0$. Virtual fermions would be on mass-shell fermions with non-physical polarization satisfying $d^2\Psi = 0$. External particles would be those with physical polarization satisfying $d\Psi = 0$, and one can say that the Feynman diagrams containing physical helicities split into products of Feynman diagrams containing only non-physical helicities in internal lines.

(d) The fermionic states at wormhole contacts should define the ground states of SUSY representation with helicity +1/2 and -1/2 rather than spin 1 or -1 as in standard realization of $\mathcal{N} = 4$ SYM used in the article. This would modify the theory but the twistorial and Grassmannian description would remain more or less as such since it depends on light-likeness and momentum conservation only.

4. 3-vertices for sparticles are replaced with 4-vertices for MEs

In $\mathcal{N} = 4$ SYM the basic vertex is on mass-shell 3-vertex which requires that for real light-like momenta all 3 states are parallel. One must allow complex momenta in order to satisfy energy conservation and light-likeness conditions. This is strange from the point of view of physics although number theoretically oriented person might argue that the extensions of rationals involving also imaginary unit are rather natural.

The complex momenta can be expressed in terms of two light-like momenta in 3-vertex with one real momentum. For instance, the three light-like momenta can be taken to be p, k, and $p - ka$ with $k = ap_R$. Here p (incoming momentum) and p_R are real light-like momenta satisfying $p \cdot p_R = 0$ but with opposite sign of energy, and a is complex number. What is remarkable that also the negative sign of energy is necessary also now.

Should one allow complex light-like momenta in TGD framework? One can imagine two options.

(a) Option I: no complex momenta. In zero energy ontology the situation is different due to the presence of a pair of MEs meaning replaced of 3-vertices with 4-vertices or 6-vertices, the allowance of negative energies in internal lines, and the fact that scattering is of sparticles is induced by that of MEs. In the simplest vertex a massive external particle with non-parallel MEs carrying non-parallel light-like momenta can decay to a pair of MEs with light-like momenta. This can be interpreted as 4-ME-vertex rather than 3-vertex (say) BFF so that complex momenta are not needed. For an incoming boson identified as wormhole contact the vertex can be seen as BFF vertex.

To obtain space-like momentum exchanges one must allow negative sign of energy and one has strong conditions coming from momentum conservation and light-likeness which allow non-trivial solutions (real momenta in the vertex are not parallel) since basically the vertices are 4-vertices. This reduces dramatically the number of graphs. Note that one can also consider vertices in which three pairs of MEs join along their ends so that 6 MEs (analog of 3-boson vertex) would be involved.

(b) Option II: complex momenta are allowed. Proceeding just formally, the $\sqrt{g_4}$ factor in Kähler action density is imaginary in Minkowskian and real in Euclidian regions. It is now clear that the formal approach is correct: Euclidian regions give rise to Kähler function and Minkowskian regions to the analog of Morse function. TGD as almost topological QFT inspires the conjecture about the reduction of Kähler action to boundary terms proportional to Chern-Simons term. This is guaranteed if the condition $j^\mu_K A_\mu = 0$ holds true: for the known extremals this is the case since Kähler current j_K is light-like or vanishing for them. This would seem that Minkowskian and Euclidian regions provide dual descriptions of physics. If so, it would not be surprising if the real and complex parts of the four-momentum were parallel and in constant proportion to each other.

This argument suggests that also the conserved quantities implied by the Noether theorem have the same structure so that charges would receive an imaginary contribution from Minkowskian regions and a real contribution from Euclidian regions (or vice versa). Four-momentum would be complex number of form $P = P_M + iP_E$. Generalized light-likeness condition would give $P_M^2 = P_E^2$ and $P_M \cdot P_E = 0$. Complexified momentum would have 6 free components. A stronger condition would be $P_M^2 = 0 = P_E^2$ so that one would have two light-like momenta "orthogonal" to each other. For both relative signs energy P_M and P_E would be actually parallel: parameterization would be in terms of light-like momentum and scaling factor. This would suggest that complex momenta do not bring in anything new and Option II reduces effectively to Option I. If one wants a complete analogy with the usual twistor approach then $P_M^2 = P_E^2 \neq 0$ must be allowed.

5. Is SUSY breaking possible or needed?

It is difficult to imagine the breaking of the proposed kind of SUSY in TGD framework, and the first guess is that all these 4 super-partners of particle have identical masses. p-Adic thermodynamics does not distinguish between these states and the only possibility is that the p-adic primes differ for the spartners. But is the breaking of SUSY really necessary? Can one really distinguish between the 8 different states of a given elementary particle using the recent day experimental methods?

(a) In electroweak and color interactions the spartners behave in an identical manner classically. The coupling of right-handed neutrinos to space-time geometry however forces the right-handed neutrinos to adopt the same direction of four-momentum as MEs has. Could some gravitational effect allow to distinguish between spartners? This would be trivially the case if the p-adic mass scales of spartners would be different. Why this should be the case remains however an open question.

(b) In the case of unbroken SUSY only spin distinguishes between spartners. Spin determines statistics and the first naive guess would be that bosonic spartners obey totally different atomic physics allowing condensation of selectrons to the ground state. Very probably this is not true: the right-handed neutrinos are de-localized to 4-D MEs and other fermions correspond to wormhole contact structures and 2-D string world sheets.

The coupling of the spin to the space-time geometry seems to provide the only possible manner to distinguish between spartners. Could one imagine a gravimagnetic effect with energy splitting proportional to the product of gravimagnetic moment and external gravimagnetic field B? If gravimagnetic moment is proportional to spin projection in the direction of B, a non-trivial effect would be possible. Needless to say this kind of effect is extremely small so that the unbroken SUSY might remain undetected.

(c) If the spin of sparticle be seen in the classical angular momentum of ME as quantum classical correspondence would suggest then the value of the angular momentum might allow to distinguish between spartners. Also now the effect is extremely small.

6. What can one say about scattering amplitudes?

One expect that scattering amplitudes factorize with the only correlation between right-handed neutrino scattering and ordinary particle scattering coming from the condition that the four-momentum of the right-handed neutrino is parallel to that of massless extremal of more general preferred extremal having interpretation as a geometric counterpart of radiation quantum. This momentum is in turn equal to the massless four-momentum associated with the space-time sheet in question such that the sum of classical four-momenta associated with the space-time sheets equals to that for all wormhole throats involved. The right-handed neutrino amplitude itself would be simply constant. This certainly satisfies the SUSY constraint and it is actually difficult to find other candidates for the amplitude. The dynamics of right-handed neutrinos would be therefore that of spectator following the leader.

Right-handed neutrino as inert neutrino?

7.5.7 Right-handed neutrino as inert neutrino?

There is a very interesting posting by Jester in Resonaances with title How many neutrinos in the sky? [C1]. Jester tells about the recent 9 years WMAP data [C7] and compares it with earlier 7 years data. In the earlier data the effective number of neutrino types was $N_{eff} = 4.34 \pm 0.87$ and in the recent data it is $N_{eff} = 3.26 \pm 0.35$. WMAP alone would give $N_{eff} = 3.89 \pm 0.67$ also in the recent data but also other data are used to pose constraints on N_{eff}.

To be precise, N_{eff} could include instead of fourth neutrino species also some other weakly interacting particle. The only criterion for contributing to N_{eff} is that the particle is in thermal equilibrium with other massless particles and thus contributes to the density of matter considerably during the radiation dominated epoch.

Jester also refers to the constraints on N_{eff} from nucleosynthesis, which show that $N_{eff} \sim 4$ us slightly favored although the entire range $[3, 5]$ is consistent with data.

It seems that the effective number of neutrinos could be 4 instead of 3 although latest WMAP data combined with some other measurements favor 3. Later a corrected version of the eprint appeared [C7] telling that the original estimate of N_{eff} contained a mistake and the correct estimate is $N_{eff} = 3.84 \pm 0.40$.

An interesting question is what $N_{eff} = 4$ could mean in TGD framework?

(a) One poses to the modes of the modified Dirac equation the following condition: electric charge is conserved in the sense that the time evolution by modified Dirac equation does not mix a mode with a well-defined em charge with those with different em charge. The implication is that all modes except pure right handed neutrino are restricted at string world sheets. The first guess is that string world sheets are minimal surfaces of space-time surface (rather than those of imbedding space). One can also consider minimal surfaces of imbedding space but with effective metric defined by the anti-commutators of the modified gamma matrices. This would give a direct physical meaning for this somewhat mysterious effective metric.

    For the neutrino modes localized at string world sheets mixing of left and right handed modes takes place and they become massive. If only 3 lowest genera for partonic 2-surfaces are light, one has 3 neutrinos of this kind. The same applies to all other fermion species. The argument for why this could be the case relies on simple observation [K13]: the genera g=0,1,2 have the property that they allow for all values of conformal moduli Z_2 as a conformal symmetry (hyper-ellipticity). For $g > 2$ this is not the case. The guess is that this additional conformal symmetry is the reason for lightness of the three lowest genera.

(b) Only purely right-handed neutrino is completely de-localized in 4-volume so that one cannot assign to it genus of the partonic 2-surfaces as a topological quantum number and it effectively gives rise to a fourth neutrino very much analogous to what is called sterile neutrino. De-localized right-handed neutrinos couple only to gravitation and in case of massless extremals this forces them to have four-momentum parallel to that of ME: only massless modes are possible. Very probably this holds true for all preferred extremals to which one can assign massless longitudinal momentum direction which can vary with spatial position.

(c) The coupling of ν_R is to gravitation alone and all electroweak and color couplings are absent. According to standard wisdom de-localized right-handed neutrinos cannot be in thermal equilibrium with other particles. This according to standard wisdom. But what about TGD?

    One should be very careful here: de-localized right-handed neutrinos is proposed to give rise to SUSY (not $\mathcal{N} = 1$ requiring Majorana fermions) and their dynamics is that of passive spectator who follows the leader. The simplest guess is that the dynamics of right handed neutrinos at the level of amplitudes is completely trivial and thus trivially supersymmetric. There are however correlations between four-momenta.

    i. The four-momentum of ν_R is parallel to the light-like momentum direction assignable to the massless extremal (or more general preferred extremal). This direct coupling to the geometry is a special feature of the modified Dirac operator and thus of sub-manifold gravity.

ii. On the other hand, the sum of massless four-momenta of two parallel pieces of preferred extremals is the - in general massive - four-momentum of the elementary particle defined by the wormhole contact structure connecting the space-time sheets (which are glued along their boundaries together since this is seems to be the only manner to get rid of boundary conditions requiring vacuum extremal property near the boundary). Could this direct coupling of the four-momentum direction of right-handed neutrino to geometry and four-momentum directions of other fermions be enough for the right handed neutrinos to be counted as a fourth neutrino species in thermal equilibrium? This might be the case!

One cannot of course exclude the coupling of 2-D neutrino at string world sheets to 4-D purely right handed neutrinos analogous to the coupling inducing a mixing of sterile neutrino with ordinary neutrinos. Also this could help to achieve the thermal equilibrium with 2-D neutrino species.

Experimental evidence for sterile neutrino?

Many physicists are somewhat disappointed to the results from LHC: the expected discovery of Higgs has been seen as the main achievement of LHC hitherto. Much more was expected. To my opinion there is no reason for disappointment. The exclusion of the standard SUSY at expected energy scale is very far reaching negative result. Also the fact that Higgs mass is too small to be stable without fine tuning is of great theoretical importance. The negative results concerning heavy dark matter candidates are precious guidelines for theoreticians. The non-QCD like behavior in heavy ion collisions and proton-ion collisions is bypassed my mentioning something about AdS/CFT correspondence and non-perturbative QCD effects. I tend to see these effects as direct evidence for M_{89} hadron physics [K40].

In any case, something interesting has emerged quite recently. Resonaances tells that the recent analysis [C6] of X-ray spectrum of galactic clusters claims the presence of monochromatic 3.5 keV photon line. The proposed interpretation is as a decay product of sterile 7 keV neutrino transforming first to a left-handed neutrino and then decaying to photon and neutrino via a loop involving W boson and electron. This is of course only one of the many interpretations. Even the existence of line is highly questionable.

One of the poorly understood aspects of TGD is right-handed neutrino, which is obviously the TGD counterpart of the inert neutrino.

(a) The old idea is that covariantly constant right handed neutrino could generate $\mathcal{N}=2$ super-symmetry in TGD Universe. In fact, all modes of induced spinor field would generate superconformal symmetries but electroweak interactions would break these symmetries for the modes carrying non-vanishing electroweak quantum numbers: they vanish for ν_R. This picture is now well-established at the level of WCW geometry [K98]: super-conformal generators are labelled angular momentum and color representations plus two conformal weights: the conformal weight assignable to the light-like radial coordinate of light-cone boundary and the conformal weight assignable to string coordinate. It seems that these conformal weights are independent. The third integer labelling the states would label genuinely Yangian generators: it would tell the poly-locality of the generator with locus defined by partonic 2-surface: generators acting on single partonic 2-surface, 2 partonic 2-surfaces, ...

(b) It would seem that even the SUSY generated by ν_R must be badly broken unless one is able to invent dramatically different interpretation of SUSY. The scale of SUSY breaking and thus the value of the mass of right-handed neutrino remains open also in TGD. In lack of better one could of course argue that the mass scale must be CP_2 mass scale because right-handed neutrino mixes considerably with the left-handed neutrino (and thus becomes massive) only in this scale. But why this argument does not apply also to left handed neutrino which must also mix with the right-handed one!

(c) One can of course criticize the proposed notion of SUSY: wonder whether fermion + extremely weakly interacting ν_R at same wormhole throat (or interior of 3-surface) can behave as single coherent entity as far spin is considered [K85]?

(d) The condition that the modes of induced spinor field have a well-defined electromagnetic charge eigenvalue [K87] requires that they are localized at 2-D string world sheets or partonic 2-surfaces: without this condition classical W boson fields would mix the em charged and neutral modes with each other. Right-handed neutrino is an exception since it has no electroweak couplings. Unless right-handed neutrino is covariantly constant, the modified gamma matrices can however mix the right-handed neutrino with the left handed one and this can induce transformation to charged mode. This does not happen if each modified gamma matrix can be written as a linear combination of either M^4 or CP_2 gamma matrices and modified Dirac equation is satisfied separately by M^4 and CP_2 parts of the modified Dirac equation.

(e) Is the localization of the modes other than covariantly constant neutrino to string world sheets a consequence of dynamics or should one assume this as a separate condition? If one wants similar localization in space-time regions of Euclidian signature - for which CP_2 type vacuum extremal is a good representative - one must assume it as a separate condition. In number theoretic formulation string world sheets/partonic 2-surfaces would be commutative/co-commutative sub-manifolds of space-time surfaces which in turn would be associative or co-associative sub-manifolds of imbedding space possessing (hyper-)octonionic tangent space structure. For this option also right-handed neutrino would be localized to string world sheets. Right-handed neutrino would be covariantly constant only in 2-D sense.

One can consider the possibility that ν_R is de-localized to the entire 4-D space-time sheet. This would certainly modify the interpretation of SUSY since the number of degrees of freedom would be reduced for ν_R.

(f) Non-covariantly constant right-handed neutrinos could mix with left-handed neutrinos but not with charged leptons if the localization to string world sheets is assumed for modes carrying non-vanishing electroweak quantum numbers. This would make possible the decay of right-handed to neutrino plus photon, and one cannot exclude the possibility that ν_R has mass 7 keV.

Could this imply that particles and their spartners differ by this mass only? Could it be possible that practically unbroken SUSY could be there and we would not have observed it? Could one imagine that sfermions have annihilated leaving only states consisting of fundamental fermions? But shouldn't the total rate for the annihilation of photons to hadrons be two times the observed one? This option does not sound plausible.

What if one assumes that given sparticle is charactrized by the same p-adic prime as corresponding particle but is dark in the sense that it corresponds to non-standard value of Planck constant. In this case sfermions would not appear in the same vertex with fermions and one could escape the most obvious contradictions with experimental facts. This leads to the notion of shadron: shadrons would be [K85] obtained by replacing quarks with dark squarks with nearly identical masses. I have asked whether so called X and Y bosons having no natural place in standard model of hadron could be this kind of creatures.

The interpretation of 3.5 keV photons as decay products of right-handed neutrinos is of course totally ad hoc. Another TGD inspired interpretation would be as photons resulting from the decays of excited nuclei to their ground state.

(a) Nuclear string model [L2] predicts that nuclei are string like objects formed from nucleons connected by color magnetic flux tubes having quark and antiquark at their ends. These flux tubes are long and define the "magnetic body" of nucleus. Quark and antiquark have opposite em charges for ordinary nuclei. When they have different charges one obtains exotic state: this predicts entire spectrum of exotic nuclei for which statistic is different from what proton and neutron numbers deduced from em charge and atomic weight would suggest. Exotic nuclei and large values of Planck constant could make also possible cold fusion [K19].

(b) What the mass difference between these states is, is not of course obvious. There is however an experimental finding [C8] (see *Analysis of Gamma Radiation from a Radon*

Source: Indications of a Solar Influence) that nuclear decay rates oscillate with a period of year and the rates correlate with the distance from Sun. A possible explanation is that the gamma rays from Sun in few keV range excite the exotic nuclear states with different decay rate so that the average decay rate oscillates [L2]. Note that nuclear excitation energies in keV range would also make possible interaction of nuclei with atoms and molecules.

(c) This allows to consider the possibility that the decays of exotic nuclei in galactic clusters generates 3.5 keV photons. The obvious question is why the spectrum would be concentrated at 3.5 keV in this case (second question is whether the energy is really concentrated at 3.5 keV: a lot of theory is involved with the analysis of the experiments). Do the energies of excited states depend on the color bond only so that they would be essentially same for all nuclei? Or does single excitation dominate in the spectrum? Or is this due to the fact that the thermal radiation leaking from the core of stars excites predominantly single state? Could $E = 3.5$ keV correspond to the maximum intensity for thermal radiation in stellar core? If so, the temperature of the exciting radiation would be about $T \simeq E/3 \simeq 1.2 \times 10^7$ K. This in the temperature around which formation of Helium by nuclear fusion has begun: the temperature at solar core is around 1.57×10^7 K.

7.6 Octo-twistors and twistor space

The basic problem of the twistor approach is that one cannot represent massive momenta in terms of twistors in an elegant manner. One can also consider generalization of the notion of spinor and twistor. I have proposed a possible representation of massive states based on the existence of preferred plane of M^2 in the basic definition of theory allowing to express four-momentum as one of two light-like momenta allowing twistor description. One could however ask whether some more elegant representation of massive M^4 momenta might be possible by generalizing the notion of twistor -perhaps by starting from the number theoretic vision.

The basic idea is obvious: in quantum TGD massive states in M^4 can be regarded as massless states in M^8 and $M^4 \times CP_2$ (recall $M^8 - H$ duality). One can therefore map any massive M^4 momentum to a light-like M^8 momentum and hope that this association could be made in a unique manner. One should assign to a massless 8-momentum an 8-dimensional spinor of fixed chirality. The spinor assigned with the light-like four-momentum is not unique without additional conditions. The existence of covariantly constant right-handed neutrino in CP_2 degrees generating the super-conformal symmetries could allow to eliminate the non-uniqueness. 8-dimensional twistor in M^8 would be a pair of this kind of spinors fixing the momentum of massless particle and the point through which the corresponding light-geodesic goes through: the set of these points forms 8-D light-cone and one can assign to each point a spinor. In $M^4 \times CP_2$ definitions makes also in the case of $M^4 \times CP_2$ and twistor space would also now be a lifting of the space of light-like geodesics.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the modified gamma matrices both in M^8 and H.

The basic challenge is to achieve twistorial description of four-momenta or even $M^4 \times CP_2$ quantum numbers: this applies both to the momenta of fundamental fermions at the lines of generalized Feynman diagrams and to the massive incoming and outcoming states identified as their composites.

(a) A rather attractive way to overcome the problem at the level of fermions propagating along the braid strands at the light-like orbits of partonic 2-surfaces relies on the assumption that generalized Feynman diagrammatics effectively reduces to a form in which all fermions in the propagator lines are massless although they can have non-physical helicity [K58]. One can use ordinary M^4 twistors. This is consistent with the idea that space-time surfaces are quaternionic sub-manifolds of octonionic imbedding space.

(b) Incoming and outgoing states are composites of massless fermions and not massless. They are however massless in 8-D sense. This suggests that they could be described using generalization of twistor formalism from M^4 to M^8 and even betterm to $M^4 \times CP_2$.

In the following two possible twistorializations are considered.

7.6.1 Two manners to twistorialize imbedding space

In the following the generalization of twistor formalism for M^8 or $M^4 \times CP_2$ will be considered in more detail. There are two options to consider.

(a) For the first option one assigns to $M^4 \times CP_2$ twistor space as a product of corresponding twistor spaces $T(M_4) = CP_3$ and the flag-manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ parameterizing the choices of quantization axes for $SU(3)$: $T_H = T(M^4) \times T(CP_2)$. Quite remarkably, M^4 and CP_2 are the only 4-D manifolds allowing twistor space with Kähler structure. The twistor space is 12-dimensional. The choice of quantization axis is certainly a physically well-define operation so that $T(CP_2)$ has physical interpretation. If all observable physical states are color singlets situation becomes more complex. If one assumes QCC for color quantum numbers Y and I_3, then also the choice of color quantization axis is fixed at the level of Kähler action from the condition that Y and I_3 have classically their quantal values.

(b) For the second option one generalizes the usual construction for M^8 regarded as tangent space of $M^4 \times CP_2$ (unless one takes $M^8 - H$ duality seriously).

The tangent space option looks like follows.

(a) One can map the points of M^8 to octonions. One can consider 2-component spinors with octonionic components and map points of M^8 light-cone to linear combinations of 2×2 Pauli sigma matrices but with octonionic components. By the same arguments as in the deduction of ordinary twistor space one finds that 7-D light-cone boundary is mapped to 7+8 D space since the octonionic 2-spinor/its conjugate can be multiplied/divided by arbitrary octonion without changing the light-like point. By standard argument this space extends to 8+8-D space. The points of M^8 can be identified as 8-D octonionic planes (analogs of complex sphere CP_1 in this space. An attractive identification is as octonionic projective space OP_2. Remarkably, octonions do not allow higher dimensional projective spaces.

(b) If one assumes that the spinors are quaternionic the twistor space should have dimension 7+4+1=12. This dimension is same as for $M^4 \times CP_2$. Does this mean that quaternionicity assumption reduces $T(M^8) = OP_2$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$? Or does it yield 12-D space $G_2/U(1) \times U(1)$, which is also natural since G_2 has 2-D Cartan algebra? Number theoretical compactification would transform $T(M^8) = G_2/U(1) \times U(1)$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$. This would not be surprising since in $M^8 - H$-duality CP_2 parametrizes (hyper)quaternionic planes containing preferred plane M^2.

Quaternionicity is certainly very natural in TGD framework. Quaternionicity for 8-momenta does not in general imply that they reduce to the observed M^4-momenta unless one identifies M^4 as one particular subspace of M^8. In $M^8 - H$ duality one in principle allows all choices of M^4: it is of course unclear whether this makes any physical difference. Color confinement could be interpreted as a reduction of M^8 momenta to M^4 momenta and would also allow the interpretational problems caused by the fact that CP_2 momenta are not possible.

(c) Since octonions can be regarded as complexified quaternions with non-commuting imaginary unit, one can say that quaternionic spinors in M^8 are "real" and thus analogous to Majorana spinors. Similar interpretation applies at the level of H. Could one can interpret the quaternionicity condition for space-time surfaces and imbedding space spinors as TGD analog of Majorana condition crucial in super string models? This would also be crucial for understanding supersymmetry in TGD sense.

7.6.2 Octotwistorialization of M^8

Consider first the twistorialization in 4-D case. In M^4 one can map light-like momoment to spinors satisfying massless Dirac equation. General point m of M^4 can be mapped to a pair of massless spinors related by incidence relation defining the point m. The essential element of this association is that mass squared can be defined as determinant of the 2×2 matrix resulting in the assignment. Light-likeness is coded to the vanishing of the determinant implying that the spinors defining its rows are linearly independent. The reduction of M^4 inner product to determinant occurs because the 2×2 matrix can be regarded as a matrix representation of complexified quaternion. Massless means that the norm of a complexified quaternion defined as the product of q and its conjugate vanishes. Incidence relation $s_1 = xs_2$ relating point of M^4 and pair of spinors defining the corresponding twistor, can be interpreted in terms of product for complexified quaternions.

The generalization to the 8-D situation is straightforward: replace quaternions with octonions.

(a) The transition to M^8 means the replacement of quaternions with octonions. Masslessness corresponds to the vanishing norm for complexified octonion (hyper-octonion).

(b) One should assign to a massless 8-momentum an 8-dimensional spinor identifiable as octonion - or more precisely as hyper-octonion obtained by multiplying the imaginary part of ordinary octonion with commuting imaginary unit j and defining conjugation as a change of sign of j or that of octonionic imaginar units.

(c) This leads to a generalization of the notion of twistor consisting of pair of massless octonion valued spinors (octonions) related by the incidence relation fixing the point of M^8. The incidence relation for Euclidian octonions says $s_1 = xs_2$ and can be interpreted in terms of triality for $SO(8)$ relating conjugate spinor octet to the product of vector octed and spinor octet. For Minkowskian subspace of complexified octonions light-like vectors and s_1 and s_2 can be taken light-like as octonions. Light like x can annihilate s_2.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the modified gamma matrices both in M^8 and H.

7.6.3 Octonionicity, $SO(1,7)$, G_2, and non-associative Malcev group

The symmetries assignable with octonions are rather intricate. First of all, octonions (their hyper-variants defining M^8) have SO(8) (SO(1,7)) as isometries. $G_2 \subset SO(7)$ acts as automorphisms of octonions and $SO(1,7) \rightarrow G_2$ clearly means breaking of Lorentz invariance.

John Baez has described in a lucid manner G_2 geometrically (`http://math.ucr.edu/home/baez/octonions/node14.html`). The basic observation is that that quaternionic sub-space is generated by two linearly independent imaginary units and by their product. By adding a fourth linearly independent imaginary unit, one can generated all octonions. From this and the fact that G_2 represents subgroup of $SO(7)$, one easily deduces that G_2 is 14-dimensional. The Lie algebra of G_2 corresponds to derivations of octonionic algebra as follows infinitesimally from the condition that the image of product is the product of images. The entire algebra $SO(8)$ is direct sum of G_2 and linear transformations generated by right and left multiplication by imaginary octonion: this gives $14 + 14 = 28 = D(SO(8))$. The subgroup $SO(7)$ acting on imaginary octonsions corresponds to the direct sum of derivations and adjoint transformations defined by commutation with imaginary octonions, and has indeed dimension $14 + 7 = 21$.

One can identify also a non-associative group-like structure.

(a) In the case of octonionic spinors this group like structure is defined by the analog of phase multiplication of spinor generalizing to a multiplication with octonionic unit expressible as linear combinations of 8 octonionic imaginary units and defining 7-sphere plays appear as analog of automorphisms $o \rightarrow uou^{-1} = uou^*$.

One can associate with these transformations a non-associative Lie group and Lie algebra like structures by defining the commutators just as in the case of matrices that is as $[a, b] = ab - ba$. One 7-D non-associative Lie group like structure with topology of 7-sphere S^7 whereas G_2 is 14-dimensional exceptional Lie group (having S^6 as coset space $S^6 = G_2/SU(3)$). This group like object might be useful in the treatment of octonionic twistors. In the case of quaternions one has genuine group acting as SO(3) rotations.

(b) Octonionic gamma matrices allow to define as their commutators octonionic sigma matrices:

$$\Sigma_{kl} = \frac{i}{2}\left[\gamma_k, \gamma_l\right] \quad . \tag{7.6.1}$$

This algebra is 14-dimensional thanks to the fact that octonionic gamma matrices are of form $\gamma_0 = \sigma_1 \otimes 1$, $\gamma_i = \sigma_2 \otimes e_i$. Due to the non-associativity of octonions this algebra does not satisfy Jacobi identity - as is easy to verify using Fano triangle - and is therefore not a genuine Lie-algebra. Therefore these sigma matrices do not define a representation of G_2 as I thought first.

This algebra has decomposition $g = h+t$, $[h, t] \subset t$, $[t, t] \subset h$ characterizing for symmetric spaces. h is the 7-D algebra generated by Σ_{ij} and identical with the non-associative Malcev algebra generated by the commutators of octonionic units. The complement t corresponds to the generators Σ_{0i}. The algebra is clearly an octonionic non-associative analog fo $SO(1, 7)$.

7.6.4 Octonionic spinors in M^8 and real complexified-quaternionic spinors in H?

This above observations about the octonionic sigma matrices raise the problem about the octonionic representation of spinor connection. In $M^8 = M^4 \times E^4$ the spinor connection is trivial but for $M^4 \times CP_2$ not. There are two options.

(a) Assume that octonionic spinor structure makes sense for M^8 only and spinor connection is trivial.

(b) An alternative option is to identify M^8 as tangent space of $M^4 \times CP_2$ possessing quaternionic structure defined in terms of octonionic variants of gamma matrices. Should one replace sigma matrices appearing in spinor connection with their octonionic analogs to get a sigma matrix algebra which is pseudo Lie algebra. Or should one map the holonomy algebra of CP_2 spinor connection to a sub-algebra of $G_2 \subset SO(7)$ and define the action of the sigma matrices as ordinary matrix multiplication of octonions rather than octonionic multiplication? This seems to be possible formally.

The replacement of sigma matrices with their octonionic counterparts seems to lead to weird looking results. Octonionic multiplication table implies that the electroweak sigma matrices associated with CP_2 tangent space reduce to M^4 sigma matrices so that the spinor connection is quaternionic. Furthermore, left-handed sigma matrices are mapped to zero so that only the neutral part of spinor connection is non-vanishing. This supports the view that only M^8 gamma matrices make sense and that Dirac equation in M^8 is just free massless Dirac equation leading naturally also to the octonionic twistorialization.

One might think that distinction between different H-chiralities is difficult to make but it turns out that quarks and leptons can be identified as different components of 2-component complexified octonionic spinors.

The natural question is what associativization of octonions gives. This amounts to a condition putting the associator $a(bc)-(ab)c$ to zero. It is enough to consider octonionic imaginary units which are different. By using the decomposition of the octonionic algebra to quaternionic sub-algebra and its complement and general structure of structure constants, one finds that quaternionic sub-algebra remains as such but the products of all imaginary units in the complement with different imaginary units vanish. This means that the complement behaves effectively as 4-D flat space-gamma matrix algebra annihilated by the quaternionic sub-algebra whose imaginary part acts like Lie algebra of $SO(3)$.

7.6.5 What the replacement of $SO(7,1)$ sigma matrices with octonionic sigma matrices could mean?

The basic implication of octonionization is the replacement of $SO(7,1)$ sigma matrices with octonionic sigma matrices. For M^8 this has no consequences since since spinor connection is trivial.

For $M^4 \times CP_2$ situation would be different since CP_2 spinor connection would be replaced with its octonionic variant. This has some rather unexpected consequences and suggests that one should not try to octonionize at the level of $M^4 \times CP_2$ but interepret gamma matrices as tensor products of quaternionic gamma matrices, which can be replaced with their matrix representations. There are however some rather intriguing observations which force to keep mind open.

Octonionic representation of 8-D gamma matrices

Consider first the representation of 8-D gamma matrices in terms of tensor products of 7-D gamma matrices and 2-D Pauli sigma matrices.

(a) The gamma matrices are given by

$$\gamma^0 = 1 \times \sigma_1 \ , \ \gamma^i = \gamma^i \otimes \sigma_2 \ , \ i = 1,..,7 \ . \tag{7.6.2}$$

7-D gamma matrices in turn can be expressed in terms of 6-D gamma matrices by expressing γ^7 as

$$\gamma^{7)}_{i+1} = \gamma^{6)}_i , i = 1,...,6 \ , \ \gamma^{7)}_1 = \gamma^{6)}_7 = \prod_{i=1}^{6} \gamma^{6)}_i \ . \tag{7.6.3}$$

(b) The octonionic representation is obtained as

$$\gamma_0 = 1 \otimes \sigma_1 \ , \ \gamma_i = e_i \otimes \sigma_2 \ . \tag{7.6.4}$$

where e_i are the octonionic units. $e_i^2 = -1$ guarantees that the M^4 signature of the metric comes out correctly. Note that $\gamma_7 = \prod \gamma_i$ is the counterpart for choosing the preferred octonionic unit and plane M^2.

(c) The octonionic sigma matrices are obtained as commutators of gamma matrices:

$$\Sigma_{0i} = je_i \times \sigma_3 \ , \ \Sigma_{ij} = jf_{ij}{}^k e_k \otimes 1 \ . \tag{7.6.5}$$

Here j is commuting imaginary unit. These matrices span G_2 algebra having dimension 14 and rank 2 and having imaginary octonion units and their conjugates as the fundamental representation and its conjugate. The Cartan algebra for the sigma matrices can be chosen to be Σ_{01} and Σ_{23} and belong to a quaternionic sub-algebra.

(d) The lower dimension $D = 14$ of the non-associative version of sigma matrix algebra algebra means that some combinations of sigma matrices vanish. All left or right handed generators of the algebra are mapped to zero: this explains why the dimension is halved from 28 to 14. From the octonionic triangle expressing the multiplication rules for octonion units [A17] one finds $e_4e_5 = e_1$ and $e_6e_7 = -e_1$ and analogous expressions for the cyclic permutations of e_4, e_5, e_6, e_7. From the expression of the left handed sigma matrix $I_L^3 = \sigma_{23} + \sigma^{30}$ representing left handed weak isospin (see the Appendix about the geometry of CP_2 [L1]) one can conclude that this particular sigma matrix and left handed sigma matrices in general are mapped to zero. The quaternionic sub-algebra $SU(2)_L \times SU(2)_R$ is mapped to that for the rotation group $SO(3)$ since in the case of Lorentz group one cannot speak of a decomposition to left and right handed subgroups. The elements of the complement of the quaternionic sub-algebra are expressible in terms of Σ_{ij} in the quaternionic sub-algebra.

Some physical implications of the reduction of $SO(7,1)$ to its octonionic counterpart

The octonization of spinor connection of CP_2 has some weird physical implications forcing to keep mind to the possibility that the octonionic description even at the level of H might have something to do with reality.

(a) If $SU(2)_L$ is mapped to zero only the right-handed parts of electro-weak gauge field survive octonionization. The right handed part is neutral containing only photon and Z^0 so that the gauge field becomes Abelian. Z^0 and photon fields become proportional to each other ($Z^0 \rightarrow sin^2(\theta_W)\gamma$) so that classical Z^0 field disappears from the dynamics, and one would obtain just electrodynamics.

(b) The gauge potentials and gauge fields defined by CP_2 spinor connection are mapped to fields in $SO(2) \subset SU(2) \times U(1)$ in quaternionic sub-algebra which in a well-defined sense corresponds to M^4 degrees of freedom and gauge group becomes $SO(2)$ subgroup of rotation group of $E^3 \subset M^4$. This looks like catastrophe. One might say that electroweak interactions are transformed to gravimagnetic interactions.

(c) In very optimistic frame of mind one might ask whether this might be a deeper reason for why electrodynamics is an excellent description of low energy physics and of classical physics. This is consistent with the fact that CP_2 coordinates define 4 field degrees of freedom so that single Abelian gauge field should be enough to describe classical physics. This would remove also the interpretational problems caused by the transitions changing the charge state of fermion induced by the classical W boson fields.

(d) Interestingly, the condition that electromagnetic charge is well-defined quantum number for the modes of the induced spinor field for $X^4 \subset H$ leads to the proposal that the solutions of the modified Dirac equation are localized to string world sheets in Minkowskian regions of space-time surface at least. For CP_2 type vacuum extremals one has massless Dirac and this allows only covariantly constant right-handed neutrino as solution. One has however only a piece of CP_2 (wormhole contact) so that holomorphic solutions annihilated by two complexified gamma matrices are possible in accordance with the conformal symmetries.

Can one assume non-trivial spinor connection in M^8

(a) The simplest option encouraged by the requirement of maximal symmetries is that it is absent. Massless 8-momenta would characterize spinor modes in M^8 and this would give physical justification for the octotwistors.

(b) If spinor connection is present at all, it reduces essentially to Kähler connection having different couplings to quarks and leptons identifiable as components of octonionic 2-spinors. It should be $SO(4)$ symmetric and since CP_2 is instant one might argue that now one has also instanton that is self-dual U(1) gauge field in $E^4 \subset M^4 \times E^4$ defining Kähler form. One can loosely say that that one has of constant electric and magnetic fields which are parallel to each other. The rotational symmetry in E^4 would break down to $SO(2)$.

(c) Without spinor connection quarks and leptons are in completely symmetric position at the level of M^8: this is somewhat disturbing. The difference between quarks and leptons in H is made possible by the fact that CP_2 does not allow standard spinor structure. Now this problem is absent. I have also consider the possibility that only leptonic spinor chirality is allowed and quarks result via a kind of anyonization process allowing them to have fractional em charges (see `http://www.tgdtheory.fi/public_html/articles/genesis.pdf`).

(d) If the solutions of the Kähler Dirac equation in Minkowskian regions are localized to two surfaces identifiable as integrable distributions of planes $M^2(x)$ and characterized by a local light-like direction defining the direction of massless momentum, they are holomorphic (in the sense of hyper-complex numbers) such that the second complexified modified gamma matrix annihilates the solution. Same condition makes sense also at the level of M^8 for solutions restricted to string world sheets and the presence or absence of spinor connection does not affect the situation.

Does this mean that the difference between quarks and leptons becomes visible only at the imbedding space level where ground states of super-conformal representations correspond to to imbedding space spinor harmonics which in CP_2 cm degrees are different for quarks and leptons?

Octo-spinors and their relation to ordinary imbedding space spinors

Octo-spinors are identified as octonion valued 2-spinors with basis

$$\begin{aligned} \Psi_{L,i} &= e_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \\ \Psi_{q,i} &= e_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad . \end{aligned} \tag{7.6.6}$$

One obtains quark and lepton spinors and conjugation for the spinors transforms quarks to leptons. Note that octospinors can be seen as 2-dimensional spinors with components which have values in the space of complexified octonions.

The leptonic spinor corresponding to real unit and preferred imaginary unit e_1 corresponds naturally to the two spin states of the right handed neutrino. In quark sector this would mean that right handed U quark corresponds to the real unit. The octonions decompose as $1 + 1 + 3 + \overline{3}$ as representations of $SU(3) \subset G_2$. The concrete representations are given by

$$\begin{array}{ll} \{1 \pm ie_1\} \ , & e_R \text{ and } \nu_R \text{ with spin } 1/2 \ , \\ \{e_2 \pm ie_3\} \ , & e_R \text{ and } \nu_L \text{ with spin -}1/2 \ , \\ \{e_4 \pm ie_5\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ , \\ \{e_6 \pm ie_7\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ . \end{array} \tag{7.6.7}$$

Instead of spin one could consider helicity. All these spinors are eigenstates of e_1 (and thus of the corresponding sigma matrix) with opposite values for the sign factor $\epsilon = \pm$.

The interpretation is in terms of vectorial isospin. States with $\epsilon = 1$ can be interpreted as charged leptons and D type quarks and those with $\epsilon = -1$ as neutrinos and U type quarks. The interpretation would be that the states with vanishing color isospin correspond to right handed fermions and the states with non-vanishing SU(3) isospin (to be not confused with QCD color isospin) and those with non-vanishing SU(3) isospin to left handed fermions.

The importance of this identification is that it allows a unique map of the candidates for the solutions of the octonionic modified Dirac equation to those of ordinary one. There are some delicacies involved due to the possibility to chose the preferred unit e_1 so that the preferred subspace M^2 can corresponds to a sub-manifold $M^2 \subset M^4$.

7.7 About the interpretation of Kähler Dirac equation

The physical interpretation of Kähler Dirac equation is not at all straightforward. The following arguments inspired by effective 2-dimensionality suggest that the modified gamma matrices and corresponding effective metric could allow dual gravitational description of the physics associated with wormhole throats that is holography. This applies in particular to condensed matter physics.

7.7.1 Three Dirac equations

To begin with, Dirac equation appears in three forms in TGD.

(a) The Dirac equation in world of classical worlds codes (WCW) for the super Virasoro conditions for the super Kac-Moody and similar representations formed by the states of wormhole contacts forming the counterpart of string like objects (throats correspond to the ends of the string. WCW Dirac operator generalizes the Dirac operator of 8-D imbedding space by bringing in vibrational degrees of freedom. This Dirac equation should give as its solutions zero energy states and corresponding M-matrices generalizing S-matrix and their collection defining the unitary U-matrix whose natural application appears in consciousness theory as a coder of what Penrose calls U-process. The ground states to which super-conformal algebras act correspond to imbedding space spinor modes in accordance with the idea that point like limit gives QFT in imbedding space.

(b) The analog of massless Dirac equation at the level of 8-D imbedding space and satisfied by fermionic ground states of super-conformal representations.

(c) Kähler Dirac equation is satisfied in the interior of space-time. In this equation the gamma matrices are replaced with modified gamma matrices defined by the contractions of canonical momentum currents $T^{\alpha}_k = \partial L/\partial_{\alpha} h^k$ with imbedding space gamma matrices Γ_k. This replacement is required by internal consistency and by super-conformal symmetries. The well-definedness of em charge implies that the modes of induced spinor field are localized at 2-D surfaces so that a connection with string theory type approach emerges.

Kähler-Dirac equation defines Dirac equation at space-time level. Consider first K-D equation in the interior of space-time surface.

(a) The condition that electromagnetic charge operator defined in terms of em charge expressed in terms of Clifford algebra is well defined for spinor modes (completely analogous to spin defined in terms of sigma matrices) leads to the proposal that induced spinor fields are necessarily localized at 2-dimensional string worlds sheets [K87]. Only the covariantly constant right handed neutrino and its modes assignable to massless extremals (at least) generating super-symmetry (super-conformal symmetries) would form an exception since electroweak couplings would vanish. Note that the modified gamma matrices possess CP_2 and this must vanish in order to have de-localization.

(b) This picture implies stringy realization of super Kac-Moody symmetry elementary particles can be identified as string like objects albeit in different sense than in string models. At light-like 3-surfaces defining the orbits of partonic 2-surfaces spinor fields carrying electroweak quantum numbers would be located at braid strands as also the notion of finite measurement resolution requires. This picture is also consistent with the puzzling observation that the solutions of the Chern-Simons Dirac equation can be localized on light-like curves inside wormhole throat orbits.

(c) Could Kähler Dirac equation provide a first principle justification for the light-hearted use of effective mass and the analog of Dirac equation in condensed manner physics? This would conform with the holographic philosophy. Partonic 2-surfaces with tangent space data and their light-like orbits would give hologram like representation of physics and the interior of space-time the 4-D representation of physics. Holography would have in the recent situation interpretation also as quantum classical correspondence between representations of physics in terms of quantized spinor fields at the light-like 3-surfaces on one hand and in terms of classical fields on the other hand.

(d) The resulting dispersion relation for the square of the Kähler-Dirac operator assuming that induced like metric, Kähler field, etc. are very slowly varying contains quadratic and linear terms in momentum components plus a term corresponding to magnetic moment coupling. In general massive dispersion relation is obtained as is also clear from the fact that Kähler Dirac gamma matrices are combinations of M^4 and CP_2 gammas so that modified Dirac mixes different M^4 chiralities (basic signal for massivation). If one takes into account the dependence of the induced geometric quantities on space-time point dispersion relations become non-local.

(e) Sound as a concept is usually assigned with a rather high level of description. Stringy world sheets could however dramatically raise the status of sound in this respect. The oscillations of string world sheets connecting wormhole throats describe non-local 2-particle interactions. Holography suggests that this interaction just "gravitational" dual for electroweak and color interactions. Could these oscillations inducing the oscillation of the distance between wormhole throats be interpreted at the limit of weak "gravitational" coupling as analogs of sound waves, and could sound velocity correspond to maximal signal velocity assignable to the effective metric?

Various arguments lead to the hypothesis that Kähler-Dirac action contains Chern-Simons-Dirac action localized at partonic orbits as additional term. This term cannot present at the space-like ends of the space-time surfaces. Also Kähler action contains Chern-Simons term and partonic orbits and reduces by field equations to Chern-Simons terms at the space-like ends of space-time surface.

(a) The variation of the Kähler-Dirac action gives rise to a boundary term, which is essentially contraction of the normal component of the vector Γ^n defined by Kähler-Dirac gamma matrices. Boundary condition gives $\sqrt{g_4}\Gamma^n\Psi = 0$. Therefore the incoming spinor modes at the boundaries of string world sheets must be massless. A further assumption is that the action of $\sqrt{g_4}\Gamma^n$ equals to that of a massless Dirac operator. By a suitable choice of coordinates this might be achieved. Thus massless Dirac equation in M^4 would emerge for on mass shell states.

(b) At parton orbits of wormhole one can assume that the spinors are generalized eigenstates of C-S-D operator reduces to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator and one would have good hopes that twistor Grassmannian approach works. In TGD based sringy variant of twistor Grassmann approach the integrals over virtual momenta as residue integrals reduce them to 3-D integrals over light-cone subject to momentum conservation constraints at vertices. Virtual fermions are massless but have unphysical polarization. This picture is discussed in detail in [K58].

7.7.2 Does energy metric provide the gravitational dual for condensed matter systems?

The modified gamma matrices define an effective metric via their anti-commutators quadratic in components of energy momentum tensor (canonical momentum densities). This effective metric vanishes for vacuum extremals. Note that the use of the modified gamma matrices guarantees among other things internal consistency and super-conformal symmetries of the theory.

If the above argument is on the right track, this effective metric should have applications in condensed matter theory. The energy metric has a natural interpretation in terms of effective light velocities which depend on direction of propagation. One can diagonalize the energy metric $g_e^{\alpha\beta}$ (contravariant form results from the anti-commutators) and one can denote its eigenvalues by (v_0, v_i) in the case that the signature of the effective metric is $(1,-1,-1,-1)$. The 3-vector v_i/v_0 has interpretation as components of effective light velocity in various directions as becomes clear by thinking the d'Alembert equation for the energy metric. This velocity field could be interpreted as that of hydrodynamic flow. The study of the extremals of Kähler action shows that if this flow is actually Beltrami flow so that the flow parameter associated with the flow lines extends to global coordinate, Kähler action reduces to a 3-D Chern-Simons action and one obtains effective topological QFT. The conserved fermion current $\overline{\Psi}\Gamma_e^\alpha\Psi$ has interpretation as incompressible hydrodynamical flow.

This would give also a nice analogy with AdS/CFT correspondence allowing to describe various kinds of physical systems in terms of higher-dimensional gravitation and black holes are introduced quite routinely to describe condensed matter systems. In TGD framework one would have an analogous situation but with 10-D space-time replaced with the interior of 4-D space-time and the boundary of AdS representing Minkowski space with the light-like 3-surfaces carrying matter. The effective gravitation would correspond to the "energy metric". One can associate with it analogs of curvature tensor, Ricci tensor and Einstein tensor using standard formulas and identify effective energy momentum tensor associated as Einstein tensor with effective Newton's constant appearing as constant of proportionality. Note however that the besides ordinary metric and "energy" metric one would have also the induced classical gauge fields having purely geometric interpretation and action would be Kähler action. This 4-D holography could provide a precise, dramatically simpler, and also a very concrete dual description. This cannot be said about model of graphene based on the introduction of 10-dimensional black holes, branes, and strings chosen in more or less ad hoc manner.

This raises questions. Could this give a general dual gravitational description of dissipative effects in terms of the "energy" metric and induced gauge fields? Does one obtain the analogs of black holes? Do the general theorems of general relativity about the irreversible evolution leading to black holes generalize to describe analogous fate of condensed matter systems caused by dissipation? Can one describe non-equilibrium thermodynamics and self-organization in this manner?

One might argue that the incompressible Beltrami flow defined by the dynamics of the preferred extremals is dissipationless and viscosity must therefore vanish locally. The failure of complete determinism for Kähler action however means generation of entropy since the knowledge about the state decreases gradually. This in turn should have a phenomenological local description in terms of viscosity, which characterizes the transfer of energy to shorter scales and eventually to radiation. The deeper description should be non-local and basically topological and might lead to quantization rules. For instance, one can imagine the quantization of the ratio η/s of the viscosity to entropy density as multiples of a basic unit defined by its lower bound (note that this would be analogous to Quantum Hall effect). For the first M-theory inspired derivation of the lower bound of η/s [D4] . The lower bound for η/s is satisfied in good approximation by what should have been QCD plasma but found to be something different (RHIC and the first evidence for new physics from LHC [K40]).

An encouraring sign comes from the observation that for so called massless extremals representing classically arbitrarily shaped pulses of radiation propagating without dissipation and dispersion along single direction the canonical momentum currents are light-like. The effective contravariant metric vanishes identically so that fermions cannot propate in the interior of massless extremals! This is of course the case also for vacuum extremals. Massless extremals are purely bosonic and represent bosonic radiation. Many-sheeted space-time decomposes into matter containing regions and radiation containing regions. Note that when wormhole contact (particle) is glued to a massless extremal, it is deformed so that CP_2 projection becomes 4-D guaranteeing that the weak form of electric magnetic duality can be satisfied. Therefore massless extremals can be seen as asymptotic regions. Perhaps one could say that dissipation corresponds to a de-coherence process creating space-time sheets consisting of matter and radiation. Those containing matter might be even seen as analogs blackholes as far as energy metric is considered.

7.7.3 Preferred extremals as perfect fluids

7.7.4 Preferred extremals as perfect fluids

Almost perfect fluids seems to be abundant in Nature. For instance, QCD plasma was originally thought to behave like gas and therefore have a rather high viscosity to entropy density ratio $x = \eta/s$. Already RHIC found that it however behaves like almost perfect fluid with x near to the minimum predicted by AdS/CFT. The findings from LHC gave additional conform the discovery [C3]. Also Fermi gas is predicted on basis of experimental observations to have at low temperatures a low viscosity roughly 5-6 times the minimal value [D2] . In the following the argument that the preferred extremals of Kähler action are perfect fluids apart from the symmetry breaking to space-time sheets is developed. The argument requires some basic formulas summarized first.

The detailed definition of the viscous part of the stress energy tensor linear in velocity (oddness in velocity relates directly to second law) can be found in [D1] .

(a) The symmetric part of the gradient of velocity gives the viscous part of the stress-energy tensor as a tensor linear in velocity. Velocity gradient decomposes to a term traceless tensor term and a term reducing to scalar.

$$\partial_i v_j + \partial_j v_i = \frac{2}{3}\partial_k v^k g_{ij} + (\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{7.7.1}$$

The viscous contribution to stress tensor is given in terms of this decomposition as

$$\sigma_{visc;ij} = \zeta \partial_k v^k g_{ij} + \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{7.7.2}$$

From $dF^i = T^{ij}S_j$ it is clear that bulk viscosity ζ gives to energy momentum tensor a pressure like contribution having interpretation in terms of friction opposing. Shear viscosity η corresponds to the traceless part of the velocity gradient often called just viscosity. This contribution to the stress tensor is non-diagonal and corresponds to momentum transfer in directions not parallel to momentum and makes the flow rotational. This term is essential for the thermal conduction and thermal conductivity vanishes for ideal fluids.

(b) The 3-D total stress tensor can be written as

$$\sigma_{ij} = \rho v_i v_j - p g_{ij} + \sigma_{visc;ij} \ . \tag{7.7.3}$$

The generalization to a 4-D relativistic situation is simple. One just adds terms corresponding to energy density and energy flow to obtain

$$T^{\alpha\beta} = (\rho - p)u^\alpha u^\beta + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{7.7.4}$$

Here u^α denotes the local four-velocity satisfying $u^\alpha u_\alpha = 1$. The sign factors relate to the concentrations in the definition of Minkowski metric $((1,-1,-1,-1))$.

(c) If the flow is such that the flow parameters associated with the flow lines integrate to a global flow parameter one can identify new time coordinate t as this flow parameter. This means a transition to a coordinate system in which fluid is at rest everywhere (comoving coordinates in cosmology) so that energy momentum tensor reduces to a diagonal term plus viscous term.

$$T^{\alpha\beta} = (\rho - p)g^{tt}\delta_t^\alpha\delta_t^\beta + pg^{\alpha\beta} - \sigma_{visc}^{\alpha\beta} \ . \tag{7.7.5}$$

In this case the vanishing of the viscous term means that one has perfect fluid in strong sense.

The existence of a global flow parameter means that one has

$$v_i = \Psi\partial_i\Phi \ . \tag{7.7.6}$$

Ψ and Φ depend on space-time point. The proportionality to a gradient of scalar Φ implies that Φ can be taken as a global time coordinate. If this condition is not satisfied, the perfect fluid property makes sense only locally.

AdS/CFT correspondence allows to deduce a lower limit for the coefficient of shear viscosity as

$$x = \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \ . \tag{7.7.7}$$

This formula holds true in units in which one has $k_B = 1$ so that temperature has unit of energy.

What makes this interesting from TGD view is that in TGD framework perfect fluid property in appropriately generalized sense indeed characterizes locally the preferred extremals of Kähler action defining space-time surface.

(a) Kähler action is Maxwell action with U(1) gauge field replaced with the projection of CP_2 Kähler form so that the four CP_2 coordinates become the dynamical variables at QFT limit. This means enormous reduction in the number of degrees of freedom as compared to the ordinary unifications. The field equations for Kähler action define the dynamics of space-time surfaces and this dynamics reduces to conservation laws for the currents assignable to isometries. This means that the system has a hydrodynamic interpretation. This is a considerable difference to ordinary Maxwell equations. Notice however that the "topological" half of Maxwell's equations (Faraday's induction law and the statement that no non-topological magnetic are possible) is satisfied.

(b) Even more, the resulting hydrodynamical system allows an interpretation in terms of a perfect fluid. The general ansatz for the preferred extremals of field equations assumes that various conserved currents are proportional to a vector field characterized by so called Beltrami property. The coefficient of proportionality depends on space-time point and the conserved current in question. Beltrami fields by definition is a vector field such that the time parameters assignable to its flow lines integrate to single global coordinate. This is highly non-trivial and one of the implications is almost topological QFT property due to the fact that Kähler action reduces to a boundary term assignable to wormhole throats which are light-like 3-surfaces at the boundaries of regions of space-time with Euclidian and Minkowskian signatures. The Euclidian regions (or wormhole throats, depends on one's tastes) define what I identify as generalized Feynman diagrams.

Beltrami property means that if the time coordinate for a space-time sheet is chosen to be this global flow parameter, all conserved currents have only time component. In

TGD framework energy momentum tensor is replaced with a collection of conserved currents assignable to various isometries and the analog of energy momentum tensor complex constructed in this manner has no counterparts of non-diagonal components. Hence the preferred extremals allow an interpretation in terms of perfect fluid without any viscosity.

This argument justifies the expectation that TGD Universe is characterized by the presence of low-viscosity fluids. Real fluids of course have a non-vanishing albeit small value of x. What causes the failure of the exact perfect fluid property?

(a) Many-sheetedness of the space-time is the underlying reason. Space-time surface decomposes into finite-sized space-time sheets containing topologically condensed smaller space-time sheets containing.... Only within given sheet perfect fluid property holds true and fails at wormhole contacts and because the sheet has a finite size. As a consequence, the global flow parameter exists only in given length and time scale. At imbedding space level and in zero energy ontology the phrasing of the same would be in terms of hierarchy of causal diamonds (CDs).

(b) The so called eddy viscosity is caused by eddies (vortices) of the flow. The space-time sheets glued to a larger one are indeed analogous to eddies so that the reduction of viscosity to eddy viscosity could make sense quite generally. Also the phase slippage phenomenon of super-conductivity meaning that the total phase increment of the super-conducting order parameter is reduced by a multiple of 2π in phase slippage so that the average velocity proportional to the increment of the phase along the channel divided by the length of the channel is reduced by a quantized amount.

The standard arrangement for measuring viscosity involves a lipid layer flowing along plane. The velocity of flow with respect to the surface increases from $v = 0$ at the lower boundary to v_{upper} at the upper boundary of the layer: this situation can be regarded as outcome of the dissipation process and prevails as long as energy is feeded into the system. The reduction of the velocity in direction orthogonal to the layer means that the flow becomes rotational during dissipation leading to this stationary situation.

This suggests that the elementary building block of dissipation process corresponds to a generation of vortex identifiable as cylindrical space-time sheets parallel to the plane of the flow and orthogonal to the velocity of flow and carrying quantized angular momentum. One expects that vortices have a spectrum labelled by quantum numbers like energy and angular momentum so that dissipation takes in discrete steps by the generation of vortices which transfer the energy and angular momentum to environment and in this manner generate the velocity gradient.

(c) The quantization of the parameter x is suggestive in this framework. If entropy density and viscosity are both proportional to the density n of the eddies, the value of x would equal to the ratio of the quanta of entropy and kinematic viscosity η/n for single eddy if all eddies are identical. The quantum would be $\hbar/4\pi$ in the units used and the suggestive interpretation is in terms of the quantization of angular momentum. One of course expects a spectrum of eddies so that this simple prediction should hold true only at temperatures for which the excitation energies of vortices are above the thermal energy. The increase of the temperature would suggest that gradually more and more vortices come into play and that the ratio increases in a stepwise manner bringing in mind quantum Hall effect. In TGD Universe the value of $\hbar$ can be large in some situations so that the quantal character of dissipation could become visible even macroscopically. Whether this a situation with large $\hbar$ is encountered even in the case of QCD plasma is an interesting question.

The following poor man's argument tries to make the idea about quantization a little bit more concrete.

(a) The vortices transfer momentum parallel to the plane from the flow. Therefore they must have momentum parallel to the flow given by the total cm momentum of the vortex. Before continuing some notations are needed. Let the densities of vortices and absorbed vortices be n and n_{abs} respectively. Denote by $v_{\parallel}$ *resp.* $v_{\perp}$ the components of cm momenta parallel to the main flow *resp.* perpendicular to the plane boundary plane. Let m be the mass of the vortex. Denote by S are parallel to the boundary plane.

(b) The flow of momentum component parallel to the main flow due to the absorbed at S is

$$n_{abs} m v_{\parallel} v_{\perp} S \ .$$

This momentum flow must be equal to the viscous force

$$F_{visc} = \eta \frac{v_{\parallel}}{d} \times S \ .$$

From this one obtains

$$\eta = n_{abs} m v_{\perp} d \ .$$

If the entropy density is due to the vortices, it equals apart from possible numerical factors to

$$s = n$$

so that one has

$$\frac{\eta}{s} = m v_{\perp} d \ .$$

This quantity should have lower bound $x = \hbar/4\pi$ and perhaps even quantized in multiples of x, Angular momentum quantization suggests strongly itself as origin of the quantization.

(c) Local momentum conservation requires that the comoving vortices are created in pairs with opposite momenta and thus propagating with opposite velocities $v_{\perp}$. Only one half of vortices is absorbed so that one has $n_{abs} = n/2$. Vortex has quantized angular momentum associated with its internal rotation. Angular momentum is generated to the flow since the vortices flowing downwards are absorbed at the boundary surface.

Suppose that the distance of their center of mass lines parallel to plane is $D = \epsilon d$, ϵ a numerical constant not too far from unity. The vortices of the pair moving in opposite direction have same angular momentum $mv \ D/2$ relative to their center of mass line between them. Angular momentum conservation requires that the sum these relative angular momenta cancels the sum of the angular momenta associated with the vortices themselves. Quantization for the total angular momentum for the pair of vortices gives

$$\frac{\eta}{s} = \frac{n\hbar}{\epsilon}$$

Quantization condition would give

$$\epsilon = 4\pi \ .$$

One should understand why $D = 4\pi d$ - four times the circumference for the largest circle contained by the boundary layer- should define the minimal distance between the vortices of the pair. This distance is larger than the distance d for maximally sized vortices of radius $d/2$ just touching. This distance obviously increases as the thickness of the boundary layer increases suggesting that also the radius of the vortices scales like d.

(d) One cannot of course take this detailed model too literally. What is however remarkable that quantization of angular momentum and dissipation mechanism based on vortices identified as space-time sheets indeed could explain why the lower bound for the ratio η/s is so small.

7.7.5 Is the effective metric one- or two-dimensional?

7.7.6 Is the effective metric effectively one- or two-dimensional?

The following argument suggests that the effective metric defined by the anti-commutators of the modified gamma matrices is effectively one- or two-dimensional. Effective one-dimensionality would conform with the observation that the solutions of the modified Dirac equations can be localized to one-dimensional world lines in accordance with the vision that finite measurement resolution implies discretization reducing partonic many-particle states to quantum superpositions of braids. This localization to 1-D curves occurs always at the 3-D orbits of the partonic 2-surfaces.

The argument is based on the following assumptions.

(a) The modified gamma matrices for Kähler action are contractions of the canonical momentum densities T^{α}_k with the gamma matrices of H.

(b) The strongest assumption is that the isometry currents

$$J^{A\alpha} = T^{\alpha}_k j^{Ak}$$

for the preferred extremals of Kähler action are of form

$$J^{A\alpha} = \Psi^A (\nabla\Phi)^{\alpha} \tag{7.7.8}$$

with a common function Φ guaranteeing that the flow lines of the currents integrate to coordinate lines of single global coordinate variables (Beltrami property). Index raising is carried out by using the ordinary induced metric.

(c) A weaker assumption is that one has two functions Φ_1 and Φ_2 assignable to the isometry currents of M^4 and CP_2 respectively.:

$$\begin{aligned} J_1^{A\alpha} &= \Psi_1^A (\nabla\Phi_1)^{\alpha} \ , \\ J_2^{A\alpha} &= \Psi_2^A (\nabla\Phi_2)^{\alpha} \ . \end{aligned} \tag{7.7.9}$$

The two functions Φ_1 and Φ_2 could define dual light-like curves spanning string world sheet. In this case one would have effective 2-dimensionality and decomposition to string world sheets [K32] . Isometry invariance does not allow more that two independent scalar functions Φ_i.

Consider now the argument.

(a) One can multiply both sides of this equation with j^{Ak} and sum over the index A labeling isometry currents for translations of M^4 and $SU(3)$ currents for CP_2. The tensor quantity $\sum_A j^{Ak} j^{Al}$ is invariant under isometries and must therefore satisfy

$$\sum_A \eta_{AB} j^{Ak} j^{Al} = h^{kl} \ , \tag{7.7.10}$$

where η_{AB} denotes the flat tangent space metric of H. In M^4 degrees of freedom this statement becomes obvious by using linear Minkowski coordinates. In the case of CP_2 one can first consider the simpler case $S^2 = CP_1 = SU(2)/U(1)$. The coset space property implies in standard complex coordinate transforming linearly under $U(1)$ that only the the isometry currents belonging to the complement of $U(1)$ in the sum contribute at the origin and the identity holds true at the origin and by the symmetric space property everywhere. Identity can be verified also directly in standard spherical coordinates. The argument generalizes to the case of $CP_2 = SU(3)/U(2)$ in an obvious manner.

(b) In the most general case one obtains

$$
\begin{aligned}
T_1^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla \Phi_1)^\alpha \equiv f_1^k (\nabla \Phi_1)^\alpha \ , \\
T_2^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla \Phi_2)^\alpha \equiv f_2^k (\nabla \Phi_2)^\alpha \ . \qquad (7.7.11)
\end{aligned}
$$

(c) The effective metric given by the anti-commutator of the modified gamma matrices is in turn is given by

$$
G^{\alpha\beta} = m_{kl} f_1^k f_1^l (\nabla \Phi_1)^\alpha (\nabla \Phi_1)^\beta + s_{kl} f_2^k f_2^l (\nabla \Phi_2)^\alpha (\nabla \Phi_2)^\beta \ . \qquad (7.7.12)
$$

The covariant form of the effective metric is effectively 1-dimensional for $\Phi_1 = \Phi_2$ in the sense that the only non-vanishing component of the covariant metric $G_{\alpha\beta}$ is diagonal component along the coordinate line defined by $\Phi \equiv \Phi_1 = \Phi_2$. Also the contravariant metric is effectively 1-dimensional since the index raising does not affect the rank of the tensor but depends on the other space-time coordinates. This would correspond to an effective reduction to a dynamics of point-like particles for given selection of braid points. For $\Phi_1 \neq \Phi_2$ the metric is effectively 2-dimensional and would correspond to stringy dynamics.

One can also develop an objection to effective 1- or 2-dimensionality. The proposal for what preferred extremals of Kähler action as deformations of the known extremals of Kähler action could be leads to a beautiful ansatz relying on generalization of conformal invariance and minimal surface equations of string model [K4]. The field equations of TGD reduce to those of classical string model generalized to 4-D context.

If the proposed picture is correct, field equations reduce to purely algebraically conditions stating that the Maxwellian energy momentum tensor for the Kähler action has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1,1) and second fundamental form H^k a tensor of type (2,0) and (0,2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

There is however an important consistency condition involved. The Maxwell energy momentum tensor for Kähler action must have vanishing covariant divergence. This is satisfied if it is linear combination of Einstein tensor and metric. This gives Einstein's equations with cosmological term in the general case. By the algebraic character of field equations also minimal surface equations are satisfied and Einstein's General Relativity would be exact part of TGD.

In the case of modified Dirac equation the result means that modified gamma matrices are contractions of linear combination of Einstein tensor and metric tensor with the induced gamma matrices so that the TGD counterpart of ordinary Dirac equation would be modified by the addition of a term proportional to Einstein tensor. The condition of effective 1- or 2-dimensionality seems to pose too strong conditions on this combination.

7.8 Do geometric invariants of preferred extremals define topological invariants of space-time surface and code for quantum physics?

The recent progress in the understanding of preferred extremals [K4] led to a reduction of the field equations to conditions stating for Euclidian signature the existence of Kähler metric.

The resulting conditions are a direct generalization of corresponding conditions emerging for the string world sheet and stating that the 2-metric has only non-diagonal components in complex/hypercomplex coordinates. Also energy momentum of Kähler action and has this characteristic (1,1) tensor structure. In Minkowskian signature one obtains the analog of 4-D complex structure combining hyper-complex structure and 2-D complex structure.

The construction lead also to the understanding of how Einstein's equations with cosmological term follow as a consistency condition guaranteeing that the covariant divergence of the Maxwell's energy momentum tensor assignable to Kähler action vanishes. This gives $T = kG + \Lambda g$. By taking trace a further condition follows from the vanishing trace of T:

$$R = \frac{4\Lambda}{k} . \tag{7.8.1}$$

That any preferred extremal should have a constant Ricci scalar proportional to cosmological constant is very strong prediction. Note that the accelerating expansion of the Universe would support positive value of Λ. Note however that both Λ and $k \propto 1/G$ are both parameters characterizing one particular preferred extremal. One could of course argue that the dynamics allowing only constant curvature space-times is too simple. The point is however that particle can topologically condense on several space-time sheets meaning effective superposition of various classical fields defined by induced metric and spinor connection.

The following considerations demonstrate that preferred extremals can be seen as canonical representatives for the constant curvature manifolds playing central role in Thurston's geometrization theorem [A33] known also as hyperbolization theorem implying that geometric invariants of space-time surfaces transform to topological invariants. The generalization of the notion of Ricci flow to Maxwell flow in the space of metrics and further to Kähler flow for preferred extremals in turn gives a rather detailed vision about how preferred extremals organize to one-parameter orbits. It is quite possible that Kähler flow is actually discrete. The natural interpretation is in terms of dissipation and self organization.

Quantum classical correspondence suggests that this line of thought could be continued even further: could the geometric invariants of the preferred extremals could code not only for space-time topology but also for quantum physics? How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge of quantum TGD. Could the correlation functions be reduced to statistical geometric invariants of preferred extemals? The latest (means the end of 2012) and perhaps the most powerful idea hitherto about coupling constant evolution is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

7.8.1 Preferred extremals of Kähler action as manifolds with constant Ricci scalar whose geometric invariants are topological invariants

An old conjecture inspired by the preferred extremal property is that the geometric invariants of space-time surface serve as topological invariants. The reduction of Kähler action to 3-D Chern-Simons terms [K4] gives support for this conjecture as a classical counterpart for the view about TGD as almost topological QFT. The following arguments give a more precise content to this conjecture in terms of existing mathematics.

(a) It is not possible to represent the scaling of the induced metric as a deformation of the space-time surface preserving the preferred extremal property since the scale of CP_2 breaks scale invariance. Therefore the curvature scalar cannot be chosen to be equal to one numerically. Therefore also the parameter $R = 4\Lambda/k$ and also Λ and k separately characterize the equivalence class of preferred extremals as is also physically clear.

Also the volume of the space-time sheet closed inside causal diamond CD remains constant along the orbits of the flow and thus characterizes the space-time surface. Λ and even $k \propto 1/G$ can indeed depend on space-time sheet and p-adic length scale hypothesis suggests a discrete spectrum for Λ/k expressible in terms of p-adic length scales: $\Lambda/k \propto 1/L_p^2$ with $p \simeq 2^k$ favored by p-adic length scale hypothesis. During cosmic evolution the p-adic length scale would increase gradually. This would resolve the problem posed by cosmological constant in GRT based theories.

(b) One could also see the preferred extremals as 4-D counterparts of constant curvature 3-manifolds in the topology of 3-manifolds. An interesting possibility raised by the observed negative value of Λ is that most 4-surfaces are constant negative curvature 4-manifolds. By a general theorem coset spaces H^4/Γ, where $H^4 = SO(1,4)/SO(4)$ is hyperboloid of M^5 and Γ a torsion free discrete subgroup of $SO(1,4)$ [A10]. It is not clear to me, whether the constant value of Ricci scalar implies constant sectional curvatures and therefore hyperbolic space property. It could happen that the space of spaces with constant Ricci curvature contain a hyperbolic manifold as an especially symmetric representative. In any case, the geometric invariants of hyperbolic metric are topological invariants.

By Mostow rigidity theorem [A15] finite-volume hyperbolic manifold is unique for $D > 2$ and determined by the fundamental group of the manifold. Since the orbits under the Kähler flow preserve the curvature scalar the manifolds at the orbit must represent different imbeddings of one and hyperbolic 4-manifold. In 2-D case the moduli space for hyperbolic metric for a given genus $g > 0$ is defined by Teichmueller parameters and has dimension $6(g-1)$. Obviously the exceptional character of $D = 2$ case relates to conformal invariance. Note that the moduli space in question plays a key role in p-adic mass calculations [K13].

In the recent case Mostow rigidity theorem could hold true for the Euclidian regions and maybe generalize also to Minkowskian regions. If so then both "topological" and "geometro" in "Topological GeometroDynamics" would be fully justified. The fact that geometric invariants become topological invariants also conforms with "TGD as almost topological QFT" and allows the notion of scale to find its place in topology. Also the dream about exact solvability of the theory would be realized in rather convincing manner.

These conjectures are the main result independent of whether the generalization of the Ricci flow discussed in the sequel exists as a continuous flow or possibly discrete sequence of iterates in the space of preferred extremals of Kähler action. My sincere hope is that the reader could grasp how far reaching these result really are.

7.8.2 Is there a connection between preferred extremals and AdS$_4$/CFT correspondence?

The preferred extremals satisfy Einstein Maxwell equations with a cosmological constant and have negative scalar curvature for negative value of Λ. 4-D space-times with hyperbolic metric provide canonical representation for a large class of four-manifolds and an interesting question is whether these spaces are obtained as preferred extremals and/or vacuum extremals.

4-D hyperbolic space with Minkowski signature is locally isometric with AdS$_4$. This suggests at connection with AdS$_4$/CFT correspondence of M-theory. The boundary of AdS would be now replaced with 3-D light-like orbit of partonic 2-surface at which the signature of the induced metric changes. The metric 2-dimensionality of the light-like surface makes possible generalization of 2-D conformal invariance with the light-like coordinate taking the role of complex coordinate at light-like boundary. AdS could represent a special case of a more general family of space-time surfaces with constant Ricci scalar satistying Einstein-Maxwell equations and generalizing the AdS_4/CFT correspondence. There is however a strong objection from cosmology: the accelerated expansion of the Universe requires positive value of Λ and favors De Sitter Space dS_4 instead of AdS_4.

These observations provide motivations for finding whether AdS_4 and/or dS_4 allows an imbedding as a vacuum extremal to $M^4 \times S^2 \subset M^4 \times CP_2$, where S^2 is a homologically trivial geodesic sphere of CP_2. It is easy to guess the general form of the imbedding by writing the line elements of, M^4, S^2, and AdS_4.

(a) The line element of M^4 in spherical Minkowski coordinates (m, r_M, θ, ϕ) reads as

$$ds^2 = dm^2 - dr_M^2 - r_M^2 d\Omega^2 \ . \tag{7.8.2}$$

(b) Also the line element of S^2 is familiar:

$$ds^2 = -R^2(d\Theta^2 + sin^2(\theta)d\Phi^2) \ . \tag{7.8.3}$$

(c) By visiting in Wikipedia one learns that in spherical coordinate the line element of AdS_4/dS_4 is given by

$$\begin{aligned} ds^2 &= A(r)dt^2 - \frac{1}{A(r)}dr^2 - r^2 d\Omega^2 \ , \\ A(r) &= 1 + \epsilon y^2 \ , \ y = \frac{r}{r_0} \ , \\ \epsilon &= 1 \ \text{for} \ AdS_4 \ , \ \epsilon = -1 \ \text{for} \ dS_4 \ . \end{aligned} \tag{7.8.4}$$

(d) From these formulas it is easy to see that the ansatz is of the same general form as for the imbedding of Schwartschild-Nordstöm metric:

$$\begin{array}{ll} m = \Lambda t + h(y) \ , & r_M = r \ , \\ \Theta = s(y) \ , & \Phi = \omega(t + f(y)) \ . \end{array} \tag{7.8.5}$$

The non-trivial conditions on the components of the induced metric are given by

$$\begin{aligned} g_{tt} &= \Lambda^2 - x^2 sin^2(\Theta) = A(r) \ , \\ g_{tr} &= \frac{1}{r_0}\left[\Lambda\frac{dh}{dy} - x^2 sin^2(\theta)\frac{df}{dr}\right] = 0 \ , \\ g_{rr} &= \frac{1}{r_0^2}\left[(\frac{dh}{dy})^2 - 1 - x^2 sin^2(\theta)(\frac{df}{dy})^2 - R^2(\frac{d\Theta}{dy})^2\right] = -\frac{1}{A(r)} \ , \\ x &= R\omega \ . \end{aligned} \tag{7.8.6}$$

By some simple algebraic manipulations one can derive expressions for $sin(\Theta)$, df/dr and dh/dr.

(a) For $\Theta(r)$ the equation for g_{tt} gives the expression

$$\begin{aligned} sin(\Theta) &= \pm\frac{P^{1/2}}{x} \ , \\ P &= \Lambda^2 - A = \Lambda^2 - 1 - \epsilon y^2 \ . \end{aligned} \tag{7.8.7}$$

The condition $0 \leq sin^2(\Theta) \leq 1$ gives the conditions

$$\begin{array}{llll} (\Lambda^2 - x^2 - 1)^{1/2} \leq & y & \leq (\Lambda^2 - 1)^{1/2} & \text{for } \epsilon = 1 \ (AdS_4) \ , \\ (-\Lambda^2 + 1)^{1/2} \leq & y & \leq (x^2 + 1 - \Lambda^2)^{1/2} & \text{for } \epsilon = -1 \ (dS_4) \ . \end{array} \tag{7.8.8}$$

Only a spherical shell is possible in both cases. The model for the final state of star considered in [K72] predicted similar layer layer like structure and inspired the proposal that stars quite generally have an onion-like structure with radii of various shells characterize by p-adic length scale hypothesis and thus coming in some powers of $\sqrt{2}$. This brings in mind also Titius-Bode law.

(b) From the vanishing of g_{tr} one obtains

$$\frac{dh}{dy} = \frac{P}{\Lambda}\frac{df}{dy} \ . \tag{7.8.9}$$

(c) The condition for g_{rr} gives

$$(\frac{df}{dy})^2 = \frac{r_0^2}{AP}[A^{-1} - R^2(\frac{d\Theta}{dy})^2] \ . \tag{7.8.10}$$

Clearly, the right-hand side is positive if $P \geq 0$ holds true and $Rd\Theta/dy$ is small. One can express $d\Theta/dy$ using chain rule as

$$(\frac{d\Theta}{dy})^2 = \frac{x^2y^2}{P(P-x^2)} \ . \tag{7.8.11}$$

One obtains

$$(\frac{df}{dy})^2 = \Lambda r_0^2 \frac{y^2}{AP}\left[\frac{1}{1+y^2} - x^2(\frac{R}{r_0})^2\frac{1}{P(P-x^2)}\right] \ . \tag{7.8.12}$$

The right hand side of this equation is non-negative for certain range of parameters and variable y. Note that for $r_0 \gg R$ the second term on the right hand side can be neglected. In this case it is easy to integrate $f(y)$.

The conclusion is that both AdS$_4$ and dS^4 allow a local imbedding as a vacuum extremal. Whether also an imbedding as a non-vacuum preferred extremal to $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere is possible, is an interesting question.

7.8.3 Generalizing Ricci flow to Maxwell flow for 4-geometries and Kähler flow for space-time surfaces

The notion of Ricci flow has played a key part in the geometrization of topological invariants of Riemann manifolds. I certainly did not have this in mind when I choose to call my unification attempt "Topological Geometrodynamics" but this title strongly suggests that a suitable generalization of Ricci flow could play a key role in the understanding of also TGD.

Ricci flow and Maxwell flow for 4-geometries

The observation about constancy of 4-D curvature scalar for preferred extremals inspires a generalization of the well-known volume preserving Ricci flow [A26] introduced by Richard Hamilton. Ricci flow is defined in the space of Riemann metrics as

$$\frac{dg_{\alpha\beta}}{dt} = -2R_{\alpha\beta} + 2\frac{R_{avg}}{D}g_{\alpha\beta} \ . \tag{7.8.13}$$

Here R_{avg} denotes the average of the scalar curvature, and D is the dimension of the Riemann manifold. The flow is volume preserving in average sense as one easily checks ($\langle g^{\alpha\beta}dg_{\alpha\beta}/dt\rangle = 0$). The volume preserving property of this flow allows to intuitively understand that the volume of a 3-manifold in the asymptotic metric defined by the Ricci flow is topological invariant. The fixed points of the flow serve as canonical representatives for the topological equivalence classes of 3-manifolds. These 3-manifolds (for instance hyperbolic 3-manifolds with constant sectional curvatures) are highly symmetric. This is easy to understand since the flow is dissipative and destroys all details from the metric.

What happens in the recent case? The first thing to do is to consider what might be called Maxwell flow in the space of all 4-D Riemann manifolds allowing Maxwell field.

(a) First of all, the vanishing of the trace of Maxwell's energy momentum tensor codes for the volume preserving character of the flow defined as

$$\frac{dg_{\alpha\beta}}{dt} = T_{\alpha\beta} \ . \tag{7.8.14}$$

Taking covariant divergence on both sides and assuming that d/dt and D_α commute, one obtains that $T^{\alpha\beta}$ is divergenceless.

This is true if one assumes Einstein's equations with cosmological term. This gives

$$\frac{dg_{\alpha\beta}}{dt} = kG_{\alpha\beta} + \Lambda g_{\alpha\beta} = kR_{\alpha\beta} + (-\frac{kR}{2} + \Lambda)g_{\alpha\beta} \ . \tag{7.8.15}$$

The trace of this equation gives that the curvature scalar is constant. Note that the value of the Kähler coupling strength plays a highly non-trivial role in these equations and it is quite possible that solutions exist only for some critical values of α_K. Quantum criticality should fix the allow value triplets (G, Λ, α_K) apart from overall scaling

$$(G, \Lambda, \alpha_K) \rightarrow (xG, \Lambda/x, x\alpha_K \ .$$

Fixing the value of G fixes the values remaining parameters at critical points. The rescaling of the parameter t induces a scaling by x.

(b) By taking trace one obtains the already mentioned condition fixing the curvature to be constant, and one can write

$$\frac{dg_{\alpha\beta}}{dt} = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \ . \tag{7.8.16}$$

Note that in the recent case $R_{avg} = R$ holds true since curvature scalar is constant. The fixed points of the flow would be Einstein manifolds [A5, A46] satisfying

$$R_{\alpha\beta} = \frac{\Lambda}{k}g_{\alpha\beta} \tag{7.8.17}$$

.

(c) It is by no means obvious that continuous flow is possible. The condition that Einstein-Maxwell equations are satisfied might pick up from a completely general Maxwell flow a discrete subset as solutions of Einstein-Maxwell equations with a cosmological term. If so, one could assign to this subset a sequence of values t_n of the flow parameter t.

(d) I do not know whether 3-dimensionality is somehow absolutely essential for getting the topological classification of closed 3-manifolds using Ricci flow. This ignorance allows me to pose some innocent questions. Could one have a canonical representation of 4-geometries as spaces with constant Ricci scalar? Could one select one particular Einstein space in the class four-metrics and could the ratio Λ/k represent topological invariant if one normalizes metric or curvature scalar suitably. In the 3-dimensional case curvature scalar is normalized to unity. In the recent case this normalization would give $k = 4\Lambda$ in turn giving $R_{\alpha\beta} = g_{\alpha\beta}/4$. Does this mean that there is only single fixed point in local sense, analogous to black hole toward which all geometries are driven by the Maxwell flow? Does this imply that only the 4-volume of the original space would serve as a topological invariant?

Maxwell flow for space-time surfaces

One can consider Maxwell flow for space-time surfaces too. In this case Kähler flow would be the appropriate term and provides families of preferred extremals. Since space-time surfaces inside CD are the basic physical objects are in TGD framework, a possible interpretation of these families would be as flows describing physical dissipation as a four-dimensional phenomenon polishing details from the space-time surface interpreted as an analog of Bohr orbit.

(a) The flow is now induced by a vector field $j^k(x,t)$ of the space-time surface having values in the tangent bundle of imbedding space $M^4 \times CP_2$. In the most general case one has Kähler flow without the Einstein equations. This flow would be defined in the space of all space-time surfaces or possibly in the space of all extremals. The flow equations reduce to

$$h_{kl}D_\alpha j^k(x,t)D_\beta h^l = \frac{1}{2}T_{\alpha\beta} \ . \tag{7.8.18}$$

The left hand side is the projection of the covariant gradient $D_\alpha j^k(x,t)$ of the flow vector field $j^k(x,t)$ to the tangent space of the space-time surface. D_{alpha} is covariant derivative taking into account that j^k is imbedding space vector field. For a fixed point space-time surface this projection must vanish assuming that this space-time surface reachable. A good guess for the asymptotia is that the divergence of Maxwell energy momentum tensor vanishes and that Einstein's equations with cosmological constant are well-defined.

Asymptotes corresponds to vacuum extremals. In Euclidian regions CP_2 type vacuum extremals and in Minkowskian regions to any space-time surface in any 6-D sub-manifold $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähler form. Symplectic transformations of CP_2 combined with diffeomorphisms of M^4 give new Lagrangian manifolds. One would expect that vacuum extremals are approached but never reached at second extreme for the flow.

If one assumes Einstein's equations with a cosmological term, allowed vacuum extremals must be Einstein manifolds. For CP_2 type vacuum extremals this is the case. It is quite possible that these fixed points do not actually exist in Minkowskian sector, and could be replaced with more complex asymptotic behavior such as limit, chaos, or strange attractor.

(b) The flow could be also restricted to the space of preferred extremals. Assuming that Einstein Maxwell equations indeed hold true, the flow equations reduce to

$$h_{kl}D_\alpha j^k(x,t)\partial_\beta h^l \quad = \quad \frac{1}{2}(kR_{\alpha\beta} - \Lambda g_{\alpha\beta}) \ . \tag{7.8.19}$$

Preferred extremals would correspond to a fixed sub-manifold of the general flow in the space of all 4-surfaces.

(c) One can also consider a situation in which $j^k(x,t)$ is replaced with $j^k(h,t)$ defining a flow in the entire imbedding space. This assumption is probably too restrictive. In this case the equations reduce to

$$(D_r j_l(x,t) + D_l j_r)\partial_\alpha h^r \partial_\beta h^l \quad = \quad kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \ . \tag{7.8.20}$$

Here D_r denotes covariant derivative. Asymptotia is achieved if the tensor $D_k j_l + D_k j_l$ becomes orthogonal to the space-time surface. Note for that Killing vector fields of H the left hand side vanishes identically. Killing vector fields are indeed symmetries of also asymptotic states.

It must be made clear that the existence of a continuous flow in the space of preferred extremals might be too strong a condition. Already the restriction of the general Maxwell flow in the space of metrics to solutions of Einstein-Maxwell equations with cosmological term might lead to discretization, and the assumption about reprentability as 4-surface in $M^4 \times CP_2$ would give a further condition reducing the number of solutions. On the other hand, one might consiser a possibility of a continuous flow in the space of constant Ricci scalar metrics with a fixed 4-volume and having hyperbolic spaces as the most symmetric representative.

Dissipation, self organization, transition to chaos, and coupling constant evolution

A beautiful connection with concepts like dissipation, self-organization, transition to chaos, and coupling constant evolution suggests itself.

(a) It is not at all clear whether the vacuum extremal limits of the preferred extremals can correspond to Einstein spaces except in special cases such as CP_2 type vacuum extremals isometric with CP_2. The imbeddability condition however defines a constraint force which might well force asymptotically more complex situations such as limit cycles and strange attractors. In ordinary dissipative dynamics an external energy feed is essential prerequisite for this kind of non-trivial self-organization patterns.

In the recent case the external energy feed could be replaced by the constraint forces due to the imbeddability condition. It is not too difficult to imagine that the flow (if it exists!) could define something analogous to a transition to chaos taking place in a stepwise manner for critical values of the parameter t. Alternatively, these discrete values could correspond to those values of t for which the preferred extremal property holds true for a general Maxwell flow in the space of 4-metrics. Therefore the preferred extremals of Kähler action could emerge as one-parameter (possibly discrete) families describing dissipation and self-organization at the level of space-time dynamics.

(b) For instance, one can consider the possibility that in some situations Einstein's equations split into two mutually consistent equations of which only the first one is independent

$$\begin{aligned} xJ^{\alpha}{}_{\nu}J^{\nu\beta} &= R^{\alpha\beta} \ , \\ L_K &= xJ^{\alpha}{}_{\nu}J^{\nu\beta} = 4\Lambda \ , \\ x &= \frac{1}{16\pi\alpha_K} \ . \end{aligned} \tag{7.8.21}$$

Note that the first equation indeed gives the second one by tracing. This happens for CP_2 type vacuum extremals.

Kähler action density would reduce to cosmological constant which should have a continuous spectrum if this happens always. A more plausible alternative is that this holds true only asymptotically. In this case the flow equation could not lead arbitrary near to vacuum extremal, and one can think of situation in which $L_K = 4\Lambda$ defines an analog of limiting cycle or perhaps even strange attractor. In any case, the assumption would allow to deduce the asymptotic value of the action density which is of utmost importance from calculational point of view: action would be simply $S_K = 4\Lambda V_4$ and one could also say that one has minimal surface with Λ taking the role of string tension.

(c) One of the key ideas of TGD is quantum criticality implying that Kähler coupling strength is analogous to critical temperature. Second key idea is that p-adic coupling constant evolution represents discretized version of continuous coupling constant evolution so that each p-adic prime would correspond a fixed point of ordinary coupling constant evolution in the sense that the 4-volume characterized by the p-adic length scale remains constant. The invariance of the geometric and thus geometric parameters of hyperbolic 4-manifold under the Kähler flow would conform with the interpretation as a flow preserving scale assignable to a given p-adic prime. The continuous evolution in question (if possible at all!) might correspond to a fixed p-adic prime. Also the hierarchy of Planck constants relates to this picture naturally. Planck constant $\hbar_{eff} = n\hbar$ corresponds to a multi-furcation generating n-sheeted structure and certainly affecting the fundamental group.

(d) One can of course question the assumption that a continuous flow exists. The property of being a solution of Einstein-Maxwell equations, imbeddability property, and preferred extremal property might allow allow only discrete sequences of space-time surfaces perhaps interpretable as orbit of an iterated map leading gradually to a fractal limit. This kind of discrete sequence might be also be selected as preferred extremals from the orbit of Maxwell flow without assuming Einstein-Maxwell equations. Perhaps the discrete p-adic coupling constant evolution could be seen in this manner and be regarded as an iteration so that the connection with fractality would become obvious too.

Does a 4-D counterpart of thermodynamics make sense?

The interpretation of the Kähler flow in terms of dissipation, the constancy of R, and almost constancy of L_K suggest an interpretation in terms of 4-D variant of thermodynamics natural in zero energy ontology (ZEO), where physical states are analogs for pairs of initial and final states of quantum event are quantum superpositions of classical time evolutions. Quantum theory becomes a "square root" of thermodynamics so that 4-D analog of thermodynamics might even replace ordinary thermodynamics as a fundamental description. If so this 4-D thermodynamics should be qualitatively consistent with the ordinary 3-D thermodynamics.

(a) The first naive guess would be the interpretation of the action density L_K as an analog of energy density $e = E/V_3$ and that of R as the analog to entropy density $s = S/V_3$. The asymptotic states would be analogs of thermodynamical equilibria having constant values of L_K and R.

(b) Apart from an overall sign factor ϵ to be discussed, the analog of the first law $de = Tds - pdV/V$ would be

$$dL_K = kdR + \Lambda \frac{dV_4}{V_4} \quad .$$

One would have the correspondences $S \to \epsilon RV_4$, $e \to \epsilon L_K$ and $k \to T$, $p \to -\Lambda$. $k \propto 1/G$ indeed appears formally in the role of temperature in Einstein's action defining a formal partition function via its exponent. The analog of second law would state the increase of the magnitude of ϵRV_4 during the Kähler flow.

(c) One must be very careful with the signs and discuss Euclidian and Minkowskian regions separately. Concerning purely thermodynamic aspects at the level of vacuum functional Euclidian regions are those which matter.

i. For CP_2 type vacuum extremals $L_K \propto E^2 + B^2$, $R = \Lambda/k$, and Λ are positive. In thermodynamical analogy for $\epsilon = 1$ this would mean that pressure is negative.

ii. In Minkowskian regions the value of $R = \Lambda/k$ is negative for $\Lambda < 0$ suggested by the large abundance of 4-manifolds allowing hyperbolic metric and also by cosmological considerations. The asymptotic formula $L_K = 4\Lambda$ considered above suggests that also Kähler action is negative in Minkowskian regions for magnetic flux tubes dominating in TGD inspired cosmology: the reason is that the magnetic contribution to the action density $L_K \propto E^2 - B^2$ dominates.

Consider now in more detail the 4-D thermodynamics interpretation in Euclidian and Minkowskian regions assuming that the the evolution by quantum jumps has Kähler flow as a space-time correlate.

(a) In Euclidian regions the choice $\epsilon = 1$ seems to be more reasonable one. In Euclidian regions $-\Lambda$ as the analog of pressure would be negative, and asymptotically (that is for CP_2 type vacuum extremals) its value would be proportional to $\Lambda \propto 1/GR^2$, where R denotes CP_2 radius defined by the length of its geodesic circle.

A possible interpretation for negative pressure is in terms of string tension effectively inducing negative pressure (note that the solutions of the modified Dirac equation indeed assign a string to the wormhole contact). The analog of the second law would require the increase of RV_4 in quantum jumps. The magnitudes of L_K, R, V_4 and Λ would be reduced and approach their asymptotic values. In particular, V_4 would approach asymptotically the volume of CP_2.

(b) In Minkowskian regions Kähler action contributes to the vacuum functional a phase factor analogous to an imaginary exponent of action serving in the role of Morse function so that thermodynamics interpretation can be questioned. Despite this one can check whether thermodynamic interpretation can be considered. The choice $\epsilon = -1$ seems to be the correct choice now. $-\Lambda$ would be analogous to a negative pressure whose gradually decreases. In 3-D thermodynamics it is natural to assign negative pressure to the magnetic flux tube like structures as their effective string tension defined by the density of magnetic energy per unit length. $-R \geq 0$ would entropy and $-L_K \geq 0$ would be the analog of energy density.

$R = \Lambda/k$ and the reduction of Λ during cosmic evolution by quantum jumps suggests that the larger the volume of CD and thus of (at least) Minkowskian space-time sheet the smaller the negative value of Λ.

Assume the recent view about state function reduction explaining how the arrow of geometric time is induced by the quantum jump sequence defining experienced time [K3]. According to this view zero energy states are quantum superpositions over CDs of various size scales but with common tip, which can correspond to either the upper or lower light-like boundary of CD. The sequence of quantum jumps the gradual increase of the average size of CD in the quantum superposition and therefore that of average value of V_4. On the other hand, a gradual decrease of both $-L_K$ and $-R$ looks physically very natural. If Kähler flow describes the effect of dissipation by quantum jumps in ZEO then the space-time surfaces would gradually approach nearly vacuum extremals with constant value of entropy density $-R$ but gradually increasing 4-volume so that the analog of second law stating the increase of $-RV_4$ would hold true.

(c) The interpretation of $-R > 0$ as negentropy density assignable to entanglement is also possible and is consistent with the interpretation in terms of second law. This interpretation would only change the sign factor ϵ in the proposed formula. Otherwise the above arguments would remain as such.

7.8.4 Could correlation functions, S-matrix, and coupling constant evolution be coded the statistical properties of preferred extremals?

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

(a) The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

(b) The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

(c) The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

    Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root " of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

(d) Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix having M-matrices as its orthonormal rows.

(e) In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

(a) General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

(b) The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \rightarrow \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

(c) What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

(d) What about Higgs field? TGD in principle allows scalar and pseudo-scalars which could be called Higgs like states. These states are however not necessary for particle massivation although they can represent particle massivation and must do so if one assumes that QFT limit exist. p-Adic thermodynamics however describes particle massivation microscopically.

> The problem is that Higgs like field does not seem to have any obvious space-time correlate. The trace of the second fundamental form is the obvious candidate but vanishes for preferred extremals which are both minimal surfaces and solutions of Einstein Maxwell equations with cosmological constant. If the string world sheets at which all spinor components except right handed neutrino are localized for the general solution ansatz of the modified Dirac equation, the corresponding second fundamental form at the level of imbedding space defines a candidate for classical Higgs field. A natural expectation is that string world sheets are minimal surfaces of space-time surface. In general they are however not minimal surfaces of the imbedding space so that one might achieve a microscopic definition of classical Higgs field and its vacuum expectation value as an average of one point correlation function over the string world sheet.

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

Part II

GENERAL THEORY

Chapter 8

Construction of Quantum Theory: Symmetries

8.1 Introduction

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the configuration space - "world of the classical worlds" (WCW) - identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

8.1.1 Physics as infinite-dimensional Kähler geometry

(a) The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

(b) WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the WCW forms what I have used to call super-symplectic algebra.

Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

(c) Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD.

(d) Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

(e) Modified Dirac equation gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the modified Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

A second breakthrough came from the realization that the well-definedness of em charge forces in the generic situation localization of the modes to 2- surfaces at which induced W fields and also Z^0 fields above weak scale vanish.

(f) The effective 2-dimensionality of the space-like 3-surfaces realizing quantum holography can be formulated as a symmetry stating that the replacement of wormhole throat by any light-like 3-surfaces parallel to it in the slicing of the space-time sheet induces only a gauge transformation of WCW Kähler function adding to it a real part of a holomorphic function of complex coordinate of WCW depending also on zero modes. This means that the Kähler metric of WCW remains invariant. It is also postulated that measurement interaction added to the modified Dirac action induces similar gauge symmetry.

(g) The study of the modified Dirac equation leads to a detailed identification of super charges of the super-conformal algebras relevant for TGD [K87]: these results represent the most recent layer in the development of ideas about supersymmetry in TGD Universe. Whereas many considerations related to supersymmetry represented earlier rely on general arguments, the results deriving from the modified Dirac equation are rather concrete and clarify the crucial role of the right-handed neutrino in TGD based realization of super-conformal symmetries. $\mathcal{N}=1$ SUSY- now almost excluded at LHC - is not possible in TGD because it requires Majorana spinors. Also $\mathcal{N}=2$ variant of the standard space-time SUSY seems to be excluded in TGD Universe. Fermionic oscillator operators for the induced spinor fields restricted to 2-D surfaces however generate large $\mathcal{N}$ SUSY and super-conformal algebra and the modes of right-handed neutrino its 4-D version.

8.1.2 p-Adic physics as physics of cognition and intentionality

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization, which involves no ad hoc elements and is inherent to the physics of TGD.

The original idea was that the notion of number theoretic braid could pose strong number theoretic conditions on physics just as p-adic thermodynamics poses on elementary particle mass spectrum. A practically oriented physicist would argue that general braids must be allowed if one wants to calculate something and that number theoretic braids represent only the intersection between the real and various p-adic physics. He could also insist that at the level of WCW various sectors must be realized in a more abstract manner - say as hierarchies of polynomials with coefficients belonging to various extensions or rationals so that one can speak about surfaces common to real and various p-adic sectors. In this view the fusion of various physics would be analogous to the completion of rationals to various number fields.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of WCW spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K65] . It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The idea is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

8.1.3 Hierarchy of Planck constants and dark matter hierarchy

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzles leads to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of CP_2 or of a causal diamond (CD

) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups.

The original intuition was the the space-time would be n-sheeted for $h_{eff} = n$. Quantum criticality expected on basis of the vacuum degeneracy of Kähler action suggests that conformal symmetries act as critical deformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Therefore one would have n conformal equivalence classes of physically equivalent space-time sheets. A hierarchy of breakings of conformal symmetry is expected on basis of ordinary catastrophe theory. These breakings would correspond to the hierarchy defined by the sub-algebras of conformal algebra or associated algebra for which conformal weights are divisible by n. This defines infinite number of inclusion hierarchies $.. \subset C(n_1) \subset C(n_3)...$ such that n_{i+1} divides n_i. These hierarchies could correspond to inclusion hierarchies of hyper-finite factors and conformal algebra acting as gauge transformations would naturally define the notion of finite measurement resolution.

This topic will not be discussed in this chapter since it is discussed in earlier chapter [K95].

8.1.4 Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

(a) There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by WCW spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.

(b) HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the modified gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane M^2 of M^4, one ends up with $M^8 - H$ duality stating that space-time surfaces can be equivalently regarded as surfaces in M^8 or $M^4 \times CP_2$. One can actually generalize M^2 to a two-dimensional Minkowskian sub-manifold of M^4. One ends up with quantum TGD by considering associative sub-algebras of the local octonionic Clifford algebra of M^8 or H. so that TGD could be seen as a generalized number theory.

This idea will not be discussed in this chapter since it has better place in the book about physics as generalized number theory [K64].

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13].

- Emergent ideas and notions [L17]
- Weak form of electric-magnetic duality [L51]
- Quantum TGD [L35]
- Identification of preferred extremals of Kaehler action [L25]
- Generalized Feynman diagrams [L19]

- The unique role of twistors in TGD [L45]
- Twistors and TGD [L47]
- Quantum theory [L36]
- TGD as ATQFT [L43]
- Vacuum functional in TGD [L48]

8.2 Symmetries

The most general expectation is that WCW can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$.

Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and H and to show that the tangent space of WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action. One must of course understand what "preferred" means.

8.2.1 General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of WCW geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional configuration space of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as a preferred extremal of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome. The proposal was that the preferred extremals are absolute minima of Kähler action.

This picture turned out to be too simple.

(a) I have already described the recent view about light-like 3-surfaces as generalized Feynman diagrams and space-time surfaces as preferred extremals of Kähler action and will not repeat what has been said.

(b) It has also become obvious that the gigantic symmetries associated with $\delta M^4_\pm \times CP_2 \subset CD \times CP_2$ manifest themselves as the properties of propagators and vertices. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of WCW to a union of configuration spaces assignable to causal diamonds CDs defined as intersections of future and past directed light-cones. The minimum assumption is that CDs label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^4_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X_l^3 as light-like 3-surface is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet must define the preferred 3-surface X_l^3.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X_l^3 of M^4 implies generalized conformal and symplectic symmetries allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

8.2.2 Light like 3-D causal determinants and effective 2-dimensionality

The light like 3-surfaces X_l^3 of space-time surface appear as 3-D causal determinants. Basic examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to WCW geometry and the interaction between the two conformal symmetries.

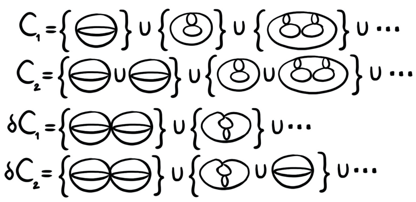

Figure 8.1: Conformal symmetry preserves angles in complex plane

(a) Field-particle duality is realized. Light-like 3-surfaces X_l^3 -generalized Feynman diagrams - correspond to the particle aspect of field-particle duality whereas the physics in the interior of space-time surface $X^4(X_l^3)$ would correspond to the field aspect. Generalized Feynman diagrams in 4-D sense could be identified as regions of space-time surface having Euclidian signature.

(b) One could also say that light-like 3-surfaces X_l^3 and the space-like 3-surfaces X^3 in the intersections of $X^4(X_l^3) \cap CD \times CP_2$ where the causal diamond CD is defined as the intersections of future and past directed light-cones provide dual descriptions.

(c) Generalized coset construction implies that the differences of super-symplectic and Super Kac-Moody type Super Virasoro generators annihilated physical states. This construction in turn led to the realization that WCW for fixed values of zero modes - in particular the values of the induced Kähler form of $\delta M_\pm^4 \times CP_2$ - allows identification as a coset space obtained by dividing the symplectic group of $\delta M_\pm^4 \times CP_2$ with Kac-Moody group, whose generators vanish at $X^2 = X_l^3 \times \delta M_\pm^4 \times CP_2$. One can say that quantum fluctuating degrees of freedom in a very concrete sense correspond to the local variant of $S^2 \times CP_2$.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M_\pm^4$ implies that the data at either X^3 or X_l^3 should be enough to determine WCW geometry. This implies that the relevant data is contained to their intersection X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light-likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory and the reduction to 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of CDs containing CDs containing.... The introduction of sub-CD:s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. What was regarded originally as a victory was that it simplifies dramatically the earlier formulas for WCW metric involving 3-dimensional integrals over $X^3 \subset M^4_+ \times CP_2$ reducing now to 2-dimensional integrals. One can of course criticize so strong form of effective 2-dimensionality as unphysical. As often happens, the later progress led to the comeback of the formulation involving 3-surfaces! The stringy picture implied by the solutions of modified Dirac action led to the 3-D picture with effective 2-dimensionality realized in terms of super conformal symmetries.

8.2.3 Magic properties of light cone boundary and isometries of WCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM^4_+, the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are paramet3rized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

(a) The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of $\delta M^4_+ \times CP_2$ the isometry group of δM^4_+ becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM^4_+ defines also symplectic structure.

    Hence any function of $\delta M^4_+ \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM^4_+ degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M^4_+ \times CP_2$, defined as the sum of light cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M^4_+ \times CP_2$ is a good candidate for the isometry group of WCW.

(b) The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of CP_2, CP_2 symplectic transformations wiykd correspond to zero modes having zero norm in the Kähler metric of WCW. This does not make sense since symplectic transformations of $\delta M^4 \times CP_2$ actually parameterize the quantum fluctuation degrees of freedom.

(c) The groups G and H, and thus WCW itself, should inherit the complex structure of the light cone boundary. The diffeomorphims of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

8.2.4 Symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

(a) The conformal algebra of WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

(b) The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

(c) The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

    The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

8.2.5 Does the symmetric space property correspond to coset construction for Super Virasoro algebras?

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t + h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{8.2.1}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough.

(a) WCW geometry allows two super-conformal symmetries. The first one corresponds to super-symplectic transformations acting at the level of imbedding space. The second one corresponds to super Kac-Moody symmetry acting as deformations of light-like 3-surfaces respecting their light-likeness.

(b) It took considerable amount of trials and errors to realize that both symplectic and Kac-Moody algebras are needed to generate the entire isometry algebra g. h is sub-algebra of this extended algebra. In general case the elements of both algebras are non-vanishing at the prefered partonic 2-surfaces considered.

(c) Strong form of holography implies that transformations located to the interior of space-like 3-surface and light-like partonic orbit define zero modes and act like gauge symmetries. The physically non-trivial transformations correspond to transformations acting non-trivially at space-like 3-surfaces. g corresponds to the algebra generated by these transformations. For preferred p3-surface - identified as (say) maximum of Kähler function - h corresponds to the elements of this algebra reducing to infinitesimal diffeomorphisms.

(d) Coset representation has five tensor factors as required by p-adic mass calculations and they correspond to color algebra, to two factors from electroweak $U(2)$, to one factor from transversal M^4 translations and one factor from symplectic algebra (note that also Hamiltonians which are products of δM^4_+ and CP_2 Hamiltonians are possible.

(e) The realization of WCW sectors with fixed values of zero modes as symmetric spaces G/H (analogous to $CP_2 = SU(3)/U(2)$) suggests that one can assign super-Virasoro algebras with G and H as a generalized coset representation for g and h so that the differences of the generators of two super Virasoro algebras annihilate the physical states for coset representations. This obviously generalizes Goddard-Olive-Kent construction [A90]. It however does not imply Equivalence Principle as believed for a long time.

8.2.6 Symplectic and Kac-Moody algebras as basic building bricks

Concerning the interpretation of the relationship between symplectic and Kac-Moody algebra there are some poorly understood points, which directly relate to what one means with precise interpretation of strong form of holography.

The basic building bricks are symplectic algebra of δCD (this includes CP_2 besides light-cone boundary) and Kac-Moody algebra assignable to the isometries of δCD [K12]. It seems however that the longheld view about the role of Kac-Moody algebra must be modified. Also the earlier realization of super-Hamiltonians and Hamiltonians seems too naive.

(a) I have been accustomed to think that Kac-Moody algebra could be regarded as a sub-algebra of symplectic algebra. p-Adic mass calculations however requires five tensor factors for the coset representation of Super Virasoro algebra naturally assigned to the coset structure G/H of a sector of WCW with fixed zero modes. Therefore Kac-Moody algebra cannot be regarded as a sub-algebra of symplectic algebra giving only single tensor factor and thus inconsistent with interpretation of p-adic mass calculations.

(b) The localization of Kac-Moody algebra generators with respect to the internal coordinates of light-like 3-surface taking the role of complex coordinate z in conformal field theory is also questionable: the most economical option relies on localization with respect to light-like radial coordinate of light-cone boundary as in the case of symplectic algebra. Kac-Moody algebra cannot be however sub-algebra of the symplectic algebra assigned with covariantly constant right-handed neutrino in the earlier approach.

(c) Right-handed covariantly constant neutrino as a generator of super symmetries plays a key role in the earlier construction of symplectic super-Hamiltonians. What raises doubts is that other spinor modes - both those of right-handed neutrino and electroweakly charged spinor modes - are absent. All spinor modes should be present and thus provide direct mapping from WCW geometry to WCW spinor fields in accordance with super-symmetry and the general idea that WCW geometry is coded by WCW spinor fields.

    Hence it seems that Kac-Moody algebra must be assigned with the modes of the induced spinor field which carry electroweak quantum numbers. If would be natural that the modes of right-handed neutrino having no weak and color interactions would generate the huge symplectic algebra of symmetries and that the modes of fermions with electroweak charges generate much smaller Kac-Moody algebra.

(d) The dynamics of Kähler action and modified Dirac action action are invisible in the earlier construction. This suggests that the definition of WCW Hamiltonians is too simplistic. The proposal is that the conserved super charges derivable as Noether charges and identifiable as super-Hamiltonians define WCW metric and Hamiltonians as their anticommutators. Spinor modes would become labels of Hamiltonians and WCW geometry relates directly to the dynamics of elementary particles.

(e) Note that light-cone boundary $\delta M^4_+ = S^2 \times R_+$ allows infinite-dimensional group of isometries consisting of conformal transformation of the sphere S^2 with conformal scaling compensated by an S^2 local scaling or the light-like radial coordinate of R_+. These isometries contain as a subgroup symplectic isometries and could act as gauge symmetries of the theory.

Gauge symmetry property means that the Kähler metric of the WCW is same for all choices of preferred X^3. Kähler function would however differ by a real part of a holomorphic function of WCW coordinates for different choices of preferred X^3.

Strong form of holography (or strong form of GCI) implies that one can take either space-like or light-like 3-surfaces as basic objects and consider the action the super-symplectic algebra also for the light-like 3-surfaces. This is possible by just parallelly translating the light-like boundary of CD so that one obtains slicing of CD by these light-like 3-surfaces. The equality of four-momenta associated with the two super-conformal representations might allow interpretation in terms of equivalence of gravitational and inertial four-momenta.

8.2.7 Comparison of TGD and stringy views about super-conformal symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

(a) In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K12] . This kind of representation applies also in Kac-Moody sector since the local transversal isometries localized in X^3_l and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

(b) A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K14] .

(c) The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an 1/2 disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

(d) If Kähler action defines the modified Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

(e) What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D\ mod\ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator $S_0 = S+S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n>0$ creates fermions whereas G_n, $n<0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{aligned}
\{G_m,G_n^\dagger\} &= 2L_{m+n}+\frac{c}{3}(m^2-\frac{1}{4})\delta_{m,-n} \ , \\
\{G_m,G_n\} &= 0 \ , \\
\{G_m^\dagger,G_n^\dagger\} &= 0 \ .
\end{aligned} \tag{8.2.2}$$

The commutators of type $[L_m,L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n<0$ and $G_n^\dagger$, $n>0$.

What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

(a) Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J=\epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K12] . Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

(b) The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em charge must be localized at string world sheets makes the connection with strings even more explicit [K87].

    One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

(c) The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X_l^3 (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M^4_\pm \times CP_2$. Clearly, wormhole throats X^3_l would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

(d) An alternative identification for TGD parts of conformal fields is inspired by $M^8 - H$ duality. Conformal fields would be fields in WCW. The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

8.3 Does modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as the exponent of Kähler function of world of classical worlds (WCW) expressible and that Kähler function reduces to Kähler action for a preferred extremal of Kähler action. One cannot however get rid of Kähler action since the gamma matrices appearing in Kähler-Dirac action are defined in terms of canonical momentum densities of Kähler action. The most one can hope is that Dirac determinant reduces to the exponent of Kähler action for preferred extremals.

8.3.1 What are the basic equations of quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

(a) Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents are conserved. An infinite hierarchy of these currents is expected and they would define fermionic counterparts for zero modes. In number theoretic vision space-time surfaces are proposed to be identifiable as associative (co-associative) surfaces. What these statements precisely mean has become clear only during this year. A rigorous proof for the equivalence of these two identifications is still lacking [?]

(b) The purely quantal equations are associated with the representations of various superconformal algebras and with the modified Dirac (Kähler-Dirac) equation. The requirement that there are deformations of the space-time surface -actually infinite number of them - giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations. The precise form of the modified Dirac equation is not however completely fixed without further input. Quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

(c) The notion of weak electric-magnetic duality generalizing the notion of electric-magnetic duality [K22] , [L9] leads to a detailed understanding of how TGD reduces to almost topological quantum field theory [K22] , [L9] . If Kähler current defines Beltrami flow [B44] it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

(d) The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD \times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation, which can be regarded as a purely classical Dirac equation. The modified Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

8.3.2 Quantum criticality and modified Dirac action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The question leading to a considerable progress in the problem was simple: Under what conditions the modified Dirac action allows to assign conserved fermionic currents with the deformations of the space-time surface? The answer was equally simple: These currents exists only if these deformations correspond to vanishing second variations of Kähler action - which is what criticality is. The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Quantum criticality and fermionic representation of conserved charges associated with second variations of Kähler action

It is rather obvious that TGD allows a far reaching generalization of conformal symmetries. The development of the understanding of conservation laws has been slow. Kähler-Dirac action provides excellent candidates for quantum counterparts of Noether charges. Unfortunately, the isometry charges vanish for Cartan algebras.

1. Conservation of the fermionic current requires the vanishing of the second variation of Kähler action

(a) The modified Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the modified Dirac action under this deformation vanishes. The vanishing of the first variation for the modified Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the modified Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J^\alpha_k \Psi \ , \\ J^\alpha_k &= \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l_\beta}\delta h^k_\beta + \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l}\delta h^l \ . \end{aligned} \tag{8.3.1}$$

Here h^k_β denote partial derivative of the imbedding space coordinate with respect to space-time coordinates. This term must vanish:

$$D_\alpha J^\alpha_k = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J^α_k does not define conserved classical charge in the general case.

(b) It is essential that the modified Dirac equation holds true so that the modified Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the modified Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J^\alpha_k \Psi \ . \tag{8.3.2}$$

Here $1/D$ is the inverse of the modified Dirac operator defining the counterpart of the fermionic propagator.

(c) The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^{\alpha} = \overline{\Psi}\Gamma^{\alpha}\Psi \ . \tag{8.3.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the modified Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance [A28, A29] . Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing modified gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^{\alpha} \ = \ \overline{\Psi}\Gamma^k J^{\alpha}_k\Psi + \overline{\Psi}\hat{\Gamma}^{\alpha}\delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^{\alpha}\Psi \ . \tag{8.3.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra [A11] .

(d) Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right-handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the modified Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

(e) The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

2. About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

(a) Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding charges are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates. Therefore one cannot represent isometry charges as fermionic bilinears. Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities but this is probably not enough. This can be seen as a problem.

i. Four-momentum and color Cartan algebra emerge naturally in the representations of super-conformal algebras. In the case of color algebra the charges in the complement of the Cartan algebra can be constructed in standard manner as extension of those for the Cartan algebra using free field representation of Kac-Moody algebras. In string theories four-momentum appears linearly in bosonic Kac-Moody generators and in Sugawara construction [A90] of super Virasoro generators as bilinears of bosonic Kac-Moody generators and fermionic super Kac-Moody generators [A11] . Also now quantized transversal parts for M^4 coordinates could define a second quantized field having interpretation as an operator acting on spinor fields of WCW. The angle coordinates conjugate to color isospin and hyper charge take the role of M^4 coordinates in case of CP_2.

ii. The understanding of the contributions to Kähler-Dirac action has been slow. It seems that what is needed is Chern-Simons Dirac action assigned to partonic orbits: this was the original proposal. The condition that the action of C-S-D operator reduces to that of massless M^4 Dirac operator. $Gamma^n\Psi = p^k\gamma_k\Psi$ would be space-time counterpart for the massless Dirac equation at the level of imbedding space. I have called this condition earlier generalized eigenvalue condition.

The assumption that C-S-D is present strongly suggests that also Kähler action contains C-S term meaning that the C-S terms from Kähler action are cancelled at partonic orbits for preferred extremals. If C-S term is present also at space-like ends of space-time surface Kähler action and therefore also Kähler function vanishes identically. At the ends of space-time surface one would therefore have $\Gamma^n\Psi = 0$ if C-S-D term is not present. Hence this assumption seems unphysical. One would have massless Dirac propagator at the fermionic lines defined by the partonic boundaries of Kähler-Dirac equation and on-mass-shell condition at the space-like ends of the space-time surface.

If this is correct interpretation then the fermionic lines identified as boundaries of string world sheets correspond to massless fermion propagators and the stringy propagators $1/L_0$ could be associated with fermion fermion scattering at wormhole contacts (see fig. **??** in the appendix of this book) . The generalized Feynman diagrammatics would be a combination of stringy and Feynman diagrammatics. External fermion lines would carry massless on-shell momenta and wormhole contacts could be seen as massive bound states of massless fermions falling into representations of super-conformal algebras assignable to wormhole contacts. This would allow stringy variant of twistor approach.

(b) The action defined by four-volume gives a first glimpse about what one can expect. In this case modified gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

(c) For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a on-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs^k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

(d) The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

(e) The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

3. Critical super algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

(a) The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the configuration space metric and thus correspond to zero modes. This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4. Zero modes do not seem to correspond to gauge degrees of freedom so that the super-conformal algebra associated with the zero modes has genuine physical content.

(b) Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

(c) The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

(d) The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

4. Connection with quantum criticality

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. For some mysterious reason I failed to realize that quantum criticality realized as the vanishing of the second variation makes possible a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality. Both the super-symmetry of D_K and conservation Dirac Noether currents for modified Dirac action have thus a connection with quantum criticality.

(a) Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

(b) The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$. The conservation of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires additional conditions to be satisfied and the holomorphy of string world sheets (partonic 2-surfaces) and associated Kähler-Dirac gamma matrices makes this possible [K87].

(c) Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the modified Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with Super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

(d) Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K21] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

(e) A breakthrough in understanding of the criticality was the discovery that the realization that the hierarchy of singular coverings of $CD \times CP_2$ needed to realize the hierarchy of Planck constants could correspond directly to a similar hierarchy of coverings forced by the factor that classical canonical momentum densities correspond to several values of the time derivatives of the imbedding space coordinates led to a considerable progress if the understanding of the relationship between criticality and hierarchy of Planck constants [K31] , [L7] . Therefore the problem which led to the geometrization program of quantum TGD, also allowed to reduce the hierarchy of Planck constants introduced on basis of experimental evidence to the basic quantum TGD. One can say that the 3-surfaces at the ends of CD *resp.* wormhole throats are critical in the sense that they are unstable against splitting to n_b *resp.* n_a surfaces so that one obtains space-time surfaces which can be regarded as surfaces in $n_a \times n_b$ fold covering of $CD \times CP_2$. This allows to understand why Planck constant is effectively replaced with $n_a n_b \hbar_0$ and explains charge fractionization.

Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

The vanishing of second variations of preferred extremals - at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

(a) The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

(b) The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that WCW metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

(c) The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \to X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

(d) Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

(e) There is a possible connection with the notion of self-organized criticality [B13] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

8.3.3 Handful of problems with a common resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose modified Dirac action (or Kähler Dirac action as solution). After that I will describe the general structures of Kähler action and Kähler Dirac action. The non-trivial terms are associated to 3-D boundary like surfaces - that is ends of space-time surface inside CD and light-like 3-surfaces at which the signature of the induced metric changes. These terms are induced as Lagrange multiplier terms guaranteeing weak form of E-M duality and quantum classical correspondence (QCC) between classical and quantal Cartan charges. The condition guaranteeing that Chern-Simons Dirac propagator reduces to ordinary massless Dirac propagator must be however assumed as a property of the modes of Kähler Dirac equation rather than forced by a separate term in the Kähler-Dirac action as thought originally.

Why modified Dirac action?

1. Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K12, K67] .

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

2. Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k{}_\alpha} L_K \ . \end{aligned} \tag{8.3.5}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{8.3.6}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^{\alpha} = \overline{\nu_R} T^{\alpha}_l \Gamma^l \Psi \tag{8.3.7}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_{\alpha} J^{\alpha k} = \overline{\nu_R} \Gamma^k T^{\alpha}_l \Gamma^l D_{\alpha} \Psi \ . \tag{8.3.8}$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned} \hat{\Gamma}^{\alpha} D_{\alpha} \Psi &= 0 \ , \\ \hat{\Gamma}^{\alpha} &= T^{\alpha}_l \Gamma^l \ . \end{aligned} \tag{8.3.9}$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi} \hat{\Gamma}^{\alpha} D_{\alpha} \Psi \ . \tag{8.3.10}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_{\mu} \hat{\Gamma}^{\mu} = 0 \tag{8.3.11}$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

3. How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_{\mu}$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_{\mu}, \hat{\Gamma}_{\nu}\} = 2 T^k_{\mu} T_{\nu k} \ . \tag{8.3.12}$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

4. Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [K67] . In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant equals to the exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X_l^3, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the symplectic currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-symplectic algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

Overall view about Kähler action and Kähler Dirac action

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

(a) The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

(b) By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

1. Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

(a) QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

(b) The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

(c) The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface if $j \cdot A = 0$ condition holds true as it does for preferred extremals. Note that weak form of electric magnetic duality is not absolutely necessary at space-like ends of the space-time surface but is favored by almost topological QFT property.

2. Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

a) What one wants?

It is could to make first clear what one really wants.

(a) What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant. It is important to notice that Γ^n brings in the dependence on metric and breaks exact topological QFT property as do also the constraint terms realizing weak form of electric magnetic duality.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignment of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

(b) If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

(c) One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

b) Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^\alpha_{C-S}D_\alpha\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{8.3.13}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

3. Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Some details about Chern-Simons Dirac equation

To avoid confusion some general comments are in order. Only the Chern-Simons Dirac operator will be considered. Modified gamma matrices contain also the contribution from the Lagrange multiplier term stating weak form of electric-magnetic duality. At space-like 3-surface one has also the contribution coming from the Lagrange multiplier terms identifying classical and quantal charges in Cartan algebra.

When C-S-D action at partonic orbits is included, one obtains what I have called generalized eigenvalue equation introduced in ad hoc manner in order to define Dirac determinant. Now Dirac determinant at least formally reduces to the same expression as in massless gauge theories. Dirac determinant could be also defined directly as the product of generalized eigenvalues $p^k\gamma_k$ defining virtual momenta propagating in fermion lines. Also the identification as hyperquaternions makes sense and the outcome is by symmetries real number or perhaps complex number.

One can of course wonder whether the Dirac determinant has anything to do with the exponent of Kähler action! Measurement interaction term states that the action of D_{C-S} modified by the contribution from em-duality constraint is identical with that of the Dirac operator of M^4 regarded as algebraic multiplication with $p^k\gamma_k$, where p^k is the four-momentum associated with the propagator line defined by the light-like orbit of parton. This simplifies the formalism enormously and gives a direct connection with similar condition posed independently in twistorial approach [K58].

One can require that the modes annihilated by Kähler-Dirac operator are eigenstates of C-S-D oeprator with generalized eigenvalues $p^k\gamma_k$ giving rise to fermion propagator Consider now the properties of eigenmodes of D_{C-S}.

(a) For $p^k = 0$ there is vacuum avoidance in the sense that Ψ must vanish in the regions where the modified gamma matrices vanish.

(b) If only CP_2 Kähler form appears in the Kähler action, the modified Dirac action defined by the Chern-Simons term is non-vanishing only when the dimension of the CP_2 projection of the 3-surface is $D(CP_2) \geq 2$ and the induced Kähler field is non-vanishing. This conforms with the properties of Kähler action.

$D(CP_2) \leq 2$ is inconsistent with the weak form of electric-magnetic duality. The extrema of Chern-Simons action have $D(CP_2) \leq 2$ and vanishing Chern-Simons density so that they would naturally represent on mass shell particles appearing as incoming and outgoing particles. This conforms with the interpretation of the basic extremals as free particles (massless extremals and cosmic strings with 2-D CP_2 projection). One could say that CP breaking is not present for free particles but unavoidably accompanies the propagator lines.

The explicit expression of D_{C-S} without constraint terms from the weak form of electric-magnetic duality is given by

$$\begin{aligned}
D &= \hat{\Gamma}^\mu D_\mu + \frac{1}{2} D_\mu \hat{\Gamma}^\mu \ , \\
\hat{\Gamma}^\mu &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\
D_\mu \hat{\Gamma}^\mu &= B_K^\alpha (J_{k\alpha} + \partial_\alpha A_k) \ , \\
B_K^\alpha &= \epsilon^{\alpha\beta\gamma}J_{\beta\gamma} \ , \ J_{k\alpha} = J_{kl}\partial_\alpha s^l \ , \ \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ .
\end{aligned} \tag{8.3.14}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma}$ does not depend on the induced metric.

The extremals of Chern-Simons action satisfy

$$B_K^\alpha (J_{kl} + \partial_l A_k)\partial_\alpha h^l = 0 \ , \ B_K^\alpha = \epsilon^{\alpha\beta\gamma}J_{\beta\gamma} \ . \tag{8.3.15}$$

For non-vanishing Kähler magnetic field B^α these equations hold true when CP_2 projection is 2-dimensional and S^2 projection is 1-dimensional or vice versa. This implies a vanishing of Chern-Simons action for both options. Consider for the simplicity the case when S^2 projection is 1-dimensional.

(a) Suppose that one can assign a global coordinate to the flow lines of the Kähler magnetic field. In this case one might hope that ordinary intuitions about motion in constant magnetic field might be helpful. The repetition of the discussion of [K31] leads to the condition $B \wedge dB = 0$ implying that a Beltrami flow for which current flows along the field lines and Lorentz forces vanishes is in question. This need not be the generic case.

(b) With this assumption the Chern-Simons Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{8.3.16}$$

(c) Consider first the general solutions of the modified Dirac equation when M^4 Dirac operator $p^k\gamma_k$ annhilates the spinor so that on mass shell massless fermion is in question. The spinor is covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 . \tag{8.3.17}$$

The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i\int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v}\Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant. Note that the dependence of these solutions on transversal coordinates of X_l^3 is arbitrary.

(d) For internal lines $p^k\gamma_k$ does not annihilate the spinor although four-momentum can be still on mass shell if the spinor has unphysical helicity. In this case the equation is modified. Again the modes can be localized to 1-D curves.

(e) The formal solution associated with a general eigenvalue can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to light-like curves indeed defines a global coordinate.

The localization is of utmost importance since and is consistent with the localization of the modes (other than right-handed neutrino) of Kähler Dirac equation at string world sheets discussed in chapter [K87]. String ends would thus define braid strands. The absence of correlation between the behaviors with respect longitudinal coordinate and transversal coordinates looked very strange at first glance. System looked like a collection of totally uncorrelated point like particles reflecting the flow of the current along flux lines.

A connection with quantum measurement theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

(a) The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

(b) One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

(c) What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

  i. The hint as how this description could be achieved comes from a long standing unanswered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the modified Dirac action? At least this question might make sense.

  ii. It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

$$\begin{aligned} J^{\alpha} &= \overline{\Psi} O \hat{\Gamma}^{\alpha} \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB}J\} \ . \end{aligned} \tag{8.3.18}$$

  Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

  iii. Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a + bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

  iv. Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

  v. There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

  vi. The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \rightarrow K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0} = kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

(d) Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

(e) Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

The formulation of quantum TGD in terms of the modified Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

(f) The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

(g) One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the modified Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z)+\overline{f(Z)}$, where Z denotes complex coordinates of WCW, the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

How to calculate Dirac determinant?

If the modes of the modified Dirac equation (or Kähler-Dirac equation) are localized to 2-D string world sheets as the well-definedness of em charge eigenvalue for the modes of induced spinor field strongly suggests, the definition of Dirac determinant could be rather simple as following argument shows.

The modes of Kähler-Dirac operator (modified Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the Chern-Simons Dirac term associated with the light-like partonic orbits with vanishing metric determinant g_4. Spinor modes at partonic orbits are assumed to be generalized eigen modes of C-S-D operator with eigenvalues $ip^k\gamma_k$, with

p^k interpreted as virtual momentum of the fermion propagating along lined defined by the string world sheet boundary. Therefore C-S-D term acts effectively as massless Dirac action in perturbation theory.

The spectrum of p^k is determined by the boundary conditions for C-S-D operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of hyper-quaternions defined by p^k. By symmetry arguments the outcome must be real.

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

Before trying to calculate Dirac determinant it is good to try to guess what the reduction to Chern Simons action could give as a result. This kind of guesses are of course highly speculative but nothing prevents from trying.

(a) Chern Simons action to which Kähler action is expected to reduce for the preferred extremals should be expressible in terms of invariants associated with string world sheets. The only invariant, which comes in mind is Kähler magnetic flux, which is zero mode and by general vision quantized as integer, rational or even algebraic number for surfaces for which parameters in their defining representations correspond to finite algebraic extensions of rationals. For instance, fluxes could belong to rationals with p-adic norm not larger than p^n and allowing realization as flux.

(b) Finite measurement resolution suggests that the Kähler magnetic fluxes defined by $J\sqrt{g_2}$, which is constant in preferred coordinates by the internal consistency of quantization of induced spinors, are quantized as integer multiplies or rationals or even algebraic numbers corresponding to the hierarchy of algebraic extensions assignable to the parameters characterizing space-time surfaces (say the coefficients of polynomials defining the space-time sheet). Therefore space-time surface itself would realize the finite measurement resolution in their dynamics as the finiteness for the number of string world sheets and natural cutoffs for the generalized eigenvalue spectrum of C-S-D operator, and the calculation of Dirac determinant using finite number of string world sheets would not be an approximation. Finite measurement resolution would be also a property of state.

(c) The value of k could depend on string world sheet so that one would obtain $K(X^3) \propto \sum_i k_i$, where the sum is sum over fluxes associated with string world sheets. Kähler function would be equal to Chern-Simons term in turn equal to the sum of Kähler fluxes over all alowed string world sheets: this looks indeed geometrically attractive.

(d) The reduction of Chern-Simons action to a sum of terms proportional to Kähler fluxes takes place if Chern-Simons action is apart from a vanishing integral of divergence proportional to the sum $\sum_i \oint_{C_i} A_\mu dx^\nu$ around the string world sheet. This form would have interpretation in terms of a coupling of charged particles at braid strands to Kähler potential so that particle picture would emerge.

(e) Since magnetic flux is conserved, one can argue that Chern-Simons term reduces to an integral of constant magnetic flux J over transverse degrees of freedom multiplied by integral over the boundary of string world sheet given by $\oint_C A_\mu(dx^\mu/ds)ds$ so that one indeed obtains the desired result. The result is non-vanishing only for monopole flux. Elementary particles indeed correspond to throats carrying monopole flux.

(f) The arguent about finite measurement resolution can be of course criticized. An alternative argument relies on idea that the sum over logariths of eigenvalues reduces to integral using as measure the transversal induced Kähler form J_T and the magnetic flux J over string world sheet. This conforms with the existence of slicing by string world sheets labelled by points of partonic 2-surface.

The formula would be

$$K \quad \propto \quad \oint J(x,y)J_T dx^1 \wedge dx^2 \quad . \tag{8.3.19}$$

This would be non-local analog for the local quadratic dependence of Kähler action on Kähler form. This decomposition might have interpretation in terms of intersections of 2-D surfaces in relative homology.

8.4 Super-conformal symmetries at space-time and configuration space level

The physical interpretation and detailed mathematical understanding of super-conformal symmetries has developed rather slowly and has involved several side tracks. In the following I try to summarize the basic picture with minimal amount of formulas with the understanding that the statement "Noether charge associated with geometrically realized Kac-Moody symmetry" is enough for the reader to write down the needed formula explicitly. Formula oriented reader might deny the value of the approach giving weight to principles. My personal experience is that piles of formulas too often hide the lack of real understanding.

8.5 WCW as a union of homogenous or symmetric spaces

In the following the vision about WCW as union of coset spaces is discussed in more detail.

8.5.1 Basic vision

The basic view about coset space construction for WCW has not changed.

(a) The idea about WCW as a union of coset spaces G/H labelled by zero modes is extremely attractive. The structure of homogenous space [A9] (`http://en.wikipedia.org/wiki/Homogenous_space`) means at Lie algebra level the decomposition $g = h \oplus t$ to sub-Lie-algebra h and its complement t such that $[h,t] \subset t$ holds true. Homogeneous spaces have G as its isometries. For symmetric space the additional condition $[t,t] \subset h$ holds true and implies the existence of involution changing at the Lie algebra level the sign of elements of t and leaving the elements of h invariant. The assumption about the structure of symmetric space [A30] (`http://en.wikipedia.org/wiki/Symmetric_space`) implying covariantly constant curvature tensor is attractive in infinite-dimensional case since it gives hopes about calculability.

An important source of intuition is the analogy with the construction of CP_2, which is symmetric space A particular choice of h corresponds to Lie-algebra elements realized as Killing vector fields which vanish at particular point of WCW and thus leave 3-surface invariant. A preferred choice for this point is as maximum or minimum of Kähler function. For this 3-surface the Hamiltonians of h should be stationary. If symmetric space property holds true then commutators of $[t,t]$ also vanish at the minimum/maximum. Note that Euclidian signature for the metric of WCW requires that Kähler function can have only maximum or minimum for given zero modes.

(b) The basic objection against TGD is that one cannot use the powerful canonical quantization using the phase space associated with configuration space - now WCW. The reason is the extreme non-linearity of the Kähler action and its huge vacuum degeneracy, which do not allow the construction of Hamiltonian formalism. Symplectic and Kähler structure must be realized at the level of WCW. In particular, Hamiltonians must be represented in completely new manner. The key idea is to construct WCW Hamiltonians as anti-commutators of super-Hamiltonians defining the contractions of WCW gamma matrices with corresponding Killing vector fields and therefore defining the matrix elements of WCW metric in the tangent vector basis defined by Killing vector fields. Super-symmetry therefre gives hopes about constructing quantum theory in which only induced spinor fields are second quantized and imbedding space coordinates are treated purely classically.

(c) It is important to understand the difference between symmetries and isometries assigned to the Kähler function. Symmetries of Kähler function do not affect it. The symmetries of Kähler action are also symmetries of Kähler action because Kähler function is Kähler action for a preferred extremal (here there have been a lot of confusion). Isometries leave invariant only the quadratic form defined by Kähler metric $g_{M\overline{N}} = \partial_M \partial_{\overline{L}} K$ but not Kähler function in general. For G/H decomposition G represents isometries and H both isometries and symmetries of Kähler function.

CP_2 is familiar example: $SU(3)$ represents isometries and $U(2)$ leaves also Kähler function invariant since it depends on the $U(2)$ invariant radial coordinate r of CP_2. The origin $r = 0$ is left invariant by $U(2)$ but for $r > 0$ $U(2)$ performs a rotation at $r = constant$ 3-sphere. This simple picture helps to understand what happens at the level of WCW.

How to then distinguish between symmetries and isometries? A natural guess is that one obtains also for the isometries Noether charges but the vanishing of boundary terms at spatial infinity crucial in the argument leading to Noether theorem as $\Delta S = \Delta Q = 0$ does not hold true anymore and one obtains charges which are not conserved anymore. The symmetry breaking contributions would now come from effective boundaries defined by wormhole throats at which the induce metric changes its signature from Minkowskian to Euclidian. A more delicate situation is in which first order contribution to ΔS vanishes and therefore also ΔQ and the contribution to ΔS comes from second variation allowing also to define Noether charge which is not conserved.

(d) The simple picture about CP_2 as symmetric space helps to understand the general vision if one assumes that WCW has the structure of symmetric space. The decomposition $g = h+t$ corresponds to decomposition of symplectic deformations to those which vanish at 3-surface (h) and those which do not (t).

For the symmetric space option, the Poisson brackets for super generators associated with t give Hamiltonians of h identifiable as the matrix elements of WCW metric. They would not vanish although they are stationary at 3-surface meaning that Riemann connection vanishes at 3-surface. The Hamiltonians which vanish at 3-surface X^3 would correspond to t and the Hamiltonians for which Killing vectors vanish and which therefore are stationary at X^3 would correspond to h. Outside X^3 the situation would of course be different. The metric would be obtained by parallel translating the metric from the preferred point of WCW to elsewhere and symplectic transformations would make this parallel translation.

For the homogenous space option the Poisson brackets for super generators of t would still give Hamiltonians identifiable as matrix elements of WCW metric but now they would be necessary those of h. In particular, the Hamiltonians of t do not in general vanish at X^3.

8.5.2 Equivalence Principle and WCW

8.5.3 EP at quantum and classical level

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

(a) The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

(b) Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K72].

    A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the modified Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

(c) A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K96].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

8.5.4 Criticism of the earlier construction

The earlier detailed realization of super-Hamiltonians and Hamiltonians can be criticized.

(a) Even after these more than twenty years it looks strange that the Hamiltonians should reduce to flux integrals over partonic 2-surfaces. The interpretation has been in terms of effective 2-dimensionality suggested strongly by strong form of general coordinate invariance stating that the descriptions based on light-like orbits of partonic 2-surfaces and space-like three surfaces at the ends of causal diamonds are dual so that only partonic 2-surfaces and 4-D tangent space data at them would matter. Strong form of holography implies effective 2-dimensionality but this should correspond gauge character for the action of symplectic generators in the interior the space-like 3-surfaces at the ends of CDs, which is something much milder.

One expects that the strings connecting partonic 2-surfaces could bring something new to the earlier simplistic picture. The guess is that imbedding space Hamiltonian assignable to a point of partonic 2-surface should be replaced with that defined as integral over string attached to the point. Therefore the earlier picture would suffer no modification at the level of general formulas.

(b) The fact that the dynamics of Kähler action and modified Dirac action are not directly involved with the earlier construction raises suspicions. I have proposed that Kähler function could allow identification as Dirac determinant [K22] but one would expect more intimate connection. Here the natural question is whether super-Hamiltonians for the modified Dirac action could correspond to Kähler charges constructible using Noether's theorem for corresponding deformations of the space-time surface and would also be identifiable as WCW gamma matrices.

8.5.5 Is WCW homogenous or symmetric space?

A key question is whether WCW can be symmetric space [A30] (`http://en.wikipedia.org/wiki/Riemannian_symmetric_space`) or whether only homogenous structure is needed. The lack of covariant constancy of curvature tensor might produce problems in infinite-dimensional context.

The algebraic conditions for symmetric space are $g = h + t$, $[h, t] \subset t$, $[t, t] \subset h$. The latter condition is the difficult one.

(a) δCD Hamiltonians should induce diffeomorphisms of X^3 indeed leaving it invariant. The symplectic vector fields would be parallel to X^3. A stronger condition is that they induce symplectic transformations for which all points of X^3 remain invariant. Now symplectic vector fields vanish at preferred 3-surface (note that the symplectic transformations are r_M local symplectic transformations of $S^2 \times CP_2$).

(b) For Kac-Moody algebra inclusion $H \subset G$ for the finite-dimensional Lie-algebra induces the structure of symmetric space. If entire algebra is involved this does not look physically very attractive idea unless one believes on symmetry breaking for both $SU(3)$, $U(2)_{ew}$, and $SO(3)$ and E_2 (here complex conjugation corresponds to the involution). If one assumes only Kac-Moody algebra as critical symmetries, the number of tensor factors is 4 instead of five, and it is not clear whether one can obtain consistency with p-adic mass calculations.

    Examples of 3-surfaces remaining invariant under $U(2)$ are 3-spheres of CP_2. They could correspond to intersections of deformations of CP_2 type vacuum extremals with the boundary of CD. Also geodesic spheres S^2 of CP_2 are invariant under $U(2)$ subgroup and would relate naturally to cosmic strings. The corresponding 3-surface would be $L \times S^2$, where L is a piece of light-like radial geodesic.

(c) In the case of symplectic algebra one can construct the imbedding space Hamiltonians inducing WCW Hamiltonians as products of elements of the isometry algebra of $S^2 \times CP_2$ for with parity under involution is well-defined. This would give a decomposition of the symplectic algebra satisfying the symmetric space property at the level imbedding space. This decomposition does not however look natural at the level of WCW since the only single point of CP_2 and light-like geodesic of δM^4_+ can be fixed by $SO(2) \times U(2)$ so that the 3-surfaces would reduce to pieces of light rays.

(d) A more promising involution is the inversion $r_M \to 1/r_M$ of the radial coordinate mapping the radial conformal weights to their negatives. This corresponds to the inversion in Super Virasoro algebra. t would correspond to functions which are odd functions of $u \equiv log(r_M/r_0)$ and h to even function of u. Stationary 3-surfaces would correspond to $u = 1$ surfaces for which $log(u) = 0$ holds true. This would assign criticality with Virasoro algebra as one expects on general grounds.

    $r_M = constant$ surface would most naturally correspond to a maximum of Kähler function which could indeed be highly symmetric. The elements with even u-parity should define Hamiltonians, which are stationary at the maximum of Kähler function. For other 3-surfaces obtained by /r_M-local) symplectic transformations the situation is different: now H is replaced with its symplectic conjugate hHg^{-1} of H is acceptable and if the conjecture is true one would obtained 3-surfaces assignable to perturbation theory around given maximum as symplectic conjugates of the maximum. The condition that H leaves X^3 invariant in poin-twise manner is certainly too strong and imply that the 3-surface has single point as CP_2 projection.

(e) One can also consider the possibility that critical deformations correspond to h and non-critical ones to t for the preferred 3-surface. Criticality for given h would hold only for a preferred 3-surface so that this picture would be very similar that above. Symplectic conjugates of h would define criticality for other 3-surfaces. WCW would decompose to a union corresponding to different criticalities perhaps assignable to the hierarchy of sub-algebras of conformal algebra labelled by integer whose multiples give the allowed conformal weights. Hierarchy of breakings of conformal symmetries would characterize this hierarchy of sectors of WCW.

    For sub-algebras of the conformal algebras (Kac-Moody and symplectic algebra) the condition $[t, t] \subset h$ cannot hold true so that one would obtain only the structure of homogenous space.

8.5.6 WCW as a union of symmetric spaces

In finite-dimensional context globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. The hope is that same holds true in infinite-dimensional context. The most one can hope of obtaining is the decomposition $C(H) = \cup_i G/H_i$ over orbits of G. One could allow also symmetry breaking in the sense that G and H depend on the orbit: $C(H) = \cup_i G_i/H_i$ but it seems that G can be chosen to be same for all orbits. What is essential is that these groups are infinite-dimensional. The basic properties of the coset space decomposition give very strong constraints on the group H, which certainly contains the subgroup of G, whose action reduces to diffeomorphisms of X^3.

If G is symplectic group of $\delta M^4_\pm \times CP_2$ then H is its subgroup, and one can wonder whether this is really consistent with the identification of H as Kac-Moody algebra assignable to light-like 3-surfaces. This raises the possibility that SKM acts as pure gauge symmetries and has nothing to do with the coset decomposition.

The improved understanding of solutions of the modified Dirac equation [K87] also leads to the realization that the direct sum of super-symplectic algebra and isometry algebra is more natural spectrum generating algebra. For super-symplectic algebra super-generators are represented in terms of contractions of covariantly constant right-handed neutrino mode with second quantized spinor field. For isometry sub-algebra super generators have representation in terms of contractions of modes of induced spinor field localized at string world sheets is a more natural identification of the fundamental conformal algebra and gives five tensor factors as required by p-adic mass calculations.

Consequences of the decomposition

If the decomposition to a union of coset spaces indeed occurs, the consequences for the calculability of the theory are enormous since it suffices to find metric and curvature tensor for single representative 3-surface on a given orbit (contravariant form of metric gives propagator in perturbative calculation of matrix elements as functional integrals over the WCW). The representative surface can be chosen to correspond to the maximum of Kähler function on a given orbit and one obtains perturbation theory around this maximum (Kähler function is not isometry invariant).

The task is to identify the infinite-dimensional groups G and H and to understand the zero mode structure of the WCW. Almost twenty (seven according to long held belief!) years after the discovery of the candidate for the Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy as well as special properties of the Kähler action.

The guess (not the first one!) would be following. G corresponds to the symplectic transformations of $\delta M^4_\pm \times CP_2$ leaving the induced Kähler form invariant. If G acts as isometries the values of Kähler form at partonic 2-surfaces (remember effective 2-dimensionality) are zero modes and WCW allows slicing to symplectic orbits of the partonic 2-surface with fixed induced Kähler form. Quantum fluctuating degrees of freedom would correspond to symplectic group and to the fluctuations of the induced metric. The group H dividing G would in turn correspond to the symplectic isometries reducing to diffeomorphisms at the 3-surfaces or possibly at partonic 2-surfaces only.

H could but not not need to corresponds to the Kac-Moody symmetries respecting light-likeness of X^3_l and acting in X^3_l but trivially at the partonic 2-surface X^2. The action of course extends also to the interior of space-like 3-surface X^3 at the boundary of CD. This coset structure was originally suggested via coset construction for super Virasoro algebras of super-symplectic and super Kac-Moody algebras.

WCW isometries as a subgroup of $Diff(\delta M^4_+ \times CP_2)$

The reduction to light cone boundary leads to the identification of the isometry group as some subgroup of for the group G for the diffeomorphisms of $\delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the the space of 3-surfaces in $\delta M^4_+ \times CP_2$. WCW is expected to decompose to a union of the coset spaces G/H_i, where H_i corresponds to some subgroup of G containing the transformations of G acting as diffeomorphisms for given X^3. Geometrically the vector fields acting as diffeomorphisms of X^3 are tangential to the 3-surface. H_i could depend on the topology of X^3 and since G does not change the topology of 3-surface each 3-topology defines separate orbit of G. Therefore, the union involves sum over all on topologies of X^3 plus possibly other 'zero modes'. Different topologies are naturally glued together since singular 3-surfaces intermediate between two 3-topologies correspond to points common to the two sectors with different topologies.

8.5.7 Isometries of WCW geometry as symplectic transformations of $\delta M^4_+ \times CP_2$

During last decade I have considered several candidates for the group G of isometries of WCW as the sub-algebra of the subalgebra of $Diff(\delta M^4_+ \times CP_2)$. To begin with let us write the general decomposition of $diff(\delta M^4_+ \times CP_2)$:

$$diff(\delta M^4_+ \times CP_2) = S(CP_2) \times diff(\delta M^4_+) \oplus S(\delta M^4_+) \times diff(CP_2) \ . \tag{8.5.1}$$

Here $S(X)$ denotes the scalar function basis of space X. This Lie-algebra is the direct sum of light cone diffeomorphisms made local with respect to CP_2 and CP_2 diffeomorphisms made local with respect to light cone boundary.

The idea that entire diffeomorphism group would act as isometries looks unrealistic since the theory should be more or less equivalent with topological field theory in this case. Consider now the various candidates for G.

(a) The fact that symplectic transformations of CP_2 and M^4_+ diffeomorphisms are dynamical symmetries of the vacuum extremals suggests the possibility that the diffeomorphisms of the light cone boundary and symplectic transformations of CP_2 could leave Kähler function invariant and thus correspond to zero modes. The symplectic transformations of CP_2 localized with respect to light cone boundary acting as symplectic transformations of CP_2 have interpretation as local color transformations and are a good candidate for the isometries. The fact that local color transformations are not even approximate symmetries of Kähler action is not a problem: if they were exact symmetries, Kähler function would be invariant and zero modes would be in question.

(b) CP_2 local conformal transformations of the light cone boundary act as isometries of δM^4_+. Besides this there is a huge group of the symplectic symmetries of $\delta M^4_+ \times CP_2$ if light cone boundary is provided with the symplectic structure. Both groups must be considered as candidates for groups of isometries. $\delta M^4_+ \times CP_2$ option exploits fully the special properties of $\delta M^4_+ \times CP_2$, and one can develop simple argument demonstrating that $\delta M^4_+ \times CP_2$ symplectic invariance is the correct option. Also the construction of WCW gamma matrices as super-symplectic charges supports $\delta M^4_+ \times CP_2$ option.

8.5.8 SUSY algebra defined by the anti-commutation relations of fermionic oscillator operators and WCW local Clifford algebra elements as chiral super-fields

Whether TGD allows space-time supersymmetry has been a long-standing question. Majorana spinors appear in $N = 1$ super-symmetric QFTs- in particular minimally super-symmetric standard model (MSSM). Majorana-Weyl spinors appear in M-theory and super string models. An undesirable consequence is chiral anomaly in the case that the numbers of left and right handed spinors are not same. For $D = 11$ and $D = 10$ these anomalies cancel which led to the breakthrough of string models and later to M-theory. The probable reason for considering these dimensions is that standard model does not predict right-handed neutrino (although neutrino mass suggests that right handed neutrino exists) so that the numbers of left and right handed Weyl-spinors are not the same.

In TGD framework the situation is different. Covariantly constant right-handed neutrino spinor acts as a super-symmetry in CP_2. One might think that right-handed neutrino in a well-defined sense disappears from the spectrum as a zero mode so that the number of right and left handed chiralities in $M^4 \times CP_2$ would not be same. For light-like 3-surfaces covariantly constant right-handed neutrino does not however solve the counterpart of Dirac equation for a non-vanishing four-momentum and color quantum numbers of the physical state. Therefore it does not disappear from the spectrum anymore and one expects the same number of right and left handed chiralities.

In TGD framework the separate conservation of baryon and lepton numbers excludes Majorana spinors and also the the Minkowski signature of $M^4 \times CP_2$ makes them impossible. The conclusion that TGD does not allow super-symmetry is however wrong. For $\mathcal{N} = 2N$ Weyl spinors are indeed possible and if the number of right and left handed Weyl spinors is same super-symmetry is possible. In 8-D context right and left-handed fermions correspond to quarks and leptons and since color in TGD framework corresponds to CP_2 partial waves rather than spin like quantum number, also the numbers of quark and lepton-like spinors are same.

The physical picture suggest a new kind of approach to super-symmetry in the sense that the anti-commutations of fermionic oscillator operators associated with the modes of the induced spinor fields define a structure analogous to SUSY algebra. This means that $\mathcal{N} = 2N$ SUSY with large N is in question allowing spins higher than two and also large fermion numbers. Recall that $\mathcal{N} \leq 32$ is implied by the absence of spins higher than two and the number of real spinor components is $N = 32$ also in TGD. The situation clearly differs from that encountered in super-string models and SUSYs and the large value of N allows to expect very powerful constraints on dynamics irrespective of the fact that SUSY is broken. Right handed neutrino modes define a sub-algebra for which the SUSY is only slightly broken by the absence of weak interactions and one could also consider a theory containing a large number of $\mathcal{N} = 2$ super-multiplets corresponding to the addition of right-handed neutrinos and antineutrinos at the wormhole throat.

Masslessness condition is essential for super-symmetry and at the fundamental level it could be formulated in terms of modified gamma matrices using octonionic representation and assuming that they span local quaternionic sub-algebra at each point of the space-time sheet. SUSY algebra has standard interpretation with respect to spin and isospin indices only at the partonic 2-surfaces so that the basic algebra should be formulated at these surfaces. Effective 2-dimensionality would require that partonic 2-surfaces can be taken to be ends of any light-like 3-surface Y_l^3 in the slicing of the region surrounding a given wormhole throat.

Super-algebra associated with the modified gamma matrices

Anti-commutation relations for fermionic oscillator operators associated with the induced spinor fields are naturally formulated in terms of the modified gamma matrices. Super-conformal symmetry suggests that the anti-commutation relations for the fermionic oscillator operators at light-like 3-surfaces or at their ends are most naturally formulated as anti-commutation relations for SUSY algebra. The resulting anti-commutation relations would fix the quantum TGD.

$$\begin{aligned} \{a^\dagger_{n\alpha}, a_{n\beta}\} &= D_{mn}D_{\alpha\beta} \ , \\ D &= (p^\mu + \sum_a Q^\mu_a)\hat{\sigma}^\mu \ . \end{aligned} \tag{8.5.2}$$

Here p^μ and Q^μ_a are space-time projections of momentum and color charges in Cartan algebra. Their action is purely algebraic. The anti-commutations are nothing but a generalization of the ordinary equal-time anti-commutation relations for fermionic oscillator operators to a manifestly covariant form. The matrix $D_{m,n}$ is expected to reduce to a diagonal form with a proper normalization of the oscillator operators. The experience with extended SUSY algebra suggest that the anti-commutators could contain additional central term proportional to $\delta_{\alpha\beta}$.

One can consider basically two different options concerning the definition of the super-algebra.

(a) If the super-algebra is defined at the 3-D ends of the intersection of X^4 with the boundaries of CD, the modified gamma matrices appearing in the operator D appearing in the anti-commutator are associated with Kähler action. If the generalized masslessness condition $D^2 = 0$ holds true -as suggested already earlier- one can hope that no explicit breaking of super-symmetry takes place and elegant description of massive states as effectively massless states making also possible generalization of twistor is possible. One must however notice that also massive representatives of SUSY exist.

(b) SUSY algebra could be also defined at 2-D ends of light-like 3-surfaces.

According to considerations of [K22] these options are equivalent for a large class of space-time sheets. If the effective 3-dimensionality realized in the sense that the effective metric defined by the modified gamma matrices is degenerate, propagation takes place along 3-D light-like 3-surfaces. This condition definitely fails for string like objects.

One can realize the local Clifford algebra also by introducing theta parameters in the standard manner and the expressing a collection of local Clifford algebra element with varying values of fermion numbers (function of CD and CP_2 coordinates) as a chiral super-field. The definition of a chiral super field requires the introduction of super-covariant derivatives. Standard form for the anti-commutators of super-covariant derivatives D_α make sense only if they do not affect the modified gamma matrices. This is achieved if p_k acts on the position of the tip of CD (rather than internal coordinates of the space-time sheet). Q_a in turn must act on CP_2 coordinates of the tip.

Super-fields associated with WCW Clifford algebra

WCW local Clifford algebra elements possess definite fermion numbers and it is not physically sensible to super-pose local Clifford algebra elements with different fermion numbers. The extremely elegant formulation of super-symmetric theories in terms of super-fields encourages to ask whether the local Clifford algebra elements could allow expansion in terms of complex theta parameters assigned to various fermionic oscillator operator in order to obtain formal superposition of elements with different fermion numbers. One can also ask whether the notion of chiral super field might make sense.

The obvious question is whether it makes sense to assign super-fields with the modified gamma matrices.

(a) Modified gamma matrices are not covariantly constant but this is not a problem since the action of momentum generators and color generators on space-time coordinates is purely algebraic.

(b) One can define the notion of chiral super-field also at the fundamental level. Chiral super-field would be continuation of the local Clifford algebra of associated with CD to a local Clifford algebra element associated with the union of CDs. This would allow elegant description of cm degrees of freedom, which are the most interesting as far as QFT limit is considered.

(c) Kähler function of WCW as a function of complex coordinates could be extended to a chiral super-field defined in quantum fluctuation degrees of freedom. It would depend on zero modes too. Does also the latter dependence allow super-space continuation? Coefficients of powers of theta would correspond to fermionic oscillator operators. Does this function define the propagators of various states associated with light-like 3-surface? WCW complex coordinates would correspond to the modes of induced spinor field so that super-symmetry would be realized very concretely.

8.5.9 Identification of Kac-Moody symmetries

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating WCW degrees of freedom contributing to the metric. The recent vision looks like follows.

(a) The recent interpretation is that these symmetries are due to the non-determinism of Kähler action and transform to each other preferred extremals with same space-like surfaces as their ends at the boundaries of causal diamond. These space-time surfaces have same Kähler action and possess same conserved quantities.

(b) The sub-algebra of conformal symmetries acts as gauge transformations of these infinite set of degenerate preferred extremals and there is finite number n of gauge equivalence classes. n corresponds to the effective (or real depending on interpretation) value of Planck constant $h_{eff} = n \times h$. The further conjecture is that the sub-algebra of conformal algebra for which conformal weights are integers divisible by n act as genuine gauge symmetries. If Kähler action reduces to a sum of 3-D Chern-Simons terms for preferred extremals, it is enough to consider the action on light-like 3-surfaces. For gauge part of algebra the algebra acts trivially at space-like 3-surfaces.

(c) A good guess is that the Kac-Moody type algebra corresponds to the sub-algebra of symplectic isometries of $\delta M^4_\pm \times CP_2$ acting on light-like 3-surfaces and having continuation to the interior.

A stronger assumption is that isometries are in question. For CP_2 nothing would change but light-cone boundary $\delta M^4_\pm = S^2 \times R_+$ has conformal transformations of S^2 as isometries. The conformal scaling is compensated by S^2-local scaling of the light like radial coordinate of R_+.

(d) This super-conformal algebra realized in terms of spinor modes and second quantized induced spinor fields would define the Super Kac-Moody algebra. The generators of this Kac-Moody type algebra have continuation from the light-like boundaries to deformations of preferred extremals and at least the generators of sub-algebra act trivially at space-like 3-surfaces.

The following is an attempt to achieve a more detailed identification of the Kac-Moody algebra is considered.

Identification of Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{8.5.3}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \to x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta} \xi^\mu + g_{\mu\beta} \partial_\alpha \xi^\mu + g_{\alpha\mu} \partial_\beta \xi^\mu \ . \tag{8.5.4}$$

Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k} \partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \quad . \tag{8.5.5}$$

This gives

$$\begin{aligned} c_A(x) & \left[D_k j_l^A + D_l j_k^A\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l \\ = & \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta} \xi^\mu + g_{\mu\beta} \partial_\alpha \xi^\mu + g_{\alpha\mu} \partial_\beta \xi^\mu \quad . \end{aligned} \tag{8.5.6}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j_l^A + D_l j_k^A = 2h_{kl} \quad . \tag{8.5.7}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta} \partial_\alpha \xi^\mu + g_{\alpha\mu} \partial_\beta \xi^\mu \quad . \tag{8.5.8}$$

A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

(a) The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k = 0 \quad . \tag{8.5.9}$$

The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r) d_A$ by a one obtains

$$d_A h_{kl} j^{Ak} \partial_r h^k = 0 \quad . \tag{8.5.10}$$

so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3- local conformal transformations of H are in question.

(b) The equation for g_{ri} gives

$$\partial_r\xi^i = \partial_r c_A h_{kl} j^{Ak} h^{ij}\partial_j h^k \ . \tag{8.5.11}$$

The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

(c) The three independent equations for g_{ij} give

$$\xi^\alpha\partial_\alpha g_{ij} + g_{kj}\partial_i\xi^k + g_{ki}\partial_j\xi^k = \partial_i c_A h_{kl} j^{Ak}\partial_j h^l \ . \tag{8.5.12}$$

These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_AJ^A, c_BJ^B]$.

$$\left[c_AJ^A, c_BJ^B\right] = c_Ac_BJ^{[A,B]} + J^A\circ c_BJ^B - J^B\circ c_AJ^A \ , \tag{8.5.13}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

(a) Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

(b) This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r\partial_{m^k} - 2m^k m^l\partial_{m^l})/2$. The commutators involving added generators are

$$\begin{aligned} \left[D,K^k\right] = -K^k \ , \quad & \left[D,P^k\right] = P^k \ , \\ \left[K^k,K^l\right] = 0 \ , \quad & \left[K^k,P^l\right] = m^{kl}D - M^{kl} \ . \end{aligned} \tag{8.5.14}$$

From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

(c) The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak}h_{kl}\partial_r h^l$$

is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

(d) Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labeled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

(e) It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$. Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3)\times E^3$ respecting the radial conformal weights would reduce to $SO(2)\times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

Hamiltonians

The action of these transformations on Kähler action is well-defined and one can deduce the conserved quantities having identification as WCW Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_\pm \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

The action of Kac-Moody algebra on spinors and fermionic representations of Kac-Moody algebra

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

(a) The basic goal is to deduce the fermionic Noether charge associated with the bosonic Kac-Moody symmetry and this can be done by a standard recipe. The first contribution to the charge comes from the transformation of modified gamma matrices appearing in the modified Dirac action associated with fermions. Second contribution comes from spinor rotation.

(b) Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors.

How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-symplectic algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

About the interpretation of super Kac-Moody symmetries

Also the light like 3-surfaces X_l^3 of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to WCW metric. In this case the symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X_l^3 so that Kac-Moody type symmetry results. Also the condition $\sqrt{g_3} = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction.

If is enough to localize only the H-isometries with respect to X_l^3, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation need not be equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them.

The following arguments support one possible identification.

(a) The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

(b) Since X_l^3-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

(c) The fact that only quarks appear in the gamma matrices of the WCW supports the view that action of the generators of X_l^3-local color transformations on WCW spinor fields represents local color transformations. If the action of X_l^3-local $SU(3)$ transformations on WCW spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N=4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only right-handed neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of WCW gamma matrices forming a subalgebra of the full super-algebra.

$N=2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N=2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N=2$ super-conformal representations are modular invariant, and that $N=2$ algebra defines so called chiral ring defining a topological quantum field theory [A49], lend a further support for the belief that $N=2$ super-conformal algebra acts in super-symplectic degrees of freedom.

The values of c and conformal weights for $N=2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ \ l=0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ \ m=-l,-l+2,....,l-2,l \ . \end{aligned} \tag{8.5.15}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k=1$ one would have $q=0,1/3,-1/3$, which brings in mind anyons. $\Delta_{l=0,m=0}=0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [A49].

For Ramond representation $L_0-c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta=c/24$ giving the condition $k=2\left[l(l+2)-m^2\right]$ (note that k must be even and that $(k,l,m)=(4,1,1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac}=\pm c/12=\pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N=4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on

WCW Hamiltonians expressible in terms of Hamiltonians of $X_l^3 \times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h = 1$ whereas T and G have conformal weights $h = 2$ and $h = 3/2$.

The experience with $N = 4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h = 1/2$ and their super-partners with $h = 0$ and realized as fermion-anti-fermion bilinears. Since G and Ψ are labeled by 2×4 spinor indices, super-partners would correspond to $2\times(3+1) = 8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

8.5.10 Coset space structure for WCW as a symmetric space

The key ingredient in the theory of symmetric spaces is that the Lie-algebra of G has the following decomposition

$$\begin{array}{l} g = h + t \ , \\ [h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ . \end{array}$$

In present case this has highly nontrivial consequences. The commutator of *any* two infinitesimal generators generating nontrivial deformation of 3-surface belongs to h and thus vanishing norm in the WCW metric at the point which is left invariant by H. In fact, this same condition follows from Ricci flatness requirement and guarantees also that G acts as isometries of WCW. This generalization is supported by the properties of the unitary representations of Lorentz group at the light cone boundary and by number theoretical considerations.

The algebras suggesting themselves as candidates are symplectic algebra of $\delta M^\pm \times CP_2$ and Kac-Moody algebra mapping light-like 3-surfaces to light-like 3-surfaces to be discussed in the next section.

The identification of the precise form of the coset space structure is however somewhat delicate.

(a) The essential point is that both symplectic and Kac-Moody algebras allow representation in terms of X_l^3-local Hamiltonians. The general expression for the Hamilton of Kac-Moody algebra is

$$H = \sum \Phi_A(x) H^A \ . \tag{8.5.16}$$

Here H^A are Hamiltonians of $SO(3) \times SU(3)$ acting in $\delta X_l^3 \times CP_2$. For symplectic algebra any Hamiltonian is allowed. If x corresponds to any point of X_l^3, one must assume a slicing of the causal diamond CD by translates of $\delta M^4_\pm$.

(b) For symplectic generators the dependence of form on r^Δ on light-like coordinate of $\delta X_l^3 \times CP_2$ is allowed. Δ is complex parameter whose modulus squared is interpreted as conformal weight. Δ is identified as analogous quantum number labeling the modes of induced spinor field.

(c) One can wonder whether the choices of the $r_M = constant$ sphere S^2 is the only choice. The Hamiltonin-Jacobi coordinate for $X^4_{X_l^3}$ suggest an alternative choice as E^2 in the decomposition of $M^4 = M^2(x)\times E^2(x)$ required by number theoretical compactification and present for known extremals of Kähler action with Minkowskian signature of induced metric. In this case $SO(3)$ would be replaced with $SO(2)$. It however seems that the radial light-like coordinate u of $X^4(X_l^3)$ would remain the same since any other curve along light-like boundary would be space-like.

(d) The vector fields for representing Kac-Moody algebra must vanish at the partonic 2-surface $X^2 \subset \delta M^4_\pm \times CP_2$. The corresponding vector field must vanish at each point of X^2:

$$j^k = \sum \Phi_A(x) J^{kl} H_l^A = 0 \ . \qquad (8.5.17)$$

This means that the vector field corresponds to $SO(2) \times U(2)$ defining the isotropy group of the point of $S^2 \times CP_2$.

This expression could be deduced from the idea that the surfaces X^2 are analogous to origin of CP_2 at which $U(2)$ vector fields vanish. WCW at X^2 could be also regarded as the analog of the origin of local $S^2 \times CP_2$. This interpretation is in accordance with the original idea which however was given up in the lack of proper realization. The same picture can be deduced from braiding in which case the Kac-Moody algebra corresponds to local $SO(2) \times U(2)$ for each point of the braid at X^2. The condition that Kac-Moody generators with positive conformal weight annihilate physical states could be interpreted by stating effective 2-dimensionality in the sense that the deformations of X_l^3 preserving its light-likeness do not affect the physics. Note however that Kac-Moody type Virasoro generators do not annihilate physical states.

(e) Kac-Moody algebra generator must leave induced Kähler form invariant at X^2. This is of course trivial since the action leaves each point invariant. The conditions of Cartan decomposition are satisfied. The commutators of the Kac-Moody vector fields with symplectic generators are non-vanishing since the action of symplectic generator on Kac-Moody generator restricted to X^2 gives a non-vanishing result belonging to the symplectic algebra. Also the commutators of Kac-Moody generators are Kac-Moody generators.

8.5.11 The relationship between super-symplectic and Super Kac-Moody algebras, Equivalence Principle, and justification of p-adic thermodynamics

The relationship between super-symplectic algebra (SS) acting at light-cone boundary and Super Kac-Moody algebra (SKM) assumed to act on light-like 3-surfaces and by continuation of the action also to the space-like 3-surfaces at the boundaries of CD has remained somewhat enigmatic due to the lack of physical insights.

Corresponding to the coset decomposition G/H of WCW there is also the sub-algebra SD of SS acting as diffeomorphisms of given 3-surface. This algebra acts as gauge algebra. It seems that SKM and SD cannot be the same algebra.

The construction of WCW gamma matrices and study of the solutions of Kähler-Dirac equation support strongly the conclusion that the construction of physical states involves the direct sum of two algebras SS and SI. The super-generators of SS are realized using only covariantly constant mode for the right-handed neutrino. The isometry sub-algebra SI is realized using all spinor modes. The direct sum $SS \oplus SI$ has the 5 tensor factors required by p-adic mass calculations. SI is Kac-Moody algebra and could be a natural identification for SKM. This forces to give up the construction of coset representation for the Super-Virasoro algebras.

This is not the only problem. The question to precisely what extent Equivalence Principle (EP) remains true in TGD framework and what might be the precise mathematical realization of EP and to wait for an answer for rather long time. Also the justification of p-adic thermodynamics for the scaling generator L_0 of Virasoro algebra - in obvious conflict with the basic wisdom that this generator should annihilate physical states - remained lacking.

One cannot still exclude the possibility that these three problems could have a common solution in terms of an appropriate coset representation. Quantum variant of EP cannot not follow from the coset representation for SS and SD. The coset representation of SS and $SI = SKM$ could however make sense and would be realized in the tensor product for the representations of SS and SI and would have the five tensor factors. Physical states would correspond to those for the direct sum $SS \oplus SI$. Since $SS \oplus SI$ acts as a spectrum generating algebra rather than gauge algebras, the condition that L_0 annihilates the physical states is not necessary. The coset representation would differ from the representation for $SS \oplus SI$ only that the states would be annihilated by the differences of the SV generators rather than their sums.

New vision about the relationship between various algebras

Consider now the new vision about the relationship between SSV, its sub-algebra acting as diffeomorphisms of 3-surface and $SKMV$.

(a) The isometries G of sub-WCW associted with given CD are symplectic transformations of $\delta CD \times CP_2$ [K12] (note that I have used the attribute "canonical" instead of "symplectic" in some contexts) reducing to diffeomorphisms at partonic 2-surfaces or at the entire 3-surfaces at the boundaries of CD. H acts a symplectic subgroup acting as diffeomorphisms of X^3 or partonic 2-surfaces. It should annihilate physical states so that SD associated with $H \subset G$ is not interesting as far as coset representations are considered.

Only the sub-algebra SI associated with symplectic isometries can provide coset representation. The representation space would be generated by the action of $SS \oplus SI$ in terms of fermionic oscillator operators and WCW isometry algebra. The same representation space allows also the representation of sums of super generators so that one has two options. $SS \oplus SI$ and $SS - SI$.

(b) Consider first the $SS \oplus SI$ option. In this case the number of tensor factors in Super-Virasoro algebra is five as required by the p-adic mass calculations. L_n annihilated physical states but there is no need for L_0 to annihilate them since symplectic algebra is not gauge algebra.

(c) Consider next the $SS - SI$ obtain, the coset representation. A generalization of the coset construction obtained by replacing finite-dimensional Lie group with infinite-dimensional symplectic group suggests itself. The differences of Super-Virasor algebra elements for SS and SI would annihilate physical states. Also the generators O_n, $n > 0$, for both algebras would annihilate the physical states so that the differences of the elements would annihilate automatically physical states for $n > 0$. For coset representation one could even require that the difference of the scaling generators L_0 annihilates the physical states.

The problem is however that the Super Virasoro algebra generators do note reduce to the sums of generators assignable to SS and SI so that one does not obtain the five tensor factors.

The coset representation motivated the proposal was that identical action of the Dirac operators assignable to G and H in coset representation could provide the long sought-for precise realization of Equivalence Principle (EP) in TGD framework. EP would state that the total inertial four-momentum and color quantum numbers assignable to G are equal to the gravitational four-momentum and color quantum numbers assignable to H. One can argue that since super-symplectic transformations correspond to the isometries of the "world of classical worlds", the assignment of the attribute "inertial" to them is natural.

This interpretation is not feasible if H corresponds acts as diffeomorphisms: the four-momentum associated with SD most naturally vanishes since it represents diffeomorphisms. If H corresponds to SI, one has the problem with the number of tensor factors. Therefore $SS \oplus SI$ seems to be the only working option.

A more feasible realization of EP quantum level is as Quantum Classical Correspondence (QCC) stating that the conserved four-momentum associated with Kähler action equals to an eigenvalue of the conserved Kähler-Dirac four-momentum having natural interpretation as gravitational four-momentum due the fact that well-defined em charge for spinor modes forces them in the generic case to string world sheets. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

Consistency with p-adic thermodynamics

The consistency with p-adic thermodynamics provides a strong reality test and has been already used as a constraint in attempts to understand the super-conformal symmetries in partonic level.

(a) The hope was that for SS/SI coset representations the p-adic thermal expectation values of the SS and SI conformal weights would be non-vanishing and identical and mass squared could be identified equivalently either as the expectation value of SI or SS scaling generator L_0. There would be no need to give up Super Virasoro conditions for $SS-SI$.

(b) There seems consistency with p-adic mass calculations for hadrons [K46] since the non-perturbative SS contributions and perturbative SKM contributions to the mass correspond to space-time sheets labeled by different p-adic primes. The earlier statement that SS is responsible for the dominating non-perturbative contributions to the hadron mass transforms to a statement reflecting $SS-SI$ duality. The perturbative quark contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SI whereas non-perturbative contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SS. Also the proposal that the exotic analogs of baryons resulting when baryon looses its valence quarks [K40] remains intact in this framework.

(c) The results of p-adic mass calculations depend crucially on the number N of tensor factors contributing to the Super-Virasoro algebra. The required number is $N=5$ and during years I have proposed several explanations for this number. This excldes the coset representation SS/SI. $SS \oplus SI$ however survives. It indeed seems that holonomic contributions related to spinor modes other than covariantly constant right-handed neutrino-that is electro-weak and spin contributions- must be regarded as contributions separate from those coming from isometries. SKM algebras in electro-weak degrees and spin degrees of of freedom, would give 2+1=3 tensor factors corresponding to $U(2)_{ew}\times SU(2)$. $SU(3)$ and $SO(3)$ (or $SO(2)\subset SO(3)$ leaving the intersection of light-like ray with S^2 invariant) would give 2 additional tensor factors. Altogether one would indeed have 5 tensor factors.

There are some further questions which pop up in mind immediately.

(a) In positive energy ontology Lorentz invariance requires the interpretation of mass squared as thermal expectation value of the conformal weight assignable to vibrational degrees of freedom. In Zero Energy Ontology (ZEO) quantum theory can be formally regarded as a square root of thermodynamics and it is possible to speak about thermal expectation value of mass squared without losing Lorentz invariance since the zero energy state corresponds to a square root of density matrix expressible as product of hermitian and unitary matrices. This implies that one can speak about thermal expectation value of mass squared rather than conformal weight. This might have some non-trivial experimental consequences since the energies of states with the same free momentum contributing to the thermal expectation value are different.

(b) The coefficient of proportionality can be however deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. The identification of the spinor partial waves is in terms of ground states of super-conformal representations.

(c) In the case of M^4 degrees of freedom it is strictly speaking not possible to talk about momentum eigen states since translations take parton out of δH_+. This would suggests that 4-momentum must be assigned with the tip of the light-cone containing the particle but this is not consistent with zero energy ontology. Hence it seems that one must restrict the translations of X^3_l to time like translations in the direction of geometric future at $\delta M^4_+ \times CP_2$. The decomposition of the partonic 3-surface X^3_l to regions $X^3_{l,i}$ carrying non-vanishing induced Kähler form and the possibility to assign $M^2(x) \subset M^4$ to the tangent space of $X^4(X^3_l)$ at points of X^3_l suggests that the points of number theoretic braid to which oscillator operators can be assigned can carry four-momentum in the plane defined by $M^2(x)$. One could assume that the four-momenta assigned with points in given region X^3_i are collinear but even this restriction is not necessary.

(d) The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \qquad (8.5.18)$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which together with the presence of preferred plane M^2 would suggest that one has

$$\begin{aligned} p^2_{i,||} &= m_i^2 \ , \\ -\sum_i p^2_{i,\perp} + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \qquad (8.5.19)$$

The masses would be reduced in bound states: $m_i^2 \to m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

How it is possible to have negative conformal weights for ground states?

p-Adic mass calculations require negative conformal weights for ground states [K37] . The only elegant solution of the problems caused by this requirement seems to be p-adic: the conformal weights are positive in the real sense but as p-adic numbers their dominating part is negative integer (in the real sense), which can be compensated by the conformal weights of Super Virasoro generators.

(a) If $\pm\lambda_i^2$ as such corresponds to a ground state conformal weight and if λ_i is real the ground state conformal weight positive in the real sense. In complex case (instanton term) the most natural formula is $h = \pm|\lambda|^2$.

(b) The first option is based on the understanding of conformal excitations in terms of CP breaking instanton term added to the modified Dirac operator. In this case the conformal weights are identified as $h = n - |\lambda_k|^2$ and the minus sign comes from the Euclidian signature of the effective metric for the modified Dirac operator. Ground state conformal weight would be non-vanishing for non-zero modes of $D(X^3_l)$. Massless bosons produce difficulties unless one has $h = |\lambda_i(1) - \lambda_i(2)|^2$, where $i = 1, 2$ refers to the two wormhole throats. In this case the difference can vanish and its non-vanishing would be due to the symmetric breaking. This scenario is assumed in p-adic mass calculations. Fermions are predicted to be always massive since zero modes of $D(X^2)$ represent super gauge degrees of freedom.

(c) In the context of p-adic thermodynamics a loop hole opens allowing λ_i to be real. In spirit of rational physics suppose that one has in natural units $h = \lambda_i^2 = xp^2 - n$, where x is integer. This number is positive and large in the real sense. In p-adic sense the dominating part of this number is $-n$ and can be compensated by the net conformal weight n of Super Virasoro generators acting on the ground state. xp^2 represents the small Higgs contribution to the mass squared proportional to $(xp^2)_R \simeq x/p^2$ ($_R$ refers to canonical identification). By the basic features of the canonical identification $p > x \simeq p$ should hold true for gauge bosons for which Higgs contribution dominates. For fermions

x should be small since p-adic mass calculations are consistent with the vanishing of Higgs contribution to the fermion mass. This would lead to the earlier conclusion that xp^2 and hence B_K is large for bosons and small for fermions and that the size of fermionic (bosonic) wormhole throat is large (small). This kind of picture is consistent with the p-adic modular arithmetics and suggests by the cutoff for conformal weights implied by the fact that both the number of fermionic oscillator operators and the number of points of number theoretic braid are finite. This solution is however tricky and does not conform with number theoretical universality.

8.6 Preferred extremals and solutions of the modified Dirac equation and super-conformal symmetries

The previous considerations concerning super-conformal symmetries and space-time SUSY have been based on general arguments. The new vision about preferred extremals and modified Dirac equation [K87] however leads to a detailed understanding of super-conformal symmetries at the level of field equations and is bound to modify the existing vision about super-conformal symmetries. One important discovery is that Einstein's equations imply the vanishing of terms proportional to Kähler current in field equations for preferred extremals and Equivalence Principle at the classical level could be realized automatically in all scales in contrast to the earlier belief. This obviously must have implications to the general vision about Super-Virasoro representations and one must be ready to modify the existing picture based on the assumption that quantum version of Equivalence Principle is realized in terms coset representations.

The very special role of right handed neutrino is also bound to have profound implications. A further important outcome is the identification of gauge potentials as duals of Kac-Moody currents at the boundaries of string world sheets: quantum gauge potentials are defined only where they are needed that is the curves defining the non-integrable phase factors. This gives also rise to the realization of the conjecture Yangian in terms of the Kac-Moody charges and commutators in accordance with the earlier conjecture.

8.6.1 Super-conformal symmetries

It is good to summarize first the basic ideas about Super-Virasoro representations. TGD allows two kinds of super-conformal symmetries.

(a) The first super-conformal symmetry is associated with $\delta M^4_\pm \times CP_2$ and corresponds to symplectic symmetries of $\delta M^4_\pm \times CP_2$. The reason for extension of conformal symmetries is metric 2-dimensionality of the light-like boundary $\delta M^4_\pm$ defining upper/lower boundary of causal diamond (CD). This super-conformal symmetry is something new and corresponds to replacing finite-dimensional Lie-group G for Kac-Moody symmetry with infinite-dimensional symplectic group. The light-like radial coordinate of $\delta M^4_\pm$ takes the role of the real part of complex coordinate z for ordinary conformal symmetry. Together with complex coordinate of S^2 it defines 3-D restriction of Hamilton-Jacobi variant of 4-D super-conformal symmetries. One can continue the conformal symmetries from light-cone boundary to CD by forming a slicing by parallel copies of $\delta M^4_\pm$. There are two possible slicings corresponding to the choices δM^4_+ and δM^4_- assignable to the upper and lower boundaries of CD. These two choices correspond to two arrows of geometric time for the basis of zero energy states in ZEO.

(b) Super-symplectic degrees of freedom determine the electroweak and color quantum numbers of elementary particles. Bosonic emergence implies that ground states assignable to partonic 2-surfaces correspond to partial waves in $\delta M^4_\pm$ and one obtains color partial waves in particular. These partial waves correspond to the solutions for the Dirac equation in imbedding space and the correlation between color and electroweak quantum numbers is not quite correct. Super-Kac-Moody generators give the compensating color for massless states obtained from tachyonic ground states guaranteeing that standard correlation is obtained. Super-symplectic degrees are therefore directly visible in particle spectrum. One can say that at the point-like limit the WCW spinors reduce to tensor products of imbedding space spinors assignable to the center of mass degrees of freedom for the partonic 2-surfaces defining wormhole throats.

I have proposed a physical interpretation of super-symplectic vibrational degrees of freedom in terms of degrees of freedom assignable to non-perturbative QCD. These degrees of freedom would be responsible for most of the baryon masses but their theoretical understanding is lacking in QCD framework.

(c) The second super-conformal symmetry is assigned light-like 3-surfaces and to the isometries and holonomies of the imbedding space and is analogous to the super-Kac-Moody symmetry of string models. Kac-Moody symmetries could be assigned to the light-like deformations of light-like 3-surfaces. Isometries give tensor factor $E^2 \times SU(3)$ and holonomies factor $SU(2)_L \times U(1)$. Altogether one has 5 tensor factors to super-conformal algebra. That the number is just five is essential for the success p-adic mass calculations [K43, K37].

The construction of solutions of the modified Dirac equation suggests strongly that the fermionic representation of the Super-Kac-Moody algebra can be assigned as conserved charges associated with the space-like braid strands at both the 3-D space-like ends of space-time surfaces and with the light-like (or space-like with a small deformation) associated with the light-like 3-surfaces. The extension to Yangian algebra involving higher multi-linears of super-Kac Moody generators is also highly suggestive. These charges would be non-local and assignable to several wormhole contacts simultaneously. The ends of braids would correspond points of partonic 2-surfaces defining a discretization of the partonic 2-surface having interpretation in terms of finite measurement resolution.

These symmetries would correspond to electroweak and strong gauge fields and to gravitation. The duals of the currents giving rise to Kac-Moody charges would define the counterparts of gauge potentials and the conserved Kac-Moody charges would define the counterparts of non-integrable phase factors in gauge theories. The higher Yangian charges would define generalization of non-integrable phase factors. This would suggest a rather direct connection with the twistorial program for calculating the scattering amplitudes implies also by zero energy ontology.

Quantization recipes have worked in the case of super-string models and one can ask whether the application of quantization to the coefficients of powers of complex coordinates or Hamilton-Jacobi coordinates could lead to the understanding of the 4-D variants of the conformal symmetries and give detailed information about the representations of the Kac-Moody algebra too.

8.6.2 What is the role of the right-handed neutrino?

A highly interesting aspect of Super-Kac-Moody symmetry is the special role of right handed neutrino.

(a) Only right handed neutrino allows besides the modes restricted to 2-D surfaces also the 4D modes de-localized to the entire space-time surface. The first ones are holomorphic functions of single coordinate and the latter ones holomorphic functions of two complex/Hamilton-Jacobi coordinates. Only ν_R has the full $D = 4$ counterpart of the conformal symmetry and the localization to 2-surfaces has interpretation as super-conformal symmetry breaking halving the number of super-conformal generators.

(b) This forces to ask for the meaning of super-partners. Are super-partners obtained by adding ν_R neutrino localized at partonic 2-surface or de-localized to entire space-time surface or its Euclidian or Minkowskian region accompanying particle identified as wormhole throat? Only the Euclidian option allows to assign right handed neutrino to a unique partonic 2-surface. For the Minkowskian regions the assignment is to many particle state defined by the partonic 2-surfaces associated with the 3-surface. Hence for spartners the 4-D right-handed neutrino must be associated with the 4-D Euclidian line of the generalized Feynman diagram.

(c) The orthogonality of the localized and de-localized right handed neutrino modes requires that 2-D modes correspond to higher color partial waves at the level of imbedding space. If color octet is in question, the 2-D right handed neutrino as the candidate for the generator of standard SUSY would combine with the left handed neutrino to form a massive neutrino. If 2-D massive neutrino acts as a generator of super-symmetries, it is is in the same role as badly broken super-symmeries generated by other 2-D modes of the induced spinor field (SUSY with rather large value of $\mathcal{N}$) and one can argue that the counterpart of standard SUSY cannot correspond to this kind of super-symmetries. The right-handed neutrinos de-localized inside the lines of generalized Feynman diagrams, could generate $\mathcal{N} = 2$ variant of the standard SUSY.

How particle and right handed neutrino are bound together?

Ordinary SUSY means that apart from kinematical spin factors sparticles and particles behave identically with respect to standard model interactions. These spin factors would allow to distinguish between particles and sparticles. But is this the case now?

(a) One can argue that 2-D particle and 4-D right-handed neutrino behave like independent entities, and because ν_R has no standard model couplings this entire structure behaves like a particle rather than sparticle with respect to standard model interactions: the kinematical spin dependent factors would be absent.

(b) The question is also about the internal structure of the sparticle. How the four-momentum is divided between the ν_R and and 2-D fermion. If ν_R carries a negligible portion of four-momentum, the four-momentum carried by the particle part of sparticle is same as that carried by particle for given four-momentum so that the distinctions are only kinematical for the ordinary view about sparticle and trivial for the view suggested by the 4-D character of ν_R.

Could sparticle character become manifest in the ordinary scattering of sparticle?

(a) If ν_R behaves as an independent unit not bound to the particle, it would continue in the original direction as particle scatters: sparticle would decay to particle and right-handed neutrino. If ν_R carries a non-negligible energy the scattering could be detected via a missing energy. If not, then the decay could be detected by the interactions revealing the presence of ν_R. ν_R can have only gravitational interactions. What these gravitational interactions are is not however quite clear since the proposed identification of gravitational gauge potentials is as duals of Kac-Moody currents analogous to gauge potentials located at the boundaries of string world sheets. Does this mean that 4-D right-handed neutrino has no quantal gravitational interactions? Does internal consistency require ν_R to have a vanishing gravitational and inertial masses and does this mean that this particle carries only spin?

(b) The cautious conclusion would be following: if de-localized ν_R and parton are uncorrelated particle and sparticle cannot be distinguished experimentally and one might perhaps understand the failure to detect standard SUSY at LHC. Note however that the 2-D fermionic oscillator algebra defines badly broken large $\mathcal{N}$ SUSY containing also massive (longitudinal momentum square is non-vanishing) neutrino modes as generators.

Taking a closer look on sparticles

It is good to take a closer look at the de-localized right handed neutrino modes.

(a) At imbedding space level that is in cm mass degrees of freedom they correspond to covariantly constant CP_2 spinors carrying light-like momentum which for causal diamond could be discretized. For non-vanishing momentum one can speak about helicity having opposite sign for ν_R and $\overline{\nu}_R$. For vanishing four-momentum the situation is delicate since only spin remains and Majorana like behavior is suggestive. Unless one has momentum continuum, this mode might be important and generate additional SUSY resembling standard $\mathcal{N} = 1$ SUSY.

(b) At space-time level the solutions of modified Dirac equation are holomorphic or anti-holomorphic.

  i. For non-constant holomorphic modes these characteristics correlate naturally with fermion number and helicity of ν_R . One can assign creation/annihilation operator to these two kinds of modes and the sign of fermion number correlates with the sign of helicity.

  ii. The covariantly constant mode is naturally assignable to the covariantly constant neutrino spinor of imbedding space. To the two helicities one can assign also oscillator operators $\{a_\pm, a_\pm^\dagger\}$. The effective Majorana property is expressed in terms of non-orthogonality of ν_R and and $\overline{\nu}_R$ translated to the the non-vanishing of the anti-commutator $\{a_+^\dagger, a_-\} = \{a_-^\dagger, a_+\} = 1$. The reduction of the rank of the 4×4 matrix defined by anti-commutators to two expresses the fact that the number of degrees of freedom has halved. $a_+^\dagger = a_-$ realizes the conditions and implies that one has only $\mathcal{N} = 1$ SUSY multiplet since the state containing both ν_R and $\overline{\nu}_R$ is same as that containing no right handed neutrinos.

  iii. One can wonder whether this SUSY is masked totally by the fact that sparticles with all possible conformal weights n for induced spinor field are possible and the branching ratio to $n = 0$ channel is small. If momentum continuum is present, the zero momentum mode might be equivalent to nothing.

What can happen in spin degrees of freedom in super-symmetric interaction vertices if one accepts this interpretation? As already noticed, this depends solely on what one assumes about the correlation of the four-momenta of particle and ν_R.

(a) For SUSY generated by covariantly constant ν_R and $\overline{\nu}_R$ there is no neutrino four-momentum involved so that only spin matters. One cannot speak about the change of direction for ν_R. In the scattering of sparticle the direction of particle changes and introduces different spin quantization axes. ν_R retains its spin and in new system it is superposition of two spin projections. The presence of both helicities requires that the transformation $\nu_R \rightarrow \overline{\nu}_R$ happens with an amplitude determined purely kinematically by spin rotation matrices. This is consistent with fermion number conservation modulo 2. $\mathcal{N} = 1$ SUSY based on Majorana spinors is highly suggestive.

(b) For SUSY generated by non-constant holomorphic and anti-holomorphic modes carrying fermion number the behavior in the scattering is different. Suppose that the sparticle does not split to particle moving in the new direction and ν_R moving in the original direction so that also ν_R or $\overline{\nu}_R$ carrying some massless fraction of four-momentum changes its direction of motion. One can form the spin projections with respect to the new spin axis but must drop the projection which does not conserve fermion number. Therefore the kinematics at the vertices is different. Hence $\mathcal{N} = 2$ SUSY with fermion number conservation is suggestive when the momentum directions of particle and ν_R are completely correlated.

(c) Since right-handed neutrino has no standard model couplings, p-adic thermodynamics for 4-D right-handed neutrino must correspond to a very low p-adic temperature $T = 1/n$. This implies that the excitations with non-vanishing conformal weights are effectively absent and one would have $\mathcal{N} = 1$ SUSY effectively.

    The simplest assumption is that particle and sparticle correspond to the same p-adic mass scale and have degenerate masses: it is difficult to imagine any good reason for why the p-adic mass scales should differ. This should have been observed -say in decay widths of weak bosons - unless the spartners correspond to large $\hbar$ phase and therefore to dark matter. Note that for the badly broken 2-D N=2 SUSY in fermionic sector this kind of almost degeneracy cannot be excluded and I have considered an explanation for the mysterious X and Y mesons in terms of this degeneracy [K40].

Why space-time SUSY is not possible in TGD framework?

LHC suggests that one does not have $\mathcal{N}=1$ SUSY in standard sense. Why one cannot have standard space-time SUSY in TGD framework. Let us begin by listing all arguments popping in mind.

(a) Could covariantly constant ν_R represents a gauge degree of freedom? This is plausible since the corresponding fermion current is non-vanishing.

(b) The original argument for absence of space-time SUSY years ago was indirect: $M^4 \times CP_2$ does not allow Majorana spinors so that $\mathcal{N}=1$ SUSY is excluded.

(c) One can however consider $\mathcal{N}=2$ SUSY by including both helicities possible for covariantly constant ν_R. For ν_R the four-momentum vanishes so that one cannot distinguish the modes assigned to the creation operator and its conjugate via complex conjugation of the spinor. Rather, one oscillator operator and its conjugate correspond to the two different helicities of right-handed neutrino with respect to the direction determined by the momentum of the particle. The spinors can be chosen to be real in this basis. This indeed gives rise to an irreducible representation of spin 1/2 SUSY algebra with right-handed neutrino creation operator acting as a ladder operator. This is however $\mathcal{N}=1$ algebra and right-handed neutrino in this particular basis behaves effectively like Majorana spinor. One can argue that the system is mathematically inconsistent. By choosing the spin projection axis differently the spinor basis becomes complex. In the new basis one would have $\mathcal{N}=2$, which however reduces to $\mathcal{N}=1$ in the real basis.

(d) Or could it be that fermion and sfermion do exist but cannot be related by SUSY? In standard SUSY fermions and sfermions forming irreducible representations of super Poincare algebra are combined to components of superfield very much like finite-dimensional representations of Lorentz group are combined to those of Poincare. In TGD framework ν_R generates in space-time interior generalization of 2-D super-conformal symmetry but covarianlty constant ν_R cannot give rise to space-time SUSY.

This would be very natural since right-handed neutrinos do not have any electroweak interactions and are are de-localized into the interior of the space-time surface unlike other particles localized at 2-surfaces. It is difficult to imagine how fermion and ν_R could behave as a single coherent unit reflecting itself in the characteristic spin and momentum dependence of vertices implied by SUSY. Rather, it would seem that fermion and sfermion should behave identically with respect to electroweak interactions.

The third argument looks rather convincing and can be developed to a precise argument.

(a) If sfermion is to represent elementary bosons, the products of fermionic oscillator operators with the oscillator operators assignable to the covariantly constant right handed neutrinos must define might-be bosonic oscillator operators as $b_n = a_n a$ and $b_n^\dagger = a_n^\dagger a^\dagger$ One can calculate the commutator for the product of operators. If fermionic oscillator operators commute, so do the corresponding bosonic operators. The commutator $[b_n, b_n^\dagger]$ is however proportional to occupation number for ν_R in $\mathcal{N}=1$ SUSY representation and vanishes for the second state of the representation. Therefore $\mathcal{N}=1$ SUSY is a pure gauge symmetry.

(b) One can however have both irreducible representations of SUSY: for them either fermion or sfermion has a non-vanishing norm. One would have both fermions and sfermions but they would not belong to the same SUSY multiplet, and one cannot expect SUSY symmetries of 3-particle vertices.

(c) For instance, γFF vertex is closely related to $\gamma\tilde{F}\tilde{\overline{F}}$ in standard SUSY. Now one expects this vertex to decompose to a product of $\gamma F\overline{F}$ vertex and amplitude for the creation of $\nu_R\tilde{\nu}_R$ from vacuum so that the characteristic momentum and spin dependent factors distinguishing between the couplings of photon to scalar and and fermion are absent. Both states behave like fermions. The amplitude for the creation of $\nu_R\tilde{\nu}_R$ from vacuum is naturally equal to unity as an occupation number operator by crossing symmetry. The presence of right-handed neutrinos would be invisible if this picture is correct. Whether this invisible label can have some consequences is not quite clear: one could argue that the decay rates of weak bosons to fermion pairs are doubled unless one introduces $1/\sqrt{2}$ factors to couplings.

Where the sfermions might make themselves visible are loops. What loops are? Consider boson line first. Boson line is replaced with a sum of two contributions corresponding to ordinary contribution with fermion and anti-fermion at opposite throats and second contribution with fermion and anti-fermion accompanied by right-handed neutrino ν_R and its antiparticle which now has opposite helicity to ν_R. The loop for ν_R decomposes to four pieces since also the propagation from wormhole throat to the opposite wormhole throat must be taken into account. Each of the four propagators equals to $a^\dagger_{1/2}a^\dagger_{-1/2}$ or its hermitian conjugate. The product of these is slashed between vacuum states and anti-commutations give imaginary unit per propagator giving $i^4 = 1$. The two contributions are therefore identical and the scaling $g \to g/\sqrt{2}$ for coupling constants guarantees that sfermions do not affect the scattering amplitudes at all. The argument is identical for the internal fermion lines.

8.6.3 WCW geometry and super-conformal symmetries

The vision about the geometry of WCW has been roughly the following and the recent steps of progress induce to it only small modifications if any.

(a) Kähler geometry is forced by the condition that hermitian conjugation allows geometrization. Kähler function is given by the Kähler action coming from space-time regions with Euclidian signature of the induced metric identifiable as lines of generalized Feynman diagrams. Minkowskian regions give imaginary contribution identifiable as the analog of Morse function and implying interference effects and stationary phase approximation. The vision about quantum TGD as almost topological QFT inspires the proposal that Kähler action reduces to 3-D terms reducing to Chern-Simons terms by the weak form of electric-magnetic duality. The recent proposal for preferred extremals is consistent with this property realizing also holography implied by general coordinate invariance. Strong form of general coordinate invariance implying effective 2-dimensionality in turn suggests that Kähler action is expressible in terms of areas of partonic 2-surfaces and string world sheets.

(b) The complexified gamma matrices of WCW come as hermitian conjugate pairs and anti-commute to the Kähler metric of WCW. Also bosonic generators of symplectic transformations of $\delta M^4_\pm \times CP_2$ a assumed to act as isometries of WCW geometry can be complexified and appear as similar pairs. The action of isometry generators coincides with that of symplectic generators at partonic 2-surfaces and string world sheets but elsewhere inside the space-time surface it is expected to be deformed from the symplectic action. The super-conformal transformations of $\delta M^4_\pm \times CP_2$ acting on the light-like radial coordinate of $\delta M^4_\pm$ act as gauge symmetries of the geometry meaning that the corresponding WCW vector fields have zero norm.

(c) WCW geometry has also zero modes which by definition do not contribute to WCW metric expect possibly by the dependence of the elements of WCW metric on zero modes through a conformal factor. In particular, induced CP_2 Kähler form and its analog for sphere $r_M = constant$ of light cone boundary are symplectic invariants, and one can define an infinite number of zero modes as invariants defined by Kähler fluxes over partonic 2-surfaces and string world sheets. This requires however the slicing of CD parallel copies of δM^4_+ or δM^4_-. The physical interpretation of these non-quantum fluctuating degrees of freedom is as classical variables necessary for the interpretation of quantum measurement theory. Classical variable would metaphorically correspond the position of the pointer of the measurement instrument.

(d) The construction receives a strong philosophical inspiration from the geometry of loop spaces. Loop spaces allow a unique Kähler geometry with maximal isometry group identifiable as Kac-Moody group. The reason is that otherwise Riemann connection does not exist. The only problem is that curvature scalar diverges since the Riemann tensor is by constant curvature property proportional to the metric. In 3-D case one would have union of constant curvature spaces labelled by zero modes and the situation is expected to be even more restrictive. The conjecture indeed is that WCW geometry exists only for $H = M^4 \times CP_2$: infinite-D Kähler geometric existence and therefore physics would be unique. One can also hope that Ricci scalar is finite and therefore zero by the constant curvature property so that Einstein's equations are satisfied.

(e) WCW Hamiltonians determined the isometry currents and WCW metric is given in terms of the anti-commutators of the Killing vector fields associated with symplectic isometry currents. The WCW Hamiltonians generating symplectic isometries correspond to the Hamiltonians spanning the symplectic group of $\delta M^4_\pm \times CP_2$. One can say that the space of quantum fluctuating degrees of freedom is this symplectic group of $\delta M^4_\pm \times CP_2$ or its subgroup or coset space: this must have very deep implications for the structure of the quantum TGD.

(f) Zero energy ontology brings in additional delicacies. Basic objects are now unions of partonic 2-surfaces at the ends of CD. Also string world sheets would naturally contribute. One can generalize the expressions for the isometry generators in a straightforward manner by requiring that given isometry restricts to a symplectic transformation at partonic 2-surfaces and string world sheets.

(g) One could criticize the effective metric 2-dimensionality forced by general consistency arguments as something non-physical. The Hamiltonians are expressed using only the data at partonic 2-surfaces: this includes also 4-D tangent space data via the weak form of electric-magnetic duality so that one has only effective 2-dimensionality. Obviously WCW geometry must have large gauge symmetries besides zero modes. The super-conformal symmetries indeed represent gauge symmetries of this kind. Effective 2-dimensionality realizing strong form of holography in turn is induced by the strong form of general coordinate invariance. Light-like 3-surfaces at which the signature of the induced metric changes must be equivalent with the 3-D space-like ends of space-time surfaces at the light-boundaries of space-time surfaces as far as WCW geometry is considered. This requires that the data from their 2-D intersections defining partonic 2-surfaces should dictate the WCW geometry. Note however that Super-Kac-Moody charges giving information about the interiors of 3-surfaces appear in the construction of the physical states.

What is the role of the right handed neutrino in this construction?

(a) In the construction of components of WCW metric as anti-commutators of super-generators only the covariantly constant right-handed neutrino appears in the super-generators analogous to super-Kac-Moody generators. All holomorphic modes of right handed neutrino characterized by two integers could in principle contribute to the WCW gamma matrices identified as fermionic super-symplectic generators anti-commuting to the metric. At the space-like ends of space-time surface the holomorphic generators would restrict to symplectic generators since the radial light-like coordinate r_M identified and complex coordinate of CP_2 allowing identification as restrictions of two complex coordinates or Hamilton-Jacobi coordinates to light-like boundary.

(b) The non-covariantly constant modes could also correspond to purely super-conformal gauge degrees of freedom. Originally the restriction to right-handed neutrino looked somewhat un-satisfactory but the recent view about Super-Kac-Moody symmetries makes its special role rather natural. One could say that WCW geometry possesses the maximal $D = 4$ supersymmetry.

(c) One can of course ask whether the Super-Kac-Moody generators assignable to the isometries of H and expressible as conserved charges associated with the boundaries of string world sheets could contribute to the WCW geometry via the anti-commutators. This option cannot be excluded but in this case the interpretation in terms of Hamiltonians is not obvious.

8.6.4 The relationship between inertial gravitational masses

The relationship between inertial and gravitational masses and Equivalence Principle have been on of the longstanding problems in TGD. Not surprisingly, the realization how GRT space-time relates to the many-sheeted space-time of TGD finally allowed to solve the problem.

ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invarance of TGD is in sharpt contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4. If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

Equivalence Principle at quantum level

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

(a) At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.

(b) The most recent view (2014) about understanding how EP emerges in TGD is described in [K72] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets associated with Käbler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).

(c) The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the modified Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

Equivalence Principle at classical level

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD spacetime and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K96].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

8.6.5 Constraints from p-adic mass calculations and ZEO

A further important physical input comes from p-adic thermodynamics forming a core element of p-adic mass calculations.

(a) The first thing that one can get worried about relates to the extension of conformal symmetries. If the conformal symmetries generalize to $D = 4$, how can one take seriously the results of p-adic mass calculations based on 2-D conformal invariance? There is no reason to worry. The reduction of the conformal invariance to 2-D one for the preferred extremals takes care of this problem. This however requires that the fermionic contributions assignable to string world sheets and/or partonic 2-surfaces - Super- Kac-Moody contributions - should dictate the elementary particle masses. For hadrons also symplectic contributions should be present. This is a valuable hint in attempts to identify the mathematical structure in more detail.

(b) ZEO suggests that all particles, even virtual ones correspond to massless wormhole throats carrying fermions. As a consequence, twistor approach would work and the kinematical constraints to vertices would allow the cancellation of divergences. This would suggest that the p-adic thermal expectation value is for the longitudinal M^2 momentum squared (the definition of CD selects $M^1 \subset M^2 \subset M^4$ as also does number theoretic vision). Also propagator would be determined by M^2 momentum. Lorentz invariance would be obtained by integration of the moduli for CD including also Lorentz boosts of CD.

(c) In the original approach one allows states with arbitrary large values of L_0 as physical states. Usually one would require that L_0 annihilates the states. In the calculations however mass squared was assumed to be proportional L_0 apart from vacuum contribution. This is a questionable assumption. ZEO suggests that total mass squared vanishes and that one can decompose mass squared to a sum of longitudinal and transversal parts. If one can do the same decomposition to longitudinal and transverse parts also for the Super Virasoro algebra then one can calculate longitudinal mass squared as a p-adic thermal expectation in the transversal super-Virasoro algebra and only states with $L_0 = 0$ would contribute and one would have conformal invariance in the standard sense.

(d) In the original approach the assumption motivated by Lorentz invariance has been that mass squared is replaced with conformal weight in thermodynamics, and that one first calculates the thermal average of the conformal weight and then equates it with mass squared. This assumption is somewhat ad hoc. ZEO however suggests an alternative interpretation in which one has zero energy states for which longitudinal mass squared of positive energy state derive from p-adic thermodynamics. Thermodynamics - or rather, its square root - would become part of quantum theory in ZEO. M-matrix is indeed product of hermitian square root of density matrix multiplied by unitary S-matrix and defines the entanglement coefficients between positive and negative energy parts of zero energy state.

(e) The crucial constraint is that the number of super-conformal tensor factors is $N = 5$: this suggests that thermodynamics applied in Super-Kac-Moody degrees of freedom assignable to string world sheets is enough, when one is interested in the masses of fermions and gauge bosons. Super-symplectic degrees of freedom can also contribute and determine the dominant contribution to baryon masses. Should also this contribution obey p-adic thermodynamics in the case when it is present? Or does the very fact that this contribution need not be present mean that it is not thermal? The symplectic contribution should correspond to hadronic p-adic length prime rather the one assignable to (say) u quark. Hadronic p-adic mass squared and partonic p-adic mass squared cannot be summed since primes are different. If one accepts the basic rules [K46], longitudinal energy and momentum are additive as indeed assumed in perturbative QCD.

(f) Calculations work if the vacuum expectation value of the mass squared must be assumed to be tachyonic. There are two options depending on whether one whether p-adic thermodynamics gives total mass squared or longitudinal mass squared.

   i. One could argue that the total mass squared has naturally tachyonic ground state expectation since for massless extremals longitudinal momentum is light-like and transversal momentum squared is necessary present and non-vanishing by the localization to topological light ray of finite thickness of order p-adic length scale. Transversal degrees of freedom would be modeled with a particle in a box.

ii. If longitudinal mass squared is what is calculated, the condition would require that transversal momentum squared is negative so that instead of plane wave like behavior exponential damping would be required. This would conform with the localization in transversal degrees of freedom.

8.6.6 The emergence of Yangian symmetry and gauge potentials as duals of Kac-Moody currents

Yangian symmetry plays a key role in $\mathcal{N} = 4$ super-symmetric gauge theories. What is special in Yangian symmetry is that the algebra contains also multi-local generators. In TGD framework multi-locality would naturally correspond to that with respect to partonic 2-surfaces and string world sheets and the proposal has been that the Super-Kac-Moody algebras assignable to string worlds sheets could generalize to Yangian.

Witten has written a beautiful exposition of Yangian algebras [B43]. Yangian is generated by two kinds of generators J^A and Q^A by a repeated formation of commutators. The number of commutations tells the integer characterizing the multi-locality and provides the Yangian algebra with grading by natural numbers. Witten describes a 2-dimensional QFT like situation in which one has 2-D situation and Kac-Moody currents assignable to real axis define the Kac-Moody charges as integrals in the usual manner. It is also assumed that the gauge potentials defined by the 1-form associated with the Kac-Moody current define a flat connection:

$$\partial_\mu j^A_\nu - \partial_\nu j^A_\nu + [j^A_\mu, j^A_\nu] = 0 \ . \tag{8.6.1}$$

This condition guarantees that the generators of Yangian are conserved charges. One can however consider alternative manners to obtain the conservation.

(a) The generators of first kind - call them J^A - are just the conserved Kac-Moody charges. The formula is given by

$$J_A = \int_{-\infty}^{\infty} dx j^{A0}(x,t) \ . \tag{8.6.2}$$

(b) The generators of second kind contain bi-local part. They are convolutions of generators of first kind associated with different points of string described as real axis. In the basic formula one has integration over the point of real axis.

$$Q^A = f^A_{BC} \int_{-\infty}^{\infty} dx \int_x^{\infty} dy j^{B0}(x,t) j^{C0}(y,t) - 2\int_{-\infty}^{\infty} j^A_x dx \ . \tag{8.6.3}$$

These charges are indeed conserved if the curvature form is vanishing as a little calculation shows.

How to generalize this to the recent context?

(a) The Kac-Moody charges would be associated with the braid strands connecting two partonic 2-surfaces - Strands would be located either at the space-like 3-surfaces at the ends of the space-time surface or at light-like 3-surfaces connecting the ends. Modified Dirac equation would define Super-Kac-Moody charges as standard Noether charges. Super charges would be obtained by replacing the second quantized spinor field or its conjugate in the fermionic bilinear by particular mode of the spinor field. By replacing both spinor field and its conjugate by its mode one would obtain a conserved c-number charge corresponding to an anti-commutator of two fermionic super-charges. The convolution involving double integral is however not number theoretically attractive whereas single 1-D integrals might make sense.

(b) An encouraging observation is that the Hodge dual of the Kac-Moody current defines the analog of gauge potential and exponents of the conserved Kac-Moody charges could be identified as analogs for the non-integrable phase factors for the components of this gauge potential. This identification is precise only in the approximation that generators commute since only in this case the ordered integral $P(exp(i\int Adx))$ reduces to $P(exp(i\int Adx))$.Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization implying that Abelian approximation works. This conforms with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

This would make possible a direct identification of Kac-Moody symmetries in terms of gauge symmetries. For isometries one would obtain color gauge potentials and the analogs of gauge potentials for graviton field (in TGD framework the contraction with M^4 vierbein would transform tensor field to 4 vector fields). For Kac-Moody generators corresponding to holonomies one would obtain electroweak gauge potentials. Note that super-charges would give rise to a collection of spartners of gauge potentials automatically. One would obtain a badly broken SUSY with very large value of $\mathcal{N}$ defined by the number of spinor modes as indeed speculated earlier [K23].

(c) The condition that the gauge field defined by 1-forms associated with the Kac-Moody currents are trivial looks unphysical since it would give rise to the analog of topological QFT with gauge potentials defined by the Kac-Moody charges. For the duals of Kac-Moody currents defining gauge potentials only covariant divergence vanishes implying that curvature form is

$$F_{\alpha\beta} = \epsilon_{\alpha\beta}[j_\mu, j^\mu] \ , \tag{8.6.4}$$

so that the situation does not reduce to topological QFT unless the induced metric is diagonal. This is not the case in general for string world sheets.

(d) It seems however that there is no need to assume that j_μ defines a flat connection. Witten mentions that although the discretization in the definition of J^A does not seem to be possible, it makes sense for Q^A in the case of $G = SU(N)$ for any representation of G. For general G and its general representation there exists no satisfactory definition of Q. For certain representations, such as the fundamental representation of $SU(N)$, the definition of Q^A is especially simple. One just takes the bi-local part of the previous formula:

$$Q^A = f^A_{BC}\sum_{i<j} J^B_i J^C_j \ . \tag{8.6.5}$$

What is remarkable that in this formula the summation need not refer to a discretized point of braid but to braid strands ordered by the label i by requiring that they form a connected polygon. Therefore the definition of J^A could be just as above.

(e) This brings strongly in mind the interpretation in terms of twistor diagrams. Yangian would be identified as the algebra generated by the logarithms of non-integrable phase factors in Abelian approximation assigned with pairs of partonic 2-surfaces defined in terms of Kac-Moody currents assigned with the modified Dirac action. Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization. This would fit nicely with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

The resulting algebra satisfies the basic commutation relations

$$\left[J^A, J^B\right] = f^{AB}_C J^C \ , \ \left[J^A, Q^B\right] = f^{AB}_C Q^C \ . \tag{8.6.6}$$

plus the rather complex Serre relations described in [B43].

8.6.7 Quantum criticality and electroweak symmetries

In the following quantum criticali and electroweak symmetries are discussed for Kähler-Dirac action.

What does one mean with quantum criticality?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

(a) What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the imbedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

(b) At more technical level one would expect criticality to corresponds to deformations of a given preferred extremal defining a vanishing second variation of Kähler Khler function or Kähler action.

   i. For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

   ii. Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A102]. Cusp catastrophe [A3] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

(c) Quantum criticality makes sense also for Kähler action.

   i. Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n\times h$ [K21] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

   ii. Also now one expects a hierarchy of criticalities and and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

iii. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

(d) I have discussed what criticality could mean for modified Dirac action [K22] .

i. I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the modified Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

ii. The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since imbedding space coordinates appear as parameters in modified Dirac action. The existence of conserved currents does not actually require the vanishing of the second variation of Kähler action as claimed earlier. It is enough that the first variation of the canonical momentum densities contracted with the imbedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation:it does not imply the vanishing of four-momentum, only the vanishing of mass. Hence conserved currents are obtained also outside the quantum criticality.

iii. It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generaic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for currents associated with the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the modified gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor. This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied.

In the following these arguments are formulated more precisely. The unexpected result is that critical deformations induce conformal scalings of the modified metric and electro-weak gauge transformations of the induced spinor connection at X^2. Therefore holomorphy brings in the Kac-Moody symmetries associated with isometries of H (gravitation and color gauge group) and quantum criticality those associated with the holonomies of H (electro-weak-gauge group) as additional symmetries.

The variation of modes of the induced spinor field in a variation of space-time surface respecting the preferred extremal property

Consider first the variation of the induced spinor field in a variation of space-time surface respecting the preferred extremal property. The deformation must be such that the deformed modified Dirac operator D annihilates the modified mode. By writing explicitly the variation of the modified Dirac action (the action vanishes by modified Dirac equation) one obtains deformations and requiring its vanishing one obtains

$$\delta\Psi = D^{-1}(\delta D)\Psi \ . \quad (8.6.7)$$

D^{-1} is the inverse of the modified Dirac operator defining the analog of Dirac propagator and δD defines vertex completely analogous to $\gamma^k \delta A_k$ in gauge theory context. The functional integral over preferred extremals can be carried out perturbatively by expressing δD in terms of δh^k and one obtains stringy perturbation theory around X^2 associated with the preferred extremal defining maximum of Kähler function in Euclidian region and extremum of Kähler action in Minkowskian region (stationary phase approximation).

What one obtains is stringy perturbation theory for calculating n-points functions for fermions at the ends of braid strands located at partonic 2-surfaces and representing intersections of string world sheets and partonic 2-surfaces at the light-like boundaries of CDs. δD- or more precisely, its partial derivatives with respect to functional integration variables - appear atthe vertices located anywhere in the interior of X^2 with outgoing fermions at braid ends. Bosonic propagators are replaced with correlation functions for δh^k. Fermionic propagator is defined by D^{-1}.

After 35 years or hard work this provides for the first time a reasonably explicit formula for the N-point functions of fermions. This is enough since by bosonic emergence [K50] these N-point functions define the basic building blocks of the scattering amplitudes. Note that bosonic emergence states that bosons corresponds to wormhole contacts with fermion and anti-fermion at the opposite wormhole throats.

What critical modes could mean for the induced spinor fields?

What critical modes could mean for the induced spinor fields at string world sheets and partonic 2-surfaces. The problematic part seems to be the variation of the modified Dirac operator since it involves gradient. One cannot require that covariant derivative remains invariant since this would require that the components of the induced spinor connection remain invariant and this is quite too restrictive condition. Right handed neutrino solutions de-localized into entire X^2 are however an exception since they have no electro-weak gauge couplings and in this case the condition is obvious: modified gamma matrices suffer a local scaling for critical deformations:

$$\delta\Gamma^\mu = \Lambda(x)\Gamma^\mu \ . \quad (8.6.8)$$

This guarantees that the modified Dirac operator D is mapped to ΛD and still annihilates the modes of ν_R labelled by conformal weight, which thus remain unchanged.

What is the situation for the 2-D modes located at string world sheets? The condition is obvious. Ψ suffers an electro-weak gauge transformation as does also the induced spinor connection so that D_μ is not affected at all. Criticality condition states that the deformation of the space-time surfaces induces a conformal scaling of Γ^μ at X^2. It might be possible to continue this conformal scaling of the entire space-time sheet but this might be not necessary and this would mean that all critical deformations induced conformal transformations of the effective metric of the space-time surface defined by $\{\Gamma^\mu, \Gamma^\nu\} = 2G^{\mu\nu}$. Thus it seems that effective metric is indeed central concept (recall that if the conjectured quaternionic structure is associated with the effective metric, it might be possible to avoid problem related to the Minkowskian signature in an elegant manner).

In fact, one can consider even more general action of critical deformation: the modes of the induced spinor field would be mixed together in the infinitesimal deformation besides infinitesimal electroweak gauge transformation, which is same for all modes. This would extend electroweak gauge symmetry. Modified Dirac equation holds true also for these deformations. One might wonder whether the conjectured dynamically generated gauge symmetries assignable to finite measurement resolution could be generated in this manner.

The infinitesimal generator of a critical deformation J_M can be expressed as tensor product of matrix A_M acting in the space of zero modes and of a generator of infinitesimal electro-weak gauge transformation $T_M(x)$ acting in the same manner on all modes: $J_M = A_M \otimes T_M(x)$. A_M is a spatially constant matrix and $T_M(x)$ decomposes to a direct sum of left- and right-handed $SU(2) \times U(1)$ Lie-algebra generators. Left-handed Lie-algebra generator can be regarded as a quaternion and right handed as a complex number. One can speak of a direct sum of left-handed local quaternion $q_{M,L}$ and right-handed local complex number $c_{M,R}$. The commutator $[J_M, J_N]$ is given by $[J_M, J_N] = [A_M, A_N] \otimes \{T_M(x), T_N(x)\} + \{A_M, A_N\} \otimes [T_M(x), T_N(x)]$. One has $\{T_M(x), T_N(x)\} = \{q_{M,L}(x), q_{N,L}(x)\} \oplus \{c_{M,R}(x), c_{N,R}(x)\}$ and $[T_M(x), T_N(x)] = [q_{M,L}(x), q_{N,L}(x)]$. The commutators make sense also for more general gauge group but quaternion/complex number property might have some deeper role.

Thus the critical deformations would induce conformal scalings of the effective metric and dynamical electro-weak gauge transformations. Electro-weak gauge symmetry would be a dynamical symmetry restricted to string world sheets and partonic 2-surfaces rather than acting at the entire space-time surface. For 4-D de-localized right-handed neutrino modes the conformal scalings of the effective metric are analogous to the conformal transformations of M^4 for $\mathcal{N} = 4$ SYMs. Also ordinary conformal symmetries of M^4 could be present for string world sheets and could act as symmetries of generalized Feynman graphs since even virtual wormhole throats are massless. An interesting question is whether the conformal invariance associated with the effective metric is the analog of dual conformal invariance in $\mathcal{N} = 4$ theories.

Critical deformations of space-time surface are accompanied by conserved fermionic currents. By using standard Noetherian formulas one can write

$$J_i^\mu = \overline{\Psi}\Gamma^\mu \delta_i \Psi + \delta_i \overline{\Psi}\Gamma^\mu \Psi \ . \tag{8.6.9}$$

Here $\delta\Psi_i$ denotes derivative of the variation with respect to a group parameter labeled by i. Since $\delta\Psi_i$ reduces to an infinitesimal gauge transformation of Ψ induced by deformation, these currents are the analogs of gauge currents. The integrals of these currents along the braid strands at the ends of string world sheets define the analogs of gauge charges. The interpretation as Kac-Moody charges is also very attractive and I have proposed that the 2-D Hodge duals of gauge potentials could be identified as Kac-Moody currents. If so, the 2-D Hodge duals of J would define the quantum analogs of dynamical electro-weak gauge fields and Kac-Moody charge could be also seen as non-integral phase factor associated with the braid strand in Abelian approximation (the interpretation in terms of finite measurement resolution is discussed earlier).

One can also define super currents by replacing $\overline{\Psi}$ or Ψ by a particular mode of the induced spinor field as well as c-number valued currents by performing the replacement for both $\overline{\Psi}$ or Ψ. As expected, one obtains a super-conformal algebra with all modes of induced spinor fields acting as generators of super-symmetries restricted to 2-D surfaces. The number of the charges which do not annihilate physical states as also the effective number of fermionic modes could be finite and this would suggest that the integer $\mathcal{N}$ for the supersymmetry in question is finite. This would conform with the earlier proposal inspired by the notion of finite measurement resolution implying the replacement of the partonic 2-surfaces with collections of braid ends.

Note that Kac-Moody charges might be associated with "long" braid strands connecting different wormhole throats as well as short braid strands connecting opposite throats of wormhole contacts. Both kinds of charges would appear in the theory.

What is the interpretation of the critical deformations?

Critical deformations bring in an additional gauge symmetry. Certainly not all possible gauge transformations are induced by the deformations of preferred extremals and a good guess is that they correspond to holomorphic gauge group elements as in theories with Kac-Moody symmetry. What is the physical character of this dynamical gauge symmetry?

(a) Do the gauge charges vanish? Do they annihilate the physical states? Do only their positive energy parts annihilate the states so that one has a situation characteristic for the representation of Kac-Moody algebras. Or could some of these charges be analogous to the gauge charges associated with the constant gauge transformations in gauge theories and be therefore non-vanishing in the absence of confinement. Now one has electro-weak gauge charges and these should be non-vanishing. Can one assign them to deformations with a vanishing conformal weight and the remaining deformations to those with non-vanishing conformal weight and acting like Kac-Moody generators on the physical states?

(b) The simplest option is that the critical Kac-Moody charges/gauge charges with non-vanishing positive conformal weight annihilate the physical states. Critical degrees of freedom would not disappear but make their presence known via the states labelled by different gauge charges assignable to critical deformations with vanishing conformal weight. Note that constant gauge transformations can be said to break the gauge symmetry also in the ordinary gauge theories unless one has confinement.

(c) The hierarchy of quantum criticalities suggests however entire hierarchy of electro-weak Kac-Moody algebras. Does this mean a hierarchy of electro-weak symmetries breakings in which the number of Kac-Moody generators not annihilating the physical states gradually increases as also modes with a higher value of positive conformal weight fail to annihilate the physical state?

 The only manner to have a hierarchy of algebras is by assuming that only the generators satisfying $n\ mod\ N = 0$ define the sub-Kac-Moody algebra annihilating the physical states so that the generators with $n\ mod\ N \neq 0$ would define the analogs of gauge charges. I have suggested for long time ago the relevance of kind of fractal hierarchy of Kac-Moody and Super-Virasoro algebras for TGD but failed to imagine any concrete realization.

 A stronger condition would be that the algebra reduces to a finite dimensional algebra in the sense that the actions of generators Q_n and Q_{n+kN} are identical. This would correspond to periodic boundary conditions in the space of conformal weights. The notion of finite measurement resolution suggests that the number of independent fermionic oscillator operators is proportional to the number of braid ends so that an effective reduction to a finite algebra is expected.

 Whatever the correct interpretation is, this would obviously refine the usual view about electro-weak symmetry breaking.

These arguments suggests the following overall view. The holomorphy of spinor modes gives rise to Kac-Moody algebra defined by isometries and includes besides Minkowskian generators associated with gravitation also SU(3) generators associated with color symmetries. Vanishing second variations in turn define electro-weak Kac-Moody type algebra.

Note that criticality suggests that one must perform functional integral over WCW by decomposing it to an integral over zero modes for which deformations of X^4 induce only an electro-weak gauge transformation of the induced spinor field and to an integral over moduli corresponding to the remaining degrees of freedom.

8.6.8 The importance of being light-like

The singular geometric objects associated with the space-time surface have become increasingly important in TGD framework. In particular, the recent progress has made clear that these objects might be crucial for the understanding of quantum TGD. The singular objects are associated not only with the induced metric but also with the effective metric defined by the anti-commutators of the modified gamma matrices appearing in the modified Dirac equation and determined by the Kähler action.

The singular objects associated with the induced metric

Consider first the singular objects associated with the induced metric.

(a) At light-like 3-surfaces defined by wormhole throats the signature of the induced metric changes from Euclidian to Minkowskian so that 4-metric is degenerate. These surfaces are carriers of elementary particle quantum numbers and the 4-D induced metric degenerates locally to 3-D one at these surfaces.

(b) Braid strands at light-like 3-surfaces are most naturally light-like curves: this correspond to the boundary condition for open strings. One can assign fermion number to the braid strands. Braid strands allow an identification as curves along which the Euclidian signature of the string world sheet in Euclidian region transforms to Minkowskian one. Number theoretic interpretation would be as a transformation of complex regions to hyper-complex regions meaning that imaginary unit i satisfying $i^2 = -1$ becomes hyper-complex unit e satisfying $e^2 = 1$. The complex coordinates $(z, \overline{z})$ become hyper-complex coordinates $(u = t + ex, v = t - ex)$ giving the standard light-like coordinates when one puts $e = 1$.

The singular objects associated with the effective metric

There are also singular objects assignable to the effective metric. According to the simple arguments already developed, string world sheets and possibly also partonic 2-surfaces are singular objects with respect to the effective metric defined by the anti-commutators of the modified gamma matrices rather than induced gamma matrices. Therefore the effective metric seems to be much more than a mere formal structure.

(a) For instance, quaternionicity of the space-time surface could allow an elegant formulation in terms of the effective metric avoiding the problems due to the Minkowski signature. This is achieved if the effective metric has Euclidian signature $\epsilon \times (1,1,1,1)$, $\epsilon = \pm 1$ or a complex counterpart of the Minkowskian signature $\epsilon(1,1,-1,-1)$.

(b) String word sheets and perhaps also partonic 2-surfaces could be understood as singularities of the effective metric. What happens that the effective metric with Euclidian signature $\epsilon \times (1,1,1,1)$ transforms to the signature $\epsilon(1,1,-1,-1)$ (say) at string world sheet so that one would have the degenerate signature $\epsilon \times (1,1,0,0)$ at the string world sheet.

What is amazing is that this works also number theoretically. It came as a total surprise to me that the notion of hyper-quaternions as a closed algebraic structure indeed exists. The hyper-quaternionic units would be given by $(1, I, iJ, iK)$, where i is a commuting imaginary unit satisfying $i^2 = -1$. Hyper-quaternionic numbers defined as combinations of these units with real coefficients do form a closed algebraic structure which however fails to be a number field just like hyper-complex numbers do. Note that the hyper-quaternions obtained with real coefficients from the basis $(1, iI, iJ, iK)$ fail to form an algebra since the product is not hyper-quaternion in this sense but belongs to the algebra of complexified quaternions. The same problem is encountered in the case of hyper-octonions defined in this manner. This has been a stone in my shoe since I feel strong disrelish towards Wick rotation as a trick for moving between different signatures.

(c) Could also partonic 2-surfaces correspond to this kind of singular 2-surfaces? In principle, 2-D surfaces of 4-D space intersect at discrete points just as string world sheets and partonic 2-surfaces do so that this might make sense. By complex structure the situation is algebraically equivalent to the analog of plane with non-flat metric allowing all possible signatures (ϵ_1, ϵ_2) in various regions. At light-like curve either ϵ_1 or ϵ_2 changes sign and light-like curves for these two kinds of changes can intersect as one can easily verify by drawing what happens. At the intersection point the metric is completely degenerate and simply vanishes.

(d) Replacing real 2-dimensionality with complex 2-dimensionality, one obtains by the universality of algebraic dimension the same result for partonic 2-surfaces and string world sheets. The braid ends at partonic 2-surfaces representing the intersection points of 2-surfaces of this kind would have completely degenerate effective metric so that the modified gamma matrices would vanish implying that energy momentum tensor vanishes as does also the induced Kähler field.

(e) The effective metric suffers a local conformal scaling in the critical deformations identified in the proposed manner. Since ordinary conformal group acts on Minkowski space and leaves the boundary of light-cone invariant, one has two conformal groups. It is not however clear whether the M^4 conformal transformations can act as symmetries in TGD, where the presence of the induced metric in Kähler action breaks M^4 conformal symmetry. As found, also in TGD framework the Kac-Moody currents assigned to the braid strands generate Yangian: this is expected to be true also for the Kac-Moody counterparts of the conformal algebra associated with quantum criticality. On the other hand, in twistor program one encounters also two conformal groups and the space in which the second conformal group acts remains somewhat mysterious object. The Lie algebras for the two conformal groups generate the conformal Yangian and the integrands of the scattering amplitudes are Yangian invariants. Twistor approach should apply in TGD if zero energy ontology is right. Does this mean a deep connection?

    What is also intriguing that twistor approach in principle works in strict mathematical sense only at signatures $\epsilon \times (1, 1, -1 - 1)$ and the scattering amplitudes in Minkowski signature are obtained by analytic continuation. Could the effective metric give rise to the desired signature? Note that the notion of massless particle does not make sense in the signature $\epsilon \times (1, 1, 1, 1)$.

These arguments provide genuine a support for the notion of quaternionicity and suggest a connection with the twistor approach.

8.6.9 Realization of large $\mathcal{N}$ SUSY in TGD

The generators large $\mathcal{N}$ SUSY algebras are obtained by taking fermionic currents for second quantized fermions and replacing either fermion field or its conjugate with its particular mode. The resulting super currents are conserved and define super charges. By replacing both fermion and its conjugate with modes one obtains c number valued currents. Therefore $\mathcal{N} = \infty$ SUSY - presumably equivalent with super-conformal invariance - or its finite $\mathcal{N}$ cutoff is realized in TGD framework and the challenge is to understand the realization in more detail.

Super-space viz. Grassmann algebra valued fields

Standard SUSY induces super-space extending space-time by adding anti-commuting coordinates as a formal tool. Many mathematicians are not enthusiastic about this approach because of the purely formal nature of anti-commuting coordinates. Also I regard them as a non-sense geometrically and there is actually no need to introduce them as the following little argument shows.

Grassmann parameters (anti-commuting theta parameters) are generators of Grassmann algebra and the natural object replacing super-space is this Grassmann algebra with coefficients of Grassmann algebra basis appearing as ordinary real or complex coordinates. This is just an ordinary space with additional algebraic structure: the mysterious anti-commuting coordinates are not needed. To me this notion is one of the conceptual monsters created by the over-pragmatic thinking of theoreticians.

This allows allows to replace field space with super field space, which is completely well-defined object mathematically, and leave space-time untouched. Linear field space is simply replaced with its Grassmann algebra. For non-linear field space this replacement does not work. This allows to formulate the notion of linear super-field just in the same manner as it is done usually.

The generators of super-symmetries in super-space formulation reduce to super translations , which anti-commute to translations. The super generators Q_α and $\overline{Q}_{\dot{\beta}}$ of super Poincare algebra are Weyl spinors commuting with momenta and anti-commuting to momenta:

$$\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\ \dot{beta}} P_\mu \ . \tag{8.6.10}$$

One particular representation of super generators acting on super fields is given by

$$\begin{aligned} D_\alpha &= i\frac{\partial}{\partial_{\theta_\alpha}} \ , \\ D_{\dot{\alpha}} &= i\frac{\partial}{\partial_{\overline{\theta}}{}_{\dot{alpha}}} + \theta^\beta \sigma^\mu_{\beta\dot{\alpha}} \partial_\mu \end{aligned} \tag{8.6.11}$$

Here the index raising for 2-spinors is carried out using antisymmetric 2-tensor $\epsilon^{\alpha\beta}$. Super-space interpretation is not necessary since one can interpret this action as an action on Grassmann algebra valued field mixing components with different fermion numbers.

Chiral superfields are defined as fields annihilated by $D_{\dot{\alpha}}$. Chiral fields are of form $\Psi(x^\mu + i\overline{\theta}\sigma^\mu\theta, \theta)$. The dependence on $\overline{\theta}_{\dot{\alpha}}$ comes only from its presence in the translated Minkowski coordinate annihilated by $D_{\dot{\alpha}}$. Super-space enthusiast would say that by a translation of M^4 coordinates chiral fields reduce to fields, which depend on θ only.

The space of fermionic Fock states at partonic 2-surface as TGD counterpart of chiral super field

As already noticed, another manner to realize SUSY in terms of representations the super algebra of conserved super-charges. In TGD framework these super charges are naturally associated with the modified Dirac equation, and anti-commuting coordinates and super-fields do not appear anywhere. One can however ask whether one could identify a mathematical structure replacing the notion of chiral super field.

In [K23] it was proposed that generalized chiral super-fields could effectively replace induced spinor fields and that second quantized fermionic oscillator operators define the analog of SUSY algebra. One would have $\mathcal{N} = \infty$ if all the conformal excitations of the induced spinor field restricted on 2-surface are present. For right-handed neutrino the modes are labeled by two integers and de-localized to the interior of Euclidian or Minkowskian regions of space-time sheet.

The obvious guess is that chiral super-field generalizes to the field having as its components many-fermions states at partonic 2-surfaces with theta parameters and their conjugates in one-one correspondence with fermionic creation operators and their hermitian conjugates.

(a) Fermionic creation operators - in classical theory corresponding anti-commuting Grassmann parameters - replace theta parameters. Theta parameters and their conjugates are not in one-one correspondence with spinor components but with the fermionic creation operators and their hermitian conjugates. One can say that the super-field in question is defined in the "world of classical worlds" (WCW) rather than in space-time. Fermionic Fock state at the partonic 2-surface is the value of the chiral super field at particular point of WCW.

(b) The matrix defined by the $\sigma^\mu\partial_\mu$ is replaced with a matrix defined by the modified Dirac operator D between spinor modes acting in the solution space of the modified Dirac equation. Since modified Dirac operator annihilates the modes of the induced spinor field, super covariant derivatives reduce to ordinary derivatives with respect the theta parameters labeling the modes. Hence the chiral super field is a field that depends on

θ_m or conjugates $\overline{\theta}_m$ only. In second quantization the modes of the chiral super-field are many-fermion states assigned to partonic 2-surfaces and string world sheets. Note that this is the only possibility since the notion of super-coordinate does not make sense now.

(c) It would seem that the notion of super-field does not bring anything new. This is not the case. First of all, the spinor fields are restricted to 2-surfaces. Second point is that one cannot assign to the fermions of the many-fermion states separate non-parallel or even parallel four-momenta. The many-fermion state behaves like elementary particle. This has non-trivial implications for propagators and a simple argument [K23] leads to the proposal that propagator for N-fermion partonic state is proportional to $1/p^N$. This would mean that only the states with fermion number equal to 1 or 2 behave like ordinary elementary particles.

How the fermionic anti-commutation relations are determined?

Understanding the fermionic anti-commutation relations is not trivial since all fermion fields except right-handed neutrino are assumed to be localized at 2-surfaces. Since fermionic conserved currents must give rise to well-defined charges as 3-D integrals the spinor modes must be proportional to a square root of delta function in normal directions. Furthermore, the modified Dirac operator must act only in the directions tangential to the 2-surface in order that the modified Dirac equation can be satisfied.

The square root of delta function can be formally defined by starting from the expansion of delta function in discrete basis for a particle in 1-D box. The product of two functions in x-space is convolution of Fourier transforms and the coefficients of Fourier transform of delta function are apart from a constant multiplier equal to 1: $\delta(x) = K\sum_n exp(inx/2\pi L)$. Therefore the Fourier transform of square root of delta function is obtained by normalizing the Fourier transform of delta function by $1/\sqrt{N}$, where $N \rightarrow \infty$ is the number of plane waves. In other words: $\sqrt{\delta(x)} = \sqrt{\frac{K}{N}}\sum_n \sum exp(inx/2\pi L)$.

Canonical quantization defines the standard approach to the second quantization of the Dirac equation.

(a) One restricts the consideration to time=constant slices of space-time surface. Now the 3-surfaces at the ends of CD are natural slices. The intersection of string world sheet with these surfaces is 1-D whereas partonic 2-surfaces have 2-D Euclidian intersection with them.

(b) The canonical momentum density is defined by

$$\begin{aligned} \Pi_\alpha &= \frac{\partial L}{\partial_t \overline{\Psi}_\alpha(x)} = \Gamma^t\Psi \ , \\ \Gamma^t &= \frac{\partial L_K}{\partial(\partial_t h^k)} \ . \end{aligned} \tag{8.6.12}$$

L_K denotes Kähler action density: consistency requires $D_\mu\Gamma^\mu = 0$, and this is guaranteed only by using the modified gamma matrices defined by Kähler action. Note that Γ^t contains also the $\sqrt{g_4}$ factor. Induced gamma matrices would require action defined by four-volume. t is time coordinate varying in direction tangential to 2-surface.

(c) The standard equal time canonical anti-commutation relations state

$$\{\Pi_\alpha, \overline{\Psi}_\beta\} = \delta^3(x,y)\delta_{\alpha\beta} \ . \tag{8.6.13}$$

Can these conditions be applied both at string world sheets and partonic 2-surfaces.

(a) Sttring world sheets do not pose problems. The restriction of the modes to string world sheets means that the square root of delta function in the normal direction of string world sheet takes care of the normal dimensions and the dynamical part of anti-commutation relations is 1-dimensional just as in the case of strings.

(b) Partonic 2-surfaces are problematic. The $\sqrt{g_4}$ factor in Γ^t implies that Γ^t approaches zero at partonic 2-surfaces since they belong to light-like wormhole throats at which the signature of the induced metric changes. Energy momentum tensor appearing in Γ^t involves to index raisins by induced metric so that it can grow without limit as one approaches partonic two-surface. Therefore it is quite possible that the limit is finite and the boundary conditions defined by the weak form of electric magnetic duality might imply that the limit is finite. The open question is whether one can apply canonical quantization at partonic 2-surfaces. One can also ask whether one can define induced spinor fields at wormhole throats only at the ends of string world sheets so that partonic 2-surface would be effectively discretized. This cautious conclusion emerged in the earlier study of the modified Dirac equation [K22].

(c) Suppose that one can assume spinor modes at partonic 2-surfaces. 2-D conformal invariance suggests that the situation reduces to effectively one-dimensional also at the partonic two-surfaces. If so, one should pose the anti-commutation relations at some 1-D curves of the partonic 2-surface only. This is the only sensical option. The point is that the action of the modified Dirac operator is tangential so that also the canonical momentum current must be tangential and one can fix anti-commutations only at some set of curves of the partonic 2-surface.

One can of course worry what happens at the limit of vacuum extremals. The problem is that Γ^t vanishes for space-time surfaces reducing to vacuum extremals at the 2-surfaces carrying fermions so that the anti-commutations are inconsistent. Should one require - as done earlier- that the anti-commutation relations make sense at this limit and cannot therefore have the standard form but involve the scalar magnetic flux formed from the induced Kähler form by permuting it with the 2-D permutations symbol? The restriction to preferred extremals, which are always non-vacuum extremals, might allow to avoid this kind of problems automatically.

In the case of right-handed neutrino the situation is genuinely 3-dimensional and in this case non-vacuum extremal property must hold true in the regions where the modes of ν_R are non-vanishing. The same mechanism would save from problems also at the partonic 2-surfaces. The dynamics of induced spinor fields must avoid classical vacuum. Could this relate to color confinement? Could hadrons be surrounded by an insulating layer of Kähler vacuum?

8.7 Trying to understand $N = 4$ super-conformal symmetry

The original idea was that $N = 4$ super-conformal symmetry is a symmetry generated by the solutions of the modified Dirac equation for the second quantized induced spinor fields. Later I was ended up with this symmetry by considering the general structure of these algebras interpreted in TGD framework. In the following the latter approach is discussed in detail.

Needless to say, a lot remains to be understood. One of the problems is that my understanding of $N = 4$ super-conformal symmetry at technical level is rather modest. There are also profound differences between these two kinds of super conformal symmetries. In TGD framework super generators carry quark or lepton number, super-symplectic and super Kac-Moody generators are identified as Hamiltonians rather than vector fields, and symplectic group is infinite-dimensional whereas the Lie groups associated with Kac-Moody algebras are finite-dimensional. On the other hand, finite measurement resolution implies discretization and cutoff in conformal weight. Therefore the naive attempt to re-interpret results of standard super-conformal symmetry to TGD framework might lead to erratic conclusions.

$N > 0$ super-conformal algebras contain besides super Virasoro generators also other types of generators and this raises the question whether it might be possible to find an algebra coding the basic quantum numbers of the induced spinor fields.

There are several variants of $N = 4$ SCAs and they correspond to the Kac-Moody algebras $SU(2)$ (small SCA), $SU(2) \times SU(2) \times U(1)$ (large SCA) and $SU(2) \times U(1)^4$. Rasmussen has found also a fourth variant based on $SU(2) \times U(1)$ Kac-Moody algebra [A81] . It seems that only minimal and maximal $N = 4$ SCAs can represent realistic options. The reduction to almost topological string theory in critical phase is probably lost for other than minimal SCA but could result as an appropriate limit for other variants.

It must be emphasized that the discussion of this section is not based on the recent view about generalization of space-time supersymmetry to TGD framework in which fermionic oscillator operators define an infinite-dimensional super-symmetry algebra [K22] . Therefore the direction connection with quantum TGD remains loose.

8.7.1 Large $N = 4$ SCA

Large $N = 4$ SCA is described in the following in detail since it might be a natural algebra in TGD framework.

The structure of large $N = 4$ SCA algebra

Large $N = 4$ super-conformal symmetry with $SU(2)_+ \times SU(2)_- \times U(1)$ inherent Kac-Moody symmetry correspond to a fundamental partonic super-conformal symmetry in TGD framework.

A concise discussion of this symmetry with explicit expressions of commutation and anti-commutation relations can be found in [A81] . The representations of SCA are characterized by three central extension parameters for Kac-Moody algebras but only two of them are independent and given by

$$\begin{aligned} k_\pm &\equiv k(SU(2)_\pm) \ , \\ k_1 &\equiv k(U(1)) = k_+ + k_- \ . \end{aligned} \tag{8.7.1}$$

The central extension parameter c is given as

$$c = \frac{6k_+k_-}{k_+ + k_-} \ . \tag{8.7.2}$$

and is rational valued as required.

A much studied $N = 4$ SCA corresponds to the special case

$$\begin{aligned} k_- &= 1 \ , \ k_+ = k+1 \ , \ k_1 = k+2 \ , \\ c &= \frac{6(k+1)}{k+2} \ . \end{aligned} \tag{8.7.3}$$

$c = 0$ would correspond to $k_+ = 0, k_- = 1, k_1 = 1$. For $k_+ > 0$ one has $k_1 = k_+ + k_- \neq k_+$.

About unitary representations of large $N = 4$ SCA

The unitary representations of large $N = 4$ SCA are briefly discussed in [A60] . The representations are labeled by the ground state conformal weigh h, SU(2) spins l_+, l_-, and U(1) charge u. Besides the inherent Kac-Moody algebra there is also "external" Kac-Moody group G involved and could correspond in TGD framework to the symplectic algebra associated with $\delta H_\pm = \delta M^4_\pm \times CP_2$ or to Kac-Moody group respecting light-likeness of light-like 3-surfaces.

Unitarity constraints apply completely generally irrespective of G so that one can apply them also in TGD framework. There are two kinds of unitary representations.

(a) Generic/long/massive representations which are ge generated from vacuum state as usual. In this case there are no null vectors.

(b) Short or massless representations have a null vector. The expression for the conformal weigt h_{short} of the null vector reads in terms of l_+, l_- and k_+, k_- as

$$h_{short} = \frac{1}{k_+ + k_-}(k_- l_+ + k_+ l_- + (l_+ - l_-)^2 + u^2) \ . \quad (8.7.4)$$

Unitarity demands that both short and long representations lie at or above $h \geq h_{short}$ and that spins lie in the range $l_\pm = 0, 1/2, ..., (k_\pm - 1)/2$.

Interesting examples of $N = 4$ SCA are provided by WZW coset models $\mathcal{W} \times U(1)$, where $\mathcal{W}$ is WZW model associated wto a quaternionic (Wolf) space. Examples based on classical groups are $\mathcal{W} = G/H = SU(n)/SU(n-1) \times U(1)$, $SO(n)/SO(n-4) \times SU(2)$, and $Sp(2n)/Sp(2n-2)$. For $n = 3$ first series gives CP_2 whereas second series gives for $n = 4$ $SO(4)/SU(2) = SU(2)$. In this case one has $k_+ = \kappa + 1$, and $k_- = \hat{c}_G$, where κ is the level of the bosonic current algebra for G and $\hat{c}_G$ is its dual Coxeter number.

8.7.2 Overall view about how different $N = 4$ SCAs could emerge in TGD framework

The basic idea is simple $N = 4$ fermion states obtained as different combinations of spin and isospin for given H-chirality of imbedding space spinor correspond to $N = 4$ multiplet. In case of leptons the holonomy group of $S^2 \times CP_2$ for given spinor chirality is $SU(2)_R \times SU(2)_R$ or $SU(2)_L \times SU(2)_R$ depending on M^4 chirality of the spinor. In case of quark one has $SU(2)_L \times SU(2)_L$ or $SU(2)_R \times SU(2)_R$. The coupling to Kähler gauge potential adds to the group $U(1)$ factor so that large $N = 4$ SCA is obtained. For covariantly constant right handed neutrino electro-weak part of holonomy group drops away as also $U(1)$ factor so that one obtains $SU(2)_L$ or $SU(2)_R$ and small $N = 4$ SCA.

How maximal $N = 4$ SCA could emerge in TGD framework?

Consider the Kac-Moody algebra $SU(2) \times SU(2) \times U(1)$ associated with the maximal $N = 4$ SCA. Besides Kac-Moody currents it contains 4 spin 1/2 fermions having an identification as quantum counterparts of leptonic spinor fields. The interpretation of the first $SU(2)$ is as rotations as rotations leaving invariant the sphere $S^2 \subset \delta M^4_\pm$. $U(2)$ has interpretation as electro-weak gauge group and as maximal linearly realized subgroup of $SU(3)$. This algebra acts naturally as symmetries of the 8-component spinors representing super partners of quaternions.

The algebra involves the integer value central extension parameters k_+ and k_- associated with the two SU(2) algebras as parameters. The value of $U(1)$ central extension parameter k is given by $k = k_+ + k_-$. The value of central extension parameter c is given by

$$c = 6k_- \frac{x}{1+x} < 6k_+ \ , \ x = \frac{k_+}{k_-} \ .$$

c can have all non-negative rational values m/n for positive values of $k_\pm$ given by $k_+ = rm, k_- = (6nr-1)m$. Unitarity might pose further restrictions on the values of c. At the limit $k_- = k$, $k_+ \to \infty$ the algebra reduces to the minimal $N = 4$ SCA with $c = 6k$ since the contributions from the second $SU(2)$ and $U(1)$ to super Virasoro currents vanish at this limit.

How small $N = 4$ SCA could emerge in TGD framework?

Consider the TGD based interpretation of the small $N = 4$ SCA.

(a) The group $SU(2)$ associated with the small $N = 4$ SCA and acting as rotations of covariantly constant right-handed neutrino spinors allows also an interpretation as a group $SO(3)$ leaving invariant the sphere S^2 of the light-cone boundary identified as $r_M = m^0$=constant surface defining generalized Kähler and symplectic structures in $\delta M^4_\pm$. Electro-weak degrees of freedom are obviously completely frozen so that $SU(2)_- \times U1$ factor indeed drops out.

(b) The choice of the preferred coordinate system should have a physical justification. The interpretation of $SO(3)$ as the isotropy group of the rest system defined by the total four-momentum assignable to the 3-surface containing partonic 2-surfaces is supported by the quantum classical correspondence. The subgroup $U(1)$ of $SU(2)$ acts naturally as rotations around the axis defined by the light ray from the tip of $M^4_\pm$ orthogonal to S^2. For $c = 0, k = 0$ case these groups define local gauge symmetries. In the more general case local gauge invariance is broken whereas global invariance remains as it should.

    In $M^2 \times E^2$ decomposition E^2 corresponds to the tangent space of S^2 at a given point and M^2 to the plane orthogonal to it. The natural assumption is that the right handed neutrino spinor is annihilated by the momentum space Dirac operator corresponding to the light-like momentum defining $M^2 \times E^2$ decomposition.

(c) For covariantly constant right handed neutrinos the dynamics would be essentially that defined by a topological quantum field theory and this kind of almost trivial dynamics is indeed associated with small $N = 4$ SCA.

1. Why $N = 4$ SUSY

$N = 2$ super-conformal invariance has been claimed to imply the vanishing of all amplitudes with more than 3 external legs for closed critical $N = 2$ strings having $c = 6, k = 1$ which is proposed to correspond to $n \to \infty$ limit [A45, A73] . Only the partition function and $2 \leq N \leq 3$ scattering amplitudes would be non-vanishing. The argument of [A45] relies on the imbedding of $N = 2$ super-conformal field theory to $N = 4$ topological string theory whereas in [A73] the Ward identities for additional unbroken symmetries associated with the chiral ring accompanying $N = 2$ super-symmetry [A49] are utilized. In fact, $N = 4$ topological string theory allows also imbeddings of $N = 1$ super strings [A45] .

The properties of $c = 6$ critical theory allowing only integral valued $U(1)$ charges and fermion numbers would conform nicely with what we know about the perturbative electro-weak physics of leptons and gauge bosons. $c = 1, k = 1$ sector with $N = 2$ super-conformal symmetry would involve genuinely stringy physics since all N-point functions would be non-vanishing and the earlier hypothesis that strong interactions can be identified as electro-weak interactions which have become strong inspired by HO-H duality [K67] could find a concrete realization.

In $c = 6$ phase $N = 2$-vertices the loop corrections coming from the presence of higher lepton genera in amplitude could be interpreted as topological mixing forced by unitarity implying in turn leptonic CKM mixing for leptons. The non-triviality of 3-point amplitudes would in turn be enough to have a stringy description of particle number changing reactions, such as single photon brehmstrahlung. The amplitude for the emission of more than one brehmstrahlung photons from a given lepton would vanish. Obviously the connection with quantum field theory picture would be extremely tight and imbeddability to a topological $N = 4$ quantum field theory could make the theory to a high degree exactly solvable.

2. Objections

There are also several reasons for why one must take the idea about the usefulness of $c = 6$ super-conformal strings from the point of view of TGD with an extreme caution.

(a) Stringy diagrams have quite different interpretation in TGD framework. The target space for these theories has dimension four and metric signature (2,2) or (0,4) and the vanishing theorems hold only for $(2, 2)$ signature. In lepton sector one might regard the covariantly constant complex right-handed neutrino spinors as generators of $N = 2$ real super-symmetries but in quark sector there are no super-symmetries.

(b) The spectrum looks unrealistic: all degrees of freedom are eliminated by symmetries except single massless scalar field so that one can wonder what is achieved by introducing the extremely heavy computational machinery of string theories. This argument relies on the assumption that time-like modes correspond to negative norm so that the target space reduces effectively to a 2-dimensional Euclidian sub-space E^2 so that only the vibrations in directions orthogonal to the string in E^2 remain. The situation changes if one assigns negative conformal weights and negative energies to the time like excitations. In the generalized coset representation used to construct physical states this is indeed assumed.

(c) The central charge has only values $c = 6k$, where k is the central extension parameter of SU(2) algebra [A39] so that it seems impossible to realize the genuinely rational values of c which should correspond to the series of Jones inclusions. One manner to circumvent the problem would be the reduction to $N = 2$ super-conformal symmetry.

(d) SU(2) Kac-Moody algebra allows to introduce only 2-component spinors naturally whereas super-quaternions allow quantum counterparts of 8-component spinors.

The $N = 2$ super-conformal algebra automatically extends to the so called small $N = 4$ algebra with four super-generators $G_\pm$ and their conjugates [A45] . In TGD framework $G_\pm$ degeneracy corresponds to the two spin directions of the covariantly constant right handed neutrinos and the conjugate of $G_\pm$ is obtained by charge conjugation of right handed neutrino. From these generators one can build up a right-handed $SU(2)$ algebra.

Hence the $SU(2)$ Kac-Moody of the small $N = 4$ algebra corresponds to the three imaginary quaternionic units and the $U(1)$ of $N = 2$ algebra to ordinary imaginary unit. Energy momentum tensor T and SU(2) generators would correspond to quaternionic units. $G_\pm$ to their super counterparts and their conjugates would define their "square roots".

What about $N = 4$ SCA with $SU(2) \times U(1)$ Kac-Moody algebra?

Rasmussen [A81] has discovered an $N = 4$ super-conformal algebra containing besides Virasoro generators and 4 Super-Virasoro generators $SU(2) \times U(1)$ Kac-Moody algebra and two spin 1/2 fermions and a scalar.

The first identification of $SU(2) \times U(1)$ is as electro-weak algebra for a given spin state. Second and more natural identification is as the algebra defined by rotation group and electromagnetic or Kähler charge acting on given charge state of fermion and naturally resulting in electro-weak symmetry breaking. Scalar might relate to Higgs field which is M^4 scalar but CP_2 vector.

There are actually two versions about Rasmussen's article [A81]: in the first version the author talks about $SU(2) \times U(1)$ Kac-Moody algebra and in the second one about $SL(2) \times U(1)$ Kac-Moody algebra.

These variants could correspond in TGD framework to two different inclusions of hyper-finite factors of type II_1.

(a) The first inclusion could be defined by $G = SL(2,R) \subset SO(3,1)$ acting on M^4 part of H-spinors (or alternatively, as Lorentz group inducing motions in the plane E^2 orthogonal to a light-like ray from the origin of light-cone M^4_+). Physically the inclusion would mean that Lorentz degrees of freedom are frozen in the physical measurement. This leaves electro-weak group $SU(2)_L \times U(1)$ as the group acting on H-spinors.

(b) The second inclusion would be defined by the electro-weak group $SU(2)_L$ so that Kac-Moody algebra $SL(2,R) \times U(1)$ remains dynamical.

8.7.3 How large $N = 4$ SCA could emerge in quantum TGD?

The discovery of the formulation of TGD as a $N = 4$ almost topological super-conformal QFT with light-like partonic 3-surfaces identified as basic dynamical objects increased considerably the understanding of super-conformal symmetries and their breaking in TGD framework. $N = 4$ super-conformal algebra corresponds to the maximal algebra with $SU(2) \times U(2)$ Kac-Moody algebra as inherent fermionic Kac-Moody algebra.

Concerning the interpretation the first guess would be that $SU(2)_+$ and $SU(2)_-$ correspond to vectorial spinor rotations in M^4 and CP_2 and $U(1)$ to Kähler charge or electromagnetic charge. For given imbedding space chirality (lepton/quark) and M^4 chirality $SU(2)$ groups are completely fixed.

There are many kinds of fermionic super generators and the role of these algebras is not yet well-understood.

Well-definedness of electromagnetic charge implies stringiness

There is also a new element not present in the original speculations. The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

Identification of super generators associated with WCW metric

The definition of the metric of "world of classical worlds" (WCW) is as anticommutators of WCW gamma matrices carrying fermion number and in one-one correspondence with the infinitesimal isometries of WCW. WCW gamma matrices can be interpreted as supergenerators but do not seem to be identifiable as super counterparts of Noether charges. Fermionic generators can be divided into those associated with symplectic transformations, isometries, or symplectic isometries.

(a) Generators of the symplectic algebra of $\delta M^4_\pm \times CP_2$ defined in terms of covariantly constant right-handed neutrino and second quantized induced spinor field. The form of current is $\overline{\nu}_R j^k_A \gamma_k \Psi$ and only leptonic Ψ contributes.

(b) Fermionic generators defined in terms of all spinor modes for the symplectic isometries by the same formulas as in the case of symplectic algebra. This algebra is Kac-Moody type algebra with radial light-like coordinate r_M of $\delta M^4_\pm$ playing the role of complex coordinate. There is conformal weight associated with r_M but also with the fermionic modes since the fermions are localized to 2-D string world sheets and labelle by integer valued conformal weight. The form of the fermionic current is $\overline{\Psi}_n j^k_A \gamma_k \Psi$ and both quark-like anbd leptonic Ψ contribute.

(c) One can also consider fermionic generators assignable as a Noether super charges to the isometries of $\delta M^4_\pm = S^2 \times R_+$, which are in 1-1 correspondence with the conformal transformations of S^2. The conformal scaling of S^2 is compensated by the S^2 dependent scaling of the light-like radial coordinate r_M. It is not completely clear whether these should be included. If not, it would be a slight dis-appointment since the metric 2-dimensionality of the $\delta M^4_\pm$ makes 4-D Minkowski space unique. Same applies to 4-D space-time since light-like 3-surfaces representing partonic 2-surfaces allow also 2-D conformal symmetries as isometries.

Supercharges accompanying conserved fermion numbers

There are also fermionic super-charges defined as super-currents serving as super counterparts of conserved fermion number in quark-like and leptonic sector.

(a) Assume that the modified Dirac operator decomposition $D = D(Y^2) + D(X^2) = D(Y^1) + D(X^1) + D(X^2)$ reflecting the dual slicings of space-time surfaces to string world sheets Y^2 and partonic 2-surfaces X^2. If the conditions guaranteing well-defined em charge hold true, when can forget the presence of X^2 and the parameters λ_k labelling spinor modes in these degrees of freedom. The highly non-trivial consistency condition possible for Kähler-Dirac action is that $D(X^2)$ vanishes at string world sheets and thus allows the localization.

(b) Y^1 represents light-like direction and also string connecting braid strands at same component of X^3_l or at two different components of X^3_l. Modified Dirac equation implies that the charges

$$\int_{X^3_l} \overline{\Psi}_n \hat{\Gamma}^v \Psi \tag{8.7.5}$$

define conserved super charges in time direction associated with Y^1 and carrying quark or lepton number. Here Ψ_n corresponds to n:th conformal excitation of Ψ and has conformal weight n (plus possible ground state conformal weight). In the case of ordinary Dirac equation essentially fermionic oscillator operators would be in question.

(c) The zero modes of $D(X^2)$ define a sub-algebra which is a good candidate for representing super gauge symmetries. If localizations to 2-D string world sheets takes place, only these transformations are present.

In particular, covariantly constant right handed neutrinos define this kind of super gauge super-symmetries. $N = 2$ super-conformal symmetry would correspond in TGD framework to covariantly constant complex right handed neutrino spinors with two spin directions forming a right handed doublet and would be exact and act only in the leptonic sector relating WCW Hamiltonians and super-Hamiltonians. This algebra extends to the so called small $N = 4$ algebra if one introduces the conjugates of the right handed neutrino spinors. This symmetry is exact if only leptonic chirality is present in theory or if free quarks carry leptonic charges.

A physically attractive realization of the braids - and more generally- of slicings of space-time surface by 3-surfaces and string world sheets, is discussed in [K32] by starting from the observation that TGD defines an almost topological QFT of braids, braid cobordisms, and 2-knots. The boundaries of the string world sheets at the space-like 3-surfaces at boundaries of CDs and wormhole throats would define space-like and time-like braids uniquely.

The idea relies on a rather direct translation of the notions of singular surfaces and surface operators used in gauge theory approach to knots [A100] to TGD framework. It leads to the identification of slicing by three-surfaces as that induced by the inverse images of $r = constant$ surfaces of CP_2, where r is $U(2)$ invariant radial coordinate of CP_2 playing the role of Higgs field vacuum expectation value in gauge theories. $r = \infty$ surfaces correspond to geodesic spheres and define analogs of fractionally magnetically charged Dirac strings identifiable as preferred string world sheets. The union of these sheets labelled by subgroups $U(2) \subset SU(3)$ would define the slicing of space-time surface by string world sheets. The choice of $U(2)$ relates directly to the choice of quantization axes for color quantum numbers characterizing CD and would have the choice of braids and string world sheets as a space-time correlate.

Identification of Kac-Moody generators

Consider next the generators of inherent Kac-Moody algebras for $SU(2) \times SU(L) \times U(1)$ and freely chosen group G.

(a) Generators of Kac-Moody algebra associated with isometries correspond Noether currents associated with the infinitesimal action of Kac-Moody algebra to the induced spinor fields. Local $SO(3) \times SU(3)$ algebra is in question and excitations should have dependence on the coordinate u in direction of Y^1. The most natural guess is that this algebra corresponds to the Kac-Moody algebra for group G.

(b) The natural candidate for the inherent Kac-Moody algebra is the holonomy algebra associated with $S^2 \times CP_2$. This algebra should correspond to a broken symmetry.

The generalized eigen modes of $D(X^2)$ labeled by λ_k should from the representation space in this case: if localization to 2-D string world sheets occurs, this space is 1-D. If Kac-Moody symmetry were not broken these representations would correspond a degeneracy associated with given value of λ_k. Electro-weak symmetry breaking is however present and coded already into the geometry of CP_2. Also $SO(3)$ symmetry is broken due to the presence of classical electro-weak magnetic fields. The broken symmetries could be formulated in terms of initial values of generalized eigen modes at X^2 defining either end of X^3_l. One can rotate these initial values by spinor rotations. Symmetry breaking would mean that the modes obtained by a rotation by angle $\phi = \pi$ from a mode with fixed eigenvalue λ_k have different eigenvalues. Four states would be obtained for a given imbedding space chirality (quark or lepton). One expects that an analog of cyclotron spectrum with cutoff results with each cyclotron state split to four states with different eigenvalues λ_k. Kac-Moody generators could be expressed as matrices acting in the space spanned by the eigen modes.

Consistency with p-adic mass calculations

The consistency with p-adic mass calculations provides a strong guide line in attempts to interpret $N = 4$ SCA. The basis ideas of p-adic mass calculations are following.

(a) Fermionic partons move in color partial waves in their cm degrees of freedom. This gives to conformal weight a vacuum contribution equal to the CP_2 contribution to mass squared. The contribution depends on electro-weak isospin and equals $h_c(U) = 2$ and $h_c(D) = 3$ for quarks and one has $h_c(\nu) = 1$ and $h_c(L) = 2$.

(b) The ground state can correspond also to non-negative value of L_0 for SKMV algebra which gives rise to a thermal degeneracy of massless states. p-Adic mass calculations require $(h_{gr}(D), h_{gr}(U)) = (0, -1,)$ and $(h_{gr}(L), h_{gr}(\nu)) = (-1, -2)$ so that the super-symplectic operator O_c screening the anomalous color charge has conformal weight $h_c = -3$ for all fermions.

The simplest interpretation is that the free parameter h appearing in the representations of the SCA corresponds to the conformal weight due to the color partial wave so that the correlation with electromagnetic charge would indeed emerge but from the correlation of color partial waves and electro-weak quantum numbers.

The requirement that ground states are null states with respect to the SCV associated with the radial light-like coordinate of $\delta M^4_\pm$ gives an additional consistency condition and $h_c = -3$ should satisfy this condition. p-Adic mass calculations do not pose non-trivial conditions on h for option 1) if one makes the identification $u = Q_{em}$ since one has $h_{short} < 1$ for all values of $k_+ + k_-$. Therefore both options 1) and 2) can be considered.

About symmetry breaking for large $N = 4$ SCA

Partonic formulation predicts that large $N = 4$ SCA is a broken symmetry, and the first guess is that breaking occurs via several steps. First a "small" $N = 4$ SCA with Kac-Moody group $SU(2)_+ \times U(1)$, where $SU(2)_+$ corresponds to ordinary rotations on spinor with fixed helicity, would result in electro-weak symmetry breaking. The next step break spin symmetry would lead to $N = 2$ SCA and the final step to $N = 0$ SCA. Several symmetry breaking scenarios are possible.

(a) The interpretation of $SU(2)_+$ in terms of right- or left- handed spin rotations and $U(1)$ as electromagnetic gauge group conforms with the general vision about electro-weak symmetry breaking in non-stringy phase. The interpretation certainly makes sense for covariantly constant right handed neutrinos for which spin direction is free. For left handed charged electro-weak bosons the action of right-handed spinor rotations is trivial so that the interpretation would make sense also now.

(b) The next step in the symmetry breaking sequence would be $N = 2$ SCA with electro-magnetic Kac-Moody algebra as inherent Kac-Moody algebra $U(1)$.

8.7.4 Relationship to super string models, M theory and WZW model

In hope of achieving more precise understanding one can try to understand the relationship of $N = 4$ super conformal symmetry as it might appear in TGD to super strings, M theory and WZW model.

Relationship to super-strings and M-theory

The (4,4) signature characterizing $N = 4$ SCA topological field theory is not a problem since in TGD framework the target space becomes a fictive concept defined by the Cartan algebra. Both $M^4 \times CP_2$ decomposition of the imbedding space and space-time dimension are crucial for the $2 + 2 + 2 + 2$ structure of the Cartan algebra, which together with the notions of WCW and generalized coset representation formed from super Kac-Moody and super-symplectic algebras guarantees $N = 4$ super-conformal invariance.

Including the 2 gauge degrees of freedom associated with M^2 factor of $M^4 = M^2 \times E^2$ the critical dimension becomes $D = 10$ and and including the radial degree of light-cone boundary the critical dimension becomes $D = 11$ of M-theory. Hence the fictive target space associated with the vertex operator construction corresponds to a flat background of super-string theory and flat background of M-theory with one light-like direction. From TGD point view the difficulties of these approaches are due to the un-necessary assumption that the fictive target space defined by the Cartan algebra corresponds to the physical imbedding space. The flatness of the fictive target space forces to introduce the notion of spontaneous compactification and dynamical imbedding space and this in turn leads to the notion of landscape.

Consistency with critical dimension of super-string models and M-theory

Mass squared is identified as the conformal weight of the positive energy component of the state rather than as a contribution to the conformal weight canceling the total conformal weight. Also the Lorentz invariance of the p-adic thermodynamics requires this. As a consequence, the pseudo 4-momentum p assignable to M^4 super Kac-Moody algebra could be always light-like or even tachyonic.

Super-symplectic algebra would generate the negative conformal weight of the ground state required by the p-adic mass calculations and super-Kac Moody algebra would generate the non-negative net conformal weight identified as mass squared. In this interpretation SKM and SC degrees of freedom are independent and correspond to opposite signs for conformal weights.

The construction is consistent with p-adic mass calculations [K37, K45] and the critical dimension of super-string models.

(a) Five Super Virasoro sectors are predicted as required by the p-adic mass calculations (the predicted mass spectrum depends only on the number of tensor factors). Super-symplectic algebra gives $Can(CP_2)$ and $Can(S^2)$. In SKM sector one has $SU(2)_L$, $U(1)$, local SU(3), SO(2) and E^2 so that 5 sectors indeed result.

(b) The Cartan algebras involved of SC is 2-dimensional and that of SKM is 7-dimensional so that 10-dimensional Cartan algebra results. This means that vertex operator construction implies generation of 10-dimensional target space which in super-string framework would be identified as imbedding space. Note however that these dimensions have Euclidian signature unlike in superstring models. SKM algebra allows also the option $SO(3) \times E(3)$ in M^4 degrees of freedom: this would mean that SKM Cartan algebra is 10-dimensional and the whole algebra 11-dimensional.

$N = 4$ super-conformal symmetry and WZW models

One can question the naive idea that the basic structure $G_{int} = SU(2) \times U(2)$ structure of $N = 4$ SCA generalizes as such to the recent framework.

(a) $N = 4$ SCA is originally associated with Majorana spinors. $N = 4$ algebra can be transformed from a real form to complex form with 2 complex fermions and their conjugates corresponding to complex H-spinors of definite chirality having spin and weak isospin. At least at formal level the complexification of $N = 4$ SCA algebra seems to make sense and might be interpreted as a direct sum of two $N = 4$ SCAs and complexified quaternions. Central charge would remain $c = 6k_+k_-/(k_+ + k_-)$ if naive complexification works. The fact that Kac-Moody algebra of spinor rotations is $G_{int} = SO(4) \times SO(4) \times U(1)$ is naturally assignable naturally to spinors of H suggests that it represents a natural generalization of $SO(4) \times U(1)$ algebra to inherent Kac-Moody algebra.

(b) One might wonder whether the complex form of $N = 4$ algebra could result from $N = 8$ SCA by posing the associativity condition.

(c) The article of Gunaydin [A68] about the representations of $N = 4$ super-conformal algebras realized in terms of Goddard-Kent-Olive construction and using gauged Wess-Zumino-Witten models forces however to question the straightforward translation of results about $N = 4$ SCA to TGD framework and it must be admitted that the situation is something confusing. Of course, there is no deep reason to believe that WZW models are appropriate in TGD framework.

   i. Gauged WZW models are constructed using super-space formalism which is not natural in TGD framework. The coset space $CP_2 \times U(2)$ where $U(2)$, could be identified as sub-algebra of color algebra or possibly as electro-weak algebra provides one such realization. Also the complexifixation of the $N = 4$ algebra is something new.

   ii. The representation involves 5-grading by the values of color isospin for $SU(3)$ and makes sense as a coset space realization for $G/H \times U(1)$ if H is chosen in such a manner that $G/H \times SU(2)$ is quaternionic space. For SU(3) one has $H = U(1)$ identifiable in terms of color hyper charge CP_2 is indeed quaternionic space. For $SU(2)$ 5-grading degenerates since spin 1/2 Lie-algebra generators are absent and H is trivial group. In M^4 degrees of gauged WZW model would be trivial.

   iii. $N = 4$ SCA results as an extension of $N = 2$ SCA using so called Freudenthal triple system. $N = 2$ SCA has realization in terms of $G/H \times U(1)$ gauged WZW theory whereas the extension to $N = 4$ SCA gives $G \times U(1)/H$ gauged WZW model: note that $SU(3) \times U(1)/H$ does not have an obvious interpretation in TGD framework.

The Kac-Moody central extension parameters satisfy the constraint $k_+ = k+1$ and $k_- = \hat{g} - 1$, where k is the central extension parameter for G. For $G = SU(3)$ one obtains $k_- = 1$ and $c = 6(k+1)/(k+2)$. $H = U(1)$ corresponding to color hyper-charge and $U(1)$ for $N = 2$ algebra corresponds to color isospin. The group $U(1)$ appearing in $SU(3) \times U(1)$ might be interpreted in terms of fermion number or Kähler charge.

iv. What looks somewhat puzzling is that the generators of second SU(2) algebra carry fermion number $F = 4I_3$. Note however that the sigma matrices of WCW with fermion number ± 2 are non-vanishing since corresponding gamma matrices anti-commute. Second strange feature is that fermionic generators correspond to 3+3 super-coordinates of the flag-manifold $SU(3)/U(1) \times U(1)$ plus 2 fermions and their conjugates. Perhaps the coset realization in CP_2 degrees of freedom is not appropriate in TGD framework and that one should work directly with the realization based on second quantized induced spinor fields.

8.7.5 The interpretation of the critical dimension $D = 4$ and the objection related to the signature of the space-time metric

The first task is to show that $D = 4$ ($D = 8$) as critical dimension of target space for $N = 2$ ($N = 4$) super-conformal symmetry makes sense in TGD framework and that the signature (2,2) ((4, 4) of the metric of the target space is not a fatal flaw. The lifting of TGD to twistor space seems the most promising manner to bring in $(2, 2)$ signature. One must of course remember that super-conformal symmetry in TGD sense differs from that in the standard sense so that one must be very cautious with comparisons at this level.

Space-time as a target space for partonic string world sheets?

Since partonic 2-surfaces are sub-manifolds of 4-D space-time surface, it would be natural to interpret space-time surface as the target space for $N = 2$ super-conformal string theory so that space-time dimension would find a natural explanation. Different Bohr orbit like solutions of the classical field equations could be the TGD counterpart for the dynamic target space metric of M-theory. Since partonic two-surfaces belong to 3-surface X^3_V, the correlations caused by the vacuum functional would imply non-trivial scattering amplitudes with CP_2 type extremals as pieces of X^3_V providing the correlate for virtual particles. Hence the theory could be physically realistic in TGD framework and would conform with perturbative character for the interactions of leptons. $N = 2$ super-conformal theory would of course not describe everything. This algebra seems to be still too small and the question remains how the functional integral over the configuration space degrees of freedom is carried out. It will be found that $N = 4$ super-conformal algebra results neatly when super Kac-Moody and super-symplectic degrees of freedom are combined.

The interpretation of the critical signature

The basic problem with this interpretation is that the signature of the induced metric cannot be (2,2) which is essential for obtaining the cancelation for $N = 2$ SCA imbedded to $N = 4$ SCA with critical dimension $D = 8$ and signature (4,4). When super-generators carry fermion number and do not reduce to ordinary gamma matrices for vanishing conformal weights, there is no need to pose the condition of the metric signature. The (4,4) signature of the target space metric is not so serious limitation as it looks if one is ready to consider the target space appearing in the calculation of N-point functions as a fictive notion.

The resolution of the problems relies on two observations.

(a) The super Kac-Moody and super-symplectic Cartan algebras have dimension $D = 2$ in both M^4 and CP_2 degrees of freedom giving total effective dimension $D = 8$.

(b) The generalized coset construction to be discussed in the sequel allows to assign opposite signatures of metric to super Kac-Moody Cartan algebra and corresponding super-symplectic Cartan algebra so that the desired signature (4,4) results. Altogether one has 8-D effective target space with signature (4,4) characterizing $N = 4$ super-conformal topological strings. Hence the number of physical degrees of freedom is $D_{phys} = 8$ as in super-string theory. Including the non-physical M^2 degrees of freedom, one has critical dimension $D = 10$. If also the radial degree of freedom associated with $\delta M^4_\pm$ is taken into account, one obtains $D = 11$ as in M-theory.

Small $N = 4$ SCA as sub-algebra of $N = 8$ SCA in TGD framework?

A possible interpretation of the small $N = 4$ super-conformal algebra would be quaternionic sub-SCA of the non-associative octonionic SCA. The $N = 4$ algebra associated with a fixed fermionic chirality would represent the fermionic counterpart for the restriction to the hyper-quaternionic sub-manifold of HO and $N = 2$ algebra in the further restriction to commutative sub-manifold of HO so that this algebra would naturally appear at the parton level. Super-affine version of the quaternion algebra can be constructed straightforwardly as a special case of corresponding octonionic algebra [A38] . The construction implies 4 fermion spin doublets corresponding and unit quaternion naturally corresponds to right handed neutrino spin doublet. The interpretation is as leptonic spinor fields appearing in Sugawara representation of Super Virasoro algebra.

A possible octonionic generalization of Super Virasoro algebra would involve 4 doublets $G^{i)}_\pm$, $i = 1, ..., 4$ of super-generators and their conjugates having interpretation as SO(8) spinor and its its conjugate. $G^{i)}_\pm$ and their conjugates $\overline{G}^{i)}_\pm$ would anti-commute to SO(8) vector octet having an interpretation as a super-affine algebra defined by the octonionic units: this would conform nicely with SO(8) triality.

One could say that the energy momentum tensor T extends to an octonionic energy momentum tensor T as real component and affine generators as imaginary components: the real part would have conformal weight $h = 2$ and imaginary parts conformal weight $h = 1$ in the proposed constructions reflecting the special role of real numbers. The ordinary gamma matrices appearing in the expression of G in Sugawara construction should be represented by units of complexified octonions to achieve non-associativity. This construction would differ from that of [A38] in that G fields would define an SO(8) octet in the proposed construction: HO-H duality would however suggest that these constructions are equivalent.

One can consider two possible interpretations for $G^{i)}_\pm$ and corresponding analogs of super Kac-Moody generators in TGD framework.

(a) Leptonic right handed neutrino spinors correspond to $G^{i)}_\pm$ generating quaternionic units and quark like left-handed neutrino spinors with leptonic charges to the remaining non-associative octonionic units. The interpretation in terms of so called mirror symmetry would be natural. What is is clear the direct sum of $N = 4$ SCAs corresponding to the Kac-Moody group $SU(2) \times SU(2)$ would be exact symmetry if free quarks and leptons carry integer charges. One might however hope of getting also $N = 8$ super-conformal algebra. The problem with this interpretation is that SO(8) transformations would in general mix states with different fermion numbers. The only way out would be the allowance of mixtures of right-handed neutrinos of both chiralities and also of their conjugates which looks an ugly option.

    In any case, the well-definedness of the fermion number would require the restriction to $N = 4$ algebra. Obviously this restriction would be a super-symmetric version for the restriction to 4-D quaternionic- or co-quaternionic sub-manifold of H.

(b) One can ask whether $G^{i)}_\pm$ and their conjugates could be interpreted as components of leptonic H-spinor field. This would give 4 doublets plus their conjugates and mean $N = 16$ super-symmetry by generalizing the interpretation of $N = 4$ super-symmetry. In this case fermion number conservation would not forbid the realization of $SO(8)$ rotations. Super-conformal variant of complexified octonionic algebra obtained by adding a commuting imaginary unit would result. This option cannot be excluded since in TGD framework complexified octonions and quaternions play a key role. The fact that only right handed neutrinos generate associative super-symmetries would mean that the remaining components $G_{\pm}^{i)}$ and their conjugates could be used to construct physical states. $N = 8$ super-symmetry would thus break down to small $N = 4$ symmetry for purely number theoretic reasons and the geometry of CP_2 would reflect this breaking.

The objection is that the remaining fermion doublets do not allow covariantly constant modes at the level of imbedding space. They could however allow these modes as induced H-spinors in some special cases which is however not enough and this option can be considered only if one accepts breaking of the super-conformal symmetry from beginning. The conclusion is that the $N = 8$ or even $N = 16$ algebra might appear as a spectrum generating algebra allowing elegant coding of the primary fermionic fields of the theory.

8.7.6 How could exotic Kac-Moody algebras emerge from Jones inclusions?

Also other Kac-Moody algebras than those associated with the basic symmetries of quantum TGD could emerge from Jones inclusions. The interpretation would be the TGD is able to mimic various conformal field theories. The discussion is restricted to Jones inclusions defined by discrete groups acting in CP_2 degrees of freedom in TGD framework but the generalization to the case of M^4 degrees of freedom is straightforward.

$\mathcal{M} : \mathcal{N} = \beta < 4$ case

The first situation corresponds to $\mathcal{M} : \mathcal{N} = \beta < 4$ for which a finite subgroup $G \subset SU(2)_L$ defines Jones inclusion $\mathcal{N}^G \subset \mathcal{M}^G$, with G commuting with the Clifford algebra elements creating physical states. $\mathcal{N}$ corresponds to a subalgebra of the entire infinite-dimensional Clifford algebra Cl for which one 8-D Clifford algebra factor identifiable as Clifford algebra of the imbedding space is replaced with Clifford algebra of M^4.

Each M^4 point corresponds to G orbit in CP_2 and the order of maximal cyclic subgroup of G defines the integer n defining the quantum phase $q = exp(i\pi/n)$. In this case the points in the covering give rise to a representation of G defining multiplets for Kac-Moody group $\hat{G}$ assignable to G via the ADE diagram characterizing G using McKay correspondence. Partonic boundary component defines the Riemann surface in which the conformal field theory with Kac Moody symmetry is defined. The formula $n = k + h_{\hat{G}}$ would determine the value of Kac-Moody central extension parameter k. The singletness of fermionic oscillator operators with respect to G would be compensated by the emergence of representations of G realized in the covering of M^4.

$\mathcal{M} : \mathcal{N} = \beta = 4$ case

Second situation corresponds to $\beta = 4$. In this case the inclusions are classified by extended ADE diagrams assignable to Kac Moody algebras. The interpretation $n = k + h_G$ assigning the quantum phase to $SU(2)$ Kac Moody algebra corresponds to the Jones inclusion $\mathcal{N}^{\hat{G}} \subset \mathcal{M}^{\hat{G}}$ of WCW spinor s for $\hat{G} = SU(2)_L$ with index $\mathcal{M} : \mathcal{N} = 4$ and trivial quantum phase $q = 1$. The Clifford algebra elements in question would be products of fermionic oscillator operators having vanishing $SU(2)_L$ quantum numbers but arbitrary $U(1)_R$ quantum numbers if the identification $\hat{G} = SU(2)_L$ is correct. Thus only right handed fermions carrying homological magnetic charge would be allowed and obviously these fermions must behave like massless particles so that $\beta < 4$ could be interpreted in terms of massivation. The ends of cosmic strings $X^2 \times S^2 \subset M^4 \times CP_2$ would represent an example of this phase having only Abelian electro-weak interactions.

According to the proposal of [K79] the finite subgroup $G \subset SU(2)$ defining the quantum phase emerges from the effective decomposition of the geodesic sphere $S^2 \subset CP_2$ to a lattice having S^2/G as the unit cell. The discrete wave functions in the lattice would give rise to $SU(2)_L \supset G$-multiplets defining the Kac Moody representations and S^2/G would represent the 2-dimensional Riemann surface in which the conformal theory in question would be defined. Quantum phases would correspond to the holonomy of S^2/G. Therefore the singletness in fermionic degrees of freedom would be compensated by the emergence of G- multiplets in lattice degrees of freedom.

8.7.7 Are both quark and lepton like chiralities needed/possible?

Before the formulation of quantum TGD based on the identification of light-like 3-surfaces as a representation of parton orbits emerged, one had to consider two different physical realizations of $N = 4$ super-conformal symmetry. The original option for which leptons and quarks correspond to different H-chiralities of the induced spinor field is consistent with the partonic picture and definitely favored so that this subsection can be regarded as as interesting side track.

On the other hand, only lepton like chiralities are needed if one can accepts a possible instability of proton. This option is mathematically the minimal but it is not at all clear whether the SU(3) associated with A_2 characterizing Jones inclusion can correspond to color SU(3). One can go further and ask whether it is even possible to have both chiralities.

Option I: $N = 4$ SCA and fractionally charged quarks

Quarks generate super affinization of quaternions, which involves in no manner the Kähler charge of quarks but for fractional quark charges only SCA in the leptonic sector is possible since covariant constancy fails. At the fundamental level one the spectrum generating algebra for quarks would thus emerge and they could appear as primary fields of $N = 4$ conformal field theory. WCW gamma matrices could be uniquely constructed in terms of the leptonic oscillator operators since they could correspond to super-generators of super-Kac Moody algebra. Furthermore, if the solutions of the modified Dirac equation generate super-conformal symmetries, it might be possible to have super-conformal symmetry acting also in the quark sector.

A possible manner to understand quarks is as a phase with $N = 2$ super-conformal symmetry with U(1) Kac-Moody algebra. Using just the requirement that the charges in the $k = 1, c = 1$ phase for $N = 2$ super-conformal symmetry are proportional to factor 1/3, one can conclude that this phase can contain ordinary quarks and fractionally charged leptons whose charge results from the phase factors depending on the sheet of the 3-fold covering of CP_2. Also phases with $n > 3$ are possible and require fractionization of both quark and lepton charges. For quarks the condition $n \ mod \ 3 = 0$ must be satisfied in this case.

Option II: $N = 4$ SCA and quarks as fractionally charged leptons

For the simplest option realizing $N = 4$ SCA only leptons are fundamental particles and quarks would be leptons in the anyonic $k = 1, c = 1, n = 3$ phase of the theory. This option would resolve elegantly the problem whether one should construct WCW gamma matrices using leptonic or quark like gamma matrices. Fermion number fractionization might in principle allow the decay of proton to positron plus pion as in GUTs. This decay might be however excluded for purely mathematical reasons. Indeed, the worlds corresponding to different value of $q = exp(i\pi/n)$ could communicate only via exchanges of bosons having a vanishing fermion number.

In the interactions between leptons and quarks the gauge bosons would penetrate to the space-time sheets corresponding to the hadrons. In $k = 1$ phase weak interactions would become strong since arbitrarily high parton vertices would become possible and strong interactions could be simply electro-weak interactions which have become strong in the anyonic phases as HO-H duality strongly suggests [K67] . By the same duality strong interactions wold have dual descriptions as non-perturbative electro-weak interactions and as color interactions.

There are objections against this picture.

(a) p-Adic mass calculations rely strongly on the fact that free quarks have fractional charges and move in CP_2 partial waves and it would be pity to lose the nice results of these calculations.

(b) This option requires that the SU(3) associated with A_2 characterizing $n = 3$ Jones inclusion produces states equivalent with triality 1 partial waves for quarks in order to reproduce the results of p-adic mass calculations. This does not seem to be the case although one can understand how effective triality 1 states results by considering 3-fold coverings of CP_2 points by M^4 points defined by the space-time surfaces in question. The essential point is that 2π rotation in CP_2 phase angle leads to a different M^4 point than original and 6π rotation brings back to the original point. This might not be however enough.

Option III: Integer charged leptons and quarks

For the third option $N = 4$ superconformal symmetry can be realized in both lepton and quark sector but by the previous arguments $N = 8$ SCA is not possible. Both imbedding space chiralities would possess leptonic quantum numbers and would be allowed as fundamental fermions. At the level of WCW the choice of either chirality to realize WCW gamma matrices would correspond to the selection of quark or lepton like chirality. This presumably leads to problems with continuity unless the two chiralities correspond to completely disjoint parts of WCW.

Finding an explanation for the experimental absence of the free integer charged quarks is the basic challenge met by the advocate of integer charged free quarks. A possible explanation could rely on the fact that also gauge bosons would be doubled. There are two options.

(a) The two kinds of gauge bosons couple to only single H-chirality. One can indeed argue that if one allows at given space-time sheet only quark or lepton like chirality then it is not possible to have quantum superpositions of fermion-anti-fermion pairs of opposite chiralities at a given space-time sheet so that bosons would couple to either quark or lepton like chirality. This would mean that leptons and free quarks would have no electro-weak interactions. Even gravitational interaction would be absent. This would however imply that ordinary hadrons should consist of fractionally charged leptons so that second chirality would not appear at all in known or experimentally testable physics.

(b) An option allowing ordinary hadrons to consist of genuine quarks is that the couplings of these two bosons are vectorial and axial with respect to H-chirality (the simplest option) and left-right permutation occurs for electro-weak couplings. This would induce a breaking of the chiral symmetry at the level of H just as the ordinary weak interactions do at the level of M^4 and the masses of integer charged quarks could differ from those of genuine leptons.

If H-vectorial and H-axial gauge bosons have same coupling strengths and masses, the diagrams representing exchanges of vectorial and axial gauge bosons would interfere to zero so that free leptons and quarks would not see each other at all. This should be true in $(c = 6, n = \infty)$ phase. This could be the case for even gravitons. On the other hand, the interactions between free quarks and hadronic quarks would be possible and would make free quarks visible so that this option seems to produce more problems than to solve them.

In $(c = 1, k = 1, n = 3)$ phase leptons and quarks should interact and this is achieved if the masses and couplings of H-vectorial or H-axial electro-weak bosons are different in this phase. It is far from clear whether this picture can be consistent with what is known about lepton-hadron interactions.

Common features of the options I and II

Consider now the common features of options I and II which on basis of the previous arguments look the only realistic ones.

(a) For both options only $c = 6$ would correspond to the integer charged world and hadrons would be represented by primary fields in this phase. Hadrons would correspond to $k = 1, c = 1$ representation for the reduced $N = 2$ conformal symmetry. Elementary fermions inside hadrons would correspond to the lowest $n = 3$ Jones inclusion having $k = 1$ which indeed corresponds to A_2 Dynkin diagram and thus SU(3). Ordinary leptons and quarks (whether fractionally charged leptons or not) would thus live in different CP_2:s (recall that the generalized imbedding space has fan like structure with different $M^4 \times CP_2$:s meeting along M^4). This would explain the impossibility to observe free fractionally charged quarks.

Anyonic color triplet leptons and fractionally charged quarks would live at the three branches of the covering of CP_2. The observation that leptonic spinors possess anomalous color hyper-charge identifiable as lepton number and that this charge corresponds to weak hyper-charge explains why the electromagnetic charge of lepton can be fractionized but not its weak isospin.

(b) An infinite hierarchy of states with fractionally charged fermions would be predicted with charges of form m/n appearing as dark matter so that the counterparts of quarks would represent only the simplest Jones inclusion. For quarks one would have $n = k + 2 \ mod \ 3 = 0$. The invisibility of free fractionally charged fermions would be equivalent with the invisibility of dark matter with scaled up value of CP_2 Planck constant in both options. For option I the phase transition transforming leptons to quarks and vice versa would require three leptons per quark in order to achieve conservation of fermion number.

(c) I have already proposed the idea that antimatter is dark matter [K59] and the obvious possibility is that matter-antimatter asymmetry corresponds to the transformation of n anti-leptons to baryon like entities consisting of n fractionally charged leptons inside which they behave like dark matter. For option II anti-leptons would correspond to baryons and antimatter would be directly observable.

Lepton-hadron interactions for various options

The interactions between leptons and quarks and their fractionally charged counterparts can be also understood. The following arguments favor option I and II over option III.

(a) Quite generally, the CP_2 type extremal representing virtual electroweak boson must tunnel between two CP_2:s in the fan formed by $M^4 \times CP_2$:s glued together along M^4 and in this process transform to hadronic weak boson. This means that also strong interactions between leptons and hadrons are generated but these interactions could be seen as secondary strong interactions occurring inside hadron in any case via the decay of photon to quark pair in turn interacting strongly with other partons.

The coupling constant characterizing the tunnelling must be such that correct results for electro-weak interactions between quarks and leptons are obtained in the lowest order. The notion of vector meson dominance meaning that weak bosons transform to strongly interacting mesons with same electro-weak quantum numbers conforms with this picture.

(b) For option II the lowest order contributions to electro-weak interactions inside hadrons could be identified as direct lepton-quark interaction and there are no obvious problems involved.

(c) For option I gauge bosons must couple to both chiralities in order to make possible the interaction between leptons and quarks. This is possible and the prediction is that gauge bosons should appear as H-vectorial and H-axial variants or their mixtures. A doubling of ordinary vector bosons is predicted. This however does not have any dramatic effects if ordinary gauge bosons correspond to H-vectorial gauge bosons and axial ones are heavy enough. Nothing new is predicted for situation in which leptons do not penetrate inside hadrons. A lepton penetrating into hadron must suffer an anyonization and becomes fractionally charged and decomposes into a triplet of leptons with fractional fermion number. This implies that lepton has strong interactions with quarks.

(d) For option III the understanding of the interactions between leptons and hadrons consisting of genuine quarks becomes a highly non-trivial problem for several reasons.

  i. The hypothesis that only fermions of fixed chirality are possible at a given space-time sheet would exclude the possibility of non-trivial interactions between leptons and hadrons. If one gives up this assumption the doubling of electro-weak interactions gives however hopes for describing the interactions. The non-observability of free quarks in $c = 6$ phase is guaranteed if the masses and couplings of H-vectorial and -axial bosons are identical in this phase. To have interactions in $k = 1$ phase, these couplings and masses must be different. This would look nice at first since one could hope of explaining strong interactions in terms of this symmetry breaking.

  ii. However, if H-vectorial and -axial couplings are different inside hadrons, the expectation is that the resulting low energy lepton-hadron electro-weak interactions are quite different from what they are known to be experimentally. The most natural guess suggested by the masslessness of gluons is that all (say) H-axial weak bosons are massless inside hadrons. However, if both H-vectorial and -axial photons are massless there would be no electromagnetic coupling between quarks and leptons and hadrons would look like em neutral particles at low energies.

  iii. The coupling constant characterizing this tunnelling should have a value making possible to reproduce the standard model picture about lepton-quark scattering. If only (say) H-vectorial ew bosons can tunnel to hadron and the amplitude A for the tunnelling equals to $A = 2$ it gives amplitude equal to $V - V + A - A = 2V - V$ between leptons then quark-lepton scattering can be reproduced correctly. This kind of transformation is however not described by a unitary S-matrix.

New view about strong interactions

The proposed picture suggests the identification of strong interactions as electro-weak interactions which have become strong in $k = 1$ anyonic phase. HO-H duality leads to the same proposal [K67] .

1. Strong interactions as electro-weak interactions in a non-perturbative phase?

Consider the situation in $k = 1, c = 1$ hadronic sector at the sheets of 3-fold covering of M^4 at which fractionally charged fermions reside. It is an experimental fact that their electro-weak interactions allow a perturbative description. One would however obtain all higher order stringy diagrams allowed by rational conformal field theories. This looks like a paradox but one can consider the possibility that electro-weak interactions give rise also to strong interactions.

For all options the non-vanishing of higher n-point functions in $k = 1, c = 1$ phase would give rise to and additional non-perturbative contribution to electro-weak interactions having a natural interpretation as strong interactions. Weak isospin and hypercharge could be interpreted also as strong isospin and hyper-charge as is indeed found to be the case experimentally. Conserved vector current hypothesis and partially conserved axial current hypothesis of the old-fashioned hadron physics indeed support this kind of duality.

For option I one can consider the possibility that H-axial bosons define the dual counterparts of gluons and are massless. H-axial electro-weak interactions would give rise also to strong interactions between quarks and anyonic leptons inside hadrons. The idea that color interactions have dual description as H-axial electro-weak interactions is admittedly rather seductive.

For option III different masses and couplings of H-vectorial and H-axial bosons inside hadrons would allow to interpret strong interactions as (say) axial weak interactions. The simplest option would be that H-axial weak bosons are massless so that strong isospin and hypercharge would correspond to their H-axial variants. The problems relating to the interaction between leptons and hadrons have been already mentioned: for instance, em interactions between leptons and quarks would vanish if they vanish in $c = 6$ phase.

2. HO-H duality and equivalence with QCD type description

One can ask how QCD type description emerges if strong interactions are non-perturbative electro-weak interactions (option II) or H-axial counterparts of them (option I). In [K67] I have discussed a possible duality suggested by the fact that space-time surfaces can be regarded as 4-surfaces in hyper-octonionic $H = M^8$ or in $H = M^4 \times CP_2$. In the first picture spinors would be octonionic spinors and correspond to two leptonic singlets and color triplet and its conjugate: there would be no trace about spin and electro-weak quantum numbers besides electro-weak hyper charge.

The absence of spin in HO description could provide a resolution of the spin puzzle of proton (quarks do not seem to contribute to the spin of proton). In H picture spinors would carry only electro-weak quantum numbers and spin besides anomalous color hypercharge. The question is whether quark like spinors in HO are equivalent with leptonic spinors in H and whether the descriptions based on (possibly) doubled electro-weak and color interactions are equivalent for many-sheeted coverings.

Chapter 9

Construction of Quantum Theory: M-matrix

9.1 Introduction

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix, and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant [K79, K21] a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the zero energy ontology applied already earlier in TGD inspired cosmology [K16] . Zero energy ontology motivated the replacement of the term 'S-matrix' with 'M-matrix'. This led to the discovery that rather stringy formulas for M-matrix elements emerge in TGD framework.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix scattered in the chapters of books about TGD and often drowned into details and plagued by side tracks. My hope is that this chapter might provide a kind of bird's eye of view and help the reader to realize how fascinating and profound and near to physics the mathematics of hyper-finite factors is. I do not pretend of having handle about the huge technical complexities and can only recommend the works of von Neumann [A77, A96, A82, A58] , Tomita [A92] , [B36, B53, B27] , the work of Powers and Araki and Woods which served as starting point for the work of Connes [A53, A52] , the work of Jones [A71] , and other leading figures in the field. What is may main contribution is fresh physical interpretation of this mathematics which also helps to make mathematical conjectures. The book of Connes [A53] available in web provides an excellent overall view about von Neumann algebras and non-commutative geometry.

9.1.1 The recent progress in Quantum TGD and identification of M-matrix

My original intention was to summarize the basic principles of Quantum TGD first. The problem is however where to start from since everything is so tightly interwoven that linear representation proceeding from principles to consequences seems impossible. Therefore it might be a good idea to try to give a summary with emphasis on what has happened during the few months in turn of 2008 to 2009 assuming that the reader is familiar with the basic concepts discussed in previous chapters. This summary gives also a bird's eye of view about what I believe M-matrix to be. Later this picture is used to answer the questions raised in the earlier version of this chapter.

Zero energy ontology

One of the key notions underlying the recent developments is zero energy ontology.

(a) Zero energy ontology leads naturally to the identification of light-like 3-surfaces interpreted as a generalization of Feynman diagrams as the most natural dynamical objects (equivalent with space-like 3-surface by holography).

(b) The fractal hierarchy of causal diamonds (CD) with light like boundaries of CD interpreted as carriers of positive and negative energy parts of zero energy state emerges naturally. If the scales of CDs come as powers of 2, p-adic length scale hypothesis follows as a consequence.

(c) The identification of M-matrix as time-like entanglement coefficients between zero energy states identified as the product of positive square root of the density matrix and unitary S-matrix emerges naturally and leads to the unification of thermodynamics and quantum theory.

(d) The identification of M-matrix in terms of Connes tensor product means that the included algebra $\mathcal{N} \subset \mathcal{M}$ acts effectively like complex numbers and does not affect the physical state. The interpretation is that $\mathcal{N}$ corresponds to zero energy states in size scales smaller than the measurement resolution and thus the insertion of this kind of zero energy state should not have any observable effects. The uniqueness of Connes tensor product gives excellent hopes that the M-matrix could be unique apart from the square root of of density matrix.

(e) The unitary U-matrix between zero energy states assignable to quantum jump has nothing to do with S-matrix measured in particle physics experiments. A possible interpretation is in terms of consciousness theory. For instance, U-matrix could make sense even for p-adic-to-real transitions interpreted as transformations of intentions to actions making sense since zero energy state is generated ('Everything is creatable from vacuum' is the basic principle of zero energy ontology) [K39] . One can express U-matrix as a collection of M-matrices labeled by zero energy states and unitaritity conditions for U-matrix boil down to orthogonality conditions for the zero energy states defined by M-matrices.

The notion of finite measurement resolution

The notion of finite measurement resolution as a basic dynamical principle of quantum TGD might be seen by a philosophically minded reader as the epistemological counterpart of zero energy ontology.

(a) As far as length scale resolution is considered, finite measurement resolution implies that only CDs above some size scale are allowed. This is not an approximation but a property of zero energy state so that zero energy states realize finite measurement resolution in their structure. One might perhaps say that quantum states represent only the information that we can becomes conscious of.

(b) In the case of angle resolution the hierarchy of Planck constants accompanied by a hierarchy of algebraic extensions of rationals by roots of unity, and realized in terms of the book like structures assigned with CD and CP_2, is a natural outcome of this thinking.

(c) Number theoretic braids implying discretization at parton level can be seen as a space-time correlate for the finite measurement resolution. Zero energy states should contain in their construction only information assignable to the points of the braids. Note however that there is also information about tangent space of space-time surface at these points so that the theory does not reduce to a genuinely discrete theory. Each choice of M^2

and geodesic spheres defines a selection of quantization axis and different choice of the number theoretic braid. Hence discreteness does not reduce to that resulting from the assumption that space-time as the arena of dynamics is discrete but reflects the limits to what we can measure, perceive, and cognize in continuous space-time. Zero energy state corresponds to wave-function in the space of these choices realized as the union of copies of the page $CD \times CP_2$. Quantum measurement must induce a localization to single point in this space unless one is ready to take seriously the notion of quantum multiverse.

(d) Finite measurement resolution allows a realization in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 (HFFs) about which the WCW Clifford algebra provides standard example. Also the factor spaces $\mathcal{M}/\mathcal{N}$ are suggestive and should correspond to quantum variants of HFFs with a finite quantum dimension. p-Adic coupling constant evolution can be understood in this framework and corresponds to the inclusions of HFFs realized as inclusions of spaces of zero energy states with two different scale cutoffs.

Number theoretical compactification and $M^8 - H$ duality

The closely related notions of number theoretical compactification and $M^8 - H$ duality have had a decisive impact on the understanding of the mathematical structure of quantum TGD.

(a) The hypothesis is that TGD allows two equivalent descriptions using either M^8- the space of hyper-octonions- or $H = M^4 \times CP_2$ as imbedding space so that standard model symmetries have a number theoretic interpretation. The underlying philosophy is that the world of classical worlds and thus H is unique so that the symmetries of H should be something very special. Number theoretical symmetries indeed fulfil this criterion.

(b) In M^8 description space-time surfaces decompose to hyper-quaternionic and co-hyperquaternionic regions. The map assigning to $X^4 \subset M^8$ the image in $X^4 \subset H$ must be a isometry and also preserve the induced Kähler form so that the Kähler action has same value in the two spaces. The isometry groups of E^4 and CP_2 are different, and the interpretation is that the low energy description of hadrons in terms of $SO(4)$ symmetry and high energy description in terms of $SU(3)$ gauge group reflect this duality.

(c) Number theoretic compactification implies very detailed conjectures about the preferred extremals of Kähler action implying dual slicings of the M^4 projection of space-time surface to string world sheets Y^2 and partonic 2-surfaces X^2 for Minkowskian signature of induced metric. This occurs for the known extremals of Kähler action of this kind [K4, K95] . These slicings allow to understand how Equivalence Principle emerges via its stringy variant in TGD framework through dimensional reduction. The tangent spaces of Y^2 and X^2 define local planes of physical and un-physical polarizations and M^2 defines also the plane for the four-momentum assignable to the braid strand so that gauge symmetries are purely number theoretical interpretation.

(d) Also a slicing of $X^4(X_l^3)$ to light-like 3-surfaces Y_l^3 parallel to X_l^3 giving equivalent space-time representations of partonic dynamics is predicted. This implies holography meaning an effective reduction of space-like 3-surfaces to 2-D surfaces. Number theoretical compactification leads also to a dramatic progress in the construction of quantum TGD in terms of the second quantized induced spinor fields. The holography seems however to be not quite simple as one might think first. Kac-Moody symmetries respecting the light-likeness of X_l^3 and leaving X^2 fixed act as gauge transformations and all light-like 3-surfaces with fixed ends and related by Kac-Moody symmetries would be geometrically equivalent in the sense that WCW Kähler metric is identical for them. These transformations would also act as zero modes of Kähler action.

(e) A physically attractive realization of the braids - and more generally- of slicings of space-time surface by 3-surfaces and string world sheets, is discussed in [K32] by starting from the observation that TGD defines an almost topological QFT of braids, braid cobordisms, and 2-knots. The boundaries of the string world sheets at the space-like

3-surfaces at boundaries of CDs and wormhole throats would define space-like and time-like braids uniquely.

The idea relies on a rather direct translation of the notions of singular surfaces and surface operators used in gauge theory approach to knots [A100] to TGD framework. It leads to the identification of slicing by three-surfaces as that induced by the inverse images of $r = constant$ surfaces of CP_2, where r is $U(2)$ invariant radial coordinate of CP_2 playing the role of Higgs field vacuum expectation value in gauge theories. $r = \infty$ surfaces correspond to geodesic spheres and define analogs of fractionally magnetically charged Dirac strings identifiable as preferred string world sheets. The union of these sheets labelled by subgroups $U(2) \subset SU(3)$ would define the slicing of space-time surface by string world sheets. The choice of $U(2)$ relates directly to the choice of quantization axes for color quantum numbers characterizing CD and would have the choice of braids and string world sheets as a space-time correlate.

WCW spinor structure

The construction of WCW ("world of classical worlds", configuration space) spinor structure in terms of second quantized induced spinor fields is certainly the most important step made hitherto towards explicit formulas for M-matrix elements.

(a) Number theoretical compactification ($M^8 - H$ duality) states that space-time surfaces can be equivalently regarded as 4-dimensional surfaces of either $H = M^4 \times CP_2$ or of 8-D Minkowski space M^8, and consisting of hyper-quaternionic and co-hyper-quaternionic regions identified as regions with Minkowskian and Euclidian signatures of induced metric. Duality preserves induced metric and Kähler form. This duality poses very strong constraints on the geometry of the preferred extremals of Kähler action implying dual slicings of the space-time surface by string worlds sheets and partonic 2-surfaces as also by light-like 1-surfaces and light-like 3-surfaces. These predictions are consistent what is known about the extremals of Kähler action. The predictions of number theoretical compactification lead to dramatic progress in the construction of configurations space spinor structure and geometry.

(b) The construction of WCW geometry and spinor structure in terms of induced spinor fields leads to the conclusion that finite measurement resolution is an intrinsic property of quantum states basically due to the vacuum degeneracy of Kähler action. This gives a justification for the notion of number theoretic braid effectively replacing light-like 3-surfaces. Hence the infinite-dimensional WCW is replaced with a finite-dimensional space $(\delta M^4_\pm \times CP_2)^n/S_n$. A possible interpretation is that the finite fermionic oscillator algebra for given partonic 2-surface X^2 represents the factor space $\mathcal{M}/\mathcal{N}$ identifiable as quantum variant of Clifford algebra. $(\delta M^4_\pm \times CP_2)^n/S_n$ would represent its bosonic analog.

(c) The isometries of the WCW corresponds to X^2 local symplectic transformations $\delta M^4_\pm \times CP_2$ depending only on the value of the invariant $\epsilon^{\mu\nu} J_{\mu\nu}$, where $J_{\mu\nu}$ can correspond to the Kähler form induced from $\delta M^4_\pm$ or CP_2. This group parameterizes quantum fluctuating degrees of freedom. Zero modes correspond to coordinates which cannot be made complex, in particular to the values of the induced symplectic form which thus behaves as a classical field so that WCW allows a slicing by the classical field patterns $J_{\mu\nu}(x)$ representing zero modes.

(d) By the effective 2-dimensionality of light-like 3-surfaces X^3_l (holography) the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. Quantum classical correspondence suggests that gauge fixing in Kac-Moody degrees of freedom takes place and implies correlation between the quantum numbers of the physical state and X^3_l or equivalently any light-like 3-surface Y^3_l parallel to X^3_l. There would be no path integral over X^3_l and only functional integral defined by WCW geometry over partonic 2-surfaces.

(e) The condition that the Noether currents assignable to the modified Dirac equation are conserved requires that space-time surfaces correspond to extremals for which second variation of Kähler action vanishes. A milder condition is that the rank of the matrix defined by the second variation of Kähler action is less than maximal. Preferred extremals of Kähler action can be identified as this kind of 4-surface and the interpretation is in terms of quantum criticality.

For given preferred extremal one expects the existence of an infinite number of deformations with a vanishing second variation of Kähler action. These deformations act as conformal gauge symmetries realizing quantum criticality at space-time level. The natural assumption is that the number, call it n, of conformal gauge equivalence classes of space-time surfaces with fixed 3-surfaces at their ends at the boundaries of CD is finite. This integer would characterize the effective value of Planck constant $h_{eff} = n \times h$.

(f) The inverse of the modified Dirac operator does not define stringy propagator since it does not depend on the quantum numbers of the state of super-conformal representation. The solution of the problem is provided by the addition of measurement interaction term to the modified Dirac action and assignable to wormhole throats or equivalently any light-like 3-surface parallel to them int the slicing of space-time sheet: this condition defines additional symmetry modifying Kähler function and Kähler action in such a manner that Kähler metric is not affected. Measurement interaction term implies that the preferred extremals of Kähler action depend on quantum numbers of the states of super-conformal representations as quantum classical correspondence requires. The coupling constants appearing in the measurement interaction term are fixed by the condition that Kähler function transforms only by a real part of a holomorphic function of complex coordinates of WCW depending also on zero modes so that Kähler metric of WCW remains unchanged. This realizes also the effective 2-dimensionality of space-like 3-surfaces but only in finite regions where the slicing by light-like 3-surfaces makes sense.

Hierarchy of Planck constants

The hierarchy of Planck constants realized as a replacement of CD and CP_2 of $CD \times CP_2$ with book like structures labeled by finite subgroups of $SU(2)$ assignable to Jones inclusions is now relatively well understood as also its connection to dark matter, charge fractionization, and anyons [K21, K51] .

(a) This notion leads also to a unique identification of number theoretical braids as intersections of CD (CP_2) projection of X^3_l and the back M^2 (the backs S^2_I and S^2_{II}) of M^4 (CP_2) book. The spheres S^2_I and $S^2_{II}$are geodesic spheres of CP_2 orthogonal to each other).

(b) The formulation of M-matrix should involve the local data from the points of number theoretic braids at partonic 2-surfaces. This data involves information about tangent space of $X^4(X^3)$ so that the theory does not reduce to 2-D theory. The hierarchy of CDs within CDs means that the improvement of measurement resolution brings in new CDs with smaller size.

(c) The points of number theoretical braids are by definition quantum critical with respect to the phase transitions changing Planck constant and meaning leakage between different pages of the books in question. This quantum criticality need not be equivalent with the quantum criticality in the sense of the degeneracy of the matrix like entity defined by the second variation of Kähler action. Note that the entire partonic 2-surface at the boundary of CD cannot be quantum critical unless it corresponds to vacuum state with only topological degrees of freedom excited (that is have as its CD (CP_2) projection at the back of CD (CP_2) book or both) since Planck constant would be ill-defined in this kind of situation.

Super-conformal symmetries

The attempts to understand super-conformal symmetries has been unavoidably a guess work and produced several alternative scenarios. The consistency with p-adic mass calculations requiring five tensor factors to Super-Virasoro algebra has been the basic experimental constraint. The work with Kähler-Dirac equation has helped dramatically in the attempts to understand of super-conformal symmetries. Also the understanding of Super-Kac-Moody symmetries acting as gauge symmetries and made possible by the non-determinism of Kähler action has helped a lot.

There have been a considerable progress also in the understanding of super-conformal symmetries [K10, K15].

(a) Super-symplectic algebra corresponds to the isometries of WCW constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW.

(b) One expects also gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

(c) An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

(d) A further step of progress relates to the understanding of the fusion rules of symplectic field theory [K8] . These fusion rules makes sense only if one allows discretization that is number theoretic braids. An infinite hierarchy of symplectic fusion algebras can be identified with nice number theoretic properties (only roots of unity appear in structure constants). Hence there are good hopes that symplecto-conformal N-point functions defining the vertices of generalized Feynman diagrams can be constructed exactly.

(e) The possible reduction of the fermionic Clifford algebra to a finite-dimensional one means that super-conformal algebras must have a cutoff in conformal weights. These algebras must reduce to finite dimensional ones and the replacement of integers with finite field is what comes first in mind.

(f) The conserved fermionic currents implied by vanishing second variations of Kähler action for preferred extremal define a hierarchy of super-conformal algebras assignable to zero modes. These currents are appear in the expression of measurement interactions added to the modified Dirac action in order to obtain stringy propagators and the coding of super-conformal quantum numbers to space-time geometry.

9.1.2 Various inputs to the construction of M-matrix

It is perhaps wise to summarize briefly the vision about M-matrix.

Zero energy ontology and interpretation of light-like 3-surfaces as generalized Feynman diagrams

(a) Zero energy ontology is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface X_l^3 connecting the light-like boundaries of a causal diamond CD identified as intersection of future and past directed light-cones. There is entire hierarchy of CDs, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to CDs corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.

(b) Light-like 3-surfaces are the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

Identification of TGD counterpart of S-matrix as time-like entanglement coefficients

(a) The TGD counterpart of S-matrix -call it M-matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of CD. One can also assign to quantum jump between zero energy states a matrix- call it U-matrix - which is unitary and assumed to be expressible in terms of M-matrices. M-matrix need not be unitary unlike the U-matrix characterizing the unitary process forming part of quantum jump. There are several good arguments suggesting that that M-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In fact, M-matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.

(b) In the general case only thermal M-matrix defines a normalizable zero energy state so that thermodynamics becomes part of quantum theory. One can assign to M-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

(a) The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

(b) Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

(c) The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

(d) There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

(a) In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

(b) This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

(c) One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

Conformal symmetries and stringy diagrammatics

The Kähler-Dirac equation has rich super-conformal symmetries helping to achieve concrete vision about the structure of M-matrix in terms of generalized Feynman diagrammatics.

Both super-conformal symmetries and the effective reduction of space-time sheet to string world sheets at Minkowskian regions as a consequence of finite measurement resolution suggest that the generalized Feynman diagrams have as vertices N-point functions of a conformal field theory assignable to the partonic 2-surfaces at which the lines of Feynman diagram meet. The vertices can be assigned with wormhole contacts with Euclidian signture of induced metric. In Minkowskian regions fundamental fermions propagate like massless particles along boundaries of string world sheets. One can say that a hybrid of Feynman and stringy diagrammatics results.

Finite measurement resolution means that this conformal theory is defined in the discrete set defined by the intersections of braids defined by boundaries of string worlds sheets with partonic two-surfaces. The presence of symplectic invariants in turn suggest a symplectic variant of conformal field theory leading to a concrete construction of symplectic fusion rules relying in crucial manner to discretization.

TGD as almost topological QFT

The idea that TGD could be regarded as almost topological QFT has been very fruitful although the hypothesis that Chern-Simons term for induced Kähler gauge potential assignable to light-like 3-surfaces identified as regions of space-time where the Euclidian signature of induced metric assignable to the interior or generalized Feynman diagram changes to Minkowskian one turned out to be too strong. The reduction of WCW and its Clifford algebra to finite dimensional structures due to finite measurement resolution however realizes this idea but in different manner.

(a) There is functional integral over the small deformations of Feynman cobordisms corresponding to the maxima of Kähler function which is finite-dimensional if finite measurement resolution is taken into account. Almost topological QFT property of quantum suggests the identification of M-matrix as a functor from the category of generalized Feynman cobordisms (generalized Feynman diagrams) to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces.

(b) The limit at which momenta vanish is well-defined for M-matrix since the modified Dirac action contains measurement interaction term and at this limit one indeed obtains topological QFT.

(c) Almost TQFT property suggests that braiding S-matrices should have important role in the construction. It is indeed possible to assign the with the lines of the generalized Feynman diagram. The reduction of quantum TGD to topological QFT should occur at quantum criticality with respect to the change of Planck constant since in this situation the M-matrix should not depend at all on Planck constant. Factoring QFTs in 1+1 dimensions give examples of this kind of theories.

Heuristic picture about generalized Feynman rules

Concerning the understanding of the relationship between HFFs and M-matrix the basic implications are following.

(a) General visions do not allow to provide explicit expressions for M-matrix elements. Therefore one must be humble and try to feed in all understanding about quantum TGD and from the quantum field theoretic picture. In particular, the dependence of M-matrix on Planck constant should be such that the addition of loop corrections as sub-CDs corresponds to an expansion in powers of $1/\hbar$ as in quantum field theory whereas for tree diagrams there is no dependence on $\hbar$.

(b) The vacuum degeneracy of Kähler action and the identification of Kähler function as Dirac determinant strongly suggest that fermionic oscillator operators define what could be interpreted as a finite quantum-dimensional Clifford algebra identifiable as a factor space $\mathcal{M}/\mathcal{N}$, $\mathcal{N} \subset \mathcal{M}$. One must be however very cautious since also an alternative option in which excitations of labeled by conformal weight are present cannot be excluded. Finite-dimensionality would mean an enormous simplification, and together with the unique identification of number theoretic braids as orbits of the end points of string world sheets this means that the dynamics is finite-quantum-dimensional conforming with the fact effective finite-dimensionality is the defining property of HFFs.

Physical states would realize finite measurement resolution in their structure so that approximation would cease to be an approximation.

(c) An interesting question is whether this means that M-matrix must be replaced with quantum M-matrix with operator valued matrix elements and whether the probabilities should be determined by taking traces of these operators having interpretation as averaging over $\mathcal{N}$ defining the degrees of freedom below measurement resolution. This kind of picture would conform with the basic properties of HFFs.

(d) To the strands of number theoretic braids one would attach fermionic propagators. Since bosons correspond to fermion pairs at the throats of wormhole contact, all propagators reduce to fermionic ones. As found, the addition of measurement interaction term fixes fermionic propagator completely and gives it a stringy character.

(e) Similar correlation function in WCW degrees of freedom would be given in lowest order -and perhaps exact - approximation in terms of the contravariant metric of the configuration space proportional to g_K^2. Besides this the exponent of Kähler action would be involved. For elementary particles it would be the exponent of Kähler action for CP_2 type vacuum extremal. In this manner something combinatorially very similar to standard perturbation theory would result and there are excellent hopes that p-adic coupling constant evolution in powers of 2 is consistent with the standard coupling constant evolution.

(f) Vertices correspond to n-point functions. The contribution depending on fermionic fields defines the quantum number dependent part of the vertices and comes from the fermion field and their conjugats attached to the ends of propagator lines identified as braid strands. Besides this there is a symplecto-conformal contribution to the vertex.

(g) The stringy variant of twistor Grassmannian approach is highly suggestive since the necessary conditions are satisfied. In particular, the fundamental fermions propagate in the internal lines effectively as massless on-mass shell states but with non-physical polarization. M^4 *resp.*CP_2 is the unique 4-D manifold *resp.* compact manifold with Minkowskian *resp.* Euclidian signature of metric allowing twistor space with Kähler structure. This suggests that a generalization of twistorialization to 8-D context makes sense. The twistor space for CP_2 is 6-dimensional flag manifold $SU(3)/U(1) \times U(1)$ parameterizing the choice of color quantization axes and has popped up earlier in TGD inspired theory of consciousness.

The expansion of M-matrix in powers of $\hbar$

One should understand how the proportionality of gauge couplings to g_K^2 emerges and how loops give rise to powers of α_K. In zero energy ontology one does not calculate M-matrix but tries to construct zero energy state in the hope that QFT wisdom yields cold help to construct Connes tensor product correctly.

(a) The basic rule of quantum field theory is that each loop gives $\alpha = g^2/4\pi$ and thus $1/\hbar$ factor whereas in tree diagrams only g^2 appears so that they correspond to the semiclassical approximation.

(b) This rule is obtained if one assumes loops correspond to a hierarchy of sub-CDs and that in loop one can distinguish one line as "base line" and other lines as radiative corrections. To each internal line one must one must assign the factor $r^{-1/2} = (\hbar_0/\hbar)^{1/2}$ and factor g_K^2 except to the portion of base line appearing in loop since otherwise double counting would result. This dictates the expansion of M-matrix in powers of $r^{-1/2}$. It would not be too surprising to have this kind of expansion.

(c) g_K^2 factor comes from the functional integral over the partonic 2-surface selected by stationary phase approximation using the exponent of Kähler action. The functional integral over the WCW degrees of freedom is carried out using contravariant Kähler metric as a propagator and this gives g_K^2 factor in the lowest non-trivial order since one must develop a perturbation theory with respect to the deformations at the partonic 2-surfaces at the ends of line.

If the analogs of radiative corrections to this functional integral vanish - as suggested by quantum criticality and required by number theoretic universality - the resulting dependence on g_K^2 is exact and completely analogous to the free field theory propagator. The numerical factors give the appropriate gauge coupling squared.

(d) Besides this one must assign to the ends of the propagator line positive and negative energy parts of quantum state representing the particle in question. These give a contribution which is zeroth order in $\hbar$. For instance, gauge bosons correspond to fermionic bilinears. Essentially fermion currents formed from spinor fields at the two light-like wormhole throats of the wormhole contact at which the signature of the induced metric changes are in question. Correct dimension requires the presence of $1/\hbar$ factor in boson state and $1/\sqrt{\hbar}$ factor in fermion state. The correlators between fermionic fields at the end points of the line are proportional to $\hbar$ so that normalization factors cancel the $\hbar$ dependence. Besides this one would expect N-points function of symplecto-conformal QFT with $N = N_{in} + N_{out}$ having no dependence on $\hbar$.

But what about the concrete Feynman rules?

The skeptic reader can say that all this is just an endless list of general principles. I dare however claim that the only manner to proceed is to try to identify the general principles first. At this moment the understanding of the fundamental variational principled of TGD understood at such level of detail that one can indeed sketch a rather concrete formulation for the generalized Feynman rules. The generalized Feynman diagrams correspond to the 4-D surfaces defined by the Euclidian regions defined by wormhole contacts plus the string world sheets connecting them and carrying spinor modes. One might also talk about combination of Feynman diagrams and stringy diagrams or even about generalization of Wilson loops. The lines of these diagrams form also braids.

(a) The boundaries of string world sheets at which the modes of induced spinor field are localized (by well-definedness of em charge) carry fermion number and are identifiable as braid strands within partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Chern-Simons term and its fermionic counterpart - Chern-Simons-Dirac (C-S-D) term - must be assigned with partonic orbits in order to obtain non-trivial fermionic propagator. Massles fermion propagator emerges if spinor modes are assumed to be eigenstates of C-S-D operator at partonic orbits with generalized eigenvalue $ip^k\gamma_k$ with p^k identified as virtual four-momentum.

(b) The fundamental interaction is the scattering of fermions at opposite wormhole throats of wormhole contact. This interaction corresponds essentially to the stringy propagator $1/L_0$ so that one obtains a combination of Feynman rules and stringy rules. The vertices correspond topologically to fusion of 4-D lines along the 3-surfaces at their ends and this means deviation from string model picture: stringy diagrams correspond at topological level to what happens when particle travels between A and B along two different routes and has nothing to do with particle decay.

(c) Physical particles are bound states of massless fundamental fermions and correspond to pairs of wormhole contacts: a pair is required since wormhole throats behave effectively as magnetic monopoles and closed flux tube consisting of pieces at the two space-time sheets and wormhole contacts is required. This resolves the infrared difficulties of twistor approach. Twistor Grassmann approach strongly suggests that the residue integral over the virtual four-momenta reduces the propagators of fundamental fermions to their inverses at mass-shell so that only non-physical fermion helicities appear as virtual fermions.

9.1.3 Topics of the chapter

The goal is to sketch an overall view about the ideas which have led to the recent view about the construction of M-matrix. First the basic philosophical ideas are discussed. These include the basic ideas behind TGD inspired theory of consciousness [K68] , the identification of p-adic physics as physics of cognition and intentionality forcing the central idea of number theoretic universality, quantum classical correspondence, and the crucial notion of zero energy ontology.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- Zero Energy Ontology (ZEO) [L52]
- Hyperfinite factors and TGD [L24]
- $M^8 - H$ duality [L28]
- Weak form of electric-magnetic duality [L51]
- Emergent ideas and notions [L17]
- WCW spinor fields [L50]
- Geometrization of fields [L20]
- Quantum theory [L36]
- Vacuum functional in TGD [L48]

9.2 Basic philosophical ideas

The ontology of quantum TGD differs dramatically from that of standard quantum field theories and these differences play a key role in the proposed approach to the construction of M-matrix.

9.2.1 Zero energy ontology

Zero energy ontology has changed profoundly the views about the construction of S-matrix and forced to introduce the separate notions of M-matrix and U-matrix. M-matrix generalizes the notion of S-matrix as used in particle physics. The unitary U-matrix is something new having a natural place in TGD inspired theory of consciousness. Therefore it it best to begin the discussion with a brief summary of zero energy ontology.

Motivations for zero energy ontology

Zero energy ontology was first forced by the finding that the imbeddings of Robertson-Walker cosmologies to $M^4 \times CP_2$ are vacuum extremals. The interpretation is that positive and negative energy parts of states compensate each other so that all quantum states have vanishing net quantum numbers. One can however assign to state quantum numbers as those of the positive energy part of the state. At space-time level zero energy state can be visualized as having positive energy part in geometric past and negative energy part in geometric future. In time scales shorter than the temporal distance between states positive energy ontology works. In longer time scales the state is analogous to a quantum fluctuation.

Zero energy ontology gives rise to a profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

The notions of U-matrix and M-matrix

Zero energy ontology implies two kinds of matrices relevant for physics: U- and M. U-matrix characterizes the unitary process associated with the quantum jump and is universal. M-matrix has interpretation in terms of time-like entanglement coefficients between positive and negative energy parts of zero energy state and seems to characterize quantum states rather than the universal quantum dynamics. Unitarity conditions can be weakened so that thermodynamic becomes part of quantum theory in the sense that M-matrix is expressible as a product of positive square root of density matrix and unitary S-matrix analogous to thermal S-matrix assignable formally to a complex time parameter. U- and M-matrix differ in many respects.

(a) M-matrix defines entanglement between positive and negative energy parts of zero energy state. This entanglement does not make sense between different number fields since the light-like 3-surface defining Feynman cobordism connecting p-adic and real partonic 2-surfaces at boundaries of CD does not make sense. Hence M-matrix is diagonal with respect to number field.

(b) If algebraic universality is accepted in its strongest form, U-matrix elements must be algebraic numbers so that in zero energy ontology U-matrix between zero energy states can have elements between different number fields. Note that the vanishing of conserved quantum numbers is absolutely essential for this. This suggests a description of intentional action as p-adic-to-real transitions in terms of U-matrix. Algebraic Universality in this sense might be too strong a condition since it becomes questionable whether one can speak at all about real and p-adic physics as distinct disciplines. A weaker form of number theoretic universality is that the real and p-adic Universes relate to the algebraic Universes based on algebraic extensions of rationals in the same manner as reals and p-adic number fields and their extensions relate to rationals and algebraics. Also in this case transitions are possible but only between the states which live in rational or algebraic sub-Universes. One might say that real and p-adic universes are like pages of a book and algebraic universes are like the back of the book making it possible for zero energy states to leak between the pages.

(c) Both options makes possible to assign U-matrix to quantum jumps transforming intention to action. The original hypothesis motivated by the stability of sensorily perceived world was that U-matrix is almost trivial but there is actually no need for this assumption. The stability of sensory perception can be understood if the ensembles formed by CDs in various scales are nearly thermal so that sensory experience which involves statistical averaging and becomes stable.

(d) From the point of view of consciousness theory the natural statement is that M-matrix corresponds to the passive aspects of conscious experience, that is perception which reduces to quantum measurement and state function reduction at the fundamental level. U-matrix would in turn correspond to active aspects of conscious experience, including volitional acts and transformations of intentions to actions.

2. How U- and M-matrices relate to each other?

The obvious objection against zero energy ontology is that the universality of S-matrix in the sense of particle physics is lost since M-matrix characterizes the time-like entanglement of zero energy state and seems therefore to be highly state dependent. It would seem that one must give up the greatest dream of theoretician. The situation is not so bad.

(a) The notion of measurement resolution realized in terms of Jones inclusions requires that the included sub-factor $\mathcal{N} \subset \mathcal{M}$ representing the degrees of freedom below measurement resolution acts effectively like complex numbers on positive and negative energy parts of the zero energy state. This requires that time-like entanglement is given in terms of highly unique Connes tensor product. M-matrix decomposes to a product of the positive square root of density matrix and unitary S-matrix and one might hope that S-matrix is essentially unique for CD with a given scale.

(b) There might be also a connection between M-matrix and universal U-matrix. U-matrix between zero energy states could reduce to a tensor product of possibly universal S-matrix and its Hermitian conjugate associated with M-matrices: the first one between positive energy parts and second one between negative energy parts of zero energy states in question. If this is the case, the same S-matrix would apply both U-process and state function reduction. One might argue that this connection is necessary since without it there would be no manner to deduce any information about U-matrix experimentally. Note that density matrix part of M-matrix can be unit matrix only for hyper-finite factors of type II_1 are in question since only in this case the trace of $S^\dagger S = Id$ equals to 1 as the normalization of zero energy states requires.

(c) M-matrices associated with different size scales for CDs coming as powers of two would also have a natural fractal structure. The matrices associated with two CDs would differ only by the effects caused by p-adic coupling constant evolution. Two subsequent M-matrices in the hierarchy would differ only by the effects caused by a change in measurement resolution (the scales defining smallest sub-CDs contributing to the calculation of M would be different). The infinite sequence of Jones inclusions for hyperfinite type II_1 factors isomorphic as von Neumann algebras could express this fractal character algebraically.

The relationship between U-matrix and M-matrix

The following represents the latest result concerning the relationship between the notions of U-matrix and M-matrix and probably provides answer to some of the questions posed in the chapter. What is highly satisfactory that U-matrix dictates M-matrix completely via unitarity conditions. A more detailed discussion can be [K39] discussing Negentropy Maximization Principle, which is the basic dynamical principle of TGD inspired theory of consciousness and states that the information content of conscious experience is maximal.

If the state function reduction associated with time-like entanglement leads always to a product of positive and negative energy states (so that there is no counterpart of bound state entanglement and negentropic entanglement possible for zero energy states: these notions are discussed below) U-matrix and can be regarded as a collection of M-matrices

$$U_{m_+n_-,r_+,s_-} = M(m_+,n_-)_{r_+,s_-} \tag{9.2.1}$$

labeled by the pairs (m_+,n_-) labelling zero energy states assumed to reduced to pairs of positive and negative energy states. M-matrix element is the counterpart of S-matrix element $S_{r,s}$ in positive energy ontology. Unitarity conditions for U-matrix read as

$$\begin{aligned}
(UU^\dagger)_{m_+n_-,r_+s_-} &= \sum_{k_+,l_-} M(m_+,n_-)_{k_+,l_-}\overline{M}(r_+,s_-)_{k_+,l_-} = \delta_{m+r_+,n_-s_-} \ , \\
(U^\dagger U)_{m_+n_-,r_+s_-} &= \sum_{k_+,l_-} \overline{M}(k_+,l_-)_{m_+,n_-}M(k_+,l_-)_{r_+,s_-} = \delta_{m+r_+,n_-s_-} \ .
\end{aligned} \tag{9.2.2}$$

The conditions state that the zero energy states associated with different labels are orthogonal as zero energy states and also that the zero energy states defined by the dual M-matrix

$$M^{\dagger}(m_+, n_-)_{k_+,l_-} \equiv \overline{M}(k_+l_-)_{m_+,n_-} \tag{9.2.3}$$

-perhaps identifiable as phase conjugate states- define an orthonormal basis of zero energy states.

When time-like binding and negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) are allowed also zero energy states with a label not implying a decomposition to a product state are involved with the unitarity condition but this does not affect the situation dramatically. As a matter fact, the situation is mathematically the same as for ordinary S-matrix in the presence of bound states. Here time-like bound states are analogous to space-like bound states and by definition are unable to decay to product states (free states). Negentropic entanglement makes sense only for entanglement probabilities, which are rationals or belong to their algebraic extensions. This is possible in what might be called the intersection of real and p-adic worlds (partonic surfaces in question have representation making sense for both real and p-adic numbers). Number theoretic entropy is obtained by replacing in the Shannon entropy the logarithms of probabilities with the logarithms of their p-adic norms. They satisfy the same defining conditions as ordinary Shannon entropy but can be also negative. One can always find prime p for which the entropy is maximally negative. The interpretation of negentropic entanglement is in terms of formations of rule or association. Schrödinger cat knows that it is better to not open the bottle: open bottle-dead cat, closed bottle-living cat and negentropic entanglement measures this information.

How the new ontology relates to the existing world view?

In the new rather Buddhistic ontology zero energy states are identified as experienced events and objective reality in the conventional sense becomes only an illusion. Before the new view can be taken seriously one must demonstrate how the illusion about positive energy reality is created and why it is so stable.

1. How the arrow of geometric time emerges?

Before one can consider this question one must have an idea about how the arrow of geometric time emerges in TGD Universe.

(a) Conscious entity- self- can be compared to a person sitting in a movie theater with an ability to put the film run in either direction. This person is curious and forces the film to run. Once she has chosen the direction she keeps it as it is since the interesting things are the things not yet known, and are contained by the part of film not yet seen. It might be also easier to run the film in another direction. Translating this to the language of quantum TGD one obtains the following description.

(b) Self has as its imbedding space correlate causal diamond CD, the basic geometric structure of zero energy ontology. The light-like space-time surfaces inside $CD \times CP_2$ define the basic unit for the "world of classical worlds" (WCW), and one can say that self corresponds to one particular sub-WCW. Geometric time is naturally assigned with CD. CD does not move anywhere in the 8-D imbedding space as the standard view about arrow of geometric time would suggest. Rather, self can be compared to the movie theater plus its conscious audience.

(c) Self is curious to know what is in the geometric past and future. Since self can induce quantum jumps shifting the quantum superposition of the space-time surfaces to either direction of the geometric time, she does it. Since the contents of consciousness are about the region of space-time surface inside CD at particular moment of subjective time, correlation between the arrows of subjective time and geometric time results. The experience about the flow of geometric time can be regarded as an illusion analogous to train illusion in which a person sitting in a stationary train has an experience of motion induced by the motion of another train which has began to move.

(d) Once a preferred direction for the arrow is chosen, geometric past corresponds to what is already known and future to the unknown so that the direction of the arrow is stabilized. The CP-breaking predicted by TGD at fundamental level [K10] might favor a preferred direction for the arrow. The generation of global arrow could involve a competition between selves, and a domino effect in the sense that the arrow for self induces that for sub-selves. Phase conjugate laser beams and self assembly in living matter seem to represent non-standard arrow of geometric time and might have interpretation in terms of local deviations from the standard arrow at some level of the scale hierarchy.

(e) One must also understand why the contents of conscious experience seem to represent time=constant snapshot of the universe. Sub-CDs are correlates for sub-selves identified as mental images. They tend to concentrate at near the light-like boundaries of CD, where the most interesting events are and generate mental images. This explains why the contents of conscious experience of self is about a narrow interval of geometric time rather than the entire 4-volume of CD.

(f) The defender of the standard view might wonder whether the self is forced to sit for all her life in the same movie theater? Does self really correspond to single CD (sub-WCW) or should one speak about a wave function in the space of CDs? CD is partially characterized by the position of the lower tip of CD in H. Also the size of CD matters as well as the choice of quantization axes. In the case of color gauge group $SU(3)$ the space for choices of quantization axes is flag-manifold, which pops up in a mysterious looking manner in the model of honeybee dance developed by topologist Barbara Shipman [A88] . Could this wave function in the space of sub-WCWs correspond to a kind of wave packet moving in H so that the direction of geometric time could emerge also in more standard manner? Or could could self expand its consciousness by growing -that is by performing quantum jumps in which the size of the CD characterizing self is scaled up but the lower tip of CD moves nowhere. Since the scales of CDs come in powers of 2, this means a testable predictions about the time scales of conscious experience [K61] .

2. How the stability of perceived reality can be understood?

Consider what the perceived stability of positive energy states, or equivalently that of zero energy states means.

(a) What we perceive consciously are time-like state function reductions for events defined by zero energy states. Quantum jumps replace zero energy states with new ones all the subjective time (this corresponds to active aspect of conscious experience) and one can ask whether this makes impossible to experience any stable Universe.

(b) Stability under quantum jumps is implied if there are statistical ensembles of CDs and corresponding zero energy states (fixed to a high degree by Connes tensor product property of time-like entanglement) in various time scales associated with CDs in H. Self experiences its sub-selves as mental images and the mental image defined by sub-self corresponds to an ensemble average over sub-selves of sub-self. Hence the stability of experienced world would reflect the stability of thermal ensemble of events guaranteed by second law of thermodynamics for zero energy states. This allows also to re-interpret the standard trick made in deducing the rates for particle reactions from S-matrix elements. The problem is that $|S_{m,n}|^2$ is proportional to a square of delta function expressing energy-momentum conservation. The trick is to interpret second delta function as space-time volume so that one ends up with the replacement of probability for a reaction with probability per four-volume interpreted as a reaction rate per volume. The density of events (CDs) per four-volume is the natural interpretation in zero energy ontology.

(c) An alternative explanation for the stability of positive energy states is due to that the U-matrix characterizing quantum jumps between zero energy states is almost trivial. This would mean that the effects of volitional action on zero energy state are very small. The event pairs would be extremely stable once they are generated (how they are generated is an unavoidable question to be addressed below). Infinite sequences of transition between states with same positive energies and same initial energies occur. What is nice that this makes it possible to test the predictions of the theory by experiencing the transition again and again.

3. Statistical physics for zero energy states

The statistical physics for zero energy states was already mentioned in the above argument. This need not be equivalent with statistical physics assignable to the zero energy states themselves and defined by the density matrix defined by M-matrix.

(a) It is natural to speak about statistical physics for an an ensemble consisting of zero energy states $|m_+n_-\rangle$ including also their time reversals $|n_+, m_-\rangle$. In the usual kinetics one deduces equilibrium values for various particle densities as ratios for the rates for transitions $m_+ \to n_+$ and their reversals $n_+ \to m_+$ so that the densities are given by $n(n_+)/n(m_+) = \sum_{n_+} \Gamma(m_+ \to n_+)/\sum_{n_+} \Gamma/(n_+ \to m_+)$. In the recent situation the same formula can be used to *define* the particle number densities in kinetic equilibrium using the proposed identification of the transition probabilities.

(b) Because of the stability of the zero energy states, one can construct many particle systems consisting of zero energy states and can speak about the density of zero energy states per volume. Also the densities $n_{+,i}$ ($n_{-,i}$) of initial (final) states of given type can be defined and $n_{+,i}$ can be identified as densities of positive energy states. Also the densities for particles contained by these states can be defined. It would seem that the new ontology can reproduce the standard ontology as something which is not necessary but to which we are accustomed and which does not produce too much harm.

(c) The sequence of quantum jumps between zero energy states defines also a sequence between initial (final) states of quantum jump. Ordinary scattering experiment involves the measurement of the quantum numbers of particles in initial and final states. In the zero energy ontology one can perform separate quantum measurements for the observables associated with positive and negative energy components of zero energy states. This measurement would give rise to the scattering event.

5. How does the quantum measurement theory generalize?

There are also important questions related to the quantum measurement theory. The zero modes associated with the interior degrees of freedom of space-time surface represent classical observables entangled with partonic observables and this entanglement is reduced in quantum jump. Negentropy Maximization Principle [K39] is the TGD based proposal for the variational principle governing the statistical dynamics of quantum jumps. NMP states that entanglement negentropy tends to be maximized in the reduction of entanglement. Number theoretic variants of Shannon entropy making sense for rationally or even algebraically entangled states can be positive so that NMP can also lead to generation of this kind of entanglement and gives rise to a highly stable bound state entanglement.

6. Is the direct creation of zero energy states from vacuum possible?

In principle generation of zero energy states from vacuum is possible. At the first glimpse this option does not seem to be consistent with the assumption that U-matrix between zero energy states is induced by S-matrices between positive and negative energy parts of zero energy states. Should we accept that we are passive spectators who just observe the already existing zero energy states. It seems that this is not necessary.

(a) Zero energy states are superpositions of state pairs with different values of conserved quantum numbers which sum up to vanishing net quantum numbers. In particular, zero energy states can contain also a part for which positive and negative energy parts have vanishing quantum numbers. Hence zero energy states can be created also from vacuum for both positive and negative energy parts of the state.

(b) There is also a correlation between positive and negative energy parts of the state meaning that also quantum numbers are correlated and conservation laws do not apply locally anymore so that zero energy state is creatable from vacuum.

(c) One can also ask whether the creation of zero energy state means a creation of entire CD or activation of CD from pure vacuum state. Or could it be that the wave function in the degrees of freedom characterizing position, size, and quantization axes characterizing of CD changes in quantum jump so that the final state wave function becomes non-vanishing in a new region of H?

The creation of zero energy states from vacuum might take place also through intentional action.

(a) The mechanism generating p-adic zero energy states as representations of intentions would be same as for the creation of genuine zero energy states. As far as quantum numbers are considered there seems to be no problems of principle involved. One can however wonder whether the notion of conserved classical quantities assignable to Kähler action makes sense p-adically since the notion of definite integral is not well-defined p-adically. A way out of the difficulty is that real and p-adic surfaces involved have same functional form in terms of algebraic functions so that real conserved quantities can be interpreted as p-adic ones when they reduce to algebraic numbers.

(b) For zero energy states, p-adic-to-real transitions and vice versa are in principle possible and I have in fact proposed a general quantum model for how intentions might be transformed to actions in this manner [K79] . In the second direction the process corresponds to a formation of cognitive representation of a zero energy physical state. The only thing that is required is that the zero energy states in question can be regarded as those possible for some algebraic extension of rationals so that they make sense both in real and p-adic context with appropriate algebraic extension of p-adic numbers.

(c) In the degrees of freedom corresponding to WCW spinors situation is very much like for reals. Rational, and more generally algebraic number based physics applies in both cases. p-Adic space-time sheets however differ dramatically from their real counterparts since they have only rational (algebraic) points in common with real space-time sheets and p-adic transcendentals are infinite as real numbers. The algebraic valued U-matrix elements for p-adic-to-real transitions can be formulated using n-point functions restricted to these rational points common to matter and mind stuff. If this picture is not terribly wrong, it would be possible to generate zero energy states from vacuum and the construction of quantum computer programs would be basically a long and tedious process involving very many intentional acts.

(d) Real-to-p-adic transitions would represent transformation of reality to cognition and would be also possible. The characteristic and perhaps the defining feature of living matter could be its highly developed ability to reconstruct reality by performing p-adic-to-real transitions and their reversals.

(e) Here an interesting aspect of the p-adic conservation laws might have some role. p-Adic integration constants are pseudo constants in the sense that a quantity having vanishing (say) time derivative can depend on a finite number of pinary digits t_n of the time coordinate $t = \sum_n t_n p^n$. Could one think that quantum jumps can generate from vacuum exact vacuum states as vacuum tensor factors of the WCW spinor , and that in subsequent quantum jumps p-adic U-matrix conserving quantum numbers only in p-adic sense transforms this state into a non-trivial zero energy state which then transforms to a real state in intentional action? Note that if conserved quantum numbers are integers they are automatically pseudo constants. p-Adic conservation laws could allow also the p-adic zero energy states to pop up directly from vacuum.

9.2.2 The anatomy of the quantum jump

In TGD framework quantum transitions correspond to a quantum jump between two different quantum histories rather than to a non-deterministic behavior of a single quantum history (understood as an evolution of Schrödinger equation). Therefore U-matrix relates to each other two quantum histories rather than the initial and final states of a single quantum history and this leads to a resolution of the basic paradox of quantum measurement theory.

To understand the philosophy behind the construction of U-matrix it is useful to notice that in TGD framework there is actually a 'holy trinity' of time developments instead of single time development encountered in ordinary quantum field theories.

(a) The classical time development is coded by the preferred extremal of Kähler action inside each causal diamond CD defining a hierarchy of time scales comings as powers of 2.

(b) The unitary "time development" defined by U associated with each quantum jump

$$\Psi_i \to U\Psi_i \to \Psi_f \quad ,$$

and defining U-matrix. One cannot however assign to the U-matrix an interpretation as a unitary time-translation operator. There is a hierarchy of time scales associated with U-matrices. U-matrices are between zero energy states and do not correspond directly to the S-matrix of particle physics, which in zero energy ontology corresponds to the matrix M defining time-like entanglement coefficients between positive and negative energy parts of zero energy state.

(c) The time development of subjective experiences by quantum jumps is identified as sequence of moments of consciousness. The value of geometric time associated with a given quantum jump is determined by the space-time locus for the contents of consciousness of the observer. The understanding of psychological time and its arrow and of the dynamics of subjective time development requires the construction of theory of consciousness [K68, K3] . A crucial role is played by zero energy ontology and by the classical non-determinism of Kähler action implying that the non-determinism of quantum jump and hence also the contents of conscious experience can be concentrated into a finite volume of the imbedding space.

Unitary process

U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution in standard quantum mechanics. U-process is however only formally analogous to Schrödinger time evolution of infinite duration since there is *no* real time evolution or translation involved.

Macro-temporal quantum coherence suggests strongly a fractal hierarchy of U-matrices defined for periods of macro-temporal quantum coherence consisting of sequences of quantum jumps defining selves. The hierarchy of these unitary S-matrices would not be only an approximation but provide exact descriptions consistent with the limitations of conscious experience. The duration of the macro-temporal quantum coherence would correspond to the time interval defining unitary time development. Also p-adic length scales would define similar hierarchy of U-matrices. The realization of zero energy ontology in terms of fractal hierarchy of causal diamonds (CDs) justifies of this expectation since one can assign to each CD U-process.

State function reduction

The selection of quantization axes, the fact that the perceived world looks classical, and the correlation of outcome of measurement with classical observables should have first level explanation if quantum measurement theory is to be more more than ad hoc construct justifying the basic rules.

1. Imbedding space correlate for the choice of the quantization axes

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes a part of quantum theory.

If one takes seriously the proposed hierarchy of Planck constants and the generalization of the imbedding space to a book like structure implied by it, the selection of quantization axes has also imbedding space correlate which means also breaking of fundamental symmetries at the level of given CD since quantization axes define physically preferred directions. Each CD would would be replaced by a union of its copies with different selection of quantization axes to guarantee symmetries at fundamental level and quantum jump would involve localization to single choice unless one is willing to accept multi-verse picture for conscious experience.

2. The outcome of the state function reduction must look classical

Quantum classical correlation requires that quantum states have classical correlates. This means that the final states of quantum jump correspond to quantum superpositions of 3-surfaces which are macroscopically equivalent so that the world of conscious experience looks classical. "Macroscopically equivalent" translates "indistinguishable in the measurement resolution available" in the recent formulation of quantum TGD.

The finiteness of the measurement resolution is a precise quantitative prediction of quantum TGD proper in its recent form and essentially due to the vacuum degeneracy of Kähler actions responsible also for the classical non-determinism. The point is that the induced spinor fields allow only finite number of zero modes for given light-like 3-surface so that anti-commutation relates can be satisfied for a finite set of points only identified as intersection of partonic 2-surface and number theoretic braid. The resulting effective discretization is much more than one might have expected but emerges very naturally in terms of zero energy ontology. The inclusions of hyper-finite factors of type II_1 (HFFs) allow a mathematical formulation of this picture in terms of quantum counterparts of WCW Clifford algebras.

A way out of the problems caused by the lack of appropriate p-adic integration measure could be that p-adic WCW spinor fields are localized to discrete subsets of the p-adic configuration space. Finite measurement resolution realized in terms of number theoretic braids implies that not only effective WCW Clifford algebra has finite quantum dimension but also the effective configuration space itself. Vacuum functional identified as the exponent of Kähler function can be defined in terms of eigenvalues of the modified Dirac operator also in p-adic context and one can consider the possibility that these eigenvalues serve as coordinates for the p-adic WCW and p-adic WCW spinor fields are localized to discrete subsets of this space. Much depends also on the representation of 3-surfaces. For instance, the representation in terms of polynomials means that the coefficients of polynomials with some additional algebraic conditions characterize the point of the p-adic WCW and one can forget the surface itself. Algebraization in terms of quantum coordinates for p-adic configuration space might also help.

State preparation

TGD inspired theory of consciousness inspires the hypothesis that the standard quantum measurement is followed by a self measurement inside self, which reduces entanglement between some subsystem and its complement in quantum fluctuating degrees of freedom. Again a measurement of the density matrix is in question. Self measurements are repeated until a completely unentangled (within measurement resolution) product state of self results: the process is equivalent with the state preparation process, which is a purely phenomenological part of standard quantum measurement theory. In well defined sense state preparation corresponds to an analysis or decay process respecting only bound state entanglement.

The dynamics of self measurement is governed by Negentropy Maximization Principle (NMP, [K39]), which specifies which subsystems are subject to quantum measurement in a given quantum jump. NMP can be regarded as a basic law for the dynamics of quantum jumps and states that the information content of conscious experience is maximized. In p-adic context NMP would dictate the dynamics of cognition. In real context, self measurement makes possible for the system to fight against thermalization by self-repair at quantum level, and might be a crucial additional element besides the many-sheeted space-time concept needed to understand how bio-systems manage to be macroscopic quantum systems (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book).

The hypothesis that bound state entanglement coefficients are in the hierarchy of extensions of rational numbers allows to use number theoretic definition of entanglement entropy. This allows to have also negative entropies and in this case NMP does not imply the reduction of entanglement in quantum jump so that there is no need to separately postulate the bound state entanglement is stable against NMP.

Classical space-time correlates for the basic steps of quantum jump

The classical space-time correlates for the basic notions of quantum measurement theory should be of crucial help in the construction of the M-matrix. The natural first expectation is that these correlates are encountered only at the level of space-time surfaces. Zero energy ontology and the generalization of the imbedding space forced by the hierarchy of Planck constants led to the conclusion that this kind of correlates emerge also at the level of imbedding space. CDs serve as correlates for selves and the fractal hierarchy of CDs allows to characterize finite measurement resolution and treat also the implications of the non-determinism of Kähler action.

1. Correlates at the level of space-time

Consider first space-time correlates for the basic steps of the quantum jump.

(a) Space-time sheets correspond to coherence regions for various classical fields obtained by inducing various geometric structures of the imbedding space to the space-time surface. They correspond also to the coherence regions of the induced spinor fields. The classical non-determinism of Kähler action and of corresponding super-symmetrically related Dirac equation makes possible to have space-time correlates for the non-determinism of quantum jump sequence leading to de-coherence. One must be however cautious with what one really means with this notion.

  i. The first guess is that de-coherence at space-time level means simply the decomposition of a space-like 3-surface into pieces during its evolution: emission of on mass shell photon by charged elementary particle is the simplest possible example here. Non-determinism must be involved in an essential manner.

  ii. At particle level M-matrices are associated with light-like 3-surfaces connecting the light-like boundaries of CD and representing generalized Feynman diagram with vertices identified as partonic 2-surfaces along with the lines represented by light-like 3-surfaces are glued together. At vertices 3-surfaces and also space-time surfaces are literally branched. State function reduction happens for the zero energy state assignable to this Feynman diagram like 3-surface. In this picture the coherence regions would correspond to connected parts of light-like 3-surfaces and the scale of the smallest CD in the hierarchy would characterize coherence length and time. De-coherence could be seen as the presence of sub-CDs and corresponding non-deterministic details of space-time surface which serve as a correlate for non-determinism of quantum jumps. At the level of M-matrix sub-CDs can be assigned to loop corrections in powers of $\hbar$.

(b) As already explained, the classical non-determinism of the Kähler action allows to represent state function reduction at classical level via stationary phase approximation. Double slit experiment serves as a good example of what could happen.

i. Before the decision to measure which slit the particle propagates through, the space-time surface representing the particle is branched (in the sense of string diagram rather than Feynman diagram) to two parts going through the slits and both branches contain classical spinor field.

ii. As the decision is made, p-adic space-time sheet representing the intention to make the measurement is transformed in quantum jump to real space-time sheets, most naturally negative energy topological light rays propagating to the geometric past and interacting with the spinor field and in such a manner that spinor field propagates only along the second branch of the space-time sheet.

iii. This is achieved if the interaction of negative energy topological light ray transforms space-time sheet to vacuum extremal for which also spinorial energy momentum tensor and various currents vanish identically. Presumably the absorption of negative energy nullifies the energy otherwise propagating along the branch in question. Conservation of various currents implies that the total probability defined by the spinor field goes to the second space-time branch.

(c) Also state preparation and NMP should have space-time correlate.

i. During state preparation process generation of de-coherence continues and involves maximal de-entanglement in quantum fluctuating degrees of freedom with the formation of bound states being exception. If join along boundaries bonds (realized in terms of magnetic flux tubes say) serve as correlates for the entanglement, the process should correspond at space-time level to the splitting of join along boundaries bonds connecting 3-sheets. 3-surface would quite literally decompose into pieces.

ii. Negentropy maximization should thus imply a non-deterministic splitting of 3-surface into pieces if standard expression for entanglement entropy is used. Generation of sub-CD:s would be equivalent correlate.

iii. If number theoretic variant of entanglement entropy is allow NMP could force formation of join along boundaries bonds. In [K11] I have considered the possibility that Kähler action indeed has an information theoretic interpretation. The non-determinism of NMP would has as a space-time correlate the non-determinism of Kähler action.

The three non-determinisms

Besides the non-determinism of quantum jump, TGD allows two other kinds of non-determinisms: the classical non-determinism basically due the vacuum degeneracy of the Kähler action and p-adic non-determinism of p-adic differential equations due to the fact that functions with vanishing p-adic derivative correspond to piecewise constant functions.

To achieve classical determinism in a generalized sense, one must generalize the definition of the 3-surfaces Y^3 (belonging to light cone boundary) by allowing also "association sequences", that is 3-surfaces which have, besides the component belonging to the light cone boundary, also disjoint components which do not belong to the light cone boundary and have mutual *time like separations*. This means the introduction of additional, one might hope typically discrete, degrees of freedom (consider non-determinism based on bifurcations as an example). It is even possible to have quantum entanglement between the states corresponding to different values of time.

The explicit quantitative realization of this vision is provided by the fractal hierarchy of CDs within CDs. To specify the zero energy state one must characterize it for all CDs with scale above measurement resolution scale. Finite resolution scale is not an approximation to reality but a basic property of zero energy states forced by the quantization of the induced spinor fields.

Without the classical and p-adic non-determinisms general coordinate invariance would reduce the theory to the light cone boundary and this would mean essentially the loss of time which occurs also in the quantization of general relativity as a consequence of general coordinate invariance. Classical and p-adic non-determinisms imply that one can have quantum jumps with non-determinism (in conventional sense) located to a finite time interval. If quantum jumps correspond to moments of consciousness, and if the contents of consciousness are determined by the locus of the non-determinism, then these quantum jumps must give rise to a conscious experience with contents located in a finite time interval.

Also p-adic space-time sheets obey their own quantum physics and are identifiable as seats of cognitive representations. p-Adic non-determinism might be the basic prerequisite for imagination and simulation.

9.3 Zero energy ontology and conformal invariance

In the following some aspects of the role of zero energy ontology and conformal invariance in the construction of M-matrix are discussed. The emphasis is on the long standing difficulties related to the realization of the analog of stringy picture about M-matrix. The general vision that emerged much after the writing the first version of this section is that vertices correspond to n-point functions of a symplecto-conformal field theory at partonic 2-surfaces. The basic deviation from string models are due to the presence of symplectic n-point functions (discussed in [K8] and due to the discretization caused by the notion of number theoretic braid. Propagators reduce to fermionic correlators assignable to the lines of the generalized Feynman diagram and the naive expectation supported by QFT like picture and effective 3-dimensionality of space-time is that the inverse of the longitudinal part of the modified Dirac operator D_K, rather than D_K itself, is in question. The problem is to understand how the analog of the stringy propagator as inverse of super Virasoro generator G is obtained. The solution of the problem is that different -one might say fundamental- representation of $1/G$ determined as the propagator associated with the longitudinal part of D_K contains a sum over virtual states labeled by integer valued conformal weights rather than only on mass shell state with ground state conformal weight just as the QFT propagator contains sum over virtual momenta.

9.3.1 M-matrix as characterizer of time-like entanglement between positive and negative energy components of zero energy state

The idea about giving up the notion of unitary S-matrix in the standard sense of the word might seem too radical and there is actually no fundamental reason forcing this in the conceptual framework provided by hyper-finite factors of type II_1. Just the opposite, the freedom to construct zero energy states rather freely could be restricted by the unitarity of the matrix determined by the entanglement coefficients. There are however both mathematical and physical reasons to believe that entanglement coefficients give rise to a thermal S-matrix which is counterpart of ordinary S-matrix but for complex time parameter.

Before continuing, it must be added that M-matrix identified as entanglement coefficients between positive and negative energy parts of zero energy states would characterize zero energy *states* and could be something totally different from the U-matrix describing unitary process associated with the *quantum jump*. If one however assumes that U-matrix reduces to a tensor product of S-matrix parts of M-matrix and its conjugate between positive energy parts and between negative energy parts of zero energy state, situation changes.

Unitarity in zero energy ontology

Quantum classical correspondence combined with the number theoretical view about conformal invariance could fix highly uniquely the dependence of M-matrix on cm degrees of freedom and on net momenta and color quantum numbers. The corner stone of the interpretation is zero energy ontology applied already earlier in classical TGD.

Unitary M-matrix is possible for zero energy ontology in case of HFFs of type II_1. The interpretation of the condition $Tr(SS^\dagger) = Tr(Id) = 1$ as a normalization condition stimulates the hope that the entanglement between positive and negative energy states in zero energy states is coded by a unitary M-matrix in the conceptual framework provided by hyper-finite type II_1 factors so that states would represent dynamics in their structure.

It must be however emphasized that unitarity is by no means obvious or necessary in zero energy ontology.

(a) What can be measured are basically the ratios of scattering rates since one must always use a clock and clock corresponds to some standard scattering occurring with rate defining the time unit used.

(b) If one gives up unitary and allows the interpretation of $M^\dagger M$ as density, thermodynamics becomes part of quantum theory. In particular, p-adic thermodynamics crucial for understanding of particle massivation could emerge in this manner.

(c) It is not obvious whether unitarity is even possible in 4-dimensional context. For TQFTs with S-matrix identified as a functor from category of ordinary cobordisms, unitary S-matrix is assignable only to trivial cobordisms for $D < 4$ [K8] , [A42] The situation might be same also for Feynman cobordisms. The whole point of holography is however that space-time is effectively 3-dimensional due to the constraint that virtual states appearing in the lines of Feynman diagram are only virtual in 3-D sense and correspond to zero modes of the modified Dirac operator in 4-D sense.

(d) There is a further strong argument in favor of identification of M-matrix as the analog of thermal S-matrix. It is quite possible that HFF of type II_1 is replaced with II_∞ factor which is a tensor product factors of type II_1 and type I. In the case of configuration degrees of freedom super-conformal symmetry might guarantee that HFF of type II_1 is in question. Imbedding space degrees of freedom however seem to give rise to factor of type I via the representations of Poincare group and color partial waves and there seems to be no natural manner to avoid this. Only thermal M-matrix would define a normalizable state so that thermodynamical states would be genuine quantum states rather than only a useful fiction of theorist.

(e) One can hope that M-matrix as analog of thermal matrix exists for general Feynman cobordisms meaning that thermodynamics and p-adic thermodynamics follow from fundamental principles somewhat like black hole temperature emerges as a property of black hole horizon. Note that for U-matrix the unitarity is necessary and U-matrix could be expressed in terms of the S-matrices associated with M-matrix.

Finite measurement resolution and the procedure leading from M-matrix to scattering rates

In standard QFT the procedure leading from from S-matrix to scattering rates breaks all rules of mathematical aesthetics. The ugliest step in this procedure involves the identification of the 4-dimensional momentum space delta function $\delta^4(0)$ as a 4-D reaction volume. Encouragingly, zero energy ontology allows to get rid of this feature and also provides a clear physical interpretation for it.

(a) In standard positive energy ontology the conservation of energy does not allow localization in time direction so that in time direction the reaction volume is necessarily infinite. In zero energy ontology causal diamonds CD define naturally finite reaction volumes. If their scales come as powers of 2 -as suggested by the geometry of CD- one can deduce p-adic length scale hypothesis from this picture in turn supported by the success of p-adic mass calculations. Additional scale hierarchy corresponds to scaled values of Planck constants so that all rational multiples of fundamental scale defined by CP_2 size are in principle possible.

(b) p-Adic length scale hierarchy assignable to the hierarchy of CDs within CDs is a good candidate for a hierarchy of Jones inclusions with increasing value of p defining an improved momentum resolution. This leads also to a vision about how p-adic coupling constant evolution for M-matrix is realized in terms of cutoff characterizing the size of the smallest sub-CD possible.

(c) In the framework of zero energy ontology one can say that there is an ensemble of CDs in $M^4 \times CP_2$ representing scattering events and reaction rates are obtained by multiplying the density of CDs with the finite reaction probabilities determined by the M-matrix. Reaction probabilities are finite since the conservation of four-momentum is a property of states in zero energy ontology and momentum space delta functions can emerge only in the restriction of the four-momentum of positive energy states to a precise value. By the finite size of CD is is however not possible to make this kind of restriction in zero energy ontology. Only in the idealization that the four-momentum of the initial state is precisely determined the square of $\delta^4(0)$ would appear and a similar limiting procedure as in the usual case would be needed but would have a clear physical interpretation.

(d) Finite length scale resolution suggests at the level of super conformal algebras to a cutoff n_{cr} for the values of conformal weight and thus mass squared. The finite number of fermionic oscillator operators indeed leads to a cutoff of conformal weight of super-conformal algebras and the replacement of integers with finite field as values of conformal weights is suggestive. The finite truncations of super conformal algebras obtained by replacing the integers n labeling the states with integers in Z/kZ would be mathematically natural and define also physically natural Jones inclusions. Prime values of k would correspond to the replacement of Z with finite field $G(k)$. p-Adic mass calculations suggests that the value of conformal weight for which the mass of the state becomes equal to Hagedorn temperature fixes n_{cr} and predicts $n_{cr} \sim log_2(p)$ [K46] . Combining this with p-adic length scale hypothesis ($p \simeq 2^k$, k integer with primes favored) would encourage the hypothesis $n_{cr} = k$.

9.3.2 Feynman rules in WCW degrees of freedom

The construction of the theory in fermionic degrees of freedom looks relatively straightforward. In WCW degrees of freedom the situation seems extremely complicated and I have not been able to find elegant formulation although a reduction to to finite-quantum-dimensional WCW is suggestive, and should reflect the fact that all points of 2-surface except the points of braid are below measurement resolution. The elegant solution could be a formulation in terms of quantize M^2 and CP_2 coordinates allowing to calculate n-point functions and here conformal field theories with string reduced to a discrete set of points representing braid is the most plausible first guess.

WCW degrees of freedom

WCW degrees of freedom can be decomposed to center of mass degrees of freedom, zero modes, and quantum fluctuating degrees of freedom contributing to the WCW metric including modular degrees of freedom.

(a) Cm degrees of freedom correspond to the position of partonic 2-surface, the definition of which should be specified precisely, perhaps as a selection of preferred braid strand. It is not sensible to assign separate four momenta to the braid strands since they are constrained to move parallel.

(b) M-matrix should reduce essentially to a Fourier transform of the N-point function assigned to the incoming and outgoing partonic two-surfaces. The decomposition $M^4 = M^2(x) \times E^2(x)$ implied by number theoretic compactification and known extremals of field equations with Minkowskian signature of the induced metric suggests that four-momentum should be in the plane $M^2(x)$ so that a correlation between space-time geometry and quantum numbers would result.

(c) Quantum field theory analogy would suggest the association of four-momenta to the propagator lines. This can be done by the introduction of Fourier transform of various correlation functions. The restriction inside CD implies small breaking of momentum conservation also induced by the restriction to the points of braids.

(d) There are also center of mass degrees of freedom associated with CP_2 and here color partial waves are necessary. Color partial waves can be assigned with partonic 2-surfaces and propagators should give correlators conserving color quantum numbers.

(e) For partonic 2-surface modular degrees of freedom characterizing the conformal equivalence class of 2-surface in the induced metric is expected to be of special importance and TGD based explanation of family replication phenomenon relies on the notion of elementary particle vacuum functional in these degrees of freedom. Therefore the reduction of the partonic 2-surface to a discrete set of points would mean the loss of crucially important information. At least the global data about topology and complex structure of X^2 must be preserved. Elementary particle vacuum functionals in modular degrees of freedom labeling the complex structures of X^2, or perhaps punctured X^2 would bring in the needed additional structure. Modular spaces have complex structure so that WCW Kähler metric could be non-trivial in these degrees of freedom. Induced Kähler form is the most important zero mode and excellent candidate for information that should not be lost in discretization.

WCW functional integral

About WCW functional integral one make only some general statements.

(a) If only braid points are specified, there is a functional integral over a huge number of 2-surfaces meaning sum of perturbative contributions from very large number of partonic 2-surfaces selected as maxima of Kähler function or by stationary phase approximation. This kind of non-perturbative contribution makes it very difficult to understand what is involved so that it seems that some restrictions must be posed. Also all information about crucial vacuum degeneracy of Kähler action would be lost as a non-local information.

(b) Induced Kähler form represents perhaps the most fundamental zero modes since it remains invariant under symplectic transformations acting as isometries of WCW. Therefore it seems natural organize WCW integral in such a manner that each choice of the induced Kähler form represents its own quantized theory and functional integral is only over deformations leaving induced Kähler form invariant.

(c) One can ask whether also the induced Kähler form of the light-cone boundary should be kept fixed so that the deformations of the partonic 2-surfaces would leave invariant both the induced areas and magnetic fluxes. The the symplectic orbits of the partonic 2-surfaces (and 3-surfaces) would therefore define a slicing of the WCW with separate quantization for each slice. It is not clear whether this restriction is consistent with conformal field theory picture.

(d) The functional integral would be over the symplectic group of CP_2 and over M^4 degrees of freedom -perhaps also in this case over the symplectic group of δM^4_+ - a rather well-defined mathematical structure. Symplectic transformations of CP_2 affect only the CP_2 part of the induced metric so that a nice separation of degrees of freedom results and the functional integral can be assigned solely to the gravitational degrees of freedom in accordance with the idea that fundamental quantum fluctuating bosonic degrees of freedom are gravitational.

(e) The WCW integration around a partonic 2-surface for which the Kähler function is maximum (it could be also selected by a stationary phase approximation) should give only tree diagrams with propagator factors proportional to g_K^2 if loop corrections to the WCW integral vanish. One could hope that there exist preferred S^2 and CP_2 coordinates such that vertex factors involving finite polynomials of S^2 and CP_2 coordinates reduce to a finite number of diagrams just as in free field theory.

Symplectic QFT

Also the symplectically invariant degrees of freedom must be treated and this leads to the notion of symplectic QFT. The explicit construction of symplectic fusion rules has been discussed in [K8] . These rules make sense only as discretized version. Discreteness can be understood also as a manifestation of finite measurement resolution: at this time it is associated with the impossibility to know the induced Kähler form at each point of partonic 2-surface. What one can measure is the Kähler flux associated with a triangle and the density of triangulation determines the measurement accuracy. The discrete set of points associated with the symplectic algebra characterizes the measurement resolution and there is an infinite hierarchy of symplectic fusion algebras corresponding to gradually increasing measurement resolution in classical sense.

An interesting question is whether the symplectic triangulation could be used to represent a hierarchy of cutoffs of super conformal algebras by introducing additional fermionic oscillators at the points of the triangulation. The M^4 coordinates at the points of symplectic triangulation of S^2_i, $i = I, II$ projection and CP_2 coordinates at the points of symplectic triangulation of S^2 could define discrete version of quantized conformal fields. The functional integral over symplectic group would mean integral over symplectic triangulations. Note that M^2 number theoretic braid is trivial as symplectic triangulation.

Fusion algebra structure constants are equal to products of three roots of unity assignable to each point of braid strand. An open question is whether these phase factors should be identified as counterparts of plane waves factors. Momentum conservation would be replaced in this approach by a weaker condition that the product of these factors equals to unity at each vertex.

In the original variant of symplectic triangulation the exact form of triangulation was left free. It would be however nice if symplectic triangulation could be fixed purely physically by the properties of the induced Kähler form since also the number of fermionic oscillator modes and number theoretical braids is fixed by the dynamics of Kähler action.

(a) A symplectically invariant manner to fix the nodes of the triangulation could be in terms of extrema of the symplectic invariant $\epsilon^{\alpha\beta}J_{\alpha\beta}$. The maxima of the magnitude of Kähler magnetic field are indeed natural observables.

(b) It is not clear whether the precise specification of the edges of the triangulation is needed or has any physical meaning. One might consider the possibility of of extremizing the fluxes but it turns out impossible to formulate this in terms of a local variational principle. The situation is analogous to finding an extremum of function in a situation when the extremum happens to be at the end of the interval so that the vanishing of derivative cannot be taken as criterion. In the recent situation one can expect that the extrema correspond to "triangles" for which symplectic area vanishes or to regions inside which $\epsilon^{\alpha\beta}J_{\alpha\beta}$ has a fixed sign.

How string model type quantization could emerge from WCW functional integral?

Conformal invariance suggests that n-point functions of conformal field theory result from the integration over WCW degrees of freedom. This means quantization of M^4 and CP_2 coordinates. The quantum variant of WCW is natural if also configuration space degrees of freedom form hyper-finite factor of type II_1 as super-conformal symmetry suggests, and could be realized through quantization of the imbedding space coordinates.

(a) There are reasons to expect that the conformal field theory in question is rational. Also number theoretic universality favors this option. The vertex operators of rational conformal field theories are constructible in terms of the vertex operators $: exp(i\alpha \cdot m) :$ plus factors for internal quantum numbers. The M^4 coordinate m is quantized using rules of string theory.

(b) In the recent case α could correspond to four-momenta assignable to the internal lines emerging from the partonic 2-surface providing a close correspondence with quantum field theory. The dynamical Kac-Moody symmetry in transversal degrees of freedom indeed suggests that this kind of factors should be included. The transversal plane to which quantized m would be restricted could be identified as the plane E^2 defined by the decomposition $M^4 = M^2 \times E^2$ characterizing given CD. The well-known tachyonity of the ground state ($\alpha \cdot \alpha = -2$)) required by vertex operator construction would not be a catastrophe if α corresponds to transversal four-momentum. The points of braid are arranged along a closed curve in X^2 in string model but in the recent case it is not clear whether the ordering remains intact.

(c) The M^4 projections of the points of number theoretic M^2 braid at X^2 can vary along light-like ray. The problem is that the variations in transversal degrees of freedom for the arguments of n-point function of M^4 coordinates vanish. The problem disappears if $S_i^2 \subset CP_2$ braids are also needed. M^2 braids would allow the description of CP_2 quantum fluctuations and CP_2 braids the description of M^4 quantum fluctuations.

(d) Also CP_2 coordinates must be quantized and the first guess is CP_2 WZW model in the point set defined by M^2 braid consisting of point at light-like ray and M^4 string model in the point set defined by CP_2 braid. These two models could allow to calculate the n-point functions for M^4 and CP_2 coordinates by performing functional integral over the symplectic group of $\delta M^4_\pm \times CP_2$.

(e) There are also factors coming from CP_2 color partial waves and $S^2 \times CP_2$ Hamiltonians depending of center of mass coordinates. The quantized M^4 coordinates would contain these degrees of freedom as center of mass term in the representations of rational conformal field as an ordered exponential. Same trick should work for $CP_2 = SU(3)/U(2)$ coordinates for braid points.

9.3.3 Rational conformal field theories and stringy scattering amplitudes

Rational conformal field theories lead to stringy scattering amplitudes as N-point functions so that there are reasons to expect that they emerge from quantum TGD.

General assumptions

Let us list first the general assumptions leading to stringy scattering amplitudes.

(a) Quantum criticality of TGD would suggest that, as far as conformal invariance is considered, all details about the microscopic dynamics can be forgotten and the amplitudes for the generation of zero energy states from vacuum can be expressed as vacuum expectation values of the products of primary fields of a rational conformal field theory at partonic 2-surfaces. The primary fields in question do not directly correspond to the M^4 local versions of fundamental super-conformal algebras creating states at the intersections of partonic causal determinants with $\delta M^4_+ \times CP_2$. Rather, they would describe the states created by these operators and possessing conformal weights consistent with rationality. Hence one can completely forget the detailed anatomy of these states and only the values of c and $\alpha = \Delta_{mn}$ matters.

(b) Since the conformal weights of primary fields are non-negative, mass squared identified as conformal weight using CP_2 mass as unit is non-negative and no problems with tachyons are encountered. The deeper reason for the non-negativity of conformal weights would be that the super-symplectic and Kac-Moody contributions to conformal weight sum up to a non-negative net result. It is important to notice that the vertex operators $V(z)$ representing Kac-Moody generators used to construct stringy scattering amplitudes have positive conformal weight $\Delta = mm'$ for $c \neq 0$ case and, as is clear on basis of Sugawara representation, they would correspond to a negative mass squared in stringy models. This would correspond to the convention $m^2 = kL_0$, $k < 0$ rather than $k > 0$, in TGD framework. It must be added that TGD mass formula is definitely not consistent with that of string models.

(c) The first guess is that the expressions for the amplitudes for creating zero energy state generalize as such an could be expressed in terms of the vacuum expectation values of n-point functions for the primary fields of rational conformal field theories. Stringy form would be obtained by the integration of the arguments over a circle of the partonic 2-surface and by using standard arguments one could fix 3 of the arguments z_i to $z = 0, 1, \infty$ in case of sphere. Apart from the normalization constant the resulting amplitude would have the general form

$$
\begin{aligned}
A(\alpha_1, ..., \alpha_n) &= \int \prod_{i=4}^{n} dz^i \langle \phi_{\alpha_1}(0), \phi_{\alpha_2}(1), \phi_{\alpha_3}(\infty)\phi_{\alpha_4}(z_4).....\phi_{\alpha_n}(z_n)\rangle \ , \\
\sum_n \alpha_n = 0 \ . & \qquad (9.3.1)
\end{aligned}
$$

Note that the conformal weights of negative energy particles are negative.

Free field representation of rational conformal field theories gives stringy amplitudes

Rational conformal field theories allow a representation of the primary fields in terms of exponentials of massless free fields $X(z)$ [A49] with the energy momentum tensor

$$
T(z) = -\frac{1}{4} : [\partial X(z)]^2 : \qquad (9.3.2)
$$

The correlation functions of $X(z)$ and $\partial X(z)$ are

$$
\begin{aligned}
\langle X(z)X(\zeta)\rangle &= -2log(z-\zeta) \ , \\
\langle \partial X(z)\partial X(\zeta)\rangle &= -\frac{2}{(z-\zeta)^2} \ . \qquad (9.3.3)
\end{aligned}
$$

$X(z)$ has the stringy expansion

$$
\begin{aligned}
X(z) &= \sqrt{2}\left(q - ip \times log(z) + i\sum \frac{a_n}{n} z^n\right) \ , \\
[q, p] &= i \ , \ [a_n, a_m] = n\delta_{n+m,0} \ . \qquad (9.3.4)
\end{aligned}
$$

There is of course no need to assume that strings are the underlying dynamical objects and z corresponds to the complex coordinate of the partonic 2-surface in TGD context.

The normal order exponentials of the free field

$$
\begin{aligned}
V_\alpha(z) &= \ : exp(i\alpha X(z) : \\
&= exp(i\sqrt{2}\alpha q)exp(i\sqrt{2}\alpha p)exp\left(\sqrt{2}\alpha\sum_{n>0}\frac{a_{-n}}{n}z^n\right) exp\left(-\sqrt{2}\alpha\sum_{n>0}\frac{a_n}{n}z^n\right) \ . \qquad (9.3.5)
\end{aligned}
$$

are also primary fields of conformal weight α^2. All primary fields of minimal models can be represented in this manner apart from possible factors relating to internal quantum numbers.

For $\alpha^2 = 1$ one obtains representation for the charged generators of ADE type Kac-Moody Lie-algebras in this manner.

The n-point function for these fields can be deduced by using Campbell-Hausdorf formula

$$: exp(i\alpha X(z)) :: exp(i\alpha X(\zeta)) := (z-\zeta)^{2\alpha\beta} : exp(i\alpha X(z) + i\beta X(\zeta)) : \ , \tag{9.3.6}$$

and is given by

$$\langle V_{\alpha_1}(z_1) V_{\alpha_2}(z_2) .... \phi_\alpha(z_n) \rangle = \prod_{i<j} (z_i - z_j)^{2\alpha_i\alpha_j} \tag{9.3.7}$$

for $\sum \alpha_i = 0$ and vanishes otherwise. Thus conformal invariance of zero energy states follows from mere internal consistency. Thus rational CFT:s and obviously also $(c = h = 0)$ case, would give the basic stringy expression for the amplitudes for creating zero energy states from vacuum.

Consider now whether and how four-momenta could appear in this formula.

(a) The number theoretic $M^4 = M^2 \times E^2$ decomposition and quantum classical correspondence are in accordance with the assignment of Kac-Moody generators with E^2 degrees of freedom. The physical interpretation would be in terms of deformations of partonic 2-surface restricted to $\delta M^4_\pm$ with one light-like coordinate so that only two degrees of freedom remain since light-like direction corresponds to Super Virasoro symmetries in the construction of WCW geometry. The generator of Kac-Moody algebra with zero norm would naturally correspond to the light-like direction along M^4_+ for super-symplectic algebra and along light-like partonic surface for Kac-Moody algebra.

One could wonder whether both of these zero norm generators could be included to the extended Dynkin diagram so that twisted affine Lie-algebra would result ($A_2^{(2)}$, $A_{2l}^{(2)}$ with $l \geq 2$, $A_{2l-1}^{(2)}$ with $l \geq 3$, $D_{l+1}^{(2)}$ with $l \geq 2$, and $E_6^{(2)}$ are possible [A49]).

(b) Suppose therefore that the formula generalizes to 4-D case simply by assigning to each component p^k of four-momentum its own quantized M^4 coordinate X^k such that oscillator operator contribution is absent in M^2 degrees of freedom, and requiring $p^k p_k = \alpha^k \alpha_k = \alpha^2$ in suitable units: α^2 is the conformal weight of the primary field. The identification of the mass squared value as conformal weight would follow automatically using this ansatz. The interpretation would differ from that adopted in string models since only the counterparts of tachyonic scattering amplitudes would be allowed as is indeed natural in zero energy ontology.

(c) If CP_2 mass is the unit of quantization the mass unit would be about 10^{-4} Planck masses. This mass scale should apply to the fundamental representations associated with the symmetries of the imbedding spaces. Physical intuition would suggest that p-adic mass squared defines the natural unit of quantization and that hadronic mass squared could be quantized in this manner. This quantization might occur for the secondary Kac-Moody representations defined by ADE series in the case of $q \neq 1$ Jones inclusions and extended ADE series in the case of $q = 1$ Jones inclusions suggested in [K21] to occur for large values of $\hbar$. The generation of multiplets of ADE quantum groups and ADE Kac Moody algebra could be made possible by the multiple coverings of M^4 defined by the space-time sheets for which points covering given point of M^4 are related by a discrete subgroup of $G_a \times G_b \subset SL(2,C) \times SU(2)$ (where one has $SU(2) \subset SU(3)$) defining the Jones inclusion. Thus one could say that TGD universal in the sense of being able to represent the quantum dynamics associated with any ADE type quantum group or Kac-Moody group.

p-Adicization favors rational values for central extension parameter and vacuum conformal weights

p-Adicization strongly suggests that the vacuum conformal weights and central extension parameter are rational numbers. Also algebraic numbers could in principle considered too: this would not give any conditions if square root allowing algebraic extension of p-adic numbers are used.

1. $N = 0$ case

For ordinary conformal algebra the null states are characterized by the conditions

$$\begin{aligned} \Delta_{mm'} &= \Delta_0 + \frac{1}{4}(\alpha_+ m + \alpha_- m')^2 \ , \quad m \ , \quad m' \geq 1 \ , \\ N &= mm' \ , \\ \Delta_0 &= \frac{1}{24}(c-1) \ , \\ \alpha_\pm &= \frac{\sqrt{1-c} \pm \sqrt{25-c}}{\sqrt{24}} \ . \end{aligned} \tag{9.3.8}$$

Thus arbitrarily high conformal weights N are possible in the construction. For $c \in (1, 25)$ the conformal weights are complex.

For ordinary conformal algebra rationality implies that the ground state conformal weight satisfies

$$\Delta_{mm'} = \frac{(mp' - m'p)^2 - (p'-p)^2}{4pp'} \ , \quad 0 < m < p \ , \quad 0 < m' < p' \ . \tag{9.3.9}$$

A more elegant expression for the central charge and weights reads as

$$\begin{aligned} c &= 1 - \frac{6}{Q(Q+1)} \ , \\ \Delta_{mm'} &= \frac{1}{4Q(Q+1)} \left[(Q(m - m' + m)^2 - 1 \right] \ , \\ Q &= \frac{p}{p'-p} \ . \end{aligned} \tag{9.3.10}$$

These conditions also imply also that the fusion rules close for a finite number of primary fields in the corresponding conformal field theory.

For $p' = p+1$ the minimal model is unitary. In this case one has $Q = p$ is integer $n \geq 3$. This range of integers characterizes also the allowed values of quantum phase characterizing Jones inclusions. Furthermore, Q is related to Kac-Moody central extension in $SU(2)_k$ theories by $Q = k + 2$.

The ground state conformal weight corresponds to $m = m' = 1$ and vanishes. The null norm state however possesses the conformal weight $mm' \geq 1$ and is therefore massive. The tachyon of string theories with conformal weight 1 is transformed in TGD framework to the absence of massless states in full accordance with the breaking of conformal invariance. $Q = p = n$ corresponds naturally to the integer labeling Jones inclusion defining both UV and IR cutoffs with respect to conformal weight. For $c = 0$ representation without breaking of conformal invariance all states are null norm states and the spectrum contains also massless particles. These representations correspond to $n = \infty$ case for Jones inclusions and to full Kac-Moody symmetry and ordinary string theory in accordance with the general picture.

Since minimal conformal field theories are in question, the number of primary fields is restricted by the conditions $0 < m < p$ and $0 < m' < p' = p+1$. By the symmetry $\Delta_{mm'} = \Delta_{p-m,p'-m'}$. If corresponding primary fields can be identified, one has $0 < m < m' < p'(= p+1)$ and $0 < m < p$.

2. Rationality for $N = 1, 2$ super-conformal algebras

The previous considerations apply on Virasoro algebra. These considerations generalize to the case of Kac-Moody algebra and also to corresponding Super algebras. In case of Super Virasoro algebra rationality requirement gives rise to different conditions on the values of c and Δ_{mn} depending in the value of N. $N = 1$ super-conformal algebra corresponds to one real super charge and one real super field and is non-physical in TGD framework. $N = 2$ case corresponds to single complex super charge and one complex super-field. In this case the Super Virasoro algebra involves also U(1) Kac-Moody algebra as inherent algebra. If these algebras are important in TGD framework, it would be natural to assign these algebras to quark and lepton type gamma matrices.

The values of the central extension parameter and conformal weights for $N = 0, 1, 2$ for unitary rational field theories at sphere are summarized by the following table [A49].

$$
\begin{array}{cccc}
c_k & 1-\frac{6}{(k+2)(k+3)} & \frac{3}{2}(1-\frac{8}{(k+2)(k+4)}) & 3(1-\frac{2}{k+2}) \\
 & & & \\
\Delta_{mm'} & \frac{\left[(k+2)m-(k+1)m'\right]^2-1}{4(k+2)(k+3)} & \frac{\left[(k+4)m-(k+2)m'\right]^2-4}{8(k+2)(k+4)} & \frac{m(m+2)-m'^2}{4(k+2)} \\
 & & & (q=\frac{mm'}{k+2} \\
m,m' & 1\leq m\leq k+1 & 1\leq m\leq k+2 & 0\leq m\leq k \\
 & 1\leq m'\leq k+2 & 1\leq m'\leq k+4 & -m\leq m'\leq m
\end{array}
\tag{9.3.11}
$$

It must be stressed that the conformal weights assignable to zero energy states are given by $\Delta_{m,m'} + mm'$ whereas in conformal field theories physical states have conformal weights $\Delta_{m,m'}$. For partonic 2-surfaces with handles modular invariance poses additional constraints since primary fields must form a closed set also under modular transformations [A49]. In the table above $q = m'/(k+2)$ corresponds to U(1) charge.

3. Rationality for $N = 4$ SCA

Large $N = 4$ super-conformal symmetry with $SU(2)_+ \times SU(2)_- \times U(1)$ inherent Kac-Moody symmetry defines the fundamental partonic super-conformal symmetry in TGD framework. In the case of SKM algebra the groups would act on induced spinors with $SU(2)_+$ representing spin rotations and $SU(2)_- \times U(1) = U(2)_{ew}$ electro-weak rotations. In super-symplectic sector the action would be geometric: $SU(2)_+$ would act as rotations on light-cone boundary and $U(2)$ as color rotations leaving invariant a preferred CP_2 point.

A concise discussion of this symmetry with explicit expressions of commutation and anti-commutation relations can be found in [A81] . The representations of SCA are characterized by three central extension parameters for Kac-Moody algebras but only two of them are independent and given by

$$
\begin{aligned}
k_\pm &\equiv k(SU(2)_\pm) \ , \\
k_1 &\equiv k(U(1)) = k_+ + k_- \ .
\end{aligned}
\tag{9.3.12}
$$

The central extension parameter c is given as

$$c = \frac{6k_+k_-}{k_+ + k_-} \ . \tag{9.3.13}$$

and is rational valued as required.

A much studied $N = 4$ SCA corresponds to the special case

$$\begin{aligned} k_- &= 1 \ , \ k_+ = k+1 \ , \ k_1 = k+2 \ , \\ c &= \frac{6(k+1)}{k+2} \ . \end{aligned} \tag{9.3.14}$$

$c = 0$ would correspond to $k_+ = 0, k_- = 1, k_1 = 1$. Central extension would be trivial in rotational degrees of freedom but non-trivial in $U(2)_{ew}$. For $k_+ > 0$ one has $k_1 = k_+ + k_- \neq k_+$. A possible interpretation is in terms of electro-weak symmetry breaking with $k_+ > 0$ signalling for the massivation of electro-weak gauge bosons.

A conjecture consistent with the general vision about the quantization of Planck constants is that k_+ and k_- relate directly to the integers n_a and n_b characterizing the values of $M^4_\pm$ and CP_2 Planck constants via the formulas $n_a = k_+ + 2$ and $n_b = k_- + 2$. This would require $k_\pm \geq 1$ for G_i a finite subgroup of $SU(2)$ ("anyonic" phases). In stringy phases with $G_i = SU(2)$ for $i = a$ or $i = b$ or for both, k_i could also vanish so that also $n_i = 2$ corresponding to A_2 ADE diagram and $SU(2)$ Kac-Moody algebra becomes possible. In the super-symplectic sector $k_+ = 0$ would mean massless gluons and $k_- = k_1$ that $U(2) \subset SU(3)$ and possibly entire $SU(3)$ represents an unbroken symmetry.

9.3.4 Objection against zero energy ontology and quantum classical correspondence

The motivation for requiring geometry and topology of space-time as correlates for quantum states is the belief that quantum measurement theory requires the representability of the outcome of quantum measurement in terms of classical physics -and if one believes in geometrization- one ends up with generalization of Einstein's vision.

There is however a counter argument against this view and second one against zero energy ontology in which one assigns eigenstates of four-momentum with causal diamonds (CDs).

(a) One can argue that momentum eigenstates for which particle regarded as a topological inhomogenuity of space-time surface, which is non-localized cannot allow a space-time correlate.

(b) Even worse, CDs have finite size so that strict four-momentum eigenstates strictly are not possible.

On the other hand, the paradoxical fact is that we are able to perceive momentum eigenstates and they look localized to us. This cannot be understood in the framework of standard Poincare symmetry.

The resolution of the objections and of the apparent paradox could rely on conformal symmetry assignable to light-like 3-surfaces implying a generalization of Poincare symmetry and other symmetries with their Kac-Moody variants for which symmetry transformations become local.

(a) Poincare group is replaced by its Kac-Moody variant so that all non-constant translations act as gauge symmetries. Translations which are constant in the interior of CD and trivial at the boundaries of CDs are physically equivalent with constant translations. Hence the latter objection can be circumvented.

(b) The same argument allows also a localization of momentum eigenstates at the boundaries of CD. In the interior the state is non-local. Classically the momentum eigenstate assigned with the partonic 2-surface is characterized by its 4-D tangent space data coding for momentum classically. The modified Dirac equation and Kähhler action indeed contain and additional term representing coupling to four-momenta of particles. Formally this corresponds only to a gauge transform linear in momentum but Kahler gauge potential has U(1) gauge symmetry only as a spin glass like degeneracy, not as a gauge symmetry so that space-time surface depends on momenta.

(c) Conscious observer corresponds in TGD inspired theory of consciousness to CD and the sensory data of the observer come from partonic 2-surfaces at the boundaries of CD and its sub-CDs. This implies classicality of sensory experience and momentum eigenstates look classical for conscious perceiver.

The usual argument resolving the paradox is based on the notion of wave packet and also this notion could be involved. The notion of finite measurement resolution is key notion of TGD and it is quite possible that one can require the localization of momentum eigenstates at the boundaries of CDs only modulo finite measurement resolution for the position of the partonic 2-surfaces.

9.3.5 Issues related to Lorentz symmetry and discrete symmetries

Lorentz invariance fixes the critical dimension of target space in super string models: 26 for bosonic string model and 10 for super-string model. This is strong argument for the claim that super string models have something to do with reality. Also in TGD framework one can ask whether Lorentz symmetric and even more- Poincare symmetric - theory is achieved.

Evidence for the breaking of Lorentz and color symmetries in TGD framework

There are several reasons consider seriously the possibility of spontaneous breaking of Poincare symmetries in TGD Universe.

(a) The choice of CD leads to a reduction of Poincare invariance to Lorentz invariance assignable to either tip of CD. Zero energy state in rest frame is a superposition of states with different masses and different energies but same vanishing 3-momenta. This induces a breaking of time translation invariance. For p-adic mass calculations the breaking of translational invariance is of order $1/p$ and thus extremely small: of order $1/M_{127} \simeq 2^{-127}$ for electron. The Lorentz boosted states are superpositions of components with same velocity but different four-momenta. This breaking of translational invariance is much weaker than that occurring in positive energy ontology if one allows superposition of states with different masses.

(b) The realization of the hierarchy of Planck constants in terms of a covering of imbedding space involves a selection of preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2_i \subset CP_2$ implying breaking of Lorentz invariance. A possible interpretation is that the fixing of quantization axes forces breaking of Poincare and color symmetries at the level of imbedding space.

(c) Number theoretical vision implies the hierarchy $M^2 \subset M^4 \subset M^8$ interpreted as inclusion hierarchy hypercomplex numbers-hyperquaternions-hyper-octonions. Number theoretical compactification is responsible for most of the progress in the understanding of quantum TGD. This hierarchy has also local variant at space-time level and this hierarchy is absolutely essential for $M^8 - H$ duality. The physical interpretation is in terms of the selection of local polarization plane and plane of four-momentum at space-level. The notion of number theoretic braid can be defined in several manners. One definition is in terms of M^4 and CP_2 projections of partonic 2-surfaces. A physically highly natural identification of braid is as boundaries of 2-D string world sheets carrying vanishing W fields and above weak scale also Z^0 fields at which the modes of induced spinor field are localized spinor modes are localized fromt the condition that em charge is well-defined for them.

(d) One could interpret $M^4 \rightarrow M^2 \times E^2$ symmetry breaking as a vanishing of the Kac-Moody central charge k in M^2 factor so that un-broken gauge invariance results. This conforms with the fact that factorizing S-matrices in M^2 correspond to finite-dimensional representations of loop group. Also the fact that only transversal degrees of freedom are quantum fluctuating degrees of freedom and contribute to WCW metric correlates with this.

(e) An interesting question is whether the breaking of Lorentz symmetry is already encountered in the hadronic scattering in quark model description, which involves the reduction of Lorentz group to $SO(1,1) \times SO(2)$ corresponding to longitudinal and transverse momenta. The selection of quantization axis in astrophysical length scales together with gigantic value of gravitational Planck constant is an especially fascinating possibility whose implications have been discussed in [K59] .

The breaking of fundamental symmetries would not take place at the level of the entire WCW if the union of copies of CDs corresponding to different selections of the quantization axes is allowed and WCW spinor fields are de-localized in the space labeling the choices of quantization axes before the decision to make the experiment. In quantum measurement a localization to fixed CD would occur unless one wants to believe to multiverse in the sense of conscious experience.

The fact that one can assign to each sector of generalized imbedding space a preferred quantization axis suggests that M-matrix identified as entanglement coefficients breaks Lorentz symmetry and color symmetry. This symmetry breaking would be interpreted as a space-time correlate for the selection of the Cartan sub-algebra of the isometry group in quantum measurement situation and would thus represent an inherent property of quantum theory, something much deeper than a trouble produced by a gauge choice as in string models. Since the interior degrees of freedom of the space-time sheets correspond to those assignable to the measurement apparatus, the breaking of Lorentz and color symmetries at space-time level would provide a space-time correlate for this symmetry breaking.

There are several instances where the spontaneous symmetry breaking makes itself manifest also at classical level.

(a) The possibility to assign almost topological quantum numbers to M^4 and CP_2 degrees of freedom (see the appendix of the book or [K35]) involves a selection of Cartan sub-algebra of the isometry group.

(b) A very general solution ansatz for the field equations based on Hamilton-Jacobi coordinates discussed in [K4] involves a local $M^2 \times E^2$ decomposition of M^4.

(c) The Abelian holonomy for the classical color fields could be interpreted in terms of the reduction of color symmetries to Cartan algebra.

Also momentum space discretization requires breaking of Lorentz invariance. Here however an interesting possibility arises. If only the phase factors defined by plane waves are observable, the explicit breaking of Lorentz and Poincare invariance is avoided. This argument generalizes also to spin and color quantum numbers since also these correspond to phase factors. Number theoretic universality implies number theoretical variant of Uncertainty Principle in the sense that if plane wave factor is algebraic number then both momentum and position cannot be simultaneously algebraic numbers as required if only algebraic extensions of rationals and p-adic numbers are allowed. Number theoretic universality allows only roots of unity as possible values of the plane wave phase, which takes the role of observable instead of position or momentum in short scales where the effects of unavoidable discretization are largest. The structure constants of symplectic fusion algebras are products of three phase factors, which are roots of unity and are assigned to the vertices of symplectic triangulation defining arguments of symplectic fields. The interpretation as plane wave factors is suggestive. Note that the 3-dimensionality of 3-space would correlate with the fact that structure constants of symplectic fusion algebra involve three algebra elements.

CPT, T and CPT breaking in zero energy ontology

CPT breaking [B3] requires the breaking of Lorentz invariance. Zero energy ontology could therefore allow a spontaneous breaking of CP and CPT. This might relate to matter antimatter asymmetry at the level of given CD.

There is some evidence that the mixing matrices for neutrinos and antineutrinos are different in the experimental situations considered [C2, C4]. This would require CPT breaking in the standard QFT framework. In TGD p-adic length scale hypothesis allowing neutrinos to reside in several p-adic mass scales. Hence one could have apparent CPT breaking if the measurement arrangements for neutrinos and antineutrinos select different p-adic length scales for them [K40] .

Could one understand the breaking of CP and T at fundamental level in TGD framework?

(a) In standard QFT framework Chern-Simons term breaks CP and T. Kähler action indeed reduces to Chern-Simons terms for the proposed ansatz for preferred extremals assuming that weak form of electric-magnetic duality holds true. This does not however need mean CP breaking. One must however add to the Kähler-Dirac action Chern-Simons Dirac term at the parton orbits in order to obtain non-trivial fermion propagator by requiring that spinor modes are generalized eigenstates of C-S-D operator with eigenvalues $p^k\gamma_k$ given by virtual momenta. One obtains thus perturbation theory and a connection with twistor Grassmannian approach. By supersymmetry one must add Chern-Simons term to Kähler action too so that it reduces to Chern-Simons terms at the space-like ends of space-time surface by weak form of electric magnetic duality. Chern-Simons Dirac terms could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.

In TGD framework one must however distinguish between space-time coordinates and imbedding space coordinates. CP breaking occurs at the imbedding space level but instanton term and Chern-Simons term are odd under P and T only at the space-time level and thus distinguish between different orientations of space-time surface. Only if one identifies P and T at space-time level with these transformations at imbedding space level, one has hope of interpreting CP and T breaking as spontaneous breaking of these symmetries for Kähler action and basically due to the weak form of electric-magnetic duality and vanishing of $j \cdot A$ term for the preferred extremals. This identification is possible for space-time regions allowing representation as graphs of maps $M^4 \rightarrow CP_2$.

(b) The GRT-QFT limit of TGD obtained by lumping together various space-time sheets to a region of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of sheets from Minkowski metric. Gauge potentials for the effective space-time would identified as sums of gauge potentials for space-time sheets. At this limit the identification of P and T at space-time level and imbedding space level would be natural. Could the resulting effective theory in Minkowski space or GRT space-time break CP and T slightly? If so, CKM matrices for quarks and fermions would emerge as a result of representing different topologies for wormhole throats with different topologies as single point like particle with additional genus quantum number.

(c) Could the breaking of CP and T relate to the generation of the arrow of time? The arrow of time relates to the fact that state function reduction can occur at either boundary of CD [K3]. Zero energy states do not change at the boundary at which reduction occurs repeatedly but the change at the other boundary and also the wave function for the position of the second boundary of CD changes in each quantum jump so that the average temporal distance between the tips of CD increases. This gives to the arrow of psychological time, and in TGD inspired theory of consciousness "self" as a counterpart of observed can be identified as sequence of quantum jumps for which the state function reduction occurs at a fixed boundary of CD. The sequence of reductions at fixed boundary breaks T-invariance and has interpretation as irreversibility. The standard view is that the irreversibility has nothing to do with breaking of T-invariance but it might be that in elementary particle scales irreversibility might manifest as small breaking of T-invariance.

Breaking of Lorentz invariance and $N = 4$ super-conformal symmetry

For $c = 0$ representations of $N = 4$ SCA critical dimension $D = 4 + 4$ should guarantee Lorentz invariance: this is indeed expected since the situation corresponds to Jones inclusion with trivial group G. One cannot however exclude the breaking of the full Lorentz and color symmetries for $c \neq 0$ representations of $N = 4$ SCA, which at the level of Jones inclusion means a change of the geometry and topology of the imbedding space and space-time.

The loss of Lorentz invariance would not be a catastrophe since M-matrix is a property of state rather than that of Universe in TGD framework. As already explained, the interpretation would be in terms of quantum measurement theory selecting a preferred Cartan subgroup for observables. This kind of breaking of course happens in the realistic experimental situation and if state describes also the measurement situation, the breaking is expected. For the scattering of zero energy states Lorentz invariance is obtained in a statistical sense.

This relates interestingly to the claimed uniqueness of super-string model if one requires unitarity and Lorentz invariance. Super string theorists might be right: only 10-D super strings might give rise to a unitary and Lorentz invariant S-matrix in perturbative sense although the perturbation series does not converge. They might be wrong in their belief that S-matrix is property of the Universe.

Whether Lorentz invariance is achieved for the stringy S-matrix characterizing entanglement between positive and negative energy states, depends on the assumptions one is ready to make about states and about what happens in state function reduction. The light cone quantization of string models involves $M^2 \times E^2$ decomposition interpreted now as a gauge choice and the scattering amplitudes are Lorentz invariant in the critical dimension. Due to the selection of preferred quantization axes the sectors of WCW are not Lorentz invariant. If zero energy states are identified as Lorentz invariant superposition of Lorentz transforms of a state in a given sector Lorentz invariance is achieved. Without this assumption it is not clear whether Lorentz invariance is achieved since zero energy ontology implies that the net Poincare quantum numbers assignable to the M-matrix elements vanish but does not imply Lorentz invariance. Similar conclusions apply in case of color quantum numbers.

A light hearted conjecture about relationship to super-strings and M-theory

$N = 4$ topological QFT can be considered as a possible candidate for the theory describing purely topological aspects of quantum TGD quantum criticality with respect to phase transitions changing Planck constant. This is just a guess to be shown wrong. The experience has taught that this kind of conjectures usually wrong: the real progress has come from understanding of TGD itself.

The (4,4) signature characterizing $N = 4$ SCA topological field theory need not be a problem since in TGD framework the target space becomes a fictive concept defined by the Cartan algebra. Both $M^4 \times CP_2$ decomposition of the imbedding space and space-time dimension are crucial for the $2+2+2+2$ structure of the Cartan algebra, which together with the notions of WCW and generalized coset representation formed from super Kac-Moody and super-symplectic algebras guarantees $N = 4$ super-conformal invariance.

Including the 2 gauge degrees of freedom associated with M^2 factor of $M^4 = M^2 \times E^2$ the critical dimension becomes $D = 10$ and and including the radial degree of light-cone boundary the critical dimension becomes $D = 11$ of M-theory. Hence the fictive target space associated with the vertex operator construction corresponds to a flat background of super-string theory and flat background of M-theory with one light-like direction. From TGD point view the difficulties of these approaches are due to the un-necessary assumption that the fictive target space defined by the Cartan algebra corresponds to the physical imbedding space. The flatness of the fictive target space forces to introduce the notion of spontaneous compactification and dynamical imbedding space and this in turn leads to the notion of landscape.

9.4 Are both symplectic and conformal field theories needed?

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light-cone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [K15] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

9.4.1 Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [K49] . In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctuation $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

(a) In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

(b) For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c^m_{kl}\Phi_m$). This intuition seems to be correct.

(c) Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s \ . \tag{9.4.1}$$

Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

(d) The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl}f(A(s_1,s_2,s))Idd\mu_s$ so that one has

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl}f(A(s_1,s_2,s))d\mu_s \ . \tag{9.4.2}$$

Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n=1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

9.4.2 Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

(a) The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1,s_2,s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1,s_2,s_3) \quad \equiv \quad A(s_1,s_2,s_3,N)-A(s_1,s_2,s_3,S) \tag{9.4.3}$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

(b) The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle (\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle \quad = \quad c_{kl}^r\int f(\Delta A(s_1,s_2,s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \tag{9.4.4}$$
$$c_{kl}^r c_{rm}\int f(\Delta A(s_1,s_2,s))f(\Delta A(s,s_3,t))d\mu_s d\mu_t \ . \tag{9.4.5}$$

Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

(c) 2-point correlation function would be given by

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1,s_2,s))d\mu_s \qquad (9.4.6)$$

(d) There is a clear difference between $n>3$ and $n=3$ cases: for $n>3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n=4$ theory is certainly well-defined, but one can argue that so are also $n>4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

(e) To sum up, the general predictions are following. Quite generally, for $f(0)=0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

9.4.3 Generalization to quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

(a) This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

(b) Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

(c) The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

(a) It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k\leq n$-simplices are known for n-simplex, the numbers of $k\leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1)=N(k-1,n)+N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons

(s_1, s_2, s_3), (s_1, s_2, s_3, X), (s_1, s_2, s_3, X, Y), and (s_1, s_2, s_3, N, S, T) also the 4-volumes of 5-polygons (s_1, s_2, s_3, X, Y), and of 6-polygon (s_1, s_2, s_3, N, S, T), $X, Y \in \{N, S, T\}$ can appear as additional arguments in the definition of 3-point function.

(b) What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of WCW. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

(c) The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

(d) This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

(a) *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum

TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [K67] .

(b) Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

(c) It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time.

(d) One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and WCW degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

(e) Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of WCW Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

(f) Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like

3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules can be treated separately. This separation indeed happens since conformal degrees of freedom correspond to quantum fluctuations contributing to the WCW metric and affecting the induced metric whereas symplectic invariants correspond to non-quantum fluctuating zero modes defining the part of quantum state not affected by quantum fluctuations parameterized by the symplectic group of $\delta M^4_\pm \times CP_2$. Also the dream about symplectic fusion rules have been realized. An explicit construction of symplectic fusion algebras is represented in [K8] .

9.5 Weak form of electric-magnetic duality and fermionic propagator

The ideas about what generalized Feynman diagrams could be have developed rather slowly and basically through trial and mostly error. Bosonic emergence implies that fermionic propagator is the fundamental object and its identification has become one of the basic challenges of TGD. For long time the belief was that a straightforward generalization of stringy propagators could make sense but it turned out that TGD requires something more original. The weak form of electric-magnetic duality meant a decisive step of progress also in the understanding of fermionic propagator. In the following the implications of weak form of electric-magnetic duality for TGD are explained by starting from classical theory and ending up with fermionic propagator.

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the modified Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

(a) Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j^\alpha_K A_\alpha$ plus and integral of the boundary term $J^{n\beta} A_\beta \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_\beta \sqrt{g_4}$ over the ends of the 3-surface.

(b) If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that

Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

(a) For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K4]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

(b) The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \ . \tag{9.5.1}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

(c) This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

(d) The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{9.5.2}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} = 0 . \qquad (9.5.3)$$

j_K is a four-dimensional counterpart of Beltrami field [B44] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K4] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

(e) There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \to A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha (j^\alpha \phi) = 0 . \qquad (9.5.4)$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q^e_\phi = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q^m_\phi = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

(f) The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the modified Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the modified Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

(g) A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

(h) Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to modified Dirac action as boundary term.

Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

9.5.1 A general solution ansatz based on almost topological QFT property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the modified Dirac equation.

Basic field equations

Consider first the equations at general level.

(a) The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T\times SO(3)\times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

(b) Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha\left[D_\beta(J^{\alpha\beta}H_A)-j^\alpha_K H^A+T^{\alpha\beta}j^l_A h_{kl}\partial_\beta h^l\right] \quad = \quad 0 \ . \tag{9.5.5}$$

The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha\left[j^\alpha_K H^A-T^{\alpha\beta}j^k_A h_{kl}\partial_\beta h^l\right] \quad = \quad 0 \ . \tag{9.5.6}$$

For energy one has $H_A=1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

(c) One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j_K^\alpha J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j_K^\alpha D_\alpha H^A = j_K^\alpha J_\alpha{}^\beta j_\beta^A + T^{\alpha\beta} H_{\alpha\beta}^k j_k^A \ . \tag{9.5.7}$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

(a) The most general assumption is that the conserved isometry currents

$$J_A^\alpha = j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \tag{9.5.8}$$

and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

(b) The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \tag{9.5.9}$$

The conservation of J_A gives

$$d*(\Psi_A d\Phi_A) = 0 \ . \tag{9.5.10}$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

(c) The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d*d\Phi_A = 0 \ , \ d\Psi_A \cdot d\Phi_A = 0 \ . \tag{9.5.11}$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j \Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p \cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

(a) The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \tag{9.5.12}$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d * d\Phi = 0 \ , \ d\Psi_A \cdot d\Phi = 0. \tag{9.5.13}$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

(b) Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A \Psi_A d\Phi_{G(A)} \ , \ d * (d\Phi_G(A)) = 0 \ , \ d\Psi_A \cdot d\Phi_G(A)) = 0 \ . \tag{9.5.14}$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [K31] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

9.5.2 Hydrodynamic picture in fermionic sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the modified Dirac equation. This would mean that the solutions of Dirac equation can be localized to lower-dimensional surface or even flow lines.

Basic objection

The obvious objection against the localization to sub-manifolds is that it is not consistent with uncertainty principle in transversal degrees of freedom. More concretely, the assumption that the mode is localized to a lower-dimensional surface of X^4 implies that the action of the transversal part of Dirac operator in question acts on delta function and gives something singular.

The situation changes if the Dirac operator in question has vanishing transversal part at the lower-dimensional surface. This is not possible for the Dirac operator defined by the induced metric but is quite possible in the case of Kähler-Dirac operator. For instance, in the case of massless extremals Kähler-Dirac gamma matrices are non-vanishing in single direction only and the solution modes could be one-dimensional. For more general preferred extremals such as cosmic strings this is not the case.

In fact, there is a strong physical argument in favor of the localization of spinor modes at 2-D string world sheets so that hydrodynamical picture would result but with flow lines replaced with fermionic string world sheets.

(a) Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

(b) The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action and requires that the part of the Kähler-Dirac operator transversal to 2-surface vanishes.

(c) This localization does not hold for cosmic string solutions which however have 2-D CP_2 projection which should have vanishing weak fields so that 4-D spinor modes with well-defined em charge are possible.

(d) A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

4-dimensional modified Dirac equation and hydrodynamical picture

In following consideration is restricted to preferred extremals for which one has decomposition to regions characterized by local light-like vector and polarization direction. In this case one has good hopes that the modes can be restricted to 1-D light-like geodesics.

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

(a) The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)} \Gamma_k \ . \end{aligned} \tag{9.5.15}$$

The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^\alpha_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \tag{9.5.16}$$

The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D modified Dirac equation would reduce everything to initial values at partonic 2-surfaces.

(b) One might hope that the conservation of these super currents for all modes is equivalent with the modified Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

(c) The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

(a) The modified gamma matrices are of form $T^{\alpha}_{k}\Gamma^k$, $T^{\alpha}_{k} = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^{α}_{k} can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h^k_l = j^{Ak}j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k}j^A_k j^k_A = T^{\alpha A}j^k_A$. One can defined gamma matrices Γ_A as $\Gamma_k j^k_A$ to get $T^{\alpha}_{k}\Gamma^k = T^{\alpha A}\Gamma_A$.

(b) This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the modified Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA}j_A D_t\Psi \quad = \quad 0 \ . \qquad (9.5.17)$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

9.5.3 Hyper-octonionic primes

Before detailed discussion of the hyper-octonionic option it is good to consider the basic properties of hyper-octonionic primes.

(a) Hyper-octonionic primes are of form

$$\Pi_p \quad = \quad (n_0, n_3, n_1, n_2, ..., n_7) \ , \ \Pi_p^2 = n_0^2 - \sum_i n_i^2 = p \ \text{ or } \ p^2 \ . \qquad (9.5.18)$$

(b) Hyper-octonionic primes have a standard representation as hyper-complex primes. The Minkowski norm squared factorizes into a product as

$$n_0^2 - n_3^2 = (n_0 + n_3)(n_0 - n_3) \ . \qquad (9.5.19)$$

If one has $n_3 \neq 0$, the prime property implies $n_0 - n_3 = 1$ so that one obtains $n_0 = n_3 + 1$ and $2n_3 + 1 = p$ giving

$$(n_0, n_3) \quad = \quad ((p+1)/2, (p-1)/2) \ . \qquad (9.5.20)$$

Note that one has $(p+1)/2$ odd for $p \ mod \ 4 = 1$) and $(p+1)/2$ even for $p \ mod \ 4 = 3$). The difference $n_0 - n_3 = 1$ characterizes prime property.

If n_3 vanishes the prime prime property implies equivalence with ordinary prime and one has $n_3^2 = p^2$. These hyper-octonionic primes represent particles at rest.

(c) The action of a discrete subgroup $G(p)$ of the octonionic automorphism group G_2 generates form hyper-complex primes with $n_3 \neq 0$ further hyper-octonionic primes $\Pi(p, k)$ corresponding to the same value of n_0 and p and for these the integer valued projection to M^2 satisfies $n_0^2 - n_3^2 = n > p$. It is also possible to have a state representing the system at rest with $(n_0, n_3) = ((p+1)/2, 0)$ so that the pseudo-mass varies in the range $[\sqrt{p}, (p+1)/2]$. The subgroup $G(n_0, n_3) \subset SU(3)$ leaving invariant the projection (n_0, n_3) generates the hyper-octonionic primes corresponding to the same value of mass for hyper-octonionic primes with same Minkowskian length p and pseudo-mass $\lambda = n \geq \sqrt{p}$.

(d) One obtains two kinds of primes corresponding to the lengths of pseudo-momenta equal to p or $\sqrt{p}$. The first kind of particles are always at rest whereas the second kind of particles can be brought at rest only if one interprets the pseudo-momentum as M^2 projection. This brings in mind the secondary p-adic length scales assigned to causal diamonds (CDs) and the primary p-adic lengths scales assigned to particles.

If the M^2 projections of hyper-octonionic primes with length $\sqrt{p}$ characterize the allowed basic momenta, ζ_D is sum of zeta functions associated with various projections which must be in the limits dictated by the geometry of the orbit of the partonic surface giving upper and lower bounds L_{max} and L_{min} on the length L. L_{min} is scaled up to $\sqrt{n_0^2-n_3^2L_{min}}$ for a given projection (n_0,n_3). In general a given M^2 projection (n_0,n_3) corresponds to several hyper-octonionic primes since $SU(3)$ rotations give a new hyper-octonionic prime with the same M^2 projection. This leads to an inconsistency unless one has a good explanation for why some basic momentum can appear several times. One might argue that the spinor mode is degenerate due to the possibility to perform discrete color rotations of the state. For hyper complex representatives there is no such problem and it seems favored. In any case, one can look how the degeneracy factors for given projection can be calculated.

(a) To calculate the degeneracy factor $D(n$ associated with given pseudo-mass value $\lambda = n$ one must find all hyper-octonionic primes Π, which can have projection in M^2 with length n and sum up the degeneracy factors $D(n,p)$ associated with them:

$$\begin{aligned} D(n) &= \sum_p D(n,p) \ , \\ D(n,p) &= \sum_{n_0^2-n_3^2=p} D(p,n_0,n_3) \ , \\ n_0^2-n_3^2 &= n \ , \ \Pi_p^2(n_0,n_3) = n_0^2-n_3^2-\sum_i n_i^2 = n-\sum_i n_i^2 = p \ . \end{aligned} \tag{9.5.21}$$

(b) The condition $n_0^2-n_3^2=n$ allows only Pythagorean triangles and one must find the discrete subgroup $G(n_0,n_3)\subset SU(3)$ producing hyper-octonions with integer valued components with length p and components (n_0,n_3). The points at the orbit satisfy the condition

$$\sum n_i^2 = p-n \ . \tag{9.5.22}$$

The degeneracy factor $D(p,n_0,n_3)$ associated with given mass value n is the number of elements of in the coset space $G(n_0,n_3,p)/H(n_0,n_3,p)$, where $H(n_0,n_3,p)$ is the isotropy group of given hyper-octonionic prime obtained in this manner. For $n_0^2-n_3^2=p^2$ $D(n_0,n_3,p)$ obviously equals to unity.

9.5.4 Three basic options for the pseudo-momentum spectrum

The calculation of the scaling factor of the Kähler function requires the knowledge of the degeneracies of the mass squared eigen values. There are three options to consider.

First option: all pseudo-momenta are allowed

If the degeneracy for pseudo-momenta in M^2 is same for all mass values- and formally characterizable by a number N telling how many 2-D pseudo-momenta reside on mass shell $n_0^2-n_3^2=m^2$. In this case zeta function would be proportional to a sum of Riemann Zetas with scaled arguments corresponding to scalings of the basic mass m to m/q.

$$\zeta_D(s) = N\sum_q \zeta(log(qx)s) \ , \ x = \frac{L_{min}}{R} \ . \tag{9.5.23}$$

This option provides no idea about the possible values of $1 \leq q \leq L_{max}/L_{min}$. The number N is given by the integral of relativistic density of states $\int dk/2\sqrt{k^2+m^2}$ over the hyperbola and is logarithmically divergent so that the normalization factor N of the Kähler function would be infinite.

Second option: All integer valued pseudo momenta are allowed

Second option is inspired by number theoretic vision and assumes integer valued components for the momenta using $m_{max} = 2\pi/L_{min}$ as mass unit. p-Adicization motivates also the assumption that momentum components using m_{max} as mass scale are integers. This would restrict the choice of the number theoretical braids.

Integer valuedness together with masses coming as integer multiples of m_{max} implies $(\lambda_0, \lambda_3) = (n_0, n_3)$ with on mass shell condition $n_0^2 - n_3^2 = n^2$. Note that the condition is invariant under scaling. These integers correspond to Pythagorean triangles plus the degenerate situation with $n_3 = 0$. There exists a finite number of pairs (n_0, n_3) satisfying this condition as one finds by expressing n_0 as $n_0 = n_3 + k$ giving $2n_3k + k^2 = p^2$ giving $n_3 < n^2/2$,$n_0 < n^2/2+1$. This would be enough to have a finite degeneracy $D(n) \geq 1$ for a given value of mass squared and ζ_D would be well defined. ζ_D would be a modification of Riemann zeta given by

$$\begin{aligned}
\zeta_D &= \sum_q \zeta_1(log(qx)s) \ , \ x = \frac{L_{min}}{R} \ , \\
\zeta_1(s) &= \sum g_n n^{-s} \ , \ g_n \geq 1 \ .
\end{aligned} \tag{9.5.24}$$

For generalized Feynman diagrams this option allows conservation of pseudo-momentum and for loops no divergences are possible since the integral over two-dimensional virtual momenta is replaced with a sum over discrete mass shells containing only a finite number of points. This option looks thus attractive but requires a regularization. On the other hand, the appearance of a zeta function having a strong resemblance with Riemann zeta could explain the finding that Riemann zeta is closely related to the description of critical systems. This point will be discussed later.

Third option: Infinite primes code for the allowed mass scales

According to the proposal of [K65] , [L8] the hyper-complex parts of hyper-octonionic primes appearing in their infinite counterparts correspond to the M^2 projections of real four-momenta. This hypothesis suggests a very detailed map between infinite primes and standard model quantum numbers and predicts a universal mass spectrum [K65] . Since pseudo-momenta are automatically restricted to the plane M^2, one cannot avoid the question whether they could actually correspond to the hyper-octonionic primes defining the infinite prime. These interpretations need not of course exclude each other. This option allows several variants and at this stage it is not possible to exclude any of these options.

(a) One must choose between two alternatives for which pseudo-momentum corresponds to hyper-complex prime serving as a canonical representative of a hyper-octonionic prime or a projection of hyper-octonionic prime to M^2.

(b) One must decide whether one allows a) only the momenta corresponding to hyper-complex primes, b) also their powers (p-adic fractality), or c) all their integer multiples ("Riemann option").

One must also decide what hyper-octonionic primes are allowed.

(a) The first guess is that all hyper-complex/hyper-octonionic primes defining length scale $\sqrt{p}L_{min} \leq L_{max}$ or $pL_{min} \leq L_{max}$ are allowed. p-Adic fractality suggests that also the higher p-adic length scales $p^{n/2}L_{min} < L_{max}$ and $p^nL_{min} < L_{max}$, $n \geq 1$, are possible. It can however happen that no primes are allowed by this criterion. This would mean vanishing Kähler function which is of course also possible since Kähler action can vanish (for instance, for massless extremals). It seems therefore safer to allow also the scale corresponding to the trivial prime $(n_0, n_3) = (1, 0)$ (1 is formally prime because it is not divisible by any prime different from 1) so that at least L_{min} is possible. This option also allows only rather small primes unless the partonic 2-surface contains vacuum regions in which case L_{max} is infinite: in this case all primes would be allowed and the exponent of Kähler function would vanish.

(b) The hypothesis that only the hyper-complex or hyper-octonionic primes appearing in the infinite hyper-octonionic prime are possible looks more reasonable since large values of p would be possible and could be identified in terms of the p-adic length scale hypothesis. All hyper-octonionic primes appearing in infinite prime would be possible and the geometry of the orbit of the partonic 2-surface would define an infinite prime. This would also give a concrete physical interpretation for the earlier hypothesis that hyper-octonionic primes appearing in the infinite prime characterize partonic 2-surfaces geometrically. One can also identify the fermionic and purely bosonic primes appearing in the infinite prime as braid strands carrying fermion number and purely bosonic quantum numbers. This option will be assumed in the following.

9.6 How to define generalized Feynman diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in zero energy ontology (ZEO) [K57] . This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [K78] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

(a) The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the "world of classical worlds" (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

(b) Zero energy ontology (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.

(c) A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also imbedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

(d) Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K78] . Modified gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D imbedding space parameterized by quaternionic space-time surfaces.

(e) A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K10]. It took long time to realize that in zero energy ontology the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

(f) A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

(g) As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K22, K78] .

(a) The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B29] automatically satisfied as in the case of ordinary Feynman diagrams.

(b) Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

(a) One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of modified Dirac operator.

(b) One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K31] in infinite-dimensional context already in the case of much simpler loop spaces [A63] .

(a) The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II¡sub¿1¡/sub¿ defining the finite measurement resolution.

(b) WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

9.6.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to to this goal is by making questions.

What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

(a) One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.

(b) Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the imbedding space. This would be true at least in the intersection of real and p-adic worlds.

(c) The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the modified Dirac action [K10] suggests however a de-localization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\overline{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

(d) Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.

In order to obtain non-trivial fermion propagator one must add to Kähler-Dirac action Chern-Simons Dirac term located at partonic orbits at which the signature of the induced metric changes. The modes of induced spinor field can be required to be generalized eigenmodes of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ with p^k identified as virtual momentum so that massless Dirac propagator is obtained. p^k is discretized by periodic boundary conditions at opposite boundaries of CD and has IR and UV cutoffs due to the finite size of CD and finite lower limit for the size of sub-CDs.

One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.

By super-symmetry one must add to Kähler action Chern-Simons term located at partonic orbits and this term must cancel the Chern-Simons term coming from Kähler action by weak form of electric-magnetic duality so that Kähler action reduces to the terms associated with space-like ends of the space-time surface. These terms reduce to Chern-Simons terms if one poses weak form of electric magnetic duality also here. The boundary condition for Kähler-Dirac equations states $\Gamma^n\Psi = 0$ so that incoming fundamental fermions are massless and there is a strong temptation to pose the additional condition $\Gamma^n\Psi = p^k\gamma_k\Psi = 0$

The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

(a) What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

(b) Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

(a) WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the modified Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.

(b) For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space , there should be no problems.

(c) If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a $sub-CD$ in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adizication would thus give a further good reason why for zero energy ontology.

(d) Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the "radial" coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$, These functions are expected to be rational functions or at least algebraic functions involving only square roots.

(e) In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that incoming stringy lines at the ends of CD correspond to fermions satisfying the stringy mass formula serving as a generalization of masslessness condition.

9.6.2 Generalized Feynman diagrams at fermionic and momentum space level

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynman diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. Zero energy ontology encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

(a) A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires ++ and −− type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to ++ or −− type lines.

(b) The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

(c) For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

(d) The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_\pm$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermion and $X_\pm$ might allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

Loop integrals are manifestly finite

One can make also more detailed observations about loops.

(a) The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_\pm$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.

(b) In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the modified Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^\alpha p_\alpha + \hat{\Gamma}^\alpha D_\alpha \ , \\ p_\alpha &= p_k \partial_\alpha h^k \ . \end{aligned} \tag{9.6.1}$$

The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is modified gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D modified Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

(c) Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.

(d) Irrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [K23] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N > 2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N-4$. For instance, a 4-vertex from which $N = 2$ states emanate is finite.

Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [B8] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_\pm$ pairs ($X_\pm$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

(a) The simplest assumption in the stringy case is that fermion-$X_\pm$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_\pm$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and modified Dirac operator.

(b) Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [K23] .

(c) If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \rightarrow F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-anti-fermion pair).

(d) The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [K18] .

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

9.6.3 Harmonic analysis in WCW as a manner to calculate WCW functional integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding "radial" coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the "radial" coordinates only and the possible generalization involves the identification the counterparts of the "radial" coordinates in the case of WCW.

Conditions guaranteeing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

(a) Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

(b) Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2 - m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

(c) Momentum conservation correlates the eigenvalue spectra of the modified Dirac operator D at propagator lines [K10] . G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

(d) The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1,2 \ . \end{aligned} \tag{9.6.2}$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \tag{9.6.3}$$

such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

(e) If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the modified Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \tag{9.6.4}$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

(a) The proposed representation of WCW Hamiltonians as flux Hamiltonians [K12, K10]

$$\begin{aligned} Q(H_A) &= \int H_A(1+K)Jd^2x \ , \\ J &= \epsilon^{\alpha\beta}J_{\alpha\beta} \ , \ J^{03}\sqrt{g_4} = KJ_{12} \ . \end{aligned} \tag{9.6.5}$$

works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (^{03} refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^/ 4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$

is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

(b) One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

(c) The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

Recall that also the construction of number theoretic braids and symplectic QFT [K14] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2 s_\pm = \int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1, s^2)}{\partial(x^1_\pm, x^2_\pm)} d^2 x_\pm \quad (9.6.6)$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J_+^{kl} J_{kl}^- \ , \\ J_\pm^{kl} &= (1 + K_\pm) \partial_\alpha s^k \partial_\beta s^l J_\pm^{\alpha\beta} \ . \end{aligned} \quad (9.6.7)$$

The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

(d) One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

(e) The quantization of the modified Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1 + K)J$ with $X\partial(s^1, s^2)/\partial(x^1_\pm, x^2_\pm)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $(1 + K)J\delta^2(x, y)$ would be replaced with $X\delta^2(s^+, s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

(f) In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

(a) In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.

(b) Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

(a) This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

(b) Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

Could one do without flux Hamiltonians?

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

(a) The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2-matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.

(b) One could of course argue that the expansions of $exp(K)$ and λ_k give in the general powers $(f_n\overline{f_n})^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.

(c) In zero energy ontology this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the modified Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

Chapter 10

Construction of Quantum Theory: More about Matrices

10.1 Introduction

This chapter is a second part of chapter representing material related to the construction of U-, M, and S-matrices. The general philosophy is discussed in the first part of the chapter and I will not repeate the discussion.

The views about M-matrix as a characterizer of time-like entanglement and M-matrix as a functor are analyzed. The role of hyper-finite factors in the construction of M-matrix is considered. One section is devoted to the possibility that Connes tensor product could define fundamental vertices. The last section is devoted to the construction of unitary U-matrix characterizing the unitary process forming part of quantum jump.

The understanding of the fundamental variational principles of TGD is so detailed that one can sketch a rather concrete formulation for the generalized Feynman rules. The generalized Feynman diagrams correspond to the 4-D surfaces defined by the Euclidian regions defined by wormhole contacts plus the string world sheets connecting them and carrying spinor modes. Fundamental fermions propagate as massless particles along the boundaries of string world sheets at which spinor modes are localized and basic interaction vertices can be identified as wormhole contacts with Virasoro generator L_0 serving as propagator mediating the interaction between fermions at opposite wormhole throats.

The last section is about the anatomy of quantum jump. The first part of the chapter began with a similar piece of text. This reflects the fact that the ideas are developing all the time so that the vision about the matrices is by no means top-down view beginning from precisely state assumption and proceeding to conclusions. The emphasis is in the relation of the new view about quantum jump and state function reduction to everyday conscious experience.

The reader wishing for a brief summary of TGD might find the three articles about TGD, TGD inspired theory of consciousness, and TGD based view about quantum biology helpful [L5, L4, L3] .

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- Quantum theory [L36]
- WCW spinor fields [L50]
- Emergent ideas and notions [L17]

- Zero Energy Ontology (ZEO) [L52]
- Hyperfinite factors and TGD [L24]
- $M^8 - H$ duality [L28]
- Weak form of electric-magnetic duality [L51]

10.2 A vision about the role of HFFs in TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFS of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by zero energy ontology and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

10.2.1 Basic facts about factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi, \xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A||||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A34] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

(a) Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

(b) A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

(c) Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

(d) For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

(a) Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.

(b) For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0, 1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.

(c) Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.

(d) Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X, μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

(a) A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.

(b) A positive linear functional is weight with $\omega(1)$ finite.

(c) A state is a weight with $\omega(1) = 1$.

(d) A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.

(e) A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0, 1, 2, ...$ For factors of type II_1 the values span the range $[0, 1]$ and for factors of type II_∞ n the range $[0, \infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

(a) Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.

(b) Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \quad , \quad L^2(\mathcal{M}) = \mathcal{H} \quad , \qquad L^1(\mathcal{M}) = \mathcal{M}_* \quad , \tag{10.2.1}$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*{}^* = \mathcal{M}$.

(c) The conjugation $x \rightarrow x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \rightarrow L^2(\mathcal{M})$ via $x \rightarrow x\Omega$. The map $S_0; x\Omega \rightarrow x^*\Omega$ is however non-isometric.

(d) Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.

(e) What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in zero energy ontology.

The basic results of Tomita-Takesaki theory are following.

(a) The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \quad , J\mathcal{M}J = \mathcal{M}' \quad .$$

(b) The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A56, A101] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

(c) $\omega \to \sigma_t^\omega = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

(a) One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

(b) The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

(c) Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a manner to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1 h_1(g_2), h_1 h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1 g_1(a_2), g_1 g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K79] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A2] and those of factors of type III by Alain Connes [A52] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [A2] .

(a) For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a)\ \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ \ , \quad h \geq 3 \ \ , \\ &b)\ \ \mathcal{M} : \mathcal{N} \geq 4 \ \ . \end{aligned} \tag{10.2.2}$$

the numbers at right hand side are known as Beraha numbers [A85] . The comments below give a rough idea about what finiteness of principal graph means.

(b) As explained in [B38] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A72] is following. The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

Connes tensor product

The inclusions The basic idea of Connes tensor product is that a sub-space generated subfactor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N} . \tag{10.2.3}$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M} \otimes \mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$ matrices.

(a) In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

(b) An alternative and more physical representation is as a state

$$\sum_n V_{mn} W_{nr} e^{mn} \otimes e^{nr}$$

in the tensor product $V \otimes W$.

(c) One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

(d) Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

10.2.2 Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A75, A56, A101] . There are good arguments showing that in HFFS of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A101] deserve to be listed since they are suggestive also from the point of view of TGD.

(a) Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O}+x)$ where $(\mathcal{O}+x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E \in \mathcal{M}(\mathcal{O})$ can be written as WW^* with $W \in \mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W = 1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.

(b) Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.

(c) The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ , \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ .$$

An infinite hierarchy of inclusions of HFFS of type III_1s is induced by set theoretic inclusions.

10.2.3 TGD and factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

(a) Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?

(b) Is the identification $M = \Delta^{it}$ sensible is quantum TGD and zero energy ontology, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.

(c) TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?

(d) What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at imbedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

(a) What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?

(b) What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?

(c) Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the modified Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the modified Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

(a) Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?

(b) Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

Zero energy ontology and factors

The first question concerns the identification of the Hilbert space associated with the factors in zero energy ontology. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

(a) The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates.

Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.

(b) The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in zero energy ontology.

(a) In zero energy ontology M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFS is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.

(b) Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \to 0$ limit. In quantum TGD the exponent of modified Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Modified Dirac action can therefore be regarded as a "square root" of Kähler action.

(c) The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in zero energy ontology based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

(d) In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the modified Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFS involves further conceptual problems.

(a) The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of modified Dirac action [K10] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.

(b) Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.

(c) Quantum criticality means that modified Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two super-conformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.

(d) The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.

(e) Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

(a) The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.

(b) The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.

(c) The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L^*_0$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

(a) The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.

(b) The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem to be however unitary because the transformation does not only modify the states but also transforms CD.

(c) Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

(d) One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

(a) In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K21] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

(b) Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

(c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

(d) The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

10.2.4 Can one identify M-matrix from physical arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

The basic action principle

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

(a) The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

    There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

(b) By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

1. Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

(a) QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

(b) The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

(c) The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface if $j \cdot A = 0$ condition holds true as it does for preferred extremals. Note that weak form of electric magnetic duality is not absolutely necessary at space-like ends of the space-time surface but is favored by almost topological QFT property.

2. Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

a) What one wants?

It is could to make first clear what one really wants.

(a) What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant. It is important to notice that Γ^n brings in the dependence on metric and breaks exact topological QFT property as do also the constraint terms realizing weak form of electric magnetic duality.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignment of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

(b) If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

(c) One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

b) Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \qquad (10.2.4)$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

3. Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Localization of the modes of Kähler-Dirac operator at string world sheets and definition of Dirac determinant

The condition that the modes of Kähler-Dirac operator have well defined electromagnetic charge eigenvalue implies that the modes are restricted to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces [K87]. In the generic case one would have a product of Dirac determinants associated with these 2-surfaces. This obviously simplifies dramatically the definition of Dirac determinant and suggests a reduction to stringy mathematics, where this kind of determinants appear routinely.

The construction of Dirac determinant could proceed in following manner.

(a) The spectrum of the Kähler Dirac (KD) operator was originally identified in terms of generalized eigenvalues. The identification coming first in mind would be in terms of conformal weights assignable to the modes of KD operator. The experience with the string models suggests that these conformal weights are integer valued, which would mean that the multiplicative contribution from given string world sheet is constant and cannot depend on 3-surface at all!

(b) The boundary conditions at the string curves at the space-like ends of space-time surface however give algebraic form of Dirac equation with the analog of Higgs coupling in algebraic form $(p^k\gamma_k + \Gamma^n)\Psi = 0$, with p^k identifiable as four-momentum of fermionic line emanating from partonic 2-surface. The normal component Γ^n (in time direction) of the vector defined by K-D gamma matrices defines the analog of Higgs vacuum expectation value, and could be covariantly constant along string curve for a suitable choice of string coordinates. $h^2 \equiv (\Gamma^n)^2$ could be interpreted as ground state conformal weight. In p-adic mass calculations ground state conformal weight must be negative half-odd integer and the time-like character of Γ^n could explain this. h^2 could have p-adically small deviation from half-odd integer value and give rise to a Higgs like additional contribution to the conformal weights.

(c) The square of the Dirac determinant would be product of eigenvalues mass squared operator assignable to the eigenvalue equation $(p^k\gamma_k+\Gamma^n)^2\Psi = \Lambda_n)\Psi$. If the eigenvalues correspond up to multiplicative factor to integer valued conformal weights, the square of Dirac determinant would be the product of corresponding mass squared values equal to conformal weight with vacuum contribution. The square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + n$, where h is expressed using unit of mass determined by CP_2 radius.

(d) One can of course ask whether it might be possible to define even the Dirac determinant itself. Here it seems that the only possible manner to proceed is number theoretic: the factors $p^k\gamma_k + \Gamma^n$ appearing in the formal Dirac determinant should be mapped to complexified octonions and the product of these factors should define Dirac determinant as complex quantity having interpretation as the product of exponents of Kähler for Euclidian and Minkowskian regions metting at wormhole throat. This would be a rather deep connection with the number theoretic approach.

(e) Since spinor modes effectively propagate as particles with momentum p^k along braid strands one could argue that one must include h^2 to the integer valued conformal weight so that the square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + nM_0^2$, M_0 the mass scale determined by CP_2 radius.

   The resulting determinant - if indeed well-defined - would depend on space-time surface and would be obtained as a perturbation from the determinant assignable to Riemann Zeta. Modulus squared for the exponent of vacuum functional would be analogous to the square of Dirac determinant associated with a massless fermion with eigenvalues of m^2 replaced with $h(n)$. The overall determinant would be product over the determinants coming from various strings and possibly also from he partonic 2-surfaces.

One must however be aware about possible objections against the hypothesis that the square of Dirac determinant gives the modulus squared for the vacuum functional.

(a) It would be exaggeration to say that Kähler function emerges from K-D action. The reason is that K-D gamma matrices appear in K-D action and internal consistency requires that an extremal of K-D action is in question. Hence it seems that Khler action and K-D action are in completely democratic position and one can wonder whether the possible connection actually gives any profound insights or means anything practical. It could only create technical challenges and one can claim that the definition of exponent of vacuum functional reducing to exponent of Chern-Simons terms looks much more practical and elegant.

(b) Kähler function corresponds to Kähler action in Euclidian space-time regions assignable to the lines of generalized Feynman diagrams. It is not clear whether one represent also the Kähler action from Minkowskian regions in this manner.

A proposal for M-matrix

This picture can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

(a) At the bosonic sector one would have converging functional integral over WCW. This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

(b) In fermionic sector Chern-Simons Dirac term in the action and the condition that spinors modes localized at string world sheets are eigenstates of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ defining virtual momentum would give effectively rise to massless Dirac action in M^4 and one would obtain massless fermionic propagators. The generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

(c) Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as gauge theory is natural.

(d) Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts.

The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

(e) The interaction vertices would correspond to the scattering of fermions at opposite wormhole throats. The natural guess is that the propagator is essentially the inverse of the scaling generator L_0 of conformal algebra. Non-locality suggests that one must product for the inverses of the super-generators G and its hermitian conjugate estimated at the two wormhole throats. There the diagrammatics would be combinations of that for QFT with massless fermions and string model diagrammatics. Topologically the vertices would be analogous to Feynman vertices: two 3-surfaces would fuse at vertices to form third. Stringy trouser diagrams would not have interpretation as decays of particle but as particle travelling two different paths.

(f) Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??**, `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg` or fig. 6, tgdgraphs in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics.

Quantum TGD as square root of thermodynamics

Zero energy ontology (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A75] [K79]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A75], [K79] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the modified Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these

4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes. Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the imbedding space [K21]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. The inclusions of these conformal algebras would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

Summarizing

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD and that different measurement interactions having interpretation in terms of different kind of quantum measurements causing wave function collapse in zero mode sector of WCW could correspond to various choices of ω. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

10.2.5 Finite measurement resolution and HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in zero energy ontology

Some clarifications concerning the notion of observable in zero energy ontology are in order.

(a) As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.

(b) Also the conjugation $A \to JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.

The formulation of quantum TGD in terms of the modified Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

(c) Zero energy ontology gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish zero energy ontology allows a symmetry breaking respecting a chosen Cartan algebra.

(d) In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFS as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

(a) Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

(b) The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

(c) The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) . \quad (10.2.5)$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle . \quad (10.2.6)$$

(d) Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^\dagger)|r_2\rangle . \quad (10.2.7)$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times Tr_N(SS^\dagger) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 . \quad (10.2.8)$$

(e) Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

(f) This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

(a) Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

(b) The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

(c) For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

(a) In zero energy ontology the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.

(b) The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.

(c) The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.

(d) In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A12] .

(e) Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S

and guarantee the symmetry. One could speak of a square root of KMS condition [A12] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.

(f) If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

(a) Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{10.2.9}$$

for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.

(b) $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.

(c) Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \tag{10.2.10}$$

for any physically reasonable choice $\mathcal{N}$.

(d) In TGD framework the condition would say that the M-matrix defined by the modified Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain modified gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A40] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

(a) The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

(b) One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

(a) Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.

(b) The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.

(c) The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

10.2.6 Questions about quantum measurement theory in zero energy ontology

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \rightarrow P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

10.2.7 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [K14] . M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results. Recall that also the weaker condition $T_p = pT_0$, p prime, would assign secondary p-adic time scales to the size scale hierarchy of CDs: $p \simeq 2^n$ would result as an outcome of some kind of "natural selection" for this option. The highly satisfactory feature would be that p-adic time scales would reflect directly the geometry of imbedding space and WCW.

p-Adic coupling constant evolution

An attractive conjecture is that the coupling constant evolution associated with CDs in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induces p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there seems to be a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

(a) The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

(b) p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

(c) In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For the weaker condition would be $T_p = pT_0$, p prime, $p \simeq 2^n$ could be seen as an outcome of some kind of "natural selection". In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

(d) The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \quad ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

10.2.8 Planar algebras and generalized Feynman diagrams

Planar algebras [A21] are a very general notion due to Vaughan Jones and a special class of them is known to characterize inclusion sequences of hyper-finite factors of type II_1 [A47] . In the following an argument is developed that planar algebras might have interpretation in terms of planar projections of generalized Feynman diagrams (these structures are metrically 2-D by presence of one light-like direction so that 2-D representation is especially natural). In [K8] the role of planar algebras and their generalizations is also discussed.

Planar algebra very briefly

First a brief definition of planar algebra.

(a) One starts from planar k-tangles obtained by putting disks inside a big disk. Inner disks are empty. Big disk contains $2k$ braid strands starting from its boundary and returning back or ending to the boundaries of small empty disks in the interior containing also even number of incoming lines. It is possible to have also loops. Disk boundaries and braid strands connecting them are different objects. A black-white coloring of the disjoint regions of k-tangle is assumed and there are two possible options (photo and its negative). Equivalence of planar tangles under diffeomorphisms is assumed.

(b) One can define a product of k-tangles by identifying k-tangle along its outer boundary with some inner disk of another k-tangle. Obviously the product is not unique when the number of inner disks is larger than one. In the product one deletes the inner disk boundary but if one interprets this disk as a vertex-parton, it would be better to keep the boundary.

(c) One assigns to the planar k-tangle a vector space V_k and a linear map from the tensor product of spaces V_{k_i} associated with the inner disks such that this map is consistent with the decomposition k-tangles. Under certain additional conditions the resulting algebra gives rise to an algebra characterizing multi-step inclusion of HFFs of type II_1.

(d) It is possible to bring in additional structure and in TGD framework it seems necessary to assign to each line of tangle an arrow telling whether it corresponds to a strand of a braid associated with positive or negative energy parton. One can also wonder whether disks could be replaced with closed 2-D surfaces characterized by genus if braids are defined on partonic surfaces of genus g. In this case there is no topological distinction between big disk and small disks. One can also ask why not allow the strands to get linked (as suggested by the interpretation as planar projections of generalized Feynman diagrams) in which case one would not have a planar tangle anymore.

General arguments favoring the assignment of a planar algebra to a generalized Feynman diagram

There are some general arguments in favor of the assignment of planar algebra to generalized Feynman diagrams.

(a) Planar diagrams describe sequences of inclusions of HFF:s and assign to them a multi-parameter algebra corresponding indices of inclusions. They describe also Connes tensor powers in the simplest situation corresponding to Jones inclusion sequence. Suppose that also general Connes tensor product has a description in terms of planar diagrams. This might be trivial.

(b) Generalized vertices identified geometrically as partonic 2-surfaces indeed contain Connes tensor products. The smallest sub-factor N would play the role of complex numbers meaning that due to a finite measurement resolution one can speak only about N-rays of state space and the situation becomes effectively finite-dimensional but non-commutative.

(c) The product of planar diagrams could be seen as a projection of 3-D Feynman diagram to plane or to one of the partonic vertices. It would contain a set of 2-D partonic 2-surfaces. Some of them would correspond vertices and the rest to partonic 2-surfaces at future and past directed light-cones corresponding to the incoming and outgoing particles.

(d) The question is how to distinguish between vertex-partons and incoming and outgoing partons. If one does not delete the disk boundary of inner disk in the product, the fact that lines arrive at it from both sides could distinguish it as a vertex-parton whereas outgoing partons would correspond to empty disks. The direction of the arrows associated with the lines of planar diagram would allow to distinguish between positive and negative energy partons (note however line returning back).

(e) One could worry about preferred role of the big disk identifiable as incoming or outgoing parton but this role is only apparent since by compactifying to say S^2 the big disk exterior becomes an interior of a small disk.

A more detailed view

The basic fact about planar algebras is that in the product of planar diagrams one glues two disks with identical boundary data together. One should understand the counterpart of this in more detail.

(a) The boundaries of disks would correspond to 1-D closed space-like stringy curves at partonic 2-surfaces along which fermionic anti-commutators vanish.

(b) The lines connecting the boundaries of disks to each other would correspond to the strands of number theoretic braids and thus to braidy time evolutions. The intersection points of lines with disk boundaries would correspond to the intersection points of strands of number theoretic braids meeting at the generalized vertex.

[Number theoretic braid belongs to an algebraic intersection of a real parton 3-surface and its p-adic counterpart obeying same algebraic equations: of course, in time direction algebraicity allows only a sequence of snapshots about braid evolution].

(c) Planar diagrams contain lines, which begin and return to the same disk boundary. Also "vacuum bubbles" are possible. Braid strands would disappear or appear in pairwise manner since they correspond to zeros of a polynomial and can transform from complex to real and vice versa under rather stringent algebraic conditions.

(d) Planar diagrams contain also lines connecting any pair of disk boundaries. Stringy decay of partonic 2-surfaces with some strands of braid taken by the first and some strands by the second parton might bring in the lines connecting boundaries of any given pair of disks (if really possible!).

(e) There is also something to worry about. The number of lines associated with disks is even in the case of k-tangles. In TGD framework incoming and outgoing tangles could have odd number of strands whereas partonic vertices would contain even number of k-tangles from fermion number conservation. One can wonder whether the replacement of boson lines with fermion lines could imply naturally the notion of half-k-tangle or whether one could assign half-k-tangles to the spinors of WCW ("world of classical worlds") whereas corresponding Clifford algebra defining HFF of type II_1 would correspond to k-tangles.

10.2.9 Miscellaneous

The following considerations are somewhat out-of-date: hence the title 'Miscellaneous'.

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A72] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A98] .

Fusion rules are indeed something more intricate that the naive product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

(a) For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

(b) The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

(c) A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A51] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k + 2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naive tensor product with something more intricate. The naivest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naive an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K14] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern-Simons action [A99] .

(a) The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

(b) Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K77] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K18] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A99] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say three-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

10.3 Number theoretic criticality and M-matrix

Number theoretic universality has been one of the basic guide lines in the construction of quantum TGD. There are two forms of the principle.

(a) The strong form of number theoretical universality states that physics for any system should effectively reduce to a physics in algebraic extension of rational numbers at the level of M-matrix so that an interpretation in both real and p-adic sense (allowing a suitable algebraic extension of p-adics) is possible. One can however worry whether this principle only means that physics is algebraic so that there would be no need to talk about real and p-adic physics at the level of M-matrix elements. It is not possible to get rid of real and p-adic numbers at the level of classical physics since calculus is a prerequisite for the basic variational principles used to formulate the theory. For this option the possibility of completion is what poses conditions on M-matrix.

(b) The weak form of principle requires only that both real and p-adic variants of physics make sense and that the intersection of these physics consist of physics associated with various algebraic extensions of rational numbers. In this rational physics would be like rational numbers allowing infinite number of algebraic extensions and real numbers and p-adic number fields as its completions. Real and p-adic physics would be completions of rational physics. In this framework criticality with respect to phase transitions changing number field becomes a viable concept. This form of principle allows also purely p-adic phenomena such as p-adic pseudo non-determinism assigned to imagination and cognition. Genuinely p-adic physics does not however allow definition of notions like conserved quantities since the notion of definite integral is lacking and only the purely local form of real physics allows p-adic counterpart.

Experience has taught that it is better to avoid too strong statements and perhaps the weak form of the principle is enough. It is however clear that number theoretical criticality could provide important insights to quantum TGD: p-adic thermodynamics is excellent example of this. In consciousness theory the transitions transforming intentions to actions and actions to cognitions would be key applications. Needless to say, zero energy ontology is absolutely essential: otherwise this kind of transitions would not make sense. The considerations in the sequel could be seen as being about conditions of number theoretical criticality if the weak form of principle is adopted.

10.3.1 Number theoretic constraints on M-matrix

Number theoretic constraints on M-matrix are non-trivial even for weaker notion of number theoretical universality.

Number theoretic criticality

Number theoretical criticality (or number theoretical universality in strong sense) requires that M-matrix elements are algebraic numbers. This is achieved naturally if the definition of M-matrix elements involves only the data associated with the number theoretic braid. Note that this data is non-local since it involves information about tangent space of X^4 at the point so that discretization happens in geometric sense but not in information theoretic sense. Note also that for algebraic surfaces finite number of points of surface allows to deduce the parameters of the polynomials involved and thus to deduce the entire surface.

If quantum version of WCW is adopted one must perform quantization for $E^2 \subset M^4$ coordinates of points S_i^2 braid and CP_2 coordinates of M^2 braid. In this kind of situation it becomes unclear whether one can speak about braiding anymore. This might make sense if each braid topology corresponds to its own quantization containing information about the fact that deformations of X_l^3 respect the braiding topology.

The partonic vertices appearing in M-matrix elements should be expressible in terms of N-point functions of some rational super-conformal field theory but with the p-adically questionable N-fold integrals over string appearing in the definition of n-point functions. The most elegant manner to proceed is to replace them with their explicit expressions if they are algebraic functions- quite generally or in number theoretical criticality. Spin chain type string discretization is an alternative, not so elegant option.

Propagators, that is correlations between partonic 2-surfaces, would be due to the interior dynamics of space-time sheets which means a deviation from super string theory. Another function of interior degrees of freedom is to provide zero modes of metric of WCW identifiable as classical degrees of freedom of quantum measurement theory entangling with quantal degrees of freedom at partonic 3-surfaces.

Number theoretical criticality poses very strong conditions on the theory.

(a) The p-adic variants of 4-D field equations associated with Kähler action make sense. Also the notion of preferred extremal makes sense in p-adic context if it corresponds to quantum criticality in the sense that second variation of Kähler action vanishes for dynamical symmetries, presumably conformal symmetries. A natural further condition is that the surface is representable in terms of algebraic equations involving only rational or algebraic coefficients and thus making sense both in real and p-adic sense. In this case also Kähler action and classical charges could exist in some algebraic extension of p-adic numbers.

(b) Also modified Dirac equation makes sense p-adically. The exponent of Kähler function defining vacuum functional is well-defined notion p-adically if the identification as product of finite number of eigenvalues of the modified Dirac operator is accepted and eigenvalues are algebraic. Also the notion of WCW metric expressible in terms of derivatives of the eigenvalues with respect to complex coordinates of WCW makes sense. In fact, the reduction of the modified Dirac equation to purely algebraic conditions implies by the localization to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) realizes algebraization in very concrete sense as holomorphy conditions.

(c) The functional integral over WCW can be defined only as an algebraic extension of real functional integral around maximum of Kähler function if the theory is integrable and gives as a result an algebraic number. One might hope that algebraic p-adicization makes sense for the maxima of Kähler function. The basic requirement is that the inverse of the matrix defined by the Kähler metric defining propagator is algebraic function of the complex coordinate of WCW. If the eigen-values of the modified Dirac operator satisfy this condition this is indeed the case.

(d) Ordinary perturbation series based on Feynman diagrams makes sense also in p-adic sense since the presence of cutoff for the size of CD implies that the number of terms if finite. One must be however cautious with momentum integrations which should reduce to finite sum due to the presence of both IR and UV cutoff implied by the finite size of CD. The formulation in terms of number theoretic braids whose intersections with partonic 2-surfaces consist of finite number of points supports the possibility of number theoretic universality.

The identification of number theoretic braids

To specify number theoretical criticality one must specify some physically preferred coordinates for $M^4 \times CP_2$ or at least $\delta M^4_\pm \times CP_2$. Number theoretical criticality requires that braid belongs to the algebraic intersection of real and p-adic variants of the partonic 2-surface so that number theoretical criticality reduces to a finite number of conditions. This is however not strong enough condition and one must specify further physical conditions.

1. What are the preferred coordinates for H?

What are the preferred coordinates of M^4 and CP_2 in which algebraicity of the points is required is not completely clear. The isometries of these spaces must be involved in the identification as well as the choice of quantization axes for given CD. In [K44] I have discussed the natural preferred coordinates of M^4 and CP_2.

(a) For M^4 linear M^4 coordinates chosen in such manner that $M^2 \times E^2$ decomposition fixing quantization axes is respected are very natural. This restricts the allowed Lorentz transformations to Lorentz boosts in M^2 and rotations in E^2 and the identification of M^2 as hyper-complex plane fixes time coordinate uniquely. E^2 coordinates are fixed apart from the action of $SO(2)$ rotation. The rationalization of trigonometric functions of angle variables allows angles associated with Pythagorean triangles as number theoretically simplest ones.

(b) The case of CP_2 is not so easy. The most obvious guess in the case of CP_2 the coordinates corresponds to complex coordinates of CP_2 transforming linearly under $U(2)$. The condition that color isospin rotations act as phase multiplications fixes the complex coordinates uniquely. Also the complex coordinates transforming linearly under $SO(3)$ rotations are natural choice for S^2 ($r_M = constant$ sphere at $\delta M^4_\pm$).

(c) Another manner to deal with CP_2 is to apply number $M^8 - H$ duality. In M^8 CP_2 corresponds to E^4 and the situation reduces to linear one and $SO(4)$ isometries help to fix preferred coordinate axis by decomposing E^4 as $E^4 = E^2 \times E^2$. Coordinates are fixed apart the action of the commuting $SO(2)$ sub-groups acting in the planes E^2. It is not clear whether the images of algebraic points of E^4 at space-time surface are mapped to algebraic points of CP_2.

2. The identification of number theoretic braids

It took some years to end up with a unique identification of number theoretic braids [K10, K51] . As a matter fact, there are several alternative identifications and it seems that all of them are needed. Consider first just braids without the attribute 'number theoretical'.

(a) Braids can be identified as lifts of the projections of X_l^3 to the quantum critical sub-manifolds M^2 or S_I^2, $i = I, II$, and in the generic case consist of 1-dimensional strands in X_l^3 These sub-manifolds are obviously in the same role as the plane to which the braid is projected to obtain a braid diagram.

(b) Braid points are always quantum critical against the change of Planck constant so that TQFT like theory characterizes the freedom remaining intact at quantum criticality. Quantum criticality in this sense need not have anything to do with the quantum criticality in the sense that the second variation of Kähler action vanishes -at least for the variations representing dynamical symmetries in the sense that only the inner product $\int(\partial L_D/\partial h^k_\alpha)\delta h^k d^4x$ (L_D denotes modified Dirac Lagrangian) without the vanishing of the integrand. This criticality leads to a generalization of the conceptual framework of Thom's catastrophe theory [K10] .

(c) It is not clear whether these three braids form some kind of trinity so that one of them is enough to formulate the theory or whether all of them are needed. Note also that one has quantum superposition over CDs corresponding to different choices of M^2 and the pair formed by S_I^2 and S_{II}^2 (note that the spheres are not independent if both appear). Quantum measurement however selects one of these choices since it defines the choice of quantization axes.

(d) One can consider also more general definition. The extrema of Kähler magnetic field strength $\epsilon^{\alpha\beta}J_{\alpha\beta}$ at X^2 define in natural manner a discrete set of points defining the nodes of symplectic triangulation. This set of extremals is same for all deformations of X_l^3 allowed in the functional integral over symplectic group although the positions of points change. For preferred symplectically invariant light-like coordinate of X_l^3 braid results. Also now geodesic spheres and M^2 would define the counterpart of the plane to which the braids are projected.

Number theoretic braids would be braids which are number theoretically critical. This means that the points of braid in preferred coordinates are algebraic points so that they can be regarded as being shared by real partonic 2-surface and its p-adic counterpart obeying same algebraic equations. The phase transitions between number fields would mean leakage via these 2-surfaces playing the role of back of a book along which real and p-adic physics representing the pages of a book are glued together. The transformation of intention to action would represent basic example of this kind of leakage and number theoretic criticality could be decisive feature of living matter. For number theoretic braids at X_l^3 whose real and p-adic variants obey same algebraic equations, only subset of algebraic points is common to real and p-adic pages of the book so that discretization of braid strand is unavoidable.

10.3.2 Physical representations of Galois groups

It would be highly desirable to have concrete physical realizations for the action of finite Galois groups. TGD indeed provides two kinds of realizations of this kind. For both options there are good hopes about the unification of number theoretical and geometric Galois programs obtained by replacing permutations with braiding homotopies and by a discretization of the continuous situation to a finite number theoretic braids having finite Galois groups as automorphisms.

Number theoretical braids and the representations of finite Galois groups as outer automorphisms of braid group algebra

Number theoretical braids [K15, K66] are in a central role in the formulation of quantum TGD based on general philosophical ideas which might apply to both physics and mathematical cognition and, one might hope, also to a good mathematics.

An attractive idea inspired by the notion of the number theoretical braid is that the symmetric group S_n might act on roots of a polynomial represented by the strands of braid and could thus be replaced by braid group B_n.

The basic philosophy underlying quantum TGD is the notion of finite resolution, both the finite resolution of quantum measurement and finite cognitive resolution [K15] . The basic implication is discretization at space-time level and finite-dimensionality of all mathematical structures which can be represented in the physical world. At space-time level the discretization means that the data involved with the definition of M-matrix comes from a subset of a discrete set of points in the intersection of real and p-adic variants of partonic 2-surface obeying same algebraic equations. Note that a finite number of braids could be enough to code for the information needed to reconstruct the entire partonic 2-surface if it is given by polynomial or rational function having coefficients as algebraic numbers. Entire WCW would be discretized in this picture. Also the reduction of the infinite braid to a finite one would conform with the spontaneous symmetry breaking S_∞ to diagonally imbedded finite Galois group imbedded diagonally.

1. Two objections

Langlands correspondence assumes the existence of finite-dimensional representations of $Gal(\overline{Q}/Q)$. In the recent situation this encourages the idea that the restrictions of mathematical cognition allow to realize only the representations of $Gal(\overline{Q}/Q)$ reducing in some sense to representations for finite Galois groups. There are two counter arguments against the idea.

(a) It is good to start from a simple abelian situation. The abelianization of $G(\overline{A}/Q)$ must give rise to multiplicative group of adeles defined as $\hat{Z} = \prod_p Z_p^\times$ where $Z_p^\times$ corresponds to the multiplicative group of invertible p-adic integers consisting of p-adic integers having p-adic norm equal to one. This group results as the inverse limit containing the information about subgroup inclusion hierarchies resulting as sequences $Z^\times/(1+pZ)^\times \subset Z^\times/(1+p^2Z)^\times \subset ..$ and expressed in terms factor groups of multiplicative group of invertible p-adic integers. Z_∞/A_∞ must give the group $\prod_p Z_p^\times$ as maximal abelian subgroup of Galois group. All smaller abelian subgroups of S_∞ would correspond to the products of subgroups of $\hat{Z}^\times$ coming as $Z_p^\times/(1+p^nZ)^\times$. Representations of finite cyclic Galois groups would be obtained by representing trivially the product of a commutator group with a subgroup of $\hat{Z}$. Thus one would obtain finite subgroups of the maximal abelian Galois group at the level of representations as effective Galois groups. The representations would be of course one-dimensional.

One might hope that the representations of finite Galois groups could result by a reduction of the representations of S_∞ to $G = S_\infty/H$ where H is normal subgroup of S_∞. Schreier-Ulam theorem [A79] however implies that the only normal subgroup of S_∞ is the alternating subgroup A_∞. Since the braid group B_∞ as a special case reduces to S_∞ there is no hope of obtaining finite-dimensional representations except abelian ones.

(b) The identification of $Gal(\overline{Q}/Q) = S_\infty$ is not consistent with the finite-dimensionality in the case of complex representations. The irreducible unitary representations of S_n are in one-one correspondence with partitions of n objects. The direct numerical inspection based on the formula for the dimension of the irreducible representation of S_n in terms of Yang tableau [A35] suggests that the partitions for which the number r of summands differs from $r = 1$ or $r = n$ (1-dimensional representations) quite generally have dimensions which are at least of order n. If d-dimensional representations corresponds to representations in $GL(d, C)$, this means that important representations correspond to dimensions $d \to \infty$ for S_∞.

Both these arguments would suggest that Langlands program is consistent with the identification $Gal(\overline{F}, F) = S_\infty$ only if the representations of $Gal(\overline{Q}, Q)$ reduce to those for finite Galois subgroups via some kind of symmetry breaking.

2. Diagonal imbedding of finite Galois group to S_∞ as a solution of problems

The idea is to imbed the Galois group acting as inner automorphisms diagonally to the m-fold Cartesian power of S_n imbedded to S_∞. The limit $m \to \infty$ gives rise to outer automorphic action since the resulting group would not be contained in S_∞. Physicist might prefer to speak about number theoretic symmetry breaking $Gal(\overline{Q}/Q) \rightarrow G$ implying that the representations are irreducible only in finite Galois subgroups of $Gal(\overline{Q}/Q)$. The action of finite Galois group G is indeed analogous to that of global gauge transformation group which belongs to the completion of the group of local gauge transformations. Note that G is necessarily finite.

Representation of finite Galois groups as outer automorphism groups of HFFs

Any finite group G has a representation as outer automorphisms of a hyper-finite factor of type II_1 (briefly HFF in the sequel) and this automorphism defines sub-factor $\mathcal{N} \subset \mathcal{M}$ with a finite value of index $\mathcal{M} : \mathcal{N}$ [A62] . Hence a promising idea is that finite Galois groups act as outer automorphisms of the associated hyper-finite factor of type II_1.

More precisely, sub-factors (containing Jones inclusions as a special case) $\mathcal{N} \subset \mathcal{M}$ are characterized by finite groups G acting on elements of $\mathcal{M}$ as outer automorphisms and leave the elements of $\mathcal{N}$ invariant whereas finite Galois group associated with the field extension K/L act as automorphisms of K and leave elements of L invariant. For finite groups the action as outer automorphisms is unique apart from a conjugation in von Neumann algebra. Hence the natural idea is that the finite subgroups of $Gal(\overline{Q}/Q)$ have outer automorphism action in group algebra of $Gal(\overline{Q}/Q)$ and that the hierarchies of inclusions provide a representation for the hierarchies of algebraic extensions. Amusingly, the notion of Jones inclusion was originally inspired by the analogy with field extensions [A62] !

It must be emphasized that the groups defining sub-factors can be extremely general and can represent much more than number theoretical information understood in the narrow sense of the word. Even if one requires that the inclusion is determined by outer automorphism action of group G uniquely, one finds that any amenable, in particular compact [A1] , group defines a unique sub-factor by outer action [A62] . It seems that practically any group works if uniqueness condition is given up.

The TGD inspired physical interpretation is that compact groups would serve as effective gauge groups defining measurement resolution by determining the measured quantum numbers. Hence the physical states differing by the action of $\mathcal{N}$ elements which are G singlets would not be indistinguishable from each other in the resolution used. The physical states would transform according to the finite-dimensional representations in the resolution defined by G.

The possibility of Lie groups as groups defining inclusions raises the question whether hyper-finite factors of type II_1 could mimic any gauge theory and one might think of interpreting gauge groups as Galois groups of the algebraic structure of this kind of theories. Also Kac-Moody algebras emerge naturally in this framework as will be discussed, and could also have an interpretation as Galois algebras for number theoretical dynamical systems obeying dynamics dictated by conformal field theory. The infinite hierarchy of infinite rationals in turn suggests a hierarchy of groups S_∞ so that even algebraic variants of Lie groups could be interpreted as Galois groups. These arguments would suggest that HFFs might be kind of Universal Math Machines able to mimic any respectable mathematical structure.

Lifting the action of Galois group to braid action in the case of number theoretic braids

The various definitions of braids were already discussed. At number theoretic quantum criticality the points of braids are obtained as solutions of polynomial equation and thus one can assign to them a Galois group permuting the points of the braid.

To make the notion of number theoretic braid more concrete, let us introduce complex coordinate w of $\delta M^4_\pm$ (assignable to the geodesic sphere S^2 and transforming by a phase rotation under $SO(2)$), the standard radial light-like coordinate r of $\delta M^4_\pm$, and Eguchi-Hanson coordinates ξ^i, $i = 1, 2$ of CP_2 and corresponding complex coordinate of the geodesic sphere S^2_i represented as $\xi^1 = \xi^2$ *resp.* $\xi^1 = \overline{\xi}^2$.

Assume that partonic 2-surface is expressible as a solution of polynomial equations

$$P_1(r,w,\xi^1,\xi^2) \ , \quad P_2(r,w,\xi^1,\xi^2) \ , \quad P_3(r,w,\xi^1,\xi^2) = 0 \ , \tag{10.3.1}$$

where P_1 and P_2 are complex valued polynomials and P_3 a real valued polynomial with coefficients which are rational numbers. The additional two conditions defining the points of number theoretic braid are $w = 0$, $\xi^1 = \xi^2$, or $\xi^1 = \overline{\xi}^2$ corresponding to 3 different number theoretic braids. Since the points of the intersection of braid with X^2 satisfy algebraic equations, their solutions are algebraic numbers and number theoretic braid results.

The solutions of these equations correspond to simultaneous roots of four polynomials each characterized by its own Galois group and the points of the number theoretic braid thus provide a geometric representation for the product of these Galois groups. By the 2-dimensionality of the geodesic spheres, it is natural to consider the replacement of the permutations representing the action of Galois group with braid group action so that a projective representation of Galois group is obtained. The action of braid group elements on induced spinor fields can be non-trivial and would induce action on physical states.

An alternative representation is based on use of ξ^2 or ξ^1 as a coordinate for X^2. Assume that X^2 can be represented locally as a graph of a map from S^2_i and use the notation $\xi = \xi^1$ to avoid confusion

$$r = P_1(\xi) \ , \quad w = P_2(\xi) \ , \quad \xi^2 = P_3(\xi) \ , \tag{10.3.2}$$

The braid is defined by the additional equation $\xi^1 = \xi^2$ or $\xi 1 = \overline{\xi}^2$. The resulting equations are very similar to that in previous case.

The conditions for extremum of $\epsilon^{\alpha\beta} J_{\alpha\beta}$ in the case of CP_2 Kähler form read as

$$\begin{array}{l} \frac{\partial J_{\xi\overline{\xi}}}{\partial \xi} = 0 \ , \\ \frac{\partial J_{\xi\overline{\xi}}}{\partial \overline{\xi}} = 0 \ , \\ J_{\xi\overline{\xi}} = 2J_{\xi^1\overline{\xi}^1} + J_{\xi^1\overline{\xi}^2}\frac{\partial P_3}{\partial \overline{\xi}} + J_{\xi^2\overline{\xi}^1}\frac{\partial P_3}{\partial \xi} + 2J_{\xi^2\overline{\xi}^2}\frac{\partial P_3}{\partial \overline{\xi}}\frac{\partial P_3}{\partial \xi} \ . \end{array} \tag{10.3.3}$$

Analogous equations are obtained for the induced Kähler form $J(\delta M^4_\pm)$ of $\delta M^4_\pm$ (or S^2). These equations are algebraic equations since the expressions for the components of the Kähler form in the complex coordinates of CP_2 and S^2 are rational functions. Hence also the extrema of Kähler magnetic fields could define number theoretic braids. What would be nice that in this case the Galois group would correspond to Galois groups of the polynomials defined by the derivatives of J and would depend on X^2 via P_3. For the option situation is more complex but it seems possible to speak about Galois groups also now.

Can one imagine a genuine physical representation of braid group analogous to that induced by the braiding defined by X^3_l?

(a) One such representation is obtained if the partonic 2-surfaces at the ends of X^3_l are identical so that the braiding induces a unique permutation of the points. This kind of assumption looks however artificial.

(b) One can consider also braidings induced by the closed paths in the spaces labeling different choices M^2 and S^2_i. In this case braid group action would permute the roots in the general case. For instance, 2π rotations in Lorentz and color group rotating quantization axes could induce non-trivial braiding permutation of the roots. This kind of rotation for subsystem containing CD in question could induce this kind of braid group action.

(c) Also the closed paths in the symplectic group of $\delta M^4_\pm \times CP_2$ would induce braiding actions and also braided Galois actions. This action is especially natural for the number theoretic braids defined by extreme of $\epsilon^{\alpha\beta}J_{\alpha\beta}$ since functional integral reduces to integral over symplectic group leaving the number and values of extrema invariant but changing the positions and therefore inducing braiding. Also closed paths in the space of coefficients of polynomials define Galois actions but in this case the rotations in general affect induced Kähler form.

Does DNA replication have counterpart at the level of fundamental physics?

The fundamental question is what happens in the vertices represented by the partonic 2-surface? The study of the 3-vertex forces to ask whether the incoming braid is replicated in a manner very much analogous to the replication of DNA. Could braid replication make it possible to make copies of classical representations of number theoretic information. Quantum representation of information by irreducible representations of Galois group would not be replicable since each incoming braid would correspond to its own irreducible representation and the choice of these representations would not be a fully deterministic process.

It seems however that this replication is too strong an assumption since the fermionic oscillator operators associated with positive energy strands need not anti-commute with those associated with negative energy strands. Therefore the n-point function can be non-trivial even if the ends of strands do not meet each other. Symplectic QFT actually predicts that this is not the case. The braid structure would however mean that partons/elementary particles might be much more complex objects than they are though to be.

In [K77] DNA has been proposed to act as a topological quantum computer using braids. Both time-like braidings (dance metaphor is good here) and their space-like counterparts induced by the braiding of threads connecting to each other braid strands (the analog is the situation in which the feet of dancers connected by threads) are involved and these braidings are dual. Similar duality - in fact first suggested by the model of DNA as topological quantum computer - holds true at the fundamental level since the stringy curves connecting braid strands and braids strands define dual braidings related in the same manner. This duality is analogous to the duality of string diagrams of hadronic string model.

Maybe even elementary particles could be seen as a kind of quantum computers and their "genome" would code at least the initial data for for the topological quantum computation program. Information processing involves besides computation also copying of data and its transfer. Particle interaction vertices would realize the copying of data and particle exchanges its communication whereas quantum computation would be carried by parton with quantum program identified with its execution (light-like 3-surfaces can be regarded either as states or processes).

Rather amazing outcome of this line of though was the discovery [K30, L2] that the states of dark nuclei in nuclear string model can be naturally associated to three kinds of groups with dimensions 64, 64, and 20: numbers of DNA codons, RNA codons, and amino-acids. Even more: there is natural realization of the analog of the genetic code with exactly the same numbers of DNAs coding for a given amino-acid as for the vertebrate genetic code. The assumptions of the model are very general which suggests that the genetic code might be realized at nuclear level and that biochemistry could provide only one particular representation of the code.

Fusion rules number theoretically

The idea that partonic 2-surfaces decompose into regions, one for each number theoretic braid, and that the number theoretic braids define representations of Galois groups permuting the strands of the braid as automorphisms in HFFs of type II_1 suggests a fresh approach to the understanding of vertices. Kind of fusion rules would certainly be in question and the the interpretation as representations of Galois groups might allow to deduce information about the fusion rules using symmetry arguments.

The first thing to notice is that in the vertex the number theoretic braids coincide so that the Galois groups G associated with incoming and outgoing braids are identical. Only in the situation in which polynomial defining G becomes reducible it might occur that some of incoming lines corresponds to a group which is product of subgroups of G but this situation is not expected to be generic.

Suppose that the number theoretic braids define irreducible projective representations of the Galois group G associated with the braid in HFF of type II_1 as outer automorphisms via diagonal imbedding of G. In vertex one expects that fusion rules for these representations mean extraction of singlet from the tensor product of these representations. This suggest a picture very similar to the fusion of representations of $SU(2)_q$ in the fusion rules of WZW theory which also can be understood in terms of braiding. If one accepts generalized McKay correspondence suggested [A13] , then the fusion rules for Galois group could have representation in terms of fusion groups for Lie group associated with it by generalized McKay correspondence.

10.4 What can one say about the braiding part of M-matrix?

M-matrix should reduce to pure braiding matrix in CD *resp.* CP_2 degrees of freedom at quantum criticality against change of Planck constant and this allows to say something non-trivial about this part of M-matrix.

10.4.1 Are factorizable QFT in M^2 and topological QFT in S^2 associated with quantum criticality?

Planck constant depends on the sector of the generalized imbedding space and is ill-defined for partonic 2-surfaces in quantum critical sub-manifolds $M^2 \times^C P_2$ and $M^4 \times S^2_i$, $i = I, II$. Maximal quantum criticality corresponds to $M^2 \times S^2_i$. S^2_I corresponds to vacuum extremal so that quantum critical partonic 2-surfaces represent vacuum extremals of Kähler action. It depends on assumptions that one is willing to make whether homologically non-trivial geodesic sphere S^2_{II} can be allowed and whether the pure gauge part of Kähler gauge potential can have M^4 part [K51] .

The natural question is what happens at criticality. Is M-matrix completely trivial or do topological degrees of freedom remain. 2-D QFTs in M^2 known as factorizing QFTs are almost trivial [B30, B45] , and generalize the topological QFTs associated with braids. Also topological QFTs at sphere with pictures - defined by braid points- are possible. The S-matrix of these theories does not depend on Planck constant [B30, B45] . Hence it is quite possible that these theories describe the situation at quantum criticality.

As explained, number theoretical braids come in 3 variants corresponding to the projections of X^3_l to M^2, S^2_I, and S^2_{II} carrying the analogs of braid diagram obtained as a projection of braid to plane so that braid points are always quantum critical. It is not clear whether these alternatives provide trinity of descriptions or whether all of them are needed. The dynamics in M^4 *resp.* CP_2 degrees of freedom should reduce to this kind of QFT in M^2 *resp.* S^2_I and to both for $M^2 \times S^2_I$.

This would mean in particular, that the S-matrix -or more generally M-matrix- does not depend on the value of $\hbar$. Since partons are 2-dimensional, one would have for $M^2 \times S^2_i$ essentially light-like geodesics as allowed solutions of field equations and thus classical theory of free massless particles. Hence factorizing QFT would be a natural description for the quantum critical dynamics at quantum criticality. This M-matrix should appear also in the full M-matrix as a factor.

10.4.2 Factorizing 2-D S-matrices and scattering at quantum criticality

In this subsection the view that the scattering in imbedding space degrees of freedom at quantum criticality could be described using a tensor product of 2-D factorizing S-matrices associated with the plane M^2 and geodesic spheres S^2_i of CP_2 defining quantization axes for a given CD and serving as critical manifolds for the phase transitions changing Planck constant realized as a leakage between different pages of CD and/or CP_2 book.

Factorizing S-matrix in M^2 as a building block of the full U-matrix

1. Why factorizability?

The known exact S-matrices in 1+1-dimensional space time are factorizing. According to [B30] there exists a strong evidence that all exact S-matrices in 1+1 dimensions are factorizing, do not allow particle production, and that the sets of the initial and final state momenta are identical.

Exactness certainly follows from infinite number of conservation laws associated with integrable systems but also finite number of them is enough. Infinite number of conservation laws are expected also in TGD since Kac Moody type symmetries are present. The conserved charges of form

$$Q_a^n = exp(n\theta_i)Q_a \ , \tag{10.4.1}$$

where n is Lorentz spin completely analogous to conformal weight imply the factorizability [A49]. These charges have interpretation as loop group generators of conformal weight n in the defining representation (where these generators are proportional to m^n) evaluated at the ray η_i of M^2 representing momenta as the positions for tips of light-cones. In the case of E^2 one obtains $exp(i\phi_n)$ where ϕ_n represent directions of momenta classically.

2. Yang-Baxter equations and Zamolodchikov algebra

Arranging the scattering particles in M^2 with respect to rapidities η_i (hyperbolic angles) such that the fastest particle is leftmost and slowest one rightmost (this is possible by the crossing symmetry and by assuming Yang-Baxter equations), the scattering can be described as a sequence of events in which particles pass by each other and can be therefore interpreted as a braiding like process with the additional feature that particles move with different velocities.

The pass-by event is described by a 2-particle S-matrix depending only on the difference $\eta_{12} = \eta_1 - \eta_2$ of their rapidities. By Uncertainty Principle, the position of the particle world line should not matter so that the world line of any particle can be shifted parallel to itself without affecting the S-matrix. This however affects the braiding. This symmetry gives rise to the celebrated Yang-Baxter equations

$$S(\eta_{12})S(\eta_{13})S(\eta_{23}) = S(\eta_{23})S(\eta_{13})S(\eta_{12}) \ . \tag{10.4.2}$$

N-particle S-matrix reduces to braiding S-matrix expressible in terms of S-matrices describing 2- particle scattering.

One can abstract the conditions on S-matrix algebraically to give what is known as Zamolodchikov algebra [A49] so that S-matrix describes the pass-by process as a generalization of the exchange operation in braiding. Posing the conditions that 2-particle S-matrix approaches unit matrix at the limit $\eta_{12} \rightarrow 0$, unitarity stating $S(\eta)S(-\eta) = 1$, real analyticity $S^{\dagger}(\eta) = S(-\overline{\eta})$ and crossing symmetry $S_{ij}^{kl}(\eta) = S_{j\bar{l}}^{\bar{i}k}(i\pi - \eta)$, one achieves axiomatization for the algebra. Sine-Gordon theory provides a basic example of an integrable system whose S-matrix satisfying these constraints.

If one poses the restriction that light cone tips belong to M^1 situation simplifies still since all particles defined by the contents of light cones would be at rest relative to each other and the S-matrix reduces to a trivial braiding matrix obtained by putting $\eta_{ij} = 0$ in above equation. The limit $\eta_{ij} \rightarrow \pm\infty$ when taken in a somewhat delicate manner gives rise to the standard form of the non-unitary braiding matrix appearing in quantum group representations as shown by Jimbo [A49].

3. Could factorizing S-matrix as tensor factor of full S-matrix make sense in TGD framework?

In a genuinely 2-D context this kind of system is of course physically somewhat uninteresting. In TGD framework the situation is different if factorizing S-matrices are interpreted as describing scattering at criticality with respect to phase transitions changing Planck constant and assignable to time like braiding since M^2 is analogous to the plane to which braid strands are projected. The basic condition is that the S-matrix elements have not dependence on Planck constant and this condition is indeed satisfied. The most general manner to satisfy this condition is by the vanishing of loop corrections to the scattering amplitudes so that only tree diagrams contribute.

By quantum classical correspondence the rapidities could be interpreted as M^2 projections of the 4-momenta of the particles created in the vertex. Since each light-cone can contain arbitrary many partons, the rapidities could be interpreted as M^2 projections of four-momenta assignable to braid strands.

The scattering matrix associated with time-like braiding would thus be almost trivial in longitudinal momentum projections but would not depend on the transversal momenta at all. The integration over all possible choices of M^2 plane would guarantee Lorentz invariance might destroy unitarity. Also the triviality in longitudinal momentum degrees for freedom looks non-physical.

Factorization of S-matrix in CP_2 degrees of freedom

The backs of CP_2 book correspond to the geodesic spheres S^2_i so that at quantum criticality one has braiding with braid projection to S^2_i, $i = I$ or $i = II$.

Center of mass degrees of freedom are unavoidable also in CP_2 degrees of freedom since Jones inclusions defined by the subgroups $G \subset SU(2) \subset SU(3)$ select preferred origin with respect to which $U(2)$ sub-group defining quantization axis acts linearly so that the choice of quantization axes means also a choice of preferred point of CP_2.

One can ask whether the complex CP_2 coordinates should be replaced with a quaternionic coordinate in such manner that the restriction to a geodesic sphere S^2_i of CP_2, $i = I, II$ or both would be the Euclidian analog of the restriction to M^2 meaning restriction to scattering in compactified complex plane and commutativity of generalized n-point functions. This was the question that I posed for years ago and the quantization of Planck constants gives an affirmative answer to this question but from quite different philosophy.

Finite measurement resolution implies the replacement of the WCW Clifford algebra with its finite-quantum-dimensional variant. Same applies to confifuration space itself and one should understand what this means.

(a) There seems to be no need for making WCW coordinates non-commutative. Rather, configuration space would be reduced to effectively finite-dimensional space obtained by replacing 2-surfaces with intersections of number theoretic braids with X^2. This would mean that the WCW Hamiltonians - representable as integrals involving the Hamiltonians of $\delta M^4_\pm \times CP_2$ and representing coordinates of configuration space - are replaced with expressions involving sums over points of the braid instead of integrals [K10] .

(b) Quantum groups should thus emerge via braidings. The observation that CP_2 parameterizes braiding matrices and that S^2 commutative braiding matrices - to be discussed below- might mean that CP_2 points represented as braiding matrices become non-commutative and that this forces the restriction of M^4 and CP_2 projections to geodesic spheres. This non-commutativity is of course something quite different from the non-commutativity in the sense of quantum groups.

(c) This non-commutativity could reflect itself in the braiding of number theoretic braids. In the case of M^2 braids the braiding tensor product of the matrices parameterized by the points of $S^2 \subset \delta M^4_{\pm}$ and CP_2, $i = I$ or II, could define the braiding as a local operation. In the case of S^2_i braid the braiding matrix could be the tensor product of braiding matrices parameterized by the point of S^2_{i+1} and M^4 point, presumably through its S^2 coordinates only. For M^2 braids the appearance of CP_2 braiding matrices would mean that the over-all braiding matrix obtained as a product of elementary braiding matrices depends on their order. In the case of Quantum Hall effect this means that non-Abelian anyons would be in question.

That this can be done is also suggested by an intriguing observations. The observation that for six-vertex model the solutions of Yang-Baxter equation are parameterized by CP_2 [A49] was one of the first intriguing observations [K77] leading to the evolution of ideas the role of quantum groups and von Neumann algebras in TGD.

1. CP_2 parameterizes R-matrices

In six-vertex model the R-matrices (counterparts of S-matrices above) have slightly different form than the S-matrices. For weak (or color) isospin 1/2 case which is fundamental also now, R-matrix is parameterized by 3 complex parameters

$$R(a,b,c) = \begin{pmatrix} a & & & \\ & b & c & \\ & c & b & \\ & & & a \end{pmatrix} . \quad (10.4.3)$$

The matrices differing by a complex scaling are physically equivalent so that a, b and c can be interpreted as complex components of fundamental representation of $SU(3)$. c can be fixed to $c = isin(\gamma)$, where $exp(i\gamma)$ can in fact be identified as the quantum phase $q = exp(i\pi/n)$ and a and b can be identified as complex CP_2 coordinates (ξ^2, ξ^2) transforming linearly under $U(2) \subset SU(3)$.

The restriction of a an b to represent points of a geodesic sphere of CP_2 going through origin implies that the matrices $R(a, b, c)$ commute. The condition for commutativity reads as

$$\begin{aligned} \Delta(a,b,c) &= \Delta(a',b',c') , \\ \Delta &= \frac{a^2+b^2-c^2}{2ab} . \end{aligned} \quad (10.4.4)$$

The solution of Yang-Baxter equation for three R-matrices reduces to the condition

$$\Delta(a,b,c) = \Delta(a',b',c') = \Delta(a'',b'',c'') . \quad (10.4.5)$$

Commutativity (in the sense of S-matrices rather than with respect to the product appearing in Yang-Baxter equations) means that the three points of CP_2 belong to the geodesic sphere identifiable also as a maximal commuting sub-manifold of CP_2 interpreted as a space obtained by gluing together three copies of quaternionic space H along sphere S^2 representing compactified complex plane for the second quaternionic space glued together just like S^2 is obtained by gluing together two complex planes along real line compactified to circle.

A canonical parameterization satisfying the commutativity conditions is given by

$$\begin{aligned} a(u) &= sinh(u+i\gamma) \ , \\ b(u) &= sinh(u) \ , \\ c(u) &= isin(\gamma) \ , \end{aligned} \tag{10.4.6}$$

where u is a complex coordinate. Using u as coordinate the Yang-Baxter equations have the same additive form as in case of M^4. In other words, one has $u'' = u' - u$. Unitarity is achieved when u is real.

These observations made already earlier [K77] suggest that the construction for M^2 generalizes to CP_2 degrees of freedom representing its Euclidian version obtained by the replacement $M^2 \subset M^4 \rightarrow S_i^2 \subset CP_2$. The commutativity of R-matrices in the case of S^2 would have interpretation in terms of space-like metric whereas in the case of M^4 Minkowski signature implies correlations and non-commutativity.

The commutativity of R-matrices at the geodesic sphere is an intriguing result. The notion of finite measurement resolution suggests that WCW Clifford algebra and WCW itself must replaced by their finite-quantum-dimensional quantum variants. For WCW Hamiltonians this might mean the replacement of CP_2 coordinates with their quantum counterparts. Could one imagine that R-matrices labeled by CP_2 points could serve as quantum representatives of CP_2 points and commutativity condition forces the restriction CP_2 projection to the braid points to S_i^2?

2. Factorizing S-matrix associated with a geodesic sphere of CP_2

Quantum classical correspondence can be applied also now as a guide line. Before continuing it is however useful to restate some facts about CP_2 and introduce notations. Assume for definiteness that CP_2 is identified as the space of right cosets $gU(2)$ of $SU(3)$ so that the natural action of $SU(3)$ is left action. The orbits of $SU(2)_L$ and $U(2)_L$ are homologically non-trivial geodesic spheres S^2 and the double coset space $SU(3)/SU(2)_L \times SU(2)_R$ of these spheres is 2-dimensional.

Also the geodesic circles $S^1_\perp$ orthogonal to a given point of S^2 are interesting as analogs of M^1 in $M^1 \times S^2$ decomposition. By the symmetry of $SU(3)/SU(2)_L \times SU(2)_R$ the actions of both $SU(3)_L$ and $SU(3)_R$ in this space are well defined, and the natural idea is that $U(1)_R$ action defines the geodesic circles $S^1_\perp$ so that electro-weak symmetry group would have a geometric counterpart.

Both $SU(2)_L \subset SU(3)$ and weak $SU(2)_L$ are represented by 4×4-dimensional R-matrices acting on fundamental fermions. The coordinate u parameterizing commuting R-matrices corresponds to the geodesic sphere $S^2 \subset CP_2$.

(a) $SU(2)_L \subset SU(3)$ quantum numbers replace M^2 momentum. Indeed, color is in TGD framework not a spin like quantum number but completely analogous to four-momentum and orbital angular momentum.

(b) The complex coordinates ξ^i, $i = 1, 2$, of CP_2 have a phase $exp[i(\pm\phi + \psi)/2]$ with ϕ assignable to isospin and ψ to hypercharge. $\xi^2 = 0$ geodesic sphere thus represents $I_3 + Y$ rotation. The classical representation for the quantization of angular momentum suggests that the direction of the total $I_3 + Y$ associated with a particular $\delta M^4_+ \times CP_2$ defines a point at S^2 parameterized in standard manner by (θ, ϕ). This fixes the value of θ via the condition

$$cos(\theta) = \frac{I_3 + Y}{\sqrt{I(I+1) + Y^2}} \tag{10.4.7}$$

when $\xi^2 = 0$ is selected as the representative geodesic sphere.

(c) The angle ϕ is the Euclidian counterpart of rapidity η so that that the classical model for the scattering would be in terms of particles rotating with different velocities along the circumference of circle. The momenta would be replaced with isospins $(I^3+Y)_k$ ordered from left to right along the circumference such that one has $(I^3+Y)_1 \geq (I^3+Y)_2 .... \geq (I^3+Y)_n$ having $\phi_1 \leq \phi_2 .... \leq \phi_n$. Unitarity requires that the parameter u is real and $\gamma_i = \phi$ identification is suggestive.

   i. In the case of M^2 the values of rapidities can be fixed by four-momenta but in the recent case there are no four-momenta and Uncertainty Principle does not encourage the fixing of the phases so that one must simply integrate over all possible values. Most naturally the convolution of the scattering amplitude with color partial waves for center of mass degrees of freedom defines this integration naturally.

   ii. On the other hand, the existence of phases in algebraic extension of p-adic numbers would suggest that ϕ_i can come only as multiples of the angle π/n defining the quantum phase q so that circle would be discretized to a circular lattice. The values of color isospin J would be restricted to $J \leq n/2$ for even n since for J and $J+n$ the wave functions differ only by a sign. For odd n one has $J \leq n$. This conforms with the fact that for the finite-dimensional representations of quantum groups associated with $q = exp(i\pi/n)$ the action of raising and lowering operators $J^n_{\pm}$ reduces to a multiplication by a complex number [A49], which can also vanish so that cyclic or semicyclic representations besides counterparts of ordinary finite-dimensional representations are obtained. Also the possibility of only $j \leq n/2$ representations of Kac Moody group fits with this picture. The groups $G \subset SU(2)$ for which n is the order of the maximal cyclic subgroup would naturally define as their orbits discrete analogs of the geodesic sphere allowing p-adicization and discrete versions of spherical harmonics. Physically the appearance of finite subgroups of SU(2) would be a direct analog for the presence of discrete subgroups of translation groups in solid state physics.

(d) This construction would allow to fix the dependence of S-matrix on the center of mass coordinates and on total color quantum numbers and the integration over the orbifold $SU(3)/SU(2)_L \times SU(2)_R$ of geodesic spheres of CP_2 would restore the exact color invariance broken by Jones inclusion.

(e) Just as the M^4 coordinates of the arguments of n-point function can be restricted to M^1, their CP_2 coordinates can be restricted to geodesic circle $S^1 \subset S^2 \subset CP_2$ implying the reduction of S-matrix to braiding S-matrix.

What about Yang-Baxter type scattering in transversal degrees of freedom?

One could also consider construction of a Yang-Baxter type scattering matrix in transversal degrees of freedom. This S-matrix cannot give rise to momentum transfers. One could argue that this is not in spirit with the basic number theoretic idea. One could however modify the idea. E^2 as the complement of hyper-complex plane in hyper-quaternionic space ($z = xiJ + yiK$) can be mapped to complex plane by $z \rightarrow iJz = x + yI$) and one can construct S-matrix for scattering in this plane. Similar argument applies in CP_2 degrees of freedom.

1. Factorizable S-matrix E^2 degrees of freedom

It is straightforward to modify the construction for CP_2 to construct S-matrix in transversal degrees of freedom. The angles ϕ_i characterizing the directions of transversal momenta would replace rapidities and particles could be ordered with respect to these angles and the intersections of projections of orbits to E^2 would define the interaction vertices. The commuting S-matrices applied in case of CP_2 parameterized by the values of u and γ could be used to define S-matrix. The values of ϕ coming as multiples of quantum angle π/n suggest themselves in p-adic context as intersections of p-adic E^2 with real one.

2. Factorizable S-matrix in $S^1 \perp S^2$ degrees of freedom

If one allows pass-by events in E^2, one must allow them also for the counterpart of E^2 in CP_2. Only the geodesic sub-manifolds representing commuting sub-algebra of quaternions and orbit of subgroup of color group are possible. This leaves only geodesic circles of CP_2 orthogonal to geodesic sphere S^2 into consideration. The reduction would be completely analogous to $M^1 \times S^2$ decomposition in the case of M^4. As noticed, the action of $U(1)_R$ groups in the space of geodesic spheres is well defined and generates these geodesic circles. The reduction of $SU(2)_L \times SU(2)_R \subset SU(3)$ to $SU(2)_L \times U(1)_R$ obviously correlates with the structure of the electro-weak gauge group.

The four-fold decomposition of H is analogous to the decomposition of 8-D spinors to four-fold tensor product of 2-D spinors. M^2 (E^2) represents classically hyperbolic (ordinary) rotations. $\xi^2 = 0$ geodesic sphere S^2 represents $I_3 + Y$ rotations and $S^1_\perp$ represents $I_3 - Y$ rotations. Every commuting isometry charge of $SO(3,1) \times SU(3)$ would thus correspond to its own tensor factor in the factorizing S-matrix.

Factorization for Kac-Moody representations

An interesting question relates to whether one should use finite-dimensional or infinite-dimensional representations of quantum Kac-Moody algebra to construct S-matrix as braiding matrix in Kac-Moody algebra. In both cases the counterpart of complex coordinate z restricted to unit circle (hyper-quaternionic M^2 coordinate restricted to $m \cdot m = 1$ hyperboloid) brings in angle (rapidity) variable.

(a) Finite-dimensional representations are obtained from those of quantum group and have vanishing central charge $k = 0$ and appear naturally in integrable 1+1-dimensional quantum theories so that the Yang-Baxter matrices are finite-dimensional, typically 2×2 matrices acting on quantum spinors. The infinite number of conservation laws have a natural interpretation in terms of elements of this algebra. Since Lorentz invariance in longitudinal degrees of freedom should not be broken by the central extension, one might argue that finite-dimensional representations are natural in this case. Also the idea that Connes tensor product makes the situation finite-dimensional fits with this interpretation. On the other hand, the breaking of Lorentz invariance might be a property of zero energy states and reflect the measurement situation as will be found and one must be cautious here.

(b) The braiding matrix for infinite-dimensional Kac-Moody representations was found by Drinfeld [A49] and has exponential form bringing in mind an exponent of Hamiltonian. The representation involves also Virasoro generator L_0. Presumably the generalization to the case of super Kac-Moody algebras exist. Neither the Kac-Moody- or quantum group R-matrix is unitary. I do not know whether a unitary R- matrix for Kac-Moody algebras is exclude by some deep reason.

The following arguments support the view that only finite-dimensional representations appear in S-matrices between zero energy states which seem to be the only possibility in TGD framework.

(a) The universality of the R-matrix for affine algebras encourages the guess non-unitarity is a universal property of Kac-Moody R-matrices containing only single continuous parameter and that unitary and thus trivial R-matrix is possible only in $q = 1$ case. This would conform with the fact that $q = 1$ also corresponds to extended ADE diagrams for Jones inclusions assignable to Kac-Moody representations.

Notice however that the non-unitary braiding R-matrix

$$\begin{pmatrix} q & & & \\ & 1 & q - q^{-1} & \\ & & 1 & \\ & & & q \end{pmatrix}$$

follows by a delicate limiting process from a unitary factorizing S-matrix at the limit $\eta_{12} \rightarrow \pm\infty$ as shown by Jimbo [A49]. Could the Kac-Moody R-matrix follow by a limiting procedure from a unitary R-matrix by allowing an additional continuous parameter analogous to rapidity to approach some limit?

(b) The invariance under isometries requires that central extension must vanish in center of mass degrees of freedom so that only finite-dimensional representations are possible.

(c) Only a finite number of degrees of freedom are observable in the sense that they appear in the S-matrix between zero energy states and this requires $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ reduction for Kac-Moody algebra leading to finite-dimensional Kac-Moody/quantum group representations.

What the reduction to braid group representations means physically?

One could choose also $M^1 \times S^2$ decomposition instead of $M^2 \times E^2$. $M^1 \times S^2$ option gives ordinary braid group representations as the limit $\eta_{ij} = 0$ meaning that the tips of light cones are at rest relative to each other. There is no convincing argument forbidding the braid group representations and they would be absolutely essential for topological quantum computation utilizing braiding S-matrices [K77] .

For CP_2 the two options correspond to (S^1, S^2) and (S^2, S^1) decompositions and are equivalent and $SU(2)_L \times U(1)_R \subset SU(3)_L \times SU(3)_R$ reduction. A reduction to braid group representation occurs always in $U(1)_R$ factor and is accompanied by a similar reduction in electro-weak degrees of freedom.

The geodesic circle $S^1 \subset S^2$ with $\theta = \pi/2$ implies $(I_3 + Y)/\sqrt{I(I+1) + Y^2} = 0$ meaning the absence of $I_3 + Y$ color rotation. The second color quantum number $I_3 - Y$ is represented by a geodesic circle $S^1_\perp$ orthogonal to S^2 and should vanish by the same argument. Quantum classical correspondence predicts that physical states correspond to $(I_3, Y) = (0, 0)$ states of color multiplets: the interpretation is as a weak form of color confinement. The vanishing of I_3 and Y implied by the weak form of color confinement means a reduction to $U(1)_L \times U(1)_R \subset SU(3)_L \times SU(3)_R$ so that S-matrix reduces to a braiding S-matrix in both $S^1_\perp$ and S^2 factors and also for electro-weak sector.

The relationship with Jones inclusions

The factorization of S-matrix to four factorizing tensor factors suggest similar structure for Jones inclusions.

1. The four basic basic types of Jones inclusions

Four kinds of Jones inclusions can be assigned with the pairs (M^2, E^2) and (S^2, S^1). Same applies in case of (M^1, S^2) and (S^1, S^1) in TGD framework.

(a) In $M^2 \times E^2$ case the discrete subgroups of $O(1, 1)$ and $O(2)$ would characterize Jones inclusions. For E^2 only $G = A_n$ or D_{2n} are possible. For M^2 the subgroups generated by powers of Lorentz boost and reflection are possible. The infinite order for these groups strongly suggests $\beta = 4$. The quantum phase $q = exp(i\pi/n)$ would emerge naturally if the action of Lorentz boosts on WCW spinor fields is unitary and reduces to a cyclic action represented by A_{n-1}. This would be very much analogous to the reduction of the quantum group representations to finite-dimensional ones as q becomes a root of unity.

(b) $M^1 \times S^2$ option allows also $G = E_6, E_8$ (tetrahedral and icosahedral groups) and $SU(2)$.

(c) For CP_2 all groups $G \subset SU(2)_L$ and $A_n \subset U(1)_R$ could define Jones inclusions. For color confined states only $G_L = A_{n_L}$ and $G_R = A_{n_R}$ are possible.

2. The type of braiding correlates with the type of Jones inclusion

Jones inclusions come in two very different types corresponding to $\beta < 4$ defined by the subgroups $G \subset SU(2)$ and $\beta = 4$ defined by $G = SU(2)$ or infinite subgroups of $SU(2)$. The two kinds of S-matrices could correspond to the two types of Jones inclusions as following arguments suggest.

(a) Constant Yang-Baxter matrices defining braid group representations emerge as intertwiners of quantum versions of Lie algebras whereas more general Yang-Baxter matrices emerge as intertwiners for the representations of quantum versions of Kac Moody algebras [A49]. Thus M^2 *resp.* S^2 would correspond to a representation of quantum Kac-Moody algebra whereas M^1 *resp.* S^1 would represent a degeneration to a purely topological braid group representation in the case of SU(2).

(b) According to the arguments of [K21] $\beta < 4$ corresponds to quantum group representations characterized by finite sub-groups $G \subset SU(2)$ whereas $\beta = 4$ representations corresponds to Kac Moody representations with monodromies of n-point functions characterized by the quantum phase q. It would seem that an equivalent characterization is as representations of quantum Kac Moody algebras.

3. Consistency with the TGD based explanation for McKay correspondence

These observations relate also interestingly to the proposal that TGD physics is universal in the sense of being able to mimic almost any physics obeying Kac Moody symmetry [K21] .

(a) McKay correspondence states that the finite subgroups $G \subset SU(2)$ characterizing $\beta < 4$ inclusions are labeled by ADE diagrams (A_n, D_{2n}, E_6 and E_8 are allowed). A concrete proposal was made for constructing the representations of the corresponding Kac-Moody algebras from these data by utilizing the new discrete degrees of freedom implied by the fact that space-time sheets define $n(G)$-fold coverings of M^4 (of CP_2 for $SU(2) \subset SL(2,C)$). The group algebra of G associated with multiple coverings of M^4 or CP_2 gave the multiplets.

The degeneration of the S-matrix to braiding S-matrix does not kill this conjecture. The point is that $n \geq 3$ condition for quantum phase excludes the Jones inclusion corresponding to A_2 (two-element subgroup of SU(2)). It would be just the representation of SU(2) realized in terms of quantum spinors which would degenerate to the braid group representation whereas other representations for which spin like degrees of freedom are represented in terms of group algebra of G are not lost.

(b) For $\beta = 4$ one obtains all extended ADE diagrams as characterizers of Jones inclusions, and an analogous construction of corresponding Kac Moody representations was proposed with quantum phase assigned with a non-trivial monodromy for n-point functions in S^2/G, S^2 a non-trivial geodesic sphere of CP_2. The natural identification would be as representations of quantum Kac-Moody algebra. All extended ADE diagrams are allowed which conforms with the fact that now SU(2) can be realized using quantum spinors. The representations of D_{2n+1} and E_7 should involve both quantum spinors and the $n(G)$-fold covering of S^2/G defining the monodromy.

(c) These proposals do not seem so speculative when one realizes that the finite dimensional representations of quantum groups can be regarded also as representations of quantum Kac-Moody algebras [A49]. As found, the generators in defining representations appear also as conserved charges in the quantum field theory models giving rise to factorizing S-matrices.

(d) According to the construction for $\beta < 4$ the dimension of CP_2 projection of the partonic 2-surface can be smaller than two: this excludes homological non-triviality. For $\beta = 4$ CP_2 projection would be homologically non-trivial geodesic sphere. This is in harmony with the assumption that geodesic circle S^1 and homologically non-trivial geodesic sphere S^2 characterize the sub-manifold of CP_2 to which the arguments of n-point functions belong for these representations.

10.4.3 Are unitarity and Lorentz invariance consistent for the quantum critical M-matrix constructed from factorizing S-matrices?

Factorizable M^2 S-matrices do not allow particle creation and the sets of initial and final state momenta are identical. The possibility to exchange internal quantum numbers possible in equal mass case could make possible momentum exchange in a very limited sense.

The extension to TGD framework brings in additional problems since the decomposition $M^4 = M^2 \times E^2$ breaks manifest Lorentz invariance. Also color invariance is broken. The question is how to achieve unitarity and Lorentz invariance simultaneously. The loss of these symmetries in case of U-matrix which characterizes universe rather than quantum state would be a catastrophe. This problem can be however circumvented.

U-matrix constructible using the proposed decomposition $M^2, E^2, S^2, S^1_\perp$ or its variant $(M^1, S^2), (S^1, S^1_\perp)$ should be unitary. Unitarity is trivial to achieve if one just restricts to a given decomposition. Since Jones inclusions have a concrete effect on imbedding space geometry and topology, one could argue that this decomposition indeed reduces Lorentz symmetry to $SO(1,1) \times SO(2)$ and color symmetry to $U(2)$ or $U(1)$.

There is a way out of the problem. One can extend the U-matrix by introducing a complete orthogonal basis of wave functions in the projective sphere P^2 labeling the choices $M^4 = M^2 \times E2$ and in the space of geodesic spheres $S^2 \subset CP_2$. The extended U-matrix is obtained by convoluting the factorizing S-matrix with this function basis. Completeness and orthonormalization of the basis reduce unitarity conditions for those of U-matrix for a fixed choice of (M^2, E^2) and $(S^2, S^1_\perp)$ pairs.

This is not a trick but corresponds to the possibility to choose the quantization axes and the wave function in question corresponds to a wave function in the space of sub-CDs and corresponding sub-WCWs defined by the different choices of quantization axes.

10.5 What can one say about U-matrix?

For some time I thought that U-matrix could be constructed using as building bricks S-matrices of factorizing QFTs but in turned out that these S-matrices can be assigned to the scattering at quantum criticality against change of Planck constant because they have no dependence on Planck constant. The realization that U-matrix could reduce to a tensor product of S-matrices associated with M-matrices characterizing zero energy states changed the situation and it seems that this is indeed the correct interpretation. The additional nice aspect of this assumption is that U-matrix can in principle be measured experimentally.

10.5.1 U-matrix as a tensor product of S-matrix part of M-matrix and its Hermitian conjugate?

U-matrix describes scattering of zero energy states and since zero energy states can be illustrated in terms of Feynman diagrams one can say that scattering of Feynman diagrams is in question. The initial and final states of the scattering are superpositions of Feynman diagrams characterizing the corresponding M-matrices which contain also the positive square root of density matrix as a factor.

The hypothesis that U-matrix is the tensor product of S-matrix part of M-matrix and its Hermitian conjugate would make U-matrix an object deducible by physical measurements. One cannot of course exclude that something totally new emerges. For instance, the description of quantum jumps creating zero energy state from vacuum might require that U-matrix does not reduce in this manner (this point was discussed already earlier). One can assign to the U-matrix a square like structure with S-matrix and its Hermitian conjugate assigned with the opposite sides of a square.

One can imagine of constructing higher level physical states as composites of zero energy states by replacing the S-matrix with M-matrix in the square like structure. These states would provide a physical representation of U-matrix. One could define U-matrix for these states in a similar manner. This kind of hierarchy could be continued indefinitely and the hierarchy of higher level U and M-matrices would be labeled by a hierarchy of n-cubes, $n = 1, 2,...$ TGD inspired theory of consciousness suggests that this hierarchy can be interpreted as a hierarchy of abstractions represented in terms of physical states. This hierarchy brings strongly in mind also the hierarchies of n-algebras and n-groups and this forces to consider the possibility that something genuinely new emerges at each step of the hierarchy. A connection with the hierarchies of infinite primes [K65] and Jones inclusions are suggestive. Below the possibility of this kind of hierarchy for Jones inclusions is considered. The discussion relates only loosely to the recent view about M-matrix and U-matrix since it was written much before the recent view about M-matrix emerged.

10.5.2 The unitarity conditions of U-matrix for HFFs of type II_1?

Zero energy ontology forced to give up the original hope the ordinary unitary S-matrix could directly correspond to U-matrix. For HFFs U-matrix could however decompose to a tensor product of unitary S-matrices acting between positive *resp.* negative parts of zero energy states. If these S-matrices are those assigned with the M-matrix for zero energy states, M-matrix would code information about U-matrix and be therefore measurable.

In the following U-matrix for HFF of type II_1 is formally treated as a matrix with discrete indices. A rigorous treatment would be by replacing indices representing 1-D projections by projections to infinite-dimensional sub-factors having non-vanishing trace.

The unitarity conditions for the scattering of zero energy states read formally as

$$\sum_{\hat{m}_+\hat{n}_-} U_{m_+n_-\rightarrow\hat{m}_+\hat{n}_-} U^*_{r_+s_-\rightarrow\hat{m}_+\hat{n}_-} = \delta_{m_+,r_+}\delta_{n_-,s_-} \quad . \tag{10.5.1}$$

The sum over the final zero energy states can be also written as a trace for the product of matrices labeled by incoming zero energy states.

$$Tr(U_{m_+n_-}U^*_{r_+s_-}) = \delta_{m_+,r_+}\delta_{n_-,s_-} \quad . \tag{10.5.2}$$

One can put $s_- = n_-$ on both sides and perform the sum over n_- to get

$$\sum_{n_-} Tr(U_{m_+n_-}U^*_{r_+n_-}) = \delta_{m_+,r_+}\sum_{n_-}\delta_{n_-,n_-} \quad . \tag{10.5.3}$$

This can be written as

$$\frac{1}{Tr(Id)}\sum_{n_-} Tr(U_{m_+n_-}U^*_{r_+n_-}) = \delta_{m_+,r_+} \quad . \tag{10.5.4}$$

For HFFs of type II_1 the sum $\sum_{n_-}\delta_{n_-,n_-}$ is equal to the trace $Tr(Id) = 1$ of the identity matrix so that one obtains

$$\sum_{n_-} Tr(U_{m_+n_-}U^*_{r_+n_-}) = \delta_{m_+,r_+} \quad . \tag{10.5.5}$$

This could be interpreted as a unitarity condition for positive and negative energy parts of the zero energy state are interpreted as incoming and outgoing state.

This result allows to consider the possibility that U-matrix between zero energy states could define also M-matrix for HFFs of type II_1. The almost triviality of U-matrix however suggests that this is not a good idea. The construction of M-matrix as time-like entanglement coefficients allowing to understand thermodynamics as part of quantum theory provides further support for this belief.

The interpretation of the result would be as a thermal expectation value of the unitarity condition in the sense of hyper-finite factors of type II_1. This averaging is necessary if one does not have any control over the scattering between zero energy states: this scattering is just a means to become conscious about the existence of the state we usually interpret as change of state.

10.5.3 U-matrix can have elements between different number fields

The argument for the number theoretical universality applies as such only to the matrix elements of U-matrix between different number fields. One can quite well consider the possibility that U matrix in the general case is non-algebraic since one can restrict the 3-surfaces contributing to this kind of transitions in such a manner that only algebraic numbers appear in the matrix elements of U. Unless this is the case, one could argue that physics reduces to purely algebraic physics so that one can forget both reals and p-adics.

This picture would conform with the idea that only those light-like 3-surfaces for which "physics is algebraic" are associated with the transitions between different number fields. One can say that these 3-surfaces would define a back of book along which leakage between different number fields occurs. For WCW spinor fields in sectors corresponding to different number fields the "overlap integral" defining U-matrix elements would involve only the 3-surfaces in the back of the book. These surfaces would be in exactly the same role as rationals and algebraic numbers in number theory. The transitions between different number fields would represent a critical phenomenon in complete analogy with the criticality against phase transitions changing the value of Planck constant. Therefore the quantum criticality of TGD Universe would have very many facets. An interesting conjecture is that these surfaces are labeled by infinite rationals and algebraics so that the analogy with number theory would be much deeper [K65] .

What the statement "physics is algebraic" means is not quite obvious.

(a) Both the field equations associated with extremals of Kähler action and modified Dirac equation represents a p-adically sensible statement. The anti-commutation relations for the finite number of eigenmodes of modified Dirac operator are algebraic. The eigenvalues of the modified Dirac operator defined by Kähler action should be algebraic for the preferred surfaces so that also the Dirac determinant defining the vacuum functional would be algebraic. Vacuum functional is conjectured to be equal to the exponent of Kähler function identifiable as Kähler action for the preferred extremal identified as 4-surface for which the second variation of Kähler action vanishes for the dynamical symmetries at least. Also these conditions are purely algebraic.

The expectation is that the number of critical deformations defining the symmetries is infinite and conformal symmetries are in question. The conformal algebras would form an infinite hierarchy of sub-algebras with generators labelled by integers proportional to an integer $n = 1, 2....$ One would have n conformal equivalence classes of space-time surfaces connecting given 3-surfaces at the boundaries of CD and n would define Planck constant $h_{eff} = n \times h$ labelling the hierarchy of dark matters (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

(b) The strongest condition would be that the values of classical charges and quantum numbers are well-defined and same for the positive and negative energy parts of quantum states assignable to given 3-surface which contribute to the transition and that real and p-adic space-time surfaces obey same algebraic equations but interpreted in different number fields. The classical conserved quantities associated with Kähler action could be defined also in p-adic case in this kind of situation and would be identical with corresponding real quantities if they are algebraic numbers.

(c) The algebraic points common to real and p-adic space-time surfaces would provide the data appearing in U so that these points much corresponds to the points of number theoretic braids which must therefore have algebraic coordinate values in preferred coordinates for M^4 and CP_2.

10.5.4 Feynman diagrams as higher level particles and their scattering as dynamics of self consciousness

The hierarchy of inclusions of hyper-finite factors of II_1 as counterpart for many-sheeted space-time lead inevitably to the idea that this hierarchy corresponds to a hierarchy of generalized Feynman diagrams for which Feynman diagrams at a given level become particles at the next level. Accepting this idea, one is led to ask what kind of quantum states these Feynman diagrams correspond, how one could describe interactions of these higher level particles, what is the interpretation for these higher level states, and whether they can be detected.

Jones inclusions as analogs of space-time surfaces

The idea about space-time as a 4-surface replicates itself at the level of operator algebra and state space in the sense that Jones inclusion can be seen as a representation of the operator algebra $\mathcal{N}$ as infinite-dimensional linear sub-space (surface) of the operator algebra $\mathcal{M}$. This encourages to think that generalized Feynman diagrams could correspond to image surfaces in II_1 factor having identification as kind of quantum space-time surfaces.

Suppose that the modular S-matrices are representable as the inner automorphisms $\Delta(\mathcal{M}_k^{it}$ assigned to the external lines of Feynman diagrams. This would mean that $\mathcal{N} \subset \mathcal{M}_k$ moves inside $calM_k$ along a geodesic line determined by the inner automorphism. At the vertex the factors $calM_k$ to fuse along $\mathcal{N}$ to form a Connes tensor product. Hence the copies of $\mathcal{N}$ move inside $\mathcal{M}_k$ like incoming 3-surfaces in H and fuse together at the vertex. Since all $\mathcal{M}_k$ are isomorphic to a universal factor $\mathcal{M}$, many-sheeted space-time would have a kind of quantum image inside II_1 factor consisting of pieces which are $d = \mathcal{M} : \mathcal{N}/2$-dimensional quantum spaces according to the identification of the quantum space as subspace of quantum group to be discussed later. In the case of partonic Clifford algebras the dimension would be indeed $d \leq 2$.

The hierarchy of Jones inclusions defines a hierarchy of S-matrices

It is possible to assign to a given Jones inclusion $\mathcal{N} \subset \mathcal{M}$ an entire hierarchy of Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset \mathcal{M}_2 ...$, $\mathcal{M}_0 = N$, $\mathcal{M}_1 = M$. A possible interpretation for these inclusions would be as a sequence of topological condensations.

This sequence also defines a hierarchy of Feynman diagrams inside Feynman diagrams. The factor $\mathcal{M}$ containing the Feynman diagram having as its lines the unitary orbits of $\mathcal{N}$ under $\Delta_{\mathcal{M}}$ becomes a parton in $\mathcal{M}_1$ and its unitary orbits under $\Delta_{\mathcal{M}_1}$ define lines of Feynman diagrams in M_1. The concrete representation for M-matrix or projection of it to some subspace as entanglement coefficients of partons at the ends of a braid assignable to the space-like 3-surface representing a vertex of a higher level Feynman diagram. In this manner quantum dynamics would be coded and simulated by quantum states.

The outcome can be said to be a hierarchy of Feynman diagrams within Feynman diagrams, a fractal structure for which many particle scattering events at a given level become particles at the next level. The particles at the next level represent dynamics at the lower level: they have the property of "being about" representing perhaps the most crucial element of conscious experience. Since net conserved quantum numbers can vanish for a system in TGD

Universe, this kind of hierarchy indeed allows a realization as zero energy states. Crossing symmetry can be understood in terms of this picture.

One might perhaps say that quantum space-time corresponds to a double inclusion and that further inclusions bring in N-parameter families of space-time surfaces.

Higher level Feynman diagrams

The lines of Feynman diagram in $\mathcal{M}_{n+1}$ are geodesic lines representing orbits of $\mathcal{M}_n$ and this kind of lines meet at vertex and scatter. The evolution along lines is determined by $\Delta_{\mathcal{M}_{n+1}}$. These lines contain within themselves $\mathcal{M}_n$ Feynman diagrams with similar structure and the hierarchy continues down to the lowest level at which ordinary elementary particles are encountered.

For instance, the generalized Feynman diagrams at the second level are ribbon diagrams obtained by thickening the ordinary diagrams in the new time direction. The interpretation as ribbon diagrams crucial for topological quantum computation and suggested to be realizable in terms of zero energy states in [K77] is natural. At each level a new time parameter is introduced so that the dimension of the diagram can be arbitrarily high. The dynamics is not that of ordinary surfaces but the dynamics induced by the $\Delta_{\mathcal{M}_n}$.

Quantum states defined by higher level Feynman diagrams

The intuitive picture is that higher level quantum states corresponds to the self reflective aspect of existence and must provide representations for the quantum dynamics of lower levels in their own structure. This dynamics is characterized by M-matrix whose elements have representation in terms of Feynman diagrams.

(a) These states correspond to zero energy states in which initial states have "positive energies" and final states have "negative energies". The net conserved quantum numbers of initial and final state partons compensate each other. Gravitational energies, and more generally gravitational quantum numbers defined as absolute values of the net quantum numbers of initial and final states do not vanish. One can say that thoughts have gravitational mass but no inertial mass.

(b) States in sub-spaces of positive and negative energy states are entangled with entanglement coefficients given by M-matrix at the level below.

To make this more concrete, consider first the simplest non-trivial case. In this case the particles can be characterized as ordinary Feynman diagrams, or more precisely as scattering events so that the state is characterized by $\hat{S} = P_{in}SP_{out}$, where S is S-matrix and P_{in} *resp.* P_{out} is the projection to a subspace of initial *resp.* final states. An entangled state with the projection of S-matrix giving the entanglement coefficients is in question.

The larger the domains of projectors P_{in} and P_{out}, the higher the representative capacity of the state. The norm of the non-normalized state $\hat{S}$ is $Tr(\hat{S}\hat{S}^{\dagger}) \leq 1$ for II_1 factors, and at the limit $\hat{S} = S$ the norm equals to 1. Hence, by II_1 property, the state always entangles infinite number of states, and can in principle code the entire S-matrix to entanglement coefficients.

The states in which positive and negative energy states are entangled by a projection of S-matrix might define only a particular instance of states for which conserved quantum numbers vanish. The model for the interaction of Feynman diagrams discussed below applies also to these more general states.

The interaction of $\mathcal{M}_n$ Feynman diagrams at the second level of hierarchy

What constraints can one pose to the higher level reactions? How Feynman diagrams interact? Consider first the scattering at the second level of hierarchy ($\mathcal{M}_1$), the first level $\mathcal{M}_0$ being assigned to the interactions of the ordinary matter.

(a) Conservation laws pose constraints on the scattering at level $\mathcal{M}_1$. The Feynman diagrams can transform to new Feynman diagrams only in such a manner that the net quantum numbers are conserved separately for the initial positive energy states and final negative energy states of the diagram. The simplest assumption is that positive energy matter and negative energy matter know nothing about each other and effectively live in separate worlds. The scattering matrix form Feynman diagram like states would thus be simply the tensor product $S \otimes S^\dagger$, where S is the S-matrix characterizing the lowest level interactions and identifiable as unitary factor of M-matrix for zero energy states. Reductionism would be realized in the sense that, apart from the new elements brought in by $\Delta_{\mathcal{M}_n}$ defining single particle free dynamics, the lowest level would determine in principle everything occurring at the higher level providing representations about representations about... for what occurs at the basic level. The lowest level would represent the physical world and higher levels the theory about it.

(b) The description of hadronic reactions in terms of partons serves as a guide line when one tries to understand higher level Feynman diagrams. The fusion of hadronic space-time sheets corresponds to the vertices $\mathcal{M}_1$. In the vertex the analog of parton plasma is formed by a process known as parton fragmentation. This means that the partonic Feynman diagrams belonging to disjoint copies of $\mathcal{M}_0$ find themselves inside the same copy of $\mathcal{M}_0$. The standard description would apply to the scattering of the initial *resp.* final state partons.

(c) After the scattering of partons hadronization takes place. The analog of hadronization in the recent case is the organization of the initial and final state partons to groups I_i and F_i such that the net conserved quantum numbers are same for I_i and F_i. These conditions can be satisfied if the interactions in the plasma phase occur only between particles belonging to the clusters labeled by the index i. Otherwise only single particle states in $\mathcal{M}_1$ would be produced in the reactions in the generic case. The cluster decomposition of S-matrix to a direct sum of terms corresponding to partitions of the initial state particles to clusters which do not interact with each other obviously corresponds to the "hadronization". Therefore no new dynamics need to be introduced.

(d) One cannot avoid the question whether the parton picture about hadrons indeed corresponds to a higher level physics of this kind. This would require that hadronic space-time sheets carry the net quantum numbers of hadrons. The net quantum numbers associated with the initial state partons would be naturally identical with the net quantum numbers of hadron. Partons and they negative energy conjugates would provide in this picture a representation of hadron about hadron. This kind of interpretation of partons would make understandable why they cannot be observed directly. A possible objection is that the net gravitational mass of hadron would be three times the gravitational mass deduced from the inertial mass of hadron if partons feed their gravitational fluxes to the space-time sheet carrying Earth's gravitational field.

(e) This picture could also relate to the suggested duality between string and parton pictures [K67] . In parton picture hadron is formed from partons represented by space-like 2-surfaces X_i^2 connected by join along boundaries bonds. In string picture partonic 2-surfaces are replaced with string orbits. If one puts positive and negative energy particles at the ends of string diagram one indeed obtains a higher level representation of hadron. If these pictures are dual then also in parton picture positive and negative energies should compensate each other. Interestingly, light-like 3-D causal determinants identified as orbits of partons could be interpreted as orbits of light like string word sheets with "time" coordinate varying in space-like direction.

Scattering of Feynman diagrams at the higher levels of hierarchy

This picture generalizes to the description of higher level Feynman diagrams.

(a) Assume that higher level vertices have recursive structure allowing to reduce the Feynman diagrams to ordinary Feynman diagrams by a procedure consisting of finite steps.

(b) The lines of diagrams are classified as incoming or outgoing lines according to whether the time orientation of the line is positive or negative. The time orientation is associated with the time parameter t_n characterizing the automorphism $\Delta^{it_n}_{\mathcal{M}_\backslash}$. The incoming and outgoing net quantum numbers compensate each other. These quantum numbers are basically the quantum numbers of the state at the lowest level of the hierarchy.

(c) In the vertices the $\mathcal{M}_{n+1}$ particles fuse and $\mathcal{M}_n$ particles form the analog of quark gluon plasma. The initial and final state particles of $\mathcal{M}_n$ Feynman diagram scatter independently and the S-matrix S_{n+1} describing the process is tensor product $S_n \otimes S_n^\dagger$. By the clustering property of S-matrix, this scattering occurs only for groups formed by partons formed by the incoming and outgoing particles $\mathcal{M}_n$ particles and each outgoing $\mathcal{M}_{n+1}$ line contains and irreducible $\mathcal{M}_n$ diagram. By continuing the recursion one finally ends down with ordinary Feynman diagrams.

10.6 The master formula for the U-matrix finally found?

In zero energy ontology U-matrix replaces S-matrix as the fundamental object characterizing the predictions of the theory. U-matrix is defined between zero energy states and its orthogonal rows define what I call M-matrices, which are analogous to thermal S-matrices of thermal QFTs. M-matrix defines the time-like entanglement coefficients between positive and negative energy parts of the zero energy state.

A dramatic development of ideas related to the construction of U-matrix has taken place during the last year. In particular, twistorialization becomes possible in zero energy ontology and leads to the generalization of the Yangian symmetry of $\mathcal{N} = 4$ SUSY to TGD framework with the replacement of finite-dimensional super-conformal group of M^4 with infinite-D super-conformal group assignable to partonic 2-surfaces. What is so beautiful is that the physical IR cutoff due to the formation of bound states of massless wormhole throats resolves the infrared divergence problem whereas UV divergences are solved by on mass shell propagation of wormhole throats for virtual particles. This also guarantees that Yangian invariance is not lost. There are excellent reasons to expect that the twistorial constructions generalize.

10.6.1 What could be the master formula for the U-matrix?

The basic challenge is however still there and boils down to a simple question represented in the title. This master formula should be something extremely simple and should generalize the formula for S-matrix defined between positive energy states and identified formally as the exponential of Hamiltonian operator. In TGD framework the notion of unitary time development is given up so that something else is required and this something else should be manifestly Lorentz invariant and characterize the interactions.

Thinking the problem from this point of view allows only one answer: replace the time evolution operator defined by the Hamiltonian with the exponent for the action containing both bosonic and fermionic term. Bosonic term is the action for preferred extremal of Kähler action, which is indeed the unique Lorentz invariant defining interactions! Fermionic term would given by and boundary term $\int d^3x \sqrt{g_4}\overline{\Psi}\Gamma^n\Psi$ guaranteing that boundary terms for the variation of Kähler-Dirac equation are compensated. Kähler-Dirac action reduces to boundary term associated with light-like three surfaces and space-like 3-surfaces at the ends of CDs. This term defines fermionic propagator.

The normal component Γ^n_{K-D} of the modified gamma matrices defined by the canonical momentum currents of Kähler action should define the inverse of massless fermionic propagator. If the action of this operator on the induced spinor mode at stringy curves satisfies

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi \ ,$$

this reduction is achieved. One can pose the condition $g_4 = constant$ as a coordinate condition on stringy curves at the boundaries of CD and the condition would correlate the spinor modes at stringy curve with incoming quantum numbers. This is extremely powerful simplification giving hopes about calculable theory. The residue integral for virtual momenta reduces the situation to integral over on mass shell momenta and only non-physical helicities contribute in internal lines. This would generalize twistorial formulas to fermionic context.

The question is whether this action should be interpreted as the counterpart of action or effective action obtained by performing path integral in presence of external sources in QFT framework. Since one restricts space-time surfaces to preferred extremals so that there is no path integral in the usual sense, the only possible interpretation as the effective action. Also the condition that one obtains fermionic propagators correctly allows only this interpretation.

Stringy considerations inspired by the localization of spinor modes to string world sheets and possibly also at partonic 2-surfaces suggests that fermion lines correspond to the analogs of stringy propagator $1/G$ defined by product $(1/G)ip^k\gamma_k(1/G^\dagger) + h.c.$, where G is nonhermitian super-conformal generator carrying fermion number and p^k virtual momentum appears as hermitian stringy propagator. As a matter fact, one can asign that super generators to the 4-fermion vertex representing interaction of two incoming fermions via wormhole contact. Super-Virasoro generators would be associated with the strings connecting fermions at opposite wormhole throats. Massive external particles identified as bound states of massless fermions would be regarded as representations of super-conformal algebras. The fact that the massless momenta assignable to the opposite wormhole throats are not exactly parallel would induce massivation as many-sheeted effect.

Majorana condition fixing space-time dimension to 10 or 11 would not be needed. For massless states this reduces to $1/p^k\gamma^k$. This representation should give in the massless sector effective QFT action.

The action interpreted as a counterpart of QFT effective action reduces to the sum of fermionic and bosonic terms. To make the representation more fluent I will mean with 3-surfaces in the following either the light-like orbits of wormhole throats at which the signature of the induced metric changes or the ends of space-time sheets at the boundaries of CDs. Note that it is possible to have CDs within CDs and these give rise to loop corrections having interpretation as zero energy states in shorter length scale. Finite measurement resolution means that one integrates over these degrees of freedom below the resolution scale. This gives rise to discrete variant of gauge coupling evolution based on scalings by factor two for CDs.

The next unpleasant question was whether this U-matrix is actually only the S-matrix appearing in the expression of a given M-matrix as a product of a hermitian square root of density matrix and unitary S-matrix having interpretation as the TGD counterpart of the ordinary S-matrix. The physical picture suggests this strongly. This observation led to a realization that the square roots of density matrices can be identified as generators of infinite-dimensional Lie-algebra of unitary matrices. Unit norm requires that hyper-finite factor of type II_1 is in question. The construction reduces to that for unitary S-matrix.

10.6.2 Universal formula for the hermitian square roots of density matrix

Zero energy ontology replaces S-matrix with M-matrix and groups M-matrices to rows of U-matrix. S-matrix appears as factor in the decomposition of M-matrix to a product of hermitian square root of density matrix and unitary S-matrix interpreted in standard sense.

$$M_i = \rho_i^{1/2} S \quad .$$

Note that one cannot drop the S-matrix factor from M-matrix since M-matrix is neither unitary nor hermitian and the dropping of S would make it hermitian. The analog of the decomposition of M-matrix to the decomposition of Schrödinger amplitude to a product of its modulus and of phase is obvious.

The interpretation is in terms of square root of thermodynamics. This interpretation should give the analogs of the Feynman rules ordinary quantum theory producing unitary matrix when one has pure quantum states so that density matrix is projector in 1-D sub-space of state space (for hyper-finite factors of type II_1 something more complex is required).

This is the case. M-matrices are in this case just the projections of S-matrix to 1-D subspaces defined by the rows of S-matrix. The state basis is naturally such that the positive energy states at the lower boundary of CD have well-defined quantum numbers and superposition of zero energy states does not contain different quantum numbers for the positive energy states. The state at the upper boundary of CD is the state resulting in the interaction of the particles of the initial state. Unitary of the resulting U-matrix reduces to that for S-matrix.

A more general situation allows square roots of density matrices which are diagonalizable hermitian matrices satisfying the orthogonality condition that the traces

$$Tr(\rho_i^{1/2}\rho_j^{1/2}) = \delta_{ij} \quad .$$

The matrices span the Lie algebra of infinite-dimensional unitary group. The hermitian square roots of M-matrices would reduce to the Lie algebra of infinite-D unitary group. This does not hold true for zero energy states.

If one however assumes that S commutes with the algebra spanned by the square roots of density matrices and allows powers of S one obtains a larger algebra completely analogous to Kac-Moody algebra in the sense that powers of S takes the role of powers of $exp(in\phi)$ in Kac-Moody algebra generators. The commutativity of S and density matrices means that the square roots of density matrices span symmetry algebra of S. A possible interpretation for the sub-space spanned by M-matrices proportional to S^n is in terms of the hierarchy of CDs. If one assumes that the size scales of CDs come as octaves 2^m of a fundamental scale then one would have $m = n$. Second possibility is that scales of CDs come as integer multiples of the CP_2 scale: in this case the interpretation of n would be as this integer: this interpretation conforms with the intuitive picture about S as TGD counterpart of time evolution operator. This interpretation could also make sense for the M-matrice associated with the hierarchy of dark matter for which the scales of CDs indeed come as integers multiples of the basic scale.

If the square roots of density matrices are required to have only non-negative eigenvalues -as I have carelessly proposed in some contexts,- only projection operators are possible for Cartan algebra so that only pure states are possible. If one allows both signs one can have more interesting density matrices and this is the only manner to obtain square root of thermodynamics. Note that the standard representation for the Cartan algebra of finite-dimensional Lie group corresponds to non-pure state. For $\rho = Id$ one obtains $M = S$ defining the ordinary S-matrix. The orthogonality of this zero energy state with respect to other ones requires

$$Tr(\rho_i^{1/2}) = 0$$

stating that $SU(N = \infty)$ Lie algebra element is in question.

The reduction of the construction of U to that of S is an enormous simplification and reduces to the problem of finding the TGD counterpart of S-matrix. Note that the finiteness of the norm of $SS^\dagger = Id$ requires that hyper-finite factor of type II_1 is in question with the defining property that the infinite-dimensional unit matrix has unit norm. This means that state function reduction is always possible only into an infinite-dimensional subspace only [K79] .

The natural guess is that the Lie algebra generated by zero energy states is just the generalization of the Yangian symmetry algebra (see `http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#Yangian`this) of$\mathcal{N} = 4$ SUSY postulated to be a symmetry algebra of TGD . The characteristic feature of the `http://en.wikipedia.org/wiki/Yangian`Yangian algebra is the multi-locality of its generators. The generators of the zero energy algebra are zero energy states and indeed form a hierarchy of multi-local objects defined by partonic 2-surfaces at upper and lower light-like boundaries of causal diamonds. Zero energy states themselves would define the symmetry algebra of the theory and the construction of quantum TGD also at the level of dynamics -not only quantum states in sense of positive energy ontology- would reduce to the construction of infinite-dimensional Lie-algebra! It is hard to imagine anything simpler!

10.6.3 The basic action principle

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

(a) The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

    There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

(b) By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

(a) QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

(b) The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

(c) The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface.

Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

1. What one wants?

It is could to make first clear what one really wants.

(a) What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignement of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

(b) If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

(c) One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

2. Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{10.6.1}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

10.6.4 A proposal for M-matrix

This picture can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

(a) At the bosonic sector one would have converging functional integral over WCW. This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

(b) In fermionic sector Chern-Simons Dirac term in the action and the condition that spinors modes localized at string world sheets are eigenstates of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ defining virtual momentum would give effectively rise to massless Dirac action in M^4 and one would obtain massless fermionic propagators. The generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

(c) Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as gauge theory is natural.

(d) Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts.

The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

(e) The interaction vertices would correspond to the scattering of fermions at opposite wormhole throats. The natural guess is that the propagator is essentially the inverse of the scaling generator L_0 of conformal algebra. Non-locality suggests that one must product for the inverses of the super-generators G and its hermitian conjugate estimated at the two wormhole throats. There the diagrammatics would be combinations of that for QFT with massless fermions and string model diagrammatics. Topologically the vertices would be analogous to Feynman vertices: two 3-surfaces would fuse at vertices to form third. Stringy trouser diagrams would not have interpretation as decays of particle but as particle travelling two different paths.

(f) Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures ??, ??, `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg` or fig. 6, tgdgraphs in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics.

10.6.5 Definition of U-matrix

The definition of U-matrix would be shockingly simple once the reduction to the construction of M- and S-matrices is accepted. Just the exponential if algebraic variant of massless M^4 Dirac action with gradient replaced with the momentum propagating in fermionic line besides Kähler action reducing to Chern-Simons term and defining the weight for the functional integral over 3-surface. What is encouraging that the resulting U-matrix would be more or less the same as the one expected on basis of heuristic considerations.

(a) The basis for bare zero energy states is obtained by using pairs of positive and negative energy states assigned to the boundaries of CD and having opposite quantum numbers. The action of the exponent of Kähler action and effective Dirac action generates from these states "dressed" states and U-matrix is defined between these stressed states and bare states. M-matrix in turn is defined by the action of L on given bare zero energy states as entanglement coefficients.

(b) U-matrix is automatically unitary in the fermionic degrees of freedom since the effective M^4 Dirac action with the inverse of the usual kinetic term on the role of kinetic term is Hermitian operator. In bosonic degrees of freedom one expects unitarity by the analogy with finite dimensional function space endowed with inner product with vacuum functional defining the weighting. This would mean a beautiful solution to the long standing problem of how to achieve unitarity.

(c) There are strong reasons to believe that a duality prevails in the sense that one can restrict the interior part of action to either the Euclidian regions of space-time surfaces defining 4-D Feynman diagram or to their Minkowskian exterior. Number theoretic vision [K67] suggests this duality and the recent considerations [K22] support the same conclusion. Obviously this duality brings in mind Wick rotation of quantum field theories.

(d) The fermionic action corresponds formally to free action so that there are no explicit interaction vertices: the situation in the geometric formulation of string theory is same. There is however no need for non-linear interaction terms which are also responsible for the divergences of quantum field theories. The interaction terms are replaced with topological interaction vertex at which the light-like 3-surfaces associated defining the orbits of partonic 2-surfaces (wormhole throats) meet like lines of the ordinary Feynman diagram.

Note that this vertex distinguishes between TGD and string models where trouser vertex is a typical vertex: in TGD framework this kind of geometric decay does not correspond to particle decay but to the propagation of particle along different paths. The conservation of quantum numbers is required at the vertices. Also massless-ness property for the particles propagating along the lines is natural in zero energy ontology and makes possible twistorialization with the constraint that physical particles are massive bound states of massless wormhole throats.

(e) The non-trivial propagation of state with total number n of fermions and anti-fermions is possible only if n contractions of the propagator appears along the line (otherwise one would obtain only quark lepton contractions forbidden by conservation laws). This implies the proportionality $1/p^n$ of the propagator so that only total fermion number $n = 1, 2$ is possible for non-vacuum wormhole throat. This proportionality was earlier deduced from the SUSY limit of TGD based on a generalization of SUSY algebra [K23] . As a consequence, wormhole contact having two throats can carry at most spin 2 and the large SUSY defined by the fermionic oscillator operators is badly broken and effectively reduced to that generated by the right-handed neutrino which is also broken.

(f) The assumption that all particles have non-vanishing mass means that given state can decay only to a virtual state with finite number of particles. This together with massless propagation along virtual lines simplifies enormously the perturbation series and is expected to imply finiteness.

(g) The integration over WCW could spoil the unitarity. Although the exponent of Kähler action is positive it could give rise to divergent integral if the Kähler action has definite sign. The reduction to Chern-Simons term does not make obvious the positivity. If one believes on Minkowskian-Euclidian duality in the sense that one can define vacuum functional either as the exponent of Kähler action for the Minkowskian or Euclidian regions, one obtains definite sign for the Kähler function since for the Euclidian signature Kähler action indeed has definite sign.

What is remarkable that in Chern-Simons term the non-analytic $1/g_K^2$ dependence on Kähler coupling strength disappears by the the weak form of electric-magnetic duality so that perturbation series with respect to the small parameter g_K^2 should make sense. One expects that this expansion gives small contributions to coupling constants determined in lowest order by bosonic emergence and involving fermionic loops.

(h) The resulting generalized Feynman diagrammatics differs from the standard one in many respects. The lines of Feynman diagrams are replaced with 3-surfaces in the sense specified above. Only a very restricted subset of loops are allowed classically by preferred extremals. The massless on mass shell property for wormhole throat momenta indeed allows very restricted phase space for loops. If all particles are massive bound states of massless wormhole throats intermediate virtual particles states with positive energies can contain only a finite number of particles so that the situation simplifies dramatically. The already mentioned collinear many-fermion states with propagator behaving like $1/p^n$, $n > 2$ are also present. Hence on can ask whether a more appropriate identification of generalized Feynman diagrams might be as counterparts of twistor diagrams.

10.6.6 What is the relationship of generalized Feynman diagrams to twistor diagrams?

The general idea about the construction of U-matrix gives strong support for the existing heuristics and provides a connection with category theoretical ideas (planar operads and generalized Feynman diagramatics [K8]) and also suggests a generalization of twistor diagrammatics. Many questions of course remain unanswered. The basic question is the relationship of generalized Feynman diagrams with twistor diagrams. There are arguments favoring also the interpretation as direct counterparts of twistor diagrams. The following series of arguments however favors Feynman diagram interpretation and leads to a precise connection between the two diagrammatics. The arguments rely on following general ideas which deserve to be restated.

(a) The realization that the hermitian square roots of density matrices form infinite-D unitary algebra and that their commutativity with universal S-matrix implies that zero energy states define the generalization of Kac-Moody algebra became only after I had realized the possibility to construct U-matrix. It is this observation which reduces the construction of U-matrix (or matrices if they form algebra) to that for S is expected to correspond directly to the ordinary S-matrix. A possible interpretation of the algebra of U-matrices is in terms of scales of CDs coming as positive integer powers of two. Another possibility more in line with the usual interpretation of S-matrix as time evolution operator is that scales of CDs come as integers and these integers correspond to powers of S.

(b) In ordinary QFT Feynman diagrams are purely algebraic objects. In TGD framework they reduce to space-time topology and geometry with Euclidian regions of space-time surfaces having interpretation as generalized Feynman diagrams. At the vertices of generalized Feynman diagrams in coming partonic 2-surfaces meet just like in ordinary Feynman diagrams which means deep difference from string theory. A more general assumption is that entire 4-D lines of generalized Feynman diagram meet at vertices. This could apply to the Euclidian regions only.

There is also a second kind of branching involved with the hierarchy of Planck constants. In Minkowskian regions similar meeting would take place for the branches of space-time sheets with same values of canonical momentum densities of Kähler action at the ends of CDs and have interpretation in terms of fractionization and hierarchy of Planck constants. The value of Planck constant for single branch would be effectively and integer multiple of the ordinary one. For the entire multi-sheeted structure describable naturally in terms of singular covering space of $M^4 \times CP_2$ it would be just the ordinary value.

(c) Zero energy ontology with massless external wormhole wormholes implies as such twistorialization of the theory although external wormhole momenta must be assumed to be massive bounds states of massless throats. This also guarantees exact Yangian symmetry and the absence of IR divergences. If also virtual wormhole throats are massless, twistorialization takes place in strong sense. This is possible only in zero energy ontology and accepting the identification of wormhole throats as basic building blocks of particles. Zero energy ontology leads also to an unexpected connection between infinite-dimensional Lie algebras and the space of allowed Hermitian square roots of density matrices multiplying unitary S-matrix in M-matrix.

(d) The notion of bosonic emergence means that bosonic propagators emerge as radiative loops for wormhole contacts (see fig. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg` or fig. 10 in the appendix of this book). The emergence generalizes to all states associated with wormhole contacts and also to flux tubes having wormhole contacts at their ends. What is nice that coupling constants emerge as normalization factors of propagators. Note that for single wormhole throat as opposed to wormhole contact having two throats bosonic propagator would result as a product of two collinear fermionic propagators and have the standard form. For states with higher total number of fermions and anti-fermions the propagator of wormhole throat behaves as p^n, $n > 2$. Here however p is replaced with what I call pseudo-momentum.

(e) Number theoretical universality suggest that at given level (CD) only finite sum of diagrams appears: otherwise there is danger that one obtains sum of rational functions which is not rational anymore. This gives strong constraints on generalized Feynman diagrams at the lowest level of the hierarchy.

(f) Category theoretical approach based on planar operad proposed for few years ago [K8] fits nicely with the twistorial construction of amplitudes interpreting radiative corrections in terms of CDs within CDs picture.

What is the correct identification of pseudo-momenta?

The eigen modes of Chern-Simons Dirac operator are characterized by generalized eigenvalues the quantities $\lambda^k\gamma_k$. I christened λ as pseudo-momentum and proposed a number theoretic quantization rules for the values of pseudo-momenta [K22]. The physical interpretation of pseudo-momenta was still open as I wrote the first version of this secion as also their relationship to massless on mass shell momenta propagating along wormhole throats associated with virtual particles. It is convenient to consider wormhole contact with two wormhole throats as a representation of incoming or virtual particle. The questions are following.

(a) Is there a summation over pseudo-momenta for wormhole throats such that the sum of pseudo-momenta equals to the total exchanged real momentum associated with the wormhole contact. The real momenta on virtual line would be massless and give strong kinematic conditions on phase space allowed in loops.

Physical propagators from wormhole contacts would result as self energy loops for pseudo-momenta and there is the danger of getting divergences unless one uses the number theoretic conditions to reduce the summation as proposed. This picture would realize the idea about the emergence of bosonic propagators as fermionic radiative corrections and also more general propagators. Coupling constants would be predicted and appear in the normalization of bosonic propagators. Note that also the integration over WCW degrees of freedom affects the values of coupling constants.

The question is how strong additional conditions the number theoretic quantization of pseudo-momenta poses on the exchanged massless real momenta depends on the strength of number theoretical conditions. Are these conditions sensible?

(b) Can one really identify pseudo-momenta as region momenta of the twistor approach as I have cautiously suggested? The above line of arguments does not encourage this interpretation. Whether the identification makes sense can be tested immediately by looking for the dependence of Grassmannian twistor amplitudes on pseudo-momenta. If it is of standard propagator form one can consider this interpretation.

Years after writing the first version of this section is has become clear that pseudomomenta as virtual momenta of massless fermions assignable to boundaries of string world sheets is natural. The fermion fields at partonic orbits are can be chosen freely and Chern-Simons Dirac action serves defines therefore the analog of Dirac action since the one can chose spinor modes to be eigenstates of C-S-D operator. The identification as region momenta is possible but now region momenta are uniquely fixed.

Connection between generalized Feynman diagrams and generalized twistor diagrams

The connection between generalized Feynman diagrams and generalized twistor diagrams should be understood.

(a) The natural manner to identify twistor diagrams for a given CD without radiative corrections given by the addition of sub-CDs would be as the diagrams obtained by connecting the points or upper and lower boundaries of CD to form a polygon. There are several manners to do this. The differences of region-momenta would give the massless momenta for each external wormhole throat. Region momenta would have nothing to do with pseudo-momenta.

(b) Twistor diagrams would represent sum for a subset of allowed generalized Feynman diagrams with massless particles in internal lines. On mass shell condition for massless wormhole throats restricts dramatically the number of contributing diagrams and the assumption that all particles have at least small mass means that particle numbers in intermediate states are finite. One however obtains infinite number of diagrams obtained as series of allowed diagrams. The problem is that although individual diagrams give rational functions, an infinite sum of them leads out from the algebraic extensions of p-adic numbers and rationals. This does not conform with number theoretic universality.

Therefore only irreducible diagrams not decomposing to series of allowed scatterings are allowed. As a consequence only finite number of diagrams are possible. The sum of these diagrams would correspond to a given basic twistor diagram. One could consider also the condition that at given length scaled determined by CD only tree diagrams are allowed. but this option looks ad hoc.

The addition of sub-CD:s would give radiative corrections from shorter length scales and the depth of the hierarchy of CDs within CDs hierarchy defines the IR and UV cutoffs and measurement resolution. If one accepts the assumption that the sizes of CD come as octaves of CP_2 time scale, there would be natural IR and UV cutoffs on the values of pseudo-momenta from p-adic length scale hypothesis so that the amplitudes should remain finite and there would no fear about the loss of number theoretic universality.

Note that in zero energy ontology cutoffs would characterize physical states themselves rather than restrictions of physicist only.

Diagrammatics based on gluing of twistor amplitudes

Radiative corrections n shorter scales than that of CD would result from the gluing of basic amplitudes for CDs within CDs.

(a) Radiative corrections could be organized in terms of twistor diagrams. The rule transforming twistor polygons to simplest Feynman diagrams is standard duality replacing polygon with external lines at vertices with a bundle of lines obtained by connecting external lines to same point in the interior of the polygon. For triangle this gives three vertex. For n-polygon this would give n-vertex which corresponds to tree diagram as a Feynman diagram.

For instance, one can understand self energy corrections in this framework in terms of two twistorial triangles with two edges of both connected by two lines. Again on mass shell massless holds true for the throats. Vertex correction corresponds to triangle triangle within triangle with vertices of the inner triangle connected to the vertices of the outer triangle.. One obtains radiative corrections from this picture.

(b) Also now one can have loops obtained as a closed ring of polygons connected to each other. There are also much more complex configurations of polygons. Unless one allow splitting of wormhole contacts the wormhole lines associated with a given wormhole throat end up to single CD.

(c) For an outgoing pair of wormhole lines from given CD the wormhole throats should have same sign of energy: this would mean that only time-like momenta can propagate between CDs so that space-like loop momenta would be possible only for the fundamental radiative corrections. This would a further strong restriction on the amplitudes and space-like momentum exchanges would come from the fundamental level involving only a finite number of diagrams.

Is this good or bad? If bad, should one be ready to assign independent CDs with the two wormhole throats? Or should the interpretation be that the wormhole contact is split and wormhole throats propagate in two different time directions? But is it possible to speak about single space-like momentum exchange if the wormhole contact is split. Note that pseudo-momentum propagator for wormhole throat would still make sense. This line of thought does not look attractive.

(d) Massless particles assigned with wormhole lines connecting the polygons and net pseudo-momenta are not on mass shell. Apart from time-likeness of net momenta, the rules for the propagators seem exactly the same as for polygons. These rules would summarize how radiative corrections from shorter scales are obtained.

10.6.7 Generalized twistor diagrams and planar operads

The resulting diagrams would have very close resemblance to structures known as planar operads [A23, A22, A57] associated with both topological quantum field theories and subfactors of von Neumann algebras. Planar operads provide a graphic representation of these structures. Since TGD corresponds to almost topological QFT and since WCW ("world of classical worlds") Clifford algebras correspond to von Neumann algebras known as hyperfinite factors of type II_1 [K79] , the natural expectation is that generalized Feynman diagrams correspond to planar operads. This is indeed what I proposed for three years ago in [K8] but with disks replaced with CDs so that a the recent view unifies several earlier visions.

An additional motivation for the operad picture came from the notion of super-symplectic analog of super-conformal field theory motivated by the assumption that the symplectic transformations of $\delta M^4_\pm \times CP_2$ act as isometries of WCW. The fusion rules of super-symplectic QFT lead to an infinite hierarchy of algebras forming an operad.

The basic structure of planar operad is very much reminiscent of generalized twistor diagrams.

(a) One has essentially disks within disks connected by lines. The modification is obvious. Replace disks within disks disks with CDs within CDs and assign to the upper *resp.* lower boundaries of CDs positive *resp.* negative energy states. Many-sheeted space-time allows locally two CDs above each other corresponding to the identification of particles as wormhole contacts.

(b) The planarity of the operad would be an obvious correlate for the absence of non-planar loops in twistor approach to $N = 4$ SUSY making it problematic. Stringy picture actually suggests the absence of non-planar diagrams. The proposed generalization of twistor diagrammatics allowing arbitrary polygons within polygons structure might be enough to compensate for the absence of non-planar diagrams.

To sum up, the recent view generalizes considerably twistor diagrammatics and gives a connection with hyper-finite factors of type II_1 and with planar operads. The identification of virtual states as composites of massless states is extremely natural in this framework. The construction is also consistent with the heuristic picture about generalized Feynman diagrams and with the earlier proposal about role of the planar operad. For these reasons I dare to expect that a big step towards precise form of the rules has been made

10.7 Anatomy of quantum jump in zero energy ontology

Consider now the anatomy of quantum jump identified as a moment of consciousness in the framework of ZEO [K39].

(a) Quantum jump begins with unitary process U described by unitary matrix assigning to a given zero energy state a quantum superposition of zero energy states. This would represent the creative aspect of quantum jump - generation of superposition of alternatives.

(b) The next step is a cascade of state function reductions proceeding from long to short scales. It starts from some CD and proceeds downwards to sub-CDs to their sub-CDs to ...... At a given step it induces a measurement of the quantum numbers of either positive or negative energy part of the quantum state. This step would represent the measurement aspect of quantum jump - selection among alternatives.

(c) The basic variational principle is Negentropy Maximization Principle (NMP) [K39] stating that the reduction of entanglement entropy in given quantum jump between two subsystems of CD assigned to sub-CDs is maximal. Mathematically NMP is very similar to the second law although states just the opposite but for individual quantum system rather than ensemble. NMP actually implies second law at the level of ensembles as a trivial consequence of the fact that the outcome of quantum jump is not deterministic.

    For ordinary definition of entanglement entropy this leads to a pure state resulting in the measurement of the density matrix assignable to the pair of CDs. For hyper-finite factors of type II_1 (HFFs) state function reduction cannot give rise to a pure state and in this case one can speak about quantum states defined modulo finite measurement resolution and the notion of quantum spinor emerges naturally. One can assign a number theoretic entanglement entropy to entanglement characterized by rational (or even algebraic) entanglement probabilities and this entropy can be negative. Negentropic entanglement can be stable and even more negentropic entanglement can be generated in the state function reduction cascade.

The irreversibility is realized as a property of zero energy states (for ordinary positive energy ontology it is realized at the level of dynamics) and is necessary in order to obtain non-trivial U-matrix. State function reduction should involve several parts. First of all it should select the density matrix or rather its Hermitian square root. After this choice it should lead to a state which prepared either at the upper or lower boundary of CD but not both since this would be in conflict with the counterpart for the determinism of quantum time evolution.

10.7.1 Generalization of S-matrix

ZEO forces the generalization of S-matrix with a triplet formed by U-matrix, M-matrix, and S-matrix. The basic vision is that quantum theory is at mathematical level a complex square roots of thermodynamics. What happens in quantum jump was already discussed.

(a) U-matrix as has its rows M-matrices , which are matrices between positive and negative energy parts of the zero energy state and correspond to the ordinary S-matrix. M-matrix is a product of a hermitian square root - call it H - of density matrix ρ and universal S-matrix S commuting with H: $[S,H]=0$. There is infinite number of different Hermitian square roots H_i of density matrices which are assumed to define orthogonal matrices with respect to the inner product defined by the trace: $Tr(H_iH_j)=0$. Also the columns of U-matrix are orthogonal. One can interpret square roots of the density matrices as a Lie algebra acting as symmetries of the S-matrix.

(b) One can consider generalization of M-matrices so that they would be analogous to the elements of Kac-Moody algebra. These M-matrices would involve all powers of S.

   i. The orthogonality with respect to the inner product defined by $\langle A|B\rangle = Tr(AB)$ requires the conditions $Tr(H_1H_2S^n)=0$ for $n\neq 0$ and H_i are Hermitian matrices appearing as square root of density matrix. H_1H_2 is hermitian if the commutator $[H_1,H_2]$ vanishes. It would be natural to assign n:th power of S to the CD for which the scale is n times the CP_2 scale.

   ii. Trace - possibly quantum trace for hyper-finite factors of type II_1) is the analog of integration and the formula would be a non-commutative analog of the identity $\int_{S^1} exp(in\phi)d\phi=0$ and pose an additional condition to the algebra of M-matrices. Since $H=H_1H_2$ commutes with S-matrix the trace can be expressed as sum $\sum_{i,j} h_is_j(i)=\sum_{i,j} h_i(j)s_j$ of products of correspondence eigenvalues and the simplest condition is that one has either $\sum_j s_j(i)=0$ for each i or $\sum_i h_i(j)=0$ for each j.

   iii. It might be that one must restrict M-matrices to a Cartan algebra for a given U-matrix and also this choice would be a process analogous to state function reduction. Since density matrix becomes an observable in TGD Universe, this choice could be seen as a direct counterpart for the choice of a maximal number of commuting observables which would be now hermitian square roots of density matrices. Therefore ZEO gives good hopes of reducing basic quantum measurement theory to infinite-dimensional Lie-algebra.

10.7.2 A concise description of quantum jump

In the following a minimalistic view about quantum jump is described. Both U-process and state preparation reduce to state function reductions to two basis for zero energy states characterized by opposite arrows of geometric time.

Unitary process and choice of the density matrix

The basic question concerning U process is which of the following two options U-process corresponds to.

(a) U-process occurs for zero energy states. U-matrix would be defined in the space of zero energy states and would represent kind of higher order scattering whereas M-matrix and S-matrix as time-like entanglement coefficients would describe what happens in a scattering experiment. This kind of possibility can be certainly considered since one can form zero energy states using zero energy states as building bricks. Entire hierarchy of zero energy states could be constructed in this manner.

(b) U-process can be said to occur for either positive or negative energy parts of zero energy states. This option is definitely minimal and in this case U-process for positive (negative) energy part of the state is dual to state function reduction for the negative (positive) energy part of the state. Furthermore, state function reduction is dual to state preparation. For this reason this option deserves to be called minimalistic.

During years I have considered both options without clearly distinguishing between them. The notion of time is very difficult concept: we do not have brain for time. Below I will consider only the minimalistic option in the hope that Nature would prefer minimalism also at this time. There is no need to emphasize how speculative these considerations are.

Consider first unitary process followed by the choice of the density matrix for the minimalistic option.

(a) There are two natural state basis for zero energy states. The states of these state basis are prepared at the upper or lower boundary of CD respectively and correspond to various M-matrices M_K^+ and M_L^-. U-process is simply a change of state basis meaning a representation of the zero energy state $M_K^\pm$ in zero energy basis $M_K^\mp$ followed by a state preparation to zero energy state $M_K^\pm$ with the state at second end fixed in turn followed by a reduction to $M_L^\mp$ to its time reverse, which is of same type as the initial zero energy state.

The state function reduction to a given M-matrix $M_K^\pm$ produces a state for the state is superposition of states which are prepared at either lower or upper boundary of CD. It does not yet produce a prepared state on the ordinary sense since it only selects the density matrix.

(b) The matrix elements of U-matrix are obtained by acting with the representation of identity matrix in the space of zero energy states as

$$I = \sum_K |K^+\rangle\langle K^+|$$

on the zero energy state $|K^-\rangle$ (the action on $|K^+\rangle$ is trivial!) and gives

$$U^+_{KL} = Tr(M_K^+ M_L^+) \ .$$

In the similar manner one has

$$U^-_{KL} = (U^{+\dagger})_{KL} = Tr(M_L^- M_K^-) = \overline{U^+_{LK}} \ .$$

These matrices are Hermitian conjugates of each other as matrices between states labelled by positive or negative energy states. The interpretation is that two unitary processes are possible and are time reversals of each other. The unitary process produces a new state only if its time arrow is different from that for the initial state. The probabilities for transitions $|K_+\rangle \to |K_-\rangle$ are given by $p_{mn} = |Tr(M_K^+ M_L^+)|^2$.

State function preparation

Consider next the counterparts of the ordinary state preparation process.

(a) The ordinary state function process can act either at the upper or lower boundary of CD and its action is thus on positive or negative energy part of the zero energy state. At the lower boundary of CD this process selects one particular prepared states. At the upper boundary it selects one particular final state of the scattering process.

(b) Restrict for definiteness the consideration to the lower boundary of CD. Denote also M_K by M. At the lower boundary of CD the selection of prepared state - that is preparation process- means the reduction

$$\sum_{m^+n^-} M^\pm_{m^+n^-}|m^+\rangle|n^-\rangle \to \sum_{n^-} M^\pm_{m^+n^-}|m^+\rangle|n^-\rangle \ .$$

The reduction probability is given by

$$p_m = \sum_{n^-} |M_{m^+n^-}|^2 = \rho_{m^+m^+} \quad .$$

For this state the lower boundary carries a prepared state with the quantum numbers of state $|m_+\rangle$. For density matrix which is unit matrix (this option giving pure state might not be possible) one has $p_m = 1$.

State function reduction process

The process which is the analog of measuring the final state of the scattering process is also needed and would mean state function reduction at the upper end of CD - to state $|n^-\rangle$ now.

(a) It is impossible to reduce to arbitrary state $|m_+\rangle|n_-\rangle$ and the reduction must at the upper end of CD must mean a loss of preparation at the lower end of CD so that one would have kind of time flip-flop!

(b) The reduction probability for the process

$$|m_+ \equiv \sum_{n^-} M_{m^+n^-}|m^+\rangle|n^-\rangle \rightarrow n_- = \sum_{m^+} M_{m^+n^-}|m^+\rangle|n^-\rangle$$

would be

$$p_{mn} = |M_{mn}^2| \quad .$$

This is just what one would expect. The final outcome would be therefore a state of type $|n^-\rangle$ and - this is very important- of the same type as the state from which the process began so that the next process is also of type U^+ and one can say that a definite arrow of time prevails.

(c) Both the preparation and reduction process involves also a cascade of state function reductions leading to a choice of state basis corresponding to eigenstates of density matrices between subsystems.

10.7.3 Questions and answers

Answering to question is the best possible manner to develop ideas in more comprehensible form. In this respect the questions of Hamed at my blog have been especially useful. Many questions below are originally made by him and inspired the objections, many of them discussed also in previous discussions. The answers to these questions have changed during latest years as the views about self and the relation between experienced time and geometric time have developed. The following answers are the most recent ones.

Question: The minimalistic option suggests very strongly that our sensory perception can be identified as quantum measurement assignable to state function reductions for upper or lower boundaries of our personal CD. Our sensory perception does not however jump between future and past boundaries of our personal CD (containing sub-CDS in turn containing)! Why?

Possible answer: The answer to this question comes from the realization that in ordinary quantum theory state function reductions leaving the reduced state invariant are possible. This must have counterpart in ZEO. In ZEO reduces zero energy states are superpositions of zero energy states associated with CDs with second boundary fixes inside light-cone boundary and the position of the second boundary of CD varying: one can speak about wave function in the moduli space of CDs. The temporal distance between the tips of CD and discrete lattice of the 3-D hyperbolic space defined by the Lorentz boosts leaving second tip invariant corresponds to the basic moduli.

The repeated state function reductions leave both the fixed boundary and parts of zero energy states associated with this boundary invariant. They however induce dispersion in the moduli space and the average temporal distance between the tips of CDs increases. This gives rise to the flow of psychological time and to the arrow of time. Self as counterpart of observer can be identified as a sequence of quantum jumps leaving the fixed boundary of CD invariant. Sensory perception gives information about varying boundary and the fixed boundary creates the experience about self as invariant not changed during quantum jumps.

Self hierarchy corresponds to the hierarchy of CDs. For instance, we perceive from day to day the - say- positive energy part of a state assignable to this very big CD. Hence the world looks rather stable.

Question: This suggests that our sensory perception actually corresponds to sequences of state function reductions to the two fixed boundaries of CDs of superposition of CDs so that our sensory inputs would alternately be about upper and lower boundaries of personal CDs. Sleep-awake cycle could correspond to a flip flop in which self falls asleep at boundary and wakes up at opposite boundary. Doesn't this lead to problems with the arrow of time?

Possible answer: If we measure time relative to the fixed boundary then the geometric time defined as the average distance between tips in superposition of CDs would increase steadily and we get older also during sleep. Hence we would experience subjective time to increase. Larger CDs than our personal CD for which the arrow of time remains fixed in the time scale of life cycle would provide the objective measure of geometric time.

Question: What is the time scale assignable to my personal CD: the typical wake-up cycle: 24 hours? Or of the order of life span. Or perhaps shorter?

Possible answer: The durations of wake-up periods for self is determined by NMP: death means that NMP favors the next state function to take place at the opposite boundary. The first naive guess is that the duration of the wake up period is of the same order of magnitude as the geometric time scale of our personal CD. In wake-up state we we would be performing state function reduction repeatedly to say "lower" boundary of our personal CD and sensory mental images as sub-CDs would be concentrated near opposite boundary. During sleep same would happen at lower boundary of CD and sensory mental images would be at opposite boundary (dreams,).

Question: Are dreams sensory perceptions with opposite arrow of time or is some sub-self in wake-up state and experiences same arrow of time as we during wake-up state? If the arrow is different in dreams, is the "now" of dreams in past and "past" in the recent of wake-upe state

Possible answer: Here I can suggest an answer based on my own subjective experiences and it would be cautious "yes".

Question: Why we do remember practically nothing about sensory perceptions during sleep period? (Note that we forget actively dream experiences).

Possible answer: That we do not have many memories about sleep and dream time existence and that these memories are unstable should relate to the change of the arrow of personal time as we wake up. Wake-up state should somehow rapidly destroy the ability to recall memories about dreams and sleep state. Wake-up memory recall means communications to geometric past, that is to the boundary of CD which remains fixed during wake-up state. In memory recall for dreams in wake-up state these communications should take place to geometric future. Memory recall of dreams would be seeing to future and much more difficult since the future is changing in each state function reduction so that dream memories are erased automatically during wake-up.

Question: Does the return to childhood at old age relate with this time flip-flop of arrow of time in the scale of life span: do we re-incarnate in biologically death at opposite end of CD with scale of life span?

Possible answer: Maybe this is the case. If this boundary corresponds to time scale of life cycle, the memories would be about childhood. Dreams are often located to the past and childhood.

10.7.4 More about the anatomy of state function reduction

In a comment to previous posting Ulla gave a link to an interesting article by George Svetlichny [J1] describing an attempt to understand free will in terms of quantum measurement. After reading of the article I found myself explaining once again to myself what state function reduction in TGD framework really means.

The proposal of Svetlichny

The basic objection against assigning free will to state function reduction in the sense of wave mechanics is that state function reduction from the point of view of outsider is like playing dice. One can of course argue that for an outsider *any* form of free will looks like throwing a dice since causally effective experience of free will is accompanied by non-determinism. We simply do cannot know what is the experience possibly associated with the state function reduction. The lesson is that we must carefully distinguish between two levels: the single particle level and ensemble level - subjective and objective. When we can say that something is random, we are talking about ensembles, not about single member of ensemble.

The author takes the objection seriously and notices that quantum measurement means a division of system to three parts: measured system, measuring system and external world and argues that in some cases this division might not be unique. The choice of this division would have interpretation as an act of free will. I leave it to the reader can decide whether this proposal is plausible or not.

TGD view about state function reduction

What can one say about the situation in TGD framework? There are several differences as compared to the standard measurement "theory", which is just certain ad hoc rules combined with Born rule, which applies naturally also in TGD framework and which I do not regard as ad hoc in infinite-D context.

In the sequel I will discuss the possible anatomy of the state function reduction part of the quantum jump.

(a) TGD ontology differs from the standard one. Space-time surfaces and quantum states as such are zombies in TGD Universe: consciousness is in the quantum jump. Conscious experience is in the change of the state of the brain, brain state as such is not conscious. Self means integration of quantum jumps to higher level quantum jumps and the hierarchy of quantum jumps and hierarchy of selves can be identified in ZEO . It has the hierarchy of CDs and space-time sheets as geometrical correlates. In TGD Universe brain and body are not conscious: rather, conscious experience is about brain and body and this leads to the illusion caused by the assimilation with the target of sensory input: I am what I perceive.

(b) In TGD framework one does not assume the division of the system to a product of measured system, measuring system, and external world before the measurement. Rather, this kind of divisions are outcomes of state function reduction which is part of quantum jump involving also the unitary process. Note that standard measurement theory is not able to say anything about the dynamics giving rise to this kind of divisions.

(c) State function reduction cascade as a part of quantum jump - this holistic view is one new element - proceeds in zero energy ontology (ZEO) from long to short length scales $CD \rightarrow sub - CDs \rightarrow ...$, and stops when Negentropy Maximization Principle (NMP [K39] defining the variational principle of consciousness is also something new) does not allow to reduce entanglement entropy for any subsystem pair of subsystem un-entangled with the external world. This is the case if the sub-system in question is such that all divisions to two parts are negentropically entangled or form an entangled bound state.

An interesting possibility is that negentropic entanglement does not correspond to bound state entanglement. The negentropically entangled particles would remain correlated by NMP rather than being in the jail defined by the interaction potential. I have proposed that this analog of love marriage could be fundamental for understanding living matter and that high energy phosphate bond central for ADP-ATP process could involve negentropic entanglement [K34].

For a given subsystem occurring in the cascade the splitting into an unentangled pair of measured and measuring system can take place if the entanglement between these subsystems is entropic. The splitting takes place for a pair with largest entanglement entropy and defines measuring and measured system.

Who measures whom? This seems to be a matter of taste and one should not talk about measuring system as conscious entity in TGD Universe, where consciousness is in quantum jump.

(d) The factorization of integer to primes is a rather precise number theoretical analogy for what happens, and the analogy might actually have a deeper mathematical meaning since Hilbert spaces with prime dimension cannot be decomposed into tensor products. Any factorization of integer to a product of primes corresponds to a cascade of state function reductions. At the first step division takes place to two integers and several alternative divisions are possible. The pair for which the reduction of entanglement entropy is largest, is preferred. The resulting two integers can be further factorized to two integers, and the process continues and eventually stops when all factors are primes and no further factorization is possible.

One could even assign to any decomposition $n = rs$ the analogs of entanglement probabilities as $p_1 = log(r)/log(n)$ and $p_2 = log(s)/log(n)$. NMP would favor the divisions to factors r and s which are as near as possible to $n/2$.

Negentropically entangled system is like prime. Note however that these systems can still make an analog of state function reduction which does not split them but increases the negentropy for all splittings of system to two parts. This would be possible only in the intersection of real and p-adic worlds, that is for living matter. My cautious proposal is that just this kind of systems - living systems - can experience free will: either in the analog of state function reduction process increasing their negentropy or in state function process reducing their entanglement with environment.

(e) In standard measurement theory observer chooses the measured observables and the theory says nothing about this process. In TGD the measured observable is the density matrix for a pair formed by any two entangled parts of sub-system division for which negentropy gain is maximal in quantum measurement defines the pair. Therefore both the measurement axis and the pair representing the target of measurement and measurer are selected in quantum jump.

(f) Quantum measurement theory assumes that measurement correlates classical long range degrees of freedom with quantal degrees of freedom. One could say that the direction of the pointer of the measurement apparatus correlates faithfully with the value of the measured microscopic observable. This requires that the entanglement is reduced between microscopic and macroscopic systems .

I have identified the "classical" degrees of freedom in TGD framework as zero modes which by definition do not contribute to the line-element of WCW although the WCW metric depends on zero modes as external parameters. The induced Kähler field represents an infinite number of zero modes whereas the Hamiltonians of the boundaries of CD define quantum fluctuating degrees of freedom.

The reduction of the entanglement between zero modes and quantum fluctuating degrees of freedom is an essential part of quantum measurement process. Also state function reductions between microscopic degrees of freedom are predicted to occur and this kind of reductions lead to de-coherence so that one can apply quantum statistical description and derive Boltzmann equations. Also state function reductions between different values of zero modes are possible are possible and one could perhaps assign "telepathic" effects with them.

The differences with respect to the standard quantum measurement theory are that several kinds of state function reductions are possible and that the division to classical and quantum fluctuating degrees of freedom has a purely geometric meaning in TGD framework.

(g) One can even imagine quantum parallel state function reduction cascades. This would make possible quantum parallel dissipation, which would be something new. My original proposal was that in hadronic physics this could make possible a state function reduction cascade proceeding in quark scales while hadronic scales would remain entangled so that one could apply statistical description to quarks as parts of a system, which is quantum coherent in hadronic length scale.

This looks nice but...! It is a pity that eventually an objection pops up against every idea irrespective how cute it looks like. The p-adic primes associated with light quarks are larger than that associated with hadron so that quarks - or rather, their magnetic bodies are larger than that hadron's magnetic body. This looks strange at first but actually conforms with Uncertainty Principle and the observation that the charge radius of proton is slightly smaller than predicted (see this, [K41]), gives support for this picture. Geometrically the situation might change if quarks are highly relativistic and color magnetic fields of quarks are dipole fields compressed to cigar like shape: Lorentz contraction could reduce the size scale of their magnetic bodies in the direction of their motion. [Note that p-adic length scale hypothesis applies in the rest system of the particle so that Lorentz contraction is in conflict with it]. Situation remains unsettled.

Further questions

There are many other interesting issues about which my understanding could be much better.

(a) In ZEO the choice of the quantization axes and would fix the moduli of the causal diamond CD: the preferred time direction defined by the line connecting the tips of CD, the spin quantization axis, etc.. This choice certainly occurs. Does it reduce to the measurement of a density matrix for some decomposition of some subsystem to a pair? Or should one simply assume state function reductions also at this level meaning localization to a sector of WCW corresponding to given CD. This would involve localization in the moduli space of CDs selecting some boost of a CD with fixed quantized proper time distance between it tips, fixed spin directions for positive and negative energy parts of zero energy states defined by light-like geodesics at its light-like boundary. Preferred complex coordinates for CP_2, etc...

(b) Zero energy states are characterized by arrow of geometric time in the sense that either positive or negative energy parts of states have well defined particles numbers and single particle numbers but not both. State function reduction is possible only for positive or negative energy part of the state but not both. This should relate very closely to the fact that our sensory percepts defined by state function reductions are mostly about the upper or lower boundary of CD, or to the fact that we do not remember the percepts made from the other boundary during sleeping period.

(c) In ZEO also quantum jumps can also lead to generation of new sub-Universes, sub-CDs carrying zero energy states. Quantum jumps can also involve phase transitions changing p-adic space-time sheets to real ones and these could serve as quantum correlates for intentional actions. Also the reverse process changing matter to thoughts is possible. These possibilities are totally unimaginable in the quantum measurement theory for systems describable by wave mechanics.

(d) There is also the notion of finite measurement resolution described in terms of inclusions of hyperfinite factors at quantum level and in terms of braids at space-time level.

To summarize, a lot of theory building is needed in order to fuse all new elements to a coherent framework. In this framework standard quantum measurement theory is only a collection of ad hoc rules and can catch only a small part of what really happens. Certainly, standard quantum measurement theory is is far from being enough for the purposes of consciousness theorist.

Chapter 11

Category Theory and Quantum TGD

11.1 Introduction

TGD predicts several hierarchical structures involving a lot of new physics. These structures look frustratingly complex and category theoretical thinking might help to build a bird's eye view about the situation. I have already earlier considered the question how category theory might be applied in TGD [K14, K9] . Besides the far from complete understanding of the basic mathematical structure of TGD also my own limited understanding of category theoretical ideas have been a serious limitation. During last years considerable progress in the understanding of quantum TGD proper has taken place and the recent formulation of TGD is in terms of light-like 3-surfaces, zero energy ontology and number theoretic braids [K76, K74] . There exist also rather detailed formulations for the fusion of p-adic and real physics and for the dark matter hierarchy. This motivates a fresh look to how category theory might help to understand quantum TGD.

The fusion rules for the symplectic variant of conformal field theory, whose existence is strongly suggested by quantum TGD, allow rather precise description using the basic notions of category theory and one can identify a series of finite-dimensional nilpotent algebras as discretized versions of field algebras defined by the fusion rules. These primitive fusion algebras can be used to construct more complex algebras by replacing any algebra element by a primitive fusion algebra. Trees with arbitrary numbers of branches in any node characterize the resulting collection of fusion algebras forming an operad. One can say that an exact solution of symplectic scalar field theory is obtained.

Conformal fields and symplectic scalar field can be combined to form symplecto-formal fields. The combination of symplectic operad and Feynman graph operad leads to a construction of Feynman diagrams in terms of n-point functions of conformal field theory. M-matrix elements with a finite measurement resolution are expressed in terms of a hierarchy of symplecto-conformal n-point functions such that the improvement of measurement resolution corresponds to an algebra homomorphism mapping conformal fields in given resolution to composite conformal fields in improved resolution. This expresses the idea that composites behave as independent conformal fields. Also other applications are briefly discussed.

Years after writing this chapter a very interesting new TGD related candidate for a category emerged. The preferred extremals would form a category if the proposed duality mapping associative (co-associative) 4-surfaces of imbedding space respects associativity (co-associativity) [K67]. The duality would allow to construct new preferred extremals of Kähler action.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http:`

//www.tgdtheory.fi/cmaphtml.html [L12]. Pdf representation of same files serving as a kind of glossary can be found at http://www.tgdtheory.fi/tgdglossary.pdf [L13].

11.2 S-matrix as a functor

John Baez's [A43] discusses in a physicist friendly manner the possible application of category theory to physics. The lessons obtained from the construction of topological quantum field theories (TQFTs) suggest that category theoretical thinking might be very useful in attempts to construct theories of quantum gravitation.

The point is that the Hilbert spaces associated with the initial and final state n-1-manifold of n-cobordism indeed form in a natural manner category. Morphisms of Hilb in turn are unitary or possibly more general maps between Hilbert spaces. TQFT itself is a functor assigning to a cobordism the counterpart of S-matrix between the Hilbert spaces associated with the initial and final n-1-manifold. The surprising result is that for $n \leq 4$ the S-matrix can be unitary S-matrix only if the cobordism is trivial. This should lead even string theorist to raise some worried questions.

In the hope of feeding some category theoretic thinking into my spine, I briefly summarize some of the category theoretical ideas discussed in the article and relate it to the TGD vision, and after that discuss the worried questions from TGD perspective. That space-time makes sense only relative to imbedding space would conform with category theoretic thinking.

11.2.1 The *-category of Hilbert spaces

Baez considers first the category of Hilbert spaces. Intuitively the definition of this category looks obvious: take linear spaces as objects in category Set, introduce inner product as additional structure and identify morphisms as maps preserving this inner product. In finite-D case the category with inner product is however identical to the linear category so that the inner product does not seem to be absolutely essential. Baez argues that in infinite-D case the morphisms need not be restricted to unitary transformations: one can consider also bounded linear operators as morphisms since they play key role in quantum theory (consider only observables as Hermitian operators). For hyper-finite factors of type II_1 inclusions define very important morphisms which are not unitary transformations but very similar to them. This challenges the belief about the fundamental role of unitarity and raises the question about how to weaken the unitarity condition without losing everything.

The existence of the inner product is essential only for the metric topology of the Hilbert space. Can one do without inner product as an inherent property of state space and reduce it to a morphism? One can indeed express inner product in terms of morphisms from complex numbers to Hilbert space and their conjugates. For any state Ψ of Hilbert space there is a unique morphisms T_Ψ from C to Hilbert space satisfying $T_\Psi(1) = \Psi$. If one assumes that these morphisms have conjugates T_Ψ^* mapping Hilbert space to C, inner products can be defined as morphisms $T_\Phi^* T_\Psi$. The Hermitian conjugates of operators can be defined with respect to this inner product so that one obtains *-category. Reader has probably realized that T_Ψ and its conjugate correspond to ket and bra in Dirac's formalism.

Note that in TGD framework based on hyper-finite factors of type II_1 (HFFs) the inclusions of complex rays might be replaced with inclusions of HFFs with included factor representing the finite measurement resolution. Note also the analogy of inner product with the representation of space-times as 4-surfaces of the imbedding space in TGD.

11.2.2 The monoidal *-category of Hilbert spaces and its counterpart at the level of nCob

One can give the category of Hilbert spaces a structure of monoid by introducing explicitly the tensor products of Hilbert spaces. The interpretation is obvious for physicist. Baez describes the details of this identification, which are far from trivial and in the theory of quantum groups very interesting things happen. A non-commutative quantum version of the tensor product implying braiding is possible and associativity condition leads to the celebrated Yang-Baxter equations: inclusions of HFFs lead to quantum groups too.

At the level of nCob the counterpart of the tensor product is disjoint union of n-1-manifolds. This unavoidably creates the feeling of cosmic loneliness. Am I really a disjoint 3-surface in emptiness which is not vacuum even in the geometric sense? Cannot be true!

This horrifying sensation disappears if n-1-manifolds are n-1-surfaces in some higher-dimensional imbedding space so that there would be at least something between them. I can emit a little baby manifold moving somewhere perhaps being received by some-one somewhere and I can receive radiation from some-one at some distance and in some direction as small baby manifolds making gentle tosses on my face!

This consoling feeling could be seen as one of the deep justifications for identifying fundamental objects as light-like partonic 3-surfaces in TGD framework. Their ends correspond to 2-D partonic surfaces at the boundaries of future or past directed light-cones (states of positive and negative energy respectively) and are indeed disjoint but not in the desperately existential sense as 3-geometries of General Relativity.

This disjointness has also positive aspect in TGD framework. One can identify the color degrees of freedom of partons as those associated with CP_2 degrees of freedom. For instance, SU(3) analogs for rotational states of rigid body become possible. 4-D space-time surfaces as preferred extremals of Kähler action connect the partonic 3-surfaces and bring in classical representation of correlations and thus of interactions. The representation as sub-manifolds makes it also possible to speak about positions of these sub-Universes and about distances between them. The habitants of TGD Universe are maximally free but not completely alone.

11.2.3 TQFT as a functor

The category theoretic formulation of TQFT relies on a very elegant and general idea. Quantum transition has as a space-time correlate an n-dimensional surface having initial final states as its n-1-dimensional ends. One assigns Hilbert spaces of states to the ends and S-matrix would be a unitary morphism between the ends. This is expressed in terms of the category theoretic language by introducing the category nCob with objects identified as n-1-manifolds and morphisms as cobordisms and *-category Hilb consisting of Hilbert spaces with inner product and morphisms which are bounded linear operators which do not however preserve the unitarity. Note that the morphisms of nCob cannot anymore be identified as maps between n-1-manifolds interpreted as sets with additional structure so that in this case category theory is more powerful than set theory.

TQFT is identified as a functor nCob $\rightarrow$ Hilb assigning to n-1-manifolds Hilbert spaces, and to cobordisms unitary S-matrices in the category Hilb. This looks nice but the surprise is that for $n \leq 4$ unitary S-matrix exists only if the cobordism is trivial so that topology changing transitions are not possible unless one gives up unitarity.

This raises several worried questions.

(a) Does this result mean that in TQFT sense unitary S-matrix for topology changing transitions from a state containing n_i closed strings to a state containing $n_f \neq n_i$ strings does not exist? Could the situation be same also for more general non-topological stringy S-matrices? Could the non-converging perturbation series for S-matrix with finite individual terms matrix fail to no non-perturbative counterpart? Could it be that M-theory is doomed to remain a dream with no hope of being fulfilled?

(b) Should one give up the unitarity condition and require that the theory predicts only the relative probabilities of transitions rather than absolute rates? What the proper generalization of the S-matrix could be?

(c) What is the relevance of this result for quantum TGD?

11.2.4 The situation is in TGD framework

The result about the non-existence of unitary S-matrix for topology changing cobordisms allows new insights about the meaning of the departures of TGD from string models.

Cobordism cannot give interesting selection rules

When I started to work with TGD for more than 28 years ago, one of the first ideas was that one could identify the selection rules of quantum transitions as topological selection rules for cobordisms. Within week or two came the great disappointment: there were practically no selection rules. Could one revive this naive idea? Could the existence of unitary S-matrix force the topological selection rules after all? I am skeptic. If I have understood correctly the discussion of what happens in 4-D case [A84] only the exotic diffeo-structures modify the situation in 4-D case.

Light-like 3-surfaces allow cobordism

In the physically interesting GRT like situation one would expect the cobordism to be mediated by a space-time surface possessing Lorentz signature. This brings in metric and temporal distance. This means complications since one must leave the pure TQFT context. Also the classical dynamics of quantum gravitation brings in strong selection rules related to the dynamics in metric degrees of freedom so that TQFT approach is not expected to be useful from the point of view of quantum gravity and certainly not the limit of a realistic theory of quantum gravitation.

In TGD framework situation is different. 4-D space-time sheets can have Euclidian signature of the induced metric so that Lorentz signature does not pose conditions. The counterparts of cobordisms correspond at fundamental level to light-like 3-surfaces, which are arbitrarily except for the light-likeness condition (the effective 2-dimensionality implies generalized conformal invariance and analogy with 3-D black-holes since 3-D vacuum Einstein equations are satisfied). Field equations defined by the Chern-Simons action imply that CP_2 projection is at most 2-D but this condition holds true only for the extremals and one has functional integral over all light-like 3-surfaces. The temporal distance between points along light-like 3-surface vanishes. The constraints from light-likeness bring in metric degrees of freedom but in a very gentle manner and just to make the theory physically interesting.

Feynman cobordism as opposed to ordinary cobordism

In string model context the discouraging results from TQFT hold true in the category of nCob, which corresponds to trouser diagrams for closed strings or for their open string counterparts. In TGD framework these diagrams are replaced with a direct generalization of Feynman diagrams for which 3-D light-like partonic 3-surfaces meet along their 2-D ends at the vertices. In honor of Feynman one could perhaps speak of Feynman cobordisms. These surfaces are singular as 3-manifolds but vertices are nice 2-manifolds. I contrast to this, in string models diagrams are nice 2-manifolds but vertices are singular as 1-manifolds (say eye-glass type configurations for closed strings).

This picture gains a strong support for the interpretation of fermions as light-like throats associated with connected sums of CP_2 type extremals with space-time sheets with Minkowski signature and of bosons as pairs of light-like wormhole throats associated with CP_2 type extremal connecting two space-time sheets with Minkowski signature of induced metric. The space-time sheets have opposite time orientations so that also zero energy ontology emerges unavoidably. There is also consistency TGD based explanation of the family replication phenomenon in terms of genus of light-like partonic 2-surfaces.

One can wonder what the 4-D space-time sheets associated with the generalized Feynman diagrams could look like? One can try to gain some idea about this by trying to assign 2-D surfaces to ordinary Feynman diagrams having a subset of lines as boundaries. In the case of

2→2 reaction open string is pinched to a point at vertex. 1→2 vertex, and quite generally, vertices with odd number of lines, are impossible. The reason is that 1-D manifolds of finite size can have either 0 or 2 ends whereas in higher-D the number of boundary components is arbitrary. What one expects to happen in TGD context is that wormhole throats which are at distance characterized by CP_2 fuse together in the vertex so that some kind of pinches appear also now.

Zero energy ontology

Zero energy ontology gives rise to a second profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

The new element would be quantum measurements performed separately for observables assignable to positive and negative energy states. These measurements would be characterized in terms of Jones inclusions. The state function reduction for the negative energy states could be interpreted as a detection of a particle reaction.

Finite temperature S-matrix defines genuine quantum state in zero energy ontology

In TGD framework one encounters two S-matrix like operators.

(a) There is U-matrix between zero energy states. This is expected to be rather trivial but very important from the point of view of description of intentional actions as transitions transforming p-adic partonic 3-surfaces to their real counterparts.

(b) The S-matrix like operator describing what happens in laboratory corresponds to the time-like entanglement coefficients between positive and negative energy parts of the state. Measurement of reaction rates would be a measurement of observables reducing time like entanglement and very much analogous to an ordinary quantum measurement reducing space-like entanglement. There is a finite measurement resolution described by inclusion of HFFs and this means that situation reduces effectively to a finite-dimensional one.

p-Adic thermodynamics strengthened with p-adic length scale hypothesis predicts particle masses with an amazing success. At first the thermodynamical approach seems to be in contradiction with the idea that elementary particles are quantal objects. Unitarity is however *not* necessary if one accepts that only relative probabilities for reductions to pairs of initial and final states interpreted as particle reactions can be measured.

The beneficial implications of unitarity are not lost if one replaces QFT with thermal QFT. Category theoretically this would mean that the time-like entanglement matrix associated with the product of cobordisms is a product of these matrices for the factors. The time parameter in S-matrix would be replaced with a complex time parameter with the imaginary part identified as inverse temperature. Hence the interpretation in terms of time evolution is not lost.

In the theory of hyper-finite factors of type III_1 the partition function for thermal equilibrium states and S-matrix can be neatly fused to a thermal S-matrix for zero energy states and one could introduce p-adic thermodynamics at the level of quantum states. It seems that this picture applies to HFFs by restriction. Therefore the loss of unitarity S-matrix might after all turn to a victory by more or less forcing both zero energy ontology and p-adic thermodynamics. Note that also the presence of factor of type I coming from imbedding space degrees of freedom forces thermal S-matrix.

Time-like entanglement coefficients as a square root of density matrix?

All quantum states do not correspond to thermal states and one can wonder what might be the most general identification of the quantum state in zero energy ontology. Density matrix formalism defines a very general formulation of quantum theory. Since the quantum states in zero energy ontology are analogous to operators, the idea that time-like entanglement coefficients in some sense define a square root of density matrix is rather natural. This would give the defining conditions

$$\begin{aligned} \rho^{+} &= SS^{\dagger} \ , \rho^{-} = S^{\dagger}S \ , \\ Tr(\rho^{\pm}) &= 1 \ . \end{aligned} \tag{11.2.1}$$

$\rho^{\pm}$ would define density matrix for positive/negative energy states. In the case HFFs of type II_1 one obtains unitary S-matrix and also the analogs of pure quantum states are possible for factors of type I. The numbers $p^{+}_{m,n} = |S^2_{m,n}|/\rho^{+}_{m,m}$ and $p^{-}_{m,n} = |S^2_{n,m}|/\rho^{-}_{m,m}$ give the counterparts of the usual scattering probabilities.

A physically well-motivated hypothesis would be that S has expression $S = \sqrt{\rho}S_0$ such that S_0 is a universal unitary S-matrix, and $\sqrt{\rho}$ is square root of a state dependent density matrix. Note that in general S is not diagonalizable in the algebraic extension involved so that it is not possible to reduce the scattering to a mere phase change by a suitable choice of state basis.

What makes this kind of hypothesis aesthetically attractive is the unification of two fundamental matrices of quantum theory to single one. This unification is completely analogous to the combination of modulus squared and phase of complex number to a single complex number: complex valued Schrödinger amplitude is replaced with operator valued one.

S-matrix as a functor and the groupoid structure formed by S-matrices

In zero energy ontology S-matrix can be seen as a functor from the category of Feynman cobordisms to the category of operators. S-matrix can be identified as a "square root" of the positive energy density matrix $S = \rho_{+}^{1/2}S_0$, where S_0 is a unitary matrix and ρ_{+} is the density matrix for positive energy part of the zero energy state. Obviously one has $SS^{\dagger} = \rho_{+}$. $S^{\dagger}S = \rho_{-}$ gives the density matrix for negative energy part of zero energy state. Clearly, S-matrix can be seen as matrix valued generalization of Schrö;dinger amplitude. Note that the "indices" of the S-matrices correspond to WCW spinor s (fermions and their bound states giving rise to gauge bosons and gravitons) and to WCW degrees of freedom. For hyper-finite factor of II_1 it is not strictly speaking possible to speak about indices since the matrix elements are traces of the S-matrix multiplied by projection operators to infinite-dimensional subspaces from right and left.

The functor property of S-matrices implies that they form a multiplicative structure analogous but not identical to groupoid [A8] . Recall that groupoid has associative product and there exist always right and left inverses and identity in the sense that ff^{-1} and $f^{-1}f$ are always defined but not identical and one has $fgg^{-1} = f$ and $f^{-1}fg = g$.

The reason for the groupoid like property is that S-matrix is a map between state spaces associated with initial and final sets of partonic surfaces and these state spaces are different so that inverse must be replaced with right and left inverse. The defining conditions for groupoid are replaced with more general ones. Also now associativity holds but the role of inverse is taken by hermitian conjugate. Thus one has the conditions $fgg^\dagger = f\rho_{g,+}$ and $f^\dagger fg = \rho_{f,-}g$, and the conditions $ff^\dagger = \rho_+$ and $f^\dagger f = \rho_-$ are satisfied. Here $\rho_\pm$ is density matrix associated with positive/negative energy parts of zero energy state. If the inverses of the density matrices exist, groupoid axioms hold true since $f_L^{-1} = f^\dagger \rho_{f,+}^{-1}$ satisfies $ff_L^{-1} = Id_+$ and $f_R^{-1} = \rho_{f,-}^{-1} f^\dagger$ satisfies $f_R^{-1} f = Id_-$.

There are good reasons to believe that also tensor product of its appropriate generalization to the analog of co-product makes sense with non-triviality characterizing the interaction between the systems of the tensor product. If so, the S-matrices would form very beautiful mathematical structure bringing in mind the corresponding structures for 2-tangles and N-tangles. Knowing how incredibly powerful the group like structures have been in physics one has good reasons to hope that groupoid like structure might help to deduce a lot of information about the quantum dynamics of TGD.

A word about nomenclature is in order. S has strong associations to unitarity and it might be appropriate to replace S with some other letter. The interpretation of S-matrix as a generalized Schrödinger amplitude would suggest Ψ-matrix. Since the interaction with Kea's M-theory blog at http://kea-monad.blogspot.com/ (M denotes Monad or Motif in this context) was led ot the realization of the connection with density matrix, also M-matrix might be considered. S-matrix as a functor from the category of Feynman cobordisms in turn suggests C or F. Or could just Matrix denoted by M in formulas be enough? Certainly it would inspire feeling of awe!

11.3 Further ideas

The work of John Baez and students has inspired also the following ideas about the role of category theory in TGD.

11.3.1 Operads, number theoretical braids, and inclusions of HFFs

The description of braids leads naturally to category theory and quantum groups when the braiding operation, which can be regarded as a functor, is not a mere permutation. Discreteness is a natural notion in the category theoretical context. To me the most natural manner to interpret discreteness is - not something emerging in Planck scale- but as a correlate for a finite measurement resolution and quantum measurement theory with finite measurement resolution leads naturally to number theoretical braids as fundamental discrete structures so that category theoretic approach becomes well-motivated. Discreteness is also implied by the number theoretic approach to quantum TGD from number theoretic associativity condition [L6] central also for category theoretical thinking as well as from the realization of number theoretical universality by the fusion of real and p-adic physics to single coherent whole.

Operads are formally single object multi-categories [A18, A87] . This object consist of an infinite sequence of sets of n-ary operations. These operations can be composed and the compositions are associative (operations themselves need not be associative) in the sense that the is natural isomorphism (symmetries) mapping differently bracketed compositions to each other. The coherence laws for operads formulate the effect of permutations and bracketing (association) as functors acting as natural isomorphisms. A simple manner to visualize the composition is as an addition of $n_1, ...n_k$ leaves to the leaves $1, ..., k$ of k-leaved tree.

An interesting example of operad is the braid operad formulating the combinatorics for a hierarchy of braids formed from braids by grouping subsets of braids having $n_1, ...n_k$ strands and defining the strands of a k-braid. In TGD framework this grouping can be identified in terms of the formation bound states of particles topologically condensed at larger space-time sheet and coherence laws allow to deduce information about scattering amplitudes. In conformal theories braided categories indeed allow to understand duality of stringy amplitudes in terms of associativity condition.

Planar operads [A57] define an especially interesting class of operads. The reason is that the inclusions of HFFs give rise to a special kind of planar operad [A22] . The object of this multi-category [A16] consists of planar k-tangles. Planar operads are accompanied by planar algebras. It will be found that planar operads allow a generalization which could provide a description for the combinatorics of the generalized Feynman diagrams and also rigorous formulation for how the arrow of time emerges in TGD framework and related heuristic ideas challenging the standard views.

11.3.2 Generalized Feynman diagram as category?

John Baez has proposed a category theoretical formulation of quantum field theory as a functor from the category of n-cobordisms to the category of Hilbert spaces [A43, A42] . The attempt to generalize this formulation looks well motivated in TGD framework because TGD can be regarded as almost topological quantum field theory in a well defined sense and braids appear as fundamental structures. It however seems that formulation as a functor from nCob to Hilb is not general enough.

In zero energy ontology events of ordinary ontology become quantum states with positive and negative energy parts of quantum states localizable to the upper and lower light-like boundaries of causal diamond (CD).

(a) Generalized Feynman diagrams associated with a given CD involve quantum superposition of light-like 3-surfaces corresponding to given generalized Feynman diagram. These superpositions could be seen as categories with 3-D light-like surfaces containing braids as arrows and 2-D vertices as objects. Zero energy states would represent quantum superposition of categories (different topologies of generalized Feynman diagram) and M-matrix defined as Connes tensor product would define a functor from this category to the Hilbert space of zero energy states for given CD (tensor product defines quite generally a functor).

(b) What is new from the point of view of physics that the sequences of generalized lines would define compositions of arrows and morphisms having identification in terms of braids which replicate in vertices. The possible interpretation of the replication is in terms of copying of information in classical sense so that even elementary particles would be information carrying and processing structures. This structure would be more general than the proposal of John Baez that S-matrix corresponds to a function from the category of n-dimensional cobordisms to the category Hilb.

(c) p-Adic length scale hypothesis follows if the temporal distance between the tips of CD measured as light-cone proper time comes as an octave of CP_2 time scale: $T = 2^n T_0$. This assumption implies that the p-adic length scale resolution interpreted in terms of a hierarchy of increasing measurement resolutions comes as octaves of time scale. A weaker condition would be $T_p = pT_0$, p prime, and would assign all p-adic time scales to the size scale hierarchy of CDs.

This preliminary picture is of course not far complete since it applies only to single CD. There are several questions. Can one allow CDs within CDs and is every vertex of generalized Feynman diagram surrounded by this kind of CD. Can one form unions of CDs freely?

(a) Since light-like 3-surfaces in 8-D imbedding space have no intersections in the generic position, one could argue that the overlap must be allowed and makes possible the interaction of between zero energy states belonging to different CDs. This interaction would be something new and present also for sub-CDs of a given CD.

(b) The simplest guess is that the unrestricted union of CDs defines the counterpart of tensor product at geometric level and that extended M-matrix is a functor from this category to the tensor product of zero energy state spaces. For non-overlapping CDs ordinary tensor product could be in question and for overlapping CDs tensor product would be non-trivial. One could interpret this M-matrix as an arrow between M-matrices of zero energy states at different CDs: the analog of natural transformation mapping two functors to each other. This hierarchy could be continued ad infinitum and would correspond to the hierarchy of n-categories.

This rough heuristics represents of course only one possibility among many since the notion of category is extremely general and the only limits are posed by the imagination of the mathematician. Also the view about zero energy states is still rather primitive.

11.4 Planar operads, the notion of finite measurement resolution, and arrow of geometric time

In the sequel the idea that planar operads or their appropriate generalization might allow to formulate generalized Feynman diagrammatics in zero energy ontology will be considered. Also a description of measurement resolution and arrow of geometric time in terms of operads is discussed.

11.4.1 Zeroth order heuristics about zero energy states

Consider now the existing heuristic picture about the zero energy states and coupling constant evolution provided by CDs.

(a) The tentative description for the increase of the measurement resolution in terms CDs is that one inserts to the upper and/or lower light-like boundary of CD smaller CDs by gluing them along light-like radial ray from the tip of CD. It is also possible that the vertices of generalized Feynman diagrams belong inside smaller CD:s and it turns out that these CD:s must be allowed.

(b) The considerations related to the arrow of geometric time suggest that there is asymmetry between upper and lower boundaries of CD. The minimum requirement is that the measurement resolution is better at upper light-like boundary.

(c) In zero energy ontology communications to the direction of geometric past are possible and phase conjugate laser photons represent one example of this.

(d) Second law of thermodynamics must be generalized in such a manner that it holds with respect to subjective time identified as sequence of quantum jumps. The arrow of geometric time can however vary so that apparent breaking of second law is possible in shorter time scales at least. One must however understand why second law holds true in so good an approximation.

(e) One must understand also why the contents of sensory experience is concentrated around a narrow time interval whereas the time scale of memories and anticipation are much longer. The proposed mechanism is that the resolution of conscious experience is higher at the upper boundary of CD. Since zero energy states correspond to light-like 3-surfaces, this could be a result of self-organization rather than a fundamental physical law.

   i. CDs define the perceptive field for self. Selves are curious about the space-time sheets outside their perceptive field in the geometric future of the imbedding space and perform quantum jumps tending to shift the superposition of the space-time sheets to the direction of geometric past (past defined as the direction of shift!). This creates the illusion that there is a time=snapshot front of consciousness moving to geometric future in fixed background space-time as an analog of train illusion.

   ii. The fact that news come from the upper boundary of CD implies that self concentrates its attention to this region and improves the resolutions of sensory experience and quantum measurement here. The sub-CD:s generated in this manner correspond to mental images with contents about this region. As a consequence, the contents of conscious experience, in particular sensory experience, tend to be about the region near the upper boundary.

iii. This mechanism in principle allows the arrow of the geometric time to vary and depend on p-adic length scale and the level of dark matter hierarchy. The occurrence of phase transitions forcing the arrow of geometric time to be same everywhere are however plausible for the reason that the lower and upper boundaries of given CD must possess the same arrow of geometric time.

iv. If this is the mechanism behind the arrow of time, planar operads can provide a description of the arrow of time but not its explanation.

This picture is certainly not general enough, can be wrong at the level of details, and at best relates to the the whole like single particle wave mechanics to quantum field theory.

11.4.2 Planar operads

The geometric definition of planar operads [A23, A18, A22, A57] without using the category theoretical jargon goes as follows.

(a) There is an external disk and some internal disks and a collection of disjoint lines connecting disk boundaries.

(b) To each disk one attaches a non-negative integer k, called the color of disk. The disk with color k has k points at each boundary with the labeling $1, 2, ...k$ running clockwise and starting from a distinguished marked point, decorated by '*'. A more restrictive definition is that disk colors are correspond to even numbers so that there are $k = 2n$ points lines leaving the disk boundary boundary. The planar tangles with $k = 2n$ correspond to inclusions of HFFs.

(c) Each curve is either closed (no common points with disk boundaries) or joins a marked point to another marked point. Each marked point is the end point of exactly one curve.

(d) The picture is planar meaning that the curves cannot intersect and diks cannot overlap.

(e) Disks differing by isotopies preserving *'s are equivalent.

Given a planar k-tangle-one of whose internal disks has color k_i- and a k_i-tangle S, one can define the tangle $T \circ_i S$ by isotoping S so that its boundary, together with the marked points and the '*'s co-indices with that of D_i and after that erase the boundary of D_i. The collection of planar tangle together with the the composition defined in this manner- is called the colored operad of planar tangles.

One can consider also generalizations of planar operads.

(a) The composition law is not affected if the lines of operads branch outside the disks. Branching could be allowed even at the boundaries of the disks although this does not correspond to a generic situation. One might call these operads branched operads.

(b) The composition law could be generalized to allow additional lines connecting the points at the boundary of the added disk so that each composition would bring in something genuinely new. Zero energy insertion could correspond to this kind of insertions.

(c) TGD picture suggests also the replacement of lines with braids. In category theoretical terms this means that besides association one allows also permutations of the points at the boundaries of the disks.

The question is whether planar operads or their appropriate generalizations could allow a characterization of the generalized Feynman diagrams representing the combinatorics of zero energy states in zero energy ontology and whether also the emergence of arrow of time could be described (but probably not explained) in this framework.

11.4.3 Planar operads and zero energy states

Are planar operads sufficiently powerful to code the vision about the geometric correlates for the increase of the measurement resolution and coupling constant evolution formulated in terms of CDs? Or perhaps more realistically, could one improve this formulation by assuming that zero energy states correspond to wave functions in the space of planar tangles or of appropriate modifications of them? It seems that the answer to the first question is almost affirmative.

(a) Disks are analogous to the white regions of a map whose details are not visible in the measurement resolution used. Disks correspond to causal diamonds (CDs) in zero energy ontology. Physically the white regions relate to the vertices of the generalized Feynman diagrams and possibly also to the initial and final states (strictly speaking, the initial and final states correspond to the legs of generalized Feynman diagrams rather than their ends).

(b) The composition of tangles means addition of previously unknown details to a given white region of the map and thus to an increase of the measurement resolution. This conforms with the interpretation of inclusions of HFFs as a characterization of finite measurement resolution and raises the hope that planar operads or their appropriate generalization could provide the proper language to describe coupling constant evolution and their perhaps even generalized Feynman diagrams.

(c) For planar operad there is an asymmetry between the outer disk and inner disks. One might hope that this asymmetry could explain or at least allow to describe the arrow of time. This is not the case. If the disks correspond to causal diamonds (CDs) carrying positive *resp.* negative energy part of zero energy state at upper *resp.* lower light-cone boundary, the TGD counterpart of the planar tangle is CD containing smaller CD:s inside it. The smaller CD:s contain negative energy particles at their upper boundary and positive energy particles at their lower boundary. In the ideal resolution vertices represented 2-dimensional partonic at which light-like 3-surfaces meet become visible. There is no inherent asymmetry between positive and negative energies and no inherent arrow of geometric time at the fundamental level. It is however possible to model the arrow of time by the distribution of sub-CD:s. By previous arguments self-organization of selves can lead to zero energy states for which the measurement resolution is better near the upper boundary of the CD.

(d) If the lines carry fermion or anti-fermion number, the number of lines entering to a given CD must be even as in the case of planar operads as the following argument shows.

   i. In TGD framework elementary fermions correspond to single wormhole throat associated with topologically condensed CP_2 type extremal and the signature of the induced metric changes at the throat.

   ii. Elementary bosons correspond to pairs of wormhole throats associated with wormhole contacts connecting two space-time sheets of opposite time orientation and modellable as a piece of CP_2 type extremal. Each boson therefore corresponds to 2 lines within CP_2 radius.

   iii. As a consequence the total number of lines associated with given CD is even and the generalized Feynman diagrams can correspond to a planar algebra associated with an inclusion of HFFs.

(e) This picture does not yet describe zero energy insertions.

   i. The addition of zero energy insertions corresponds intuitively to the allowance of new lines inside the smaller CD:s not coming from the exterior. The addition of lines connecting points at the boundary of disk is possible without losing the basic geometric composition of operads. In particular one does not lose the possibility to color the added tangle using two colors (colors correspond to two groups G and H which characterize an inclusion of HFFs [A57]).

   ii. There is however a problem. One cannot remove the boundaries of sub-CD after the composition of CDs since this would give lines beginning from and ending to the interior of disk and they are invisible only in the original resolution. Physically this is of course what one wants but the inclusion of planar tangles is expected to fail in its original form, and one must generalize the composition of tangles to that of CD:s so that the boundaries of sub-CD:s are not thrown away in the process.

iii. It is easy to see that zero energy insertions are inconsistent with the composition of planar tangles. In the inclusion defining the composition of tangles both sub-tangle and tangle induce a color to a given segment of the inner disk. If these colors are identical, one can forget the presence of the boundary of the added tangle. When zero energy insertions are allowed, situation changes as is easy to see by adding a line connecting points in a segment of given color at the boundary of the included tangle. There exists no consistent coloring of the resulting structure by using only two colors. Coloring is however possible using four colors, which by four-color theorem is the minimum number of colors needed for a coloring of planar map: this however requires that the color can change as one moves through the boundary of the included disk - this is in accordance with the physical picture.

iv. Physical intuition suggests that zero energy insertion as an improvement of measurement resolution maps to an improved color resolution and that the composition of tangles generalizes by requiring that the included disk is colored by using new nuances of the original colors. The role of groups in the definition of inclusions of HFFs is consistent with idea that G and H describe color resolution in the sense that the colors obtained by their action cannot be resolved. If so, the improved resolution means that G and H are replaced by their subgroups $G_1 \subset G$ and $H_1 \subset H$. Since the elements of a subgroup have interpretation as elements of group, there are good hopes that by representing the inclusion of tangles as inclusion of groups, one can generalize the composition of tangles.

(f) Also CD:s glued along light-like ray to the upper and lower boundaries of CD are possible in principle and -according the original proposal- correspond to zero energy insertions according. These CD:s might be associated with the phase transitions changing the value of $\hbar$ leading to different pages of the book like structure defined by the generalized imbedding space.

(g) p-Adic length scale hypothesis is realized if the hierarchy of CDs corresponds to a hierarchy of temporal distances between tips of CDs given as $a = T_n = 2^{-n}T_0$ using light-cone proper time.

(h) How this description relates to braiding? Each line corresponds to an orbit of a partonic boundary component and in principle one must allow internal states containing arbitrarily high fermion and anti-fermion numbers. Thus the lines decompose into braids and one must allow also braids of braids hierarchy so that each line corresponds to a braid operad in improved resolution.

11.4.4 Relationship to ordinary Feynman diagrammatics

The proposed description is not equivalent with the description based on ordinary Feynman diagrams.

(a) In standard physics framework the resolution scale at the level of vertices of Feynman diagrams is something which one is forced to pose in practical calculations but cannot pose at will as opposed to the measurement resolution. Light-like 3-surfaces can be however regarded only locally orbits of partonic 2-surfaces since generalized conformal invariance is true only in 3-D patches of the light-like 3-surface. This means that light-like 3-surfaces are in principle the fundamental objects so that zero energy states can be regarded only locally as a time evolutions. Therefore measurement resolution can be applied also to the distances between vertices of generalized Feynman diagrams and calculational resolution corresponds to physical resolution. Also the resolution can be better towards upper boundary of CD so that the arrow of geometric time can be understood. This is a definite prediction which can in principle kill the proposed scenario.

(b) A further counter argument is that generalized Feynman diagrams are identified as light-like 3-surfaces for which Kähler function defined by a preferred extremal of Kähler action is maximum. Therefore one cannot pose any ad hoc rules on the positions of the vertices. One can of course insist that maximum of Kähler function with the constraint posed by $T_n = 2^n T_0$ (or $T_p = pT_0$) hierarchy is in question.

It would be too optimistic to believe that the details of the proposal are correct. However, if the proposal is on correct track, zero energy states could be seen as wave functions in the operad of generalized tangles (zero energy insertions and braiding) as far as combinatorics is involved and the coherence rules for these operads would give strong constraints on the zero energy state and fix the general structure of coupling constant evolution.

11.5 Category theory and symplectic QFT

Besides the counterpart of the ordinary Kac-Moody invariance quantum TGD possesses so called super-symplectic conformal invariance. This symmetry leads to the proposal that a symplectic variant of conformal field theory should exist. The n-point functions of this theory defined in S^2 should be expressible in terms of symplectic areas of triangles assignable to a set of n-points and satisfy the duality rules of conformal field theories guaranteeing associativity. The crucial prediction is that symplectic n-point functions vanish whenever two arguments co-incide. This provides a mechanism guaranteeing the finiteness of quantum TGD implied by very general arguments relying on non-locality of the theory at the level of 3-D surfaces.

The classical picture suggests that the generators of the fusion algebra formed by fields at different point of S^2 have this point as a continuous index. Finite quantum measurement resolution and category theoretic thinking in turn suggest that only the points of S^2 corresponding the strands of number theoretic braids are involved. It turns out that the category theoretic option works and leads to an explicit hierarchy of fusion algebras forming a good candidate for so called little disk operad whereas the first option has difficulties.

11.5.1 Fusion rules

Symplectic fusion rules are non-local and express the product of fields at two points s_k an s_l of S^2 as an integral over fields at point s_r, where integral can be taken over entire S^2 or possibly also over a 1-D curve which is symplectic invariant in some sense. Also discretized version of fusion rules makes sense and is expected serve as a correlate for finite measurement resolution.

By using the fusion rules one can reduce n-point functions to convolutions of 3-point functions involving a sequence of triangles such that two subsequent triangles have one vertex in common. For instance, 4-point function reduces to an expression in which one integrates over the positions of the common vertex of two triangles whose other vertices have fixed. For n-point functions one has n-3 freely varying intermediate points in the representation in terms of 3-point functions.

The application of fusion rules assigns to a line segment connecting the two points s_k and s_l a triangle spanned by s_k, s_l and s_r. This triangle should be symplectic invariant in some sense and its symplectic area A_{klm} would define the basic variable in terms of which the fusion rule could be expressed as $C_{klm} = f(A_{klm})$, where f is fixed by some constraints. Note that A_{klm} has also interpretations as solid angle and magnetic flux.

11.5.2 What conditions could fix the symplectic triangles?

The basic question is how to identify the symplectic triangles. The basic criterion is certainly the symplectic invariance: if one has found N-D symplectic algebra, symplectic transformations of S^2 must provide a new one. This is guaranteed if the areas of the symplectic triangles remain invariant under symplectic transformations. The questions are how to realize this condition and whether it might be replaced with a weaker one. There are two approaches to the problem.

Physics inspired approach

In the first approach inspired by classical physics symplectic invariance for the edges is interpreted in the sense that they correspond to the orbits of a charged particle in a magnetic field defined by the Kähler form. Symplectic transformation induces only a $U(1)$ gauge transformation and leaves the orbit of the charged particle invariant if the vertices are not affected since symplectic transformations are not allowed to act on the orbit directly in this approach. The general functional form of the structure constants C_{klm} as a function $f(A_{klm})$ of the symplectic area should guarantee fusion rules.

If the action of the symplectic transformations does not affect the areas of the symplectic triangles, the construction is invariant under general symplectic transformations. In the case of uncharged particle this is not the case since the edges are pieces of geodesics: in this case however fusion algebra however trivializes so that one cannot conclude anything. In the case of charged particle one might hope that the area remains invariant under general symplectic transformations whose action is induced from the action on vertices. The equations of motion for a charged particle involve a Kähler metric determined by the symplectic structure and one might hope that this is enough to achieve this miracle. If this is not the case - as it might well be - one might hope that although the areas of the triangles are not preserved, the triangles are mapped to each other in such a manner that the fusion algebra rules remain intact with a proper choice of the function $f(A_{klm})$. One could also consider the possibility that the function $f(A_{klm})$ is dictated from the condition that the it remains invariant under symplectic transformations. It however turns that this approach does not work as such.

Category theoretical approach

The second realization is guided by the basic idea of category theoretic thinking: the properties of an object are determined its relationships to other objects. Rather than postulating that the symplectic triangle is something which depends solely on the three points involved via some geometric notion like that of geodesic line of orbit of charged particle in magnetic field, one assumes that the symplectic triangle reflects the properties of the fusion algebra, that is the relations of the symplectic triangle to other symplectic triangles. Thus one must assign to each triplet (s_1, s_2, s_3) of points of S^2 a triangle just from the requirement that braided associativity holds true for the fusion algebra.

All symplectic transformations leaving the N points fixed and thus generated by Hamiltonians vanishing at these points would give new gauge equivalent realizations of the fusion algebra and deform the edges of the symplectic triangles without affecting their area. One could even say that symplectic triangulation defines a new kind geometric structure in S^2. The quantum fluctuating degrees of freedom are parameterized by the symplectic group of $S^2 \times CP_2$ in TGD so that symplectic the geometric representation of the triangulation changes but its inherent properties remain invariant.

The elegant feature of category theoretical approach is that one can in principle construct the fusion algebra without any reference to its geometric realization just from the braided associativity and nilpotency conditions and after that search for the geometric realizations. Fusion algebra has also a hierarchy of discrete variants in which the integral over intermediate points in fusion is replaced by a sum over a fixed discrete set of points and this variant is what finite measurement resolution implies. In this case it is relatively easy to see if the geometric realization of a given abstract fusion algebra is possible.

The notion of number theoretical braid

Braids -not necessary number theoretical- provide a realization discretization as a space-time correlate for the finite measurement resolution. The notion of braid was inspired by the idea about quantum TGD as almost topological quantum field theory. Although the original form of this idea has been buried, the notion of braid has survived: in the decomposition of space-time sheets to string world sheets, the ends of strings define representatives for braid strands at light-like 3-surfaces.

The notion of number theoretic universality inspired the much more restrictive notion of number theoretic braid requiring that the points in the intersection of the braid with the partonic 2-surface correspond to rational or at most algebraic points of H in preferred coordinates fixed by symmetry considerations. The challenge has been to find a unique identification of the number theoretic braid or at least of the end points of the braid. The following consideration suggest that the number theoretic braids are not a useful notion in the generic case but make sense and are needed in the intersection of real and p-adic worlds which is in crucial role in TGD based vision about living matter [K39] .

It is only the braiding that matters in topological quantum field theories used to classify braids. Hence braid should require only the fixing of the end points of the braids at the intersection of the braid at the light-like boundaries of CDs and the braiding equivalence class of the braid itself. Therefore it is enough is to specify the topology of the braid and the end points of the braid in accordance with the attribute "number theoretic". Of course, the condition that all points of the strand of the number theoretic braid are algebraic is impossible to satisfy.

The situation in which the equations defining X^2 make sense both in real sense and p-adic sense using appropriate algebraic extension of p-adic number field is central in the TGD based vision about living matter [K39] . The reason is that in this case the notion of number entanglement theoretic entropy having negative values makes sense and entanglement becomes information carrying. This motivates the identification of life as something in the intersection of real and p-adic worlds. In this situation the identification of the ends of the number theoretic braid as points belonging to the intersection of real and p-adic worlds is natural. These points -call them briefly algebraic points- belong to the algebraic extension of rationals needed to define the algebraic extension of p-adic numbers. This definition however makes sense also when the equations defining the partonic 2-surfaces fail to make sense in both real and p-adic sense. In the generic case the set of points satisfying the conditions is discrete. For instance, according to Fermat's theorem the set of rational points satisfying $X^n + Y^n = Z^n$ reduces to the point $(0, 0, 0)$ for $n = 3, 4, ...$ Hence the constraint might be quite enough in the intersection of real and p-adic worlds where the choice of the algebraic extension is unique.

One can however criticize this proposal.

(a) One must fix the the number of points of the braid and outside the intersection and the non-uniquencess of the algebraic extension makes the situation problematic. Physical intuition suggests that the points of braid define carriers of quantum numbers assignable to second quantized induced spinor fields so that the total number of fermions anti-fermions would define the number of braids. In the intersection the highly non-trivial implication is that this number cannot exceed the number of algebraic points.

(b) In the generic case one expects that even the smallest deformation of the partonic 2-surface can change the number of algebraic points and also the character of the algebraic extension of rational numbers needed. The restriction to rational points is not expected to help in the generic case. If the notion of number theoretical braid is meant to be practical, must be able to decompose WCW to open sets inside which the numbers of algebraic points of braid at its ends are constant. For real topology this is expected to be impossible and it does not make sense to use p-adic topology for WCW whose points do not allow interpretation as p-adic partonic surfaces.

(c) In the intersection of real and p-adic worlds which corresponds to a discrete subset of WCW, the situation is different. Since the coefficients of polynomials involved with the definition of the partonic 2-surface must be rational or at most algebraic, continuous deformations are not possible so that one avoids the problem.

(d) This forces to ask the reason why for the number theoretic braids. In the generic case they seem to produce only troubles. In the intersection of real and p-adic worlds they could however allow the construction of the elements of M-matrix describing quantum transitions changing p-adic to real surfaces and vice versa as realizations of intentions and generation of cognitions. In this the case it is natural that only the data from the intersection of the two worlds are used. In [K39] I have sketched the idea about number theoretic quantum field theory as a description of intentional action and cognition.

There is also the the problem of fixing the interior points of the braid modulo deformations not affecting the topology of the braid.

(a) Infinite number of non-equivalent braidings are possible. Should one allow all possible braidings for a fixed light-like 3-surface and say that their existence is what makes the dynamics essentially three-dimensional even in the topological sense? In this case there would be no problems with the condition that the points at both ends of braid are algebraic.

(b) Or should one try to characterize the braiding uniquely for a given partonic 2-surfaces and corresponding 4-D tangent space distributions? The slicing of the space-time sheet by partonic 2-surfaces and string word sheets suggests that the ends of string world sheets could define the braid strands in the generic context when there is no algebraicity condition involved. This could be taken as a very natural manner to fix the topology of braid but leave the freedom to choose the representative for the braid. In the intersection of real and p-adic worlds there is no good reason for the end points of strands in this case to be algebraic at both ends of the string world sheet. One can however start from the braid defined by the end points of string world sheets, restrict the end points to be algebraic at the end with a smaller number of algebraic points and and then perform a topologically non-trivial deformation of the braid so that also the points at the other end are algebraic? Non-trivial deformations need not be possible for all possible choices of algebraic braid points at the other end of braid and different choices of the set of algebraic points would give rise to different braidings. A further constraint is that only the algebraic points at which one has assign fermion or anti-fermion are used so that the number of braid points is not always maximal.

(c) One can also ask whether one should perform the gauge fixing for the strands of the number theoretic braid using algebraic functions making sense both in real and p-adic context. This question does not seem terribly relevant since since it is only the topology of the braid that matters.

Symplectic triangulations and braids

The identification of the edges of the symplectic triangulation as the end points of the braid is favored by conceptual economy. The nodes of the symplectic triangulation would naturally correspond to the points in the intersection of the braid with the light-like boundaries of CD carrying fermion or anti-fermion number. The number of these points could be arbitrarily large in the generic case but in the intersection of real and p-adic worlds these points correspond to subset of algebraic points belonging to the algebraic extension of rationals associated with the definition of partonic 2-surfaces so that the sum of fermion and anti-fermion numbers would be bounded above. The presence of fermions in the nodes would be the physical prerequisite for measuring the phase factors defined by the magnetic fluxes. This could be understood in terms of gauge invariance forcing to assign to a pair of points of triangulation the non-integrable phase factor defined by the Kähler gauge potential.

The remaining problem is how uniquely the edges of the triangulation can be determined.

(a) The allowance of all possible choices for edges would bring in an infinite number of degrees of freedom. These curves would be analogous to freely vibrating strings. This option is not attractive. One should be able to pose conditions on edges and whatever the manner to specify the edges might be, it must make sense also in the intersection of real and p-adic worlds. In this case the total phase factor must be a root of unity in the algebraic extension of rationals involved and this poses quantization rules analogous to those for magnetic flux. The strongest condition is that the edges are such that the non-integrable phase factor is a root of unity for each edge. It will be found that similar quantization is implied also by the associativity conditions and this justifies the interpretation of phase factors defining the fusion algebra in terms of the Kähler magnetic fluxes. This would pose strong constraints on the choice of edges but would not fix completely the phase factors, and it seems that one must allow all possible triangulations consistent with this condition and the associativity conditions so that physical state is a quantum superposition over all possible symplectic triangulations characterized by the fusion algebras.

(b) In the real context one would have an infinite hierarchy of symplectic triangulations and fusion algebras satisfying the associativity conditions with the number of edges equal to the total number N of fermions and anti-fermions. Encouragingly, this hierarchy corresponds also to a hierarchy of $\mathcal{N} = N$ SUSY algebras [K23] (large values of $\mathcal{N}$ are not a catastrophe in TGD framework since the physical content of SUSY symmetry is not the same as that in the standard approach). In the intersection of real and p-adic worlds the value of $\mathcal{N}$ would be bounded by the total number of algebraic points. Hence the notion of finite measurement resolution, cutoff in $\mathcal{N}$ and bound on the total fermion number would make physics very simple in the intersection of real and p-adic worlds.

Two kinds of symplectic triangulations are possible since one can use the symplectic forms associated with CP_2 and $r_M = constant$ sphere S^2 of light-cone boundary. For a given collection of nodes the choices of edges could be different for these two kinds of triangulations. Physical state would be proportional to the product of the phase factors assigned to these triangulations.

11.5.3 Associativity conditions and braiding

The generalized fusion rules follow from the associativity condition for n-point functions modulo phase factor if one requires that the factor assignable to n-point function has interpretation as n-point function. Without this condition associativity would be trivially satisfied by using a product of various bracketing structures for the n fields appearing in the n-point function. In conformal field theories the phase factor defining the associator is expressible in terms of the phase factor associated with permutations represented as braidings and the same is expected to be true also now.

(a) Already in the case of 4-point function there are three different choices corresponding to the 4 possibilities to connect the fixed points s_k and the varying point s_r by lines. The options are (1-2, 3-4), (1-3,2-4), and (1-4,2-3) and graphically they correspond to s-, t-, and u-channels in string diagrams satisfying also this kind of fusion rules. The basic condition would be that same amplitude results irrespective of the choice made. The duality conditions guarantee associativity in the formation of the n-point amplitudes without any further assumptions. The reason is that the writing explicitly the expression for a particular bracketing of n-point function always leads to some bracketing of one particular 4-point function and if duality conditions hold true, the associativity holds true in general. To be precise, in quantum theory associativity must hold true only in projective sense, that is only modulo a phase factor.

(b) This framework encourages category theoretic approach. Besides different bracketing there are different permutations of the vertices of the triangle. These permutations can induce a phase factor to the amplitude so that braid group representations are enough. If one has representation for the basic braiding operation as a quantum phase $q = exp(i2\pi/N)$, the phase factors relating different bracketings reduce to a product of these phase factors since $(AB)C$ is obtained from $A(BC)$ by a cyclic permutation involving to permutations represented as a braiding. Yang-Baxter equations express the reduction of associator to braidings. In the general category theoretical setting associators and braidings correspond to natural isomorphisms leaving category theoretical structure invariant.

(c) By combining the duality rules with the condition that 4-point amplitude vanishes, when any two points co-incide, one obtains from $s_k = s_l$ and $s_m = s_n$ the condition stating that the sum (or integral in possibly existing continuum version) of $U^2(A_{klm})|f|^2(x_{kmr})$ over the third point s_r vanishes. This requires that the phase factor U is non-trivial so that Q must be non-vanishing if one accepts the identification of the phase factor as Bohm-Aharonov phase.

(d) Braiding operation gives naturally rise to a quantum phase. A good guess is that braiding operation maps triangle to its complement since only in this manner orientation is preserved so that area is A_{klm} is mapped to $A_{klm} - 4\pi$. If the f is proportional to the exponent $exp(-A_{klm}Q)$, braiding operation induces a complex phase factor $q = exp(-i4\pi Q)$.

(e) For half-integer values of Q the algebra is commutative. For $Q = M/N$, where M and N have no common factors, only braided commutativity holds true for $N \geq 3$ just as for quantum groups characterizing also Jones inclusions of HFFs. For $N = 4$ anti-commutativity and associativity hold true. Charge fractionization would correspond to non-trivial braiding and presumably to non-standard values of Planck constant and coverings of M^4 or CP_2 depending on whether S^2 corresponds to a sphere of light-cone boundary or homologically trivial geodesic sphere of CP_2.

11.5.4 Finite-dimensional version of the fusion algebra

Algebraic discretization due to a finite measurement resolution is an essential part of quantum TGD. In this kind of situation the symplectic fields would be defined in a discrete set of N points of S^2: natural candidates are subsets of points of p-adic variants of S^2. Rational variant of S^2 has as its points points for which trigonometric functions of θ and ϕ have rational values and there exists an entire hierarchy of algebraic extensions. The interpretation for the resulting breaking of the rotational symmetry would be a geometric correlate for the choice of quantization axes in quantum measurement and the book like structure of the imbedding space would be direct correlate for this symmetry breaking. This approach gives strong support for the category theory inspired philosophy in which the symplectic triangles are dictated by fusion rules.

General observations about the finite-dimensional fusion algebra

(a) In this kind of situation one has an algebraic structure with a finite number of field values with integration over intermediate points in fusion rules replaced with a sum. The most natural option is that the sum is over all points involved. Associativity conditions reduce in this case to conditions for a finite set of structure constants vanishing when two indices are identical. The number $M(N)$ of non-vanishing structure constants is obtained from the recursion formula $M(N) = (N-1)M(N-1) + (N-2)M(N-2) + ... + 3M(3) = NM(N-1)$, $M(3) = 1$ given $M(4) = 4$, $M(5) = 20$, $M(6) = 120$,... With a proper choice of the set of points associativity might be achieved. The structure constants are necessarily complex so that also the complex conjugate of the algebra makes sense.

(b) These algebras resemble nilpotent algebras ($x^n = 0$ for some n) and Grassmann algebras ($x^2 = 0$ always) in the sense that also the products of the generating elements satisfy $x^2 = 0$ as one can find by using duality conditions on the square of a product $x = yz$ of two generating elements. Also the products of more than N generating elements necessary vanish by braided commutativity so that nilpotency holds true. The interpretation in terms of measurement resolution is that partonic states and vertices can involve at most N fermions in this measurement resolution. Elements anti-commute for $q = -1$ and commute for $q = 1$ and the possibility to express the product of two generating elements as a sum of generating elements distinguishes these algebras from Grassman algebras. For $q = -1$ these algebras resemble Lie-algebras with the difference that associativity holds true in this particular case.

(c) I have not been able to find whether this kind of hierarchy of algebras corresponds to some well-known algebraic structure with commutativity and associativity possibly replaced with their braided counterparts. Certainly these algebras would be category theoretical generalization of ordinary algebras for which commutativity and associativity hold true in strict sense.

(d) One could forget the representation of structure constants in terms of triangles and think these algebras as abstract algebras. The defining equations are $x_i^2 = 0$ for generators plus braided commutativity and associativity. Probably there exists solutions to these conditions. One can also hope that one can construct braided algebras from commutative and associative algebras allowing matrix representations. Note that the solution the conditions allow scalings of form $C_{klm} \to \lambda_k \lambda_l \lambda_m C_{klm}$ as symmetries.

Formulation and explicit solution of duality conditions in terms of inner product

Duality conditions can be formulated in terms of an inner product in the function space associated with N points and this allows to find explicit solutions to the conditions.

(a) The idea is to interpret the structure constants C_{klm} as wave functions C_{kl} in a discrete space consisting of N points with the standard inner product

$$\langle C_{kl}, C_{mn} \rangle = \sum_r C_{klr} \overline{C}_{mnr} \quad . \tag{11.5.1}$$

(b) The associativity conditions for a trivial braiding can be written in terms of the inner product as

$$\langle C_{kl}, \overline{C}_{mn} \rangle = \langle C_{km}, \overline{C}_{ln} \rangle = \langle C_{kn}, \overline{C}_{ml} \rangle \quad . \tag{11.5.2}$$

(c) Irrespective of whether the braiding is trivial or not, one obtains for $k = m$ the orthogonality conditions

$$\langle C_{kl}, \overline{C}_{kn} \rangle = 0 \quad . \tag{11.5.3}$$

For each k one has basis of $N-1$ wave functions labeled by $l \neq k$, and the conditions state that the elements of basis and conjugate basis are orthogonal so that conjugate basis is the dual of the basis. The condition that complex conjugation maps basis to a dual basis is very special and is expected to determine the structure constants highly uniquely.

(d) One can also find explicit solutions to the conditions. The most obvious trial is based on orthogonality of function basis of circle providing representation for Z_{N-2} and is following:

$$C_{klm} = E_{klm} \times exp(i\phi_k + \phi_l + \phi_m) \quad , \quad \phi_m = \frac{n(m)2\pi}{N-2} \quad . \tag{11.5.4}$$

Here E_{klm} is non-vanishing only if the indices have different values. The ansatz reduces the conditions to the form

$$\sum_r E_{klr} E_{mnr} exp(i2\phi_r) = \sum_r E_{kmr} E_{lnr} exp(i2\phi_r) = \sum_r E_{knr} E_{mlr} exp(i2\phi_r) \tag{11.5.5}$$

In the case of braiding one can allow overall phase factors. Orthogonality conditions reduce to

$$\sum_r E_{klr}E_{knr}exp(i2\phi_r) = 0 \ . \tag{11.5.6}$$

If the integers $n(m)$, $m \neq k,l$ span the range $(0, N-3)$ ortogonality conditions are satisfied if one has $E_{klr} = 1$ when the indices are different. This guarantees also duality conditions since the inner products involving k,l,m,n reduce to the same expression

$$\sum_{r\neq k,l,m,n} exp(i2\phi_r) \ . \tag{11.5.7}$$

(e) For a more general choice of phases the coefficients E_{klm} must have values differing from unity and it is not clear whether the duality conditions can be satisfied in this case.

Do fusion algebras form little disk operad?

The improvement of measurement resolution means that one adds further points to an existing set of points defining a discrete fusion algebra so that a small disk surrounding a point is replaced with a little disk containing several points. Hence the hierarchy of fusion algebras might be regarded as a realization of a little disk operad [A14] and there would be a hierarchy of homomorphisms of fusion algebras induced by the fusion. The inclusion homomorphism should map the algebra elements of the added points to the algebra element at the center of the little disk.

A more precise prescription goes as follows.

(a) The replacement of a point with a collection of points in the little disk around it replaces the original algebra element ϕ_{k_0} by a number of new algebra elements ϕ_K besides already existing elements ϕ_k and brings in new structure constants C_{KLM}, C_{KLk} for $k \neq k_0$, and C_{Klm}.

(b) The notion of improved measurement resolution allows to conclude

$$C_{KLk} = 0 \ , \ k \neq k_0 \ , \ C_{Klm} = C_{k_0lm} \ . \tag{11.5.8}$$

(c) In the homomorphism of new algebra to the original one the new algebra elements and their products should be mapped as follows:

$$\begin{array}{l} \phi_K \rightarrow \phi_{k_0} \ , \\ \phi_K\phi_L \rightarrow \phi_{k_0}^2 = 0 \ , \ \ \phi_K\phi_l \rightarrow \phi_{k_0}\phi_l \ . \end{array} \tag{11.5.9}$$

Expressing the products in terms of structure constants gives the conditions

$$\sum_M C_{KLM} = 0 \ , \ \ \sum_r C_{Klr} = \sum_r C_{k_0lr} = 0 \ . \tag{11.5.10}$$

The general ansatz for the structure constants based on roots of unity guarantees that the conditions hold true.

(d) Note that the resulting algebra is more general than that given by the basic ansatz since the improvement of the measurement resolution at a given point can correspond to different value of N as that for the original algebra given by the basic ansatz. Therefore the original ansatz gives only the basic building bricks of more general fusion algebras. By repeated local improvements of the measurement resolution one obtains an infinite hierarchy of algebras labeled by trees in which each improvement of measurement resolution means the splitting of the branch with arbitrary number N of branches. The number of improvements of the measurement resolution defining the height of the tree is one invariant of these algebras. The fusion algebra operad has a fractal structure since each point can be replaced by any fusion algebra.

How to construct geometric representation of the discrete fusion algebra?

Assuming that solutions to the fusion conditions are found, one could try to find whether they allow geometric representations. Here the category theoretical philosophy shows its power.

(a) Geometric representations for C_{klm} would result as functions $f(A_{klm})$ of the symplectic area for the symplectic triangles assignable to a set of N points of S^2.

(b) If the symplectic triangles can be chosen freely apart from the area constraint as the category theoretic philosophy implies, it should be relatively easy to check whether the fusion conditions can be satisfied. The phases of C_{klm} dictate the areas A_{klm} rather uniquely if one uses Bohm-Aharonov ansatz for a fixed the value of Q. The selection of the points s_k would be rather free for phases near unity since the area of the symplectic triangle associated with a given triplet of points can be made arbitrarily small. Only for the phases far from unity the points s_k cannot be too close to each other unless Q is very large. The freedom to chose the points rather freely conforms with the general view about the finite measurement resolution as the origin of discretization.

(c) The remaining conditions are on the moduli $|f(A_{klm})|$. In the discrete situation it is rather easy to satisfy the conditions just by fixing the values of f for the particular triangles involved: $|f(A_{klm})| = |C_{klm}|$. For the exact solution to the fusion conditions $|f(A_{klm})| = 1$ holds true.

(d) Constraints on the functional form of $|f(A_{klm})|$ for a fixed value of Q can be deduced from the correlation between the modulus and phase of C_{klm} without any reference to geometric representations. For the exact solution of fusion conditions there is no correlation.

(e) If the phase of C_{klm} has A_{klm} as its argument, the decomposition of the phase factor to a sum of phase factors means that the A_{klm} is sum of contributions labeled by the vertices. Also the symplectic area defined as a magnetic flux over the triangle is expressible as sum of the quantities $\int A_\mu dx^\mu$ associated with the edges of the triangle. These fluxes should correspond to the fluxes assigned to the vertices deduced from the phase factors of $\Psi(s_k)$. The fact that vertices are ordered suggest that the phase of $\Psi(s_j)$ fixes the value of $\int A_\mu dx^\mu$ for an edge of the triangle starting from s_k and ending to the next vertex in the ordering. One must find edges giving a closed triangle and this should be possible. The option for which edges correspond to geodesics or to solutions of equations of motion for a charged particle in magnetic field is not flexible enough to achieve this purpose.

(f) The quantization of the phase angles as multiples of $2\pi/(N-2)$ in the case of N-dimensional fusion algebra has a beautiful geometric correlate as a quantization of symplecto-magnetic fluxes identifiable as symplectic areas of triangles defining solid angles as multiples of $2\pi/(N-2)$. The generalization of the fusion algebra to p-adic case exists if one allows algebraic extensions containing the phase factors involved. This requires the allowance of phase factors $exp(i2\pi/p)$, p a prime dividing $N-2$. Only the exponents $exp(i\int A_\mu dx^\mu) = exp(in2\pi/(N-2))$ exist p-adically. The p-adic counterpart of the curve defining the edge of triangle exists if the curve can be defined purely algebraically (say as a solution of polynomial equations with rational coefficients) so that p-adic variant of the curve satisfies same equations.

Does a generalization to the continuous case exist?

The idea that a continuous fusion algebra could result as a limit of its discrete version does not seem plausible. The reason is that the spatial variation of the phase of the structure constants increases as the spatial resolution increases so that the phases $exp(i\phi(s)$ cannot be continuous at continuum limit. Also the condition $E_{klm} = 1$ for $k \neq l \neq m$ satisfied by the explicit solutions to fusion rules fails to have direct generalization to continuum case.

To see whether the continuous variant of fusion algebra can exist, one can consider an approximate generalization of the explicit construction for the discrete version of the fusion algebra by the effective replacement of points s_k with small disks which are not allowed to intersect. This would mean that the counterpart $E(s_k, s_l, s_m)$ vanishes whenever the distance between two arguments is below a cutoff a small radius d. Puncturing corresponds physically to the cutoff implied by the finite measurement resolution.

(a) The ansatz for C_{klm} is obtained by a direct generalization of the finite-dimensional ansatz:

$$C_{klm} = \kappa_{s_k,s_l,s_m}\Psi(s_k)\Psi(s_l)\Psi(s_m) \ . \tag{11.5.11}$$

where κ_{s_k,s_l,s_m} vanishes whenever the distance of any two arguments is below the cutoff distance and is otherwise equal to 1.

(b) Orthogonality conditions read as

$$\Psi(s_k)\Psi(s_l)\int \kappa_{s_k,s_l,s_r}\kappa_{s_k,s_n,s_r}\Psi^2(s_m)d\mu(s_r) = \Psi(s_k)\Psi(s_l)\int_{S^2(s_k,sl,s_n)}\Psi^2(s_r)d\mu(s_r) = 0 \tag{11.5.12}$$

The resulting condition reads as

$$\int_{S^2(s_k,sl,s_n)}\Psi^2(s_r)d\mu(s_r) = 0 \tag{11.5.13}$$

This condition holds true for any pair s_k, s_l and this might lead to difficulties.

(c) The general duality conditions are formally satisfied since the expression for all fusion products reduces to

$$\begin{aligned}
& \Psi(s_k)\Psi(s_l)\Psi(s_m)\Psi(s_n)X \ , \\
X &= \int_{S^2}\kappa_{s_k,s_l,s_m,s_n}\Psi(s_r)d\mu(s_r) \\
&= \int_{S^2(s_k,s_l,s_m,s_n)}\Psi(s_m)d\mu(s_r) \\
&= -\int_{D^2(s_i)}\Psi^2(s_r)d\mu(s_r) \ , \ \ i = k,l,s,m \ .
\end{aligned} \tag{11.5.14}$$

These conditions state that the integral of Ψ^2 any disk of fixed radius d is same: this result follows also from the orthogonality condition. This condition might be difficult to satisfy exactly and the notion of finite measurement resolution might be needed. For instance, it might be necessary to restrict the consideration to a discrete lattice of points which would lead back to a discretized version of algebra. Thus it seems that the continuum generalization of the proposed solution to fusion rules does not work.

11.6 Could operads allow the formulation of the generalized Feynman rules?

The previous discussion of symplectic fusion rules leaves open many questions.

(a) How to combine symplectic and conformal fields to what might be called symplecto-conformal fields?

(b) The previous discussion applies only in super-symplectic degrees of freedom and the question is how to generalize the discussion to super Kac-Moody degrees of freedom. One must of course also try to identify more precisely what Kac-Moody degrees of freedom are!

(c) How four-momentum and its conservation in the limits of measurement resolution enters this picture? Could the phase factors assocaited with the symplectic triangulation carry information about four-momentum?

(d) At least two operads related to measurement resolution seem to be present: the operads formed by the symplecto-conformal fields and by generalized Feynman diagrams. For generalized Feynman diagrams causal diamond (CD) is the basic object whereas disks of S^2 are the basic objects in the case of symplecto-conformal QFT with a finite measurement resolution. Could these two different views about finite measurement resolution be more or less equivalent and could one understand this equivalence at the level of details.

(e) Is it possible to formulate generalized Feynman diagrammatics and improved measurement resolution algebraically?

11.6.1 How to combine conformal fields with symplectic fields?

The conformal fields of conformal field theory should be somehow combined with symplectic scalar field to form what might be called symplecto-conformal fields.

(a) The simplest thing to do is to multiply ordinary conformal fields by a symplectic scalar field so that the fields would be restricted to a discrete set of points for a given realization of N-dimensional fusion algebra. The products of these symplecto-conformal fields at different points would define a finite-dimensional algebra and the products of these fields at same point could be assumed to vanish.

(b) There is a continuum of geometric realizations of the symplectic fusion algebra since the edges of symplectic triangles can be selected rather freely. The integrations over the coordinates z_k (most naturally the complex coordinate of S^2 transforming linearly under rotations around quantization axes of angular momentum) restricted to the circle appearing in the definition of simplest stringy amplitudes would thus correspond to the integration over various geometric realizations of a given N-dimensional symplectic algebra.

Fusion algebra realizes the notion of finite measurement resolution. One implication is that all n-point functions vanish for $n > N$. Second implication could be that the points appearing in the geometric realizations of N-dimensional symplectic fusion algebra have some minimal distance. This would imply a cutoff to the multiple integrals over complex coordinates z_k varying along circle giving the analogs of stringy amplitudes. This cutoff is not absolutely necessary since the integrals defining stringy amplitudes are well-defined despite the singular behavior of n-point functions. One can also ask whether it is wise to introduce a cutoff that is not necessary and whether fusion algebra provides only a justification for the $1 + i\epsilon$ prescription to avoid poles used to obtain finite integrals.

The fixed values for the quantities $\int A_\mu dx^\mu$ along the edges of the symplectic triangles could indeed pose a lower limit on the distance between the vertices of symplectic triangles. Whether this occurs depends on what one precisely means with symplectic triangle.

(a) The conformally invariant condition that the angles between the edges at vertices are smaller than π for triangle and larger than π for its conjugate is not enough to exclude loopy edges and one would obtain ordinary stringy amplitudes multiplied by the symplectic phase factors. The outcome would be an integral over arguments $z_1, z_2, ..z_n$ for standard stringy n-point amplitude multiplied by a symplectic phase factor which is piecewise constant in the integration domain.

(b) The condition that the points at different edges of the symplectic triangle can be connected by a geodesic segment belonging to the interior of the triangle is much stronger and would induce a length scale cutoff since loops cannot be used to create large enough value of $\int A_\mu dx^\mu$ for a given side of triangle. Symplectic invariance would be obtained for small enough symplectic transformations. How to realize this cutoff at the level of calculations is not clear. One could argue that this problem need not have any nice solution and since finite measurement resolution requires only finite calculational resolution, the approximation allowing loopy edges is acceptable.

(c) The restriction of the edges of the symplectic triangle within a tubular neighborhood of a geodesic -more more generally an orbit of charged particle - with thickness determined by the length scale resolution in S^2 would also introduce the length scale cutoff with symplectic invariance within measurement resolution.

Symplecto-conformal should form an operad. This means that the improvement of measurement resolution should correspond also to an algebra homomorphism in which super-symplectic symplecto-conformal fields in the original resolution are mapped by algebra homomorphism into fields which contain sum over products of conformal fields at different points: for the symplectic parts of field the products reduces always to a sum over the values of field. For instance, if the field at point s is mapped to an average of fields at points s_k, nilpotency condition $x^2 = 0$ is satisfied.

11.6.2 Symplecto-conformal fields in Super-Kac-Moody sector

The picture described above applies only in super-symplectic degrees of freedom. The vertices of generalized Feynman diagrams are absent from the description and CP_2 Kähler form induced to space-time surface which is absolutely essential part of quantum TGD is nowhere visible in the treatment.

How should one bring in Super Kac-Moody (SKM) algebra? The condition that the basic building bricks are same for the treatment of these degrees of freedom is a valuable guideline.

What does SKM algebra mean?

The first thing to consider is what SKM could mean. The recent view is that symplectic algebra corresponds to symplectic transformations for the boundary of causal diamond CD which looks locally like $\delta M^4_\pm \times CP_2$. For this super-algebra fermionic generators would be contractions of covariantly constant right-handed neutrino with the second quantized induced spinor field to which the contraction$j^k_A \Gamma_k$ of symplectic vector field with gamma matrices acts. For SKM algebra corresponding generators would be similar contractions of other spinor modes but involving only Killing vectors fields that is symplectic isometries.

The recent view about quantum criticality strongly suggests that the conformal symmetries act as almost gauge symmetries producing from a given preferred extremal new ones with same action and conserved charges. "Almost" means that sub-algebra of conformal algebra annihilates the physical states. The subalgebras in question form a fractal hierarchy and are isomorphic with the conformal algebra itself. They contain generators for which the conformal weight is multiple of integer n characterizing also the value of Planck constant given by $h_{eff} = n \times h$. n defines the number of conformal equivalence classes of space-time surfaces connecting fixed 3-surfaces at the boundaries of CD (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Since Kähler action reduces for the general ansatz for the preferred extremals to 3-D Chern-Simons terms, the action of the conformal symmetries reduces also to the 3-D space-like surfaces where it is trivial by definition and to non-trivial action to the light-like 3-surfaces at which the signature of the induced metric changes: I have used to call this surface partonic orbits.

It must be however observed that one can consider also the possibility that SKM algebra corresponds to the isometries of $\delta M^4 \pm \times CP_2$ continued to the space-time surface by field equations. These isometries are conformal transformations of S^2 ($\delta M^4_{\pm} = S^2 \times R_+$) with conformal scaling compensated by the local scaling of the light-like radial coordinate r_M to guarantee that the metric reducing to that for S^2 apart from conformal scaling factor R^2_M remains invariant. If this is the case the SKM contains also other than symplectic isometries.

Attempt to formulate symplectic triangulation for SKM algebra

The analog of symplectic triangulation for SKM algebra obviously requires that SKM algebra corresponds to symplectic isometries rather than including all $\delta M^4_{\pm} = S^2 \times R_+$ isometries in one-one correspondence with conformal transformations of S^2.

(a) In the transition from super-symplectic to SKM degrees of freedom the light-cone boundary is naturally replaced with the light-like 3-surface X^3 representing the light-like random orbit of parton and serving as the basic dynamical object of quantum TGD. The sphere S^2 of light-cone boundary is in turn replaced with a partonic 2-surface X^2. This suggests how to proceed.

(b) In the case of SKM algebra the symplectic fusion algebra is represented geometrically as points of partonic 2-surface X^2 by replacing the symplectic form of S^2 with the induced CP_2 symplectic form at the partonic 2-surface and defining $U(1)$ gauge field. This gives similar hierarchy of symplecto-conformal fields as in the super-symplectic case. This also realizes the crucial aspects of the classical dynamics defined by Kähler action. In particular, for vacuum 2-surfaces symplectic fusion algebra trivializes since Kähler magnetic fluxes vanish identically and 2-surfaces near vacua require a large value of N for the dimension of the fusion algebra since the available Kähler magnetic fluxes are small.

(c) In super-symplectic case the projection along light-like ray allows to map the points at the light-cone boundaries of CD to points of same sphere S^2. In the case of light-like 3-surfaces light-like geodesics representing braid strands allow to map the points of the partonic two-surfaces at the future and past light-cone boundaries to the partonic 2-surface representing the vertex. The earlier proposal was that the ends of strands meet at the partonic 2-surface so that braids would replicate at vertices. The properties of symplectic fields would however force identical vanishing of the vertices if this were the case. There is actually no reason to assume this condition and with this assumption vertices involving total number N of incoming and outgoing strands correspond to symplecto-conformal N-point function as is indeed natural. Also now Kähler magnetic flux induces cutoff distance.

(d) SKM braids reside at light-like 3-surfaces representing lines of generalized Feynman diagrams. If super-symplectic braids are needed at all, they must be assigned to the two light-like boundaries of CD meeting each other at the sphere S^2 at which future and past directed light-cones meet.

11.6.3 The treatment of four-momentum

Four-momentum enjoys a special role in super-symplectic and SKM representations in that it does not correspond to a quantum number assignable to the generators of these algebras. It would be nice if the somewhat mysterious phase factors associated with the representation of the symplectic algebra could code for the four-momentum - or rather the analogs of plane waves representing eigenstates of four-momentum at the points associated with the geometric representation of the symplectic fusion algebra.

Also the vision about TGD as almost topological QFT suggests that the symplectic degrees of freedom added to the conformal degrees of freedom defining alone a mere topological QFt somehow code for the physical degrees of freedom should and also four-momentum. If so, the symplectic triangulation might somehow code for four-momentum.

The representation of longitudinal momentum in terms of phase factors

The following argument suggests that S^2 and X^2 triangulations cannot naturally represent four-momentum and that one needs extension into 3-D light-like triangulation to achieve this.

(a) The basic question is whether four-momentum could be coded in terms of non-integrable phase factors appearing in the representations of the symplectic fusion algebras.

(b) In the symplectic case S^2 triangulation suggests itself as a representation of angular momentum only: it would be kind of spin network. In the SKM case X^2 would suggest representation of color hyper charge and isospin in terms of phases since CP_2 symmetries act non-trivially in Chern-Simons action. Does this mean that symplectic and SKM triangulations must be extended so that they are 3-D and defined for space-like 3-surface and the light-like orbit of partonic 2-surface. This would give additional phase factors assignable to presumably light-like edges. Ligh-like momentum would be natural and the recent twistorial formulation of quantum TGD indeed assigns massless momenta to fermion lines.

Suppose that one has 3-D light-like triangulation eith at δCD or at light-like orbits of partonic 2-surface. Consider first coding of four-momentum assuming only Kähler gauge potential of CP_2 possibly having M^4 part which is pure gauge.

(a) Four different phase factors are needed if all components of four-momentum are to be coded. Both number theoretical vision about quantum TGD and the realization of the hierarchy of Planck constants assign to each point of space-time surface the same plane $M^2 \subset M^4$ having as the plane of non-physical polarizations. This condition allows to assign to a given light-like partonic 3-surface unique extremal of Kähler action defining the Kähler function as the value of Kähler action.

Also p-adic mass calculations support the view that the physical states correspond to eigen states for the components of longitudinal momentum only (also the parton model for hadrons assumes this). This encourages to think that only M^2 part of four-momentum is coded by the phase factors. Transversal momentum squared would be a well defined quantum number and determined from mass shell conditions for the representations of super-symplectic (or equivalently SKM) conformal algebra much like in string model.

(b) The phase factors associated with the 3-D symplectic fusion algebra in $S^2 \times R_+$ mean a deviation from conformal n-point functions, and the innocent question is whether these phase factors could be identified as plane-wave phase factors in S^2 could be associated with the transversal part of the four-momentum so that the n-point functions would be strictly analogous with stringy amplitudes. Alternative, and perhaps more natural, interpretation is in terms of spin and angular momentum.

(c) Suppose one allows a gauge transformation of Kähler gauge potential inducing a pure gauge M^4 component to the Kähler gauge potential expressible as scalar function of M^4 coordianates. This kind of term might allow to achieve the vanishing of $j^\alpha A_\alpha$ term of at least its integral over space-time surface in Kähler action implying reduction of Kähler action to Chern-Simons terms if weak form of electric magnetic duality holds true. The scalar function can be interpreted as integral of a position dependent momentum along curve defined by $S^2 \times R_+$ triangulation and gives hopes of coding four-momentum in terms of Kähler gauge potential.

In fact, the identification of the phase factors $exp(i \int A_\mu dx^\mu/\hbar)$ along a path as phase factors $exp(ip_{L,k}\Delta m^k)$ defined by the ends of the path and associated with the longitudinal part of four-momentum would correspond to an integral form of covariant constancy condition $\frac{dx^\mu}{ds}(\partial_\mu - iA_\mu)\Psi = 0$ along the edge of the symplectic triangle of more general path.

(d) For the SKM triangulation associated with the light-like orbit X_l^3 of partonic 2-surface analogous phase factor would come from the integral along the (most naturally) light-like curve defining braid strand associated with the point in question. A geometric representation for the two projections of the four-momentum would thus result in SKM degrees of freedom and apart from the non-uniqueness related to the multiples of a 2π the components of M^2 momentum could be deduced from the phase factors. If one is satisfied with the projection of momentum in M^2, this is enough.

(e) Neither of these phase factors is able to code all components of four-momentum. One might however hope that together they could give enough information to deduce the four-momentum if it is assumed to correspond to the rest system.

(f) The phase factors assignable to the symplectic triangles in S^2 and X^2have nothing to do with momentum. Because the space-like phase factor $exp(iS_z\Delta\phi/\hbar)$ associated with the edge of the symplectic triangle is completely analogous to that for momentum, one can argue that the symplectic triangulation could define a kind of spin network utilized in discretized approaches to quantum gravity. The interpretation raises the question about the interpretation of the quantum numbers assignable to the Lorentz invariant phase factors defined by the CP_2 Kähler gauge potential.

The quantum numbers associated with phase factors for CP_2 parts of Kähler gauge potentials

Suppose that it is possible to assign two independent and different phase factors to the same geometric representation, in other words have two independent symplectic fields with the same geometric representation. The product of two symplectic fields indeed makes sense and satisfies the defining conditions. One can define prime symplectic algebras and decompose symplectic algebras to prime factors. Since one can allow permutations of elements in the products it becomes possible to detect the presence of product structure experimentally by detecting different combinations for products of phases caused by permutations realized as different combinations of quantum numbers assigned with the factors. The geometric representation for the product of n symplectic fields would correspond to the assignment of n edges to any pair of points. The question concerns the interpretation of the phase factors assignable to the CP_2 parts of Kähler gauge potentials of S^2 and CP_2 Kähler form.

(a) The natural interpretation for the two additional phase factors would be in terms of color quantum numbers. Color hyper charge and isospin are mathematically completely analogous to the components of four-momentum so that a possible identification of the phase factors is as a representation of these quantum numbers. The representation of plane waves as phase factors $exp(ip_k\Delta m^k/\hbar)$ generalizes to the representation $exp(iQ_A\Delta\Phi^A/\hbar)$, where Φ_A are the angle variables conjugate to the Hamiltonians representing color hyper charge and isospin. This representation depends on end points only so that the crucial symplectic invariance with respect to the symplectic transformations respecting the end points of the edge is not lost ($U(1)$ gauge transformation is induced by the scalar j^kA_k, where j^k is the symplectic vector field in question).

(b) One must be cautious with the interpretation of the phase factors as a representation for color hyper charge and isospin since a breaking of color gauge symmetry would result since the phase factors associated with different values of color isospin and hypercharge would be different and could not correspond to same edge of symplectic triangle. This is questionable since color group itself represents symplectic transformations. The construction of CP_2 as a coset space $SU(3)/U(2)$ identifies $U(2)$ as the holonomy group of spinor connection having interpretation as electro-weak group. Therefore also the interpretation of the phase factors in terms of em charge and weak charge can be considered. In TGD framework electro-weak gauge potential indeed suffer a non-trivial gauge transformation under color rotations so that the correlation between electro-weak quantum numbers and non-integrable phase factors in Cartan algebra of the color group could make sense. Electro-weak symmetry breaking would have a geometric correlate in the sense that different values of weak isospin cannot correspond to paths with same values of phase angles $\Delta\Phi^A$ between end points.

(c) If the phase factors associated with the M^4 and CP_2 are assumed to be identical, the existence of geometric representation is guaranteed. This however gives constraints between rest mass, spin, and color (or electro-weak) quantum numbers.

Some general comments

Some further comments about phase factors are in order.

(a) By number theoretical universality the plane wave factors associated with four-momentum must have values coming as roots of unity (just as for a particle in box consisting of discrete lattice of points). At light-like boundary the quantization conditions reduce to the condition that the value of light-like coordinate is rational of form m/N, if N:th roots of unity are allowed.

(b) In accordance with the finite measurement resolution of four-momentum, four-momentum conservation is replaced by a weaker condition stating that the products of phase factors representing incoming and outgoing four-momenta are identical. This means that positive and negative energy states at opposite boundaries of CD would correspond to complex conjugate representations of the fusion algebra. In particular, the product of phase factors in the decomposition of the conformal field to a product of conformal fields should correspond to the original field value. This would give constraints on the trees physically possible in the operad formed by the fusion algebras. Quite generally, the phases expressible as products of phases $exp(in\pi/p)$, where $p \leq N$ is prime must be allowed in a given resolution and this suggests that the hierarchy of p-adic primes is involved. At the limit of very large N exact momentum conservation should emerge.

(c) Super-conformal invariance gives rise to mass shell conditions relating longitudinal and transversal momentum squared. The massivation of massless particles by Higgs mechanism and p-adic thermodynamics pose additional constraints to these phase factors.

11.6.4 What does the improvement of measurement resolution really mean?

To proceed one must give a more precise meaning for the notion of measurement resolution. Two different views about the improvement of measurement resolution emerge. The first one relies on the replacement of braid strands with braids applies in SKM degrees of freedom and the homomorphism maps symplectic fields into their products. The homomorphism based on the averaging of symplectic fields over added points consistent with the extension of fusion algebra described in previous section is very natural in super-symplectic degrees of freedom. The directions of these two algebra homomorphisms are different. The question is whether both can be involved with both super-symplectic and SKM case. Since the end points of SKM braid strands correspond to both super-symplectic and SKM degrees of freedom, it seems that division of labor is the only reasonable option.

(a) Quantum classical correspondence requires that measurement resolution has a purely geometric meaning. A purely geometric manner to interpret the increase of the measurement resolution is as a replacement of a braid strand with a braid in the improved resolution. If one assigns the phase factor assigned with the fusion algebra element with four-momentum, the conservation of the phase factor in the associated homomorphism is a natural constraint. The mapping of a fusion algebra element (strand) to a product of fusion algebra elements (braid) allows to realize this condition. Similar mapping of field value to a product of field values should hold true for conformal parts of the fields. There exists a large number equivalent geometric representations for a given symplectic field value so that one obtains automatically an averaging in conformal degrees of freedom. This interpretation for the improvement of measurement resolution looks especially natural for SKM degrees of freedom for which braids emerge naturally.

(b) One can also consider the replacement of symplecto-conformal field with an average over the points becoming visible in the improved resolution. In super-symplectic degrees of freedom this looks especially natural since the assignment of a braid with light-cone boundary is not so natural as with light-like 3-surface. This map does not conserve the phase factor but this could be interpreted as reflecting the fact that the values of the light-like radial coordinate are different for points involved. The proposed extension of the symplectic algebra proposed in the previous section conforms with this interpretation.

(c) In the super-symplectic case the improvement of measurement resolution means improvement of angular resolution at sphere S^2. In SKM sector it means improved resolution for the position at partonic 2-surface. This generalizes also to the 3-D symplectic triangulations. For SKM algebra the increase of the measurement resolution related to the braiding takes place inside light-like 3-surface. This operation corresponds naturally to an addition of sub-CD inside which braid strands are replaced with braids. This is like looking with a microscope a particular part of line of generalized Feynman graph inside CD and corresponds to a genuine physical process inside parton. In super-symplectic case the replacement of a braid strand with braid (at light-cone boundary) is induced by the replacement of the projection of a point of a partonic 2-surface to S^2 with a a collection of points coming from several partonic 2-surfaces. This replaces the point s of S^2 associated with CD with a set of points s_k of S^2 associated with sub-CD. Note that the solid angle spanned by these points can be rather larger so that zoom-up is in question.

(d) The improved measurement resolution means that a point of S^2 (X^2) at boundary of CD is replaced with a point set of S^2 (X^2) assignable to sub-CD. The task is to map the point set to a small disk around the point. Light-like geodesics along light-like X^3 defines this map naturally in both cases. In super-symplectic case this map means scaling down of the solid angle spanned by the points of S^2 associated with sub-CD.

11.6.5 How do the operads formed by generalized Feynman diagrams and symplecto-conformal fields relate?

The discussion above leads to following overall view about the situation. The basic operation for both symplectic and Feynman graph operads corresponds to an improvement of measurement resolution. In the case of planar disk operad this means to a replacement of a white region of a map with smaller white regions. In the case of Feynman graph operad this means better space-time resolution leading to a replacement of generalized Feynman graph with a new one containing new sub-CD bringing new vertices into daylight. For braid operad the basic operation means looking a braid strand with a microscope so that it can resolve into a braid: braid becomes a braid of braids. The latter two views are equivalent if sub-CD contains the braid of braids.

The disks D^2 of the planar disk operad has natural counterparts in both super-symplectic and SKM sector.

(a) For the geometric representations of the symplectic algebra the image points vary in continuous regions of S^2 (X^2) since the symplectic area of the symplectic triangle is a highly flexible constraint. Posing the condition that any point at the edges of symplectic triangle can be connected to any another edge excludes symplectic triangles with loopy sides so that constraint becomes non-trivial. In fact, since two different elements of the symplectic algebra cannot correspond to the same point for a given geometric representation, each element must correspond to a connected region of S^2 (X^2). This allows a huge number of representations related by the symplectic transformations S^2 in super-symplectic case and by the symplectic transformations of CP_2 in SKM case. In the case of planar disk operad different representations are related by isotopies of plane.

This decomposition to disjoint regions naturally correspond to the decomposition of the disk to disjoint regions in the case of planar disk operad and Feynman graph operad (allowing zero energy insertions). Perhaps one might say that N-dimensional elementary symplectic algebra defines an N-coloring of S^2 (S^2) which is however not the same thing as the 2-coloring possible for the planar operad. TGD based view about Higgs mechanism leads to a decomposition of partonic 2-surface X^2 (its light-like orbit X^3) into conformal patches. Since also these decompositions correspond to effective discretizations of X^2 (X^3), these two decompositions would naturally correspond to each other.

(b) In SKM sector disk D^2 of the planar disk operad is replaced with the partonic 2-surface X^2 and since measurement resolution is a local notion, the topology of X^2 does not matter. The improvement of measurement resolution corresponds to the replacement of braid strand with braid and homomorphism is to the direction of improved spatial resolution.

(c) In super-symplectic case D^2 is replaced with the sphere S^2 of light-cone boundary. The improvement of measurement resolution corresponds to introducing points near the original point and the homomorphism maps field to its average. For the operad of generalized Feynman diagrams CD defined by future and past directed light-cones is the basic object. Given CD can be indeed mapped to sphere S^2 in a natural manner. The light-like boundaries of CDs are metrically spheres S^2. The points of light-cone boundaries can be projected to any sphere at light-cone boundary. Since the symplectic area of the sphere corresponds to solid angle, the choice of the representative for S^2 does not matter. The sphere defined by the intersection of future and past light-cones of CD however provides a natural identification of points associated with positive and negative energy parts of the state as points of the same sphere. The points of S^2 appearing in n-point function are replaced by point sets in a small disks around the n points.

(d) In both super-symplectic and SKM sectors light-like geodesic along X^3 mediate the analog of the map gluing smaller disk to a hole of a disk in the case of planar disk operad defining the decomposition of planar tangles. In super-symplectic sector the set of points at the sphere corresponding to a sub-CD is mapped by SKM braid to the larger CD and for a typical braid corresponds to a larger angular span at sub-CD. This corresponds to the gluing of D^2 along its boundaries to a hole in D^2 in disk operad. A scaling transformation allowed by the conformal invariance is in question. This scaling can have a non-trivial effect if the conformal fields have anomalous scaling dimensions.

(e) Homomorphisms between the algebraic structures assignable to the basic structures of the operad (say tangles in the case of planar tangle operad) are an essential part of the power of the operad. These homomorphisms associated with super-symplectic and SKM sector code for two views about improvement of measurement resolution and might lead to a highly unique construction of M-matrix elements.

The operad picture gives good hopes of understanding how M-matrices corresponding to a hierarchy of measurement resolutions can be constructed using only discrete data.

(a) In this process the n-point function defining M-matrix element is replaced with a superposition of n-point functions for which the number of points is larger: $n \rightarrow \sum_{k=1,\ldots,m} n_k$. The numbers n_k vary in the superposition. The points are also obtained by downwards scaling from those of smaller S^2. Similar scaling accompanies the composition of tangles in the case of planar disk operad. Algebra homomorphism property gives constraints on the compositeness and should govern to a high degree how the improved measurement resolution affects the amplitude. In the lowest order approximation the M-matrix element is just an n-point function for conformal fields of positive and negative energy parts of the state at this sphere and one would obtain ordinary stringy amplitude in this approximation.

(b) Zero energy ontology means also that each addition in principle brings in a new zero energy insertion as the resolution is improved. Zero energy insertions describe actual physical processes in shorter scales in principle affecting the outcome of the experiment in longer time scales. Since zero energy states can interact with positive (negative) energy particles, zero energy insertions are not completely analogous to vacuum bubbles and cannot be neglected. In an idealized experiment these zero energy states can be assumed to be absent. The homomorphism property must hold true also in the presence of the zero energy insertions. Note that the Feynman graph operad reduces to planar disk operad in absence of zero energy insertions.

11.7 Possible other applications of category theory

It is not difficult to imagine also other applications of category theory in TGD framework.

11.7.1 Categorification and finite measurement resolution

I read a very stimulating article by John Baez with title Categorification [A41] about the basic ideas behind a process called categorification. The process starts from sets consisting of elements. In the following I describe the basic ideas and propose how categorification could be applied to realize the notion of finite measurement resolution in TGD framework.

What categorification is?

In categorification sets are replaced with categories and elements of sets are replaced with objects. Equations between elements are replaced with isomorphisms between objects: the right and left hand sides of equations are not the same thing but only related by an isomorphism so that they are not tautologies anymore. Functions between sets are replaced with functors between categories taking objects to objects and morphisms to morphisms and respecting the composition of morphisms. Equations between functions are replaced with natural isomorphisms between functors, which must satisfy certain coherence laws representable in terms of commuting diagrams expressing conditions such as commutativity and associativity.

The isomorphism between objects represents equation between elements of set replaces identity. What about isomorphisms themselves? Should also these be defined only up to an isomorphism of isomorphism? And what about functors? Should one continue this replacement ad infinitum to obtain a hierarchy of what might be called n-categories, for which the process stops after n:th level. This rather fuzzy buisiness is what mathematicians like John Baez are actually doing.

Why categorification?

There are good motivations for the categofication. Consider the fact that natural numbers. Mathematically oriented person would think number '3' in terms of an abstract set theoretic axiomatization of natural numbers. One could also identify numbers as a series of digits. In the real life the representations of three-ness are more concrete involving many kinds of associations. For a child '3' could correspond to three fingers. For a mystic it could correspond to holy trinity. For a Christian "faith,hope,love". All these representations are isomorphic representation of threeness but as real life objects three sheeps and three cows are not identical.

We have however performed what might be called decategorification: that is forgitten that the isomorphic objects are not equal. Decatecorification was of course a stroke of mathematical genius with enormous practical implications: our information society represents all kinds of things in terms of numbers and simulates successfully the real world using only bit sequences. The dark side is that treating people as mere numbers can lead to a rather cold society.

Equally brilliant stroke of mathematical genius is the realization that isomorphic objects are not equal. Decategorization means a loss of information. Categorification brings back this information by bringing in consistency conditions known as coherence laws and finding these laws is the hard part of categorization meaning discovery of new mathematics. For instance, for braid groups commutativity modulo isomorphisms defines a highly non-trivial coherence law leading to an extremely powerful notion of quantum group having among other things applications in topological quantum computation.

The so called associahedrons [A50] emerging in n-category theory could replace space-time and space as fundamental objects. Associahedrons are polygons used to represent geometrically associativity or its weaker form modulo isomorphism for the products of n objects bracketed in all possible manners. The polygon defines a hierarchy containing sub-polygons as its edges containing.... Associativity states the isomorphy of these polygons. Associahedrons and related geometric representations of category theoretical arrow complexes in terms or simplexes allow a beautiful geometric realization of the coherence laws. One could perhaps say that categories as discrete structures are not enough: only by introducing the continuum allowing geometric representations of the coherence laws things become simple.

No-one would have proposed categorification unless it were demanded by practical needs of mathematics. In many mathematical applications it is obvious that isomorphism does not mean identity. For instance, in homotopy theory all paths deformable to each other in continuous manner are homotopy equivalent but not identical. Isomorphism is now homotopy. These paths can be connected and form a groupoid. The outcome of the groupoid operation is determined up to homotopy. The deformations of closed path starting from a given point modulo homotopies form homotopy group and one can interpret the elements of homotopy group as copies of the point which are isomorphic. The replacement of the space with its universal covering makes this distinction explicit. One can form homotopies of homotopies and continue this process ad infinitum and obtain in this manner homotopy groups as characterizes of the topology of the space.

Cateforification as a manner to describe finite measurement resolution?

In quantum physics gauge equivalence represents a standard example about equivalence modulo isomorphisms which are now gauge transformations. There is a practical strategy to treat the situation: perform a gauge choice by picking up one representative amongst infinitely many isomorphic objects. At the level of natural numbers a very convenient gauge fixing would correspond the representation of natural number as a sequence of decimal digits rather than image of three cows.

In TGD framework a excellent motivation for categorification is the need to find an elegant mathematical realization for the notion of finite measurement resolution. Finite measurement resolutions (or cognitive resolutions) at various levels of information transfer hierarchy imply accumulation of uncertainties. Consider as a concrete example uncertainty in the determination of basic parameters of a mathematical model. This uncertainty is reflected to final outcome as via a long sequence of mathematical maps and additional uncertainties are produced by the approximations at each step of this process.

How could onbe describe the finite measurement resolution elegantly in TGD Universe? Categorification suggests a natural method. The points equivalent with measurement resolution are isomorphic with each other. A natural guess inspired by gauge theories is that one should perform a gauge choice as an analog of decategorification. This allows also to avoid continuum of objects connected by arrows not n spirit with the discreteness of category theoretical approach.

(a) At space-time level gauge choice means discretization of partonic 2-surfaces replacing them with a discrete set points serving as representatives of equivalence classes of points equivalent under finite measurement resolution. An especially interesting choice of points is as rational points or algebraic numbers and emerges naturally in p-adicization process. One can also introduce what I have called symplectic triangulation of partonic 2-surfaces with the nodes of the triangulation representing the discretization and carrying quantum numbers of various kinds.

(b) At the level of "world classical worlds" (WCW) this means the replacement of the subgroup if the symplectic group of $\delta M^4 \times CP_2$ -call it G - permuting the points of the symplectic triangulation with its discrete subgroup obtained as a factor group G/H , where H is the normal subgroup of G leaving the points of the symplectic triangulation fixed. One can also consider subgroups of the permutation group for the points of the triangulation. One can also consider flows with these properties to get braided variant of G/H. It would seem that one cannot regard the points of triangulation as isomorphic in the category theoretical sense. This because, one can have quantum superpositions of states located at these points and the factor group acts as the analog of isometry group. One can also have many-particle states with quantum numbers at several points. The possibility to assign quantum numbers to a given point becomes the physical counterpart for the axiom of choice.

The finite measurement resolution leads to a replacement of the infinite-dimensional world of classical worlds with a discrete structure. Therefore operation like integration over entire "world of classical worlds" is replaced with a discrete sum.

(c) What suggests itself strongly is a hierarchy of n-categories as a proper description for the finite measurement resolution. The increase of measurement resolution means increase for the number of braid points. One has also braids of braids of braids structure implied by the possibility to map infinite primes, integers, and rationals to rational functions of several variables and the conjecture possibility to represent the hierarchy of Galois groups involved as symplectic flows. If so the hierarchy of n-categories would correspond to the hierarchy of infinite primes having also interpretation in terms of repeated second quantization of an arithmetic SUSY such that many particle states of previous level become single particle states of the next level.

The finite measurement resolution has also a representation in terms of inclusions of hyperfinite factors of type II_1 defined by the Clifford algebra generated by the gamma matrices of WCW [K79]

(a) The included algebra represents finite measurement resolution in the sense that its action generates states which are cannot be distinguished from each other within measurement resolution used. The natural conjecture is that this indistuinguishability corresponds to a gauge invariance for some gauge group and that TGD Universe is analogous to Turing machine in that almost any gauge group can be represented in terms of finite measurement resolution.

(b) Second natural conjecture inspired by the fact that symplectic groups have enormous representabive power is that these gauge symmetries allow representation as subgroups of the symplectic group of $\delta M^4 \times CP_2$. A nice article about universality of symplectic groups is the article The symplectification of science by Mark. J. Gotay [A31].

(c) An interesting question is whether there exists a finite-dimensional space, whose symplectomorphisms would allow a representation of any gauge group (or of all possible Galois groups as factor groups) and whether $\delta M^4 \times CP_2$ could be a space of this kind with the smallest possible dimension.

11.7.2 Inclusions of HFFs and planar tangles

Finite index inclusions of HFFs are characterized by non-branched planar algebras for which only an even number of lines can emanate from a given disk. This makes possible a consistent coloring of the k-tangle by black and white by painting the regions separated by a curve using opposite colors. For more general algebras, also for possibly existing branched tangle algebras, the minimum number of colors is four by four-color theorem. For the description of zero energy states the 2-color assumption is not needed so that the necessity to have general branched planar algebras is internally consistent. The idea about the inclusion of positive energy state space into the space of negative energy states might be consistent with branched planar algebras and the requirement of four colors since this inclusion involves also conjugation and is thus not direct.

In [A23] if was proposed that planar operads are associated with conformal field theories at sphere possessing defect lines separating regions with different color. In TGD framework and for branched planar algebras these defect lines would correspond to light-like 3-surfaces. For fermions one has single wormhole throat associated with topologically condensed CP_2 type extremal and the signature of the induced metric changes at the throat. Bosons correspond to pairs of wormhole throats associated with wormhole contacts connecting two space-time sheets modellable as a piece of CP_2 type extremal. Each boson thus corresponds to 2 lines within CP_2 radius so that in purely bosonic case the planar algebra can correspond to that associated with an inclusion of HFFs.

11.7.3 2-plectic structures and TGD

Chris Rogers and Alex Hoffnung have demonstrated [A83] that the notion of symplectic structure generalizes to n-plectic structure and in $n = 2$ case leads to a categorification of Lie algebra to 2-Lie-algebra. In this case the generalization replaces the closed symplectic 2-form with a closed 3-form ω and assigns to a subset of one-forms defining generalized Hamiltonians vector fields leaving the 3-form invariant.

There are two equivalent definitions of the Poisson bracket in the sense that these Poisson brackets differ only by a gradient, which does not affect the vector field assignable to the Hamiltonian one-form. The first bracket is simply the Lie-derivate of Hamiltonian one form G with respect to vector field assigned to F. Second bracket is contraction of Hamiltonian one-forms with the three-form ω. For the first variant Jacobi identities hold true but Poisson bracket is antisymmetric only modulo gradient. For the second variant Jacobi identities hold true only modulo gradient but Poisson bracket is antisymmetric. This modulo property is in accordance with category theoretic thinking in which commutativity, associativity, antisymmetry,... hold true only up to isomorphism.

For 3-dimensional manifolds n=2-plectic structure has the very nice property that *all* one-forms give rise to Hamiltonian vector field. In this case any 3-form is automatically closed so that a large variety of 2-plectic structures exists. In TGD framework the natural choice for the 3-form ω is as Chern-Simons 3-form defined by the projection of the Kähler gauge potential to the light-like 3-surface. Despite the fact the induced metric is degenerate, one can deduce the Hamiltonian vector field associated with the one-form using the general defining conditions

$$i_{v_F}\omega \quad = \quad dF \qquad (11.7.1)$$

since the vanishing of the metric determinant appearing in the formal definition cancels out in the expression of the Hamiltonian vector field.

The explicit formula is obtained by writing ω as

$$\begin{aligned} &\omega = K\epsilon_{\alpha\beta\gamma} \times \epsilon^{\mu\nu\delta} A_\mu J_{\nu\delta}\sqrt{g} \quad = \epsilon_{\alpha\beta\gamma} \times C-S \quad , \\ &C-S = KE^{\alpha\beta\gamma} A_\alpha J_{\beta\gamma} \quad . \end{aligned} \qquad (11.7.2)$$

Here $E^{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma}$ holds true numerically and metric determinant, which vanishes for light-like 3-surfaces, has disappeared.

The Hamiltonian vector field is the curl of F divided by the Chern-Simons action density $C-S$:

$$v_F^\alpha = \frac{1}{2} \times \frac{\epsilon^{\alpha\beta\gamma}(\partial_\beta F_\gamma - \partial_\gamma F_\beta)\sqrt{g}}{C-S\sqrt{g}} = \frac{1}{2} \times \frac{E^{\alpha\beta\gamma}(\partial_\beta F_\gamma - \partial_\gamma F_\beta)}{C-S} \quad . \qquad (11.7.3)$$

The Hamiltonian vector field multiplied by the dual of 3-form multiplied by the metric determinant has a vanishing divergence and is analogous to a vector field generating volume preserving flow. and the value of Chern Simons 3-form defines the analog of the metric determinant for light-like 3-surfaces. The generalized Poisson bracket for Hamiltonian 1-forms defined in terms of the action of Hamiltonian vector field on Hamiltonian as $J_1^\beta D_\beta F_2\alpha - J_2^\beta D_\beta H_2\alpha$ is Hamiltonian 1-form. Here J_i denotes the Hamiltonian vector field associated with F_i. The bracked unique apart from gradient. The corresponding vector field is the commutator of the Hamiltonian vector fields.

The objection is that gauge invariance is broken since the expression for the vector field assigned to the Hamiltonian one-form depends on gauge. In TGD framework there is no need to worry since Kähler gauge potential has unique natural expression and the $U(1)$ gauge transformations of Kähler gauge potential induced by symplectic transformations of CP_2 are not genuine gauge transformations but dynamical symmetries since the induced metric changes and space-time surface is deformed. Another important point is that Kähler gauge potential for a given CD has M^4 part which is "pure gauge" constant Lorentz invariant vector and proportional to the inverse of gravitational constant G. Its ratio to CP_2 radius squared is determined from electron mass by p-adic mass calculations and mathematically by quantum criticality fixing also the value of Kähler coupling strength.

11.7.4 TGD variant for the category nCob

John Baez has suggested that quantum field theories could be formulated as functors from the category of n-cobordisms to the category of Hilbert spaces [A43, A42] . In TGD framework light-like 3-surfaces containing the number theoretical braids define the analogs of 3-cobordisms and surface property brings in new structure. The motion of topological condensed 3-surfaces along 4-D space-time sheets brings in non-trivial topology analogous to braiding and not present in category nCob.

Intuitively it seems possible to speak about one-dimensional orbits of wormhole throats and -contacts (fermions and bosons) in background space-time (homological dimension). In this case linking or knotting are not possible since knotting is co-dimension 2 phenomenon and only objects whose homological dimensions sum up to $D-1$ can get linked in dimension D. String like objects could topologically condense along wormhole contact which is string like object. The orbits of closed string like objects are homologically co-dimension 2 objects and could get knotted if one does not allow space-time sheets describing un-knotting. The simplest examples are ordinary knots which are not allowed to evolve by forming self intersections. The orbits of point like wormhole contact and closed string like wormhole contact can get linked: a point particle moving through a closed string is basic dynamical example. There is no good reason preventing unknotting and unlinking in absolute sense.

11.7.5 Number theoretical universality and category theory

Category theory might be also a useful tool to formulate rigorously the idea of number theoretical universality and ideas about cognition. What comes into mind first are functors real to p-adic physics and vice versa. They would be obtained by composition of functors from real to rational physics and back to p-adic physics or vice versa. The functors from real to p-adic physics would provide cognitive representations and the reverse functors would correspond to the realization of intentional action. The functor mapping real 3-surface to p-adic 3-surfaces would be simple: interpret the equations of 3-surface in terms of rational functions with coefficients in some algebraic extension of rationals as equations in arbitrary number field. Whether this description applies or is needed for 4-D space-time surface is not clear.

At the Hilbert space level the realization of these functors would be quantum jump in which quantum state localized to p-adic sector tunnels to real sector or vice versa. In zero energy ontology this process is allowed by conservation laws even in the case that one cannot assign classical conserved quantities to p-adic states (their definition as integrals of conserved currents does not make sense since definite integral is not a well-defined concept in p-adic physics). The interpretation would be in terms of generalized M-matrix applying to cognition and intentionality. This M-matrix would have values in the field of rationals or some algebraic extension of rationals. Again a generalization of Connes tensor product is suggestive.

11.7.6 Category theory and fermionic parts of zero energy states as logical deductions

Category theory has natural applications to quantum and classical logic and theory of computation [A42] . In TGD framework these applications are very closely related to quantum TGD itself since it is possible to identify the positive and negative energy pieces of fermionic part of the zero energy state as a pair of Boolean statements connected by a logical deduction, or rather- quantum superposition of them. An alternative interpretation is as rules for the behavior of the Universe coded by the quantum state of Universe itself. A further interpretation is as structures analogous to quantum computation programs with internal lines of Feynman diagram would represent communication and vertices computational steps and replication of classical information coded by number theoretical braids.

11.7.7 Category theory and hierarchy of Planck constants

Category theory might help to characterize more precisely the proposed geometric realization of the hierarchy of Planck constants explaining dark matter as phases with non-standard value of Planck constant. The situation is topologically very similar to that encountered for generalized Feynman diagrams. Singular coverings and factor spaces of M^4 and CP_2 are glued together along 2-D manifolds playing the role of object and space-time sheets at different vertices could be interpreted as arrows going through this object.

Part III

TWISTORS AND TGD

Chapter 12

Yangian Symmetry, Twistors, and TGD

12.1 Introduction

Lubos Motl [B49] told for some time ago about last impressive steps in the understanding of $\mathcal{N} = 4$ maximally supersymmetric YM theory (SYM) possessing 4-D super-conformal symmetry. This theory is related by AdS/CFT duality to certain string theory in $AdS_5 \times S^5$ background. Second stringy representation was discovered by Witten and based on 6-D Calabi-Yau manifold defined by twistors.

In the following I will discuss briefly the notion of Yangian symmetry and suggest its generalization in TGD framework by replacing conformal algebra with appropriate super-conformal algebras. Also a possible realization of twistor approach and the construction of scattering amplitudes in terms of Yangian invariants defined by Grassmannian integrals is considered in TGD framework and based on the idea that in zero energy ontology one can represent massive states as bound states of massless particles. There is also a proposal for a physical interpretation of the Cartan algebra of Yangian algebra allowing to understand at the fundamental level how the mass spectrum of n-particle bound states could be understood in terms of the n-local charges of the Yangian algebra. The study of modified Dirac equation leads to a detailed proposal for the generators of Yangian algebras [K87]: the proposal is discussed also in this chapter.

Twistors were originally introduced by Penrose to characterize the solutions of Maxwell's equations. Kähler action is Maxwell action for the induced Kähler form of CP_2. The preferred extremals allow a very concrete interpretation in terms of modes of massless non-linear field. Both conformally compactified Minkowski space identifiable as so called causal diamond and CP_2 allow a description in terms of twistors.

12.1.1 Background

I am outsider as far as concrete calculations in $\mathcal{N} = 4$ SUSY are considered and the following discussion of the background probably makes this obvious. My hope is that the reader had patience to not care about this and try to see the big pattern.

The developments began from the observation of Parke and Taylor [B51] that n-gluon tree amplitudes with less than two negative helicities vanish and those with two negative helicities have unexpectedly simple form when expressed in terms of spinor variables used to represent light-like momentum. In fact, in the formalism based on Grassmanian integrals the reduced tree amplitude for two negative helicities is just "1" and defines Yangian invariant. The article *Perturbative Gauge Theory As a String Theory In Twistor Space* [B58] by Witten led to so called Britto-Cachazo-Feng-Witten (BCFW) recursion relations for tree level amplitudes

[B52, B35, B52] allowing to construct tree amplitudes using the analogs of Feynman rules in which vertices correspond to maximally helicity violating tree amplitudes (2 negative helicity gluons) and propagator is massless Feynman propagator for boson. The progress inspired the idea that the theory might be completely integrable meaning the existence of infinite-dimensional un-usual symmetry. This symmetry would be so called Yangian symmetry [K80] assigned to the super counterpart of the conformal group of 4-D Minkowski space.

Drumond, Henn, and Plefka represent in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory* [B39] an argument suggesting that the Yangian invariance of the scattering amplitudes ins an intrinsic property of planar $\mathcal{N}=4$ super Yang Mills at least at tree level.

The latest step in the progress was taken by Arkani-Hamed, Bourjaily, Cachazo, Carot-Huot, and Trnka and represented in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory* [B33] . At the same day there was also the article of Rutger Boels entitled *On BCFW shifts of integrands and integrals* [B25] in the archive. Arkani-Hamed *et al* argue that a full Yangian symmetry of the theory allows to generalize the BCFW recursion relation for tree amplitudes to all loop orders at planar limit (planar means that Feynman diagram allows imbedding to plane without intersecting lines). On mass shell scattering amplitudes are in question.

12.1.2 Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K80] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B43] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N}=4$ SUSY). One of the conditions conditions is that the tensor product $R\otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n=0$ and $n=1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n=1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n+1$-local in the sense that they involve $n+1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- Generalized Feynman diagrams [L19]
- The unique role of twistors in TGD [L45]
- Twistors and TGD [L47]

12.2 How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, I have nothing to say. I am just perplexed. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

(a) The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A11] and Virasoro algebras [A29] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

(b) The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

(c) This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

(a) At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N}=4$ SUSY would allow only the adjoint.

(b) Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. Super-symplectic algebra is realized in terms of second quantized spinor fields and covariantly constant modes of right-handed neutrino. Symplectic group has as sub-group symplectic isometries and the Super-Kac-Moody algebra associated with this group and represented in terms of spinor modes localized to string world sheets plays also a key role in TGD.

(c) The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

(d) Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

12.2.1 Is there any hope about description in terms of Grassmannians?

At technical level the successes of the twistor approach rely on the observation that the amplitudes can be expressed in terms of very simple integrals over sub-manifolds of the space consisting of k-dimensional planes of n-dimensional space defined by delta function appearing in the integrand. These integrals define super-conformal Yangian invariants appearing in twistorial amplitudes and the belief is that by a proper choice of the surfaces of the twistor space one can construct all invariants. One can construct also the counterparts of loop corrections by starting from tree diagrams and annihilating pair of particles by connecting the lines and quantum entangling the states at the ends in the manner dictated by the integration over loop momentum. These operations can be defined as operations for Grassmannian integrals in general changing the values of n and k. This description looks extremely powerful and elegant and -most importantly- involves only the external momenta.

The obvious question is whether one could use similar invariants in TGD framework to construct the momentum dependence of amplitudes.

(a) The first thing to notice is that the super algebras in question act on infinite-dimensional representations and basically in the world of classical worlds assigned to the partonic 2-surfaces correlated by the fact that they are associated with the same space-time surface. This does not promise anything very practical. On the other hand, one can hope that everything related to other than M^4 degrees of freedom could be treated like color degrees of freedom in $\mathcal{N}=4$ SYM and would boil down to indices labeling the quantum states. The Yangian conditions coming from isometry quantum numbers, color quantum numbers, and electroweak quantum numbers are of course expected to be highly non-trivial and could fix the coefficients of various singlets resulting in the tensor product of incoming and outgoing states.

(b) The fact that incoming particles can be also massive seems to exclude the use of the twistor space. The following observation however raises hopes. The Dirac propagator for wormhole throat is massless propagator but for what I call pseudo momentum. It is still unclear how this momentum relates to the actual four-momentum. Could it be actually equal to it? The recent view about pseudo-momentum does not support this view but it is better to keep mind open. In any case this finding suggests that twistorial approach could work in in more or less standard form. What would be needed is a representation for massive incoming particles as bound states of massless partons. In particular, the massive states of super-conformal representations should allow this kind of description.

Could zero energy ontology allow to achieve this dream?

(a) As far as divergence cancellation is considered, zero energy ontology suggests a totally new approach producing the basic nice aspects of QFT approach, in particular unitarity and coupling constant evolution. The big idea related to zero energy ontology is that all virtual particle particles correspond to wormhole throats, which are pairs of on mass shell particles. If their momentum directions are different, one obtains time-like continuum of virtual momenta and if the signs of energy are opposite one obtains also space-like virtual momenta. The on mass shell property for virtual partons (massive in general) implies extremely strong constraints on loops and one expect that only very few loops remain and that they are finite since loop integration reduces to integration over much lower-dimensional space than in the QFT approach. There are also excellent hopes about Cutkoski rules.

(b) Could zero energy ontology make also possible to construct massive incoming particles from massless ones? Could one construct the representations of the super conformal algebras using only massless states so that at the fundamental level incoming particles would be massless and one could apply twistor formalism and build the momentum dependence of amplitudes using Grassmannian integrals.

    One could indeed construct on mass shell massive states from massless states with momenta along the same line but with three-momenta at opposite directions. Mass squared is given by $M^2 = 4E^2$ in the coordinate frame, where the momenta are opposite and of same magnitude. One could also argue that partonic 2-surfaces carrying quantum numbers of fermions and their superpartners serve as the analogs of point like massless particles and that topologically condensed fermions and gauge bosons plus their superpartners correspond to pairs of wormhole throats. Stringy objects would correspond to pairs of wormhole throats at the same space-time sheet in accordance with the fact that space-time sheet allows a slicing by string worlds sheets with ends at different wormhole throats and defining time like braiding.

The weak form of electric magnetic duality indeed supports this picture. To understand how, one must explain a little bit what the weak form of electric magnetic duality means.

(a) Elementary particles correspond to light-like orbits of partonic 2-surfaces identified as 3-D surfaces at which the signature of the induced metric of space-time surface changes from Euclidian to Minkowskian and 4-D metric is therefore degenerate. The analogy with black hole horizon is obvious but only partial. Weak form of electric-magnetic duality states that the Kähler electric field at the wormhole throat and also at space-like 3-surfaces defining the ends of the space-time surface at the upper and lower light-like boundaries of the causal diamond is proportonial to Kähler magnetic field so that Kähler electric flux is proportional Kähler magnetic flux. This implies classical quantization of Kähler electric charge and fixes the value of the proportionality constant.

(b) There are also much more profound implications. The vision about TGD as almost topological QFT suggests that Kähler function defining the Kähler geometry of the "world of classical worlds" (WCW) and identified as Kähler action for its preferred extremal reduces to the 3-D Chern-Simons action evaluted at wormhole throats and possible boundary components. Chern-Simons action would be subject to constraints.

Wormhole throats and space-like 3-surfaces would represent extremals of Chern-Simons action restricted by the constraint force stating electric-magnetic duality (and realized in terms of Lagrange multipliers as usual).

If one assumes that Kähler current and other conserved currents are proportional to current defining Beltrami flow whose flow lines by definition define coordinate curves of a globally defined coordinate, the Coulomb term of Kähler action vanishes and it reduces to Chern-Simons action if the weak form of electric-magnetic duality holds true. One obtains almost topological QFT. The absolutely essential attribute "almost" comes from the fact that Chern-Simons action is subject to constraints. As a consequence, one obtains non-vanishing four-momenta and WCW geometry is non-trivial in M^4 degrees of freedom. Otherwise one would have only topological QFT not terribly interesting physically.

Consider now the question how one could understand stringy objects as bound states of massless particles.

(a) The observed elementary particles are not Kähler monopoles and there much exist a mechanism neutralizing the monopole charge. The only possibility seems to be that there is opposite Kähler magnetic charge at second wormhole throat. The assumption is that in the case of color neutral particles this throat is at a distance of order intermediate gauge boson Compton length. This throat would carry weak isospin neutralizing that of the fermion and only electromagnetic charge would be visible at longer length scales. One could speak of electro-weak confinement. Also color confinement could be realized in analogous manner by requiring the cancellation of monopole charge for many-parton states only. What comes out are string like objects defined by Kähler magnetic fluxes and having magnetic monopoles at ends. Also more general objects with three strings branching from the vertex appear in the case of baryons. The natural guess is that the partons at the ends of strings and more general objects are massless for incoming particles but that the 3-momenta are in opposite directions so that stringy mass spectrum and representations of relevant super-conformal algebras are obtained. This description brings in mind the description of hadrons in terms of partons moving in parallel apart from transversal momentum about which only momentum squared is taken as observable.

(b) Quite generally, one expects for the preferred extremals of Kähler action the slicing of space-time surface with string world sheets with stringy curves connecting wormhole throats. The ends of the stringy curves can be identified as light-like braid strands. Note that the strings themselves define a space-like braiding and the two braidings are in some sense dual. This has a concrete application in TGD inspired quantum biology, where time-like braiding defines topological quantum computer programs and the space-like braidings induced by it its storage into memory. Stringlike objects defining representations of super-conformal algebras must correspond to states involving at least two wormhole throats. Magnetic flux tubes connecting the ends of magnetically charged throats provide a particular realization of stringy on mass shell states. This would give rise to massless propagation at the parton level. The stringy quantization condition for mass squared would read as $m^2 = n$ in suitable units for the representations of super-conformal algebra associated with the isometries. For pairs of throats of the same wormhole contact stringy spectrum does not seem plausible since the wormhole contact is in the direction of CP_2. One can however expect generation of small mass as deviation of vacuum conformal weight from half integer in the case of gauge bosons.

If this picture is correct, one might be able to determine the momentum dependence of the scattering amplitudes by replacing free fermions with pairs of monopoles at the ends of string and topologically condensed fermions gauge bosons with pairs of this kind of objects with wormhole throat replaced by a pair of wormhole throats. This would mean suitable number of doublings of the Grassmannian integrations with additional constraints on the incoming momenta posed by the mass shell conditions for massive states.

12.2.2 Could zero energy ontology make possible full Yangian symmetry?

The partons in the loops are on mass shell particles have a discrete mass spectrum but both signs of energy are possible for opposite wormhole throats. This implies that in the rules for constructing loop amplitudes from tree amplitudes, propagator entanglement is restricted to that corresponding to pairs of partonic on mass shell states with both signs of energy. As emphasized in [B33] , it is the Grassmannian integrands and leading order singularities of $\mathcal{N} = 4$ SYM, which possess the full Yangian symmetry. The full integral over the loop momenta breaks the Yangian symmetry and brings in IR singularities. Zero energy ontologist finds it natural to ask whether QFT approach shows its inadequacy both via the UV divergences and via the loss of full Yangian symmetry. The restriction of virtual partons to discrete mass shells with positive or negative sign of energy imposes extremely powerful restrictions on loop integrals and resembles the restriction to leading order singularities. Could this restriction guarantee full Yangian symmetry and remove also IR singularities?

12.2.3 Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

12.2.4 What about the selection of preferred $M^2 \subset M^4$?

The puzzling aspect of the proposed picture is the restriction of the pseudo-momenta to M^2 and quite generally the the selection of preferred plane $M^2 \subset M^4$. This selection is one the key aspects of TGD but is not too well understood. Also the closely related physical interpretation of the 2-D pseudo-momenta in M^2 is unclear.

The avatars of $M^2 \subset M^4$ in quantum TGD

The choice of preferred plane $M^2 \subset M^4$ pops u again and again in quantum TGD.

(a) There are very strong reasons to believe that the solutions of field equations for the preferred extremals assign M^2 to each point of space-time surface and the interpretation is as the plane of non-physical polarizations. One can also consider the possibility that M^2 depends on the point of space-time surface but that the different choices integrate to 2-D surface analogous to string world sheet - very naturally projection of stringy worlds sheets defining the slicing of the space-time surface.

(b) The number theoretic vision- in particular $M^8 - H$ duality ($H = M^4 \times CP_2$) providing a purely number theoretic interpretation for the choice $H = M^4 \times CP_2$ - involves also the selection of preferred M^2. The duality states that the surfaces in H can be regarded equivalently as surfaces in M^8. The induced metric and Kähler form are identical as also the value of Kähler function. The description of the duality is following.

i. The points of space-time surface in $M^8 = M^4 \times E^4$ in M^8 are mapped to points of space-time surface in $M^4 \times CP_2$. The M^4 part of the map is just a projection.

ii. CP_2 part of the map is less trivial. The idea is that M^8 is identified as a sub-space of complexified octonions obtained by adding commutative imaginary unit, I call this sub-space hyper-octonionic. Suppose that space-time surface is hyper-quaternionic (in appropriate sense meaning that one can attach to its each point a hyper-quaternionic plane, not necessary tangent plane). Assume that it also contains a preferred hypercomplex plane M^2 of M^8 at each point -or more generally a varying plane M^2 planes whose distribution however integrates to form 2-surface analogous to string world sheet. The interpretation is as a preferred plane of non-physical polarizations so that basic aspect of gauge symmetry would have a number theoretic interpretation. Note that one would thus have a local hierarchy of octonionic, quaternionic, and complex planes.

iii. Under these assumptions the tangent plane (if action is just the four-volume or its generalization in the case of Kähler action) is characterized by a point of $CP_2 = SU(3)/U(2)$ where $SU(3)$ is automorphism group of octonions respecting preferred plane M^2 of polarizations and $U(2)$ is automorphism group acting in the hyper-quaternionic plane. This point can be identified as a point of CP_2 so that one obtains the duality.

(c) Also the definition of CDs and the proposed construction of the hierarchy of Planck constants involve a choice of preferred M^2, which corresponds to the choice of rest frame and quantization axis of angular momentum physically. Therefore the choice of quantization axis would have direct correlates both at the level of CDs and space-time surface. The vector between the tips of CD indeed defines preferred direction of time and thus rest system. Similar considerations apply in the case of CP_2.

(d) Preferred M^2 -but now at this time at momentum space level - appears as the plane of pseudo-momenta associated with the generalized eigen modes of the modified Dirac equation associated with Chern-Simons action. Internally consistency requires a restriction to this plane. This looks somewhat mysterious since this would mean that all exchanged virtual momenta would be in M^2 if the choice is same for all lines of the generalized Feynman graph. This would restrict momentum exchanges in particle reactions to single dimension and does not make sense. One must however notice that in the description of hadronic reactions in QCD picture one makes a choice of longitudinal momentum direction and considers only longitudinal momenta. It would seem that the only possibility is that the planes M^2 are independent for independent exchanged momenta. For instance, in $2 \to 2$ scattering the exchange would be in plane defined by the initial and final particles of the vertex. There are also good arguments for a number theoretic quantization of the momenta in M^2.

The natural expectation from $M^8 - H$ duality is that the selection of preferred M^2 implies a reduction of symmetries to those of $M^2 \times E^6$ and $M^2 \times E^2 \times CP_2$. Could the equivalence of M^8 and H descriptions force the reduction of M^4 momentum to M^2 momentum implied also by the generalized eigen value equation for the modified Dirac operator at wormhole throats?

The moduli space associated with the choice of M^2

Lorentz invariance requires that one must have moduli space of CDs with fixed tips defined as $SO(3,1)/SO(1) \times SO(2)$ characterizing different choices of M^2. Maximal Lorentz invariance requires the association of this moduli space to all lines of the generalized Feynman graph. It is easy to deduce that this space is actually the hyperboloid of 5-D Minkowski space. The moduli space is 4-dimensional and has Euclidian signature of the metric. This follows from the fact that $SO(3,1)$ has Euclidian signature as a surface in the four-fold Cartesian power $H(1,3)^4$ of Lobatchevski space with points identified as four time-like unit vectors defining rows of the matrix representing Lorentz transformation. This surface is defined by the 6 orthogonality conditions for the rows of the Lorentz transformation matrix constraints stating the orthogonality of the 4 unit vectors. The Euclidian signature fixes the identification of the moduli space as $H(1,5)$ having Euclidian signature of metric. The 10-D isometry group $SO(1,5)$ of the moduli space acts as symmetries of 5-D Minkowski space (note that the conformal group of M^4 is $SO(2,4)$. The non-compactness of this space does not favor the idea of wave function in moduli degrees of freedom.

Concerning the interpretation of pseudo-momenta it is best to be cautious and make only questions. Should one assume that M^2 for the exchanged particle is fixed by the initial and final momenta of the particle emitting it? How to fix in this kind of situation a unique coordinate frame in which the number theoretic quantization of exchanged momenta takes place? Could it be the rest frame for the initial state of the emitting particle so that one should allow also boosts of the number theoretically preferred momenta? Should one only assume the number theoretically preferred mass values for the exchanged particle but otherwise allow the hyperbolic angle characterizing the energy vary freely?

12.2.5 Does M^8-H duality generalize the duality between twistor and momentum twistor descriptions?

M^8-H duality is intuitively analogous to the duality of elementary wave mechanics meaning that one can use either x-space or momentum space to describe particles. M^8 is indeed the tangent space of H and one could say that M^8-H duality assigns to a 4-surface in H its "momentum" or tangent as a 4-surface in M^8. The more concrete identification of M^8 as cotangent bundle of H so that its points would correspond to 8-momenta: this very naive picture is of course not correct.

M^8-H duality suggests that the descriptions using isometry groups of $M^4\times E^4$ and $M^4\times CP_2$ -or as the special role of M^2 suggests - those of $M^2\times E^6$ and $M^2\times E^2\times CP_2$ should be equivalent. The interpretation in hadron physics context would be that $SO(4)$ is the counterpart of color group in low energy hadron physics acting on strong isospin degrees of freedom and $SU(3)$ that of QCD description useful at high energies. $SO(4)$ is indeed used in old fashioned hadron physics when quarks and gluons had not yet been introduced. Skyrme model is one example.

The obvious question is whether the duality between descriptions based on twistors and momentum space twistors generalizes to M^8-H duality. The basic objection is that the charges and their duals should correspond to the same Lie algebra- or rather Kac-Moody algebra. This is however not the case. For the massless option one has $SO(2)\times SU(3)$ at H-side and $SO(2)\times SO(4)$ or $SO(6)$ and M^8 side.This suggests that M^8-H duality is analogous to the duality between descriptions using twistors and momentum space twistors and transforms the local currents J_0 to non-local currents J_1 and vice versa. This duality would be however be more general in the sense that would relate Yangian symmetries with different Kac-Moody groups transforming locality to non-locality and vice versa. This interpretation is consistent with the fact that the groups $SO(2)\times SO(4)$, $SO(6)$ and $SO(2)\times SU(3)$ have same rank and the standard construction of Kac-Moody generators in terms of exponentials of the Cartan algebra involves only different weights in the exponentials.

If M^8-H duality has something to do with the duality between descriptions using twistors and momentum space twistors involved with Yangian symmetry, it should be consistent with the basic aspects of the latter duality. The following arguments provide support for this.

(a) $SO(4)$ should appear as a dynamical symmetry at $M^4\times CP_2$ side and $SU(3)$ at M^8 side (where it indeed appears as both subgroup of isometries and as tangent space group respecting the choice of M^2. One could consider the breaking of $SO(4)$ to the subgroup corresponding to vectorial transformations and interpreted in terms of electroweak vectorial $SU(2)$: this would conform with conserved vector current hypothesis and partially conserved axial current hypothesis. The $U(1)$ factor assignable to Kähler form is also present and allows Kac-Moody variant and an extension to Yangian.

(b) The heuristics of twistorial approach suggests that the roles of currents J_0 and their non-local duals J_1 in Minkowski space are changed in the transition from H description to M^8 description in the sense that the non-local currents J_1 in H description become local currents in 8-momentum space (or 4-momentum +strong isospin) in M^8 description and J_0 becomes non-local one. In the case of hadron physics the non-local charges assignable to hadrons as collections of partons would become local charges meaning that one can assign them to partonic 2-surfaces at boundaries of CDs assigned to M^8: this says that hadrons are the only possible final states of particle reactions. By the locality it would be impossible decompose momentum and strong isospin to a collection of momenta and strong isospins assigned to partons.

(c) In H description it would be impossible to do decompose quantum numbers to those of quarks and gluons at separate uncorrelated partonic 2-surfaces representing initial and final states of particle reaction. A possible interpretation would be in terms of monopole confinement accompanying electroweak and color confinement: single monopole is not a particle. In $M^4 \times E^4$ monopoles must be also present since induced Kähler forms are identical. The Kähler form represents magnetic monopole in E^4 and breaks its translational symmetry and also selects unique $M^4 \times E^4$ decomposition.

(d) Since the physics should not depend on its description, color should be confined also now. Indeed, internal quantum numbers should be assigned in M^8 picture to a wave function in $M^2 \times E^6$ and symmetries would correspond to $SO(1,1) \times SO(6)$ or - if broken- to those of $SO(1,1) \times G$, $G = SO(2) \times SO(4)$ or $G = SO(3) \times SO(3)$. Color would be completely absent in accordance with the idea that fundamental observable objects are color singlets. Instead of color one would have $SO(4)$ quantum numbers and $SO(4)$ confinement: note that the rank of this group crucial for Kac-Moody algebra construction is same as that of $SU(3)$.

    It is not clear whether the numbers of particle states should be same for $SO(4)$ and $SU(3)$. If so, quark triplet should correspond to doublet and singlet for strong vectorial isospin in M^8 picture. Gluons would correspond to $SU(2)_V$ multiplets contained by color octet and would therefore contain also other representations than adjoint. This could make sense in composite particle interpretation.

(e) For $M^2 \times E^2$ longitudinal momentum and helicity would make sense and one could speak of massless strong isospin at M^8 side and massless color at H-side: note that massless color is the only possibility. For $M^2 \times SO(6)$ option one would have 15-D adjoint representation of $SO(6)$ decomposing as $3 \times 3 + 3 \times 1 + 1 \times 3$ under $SO(3) \times SO(3)$. This could be interpreted in terms of spin and vectorial isospin for massive particles so that the multiplets would relate to weak gauge bosons and Higgs boson singlet and triplet plus its pseudo-scalar variant. For 4-D representation of $SO(6)$ one would have 2×2 decomposition having interpretation in terms of spin and vectorial isospin.

    Massive spin would be associated as a local notion with $M^2 \times E^3$ and would be essentially 5-D concept. At H side massive particle would make sense only as a non-local notion with four-momentum and mass represented as a non-local operator.

These arguments indeed encourage to think that $M^8 - H$ duality could be the analog for the duality between the descriptions in terms of twistors and momentum twistors. In this case the Kac-Moody algebras are however not identical since the isometry groups are not identical.

12.3 Some mathematical details about Grasmannian formalism

In the following I try to summarize my amateurish understanding about the mathematical structure behind the Grassmann integral approach. The representation summarizes what I have gathered from the articles of Arkani-Hamed and collaborators [B31, B33] . These articles are rather sketchy and the article of Bullimore provides additional details [B28] related to soft factors. The article of Mason and Skinner provides excellent introduction to super-twistors [B39] and dual super-conformal invariance. I apologize for unavoidable errors.

Before continuing a brief summary about the history leading to the articles of Arkani-Hamed and others is in order. This summary covers only those aspects which I am at least somewhat familiar with and leaves out many topics about existence which I am only half-conscious.

(a) It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda_a\tilde{\lambda}_{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned}
\langle\lambda,\mu\rangle &= \epsilon_{ab}\lambda^a\mu^b \ , \\
\left[\tilde{\lambda},\tilde{\mu}\right] &= \epsilon_{a'b'}\lambda^{a'}\mu^{b'} \ , \\
p\cdot q &= \langle\lambda,\mu\rangle\left[\tilde{\lambda},\tilde{\mu}\right] \ , \ (q_{aa'} = \mu_a\tilde{\mu}_{a'}) \ .
\end{aligned} \tag{12.3.1}$$

If the particle has spin one can assign it a positive or negative helicity $h = \pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a $(\mu_{a'})$ not parallel to λ_a $(\mu_{a'})$ so that one can write for the polarization vector

$$\begin{aligned}
\epsilon_{aa'} &= \frac{\mu_a\tilde{\lambda}_{a'}}{\langle\mu,\lambda\rangle} \ , \ \text{positive helicity} \ , \\
\epsilon_{aa'} &= \frac{\lambda_a\tilde{\mu}_{a'}}{\left[\tilde{\mu},\tilde{\lambda}\right]} \ , \ \text{negative helicity} \ .
\end{aligned} \tag{12.3.2}$$

In the case of momentum twistors the μ part is determined by different criterion to be discussed later.

(b) Tree amplitudes are considered and it is convenient to drop the group theory factor $Tr(T_1T_2\cdots T_n)$. The starting point is the observation that tree amplitude for which more than $n-2$ gluons have the same helicity vanish. MHV amplitudes have exactly $n-2$ gluons of same helicity- taken by a convention to be negative- have extremely simple form in terms of the spinors and reads as

$$A_n = \frac{\langle\lambda_x,\lambda_y\rangle^4}{\prod_{i=1}^n\langle\lambda_i,\lambda_{i+1}\rangle} \tag{12.3.3}$$

When the sign of the helicities is changed $\langle..\rangle$ is replaced with $[..]$.

(c) The article of Witten [B58] proposed that twistor approach could be formulated as a twistor string theory with string world sheets "living" in 6-dimensional CP_3 possessing Calabi-Yau structure and defining twistor space. In this article Witten introduced what is known as half Fourier transform allowing to transform momentum integrals over light-cone to twistor integrals. This operation makes sense only in space-time signature $(2,2)$. Witten also demonstrated that maximal helicity violating (MHV) twistor amplitudes (two gluons with negative helicity) with n particles with $k+2$ negative helicities and l loops correspond in this approach to holomorphic 2-surfaces defined by polynomials defined by polynomials of degree $D = k-1+l$, where the genus of the surface satisfies $g \leq l$. AdS/CFT duality provides a second stringy approach to $\mathcal{N} = 4$ theory allowing to understand the scattering amplitudes in terms of Wilson loops with light-like edges: about this I have nothing to say. In any case, the generalization of twistor string theory to TGD context is highly attractive idea and will be considered later.

(d) In the article [B35] Cachazo, Svrcek, and Witten propose the analog of Feynman diagrammatics in which MHV amplitudes can be used as analogs of vertices and ordinary $1/P^2$ propagator as propagator to construct tree diagrams with arbitrary number of negative helicity gluons. This approach is not symmetric with respect to the change of the sign of helicities since the amplitudes with two positive helicities are constructed as tree diagrams. The construction is non-trivial because one must analytically continue the on mass shell tree amplitudes to off mass shell momenta. The problem is how to assign a twistor to these momenta. This is achieved by introducing an arbitrary twistor $\eta^{a'}$ and defining λ_a as $\lambda_a = p_{aa'}\eta^{a'}$. This works for both massless and massive case. It however leads to a loss of the manifest Lorentz invariance. The paper however argues and the later paper [B52, B52] shows rigorously that the loss is only apparent. In this paper also BCFW recursion formula is introduced allowing to construct tree amplitudes recursively by starting from vertices with 2 negative helicity gluons. Also the notion which has become known as BCFW bridge representing the massless exchange in these diagrams is introduced. The tree amplitudes are not tree amplitudes in gauge theory sense where correspond to leading singularities for which 4 or more lines of the loop are massless and therefore collinear. What is important that the very simple MHV amplitudes become the building blocks of more complex amplitudes.

(e) The nex step in the progress was the attempt to understand how the loop corrections could be taken into account in the construction BCFW formula. The calculation of loop contributions to the tree amplitudes revealed the existence of dual super-conformal symmetry which was found to be possessed also by BCFW tree amplitudes besides conformal symmetry. Together these symmetries generate infinite-dimensional Yangian symmetry [B39] .

(f) The basic vision of Arkani-Hamed and collaborators is that the scattering amplitudes of $\mathcal{N} = 4$ SYM are constructible in terms of leading order singularities of loop diagrams. These singularities are obtained by putting maximum number of momenta propagating in the lines of the loop on mass shell. The non-leading singularities would be induced by the leading singularities by putting smaller number of momenta on mass shell are dictated by these terms. A related idea serving as a starting point in [B31] is that one can define loop integrals as residue integrals in momentum space. If I have understood correctly, this means that one an imagine the possibility that the loop integral reduces to a lower dimensional integral for on mass shell particles in the loops: this would resemble the approach to loop integrals based on unitarity and analyticity. In twistor approach these momentum integrals defined as residue integrals transform to residue integrals in twistor space with twistors representing massless particles. The basic discovery is that one can construct leading order singularities for n particle scattering amplitude with $k+2$ negative helicities as Yangian invariants $Y_{n,k}$ for momentum twistors and invariants constructed from them by canonical operations changing n and k. The correspondence $k = l$ does not hold true for the more general amplitudes anymore.

12.3.1 Yangian algebra and its super counterpart

The article of Witten [B43] gives a nice discussion of the Yangian algebra and its super counterpart. Here only basic formulas can be listed and the formulas relevant to the super-conformal case are given.

Yangian algebra

Yangian algebra $Y(G)$ is associative Hopf algebra. The elements of Yangian algebra are labelled by non-negative integers so that there is a close analogy with the algebra spanned by the generators of Virasoro algebra with non-negative conformal weight. The Yangian symmetry algebra is defined by the following relations for the generators labeled by integers $n = 0$ and $n = 1$. The first half of these relations discussed in very clear manner in [B43] follows uniquely from the fact that adjoint representation of the Lie algebra is in question

$$\left[J^A,J^B\right]=f_C^{AB}J^C \quad , \quad \left[J^A,J^{(1)B}\right]=f_C^{AB}J^{(1)C} \quad . \tag{12.3.4}$$

Besides this Serre relations are satisfied. These have more complex and read as

$$\begin{aligned}
&\left[J^{(1)A},\left[J^{(1)B},J^C\right]\right]+\left[J^{(1)B},\left[J^{(1)C},J^A\right]\right]+\left[J^{(1)C},\left[J^{(1)A},J^B\right]\right] \\
&\qquad =\frac{1}{24}f^{ADK}f^{BEL}f^{CFM}f_{KLM}\{J_D,J_E,J_F\} \quad , \\
&\left[\left[J^{(1)A},J^{(1)B}\right],\left[J^C,J^{(1)D}\right]\right]+\left[\left[J^{(1)C},J^{(1)D}\right],\left[J^A,J^{(1)B}\right]\right] \\
&\qquad =\frac{1}{24}f^{AGL}f^{BEM}f_K^{CD} \\
&\qquad +f^{CGL}f^{DEM}f_K^{AB})f^{KFN}f_{LMN}\{J_G,J_E,J_F\} \quad .
\end{aligned} \tag{12.3.5}$$

The indices of the Lie algebra generators are raised by invariant, non-degenerate metric tensor g_{AB} or g^{AB}. $\{A,B,C\}$ denotes the symmetrized product of three generators.

Repeated commutators allow to generate the entire algebra whose elements are labeled by non-negative integer n. The generators obtain in this manner are n-local operators arising in $(n-1)$-commutator of $J^{(1)}$:s. For $SU(2)$ the Serre relations are trivial. For other cases the first Serre relation implies the second one so the relations are redundant. Why Witten includes it is for the purposed of demonstrating the conditions for the existence of Yangians associated with discrete one-dimensional lattices (Yangians exists also for continuum one-dimensional index).

Discrete one-dimensional lattice provides under certain consistency conditions a representation for the Yangian algebra. One assumes that each lattice point allows a representation R of J^A so that one has $J^A=\sum_i J_i^A$ acting on the infinite tensor power of the representation considered. The expressions for the generators J^{1A} are given as

$$J^{(1)A} \quad = \quad f_{BC}^A\sum_{i<j}J_i^BJ_j^C \quad . \tag{12.3.6}$$

This formula gives the generators in the case of conformal algebra. This representation exists if the adjoint representation of G appears only one in the decomposition of $R\otimes R$. This is the case for SU(N) if R is the fundamental representation or is the representation of by k^{th} rank completely antisymmetric tensors.

This discussion does not apply as such to $\mathcal{N}=4$ case the number of lattice points is finite and corresponds to the number of external particles so that cyclic boundary conditions are needed guarantee that the number of lattice points reduces effectively to a finite number. Note that the Yangian in color degrees of freedom does not exist for SU(N) SYM.

As noticed, Yangian algebra is a Hopf algebra and therefore allows co-product. The co-product Δ is given by

$$\begin{aligned}
\Delta(J^A) &= J^A\otimes 1+1\otimes J^A \quad , \\
\Delta(J^{(1)A}) &= J^{(1)A}\otimes 1+1\otimes J^{(1)A}+f_{BC}^AJ^B\otimes J^C \ per,
\end{aligned} \tag{12.3.7}$$

Δ allows to imbed Lie algebra to the tensor product in non-trivial manner and the non-triviality comes from the addition of the dual generator to the trivial co-product. In the case that the single spin representation of $J^{(1)A}$ is trivial, the co-product gives just the expression of the dual generator using the ordinary generators as a non-local generator. This is assumed in the recent case and also for the generators of the conformal Yangian.

Super-Yangian

Also the Yangian extensions of Lie super-algebras make sense. From the point of physics especially interesting Lie super-algebras are $SU(m|m)$ and $U(m|m)$. The reason is that $PSU(2,2|4)$ (P refers to "projective") acting as super-conformal symmetries of $\mathcal{N}=4$ SYM and this super group is a real form of $PSU(4|4)$. The main point of interest is whether this algebra allows Yangian representation and Witten demonstrated that this is indeed the case [B43] .

These algebras are Z_2 graded and decompose to bosonic and fermionic parts which in general correspond to n- and m-dimensional representations of $U(n)$. The representation associated with the fermionic part dictates the commutation relations between bosonic and fermionic generators. The anti-commutator of fermionic generators can contain besides identity also bosonic generators if the symmetrized tensor product in question contains adjoint representation. This is the case if fermions are in the fundamental representation and its conjugate. For $SU(3)$ the symmetrize tensor product of adjoint representations contains adjoint (the completely symmetric structure constants d_{abc}) and this might have some relevance for the super $SU(3)$ symmetry.

The elements of these algebras in the matrix representation (no Grassmann parameters involved) can be written in the form

$$x = \begin{pmatrix} a & b \\ c & d \end{pmatrix} .$$

a and d representing the bosonic part of the algebra are $n\times n$ matrices and $m\times m$ matrices corresponding to the dimensions of bosonic and fermionic representations. b and c are fermionic matrices are $n\times m$ and $m\times n$ matrices, whose anti-commutator is the direct sum of $n\times n$ and $n\times n$ matrices. For $n=m$ bosonic generators transform like Lie algebra generators of $SU(n)\times SU(n)$ whereas fermionic generators transform like $n\otimes\overline{n}\oplus\overline{n}\otimes n$ under $SU(n)\times SU(n)$. Supertrace is defined as $Str(x)=Tr(a)-Tr(b)$. The vanishing of Str defines $SU(n|m)$. For $n\neq m$ the super trace condition removes identity matrix and $PU(n|m)$ and $SU(n|m)$ are same. That this does not happen for $n=m$ is an important delicacy since this case corresponds to $\mathcal{N}=4$ SYM. If any two matrices differing by an additive scalar are identified (projective scaling as now physical effect) one obtains $PSU(n|n)$ and this is what one is interested in.

Witten shows that the condition that adjoint is contained only once in the tensor product $R\otimes\overline{R}$ holds true for the physically interesting representations of $PSU(2,2|4)$ so that the generalization of the bilinear formula can be used to define the generators of $J^{(1)A}$ of super Yangian of $PU(2,2|4)$. The defining formula for the generators of the Super Yangian reads as

$$\begin{aligned} J_C^{(1)} &= g_{CC'}J^{(1)C'} = g_{CC'}f^{C'}_{AB}\sum_{i<j}J_i^AJ_j^B \\ &= g_{CC'}f^{C'}_{AB}g^{AA'}g^{BB'}\sum_{i<j}J^i_{A'}J^j_{B'} \ . \end{aligned} \tag{12.3.8}$$

Here $g_{AB}=Str(J_AJ_B)$ is the metric defined by super trace and distinguishes between $PSU(4|4)$ and $PSU(2,2|4)$. In this formula both generators and super generators appear.

Generators of super-conformal Yangian symmetries

The explicit formula for the generators of super-conformal Yangian symmetries in terms of ordinary twistors is given by

$$\begin{aligned} j^A_B &= \sum_{i=1}^{n} Z_i^A \partial_{Z_i^B} \quad , \\ j^{(1)A}_B &= \sum_{i<j} (-1)^C \left[Z_i^A \partial_{Z_j^C} Z_j^C \partial_{Z_j^B} \right] \quad . \end{aligned} \tag{12.3.9}$$

This formula follows from completely general formulas for the Yangian algebra discussed above and allowing to express the dual generators $j_N^{(1)}$ as quadratic expression of j_N involving structures constants. In this rather sketchy formula twistors are ordinary twistors. Note however that in the recent case the lattice is replaced with its finite cutoff corresponding to the external particles of the scattering amplitude. This probably corresponds to the assumption that for the representations considered only finite number of lattice points correspond to non-trivial quantum numbers or to cyclic symmetry of the representations.

In the expression for the amplitudes the action of transformations is on the delta functions and by partial integration one finds that a total divergence results. This is easy to see for the linear generators but not so for the quadratic generators of the dual super-conformal symmetries. A similar formula but with j^A_B and $j^{(1)A}_B$ interchanged applies in the representation of the amplitudes as Grassmann integrals using ordinary twistors. The verification of the generalization of Serre formula is also straightforward.

12.3.2 Twistors and momentum twistors and super-symmetrization

In [B39] the basics of twistor geometry are summarized. Despite this it is perhaps good to collect the basic formulas here.

Conformally compactified Minkowski space

Conformally compactified Minkowski space can be described as $SO(2,4)$ invariant (Klein) quadric

$$T^2 + V^2 - W^2 - X^2 - Y^2 - Z^2 = 0 \quad . \tag{12.3.10}$$

The coordinates (T, V, W, X, Y, Z) define homogenous coordinates for the real projective space RP^5. One can introduce the projective coordinates $X_{\alpha\beta} = -X_{\beta\alpha}$ through the formulas

$$\begin{array}{lll} X_{01} = W - V \ , & X_{02} = Y + iX \ , & X_{03} = \frac{i}{\sqrt{2}} T - Z \ , \\ X_{12} = -\frac{i}{\sqrt{2}}(T + Z) \ , & X_{13} = Y - iX \ , & X_{23} = \frac{1}{2}(V + W) \ . \end{array} \tag{12.3.11}$$

The motivation is that the equations for the quadric defining the conformally compactified Minkowski space can be written in a form which is manifestly conformally invariant:

$$\epsilon^{\alpha\beta\gamma\delta} X_{\alpha\beta} X_{\gamma\delta} \quad = \quad 0 \ per. \tag{12.3.12}$$

The points of the conformally compactified Minkowski space are null separated if and only if the condition

$$\epsilon^{\alpha\beta\gamma\delta}X_{\alpha\beta}Y_{\gamma\delta} = 0 \tag{12.3.13}$$

holds true.

Correspondence with twistors and infinity twistor

One ends up with the correspondence with twistors by noticing that the condition is equivalent with the possibility to expression $X_{\alpha\beta}$ as

$$X_{\alpha\beta} = A_{[\alpha}B_{\beta]} \ , \tag{12.3.14}$$

where brackets refer to antisymmetrization. The complex vectors A and B define a point in twistor space and are defined only modulo scaling and therefore define a point of twistor space CP_3 defining a covering of 6-D Minkowski space with metric signature $(2,4)$. This corresponds to the fact that the Lie algebras of $SO(2,4)$ and $SU(2,2)$ are identical. Therefore the points of conformally compactified Minkowski space correspond to lines of the twistor space defining spheres CP_1 in CP_3.

One can introduce a preferred scale for the projective coordinates by introducing what is called infinity twistor (actually a pair of twistors is in question) defined by

$$I_{\alpha\beta} = \begin{pmatrix} \epsilon^{A'B'} & 0 \\ 0 & 0 \end{pmatrix} \ . \tag{12.3.15}$$

Infinity twistor represents the projective line for which only the coordinate X_{01} is non-vanishing and chosen to have value $X_{01} = 1$.

One can define the contravariant form of the infinite twistor as

$$I^{\alpha\beta} = \epsilon^{\alpha\beta\gamma\delta}I_{\gamma\delta} = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon^{AB} \end{pmatrix} \ . \tag{12.3.16}$$

Infinity twistor defines a representative for the conformal equivalence class of metrics at the Klein quadric and one can express Minkowski distance as

$$(x-y)^2 = \frac{X^{\alpha\beta}Y_{\alpha\beta}}{I_{\alpha\beta}X^{\alpha\beta}I_{\mu\nu}Y^{\mu\nu}} \ . \tag{12.3.17}$$

Note that the metric is necessary only in the denominator. In twistor notation the distance can be expressed as

$$(x-y)^2 = \frac{\epsilon(A,B,C,D)}{\langle AB\rangle\langle CD\rangle} \ . \tag{12.3.18}$$

Infinite twistor $I_{\alpha\beta}$ and its contravariant counterpart project the twistor to its primed and unprimed parts usually denoted by $\mu^{A'}$ and λ^A and defined spinors with opposite chiralities.

Relationship between points of M^4 and twistors

In the coordinates obtained by putting $X_{01} = 1$ the relationship between space-time coordinates $x^{AA'}$ and $X^{\alpha\beta}$ is

$$X_{\alpha\beta} = \begin{pmatrix} -\frac{1}{2}\epsilon^{A'B'}x^2 & -ix^{A'}_{\ B} \\ ix_{A}^{\ B'} & \epsilon_{A,B} \end{pmatrix} \quad , \quad X^{\alpha\beta} = \begin{pmatrix} \epsilon_{A'B}{}'x^2 & -ix_{\ A'}^{B} \\ ix^{A}_{B'} & -\frac{1}{2}\epsilon^{AB}x^2 \end{pmatrix} \quad , \tag{12.3.19}$$

If the point of Minkowski space represents a line defined by twistors (μ_U, λ_U) and (μ_V, λ_V), one has

$$x^{AC'} = i\frac{(\mu_V\lambda_U - \mu_U\lambda_V)^{AC'}}{\langle UV\rangle} \tag{12.3.20}$$

The twistor μ for a given point of Minkowski space in turn is obtained from λ by the twistor formula by

$$\mu^{A'} \quad = \quad -ix^{AA'}\lambda_A \quad . \tag{12.3.21}$$

Generalization to the super-symmetric case

This formalism has a straightforward generalization to the super-symmetric case. CP_3 is replaced with $CP_{3|4}$ so that Grassmann parameters have four components. At the level of coordinates this means the replacement $[W_I] = [W_\alpha, \chi_\alpha]$. Twistor formula generalizes to

$$\mu^{A'} = -ix^{AA'}\lambda_A \quad , \quad \chi_\alpha = \theta_\alpha^A\lambda_A \quad . \tag{12.3.22}$$

The relationship between the coordinates of chiral super-space and super-twistors generalizes to

$$(x,\theta) \quad = \quad \left(i\frac{(\mu_V\lambda_U - \mu_U\lambda_V)}{\langle UV\rangle}, \frac{(\chi_V\lambda_U - \chi_U\lambda_V)}{\langle UV\rangle}\right) \tag{12.3.23}$$

The above formulas can be applied to super-symmetric variants of momentum twistors to deduce the relationship between region momenta x assigned with edges of polygons and twistors assigned with the ends of the light-like edges. The explicit formulas are represented in [B39] . The geometric picture is following. The twistors at the ends of the edge define the twistor pair representing the region momentum as a line in twistor space and the intersection of the twistor lines assigned with the region momenta define twistor representing the external momenta of the graph in the intersection of the edges.

Basic kinematics for momentum twistors

The super-symmetrization involves replacement of multiplets with super-multiplets

$$\Phi(\lambda,\tilde{\lambda},\eta) = G^{+}(\lambda,\tilde{\lambda}) + \eta_i\Gamma^a\lambda,\tilde{\lambda}) + \cdots + \epsilon_{abcd}\eta^a\eta^b\eta^c\eta^d G^{-}(\lambda,\tilde{\lambda}) \ . \tag{12.3.24}$$

Momentum twistors are dual to ordinary twistors and were introduced by Hodges. The light-like momentum of external particle a is expressed in terms of the vertices of the closed polygon defining the twistor diagram as

$$p_i^{\mu} = x_i^{\mu} - x_{i+1}^{\mu} = \lambda_i\tilde{\lambda}_i \ \ , \ \ \theta_i - \theta_{i+1} = \lambda_i\eta_i \ . \tag{12.3.25}$$

One can say that massless momenta have a conserved super-part given by $\lambda_i\eta_i$. The dual of the super-conformal group acts on the region momenta exactly as the ordinary conformal group acts on space-time and one can construct twistor space for dua region momenta.

Super-momentum conservation gives the constraints

$$\sum p_i = 0 \ \ , \ \ \sum \lambda_i\eta_i = 0 \ . \tag{12.3.26}$$

The twistor diagrams correspond to polygons with edges with lines carrying region momenta and external massless momenta emitted at the vertices.

This formula is invariant under overall shift of the region momenta x_a^{μ}. A natural interpretation for x_a^{μ} is as the momentum entering to the the vertex where p_a is emitted. Overall shift would have interpretation as a shift in the loop momentum. x_a^{μ} in the dual coordinate space is associated with the line $Z_{a-1}Z_a$ in the momentum twistor space. The lines $Z_{a-1}Z_a$ and Z_aZ_{a+1} intersect at Z_a representing a light-like momentum vector p_a^{μ}.

The brackets $\langle abcd\rangle \equiv \epsilon_{IJKL}Z_a^IZ_b^JZ_c^KZ_d^L$ define fundamental bosonic conformal invariants appearing in the tree amplitudes as basic building blocks. Note that Z_a define points of 4-D complex twistor space to be distinguished from the projective twistor space CP_3. Z_a define projective coordinates for CP_3 and one of the four complex components of Z_a is redundant and one can take $Z_a^0 = 1$ without a loss of generality.

12.3.3 Brief summary of the work of Arkani-Hamed and collaborators

The following comments are an attempt to summarize my far from complete understanding about what is involved with the representation as contour integrals. After that I shall describe in more detail my impressions about what has been done.

Limitations of the approach

Consider first the limitations of the approach.

(a) The basis idea is that the representation for tree amplitudes generalizes to loop amplitudes. On other words, the amplitude defined as a sum of Yangian invariants expressed in terms of Grassmann integrals represents the sum of loops up to some maximum loop number. The problem is here that shifts of the loop momenta are essential in the UV regularization procedure. Fixing the coordinates $x_1, \cdots, x_n$ having interpretation as momenta associated with lines in the dual coordinate space allows to eliminate the non-uniqueness due to the common shift of these coordinates.

(b) It is not however not possible to identify loop momentum as a loop momentum common to different loop integrals unless one restricts to planar loops. Non-planar diagrams are obtained from a planar diagram by permuting the coordinates x_i but this means that the unique coordinate assignment is lost. Therefore the representation of loop integrands as Grassmann integrals makes sense only for planar diagrams. From TGD point of view one could argue that this is one good reason for restricting the loops so that they are for on mass shell particles with non-parallel on mass shell four-momenta and possibly different sign of energies for given wormhole contact representing virtual particle.

(c) IR regularization is needed even in $\mathcal{N} = 4$ for SYM given by "moving out on the Coulomb branch theory" so that IR singularities remain the problem of the theory.

What has been done?

The article proposes a generalization of the BCFW recursion relation for tree diagrams of $\mathcal{N} = 4$ for SYM so that it applies to planar diagrams with a summation over an arbitrary number of loops.

(a) The basic goal of the article is to generalize the recursion relations of tree amplitudes so that they would apply to loop amplitudes. The key idea is following. One can formally represent loop integrand as a contour integral in complex plane whose coordinate parameterizes the deformations $Z_n \rightarrow Z_n + \epsilon Z_{n-1}$ and re-interpret the integral as a contour integral with oppositely oriented contour surrounding the rest of the complex plane which can be imagined also as being mapped to Riemann sphere. What happens only the poles which correspond to lower number of loops contribute this integral. One obtains a recursion relation with respect to loop number. This recursion seems to be the counterpart for the recursive construction of the loops corrections in terms of absorptive parts of amplitudes with smaller number of loop using unitarity and analyticity.

(b) The basic challenge is to deduce the Grassmann integrands as Yangian invariants. From these one can deduce loop integrals by integration over the four momenta associated with the lines of the polygonal graph identifiable as the dual coordinate variables x_a. The integration over loop momenta can induce infrared divergences breaking Yangian symmetry. The big idea here is that the operations described above allow to construct loop amplitudes from the Yangian invariants defining tree amplitudes for a larger number of particles by removing external particles by fusing them to form propagator lines and by using the BCFW bridge to fuse lower-dimensional invariants. Hence the usual iterative procedure (bottom-up) used to construct scattering amplitudes is replaced with a recursive procedure (top-down). Of course, once lower amplitudes has been constructed they can be used to construct amplitudes with higher particle number.

(c) The first guess is that the recursion formula involves the same lower order contributions as in the case of tree amplitudes. These contributions have interpretation as factorization of channels involving single particle intermediate states. This would however allow to reduce loop amplitudes to 3-particle loop amplitudes which vanish in $\mathcal{N} = 4$ SYM by the vanishing of coupling constant renormalization. The additional contribution is necessary and corresponds to a source term identifiable as a "forward limit" of lower loop integrand. These terms are obtained by taking an amplitude with two additional particles with opposite four-momenta and forming a state in which these particles are entangled with respect to momentum and other quantum numbers. Entanglement means integral over the massless momenta on one hand. The insertion brings in two momenta x_a and x_b and one can imagine that the loop is represented by a branching of propagator line. The line representing the entanglement of the massless states with massless momentum define the second branch of the loop. One can of course ask whether only massless momentum in the second branch. A possible interpretation is that this state is expressible by unitarity in terms of the integral over light-like momentum.

(d) The recursion formula for the loop amplitude $M_{n,k,l}$ involves two terms when one neglects the possibility that particles can also suffer trivial scattering (cluster decomposition). This term basically corresponds to the Yangian invariance of n arguments identified as Yangian invariant of $n-1$ arguments with the same value of k.

i. The first term corresponds to single particle exchange between particle groups obtained by splitting the polygon at two vertices and corresponds to the so called BCFW bridge for tree diagrams. There is a summation over different splittings as well as a sum over loop numbers and dimensions k for the Grassmann planes. The helicities in the two groups are opposite.

ii. Second term is obtained from an amplitude obtained by adding of two massless particles with opposite momenta and corresponds to $n+2, k+1, l-1$. The integration over the light-like momentum together with other operations implies the reduction $n+2 \rightarrow n$. Note that the recursion indeed converges. Certainly the allowance of added zero energy states with a finite number of particles is necessary for the convergence of the procedure.

12.3.4 The general form of Grassmannian integrals

If the recursion formula proposed in [B33] is correct, the calculations reduce to the construction of N^kMHV (super) amplitudes. MHV refers to maximal helicity violating amplitudes with 2 negative helicity gluons. For N^kMHV amplitude the number of negative helicities is by definition $k+2$ [B31] . Note that the total right handed R-charge assignable to 4 super-coordinates η_i of negative helicity gluons can be identified as $R = 4k$. BCFW recursion formula [B52, B52] allows to construct from MHV amplitudes with arbitrary number of negative helicities.

The basic object of study are the leading singularities of color-stripped n-particle N^kMHV amplitudes. The discovery is that these singularities are expressible in terms Yangian invariants $Y_{n,k}(Z_1, \cdots, Z_n)$, where Z_i are momentum super-twistors. These invariants are defined by residue integrals over the compact $nk-1$-dimensional complex space $G(n,k) = U(n)/U(k) \times U(n-k)$ of k-planes of complex n-dimensional space. n is the number of external massless particles, k is the number negative helicity gluons in the case of N^kMHV amplitudes, and Z_a, $i = 1, \cdots, n$ denotes the projective 4-coordinate of the super-variant $CP^{3|4}$ of the momentum twistor space CP_3 assigned to the massless external particles is following. $Gl(n)$ acts as linear transformations in the n-fold Cartesian power of twistor space. Yangian invariant $Y_{n,k}$ is a function of twistor variables Z^a having values in super-variant $CP_{3|3}$ of momentum twistor space CP_3 assigned to the massless external particles being simple algebraic functions of the external momenta.

It is also possible to define N^kMHV amplitudes in terms of Yangian invariants $L_{n,k+2}(W_1, \cdots, W_n)$ by using ordinary twistors W_a and identical defining formula. The two invariants are related by the formula $L_{n,k+2}(W_1, \cdots, W_n) = M^{tree}_{MHV} \times Y_{n,k}(Z_1, \cdots, Z_n)$. Here M^{tree}_{MHV} is the tree contribution to the maximally helicity violating amplitude for the scattering of n particles: recall that these amplitudes contain two negative helicity gluons whereas the amplitudes containing a smaller number of them vanish [B35] . One can speak of a factorization to a product of n-particle amplitudes with $k-2$ and 2 negative helicities as the origin of the duality. The equivalence between the descriptions based on ordinary and momentum twistors states the dual conformal invariance of the amplitudes implying Yangian symmetry. It has been conjectured that Grassmannian integrals generate all Yangian invariants.

The formulas for the Grassmann integrals for twistors and momentum twistors appearing in the expressions of N^kMHV amplitudes are given by following expressions.

(a) The integrals $L_{n,k}(W_1, \cdots, W_n)$ associated with $N^{k-2}MHV$ amplitudes in the description based on ordinary twistors correspond to k negative helicities and are given by

$$
\begin{aligned}
L_{n,k}(W_1,\cdots,W_n) &= \frac{1}{Vol(GL(2))}\int \frac{d^{k\times n}C_{\alpha a}}{(1\cdots k)(2\cdots k+1)\cdots(n1\cdots k-1)}\times \\
&\times \prod_{\alpha=1}^{k} d^{4|4}Y_\alpha \prod_{i=1}^{n}\delta^{4|4}(W_i-C_{\alpha i}Y_\alpha) \ .
\end{aligned}
\tag{12.3.27}
$$

Here $C_{\alpha a}$ denote the $n\times k$ coordinates used to parametrize the points of $G_{k,n}$.

(b) The integrals $Y_{n,k}(W_1,\cdots,W_n)$ associated with N^kMHV amplitudes in the description based on momentum twistors are defined as

$$
Y_{n,k}(Z_1,\cdots,Z_n) = \frac{1}{Vol(GL(k))}\times\int \frac{d^{k\times n}C_{\alpha a}}{(1\cdots k)(2\cdots k+1)\cdots(n1\cdots k-1)}\times\prod_{\alpha=1}^{k}\delta^{4|4}(C_{\alpha a}Z_a) \ .
\tag{12.3.28}
$$

The possibility to select $Z_a^0=1$ implies $\sum_k C_{\alpha k}=0$ allowing to eliminate $C_{\alpha n}$ so that the actual number of coordinates Grassman coordinates is $nk-1$. As already noticed, $L_{n,k+2}(W_1,\cdots,W_n)=M^{tree}_{MHV}\times Y_{n,k}(Z_1,\cdots,Z_n)$. Momentum twistors are obviously calculationally easier since the value of k is smaller by two units.

The $4k$ delta functions reduce the number of integration variables of contour integrals from nk to $(n-4)k$ in the bosonic sector (the definition of delta functions involves some delicacies not discussed here). The n quantities $(m,\cdots m+k)$ are $k\times k$-determinants defined by subsequent columns from m to $m+k-1$ of the $k\times n$ matrix defined by the coordinates $C_{\alpha a}$ and correspond geometrically to the k-volumes of the k-dimensional parallel-pipeds defined by these column vectors. The fact that the scalings of twistor space coordinates Z_a can be compensated by scalings of $C_{\alpha a}$ deforming integration contour but leaving the residue integral invariant so that the integral depends on projective twistor coordinates only.

Since the integrand is a rational function, a multi-dimensional residue calculus allows to deduce the values of these integrals as residues associated with the poles of the integrand in a recursive manner. The poles correspond to the zeros of the $k\times k$ determinants appearing in the integrand or equivalently to singular lower-dimensional parallel-pipeds. It can be shown that local residues are determined by $(k-2)(n-k-2)$ conditions on the determinants in both cases. The value of the integral depends on the explicit choice of the integration contour for each variable $C_{\alpha a}$ left when delta functions are taken into account. The condition that a correct form of tree amplitudes is obtained fixes the choice of the integration contours.

For the ordinary twistors W the residues correspond to projective configurations in CP_{k-1}, or more precisely in the space $CP^n_{k-1}/Gl(k)$, which is $(k-1)n-k^2$-dimensional space defining the support for the residues integral. $Gl(k)$ relates to each other different complex coordinate frames for k-plane and since the choice of frame does not affect the plane itself, one has $Gl(k)$ gauge symmetry as well as the dual $Gl(n-k)$ gauge symmetry.

CP_{k-1} comes from the fact that C_{α_k} are projective coordinates: the amplitudes are indeed invariant under the scalings $W_i\rightarrow t_iW_i$, $C_{\alpha i}\rightarrow tC_{\alpha i}$. The coset space structure comes from the fact that $Gl(k)$ is a symmetry of the integrand acting as $C_{\alpha i}\rightarrow \Lambda_\alpha^{\ \beta}C_{\beta_i}$. This analog of gauge symmetry allows to fix k arbitarily chosen frame vectors $C_{\alpha i}$ to orthogonal unit vectors. For instance, one can have $C_{\alpha i}=\delta_{\alpha_i}$ for $\alpha=i\in 1,\cdots,k$. This choice is discussed in detail in [B31] . The reduction to CP_{k-1} implies the reduction of the support of the integral to line in the case of MHV amplitudes and to plane in the case of NMHV as one sees from the expression $d\mu=\prod_\alpha d^{4|4}Y_\alpha\prod_{i=1}^n\delta^{4|4}(W_i-C_{\alpha i}Y_\alpha)$. For $(i_1,\cdots,i_k)=0$ the vectors $i_1,..i_k$ belong to $k-2$-dimensional plane of CP_{k-1}. In the case of $NMHV$ (N^2MHV) amplitudes this translates at the level of twistors to the condition that the corresponding twistors $\{i_1,i_2,i_3\}$ ($\{i_1,i_2,i_3,i_4\}$) are collinear (in the same plane) in twistor space. This can be understood from the fact that the delta functions in $d\mu$ allow to express W_i in terms of $k-1$ Y_α:s in this case.

The action of conformal transformations in twistor space reduces to the linear action of $SU(2,2)$ leaving invariant Hermitian sesquilinear form of signature $(2,2)$. Therefore the conformal invariance of the Grassmannian integral and its dual variant follows from the possibility to perform a compensating coordinate change for $C_{\alpha a}$ and from the fact that residue integral is invariant under small deformations of the integration contour. The above described relationship between representations based on twistors and momentum twistors implies the full Yangian invariance.

12.3.5 Canonical operations for Yangian invariants

General l-loop amplitudes can be constructed from the basic Yangian invariants defined by N^kMHV amplitudes by various operations respecting Yangian invariance apart from possible IR anomalies. There are several operations that one can perform for Yangian invariants $Y_{n,k}$ and all these operations appear in the recursion formula for planar all loop amplitudes. These operations are described in [B33] much better than I could do it so that I will not go to any details. It is possible to add and remove particles, to fuse two Yangian invariants, to merge particles, and to construct from two Yangian invariants a higher invariant containing so called BCFW bridge representing single particle exchange using only twistorial methods.

Inverse soft factors

Inverse soft factors add to the diagram a massless collinear particles between particles a and b and by definition one has

$$O_{n+1}(a,c,b,\cdots) = \frac{\langle ab\rangle}{\langle ac\rangle\langle cb\rangle}O_n(a'b') \ . \tag{12.3.29}$$

At the limit when the momentum of the added particle vanishes both sides approach the original amplitude. The right-handed spinors and Grassmann parameters are shifted

$$\begin{array}{ll}\tilde{\lambda}'_a = \tilde{\lambda}_a + \frac{\langle cb\rangle}{\langle ab\rangle}\tilde{\lambda}_c \ , & \tilde{\lambda}'_b = \tilde{\lambda}_b + \frac{\langle ca\rangle}{\langle ba\rangle}\tilde{\lambda}_c \ , \\ \eta'_a = \eta_a + \frac{\langle cb\rangle}{\langle ab\rangle}\eta_c \ , & \eta'_b = \eta_b + \frac{\langle ca\rangle}{\langle ba\rangle}\eta_c \ . \end{array} \tag{12.3.30}$$

There are two kinds of inverse soft factors.

(a) The addition of particle leaving the value k of negative helicity gluons unchanged means just the re-interpretation

$$Y'_{n,k}(Z_1,\cdots,Z_{n-1},Z_n) = Y_{n-1,k}(Z_1,\cdots,Z_{n-1}) \tag{12.3.31}$$

without actual dependence on Z_n. There is however a dependence on the momentum of the added particle since the relationship between momenta and momentum twistors is modified by the addition obtained by applying the basic rules relating region super momenta and momentum twistors (light-like momentum determines λ_i and twistor equations for x_i and λ_i,η_i determine (μ_i,χ_i)) is expressible assigned to the external particles [B28] . Modifications are needed only for the new vertex and its neighbors.

(b) The addition of a particle increasing k with single unit is a more complex operation which can be understood in terms of a residue of $Y_{n,k}$ proportional to $Y_{n-1,k-1}$ and Yangian invariant $[z_1\cdots z_5]$ with five arguments constructed from basic Yangian invariants with four arguments. The relationship between the amplitudes is now

$$Y'_{n,k}(..,Z_{n-1}Z_n,Z_1\cdots) \quad = \quad [n-2\ n-1\ n\ 1\ 2]\times Y_{n-1,k-1}(\cdots\hat{Z}_{n-1},\hat{Z}_1) \tag{12.3.32}$$

Here

$$[abcde] \quad = \quad \frac{\delta^{0|4}(\eta_a\langle bcde\rangle + cyclic)}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle} \ . \tag{12.3.33}$$

denoted also by $R(a,b,c,d,e)$ is the fundamental R-invariant appearing in one loop corrections of MHV amplitudes and will appears also in the recursion formulas. $\langle abcd\rangle$ is the fundamental super-conformal invariant associated with four super twistors defined in terms of the permutation symbol.

$\hat{Z}_{n-1},\hat{Z}_1$ are deformed momentum twistor variables. The deformation is determined from the relationship between external momenta, region momenta and momentum twistor variables. $\hat{Z}^1$ is the intersection $\hat{Z}^1 = (n-2\ n-1\ 2)\cap(12)$ of the the line (12) with the plane $(n-2\ n-1\ 2)$ and $\hat{Z}^{n-1}$ the intersection $\hat{Z}^1 = (12n)\cap(n-2\ n-1)$ of the the line $(n-2\ n-1)$ with the plane $(12n)$. The interpretation for the intersections at the level of ordinary Feynman diagrams is in terms of the collinearity of the four-momenta involved with the underlying box diagram with parallel on mass shell particles. These result from unitarity conditions obtained by putting maximal number of loop momenta on mass shell to give the leading singularities.

The explicit expressions for the momenta are

$$\begin{aligned}
\hat{Z}^1 \quad &\equiv \quad (n-2\ n-1\ 2)\cap(12)Z_1 = \langle 2\ n-2\ n-1\ n\rangle + Z_2\langle n-2\ n-1\ n\ 1\rangle \ , \\
\hat{Z}^{n-1} \quad &\equiv \quad (12n)\cap(n-2\ n-1) = Z_{n-2}\langle n-2\ n-1\ n\ 2\rangle + Z_{n-1}\langle n\ 1\ 2\ n-2\rangle \ .
\end{aligned} \tag{12.3.34}$$

These intersections also appear in the expressions defining the recursion formula.

Removal of particles and merge operation

Particles can be also removed. The first manner to remove particle is by integrating over the twistor variable characterizing the particle. This reduces k by one unit. Merge operation preserves the number of loops but removes a particle particle by identifying the twistor variables of neighboring particles. This operation corresponds to an integral over on mass shell loop momentum at the level of tree diagrams and by Witten's half Fourier transform can be transformed to twistor integral.

The product

$$Y'(Z_1,\cdots Z_n) \quad = \quad Y_1(Z_1,\cdots Z_m)\times Y_2(Z_{m+1},\cdots Z_n) \tag{12.3.35}$$

of two Yangian invariants is again a Yangian invariant. This is not quite trivial since the dependence of region momenta and momentum twistors on the momenta of external particles makes the operation non-trivial.

Merge operation allows to construct more interesting invariants from the products of Yangian invariants. One begins from a product of Yangian invariants (Yangian invariant trivially) represented cyclically as points of circle and identifies the last twistor argument of given invariant with the first twistor argument of the next invariant and performs integrals over the momentum twistor variables appearing twice. The soft k-increasing and preserving operations can be described also in terms of this operation for Yangian invariants such that the second invariant corresponds to 3-vertex. The cyclic merge operation applied to four MHV amplitudes gives NMHV amplitudes associated with on mass shell momenta in box diagrams. By applying similar operation to NMHV amplitudes and MHV amplitudes one obtains 2-loop amplitudes. In [B33] examples about these operations are described.

BCFW bridge

BCFW bridge allows to build general tree diagrams from MHV tree diagrams [B52, B52] and recursion formula of [B33] generalizes this to arbitrary diagrams. At the level of Feynman diagrams it corresponds to a box diagram containing general diagrams labeled by L and R and MHV and $\overline{MHV}$ 3-vertices ($\overline{MHV}$ 3-vertex allows expression in terms of MHV diagrams) with the lines of the box on mass shell so that the three momenta emanating from the vertices are parallel and give rise to a one-loop leading singularity.

At the level of Feynman diagrams BCFW bridge corresponds to so called "two-mass hard" leading singularities associated with box diagrams with light-like momenta at the four lines of the diagram [B31] . The motivation for the study of these diagrams comes from the hypothesis the leading order singularities obtained by putting as many particles as possible on mass shell contain the data needed to construct scattering amplitudes of $\mathcal{N} = 4$ SYM completely. This representation of the leading singularities generalizes to arbitrary loops. The recent article is a continuation of this program to planar amplitudes.

Also BCFW bridge allows an interpretation as a particular kind fusion for Yang invariants and involves all the basic operations. One starts from the amplitudes $Y^L_{n_L,k_L}$ and $Y^R_{n_R,k_R}$ and constructs an amplitude $Y'_{n_L+n_R,k_L+k_R+1}$ representing the amplitude which would correspond to a generalization of the MHV diagrams with the two tree diagrams connected by the MHV propagator (BFCW bridge) replaced with arbitrary loop diagrams. Particle "1" *resp.* "j+1" is added by the soft k-increasing factor to Y_{n_L+1,k_L+1} *resp.* Y_{n_R+1,k_R+1} giving amplitude with $n+2$ particles and with k-charge equal to k_L+k_R+2. The subsequent operations must reduce k-charge by one unit. First repeated "1" and "j+1" are identified with their copies by k conserving merge operation, and after that one performs an integral over the twistor variable Z^I associated with the internal line obtained and reducing k by one unit. The soft k-increasing factors bring in the invariants $[n-1\ n\ 1\ I\ j+2]$ associated with Y_L and $[1\ I\ j+1\ j\ j-1]$ associated with Y_R. The integration contour is chosen so that it selects the pole defined by $\angle n-1\ n\ 1\ I\rangle$ in the denominator of $[n-1\ n\ 1\ I\ j+2]$ and the pole defined by $\langle 1\ I\ j+1\ j\rangle$ in the denominator of $[1\ I\ j+1\ j\ j-1]$.

The explicit expression for the BCFW bridge is very simple:

$$\begin{aligned} (Y_L \otimes_{BFCW} Y_R)(1,\cdots,n) &= [n-1\ n\ 1\ j\ j+1] \times Y_R(1,\cdots,j,I) Y_L(I,j+1,\cdots,n-1,\hat{n}) \ , \\ \hat{n} &= (n-1\ n) \cap (j\ j+1\ 1) \ , \ I = (j\ j+1) \cap (n-1\ n\ 1) \end{aligned} \quad (12.3.36)$$

Single cuts and forward limit

Forward limit operation is used to increase the number of loops by one unit. The physical picture is that one starts from say 1-loop amplitude and cuts one line by assigning to the pieces of the line opposite light-like momenta having interpretation as incoming and outgoing particles. The resulting amplitude is called forward limit. The only reasonable interpretation seems to be that the loop integration is expressed by unitarity as forward limit meaning cutting of the line carrying the loop momentum. This operation can be expressed in a manifestly Yangian invariant way as entangled removal of two particles with the merge operation meaning the replacement $Z_n \rightarrow Z_{n-1}$. Particle $n+1$ is added adjacent to A, B as a k-increasing inverse soft factor and then A and B are removed by entangled integration, and after this merge operation identifies $n+1$ and 1.

Forward limit is crucial for the existence of loops and for Yangian invariants it corresponds to the poles arising from $\langle (AB)_q Z_n(z) Z_1)\rangle$ the integration contour $Z_n + zZ_{n-1}$ around Z_b in the basic formula $M = \oint (dz/z) M_n$ leading to the recursion formula. A and B denote the momentum twistors associated with opposite light-like momenta. In the generalized unitarity conditions the singularity corresponds to the cutting of line between particles n and 1 with momenta q and $-q$, summing over the multiplet of stats running around the loop. Between particles n_2 and 1 one has particles $n-1$, n with momenta $q, -q$. $q = x_1 - x_n = -x_n + x_{n-1}$ giving $x_1 = x_{n-1}$. Light-likeness of q means that the lines $(71) = (76)$ and (15) intersect.

At the forward limit giving rise to the pole Z_6 and Z_7 approach to the intersection point $(76) \cap (15)$. In a generic gauge theories the forward limits are ill-defined but in super-symmetric gauge theories situation changes.

The corresponding Yangian operation removes two external particles with opposite four-momenta and involves integration over two twistor variables Z_a and Z_b and gives rise to the following expression

$$\int_{GL(2)} Y(\cdots, Z_n, Z_A, Z_B, Z_1, \cdots) \ . \tag{12.3.37}$$

The integration over $GL(2)$ corresponds to integration over twistor variables associated Z_A and Z_B. This operation allows addition of a loop to a given amplitude. The line Z_aZ_b represents loop momentum on one hand and the dual x-coordinate identified as momentum propagating along the line on the other hand.

The integration over these variables is equivalent to an integration over loop momentum as the explicit calculation of [B33] (see pages 12-13) demonstrates. If the integration contours are products in the product of twistor spaces associated with a and b the and gives lower order Yangian invariant as answer. It is however also possible to choose the integration contour to be entangled in the sense that it cannot be reduced to a product of integration contours in the Cartesian product of twistor spaces. In this case the integration gives a loop integral. In the removal operation Yangian invariance can be broken by IR singularities associated with the integration contour and the procedure does not produce genuine Yangian invariant always.

What is highly interesting from TGD point of view is that this integral can be expressed as a contour integral over $CP_1 \times CP_1$ combined with integral over loop momentum. If TGD vision about generalized Feynman graps in zero energy ontology is correct, the loop momentum integral is discretized to an an integral over discrete mass shells and perhaps also to a sum over discretized momenta and one can therefore avoid IR singularities.

12.3.6 Explicit formula for the recursion relation

Recall that the recursion formula is obtained by considering super-symmetric momentum-twistor deformation $Z_n \to Z_n + zZ_{n-1}$ and by integrating over z to get the identity

$$M_{n,k,l} \quad = \quad \oint \frac{dz}{z} \hat{M}_{n,k,l}(z) \ . \tag{12.3.38}$$

This integral equals to integral with reversed integration contour enclosing the exterior of the contour. The challenge is to deduce the residues contributing to the residue integral and the claim of [B33] is that these residues reduce to simple basic types.

(a) The first residue corresponds to a pole at infinity and reduces the particle number by one giving a contribution $M_{n-1,k,l}(1, \cdots, n-1)$ to $M_{n,k,l}(1, \cdots, n-1, n)$. This is not totally trivial since the twistor variables are related to momenta in different manner for the two amplitudes. This gives the first contribution to the right hand side of the formula below.

(b) Second pole corresponds to the vanishing of $\langle Z_n(z) Z_1 Z_j Z_{j+1} \rangle$ and corresponds to the factorization of channels. This gives the second BCFW contribution to the right hand side of the formula below. These terms are however not enough since the recursion formula would imply the reduction to expressions involving only loop corrections to 3-loop vertex which vanish in $\mathcal{N} = 4$ SYM.

(c) The third kind of pole results when $\langle (AB)_q Z_n(z) Z_1\rangle$ vanishes in momentum twistor space. $(AB)_q$ denotes the line in momentum twistor space associated with q:th loop variable.

The explicit formula for the recursion relation yielding planar all loop amplitudes is obtained by putting all these pieces together and reads as

$$
\begin{array}{lll}
M_{n,k,l}(1,\cdots,n) & = & M_{n-1,k,l}(1,\cdots,n-1) \\
 & + & \sum_{n_L,k_L,l_L;j} [j\ j+1\ n-1\ n\ 1]\, M^R_{n_R,k_R,l_R}(1,\cdots,j,I_j)\times M^L_{n_L,k_L,l_L}(I_j,j+1,\cdots,\hat{n}_j) \\
 & + & \int_{GL(2)} [AB\ n-1\ n\ 1]\, M_{n+2,k+1,n,k-1}(1,\cdots,\hat{n}_{AB},\hat{A},B)\ , \\
n_L & + & n_R=n+2\ ,\ k_L+k_R=k-1\ ,\ l_R+l_L=l\ .
\end{array}
\tag{12.3.39}
$$

The momentum super-twistors are given by

$$
\begin{array}{ll}
\hat{n}_j=(n-1\ n)\cap(j\ j+1\ 1)\ , & I_j=(j\ j+1\ 1)\cap(n-1\ n\ 1)\ , \\
\hat{n}_{AB}=(n-1\ n)\cap(AB\ 1)\ , & \hat{A}=(AB)\cap(n-1\ n\ 1)\ .
\end{array}
\tag{12.3.40}
$$

The index l labels loops in $n+2$-particle amplitude and the expression is fully symmetrized with equal weight for all loop integration variables $(AB)_l$. A and B are removed by entangled integration meaning that $GL(2)$ contour is chosen to encircle points where both points A,B on the line (AB) are located at the intersection of the line (AB) with the plane $(n-1\ n\ 1)$. GL(2) integral can be done purely algebraically in terms of residues.

In [B33] and [B28] explicit calculations for N^kMHV amplitudes are carried out to make the formulas more concrete. For N^1MHV amplitudes second line of the formula vanishes and the integrals are rather simple since the determinants are 1×1 determinants.

12.4 Could the Grassmannian program be realized in TGD framework?

In the following the TGD based modification of the approach based on zero energy ontology is discussed in some detail. It is found that pseudo-momenta are very much analogous to region momenta and the approach leading to discretization of pseudo-mass squared for virtual particles - and even the discretization of pseudo-momenta - is consistent with the Grassmannian approach in the simple case considered and allow to get rid of IR divergences. Also the possibility that the number of generalized Feynman diagrams contributing to a given scattering amplitude is finite so that the recursion formula for the scattering amplitudes would involve only a finite number of steps (maximum number of loops) is considered. One especially promising feature of the residue integral approach with discretized pseudo-momenta is that it makes sense also in the p-adic context in the simple special case discussed since residue integral reduces to momentum integral (summation) and lower-dimensional residue integral.

12.4.1 What Yangian symmetry could mean in TGD framework?

The loss of the Yangian symmetry in the integrations over the region momenta x^a ($p^a = x^{a+1}-x^a$) assigned to virtual momenta seems to be responsible for many ugly features. It is basically the source of IR divergences regulated by "moving out on the Coulomb branch theory" so that IR singularities remain the problem of the theory. This raises the question whether the loss of Yangian symmetry is the signature for the failure of QFT approach and whether the restriction of loop momentum integrations to avoid both kind of divergences might be a royal road beyond QFT. In TGD framework zero energy ontology indeed leads to to a concrete proposal based on the vision that virtual particles are something genuinely real.

The detailed picture is of course far from clear but to get an idea about what is involved one can look what kind of assumptions are needed if one wants to realize the dream that only a finite number of generalized Feynman diagrams contribute to a scattering amplitude which is Yangian invariant allowing a description using a generalization of the Grassmannian integrals.

(a) Assume the bosonic emergence and its super-symmetric generalization holds true. This means that incoming and outgoing states are bound states of massless fermions assignable to wormhole throats but the fermions can opposite directions of three-momenta making them massive. Incoming and outgoing particles would consist of fermions associated with wormhole throats and would be characterized by a pair of twistors in the general situation and in general massive. This allows also string like mass squared spectrum for bound states having fermion and anti-fermion at the ends of the string as well as more general n-particle bound states. Hence one can speak also about the emergence of string like objects. For virtual particles the fermions would be massive and have discrete mass spectrum. Also super partners containing several collinear fermions and anti-fermions at a given throat are possible. Collinearity is required by the generalization of SUSY. The construction of these states bring strongly in mind the merge procedure involving the replacement $Z^{n+1} \rightarrow Z^n$.

(b) The basic question is how the momentum twistor diagrams and the ordinary Feynman diagrams behind them are related to the generalized Feynman diagrams.

   i. It is good to start from a common problem. In momentum twistor approach the relationship of region momenta to physical momenta remains somewhat mysterious. In TGD framework one can assign to the space-like 3-surfaces at the ends of CD four-momenta obeying stringy mass squared formula and to the fermion lines at light-like partonic orbits massless virtual momenta. Residue integration over virtual momenta leaves massless on mass shell momenta. The identification of these momenta as the TGD counterpart of the region momentum x looks like a natural first guess.

   ii. The identification $x_{a+1} - x_a = p_a$ with p_a representing light-like physical four-momentum generalizes in obvious manner. Also the identification of the light-like momentum of the external parton as pseudo-momentum looks natural. What is important is that this does not require the identification of the pseudo-momenta propagating along internal lines of generalized Feynman diagram as actual physical momenta since pseudo-momentum just like x is fixed only apart from an overall shift. The identification allows the physical four-momenta associated with the wormhole throats to be always on mass shell and massless: if the sign of the physical energy can be also negative space-like momentum exchanges become possible.

   iii. The pseudo-momenta and light-like physical massless momenta at the lines of generalized Feynman diagrams on one hand, and region momenta and the light-like momenta associated with the collinear singularities on the other hand would be in very similar mutual relationship. Partonic 2-surfaces can carry large number of collinear light-like fermions and bosons since super-symmetry is extended. Generalized Feynman diagrams would be analogous to momentum twistor diagrams if this picture is correct and one could hope that the recursion relations of the momentum twistor approach generalize.

(c) The discrete mass spectrum for four-momentum would in the momentum twistor approach mean the restriction of x to discrete mass shells, and the obvious reason for worry is that this might spoil the Grassmannian approach relying heavily on residue integrals and making sense also p-adically. It seems however that there is no need to worry. In [B33] the $M_{6,4,l=0}(1234AB)$ the integration over twistor variables z_A and z_B

using "entangled" integration contour leads to 1-loop MHV amplitude N^pMHV, $p = 1$. The parameterization of the integration contour is $z_A = (\lambda_A, x\lambda_A)$, $z_B = (\lambda_B, x\lambda_B)$, where x is the M^4 coordinate representing the loop momentum. This boils down to an integral over $CP_1 \times CP_1 \times M^4$ [B33] . The integrals over spheres CP_1s are contour integrals so that only an ordinary integral over M^4 remains. The reduction to this kind of sums occurs completely generally thanks to the recursion formula.

(d) The obvious implication of the restriction of the four-momenta x on massive mass shells is the absence of IR divergences and one might hope that under suitable assumptions one achieves Yangian invariance. The first question is of course whether the required restriction of x to mass shells in z_A and z_B or possibly even algebraic discretization of momenta is consistent with the Yangian invariance. This seems to be the case: the integration contour reduces to entangled integration contour in $CP_1 \times CP_1$ not affected by the discretization and the resulting loop integral differs from the standard one by the discretization of masses and possibly also momenta with massless states excluded. Whether Yangian invariance poses also conditions on mass and momentum spectrum is an interesting question.

(e) One can consider also the possibility that the incoming and outgoing particles - in general massive and to be distinguished from massless fermions appearing as their building blocks- have actually small masses presumably related to the IR cutoff defined by the size scale of the largest causal diamond involved. p-Adic thermodynamics could be responsible for this mass. Also the binding of the wormhole throats can give rise to a small contribution to vacuum conformal weight possibly responsible for gauge boson masses. This would imply that a given n-particle state can decay to N-particle states for which N is below some limit. The fermions inside loops would be also massive. This allows to circumvent the IR singularities due to integration over the phase space of the final states (say in Coulomb scattering).

(f) The representation of the off mass shell particles as pairs of wormhole throats with non-parallel four-momenta (in the simplest case only the three-momenta need be in opposite directions) makes sense and that the particles in question are on mass shell with mass squared being proportional to inverse of a prime number as the number theoretic vision applied to the modified Dirac equation suggests. On mass shell property poses extremely powerful constraints on loops and when the number of the incoming momenta in the loop increases, the number of constraints becomes larger than the number of components of loop momentum for the generic values of the external momenta. Therefore there are excellent hopes of getting rid of UV divergences.

A stronger assumption encouraged by the classical space-time picture about virtual particles is that the 3-momenta associated with throats of the same wormhole contact are always in same or opposite directions. Even this allows to have virtual momentum spectrum and non-trivial mass spectrum for them assuming that the three momenta are opposite.

(g) The best that one can hope is that only a finite number of generalized Feynman diagrams contributes to a given reaction. This would guarantee that amplitudes belong to a finite-dimensional algebraic extension of rational functions with rational coefficients since finite sums do not lead out from a finite algebraic extension of rationals. The first problem are self energy corrections. The assumption tht the mass non-renormalization theorems of SUSYs generalize to TGD framework would guarantee that the loops contributing to fermionic propagators (and their super-counterparts) do not affect them. Also the iteration of more complex amplitudes as analogs of ladder diagrams representing sequences of reactions $M \to M_1 \to M_2 \cdots . \to N$ such that at each M_n in the sequence can appear as on mass shell state could give a non-vanishing contribution to the scattering amplitude and would mean infinite number of Feynman diagrams unless these amplitudes vanish. If N appears as a virtual state the fermions must be however massive on mass shell fermions by the assumption about on-mass shell states and one can indeed imagine a situation in which the decay $M \to N$ is possible when N consists of states made of massless fermions is possible but not when the fermions have non-vanishing masses. This situation seems to be consistent with unitarity. The implication would be that the recursion formula for the all loop amplitudes for a given reaction would give vanishing result for some critical value of loops.

Already these assumptions give good hopes about a generalization of the momentum Grassmann approach to TGD framework. Twistors are doubled as are also the Grassmann variables and there are wave functions correlating the momenta of the the fermions associated with the opposite wormhole throats of the virtual particles as well as incoming gauge bosons which have suffered massivation. Also wave functions correlating the massless momenta at the ends of string like objects and more general many parton states are involved but do not affect the basic twistor formalism. The basic question is whether the hypothesis of unbroken Yangian symmetry could in fact imply something resembling this picture. The possibility to discretize integration contours without losing the representation as residue integral quite generally is basic prerequisite for this and should be shown to be true.

12.4.2 How to achieve Yangian invariance without trivial scattering amplitudes?

In $\mathcal{N}=4$ SYM the Yangian invariance implies that the MHV amplitudes are constant as demonstrated in [B33] . This would mean that the loop contributions to the scattering amplitudes are trivial. Therefore the breaking of the dual super-conformal invariance by IR singularities of the integrand is absolutely essential for the non-triviality of the theory. Could the situation be different in TGD framework? Could it be possible to have non-trivial scattering amplitudes which are Yangian invariants. Maybe! The following heuristic argument is formulated in the language of super-twistors.

(a) The dual conformal super generators of the super-Lie algebra $U(2,2)$ acting as super vector fields reducing effectively to the general form $J=\eta_a^K\partial/\partial Z_a^J$ and the condition that they annihilate scattering amplitudes implies that they are constant as functions of twistor variables. When particles are replaced with pairs of wormhole throats the super generators are replaced by sums J_1+J_2 of these generators for the two wormhole throats and it might be possible to achieve the condition

$$(J_1+J_2)M=0 \tag{12.4.1}$$

with a non-trivial dependence on the momenta if the super-components of the twistors associated with the wormhole throats are in a linear relationship. This should be the case for bound states.

(b) This kind of condition indeed exists. The condition that the sum of the super-momenta expressed in terms of super-spinors λ reduces to the sum of real momenta alone is not usually posed but in the recent case it makes sense as an additional condition to the super-components of the the spinors λ associated with the bound state. This quadratic condition is exactly of the same general form as the one following from the requirement that the sum of all external momenta vanishes for scattering amplitude and reads as

$$X = \lambda_1\eta_1+\lambda_2\eta_2=0 \ . \tag{12.4.2}$$

The action of the generators $\eta_1\partial_{\lambda_1}+\eta_2\partial_{\lambda_2}$ forming basic building blocks of the super generators on $p_1+p_2=\lambda_1\tilde{\lambda}_1+\lambda_2\tilde{\lambda}_2$ appearing as argument in the scattering amplitude in the case of bound states gives just the quantity X, which vanishes so that one has super-symmetry. The generalization of this condition to n-parton bound state is obvious.

(c) The argument does not apply to free fermions which have not suffered topological condensation and are therefore represented by CP_2 type vacuum extremal with single wormhole throat. If one accepts the weak form of electric-magnetic duality, one can circumvent this difficulty. The free fermions carry Kähler magnetic charge whereas physical fermions are accompanied by a bosonic wormhole throat carrying opposite Kähler magnetic charge and opposite electroweak isospin so that a ground state of string like object with size of order electroweak length scale is in question. In the case of quarks the Kähler magnetic charges need not be opposite since color confinement could involve Kähler magnetic confinement: electro-weak confinement holds however true also now. The above argument generalizes as such to the pairs formed by wormhole throats at the ends of string like object. One can of course imagine also more complex hybrids of these basic options but the general idea remains the same.

Note that the argument involves in an essential manner non-locality , which is indeed the defining property of the Yangian algebra and also the fact that physical particles are bound states. The massivation of the physical particles brings in the IR cutoff.

12.4.3 Could recursion formula allow interpretation in terms of zero energy ontology?

The identification of pseudo-momentum as a counterpart of region momentum suggests that generalized Feynman diagrams could be seen as a generalization of momentum twistor diagrams. Of course, the generalization from $\mathcal{N} = 4$ SYM to TGD is an enormous step in complexity and one must take all proposals in the following with a big grain of salt. For instance, the replacement of point-like particles with wormhole throats and the decomposition of gauge bosons to pairs of wormhole throats means that naive generalizations are dangerous.

With this in firmly in mind one can ask whether the recursion formula could allow interpretation in terms of zero energy states assigned to causal diamonds (CDs) containing CDs containing $\cdots$. In this framework loops could be assigned with sub-CDs.

The interpretation of the leading order singularities forming the basic building blocks of the twistor approach in zero ontology is the basic source of questions. Before posing these questions recall the basic proposal that partonic fermions are massless but opposite signs of energy are possible for the opposite throats of wormhole contacts. Partons would be on mass shell but besides physical states identified as bound states formed from partons also more general intermediate states would be possible but restricted by momentum conservation and mass shell conditions for partons at vertices. Consider now the questions.

(a) Suppose that the massivation of virtual fermions and their super partners allows only ladder diagrams in which the intermediate states contain on mass shell massless states. Should one allow this kind of ladder diagrams? Can one identify them in terms of leading order singularities? Could one construct the generalized Feynman diagrams from Yangian invariant tree diagrams associated with the hierarchy of sub-CDs and using BCFW bridges and entangled pairs of massless states having interpretation as box diagrams with on mass shell momenta at microscopic level? Could it make sense to say that scattering amplitudes are represented by tree diagrams inside CDs in various scales and that the fermionic momenta associated with throats and emerging from sub-CDs are always massless?

(b) Could BCFW bridge generalizes as such and could the interpretation of BCFW bridge be in terms of a scattering in which the four on mass shell massless partonic states (partonic throats have arbitrary fermion number) are exchanged between four sub-CDs. This admittedly looks somewhat artificial.

(c) Could the addition of 2-particle zero energy state responsible for addition of loop in the recursion relations and having interpretation in terms of the cutting of line carrying loop momentum correspond to an addition of sub-CD such that the 2-particle zero energy state has its positive and negative energy part on its past and future boundaries? Could this mean that one cuts a propagator line by adding CD and leaves only the portion of the line within CD. Could the reverse operation mean to the addition of zero energy "thermally entangled" states in shorter time and length scales and assignable as a zero energy state to a sub-CD. Could one interpret the Cutkosky rule for propagator line in terms of this cutting or its reversal. Why only pairs would be needed in the recursion formula? Why not more general states? Does the recursion formula imply that they are included? Does this relate to the fact that these zero energy states have interpretation as single particle states in the positive energy ontology and that the basic building block of Feynman diagrams is single particle state? Could one regard the unitarity as an identity which states that the discontinuity of T-matrix characterizing zero energy state over cut is expressible in terms of $TT^\dagger$ and T matrix is the relevant quantity?

Maybe it is again dangerous to try to draw too detailed correspondences: after all, point like particles are replaced by partonic two-surfaces in TGD framework.

(d) If I have understood correctly the genuine l-loop term results from $l-1$-loop term by the addition of the zero energy pair and integration over GL(2) as a representative of loop integral reducing $n+2$ to n and calculating the added loop at the same time [B33] . The integrations over the two momentum twistor variables associated with a line in twistor space defining off mass shell four-momentum and integration over the lines represent the integration over loop momentum. The reduction to $GL(2)$ integration should result from the delta functions relating the additional momenta to $GL(2)$ variables (note that GL(2) performs linear transformations in the space spanned by the twistors Z_A and Z_B and means integral over the positions of Z_A an Z_B). The resulting object is formally Yangian invariant but IR divergences along some contours of integration breaks Yangian symmetry.

The question is what happens in TGD framework. The previous arguments suggests that the reduction of the the loop momentum integral to integrals over discrete mass shells and possibly to a sum over their discrete subsets does not spoil the reduction to contour integrals for loop integrals in the example considered in [B33] . Furthermore, the replacement of mass continuum with a discrete set of mass shells should eliminate IR divergences and might allow to preserve Yangian symmetry. One can however wonder whether the loop corrections with on mass shell massless fermions are needed. If so, one would have at most finite number of loop diagrams with on mass shell fermionic momenta and one of the TGD inspired dreams already forgotten would be realized.

12.4.4 What about unitarity?

The approach of Arkani-Hamed and collaborators means that loop integral over four-momenta are replaced with residue integrals around a small sphere $p^2 = \epsilon$. This is very much reminiscent of my own proposal for a few years ago based on the idea that the condition of twistorialization forces to accept only massless virtual states [K78, K50] . I of course soon gave up this proposal as too childish.

This idea seems to however make a comeback in a modified form. At this time one would have only massive and quantized pseudo-momenta located at discrete mass shells. Can this picture be consistent with unitarity?

Before trying to answer this question one must make clear what one could assume in TGD framework.

(a) Physical particles are in the general case massive and consist of collinear fermions at wormhole throats. External partons at wormhole throats must be massless to allow twistorial interpretation. Therefore massive states emerge. This applies also to stringy states.

(b) The simplest assumption generalizing the childish idea is that on mass shell massless states for partons appear as both virtual particles and external particles. Space-like virtual momentum exchanges are possible if the virtual particles can consist of pairs of positive and negative energy fermions at opposite wormhole throats. Hence also partons at internal lines should be massless and this raises the question about the identification of propagators.

(c) Generalized eigenvalue equation for Chern-Simons Dirac operator guarantees that one can assign ordinary massless fermionic propagators with fermionic lines identified as boundaries of string world sheets which in turn follow from the condition that electromagnetic charge is well-defined for the modes of the induced spinor field. The generalized eigen mode property implies that virtual elementary fermions have in general non-light-like and possibly quantized pseudo-momenta whereas external elementary fermions have light-like pseudo-momenta from the boundary conditions at space-like ends of space-time surfaces. The (in general) massive pseudo-momentum assigned with the Dirac propagator of a parton line could be identified with the virtual momentum assignable to the fermion line. The region momenta introduced in Grassmannian approach are something analogous.

As already explained, this brings in mind is the identification of this pseudo-momentum as the counterpart of the region momentum of momentum twistor diagrams so that the external massless fermionic momenta would be differences of the pseudo-momenta. Indeed, since region momenta are determined apart from a common shift, they need not correspond to real momenta. Same applies to pseudo-momenta and one could assume that both internal and external fermion lines carry light-like pseudo-momenta and that external pseudo-momenta are equal to real momenta.

(d) This picture has natural correspondence with twistor diagrams. For instance, the region momentum appearing in BCFW bridge defining effective propagator is in general massive although the underlying Feynman diagram would contain online massless momenta. In TGD framework massless lines of Feynman graphs associated with singularities would correspond to real momenta of massless fermions at wormhole throats. Also other canonical operations for Yangian invariants involve light-like momenta at the level of Feynman diagrams and would in TGD framework have a natural identification in terms of partonic momenta. Hence partonic picture would provide a microscopic description for the lines of twistor diagrams.

Let us assume being virtual particle means only that the discretized pseudo-momentum is on shell but massive whereas all real momenta of partons are light-like, and that negative partonic energies are possible. Can one formulate Cutkosky rules for unitarity in this framework? What could the unitarity condition

$$iDisc(T - T^\dagger) = -TT^\dagger$$

mean now? In particular, are the cuts associated with mass shells of physical particles or with mass shells of pseudo-momenta? Could these two assignments be equivalent?

(a) The restriction of the partons to be massless but having both signs of energy means that the spectrum of intermediate states contains more states than the external states identified as bound states of partons with the same sign of energy. Therefore the summation over intermediate states does not reduce to a mere summation over physical states but involves a summation over states formed from massless partons with both signs of energy so that also space-like momentum exchanges become possible.

(b) The understanding of the unitarity conditions in terms of Cutkosky rules would require that the cuts of the loop integrands correspond to mass shells for the virtual states which are also physical states. Therefore real momenta have a definite sign and should be massless. Besides this bound state conditions guaranteeing that the mass spectrum for physical states is discrete must be assumed. With these assumptions the unitary cuts would not be assigned with the partonic light-cones but with the mass shells associated of physical particles.

(c) There is however a problem. The pseudo-momenta of partons associated with the external partons are assumed to be light-like and equal to the physical momenta.

   i. If this holds true also for the intermediate physical states appearing in the unitarity conditions, the pseudo-momenta at the cuts are light-like and cuts must be assigned with pseudo-momentum light-cones. This could bring in IR singularities and spoil Yangian symmetry. The formation of bound states could eliminate them and the size scale of the largest CD involved would bring in a natural IR cutoff as the mass scale of the lightest particle. This assumption would however force to give up the assumption that only massive pseudo-momenta appear at the lines of the generalized Feynman diagrams.

ii. On the other hand, if pseudo-momenta are not regarded as a property of physical state and are thus allowed to be massive for the real intermediate states in Cutkosky rules, the cuts at parton level correspond to on mass shell hyperboloids and IR divergences are absent.

12.5 Comparing twistor revolution with TGD revolution

Lubos Motl saved my Sunday by giving a link to an excellent talk by Nima Arkani-Hamed about the latest twistorial breakthroughs. Lubos Motl talks about "minirevolution" but David Gross uses a more appropriate expression "uprising". I would prefer to speak about revolution inducing at the sociological level a revolt. One must give up QFT in fixed space-time and string theory, and replace them with a theory whose name Nima guesses to be just "T".

For some time ago Lubos Motl told about the latest articles from Nima and collaborators: A Note on Polytopes for Scattering Amplitudes and Local Integrals for Planar Scattering Amplitudes.

Soon after this Lubos Motl gave a link to a video in which Witten talked about knot invariants. This talk was very inspiring and led to TGD based vision about how to calculate invariants of braids, braid cobordisms, and 2-knots in TGD framework and the idea that TGD could be seen as symplectic QFT for calculating these invariances among other things. Much of work was just translation of the basic ideas involved to TGD framework.

One crucial observation was that one can assign to the symplectic group of $\delta M^4_+ \times CP_2$ gerbe gauge potentials generalizing ordinary gauge potentials in terms of which one can define infinite number of classical 2-fluxes allowing to generalize Wilson loop to a Wilson surface. Most importantly, a unique identification for the decomposition of space-time surface to string world sheets identified as singularities of induce gauge fields and partonic 2-surfaces emerged and one can see the two decompositions as dual descriptions. TGD as almost topological QFT concretized to a symplectic QFT for knots, braids, braid cobordisms, and 2-knots. These ideas are documented in the chapter Knots and TGD of "TGD: Physics as Infinite-Dimensional Geometry" [K32] . I did not realize the obvious connection with twistor approach as I wrote the new chapter.

In his rather energetic lecture Nima emphasized how the Yangian symmetry originally discovered in 2-D QFTs, algebraic geometry, twistor theory, and string theory fuse to something bigger called "T". I realized that the twistorial picture developed in the earlier postings integrates nicely with the braidy vision inspired by Witten's talk and that one could understand in TGD framework why twistor description, Yangian symmetry of 2-D integrable systems, and algebraic geometry picture are so closely related. In particular, the dual conformal symmetries of twistor approach could be understood in terms of duality between partonic 2-surfaces and string world sheets expressing the strong form of holography. Also a generalization for the dual descriptions provided by super Wilson loop and ordinary scattering amplitude in $\mathcal{N} = 4$ SUSY in terms of Wilson sheets suggests itself among many other things. Also a rather obvious solution to the problem posed by non-planar diagrams to twistor approach suggests itself. Planar diagrams are simply not present and parton-string duality and huge symmetries of TGD give good reasons for why this should be the case.

12.5.1 The declaration of revolution by Nima from TGD point of view

At first look Nima's program is a declaration of revolution against all sacred principles. Nima dooms space-time, wants to get rid of QFT, does not even explicitly care about unitarity, and wants to throw Feynman diagrams to paper basket. Nima does not even respect string theory and sees it only as one particular- possibly not the best- manner to describe the underlying simplicity.

Give up space-time

In many respects I agree with Nima about the fate of space-time of QFT. I however see Nima's view a little bit exaggerated: one can perhaps compute scattering amplitudes without Minkowski space but one cannot translate the results of computations to the language of experiments without bringing in frequencies and wavelengths, classical fields, and therefore also space-time. Quantum classical correspondence: this is needed and this brings space-time unavoidably into the picture. Space-time surface serves as a dynamical correlate for quantum dynamics- generalized Bohr orbit required by General Coordinate Invariance and strong form of holography. The enormously important implication is absence of Feynman graphs in ordinary sense since their is no path integral over space-time surface but just single surface: the preferred extremal of Kähler action is enough (forgetting the delicacies caused by the failure of classical determinism in standard sense for Kähler action allowing to realize also the space-time correlates of quantum jump sequences).

Nima uses black hole based arguments to demonstrate that local observables are not operationally defined in neither gravitational theories nor quantum field theories and concludes that space-time is doomed. What would remain would be 4-D space-time regarded as a boundary of higher dimensional space-time (AdS/CFT correspondence). I think that this is quite too complex and that the reduction in degrees of freedom is much more radical: the landscape misery is after all basically due to the exponential inflation in the number of degrees of freedom due to the fatal mistake of making 10-D or 11-D target space dynamical.

What remains in TGD are boundaries of space-time surfaces at the upper and lower ends of causal diamonds $CD \times CP_2$ (briefly CD) and wormhole throats at which the signature of induced metric changes from Euclidian to Minkowskian (recall that Euclidian regions represent generalized Feynman diagrams). CD is essentially a representation of Penrose diagram which fits nicely with twistor approach. Strong form of holography implies that partonic 2-surfaces (or dual string world sheets) and 4-D tangent space data a them are enough as basic particle physics objects. The rest of space-time is needed to realized quantum classical correspondence essential for quantum measurement theory.

The basic message of TGD is that quantum superpositions of space-time surfaces are relevant for physics in all scales. Particles are the dynamical space-time quanta. There is however higher-dimensional space-time which is fixed and rigid $H = M^4 \times CP_2$ and is needed for the symmetries of the theory and guarantees the Kähler geometric existence of the world of classical worlds (WCW). This simplifies the situation enormously: instead of 10- or 11-D dynamical space-time one has just 4-D space-time and 2-D surfaces plus 4-D tangent space data. Holography is what we experience it to be: we see only 2-D surfaces. And physics is experimental science although some super string theorists might argue something else!

Give up fields

Nima argues also that fields are doomed too. I must say that I do not like this Planck length mysticism: it assumes quite too much and in TGD framework something new emerge already in CP_2 scale about 10^4 longer than Planck scale. According to Nima all this pain with Feynman diagrams would be due to the need to realize unitary representations of Poincare group in terms of fields. For massless particles one is forced to assume gauge invariance to eliminate the unphysical polarizations. Nima sees gauge invariance as the source of all troubles. Here I do not completely agree with Nima. The unitary time evolution in fixed space-time translated to the path integral over classical fields is what leads to the combinatorial nightmare of summing over Feynman diagrams and plagues also ϕ^4 theory. Amusingly, as Nima emphasizes all this has been known for 60 years. It is easy to understand that the possibility to realize unitarity elegantly using Feynman diagrams led to the acceptance of this approach as the only possible one.

In TGD framework the geometry of sub-manifolds replaces fields: the dynamics of partonic 2-surfaces identified as throats of light-like wormhole contacts containing fermions at them gives rise to bosons as bound states of fermions and anti-fermions. There is no path integral over space-time surfaces, just functional integral over partonic 2-surfaces so that path integral disappears. In zero energy ontology this means that incoming states are bound states of massless fermions and anti-fermions at wormhole throats and virtual states consist also of massless fermions but without the bound state constraint. This means horribly strong kinematic constraints on vertices defined by partonic 2-surfaces and UV finiteness and IR finiteness are automatic outcome of the theory. Massivation guaranteeing IR finiteness is consistent with massless-ness of fundamental particles since massive states are bound states of massless particles.

Nima talks also about emergence as something fundamental and claims that also space-time emerges. In TGD framework emergence has very concrete meaning. All particles are bound states of massless fermions and the additional purely bosonic degrees of freedom correspond to vibrational degrees of freedom for partonic 2-surfaces.

What is lacking from the program of Nima is the vision about physics as a geometry of worlds of classical worlds [K56] and physics as generalized number theory [K64] . This is what makes the higher-D imbedding space unique and allows the geometrization of quantum physics and identification of standard model symmetries as number theoretical symmetries. Infinite-dimensional geometry is unique just from the requirement that it exists!

12.5.2 Basic results of twistor approach from TGD point of view

The basic ideas of twistor approach are remarkably consistent with the basic picture of TGD.

Only on mass-shell amplitudes appear in the recursion formula

What is striking that the recursion formula of Nima and collaborators for the integrands of the planar amplitudes of $\mathcal{N}=4$ SUSY involve only on mass shell massless particles in the role of intermediate states. This is in sharp conflict with not only Feynman diagrammatic intuition but also with the very path integral ideology motivated by the need to realize unitary time development.

As already mentioned, in ZEO (zero energy ontology) all states- both on mass shell and off mass shell are composites of massless states assigned to 2-D partonic surfaces. Path integral is indeed replaced with generalized Bohr orbits and one obtains only very few generalized Feynman diagrams. What remains is functional integral over 3-surfaces, or even less over partonic 2-surfaces with varying tangent space data.

A further simplification is that as a result of the dynamics of preferred extremals many particle states correspond to discrete sets of points at partonic 2-surfaces serving as the ends of orbits of braid strands and possibly also 2-knots and functional integral involves integral over different configurations of these points [K22] . The physical interpretation is as a realization of finite measurement resolution as a property of dynamics itself. The string word sheets are uniquely identified as inverse images under imbedding map of space-time surface to $H = M^4 \times CP_2$ of homologically non-trivial geodesic sphere of CP_2 defining homological magnetic monopole. Holography in its strongest sense states that all information about non-trivial 2-homology if space-time surface and knottedness of the string world sheets is coded to the data at partonic 2-surfaces. For details see the chapter Knots and TGD of "TGD: Physics as Infinite-Dimensional Geometry [K32] .

Twistors and algebraic geometry connection emerge naturally in TGD framework

$H = M^4 \times CP_2$ and the reduction of all on mass shell states to bound states of massless states imply that twistor approach is the natural description of scattering amplitudes in TGD framework.

What is new that one must convolute massless theories in the sense that opposite throats of CP_2 sized wormhole contacts carry massless states. This allows to get rid of IR divergencies and realized exact Yangian symmetry by a purely physical mechanism making particle states massive.

An important implication is that even photon, gluons, and graviton have small masses and that in TGD framework all components of Higgs field are eaten by electroweak gauge bosons. Also gluons have colored scalar and pseudo-scalar counterparts and already now there are some hints at LHC for pseudo-scalar gluons. The discovery of Higgs can of course kill this idea anytime.

The connection with twistors allows to understand how algebraic geometry of projective spaces emerges in TGD framework and one indeed ends up to an alternative formulation of quantum TGD with space-time surfaces in H replaced with holomorphic 6-surfaces of $CP_3 \times CP_3$, which are sphere bundles and there effectively 4-D. The equations determining the 6-surfaces are dictated by rather general constraints.

Dual descriptions in terms of QFT and strings

The connections of $\mathcal{N} = 4$ SUSY with 2-D integrable systems and the possibly of both stringy and QFT descriptions characterized by dual conformal symmetries giving rise to Yangian invariance reduce in TGD framework to the duality between descriptions based on string world sheets and partonic 2-surfaces.

(a) The connection with string description emerges from the basic TGD in the sense that one can localize the solutions of the modified Dirac equation [K22] at braid strands located at the light-like 3-D wormhole throats. Similar localization to string world sheets defined in the above described manner holds true in space-time interior. The solutions of the modified Dirac equation localized to braid strands (and to string world sheets in space-time interior) are characterized by what I called pseudo momenta not directly identifiable as momenta. The natural identification is as region momenta of the twistor approach. Recall that the twistorialization of region momenta leads to the momentum twistor approach making dual conformal invariance manifest.

(b) The strange looking localization of fermions at braid strands makes mathematically sense only because the classical dynamics of preferred extremals reduces to hydrodynamics such that the flow parameters for flow lines integrate to global coordinates. So called Beltrami flows are in question and mean that preferred extremals have interpretation as perfect fluid flows for which dissipation is minimal [K22] . This property implies also the almost topological QFT property of TGD meaning that Kähler action reduces to Chern-Simons action localized at light-like wormhole throats and space-like 3-surfaces at the ends of CDs.

(c) The mathematical motivation on braid strands comes from the fact that this allows to avoid delta functions in the anti-commutators of fermionic oscillator operators at partonic 2-surfaces and therefore also the basic quadratic divergences of quantum field theories. Oscillator algebra has countable -perhaps even finite number- of generators and the loss of complete locality is in terms of finite measurement resolution. The larger the number of braid points selected at partonic 2-surface, the larger the number string world sheets and the higher the complexity of space-time surface. This obviously means a concrete realization of holography. The oscillator algebra has interpretation as SUSY algebra with arbitrarily large N fixed by the number of braid points. This SUSY symmetry is dynamical and badly broken. For right handed neutrino the breaking is smallest but also in this case the mixing of left- and right handed M^4 chiralities in modified Dirac equation implies non-conservation of R-parity as well as particle massivation and also the absence of lightest stable SUSY partner, which means that one particular dark matter candidate is out of game.

(d) The big difference between TGD and string models is that super generators do not correspond to Majorana spinors: this is indeed impossible for $M^4 \times CP_2$ since it would mean non-conservation of baryon and lepton numbers. I believed for a long time that stringy propagators emerge from TGD and the long standing painful question was what about stringy propagator defined by the inverse $1/G$ of the hermitian super generator in string models. In TGD $1/G$ cannot define stringy propagator since G carries fermion number. The reduction of strings to pairs of massless particles saves the situation and ordinary massless propagator for the counterparts of region momenta gives well defined propagators for on mass shell massless states! Stringy states reduce to bound states of massless particles in accordance with emergence philosophy. Nothing is scared these days!

Connection with integrable 2-D discrete systems

Twistor approach has revealed a striking connection between 2-D integrable systems and $\mathcal{N} = 4$ SUSY. For instance, one can calculate the anomalous dimensions of $\mathcal{N} = 4$ SUSY from an integrable model for spin chain in 2 dimensions without ever mentioning Feynman diagrams.

The description in terms of partonic 2-surfaces mean a direct connection with braids appearing in 2-D integrable thermodynamical systems and the description in terms of string world sheets means connection with integrable theories in 2-D Minkowski space. Both theories involve Yangian symmetry [A36] for which there exists a hierarchy of non-local conserved charged. Super-conformal invariance and its dual crucial for Yangian symmetry correspond to partonic 2-surfaces and string world sheets. The symmetry algebra is extended dramatically. In $\mathcal{N} = 4$ SUSY one has Yangian of conformal algebra of M^4. In TGD this algebra is generalized to include the super Kac-Moody algebra associated with isometries of the imbedding space, the super-conformal variant of the symplectic algebra of $\delta M^4 \times CP_2$, and also conformal transformations of M^4 mapping given boundary of CD to itself.

This allows also to understand and generalize the duality stating that QFT amplitudes for $\mathcal{N} = 4$ SUSY have interpretation as supersymmetric Wilson loops in dual Minkowski space. The ends of braid strands indeed define Wilson loops. In TGD framework work one must however generalize Wilson loops to Wilson sheets [K32] and the circulations of gauge potentials are replaced with fluxes of gerbe gauge potentials associated with the symplectic group of $\delta M^4_+ \times CP_2$. As noticed, dual conformal symmetries correspond to duality of partonic 2-surfaces and string world sheets implies by the 2-D holography for string world sheets.

12.5.3 Could planar diagrams be enough in the theory transcending $\mathcal{N} = 4$ SUSY?

Twistor approach as it appears in $\mathcal{N} = 4$ SYM is of course not the final solution.

(a) $\mathcal{N} = 4$ SUSY is not enough for the purposes of LHC.

(b) The extremely beautiful Yangian symmetry fails as one performs integration to obtain the scattering amplitudes and generates IR singularities. ZEO provides an elegant solution to this problem by replacing physical on mass shell particles with bound states of massless particles. Also string like objects emerge as this kind of states.

(c) Only planar diagrams allow to assign to the sum of Feynman diagrams a single integrand defining the twistor diagram. Something definitely goes wrong unless one is able to treat the non-planar diagrams. The basic problem is that one cannot assign common loop momentum variables to all diagrams simultaneously and this is due to the tricky character of Feynman diagrams. It is difficult to integrate without integrand!

The easy-to-guess question is whether the sum over the non-planar diagrams vanishes or whether they are just absent in a theory transcending $\mathcal{N} = 4$ SUSY and QFTs. Let N denote the number of colors of the SUSY. For $N \to \infty$ limit with g^2N fixed only planar diagrams survive in this kind of theory and one obtains a string model like description as conjectured long time ago by 't Hooft [B57]. This argument led later to AdS/CFT duality.

The stringy diagrams in TGD framework could correspond to planar diagrams of $\mathcal{N} = 4$ QFT. Besides this one would have s functional integral over partonic 2-surfaces.

(a) The description would be either in terms of partonic 2-surfaces or string world sheets with both determined uniquely in terms of a slicing of space-time surface with physical states characterized in terms of string world sheets in finite measurement resolution.

(b) $N \rightarrow \infty$ limit could in TGD framework be equivalent with two replacements. The color group with the infinite-D symplectic group of $\delta M^4_\pm \times CP_2$ and symplectic group and isometry group of H are replaced with their conformal variants.

(c) Could $g^2N = constant$ be equivalent with the use of hyper-finite factors of type II_1 [K79] for which the trace of the unit matrix equals to 1 instead of $\mathcal{N} = \infty$. These factors characterize the spinor structure of WCW identifiable in terms of Clifford algebra defined by infinite-D fermionic oscillator algebra defined by second quantized fermions at partonic 2-surfaces.

12.5.4 Stringy variant of twistor Grassmannian approach

The basic problem of the twistor approach is that one cannot represent massive momenta in terms of twistors in elegant manner. One can imagin two manners to circumvent this problem. The first one is modification of the notion of massless to masslessness in 8-D sense. One can indeed imagine an 8-D generalization of the twistor approach of Penrose based on the notion of octonionic spinor [K99]. The status of octonionic spinors remains uncertain.

One can also consider a stringy variant of twistor Grassmannian approach [K58] in which fundamental fermions (as opposed to elementary fermions) are massless. Since this approach looks more promising it is briefly summarized below.

(a) The approach is motivated by the stringy picture of elementary particles forced by the well-definedness of em charge for the modes of induced spinor field, and the assumption that elementary particles can be seen as bound states of massless fermions associated with the orbits of string ends at light-like orbits of partonic 2-surfaces. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet.

This brings the desired purely physical IR cutoff expected to cancel IR divergences. The fermions are massless and on-shell, and one assigns the inverse of massless propagator to the line which corresponds to non-physical helicity. This picture follows from Feynman graph approach if one can perform residue integral over virtual fermion momenta.

(b) In order to obtain non-trivial fermion propagator one must add to Kähler-Dirac action Chern-Simons Dirac term located at partonic orbits at which the signature of the induced metric changes. The modes of induced spinor field can be required to be generalized eigenmodes of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ with p^k identified as virtual momentum so that massless Dirac propagator is obtained. By super-symmetry one must add to Kähler action Chern-Simons term located at partonic orbits and this term must cancel the Chern-Simons term coming from Kähler action by weak form of electric-magnetic duality so that only the Chern-Simons terms associated with space-like ends of the space-time surface remain. These terms reduce to Chern-Simons terms only if one poses weak form of electric magnetic duality also here. This is not necessary.

(c) The quantum numbers characterizing zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse. Thermodynamics would naturally couple to the space-time geometry via the thermodynamical or quantum averages of the quantum numbers.

(d) The basic vertex is essentially four-fermion vertex although two light-like momenta combine to form virtual bosonic wormhole contact with total four-momentum which can be also space-like. BCFW recursion formula is expected to hold for the diagrams when one interprets these lines as virtual bosons. This picture could be seen as reduction of bosonic lines fermion lines and replacement of point like elementary particles with stringy structures formed by pairs wormhole contacts (see fig. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg` or fig. 10 in the appendix of this book).

(e) Contrary to the original over-optimistic assumptions the logarithmic UV divergences do not cancel unless on assumes stringy picture. This means that one assigns to the ends of the fermion line the analog of super-conformal propagator and its Hermitian conjugate. The analog of super-conformal propagator is defined by the inverse G/L_0 of super-generator $G^\dagger$. This assignment allows to circumvent the problem due to the fact that G carries fermion number in TGD framework.

(f) What is of special interest is that M^4 and CP_2 are the only 4-D manifolds allowing twistor space with Kähler structure. For CP_2 the twistor space has interpretation as the space $SU(3)/U(1)\times U(1)$ for the choices of quantization axes for color quantum numbers. This kind of twistor space can be assigned even with WCW but it is not clear what the physical interpretation and mathematical role of this twistor space is.

12.5.5 Motives and twistors

Nima mentions at the end of his talk motives [B9] . I know about this abstract branch of algebraic geometry only that it is an attempt to build a universal cohomology theory, which in turn is an algebraic approach to topology allowing to linearize highly non-linear situations encountered typically in algebraic geometry where topology is replaced with holomorphy which is must more stringent property and allows richer structures.

(a) Physics as generalized number theory vision involving also fusion of real and p-adic number fields to a larger super structure brings algebraic geometry to the core of TGD. The partonic 2-surfaces allowing interpretation as inhabitants of the intersection of real and p-adic worlds serve as correlates for living matter in TGD Universe. They are algebraic surfaces allowing in preferred coordinates a representation in terms of polynomials with rational coefficients. Motives would be needed to understand the cohomology of these surfaces. One encounters all kinds of problems such as counting the number of rational points in the intersection of p-adic and real variants of the surface and for algebraic surfaces this reduces to the counting of rational points for real 2-surface about which algebraic geometers know a lot of. For instance, surfaces of form $x^n+y^n+z^n=0$ for $n\geq 3$ appearing in Fermat's theorem are child's play since they allow only origin as a common point.

(b) As cautiously concluded in "Knots and TGD" [K32] , the intersection form for string world sheets defines a representation of the second relative homology of space-time surface and by Poincare duality also second cohomology. "Relative" is with respect to ends of space-time at the boundaries of CDs and light-like wormhole throats. The intersection form characterizing the collection of self-intersection points at which the braid strands are forced to go through each other is almost enough to characterize connected 4-manifolds topologically by Donaldson theorem [A4] .

(c) String world sheets define a violent unknotting procedure based on reconnections for braid strands- basic stringy vertex for closed strings- and in this manner knot invariant in the same manner as the recursion allowing to calculate the value of Jones polynomial for a given knot. Quantum TGD gives as a by-product rise to a symplectic QFT describing braids, their cobordisms, and 2-knots. It would not be surprising if the M-matrix elements would have also interpretation as symplectic covariants providing information about the topology of the space-time surface. The 2-braid theory associated with space-time surface would also characterize its topology just as ordinary knots can characterize topology of 3-manifolds.

To sum up, TGD suggests a surprisingly stringy but at the same time incredibly simple generalization of string model in which the discoveries made possible by the twistor approach to $\mathcal{N} = 4$ SUSY find a natural generalization. Nima has realized that much more than a mere discovery of computational recipes is involved and indeed talks about T-theory. I feel that the lonely "T" is desperately yearning for the company of "G" and "D"!

12.5.6 Reducing non-planar diagrams to planar ones by a generalization of algorithm for calculating knot invariants?

I have been listening some lectures in Strings 2001. The lectures related to progress in the calculation of gauge theory and super-gravity amplitudes are really electrifying: one really feels the sparking enthusiasm of the speakers. Besides twistor revolution there is also other amazing progress taking place in QFT side.

At this morning I started to listen the talk of Henrik Johansson about Lie algebra structures in YM and gravitational amplitudes. I have already earlier written about the finding that there is a symmetry between kinematical numerators of the amplitudes involving polarizations and momenta on one hand and color factors on the other hand, and that one can in well defined sense express gravitational scattering amplitudes in terms of squares of YM amplitudes. This holds true for on mass shell amplitudes. The reduction of the gravitational amplitudes to squares of YM amplitudes would be incredible simplification: even 3-graviton off mass shell vertex contains about 100 terms! As a matter fact, gravitation is a gauge theory too with gauge group replaced with Poincare group so that it would not be totally surprising that this kind of duality between kinematics would hold true.

This duality is not however the topic of this posting. As Johansson was explaining the Jacobi identity for the kinematical Lie algebra I got Eureka experience. What the kinematic Jacobi identity states is following:

The numerator for four-point amplitude with twisted legs in s-channel is expressible as a **difference** *of planar s- and t-channel amplitudes.*

If you did not get the association to twistor program already from this sentence, recall that the basic problem of twistor approach are non-planar diagrams. For them one cannot order the loop momenta in such a manner that the ordering would be universal and depend only on the number of loops as it is for planar diagrams without crossings. Hence one is not able to combine all diagrams to single integrand and this is related to the tricks one is forced to apply to make the loop integrals finite: same identification of loop momenta for all diagrams is not possible if one wants finiteness.

What one needs for a generalizaton of twistor approach to apply to non-planar diagrams is a universal identification of the loop momenta by cancelling all crossings: the amplitude itself need not be equal to the difference of the amplitudes obtained by reconnecting in two manners but could be something more general. This operation would be performed for internal lines only. For external lines it tells that the amplitudes changes possible sign when external lines are permuted. For braid statistics a more phase factor would result.

The duality of old-fashioned string models says that the difference of s- and t-channel amplitudes vanishes so that one can say that amplitudes with twisted legs vanish. Also at large N (number of colors) limit of $\mathcal{N} = 4$ SUSY these differences vanish and YM theory behaves like string theory and planar twistor approach should give exact answers at this limit. In TGD framework the effective replacement of gauge group with infinite-dimensional symplectic group could have the same effect. But what about finite values of N in super YM theories?

Could one generalize the twistor approach so that one could calculate **all** *amplitudes by recursion- not only the planar ones?*

Alert reader has of course answered already but I try to explain for non-specialists (with me included). If one has worked with braids and knots, one realizes that the expression for the amplitude as difference of planar amplitudes is analogous to what you get in elementary un-knotting operation for braids annihilating one crossing in the knot diagram! In the process you form the difference of two possible reconnections at the crossing point. If you interpret the process as time evolution, it corresponds to two vertices in which interiors of strings touch each other and reconnect in a new manner. In the construction of Jones polynomial as a knot invariant the repeated application of these un-twisting operations eventually leads to un-knot and you get as an outcome the knot invariant. Also non-planar Feynman diagram is like a knot diagram and the outcome of similar procedure should consists of only planar amplitudes.

For Feynman diagrams one cannot distinguish between upper and lower crossings of the lines. This could be interpreted by saying that both crossings give the same contribution. This is the case if untwisting gives the difference of numerators in both color and momentum degrees of freedom so that the signs cancel and the integrals of both contributions are identical despite the fact that the propagator denominators are not identical. The most general outcome would be a term proportional to the sum of the four planar contributions and one could perhaps treat the situation using twistorial methods. Proportionality coefficient could depend on dimensionless Lorentz scalars constructed from the incoming momenta of the sub-diagram with crossing and dictated to high degree by conformal invariance. Professional could probably demonstrate in five minutes that the conjecture cannot hold true.

Especially, if you have written N times "Quantum TGD as almost topological QFT ..." you get at the large N limit the vibe in your spine. Because the combinatorics of an almost topological QFT must be that of a topological QFT and because braids are basic building brick of TGD amplitudes, it should be possible to reduce all non-planar amplitudes -both those of TGD and those of $\mathcal{N} = 4$ SUSY and even other gauge theories - by a repeated un-twisting to planar amplitudes. A generalization of the basic algorithm of knot theory would become part of twistorial Feynman diagrammatics and could perhaps also be used to *define* the integrand including also the loops with crossings!

If the proposal can be realized in some sense, the rules for calculating the twistor amplitudes would be simple.

(a) You - or your knot theoretical friend- must first patiently unknot the Feynman diagrams involved by eliminating all twists using the basic formula allowing to express twisted sub-amplitude with a difference of un-twisted sub-amplitudes. You might even dream that he gives you explicit formulas for the outcome to get rid of your continual requests for help.

(b) At the end of the day you get just planar diagrams and you can apply the general recursive formulas of Nima and others working for all numbers of external particles and all numbers of loops to get the *integrand*, which you should be able to integrate.

(c) Unfortunately you are not! But you can knock the door of Goncharov and ask whether he could kindly perform the integral using his magic Symbolic Integration Machine [A67] about which Anastasia Volovich tells in her talk "Symblifying N=4 SUSY Scattering Amplitudes".

Is this idea just a passign daydream? Or morning dream- my hungry cat forced me to wake up at 3 a'clock so that I might be hallucinating in half-sleeping state. A specialist could immediately tell where this crazy idea of Europe's (if not World's) worst Feynman diagrammatician fails.

12.5.7 Langlands duality, electric-magnetic duality, S-duality, finite measurement resolution, and quantum Yangian symmetry

The arguments represented in the chapter "Langlands program and TGD" [K33] support the view that in TGD Universe number theoretic and geometric Langlands conjectures could be understood very naturally. The reader is warmly recommended to consult to this chapter for a more detailed representation.

What is important is that the discussion improves considerably the understanding about TGD proper. Same can be said about other attempts to apply TGD approach to the problems of modern mathematics to which topological quantumf field theories have been applied [K65, K82, K33]. In particular, a connection of Langlands conjectures and Yangian symmetry emerges. The group G *resp.* its Langlands dual ^LG would define what might be called twisted quantum Yangian associated with G *resp.* ^LG. The Lie group G *resp.* ^LG corresponds to the description of TGD in terms of partonic 2-surfaces *resp.* string world sheets made possible by strong form of holography in turn implied by strong form of general coordinate invariance implying also electric-magnetic duality and S-duality. Another new result is the identification of the gauge group G as a group defining the measurement resolution in the approach based on hyperfinite factors of type II_1 and proposal for the concrete representation of the corresponding Kac-Moody algebra. A further unexpected outcome are S-dual descriptions of TGD in terms of open string world sheets and partonic 2-surfaces in the moduli spaces of each other. Besides TGD based view about space-time, zero energy ontology and the notion of finite measurement resolution are the basic new notions as compared with the approach of Witten and Kapustin [A74] to the geometric Langlands duality.

(a) Zero energy ontology (ZEO) and the related notion of causal diamond CD (CD is a short hand for the cartesian product of causal diamond of M^4 and of CP_2). ZEO leads to the notion of partonic 2-surfaces at the light-like boundaries of CD and to the notion of string world sheet. These notions are central in the recent view about TGD. One can assign to the partonic 2-surfaces a conformal moduli space having as additional coordinates the positions of braid strand ends (punctures). By electric-magnetic duality this moduli space must correspond closely to the moduli space of string world sheets.

(b) Electric-magnetic duality realized in terms of string world sheets and partonic 2-surfaces. The group G and its Langlands dual ^LG would correspond to the time-like and space-like braidings. Duality predicts that the moduli space of string world sheets is very closely related to that for the partonic 2-surfaces. The strong form of 4-D general coordinate invariance implying electric-magnetic duality and S-duality as well as strong form of holography indeed predicts that the collection of string world sheets is fixed once the collection of partonic 2-surfaces at light-like boundaries of CD and its sub-CDs is known.

(c) The proposal is that finite measurement resolution is realized in terms of inclusions of hyperfinite factors of type II_1 at quantum level and represented in terms of confining effective gauge group [K79]. This effective gauge group could be some associate of G: gauge group, Kac-Moody group or its quantum counterpart, or so called twisted quantum Yangian strongly suggested by twistor considerations. At space-time level the finite measurement resolution would be represented in terms of braids at space-time level. The braids come in two varieties correspond to braids assignable to space-like surfaces at the two light-like boundaries of CD and with light-like 3-surfaces at which the signature of the induced metric changes and which are identified as orbits of partonic 2-surfaces connecting the future and past boundaries of CDs.

    There are several steps leading from G to its twisted quantum Yangian. The first step replaces point like particles with partonic 2-surfaces: this brings in Kac-Moody character. The second step brings in finite measurement resolution meaning that Kac-Moody type algebra is replaced with its quantum version. The third step brings in zero energy ontology: one cannot treat single partonic surface or string world sheet as independent unit: always the collection of partonic 2-surfaces and corresponding string worlds sheets defines the geometric structure so that multi-locality and therefore quantum Yangian algebra with multilocal generators is unavoidable. Also ZEO forces multi-locality since zero energy states defining orthonormal M-matrices are define multi-local Kac-Moody type algebra with integer powers of $S-$matrix defining the exponent of phase factor assignable with power z^n in the loop algebra generator.

(d) In finite measurement resolution geometric Langlands duality and number theoretic Langlands duality are very closely related since partonic 2-surface is effectively replaced with the punctures representing the ends of braid strands and the orbit of this set under a discrete subgroup of G defines effectively a collection of "rational" 2-surfaces. The number of the "rational" surfaces in geometric Langlands conjecture replaces the number of rational points of partonic 2-surface in its number theoretic variant. The ability to compute both these numbers is very relevant for quantum TGD.

(e) The natural identification of the associate of G is quantum Yangian of Kac-Moody type group associated with Minkowskian open string model assignable to string world sheet representing a string moving in the moduli space of partonic 2-surface. The dual group corresponds to Euclidian string model with partonic 2-surface representing string orbit in the moduli space of the string world sheets. The Kac-Moody algebra assigned with simply laced G is obtained using the standard tachyonic free field representation obtained as ordered exponentials of Cartan algebra generators identified as transversal parts of M^4 coordinates for the braid strands. The importance of the free field representation generalizing to the case of non-simply laced groups in the realization of finite measurement resolution in terms of Kac-Moody algebra cannot be over-emphasized (note that in string models and conformal field theories this realization of vertex operators in terms of free fields is of comparable importance).

(f) Langlands duality involves besides harmonic analysis side also the number theoretic side. Galois groups (collections of them) defined by infinite primes and integers having representation as symplectic flows defining braidings. I have earlier proposed that the hierarchy of these Galois groups define what might be regarded as a non-commutative homology and cohomology. Also G has this kind of representation which explains why the representations of these two kinds of groups are so intimately related. This relationship could be seen as a generalization of the MacKay correspondence between finite subgroups of $SU(2)$ and simply laced Lie groups.

(g) Symplectic group of the light-cone boundary acting as isometries of the WCW geometry [K12] allowing to represent projectively both Galois groups and symmetry groups as symplectic flows so that the non-commutative cohomology would have braided representation. This leads to braided counterparts for both Galois group and effective symmetry group.

(h) The moduli space for Higgs bundle playing central role in the approach of Witten and Kapustin to geometric Landlands program is in TGD framework replaced with the conformal moduli space for partonic 2-surfaces. It is not however possible to speak about Higgs field although moduli defined the analog of Higgs vacuum expectation value. Note that in TGD Universe the most natural assumption is that all Higgs like states are "eaten" by gauge bosons so that also photon and gluons become massive. This mechanism would be very general and mean that massless representations of Poincare group organize to massive ones via the formation of bound states. It might be however possible to see the contribution of p-adic thermodynamics depending on genus as analogous to Higgs contribution since the conformal moduli are analogous to vacuum expectation of Higgs field.

12.5.8 About the structure of the Yangian algebra

The attempt to understand Langlands conjecture in TGD framework led to a completely unexpected progress in the understanding of the Yangian symmetry expected to be the basic symmetry of quantum TGD and the following vision suggesting how conformal field theory could be generalized to four-dimensional context is a fruit of this work.

The structure of the Yangian algebra is quite intricate and in order to minimize confusion easily caused by my own restricted mathematical skills it is best to try to build a physical interpretation for what Yangian really is and leave the details for the mathematicians.

(a) The first thing to notice is that Yangian and quantum affine algebra are two different quantum deformations of a given Lie algebra. Both rely on the notion of R-matrix inducing a swap of braid strands. R-matrix represents the projective representations of the permutation group for braid strands and possible in 2-dimensional case due to the non-commutativity of the first homotopy group for 2-dimensional spaces with punctures. The R-matrix $R_q(u,v)$ depends on complex parameter q and two complex coordinates u,v. In integrable quantum field theories in M^2 the coordinates u,v are real numbers having identification as exponentials representing Lorenz boosts. In 2-D integrable conformal field theory the coordinates u,v have interpretation as complex phases representing points of a circle. The assumption that the coordinate parameters are complex numbers is the safest one.

(b) For Yangian the R-matrix is rational whereas for quantum affine algebra it is trigonometric. For the Yangian of a linear group quantum deformation parameter can be taken to be equal to one by a suitable rescaling of the generators labelled by integer by a power of the complex quantum deformation parameter q. I do not know whether this true in the general case. For the quantum affine algebra this is not possible and in TGD framework the most interesting values of the deformation parameter correspond to roots of unity.

Slicing of space-time sheets to partonic 2-surfaces and string world sheets

The proposal is that the preferred extremals of Kähler action are involved in an essential manner the slicing of the space-time sheets by partonic 2-surfaces and string world sheets. Also an analogous slicing of Minkowski space is assumed and there are infinite number of this kind of slicings defining what I have called Hamilton-Jaboci coordinates [K4]. What is really involved is far from clear. For instance, I do not really understand whether the slicings of the space-time surfaces are purely dynamical or induced by special coordinatizations of the space-time sheets using projections to special kind of sub-manifolds of the imbedding space, or are these two type of slicings equivalent by the very property of being a preferred extremal. Therefore I can represent only what I think I understand about the situation.

(a) What is needed is the slicing of space-time sheets by partonic 2-surfaces and string world sheets. The existence of this slicing is assumed for the preferred extremals of Kähler action [K4]. Physically the slicing corresponds to an integrable decomposition of the tangent space of space-time surface to 2-D space representing non-physical polarizations and 2-D space representing physical polarizations and has also number theoretical meaning.

(b) In zero energy ontology the complex coordinate parameters appearing in the generalized conformal fields should correspond to coordinates of the imbedding space serving also as local coordinates of the space-time surface. Problems seem to be caused by the fact that for string world sheets hyper-complex coordinate is more natural than complex coordinate. Pair of hyper-complex and complex coordinate emerge naturally as Hamilton-Jacobi coordinates for Minkowski space encountered in the attempts to understand the construction of the preferred extremals of Kähler action.

Also the condition that the flow lines of conserved isometry currents define global coordinates lead to the to the analog of Hamilton-Jacobi coordinates for space-time sheets [K4]. The physical interpretation is in terms of local polarization plane and momentum plane defined by local light-like direction. What is so nice that these coordinates are highly unique and determined dynamically.

(c) Is it really necessary to use two complex coordinates in the definition of Yangian-affine conformal fields? Why not to use hyper-complex coordinate for string world sheets? Since the inverse of hyper-complex number does not exist when the hyper-complex number is light-like, hyper-complex coordinate should appear in the expansions for the Yangian generalization of conformal field as positive powers only. Intriguingly, the Yangian algebra is "one half" of the affine algebra so that only positive powers appear in the expansion. Maybe the hyper-complex expansion works and forces Yangian-affine instead of doubly affine structure. The appearance of only positive conformal weights in Yangian sector could also relate to the fact that also in conformal theories this restriction must be made.

(d) It seems indeed essential that the space-time coordinates used can be regarded as imbedding space coordinates which can be fixed to a high degree by symmetries: otherwise problems with general coordinate invariance and with number theoretical universality would be encountered.

(e) The slicing by partonic 2-surfaces could (but need not) be induced by the slicing of CD by parallel translates of either upper or lower boundary of CD in time direction in the rest frame of CD (time coordinate varying in the direction of the line connecting the tips of CD). These slicings are not global. Upper and lower boundaries of CD would definitely define analogs of different coordinate patches.

Physical interpretation of the Yangian of quantum affine algebra

What the Yangian of quantum affine algebra or more generally, its super counterpart could mean in TGD framework? The key idea is that this algebra would define a generalization of super conformal algebras of super conformal field theories as well as the generalization of super Virasoro algebra. Optimist could hope that the constructions associated with conformal algebras generalize: this includes the representation theory of super conformal and super Virasoro algebras, coset construction, and vertex operator construction in terms of free fields. One could also hope that the classification of extended conformal theories defined in this manner might be possible.

(a) The Yangian of a quantum affine algebra is in question. The heuristic idea is that the two R-matrices - trigonometric and rational- are assignable to the swaps defined by space-like braidings associated with the braids at 3-D space-like ends of space-time sheets at light-like boundaries of CD and time like braidings associated with the braids at 3-D light-like surfaces connecting partonic 2-surfaces at opposite light-like boundaries of CD. Electric-magnetic duality and S-duality implied by the strong form of General Coordinate Invariance should be closely related to the presence of two R-matrices. The first guess is that rational R-matrix is assignable with the time-like braidings and trigonometric R-matrix with the space-like braidings. Here one must or course be very cautious.

(b) The representation of the collection of Galois groups associated with infinite primes in terms of braided symplectic flows for braid of braids of .... braids implies that there is a hierarchy of swaps: swaps can also exchange braids of ...braids. This would suggest that at the lowest level of the braiding hierarchy the R-matrix associated with a Kac-Moody algebra permutes two braid strands which decompose to braids. There would be two different braided variants of Galois groups.

(c) The Yangian of the affine Kac-Moody algebra could be seen as a 4-D generalization of the 2-D Kac-Moody algebra- that is a local algebra having representation as a power series of complex coordinates defined by the projections of the point of the space-time sheet to geodesic spheres of light-cone boundary and geodesic sphere of CP_2.

(d) For the Yangian the generators would correspond to polynomials of the complex coordinate of string world sheet and for quantum affine algebra to Laurent series for the complex coordinate of partonic 2-surface. What the restriction to polynomials means is not quite clear. Witten sees Yangian as one half of Kac-Moody algebra containing only the generators having $n \geq 0$. This might mean that the positivity of conformal weight for physical states essential for the construction of the representations of Virasoro algebra would be replaced with automatic positivity of the conformal weight assignable to the Yangian coordinate.

(e) Also Virasoro algebra should be replaced with the Yangian of Virasoro algebra or its quantum counterpart. This construction should generalize also to Super Virasoro algebra. A generalization of conformal field theory to a theory defined at 4-D space-time surfaces using two preferred complex coordinates made possible by surface property is highly suggestive. The generalization of conformal field theory in question would have two complex coordinates and conformal invariance associated with both of them. This would therefore reduce the situation to effectively 2-dimensional one rather than 3-dimensional: this would be nothing but the effective 2-dimensionality of quantum TGD implied by the strong form of General Coordinate Invariance.

(f) This picture conforms with what the generalization of $D = 4$ $\mathcal{N} = 4$ SYM by replacing point like particles with partonic 2-surfaces would suggest: Yangian is replaced with Yangian of quantum affine algebra rather than quantum group. Note that it is the finite measurement resolution alone which brings in the quantum parameters q_1 and q_2. The finite measurement resolution might be relevant for the elimination of IR divergences.

How to construct the Yangian of quantum affine algebra?

The next step is to try to understand the construction of the Yangian of quantum affine algebra.

(a) One starts with a given Lie group G. It could be the group of isometries of the imbedding space or subgroup of it or even the symplectic group of the light-like boundary of $CD \times CP_2$ and thus infinite-dimensional. It could be also the Lie group defining finite measurement resolution with the dimension of Cartan algebra determined by the number of braid strands.

(b) The next step is to construct the affine algebra (Kac-Moody type algebra with central extension). For the group defining the measurement resolution the scalar fields assigned with the ends of braid strands could define the Cartan algebra of Kac-Moody type algebra of this group. The ordered exponentials of these generators would define the charged generators of the affine algebra.

For the imbedding space isometries and symplectic transformations the algebra would be obtained by localizing with respect to the internal coordinates of the partonic 2-surface. Note that also a localization with respect to the light-like coordinate of light-cone boundary or light-like orbit of partonic 2-surface is possible and is strongly suggested by the effective 2-dimensionality of light-like 3-surfaces allowing extension of conformal algebra by the dependence on second real coordinate. This second coordinate should obviously correspond to the restriction of second complex coordinate to light-like 3-surface. If the space-time sheets allow slicing by partonic 2-surfaces and string world sheets this localization is possible for all 2-D partonic slices of space-time surface.

(c) The next step is quantum deformation to quantum affine algebra with trigonometric R-matrix $R_{q_1}(u, v)$ associated with space-like braidings along space-like 3-surfaces along the ends of CD. u and v could correspond to the values of a preferred complex coordinate of the geodesic sphere of light-cone boundary defined by rotational symmetry. It choice would fix a preferred quantization axes for spin.

(d) The last step is the construction of Yangian using rational R-matrix $R_{q_2}(u, v)$. In this case the braiding is along the light-like orbit between ends of CD. u and v would correspond to the complex coordinates of the geodesic sphere of CP_2. Now the preferred complex coordinate would fix the quantization axis of color isospin.

These arguments are of course heuristic and do not satisfy any criteria of mathematical rigor and the details could of course change under closer scrutinity. The whole point is in the attempt to understand the situation physically in all its generality.

How 4-D generalization of conformal invariance relates to strong form of general coordinate invariance?

The basic objections that one can rise to the extension of conformal field theory to 4-D context come from the successes of p-adic mass calculations. p-Adic thermodynamics relies heavily on the properties of partition functions for super-conformal representations. What happens when one replaces affine algebra with (quantum) Yangian of affine algebra? Ordinary Yangian involves the original algebra and its dual and from these higher multi-local generators are constructed. In the recent case the obvious interpretation for this would be that one has Kac-Moody type algebra with expansion with respect to complex coordinate w for partonic 2-surfaces and its dual algebra with expansion with respect to hyper-complex coordinate of string world sheet.

p-Adic mass calculations suggest that the use of either algebra is enough to construct single particle states. Or more precisely, local generators are enough. I have indeed proposed that the multi-local generators are relevant for the construction of bound states. Also the strong form of general coordinate invariance implying strong form of holography, effective 2-dimensionality, electric-magnetic duality and S-duality suggests the same. If one could construct the states representing elementary particles solely in terms of either algebra, there would be no danger that the results of p-adic mass calculations are lost. Note that also the necessity to restrict the conformal weights of conformal representations to be non-negative would have nice interpretation in terms of the duality.

12.6 Twistor revolution and TGD

Lubos Motl wrote a nice summary about the talk of Nima Arkani Hamed about twistor revolution in Strings 2012 and gave also a link to the talk [B21]. It seems that Nima and collaborators are ending to a picture about scattering amplitudes which strongly resembles that provided by generalized Feynman diagrammatics in TGD framework

TGD framework is much more general than $\mathcal{N} = 4$ SYM and is to it same as general relativity for special relativity whereas the latter is completely explicit. Of course, I cannot hope that TGD view could be taken seriously - at least publicly. One might hope that these approaches could be combined some day: both have a lot to give for each other. Below I compare these approaches.

The recent approach below emerges from the study of preferred extremals of Kähler and solutions of the modified Dirac equations so that it begins directly from basic TGD whereas the approaches hitherto have been based on general arguments and the precise role of right-handed neutrino has remained enigmatic. Chapters "Construction of quantum TGD: Symmetries" [K15] and "The recent vision about preferred extremals and solutions of the modified Dirac equation" [K87] contain section explaining how super-conformal and Yangian algebras crucial for the Grassmannian approach emerge from the basic TGD.

12.6.1 The origin of twistor diagrammatics

In TGD framework zero energy ontology forces to replace the idea about continuous unitary evolution in Minkowski space with something more general assignable to causal diamonds (CDs), and S-matrix is replaced with a square root of density matrix equal to a hermitian l square root of density matrix multiplied by unitary S-matrix. Also in twistor approach unitarity has ceased to be a star actor. In p-Adic context continuous unitary time evolution fails to make sense also mathematically.

Twistor diagrammatics involves only massless on mass shell particles on both external and internal lines. Zero energy ontology (ZEO) requires same in TGD: wormhole lines carry parallelly moving massless fermions and anti-fermions. The mass shell conditions at vertices are enormously powerful and imply UV finiteness. Also IR finiteness follows if external particles are massive.

What one means with mass is however a delicate matter. What does one mean with mass? I have pondered 35 years this question and the recent view is inspired by p-adic mass calculations and ZEO, and states that observed mass is in a well-defined sense expectation value of longitudinal mass squared for all possible choices of $M^2 \subset M^4$ characterizing the choices of quantization axis for energy and spin at the level of "world of classical worlds" (WCW) assignable with given causal diamond CD.

The choice of quantization axis thus becomes part of the geometry of WCW. All wormhole throats are massless but develop non-vanishing longitudinal mass squared. Gauge bosons correspond to wormhole contacts and thus consist of pairs of massless wormhole throats. Gauge bosons could develop 4-D mass squared but also remain massless in 4-D sense if the throats have parallel massless momenta. Longitudinal mass squared is however non-vanishing and p-adic thermodynamics predicts it.

12.6.2 The emergence of 2-D sub-dynamics at space-time level

Nima et al introduce ordering of the vertices in 4-D case. Ordering and related braiding are however essentially 2-D notions. Somehow 2-D theory must be a part of the 4-D theory also at space-time level, and I understood that understanding this is the challenge of the twistor approach at this moment.

The twistor amplitude can be represented as sum over the permutations of n external gluons and all diagrams corresponding to the same permutation are equivalent. Permutations are more like braidings since they carry information about how the permutation proceeded as a homotopy. Yang-Baxter equation emerges and states associativity of the braid group. The allowed braidings are minimal braidings in the sense that the repetitions of permutations of two adjacent vertices are not considered to be separate. Minimal braidings reduce to ordinary permutations. Nima also talks about affine braidings which I interpret as analogs of Kac-Moody algebras meaning that one uses projective representations which for Kac-Moody algebra mean non-trivial central extension. Perhaps the condition is that the square of a permutation permuting only two vertices which each other gives only a non-trivial phase factor. Lubos suggests an alternative interpretation which would select only special permutations and cannot be therefore correct.

There are rules of identifying the permutation associated with a given diagram involving only basic 3-gluon vertex with white circle and its conjugate. Lubos explains this "Mickey Mouse in maze" rule in his posting in detail: to determine the image $p(n)$ of vertex n in the permutation put a mouse in the maze defined by the diagram and let it run around obeying single rule: if the vertex is black turn to the right and if the vertex is white turn to the left. The mouse cannot remain in a loop: if it would do so, the rule would force it to run back to n after single full loop and one would have a fixed point: $p(n) = n$. The reduction in the number of diagrams is enormous: the infinity of different diagrams reduces to $n!$ diagrams!

What happens in TGD framework?

(a) In TGD framework string world sheets and partonic 2-surfaces (or either or these if they are dual notions as conjectured) at space-time surface would define the sought for 2-D theory, and one obtains indeed perturbative expansion with fermionic propagator defined by the inverse of the modified Dirac operator and bosonic propagator defined by the correlation function for small deformations of the string world sheet. The vertices of twistor diagrams emerge as braid ends defining the intersections of string world sheets and partonic 2-surfaces.

String model like description becomes part of TGD and the role of string world sheets in X^4 is highly analogous to that of string world sheets connecting branes in $AdS^5 \times S^5$ of $\mathcal{N} = 4$ SYM. In TGD framework 10-D $AdS^5 \times S^5$ is replaced with 4-D space-time surface in $M^4 \times CP_2$. The meaning of the analog of AdS^5 duality in TGD framework should be understood. In particular, it could it be that the descriptions involving string world sheets on one hand and partonic 2-surfaces - or 3-D orbits of wormhole throats defining the generalized Feynman diagram- on the other hand are dual to each other. I have conjectured something like this earlier but it takes some time for this kind of issues to find their natural answer.

(b) As described in the article, string world sheets and partonic 2-surfaces emerge directly from the construction of the solutions of the modified Dirac equation by requiring conservation of em charge. This result has been conjectured already earlier but using other less direct arguments. 2-D "string world sheets" as sub-manifolds of the space-time surface make the ordering possible, and guarantee the finiteness of the perturbation theory involving n-point functions of a conformal QFT for fermions at wormhole throats and n-point functions for the deformations of the space-time surface. Conformal invariance should dictate these n-point functions to a high degree. In TGD framework the fundamental 3-vertex corresponds to joining of light-like orbits of three wormhole contacts along their 2-D ends (partonic 2-surfaces).

12.6.3 The emergence of Yangian symmetry

Yangian symmetry associated with the conformal transformations of M^4 is a key symmetry of Grassmannian approach. Is it possible to derive it in TGD framework?

(a) TGD indeed leads to a concrete representation of Yangian algebra as generalization of color and electroweak gauge Kac-Moody algebra using general formula discussed in Witten's article about Yangian algebras (see the article).

(b) Article discusses also a conjecture about 2-D Hodge duality of quantized YM gauge potentials assignable to string world sheets with Kac-Moody currents. Quantum gauge potentials are defined only where they are needed - at string world sheets rather than entire 4-D space-time.

(c) Conformal scalings of the effective metric defined by the anti-commutators of the modified gamma matrices emerge as realization of quantum criticality. They are induced by critical deformations (second variations not changing Kähler action) of the space-time surface. This algebra can be generalized to Yangian using the formulas in Witten's article (see the article).

(d) Critical deformations induce also electroweak gauge transformations and even more general symmetries for which infinitesimal generators are products of U(n) generators permuting n modes of the modified Dirac operator and infinitesimal generators of local electro-weak gauge transformations. These symmetries would relate in a natural manner to finite measurement resolution realized in terms of inclusions of hyperfinite factors with included algebra taking the role of gauge group transforming to each other states not distinguishable from each other.

(e) How to end up with Grassmannian picture in TGD framework? This has inspired some speculations in the past. From Nima's lecture one however learns that Grassmannian picture emerges as a convenient parameterization. One starts from the basic 3-gluon vertex or its conjugate expressed in terms of twistors. Momentum conservation implies that with the three twistors λ_i or their conjugates are proportional to each other (depending on which is the case one assigns white or black dot with the vertex). This constraint can be expressed as a delta function constraint by introducing additional integration variables and these integration variables lead to the emergence of the Grassmannian $G_{n,k}$ where n is the number of gluons, and k the number of positive helicity gluons.

   Since only momentum conservation is involved, and since twistorial description works because only massless on mass shell virtual particles are involved, one is bound to end up with the Grassmannian description also in TGD.

12.6.4 The analog of AdS^5 duality in TGD framework

The generalization of AdS^5 duality of $\mathcal{N} = 4$ SYMs to TGD framework is highly suggestive and states that string world sheets and partonic 2-surfaces play a dual role in the construction of M-matrices. Some terminology first.

(a) Let us agree that string world sheets and partonic 2-surfaces refer to 2-surfaces in the slicing of space-time region defined by Hermitian structure or Hamilton-Jacobi structure.

(b) Let us also agree that *singular* string world sheets and partonic 2-surfaces are surfaces at which the *effective* metric defined by the anti-commutators of the modified gamma matrices degenerates to effectively 2-D one.

(c) Braid strands at wormhole throats in turn would be loci at which the *induced* metric of the string world sheet transforms from Euclidian to Minkowskian as the signature of induced metric changes from Euclidian to Minkowskian.

AdS^5 duality suggest that string world sheets are in the same role as string world sheets of 10-D space connecting branes in AdS^5 duality for $\mathcal{N} = 4$ SYM. What is important is that there should exist a duality meaning two manners to calculate the amplitudes. What the duality could mean now?

(a) Also in TGD framework the first manner would be string model like description using string world sheets. The second one would be a generalization of conformal QFT at light-like 3-surfaces (allowing generalized conformal symmetry) defining the lines of generalized Feynman diagram. The correlation functions to be calculated would have points at the intersections of partonic 2-surfaces and string world sheets and would represent braid ends.

(b) General Coordinate Invariance (GCI) implies that physics should be codable by 3-surfaces. Light-like 3-surfaces define 3-surfaces of this kind and same applies to space-like 3-surfaces. There are also preferred 3-surfaces of this kind. The orbits of 2-D wormhole throats at which 4-metric degenerates to 3-dimensional one define preferred light-like 3-surfaces. Also the space-like 3-surfaces at the ends of space-time surface at light-like boundaries of causal diamonds (CDs) define preferred space-like 3-surfaces. Both light-like and space-like 3-surfaces should code for the same physics and therefore their intersections defining partonic 2-surfaces plus the 4-D tangent space data at them should be enough to code for physics. This is strong form of GCI implying effective 2-dimensionality. As a special case one obtains singular string world sheets at which the effective metric reduces to 2-dimensional and singular partonic 2-surfaces defining the wormhole throats. For these 2-surfaces situation could be especially simple mathematically.

(c) The guess inspired by strong GCI is that string world sheet -partonic 2-surface duality holds true. The functional integrals over the deformations of 2 kinds of 2-surfaces should give the same result so that functional integration over either kinds of 2-surfaces should be enough. Note that the members of a given pair in the slicing intersect at discrete set of points and these points define braid ends carrying fermion number. Discretization and braid picture follow automatically.

(d) Scattering amplitudes in the twistorial approach could be thus calculated by using *any* pair in the slicing - or only either member of the pair if the analog of AdS^5 duality holds true as argued. The possibility to choose any pair in the slicing means general coordinate invariance as a symmetry of the Kähler metric of WCW and of the entire theory suggested already early: Kähler functions for difference choices in the slicing would differ by a real part of holomorphic function and give rise to same Kähler metric of "world of classical worlds" (WCW). For a general pair one obtains functional integral over deformations of space-time surface inducing deformations of 2-surfaces with only other kind 2-surface contributing to amplitude. This means the analog of stringy QFT: Minkowskian or Euclidian string theory depending on choice.

(e) For singular string world sheets and partonic 2-surfaces an enormous simplification results. The propagators for fermions and correlation functions for deformations reduce to 1-D instead of being 2-D: the propagation takes place only along the light-like lines at which the string world sheets with Euclidian signature (inside CP_2 like regions) change to those with Minkowskian signature of induced metric. The local reduction of space-time dimension would be very real for particles moving along sub-manifolds at which higher dimensional space-time has reduced metric dimension: they cannot get out from lower-D sub-manifold. This is like ending down to 1-D black hole interior and one would obtain the analog of ordinary Feynman diagrammatics. This kind of Feynman diagrammatics involving only braid strands is what I have indeed ended up earlier so that it seems that I can trust good intuition combined with a sloppy mathematics sometimes works;-).

These singular lines represent orbits of point like particles carrying fermion number at the orbits of wormhole throats. Furthermore, in this representation the expansions coming from string world sheets and partonic 2-surfaces are identical automatically. This follows from the fact that only the light-like lines connecting points common to singular string world sheets and singular partonic 2-surfaces appear as propagator lines!

(f) The TGD analog of AdS^5 duality of $\mathcal{N}=4$ SUSYs would be trivially true as an identity in this special case, and the good guess is that it is true also generally. One could indeed use integral over either string world sheets or partonic 2-sheets to deduce the amplitudes.

What is important to notice that singularities of Feynman diagrams crucial for the Grassmannian approach of Nima and others would correspond at space-time level 2-D singularities of the effective metric defined by the modified gamma matrices defined as contractions of canonical momentum currents for Kähler action with ordinary gamma matrices of the imbedding space and therefore directly reflecting classical dynamics.

12.6.5 Problems of the twistor approach from TGD point of view

Twistor approach has also its problems and here TGD suggests how to proceed. Signature problem is the first problem.

(a) Twistor diagrammatics works in a strict mathematical sense only for $M^{2,2}$ with metric signature (1,1,-1,-1) rather than M^4 with metric signature (1,-1,-1,-1). Metric signature is wrong in the physical case. This is a real problem which must be solved eventually.

(b) Effective metric defined by anti-commutators of the modified gamma matrices (to be distinguished from the induced gamma matrices) could solve that problem since it would have the correct signature in TGD framework (see the article). String world sheets and partonic 2-surfaces would correspond to the 2-D singularities of this effective metric at which the even-even signature (1,1,1,1) changes to even-even signature (1,1,-1,-1). Space-time at string world sheet would become locally 2-D with respect to effective metric just as space-time becomes locally 3-D with respect to the induced metric at the light-like orbits of wormhole throats. String world sheets become also locally 1-D at light-like curves at which Euclidian signature of world sheet in induced metric transforms to Minkowskian.

(c) Twistor amplitudes are indeed singularities and string world sheets implied in TGD framework by conservation of em charge would represent these singularities at space-time level. At the end of the talk Nima conjectured about lower-dimensional manifolds of space-time as representation of space-time singularities. Note that string world sheets and partonic 2-surfaces have been part of TGD for years. TGD is of course to $\mathcal{N}=4$ SYM what general relativity is for the special relativity. Space-time surface is dynamical and possesses induced and effective metrics rather than being flat.

Second limitation is that twistor diagrammatics works only for planar diagrams. This is a problem which must be also fixed sooner or later.

(a) This perhaps dangerous and blasphemous statement that I will regret it some day but I will make it;-). Nima and others have not yet discovered that $M^2 \subset M^4$ must be there but will discover it when they begin to generalize the results to non-planar diagrams and realize that Feynman diagrams are analogous to knot diagrams in 2-D plane (with crossings allowed) and that this 2-D plane must correspond to $M^2 \subset M^4$. The different choices of causal diamond CD correspond to different choices of M^2 representing choice of quantization axes 4-momentum and spin. The integral over these choices guarantees Lorentz invariance. Gauge conditions are modified: longitudinal M^2 projection of massless four-momentum is orthogonal to polarization so that three polarizations are possible: states are massive in longitudinal sense.

(b) In TGD framework one replaces the lines of Feynman diagrams with the light-like 3-surfaces defining orbits of wormhole throats. These lines carry many fermion states defining braid strands at light-like 3-surfaces. There is internal braiding associated with these braid strands. String world sheets connect fermions at different wormhole throats with space-like braid strands. The M^2 projections of generalized Feynman diagrams with 4-D "lines" replaced with genuine lines define the ordinary Feynman diagram as the analog of braid diagram. The conjecture is that one can reduce non-planar diagrams to planar diagrams using a procedure analogous to the construction of knot invariants by un-knotting the knot in Alexandrian manner by allowing it to be cut temporarily.

(c) The permutations of string vertices emerge naturally as one constructs diagrams by adding to the interior of polygon sub-polygons connected to the external vertices. This corresponds to the addition of internal partonic two-surfaces. There are very many equivalent diagrams of this kind. Only permutations matter and the permutation associated with a given diagram of this kind can be deduced by the Mickey-Mouse rule described explicitly by Lubos. A connection with planar operads is highly suggestive and also conjecture already earlier in TGD framework.

12.6.6 Could $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM be a part of TGD after all?

Whether right-handed neutrinos generate a supersymmetry in TGD has been a long standing open question. $\mathcal{N} = 1$ SUSY is certainly excluded by fermion number conservation but already $\mathcal{N} = 2$ defining a "complexification" of $\mathcal{N} = 1$ SUSY is possible and could generate right-handed neutrino and its antiparticle. These states should however possess a non-vanishing light-like momentum since the fully covariantly constant right-handed neutrino generates zero norm states. So called massless extremals (MEs) allow massless solutions of the modified Dirac equation for right-handed neutrino in the interior of space-time surface, and this seems to be case quite generally in Minkowskian signature for preferred extremals. This suggests that particle represented as magnetic flux tube structure with two wormhole contacts sliced between two MEs could serve as a starting point in attempts to understand the role of right handed neutrinos and how $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM emerges at the level of space-time geometry. The following arguments inspired by the article of Nima Arkani-Hamed et al [B32] about twistorial scattering amplitudes suggest a more detailed physical interpretation of the possible SUSY associated with the right-handed neutrinos.

The fact that right handed neutrinos have only gravitational interaction suggests a radical re-interpretation of SUSY: no SUSY breaking is needed since it is very difficult to distinguish between mass degenerate spartners of ordinary particles. In order to distinguish between different spartners one must be able to compare the gravitomagnetic energies of spartners in slowly varying external gravimagnetic field: this effect is extremely small.

Scattering amplitudes and the positive Grassmannian

The work of Nima Arkani-Hamed and others represents something which makes me very optimistic and I would be happy if I could understand the horrible technicalities of their work. The article Scattering Amplitudes and the Positive Grassmannian by Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, and Trnka [B32] summarizes the recent situation in a form, which should be accessible to ordinary physicist. Lubos has already discussed the article. The following considerations do not relate much to the main message of the article (positive Grassmannians) but more to the question how this approach could be applied in TGD framework.

1. All scattering amplitudes have on shell amplitudes for massless particles as building bricks

The key idea is that all planar amplitudes can be constructed from on shell amplitudes: all virtual particles are actually real. In zero energy ontology I ended up with the representation of TGD analogs of Feynman diagrams using only mass shell massless states with both positive and negative energies. The enormous number of kinematic constraints eliminates UV and IR divergences and also the description of massive particles as bound states of massless ones becomes possible.

In TGD framework quantum classical correspondence requires a space-time correlate for the on mass shell property and it indeed exists. The mathematically ill-defined path integral over all 4-surfaces is replaced with a superposition of preferred extremals of Kähler action analogous to Bohr orbits, and one has only a functional integral over the 3-D ends at the light-like boundaries of causal diamond (Euclidian/Minkowskian space-time regions give real/imaginary Chern-Simons exponent to the vacuum functional). This would be obviously the deeper principle behind on mass shell representation of scattering amplitudes that Nima and others are certainly trying to identify. This principle in turn reduces to general coordinate invariance at the level of the world of classical worlds.

Quantum classical correspondence and quantum ergodicity would imply even stronger condition: the quantal correlation functions should be identical with classical correlation functions for any preferred extremal in the superposition: all preferred extremals in the superposition would be statistically equivalent [K87]. 4-D spin glass degeneracy of Kähler action however suggests that this is is probably too strong a condition applying only to building bricks of the superposition.

Minimal surface property is the geometric counterpart for masslessness and the preferred extremals are also minimal surfaces: this property reduces to the generalization of complex structure at space-time surfaces, which I call Hamilton-Jacobi structure for the Minkowskian signature of the induced metric. Einstein Maxwell equations with cosmological term are also satisfied.

2. Massless extremals and twistor approach

The decomposition $M^4 = M^2 \times E^2$ is fundamental in the formulation of quantum TGD, in the number theoretical vision about TGD, in the construction of preferred extremals, and for the vision about generalized Feynman diagrams. It is also fundamental in the decomposition of the degrees of string to longitudinal and transversal ones. An additional item to the list is that also the states appearing in thermodynamical ensemble in p-adic thermodynamics correspond to four-momenta in M^2 fixed by the direction of the Lorentz boost. In twistor approach to TGD the possibility to decompose also internal lines to massless states at parallel space-time sheets is crucial.

Can one find a concrete identification for $M^2 \times E^2$ decomposition at the level of preferred extremals? Could these preferred extremals be interpreted as the internal lines of generalized Feynman diagrams carrying massless momenta? Could one identify the mass of particle predicted by p-adic thermodynamics with the sum of massless classical momenta assignable to two preferred extremals of this kind connected by wormhole contacts defining the elementary particle?

Candidates for this kind of preferred extremals indeed exist. Local $M^2 \times E^2$ decomposition and light-like longitudinal massless momentum assignable to M^2 characterizes "massless extremals" (MEs, "topological light rays"). The simplest MEs correspond to single space-time sheet carrying a conserved light-like M^2 momentum. For several MEs connected by wormhole contacts the longitudinal massless momenta are not conserved anymore but their sum defines a time-like conserved four-momentum: one has a bound states of massless MEs. The stable wormhole contacts binding MEs together possess Kähler magnetic charge and serve as building bricks of elementary particles. Particles are necessary closed magnetic flux tubes having two wormhole contacts at their ends and connecting the two MEs.

The sum of the classical massless momenta assignable to the pair of MEs is conserved even when they exchange momentum. Quantum classical correspondence requires that the conserved classical rest energy of the particle equals to the prediction of p-adic mass calculations. The massless momenta assignable to MEs would naturally correspond to the massless momenta propagating along the internal lines of generalized Feynman diagrams assumed in zero energy ontology. Masslessness of virtual particles makes also possible twistor approach. This supports the view that MEs are fundamental for the twistor approach in TGD framework.

3. Scattering amplitudes as representations for braids whose threads can fuse at 3-vertices

Just a little comment about the content of the article. The main message of the article is that non-equivalent contributions to a given scattering amplitude in $\mathcal{N}=4$ SYM represent elements of the group of permutations of external lines - or to be more precise - decorated permutations which replace permutation group S_n with $n!$ elements with its decorated version containing $2^n n!$ elements. Besides 3-vertex the basic dynamical process is permutation having the exchange of neighboring lines as a generating permutation completely analogous to fundamental braiding. BFCW bridge has interpretation as a representations for the basic braiding operation.

This supports the TGD inspired proposal (TGD as almost topological QFT) that generalized Feynman diagrams are in some sense also knot or braid diagrams allowing besides braiding operation also two 3-vertices [K32]. The first 3-vertex generalizes the standard stringy 3-vertex but with totally different interpretation having nothing to do with particle decay: rather particle travels along two paths simultaneously after $1 \rightarrow 2$ decay. Second 3-vertex generalizes the 3-vertex of ordinary Feynman diagram (three 4-D lines of generalized Feynman diagram identified as Euclidian space-time regions meet at this vertex). The main idea is that in TGD framework knotting and braiding emerges at two levels.

(a) At the level of space-time surface string world sheets at which the induced spinor fields (except right-handed neutrino [K87]) are localized due to the conservation of electric charge can form 2-knots and can intersect at discrete points in the generic case. The boundaries of strings world sheets at light-like wormhole throat orbits and at space-like 3-surfaces defining the ends of the space-time at light-like boundaries of causal diamonds can form ordinary 1-knots, and get linked and braided. Elementary particles themselves correspond to closed loops at the ends of space-time surface and can also get knotted (possible effects are discussed in [K32]).

(b) One can assign to the lines of generalized Feynman diagrams lines in M^2 characterizing given causal diamond. Therefore the 2-D representation of Feynman diagrams has concrete physical interpretation in TGD. These lines can intersect and what suggests itself is a description of non-planar diagrams (having this kind of intersections) in terms of an algebraic knot theory. A natural guess is that it is this knot theoretic operation which allows to describe also non-planar diagrams by reducing them to planar ones as one does when one constructs knot invariant by reducing the knot to a trivial one. Scattering amplitudes would be basically knot invariants.

"Almost topological" has also a meaning usually not assigned with it. Thurston's geometrization conjecture stating that geometric invariants of canonical representation of manifold as Riemann geometry, defined topological invariants, could generalize somehow. For instance, the geometric invariants of preferred extremals could be seen as topological or more refined invariants (symplectic, conformal in the sense of 4-D generalization of conformal structure). If quantum ergodicity holds true, the statistical geometric invariants defined by the classical correlation functions of various induced classical gauge fields for preferred extremals could be regarded as this kind of invariants for sub-manifolds. What would distinguish TGD from standard topological QFT would be that the invariants in question would involve length scale and thus have a physical content in the usual sense of the word!

Could $\mathcal{N}$ =2 or $\mathcal{N}=4$ SUSY have something to do with TGD?

$\mathcal{N}=4$ SYM has been the theoretical laboratory of Nima and others. $\mathcal{N}=4$ SYM is definitely a completely exceptional theory, and one cannot avoid the question whether it could in some sense be part of fundamental physics. In TGD framework right handed neutrinos have remained a mystery: whether one should assign space-time SUSY to them or not. Could they give rise to something resembling $\mathcal{N}=2$ or $\mathcal{N}=4$ SUSY with fermion number conservation?

1. Earlier results

My latest view is that *fully* covariantly constant right-handed neutrinos decouple from the dynamics completely. I will repeat first the earlier arguments which consider only fully covariantly constant right-handed neutrinos.

(a) $\mathcal{N} = 1$ SUSY is certainly excluded since it would require Majorana property not possible in TGD framework since it would require superposition of left and right handed neutrinos and lead to a breaking of lepton number conservation. Could one imagine SUSY in which both MEs between which particle wormhole contacts reside have $\mathcal{N} = 2$ SUSY which combine to form an $\mathcal{N} = 4$ SUSY?

(b) Right-handed neutrinos which are covariantly constant right-handed neutrinos in both M^4 degrees of freedom cannot define a non-trivial theory as shown already earlier. They have no electroweak nor gravitational couplings and carry no momentum, only spin.

The fully covariantly constant right-handed neutrinos with two possible helicities at given ME would define representation of SUSY at the limit of vanishing light-like momentum. At this limit the creation and annihilation operators creating the states would have vanishing anti-commutator so that the oscillator operators would generate Grassmann algebra. Since creation and annihilation operators are hermitian conjugates, the states would have zero norm and the states generated by oscillator operators would be pure gauge and decouple from physics. This is the core of the earlier argument demonstrating that $\mathcal{N} = 1$ SUSY is not possible in TGD framework: LHC has given convincing experimental support for this belief.

2. Could massless right-handed neutrinos covariantly constant in CP_2 degrees of freedom define $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY?

Consider next right-handed neutrinos, which are covariantly constant in CP_2 degrees of freedom but have a light-like four-momentum. In this case fermion number is conserved but this is consistent with $\mathcal{N} = 2$ SUSY at both MEs with fermion number conservation. $\mathcal{N} = 2$ SUSYs could emerge from $\mathcal{N} = 4$ SUSY when one half of SUSY generators annihilate the states, which is a basic phenomenon in supersymmetric theories.

(a) At space-time level right-handed neutrinos couple to the space-time geometry - gravitation - although weak and color interactions are absent. One can say that this coupling forces them to move with light-like momentum parallel to that of ME. At the level of space-time surface right-handed neutrinos have a spectrum of excitations of four-dimensional analogs of conformal spinors at string world sheet (Hamilton-Jacobi structure).

For MEs one indeed obtains massless solutions depending on longitudinal M^2 coordinates only since the induced metric in M^2 differs from the light-like metric only by a contribution which is light-like and contracts to zero with light-like momentum in the same direction. These solutions are analogs of (say) left movers of string theory. The dependence on E^2 degrees of freedom is holomorphic. That left movers are only possible would suggest that one has only single helicity and conservation of fermion number at given space-time sheet rather than 2 helicities and non-conserved fermion number: two real Majorana spinors combine to single complex Weyl spinor.

(b) At imbedding space level one obtains a tensor product of ordinary representations of $\mathcal{N} = 2$ SUSY consisting of Weyl spinors with opposite helicities assigned with the ME. The state content is same as for a reduced $\mathcal{N} = 4$ SUSY with four $\mathcal{N} = 1$ Majorana spinors replaced by two complex $\mathcal{N} = 2$ spinors with fermion number conservation. This gives 4 states at both space-time sheets constructed from ν_R and its antiparticle. Altogether the two MEs give 8 states, which is one half of the 16 states of $\mathcal{N} = 4$ SUSY so that a degeneration of this symmetry forced by non-Majorana property is in question.

3. Is the dynamics of $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM possible in right-handed neutrino sector?

Could $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM be a part of quantum TGD? Could TGD be seen a fusion of a degenerate $\mathcal{N} = 4$ SYM describing the right-handed neutrino sector and string theory like theory describing the contribution of string world sheets carrying other leptonic and quark spinors? Or could one imagine even something simpler?

What is interesting that the net momenta assigned to the right handed neutrinos associated with a pair of MEs would correspond to the momenta assignable to the particles and obtained by p-adic mass calculations. It would seem that right-handed neutrinos provide a representation of the momenta of the elementary particles represented by wormhole contact structures. Does this mimicry generalize to a full duality so that all quantum numbers and even microscopic dynamics of defined by generalized Feynman diagrams (Euclidian space-time regions) would be represented by right-handed neutrinos and MEs? Could a generalization of $\mathcal{N} = 4$ SYM with non-trivial gauge group with proper choices of the ground states helicities allow to represent the entire microscopic dynamics?

Irrespective of the answer to this question one can compare the TGD based view about supersymmetric dynamics with what I have understood about $\mathcal{N} = 4$ SYM.

(a) In the scattering of MEs induced by the dynamics of Kähler action the right-handed neutrinos play a passive role. Modified Dirac equation forces them to adopt the same direction of four-momentum as the MEs so that the scattering reduces to the geometric scattering for MEs as one indeed expects on basic of quantum classical correspondence. In ν_R sector the basic scattering vertex involves four MEs and could be a re-sharing of the right-handed neutrino content of the incoming two MEs between outgoing two MEs respecting fermion number conservation. Therefore $\mathcal{N} = 4$ SYM with fermion number conservation would represent the scattering of MEs at quantum level.

(b) $\mathcal{N} = 4$ SUSY would suggest that also in the degenerate case one obtains the full scattering amplitude as a sum of permutations of external particles followed by projections to the directions of light-like momenta and that BCFW bridge represents the analog of fundamental braiding operation. The decoration of permutations means that each external line is effectively doubled. Could the scattering of MEs can be interpreted in terms of these decorated permutations? Could the doubling of permutations by decoration relate to the occurrence of pairs of MEs?

   One can also revert these questions. Could one construct massive states in $\mathcal{N} = 4$ SYM using pairs of momenta associated with particle with integer label k and its decorated copy with label $k + n$? Massive external particles obtained in this manner as bound states of massless ones could solve the IR divergence problem of $\mathcal{N} = 4$ SYM.

(c) The description of amplitudes in terms of leading singularities means picking up of the singular contribution by putting the fermionic propagators on mass shell. In the recent case it would give the inverse of massless Dirac propagator acting on the spinor at the end of the internal line annihilating it if it is a solution of Dirac equation.

   The only way out is a kind of cohomology theory in which solutions of Dirac equation represent exact forms. Dirac operator defines the exterior derivative d and virtual lines correspond to non-physical helicities with $d\Psi \neq 0$. Virtual fermions would be on mass-shell fermions with non-physical polarization satisfying $d^2\Psi = 0$. External particles would be those with physical polarization satisfying $d\Psi = 0$, and one can say that the Feynman diagrams containing physical helicities split into products of Feynman diagrams containing only non-physical helicities in internal lines.

(d) The fermionic states at wormhole contacts should define the ground states of SUSY representation with helicity +1/2 and -1/2 rather than spin 1 or -1 as in standard realization of $\mathcal{N} = 4$ SYM used in the article. This would modify the theory but the twistorial and Grassmannian description would remain more or less as such since it depends on light-likeneness and momentum conservation only.

4. 3-vertices for sparticles are replaced with 4-vertices for MEs

In $\mathcal{N} = 4$ SYM the basic vertex is on mass-shell 3-vertex which requires that for real light-like momenta all 3 states are parallel. One must allow complex momenta in order to satisfy energy conservation and light-likeness conditions. This is strange from the point of view of physics although number theoretically oriented person might argue that the extensions of rationals involving also imaginary unit are rather natural.

The complex momenta can be expressed in terms of two light-like momenta in 3-vertex with one real momentum. For instance, the three light-like momenta can be taken to be p, k, and $p - ka$ with $k = ap_R$. Here p (incoming momentum) and p_R are real light-like momenta satisfying $p \cdot p_R = 0$ but with opposite sign of energy, and a is complex number. What is remarkable that also the negative sign of energy is necessary also now.

Should one allow complex light-like momenta in TGD framework? One can imagine two options.

(a) Option I: no complex momenta. In zero energy ontology the situation is different due to the presence of a pair of MEs meaning replaced of 3-vertices with 4-vertices or 6-vertices, the allowance of negative energies in internal lines, and the fact that scattering is of sparticles is induced by that of MEs. In the simplest vertex a massive external particle with non-parallel MEs carrying non-parallel light-like momenta can decay to a pair of MEs with light-like momenta. This can be interpreted as 4-ME-vertex rather than 3-vertex (say) BFF so that complex momenta are not needed. For an incoming boson identified as wormhole contact the vertex can be seen as BFF vertex.

    To obtain space-like momentum exchanges one must allow negative sign of energy and one has strong conditions coming from momentum conservation and light-likeness which allow non-trivial solutions (real momenta in the vertex are not parallel) since basically the vertices are 4-vertices. This reduces dramatically the number of graphs. Note that one can also consider vertices in which three pairs of MEs join along their ends so that 6 MEs (analog of 3-boson vertex) would be involved.

(b) Option II: complex momenta are allowed. Proceeding just formally, the $\sqrt{g_4}$ factor in Kähler action density is imaginary in Minkowskian and real in Euclidian regions. It is now clear that the formal approach is correct: Euclidian regions give rise to Kähler function and Minkowskian regions to the analog of Morse function. TGD as almost topological QFT inspires the conjecture about the reduction of Kähler action to boundary terms proportional to Chern-Simons term. This is guaranteed if the condition $j^\mu_K A_\mu = 0$ holds true: for the known extremals this is the case since Kähler current j_K is light-like or vanishing for them. This would seem that Minkowskian and Euclidian regions provide dual descriptions of physics. If so, it would not be surprising if the real and complex parts of the four-momentum were parallel and in constant proportion to each other.

    This argument suggests that also the conserved quantities implied by the Noether theorem have the same structure so that charges would receive an imaginary contribution from Minkowskian regions and a real contribution from Euclidian regions (or vice versa). Four-momentum would be complex number of form $P = P_M + iP_E$. Generalized light-likeness condition would give $P_M^2 = P_E^2$ and $P_M \cdot P_E = 0$. Complexified momentum would have 6 free components. A stronger condition would be $P_M^2 = 0 = P_E^2$ so that one would have two light-like momenta "orthogonal" to each other. For both relative signs energy P_M and P_E would be actually parallel: parameterization would be in terms of light-like momentum and scaling factor. This would suggest that complex momenta do not bring in anything new and Option II reduces effectively to Option I. If one wants a complete analogy with the usual twistor approach then $P_M^2 = P_E^2 \neq 0$ must be allowed.

5. Is SUSY breaking possible or needed?

It is difficult to imagine the breaking of the proposed kind of SUSY in TGD framework, and the first guess is that all these 4 super-partners of particle have identical masses. p-Adic thermodynamics does not distinguish between these states and the only possibility is that the p-adic primes differ for the spartners. But is the breaking of SUSY really necessary? Can one really distinguish between the 8 different states of a given elementary particle using the recent day experimental methods?

(a) In electroweak and color interactions the spartners behave in an identical manner classically. The coupling of right-handed neutrinos to space-time geometry however forces the right-handed neutrinos to adopt the same direction of four-momentum as MEs has. Could some gravitational effect allow to distinguish between spartners? This would be trivially the case if the p-adic mass scales of spartners would be different. Why this should be the case remains however an open question.

(b) In the case of unbroken SUSY only spin distinguishes between spartners. Spin determines statistics and the first naive guess would be that bosonic spartners obey totally different atomic physics allowing condensation of selectrons to the ground state. Very probably this is not true: the right-handed neutrinos are de-localized to 4-D MEs and other fermions correspond to wormhole contact structures and 2-D string world sheets.

The coupling of the spin to the space-time geometry seems to provide the only possible manner to distinguish between spartners. Could one imagine a gravimagnetic effect with energy splitting proportional to the product of gravimagnetic moment and external gravimagnetic field B? If gravimagnetic moment is proportional to spin projection in the direction of B, a non-trivial effect would be possible. Needless to say this kind of effect is extremely small so that the unbroken SUSY might remain undetected.

(c) If the spin of sparticle be seen in the classical angular momentum of ME as quantum classical correspondence would suggest then the value of the angular momentum might allow to distinguish between spartners. Also now the effect is extremely small.

6. What can one say about scattering amplitudes?

One expect that scattering amplitudes factorize with the only correlation between right-handed neutrino scattering and ordinary particle scattering coming from the condition that the four-momentum of the right-handed neutrino is parallel to that of massless extremal of more general preferred extremal having interpretation as a geometric counterpart of radiation quantum. This momentum is in turn equal to the massless four-momentum associated with the space-time sheet in question such that the sum of classical four-momenta associated with the space-time sheets equals to that for all wormhole throats involved. The right-handed neutrino amplitude itself would be simply constant. This certainly satisfies the SUSY constraint and it is actually difficult to find other candidates for the amplitude. The dynamics of right-handed neutrinos would be therefore that of spectator following the leader.

Right-handed neutrino as inert neutrino?

12.6.7 Right-handed neutrino as inert neutrino?

There is a very interesting posting by Jester in Resonaances with title How many neutrinos in the sky? [C1]. Jester tells about the recent 9 years WMAP data [C7] and compares it with earlier 7 years data. In the earlier data the effective number of neutrino types was $N_{eff} = 4.34 \pm 0.87$ and in the recent data it is $N_{eff} = 3.26 \pm 0.35$. WMAP alone would give $N_{eff} = 3.89 \pm 0.67$ also in the recent data but also other data are used to pose constraints on N_{eff}.

To be precise, N_{eff} could include instead of fourth neutrino species also some other weakly interacting particle. The only criterion for contributing to N_{eff} is that the particle is in thermal equilibrium with other massless particles and thus contributes to the density of matter considerably during the radiation dominated epoch.

Jester also refers to the constraints on N_{eff} from nucleosynthesis, which show that $N_{eff} \sim 4$ us slightly favored although the entire range $[3, 5]$ is consistent with data.

It seems that the effective number of neutrinos could be 4 instead of 3 although latest WMAP data combined with some other measurements favor 3. Later a corrected version of the eprint appeared [C7] telling that the original estimate of N_{eff} contained a mistake and the correct estimate is $N_{eff} = 3.84 \pm 0.40$.

An interesting question is what $N_{eff} = 4$ could mean in TGD framework?

(a) One poses to the modes of the modified Dirac equation the following condition: electric charge is conserved in the sense that the time evolution by modified Dirac equation does not mix a mode with a well-defined em charge with those with different em charge. The implication is that all modes except pure right handed neutrino are restricted at string world sheets. The first guess is that string world sheets are minimal surfaces of space-time surface (rather than those of imbedding space). One can also consider minimal surfaces of imbedding space but with effective metric defined by the anti-commutators of the modified gamma matrices. This would give a direct physical meaning for this somewhat mysterious effective metric.

For the neutrino modes localized at string world sheets mixing of left and right handed modes takes place and they become massive. If only 3 lowest genera for partonic 2-surfaces are light, one has 3 neutrinos of this kind. The same applies to all other fermion species. The argument for why this could be the case relies on simple observation [K13]: the genera g=0,1,2 have the property that they allow for all values of conformal moduli Z_2 as a conformal symmetry (hyper-ellipticity). For $g > 2$ this is not the case. The guess is that this additional conformal symmetry is the reason for lightness of the three lowest genera.

(b) Only purely right-handed neutrino is completely de-localized in 4-volume so that one cannot assign to it genus of the partonic 2-surfaces as a topological quantum number and it effectively gives rise to a fourth neutrino very much analogous to what is called sterile neutrino. De-localized right-handed neutrinos couple only to gravitation and in case of massless extremals this forces them to have four-momentum parallel to that of ME: only massless modes are possible. Very probably this holds true for all preferred extremals to which one can assign massless longitudinal momentum direction which can vary with spatial position.

(c) The coupling of ν_R is to gravitation alone and all electroweak and color couplings are absent. According to standard wisdom de-localized right-handed neutrinos cannot be in thermal equilibrium with other particles. This according to standard wisdom. But what about TGD?

One should be very careful here: de-localized right-handed neutrinos is proposed to give rise to SUSY (not $\mathcal{N} = 1$ requiring Majorana fermions) and their dynamics is that of passive spectator who follows the leader. The simplest guess is that the dynamics of right handed neutrinos at the level of amplitudes is completely trivial and thus trivially supersymmetric. There are however correlations between four-momenta.

   i. The four-momentum of ν_R is parallel to the light-like momentum direction assignable to the massless extremal (or more general preferred extremal). This direct coupling to the geometry is a special feature of the modified Dirac operator and thus of sub-manifold gravity.

   ii. On the other hand, the sum of massless four-momenta of two parallel pieces of preferred extremals is the - in general massive - four-momentum of the elementary particle defined by the wormhole contact structure connecting the space-time sheets (which are glued along their boundaries together since this is seems to be the only manner to get rid of boundary conditions requiring vacuum extremal property near the boundary). Could this direct coupling of the four-momentum direction of right-handed neutrino to geometry and four-momentum directions of other fermions be enough for the right handed neutrinos to be counted as a fourth neutrino species in thermal equilibrium? This might be the case!

One cannot of course exclude the coupling of 2-D neutrino at string world sheets to 4-D purely right handed neutrinos analogous to the coupling inducing a mixing of sterile neutrino with ordinary neutrinos. Also this could help to achieve the thermal equilibrium with 2-D neutrino species.

Experimental evidence for sterile neutrino?

Many physicists are somewhat disappointed to the results from LHC: the expected discovery of Higgs has been seen as the main achievement of LHC hitherto. Much more was expected.

To my opinion there is no reason for disappointment. The exclusion of the standard SUSY at expected energy scale is very far reaching negative result. Also the fact that Higgs mass is too small to be stable without fine tuning is of great theoretical importance. The negative results concerning heavy dark matter candidates are precious guidelines for theoreticians. The non-QCD like behavior in heavy ion collisions and proton-ion collisions is bypassed my mentioning something about AdS/CFT correspondence and non-perturbative QCD effects. I tend to see these effects as direct evidence for M_{89} hadron physics [K40].

In any case, something interesting has emerged quite recently. Resonaances tells that the recent analysis [C6] of X-ray spectrum of galactic clusters claims the presence of monochromatic 3.5 keV photon line. The proposed interpretation is as a decay product of sterile 7 keV neutrino transforming first to a left-handed neutrino and then decaying to photon and neutrino via a loop involving W boson and electron. This is of course only one of the many interpretations. Even the existence of line is highly questionable.

One of the poorly understood aspects of TGD is right-handed neutrino, which is obviously the TGD counterpart of the inert neutrino.

(a) The old idea is that covariantly constant right handed neutrino could generate $\mathcal{N}=2$ super-symmetry in TGD Universe. In fact, all modes of induced spinor field would generate superconformal symmetries but electroweak interactions would break these symmetries for the modes carrying non-vanishing electroweak quantum numbers: they vanish for ν_R. This picture is now well-established at the level of WCW geometry [K98]: super-conformal generators are labelled angular momentum and color representations plus two conformal weights: the conformal weight assignable to the light-like radial coordinate of light-cone boundary and the conformal weight assignable to string coordinate. It seems that these conformal weights are independent. The third integer labelling the states would label genuinely Yangian generators: it would tell the poly-locality of the generator with locus defined by partonic 2-surface: generators acting on single partonic 2-surface, 2 partonic 2-surfaces, ...

(b) It would seem that even the SUSY generated by ν_R must be badly broken unless one is able to invent dramatically different interpretation of SUSY. The scale of SUSY breaking and thus the value of the mass of right-handed neutrino remains open also in TGD. In lack of better one could of course argue that the mass scale must be CP_2 mass scale because right-handed neutrino mixes considerably with the left-handed neutrino (and thus becomes massive) only in this scale. But why this argument does not apply also to left handed neutrino which must also mix with the right-handed one!

(c) One can of course criticize the proposed notion of SUSY: wonder whether fermion + extremely weakly interacting ν_R at same wormhole throat (or interior of 3-surface) can behave as single coherent entity as far spin is considered [K85]?

(d) The condition that the modes of induced spinor field have a well-defined electromagnetic charge eigenvalue [K87] requires that they are localized at 2-D string world sheets or partonic 2-surfaces: without this condition classical W boson fields would mix the em charged and neutral modes with each other. Right-handed neutrino is an exception since it has no electroweak couplings. Unless right-handed neutrino is covariantly constant, the modified gamma matrices can however mix the right-handed neutrino with the left handed one and this can induce transformation to charged mode. This does not happen if each modified gamma matrix can be written as a linear combination of either M^4 or CP_2 gamma matrices and modified Dirac equation is satisfied separately by M^4 and CP_2 parts of the modified Dirac equation.

(e) Is the localization of the modes other than covariantly constant neutrino to string world sheets a consequence of dynamics or should one assume this as a separate condition? If one wants similar localization in space-time regions of Euclidian signature - for which CP_2 type vacuum extremal is a good representative - one must assume it as a separate condition. In number theoretic formulation string world sheets/partonic 2-surfaces would be commutative/co-commutative sub-manifolds of space-time surfaces which in turn would be associative or co-associative sub-manifolds of imbedding space possessing (hyper-)octonionic tangent space structure. For this option also right-handed neutrino would be localized to string world sheets. Right-handed neutrino would be covariantly constant only in 2-D sense.

One can consider the possibility that ν_R is de-localized to the entire 4-D space-time sheet. This would certainly modify the interpretation of SUSY since the number of degrees of freedom would be reduced for ν_R.

(f) Non-covariantly constant right-handed neutrinos could mix with left-handed neutrinos but not with charged leptons if the localization to string world sheets is assumed for modes carrying non-vanishing electroweak quantum numbers. This would make possible the decay of right-handed to neutrino plus photon, and one cannot exclude the possibility that ν_R has mass 7 keV.

Could this imply that particles and their spartners differ by this mass only? Could it be possible that practically unbroken SUSY could be there and we would not have observed it? Could one imagine that sfermions have annihilated leaving only states consisting of fundamental fermions? But shouldn't the total rate for the annihilation of photons to hadrons be two times the observed one? This option does not sound plausible.

What if one assumes that given sparticle is charactrized by the same p-adic prime as corresponding particle but is dark in the sense that it corresponds to non-standard value of Planck constant. In this case sfermions would not appear in the same vertex with fermions and one could escape the most obvious contradictions with experimental facts. This leads to the notion of shadron: shadrons would be [K85] obtained by replacing quarks with dark squarks with nearly identical masses. I have asked whether so called X and Y bosons having no natural place in standard model of hadron could be this kind of creatures.

The interpretation of 3.5 keV photons as decay products of right-handed neutrinos is of course totally ad hoc. Another TGD inspired interpretation would be as photons resulting from the decays of excited nuclei to their ground state.

(a) Nuclear string model [L2] predicts that nuclei are string like objects formed from nucleons connected by color magnetic flux tubes having quark and antiquark at their ends. These flux tubes are long and define the "magnetic body" of nucleus. Quark and antiquark have opposite em charges for ordinary nuclei. When they have different charges one obtains exotic state: this predicts entire spectrum of exotic nuclei for which statistic is different from what proton and neutron numbers deduced from em charge and atomic weight would suggest. Exotic nuclei and large values of Planck constant could make also possible cold fusion [K19].

(b) What the mass difference between these states is, is not of course obvious. There is however an experimental finding [C8] (see *Analysis of Gamma Radiation from a Radon Source: Indications of a Solar Influence*) that nuclear decay rates oscillate with a period of year and the rates correlate with the distance from Sun. A possible explanation is that the gamma rays from Sun in few keV range excite the exotic nuclear states with different decay rate so that the average decay rate oscillates [L2]. Note that nuclear excitation energies in keV range would also make possible interaction of nuclei with atoms and molecules.

(c) This allows to consider the possibility that the decays of exotic nuclei in galactic clusters generates 3.5 keV photons. The obvious question is why the spectrum would be concentrated at 3.5 keV in this case (second question is whether the energy is really concentrated at 3.5 keV: a lot of theory is involved with the analysis of the experiments). Do the energies of excited states depend on the color bond only so that they would be essentially same for all nuclei? Or does single excitation dominate in the spectrum? Or is this due to the fact that the thermal radiation leaking from the core of stars excites predominantly single state? Could $E = 3.5$ keV correspond to the maximum intensity for thermal radiation in stellar core? If so, the temperature of the exciting radiation would be about $T \simeq E/3 \simeq 1.2 \times 10^7$ K. This in the temperature around which formation of Helium by nuclear fusion has begun: the temperature at solar core is around 1.57×10^7 K.

12.6.8 Still one attempt understand generalized Feynman diagrams

The only manner to develop the understanding about generalized Feynman diagrams is to articulate the basic questions again and again in the hope that something new might emerge. There are many questions to be answered.

What Grasmannian twistorialization means when imbedding space spinor fields are the fundamental objects. How does ZEO make twistorialization possible? How twistorialization emerges from the functional integral in WCW from the proposed stringy construction of spinor modes.

One must also understand in detail the realization of super-conformal symmetries and how n-point functions of conformal field theory are associated with scattering amplitudes, and how cm degrees of freedom described using imbedding space spinor harmonics are treated in the scattering amplitudes. Also the braiding and knotting should be understood. The challenge is to find a universal form for the vertices and to identify the propagators. Also the modular degrees of freedom of partonic 2-surfaces explaining family replication phenomenon should be taken into account.

Zero energy ontology, twistors, and Grassmannian description?

In ZEO also virtual wormhole throats are massless particles and four-momentum conservation at vertices identifiable as partonic 2-surfaces at which wormhole throats meet expressed in terms of twistors leads to Grassmannian formulation automatically. This feature is thus not specific to $\mathcal{N}=4$ SYM.

Momentum conservation and massless on mass-shell conditions at vertices defined as partonic 2-surfaces at which the orbits of wormhole contacts meet, are extremely restrictive, and one has good hopes that huge reduction in the number of twistorial diagrams takes place and could even lead to finite number of diagrams (number theoretic arguments favor this).

Realization of super-conformal algebra

Thanks to the advances in the construction of preferred extremals and solutions of the modified Dirac equation there has been considerable progress in the understanding of super-conformal invariance and its 4-D generalization [K87].

(a) In ordinary SYM ground states correspond to both maximal helicites or only second maximal helicity of super multiplet ($\mathcal{N}=4$ case). Now these ground states are replaced by the modes of imbedding space spinor fields assignable to center of mass degrees of freedom for partonic 2-surfaces. The light-like four-momenta of these modes can be expressed in terms of twistor variables. Spin-statistics connection seems to require that the total number of fermions and anti-fermions associated with given wormhole throat is always odd.

(b) Super-algebra consists of oscillator operators with non-vanishing quark or lepton number. By conformal invariance fermionic oscillator operators obey 1-D anti-commutation relations. The integral over CD boundary defines a bi-linear form analogous to inner product. If a reduction to single particle level takes place, the vertex is expressible as a matrix element between two fermion-anti-fermion states: the first one assignable to the incoming and outgoing wormhole throats one and second to the virtual boson identified as wormhole contact on one hand. The exchange boson entangled fermion-anti-fermion state represented by a bi-local generalization of the gauge current. This picture applying to gauge boson exchanges generalizes in rather obvious manner.

(c) Unitary demands correlation between fermionic oscillator operators and spinor harmonics of imbedding space as following argument suggets. The bilocal generalization of gauge current defines a "norm" for spinor modes as generalization of what in QFT regarded as charge. On basis of experience with Dirac spinors one expect that this norm is not positive definite. This "norm" must be consistent with the unitarity of the scattering amplitude and the experience with QFT suggests a correlation between creation/annihilation operator character of fermionic oscillator operators and the sign of the "norm" in imbedding space degrees of freedom.

(d) The modes with negative norm should correspond to negative energy fermions and annihilation operators and modes with positive norm to positive energy fermions and creation operators. Therefore the anti-commutators of fermionic oscillator operators must be linear in four-momentum or its longitudinal projection and thus proportional to $p^k\gamma_k$ or $p^k_L\gamma_k$.

On the other hand, the primary anti-commutators for the induced spinor fields are proportional to the modified gamma matrix in a direction normal to the 1-D quantization curve at the boundary of string world sheet or at the partonic 2-surface. These two anti-commutators should be consistent.

i. Does the functional integral somehow lead from the primordial anti-commutators to the anti-commutators involving longitudinal momentum and perhaps 1-D delta function in the intersection of M^2 with CD boundary (light-like line)?

ii. Or does the connection between the two quantizations emerge as boundary conditions stating that the normal component of modified gamma matrix at the boundary and along string world sheet equals to $p^k_L\gamma_k$? This would also realize quantum classical correspondence in the sense that the longitudinal momentum is reflected in the geometry of the space-time sheet. Quaternionic space-time surfaces indeed contain integrable distribution of $M^2(x) \subset M^4$ at their tangent spaces. The restriction to braid strands would mean that the condition indeed makes sense. Note that braid strands should correspond to same $M^2(x)$.

How conformal time evolution corresponds to physical time evolution?

The only internally consistent option is conformally invariant meaning that induced spinor fields anti-commute only along as set of 1-D curves belonging to partonic 2-surfaces. This means that one can speak about conformal time evolution at partonic 2-surface.

This suggests a huge simplification of the conformal dynamics.

(a) Conformal time evolution can be translated to time evolution along light-like orbit of wormhole throat by projecting the intersections of this surface with shifted light-cone boundary to the upper or lower light-like boundary of CD: whether it is upper or lower boundary of CD depends on the arrow of imbedding space time associated with the zero energy state. All partonic 2-surfaces would be mapped to same light-cone boundary. The orbits of braid strands at wormhole throat project to orbits at light-cone boundary in question and can be further projected to the sphere $r_M = constant$ at light-boundary. 3-D dynamics would project to simplest possible stringy 2-D dynamics (spherical string orbit) and dictated by conformal invariance.

(b) The conformal field theory in question is for conformal fermionic fields realized in terms of fermionic oscillator operators and n-point functions correspond to fermionic n-point functions. The non-triviality of dynamics in these degrees of freedom follows from the non-triviality of the conformal field theory. The entire collection of partonic 2-surfaces at the ends of CD would reduce to its projection to S^2.

(c) One can try to build a geometric view about the situation using as a guideline conformal Hamiltonian quantum evolution. Time=constant slices would correspond to 1-D curve or collection of them. At these slices fermionic oscillator operators would satisfy the conformal anti-commutation relations. This kind of slice would be associated with both ends of CD. Braid strands would connect these 1-D slices as kind of hairs. One can however ask whether there is any need to restrict the end points of braid strands to line on a curve at which fermionic oscillator operators satisfy stringy anti-commutation relations.

What happens in 3-vertices?

The vision is that only 3-vertices are needed. Idealize particles as wormhole contacts (in reality pair of wormhole contacts connected by a flux tube would describe elementary particles). A very convenient visualization of wormhole contact is as a very short string like object with throats at its ends so that stringy diagrammatics allows to identify the vertices as the analogs of open string vertices. One can even consider the possibility that string theory amplitudes define the vertices. This would conform with the p-adic mass calculations applying conformal invariance in CP_2 scale. Note also that partonic 2-surfaces are effectively replaced by braids so that very stringy picture results.

(a) Consider a three vertex representing the emission of boson by incoming fermion (FFB) or by incoming boson (BBB) described as wormhole contact such that throats carry fermion and anti-fermion number in the bosonic case. In the fermionic the first throat carries fermion and second one represents vacuum state. The exchanged boson can be regarded as fermion anti-fermion pair such that second fermion travels to future and second one to the past in the vertex. 3-vertex would reduce to two 2-vertices representing the transformation of fermion line from incoming line to exchanged line or from latter to outgoing line.

(b) The minimal option is that the same vertex describes the situation if both cases. Essentially a combination of incoming free fermions to boson like state is in question and corresponds in string picture an exchange of open string between open strings. If so, second wormhole throat is passive and suffers forward scattering in the vertex. The fermion and anti-fermion of the exchanged virtual boson (the light-like momenta of wormhole throats need not be collinear for virtual bosons and also the sign of energy can be different form them) would suffer scattering before the transformation to fermions belonging to incoming and outgoing wormhole contact.

One expects the vertices to factorize into products of two kinds of factors: the inner products of fermionic Fock states defined by conformal n-point functions at sphere of light-cone boundary, and the bi-linear forms for the modes of imbedding space spinor fields involving integral over cm degrees of freedom and allowing twistorialization by previous arguments. Let us continue with the simple example in which wormhole throats carry fermion number 0 or 1.

(a) If second wormhole throat is passive, it is enough to construct only FFB vertex, with B identified as a wormhole contact carrying fermion and anti-fermion. One has 4 fermions altogether, and one expects that in cm degrees of freedom incoming and outgoing fermion are un-correlated whereas the fermions of the boson exchange are correlated and the correlation is expressible as the analog of gauge current.

(b) This suggests a sum over bi-local counterparts of electro-weak and color gauge currents at opposite ends of the exchanged line. Bi-local gauge currents would contain a spinor mode from both wormhole throats, and the strict locality of M^4 gauge currents would be replaced with a bi-locality in CP_2 scale.

(c) The current assignable to a particular boson exchange must involve the matrix element of corresponding charge matrix between spinor modes besides the quantity. Is it possible to find a general expression for the sum over current - current interaction terms? If this is the case, there would be no need to perform the summation over bosonic exchanges explicitly. One would have the analog for the $\sum_n |n\rangle\langle n|$ in propagator line but summation allowing the momenta of fermion and anti-fermion to be arbitrary massless momenta rather than summing up to the on mass shell momentum of boson. The counterpart of gauge coupling should be universal and naturally given by Kähler coupling.

(d) The TGD counterparts of scalar and pseudo-scalar bosons would be vector bosons with polarization in CP_2 direction and they could be also seen both as Higgs like states and Euclidian pions assignable to wormhole contacts. Genuine H-scalars are excluded implied by 8-D chiral symmetry implying also separate conservation of B and L.

In the general case the wormhole throats carry arbitrary odd fermion number but for fermion numbers $n > 1$ at any wormhole throat exotic super-partner with propagator decaying faster than $1/p^2$ is in question. Furthermore, wormhole contact is accompanied by second wormhole contact since the flux lines of monopole flux must closed. Therefore one has a pair of "long" string like flux tubes connected by short flux tubes at their ends. Its length is given by weak length scale quite generally or possibly by Compton length. The other end of the long flux tube can also contain fermions at both flux tubes.

The identification of propagators

A natural guess is that the propagator for single fermion state is just the longitudinal Dirac propagator D_{p_L} for a massless fermion in $M^4 \supset M^2$. For states, which by statistics constraint always contain an odd number $M = 2N + 1$ of fermions and anti-fermions, the propagator would be M:th power of fermionic longitudinal propagator so that it would reduce to $p_L^{-2N} D_{p_L}$ meaning that only the single fermion states would be behave like ordinary elementary particles. States with higher fermion number would represent radiative corrections reflecting the non-point-like nature of partons. Longitudinal mass squared would be equal to the sum of the contribution from CP_2 degrees of freedom and the integer valued conformal contribution from spinor harmonics. The M^4 momenta associated with wormhole throats would be light-like. In the prescription using fermionic longitudinal propagators assigned to the braid strands, braid strands are analogous to the edges of polygons appearing in twistor Grassmannian approach.

Some open questions

A long list of open questions remains without a final answer. Consider first twistor Grassmannian approach.

(a) Does this prescription follow from quantum criticality? Recall that quantum criticality formulated in terms of preferred extremals and modified Dirac equation leads to a stringy perturbation theory involving fermionic propagator defined by the modified Dirac operator and functional integral over WCW for the deformations of space-time surface preserving the preferred extremal property [K87]. This propagator could be called space-time propagator to distinguish it from the imbedding space propagator associated with the longitudinal momentum.

(b) One expects that one still has topological Feynman diagrammatic expansion (besides that defined by functional integral over small deformations of space-time surface with given topology) involving in principle an arbitrary number of vertices defined by the intermediate partonic 2-surfaces. Momentum conservation and massless on mass-shell conditions however pose powerful restrictions on the allowed diagrams, and one might hope that the simplicity of the outcome is comparable to Grassmannian twistor approach for $\mathcal{N} = 4$ SYM. One can even hope that the number of contributing diagrams is finite. The important point would be that Grassmannian diagrams would give the outcome of the functional integral over 3-surfaces. Twistorial Grassmann representation is the first guess hitherto for the explicit outcome of the functional integral over WCW.

(c) The lines of Feynman graph are replaced with braids. A new element is that braid strands are braided as curves inside light-like 3-surfaces defined by the orbit of the wormhole throat. Twistorial construction applies only to the planar amplitudes of $\mathcal{N} = 4$ SYM. Can one imagine TGD counterparts for non-planar amplitudes in TGD framework or does the stringy picture imply that they are completely absent?

   A possible answer to the question is based on the M^2 projection of the lines of braid strands (or on the projection to the 2-surface defined by an integrable distribution of tangent planes $M^2(x)$). For non-planar diagrams the projections intersect and the intersection cannot be eliminated by a small deformation. It does not make sense to say that line goes over or below the second line. One can speak only about crossings. In the theory or algebraic knots [A78] algebraic knots with crossings are possible [K32]. Could algebraic knot theory allow to reduce non-planar diagrams to sums of planar diagrams?

(d) Does one obtain Yangian symmetry using longitudinal propagators and by integrating over the moduli labeling among other things the choices of the preferred plane $M^2 \subset M^4$ or integrable distribution of preferred planes $M^2(x) \subset M^4$? The integral over the choices $M^2 \subset M^4$ gives formally a Lorentz invariant outcome. Does it also give rise to physically acceptable scattering amplitudes? Are the gauge conditions for the incoming gauge boson states formulated in terms of longitudinal momentum and thus allowing also the third polarization physical? Can one apply this gauge condition also to the virtual boson like exchanges?

(e) It is still somewhat unclear whether one should assume single global choice of M^2 or an integrable distribution of $M^2(x)$.

   i. The choice of $M^2(x)$ must be same for all braid strands of given partonic 2-surface and remain constant along braid strand and therefore be same also at second end of the strand. Otherwise the fermionic propagator would vary along braid strand. A possible additional condition on braids is that braid strands correspond to the same choice of $M^2(x)$. In quantum measurement theory this corresponds to the choice of same spin quantization axes for all fermions inside parton and is physically extremely natural condition. The implication is that one can indeed assign a fixed M^2 with CD and choice of braid strands via boundary conditions. The simplest boundary conditions would require $M^2(x)$ to be constant at light-like 3-surfaces and at the ends of space-time surface at boundaries of CD. This is in spirit with holography stating that quantum measurements can be carried out only at these 3-surfaces (or at least those at the ends of CD).

   ii. One cannot exclude the possibility that $M^2(x)$ does not depend on x for a particular space-time sheet and even entire CD although this looks rather strong a restriction. On the other hand, one can ask whether the preferred M^2 assigned with CD should be generalized to an integrable distribution $M^2(x)$ assigned with CD such that $M^2(x)$ is contained in the tangent space of preferred Minkowskian extremal.

   iii. Is the functional integral over integrable distributions $M^2(x)$ needed? It would be analogous to a functional integral over string world sheets. It is enough to integrate over Lorentz transforms of a given distribution $M^2(x)$ to achieve Lorentz invariance. This because the choice of the integrable distribution of $M^2(x)$ for space-time surface reduces effectively to the choice of M^2 for the disconnected pieces of generalized Feynman diagram. Physical intuition suggests that a particular choice of $M^2(x)$ corresponds to fixing of zero modes of WCW and is essentially fixing of classical variables needed to fix quantization axes. The fixing of value distributions of induced Kähler fields n 4-D sense at partonic 2-surfaces would be similar fixation of zero modes.

   iv. If only M^2 momentum makes it visible in anti-commutators, how the other components of four-momentum can make themselves visible in dynamics? This is possible via momentum conservation at vertices making possible twistor Grassmannian approach. The dynamics in transversal momenta would be dictated completely by the conservation laws.

There are also other challenges.

(a) Family replication phenomenon has TGD based explanation in terms of the conformal moduli of partonic 2-surfaces. How conformal moduli should be taken into account in the Feynman diagrammatics? Phenomena like topological mixing inducing in turn the mixing of partonic 2-topologies responsible for CKM mixing in TGD Universe should be understood in this description.

(b) Number theoretical universality requires that also the p-adic variants of the amplitudes should make sense. One could even require that the amplitudes decompose to products of parts belonging to different number fields [K83]. If one were able to formulate this vision precisely, it would provide powerful constraints on the amplitudes. For instance, a reduction of the amplitudes to a sum over finite number of generalized Feynman diagrams is plausible since this would guarantee that individual contributions which must give rise to algebraic numbers for algebraic 4-momenta, would sum up to an algebraic number.

Chapter 13

Some Fresh Ideas about Twistorialization of TGD

13.1 Introduction

I found from web a thesis by Tim Adamo titled "Twistor actions for gauge theory and gravity" [B18]. The work considers formulation of $N = 4$ SUSY gauge theory directly in twistor space instead of Minkowski space. The author is able to deduce MHV formalism, tree level amplitudes, and planar loop amplitudes from action in twistor space. Also local operators and null polygonal Wilson loops can be expressed twistorially. This approach is applied also to general relativity: one of the challenges is to deduce MHV amplitudes for Einstein gravity. The reading of the article inspired a fresh look on twistors and a possible answer to several questions (I have written two chapters about twistors and TGD [K78, K80] giving a view about development of ideas).

Both M^4 and CP_2 are highly unique in that they allow twistor structure and in TGD one can overcome the fundamental "googly" problem of the standard twistor program preventing twistorialization in general space-time metric by lifting twistorialization to the level of the imbedding space containg M^4 as a Cartesian factor. Also CP_2 allows twistor space identifiable as flag manifold $SU(3)/U(1) \times U(1)$ as the self-duality of Weyl tensor indeed suggests. This provides an additional "must" in favor of sub-manifold gravity in $M^4 \times CP_2$. Both octonionic interpretation of M^8 and triality possible in dimension 8 play a crucial role in the proposed twistorialization of $H = M^4 \times CP_2$. It also turns out that $M^4 \times CP_2$ allows a natural twistorialization respecting Cartesian product: this is far from obvious since it means that one considers space-like geodesics of H with light-like M^4 projection as basic objects. p-Adic mass calculations however require tachyonic ground states and in generalized Feynman diagrams fermions propagate as massless particles in M^4 sense. Furthermore, light-like H-geodesics lead to non-compact candidates for the twistor space of H. Hence the twistor space would be 12-dimensional manifold $CP_3 \times SU(3)/U(1) \times U(1)$.

The recent view about basic variational principles of TGD is precise enough to conclude what the generalized Feynman diagrammatics looks like. The basic outcome is that fundamental fermionic propagators are massless and associated with the boundaries of string world sheets at which spinor modes are in the generic situation localized. The fundamental interaction vertices correspond to wormhole contacts and the propagator mediating interaction between fermions is inverse of the scaling generator L_0 so that a combination of stringy and Feynman diagrammatics is in question. Physical particles are bound states of massless fundamental fermions assignable to the throats to the two wormhole contacts through which a closed flux tube carrying monopole Kähler magnetic flux traverses. This suggest strong strongly stringy variant of twistor Grassmannian approach and solution of IR difficulties in terms of new particle concept.

Generalisation of 2-D conformal invariance extending to infinite-D variant of Yangian symmetry; light-like 3-surfaces as basic objects of TGD Universe and as generalised light-like geodesics; light-likeness condition for momentum generalized to the infinite-dimensional context via super-conformal algebras. These are the facts inspiring the question whether also the "world of classical worlds" (WCW) could allow twistorialization. It turns out that center of mass degrees of freedom (imbedding space) allow natural twistorialization: twistor space for $M^4 \times CP_2$ serves as moduli space for choice of quantization axes in Super Virasoro conditions. Contrary to the original optimistic expectations it turns out that although the analog of incidence relations holds true for Kac-Moody algebra, twistorialization in vibrational degrees of freedom does not look like a good idea since incidence relations force an effective reduction of vibrational degrees of freedom to four.

The Grassmannian formalism for scattering amplitudes is expected to generalize for generalized Feynman diagrams: the basic modification is due to the possible presence of CP_2 twistorialization and the fact that 4-fermion vertex -rather than 3-boson vertex- and its super counterparts define now the fundamental vertices. Both QFT type BFCW and stringy BFCW can be considered.

(a) For QFT type BFCW BFF and BBB vertices would be an outcome of bosonic emergence (bosons idealized as wormhole contacts) and 4-fermion vertex is proportional to factor with dimensions of inverse mass squared and naturally identifiable as proportional to the factor $1/p^2$ assignable to each boson line. This predicts a correct form for the bosonic propagators for which mass squared is in general non-vanishing unlike for fermion lines. The usual BFCW construction would emerge naturally in this picture. There is however a problem: the emergent bosonic propagator diverges or vanishes depending on whether one assumes SUSY at the level of single wormhole throat or not. By the special properties of $\mathcal{N} = 4$ SUSY generated by right handed neutrino the SUSY cannot be applied to single wormhole throat but only to a pair of wormhole throats.

(b) This as also the fact that physical particles are necessarily pairs of wormhole contacts connected by fermionic strings forces stringy variant of BFCW avoiding the problems caused by non-planar diagrams. Now boson line BFCW cuts are replaced with stringy cuts and loops with stringy loops. By generalizing the earlier QFT twistor Grassmannian rules one ends up with their stringy variants in which super Virasoro generators $G, G^\dagger$ and L bringing in CP¡sub¿2¡/sub¿ scale appear in propagator lines: most importantly, the fact that G and $G^\dagger$ carry fermion number in TGD framework ceases to be a problem since a string world sheet carrying fermion number has $1/G$ and $1/G^\dagger$ at its ends. Twistorialization applies because all fermion lines are light-like.

(c) A more detailed analysis of the properties of right-handed neutrino demonstrates that modified gamma matrices in the modified Dirac action mix right and left handed neutrinos but that this happens markedly only in very short length scales comparable to CP_2 scale. This makes neutrino massive and also strongly suggests that SUSY generated by right-handed neutrino emerges as a symmetry at very short length scales so that spartners would be very massive and effectively absent at low energies. Accepting CP_2 scale as cutoff in order to avoid divergent gauge boson propagators QFT type BFCW makes sense. The outcome is consistent with conservative expectations about how QFT emerges from string model type description.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- The unique role of twistors in TGD [L45]
- Twistors and TGD [L47]

13.2 Basic results and problems of twistor approach

The author describes both the basic ideas and results of twistor approach as well as the problems.

13.2.1 Basic results

There are three deep results of twistor approach besides the impressive results which have emerged after the twistor resolution.

(a) Massless fields of arbitrary helicity in 4-D Minkowski space are in 1-1 correspondence with elements of Dolbeault cohomology in the twistor space CP_3. This was already the discovery of Penrose..The connection comes from Penrose transform. The light-like geodesics of M^4 correspond to points of 5-D sub-manifold of CP_3 analogous to light-cone boundary. The points of M^4 correspond to complex lines (Riemann spheres) of the twistor space CP_3: one can imagine that the point of M^4 corresponds to all light-like geodesics emanating from it and thus to a 2-D surface (sphere) of CP_3. Twistor transform represents the value of a massless field at point of M^4 as a weighted average of its values at sphere of CP_3. This correspondence is formulated between open sets of M^4 and of CP_3. This fits very nicely with the needs of TGD since causal diamonds which can be regarded as open sets of M^4 are the basic objects in zero energy ontology (ZEO).

(b) Self-dual instantons of non-Abelian gauge theories for SU(n) gauge group are in one-one correspondence with holomorphic rank-N vector bundles in twistor space satisfying some additional conditions. This generalizes the correspondence of Penrose to the non-Abelian case. Instantons are also usually formulated using classical field theory at four-sphere S^4 having Euclidian signature.

(c) Non-linear gravitons having self-dual geometry are in one-one correspondence with spaces obtained as complex deformations of twistor space satisfying certain additional conditions. This is a generalization of Penrose's discovery to the gravitational sector.

Complexification of M^4 emerges unavoidably in twistorial approach and Minkowski space identified as a particular real slice of complexified M^4 corresponds to the 5-D subspace of twistor space in which the quadratic form defined by the SU(2,2) invariant metric of the 8-dimensional space giving twistor space as its projectivization vanishes. The quadratic form has also positive and negative values with its sign defining a projective invariant, and this correspond to complex continuations of M^4 in which positive/negative energy parts of fields approach to zero for large values of imaginary part of M^4 time coordinate.

Interestgingly, this complexification of M^4 is also unavoidable in the number theoretic approach to TGD: what one must do is to replace 4-D Minkowski space with a 4-D slice of 8-D complexified quaternions. What is interesting is that real M^4 appears as a projective invariant consisting of light-like projective vectors of C^4 with metric signature (4,4). Equivalently, the points of M^4 represented as linear combinations of sigma matrices define hermitian matrices.

13.2.2 Basic problems of twistor approach

The best manner to learn something essential about a new idea is to learn about its problems. Difficulties are often put under the rug but the thesis is however an exception in this respect. It starts directly from the problems of twistor approach. There are two basic challenges.

(a) Twistor approach works as such only in the case of Minkowski space. The basic condition for its applicability is that the Weyl tensor is self-dual. For Minkowskian signature this leaves only Minkowski space under consideration. For Euclidian signature the conditions are not quite so restrictive. This looks a fatal restriction if one wants to generalize the result of Penrose to a general space-time geometry. This difficulty is known as "googly" problem.

According to the thesis MHV construction of tree amplitudes of $\mathcal{N} = 4$ SYM based on topological twistor strings in CP_3 meant a breakthrough and one can indeed understand also have analogs of non-self-dual amplitudes. The problem is however that the gravitational theory assignable to topological twistor strings is conformal gravity, which is generally regarded as non-physical. There have been several attempts to construct the on-shell scattering amplitudes of Einstein's gravity theory as subset of amplitudes of conformal gravity and also thesis considers this problem.

(b) The construction of quantum theory based on twistor approach represents second challenge. In this respect the development of twistor approach to $\mathcal{N} = 4$ SYM meant a revolution and one can indeed construct twistorial scattering amplitudes in M^4.

13.3 TGD inspired solution of the problems of the twistor approach

TGD suggests an alternative solution to the problems of twistor approach. Space-times are 4-D surfaces of $M^4 \times CP_2$ so that one obtains automatically twistor structure at the level of M^4 - that is imbedding space.

It seems natural to interpret twistor structure from the point of view of Zero Energy Ontology (ZEO). The two tips of CD are accompanied by light-cone boundaries and define a pair of 2-spheres in CP_3 since the light-like rays associated with the tips are mapped to points of twistor space. M^4 coordinates for the tips serve as moduli for the space of CDs and can be mapped to pairs of twistor spheres. The points of partonic 2-surfaces at the boundaries of CD reside at light-like geodesics and the conformal invariance with respect to radial coordinate emanating from the tip of CD suggests that the position at light-like geodesic does not matter. Therefore the points of partonic 2-surfaces can be mapped to union of spheres of twistor space.

13.3.1 Twistor structure for space-time surfaces?

Induction procedure is the core element of sub-manifold gravity. Could one induce the the twistor structure of M^4 to the space-time surface? Would it have any useful function? This idea does not look attractive.

(a) Twistor structure assigns to a given point of M^4 a sphere of CP_3 having interpretation as a sphere parametrizing the light-like geodesics emanating from the point. The X^4 counterpart of this assignment would be obtained simply by mapping the M^4 projection of space-time point to a sphere of twistor space in standard manner. This could make sense if the M^4 projection of space-time surface 4-dimensional but not necessary when the M^4 projection is lower-dimensional - say for cosmic strings.

(b) Twistor structure assigns to a light-like geodesic of M^4 a point of CP_3. Should one try to generalize this correspondence to the light-like geodesics of space-time surface? Light-like geodesic corresponds to its light-like tangent vectors at x whose direction as imbedding space vector depends now on the point x of the geodesic. The M^4 projection for the tangent vector of light-like geodesics of space-time surface in general time-like vector of M^4 so that one should map time-like M^4 ray to CP_3. Twistor spheres associated with the two points of this geodesic do not intersect so that one cannot define the image point in CP_3 as an intersection of twistor spheres. One could consider the lifts of the light-like geodesics of M^4 to X^4 and map their M^4 projections to the points of CP_3? This looks however somewhat trivial and physically uninteresting.

13.3.2 Could one assign twistor space to CP_2?

Can one assign a twistor space to CP_2? Could this property of CP_2 make it physically special? The necessary condition is satisfied: the Weyl tensor of CP_2 is self-dual.

CP_2 twistor space as flag manifold

CP_2 indeed allows a twistor structure as one learns from rather technical article about twistor structures (`http://www.ams.org/journals/tran/2004-356-03/S0002-9947-03-03157-X/S0002-9947-03-03157-X.pdf`). The twistor space associated with CP_2 is six-dimensional flag manifold (`http://en.wikipedia.org/wiki/Flag_manifold`) [A6] $F(1,2,3) = U(3)/\times U(1) \times U(1) \times U(1) = SU(3)/U(1) \times U(1)$ [A44] (`http://www.ams.org/journals/tran/2004-356-03/S0002-9947-03-03157-X/S0002-9947-03-03157-X.pdf`).

This flag manifold has interpretation as the space of all possible choices of quantization axes for color hyper charge and isospin. Note that the earlier proposal [K80] that the analog of twistor space for CP_2 is CP_3 is wrong.

The twistor space assignable to M^4 can be interpreted as a flag manifold consisting of 2-planes associated with 8-D complexified Minkowski space as is clear from interpretation as projection space CP_3. It might also have an interpretation as the space of the choices of quantization axes. For M^4 light-like vector defines a unique time-like 2-plane M^2 and the direction of the associated 3-vector defines quantization axes of spin whereas the sum of the light-like vector and its dual has only time component and defines preferred time coordinate and thus quantization axes for energy. In fact, the choice of $M^1 \subset M^2 \subset M^4$ defining flag is in crucial role in the number theoretic vision and also in the proposed construction of preferred extremals: the local choice of M^2 would define the plane of unphysical polarizations and as its orthogonal complement the plane of physical polarizations.

Amusingly, the flag manifold $SU(3)/U(1)\times U(1)$ associated with $SU(3)$ made its first appearance in TGD long time ago and in rather unexpected context. The mathematician Barbara Shipman discovered that the the dance of honeybees can be described in terms of this flag manifold [A88] and made the crazy proposal that quark level physics is somehow related to the honeybee dance. TGD indeed predicts scaled variants of also quarks and QCD like physics and in biology the presence of 4 Gaussian Mersenne primes in the length scale range 10 nm- 2.5 μm [K7] suggests that these QCDs might be realized in the new physics of living cell [K28].

In TGD inspired theory of consciousness the choice of quantization axis represents a higher level state function reduction and contributes to conscious experience - one can indeed speak about flag manifold qualia. It will be found that the choice of quantization axis is also unavoidable in the conditions stating the light-likeness of 3-surfaces and leading to a generalization of Super Virasoro algebra so that the twistor space of H emerges naturally from basic TGD.

What is the interpretation of the momentum like color quantum numbers?

There is a rather obvious objection against the notion of momentum like quantum numbers in CP_2 degrees of freedom. If the propagator is proportional to $1/(p^2 - Y^2 - I_3^2)$, where Y and I_3 are assigned to quark, a strong breaking of color symmetry results. The following argument demonstrates that this is not the case and also gives an interpretation for the notion of anomalous hyper-charge assignable to CP_2 spinors.

(a) Induced spinors do not form color triplets: this is the property of only physical states involving several wormhole throats and the action of super generators and spinor harmonics in cm mass degrees of freedom to which one can assign imbedding space spinor harmonics to be distinguished from second quantize induced spinors appearing in propagator lines. Color is analogous to rigid body angular momentum and one can speak of color partial waves. The total color quantum numbers are dictated by the cm color quantum numbers plus those associated with the Super Virasoro generators used to create the state [K37] and which also help to correct the wrong correlation between color and electroweak quantum numbers between spinor harmonics.

(b) Since CP_2 is projective space the standard complex coordinates are ratios of complex coordinates of C^3: $\{\xi^i = z^i/z_k \ , \ i \neq k\}$, where k corresponds to one of the complex coordinates z^i for given coordinate patch (there are three coordinate patches). For instance, for $k=3$ the coordinates are $(\xi^1,\xi^2)z^1/z^3,z^2/z^3)$. The coordinates z^i triplet representation of $SU(3)$ so that $\{\xi^i, i\neq k\}$ carries anomalous color quantum numbers given by the negatives of the z^k.

(c) Also the spinors carry anomalous Y and I_3, which are negative to anomalous color quantum numbers of CP_2 coordinates from the fact that spinors and z^i/z_k form color triplet. These quantum numbers are same for all spinor components inside given CP_2 coordinate patch so that no breaking of color symmetry results in a given patch. The color momentum would appear in the Dirac operator assignable to super Virasoro generators and define most naturally the contribution to region momentum. The "8-momenta" of external lines would be differences of region momenta and their color part would vanish for single fermion states associated with wormhole throat orbits.

13.3.3 Could one assign twistor space to $M^4 \times CP_2$?

The twistorialization of TGD could be carried by identifying the twistor counterpart of the imbedding space $H = M^4 \times CP_2$. The first guess that comes in mind is that the twistor space is just the product of twistor spaces for M^4 and CP_2. The next thought is that one could identify the counterpart of twistor space in 8-D context as the space of light-like geodesics of H. Since light-like geodesics in CP_2 couple M^4 and CP_2 degrees of freedom and since the M^4 projection of the light-like geodesic is in general time-like, this would allow the treatment of also massive states if the 8-D mass defined as eigenvalue of d'Alembertian vanishes. It however turns that the first thought is consistent with the general TGD based view and that second option yields twistor spaces which are non-compact.

In the following two attempts to identify the twistor space as light-like geodesics is made. I apologize my rudimentary knowledge about the matters involved.

(a) If the dimension of the twistor space is same as that for the projective complexifications of M^8 one would dhave $D=14$. This is also the dimension of projective complexification of octonsions whose importance is suggested by number theoretical considerations. If the twistorialization respects cartesian products then the dimension would be $D=12$.

(b) For M^8 at least the twistor space should have local structure given by $X^8 \times S^6$, where S^6 parametrizes direction vectors in 8-D light-cone. The conformal boundary of the space of light-like geodesics correspond to light-like geodesics of M^4 and this suggests that the conformal boundary of twistor space is $CP_3 \times CP_2$ with dimension $D=10$.

One can consider several approaches to the identification of the twistor space. One could start from the condition that twistor space describes projective complexification of $M^4 \times CP_2$, from the direct study of light-like geodesics in H, from the definition as flag manifold characterizing the choices of quantization axes for the isometry group of H.

(a) The first guess of a category theorist would be that twistorialization commutes with Cartesian products if isometry group decomposes into factors leaving the factors invariant. The naive identification would be as the twelve-dimensional space $CP_3 \times F(1,2,3)$, $F(1,2,3) = SU(3)/U(1)\times U(1)$. The points of H would in turn be mapped to products $S^2 \times S^3 \subset CP_3 \times SU(3)/U(1) \times U(1)$, which are 5-dimensional objects.

One can criticize this proposal. The points of this space could be interpreted as 2-dimensional objects defined as products of light-like geodesics and geodesic circles of CP_2. They could be also interpreted as space-like geodesics with light-like M^4 projection. Why should space-like geodesics replace light-like geodesics of H with light-like projection?

The experience with TGD however suggests that this could be the physical option. p-Adic mass calculations require tachyonic ground states and the action of conformal algebras gives vanishing conformal weight for the physical states. Also massless extremals are characterized by longitudinal space M^2 in which momentum projection is light-like whereas the entire momentum for Fourier components in the expansion of imbedding space coordinates are space-like. This has led to the proposal that it is light-like M^2 projection of momentum that matters. Also the recent vision about generalized Feynman diagrams is that fermions propagate as massless particles in M^4 sense and that massive particles are bound states of massless particles: many-sheeted space-time makes possible to realize this picture. Also the construction of the analog of Super Virasoro algebra for light-like 3-surface leads naturally to the product of twistor spaces as moduli space.

(b) The second approach is purely group theoretical and would identify twistor space as the space for the choices of quantization axes for the isometries which form now a product of Poincare group and color group. In the case of Poincare group energy and spin are the observabels and in the case of color group one has isospin and color hypercharge. The twistor space in the case of time-like M^4 projections of 8-momentum is obtained as coset space $P/SO(2) \times SU(3)/U(1) \times U(1) = M^4 \times SO(3,1)/M^1 \times SO(2) \times SU(3)/U(1) \times U(1) = E^3 \times SO(3,1)/SO(2) \times SU(3)/U(1) \times U(1)$. The dimension is the expected $D = 14$. In Euclidian sector one would have $E^4 \times SO(4)/SO(2) \times SO(2) \times SU(3)/U(1) \times U(1)$ having also dimension $D = 14$. The twistor space would not be compact and this is very undesired feature.

Ordinary twistors define flag manifold for projectively complexified M^4. If this is the case also now one obtains just the naively expected 12-dimensional $CP_3 \times SU(3)/U(1) \times U(1)$ with two spheres replaced with $S^2 \times S^3$. This option corresponds to the "tachyonic" identification of geodesics of H defining the twistor space as geodesics having light-like M^4 projection and space-like CP_2 projection.

(c) One can consider also the space of light-like H-geodesics. Locally the light-like geodesics for which M^4 projection is not space like geodesic can be parametrized by their position defined as intersection with arbitrary time-like hyper-plane $E^3 \subset M^4$. Tangent vector characterizes the geodesic completely since CP_2 geodesics can be characterized by their tangent vector. Hence the situation reduces locally to that in M^8 and light-likeness and projective invariance mean that the sphere S^6 parametrizes the moduli for light-like geodesics at given point of E^3. Hence the parameter space would be at least locally $E^3 \times S^6$. S^6 would be the counterpart of S^2 for ordinary twistors. An important special case are light-like geodesics reducing to light-like geodesics of M^4. These are parameterized by $X^5 \times CP_2$, where X^5 is the space of light-like geodesics in M^4 and defines the analog of light-cone in twistor space CP_3. Therefore the dimension of twistor space must be higher than 10. For M^4 the twistor space has same dimension as projective complexification of M^4.

One can study the light-like geodesics of H directly. The equation of light-like geodesic of H in terms of curve parameter s can be written as $m^k = v^k s, \phi = \omega s$, $v_k v^k = 1$ for time-like M^4 projection and $v^k v_k = 0$ for light-like M^4 projection. For time-like M^4 projection light-likeness gives $1 - R^2\omega^2 = 0$ fixing the value of ω to $\omega = 1/R$; therefore CP_2 part of the geodesic is characterized by giving unit vector characterizing its direction at arbitrarily chosen point of CP_2 and the moduli sopace space is 3-dimensional S^3. For light-like M^4 projection one obtains $\omega = 0$ so that the CP_2 projection contracts to a point. The hyperbolic space H^3 or Lobatchevski space (mass shell) parametrizing the space of unit four-velocities and S^3 gives the possible directions of velocity at given point of CP_2.

The space of light-like geodesics in H could be therefore regarded as a singular bundle like structure. The interior of the bundle has the space $X^6 = E^3 \times H^3$ of time-like geodesics of M^4 as base and S^3 perhaps identifiable as subspace of flag-manifold $SU(3)/U(1) \times U(1)$ of CP_2 defining CP_2 twistors as fiber. This space could be 9-dimensional subspace of $D = 14$ twistor space and consistency with $D = 14$ obtained from previous argument. Boundary consists of light-like geodesics of M^4 - that is 5-D subspace of twistor space CP_3 and fiber reduces to CP_2. The bundle structure seems trivial apart the singular boundary. Again there are good reasons to believe that the twistor space is non-compact which is a highly undesirable feature.

The cautious conclusion is that category theorist is right, and that one must take seriously p-adic mass calculations and generalized Feynman diagrams: the twistor space in question corresponds to space-like geodesics of H with light-like M^4 projection and reduces to the product of twistor spaces of M^4 and CP_2.

I have earlier speculated about twistorial formulation of TGD assuming that the analog of twistor space for $M^4 \times CP_2$ is $CP_3 \times CP_3$ and also noticed the analogy with F-theory [K80]. In the same chapter I have also considered an explicit proposal for the realization of the 10-D counterparts of space-time surfaces as 6-dimensional holomorphic surfaces in $CP_3 \times CP_3$ speculated to be Calabi-Yau manifolds. These speculations can be repeated for $CP_3 \times F(1,2,6)$ but with space-time surfaces mapped to 9-D surfaces having interpretation as $S^2 \times S^3$ bundles with space-time surface as a base space. Light-like 3-surfaces would be mapped to 8-D surfaces. Whether they could allow the identification as 4-complex-dimensional Calabi-Yau manifolds with structure group SU(4) as a structure group and Kähler metric with global holonomy contained in SU(4) is a question that mathematician might be able to answer immediately.

13.3.4 Three approaches to incidence relations

The algebraic realization of incidence relations involves spinors. The 2-dimensional character of the spinors and the possibility to interpret 2×2 Pauli sigma matrices as matrix representation of units of complexified quaternions with additional imaginary unit commuting with quaternionic imaginary units seem to be essential. How could one generalize the incidence relations to 8-D context?

One can consider three approaches to the generalization of the incidence relations defining algebraically the correspondence between bi-spinors and light-like vectors.

(a) The simplest approach assumes that twistor space is Cartesian product of those associated with M^4 and CP_2 separately so that nothing new should emerge besides the quantization of Y_3 and I_3. The incidence relations for Minkowskian and Euclidian situation are discussed in detail later in the section. It might well be that this is all that is needed.

(b) Second approach is based on triality for the representations of $SO(1,7)$ realized for 8-D spaces.

(c) Third approach relies on octonionic representations of sigma matrices and replaces $SO(1,7)$ with the octonionic automorphism group G_2.

The first approach will be discussed in detail at the end of the section.

The approach to incidence relations based on triality

Second approach to incidence relations is based on the notion of triality serving as a special signature of 8-D imbedding space.

(a) The triality symmetry making 8-D spaces unique states there are 3 8-D representations of SO(8) or SO(1,7) related by triality. They correspond complexified vector representation and spinor representations together with its conjugate. Could ordinary 8-D gamma matrices define sigma matrices obtained simply by multiplying them by γ^0 so that one obtains unit matrix and analogs of 3-D sigma matrices. Sigma matrices defined in this manner span an algebra which has dimension $d_1 = 2^{D-1}$ corresponding to the even part of 8-D Clifford algebra.

This dimension should be equal to the real dimension of the complex $D \times D$ matrix algebra given by $d_2 = 2 \times D \times D$. For $D = 8$ one one indeed has $d_1 = 128 = d_2$! Hence triality symmetry seems to allow the realization of the incidence relations for 8-vectors and 8-spinors and their conjugates! Could this realize the often conjectured role of triality symmetry as the holy trinity of physics? Note that for the Pauli sigma matrices the situation is different. They correspond to complexified quaternions defining 8-D algebra with dimension $d_1 = 8$, which is same as the dimension d_2 for $D = 2$ assignable to the two 2-spinors.

(b) There is however a potential problem. For $D = 4$ the representations of points of complexified M^4 span the entire sigma matrix algebra (complexified quaternions). For $D = 8$ complexified points define 16-D algebra to be contrasted with 128 dimensional algebra spanned by sigma matrices. Can this lead to difficulties?

(c) Vector $x^k\sigma_k$ would have geometric interpretation as the tangent vector of the light-like geodesic at some reference point - most naturally defined by the intersection with $X^3 \times CP_2$, where X^3 is 3-D subspace of M^4. X^3 could correspond to time=constant slice E^3. Zero energy ontology would suggests either of the 3-D light-like boundaries of CD: this would give only subspace of full twistor space.

Geometrically the incidence relation would in the 8-D case state that two 6-spheres of 12-D twistor space define as their intersection light-like line of M^8. Here one encounters an unsolved mathematical problem. Generalizing from the ordinary twistors, one might guess that complex structure of 6-sphere could be be crucial for defining complex structure of twistor space. 6-sphere allows almost complex structures induced by octonion structure. These structures are not integrable (do not emerge as a side product of complex manifold structure) and an open problem is whether S^6 admits complex structure (`http://www.math.bme.hu/~etesi/s6-spontan.pdf`) [A61]. From the reference one however learns that S^6 allows twistor structure presumably identified in terms of the space of geodesics.

The approach to incidence relations based on octonionic variant of Clifford algebra

Third approach is purely number theoretical being based on octonions. Only sigma matrices are needed in the definition of twistors and incidence relations. In the case of sigma matrices the replacement of the ordinary sigma matrices with abstract quaternion units makes sense. One could replace bi-spinors with complexified quaternions and identify the two spinors in their matrix representation as the two columns or rows of the matrix.

The octonionic generalization would replace sigma matrices with octonionic units. The non-associativity of octonions however implies that matrix representation does not exist anymore. Only quaternionic subspaces of octonions allow matrix representation and the basic dynamical principle of number theoretic vision is that space-time surfaces are associative in the sense that the tangent space is quaternionic and contains preferred complex subspace. In the purely octonionic context there seems to be no manner to distinguish between vector x and spinor and its conjugate. The distinction becomes possible only in quaternionic subspaces in which 8-D spinors reduces to 4-D spinors and one can use matrix representation to identify vector and and spinor and its conjugate.

In [K78] I have considered also the proposal for the construction of the octonionic gamma matrices (they are not necessary in the twistorial construction). Now octonions alone are not enough since unit matrix does not allow identification as gamma matrix. The proposal constructs gamma matrices as tensor products of σ_3 and octonion units defining octonionic counterpart of the Clifford algebra realized usually in terms of gamma matrices.

Light-likeness condition corresponds to the vanishing of the determinant for the matrix defined by the components of light-like vector. Can one generalize this condition to the octonionic representation? The problem is that matrix representation is lacking and therefore also the notion of determinant is problematic. The vanishing of determinant is equivalent with the existence of vectors annihilated by the matrix. This condition makes sense also now and would say that x as octonion with complexified components produces zero in multiplication with some complexified octonion. This is certainly true for some complexified octonions which are not number field since there exist complexified octonions having no inverse. It is of course easy to construct such octonions and they correspond to light-like 8-vectors having no inverse.

The multiplication of octonionic spinors by octonionic units would appear in the generalization of the incidence relation $\mu^{A\prime} = x^{AA'}\lambda_A$ by replacing spinors and 8-coordinate with complex octonions. This would allow to assign to the tangent vector of light-like geodesic at given point of X^4 a generalized twistor defined by a pair of complexified 8-component octonionic spinors. It is however impossible to make distinction between these three objects unless one restricts to quaternionic spinors and vectors and uses matrix representation for quaternions.

13.3.5 Are four-fermion vertices of TGD more natural than 3-vertices of SYM?

There are some basic differences between TGD and super Yang-Mills theory (SYM) and it is interesting to compare the two situations from the perspective of both momentum space and twistor space. Here the miminal approach to incidence relations assuming cartesian product $CP_3 \times SU(3)/U(1) \times U(1)$ is starting point but the dimension of spinor space is allowed to be free.

(a) In SYM the basic vertex is 3-vertex. Momentum conservation for three massless real momenta requires that the momenta are parallel. This implies that for on mass shell states the vertex is highly singular and this in turn is source of IR divergences. The three twistor pairs would be for real on mass shell states proportional to each other. In twistor formulation one however allows complex light-like momenta and this requires that either λ_i are or $\hat{\lambda}_i$ are collinear. The condition $\lambda_i = \pm(\hat{lambda_i})^*$ implies that twistors are collinear.

(b) In TGD framework physical states correspond to collections of wormhole contacts carrying fermion and anti-fermions at the throats. The simplest states are fermions having fermion number at either throat. For bosons one has fermion and anti-fermion at opposite throats. External particles are bound states of massless particles. 4-fermion vertex is fundamental one and replaces BFF vertex.

The basic 4-vertex represents a situation in which there are incoming wormhole contacts which in vertex emit a wormhole contact. For boson exchange incoming fermion and anti-fermion combine to form the exchanged boson consisting from the fermion and anti-fermion at opposite throats of the wormhole contact. All fundamental fermions are massless in real sense also inside internal lines and only the sum of the massless four-momenta is off mass shell. The momentum of the exchanged wormhole contact can be also space-like if energies of fermion and anti-fermion have opposite signs. The four-fermionic interaction corresponds essentially to the of the stringy propagator defined by the inverse of the scaling generator L_0 of super-Virasoro algebra and is associated with the wormhole contact and gives in massless sector of super-conformal representations massless exchange. Exchange wormhole contacts carrying fermion and anti-fermion at the throats correspond to fundamental boson exchanges. The outcome is essentially a hybrid of Feynman diagrammatics of QFTs and string models.

The real on mass shell property reduces the number of allow diagrams dramatically and strongly suggests the absence of both UV and IR divergences. Without further conditions ladder diagrams involving arbitrary number of loops representing massess exchanges are possible but simple power counting argument demonstrates that no divergences are generated from these loops.

(c) $\mathcal{N} = 4$ SUSY *as such* is not present so that super-twistors might not needed. SUSY is at WCW level replaced with conformal supersymmetry. Right-handed neutrino represents the least broken SUSY and the considerations related to the realization of super-conformal algebra and WCW gamma matrices as fermion number carrying objects suggest that the analogy of $\mathcal{N} = 4$ SUSY with conserved fermion number based on covariantly constant right-handed neutrino spinors emerges from TGD.

Consider now the basic formula for the 3-vertex appearing in gauge theories forgetting the complications due to SUSY.

(a) The vertex contains determinants of 2×2 matrices defined by pairs (λ_i, λ_j) and $(\hat{\lambda}_i, \hat{\lambda}_j)$, $i = 1,2,3$. $\hat{\lambda}' = -(\lambda^\alpha)^*$ holds true in Minkowskian signature. These determinants define antisymmetric Lorentz invariant "inner products" based on the 2-dimensional permutation symbol $\epsilon_{\alpha\alpha'}$ defining the Lorentz invariant bilinear for spinors. This form should generalize to the analog of Kähler form.

(b) Second essential element is the expression for momentum conservation in terms of the spinors λ and $\hat{\lambda}$. The momentum conservation condition $\sum_k p_k = 0$ combined with the basic identification

$$p^{\alpha\alpha'} = \lambda^\alpha \hat{\lambda}^{\alpha'} \qquad (13.3.1)$$

equivalent with incidence relations gives

$$\sum_{k=1,\ldots,n} \lambda_k^\alpha \hat{\lambda}_k^{\alpha'} = 0 \ . \qquad (13.3.2)$$

The key idea is to interpret λ_k^α and $\hat{\lambda}_k^{\alpha'}$ as vectors in n-dimensional space which is Grassmannian $G(2,n)$ since from a given solution to the conditions one obtains a new one by scaling the spinors λ_i and $\hat{\lambda}_j$ by scaling factors, which are inverses of each other. The conditions state that the 2-planes spanned by the λ^α and $\hat{\lambda}^{\alpha'}$ as complex 3-vectors are orthogonal. The conservation conditions can be satisfied only for 3-vectors.

Since the expression of momentum conservation as orthogonality conditions is a crucial element in the construction of twistor amplitudes it is good to look in detail what the conditions mean. For future purposes it is convenient to consider N-spinors instead of 2-spinors.

(a) The number of these vectors is 2+2 for 2-spinors. For N-component spinors it is $N+N = 2N$. The number of conditions to be satisfied is $2N \times N - N$rather than $2N^2$: the reduction comes from the factor the condition $\hat{\lambda}^{\alpha'} = -(\lambda^\alpha)^*$ holding for real four-momenta in M^4 case. For complex light-like momenta the number of conditions is $2N^2 = 8$.

(b) For $N = 2$ and $n = 3$ with real masses one obtains 6 conditions and 6 independent components so that the conditions allow to solve the constraint uniquely (apart from complex scalings). All momenta are light-like and parallel. For complex masses one has 8 conditions and 12 independent spinor components and conditions imply that either λ_i or $\hat{\lambda}_i$ are parallel so that one has 4 complex spinors . For $n > 3$ the number of conditions is smaller than the total number of spinor components in accordance with the fact that momentum conservation conditions allow continuum of solutions. 3-vertex is the generating vertex in twistor formulation of gauge theories. For $N > 2$ the number conditions is larger than available spinor components and the situation reduces to $N = 2$ for solutions.

(c) Euclidian spinors appear in CP_2 degrees of freedom. In $N = 2$ case spinors are complex, "momentum" having anomalous isospin and hyper-charge of CP_2 spinor as components is not light-like, and massless Dirac equation is not satisfied. Hence number of orthogonality conditions is $2 \times N^2 = 8$ whereas the total number of spinor components is $3\times 2 + 3\times 2 = 12$ as for complex massless momenta. Orthogonality conditions can be satisfied. For $N > 2$ the real dimension of the sub-paces spanned by spinors is at most 3 and orthogonality condition can be satisfied if N reduces effectively to $N = 2$.

Similar discussion applies for 4-fermion vertex in the case of TGD.

(a) Consider first M^4 case ($N = 2$) for $n = 4$-vertex. The momentum conservation conditions imply that fourth momentum is the negative of the sum of the three other and massless. For real momenta the number of conditions on spinors is also now $2 \times N^2 - N = 6$ for $N = 2$. The number of spinor components is now $n \times N = 4 \times N = 8$ so that 2 spinor components characterizing the virtual on mass shell momentum of the second fermion composing the boson remains free in the vertex.

(b) In CP_2 degrees of freedom and for $n = 4, N = 2$ the number of orthogonality conditions is $2N^2 = 8$ and the total number of spinor components is $2 \times n \times N = 16$ so that 8 spinor components remain free. The quantization of anomalous hyper-charge and isospin however discretizes the situation as suggested by number theoretic arguments. Also in M^4 degrees of freedom discretisation of four-momenta is suggestive.

(c) For $N > 2$ the situation reduces effectively to $N = 2$ for the solutions to the conditions for both Minkowskian and Euclidian signature.

13.4 Emergence of $M^4 \times CP_2$ twistors at the level of WCW

One could imagine even more dramatic generalization of the notion of twistor, which conforms with the general vision about TGD and twistors. The orbits of partonic 2-surfaces are light-like surfaces and generalize the notion of light-like geodesics. In TGD framework the replacement of point like particle with partonic 2-surface plus 4-D tangent space data suggests strongly that the Yangian algebra defined by finite-dimensional conformal algebra of M^4 generalizes to that defined by the infinite-dimensional conformal algebra associated with all symmetries of WCW.

The twistorialization should give twistorialization of $M^4 \times CP_2$ at point-like limit defined by $CP_2 \times SU(3)/U(1) \times U(1)$. In the following it will be found that this is indeed the case and that twistorialization can be seen as a representation for a choice of quantization axes characterized by appropriate flag manifold.

13.4.1 Concrete realization for light-like vector fields and generalized Virasoro conditions from light-likeness

The points of WCW correspond to partonic two-surfaces plus 4-D tangent space data. It is attractive to identify the tangent space data in terms of light-like vector fields defined at the partonic 2-surfaces at the ends of light-like 3-surface defining a like of generalized Feynman diagrams so that their would define light-like vector field in the piece of WCW defined by single line of generalized Feynman diagrams. It is also natural to continue these light-like vector fields to light-like vector fields defined at entire light-like 3-surface - call it X^3.

To get some grasp about the situation one can start from a simpler situation, CP_2 type vacuum extremals with 1-D light-like curve as M^4 projection. The light-likeness condition reads as

$$m_{kl}\frac{dm^k}{ds}\frac{dm^l}{ds} = 0 \ , \tag{13.4.1}$$

One can use the expansion

$$\begin{aligned} m^k &= m_{k,0} + p_0^k s + \sum_{n,i} a_{n,i}\frac{\epsilon_i^k}{\sqrt{n}}s^n \ , \\ \epsilon_i \cdot \epsilon_j &= -P_{ij}^2 \ . \end{aligned} \tag{13.4.2}$$

Here orthonormalized polarization vectors ϵ_i define 2-D transversal space orthogonal to the longitudinal space $M^2 \subset M^4$ and characterized by the projection operator P^2. M^2 can be fixed by a light-like vector and corresponds to the real section of the twistor space naturally. These conditions are familiar from string (complex coordinate is replaced with s). Here ϵ_i are polarization vectors orthogonal to each other. One obtains the Virasoro conditions

$$L_n = p\cdot p + 2\sum_m a_{n-m}a_m\sqrt{n-k}\sqrt{k} = 0 \tag{13.4.3}$$

expressing the invariance of light-likeness condition with respect to diffeomorphisms acting on coordinate s. For $n = 0$ one obtains the Virasoro conditions. This can be regarded as restriction of conformal invariance from string world sheets emerging from the modified Dirac equation at their ends at light-like 3-surfaces.

The generalization of these conditions is rather obvious. Instead of functions $m_n^k = \epsilon_n^k s^n$ one considers functions

$$\begin{aligned}
m_{n,\alpha}^k &= m^0 + p_0^k s + \sum_{n,i} a_{n,i,\alpha}\epsilon_i^k \frac{s^n}{\sqrt{n}} f_\alpha(x^T) + \sum_{n,i} b_{n,i,\alpha} c_i^k \frac{s^n}{\sqrt{n}} g_\alpha(x^T) \ , \\
s_{n,\alpha}^k &= s_0^k + J_0^k s + c_i^k s^n g_\alpha(x^T) \ , \\
c_i^k \cdot c_j^k &= -\delta_{ij} \ .
\end{aligned} \tag{13.4.4}$$

where s^k denotes CP_2 coordinates. The tangent vecotor J^k characterizes a geodesic line in CP_2 degrees of freedom. There is no reason to restrict the polarization directions in CP_2 degrees of freedom so that the projection operator is flat Eucldian 4-D metric. $\{f_\alpha\}$ is a complete basis of functions of the transversal coordinates for the $s = constant$ slice defined the partonic 2-surface at given position of its orbit. One can assume that the modes are orthogonal in the inner product defined by the imbedding space metric and the integral over partonic 2-surface in measure defined by the $\sqrt{g_2}$ for the 2-D induced metric at the partonic 2-surface

$$\langle f_\alpha, f_\beta\rangle = \delta_{\alpha\beta} \ . \tag{13.4.5}$$

.

The space of functions f_α is assumed to be closed under product so that they satisfy the multiplication table

$$f_\alpha f_\beta = c_{\alpha\beta}^\gamma f_\gamma \ . \tag{13.4.6}$$

.

This representation allows to generalize the light-likeness conditions to 3-D form

$$L_{n,\alpha} = p_k p^k + J_k J^k + \sum_{k,\alpha,\beta} \left[2a_{n-k,\alpha}a_{k,\alpha} + 4b_{n-k,\alpha}b_{k,\alpha}\right]\sqrt{n-k}\sqrt{k} = 0 \ . \tag{13.4.7}$$

These equations define a generalization of Virasoro conditions to 3-D light-like surfaces. The center of mass part now corresponds to conserved color charge vector associated with CP_2 geodesic. One can also write variants of these conditions by performing complexification for functions f_α.

13.4.2 Is it enough to use twistor space of $M^4 \times CP_2$?

The following argument suggests that Virasoro conditions require naturally the integration over the twistor space for $M^4 \times CP_2$ but that twistorialization in vibrational degrees of freedom is not needed.

The basic problem of Virasoro conditions is that four-momentum in cm degrees of freedom is time-like in the general case. It is very difficult to accept the generalization of the twistor space to $E^3 \times SO(3,1)/SO(2) \times SO(1,1) \times SU(3)/U(1) \times U(1)$ in cm degrees of freedom? The idea about straightforward generalization twistor space to vibrational degrees of freedom seems to lead to grave difficulties. It however seems that a loophole, in fact two of them, exist and is based on the notion of momentum twistors.

(a) The key observation is that the selection of M^2 in the Virasoro conditions reduces to a fixing of light-like vector in given M^4 coordinates fixing $M^2 \subset M^4$. This choices defines a twistor in the real section of the twistor space. Could twistors emerge through this kind of condition? In the quantization of the theory which must somehow appear also in TGD framework, the selection of quantization axes must be made and means selection of point of a flag manifold defining the twistor spaces associated with M^4 and CP_2. In quasiclassical picture only the components of the tangent vector in CP_2 degrees of freedom have well-defined isospin and hypercharge so that J_k would be a linear combination of I_3 and Y. Standard complex coordinates transforming linearly at their origin under U(2) indeed have this property.

    Could the integration over twistor space mean in WCW context an integration over the possible choices of the quantization axes necessary in order to preserve isometries as symmetries? Four-momenta of external lines itself could be assumed to be massless as conformal invariance strongly suggests.

(b) Consider now the problem. Virasoro conditions require that M^4 momentum is massive. This is not consistent with twistorialization. Momentum twistors for which external light-like momenta characterizing external lines are differences $p_i = x_i - x_{i-1}$ of the "region momenta" x_i assigned with the twistor lines [B28] (http://arxiv.org/pdf/1008.3110v1.pdf) might solve the problem. In the recent case region momenta x_i would correspond to those appearing in Virasoro conditions and light-like momenta of outgoing lines would correspond to their differences. Similar identification would apply to color iso-spin and hyper-charge. For SYM massless real momenta in the condition $p_i = x_i - x_{i-1}$ implies that all three momenta are parallel, which is a catastrophic result. In the TGD based twistor approach region momenta can be however real and massless : this would give rise to dual conformal invariance leading to Yangian symmetries. In this picture Super Virasoro conditions would separate completely from twistorialization and apply in overall cm degrees of freedos: this is indeed what has been assumed hitherto.

    It is easy to see that that region momenta can be real and light-like in TGD framework. A generalization of the condition $p_i = x_i - x_{i-1}$ from 3-vertex to 4-fermion vertex is needed (4-particle vertex requires super-symmetrization but this is not essential for the argument). 4-fermion vertex involves interaction between 2-fermions via Euclidian wormhole contact (this will be discussed later) inducing their scattering. For massless external fermion second internal line is a wormhole contact carrying massless fermion and anti-fermion at its opposite throats. The region momentum associated with this line can be defined as sum of the light-like region momenta associated with the throats. If the external particle is boson like carrying - in general non-parallel - light-like momenta at its throats, then p_i is sum of their light-like momenta.

Concerning the identification of region momenta, one could consider also another option inspired by the vision that also the fermions propagating in the internal lines are massless.

(a) For this option also region momenta are light-like in accordance with the idea about twistor diagrams as null polygons and the idea about light-light on mass shell propagation also on internal lines. One can consider two options for the fermionic propagator.

i. In twistor description the inverse of the full massless Dirac propagator would appear in the line in twistor formalism and this would leave only non-physical helicities making the lines virtual: the interpretation would be as a residue of $1/p^2$ pole.

ii. The M^2 projection of the light-like momentum associated with the corresponding internal line would be time-like. In CP_2 degrees of freedom J^k could be replaced by its projection to the plane spanned by isospin and hypercharge. The values of the sum of transverse E^2 momentum squared and in cm and vibrational degrees of freedom would be identical.
Indeed, one possible option considered already earlier is that M^4 momentum is always light-like and only its longitudinal M^2 part is precisely defined for quantum states (as for partons inside hadron). The original argument was that if only the M^2 part of momentum appears in the propagators, one can have on mass shell massless particles without diverging propagators: in twistorial approach one gets rid of the ordinary propagators in the case gauge fields. The integration over different choices of M^2 associated with the internal line and having interpretation as integration over light-like virtual momenta would guarantee overall Lorentz invariance. This would allow also the use of the M^2 part of four-momentum - an option cautiously considered for generalized Feynman diagrams - without losing isometries as symmetries.

(b) The fermion propagator could also contain CP_2 contribution. Since only Cartan algebra charges can be measured simultaneously, J^k would correspond to a superposition of color hypercharge and isospin generators. The flag manifold $SU(3)/U(1) \times U(1)$ would characterize possible choices of quantization axes for CP_2. Also in the case of CP_2 only the "polarization directions" orthogonal to the plane defined by I_3 and Y could be allowed and it might be possible to speak about CP_2 polarization perhaps related to Higgs field. The dimension of $M^4 \times CP_2$ in vibrational degrees of freedom would effectively reduce to 4. Number theoretically this could correspond to the choice of quaternionic subspace of the octonionic tangent space.

What can one conclude?

(a) Since the choice of quantization axis is same for all modes and forces them to a space orthogonal to that defined by quantization axes, one can say that all modes are characterized by the twistor space for $M^4 \times CP_2$ and there is no need to consider infinite-dimensional generalization of the twistor space only $M^4 \times CP_2$ twistors would be needed and would have interpretation as the integration over the choices of quantization axes is natural part of quantum TGD.

(b) The use of ordinary massless Dirac operator is very attractive option since it gives the inverse of massless Dirac operator as effective propagator in twistor formalism and requires that only non-physical helicities propagate. Massless on mass shell propagation is possible only for fermions as fundamental particles. If one wants also CP_2 contribution to the propagator then restriction to $I_3 - Y$ plane might be necessary. This option does not look too promising.

(c) From the TGD point of view twistor approach to gauge theory in M^4 would not describe not much more than the physics related to the choice of quantization axes in M^4. The physics described by gauge theories is indeed in good approximation to that assignable to cm degrees of freedom. The remaining part of the physics in TGD Universe - maybe the most interesting part of it involving WCW integration - would be described in terms of infinite-dimensional super-conformal algebras.

13.4.3 Super counterparts of Virasoro conditions

Although super-conformal algebras have been applied successfully in p-adic mass calculations, many aspects related to super Virasoro conditions remain still unclear. p-Adic mass calculations require only that there are 5 super-conformal tensor factors and leaves a lot of room for imagination.

(a) There are two super conformal algebras. The first one is the super-symplectic algebra assignable to the space-like 3-surface and acts at the level of imbedding spaceand is induced by Hamiltonians of $\delta M^4_\pm \times CP_2$. Second algebra is Super Kac-Moody algebra acting on light-like 3-surfaces as deformations respecting their light-likeness and is also assignable to partonic 2-surfaces and their 4-D tangent space. Do these algebras combine to single algebra or do they define separate Super Virasoro conditions? p-Adic mass calculations assume that the direct sum is in question and can be localized to partonic 2-surfaces by strong form of holography. This makes the application of p-adic thermodynamics [K37] sensical .

(b) Do the Super Virasoro conditions apply only in over all cm degrees of freedom so that spinors are imbedding space spinors. They would thus apply at the level of the entire 3-surfaces assigned to external elementary particles and containing at least two wormhole contacts. In this case the resulting massive states would be bound states of massless fermions with non-parallel light-like momenta and the resulting massivation could be consistent with conformal invariance.

This is roughly the recent picture about the situation. One can however consider also alternatives.

(a) Could the Super Virasoro conditions apply to individual partonic 2-surfaces or even at the lines of generalized Feynman diagrams but in this case involve only the longitudinal part of massless M^4 momentum?

(b) Could Super-Virasoro conditions be satisfied at partonic 2-surfaces defining vertices in the sense that the sum of incoming super Virasoro generators annihilate the vertex identified. In cm degrees of freedom this condition would be satisfied in cm degrees of freedom momentum conservation holds true. In vibrational degrees of freedom the condition is non-trivial but in principle can be satisfied. The fermionic oscillator operators at incoming legs are related linearly to each other and the problem is to solve this relationship. In the case of N-S generators the same applies. For Virasoro generators the conditions are satisfied if the Virasoro algebras of lines annihilate the state associated with them separately.

These options do look too plausible and would make the situation un-necessarily complex.

How the cm parts of WCW gamma matrices could carry fermion number?

Super counterparts of Virasoro conditions must be satisfied for the entire 3-surface or less probably for the light-like lines of generalized Feynman diagram. These conditions look problematic, and I have considered earlier several solutions to the problem with a partial motivation coming from p-adic thermodynamics.

The problem is following.

(a) In Ramond representation super generators are labeled by integers and string models suggest that super generator G_0 and its hermitian conjugate have ordinary Dirac operator as its cm term and vibrational part has fermion number ± 1. This does not conform with the non-hermiticity of G_0 and looks non-sensical and it seems difficult to satisfy the super Virasoro conditions in non-trivial manner.

(b) There exist a mechanism providing the cm part of G_0 with fermion number? Right-handed neutrino is exceptional: it is de-localized into entire X^4 as opposed to other spinor components localized to string world sheets and has covariantly constant zero modes with vanishing momentum. These modes seem to provide the only possible option that one can imagine. The fermion number carrying gamma matrices in cm degrees of freedom of H would be defined as $\Gamma^\alpha = \gamma^\alpha \Psi_\nu$ and $\Gamma^{\alpha\dagger} = \overline{\Psi}_{\nu_R}\gamma^\alpha$, where Ψ_{ν_R} represents covariantly constant right-handed neutrino. The anti-commutator gives imbedding space metric as required. Right-handed neutrino would have a key role in the mathematical structure of the theory.

(c) For Neveu-Scwartz representation WCW gamma matrices and super generators are labeled by half odd integers and in this case all generators would have fermion number ± 1. The squares of super generators give rise to Virasoro generators L_n and L_0 should be essentially the mass squared operator as $G_{1/2}G_{-1/2} + hc.$. This operator should give the d'Alembertian in $M^4 \times CP_2$ or its longitudinal part. This is quite possible but it seems that Ramond option is the physical one.

The two spin states of covariantly constant right handed neutrino and its antiparticle could provide a fermion number conserving TGD analog of $\mathcal{N} = 4$ SUSY since the four oscillator operators for Ψ_{ν_R} would define the analogs of the four theta parameters.

What is the nature of the possible space-time supersymmetry generated by the right-handed neutrino? Do different super-partners have different mass as seems clear if different super-partners can be distinguished by their interactions. If they have different masses do they obey same mass formula but with different p-adic prime defining the mass scale? This problem is discussed the article [L11] and in the chapter [K59].

About the SUSY generated by covariantly constant right-handed neutrinos

The interpretation of covariantly constant right-handed neutrinos (ν_R in what follows) in $M^4 \times CP_2$ has been a continual head-ache. Should they be included to the spectrum or not. If not, then one has no fear/hope about space-time SUSY of any kind and has only conformal SUSY. First some general obsrevations.

(a) In TGD framework right-handed neutrinos differ from other electroweak charge states of fermions in that the solutions of the modified Dirac equation for them are de-localized at entire 4-D space-time sheets whereas for other electroweak charge states the spinors are localized at string world sheets [K87].

(b) Since right-handed neutrinos are in question so that right-handed neutrino are in 1-1 correspondence with complex 2-component Weyl spinors, which are eigenstates of γ_5 with eigenvalue say $+1$ (I never remember whether $+1$ corresponds to right or left handed spinors in standard conventions).

(c) The basic question is whether the fermion number associated with covariantly constant right-handed neutrinos is conserved or conserved only modulo 2. The fact that the right-handed neutrino spinors and their conjugates belong to unitarily equivalent pseudoreal representations of SO(1,3) (by definition unitarily equivalent with its complex conjugate) suggests that generalized Majorana property is true in the sense that the fermion number is conserved only modulo 2. Since ν_R decouples from other fermion states, it seems that lepton number is conserved.

(d) The conservation of the number of right-handed neutrinos in vertices could cause some rather obvious mathematical troubles if the right-handed neutrino oscillator algebras assignable to different incoming fermions are identified at the vertex. This is also suggested by the fact that right-handed neutrinos are de-localized.

(e) Since the ν_R:s are covariantly constant complex conjugation should not affect physics. Therefore the corresponding oscillator operators would not be only hermitian conjugates but hermitian apart from unitary transformation (pseudo-reality). This would imply generalized Majorana property.

(f) A further problem would be to understand how these SUSY candidates are broken. Different p-adic mass scale for particles and super-partners is the obvious and rather elegant solution to the problem but why the addition of right-handed neutrino should increase the p-adic mass scale beyond TeV range?

If the ν_R:s are included, the pseudor3al analog of $\mathcal{N} = 1$ SUSY assumed in the minimal extensions of standard model or the analog of $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY $\mathcal{N} = 2$ or even $\mathcal{N} = 4$ SUSY is expected so that SUSY type theory might describe the situation. The following is an attempt to understand what might happen. The earlier attempt was made in [K59].

1. Covariantly constant right-handed neutrinos as limiting cases of massless modes

For the first option covariantly constant right-handed neutrinos are obtained as limiting case for the solutions of massless Dirac equation. One obtains 2 complex spinors satisfying Dirac equation $n^k\gamma_k\Psi = 0$ for some momentum direction n^k defining quantization axis for spin. Second helicity is unphysical: one has therefore one helicity for neutrino and one for antineutrino.

(a) If the oscillator operators for ν_R and its conjugate are hermitian conjugates, which anti-commute to zero (limit of anti-commutations for massless modes) one obtains the analog of $\mathcal{N} = 2$ SUSY.

(b) If the oscillator operators are hermitian or pseudohermitian, one has pseudoreal analog of $\mathcal{N} = 1$ SUSY. Since ν_R decouples from other fermion states, lepton number and baryon number are conserved.

Note that in TGD based twistor approach four-fermion vertex is the fundamental vertex and fermions propagate as massless fermions with non-physical helicity in internal lines. This would suggest that if right-handed neutrinos are zero momentum limits, they propagate but give in the residue integral over energy twistor line contribution proportional to $p^k\gamma_k$, which is non-vanishing for non-physical helicity in general but vanishes at the limit $p^k \to 0$. Covariantly constant right-handed neutrinos would therefore decouple from the dynamics (natural in continuum approach since they would represent just single point in momentum space). This option is not too attractive.

2. Covariantly constant right-handed neutrinos as limiting cases of massless modes

For the second option covariantly constant neutrinos have vanishing four-momentum and both helicities are allowed so that the number of helicities is 2 for both neutrino and antineutrino.

(a) The analog of $\mathcal{N} = 4$ SUSY is obtained if oscillator operators are not hermitian apart from unitary transformation (pseudo reality) since there are 2+2 oscillator operators.

(b) If hermiticity is assumed as pseudoreality suggests, $\mathcal{N} = 2$ SUSY with right-handed neutrino conserved only modulo two in vertices obtained.

(c) In this case covariantly constant right-handed neutrinos would not propagate and would naturally generate SUSY multiplets.

3. Could twistor approach provide additional insights?

Concerning the quantization of ν_R:s, it seems that the situation reduces to the oscillator algebra for complex M^4 spinors since CP_2 part of the H-spinor is spinor is fixed. Could twistor approach provide additional insights?

As discussed, M^4 and CP_2 parts of H-twistors can be treated separately and only M^4 part is now interesting. Usually one assigns to massless four-momentum a twistor pair $(\lambda^a, \hat{\lambda}^{a'})$ such that one has $p^{aa'} = \lambda^a\hat{\lambda}^{a'}$. Dirac equation gives $\lambda^a = \pm(\hat{\lambda}^{a'})^*$, where $\pm$ corresponds to positive and negative frequency spinors.

(a) The first - presumably non-physical - option would correspond to limiting case and the twistors λ and $\hat{\lambda}$ would both approach zero at the $p^k \to 0$ limit, which again would suggest that covariantly constant right-handed neutrinos decouple completely from dynamics.

(b) For the second option one could assume that either λ or $\hat{\lambda}$ vanishes. In this manner one obtains 2 spinors λ_i, $i = 1, 2$ and their complex conjugates $\hat{\lambda}_i$ as representatives for the super-generators and could assign the oscillator algebra to these. Obviously twistors would give something genuinely new in this case. The maximal option would give 2 anti-commuting creation operators and their hermitian conjugates and the non-vanishing anti-commutators would be proportional to $\delta_{a,b}\lambda_i^a(\lambda^b)_j^*$ and $\delta_{a,b}\hat{\lambda}_i^{a'}(\hat{\lambda}^{b'})_j^*$. If the oscillator operators are hermitian conjugates of each other and (pseudo-)hermitian, the anti-commutators vanish.

An interesting challenge is to deduce the generalization of conformally invariant part of four-fermion vertices in terms of twistors associated with the four-fermions and also the SUSY extension of this vertex.

Are fermionic propagators defined at the space-time level, imbedding space level, or WCW level?

There are also questions related to the fermionic propagators. Does the propagation of fermions occur at space-time level, imbedding space level, or WCW level?

(a) Space-time level the propagator would defined by the modified Dirac operator. This description seems to correspond to ultramicroscopic level integrated out in twistorial description.

(b) At imbedding space level allowing twistorial description the lines of generalized Feynman diagram would be massless in the usual sense and involve only the fermionic propagators defined by the twistorial "8-momenta" defining region momenta in twistor approach.This allows two options.

   i. Only the projection to M^2 and preferred $I_3 - Y$ plane of the momenta would be contained by the propagator. The integration over twistor space would be necessary to guarantee Lorentz invariance.

   ii. M^4 helicity for internal lines would be "wrong" so that M^4 Dirac operator would not annihilate it. For ordinary Feynman diagrams the propagator would be $p^k\gamma_k/p^2$ and would diverge but for twistor diagrams only its inverse $p^k\gamma_k$ would appear and would be well-defined. This option looks attractive from twistor point of view.

(c) If WCW level determines the sermonic propagator as in string models, bosonic propagator would naturally correspond to $1/L_0$. The generalization of the fermionic propagator could be defined as G/L_0, where the super generator G contains the analog of ordinary Dirac operator as cm part. The square of G would give L_0 allowing to define the generalization of bosonic propagator. The inverse of the fermionic propagator would carry fermion number.

   This is good enough reason for excluding WCW level propagator and for assuming that the fermion propagators defined at imbedding space level appear in the generalized Feynman diagrams and Super Virasoro algebra are applied only in particle states as done in p-adic mass calculations.

The conclusion is that the original picture about fermion propagation is the only possible one. If one requires that ordinary Feynman diagrams make sense then only the M^2 part of 4-momentum can appear in the propagator. If one assumes that only twistor formalism is needed then propagator is replaced with its inverse in fermionic lines and if polarization is "wrong" the outcome is non-vanishing. This situation has interpretation in terms of homology theory. One could also the interpret the situation in terms of residue calculus picking up $p^k\gamma_k$ as the residue of the pole of $1/(p^2 + i\epsilon)$.

13.4.4 What could 4-fermion twistor amplitudes look like?

What can one conclude about 4-fermion twistor amplitudes on basis of $\mathcal{N} = 4$ amplitudes? Instead of 3-vertices as in SYM, one has 4-fermion vertices as fundamental vertices and the challenge is to guess their general form. The basis idea is that $\mathcal{N} = 4$ SYM amplitudes could give as special case the n-fermion amplitudes and their supersymmetric generalizations.

A attempt to understand the physical picture

One must try to identify the physical picture first.

(a) Elementary particles consist of pairs of wormhole contacts connecting two space-time sheets. The throats are connected by magnetic fluxes running in opposite directions so that a closed monopole flux loop is in question. One can assign to the ordinary fermions open string world sheets whose boundary belong to the light-like 3-surfaces assignable to these two wormhole contacts. The question is whether one can restrict the consideration to single wormhole contact or should one describe the situation as dynamics of the open string world sheets so that basic unit would involve two wormhole contacts possibly both carrying fermion number at their throats.

Elementary particles are bound states of massless fermions assignable to wormhole throats. Virtual fermions are massless on mass shell particles with unphysical helicity. Propagator for wormhole contact as bound state - or rather entire elementary particle would be from p-adic thermodynamics expressible in terms of Virasoro scaling generator as $1/L_0$ in the case of boson. Super-symmetrization suggests that one should replace L_0 by G_0 in the wormhole contact but this leads to problems if G_0 carries fermion number. This might be a good enough motivation for the twistorial description of the dynamics reducing it to fermion propagator along the light-like orbit of wormhole throat. Super Virasoro algebra would emerged only for the bound states of massless fermions.

(b) Suppose that the construction of four-fermion vertices reduces to the level of single wormhole contact. 4-fermion vertex involves wormhole contact giving rise to something analogous to a boson exchange along wormhole contact. This kind of exchange might allow interpretation in terms of Euclidian correlation function assigned to a deformation of CP_2 type vacuum extremal with Euclidian signature.

A good guess for the interaction terms between fermions at opposite wormhole contacts is as current-current interaction $j^{\alpha}(x)j_{\alpha}(y)$, where x and y parametrize points of opposite throats. The current is defined in terms of induced gamma matrices as $\overline{\Psi}\Gamma^{\alpha}\Psi$ and one functionally integrates over the deformations of the wormhole contact assumed to correspond in vacuum configuration to CP_2 type vacuum extremal metrically equivalent with CP_2 itself. One can expand the induced gamma matrix as a sum of CP_2 gamma matrix and contribution from M^4 deformation $\Gamma_{\alpha} = \Gamma_{\alpha}^{CP_2} + \partial_{\alpha} m^k \gamma_k$. The transversal part of M^4 coordinates orthogonal to $M^2 \subset M^4$ defines the dynamical part of m^k so that one obtains strong analogy with string models and gauge theories.

(c) The deformation Δm^k can be expanded in terms of CP_2 complex coordinates so that the modes have well defined color hyper-charge and isospin. There are two options to be considered.

   i. One could use CP_2 spherical harmonics defined as eigenstates of CP_2 scalar Laplacian D^2. The scale of eigenvalues would be $1/R^2$, where R is CP_2 radius of order 10^4 Planck lengths. The spherical harmonics are in general not holomorphic in CP_2 complex coordinates ξ_i, $i = 1, 2$. The use of CP_2 spherical harmonics is however not necessary since wormhole throats mean that wormhole contact involves only a part of CP_2 is involved.

   ii. Conformal invariance suggests the use of holomorphic functions $\xi_1^m \xi_2^n$ as analogs of z^n in the expansion. This would also be the Euclidian analog for the appearance of massless spinors in internal lines. Holomorphic functions are annihilated by the ordinary scalar Laplacian. For conformal Laplacian they correspond to the same eigenvalue given by the constant curvature scalar R of CP_2. This might have interpretation as a spontaneous breaking of conformal invariance.

   The holomorphic basis z^n reduces to phase factors $exp(in\phi)$ at unit circle and can be orthogonalized. Holomorphic harmonics reduce to phase factors $exp(im\phi_1)exp(in\phi_2)$ and torus defined by putting the moduli of ξ_i constant and can thus be orthogonalized. Inner product for the harmonics is however defined at partonic 2-surface. Since partonic 2-surfaces represent Kähler magnetic monopoles they have 2-dimensional CP_2 projection. The phases $exp(im\phi_i)$ could be functionally independent and a reduction of inner product to integral over circle and reduction of phase factors to powers $exp(in\phi)$ could take place and give rise to the analog of ordinary conformal invariance at partonic 2-surface. This does not mean that separate conservation of I_3 and Y is broken for propagator.

iii. Holomorphic harmonics are very attractive but the problem is that it is annihilated by the ordinary Laplacian. Besides ordinary Laplacian one can however consider conformal Laplacian [A37] (`http://en.wikipedia.org/wiki/Laplace_operators_in_differential_geometry#Conformal_Laplacian`) defined as

$$D_c^2 = -6D^2 + R \ , \tag{13.4.8}$$

and relating the curvature scalars of two conformally scaled metrics. The overall scale factor and also its sign is just a convention. This Laplacian has the same eigenvalue for all conformal harmonics. The interpretation would be in terms of a breaking of conformal invariance due to CP_2 geometry: this could also relate closely to the necessity to assume tachyonic ground state in the p-adic mass calculations [K37].

The breaking of conformal invariance is necessary in order to avoid infrared divergences. The replacement of M^4 massless propagators with massive CP_2 bosonic propagators in 4-fermion vertices brings in the needed breaking of conformal invariance. Conformal invariance is however retained at the level of M^4 fermion propagators and external lines identified as bound states of massless states.

How to identify the bosonic correlation function inside wormhole contacts?

The next challenge is to identify the correlation function for the deformation δm^k inside wormhole contacts.

Conformal invariance suggests the identification of the analog of propagator as a correlation function fixed by conformal invariance for a system defined by the wormhole contact. The correlation function should depend on the differences $\xi_i = \xi_{i,1} - \xi_{i,2}$ of the complex CP_2 coordinates at the points $\xi_{i,1)}$ and $\xi_{i,2}$ of the opposite throats and transforms in a simple manner under scalings of ξ_i. The simplest expectation is that the correlation function is power r^{-n}, where $r = \sqrt{|\xi_1|^2 + |\xi_2|^2}$ is U(2) invariant coordinate distance. The correlation function can be expanded as products of conformal harmonics or ordinary harmonics of CP_2 assignable to $\xi_{i,1}$ and $\xi_{i,2}$ and one expects that the values of Y and I_3 vanish for the terms in the expansions: this just states that Y and I_3 are conserved in the propagation.

Second approach relies on the idea about propagator as the inverse of some kind of Laplacian. The approach is not in conflict with the general conformal approach since the Laplacian could occur in the action defining the conformal field theory. One should try to identify a Laplacian defining the propagator for δm^k inside Euclidian regions.

(a) The propagator defined by the ordinary Laplacian D^2 has infinite value for all conformal harmonics appearing in the correlation function. This cannot be the case.

(b) If the propagator is defined by the conformal Laplacian D_c^2 of CP_2 multiplied by some numerical factor it gives fro a given model besides color quantum numbers conserving delta function a constant factor nR^2 playing the same role as weak coupling strength in the four-fermion theory of weak interactions. Propagator in CP_2 degrees of freedom would give a constant contribution if the total color quantum numbers for vanish for wormhole throat so that one would have four-fermion vertex.

This option does not look physically attractive. If four-fermion vertex involves always wormhole contact carrying fermion and anti-fermion at its throats, the interpretation as effective boson exchange is possible and one can assume that the vertex contains instead of L^2 a factor proportional to $1/p^2$. It will be shows later that this description leads to gauge theory like picture. A further possibility is that L^2 is replaced by p-adic length scale square L_p^2 associated with p^2. This would discretize coupling constant evolution.

(c) One can consider also a third - perhaps artificial option - motivated for Dirac spinors by the need to generalize Dirac operator to contain only I_3 and Y. Holomorphic partial waves are also eigenstates of a modified Laplacian D_C^2 defined in terms of Cartan algebra as

$$D_C^2 \equiv \frac{aY^2 + bI_3^2}{R^2} \ , \tag{13.4.9}$$

where a and b suitable numerical constants and R denotes the CP_2 radius defined in terms of the length $2\pi R$ of CP_2 geodesic circle. The value of a/b is fixed from the condition $Tr(Y^2) = Tr(I_3^2)$ and spectra of Y and I_3 given by $(2/3, -1/3, -1/3)$ and $(0, 1/2, -1/2)$ for triplet representation. This gives $a/b = 9/20$ so that one has

$$D_C^2 = (\frac{9}{20}Y^2 + I_3^2) \times \frac{a}{R^2} \ . \tag{13.4.10}$$

In the fermionic case this kind of representation is well motivated since fermionic Dirac operator would be $Y^k e_k^A \gamma_A + I_3^k e_k^A \gamma_A$, where the vierbein projections $Y^k e_k^A$ $Y^k e_k^A$ and $I_3^k e_k^A$ of Killing vectors represent the conserved quantities along geodesic circles and by semiclassical quantization argument should correspond to the quantized values of Y and I_3 as vectors in Lie algebra of $SU(3)$ and thus tangent vectors in the tangent space of CP_2 at the point of geodesic circle along which these quantities are conserved. In the case of S^2 one would have Killing vector field L_z at equator.

Two general remarks are in order.

(a) That a theory containing only fermions as fundamental elementary particles would have four-fermion vertex with dimensional coupling as a basic vertex at twistor level, would not be surprising. As a matter of fact, Heisenberg suggested for long time ago a unified theory based on use of only spinors and this kind of interaction vertex. A little book about this theory actually inspired me to consider seriously the fascinating challenge of unification.

(b) A common problem of all these options seems to be that the 4-fermion coupling strength is of order R^2 - about 10^8 times gravitational coupling strength and quite too weak if one wants to understand gauge interactions. It turns out however that color partial waves for the deformations of space-time surface propagating in loops can increase R^2 to the square $L_p^2 = pR^2$ of p-adic length scale. For D_C^2 assumed to serve as a propagator of an effective action of a conformal field theory one can argue that large renormalization effects from loops increase R^2 to something of order pR^2.

Do color quantum numbers propagate and are they conserved in vertices?

The basic questions are whether one can speak about conservation of color quantum numbers in vertices and their propagation along the internal lines and the closed magnetic flux loops assigned with the elementary particles having size given by p-adic length scale and having wormhole contacts at its ends. p-Adic mass calculations predict that in principle all color partial waves are possible in cm degreees of freedom: this is a description at the level of imbedding space and its natural counterpart at space-time level would be conformal harmonics for induced spinor fields and allowance of all of them in generalized Feynman diagrams.

(a) The analog of massless propagation in Euclidian degrees of freedom would correspond naturally to the conservation of Y and I_3 along propagator line and conservation of Y and I_3 at vertices. The sum of fermionic and bosonic color quantum numbers assignable to the color partial waves woul be conserved. For external fermions the color quantum numbers are fixed but fermions in internal lines could move also in color excited states.

(b) One can argue that the correlation function for the M^4 coordinates for points at the ends of fermionic line do not correlate as functions of CP_2 coordinates since the distance between partonic 2-surface is much longer than CP_2 scale but do so as functions of the string world sheet coordinates as stringy description strongly suggests and that stringy correlation function satisfying conformal invariance gives this correlation. One can however counter argue that for hadrons the color correlations are different in hadronic length scale. This in turn suggests that the correlations are non-trivial for both the wormhole magnetic flux tubes assignable to elementary particles and perhaps also for the internal fermion lines.

(c) I_3 and Y assignable to the exchanged boson should have interpretation as an exchange of quantum numbers between the fermions at upper and lower throat or change of color quantum numbers in the scattering of fermion. The problem is that induced spinors have constant anomalous Y and I_3 in given coordinate patch of CP_2 so that the exchange of these quantum numbers would vanish if upper and lower coordinate patches are identical. Should one expand also the induced spinor fields in Euclidian regions using the harmonics or their holomorphic variants as suggested by conformal invariance?

The color of the induced spinor fields as analog of orbital angular momentum would realized as color of the holomorphic function basis in Euclidian regions. If the fermions in the internal lines cannot carry anomalous color, the sum over exchanges trivializes to include only a constant conformal harmonic. The allowance of color partial waves would conform with the idea that all color partial waves are allowed for quarks and leptons at imbedding space level but define very massive bound states of massless fermions.

(d) The fermion vertex would be a sum over the exchanges defined by spherical harmonics or - more probably - by their holomorphic analogs. For both the spherical and conformal harmonic option the 4-fermion coupling strength would be of order R^2, where R is CP_2 length. The coupling would be extremely weak - about 10^8 times the gravitational coupling strength G if the coupling is of order one. This is definitely a severe problem: one would want something like L_p^2, where p is p-adic prime assignable to the elementary particle involved.

This problem provides a motivation for why a non-trivial color should propagate in internal lines. This could amplify the coupling strength of order R^2 to something of order $L_p^2 = pR^2$. In terms of Feynman diagrams the simplest color loops are associated with the closed magnetic flux tubes connecting two elementary wormhole contacts of elementary particle and having length scale given by p-adic length scale L_p. Recall that $\nu_L \overline{\nu}_R$ pair or its conjugate neutralizes the weak isospin of the elementary fermion. The loop diagrams representing exchange of neutrino and the fermion associated with the two different wormhole contacts and thus consisting of fermion lines assignable to "long" strings and boson lines assignable to "short strings" at wormhole contacts represent first radiative correction to 4-fermion diagram. They would give sum over color exchanges consistent with the conservation of color quantum numbers at vertices. This sum, which in 4-D QFT gives rise to divergence, could increase the value of four-fermion coupling to something of order $L_p^2 = kpR^2$ and induce a large scaling factor of D_C^2.

(e) Why known elementary fermions correspond to color singlets and triplets? p-Adic mass calculations provide one explanation for this: colored excitations are simply too massive. There is however evidence that leptons possess color octet excitations giving rise to light mesonlike states. Could the explanation relate to the observation that color singlet and triplet partial waves are special in the sense that they are apart from the factor $1/\sqrt{1+r^2}$, $r^2 = \sum \xi_i \overline{\xi}_i$ for color triplet holomorphic functions?

Why twistorialization in CP_2 degrees of freedom?

A couple of comments about twistorialization in CP_2 degrees of freedom are in order.

i. Both M^4 and CP_2 twistors could be present for the holomorphic option. M^4 twistors would characterize fermionic momenta and CP_2 twistors to the quantum numbers assignable to deformations of CP_2 type vacuum extremals. CP_2 twistors would be discretized since I_3 and Y have discrete spectrum and it is not at all clear whether twistorialization is useful now. There is excellent motivation for the integration over the flag-manifold defining the choices of color quantization axes. The point is that the choice of conformal basis with well-defined Y and I_3 breaks overall color symmetry $SU(3)$ to $U(2)$ and an integration over all possible choices restores it.

ii. Four-fermion vertex has a singularity corresponding to the situation in which p_1, p_2 and $p_1 + p_2$ assignable to emitted virtual wormhole throat are collinear and thus all light-like. The amplitude must develop a pole as $p_3 + p_3 = p_1 + p_2$ becomes massless. These wormhole contacts would behave like virtual boson consisting of almost collinear pair of fermion and anti-fermion at wormhole throats.

Reduction of scattering amplitudes to subset of $\mathcal{N} = 4$ scattering amplitudes

$\mathcal{N} = 4$ SUSY provides quantitative guidelines concerning the actual construction of the scattering amplitudes.

(a) For single wormhole contact carrying one fermion, one obtains two $\mathcal{N} = 2$ SUSY multiplets from fermions by adding to ordinary one-fermion state right-handed neutrino, its conjugate with opposite spin, or their pair. The net spin projections would be $0, 1/2, 1$ with degeneracies (1,2,1) for fermion helicity 1/2 and $(0, -1/2, -1)$ with same degeneracies for fermion helicity -1/2. These $\mathcal{N} = 2$ multiplets can be imbedded to the $\mathcal{N} = 4$ multiplet containing 2^4 states with spins $(1, 1/2, 0, -1/2, -1)$ and degeneracies given by $(1, 4, 6, 4, 1)$. The amplitudes in $\mathcal{N} = 2$ case could be special cases of $\mathcal{N} = 4$ amplitudes in the same manner as they amplitudes of gauge theories are special cases of those of super-gauge theories. The only difference would be that propagator factors $1/p^2$ appearing in twistorial construction would be replaced by propagators in CP_2 degrees of freedom.

(b) In twistor Grassmannian approach to planar SYM one obtains general formulas for n-particle scattering amplitudes with k positive (or negative helicities) in terms of residue integrals in Grassmann manifold $G(n, k)$. 4-particle scattering amplitudes of TGD, that is 4-fermion scattering amplitudes and their super counterparts would be obtained by restricting to $\mathcal{N} = 2$ sub-multiplets of full $\mathcal{N} = 4$ SYM. The only non-vanishing amplitudes correspond for $n = 4$ to $k = 2 = n - 2$ so that they can be regarded as either holomorphic or anti-holomorphic in twistor variables, an apparent paradox understandable in terms of additional symmetry as explained and noticed by Witten. Four-particle scattering amplitude would be obtained by replacing in Feynman graph description the four-momentum in propagator with CP_2 momentum defined by I_3 and Y for the particle like entity exchanged between fermions at opposite wormhole throats. Analogous replacement should work for twistorial diagrams.

(c) In fact, single fermion per wormhole throat implying 4-fermion amplitudes as building blocks of more general amplitudes is only a special case although it is expected to provide excellent approximation in the case of ordinary elementary particles. Twistorial approach could allow the treatment of also $n > 4$-fermion case using subset of twistorial n-particle amplitudes with Euclidian propagator. One cannot assign right-handed neutrino to each fermion separately but only to the elementary particle 3-surface so that the degeneration of states due to SUSY is reduced dramatically. This means strong restrictions on allowed combinations of vertices.

Some words of critism is in order.

(a) Should one use CP_2 twistors everywhere in the 3-vertices so that only fermionic propagators would remain as remnants of M^4? This does not look plausible. Should one use include to 3-vertices both M^4 and CP_2 type twistorial terms? Do CP_2 twistorial terms trivialize as a consequence of quantization of Y and I_3?

(b) Nothing has been said about modified Dirac operator. The assumption has been that it disappears in the functional integration and the outcome is twistor formalism. The above argument however implies functional integration over the deformations of CP_2 type vacuum extremals.

13.5 Conclusions

The conclusions of these lengthy considerations are following.

(a) Twistorialization takes place naturally at the level of imbedding space and twistor space is Cartesian product of those associated with M^4 and CP_2. The twistor space has interpretation as a flag manifold characterizing the choices of quantization axes for longitudinal momentum components and spin and for isospin and hyper-charge. The integration over twistor space guarantees Lorentz invariance and color invariance.

(b) The Super Virasoro conditions apply only to the entire physical states associated with particle like 3-surfaces containing in general several partonic 2-surfaces. These states can be regarded as bound states of in general non-parallelly propagating massless fermions. Virtual fermions are massless but possess wrong polarization and residue integral replaces fermion propagator with its inverse making sense mathematically. The light-likeness conditions for light-like 3-surfaces allow to deduce the general form of Virasoro conditions. Covariantly constant right-handed neutrinos could define the fermion number conserving analog of $\mathcal{N} = 4$ SUSY.

(c) Apart from CP_2 twistorialization the resulting formalism resembles closely the Grassmannian twistor formalism with one important exception. The 3-vertex of gauge theories is replaced with fermionic 4-vertex which is non-vanishing also for non-parallel on mass shell real momenta and thus avoids the IR singularity of gauge theory vertex.

(d) At the level of WCW twistorial incidence relations have an analogy following from expressibility of Kac-Moody generators as sums of bosonic parts analogous to M^4 coordinates and fermionic parts bilinear in fermionic operators creating WCW spinors and thus analogous to spinors. The attempt to generalize four-momentum conservation to quadratic conditions for WCW spinors fails.

Twistor formalism allows to construct the analogs of Feynman rules for QFT limit of TGD. This process has been rather tortuous and has involved several unpleasant surprises and there are still many open problems.

(a) The generalization of the BFCW recursion relation using 4-fermion vertex with fermions of internal lines massless in real sense and possessing unphysical helicity can be considered. Bosonic emergence is essential element of the construction and suggests a construction very similar to that in gauge theories involving only BFF and BBB vertices as fundamental vertices. This approach however encounters a serious difficulty: contrary to the original optimistic expectations, the fermionic loop defining bosonic wormhole propagator diverges without SUSY but vanishes with SUSY.

(b) The only manner to circumvent the problem is to begin from stringy propagators for real elementary particles identified as pairs of wormhole contacts as required by the Kähler magnetic charges of wormhole throats. Since SUSY is associated with the entire space-time sheet, it does not apply to individual wormhole throat lines separately and does not imply the vanishing of bosonic wormhole throat propagators. As a matter of fact, one cannot even define these propagators since string is the basic object. Stringy propagators in turn remain finite. The challenge is to generalize the BFCW recursion relations. The natural guess is that BFCW cuts are performed for the string world sheets by making some momenta complex. Loops would correspond to stringy loops. In the stringy approach the problems due to non-planarity disappear. There is no specific reason to except the vanishing of stringy loops.

(c) The generalization to gravitational sector is not a problem in sub-manifold gravity since M^4 - the only space-time geometry with Minkowski signature allowing twistor structure - appears as the Cartesian factor of the imbedding space. Furthermore, CP_2 is the only Euclidian 4-D Kähler manifold allowing twistor space with Kähler structure. The analog of twistorial construction in CP_2 degrees of freedom based on the notion of flag manifold can be considered but the situation remains unclear. Graviton as stringy object is geometrically very similar with ordinary elementary particles.

(d) Discrete p-adic coupling constant evolution with local RG invariance is very attractive notion giving a very profound role for the p-adicity but not required by the stringy BFCW. Positivity of Grassmannian - assuming that amplitudes reduce to something proportional to amplituhehdron volume - might be necessary in order to achieve number theoretical universality.

Part IV

MISCELLANEOUS TOPICS

Chapter 14

Does the QFT Limit of TGD Have Space-Time Super-Symmetry?

14.1 Introduction

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the modified Dirac action [K10, K15] . The introduction of a measurement interaction term to the action allows to understand how stringy propagator results and provides profound insights about physics predicted by TGD. Also an old anomalous particle production event [C10] that I learned of in the blog of Tommaso Dorigo [C5] having interpretation in terms of super-symmetry forced to reconsider the possibility of space-time super-symmetry in TGD [K40] .

The appearance of the momentum and color quantum numbers in the measurement interaction couples space-time degrees of freedom to quantum numbers and allows also to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation $\mathcal{N} = 2N$ SUSY algebra or fermionic part of super-conformal algebra with infinite number of oscillator operators results. The addition of fermion in particular mode would define particular super-symmetry. Zero energy ontology implies that fermions as wormhole throats correspond to chiral super-fields assignable to positive or negative energy SUSY algebra whereas bosons as wormhole contacts with two throats correspond to the direct sum of positive and negative energy algebra and to fields which are chiral or antichiral with respect to both positive and negative energy theta parameters. This super-symmetry is badly broken due to the dynamics of the modified Dirac operator which also mixes M^4 chiralities inducing massivation. Since right-handed neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

(a) In TGD framework super-symmetry means addition of fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.

(b) The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The the general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in D=10 and D=11 as the only possible candidates for TOE after it turned out that chiral anomalies cancel.

(c) The massivation of particles is the basic problem of both SUSYs and twistor approach. The fact that particles which are massive in M^4 sense can be interpreted as massless particles in $M^4 \times CP_2$ suggests a manner to understand super-symmetry breaking and massivation in TGD framework. In particular, the massive particle can be put in short representations of SUSY even when the massivation is by p-adic thermodynamics. The octonionic realization of twistors is a very attractive possibility in this framework and quaternionicity condition guaranteeing associativity leads to twistors which are almost equivalent with ordinary 4-D twistors.

It seems possible to formulate even quantum TGD proper in terms of super-field defined in the world of classical worlds (WCW). Super-fields would provide in this framework an elegant book-keeping apparatus for the elements of local Clifford algebra of WCW extended to fields in the $M^4 \times CP_2$ whose points label the positions of the tips of the causal diamonds CDs). What the actual construction of SUSY QFT limit means depends on how strong approximations one wants to make.

(a) The minimal approach to SUSY QFT limit is based on an approximation assuming only the super-multiplets generated by right-handed neutrino or both right-handed neutrino and its antineutrino. The assumption that right-handed neutrino has fermion number opposite to that of the fermion associated with the wormhole throat implies that bosons correspond to $\mathcal{N} = (1,1)$ SUSY and fermions to $\mathcal{N} = 1$ SUSY identifiable also as a short representation of $\mathcal{N} = (1,1)$ SUSY algebra trivial with respect to positive or negative energy algebra. This means a deviation from the standard view but the standard SUSY gauge theory formalism seems to apply in this case.

(b) A more ambitious approach would put the modes of induced spinor fields up to some cutoff into super-multiplets. At the level next to the one described above the lowest modes of the induced spinor fields would be included. The very large value of $\mathcal{N}$ means that $\mathcal{N} \leq \ni\in$ SUSY cannot define the QFT limit of TGD for higher cutoffs. One should generalize SUSYs gauge theories to arbitrary value of $\mathcal{N}$ but there are reasons to expect that the formalism becomes rather complex. More ambitious approach working at TGD however suggest a more general manner to avoid this problem.

   i. One of the key predictions of TGD is that gauge bosons and Higgs can be regarded as bound states of fermion and anti-fermion located at opposite throats of a wormhole contact. This implies bosonic emergence meaning that it QFT limit can be defined in terms of Dirac action. The resulting theory was discussed in detail in [K50] and it was shown that bosonic propagators and vertices can be constructed as fermionic loops so that all coupling constant follow as predictions. One must however pose cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops in order to obtain finite theory and to avoid massivation of bosons. The resulting coupling constant evolution is consistent with low energy phenomenology if the cutoffs in hyperbolic angle as a function of p-adic length scale is chosen suitably.

   ii. The generalization of bosonic emergence that the TGD counterpart of SUSY is obtained by the replacement of Dirac action with action for chiral super-field coupled to vector field as the action defining the theory so that the propagators of bosons and all their super-counterparts would emerge as fermionic loops.

   iii. The huge super-symmetries give excellent hopes about the cancelation of infinities so that this approach would work even without the cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops. Cutoffs have a physical motivation in zero energy ontology but it could be an excellent approximation to take them to infinity. Alternatively, super-symmetric dynamics provides cutoffs dynamically.

(c) The intriguing formal analogy of the Kähler potential and super-potential with the Kähler function defining the Kähler metric of WCW and determined up to a real part of analytic function of the complex coordinates of WCW. This analogy suggests that the action defining the SUSY-Kähler potential- is identifiable as the Kähler function defining WCW Kähler metric at its maximum. Super-potential in turn would correspond to a holomorphic function defining the modification of Kähler function due and the space-time sheet due to measurement interaction. This beautiful correspondence would make WCW geometry directly visible in the properties of QFT limit of TGD.

(d) The condition that $\mathcal{N} = \infty$ variants for chiral and vector superfields exist fixes completely the identification of these fields in zero energy ontology.

i. In this framework chiral fields are generalizations of induced spinor fields and vector fields those of gauge potentials obtained by replacing them with their super-space counterparts. Chiral condition reduces to analyticity in theta parameters thanks to the different definition of hermitian conjugation in zero energy ontology (θ is mapped to a derivative with respect to theta rather than to $\overline{\theta}$) and conjugated super-field acts on the product of all theta parameters.

ii. Chiral action is a straightforward generalization of the Dirac action coupled to gauge potentials. The counterpart of YM action can emerge only radiatively as an effective action so that the notion emergence is now unavoidable and indeed basic prediction of TGD.

iii. The propagators associated with the monomials of n theta parameters behave as $1/p^n$ so that only $J = 0, 1/2, 1$ states propagate in normal manner and correspond to normal particles. The presence of monomials with number of thetas higher than 2 is necessary for the propagation of bosons since by the standard argument fermion and scalar loops cancel each other by super-symmetry. This picture conforms with the identification of graviton as a bound state of wormhole throats at opposite ends of string like object. A second element essential for the finiteness of the theory is that the super-vector bosons emitted by chiral particles move collinearly as indeed required by the wormhole contact picture. Therefore these emission vertices are local in momentum space.

iv. This formulation allows also to use modified gamma matrices in the measurement interaction defining the counterpart of super variant of Dirac operator. Poincare invariance is not lost since momenta and color charges act on the tip of CD rather than the coordinates of the space-time sheet. Hence what is usually regarded as a quantum theory in the background defined by classical fields follows as exact theory. This feeds all data about space-time sheet associated with the maximum of Kähler function. In this approach WCW as a Kähler manifold is replaced by a cartesian power of CP_2, which is indeed quaternionic Kähler manifold. The replacement of light-like 3-surfaces with number theoretic braids when finite measurement resolution is introduced, leads to a similar replacement.

v. Quantum TGD as a "complex square root" of thermodynamics approach suggests that one should take a superposition of the amplitudes defined by the points of a coherence region (identified in terms of the slicing associated with a given wormhole throat) by weighting the points with the Kähler action density. The situation would be highly analogous to a spin glass system since the modified gamma matrices defining the propagators would be analogous to the parameters of spin glass Hamiltonian allowed to have a spatial dependence. This would predict the proportionality of the coupling strengths to Kähler coupling strength and bring in the dependence on the size of CD coming as a power of 2 and give rise to p-adic coupling constant evolution. Since TGD Universe is analogous to 4-D spin glass, also a sum over different preferred extremals assignable to a given coherence regions and weighted by $exp(K)$ is probably needed.

vi. In TGD Universe graviton is necessarily a bi-local object and the emission and absorption of graviton are bi-local processes involving two wormhole contacts: a pair of particles rather than single particle emits graviton. This is definitely something new and defies a description in terms of QFT limit using point like particles. Graviton like states would be entangled states of vector bosons at both ends of stringy curve so that gravitation could be regarded as a square of YM interactions in rather concrete sense. The notion of emergence would suggest that graviton propagator is defined by a bosonic loop. Since bosonic loop is dimensionless, IR cutoff defined by the largest CD present must be actively involved. At QFT limit one can hope a description as a bi-local process using a bi-local generalization of the QFT limit. It turns out that surprisingly simple candidate for the bi-local action exists.

The plan of the chapter reflects partially my own selfish needs. I have to learn space-time super-symmetry at the level of the basic formalism and the best manner to do it is to write it out.

(a) The chapter begins with a brief summary of the basic concepts of SUSYs without doubt revealing my rather fragmentary knowledge about these theories. My only excuse is that I really thought that space-time super-symmetries and the formalism of SUSY theories do not generalize in TGD framework.

(b) Just learning the basics led to amazing findings. First, the anti-commutation relations of the fermionic oscillator operators for the modified Dirac action can be formulated as a generalized SUSY algebra for space-time super-symmetries with large or even infinite value of $\mathcal{N} = 2N$. Secondly, the notion of super-field allows an elegant formulation for the local Clifford algebra of WCW. And thirdly, Kähler potential and super-potential have interpretation in terms of the Kähler function characterizing WCW geometry. I can now grasp why SUSY afficionados are so fascinated about their brain child.

(c) Twistors have indeed become a part of the calculational arsenal of SUSY gauge theories, and TGD leads to a proposal how to avoid the problems caused by massive particles by using the notion of masslessness in 8-D sense and the notion of induced octo-twistor. At QFT limit the idea is simple: massless free particles correspond to geodesics of $M^4 \times CP_2$ and in QFT formulation one keeps just the knowledge that particle moves along geodesic circle $S^1 \times CP_2$.

(d) SUSY algebras at the level of quantum TGD proper and its QFT limit are discussed and the conditions guaranteeing that standard SUSY formalism applies are discussed: in this theory fermions *resp.* bosons correspond to $\mathcal{N} = 1$ *resp.* $\mathcal{N} = (1, 1)$ SUSY.

(e) Finally, SUSY QFT limit of quantum TGD based on the generalization of the bosonic emergence [K50] is proposed. The generalization of SUSY YM action emerges radiatively through super-symmetric fermion loops in this framework and the counterpart of chiral action is the fundamental action. The first approach applying only for small values of $\mathcal{N}$ relies on the replacement of the Dirac action coupled to gauge potentials with the Kähler potential defined by WCW Kähler function at its maximum. Second approach is inspired by $\mathcal{N} = \infty$ case and based on different definition of super-fields.

This chapter is a fourth one in a series containing two chapters about twistors [K78, K80] and a chapter about bosonic emergence [K50] . At this moment the chapter about the generalization of twistor Grassmannian approach [B33] and Yangian symmetry [A36] to TGD framework [K80] represents the most realistic view about what quantum TGD might be. Although a lot of cognitive dust is present, this chapter together with the chapters [K78, K50] might be helpful for the reader trying to get a better understanding about my motivations and goals. There is also a connection with the topological explanation of family replication phenomenon: by combining the assumption that SU(3) acts as dynamical symmetry acting on fermion families for vertices allows only BFF type vertices and their super-symmetric generalizations at fundamental level [K13]. Also bosonic emergence allows only BFF type vertices: this simplifies enormously the construction of M-matrix. Right-handed neutrinos have been the longstanding poorly understood issue of TGD and one can develop arguments that $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY emerges naturally in TGD framework and corresponds to the addition of a collinear right-handed neutrino and and antineutrino to the state representing massless particle. It must be emphasized that also this chapter is like a lab note book, a story telling how ideas have evolved and continue to evolve.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L12]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L13]. The topics relevant to this chapter are given by the following list.

- SUSY and TGD [L39]

14.2 SUSY briefly

The Tasi 2008 lectures by Yuri Shirman [B55] provide a modern introduction to 4-dimensional $\mathcal{N} = 1$ super-symmetry and super-symmetry breaking. In TGD framework the super-symmetry is 8-dimensional super-symmetry induced to 4-D space-time surface and one $\mathcal{N} = 2N$ can be large so that this introduction is quite not enough for the recent purposes. This section provides only a brief summary of the basic concepts related to SUSY algebras and SUSY QFTs and the breaking of super-symmetry is mentioned only by passign. I have also listed the crucial basic facts about $\mathcal{N} > 1$ super-symmetry [B4, B16] with emphasis in demonstrating that for 8-D super-gravity with one time-dimension super-charges are non-Hermitian and that Majorana spinors are absent as required by quantum TGD.

14.2.1 Weyl fermions

Gamma matrices in chiral basis.

$$\begin{array}{llll}
\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \ , & \gamma_5 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix} \ , & & \\
\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ , & \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , & \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , & \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ , \\
\overline{\sigma}^0 = \sigma^0 \ , & \overline{\sigma}^i = -\sigma^i \ . & &
\end{array} \tag{14.2.1}$$

Note that Pauli sigma matrices can be interpreted as matrix representation for hyper-quaternion units.

Dirac spinors can be expressed in terms of Weyl spinors as

$$\Psi = \begin{pmatrix} \eta^\alpha \\ \overline{\chi}^*_{\dot\alpha} \end{pmatrix} \ . \tag{14.2.2}$$

Note that does not denote complex conjugation and that complex conjugation transforms non-dotted and dotted indices to each other. η and $\overline{\chi}$ are both left handed Weyl spinors and transform according to complex conjugate representations of Lorentz group and one can interpret $\overline{\chi}$ as representing that charge conjugate of right handed Dirac fermion.

Spinor indices can be lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot\alpha\dot\beta}$ and one has

$$\begin{array}{ll}
\eta^\alpha \eta_\alpha = 0 \ , & \overline{\chi}^*_{\dot\alpha}\overline{\chi}^*_{\dot\alpha} = 0 \ per, \\
\eta\overline{\chi} = \overline{\chi}\eta = \epsilon^{\alpha\beta}\eta_\alpha\overline{\chi}_\beta \ , & \eta^*\overline{\chi}^* = \overline{\chi}^*\eta^* = \epsilon^{\alpha\beta}\eta^*_\alpha\overline{chi}^*_\beta \ .
\end{array} \tag{14.2.3}$$

Left-handed and right handed spinors can be combined to Lorentz vectors as

$$\eta^*_{\dot{\alpha}}\sigma^{\mu\dot{\alpha}\alpha}\eta_\alpha = -\eta^{*\alpha}\sigma^\mu_{\alpha\dot{\alpha}}\eta^{*\dot{\alpha}} \quad . \tag{14.2.4}$$

The SUSY algebra at QFT limit differs from the SUSY algebra defining the fundamental anti-commutators of the fermionic oscillator operators for the induced spinor fields since the modified gamma matrices defined by the Kähler action are replaced with ordinary gamma matrices. This is quite a dramatic difference and raises two questions.

The Dirac action

$$L = i\overline{\Psi}\partial_\mu\gamma^\mu\Psi - m\overline{\Psi}\Psi \tag{14.2.5}$$

for a massive particle reads in Weyl representation as

$$L = i\eta^*\partial_\mu\sigma^\mu\eta + i\overline{\chi}^*\partial_\mu\overline{\sigma}^\mu\overline{\chi} - m\overline{\chi}\eta - m\overline{\chi}^*\eta^* \quad . \tag{14.2.6}$$

14.2.2 SUSY algebras

In the following 4-D SUSY algebras are discussed first following the representation of [B55] . After that basic results about higher-dimensional SUSY algebras are listed with emphasis on 8-D case.

$D = 4$ SUSY algebras

Poincare SUSY algebra contains as super-generators transforming as Weyl spinors transforming in complex conjugate representations of Lorentz group. The basic anti-commutation relations of Poincare SUSY algebra in Weyl fermion basis can be expressed as

$$\begin{aligned}
\{Q_\alpha, Q_{\dot{\beta}}\} &= 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu \quad , \\
\{Q_\alpha, Q_\beta\} &= \{Q_{\dot{\alpha}}, Q_{\dot{\beta}}\} = 0 \quad , \\
[Q_\alpha, P_\mu] &= [Q_{\dot{\alpha}}, P_\mu] = 0 \quad .
\end{aligned} \tag{14.2.7}$$

By taking a trace over spinor indices one obtains expression for energy as $P^0 = \sum_i Q_i\overline{Q}_i + \overline{Q}_iQ_i$. Since super-generators must annihilated super-symmetric ground states, the energy must vanish for them.

This algebra corresponds to simplest $\mathcal{N} = 1$ SUSY in which only left-handed fermion appears. For $\mathcal{N} = 1$ SUSY the super-charges are are hermitian whereas in TGD framework supercharges carry fermion number. This implies that super-charges come in pairs of super charge so that $\mathcal{N} = 2N$ must hold true and its hermitian conjugate and only the second half of super-charges can annihilate vacuum state. Weyl spinors must also come as pairs of right- and left-handed spinors.

The construction generalizes in a straightforward manner to allow arbitrary number of fermionic generators. The most general anti-commutation relations in this case are

$$\begin{aligned}
\{Q_{i\alpha}, Q^j_{\dot{\beta}}\} &= 2\delta^j_i\sigma^\mu_{\alpha\dot{\beta}}P_\mu \quad , \\
\{Q_{i\alpha}, Q_{j\beta}\} &= \epsilon_{\alpha\beta}Z_{ij}, \\
\{Q_{\dot{\alpha}}, Q_{\dot{\beta}}\} &= \epsilon^{\dot{\alpha}\dot{\beta}}Z^*_{ij} \quad .
\end{aligned} \tag{14.2.8}$$

The complex constants are called central charges because they commute with all generators of the super-Poincare group.

Higher-dimensional SUSY algebras

The character of supersymmetry is sensitive to the dimension D of space-time and to the signature of the space-time metric higher dimensions [B4] . The available spinor representations depend on k; the maximal compact subgroup of the little group of the Lorentz that preserves the momentum of a massless particle is $Spin(d-1) \times Spin(D-d-1)$, where d is the number of spatial dimensions $D-d$ is the number time dimensions and k is defined as $k = 2d - D$. Due to the mod 8 Bott periodicity of the homotopy groups of the Lorentz group, really we only need to consider $k = 2d - D$ modulo 8. In TGD framework one has $D = 8$, $d = 7$ and $k = 6$.

For any value of k there is a Dirac representation, which is always of real dimension $N = [2^{1+[(2d-k)/2]}$ where $[x]$ is the greatest integer less than or equal to x. For TGD this of course gives $2^5 = 32$ corresponding to complex 8-component quark and lepton like spinors. For $-2 \leq k \leq 2$ not realized in TGD there is a real Majorana spinor representation, whose dimension is $N/2$. When k is even (TGD) there is a Weyl spinor representation, whose real dimension is $N/2$. For $k\ mod\ 8 = 0$ (say in super-string models) there is a Majorana-Weyl spinor, whose real dimension is $N/4$. For $3 \leq k \leq 5$ so called symplectic Majorana spinor with dimension $D/2$ and for $k = 4$ symplectic Weyl-Majorana spinors with dimension $D/4$ is possible. The matrix Γ_{D+1} defined as the product of all gamma matrices has eigenvalues $\pm(-1)^{-k/2}$. The eigenvalue of Γ_{D+1} is the chirality of the spinor. CPT theorem implies that the for $D\ mod\ 4 = 0$ the numbers of left and right handed super-charges are same. For $D\ mod\ 4 = 2$ the numbers of left and right handed chiralities can be different and corresponding SUSYs are classified by $\mathcal{N} = (\mathcal{N}_L, \mathcal{N}_R)$, where $\mathcal{N}_L$ and $\mathcal{N}_R$ are the numbers of left and right handed super charges. Note that in TGD the chiralities are ± 1 and correspond to quark and leptons like spinors.

TGD does not allow super-symmetry with Majorana particles. It is indeed possible to have non-hermitian super-charges [B16] in dimension $D = 8$. In $D = 8$ SUGRA with one time dimension super-charges ar non-hermitian and Majorana particles are absent. Also in $D = 4$ SUGRA predicts super-charges are non-hermitian super-charges but Majorana particles are present.

(a) $D = 8$ super-gravity corresponds to $\mathcal{N} = 2$ and allows complex super-charges $Q^i_\alpha \in 8$ and their hermitian conjugates $\overline{Q}^i_\alpha \in \overline{8}$. The group of R symmetries is $U(2)$. Bosonic fields consists the metric g_{mn}, seven real scalars, six vectors, three 2-form fields and one 3-form field. Fermionic fields consist of two Weyl (left) gravitini $\psi^{\alpha i}$, six Weyl (right) spinors plus their hermitian conjugates of opposite chirality. There are no Majorana fermions.

(b) $D = 4, \mathcal{N} = 8$ SUGRA is second example allowing complex non-hermitian super-charges. The supercharges $Q^i_\alpha \in 2$ and their hermitian conjugates $\overline{Q}^i_{\dot\alpha} \in \overline{2}$. R-symmetry group is $U(8)$. Bosonic fields are metric g_{mn}, 70 real scalars and 28 vectors. Fermionic fields are 8 Majorana gravitini $\Psi^{a,i}_m$ and 56 Majorana spinors.

For $\mathcal{N} = 2N$ and at least $D = 8$ with one time dimension the super charges can be assumed to come in hermitian conjugate pairs and the non-vanishing anti-commutators can be expressed as

$$\begin{aligned} \{Q^\dagger_{i\alpha}, Q^j_{\dot\beta}\} &= 2\delta^j_i \sigma^\mu_{\alpha\dot\beta} P_\mu \ , \\ \{Q^\dagger_{i\alpha}, Q_{j\beta}\} &= \epsilon_{\alpha\beta} Z_{ij} , \\ \{Q^\dagger_{\dot\alpha}, Q_{\dot\beta}\} &= \epsilon^{\dot\alpha\ dot\beta} Z^*_{ij} \ . \end{aligned} \tag{14.2.9}$$

In this case Z_{ij} is anti-hermitian matrix. 8-D chiral invariance (separate conservation of lepton and quark numbers) suggests strongly that that the condition $Z_{ij} = 0$ must hold holds true. A given pair of super-charges is analogous to creation and annihilation operators for a given fermionic chirality. In TGD framework opposite chiralities correspond to quark and lepton like spinors.

Representations of SUSY algebras in dimension $D = 4$

The physical components of super-fields correspond to states in the irreducible representations of SUSY algebras. The representations can be constructed by using the basic anti-commutation relations for $Q_{i\alpha}$ and $Q_{j\dot{\alpha}}$, $i, j \in \{1, ..., \mathcal{N}\}$, $\alpha, \dot{\alpha} \in \{1, 2\}$. The representations can be classified to massive and massless ones. Also the presence of central charges affects the situation. A given irreducible representation is characterized by its ground state and R-parity assignments distinguish between representations with the same spin content, say fermion and its scalar super-partner and Higss with its fermionic super-partner.

(a) In the massive case one obtains in the rest system just fermionic creation operators and $2^{\mathcal{N}}$ annihilation operators. The number of states created from a vacuum state with spin s_0 is $2\mathcal{N}$ and maximum spin is $s_0 + \mathcal{N}/2$. For instance, for $\mathcal{N} = 1$ and $s_0 = 0$ one obtains for 4 states with spins $J \leq 1/2$. Renormalizability requires massive matter to have $s \leq 1/2$ so that only $\mathcal{N} = 1$ is possible in this case. For particles massless at fundamental level and getting their masses by symmetry breaking this kind of restriction does not apply.

(b) In the massless case only one half of fermionic oscillator operators have vanishing anti-commutators corresponding to the fact that for massless state only the second helicity is physical. This implies that the number of states is only $2^{\mathcal{N}}$ and the helicities vary from λ_0 to $\lambda_0 + \mathcal{N}/2$. For $\mathcal{N} = 1$ the representation is 2-dimensional.

(c) In the presence of central charges $Z_{ij} = -Z_{ji}$ the representations are in general massive (Z_{ij} has dimensions of mass), $U(N)$ acts as symmetries of Z, and since Z^2 is symmetric its diagonalizability implies that Z matrix can be cast by a unitary transformation into a direct sum of 2-D antisymmetric real matrices multiplied by constants Z_i. Therefore the super-algebra can be cast in diagonal form with anti-commutators proportional to $M \pm Z_m$ with $M - Z_m \geq 0$ by unitarity. This implies the celebrated Bogomol'nyi bound $M \geq max\{Z_n\}$. For this value of varying mass parameter it is possible to have reduction of the dimension of the representation by one half. If the eigenvalues Z_n are identical the number of states is reduced to that for a massless representation. This multiplet is known as short BPS multiplet. Although BPS multiplets are massive (mass is expressible in terms of Higgs expectation value) they form multiplets shorter than the usual massive SUSY multiplets.

14.2.3 Super-space

The heuristic view about super-space [B15] is as a manifold with D local bosonic coordinates x^μ and $\mathcal{N}D/2$ complex anti-commuting spinor coordinates θ_i^α and their complex conjugates $\overline{\theta}^i_{\dot{\alpha}} = (\theta_i^\alpha)^*$. For $\mathcal{N} = 1$, which is relevant to minimally super-symmetric standard model (MSSM), the spinors θ can also chosen to be real that is Majorana spinors, so that one has 4 bosonic and four real coordinates. In TGD framework one must however use Weyl spinors.

The anti-commutation relations for the super-coordinates are

$$\{\theta_\alpha, \theta_\beta\} = \{\theta_{\dot{\alpha}}, \theta_{\dot{\beta}}\} = \{\theta_\alpha, \theta_{\dot{\beta}}\} = 0 \ . \tag{14.2.10}$$

The integrals over super-space in 4-D $\mathcal{N} = 1$ case are defined by the following formal rules which actually state that super-integration is formally analogous to derivation.

$$\begin{aligned}
\int d\theta = \int d\overline{\theta} = \int d\theta\overline{\theta} = \int d\overline{\theta}\theta = 0 \ , \\
\int d\theta^\alpha d\theta_\beta = \delta^\alpha_\beta \ , \ \int d\overline{\theta}_{\dot{\alpha}} d\overline{\theta}_{\dot{\beta}} = \delta^{\dot{\beta}}_{\dot{\alpha}} \ , \\
\int d^2\theta\theta^2 = \int d^2\overline{\theta}\overline{\theta}^2 \ , \ \int d^4\theta\theta^2\overline{\theta}^2 = 1 \ .
\end{aligned} \tag{14.2.11}$$

Here the shorthand notations

$$d^2\theta \equiv -\frac{1}{4}\epsilon_{\alpha\beta}d\theta^{\alpha}d\theta^{\beta} \quad ,$$
$$d^2\overline{\theta} \equiv -\frac{1}{4}\epsilon^{\dot{\alpha}\dot{\beta}}d\theta_{\dot{\alpha}}d\ theta_{\dot{\beta}} \quad ,$$
$$d^4\theta \equiv d^2\theta d^2\overline{\theta} \quad . \tag{14.2.12}$$

are used.

The generalization of the formulas to $D > 4$ and $\mathcal{N} > 1$ cases is trivial. In infinite-dimensional case relevant for the super-symmetrization of the WCW geometry in terms of local Clifford algebra of WCW to be proposed later the infinite number of complex theta parameters poses technical problems unless one defines super-space functions properly.

Chiral super-fields

Super-multiplets can be expressed as single super-field define in super-space. Super-field can be expanded as a Taylor series with respect to the theta parameters. In 4-dimensional $\mathcal{N} = 1$ case one has

$$\Phi(x^{\mu},\theta,\overline{\theta}) = \phi(x^{\mu}) + \theta\eta(x^{\mu}) + \overline{\theta}\eta^{\dagger}(x^{\mu}) + \overline{\theta\ sigma}^{\alpha}\theta V_{\alpha}(x^{\mu}) + \theta^2F(x^{\mu}) + \overline{\theta^2 F}(x^{\mu}) ... + \theta^2\overline{\theta}^2D(x^{\mu}) \tag{14.2.13}$$

The action of super-symmetries on super-fields can be expressed in terms of super-covariant derivatives defined as

$$D_{\alpha} = \frac{\partial}{\partial\theta^{\alpha}} - i\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{\theta}^{\ dot\alpha}\frac{\partial}{\partial_{\mu}} \quad , \quad \overline{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} + i\theta^{\alpha}\sigma^{\mu}_{\alpha\ dot\alpha}\frac{\partial}{\partial_{\mu}} \quad . \tag{14.2.14}$$

This allows very concise realization of super-symmetries.

General super-field defines a reducible representation of super-symmetry. One can construct irreducible representations of super-fields a pair of chiral and antichiral super-fields by posing the condition

$$\overline{D}_{\dot{\alpha}}\Phi = 0 \quad , \quad D_{\alpha}\Phi^{\dagger} = 0 \quad . \tag{14.2.15}$$

The hermitian conjugate of chiral super-field is anti-chiral.

Chiral super-fields can be expressed in the form

$$\Phi = \Phi(\theta, y^{\mu}) \quad , \quad y^{\mu} = x^{\mu} + i\overline{\theta}\sigma^{\mu}\theta \quad , y^{\mu\dagger} = x^{\mu} - i\overline{\theta}\sigma^{\mu}\theta \quad . \tag{14.2.16}$$

These formulas generalize in a rather straightforward manner to $D > 4$ and $\mathcal{N} > 1$ case.

It is easy to check that any analytic function of a chiral super-field, call it $W(\Phi)$, is a chiral super-field. In super-symmetries its θ^2 component transforms by a total derivative so that the action defined by the super-space integral of $W(\phi)$ is invariant under super-symmetries. This allows to construct super-symmetric actions using $W(\Phi)$ and $W(\Phi^{\dagger})$. The so called super-potential is defined using the sum of $W(\Phi) + W(\Phi^{\dagger})$.

Analytic functions of does not give rise to kinetic terms in the action. The observation $\theta^2\overline{\theta}^2$ component of a real function of chiral super-fields transforms also as total derivative under super-symmetries allows to circumvent this problem by introducing the notion of Kähler potential $K(\Phi, \Phi^\dagger)$ as a real function of chiral super-field and its conjugate. In he simplest case one has

$$K = \sum_i \Phi_i^\dagger \Phi_i \ . \qquad (14.2.17)$$

$L_K = \int K d^4\theta$ gives rise to simples super-symmetric action for left-handed fermion and its scalar super-partner.

Kähler potential allows an interpretation as a Kähler function defining the Kähler metric for the manifold defined by the scalars ϕ_i. This Kähler metric depends in the general case on ϕ_i and appears in the kinetic term of the super-symmetric action. Super-potential in turn can be interpreted as a counterpart of real part of a complex function which can be added to the Kähler function without affect the Kähler metric. This geometric interpretation suggests that in TGD framework every complex coordinate ϕ_i of WCW defines a chiral super-field whose bosonic part.

Wess-Zumino model as simple example

Wess-Zumino model without interaction term serves as a simple illustration of above formal considerations. The action density of Wess-Zumino Witten model can be deduced by integration Kähler potential $K = \Phi^\dagger\Phi$ for chiral super fields over theta parameters. The result is

$$L = \partial_u \phi^* \partial^\mu \phi + i\eta^* \partial^\mu \eta + F^* F \ . \qquad (14.2.18)$$

The action of super-symmetry

$$\delta\Phi = \epsilon^\alpha D_\alpha \Phi \ , \quad \delta\Phi^\dagger = \overline{\epsilon}^{\dot{\alpha}} \overline{D}_{\dot{\alpha}} \Phi \ , \quad \epsilon_{\dot{\alpha}} = \epsilon^{*\alpha} \qquad (14.2.19)$$

gives the transformation formulas

$$\delta\phi = \epsilon^\alpha \eta_\alpha \ , \quad \delta\eta = -i\eta^{*\dot{\alpha}} \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \phi + \epsilon_\alpha F \ , \quad \delta F = -i\epsilon_{\dot{\alpha}} \overline{\sigma}^{\mu\dot{\alpha}\alpha} \ partial_\mu \eta_\alpha \qquad (14.2.20)$$

plus their hermitian conjugates. The corresponding Noether current is indeed hermitian since the transformation parameters ϵ^α and $\overline{\epsilon}_{\dot{\alpha}} = \epsilon^{*\alpha}$ appear in it and cannot be divided away. This conserved current has as such no meaning and the statement that ground state is annihilated by the corresponding super-charge means that vacuum field configuration rather than Fock vacuum remains invariant under supersymmetries. Rather, the breaking of super-symmetry by adding a super-potential implies that F develops vacuum expectation and the vacuum solution $(\phi = 0, \eta = 0, F = constant)$ of field equations is not anymore invariant super super-symmetries.

The non-hermitian parts of the super current corresponding to different fermion numbers are separately conserved and corresponding super-charges are non-Hermitian and together with other charges define a super-algebra which to my best understanding is not equivalent with the super-algebra defined by allowing the presence of anti-commuting parameters ϵ. The situation is similar in TGD where one class of non-hermitian super-currents correspond to the modes of the induced spinor fields contracted with $\overline{\Psi}$ and their conjugates. The octonionic solution ansatz for the induced spinor field allows to express the solutions in terms of two complex scalar functions so that the super-currents in question would be analogous to those of $\mathcal{N}=2$ SUSY and one might see the super-symmetry of quantum TGD extended super-symmetry obtained from the fundamental $\mathcal{N}=2$ super-symmetry.

Vector super-fields and supersymmetric variant of YM action

Chiral super-fields allow only the super-symmetrization of Dirac action. The super-symmetrization of YM action requires the notion of a hermitian vector super field $V=V^\dagger$, whose components correspond to vector bosons, their super-counterparts and additional degrees of freedom which cannot be dynamical. These degrees of freedom correspond gauge degrees of freedom.

In the Abelian case the gauge symmetries are realized as $V\to V+\Lambda+\Lambda^\dagger$, where Λ is a chiral super-field. These symmetries induce gauge transformations of the vector potential. Their action on chiral super-fields is $\Phi\to exp(-q\Lambda)\Phi$, $\Phi^\dagger\to\Phi^\dagger exp(-\Lambda^\dagger)$. In non-Abelian case the realization is as $exp(V)\to exp(-\Lambda^\dagger)exp(V)exp(\Lambda)$ so that the modified Kähler potential $K(\Phi^\dagger, exp(qV)\Phi)$ remains invariant.

One can assign to V a gauge invariant chiral spinor super-field as

$$
\begin{aligned}
W_\alpha &= -\frac{1}{4}\overline{D}^2(e^V D_\alpha e^{-V}) \ , \\
\overline{D}^2 &= \epsilon^{\dot{\alpha}\dot{\beta}}\overline{D}_{\dot{alpha}}\overline{D}_{\dot{\beta}}
\end{aligned}
\tag{14.2.21}
$$

defining the analog of gauge field. $\overline{D}^2$ eliminates all terms the exponent of $\overline{\theta}$ is higher than that of θ since these would spoil the chiral super-field property (the anti-commutativity of super-covariant derivatives $\overline{D}_{\dot{\alpha}}$ makes this obvious). D_α in turn eliminates from the resulting scalar part so that one indeed has chiral spinor super-field. In higher dimensions and for larger value of $\mathcal{N}$ the definition of W_α must be modified in order to achieve this: what is needed is the product of all derivatives $\overline{D}_{i\dot{\alpha}}$.

The analytic functions of chiral spinor super-fields are chiral super-fields and θ^2 component of $W^\alpha W_\alpha$ transforms as a total derivatives. The super-symmetric Lagrangian of U(1) theory can be written as

$$
L = \frac{1}{4g^2}\left(\int d^2\theta W^\alpha W_\alpha + \int d^2\overline{\theta} W^\dagger_{\dot{\alpha}} W^\dagger_{\dot{\alpha}}\right) \ . \tag{14.2.22}
$$

Note that in standard form of YM action $1/2g^2$ appears.

R-symmetry

R-symmetry is an important concomitant of super-symmetry. In $\mathcal{N}=1$ case R-symmetry performs a phase rotation $\theta\to e^{i\alpha}\theta$ for the super-space coordinate θ and an opposite phase rotation for the differential $d\theta$. For $\mathcal{N}>1$ R-symmetries are $U(N)$ rotations. R-symmetry is an additional symmetry of the Lagrangian terms due to Kähler potential since both $d^4\theta$ (and its generalization) as well as Kähler potential are real. Also super-symmetric YM action is R-invariant. R-symmetry is a symmetry of if super-potential W only if it has super-charge $Q_R=2$ ($Q_R=2\mathcal{N}$) in order to compensate the super-charge of $d^{2\mathcal{N}}\theta$.

14.2.4 Non-renormalization theorems

Super-symmetry gives powerful constraints on the super-symmetric Lagrangians and leads to non-renormalization theorems.

The following general results about renormalization of supersymmetric gauge theories hold true (see [B55] , where heuristic justification of the non-renormalization theorems and explicit formulas are discussed).

(a) Super-potential is not affected by the renormalization.

(b) Kähler potential is subject to wave function renormalization in all orders. The renormalization depends on the parameters with dimensions of mass. In particular, quadratic divergences to masses cancel.

(c) Gauge coupling suffers renormalization only by a constant which corresponds to one-loop renormalization. Any renormalization beyond one loop is due to wave function renormalization of the Kähler potential and it is possible to calculate the beta function exactly.

It is interesting to try to see these result from TGD perspective.

(a) In TGD framework super-potential interpreted as defining the modification of WCW Kähler function, which does not affect Kähler metric and would reflect measurement interaction. The non-renormalization of W would mean that the measurement interaction is not subject to renormalization. The interpretation is in terms of quantum criticality which does not allow renormalization of the coefficients appearing in the measurement interaction term since otherwise Kähler metric of WCW would be affected.

(b) The wave function renormalization of Kähler potential would correspond in TGD framework scaling of the WCW Kähler metric. Quantum criticality requires that Kähler function remains invariant. Also since no parameters with dimensions of mass are available, there is temptation to conclude that wave function renormalization is trivial.

(c) Only the gauge coupling would be suffer renormalization. If one however believes in the generalization of bosonic emergence it is Kähler function which defines the SUSY QFT limit of TGD so that gauge couplings follow as predictions and their renormalization is a secondary -albeit real- effect having interpretation in terms of the dependence of the gauge coupling on the p-adic length scale. The conclusion would be that at the fundamental level the quantum TGD is RG invariant.

14.3 Does TGD allow the counterpart of space-time super-symmetry?

The question whether TGD allows space-time super-symmetry or something akin to it has been a longstanding problem. A considerable progress in the respect became possible with the better understanding of the modified Dirac equation. At the same time I learned about almost 15 year old striking $ee\gamma\gamma+\not{E}$ detected by CDF collaboration [C10, C9] from Tommaso Dorigo's blog [C5] .

14.3.1 Basic data bits

Let us first summarize the data bits about possible relevance of super-symmetry for TGD before the addition of the 3-D measurement interaction term to the modified Dirac action [K10, K22].

(a) Right-handed covariantly constant neutrino spinor ν_R defines a super-symmetry in CP_2 degrees of freedom in the sense that Dirac equation is satisfied by covariant constancy and there is no need for the usual ansatz $\Psi = D\Psi_0$ giving $D^2\Psi = 0$. This super-symmetry allows to construct solutions of Dirac equation in CP_2 [A91, A69, A80, A66] .

(b) In $M^4 \times CP_2$ this means the existence of massless modes $\Psi = \not{p}\Psi_0$, where Ψ_0 is the tensor product of M^4 and CP_2 spinors. For these solutions M^4 chiralities are not mixed unlike for all other modes which are massive and carry color quantum numbers depending on the CP_2 chirality and charge. As matter fact, covariantly constant right-handed neutrino spinor mode is the only color singlet. The mechanism leading to non-colored states for fermions is based on super-conformal representations for which the color is neutralized [K37, K45] . The negative conformal weight of the vacuum also cancels the enormous contribution to mass squared coming from mass in CP_2 degrees of freedom.

(c) Right-handed covariantly constant neutrino allows to construct the gamma matrices of the world of classical worlds (WCW) as fermionic counterparts of Hamiltonians of WCW. This gives rise super-symplectic symmetry algebra having interpretation also as a conformal algebra. Also more general super-conformal symmetries exist.

(d) Space-time (in the sense of Minkowski space M^4) super-symmetry in the conventional sense of the word is impossible in TGD framework since it would require require Majorana spinors. In 8-D space-time with Minkowski signature of metric Majorana spinors are definitely ruled out by the standard argument leading to super string model. Majorana spinors would also break separate conservation of lepton and baryon numbers in TGD framework.

14.3.2 Could one generalize super-symmetry?

Could one then consider a more general space-time super-symmetry with "space-time" identified as space-time surface rather than Minkowski space?

(a) The TGD variant of the super-symmetry could correspond quite concretely to the addition to fermion and boson states right-handed neutrinos. Since right-handed neutrinos do not have electro-weak interactions, the addition might not appreciably affect the mass formula although it could affect the p-adic prime defining the mass scale.

(b) The problem is to understand what this addition of the right-handed neutrino means. To begin with, notice that in TGD Universe fermions reside at light-like 3-surfaces at which the signature of induced metric changes. Bosons correspond to pairs of light-like wormhole throats with wormhole contact having Euclidian signature of the induced metric. It is essential that either fermion or anti-fermion in the boson state carries what might be called un-physical polarization in the standard conceptual framework. Only in this manner the helicities can come out correctly. The assumption that the bosonic wormhole throats correspond to positive and negative energy space-time sheets realizes this constraint in the framework of zero energy ontology.

(c) The super-symmetry as an addition to the fermion state a second wormhole throats carrying right handed neutrino quantum numbers does not make sense since the resulting state cannot be distinguished from gauge boson or Higgs type particle. The light-like 3-surfaces can however carry fermion numbers up to the number of modes of the induced spinor field, which is expected to be infinite inside string like objects having wormhole throats at ends and finite when one has space time sheets containing the throats [K22] . In very general sense one could say that each mode defines a very large broken N-super-symmetry with the value of N depending on state and light-like 3-surface. The breaking of this super-symmetry would come from electro-weak - , color - , and gravitational interactions. Right-handed neutrino would by its electro-weak and color inertness define a minimally broken super-symmetry.

(d) What this addition of the right handed neutrinos or more general fermion modes could precisely mean? One cannot assign fermionic oscillator operators to right handed neutrinos which are covariantly constant in both M^4 and CP_2 degrees of freedom since the modes with vanishing energy (frequency) cannot correspond to fermionic oscillator operator creating a physical state since one would have $a = a^\dagger$. The intuitive view is that all the spinor modes move in an exactly collinear manner -somewhat like quarks inside hadron do approximately.

14.3.3 Modified Dirac equation briefly

The answer to the question what "collinear motion" means mathematically emerged from the recent progress in the understanding of the modified Dirac equation.

(a) The modified Dirac action involves two terms. Besides the original 4-D modified Dirac action there is measurement interaction. This term correlates space-time geometry with quantum numbers assignable to super-conformal representations and is also necessary to obtain almost-stringy propagator.

(b) The modified Dirac equation (or Kähler Dirac equation) reads as

$$D_K\Psi = 0 \ . \tag{14.3.1}$$

in the interior of space-time surface. The boundary variation of K-D equation gives the term

$$\Gamma^n\Psi = 0 \tag{14.3.2}$$

space-like ends. Clearly, Kähler-Dirac gamma matrix Γ^n in normal direction must be light-like which motives the proposal that the action of Γ^n in suitable coordinates is equal to that of algebraic variant of massless Dirac operator $p^k\gamma_k$. p^k would define the massless momentum assignable to the incoming state associated with string curve in question (the modes of Kähler-Dirac equation are restricted to string world sheets in the generic case).

(c) In order to obtain non-trivial fermion propagator one must add to Kähler-Dirac action Chern-Simons Dirac term located at partonic orbits at which the signature of the induced metric changes. The modes of induced spinor field can be required to be generalized eigenmodes of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ with p^k identified as virtual momentum so that massless Dirac propagator is obtained. By super-symmetry one must add to Kähler action Chern-Simons term located at partonic orbits and this term must cancel the Chern-Simons term coming from Kähler action by weak form of electric-magnetic duality so that only the Chern-Simons terms associated with space-like ends of the space-time surface remain. These terms reduce to Chern-Simons terms only if one poses weak form of electric magnetic duality also here. This is not necessary.

(d) The quantum numbers characterizing zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse. Thermodynamics would naturally couple to the space-time geometry via the thermodynamical or quantum averages of the quantum numbers.

14.3.4 TGD counterpart of space-time super-symmetry

This picture allows to define more precisely what one means with the approximate super-symmetries in TGD framework.

(a) One can in principle construct many-fermion states containing both fermions and anti-fermions at given light-like 3-surface. The four-momenta of states related by super-symmetry need not be same. Super-symmetry breaking is present and has as the space-time correlate the deviation of the modified gamma matrices from the ordinary M^4 gamma matrices. In particular, the fact that $\hat{\Gamma}^\alpha$ possesses CP_2 part in general means that different M^4 chiralities are mixed: a space-time correlate for the massivation of the elementary particles.

(b) For right-handed neutrino super-symmetry breaking is expected to be smallest but also in the case of the right-handed neutrino mode mixing of M^4 chiralities takes place and breaks the TGD counterpart of super-symmetry.

(c) The fact that all helicities in the state are physical for a given light-like 3-surface has important implications. For instance, the addition of a right-handed antineutrino to right-handed (left-handed) electron state gives scalar (spin 1) state. Also states with fermion number two are obtained from fermions. For instance, for e_R one obtains the states $\{e_R, e_R\nu_R\overline{\nu}_R, e_R\overline{\nu}_R, e_R\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,0,1)$. For e_L one obtains the states $\{e_L, e_L\nu_R\overline{\nu}_R, e_L\overline{\nu}_R, e_L\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,1,0)$. In the case of gauge boson and Higgs type particles -allowed by TGD but not required by p-adic mass calculations- gauge boson has 15 super partners with fermion numbers $[2,1,0,-1,-2]$.

The cautious conclusion is that the recent view about quantum TGD allows the analog of super-symmetry which is necessary broken and for which the multiplets are much more general than for the ordinary super-symmetry. Right-handed neutrinos might however define something resembling ordinary super-symmetry to a high extent. The question is how strong prediction one can deduce using quantum TGD and proposed super-symmetry.

(a) For a minimal breaking of super-symmetry only the p-adic length scale characterizing the super-partner differs from that for partner but the mass of the state is same. This would allow only a discrete set of masses for various super-partners coming as half octaves of the mass of the particle in question. A highly predictive model results.

(b) The quantum field theoretic description should be based on QFT limit of TGD formulated in terms of bosonic emergence [K50] . This formulation should allow to calculate the propagators of the super-partners in terms of fermionic loops.

(c) This TGD variant of space-time super-symmetry resembles ordinary super-symmetry in the sense that selection rules due to the right-handed neutrino number conservation and analogous to the conservation of R-parity hold true. The states inside super-multiplets have identical electro-weak and color quantum numbers but their p-adic mass scales can be different. It should be possible to estimate reaction reaction rates using rules very similar to those of super-symmetric gauge theories.

(d) It might be even possible to find some simple generalization of standard super-symmetric gauge theory to get rough estimates for the reaction rates. There are however problems. The fact that spins $J = 0,1,2,3/2,2$ are possible for super-partners of gauge bosons forces to ask whether these additional states define an analog of non-stringy strong gravitation. Note that graviton in TGD framework corresponds to a pair of wormhole throats connected by flux tube (counterpart of string) and for gravitons one obtains 2^8-fold degeneracy.

14.3.5 Experimental indication for space-time super-symmetry

There is experimental indication for super-symmetry dating back to 1995 [C10] . The event involves $e^+e^-\gamma\gamma$ plus missing transverse energy $\not{E}_T$. The electron-positron pair has transversal energies $E_T = (36,59)$ GeV GeV and invariant mass $M_{ee} = 165$ GeV. The two photons have transversal energies (30,38) GeV. The missing transverse energy is $\not{E}_T = 53$ GeV. The cross sections for these events in standard model are too small to be observed. Statistical fluctuation could be in question but one could also consider the event as an indication for super-symmetry.

In [C9] an explanation of the event in terms of minimal super-symmetric standard model (MSSM) was proposed.

(a) The collision of proton and antiproton would induce an annihilation of quark and antiquark to selectron pair $\tilde{e}^-\tilde{e}^+$ via virtual photon or Z^0 boson with the mass of $\tilde{e}$ in the range (80,130) GeV (the upper bound comes from the total energy of the particles involved.

(b) $\tilde{e}^{\pm}$ would in turn decay to $e^{\pm}$ and neutralino χ_2^0 ad χ_2^0 to the lightest super-symmetric particle χ_1^0 and photon. The neutralinos are in principle mixtures of the super partners associated with γ, Z^0, and neutral higgs h (there are two of them in minimal super-symmetric generalization of standard model). The highest probability for the chain is obtained if χ_2^0 is gluino and χ_1^0 is higgsino.

(c) The kinematics of the event allows to deduce the bounds

$$\begin{aligned}
80 &< m(\tilde{e})/GeV < 130 \ , \\
38 &\leq m(\chi_2^0)/GeV \leq min\left[1.12m(\tilde{e})/GeV - 37,95 + 0.17m(\chi_1^0)/GeV\right] \ , \\
m(\chi_1^0)/GeV &\leq m(\chi_2^0)/GeV \leq min\left[1.4m(\tilde{e})/GeV - 105, 1.6m(\chi_2^0)/GeV - 60\right] \ .
\end{aligned} \tag{14.3.3}$$

(d) Sfermion production rate depends only on masses of the sfermions, so that slepton production cross section decouples from the analysis of particular scenarios. The cross section is at the level of $\sigma = 10$ fb and consistent with data (one event!). The parameters of MSSM are super-symmetric soft-breaking parameters, super-potential parameters, and the parameter $tan(\beta)$. This allows to derive more stringent limits on the masses and parameters of MSSM.

Consider now the explanation of the event in TGD framework.

(a) By the properties of super-partners the production rate for $\tilde{e}^-\tilde{e}^+$ is predicted to be same as in MSSM for $\tilde{e} = e_R\overline{\nu}_R$. Same order of magnitude is predicted also for more exotic super-partners such as $e_L\overline{\nu}_R$ with spin 1.

(b) In TGD framework it is safest to use just the kinematical bounds on the masses and p-adic length scale hypothesis. If super-symmetry breaking means same mass formula from p-adic thermodynamics but in a different p-adic mass scale, $m(\tilde{e})$ is related by a power of $\sqrt{2}$ to $m(e)$. Using $m(\tilde{e}) = 2^{(127-k(\tilde{e}))/2}m(e)$ one finds that the mass range $[80, 130]$ GeV allows two possible masses for selectron corresponding to $p \simeq 2^k$, $k = 91$ with $m(\tilde{e}) = 131.1$ GeV and $k = 92$ with $m(\tilde{e}) = 92.7$ GeV. The bounds on $m(Z)$ leave only the option $m(\tilde{Z}) = m(Z) = 91.2$ GeV and $m(\tilde{e}) = 131.1$ GeV.

(c) The indirect determinations of Higgs masses from experimental data seem to converge to two different values. The first one would correspond to $m(h) = 129$ GeV and $k(h) = 94$ and second one to $m(h) = 91$ GeV with $k(h) = 95$ [K13, K46] . The fact that already the TGD counterpart for the Gell-Mann-Okubo mass formula in TGD framework requires quarks to exist at several p-adic mass scales [K46] , suggests that Higgs can exist in both of these mass scales depending on the experimental situation. The mass of Higgsino would correspond to some half octave of $m(h)$. Note that the model allows to conclude that Higgs indeed exists also in TGD Universe although it does not seem to play the same role in particle massivation as in the standard model. The bounds allow only $k(\tilde{h}) = k(h)+3 = 97$ and $m(\tilde{h}) = 45.6$ GeV for $m(h) = 129$ GeV . The same same mass is obtained for $m(h) = 91$ GeV. Therefore the kinematic limits plus super-symmetry breaking at the level of p-adic mass scale fix completely the masses of the super-particles involved in absence of mixing effects for sneutralinos. To sum up, the masses of sparticles involved are predicted to be

$$m(\tilde{e}) = 131 \ \ GeV \ \ , \ \ \ m(\tilde{Z}^0) = 91.2 \ \ GeV \ \ , \ \ \ m(\tilde{h}) = 45.6 \ \ GeV \ \ . \qquad (14.3.4)$$

14.4 SUSY algebra of fermionic oscillator operators and WCW local Clifford algebra elements as super-fields

Whether TGD allows space-time supersymmetry has been a long-standing question. Majorana spinors appear in $N = 1$ super-symmetric QFTs- in particular minimally super-symmetric standard model (MSSM). Majorana-Weyl spinors appear in M-theory and super string models. An undesirable consequence is chiral anomaly in the case that the numbers of left and right handed spinors are not same. For $D = 11$ and $D = 10$ these anomalies cancel which led to the breakthrough of string models and later to M-theory. The probable reason for considering these dimensions is that standard model does not predict right-handed neutrino (although neutrino mass suggests that right handed neutrino exists) so that the numbers of left and right handed Weyl-spinors are not the same.

In TGD framework the situation is different. Covariantly constant right-handed neutrino spinor acts as a super-symmetry in CP_2. One might think that right-handed neutrino in a well-defined sense disappears from the spectrum as a zero mode so that the number of right and left handed chiralities in $M^4 \times CP_2$ would not be same. For light-like 3-surfaces covariantly constant right-handed neutrino does not however solve the counterpart of Dirac equation for a non-vanishing four-momentum and color quantum numbers of the physical state. Therefore it does not disappear from the spectrum anymore and one expects the same number of right and left handed chiralities.

In TGD framework the separate conservation of baryon and lepton numbers excludes Majorana spinors and also the the Minkowski signature of $M^4 \times CP_2$ makes them impossible. The conclusion that TGD does not allow super-symmetry is however wrong. For $\mathcal{N} = 2N$ Weyl spinors are indeed possible and if the number of right and left handed Weyl spinors is same super-symmetry is possible. In 8-D context right and left-handed fermions correspond to quarks and leptons and since color in TGD framework corresponds to CP_2 partial waves rather than spin like quantum number, also the numbers of quark and lepton-like spinors are same.

The physical picture suggest a new kind of approach to super-symmetry in the sense that the anti-commutations of fermionic oscillator operators associated with the modes of the induced spinor fields define a structure analogous to SUSY algebra. This means that $\mathcal{N} = 2N$ SUSY with large N is in question allowing spins higher than two and also large fermion numbers. Recall that $\mathcal{N} \leq 32$ is implied by the absence of spins higher than two and the number of real spinor components is $N = 32$ also in TGD. The situation clearly differs from that encountered in super-string models and SUSYs and the large value of N allows to expect very powerful constraints on dynamics irrespective of the fact that SUSY is broken. Right handed neutrino modes define a sub-algebra for which the SUSY is only slightly broken by the absence of weak interactions and one could also consider a theory containing a large number of $\mathcal{N} = 2$ super-multiplets corresponding to the addition of right-handed neutrinos and antineutrinos at the wormhole throat.

Masslessness condition is essential for super-symmetry and at the fundamental level it could be formulated in terms of modified gamma matrices using octonionic representation and assuming that they span local quaternionic sub-algebra at each point of the space-time sheet. SUSY algebra has standard interpretation with respect to spin and isospin indices only at the partonic 2-surfaces so that the basic algebra should be formulated at these surfaces. Effective 2-dimensionality would require that partonic 2-surfaces can be taken to be ends of any light-like 3-surface Y_l^3 in the slicing of the region surrounding a given wormhole throat.

14.4.1 Super-algebra associated with the modified gamma matrices

Anti-commutation relations for fermionic oscillator operators associated with the induced spinor fields are naturally formulated in terms of the modified gamma matrices. Super-conformal symmetry suggests that the anti-commutation relations for the fermionic oscillator operators at light-like 3-surfaces or at their ends are most naturally formulated as anti-commutation relations for SUSY algebra. The resulting anti-commutation relations would fix the quantum TGD. Lepton and quark like spinors are now the counterparts of right and left handed Weyl spinors. Spinors with dotted and un-dotted indices correspond to conjugate representations of $SO(3,1) \times SU(4)_L \times SU(2)_R$. The anti-commutation relations make sense for sigma matrices identified as 6-dimensional matrices $1_6, \gamma_7, \gamma_1, ...\gamma_6$.

In leptonic sector one would have the anti-commutation relations

$$\begin{aligned} \{a^\dagger_{m\dot{\alpha}}, a^n_\beta\} &= 2\delta^n_m D_{\dot{\alpha}\beta} \ , \\ D &= (p_\mu + \sum_a Q^a_\mu)\sigma^\mu \ . \end{aligned} \tag{14.4.1}$$

In quark sector σ^μ is replaced with $\overline{\sigma}^\mu$ obtained by changing the signs of space-like sigma matrices. p_μ and Q^a_μ are the projections of momentum and color charges in Cartan algebra to the space-time surface. The action of these charges is on the position of the tip of CD and therefore purely algebraic as far as space-time coordinates are considered. The anti-commutation relations define a generalization of the ordinary equal-time anti-commutation relations for fermionic oscillator operators to a manifestly covariant form. Extended SUSY algebra suggest that the anti-commutators could contain additional central charge term proportional to $\delta_{\alpha\beta}$ but the 8-D chiral invariance excludes this term.

In the octonionic representation of the sigma matrices matrix indices cannot be present at the right handed side without additional conditions. Octonionic units however allow a representation as matrices defined by the structure constants failing only when products of more than two octonions are considered. For the quaternionic sub-algebra this does not occur. Both spinor modes and and gamma matrices must belong to the local hyper-quaternionic sub-algebra. Octonionic representation reduces $SO(7,1)$ so G_2 as a tangent space group. Similar reduction for 7-dimensional compact space takes place also M-theory.

One can consider basically two different options concerning the definition of the super-algebra. If the super-algebra is defined at the 3-D ends of the intersection of X^4 with the boundaries of CD, the modified gamma matrices appearing in the operator D appearing in the anti-commutator are associated with Kähler action. If the generalized masslessness condition $D^2 = 0$ holds true -as suggested already earlier- one can hope that no explicit breaking of super-symmetry takes place and elegant description of massive states as effectively massless states making also possible generalization of twistor is possible. One must however notice that also massive representatives of SUSY exist. SUSY algebra could be also defined at 2-D ends of light-like 3-surfaces. According to considerations of [K22] these options are equivalent if the effective metric defined by the modified gamma matrices is degenerate so that space-time sheet is effectively 3-dimensional. In this case propagation takes place along 3-D light-like 3-surfaces. This condition fails for string like objects.

One can realize the local Clifford algebra in terms of super fields by introducing theta parameters in the standard manner and the expressing a collection of local Clifford algebra element with varying values of fermion numbers (function of CD and CP_2 coordinates) as a chiral super-field. The definition of a chiral super field requires the introduction of super-covariant derivatives.

Standard form for the anti-commutators of super-covariant derivatives D_α make sense only if the momentum and color charges do not act as differential operators acting on space-time coordinates and thus affecting the modified gamma matrices. This is achieved since p_k and Q_a act on the position of the tip of CD in $M^4 \times CP_2$ (rather than internal coordinates of the space-time sheet).

14.4.2 Super-fields associated with WCW Clifford algebra

WCW local Clifford algebra elements possess definite fermion numbers and it is not physically sensible to super-pose local Clifford algebra elements with different fermion numbers. The extremely elegant formulation of super-symmetric theories in terms of super-fields encourages to ask whether the local Clifford algebra elements could allow expansion in terms of complex theta parameters assigned to various fermionic oscillator operator in order to obtain formal superposition of elements with different fermion numbers. One can also ask whether the notion of chiral super field might make sense.

The obvious question is whether it makes sense to assign super-fields with the modified gamma matrices.

(a) As already noticed, modified gamma matrices are not covariantly constant but this is not a problem since the action of momentum generators and color generators space-time coordinates is purely algebraic.

(b) One can define the notion of super-field also at the fundamental level. Chiral super-field would be continuation of the local Clifford algebra of associated with CD to a local Clifford algebra element associated with the union of CDs. This would allow elegant description of cm degrees of freedom, which are the most interesting as far as QFT limit is considered.

(c) In particular, the Kähler function of WCW as a function of complex coordinates can be extended to a chiral super-field defined in quantum fluctuation degrees of freedom. It would depend on zero modes too. Does also the latter dependence allow super-space continuation? Coefficients of powers of theta would correspond to fermionic oscillator operators. Does this function define the propagators of various states associated with light-like 3-surface? WCW complex coordinates would correspond to the modes of induced spinor field so that super-symmetry would be realized very concretely.

(d) Quantum criticality implies infinite number of conserved super-currents assignable to zero modes and it seems that similar coding makes sense also for the dependence of Kähler function on zero modes.

The really elegant feature of the super-field concept is that it allows to code the Taylor polynomial of a function at given point -essentially non-local data- to a purely local data about super field. The coding of the Taylor expansion of WCW Kähler function at maximum would represent only one example of this expansion.

The obvious idea is that the exponent of the super-space Kähler function defines the vacuum functional of the theory determining all interaction vertices. In this interpretation the scalar components ϕ_i of infinite-component chiral field would correspond to complex coordinates of WCV. Also zero modes might allow super-symmetrization by using the fermionic currents implied by quantum criticality.

It is not clear whether vector super-fields make sense in this framework or are needed.

(a) Zero energy ontology and the identification of gauge bosons as wormhole contacts encourage the identification of both fermions and bosons as chiral super-fields. One could assign to fermions either the positive or negative energy variant of $\mathcal{N}$ super-algebra having possibly infinite number of generators and to bosons the direct sum of these super-algebras so that one has positive and negative energy fermions F_+ and F_- with $\mathcal{N}$ super-symmetry and bosons B_{+-} with $(\mathcal{N},\mathcal{N})$ super-symmetry. B_{+-} would be anti-chiral with respect to θ_+ and chiral with respect to θ_- and hermiticity condition can be considered as an additional condition with hermitian conjugation mapping θ_+ to θ_-. The fundamental action would reduce to the integral over θ_+ and θ_- and their conjugates in the product of $\overline{F}_- B_{-+} F_+$. Gauge symmetries are consistent with this guess if realized by regarding both B and F fields are chiral super-fields. Bosonic emergence would suggest that no kinetic term is needed for bosons.

(b) One can consider also the possibility of Hermitian vector field V as local Clifford algebra element by using the same basic definition as used in super-symmetric quantum field theories. The c-number part of V could be interpreted in terms of the spinor connection of WCW: this part cannot be dynamical. It is not however clear whether the definition of corresponding chiral super-field is sensible in the infinite-dimensional context. Also one can ask whether this kind of field is needed at the fundamental level since bosons and their super-partners in TGD framework are identified as pairs of wormhole throats. Super-Kähler function $K = K(\Phi^\dagger, \Phi)$ ($K = K(\Phi^\dagger, exp(-V)\Phi)$) would be a function of chiral super-field Φ, its conjugate $\Phi^\dagger$ (, and vector super-field V). A profound generalization of the physics as geometry idea would be the outcome.

At QFT limit the super fields depend on the point of M^4. The dependence on the point h of $M^4 \times CP_2$ makes sense also in WCW context since h can be interpreted as position of the either tip of CD. A given value of Φ at given point h of H fixes the WCW coordinates characterizing the light-like 3-surface X_l^3 inside CD with tip at h. Constant values of Φ analogous to vacuum expectation of Higgs means that X_l^3 is same for all CDs. The quantum field character of Φ codes for the fact that one has actually quantum superposition of space-time surfaces. The functional integral around a given maximum of Kähler function replaces this superposition effectively with single space-time surface.

14.5 SUSY algebra at QFT limit

The first expectation is that QFT limit TGD corresponds to a situation in which space-time surfaces are representable as a graph for some map $M^4 \rightarrow CP_2$. This assumption is not actually needed in zero energy ontology since M^4 labels the positions of either tip of CD rather than points of the space-time sheet. The position of the other tip of CD relative to the first one could be interpreted in terms of Robertson-Walker coordinates for quantum cosmology [K60] . Second intuitively plausible idea is that particle space-time sheets with Euclidian induced metric are replaced with world-lines. Actually the replacement of partonic 2-surfaces with points is needed and even this assumption can be given up in one formulation of QFT limit feeding information about partonic 2-surfaces to the theory. What is essential that only perturbations around single maximum of Kähler function are considered. If several maxima are important, one must include a weighting defined by the values of the exponent of Kähler function.

14.5.1 Minimum information about space-time sheet and particle quantum numbers needed to formulate SUSY algebra

The basic problem is how to feed just the essential information about quantum states and space-time surfaces to the definition of the QFT limit.

(a) The information about quantum numbers of particles feeded to the measurement interaction must be feeded also to the QFT action. It is natural to start from the classical description of point like particles in H in terms of light-like curves of H reducing to light-like geodesic lines for free particles. Momentum and color charges serve as natural quantum numbers. The conserved color charges associated with CP_2 geodesics need not correspond to the usual color charges since they correspond to center of mass rotational motion in CP_2 degrees of freedom. Ordinary color charges correspond to the spinorial partial wavs assignable to CP_2 type extremals.

(b) Should one interpret QFT limit as a QFT in X^4 representable as a graph for a map $M^4 \rightarrow CP_2$, or in M^4, or perhaps in $M^4 \times CP_2$? In zero energy ontology the proper interpretation is in terms of QFT in M^4 labeling the tips of CDs so that no restrictions on space-time sheets need to be posed. Furthermore, by quantum classical correspondence the space-time sheet surrounding given wormhole throat depends on the four-momentum assigned so that Poincare invariant theory in M^4 is the only logically consistent option.

Minimal extension to $M^4 \times CP_2$ is required in order to take into account the geodesic motion in CP_2 degrees of freedom.

(c) What information about space-time surface is needed?

i. One can in principle feed all information about space-time sheet without losing Poincare invariance since momentum operators do not act on space-time coordinates. The description becomes however in-practical even if one restricts the consideration to the maxima of Kähler function.

ii. The minimal approach would use only cm degrees of freedom for the tip of the CD associated with the particle and feed minimum information about light-like 3-surface inside the CD.

iii. The information about partonic two-surfaces X^2 defined as intersections of 3-D light-like wormhole throats with the boundary of CD characterizes elementary particles, and it would be natural to feed this information to the theory by replacing M^4 gamma matrices with modified gamma matrices. This would feed in also the information about hyper-quaternionicity making possible to generalize the notion of twistor. This information would be coded by the partial derivatives of the imbedding space coordinates at X^2, and would be needed only at the partonic 2-surfaces X^2 defining the generalized vertices.

iv. Some information about zero modes characterized by the induced Kähler form invariant under quantum fluctuations assignable to Hamiltonians of $\delta M^4_{\pm} \times CP_2$ at boundaries of CD is certainly needed: here the identification of Kähler potential as the Kähler function of WCW is highly attractive hypothesis.

14.5.2 The physical picture behind the realization of SUSY algebra at point like limit

The challenge is to deduce SUSY algebra in the approximation that partonic 2-surfaces are replaced by points. The basic physical constraint on the realization of the SUSY algebra come from the condition that one must be able to describe also massive particles as members of SUSY multiplets. This should make possible also twistorialization in terms of octonionic gamma matrices reducing to quaternionic ones using representation of octonion units in terms of the structure constants of the octonionic algebra. The general structure of modified Dirac action suggests how to proceed. $p^k\gamma_k$ should be replaced with a simplified version of its 8-D variant in $M^4 \times CP_2$ and the CP_2 part of this operator should describe the massivation.

(a) Since light-like 3-surfaces contract to light-like curves at point like limit and since only CP_2 gamma matrices contribute to Chern-Simons Dirac action, it is natural to assume that the CP_2 projection of the light-like curve describing the particle characterizes the situation. The interpretation of the curve is in terms of center of mass motion of the topologically condensed space-time sheet describing the particle. For particles which are massless in M^4, the CP_2 projection must contract to a point. For massive particles the projection is a curve in CP_2.

(b) The generalization of the Dirac operator appearing in commutation relations reads as

$$\begin{aligned} p^k\gamma_k \quad &\rightarrow \quad D = p^k\gamma_k + Q\gamma_k\frac{ds^k}{ds} \ , \\ s_{kl}\frac{ds^k}{dt}\frac{ds^l}{dt} &= 1 \ . \end{aligned} \tag{14.5.1}$$

Mass shell condition fixes the value of Q

$$Q \ = \ \pm m \ . \tag{14.5.2}$$

For geodesic circle the angle coordinate to be angle parameterizing the geodesic circle is the natural variable and the gamma matrices can be taken to be just single constant gamma matrix along the geodesic circle.

(c) Imbedding space spinors have anomalous color charge equal to -1 unit for lepton and 1/3 units for quarks. Mass shell condition is satisfied if Q is proportional to anomalous hyper-charge and mass of the particle in turn determined by p-adic thermodynamics.

(d) The geometric interpretation would be that in topological condensation the color rotational degrees of freedom of the particle are reduced. If the light-like 3-surfaces contains the geodesic the geodesic circle S^1, color rotational degrees of freedom are not lost completely and color hypercharge remains a good quantum number in these degrees of freedom. It is however important to notice that anomalous color hypercharge has nothing to do with ordinary color quantum numbers.

(e) Particle mass m should relate closely to the frequencies characterizing general extremals. Quite generally, one can write in cylindrical coordinates the general expressions of CP_2 angle variables Ψ and Φ as $(\Psi,\Phi) = (\omega_1 t + k_1 z + n_1\phi..., \omega_2 t + k_2 z + n_2\phi...)$. Here ... denotes Fourier expansion [L1] , [L1] : this corresponds to Cartan algebra of Poincare group with energy, one momentum component and angular momentum defining the quantum numbers. One can say that the frequencies define a warping of M^4 for $(\Psi,\Phi) = (\omega_1 t, \omega_2 t)$. The frequencies characterizing the warping of the canonically imbedded M^4 should closely relate to the mass of the particle. This raises the question whether the replacement of S^1 with $S^1 \times S^1$ is appropriate.

(f) Twistor description is also required. Generalization of ordinary twistors to octotwistor with quaternionicity condition as constraint allows to describe massive particles using almost-twistors. For massive particle the unit octonion corresponding to momentum in rest frame, the octonion defined by the polarization vector $\epsilon_k\gamma_k$, and the tangent vector $\gamma_k ds^k/ds$ (analog of polarization vector in CP_2) generate quaternionic sub-algebra. For massless particle momentum and polarization generate quaternionic sub-algebra as M^4 tangent space.

The SUSY algebra at QFT limit differs from the SUSY algebra defining the fundamental anti-commutators of the fermionic oscillator operators for the induced spinor fields since the modified gamma matrices defined by the Kähler action are replaced with ordinary gamma matrices. This is quite a dramatic difference and raises two questions. The first question "Why not replace the anti-commutation relations with those for the actual fermionic oscillator algebra?" has been already answered.

One can also wonder why not to replace Kähler action with the action defined by the 4-D volume in the induced metric? After all, apart from almost vacuum extremals 4-volume action has almost the same basic extremals (CP_2 type extremals, restricted subset of massless extremals, string like objects). The modified gamma matrices for volume action are just induced gamma matrices reducing to M^4 gamma matrices for canonically imbedded M^4 so that the proposed form of the super-algebra in this framework can be seen as a well-motivated approximation. Also super-symmetry breaking induced by the mixing of M^4 chiralities is expected to occur. There are however arguments in favor Kähler action.

(a) Four-volume option has obvious shortcomings. Only very small space-time sheets are possible since vacuum functional decreases exponentially as a function of four-volume so that the Planck constant allowing space-time sheet with a given four-volume would scale like 1/four-volume. As a matter fact, also for Kähler action large value of Planck constant is required and this explains why string like objects correspond to a macroscopic quantum phase. Classical gauge fields would be completely absent from the space-time dynamics. The notion of effective 3-dimensionality would make no sense and the slicings by light-like 3-surfaces are not restricted to a finite volume surrounding the wormhole throat at QFT limit. Hence the value of $\mathcal{N}$ is expected to be infinite and one can hope of obtaining the SUSY QFT limit with a finite value of $\mathcal{N}$ only as an approximation.

(b) For CP_2 type extremals and string like objects the two actions are expected to give rise to a rather similar theory. Canonically imbedded M^4 and its small deformations are an exception. The good news is that for the small deformations of M^4 one can expect finite value of $\mathcal{N}$ as an exact result rather than approximation.

(c) The information about the zero modes - including vacuum degeneracy - is actually not lost as one replaces modified gamma matrices with the ordinary ones in anti-commutations since the Kähler potential defining the action principle (assuming bosonic emergence) carries information about zero modes. The Kähler potential carries also information about quantum fluctuating degrees of freedom coded by the super-potential at the maximum of Kähler function (with measurement interaction controllable by the experimenter included and affecting only the super-potential and thus the maximum of Kähler function but not WCW metric).

(d) The quantum criticality of the Kähler action distinguishes Kähler action from four-volume. It predicts inclusion hierarchies of super-conformal algebras assignable to the zero modes. In each breaking of the super-conformal symmetry the rank of the WCW Kähler metric is reduced as quantum fluctuating degrees of freedom are transformed to zero modes. Some components of the inverse of the Kähler metric appearing in the Kähler potential diverge as a consequence and the corresponding complex coordinates of WCW transform to zero modes. This picture conforms with the view about SUSY breaking as a reduction of the rank of the Kähler metric defined by Kähler potential.

14.5.3 Explicit form of the SUSY algebra at QFT limit

The explicit form of the SUSY algebra follows from the proposed picture.

(a) Spinor modes at X^2 correspond to the generators of the algebra. Effective 2-D property implies that spinor modes at partonic 2-surface can be assumed to have well-defined weak isospin and spin and be proportional to constant spinors.

(b) The anti-commutators of oscillator operators define SUSY algebra. In leptonic sector one has

$$\begin{aligned} \{a^{\dagger}_{m\dot{\alpha}}, a^n_{\beta}\} &= \delta^n_m D_{\dot{\alpha}\beta} \ , \\ D &= (p^k\sigma_k + Q^a\sigma_a) \ . \end{aligned} \tag{14.5.3}$$

Q^a denote color charges. The notions are same as in the case of WCW Clifford algebra. In quark sector one has opposite chirality and σ is replaced with $\hat{\sigma}$. Both the ordinary and octonionic representations of sigma matrices are possible.

14.5.4 How the representations of SUSY in TGD differ from the standard representations?

The minimal super-sub-algebra generated by right-handed neutrino and antineutrino are the most interesting at low energies, and it is interesting to compare the naturally emerging representations of SUSY to the standard representations appearing in super-symmetric YM theories.

The basic new element is that it is possible to have short representations of SUSY algebra for massive states since particles are massless in 8-D sense. The mechanism causing the massivation remains open and p-adic thermodynamics can be responsible for it. Higgs mechanism could however induce small corrections to the masses.

The SUSY representations of SYM theories are constructed from $J = 0$ ground state (chiral multiplet for $\mathcal{N} = 1$ hyper-multiplet for $\mathcal{N} = 2$: more logical naming convention would be just scalar multiplet) and $J = 1/2$ ground state for vector multiplet in both cases. $\mathcal{N} = 2$ multiplet decomposes to vector and chiral multiplets of $\mathcal{N} = 1$ SUSY. Hyper-multiplet decomposes into two chiral multiplets which are hermitian conjugates of each other. The group of R-symmetries is $SU(2)_R \times U(1)_R$. In TGD framework the situation is different for two reasons.

(a) The counterparts of ordinary fermions are constructed from $J = 1/2$ ground state with standard electro-weak quantum numbers associated with wormhole throat rather than $J = 0$ ground state.

(b) The counterparts of ordinary bosons are constructed from $J = 0$ and $J = 1$ ground states assigned to wormhole contacts with the electroweak quantum numbers of Higgs and electroweak gauge bosons. If one poses no restrictions on bound states, the value of $\mathcal{N}$ is effectively doubled from that for representation associated with single wormhole throat.

These differences are allowed by general SUSY symmetry which allow the ground state to have arbitrary quantum numbers. Standard SYM theories however correspond to different representations so that the formalism used does not apply as such.

Consider first the states associated with single wormhole throat. The addition of right-handed neutrinos and their antineutrinos to a state with the constraint that $p^k\gamma^k$ annihilates the state at partonic 2-surface X^2 would mean that the helicities of the two super-symmetry generators are opposite. In this respect the situation is same as in the case of ordinary SUSY.

(a) If one starts from $J = 0$ ground state, which could correspond to a bosonic state generated by WCW Hamiltonian and carrying $SO(2) \times SU(3)_c$ quantum numbers one obtains the counterparts of chiral/hyper- multiplets. These states have however vanishing electro-weak quantum numbers and do not couple to ordinary quarks neither.

(b) If one starts $J = 1/2$ ground state one obtains the analog of the vector multiplet as in SYM but but belonging to a fundamental representation of rotation group and weak isospin group rather than to adjoint representation. For $\mathcal{N} = 1$ one obtains the analog of vector chiral multiplet but containing spins $J = 1/2$ and $J = 1$. For $\mathcal{N} = 2$ on obtains two chiral multiplets with $(J, F, R) = (1, 2, 1)$ and $(J, F, R) = (1/2, 1, 0)$ and $(J, F, R) = (0, 0, -1)$ and $(-1/2, 1, 0) = (0, 0, 0)$.

(c) It is possible to have standard SUSY multiplet if one assumes that the added neutrino has always fermionic number opposite that the fermion in question. In this case on obtains $\mathcal{N} = 1$ scalar multiplet. This option could be defended by stability arguments and by the fact that it does not put right-handed neutrino itself to a special role.

For the states associates with wormhole contact zero energy ontology allows to consider two non-equivalent options. The following argument supports the view that gauge bosons are obtained as wormhole throats only if the throats correspond to different signs of energy.

(a) For the first option the both throats correspond to positive energies so that spin 1 bosons are obtained only if the fermion and anti-fermion associated with throats have opposite M^4 chirality in the case that they are massless (this is important!). This looks somewhat strange but reflects the fact that $J = 1$ states constructed from fermion and anti-fermion with same chirality and parallel 4-momenta have longitudinal polarization. If the ground state has longitudinal polarization the spin of the state is due to right-handed neutrinos alone: in this case however spin 1 states would have fermion number 2 and -2.

(b) If the throats correspond to positive and negative energies the momenta are related by time reflection and physical polarizations for the negative energy anti-fermion corespond to non-physical polarizations of positive energy anti-fermion. In this case physical polarizations are obtained.

If one assumes that the signs of the energy are opposite for the wormhole throats, the following picture emerges.

(a) If fermion and anti-fermion correspond to $\mathcal{N} = 2$-dimensional representation of super-symmetry, one expects $2\mathcal{N} = 4$ gauge boson states obtained as a tensor product of two hyper-multiplets if bound states with all possible quantum number combinations are possible. Taking seriously the idea that only the bound states of fermion and anti-fermion are possible, one is led to consider the idea that the wormhole throats carry representations of $\mathcal{N} = 1$ super-symmetry generated by M^4 Weyl spinors with opposite chiralities at the two wormhole throats (right-handed neutrino and its antineutrino). This would give rise to a vector representation and eliminate a large number of exotic quantum number combinations such as the states with fermion number equal to two and also spin two states. This idea makes sense a also for a general value of $\mathcal{N}$. Bosonic representation could be also seen as the analog of short representation for $\mathcal{N} = 2N$ super-algebra reducing to a long representation $\mathcal{N} = N$. Short representations occur quite generally for the massive representations of SUSY and super-conformal algebras when 2^r generators annihilate the states [B23] .

Note that in TGD framework the fermionic states of vector and hyper multiplets related by $U(2)_R$ R-symmetry differ by a $\nu_R\overline{\nu}_R$ pair whose members are located at the opposite throats of the wormhole contact.

(b) If no restrictions on the quantum numbers of the boson like representation are posed, zero energy ontology allows to consider also an alternative interpretation. $\mathcal{N} = 4$ (or more generally, $\mathcal{N} = 2N$-) super-algebra could be interpreted as a direct sum of positive and negative energy super-algebras assigned to the opposite wormhole throats. Boson like multiplets could be interpreted as a long representation of the full algebra and fermionic representations as short representations with states annihilated either by the positive or negative energy part of the super-algebra. The central charges Z_{ij} must vanish in order to have a trivial representations with $p^k = 0$. This is expected since the representations are massless in the generalized sense.

(c) Standard $\mathcal{N} = 2$ multiplets are obtained if one assume that right-handed neutrino has always opposite fermion number than the fermion at the throat. The arguments in favor of this option have been already given.

14.6 Super-symmetric QFT limit of TGD

The definition of the SUSY QFT limit of TGD involves several challenges. A generalization of the super-space concept is needed to cope with $\mathcal{N} > 1$ symmetry and the notions of chiral and vector super-fields must be defined precisely. The previous findings about the super-multiplets assignable to fermions and bosons suggest that standard formalism does not generalize as such. Accordingly, two lines of approach are studied in this section. The first one relies on the generalization of the standard definitions chiral and vector super-fields applied in TGD framework, and works in practice only for $\mathcal{N} = (1,0)$ and $\mathcal{N} = (0,1)$ in fermionic sector and $\mathcal{N} = (1,1)$ in bosonic sector (notation is motivated by zero energy ontology). Second approach relies on a new view about super fields forced by the condition that the formalism makes sense for $\mathcal{N} = \infty$.

14.6.1 Basic concepts and ideas

A brief overview about basic concepts and ideas to be discussed in this section is in order before going to the details.

The notion of super-space

(a) Majorana spinors do not make sense in TGD framework but the use of Weyl spinors as spinors with definite H-chirality is possible. It is possible to use spinor of fixed chirality only since leptons and charge conjugates of quarks can be regarded as having same H-chirality. By hyper-quaternionicity the octonionic gamma matrices allow a matrix representation in terms of octonionic structure constants so that also octonionic formulation makes sense. The pair $\{a_m^\dagger, a_m\}$ of oscillator operators corresponds to the pair $(\overline{\theta}_{m\dot{\alpha}}, \theta_m^\alpha)$.

(b) A non-trivial question relates to the identification of the super-space. The first candidate is $M^{1,\mathcal{N}}$, $M = M^4 \times S^1$, S^1 a geodesic circle of CP_2. Since the gamma matrices of S^1 must be expressed in terms of H gamma matrices one can however argue that effectively super-space corresponds induced from $M^{1,\mathcal{N}}$, $M = H$. This and the condition of hyper-quaternionicity suggest the notion of induced super-symmetry suggests itself meaning that $D=4$ holds true effectively. The value of $\mathcal{N}$ would be naturally $\mathcal{N}=2$ for fermions and $\mathcal{N}=4$ for bosons if one restricts the consideration to right-handed neutrino and antineutrino modes since CP_2 spinor indices are effectively frozen in this case. If the numbers of quark and lepton like modes are different, one has $\mathcal{N}=(\mathcal{N}_1,\mathcal{N}_2)$ super-symmetry, and the axial anomaly with respect to H-chirality is possible. The different couplings of lepton and quark fields to Kähler gauge potential should take care of the anomaly.

Super-covariant derivatives

Consider next the definition of super-covariant derivatives.

(a) The dotted and un-dotted indices $D_{m,\alpha}$ *resp.* $D_{m,\dot{\alpha}}$ label the spin and weak isospin indices of quark *resp.* lepton like spinors. The indices m label the spinor modes associated with quarks and leptons for the space-time sheet whose zero modes are coded by the induced CP_2 Kähler form $J_{\alpha\beta}$. Also now leptons and antiquarks can be regarded as two induces spinor field with with same chirality so that one has $\mathcal{N}=2N$ super-symmetry.

(b) Super-covariant derivatives can be defined by modifying the usual definitions in rather obvious manner.

$$D_{m,\alpha}=\frac{\partial}{\partial\theta^{m,\alpha}}-iQ_{\alpha\dot{\alpha}}\overline{\theta}_m^{\dot{alpha}} \quad , \quad \overline{D}_{m\dot{\alpha}}=-\frac{\partial}{\partial\overline{\theta}^{m\dot{\alpha}}}+i\theta_m^{\alpha}Q_{\alpha\ \dot{alpha}} \quad , \quad Q=\sigma^A(p_A+Q_A) \quad (14.6.1)$$

(c) The anti-commutations for given a H-chirality can be written as

$$\begin{array}{ll} \{D_{m\alpha},D_{n\beta}\}=0 \ , & \{\overline{D}_{m\alpha},\overline{D}_{n\beta}\}=0 \ , \\ \{D_{m\alpha},\overline{D}_{n\dot{\beta}}\}=\delta_m^n D_{\alpha\ \dot{beta}} \ . & \end{array} \quad (14.6.2)$$

Identification of the super-fields: conventional approach

Also now super-field can be defined in terms of the Taylor expansion with respect to theta parameters. Chiral super-fields satisfy the usual conditions given by

$$D_{m\alpha}\Phi=0 \ , \quad \overline{D}_{m\dot{\alpha}}\Phi=0 \ . \quad (14.6.3)$$

The differences from standard SUSY are due to the fact that fermions have ground state which is not scalar but $J=1/2$ particle whereas bosons correspond to $J=1$ ground states and wormhole throats so that $\mathcal{N}_B=2\mathcal{N}_F$ holds true. This means that $J=0$ chiral field must be replaced with spin $J=1/2$ and $\mathcal{N}=2$ chiral superfield in the case of fermions and spin 1/2 vector field must be replaced with spin $J=1$ $\mathcal{N}_B=4$ analog of vector super-field unless one poses additional conditions of the allowed bound states to reduce $\mathcal{N}_B$ to $\mathcal{N}_B=2$.

As found, the assumption that right handed neutrino has fermion number opposite to the fermion state assigned to the throat reduces the fermionic super-symmetry to $\mathcal{N}=1$ and bosonic super-symmetry to $\mathcal{N}=2$ and fermionic states can be regarded as short representations of $\mathcal{N}=2$ super-symmetry natural in zero energy ontology. With these assumptions standard formalism works as such.

(a) The chiral super-field can be written as

$$\Phi=\Phi(\theta_m,H^k) \ , \quad H^k=h^k+i\overline{\theta}_m\overline{Q}\theta^m \ , \quad h^k\equiv(m^k,\phi) \ . \quad (14.6.4)$$

Here the sum is over both lepton and antiquark plus modes mode the induced spinor field.

(b) Vector super-fields describe gauge bosons and their super-counterparts. $V = V^\dagger$ is satisfied. The definition of vector super field is as usual. One starts from super field V and defines super gauge transformations as transformations $V \rightarrow V + \Lambda + \Lambda^\dagger$, where Λ is chiral spinor super-field.

$$W_{m\alpha} = XD_{m,\alpha}V \ \ , \ \ \overline{W}_{m\alpha} = \overline{XD}_{m,\alpha}V \ \ , \ X \ \ = \ \ -\frac{1}{2^{\mathcal{N}}}\prod_{m,\alpha}\overline{D}_{m\alpha} \ \ . \tag{14.6.5}$$

defines a gauge invariant quantity analogous to gauge field. Chiral super-fields transform as $\Phi \rightarrow exp(\Lambda)\Phi$, $\Phi^\dagger \rightarrow \Phi^\dagger exp(\Lambda^\dagger)$.
The quantity

$$L \ \ = \ \ \Phi^\dagger exp(-V)\Phi \ \ . \tag{14.6.6}$$

is gauge invariant and defines a generalization of Dirac Lagrangian. This action can be regarded as a particular Kähler potential.

(c) For ordinary SUSYs Kähler potential can be very general real function of super-fields and the space of super-fields defines Kähler manifold. Also super-potential which is sum of holomorphic function of chiral super-fields and its conjugate is possible and corresponds to the addition of real part of complex function to Kähler potential is possible. These terms are make possible breaking of super-symmetry by a generation of vacuum expectation values of some scalar fields.

Identification of the super-fields: the approach inspired by $\mathcal{N} = \infty$ case

The standard approach does not work at all for $\mathcal{N} = \infty$ and becomes highly questionable also for the values of $\mathcal{N}$, which are large. Zero energy ontology and the identification of fermions as wormhole throats and bosons as wormhole contacts inspires a new manner to see super fields. Positive energy chiral fields correspond to analytic functions of θ alone with no dependence on $\overline{\theta}$. Negative energy chiral field is obtained as hermitian conjugate of this field. Hermitian conjugation maps θ to $\partial_\theta D$ in positive energy chiral super field and the resulting operator acts on the product X of all theta parameters. Note that the presence of D is essential for obtaining the generalization of Dirac action. Note that in this approach there is no need to introduce super-derivatives and θs and $\partial_\theta D$s define the representation of the space-time super-symmetry algebra. Super vector fields are defined as hermitian operators of form $V_k = V_k(\theta, \partial_\theta)$ acting on chiral super-fields, and the generalization of chiral action with coupling to super vector fields is obtained by the minimal substitution $D \rightarrow D + V$.

SUSY breaking

The general vision about breaking of super-symmetry would be following.

(a) The effective dimension of space-time as it appears in the anti-commutators of super-generators in $D = 5$. Since the number of components of 5-D Weyl spinors is 4, the number κ of supergenerators is given by $\kappa = 4\mathcal{N}$.

(b) One obtains a hierarchy of SUSY breakings. It is possible to decompose the full SUSY action to a sum of actions with a smaller value of $\mathcal{N}$ by integrating over theta parameters associated with the higher modes of the induced spinor field. It is also possible to take into account only finite number of spinor modes. The presence of higher modes poses strong constraints on the coupling parameters of the SUSY action.

(c) Taking into account only right-handed neutrino and its antineutrino, the number of real supercharges is $\kappa = 8$ so that if $D = 5$ is the effective space-time dimension $\mathcal{N} = (1, 1)$ SUSY is obtained at the lowest level in good approximation due to the electro-weak inertness of right-handed neutrinos.

14.6.2 About super-field formalism in $\mathcal{N}=2$ case

For SUSY limit of quantum TGD assuming that only right-handed neutrinos and antineutrinos appear as generators of super-symmetries and that the added right-handed neutrino has fermion number opposite that that of the fermion of the throat corresponds to $\mathcal{N}=2$ for gauge bosons and Higgs and to $\mathcal{N}=1$ equivalently $\mathcal{N}=2$ short representation for fermions. For this option super-field formalism guarantees also the conservation of fermion numbers automatically. With these assumptions it is of considerable interest to summarize the basic facts of $\mathcal{N}=2$ super-fields.

(a) $\mathcal{N}=2$ super-multiplets are known as vector multiplet assigned to gauge bosons and their partners and hyper multiplet assigned with matter. Vector multiplet contains two Weyl fermions and vector boson and scalar in adjoint representations. The two fermionic states transform non-trivially under the R-symmetry group $SU(2)_R\times U(1)_R$. Vector multiplet decomposes under $\mathcal{N}=1$ supersymmetry to vector multiplet and chiral multiplet. Hyper-multiplet consists of two Weyl fermion and complex bosons and SU_R mixes the two fermions. Two $\mathcal{N}=1$ multiplets are in question.

(b) A pedagogical representation for the generalization of $\mathcal{N}=1$ SYM action to $\mathcal{N}=2$ case can be found in the article of Adel Bilal [B22] . This action includes only $\mathcal{N}=2$ super partners of gauge boson which are all in the adjoint representation of the gauge group. $\mathcal{N}=2$ vector multiplet decomposes to $\mathcal{N}=1$ vector multiplet and chiral multiplet and the $\mathcal{N}=1$ reduction of the action gives sum of $\mathcal{N}=1$ YM action and Kähler potential. $\mathcal{N}=2$ symmetry allows no super-potential for vector multiplet. The super YM action is determined by a holomorphic function known as pre-potential fixed completely by renormalizability to be quadratic function of $\mathcal{N}=2$ vector super-field Ψ.

(c) The Lagrangian of $\mathcal{N}=2$ SUSY YM theory reduced to $\mathcal{N}=1$ notation reads as

$$L = \frac{1}{4\pi}Im\left[\int d^4\theta\frac{\partial\mathcal{F}(A)}{\partial A}+\int d^2\theta\frac{1}{2}\frac{\partial^2\mathcal{F}(A)}{\partial A^2}W_\alpha W^\alpha\right] . \quad (14.6.7)$$

A denotes $\mathcal{N}=1$ chiral multiplet in $\mathcal{N}=2$ vector multiplet whose scalar component is denoted by a.

(d) $\mathcal{N}=2$ supersymmetry implies that the Kähler potential and Kähler metric associated with the vector multiplet can be written in terms of single holomorphic function $\mathcal{F}(A)$ known as prepotential as

$$\begin{aligned} K &= Im(\frac{\partial\mathcal{F}(A)}{\partial A}\overline{A}) \ , \\ ds^2 &= Im(\frac{\partial^2\mathcal{F}(A)}{\partial^2 a^2})dad\overline{a} \ . \end{aligned} \quad (14.6.8)$$

(e) In the classical theory tree level Lagrangian allows to deduce $\mathcal{F}(A)$ as

$$\mathcal{F}(A) = \frac{1}{2}\times\tau_{cl}A^2 \ , \ \tau_{cl}=\frac{\theta}{2\pi\hbar}+i\frac{4\pi}{g^2} \ . \quad (14.6.9)$$

Here τ unifies gauge coupling strength and θ parameter associated with the instanton term to single complex parameter and the holomorphy of $\mathcal{F}(a)$ poses very powerful constraints on the theory.

The expression of the scalar potential associated with vector multiplet reads as

$$V(\phi) = \frac{1}{g^2}Tr(\phi,\phi^\dagger]^2) \ . \quad (14.6.10)$$

Scalar potential vanishes in the sub-space defined by the Cartan algebra of gauge group so that scalar potential has r-dimensional sub-manifold of vanishing extrema, where

r is the rank of the Cartan sub-algebra. Radiative corrections affect V so that the the vacuum degeneracy disappears. Note that vacuum degeneracy is analogous to the vacuum degeneracy of Kähler action in TGD.

There are very strong constraints on the moduli space defined by scalars [B7] .

(a) For $D = 4$ and $\mathcal{N} = 2$ the moduli space associated with the vector multiplet (so called Coulomb branch) contains one complex scalar defining so calling special Kähler manifold [B56] . The moduli space associated with hyper-multiplet (so called Higgs branch) contains two scalars and is Hyper-Kähler manifold [B19] . For $\mathcal{N} > 2$ the moduli spaces are symmetric spaces. The article of Antoine van Proyen discusses vector multiplets in $\mathcal{N} = 2$ supersymmetry and associated moduli spaces for the scalar fields appearing in the theory and fixed to a high degree by super-symmetry.

(b) This picture conforms with the view that WCW is infinite-dimensional symmetric space with Hyper-Kähler structure and corresponds to the moduli space of hypermultiplet.

In TGD framework vector multiplets are associated with wormhole contacts. They do not represent fundamental degrees of freedom and describe at QFT limit phenomenologically bound states since it is the total momentum and color charge which appear in the modified Dirac equation in regions surrounding both wormhole throats. With above described assumptions about super-symmetry the bosonic multiplets are $\mathcal{N} = 2$ multiplets whereas fermionic ones are short variants of them. Zero energy ontology plays an essential role.

14.6.3 Electric magnetic duality, monopole condensation and confinement from TGD point view

$\mathcal{N} = 2$ SYM theory was studied by Seiberg and Witten in their seminal paper *Electric-magnetic duality, monopole condensation, and confinement in* $\mathcal{N} = 2$ *super-symmetric Yang-Mills theory* [B54] and it is interesting to try to see the results of Seiberg and Witten from TGD point of view. Electric magnetic duality conjecture of Olive and Montonen was inspired by the observation about the upper bound form the masses of dyons deduced by Prasad and Somerfield and Bogomol'nyi (BPS) and reading as

$$M \geq \sqrt{|Z|} \ , \quad Z = v(n_e + i\frac{n_m}{\alpha}) \ . \tag{14.6.11}$$

Here v denotes Higgs expectation value, $\alpha = g^2/4\pi\hbar$ is gauge coupling strength, and n_e and n_m characterize the electric and magnetic charges of the dyon. States for which equality holds true in above formula are called BPS saturated and they correspond to massive representation of SUSY with the same number of states as appearing in massless representations. The observation inspiring the duality conjecture was that the formula is symmetric under $n_e \leftrightarrow n_m$, $v \leftrightarrow v/\alpha$, and $\alpha \leftrightarrow 1/\alpha$. Electric magnetic duality implies that the strong coupling phase for ordinary particles can be understood as a weak coupling phase for monopoles.

Witten demonstrated that in the original sense this duality can hold true for $\mathcal{N} = 4$ theories since only in this case the electrons and monopoles have same quantum number spectrum but in the case of $\mathcal{N} = 2$ theories it can hold true at the low energy Abelian limit of the theory and between particle and special class of dyons with vanishing electric charges (neutral magnetic monopoles). The duality of Montonen and Olive actually generalizes in the sense that all transformation of the group $SL(2, Z)$ acting on the complex coupling strength

$$\tau = i\frac{4\pi\hbar}{g^2} + \frac{\theta}{2\pi}$$

transform different phases of the theory to each other. These transformations also transform dyons with charges (n_m, n_e) to each other.

An interesting question is whether electric magnetic duality and color confinement based on the condensate of magnetic monopoles could have counterparts in TGD framework.

(a) The counterpart for the electric phase corresponds to wormhole throats surrounded by a slicing by light-like 3-surfaces with boundary so that momentum and color charges can be assigned to single wormhole throat and $\mathcal{N}$ for the entire super-algebra is finite.

(b) Also the counterpart of the magnetic phase exists. Topological magnetic monopoles make sense in TGD framework since the topology of CP_2 allows wormhole throats carrying homological magnetic charges. Monopole phase could exist in the sense that the outer boundary of the space-time sheet carries the neutralizing magnetic charge or that the wormhole throats feed the magnetic fluxes to large space-time sheets. Also the deformations of string like objects $X^2 \times Y^2 \subset M^4 \times CP_2$ carry naturally magnetic fluxes along them and could feed them to larger space-time sheets through magnetically charged wormhole throats defining bosonic super-multiplet. Also fermions with opposite magnetic charges can topologically condense at string like object and effectively serve as its ends. The same momentum and color charge would be associated with all wormhole throats associated with a given string like object having an interpretation as hadron-like object so that a color confined perturbative phase would be in question. The value of $\mathcal{N}$ for the entire super-algebra is infinite for string like objects and the description in terms of super-conformal algebra seems to be more appropriate than QFT description. In this sense one would have genuinely non-perturbative phase.

(c) As Witten shows, in $\mathcal{N} = 2$ theory electric magnetic duality of Montonen and Olive fails in $\mathcal{N} = 2$ SUSY because the number of states for electron multiplet is 4 and contains spin 1 state whereas monopole states have $J \leq 1/2$. In TGD framework the reduction of $\mathcal{N} = 2$ symmetry to $\mathcal{N} = 1$ symmetry for massless fermions changes the situation so that a natural conjecture is that electric magnetic duality actually holds true in TGD framework. If the conjecture is really true, it could be seen as a support for zero energy ontology as also for the identification of fermions as wormhole throats and bosons as wormhole contacts.

(d) The mapping of the coupling constant to its inverse cannot apply to the Kähler coupling strength fixed by the quantum criticality but makes sense for the color coupling strength. If one accepts holography, then light-like 3-surfaces are fundamental objects and whether one can regard them as magnetic monopoles or not, depends on the space-time sheets assigned to them. This assignment could change in a phase transition transforming the space-time sheets surrounding the wormhole throats so that particles would transform to monopole like entities. More generally, the basic objects would be dyons and the phase transitions would be characterized by $Sl(2, Z)$.

(e) One can of course, ask whether the inverse of Kähler coupling strength, which is analogous to the inverse of critical temperature and with CP breaking theta angle added to it as an imaginary part, could have a discrete spectrum of values identifiable as the orbit of $Sl(2, Z)$.

14.6.4 Interpretation of Kähler potential and super-potential terms in TGD framework

TGD suggests the interpretation of Kähler potential and superpotential in terms of WCW geometry.

(a) The Kähler potential could be interpreted in terms of WCW Kähler function. If K is quadratic in chiral super-field only the dependence on zero modes is possible. This is what is required since integration over quantum fluctuating degrees of freedom is carried out at QFT limit. Maximum of WCW Kähler function defines Kähler potential. Spinor modes more or less in 1-1 correspondence with coordinates of WCW. Could Kähler potential define the Kähler potential of WCW which would thus make itself directly visible at space-time level.

(b) Super-potential term could be interpreted as counterpart for the addition of a real part of holomorphic function to Kähler function. This part would not affect WCW metric but could characterize different measurement interactions. Separate conservation of lepton and quark numbers require that super-potential is sum over lepton and quark contributions. R-parity conservation allows only quadratic super-potential. By quantum criticality the moduli space for the super-potential could correspond to the modifications of Kähler function not affecting Kähler metric but affecting the maximum of Kähler function and thus space-time sheet. This would be counterpart for the non-renormalization theorems of super-potential in SUSYs.

14.6.5 Generalization of bosonic emergence

Generalization of the bosonic emergence. The propagators for wormhole contacts carrying manyfermion states at wormhole throats are induced by propagators assignable to single throats as radiative corrections. Dirac action is replaced with $K = \Phi^\dagger exp(V)\Phi$, where Φ is chiral super field.

Different measurement interactions correspond to different super-potentials.

By previous arguments bosonic emergence would mean that the super-variant of WCW Kähler function defines the super-symmetric action principle at QFT limit so that one can say that the geometrization of quantum physics takes place in a very concrete sense also at QFT limit. This would be quite an elegant physical manifestation of the underlying infinite-dimensional geometry.

14.6.6 Is $\mathcal{N} > 8$ super-symmetry internally consistent?

The standard wisdom says that $\mathcal{N} = 8$ is absolute upper bound for the super-symmetry (spins larger than 2 are not regarded as physical). In TGD $\mathcal{N} = 8$ emerges naturally for space-time surfaces due to the dimension $D = 8$ of imbedding space and the fact that imbedding space spinors with a given H-chirality (quarks and leptons which color appearing as partial waves in CP_2 have 8 complex components. One obtains $\mathcal{N} = 8$ without restrictions if one considers only the super-algebra defined by the oscillator operators associated with the lowest modes of these spinor fields obtained as a solutions of the Kähler-Dirac equation. These solutions are holomorphic spinors localized at 2-D surfaces (string world sheets and possibly also partonic 2-surfaces).

On order to obtain non-trivial fermionic propagator one must add Chern-Simons Dirac term associated with partonic orbits at which the signature of the induced metric changes. The generalized eigenvalues of C-S-D operator are equal to the virtual momenta $p^k\gamma_k$ and one obtains massless Dirac propagator at boundaries of the string world sheets.

Quantum classical correspondence suggests also fermionic measurement interaction term which is supersymmetric counterpart of measurement interaction term associated with the Kähler action and stating that classical conserved charges in Cartan algebra are equal to their quantum counterparts for the space-time surfaces appearing in quantum superposition. This makes sense if classical charges parametrize zero modes and would realize state function collapse at the level of WCW.

It is also possible to consider the super-symmetry generated by all modes of the induced spinor fields and thus with a quite large (even infinite for string like objects) number $\mathcal{N}$ of super generators. This super-symmetry is broken as all super-symmetries in TGD framework. This means that rather high spins are present in the analogs of scalar and vector multiplets and the Kähler potential (expected to be closely related to the Kähler function of the world of the classical worlds (WCW)) describing interaction of chiral multiplet with a vector multiplet can be constructed also for any value of $\mathcal{N}$ - at least formally. If one believes on the generalization of the bosonic emergence, one expects that bosonic part of the action is generated radiatively as one functionally integrates over the fields appearing in the chiral multiplet.

The standard wisdom says that is is not possible to construct interactions for higher spin fields. Is this really true? Why wouldn't the analogs of scalar (chiral/hyper) and vector multiplets make sense for higher values of N? Why would it be impossible to define an spin

1/2 chiral super-field associated with the vector multiplet and therefore the super-symmetric analog of YM action using standard formulas? Why the standard coupling to chiral multiplet would not make sense?

One objection against higher spins is of course the lack of the geometric interpretation. Spin 1 and Spin 2 fields allow it. Can one then imagine any geometric interpretation for higher spin components of super-fields? John Baez and others [A32] are busily developing non-Abelian generalizations of group theory, categories and geometry and speak about things that they call n-groups, n-categories, and n-geometries. Could the generalization of ordinary geometry to n-geometry in which parallel translations are performed for higher dimensional objects rather than points provide a natural interpretation for gauge fields assigned to higher spins? One would have natural hierarchy. Parallel translations of points would give rise curves, parallel translations of curves would give rise to surfaces, and so on. As as a special case the entire hierarchy of these parallel translations would be induced by ordinary parallel translation.

14.6.7 Super-fields in TGD framework

In the case of infinite-dimensional super-space the definition of the super-fields is not quite straightforward since the super-space integrals of finite polynomials of theta parameters always vanish so that the construction of super-symmetric action as an integral over super-space would give a trivial result. For chiral fields the integrals are formally non-vanishing but in the case that the super-field reduces to a finite polynomial of theta at $y^\mu = 0$ the non-vanishing terms in real Lagrangian involve the action of an infinite number of operators $\overline{D}_{\dot\alpha}$ implying the proportionality to an infinite power of momentum which vanishes for massless states. It seems that one should be able to add in a natural manner terms which are obtained as theta derivatives of the product of all theta parameters and that the action should consist of the products of the terms associated with mononomials of theta and monomials of derivatives with respect to theta parameters acting on the infinite product of theta parameters, call it X.

The fact that positive *resp.* negative energy vacuum is analogous to Dirac sea with negative *resp.* positive energy states filled suggests a remedy to the situation. This would mean that positive energy chiral field is just like its ordinary counterpart whereas negative energy chiral fields would be obtained by applying a polynomial of derivatives of theta to the product $X = \prod \theta_{i\alpha}$ of all theta parameters. The theta integral of X is by definition equal to 1. In integral over theta parameters the monomials of theta associated with positive energy chiral field and negative energy chiral field would combine together and one would obtain desired action. In the following this approach is sketched. Devil lies in the details and detailed checks that everything works are not yet done.

TGD variants of chiral super fields

Consider first the construction of chiral super-fields and of the super-counterpart of Dirac action.

(a) Wormhole throats carry a collection of *collinearly* moving fermions. This suggests that kinetic terms behave positive powers of Dirac operator with one power for each theta parameter.

(b) One must be careful with dimensions. The counterpart of Dirac operator is $D = \sigma^k(p_k + Q_k)/M$. The mass parameter M must be included for dimensional reasons and changes only the normalization of the theta parameters from that used earlier and changes the anti-commutation relations of the super-algebra in an obvious manner. The value of M is determined by quantum criticality. The first guess for the value of M is of order CP_2 mass defined as $m(CP_2) = n\hbar_0/R$, where R is the length of CP_2 geodesic and n is a numerical constant. The proposal for a bi-local QFT limit describing gravitational interaction leads to the conclusion that gravitational constant is proportional to $1/M^2$.

(c) In the case of single wormhole throat one can speak about positive and negative energy chiral fields. Positive energy chiral fields are constructed as polynomials, and more generally, as Taylor series whereas negative energy chiral fields are obtained by mapping positive energy chiral fields to an operator in which each theta parameter θ is replaced with

$$\partial_\theta D = \partial_\theta \sigma^k \frac{(p_k + Q_k)}{M} \quad . \tag{14.6.12}$$

This operator acts in the product X of all theta parameters to give the negative energy counterpart of chiral field. The inclusion of sigma-matrices is necessary in order to obtain chiral symmetry at the level of H, in particular the counterpart of Dirac action. In the integral over all theta parameters defining the Lagrangian density the terms corresponding to mononomials $M(\theta, x)$ and their conjugates $M(\partial_{\overline{\theta}} D^{\rightarrow}, x)$ are paired and theta integrals can be carried out easily. Here $\rightarrow$ tells that the spatial derivatives appearing in D are applied to M.

(d) There is an asymmetry between positive and negative energy states and the experience with the ordinary Dirac action $\overline{\Psi} D^{\rightarrow} \Psi - \overline{\Psi} D^{\leftarrow} \Psi$ suggests that one should add a term in which θ parameters are replaced with $-D\theta$ so that space-time derivatives act on the positive energy chiral field and partial derivative $\partial_{\overline{\theta}}$ appear as such. The most plausible interpretation is that the negative energy chiral field is obtained by replacing θs in the positive energy chiral field with ∂_θs and allowing to act on X. The addition of D would thus give rise to the generalization of the kinetic term.

(e) Chiral condition can be posed and one can express positive energy chiral field in as an infinite powers series containing all finite powers of theta parameters whereas negative energy chiral field contains only infinite powers of θ. The interpretation is in terms of different Dirac vacuum. What one means which super-covariant derivatives is not quite clear.

   i. The usual definition of super covariant derivatives would be as

$$D_{i\alpha} = \partial_{i\alpha} + i(\overline{\theta} D)_{i\alpha} \quad , \quad \overline{D}_{i\dot{\alpha}} = \partial_{\theta_{i\dot{\alpha}}} + i(D\theta)_{i\dot{\alpha}} \quad . \tag{14.6.13}$$

   ii. A definition giving rise to the same anti-commutators would be as

$$D_{i\alpha} = \partial_{i\alpha} \quad , \quad \overline{D}_{i\dot{\alpha}} = \partial_{i\dot{\alpha}} + 2i(D\theta)_{\dot{\alpha}} \tag{14.6.14}$$

   In the recent case $\overline{D}$ does not appear at all in the chiral action since for negative energy chiral field conjugation does not correspond to $\theta \rightarrow \overline{\theta}$ but to $\theta \rightarrow \partial_\theta$ and $1 \rightarrow X$. Hence the simplest theory would result using $D_\alpha = \partial_{i\alpha}$.

   iii. If one includes into the product of X of theta parameters only θs but not their conjugates, the two definitions are equivalent since the powers of $\overline{\theta} D\theta$ give nothing in theta integration. This definition of X is be possible using the definition of hermitian conjugation appropriate also for $\mathcal{N} = \infty$. This formalism of course works also for a finite value of $\mathcal{N}$.

Consider now the resulting action obtained by performing the theta integrations. The interesting question is what form of the super-covariant derivatives one should use. The following considerations suggests that the two alternatives give almost identical -if not identical- results but that the simpler definition $D_\alpha = \partial_{i\alpha}$ is much more elegant.

(a) For $D_\alpha = \partial_{i\alpha}$ the propagators are just inverses of D^d where d is the number of theta parameters in the monomial defining the super-field component in question so that the Feynman rules for calculating bosonic propagators and vertices are very simple. Only the spinor and vector terms corresponding to degree $d = 1$ and $d = 2$ in theta parameters behave in the expected manner. This conforms with the collinearity. In particular, for spin 2 states the propagator would behave like p^{-4} for large momenta. This conforms with the prediction that graviton cannot correspond to singlet wormhole throat but to a string like object consisting of a superposition of pairs of wormhole contacts and of wormhole throats. If this expansion makes sense, higher spin propagators would behave as increasingly higher inverse powers of momentum and would not contribute much to the high energy physics. At energies much smaller than mass scale they would give rise to contact terms proportional to a negative power of mass dictated by the number of thetas.

(b) For $D_\alpha = \partial_{i\alpha} + i(\overline{\theta}D)_\alpha$ the situation is considerably more complex although the basic contribution to the propagators is same. The chiral field property using the standard definition means that propagator is multiplied by an infinite geometric series in D^2 coming from the contractions of $\overline{\theta}D\theta$ in positive energy chiral super-field as they are contracted with corresponding terms $\partial_{\overline{\theta}}D\partial\theta$ appearing in negative energy chiral super-field acting on X. The summation can be done by Feynman rules for a "free field theory" in which incoming particles correspond to θ parameters and outgoing particles to partial derivatives with respect to theta parameters. The rule is that any theta parameter can be connected to any derivative with respect to theta parameter and any pair of theta parameters and its conjugate connected in this manner gives D^2/M^2 as a result. For the N:th power of $\overline{\theta}D\theta$ a given theta can be connected to $\partial\theta$ in $N!$ manner and same applies to its conjugate. Hence the $1/N!$ factors coming from the expansion of plane wave $exp(ip\cdot m + i\overline{\theta}D\theta)$ cancel each other and one obtains geometric series.

   i. These rules assign to the power series $exp(i\overline{\theta}D\theta)$ an over-all factor

$$Y = \mathcal{K}Y_1 \ , \ Y_1 = \frac{1-(\frac{p^2-m^2}{M^2})^{K+1}}{1-\frac{p^2-m^2}{M^2}} \ . \tag{14.6.15}$$

   The integer K results when one truncates the SUSY to a SUSY with finite value of $\mathcal{N}$. The value of K depends somewhat on the number of theta parameters associated with the field component but approaches infinite value for $\mathcal{N} = \infty$.

   ii. Propagator is inversely proportional to Y. This factor appears also in vertices and since the propagators and vertices defining the bosonic action involve always chiral loops with the same number of chiral field propagators and incoming vector super-fields the factors $\mathcal{N}$ cancel out neatly.

   iii. For $p^2 \ll M^2$ the factor Y_1 equals to unity in good approximation. For $|p^2-m^2| \gg M^2$ Y_1 diverges at the limit $\mathcal{K} \to \infty$ and propagator vanishes for $|p^2-m^2| \gg M^2$, which raises the hope about dynamical cutoff guaranteeing UV finiteness. Vertices however contain a similar factor. Again the fact that the loops defining the bosonic vertices and propagators contain same number of vertices and propagators implies that these factors cancel each other.

   iv. The overall result seems to be a presence of infinite factors which however cancel completely in the expressions for bosonic vertices and propagators and introduce only a small effect at low energies. For $D_\alpha = \partial_{i\alpha}$ all these complications are avoided. It should not be difficult to decide between these options.

TGD variant of vector super field

Chiral super-fields are certainly not all that is needed. Also interactions must be included, and this raises the question about the TGD counterpart of the vector super-field.

(a) The counterpart of the chiral action would be a generalization of the Dirac action coupled to a gauge potential obtained by adding the super counterpart of the vector potential to the proposed super counterpart of Dirac action. The generalization of the vector potential would be the TGD counterpart of the vector super field. Vector particle include M^4 scalars since Higgs behaves as CP_2 vector and H-scalars are excluded by chiral invariance.

(b) Since bosons are bound states of positive and negative energy fermions at opposite wormhole throats it seems that vector super field must correspond to an operator slashed between positive and negative energy super-fields rather than ordinary vector super-field. The first guess is that vector super-field is an operator expressible as a Taylor series in which positive energy fermions correspond to the powers of θ_α and negative energy fermions correspond to the powers of derivatives ∂_{θ_α}. Naively, D in $\partial_\theta D$ is replaced by $D + V$. Vector super-field must be hermitian ($V = V^\dagger$) with hermitian conjugation defined so that it maps theta parameters to the partial derivatives ∂_θ and performs complex conjugation. A better guess is that D appearing in the definition of the kinetic term is replaced with $D + V$ where V is a hermitian super-field. This definition would be direct generalization of the minimal substitution rule.

(c) It is important to notice that the gauge bosons appearing in the covariant derivatives have *same momentum* so that the interaction terms are local in momentum space rather than x-space. This conforms with the view that the N virtual bosons emitted in the $N+2$ vertex propagate along single wormhole throat. For bosonic emergence these couplings give rise to exchanges of N collinear vector particles between two fermion lines behaving like $(k^2 - m^2)^{-2N}$ and thus approaching rapidly to zero in UV and contact interaction in IR. For $m = 0$ one obtains series of interactions corresponding to potentials of form $V \propto r^{2n+1}$. For massive case these interactions are screened by the Yukawa factor $exp(-mr/\hbar)$. Confining linear interaction potential in QCD could result in this manner for $m = 0$ and $N = 2$. If the coupling were local in x-space the exchange would involve $N - 1$ free loop momenta giving loop integral diverging like λ^{2N-4} as a function of momentum cutoff. Already for $d = 2$ one would obtain logarithmic divergence.

(d) It is difficult to imagine how a kinetic term for the vector super-field could be defined. This supports the idea that bosonic propagators and vertices emerge as one performs functional integral over components of the chiral fields.

(e) There is also the question about gauge invariance. The super-field generalization of the non-Abelian gauge transformation formula looks more like the generalization of Dirac action to its super-counterpart: $D \to D + V$ everywhere. Her V is the contraction of sigma matrices with super-field V_k, which is vector field in M^4 having also S^1 component which does not depend on S^1 coordinate. Positive energy chiral field would transform as $\Phi_+ \to exp(\Lambda)\Phi$, where λ is a chiral field. The negative energy chiral field would transform as $\Phi_- \to \Phi_- exp(\Lambda^\dagger)$ with hermitian conjugation involving also the map of thetas to their derivatives. Each theta parameter would represent a fermion transforming under gauge symmetries in a manner dictated by its electro-weak quantum numbers (the inclusion of color quantum numbers is not quite trivial: probably they must be included as a label for quark modes). As in the case of Dirac action, the transformation formula for vector super-field would be dictated by the requirement that the derivatives of Λ coming from $exp(\Lambda)$ are canceled by the derivative terms in the transformation formula for the vector super field. The resulting transformation formulas are identical with standard ones formally since the only new thing is that both V_k and and gauge group element g are super-fields.

How to feed information about classical physics of space-time sheet to chiral and vector super-fields?

The new view about super fields need not be consistent with the geometric interpretation assigned to the chiral multiplets in the standard SUSY without some modifications.

(a) The geometric interpretation of Kähler potential and super-potential are very attractive features of ordinary SUSY. The most general interpretation in TGD framework would be as the WCW Kähler function K and holomorphic function f, whose real part added to K does not affect its metric but changes the maxima of Kähler function.

(b) In standard SUSYs the scalar parts of chiral fields give rise to Higgs expectation values and internal consistency arguments force the manifold of Higgs expectation values to be Kähler or even quaternionic Kähler manifold with coordinates interpreted as Higgs field. In TGD framework Higgs is CP_2 vector which brings in additional constraint. CP_2 is quaternionic Kähler manifold but CP_2 coordinates do not allow interpretation as Higgs field. CP_2 gamma matrices induced to S^1 giving rise to a constant gamma matrix could be however identified as S^1 component of vector potential identifiable as Higgs vacuum expectation contributing to the mass of a given particle besides the dominating contribution coming from p-adic thermodynamics.

(c) In the model based on ordinary $\mathcal{N} = 1$ SUSY each particle would correspond to its own $\mathcal{N} = 1$ multiplet so that a Cartesian power of CP_2s would define the quaternionic manifold. This conforms with the geometric picture provided by the replacement of light-like 3-surfaces with braid strands in which a Cartesian power of $\delta M^4_\pm \times CP_2$ effectively replaces WCW.

(d) The new view about super fields requires the replacement of the constant S^1 component of the super gauge potential with a diagonal matrix whose eigenvalues depend on the mode of spinor field characterized by the theta parameter. Quite generally, super-symmetry breaking results from the replacement of the mass parameter m in D with a diagonal operator whose eigenvalues m_k give the masses assignable to the modes depending on the spinor mode. Mathematically the genuine S^1 mass term determined by p-adic thermodynamics can be distinguished from a small Higgs expectation coded by S^1 vector potential by comparing particles with different charges.

The induced Kähler form defining zero modes is so essential for quantum TGD at the fundamental level that the coding of at least part of this information to the chiral action is a highly desirable feature. This seems possible.

(a) The overall renormalization factor of the chiral super field cannot carry the information about the geometry of the space-time sheet. The vector super-field vertices involving vector particles are obtained as chiral loops an the normalization factors from the vertices involving N vectors and 2 chiral particles cancel their inverses associated with the chiral propagators. Hence the possible renormalization of the generalized Dirac action has no physical implications. This is one of the nice outcomes of emergence concept.

(b) One can however add to the induced gauge potentials associated with the space-time sheet to the super gauge potential as its classical parts. It is important to notice that X^4 coordinates would appear as parameters constant with respect to p_k since p_k would correspond to M^4 coordinate for the tip of CD rather than space-time coordinate. The standard interpretation would be as slowly varying background fields.

(c) The information about vacuum degeneracy coded by the modified gamma matrices could be coded by replacing the operator D with that appearing in the modified Dirac action and assigned to the maximum of Kähler function. Note that this would bring in two color charges Q_i. The modified gamma matrices appearing in it would behave as constants with respect to p_k and Q_k. Somewhat surprisingly, zero energy ontology would make it possible to feed all information about the classical physics of the space-time sheet without losing Poincare invariance.

14.6.8 Could QFT limit be finite?

Could the resulting theory be finite without hyperbolic and mass scale cutoffs in UV region? Consider first general arguments without any resort to the proposed definition of TGD counterparts of super-fields.

(a) Non-renormalization theorems allow to expect that a cancelation of quadratic infinities takes place as a consequence of super-symmetry. Cancelation of quadratic divergences in the bosonic propagators means that there is no need to assume that hyperbolic cutoffs are different for time-like and space-like momenta.

(b) There are arguments suggesting that $N = 8$ SUGRA is UV finite. Since the number of super-symmetries in quantum TGD is even higher than in $N = 8$ super-gravity, the theory might be also UV finite. If infinities cancel, the theory without UV cutoff for the mass scale and hyperbolic angle could provide an excellent approximation to the theory. Also the standard prescription for calculating loop integrals might make sense if this is the case. Geometric arguments support the presence of the cutoffs but one must remain critical.

(c) Super-symmetry alone does not guarantee finiteness since it is possible to define extremely general SUSY actions in terms of integrals of functions of super-space integrated over super-space. Chiral action should have some additional symmetries not possessed by super-symmetric counter-terms. Chiral action is quadratic in chiral super-field meaning the absence of self couplings of the chiral super field. Linear superposition of the solutions is certainly a very special symmetry and very essential for the perturbation theory. QFT limit is obtained by integrating over the quantum fluctuations in WCW degrees of freedom for a a maximum of Kähler function. Therefore Kähler potential naturally corresponds to WCW Kähler potential at its maximum and depends only on zero modes. This would conform with the fact that only second derivatives of Kähler potential (Kähler metric) appear in the Kähler potential. Also R-parity arguments favor this form.

(d) Also for the proposed TGD inspired identifications of super-fields, the cancelation of UV divergences should be essentially algebraic and due to the cancelation of chiral contributions from the loops contributing to the vector super-field propagators and vertices. Also for the emerging bosonic effective action same mechanism should be at work. The renormalization theorems state that the only renormalizations in SUSYs are wave function renormalizations. In the case of bosonic propagators loops therefore mean only the renormalization of the propagator. In the recent case only the chiral loops are included so that the situation is analogous to Abelian YM theory or $\mathcal{N}=4$ super YM theory, where the beta functions for gauge couplings vanish. Hence one might hope that also now wave function renormalization is the only effect so that the radiatively generated contribution should be proportional to the standard form of the vector propagator. The worst that can occur is logarithmically diverging renormalization of the propagator which occur in many SUSYs. The challenge is to show that logarithmic divergences possibly coming from the θ^d, $d=1,2$, parts of the chiral super-field cancel. The condition for this cancelation is purely algebraic since the coupling to $k=2$ part is gradient coupling so that the leading divergences have same form.

(e) It could happen that the contributions from $d\leq 2$ cancel exactly as they do in SUSYs but the contributions from the field components with $d>2$ give a non-vanishing and certainly finite contribution. If this were the case then the exotic chiral field components with propagators behaving like $1/p^d$, $d>2,...$ would make possible the propagation for the components of the vector super-field.

14.6.9 Can one understand p-adic coupling constant evolution as a prediction of QFT limit?

The precise formulation of the p-adic coupling constant evolution is one the basic challenges of quantum TGD. The best that one can hope is the deduction of p-adic thermodynamics and p-adic length scale hypothesis as well as p-adic coupling constant evolution from QFT limit alone.

For the simple form of the QFT limit involving $M^4\times S^1$ gamma matrices in the definition of D, p-adic length scale could make itself visible via the mass parameters and the contribution of the Higgs field appearing as S^1 part of the gauge potential. This is of course just feeding in the results of p-adic thermodynamics. Gauge couplings predicted by the emergence would depend on these parameters but the coupling constant evolution would reflect only the effects of mass parameters on it.

4-D spin glass analogy is one of the basic visions about the physics of quantum TGD. In the theory of spin glasses ultra-metric topology possessed also by p-adic number fields emerges as a topology of the energy landscape consisting of the minima of free energy. In TGD framework the space for the maxima of Kähler function could obey ultra-metric topology with the value of prime p fixed by the scale of CD in question and given by a power of 2. Therefore one has good hopes that for small enough sub-CDs of a given CD the failure of the strict non-determinism implies p-adic coupling constant evolution. p-Adic thermodynamics determining particle masses cannot of course follow from QFT limit since it relates to the space-like space-time regions (locally CP_2 type vacuum extremals) defining the generalized Feynman diagrams. By combining this vision and the QFT limit with maximal information feed about the space-time sheet gives hopes about achieving more ambitious goals.

(a) If one replaces D with the actual measurement interaction term, all information about the space-time sheet within a given CD is feeded via modified gamma matrices $\hat{\Gamma}^{\alpha} = \Gamma^k \partial L_K / \partial_{\alpha} h^k$ in as effective slowly varying background fields. The propagators reflect directly the local space-time dynamics, and one obtains a distribution of scattering amplitudes as a function of the point of the space-time sheet within a given CD. A 4-D distribution for the values of gauge coupling constants is predicted whereas 1-D evolution or even discrete p-adic evolution would be quite enough.

(b) Space-time sheets decompose into connected Minkowskian regions surrounding wormhole throats (basins for the local slicings by light-like 3-surfaces parallel to the throats) and these regions naturally correspond to coherence regions at QFT limit. A space-time integral defining a quantum superposition of the amplitudes associated with various points of the coherence region looks like a physically natural mathematical object to consider. Only the kinetic terms for vector super-field would involve the weighting over the coherence region. Ideal weighting depends on zero modes only and therefore cannot depend on the induced metric.

   i. If the weighting is defined by Kähler action density, normalization is not required and would lead to difficulties when the Kähler action vanishes. This implies the proportionality of emerging bosonic kinetic terms to $1/g_K^2$ so that propagators and also gauge coupling strengths included by definition to the propagators are proportional to g_K^2. The basic property of emergence is that the dependence of Feynman diagrams on $1/g_K^2$ coming from the modified gamma matrices cancels out in perturbation theory. This is a nice feature consistent with the idea that the propagator for the small deformations of 3-surfaces corresponds to the WCW contravariant Kähler metric proportional to g_K^2 [K12] . The counterparts of the gauge couplings identified in terms of the inverses of propagators for the vector super-field components obtained in this manner would depend on the p-adic length scale $L_p \propto \sqrt{p}$, $p \simeq 2^k$, for the smallest CD containing the coherence region. One can criticize this weighting scheme. The ideal weighting scheme should depend on zero modes ($J_{\alpha\beta}$) only but this weighting scheme depends on the induced metric. Huge amount of information is needed and a concrete connection with the view about generalized Feynman graphs is lacking. Note also that the properties of elementary particles reflecting themselves at the level of propagators would depend on the macroscopic field patterns of Kähler electric field.

   ii. The idea that light-like 3-surfaces meet at partonic two-surfaces X^2 identified as their intersections of wormhole throats with the light-like boundaries of CDs would suggest only 2-D weighting over partonic 2-surfaces with Kähler action replaced by magnetic flux density $\frac{1}{g_K} J\sqrt{g_2}$, $J = J^{\alpha\beta}\epsilon_{\alpha\beta}$. The dependence on the induced metric is only apparent. $1/g_K$ factor must be included to get dimensions correctly. This weighting depends on zero modes only. The dependence of the coupling constants on the p-adic size scale of CD comes out naturally, and only the information from the scales relevant to elementary particles affect propagators and couplings. As in the previous case, non-trivial interference effects are possible since the sign of J varies. The only information about the preferred extremals of Kähler action is about the derivatives $\partial_{\alpha} h^k$ at X^2 appearing in $\hat{\Gamma}^{\alpha}$. In the calculation of the super-vector field propagators the information about the modified gamma matrices at *both* ends of X_l^3 is needed and the weighting would be over the both ends.

   iii. One must decide whether to perform the weighting for the kinetic term of vector super-field action or for the loop integrals defining the corresponding propagators.

Spin glass analogy would suggest the first option. The weighting the kinetic term would be proportional to x^2/g_K^2, where x is a numerical parameter characterizing the net result of the weighting. The emerging propagators would be proportional to g_K^2/x^2 and g_K^2 proportionality is indeed what one expects. For the latter option propagator would be proportional to x^2/g_K^2, which does not make sense unless one considers the rather remote possibility that electric-magnetic duality relates the two weightings. The value of x is expected to be smallest for homologically trivial partonic 2-surfaces (Kähler magnetic charge vanishes). Gauge coupling strengths would be therefore smallest for magnetic monopoles, which looks somewhat counter-intuitive if one thinks in terms of electric-magnetic duality. On the other hand, since the propagator for the deformations of 3-surface is contravariant Kähler metric of WCW becoming singular near vacuum extremals and since the kinetic term of Kähler action approaches zero near vacuum extremals, one expects gauge couplings to grow large near vacuum extremals since they are inversely proportional to the scale of the kinetic term. Also asymptotic freedom conforms with this result since in very short length scales magnetically charged string like objects are expected to replace space-time sheets as basic objects whereas long length scales correspond to nearly vacuum extremals.

A cautious conclusion is that the weighting scheme based on Kähler magnetic magnetic flux is the correct choice.

(c) p-Adic coupling constant evolution for the propagator is obtained in a manner consistent with what has been discussed in [K50] . If virtual boson momenta in a given half octave of masses labeled by integer k correspond to CD labeled by this integer. Since the coherence region surrounding the propagating wormhole throat is contained inside a CD characterized by this size scale, the scale of CD indeed defines the p-adic length scale in question. Since the propagator by definition is proportional to the coupling strength also coupling constant evolution is coded in this manner. The difference to earlier picture is that α_K proportionality means that the loops defining propagator must be of order one or larger. In the model based on hyperbolic cutoff the cutoff guaranteed the desired value.

(d) The failure of the strict determinism for Kähler action suggests that for the practical purposes the coherence regions must be replaced with an ensemble of local preferred extremals of Kähler action. The dependence of the modified gamma matrices defined by the Kähler action on the space-time point is analogous to a similar dependence of the coupling constant parameters of the spin glass Hamiltonian. The vacuum functional $exp(K)$ for the coherence region defines the counterpart for the real square root of the density matrix and the sum over the preferred extremals weighted by $exp(K)$ for the coherence region defines the analog of statistical average.

14.6.10 Is the QFT type description of gravitational interactions possible?

In TGD Universe graviton is necessarily a bi-local object and the emission and absorption of graviton are bi-local processess involving two wormhole contacts: a pair of particles rather than single particle emits graviton. This is definitely something new and defies a description in terms of QFT limit using point like particles. Graviton like states would be entangled states of vector bosons at both ends of string so that gravitation could be regarded as a square of YM interactions in rather concrete sense. The notion of emergence would suggest that graviton propagator is defined by a bosonic loop. Since bosonic loop is dimensionless, IR cutoff defined by the largest CD present must be actively involved.

The connection with strings is via the assignment of wormhole contacts at the ends of a stringy curve. Stringy diagrams would not however describe graviton emission. Rather, a generalization of the vertex of Feynman diagram would be in question in the sense that three string world sheets would be glued together along their 1-dimensional ends in the vertex. This generalizes similar description for gauge interactions using Feynman diagrams. In the microscopic description point like particles are replaced with 2-D partonic surfaces so that in gravitational case one has stringy 3-surfaces at vertices.

At QFT limit one can hope a description as a bi-local process using a bi-local generalization of the QFT limit so that stringy degrees of freedom need not be described explicitly. There are hopes about success, since these degrees of freedom have been taken into account in the spectrum of modes of the induced spinor field and reflect themselves as quantum numbers labeling fermionic oscillator operators. Also modified gamma matrices feed information about space-time surface to the theory.

What one really means with strings?

Before continuing is it is good take critical attitude to the proposed picture. What one really means with string is the first question.

(a) Stringy curves appear in in the slicing of the space-time sheet around wormhole throat to light-like 3-surfaces labeled by the points of string. Hamilton-Jacobi coordinates [K4] suggest that the M^4 projections of these curves light-like so that the curves would be space-like.

(b) For string like objects obtained as deformations of cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ one can assign Kähler magnetic flux flowing along the stringy curves. These curves should define a special class of stringy curves.

(c) If the basin for the slicing by light-like 3-surfaces for a given wormhole throat has an outer boundary at which induced Kähler form vanishes (it is not obvious that this can be the case), one can ask whether stringy curves effectively end at the boundary of the basin or what happens? Magnetic flux conservation does not allow to assign magnetic flux to the stringy curves now. The analogy with field lines gravitational scalar potential suggests a possible answer. All wormhole throats would act as sources for these lines identifiable as field lines of a gradient vector field. Basins would not actually have any boundaries since the extrema of the potential would consist in the generic case of a discrete set of points. Whether stringy curves really have something to do with field lines of a gradient of gravitational potential must be however left an open question.

(d) Despite the emergence of stringy picture, string model as such does not seem to help much since the graviton emission vertex is completely different from that in string models.

A physically attractive realization of the braids - and more generally- of slicings of space-time surface by 3-surfaces and string world sheets, is discussed in [K32] by starting from the observation that TGD defines an almost topological QFT of braids, braid cobordisms, and 2-knots. The boundaries of the string world sheets at the space-like 3-surfaces at boundaries of CDs and wormhole throats would define space-like and time-like braids uniquely.

The idea relies on a rather direct translation of the notions of singular surfaces and surface operators used in gauge theory approach to knots [A100] to TGD framework. It leads to the identification of slicing by three-surfaces as that induced by the inverse images of $r = constant$ surfaces of CP_2, where r is $U(2)$ invariant radial coordinate of CP_2 playing the role of Higgs field vacuum expectation value in gauge theories. $r = \infty$ surfaces correspond to geodesic spheres and define analogs of fractionally magnetically charged Dirac strings identifiable as preferred string world sheets. The union of these sheets labelled by subgroups $U(2) \subset SU(3)$ would define the slicing of space-time surface by string world sheets. The choice of $U(2)$ relates directly to the choice of quantization axes for color quantum numbers characterizing CD and would have the choice of braids and string world sheets as a space-time correlate.

What one really means with gravitons?

One can also ask what one really means with graviton. The identification of graviton is indeed far from obvious.

(a) Wormhole throats and contacts allow $J = 2$ states but they couple only to states which corresponds to $d \geq 2$ monomials of theta so that couplings to the fermions are absent.

(b) TGD predicts a hierarchy of string like objects of all possible sizes and these are good candidates for graviton like states. The hierarchy of Planck constants and the huge values of gravitational Planck constant suggests that gigantic gravitons identifiable as stringy curves connecting particles at astrophysical distances are possible. The emission of dark graviton would be bi-local process in astrophysical length scales and would look locally like an emission of gauge boson.

(c) One can of course argue it is not clear whether stringy gravitons represent hadron like objects responsible for strong gravitation below relevant p-adic length scale rather than genuine gravitons. For instance, the identification of elementary particles in terms of CP_2 type extremals forces to ask whether gravitons could correspond to pieces of CP_2 type extremals connecting positive and negative energy space-time sheets with a wormhole contact having two pairs of wormhole throats so that spin two states would become possible. If this generalization is accepted, one must also accept the possibility of wormhole contacts with arbitrary number of throat pairs. One can also wonder what is the origin of Planck length which is roughly 10^4 times shorter than CP_2 length. For instance, could it have purely geometric interaction characterizing the distance between these wormhole contacts?

    With this identification graviton emission at elementary particle level could be seen as a creation of a virtual wormhole throat pair inside wormhole contact formed by fermion and anti-fermion and making possible emission of graviton. One can also consider a distribution of wormhole throat pairs inside wormhole created in this manner in which case $1/G_N$ would characterize the probability for the appearance of wormhole throat pair.

(d) Graviton must be generalized to a super-field and bi-locality suggests that this field is a bi-local composite of super gauge fields in some sense. Ordinary graviton would be only single component of this field.

To sum up, if is far from clear what graviton precisely is and gauge-theory-gravitation correspondence suggests that there is a rich spectrum of graviton like states. Despite this one can characterize rather precisely what the description of gravitational interaction at QFT limit must be by using general symmetry principles and basic structure of quantum TGD.

Could bi-local QFT allow to describe gravitation as a square of gauge interactions?

The key question is whether one can generalize the formalism of QFT limit to describe also gravitational interactions. The first guess is that in some sense gravitation is a square of YM interactions. This statement has a precise content in some string theories. Also the scattering amplitudes of $N = 8$ super-gravity allow a construction in terms of $N = 4$ SYM amplitudes. In the recent case gravitation as a square of YM theory would mean that graviton propagator emerges from vector super-field propagators assignable at the ends of the gravitonic string. Vector propagators would in turn emerge from chiral super-field propagators.

The bi-local character of the basic process suggests that a bi-local generalization of QFT limit is needed to describe gravitation. In fact, at the long length scale limit of the theory the appearance of second derivatives in the curvature scalar could be seen as a signature of bi-locality at fundamental level. Bi-locality brings in the notion of distance and the metric description of gravitation indeed assumes that distances are dynamical. Note that also the typical experimental arrangements for detecting gravitons are bi-local (typically the variation of the distance between the ends of a metal bar is measured).

(a) Bi-locality would suggest that one has a pairing of the chiral actions to a bi-local action. Whether the vector bosons at the ends of graviton string move collinearly or not is a non-trivial question. Experimentation with the candidates for the bilinear gravitational action shows that the simplest theory results when collinearity assumption is given up. It is also far from clear whether collinearity assumption allows any internally consistent mathematical realization: the problem is that by collinearity graviton propagator becomes proportional to $1/p^4$ and one should somehow eliminate one $1/p^2$ factor. If one gives up collinearity, one obtains a bosonic loop integral with vector boson momenta $p-k$ and k. Graviton kinetic term emerges from a loop of two bosons and is therefore dimensionless so that IR cutoff L is necessary in order to obtain p^2L^2 type kinetic term and finiteness. The IR cutoff comes naturally as the size scale L of the largest CD involved and appears also as scaling factor of the action by purely dimensional reasons and disappears naturally from the interaction vertices.

(b) Graviton would correspond to a bi-local composite of super gauge fields acting as operators $V(\theta_i, \partial_{\theta_i})$, $i = 1, 2$, on the chiral super-fields at the ends of the string and graviton propagation should reduce to vector boson propagation just as vector boson propagation reduces to fermion pair propagation. General gauge invariance at the level of space-time sheet is not a problem. At the level of M^4, whose coordinates label the positions for the tips of CD the possibility to choose preferred M^4 coordinates guarantees general coordinate invariance trivially. The elimination of non-physical graviton polarizations for massless gravitons is achieved by the ordinary gauge invariance. In this conceptual framework elimination of non-physical graviton polarization does not have obvious connection with general coordinate invariance. The properties of the slicings by light-like 3-surfaces suggest this connection.

(c) At the point like limit the emission of gravitons is described by an interaction term of form $T^{\alpha\beta}\delta g_{\alpha\beta}$. This expression should have a bi-local gauge invariant generalization. The energy momentum tensor for Dirac action suggests the following remarkably simple expression for the interaction action L_{gr} in super space.

$$L_{gr} = K\overline{\Psi}D^A\Psi \times \overline{\Psi}D_A\Psi \quad . \tag{14.6.16}$$

A summation over the contracted index pairs is understood in the formula. The theta parameters associated with the two actions are regarded as independent Grassmann variables.

(d) If the vector boson momenta at the ends of graviton string vary freely apart from the constraint that they sum up to the momentum of the virtual graviton, K must be of form $K = kL^2$ to compensate the dimension $1/L^2$ coming from the two D_As. L naturally corresponds to IR cutoff defined by the size of the largest CD involved. The bosonic loop giving graviton propagator at the IR limit is dimensionless so that the resulting propagator must be proportional to $1/(p^2L^2)$ so that the powers of L cancel each other in the propagators. $D_A = p_A + Q_A + V_A$ is the covariant derivative corresponding to a particular momentum component. Note that also color charge (color hyper charge or isospin) is included and is present for massive particles. Since only covariant derivatives appear, the expression is manifestly super gauge invariant.

(e) The integration over the theta parameters gives factors $D = \hat{\Gamma}^A D_A/M$ for each integrated pair of theta parameters. Here M is a parameter with dimensions of mass to make D dimensionless and CP_2 is the most natural guess for its value. The resulting action for graviton has the following form

$$\begin{aligned}
L_{gr} &= \frac{K}{M^2}\overline{\Psi}O^A\Psi \times \overline{\Psi}O_A\Psi \quad , \\
O^A &= \frac{D^{\rightarrow}D^{A\rightarrow} - D^{\leftarrow}D^{A\ leftarrow}}{M} \quad , \\
D^{\leftarrow} &= \frac{D_A^{\leftarrow}\hat{\Gamma}^A}{M} \quad , \quad D^{\rightarrow} = \frac{\hat{\Gamma}^A D_A^{\rightarrow}}{M} \quad , \quad D = \frac{\hat{\Gamma}^A D_A}{M} \quad .
\end{aligned} \tag{14.6.17}$$

The $1/p^2L^2$ from the IR cutoff for the loop integral defining the emergent graviton propagator given by a dimensionless bosonic loop cancels L^2 from factor kL^2/M^2 and kM^2 should be proportional to G_N.

(f) The tensor structure of the graviton vertex resulting from DD^A in the lowest order seems to be correct. $\hat{\Gamma}^A D^{B\rightarrow}$ *resp.* $\hat{\Gamma}^A D^{B\leftarrow}$ is the analog of energy momentum tensor and in the lowest order gives rise to the desired proportionality of the gravitational coupling to momentum. Since graviton is emitted by a pair of particles the proportionality to the momenta of both particles is natural. Note that only the momenta associated with the vector bosons defining the emitted graviton-like particle are collinear, not the momenta of the emitting particles at the ends of the string. The bilinear $D_B \otimes D_C$ is the counterpart of $\delta g_{\alpha\beta}$ and polarization tensor of graviton. Both D_B and D_C are contracted with the analog of the energy momentum tensor.

(g) Ordinary graviton must correspond to electro-weak $U(1)$ for which coupling is to fermion number. The mixing of $U(1)_Y$ and $U(1) \subset SU(2)$ should be completely absent for gravitons. In other words, the corresponding value of Weinberg angle must vanish: $sin^2(\theta_{W,gr}) = 0$ implying $m_Z = m_W$ in gravitonic propagation. The graviton analogs formed from massless gluons would have a finite interaction range by confinement and weak gravitons would be massive so that no dramatic new effects are predicted.

When one feeds the information about space-time surface into the theory additional complications arise since the modified gamma matrices $\hat{\Gamma}^\alpha(x) = \Gamma^k \partial L_K/\partial_\alpha h^k$ would replace the $M^4 \times S^1$ gamma matrices and one must integrate over the points x. As found, the condition that weighting scheme depends on zero modes only (is symplectic invariant at the level of WCW) fixes it uniquely to a weighting by magnetic flux at the intersections of light-like wormhole throats with the light-like boundaries of CD relevant for the p-adic length scale defined by the virtual momentum squared.

(a) Bi-locality suggests that both ends of string correspond to their own partonic 2-surfaces so that both ends involve weighting by the Kähler magnetic flux $J\sqrt{g}/g_K$. $J = J^{\alpha\beta}\epsilon_{\alpha\beta}$. Since J vanishes for vacuum regions and since also its sign varies, this is expected to bring in four identical reduction factors -call them x- to the kinetic term of graviton.

(b) The defining property of the emergence is that the variation of the scale of $\hat{\Gamma}^A$ is compensated by the variation of the scale of the propagator so that the proportionality of $\hat{\Gamma}^A$ to $1/g_K^2$ is not seen in scattering amplitudes.

(c) Again one must decide whether the weighting is performed for the calculation of the propagator or whether one uses the bosonic propagators already calculated with corresponding weightings at the ends of lines. Only the latter option conforms with the idea about gravitation as square of gauge interactions predicting α_K^2-proportionality for the graviton propagator. From the general proportionality of bosonic propagators to $g_K^2/x^2 \propto \alpha_K$ one has the order of magnitude estimate

$$16\pi G_N \sim \frac{kg_K^4}{x^4M^2} \sim \frac{k\alpha_K^2}{M^2} \ . \tag{14.6.18}$$

$16\pi G$ comes from the fact that it appears in the linearized Einstein's equations as the coefficient of energy momentum tensor. If k does not depend on $\hbar$ then $\alpha_K \propto 1/\hbar$ and $M \propto \hbar/R$ correctly predicts that G_N does not depend on $\hbar$. Using $\alpha_K \simeq 1/137$ and $M = m(CP_2) = .2437 \times 10^{-3} m_{Pl}$ [K37] one obtains the very rough estimate $k = (m(CP_2)/\alpha_K m_{Pl})^2 = .056$.

(d) Note that the solutions of field equations in the static limit when the situation resembles formally electrostatics, gravitational coupling strength is estimated classically to be of order CP_2 length squared [K4] . Since the value of CP_2 mass (and thus length) is firmly fixed by elementary particle mass calculations [K37] , this results could be seen as a a serious objection against TGD. One could say that the weighting provides a "screening mechanism" reducing the naive value of the gravitational coupling strength.

This picture allows to interpret the cutoff for $\mathcal{N}$ as a cutoff for the maximal number of points of the partonic 2-surface carrying fermionic quantum numbers: essentially a cutoff in measurement resolution is in question. The super-symmetric excitations of graviton can be interpreted microscopically as multi-string states but looking like a single string in the spatial measurement resolution provided by single partonic 2-surface.

Could one apply the formalism at fundamental level?

There are good motivations for asking whether this formalism - when appropriately generalized- could apply to the basic quantum TGD.

(a) Only the data about partonic 2-surfaces are feeded into the vertices so that the assumption that space-time sheets are representable as graphs for maps from M^4 to CP_2 is not actually needed. The information about the interior topology of the space-time sheet is un-necessary and the effective 2-dimensionality simplifies the situation enormously. Note however that the initial values of derivatives of H-coordinates at partonic 2-surfaces are needed.

(b) The cutoff for $\mathcal{N}$ has interpretation as a cutoff for the maximal number of points of the partonic 2-surface carrying fermionic quantum numbers: essentially a cutoff in measurement resolution is in question. Already $\mathcal{N} = 2$ cutoff is expected to be a good approximation since higher theta monomials give rise to short range forces.

(c) The super-symmetric excitations of graviton can be interpreted microscopically as multi-string states but looking like a single string in the spatial measurement resolution provided by single partonic 2-surface. Therefore strings split to space-like braids at fundamental level. The duality between these space-like braids and light-like braids at light-like 3-surfaces might be important.

(d) The generalization should take into account the fact theta parameters correspond to different points of X^2 and that the wormhole throats associated with the bosonic wormhole contacts are not one and same thing. These effects are expected to be small unless the size of the wormhole is very large as it is for anyon-like wormhole throats with macroscopic size containing states with a high fermion number [K51] . Also global data such as the moduli characterizing the conformal equivalence class of partonic 2-surface are needed in order to describe family replication phenomenon at the fundamental level. The description of color quantum numbers at fundamental level introduces additional complications. The functional integral over WCW must be performed and gives rise to non-perturbative effects when a large number of maxima of Kähler function must be included.

14.7 A more detailed summary of Feynman diagrammatics for emergence

The material of this section is about emergence and unfortunately somewhat obsolete. I ended up with the notion of bosonic emergence years before the realization of what generalized Feynman diagrams could really mean what QFT limit of TGD could be. In the recent approach to generalized Feynman diagrams fundamental fermions are massless, and in twistor Grassmannian approach one can hope that residue integration reduces the internal fermion lines to massless on-mass shell lines with non-physical helicity. The integration over loops carrying only light-like momenta would be a remnant of the idea of bosonic emergence. The propagators mediating interaction between fundamental fermions would be stringy propagators assignable to the wormhole contacts, whose throats would carry the fermions. Wormhole contacts with fermion and anti-fermion at opposite throats could be seen as fundamental bosons - to be distinguished from elementary bosons. One could also now say that bosonic propagators indeed emerge. Actuall all elementary particles would emerge from fundamental fermions. Hence on can see bosonic emergence as a kind of clumsy precognition.

In the following the Feynman diagrammatics for Dirac action coupled to gauge potential is sketched briefly and some comments on generalization to the super-symmetric case are made.

14.7.1 Emergence in absence of super-symmetry

The resulting Feynman diagrammatics deserves some more detailed comments.

(a) Consider first the exponent of the action $exp(iS_c)$ resulting in fermionic path integral. The exponent

$$exp[i\int dx^4d^4y\overline{\xi}(x)G_F(x-y)\xi(y)] = exp[i\int d^4k\overline{\xi}(-k)G_F(k)\xi(k)]$$

is combinatorially equivalent with the sum over n-point functions of a theory representing free fermions constructed using Wick's rules that is by connecting n Grassmann spinors and their conjugates in all possible ways by the fermion propagator G_F.

(b) The action of

$$exp\left[i\int d^4x\frac{\delta}{\delta\overline{\xi}(x)}\gamma\cdot A(x)\frac{\delta}{\delta\xi(x)}\right] = exp\left[i\int d^4kd^4k_1\frac{\delta}{\delta\overline{\xi}(k-k_1)}\gamma\cdot A(-k)\frac{\delta}{\delta\xi(k_1)}\right]$$

on diagrams consisting of n free fermion lines gives sum over all diagrams obtained by connecting fermion and anti-fermion ends of two fermion lines and inserting to the resulting vertex $A(-k)$ such that momentum is conserved. This gives sum over all closed and open fermion lines containing $n\geq 2$ boson insertions. The diagram with single gauge boson insertion gives a term proportional to $A_\mu(k=0)\cdot\int d^4kk^\mu k^{-2}$, which vanishes.

(c) S_c as obtained in the fermionic path integral is the generating functional for connected many-fermion diagrams in an external gauge boson field and represented as sum over diagrams in which one has either closed fermion loop or open fermion line with $n\geq 2$ bosons attached to it. The two parts of S_c have interpretation as the counterparts of YM action for gauge bosons and Dirac action for fermions involving arbitrary high gauge invariant n-boson couplings besides the standard coupling. An expansion in powers of $\gamma^\mu D_\mu$ is suggestive. Arbitrary number of gauge bosons can appear in the bosonic vertices defined by the closed fermion loops and gauge invariance must pose strong constraints on the bosonic part of the action if expressible in terms of bosonic gauge invariants. The closed fermion loop with $n=2$ gauge boson insertions defines the bosonic kinetic term and bosonic propagator. The sign of the kinetic terms comes out correctly thanks to the minus sign assigned to the fermion loop.

(d) Feynman diagrammatics is constructed for S_c using standard Feynman rules. In ordinary YM theory ghosts are needed for gauge fixing and this seems to be the case also now.

(e) One can consider also the presence of Higgs bosons. Also the Higgs propagator would be generated radiatively and would be massless for massless fermions as the study of the fermionic self energy diagram shows. Higgs would be necessary CP_2 vector in $M^4\times CP_2$ picture and E^4 vector in $M^8=M^4\times E^4$ picture. It is not clear whether one can describe Higgs simply as an M^4 scalar. Note that TGD allows in principle Higgs boson but - according to the recent view - it does not play a role in particle massivation.

14.7.2 Some differences from standard Feynman diagrammatics

The diagrammatics differs from the Feynman diagrammatics of standard gauge theories in some respects.

(a) 1-P irreducible self energy insertions involve always at least one gauge boson line since the simplest fermionic loop has become the inverse of the bosonic propagator. Fermionic self energy loops in gauge theories tends to spoil asymptotic freedom in gauge theories. In the recent case the lowest order self-energy corrections to the propagators of non-abelian gauge bosons correspond to bosonic loops since fermionic loops define propagators. Hence asymptotic freedom is suggestive.

(b) The only fundamental vertex is $AF\overline{F}$ vertex. As already found, there seems no point in attaching to the vertex an explicit gauge coupling constant g. If this is however done n-boson vertices defined by loops are proportional to g^n. In gauge theories n-boson vertices are proportional to g^{n-2} so that a formal consistency with the gauge theory picture is achieved for $g = 1$. In each internal boson line the g^2 factor coming from the ends of the bosonic propagator line is canceled by the g^{-2} factor associated with the bosonic propagator. In S-matrix the division of the bosonic propagator from the external boson lines implies g^n proportionality of an n-point function involving n gauge bosons. This means asymmetry between fermions and bosons unless one has $g = 1$. $g = 1$ above means $g = \sqrt{\hbar_0}$. Since fermionic propagator is proportional to $\hbar_0^0$ and since loop integral involves the factor $1/\hbar_0$, the dimensions of bosonic propagator and radiatively generated vertices come out correctly. The counterparts of gauge coupling constants could be identified from the amplitudes for 2-fermion scattering by comparison with the predictions of standard gauge theories. The small value of effective gauge coupling g obtained in this manner would correspond to a large deviation of the normalization factor of the radiatively generated boson propagator from its standard value.

(c) Furry's theorem holding true for Abelian gauge theories implies that all closed loops with an odd number of Abelian gauge boson insertions vanish. This conforms with the expectation that 3-vertices involving Abelian gauge bosons must vanish by gauge invariance. In the non-abelian case Furry's theorem does not hold true so that non-Abelian 3-boson vertices are obtained.

14.7.3 Generalization of the formalism to the super-symmetric case

In principle the generalization of the formula of generalized Feynman diagrammatics to super-symmetric case at QFT limit of TGD is straightforward.

(a) Consider first the standard formalism making sense only for $\mathcal{N} = 1$ case in TGD framework. In this case the Kähler potential $K(\Phi^\dagger, exp(-V)\Phi)$ replaces Dirac action coupled to gauge potentials and the functional integral at the first step is over super-fields assigned to the fermions. Theta integrations gives the Lagrangian as function of components of super-fields and the free-field functional integral over the fields appearing in Φ and $\Phi^\dagger$ gives the action as a functional of gauge boson fields and their super-partners appearing in V. All vertices and propagators are expressible in terms of loops of fermions and their super-partners and gauge couplings and their evolution follow as predictions.

(b) For the option based on TGD inspired generalization of super-fields the fundamental action is the generalization of Dirac action. As already found the propagators for chiral field components with d theta parameters behave as k^{-d} so that they do not induce divergences in fermionic loops. Also bosonic propagators for field components involving d thetas behave in similar manner. The possible divergences in the bosonic propagators would vanish by the same mechanism as in ordinary super-symmetry.

In the earlier approach the elimination of UV divergences required the introduction of cutoffs in mass squared and hyperbolic angle characterizing velocity of virtual fermion in the rest system of the virtual boson. The requirement that quadratic divergences are absent in the inverses of the propagators forced the hyperbolic cutoffs to be different for time-like and space-like momenta. The justification for hyperbolic cutoff was in terms of a geometric argument: the causal diamond (CD) characterizing virtual fermion must remain inside the CD defining the IR cutoff. Super-symmetry could imply this cutoff in a smooth manner by the cancelation of the divergences associated with particles and their super-partners as already noticed.

14.8 Could $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM be a part of TGD after all?

Whether right-handed neutrinos generate a supersymmetry in TGD has been a long standing open question. $\mathcal{N} = 1$ SUSY is certainly excluded by fermion number conservation but already $\mathcal{N} = 2$ defining a "complexification" of $\mathcal{N} = 1$ SUSY is possible and could generate right-handed neutrino and its antiparticle. These states should however possess a non-vanishing light-like momentum since the fully covariantly constant right-handed neutrino generates zero norm states. So called massless extremals (MEs) allow massless solutions of the modified Dirac equation for right-handed neutrino in the interior of space-time surface, and this seems to be case quite generally in Minkowskian signature for preferred extremals. This suggests that particle represented as magnetic flux tube structure with two wormhole contacts sliced between two MEs could serve as a starting point in attempts to understand the role of right handed neutrinos and how $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SYM emerges at the level of space-time geometry. The following arguments inspired by the article of Nima Arkani-Hamed et al [B32] about twistorial scattering amplitudes suggest a more detailed physical interpretation of the possible SUSY associated with the right-handed neutrinos.

The fact that right handed neutrinos have only gravitational interaction suggests a radical re-interpretation of SUSY: no SUSY breaking is needed since it is very difficult to distinguish between mass degenerate spartners of ordinary particles. In order to distinguish between different spartners one must be able to compare the gravitomagnetic energies of spartners in slowly varying external gravimagnetic field: this effect is extremely small.

14.8.1 Scattering amplitudes and the positive Grassmannian

The work of Nima Arkani-Hamed and others represents something which makes me very optimistic and I would be happy if I could understand the horrible technicalities of their work. The article Scattering Amplitudes and the Positive Grassmannian by Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, and Trnka [B32] summarizes the recent situation in a form, which should be accessible to ordinary physicist. Lubos has already discussed the article. The following considerations do not relate much to the main message of the article (positive Grassmannians) but more to the question how this approach could be applied in TGD framework.

All scattering amplitudes have on shell amplitudes for massless particles as building bricks

The key idea is that all planar amplitudes can be constructed from on shell amplitudes: all virtual particles are actually real. In zero energy ontology I ended up with the representation of TGD analogs of Feynman diagrams using only mass shell massless states with both positive and negative energies. The enormous number of kinematic constraints eliminates UV and IR divergences and also the description of massive particles as bound states of massless ones becomes possible.

In TGD framework quantum classical correspondence requires a space-time correlate for the on mass shell property and it indeed exists. The mathematically ill-defined path integral over all 4-surfaces is replaced with a superposition of preferred extremals of Kähler action analogous to Bohr orbits, and one has only a functional integral over the 3-D ends at the light-like boundaries of causal diamond (Euclidian/Minkowskian space-time regions give real/imaginary Chern-Simons exponent to the vacuum functional). This would be obviously the deeper principle behind on mass shell representation of scattering amplitudes that

Nima and others are certainly trying to identify. This principle in turn reduces to general coordinate invariance at the level of the world of classical worlds.

Quantum classical correspondence and quantum ergodicity would imply even stronger condition: the quantal correlation functions should be identical with classical correlation functions for any preferred extremal in the superposition: all preferred extremals in the superposition would be statistically equivalent [K87]. 4-D spin glass degeneracy of Kähler action however suggests that this is is probably too strong a condition applying only to building bricks of the superposition.

Minimal surface property is the geometric counterpart for masslessness and the preferred extremals are also minimal surfaces: this property reduces to the generalization of complex structure at space-time surfaces, which I call Hamilton-Jacobi structure for the Minkowskian signature of the induced metric. Einstein Maxwell equations with cosmological term are also satisfied.

Massless extremals and twistor approach

The decomposition $M^4 = M^2 \times E^2$ is fundamental in the formulation of quantum TGD, in the number theoretical vision about TGD, in the construction of preferred extremals, and for the vision about generalized Feynman diagrams. It is also fundamental in the decomposition of the degrees of string to longitudinal and transversal ones. An additional item to the list is that also the states appearing in thermodynamical ensemble in p-adic thermodynamics correspond to four-momenta in M^2 fixed by the direction of the Lorentz boost. In twistor approach to TGD the possibility to decompose also internal lines to massless states at parallel space-time sheets is crucial.

Can one find a concrete identification for $M^2 \times E^2$ decomposition at the level of preferred extremals? Could these preferred extremals be interpreted as the internal lines of generalized Feynman diagrams carrying massless momenta? Could one identify the mass of particle predicted by p-adic thermodynamics with the sum of massless classical momenta assignable to two preferred extremals of this kind connected by wormhole contacts defining the elementary particle?

Candidates for this kind of preferred extremals indeed exist. Local $M^2 \times E^2$ decomposition and light-like longitudinal massless momentum assignable to M^2 characterizes "massless extremals" (MEs, "topological light rays"). The simplest MEs correspond to single space-time sheet carrying a conserved light-like M^2 momentum. For several MEs connected by wormhole contacts the longitudinal massless momenta are not conserved anymore but their sum defines a time-like conserved four-momentum: one has a bound states of massless MEs. The stable wormhole contacts binding MEs together possess Kähler magnetic charge and serve as building bricks of elementary particles. Particles are necessary closed magnetic flux tubes having two wormhole contacts at their ends and connecting the two MEs.

The sum of the classical massless momenta assignable to the pair of MEs is conserved even when they exchange momentum. Quantum classical correspondence requires that the conserved classical rest energy of the particle equals to the prediction of p-adic mass calculations. The massless momenta assignable to MEs would naturally correspond to the massless momenta propagating along the internal lines of generalized Feynman diagrams assumed in zero energy ontology. Masslessness of virtual particles makes also possible twistor approach. This supports the view that MEs are fundamental for the twistor approach in TGD framework.

Scattering amplitudes as representations for braids whose threads can fuse at 3-vertices

Just a little comment about the content of the article. The main message of the article is that non-equivalent contributions to a given scattering amplitude in $\mathcal{N} = 4$ SYM represent elements of the group of permutations of external lines - or to be more precise - decorated permutations which replace permutation group S_n with $n!$ elements with its decorated version containing $2^n n!$ elements. Besides 3-vertex the basic dynamical process is permutation having the exchange of neighboring lines as a generating permutation completely analogous to fundamental braiding. BFCW bridge has interpretation as a representations for the basic braiding operation.

This supports the TGD inspired proposal (TGD as almost topological QFT) that generalized Feynman diagrams are in some sense also knot or braid diagrams allowing besides braiding operation also two 3-vertices [K32]. The first 3-vertex generalizes the standard stringy 3-vertex but with totally different interpretation having nothing to do with particle decay: rather particle travels along two paths simultaneously after $1 \rightarrow 2$ decay. Second 3-vertex generalizes the 3-vertex of ordinary Feynman diagram (three 4-D lines of generalized Feynman diagram identified as Euclidian space-time regions meet at this vertex). The main idea is that in TGD framework knotting and braiding emerges at two levels.

(a) At the level of space-time surface string world sheets at which the induced spinor fields (except right-handed neutrino [K87]) are localized due to the conservation of electric charge can form 2-knots and can intersect at discrete points in the generic case. The boundaries of strings world sheets at light-like wormhole throat orbits and at space-like 3-surfaces defining the ends of the space-time at light-like boundaries of causal diamonds can form ordinary 1-knots, and get linked and braided. Elementary particles themselves correspond to closed loops at the ends of space-time surface and can also get knotted (possible effects are discussed in [K32]).

(b) One can assign to the lines of generalized Feynman diagrams lines in M^2 characterizing given causal diamond. Therefore the 2-D representation of Feynman diagrams has concrete physical interpretation in TGD. These lines can intersect and what suggests itself is a description of non-planar diagrams (having this kind of intersections) in terms of an algebraic knot theory. A natural guess is that it is this knot theoretic operation which allows to describe also non-planar diagrams by reducing them to planar ones as one does when one constructs knot invariant by reducing the knot to a trivial one. Scattering amplitudes would be basically knot invariants.

"Almost topological" has also a meaning usually not assigned with it. Thurston's geometrization conjecture stating that geometric invariants of canonical representation of manifold as Riemann geometry, defined topological invariants, could generalize somehow. For instance, the geometric invariants of preferred extremals could be seen as topological or more refined invariants (symplectic, conformal in the sense of 4-D generalization of conformal structure). If quantum ergodicity holds true, the statistical geometric invariants defined by the classical correlation functions of various induced classical gauge fields for preferred extremals could be regarded as this kind of invariants for sub-manifolds. What would distinguish TGD from standard topological QFT would be that the invariants in question would involve length scale and thus have a physical content in the usual sense of the word!

14.8.2 Could $\mathcal{N}$ =2 or $\mathcal{N} = 4$ SUSY have something to do with TGD?

$\mathcal{N} = 4$ SYM has been the theoretical laboratory of Nima and others. $\mathcal{N} = 4$ SYM is definitely a completely exceptional theory, and one cannot avoid the question whether it could in some sense be part of fundamental physics. In TGD framework right handed neutrinos have remained a mystery: whether one should assign space-time SUSY to them or not. Could they give rise to something resembling $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY with fermion number conservation?

Earlier results

My latest view is that *fully* covariantly constant right-handed neutrinos decouple from the dynamics completely. I will repeat first the earlier arguments which consider only fully covariantly constant right-handed neutrinos.

(a) $\mathcal{N}=1$ SUSY is certainly excluded since it would require Majorana property not possible in TGD framework since it would require superposition of left and right handed neutrinos and lead to a breaking of lepton number conservation. Could one imagine SUSY in which both MEs between which particle wormhole contacts reside have $\mathcal{N}=2$ SUSY which combine to form an $\mathcal{N}=4$ SUSY?

(b) Right-handed neutrinos which are covariantly constant right-handed neutrinos in both M^4 degrees of freedom cannot define a non-trivial theory as shown already earlier. They have no electroweak nor gravitational couplings and carry no momentum, only spin.

The fully covariantly constant right-handed neutrinos with two possible helicities at given ME would define representation of SUSY at the limit of vanishing light-like momentum. At this limit the creation and annihilation operators creating the states would have vanishing anti-commutator so that the oscillator operators would generate Grassmann algebra. Since creation and annihilation operators are hermitian conjugates, the states would have zero norm and the states generated by oscillator operators would be pure gauge and decouple from physics. This is the core of the earlier argument demonstrating that $\mathcal{N}=1$ SUSY is not possible in TGD framework: LHC has given convincing experimental support for this belief.

Could massless right-handed neutrinos covariantly constant in CP_2 degrees of freedom define $\mathcal{N}=2$ or $\mathcal{N}=4$ SUSY?

Consider next right-handed neutrinos, which are covariantly constant in CP_2 degrees of freedom but have a light-like four-momentum. In this case fermion number is conserved but this is consistent with $\mathcal{N}=2$ SUSY at both MEs with fermion number conservation. $\mathcal{N}=2$ SUSYs could emerge from $\mathcal{N}=4$ SUSY when one half of SUSY generators annihilate the states, which is a basic phenomenon in supersymmetric theories.

(a) At space-time level right-handed neutrinos couple to the space-time geometry - gravitation - although weak and color interactions are absent. One can say that this coupling forces them to move with light-like momentum parallel to that of ME. At the level of space-time surface right-handed neutrinos have a spectrum of excitations of four-dimensional analogs of conformal spinors at string world sheet (Hamilton-Jacobi structure).

For MEs one indeed obtains massless solutions depending on longitudinal M^2 coordinates only since the induced metric in M^2 differs from the light-like metric only by a contribution which is light-like and contracts to zero with light-like momentum in the same direction. These solutions are analogs of (say) left movers of string theory. The dependence on E^2 degrees of freedom is holomorphic. That left movers are only possible would suggest that one has only single helicity and conservation of fermion number at given space-time sheet rather than 2 helicities and non-conserved fermion number: two real Majorana spinors combine to single complex Weyl spinor.

(b) At imbedding space level one obtains a tensor product of ordinary representations of $\mathcal{N}=2$ SUSY consisting of Weyl spinors with opposite helicities assigned with the ME. The state content is same as for a reduced $\mathcal{N}=4$ SUSY with four $\mathcal{N}=1$ Majorana spinors replaced by two complex $\mathcal{N}=2$ spinors with fermion number conservation. This gives 4 states at both space-time sheets constructed from ν_R and its antiparticle. Altogether the two MEs give 8 states, which is one half of the 16 states of $\mathcal{N}=4$ SUSY so that a degeneration of this symmetry forced by non-Majorana property is in question.

Is the dynamics of $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM possible in right-handed neutrino sector?

Could $\mathcal{N}=2$ or $\mathcal{N}=4$ SYM be a part of quantum TGD? Could TGD be seen a fusion of a degenerate $\mathcal{N}=4$ SYM describing the right-handed neutrino sector and string theory like theory describing the contribution of string world sheets carrying other leptonic and quark spinors? Or could one imagine even something simpler?

What is interesting that the net momenta assigned to the right handed neutrinos associated with a pair of MEs would correspond to the momenta assignable to the particles and obtained by p-adic mass calculations. It would seem that right-handed neutrinos provide a representation of the momenta of the elementary particles represented by wormhole contact structures. Does this mimicry generalize to a full duality so that all quantum numbers and even microscopic dynamics of defined by generalized Feynman diagrams (Euclidian space-time regions) would be represented by right-handed neutrinos and MEs? Could a generalization of $\mathcal{N}=4$ SYM with non-trivial gauge group with proper choices of the ground states helicities allow to represent the entire microscopic dynamics?

Irrespective of the answer to this question one can compare the TGD based view about supersymmetric dynamics with what I have understood about $\mathcal{N}=4$ SYM.

(a) In the scattering of MEs induced by the dynamics of Kähler action the right-handed neutrinos play a passive role. Modified Dirac equation forces them to adopt the same direction of four-momentum as the MEs so that the scattering reduces to the geometric scattering for MEs as one indeed expects on basic of quantum classical correspondence. In ν_R sector the basic scattering vertex involves four MEs and could be a re-sharing of the right-handed neutrino content of the incoming two MEs between outgoing two MEs respecting fermion number conservation. Therefore $\mathcal{N}=4$ SYM with fermion number conservation would represent the scattering of MEs at quantum level.

(b) $\mathcal{N}=4$ SUSY would suggest that also in the degenerate case one obtains the full scattering amplitude as a sum of permutations of external particles followed by projections to the directions of light-like momenta and that BCFW bridge represents the analog of fundamental braiding operation. The decoration of permutations means that each external line is effectively doubled. Could the scattering of MEs can be interpreted in terms of these decorated permutations? Could the doubling of permutations by decoration relate to the occurrence of pairs of MEs?

One can also revert these questions. Could one construct massive states in $\mathcal{N}=4$ SYM using pairs of momenta associated with particle with integer label k and its decorated copy with label $k+n$? Massive external particles obtained in this manner as bound states of massless ones could solve the IR divergence problem of $\mathcal{N}=4$ SYM.

(c) The description of amplitudes in terms of leading singularities means picking up of the singular contribution by putting the fermionic propagators on mass shell. In the recent case it would give the inverse of massless Dirac propagator acting on the spinor at the end of the internal line annihilating it if it is a solution of Dirac equation.

The only way out is a kind of cohomology theory in which solutions of Dirac equation represent exact forms. Dirac operator defines the exterior derivative d and virtual lines correspond to non-physical helicities with $d\Psi \neq 0$. Virtual fermions would be on mass-shell fermions with non-physical polarization satisfying $d^2\Psi = 0$. External particles would be those with physical polarization satisfying $d\Psi = 0$, and one can say that the Feynman diagrams containing physical helicities split into products of Feynman diagrams containing only non-physical helicities in internal lines.

(d) The fermionic states at wormhole contacts should define the ground states of SUSY representation with helicity $+1/2$ and -1/2 rather than spin 1 or -1 as in standard realization of $\mathcal{N}=4$ SYM used in the article. This would modify the theory but the twistorial and Grassmannian description would remain more or less as such since it depends on light-likeness and momentum conservation only.

3-vertices for sparticles are replaced with 4-vertices for MEs

In $\mathcal{N}=4$ SYM the basic vertex is on mass-shell 3-vertex which requires that for real light-like momenta all 3 states are parallel. One must allow complex momenta in order to satisfy energy conservation and light-likeness conditions. This is strange from the point of view of physics although number theoretically oriented person might argue that the extensions of rationals involving also imaginary unit are rather natural.

The complex momenta can be expressed in terms of two light-like momenta in 3-vertex with one real momentum. For instance, the three light-like momenta can be taken to be p, k, and $p - ka$ with $k = ap_R$. Here p (incoming momentum) and p_R are real light-like momenta satisfying $p \cdot p_R = 0$ but with opposite sign of energy, and a is complex number. What is remarkable that also the negative sign of energy is necessary also now.

Should one allow complex light-like momenta in TGD framework? One can imagine two options.

(a) Option I: no complex momenta. In zero energy ontology the situation is different due to the presence of a pair of MEs meaning replaced of 3-vertices with 4-vertices or 6-vertices, the allowance of negative energies in internal lines, and the fact that scattering is of sparticles is induced by that of MEs. In the simplest vertex a massive external particle with non-parallel MEs carrying non-parallel light-like momenta can decay to a pair of MEs with light-like momenta. This can be interpreted as 4-ME-vertex rather than 3-vertex (say) BFF so that complex momenta are not needed. For an incoming boson identified as wormhole contact the vertex can be seen as BFF vertex.

    To obtain space-like momentum exchanges one must allow negative sign of energy and one has strong conditions coming from momentum conservation and light-likeness which allow non-trivial solutions (real momenta in the vertex are not parallel) since basically the vertices are 4-vertices. This reduces dramatically the number of graphs. Note that one can also consider vertices in which three pairs of MEs join along their ends so that 6 MEs (analog of 3-boson vertex) would be involved.

(b) Option II: complex momenta are allowed. Proceeding just formally, the $\sqrt{g_4}$ factor in Kähler action density is imaginary in Minkowskian and real in Euclidian regions. It is now clear that the formal approach is correct: Euclidian regions give rise to Kähler function and Minkowskian regions to the analog of Morse function. TGD as almost topological QFT inspires the conjecture about the reduction of Kähler action to boundary terms proportional to Chern-Simons term. This is guaranteed if the condition $j^\mu_K A_\mu = 0$ holds true: for the known extremals this is the case since Kähler current j_K is light-like or vanishing for them. This would seem that Minkowskian and Euclidian regions provide dual descriptions of physics. If so, it would not be surprising if the real and complex parts of the four-momentum were parallel and in constant proportion to each other.

This argument suggests that also the conserved quantities implied by the Noether theorem have the same structure so that charges would receive an imaginary contribution from Minkowskian regions and a real contribution from Euclidian regions (or vice versa). Four-momentum would be complex number of form $P = P_M + iP_E$. Generalized light-likeness condition would give $P_M^2 = P_E^2$ and $P_M \cdot P_E = 0$. complexified momentum would have 6 free components. A stronger condition would be $P_M^2 = 0 = P_E^2$ so that one would have two light-like momenta "orthogonal" to each other. For both relative signs energy P_M and P_E would be actually parallel: parameterization would be in terms of light-like momentum and scaling factor. This would suggest that complex momenta do not bring in anything new and Option II reduces effectively to Option I. If one wants a complete analogy with the usual twistor approach then $P_M^2 = P_E^2 \neq 0$ must be allowed.

Is SUSY breaking possible or needed?

It is difficult to imagine the breaking of the proposed kind of SUSY in TGD framework, and the first guess is that all these 4 super-partners of particle have identical masses. p-Adic thermodynamics does not distinguish between these states and the only possibility is that the p-adic primes differ for the spartners. But is the breaking of SUSY really necessary? Can one really distinguish between the 8 different states of a given elementary particle using the recent day experimental methods?

(a) In electroweak and color interactions the spartners behave in an identical manner classically. The coupling of right-handed neutrinos to space-time geometry however forces the right-handed neutrinos to adopt the same direction of four-momentum as MEs has. Could some gravitational effect allow to distinguish between spartners? This would be trivially the case if the p-adic mass scales of spartners would be different. Why this should be the case remains however an open question.

(b) In the case of unbroken SUSY only spin distinguishes between spartners. Spin determines statistics and the first naive guess would be that bosonic spartners obey totally different atomic physics allowing condensation of selectrons to the ground state. Very probably this is not true: the right-handed neutrinos are de-localized to 4-D MEs and other fermions correspond to wormhole contact structures and 2-D string world sheets.

The coupling of the spin to the space-time geometry seems to provide the only possible manner to distinguish between spartners. Could one imagine a gravimagnetic effect with energy splitting proportional to the product of gravimagnetic moment and external gravimagnetic field B? If gravimagnetic moment is proportional to spin projection in the direction of B, a non-trivial effect would be possible. Needless to say this kind of effect is extremely small so that the unbroken SUSY might remain undetected.

(c) If the spin of sparticle be seen in the classical angular momentum of ME as quantum classical correspondence would suggest then the value of the angular momentum might allow to distinguish between spartners. Also now the effect is extremely small.

What can one say about scattering amplitudes?

One expect that scattering amplitudes factorize with the only correlation between right-handed neutrino scattering and ordinary particle scattering coming from the condition that the four-momentum of the right-handed neutrino is parallel to that of massless extremal of more general preferred extremal having interpretation as a geometric counterpart of radiation quantum. This momentum is in turn equal to the massless four-momentum associated with the space-time sheet in question such that the sum of classical four-momenta associated with the space-time sheets equals to that for all wormhole throats involved. The right-handed neutrino amplitude itself would be simply constant. This certainly satisfies the SUSY constraint and it is actually difficult to find other candidates for the amplitude. The dynamics of right-handed neutrinos would be therefore that of spectator following the leader.

14.8.3 Right-handed neutrino as inert neutrino?

There is a very interesting posting by Jester in Resonaances with title How many neutrinos in the sky? [C1]. Jester tells about the recent 9 years WMAP data [C7] and compares it with earlier 7 years data. In the earlier data the effective number of neutrino types was $N_{eff}=4.34\pm 0.87$ and in the recent data it is $N_{eff}=3.26\pm 0.35$. WMAP alone would give $N_{eff}=3.89\pm 0.67$ also in the recent data but also other data are used to pose constraints on N_{eff}.

To be precise, N_{eff} could include instead of fourth neutrino species also some other weakly interacting particle. The only criterion for contributing to N_{eff} is that the particle is in thermal equilibrium with other massless particles and thus contributes to the density of matter considerably during the radiation dominated epoch.

Jester also refers to the constraints on N_{eff} from nucleosynthesis, which show that $N_{eff}\sim 4$ us slightly favored although the entire range $[3,5]$ is consistent with data.

It seems that the effective number of neutrinos could be 4 instead of 3 although latest WMAP data combined with some other measurements favor 3. Later a corrected version of the eprint appeared [C7] telling that the original estimate of N_{eff} contained a mistake and the correct estimate is $N_{eff}=3.84\pm 0.40$.

An interesting question is what $N_{eff}=4$ could mean in TGD framework?

(a) One poses to the modes of the modified Dirac equation the following condition: electric charge is conserved in the sense that the time evolution by modified Dirac equation does not mix a mode with a well-defined em charge with those with different em charge. The implication is that all modes except pure right handed neutrino are restricted at string world sheets. The first guess is that string world sheets are minimal surfaces of space-time surface (rather than those of imbedding space). One can also consider minimal surfaces of imbedding space but with effective metric defined by the anti-commutators of the modified gamma matrices. This would give a direct physical meaning for this somewhat mysterious effective metric.

For the neutrino modes localized at string world sheets mixing of left and right handed modes takes place and they become massive. If only 3 lowest genera for partonic 2-surfaces are light, one has 3 neutrinos of this kind. The same applies to all other fermion species. The argument for why this could be the case relies on simple observation [K13]: the genera g=0,1,2 have the property that they allow for all values of conformal moduli Z_2 as a conformal symmetry (hyper-ellipticity). For $g > 2$ this is not the case. The guess is that this additional conformal symmetry is the reason for lightness of the three lowest genera.

(b) Only purely right-handed neutrino is completely de-localized in 4-volume so that one cannot assign to it genus of the partonic 2-surfaces as a topological quantum number and it effectively gives rise to a fourth neutrino very much analogous to what is called sterile neutrino. De-localized right-handed neutrinos couple only to gravitation and in case of massless extremals this forces them to have four-momentum parallel to that of ME: only massless modes are possible. Very probably this holds true for all preferred extremals to which one can assign massless longitudinal momentum direction which can vary with spatial position.

(c) The coupling of ν_R is to gravitation alone and all electroweak and color couplings are absent. According to standard wisdom de-localized right-handed neutrinos cannot be in thermal equilibrium with other particles. This according to standard wisdom. But what about TGD?

One should be very careful here: de-localized right-handed neutrinos is proposed to give rise to SUSY (not $\mathcal{N} = 1$ requiring Majorana fermions) and their dynamics is that of passive spectator who follows the leader. The simplest guess is that the dynamics of right handed neutrinos at the level of amplitudes is completely trivial and thus trivially supersymmetric. There are however correlations between four-momenta.

i. The four-momentum of ν_R is parallel to the light-like momentum direction assignable to the massless extremal (or more general preferred extremal). This direct coupling to the geometry is a special feature of the modified Dirac operator and thus of sub-manifold gravity.

ii. On the other hand, the sum of massless four-momenta of two parallel pieces of preferred extremals is the - in general massive - four-momentum of the elementary particle defined by the wormhole contact structure connecting the space-time sheets (which are glued along their boundaries together since this is seems to be the only manner to get rid of boundary conditions requiring vacuum extremal property near the boundary). Could this direct coupling of the four-momentum direction of right-handed neutrino to geometry and four-momentum directions of other fermions be enough for the right handed neutrinos to be counted as a fourth neutrino species in thermal equilibrium? This might be the case!

One cannot of course exclude the coupling of 2-D neutrino at string world sheets to 4-D purely right handed neutrinos analogous to the coupling inducing a mixing of sterile neutrino with ordinary neutrinos. Also this could help to achieve the thermal equilibrium with 2-D neutrino species.

Experimental evidence for sterile neutrino?

Many physicists are somewhat disappointed to the results from LHC: the expected discovery of Higgs has been seen as the main achievement of LHC hitherto. Much more was expected. To my opinion there is no reason for disappointment. The exclusion of the standard SUSY at expected energy scale is very far reaching negative result. Also the fact that Higgs mass is too small to be stable without fine tuning is of great theoretical importance. The negative results concerning heavy dark matter candidates are precious guidelines for theoreticians. The non-QCD like behavior in heavy ion collisions and proton-ion collisions is bypassed my mentioning something about AdS/CFT correspondence and non-perturbative QCD effects. I tend to see these effects as direct evidence for M_{89} hadron physics [K40].

In any case, something interesting has emerged quite recently. Resonaances tells that the recent analysis [C6] of X-ray spectrum of galactic clusters claims the presence of monochromatic 3.5 keV photon line. The proposed interpretation is as a decay product of sterile 7 keV neutrino transforming first to a left-handed neutrino and then decaying to photon and neutrino via a loop involving W boson and electron. This is of course only one of the many interpretations. Even the existence of line is highly questionable.

One of the poorly understood aspects of TGD is right-handed neutrino, which is obviously the TGD counterpart of the inert neutrino.

(a) The old idea is that covariantly constant right handed neutrino could generate $\mathcal{N} = 2$ super-symmetry in TGD Universe. In fact, all modes of induced spinor field would generate superconformal symmetries but electroweak interactions would break these symmetries for the modes carrying non-vanishing electroweak quantum numbers: they vanish for ν_R. This picture is now well-established at the level of WCW geometry [K98]: super-conformal generators are labelled angular momentum and color representations plus two conformal weights: the conformal weight assignable to the light-like radial coordinate of light-cone boundary and the conformal weight assignable to string coordinate. It seems that these conformal weights are independent. The third integer labelling the states would label genuinely Yangian generators: it would tell the poly-locality of the generator with locus defined by partonic 2-surface: generators acting on single partonic 2-surface, 2 partonic 2-surfaces, ...

(b) It would seem that even the SUSY generated by ν_R must be badly broken unless one is able to invent dramatically different interpretation of SUSY. The scale of SUSY breaking and thus the value of the mass of right-handed neutrino remains open also in TGD. In lack of better one could of course argue that the mass scale must be CP_2 mass scale because right-handed neutrino mixes considerably with the left-handed neutrino (and thus becomes massive) only in this scale. But why this argument does not apply also to left handed neutrino which must also mix with the right-handed one!

(c) One can of course criticize the proposed notion of SUSY: wonder whether fermion + extremely weakly interacting ν_R at same wormhole throat (or interior of 3-surface) can behave as single coherent entity as far spin is considered [K85]?

(d) The condition that the modes of induced spinor field have a well-defined electromagnetic charge eigenvalue [K87] requires that they are localized at 2-D string world sheets or partonic 2-surfaces: without this condition classical W boson fields would mix the em charged and neutral modes with each other. Right-handed neutrino is an exception since it has no electroweak couplings. Unless right-handed neutrino is covariantly constant, the modified gamma matrices can however mix the right-handed neutrino with the left handed one and this can induce transformation to charged mode. This does not happen if each modified gamma matrix can be written as a linear combination of either M^4 or CP_2 gamma matrices and modified Dirac equation is satisfied separately by M^4 and CP_2 parts of the modified Dirac equation.

(e) Is the localization of the modes other than covariantly constant neutrino to string world sheets a consequence of dynamics or should one assume this as a separate condition? If one wants similar localization in space-time regions of Euclidian signature - for which CP_2 type vacuum extremal is a good representative - one must assume it as a separate condition. In number theoretic formulation string world sheets/partonic 2-surfaces would be commutative/co-commutative sub-manifolds of space-time surfaces which in turn would be associative or co-associative sub-manifolds of imbedding space possessing (hyper-)octonionic tangent space structure. For this option also right-handed neutrino would be localized to string world sheets. Right-handed neutrino would be covariantly constant only in 2-D sense.

One can consider the possibility that ν_R is de-localized to the entire 4-D space-time sheet. This would certainly modify the interpretation of SUSY since the number of degrees of freedom would be reduced for ν_R.

(f) Non-covariantly constant right-handed neutrinos could mix with left-handed neutrinos but not with charged leptons if the localization to string world sheets is assumed for modes carrying non-vanishing electroweak quantum numbers. This would make possible the decay of right-handed to neutrino plus photon, and one cannot exclude the possibility that ν_R has mass 7 keV.

Could this imply that particles and their spartners differ by this mass only? Could it be possible that practically unbroken SUSY could be there and we would not have observed it? Could one imagine that sfermions have annihilated leaving only states consisting of fundamental fermions? But shouldn't the total rate for the annihilation of photons to hadrons be two times the observed one? This option does not sound plausible.

What if one assumes that given sparticle is charactrized by the same p-adic prime as corresponding particle but is dark in the sense that it corresponds to non-standard value of Planck constant. In this case sfermions would not appear in the same vertex with fermions and one could escape the most obvious contradictions with experimental facts. This leads to the notion of shadron: shadrons would be [K85] obtained by replacing quarks with dark squarks with nearly identical masses. I have asked whether so called X and Y bosons having no natural place in standard model of hadron could be this kind of creatures.

The interpretation of 3.5 keV photons as decay products of right-handed neutrinos is of course totally ad hoc. Another TGD inspired interpretation would be as photons resulting from the decays of excited nuclei to their ground state.

(a) Nuclear string model [L2] predicts that nuclei are string like objects formed from nucleons connected by color magnetic flux tubes having quark and antiquark at their ends. These flux tubes are long and define the "magnetic body" of nucleus. Quark and antiquark have opposite em charges for ordinary nuclei. When they have different charges one obtains exotic state: this predicts entire spectrum of exotic nuclei for which statistic is different from what proton and neutron numbers deduced from em charge and atomic weight would suggest. Exotic nuclei and large values of Planck constant could make also possible cold fusion [K19].

(b) What the mass difference between these states is, is not of course obvious. There is however an experimental finding [C8] (see *Analysis of Gamma Radiation from a Radon Source: Indications of a Solar Influence*) that nuclear decay rates oscillate with a period of year and the rates correlate with the distance from Sun. A possible explanation is that the gamma rays from Sun in few keV range excite the exotic nuclear states with different decay rate so that the average decay rate oscillates [L2]. Note that nuclear excitation energies in keV range would also make possible interaction of nuclei with atoms and molecules.

(c) This allows to consider the possibility that the decays of exotic nuclei in galactic clusters generates 3.5 keV photons. The obvious question is why the spectrum would be concentrated at 3.5 keV in this case (second question is whether the energy is really concentrated at 3.5 keV: a lot of theory is involved with the analysis of the experiments). Do the energies of excited states depend on the color bond only so that they would be essentially same for all nuclei? Or does single excitation dominate in the spectrum? Or is this due to the fact that the thermal radiation leaking from the core of stars excites predominantly single state? Could $E = 3.5$ keV correspond to the maximum intensity for thermal radiation in stellar core? If so, the temperature of the exciting radiation would be about $T \simeq E/3 \simeq 1.2 \times 10^7$ K. This in the temperature around which formation of Helium by nuclear fusion has begun: the temperature at solar core is around 1.57×10^7 K.

Chapter 15

Coupling Constant Evolution in Quantum TGD

15.1 Introduction

In quantum TGD two kinds of discrete coupling constant evolutions emerge. p-Adic coupling constant evolution is with respect to the discrete hierarchy of p-adic length scales and p-adic length scale hypothesis suggests that only the length scales coming as half octaves of a fundamental length scale are relevant here. Second coupling constant evolution corresponds to hierarchy of Planck constants requiring a generalization of the notion of imbedding space. One can assign this evolution with angle resolution in number theoretic approach.

The notion of zero energy ontology allows to justify p-adic length scale hypothesis and formulate the discrete coupling constant evolution at fundamental level. WCW would consists of sectors which correspond to causal diamonds (CDs) identified as intersections of future and past directed light-cones. If the sizes of CDs come in powers of 2^n, p-adic length scale hypothesis emerges, and coupling constant evolution is discrete provided RG invariance holds true inside CDs for space-time evolution of coupling constants defined in some sense to be defined. In this chapter arguments supporting this conclusion are given by starting from a detailed vision about the basic properties of preferred extremals of Kähler action.

How to calculate or even "understand" the correlation functions and coupling constant evolution has remained a basic unresolved challenge. The latest (means the end of 2012) and perhaps the most powerful idea hitherto is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

The chapter decomposes into two parts. In the first part basic notions are introduced and a general vision about p-adic coupling constant evolution is introduced. After that a general formulation of coupling constant evolution at space-time level and related interpretational issues are considered. In the second part quantitative predictions involving some far from rigorous arguments, which I however dare to take rather seriously, are discussed. It must be emphasized that this chapter like many others is more like a still continuing story about development of ideas - not a brief summary about a solution of a precisely defined problem. There are many ad hoc ideas and conflicting views. These books are just lab note books - nothing more.

15.1.1 Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

Physics as infinite-dimensional Kähler geometry

(a) The basic idea is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

(b) WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. WCW gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of the configuration space forms what I have used to called super-symplectic algebra.

Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCdthey define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. The assumption that the commutator algebra of these super-symplectic and super Kac-Moody algebras annihilates physical states gives rise to Super Virasoro conditions which could be regarded as analogs of configuration space Dirac equation.

Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

(c) WCW spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds.

p-Adic physics as physics of cognition and intentionality

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of configuration space spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K65] . It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The ideas is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points ((hyper-)quaternions, (hyper-)octonions) become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

Hierarchy of Planck constants and dark matter hierarchy

The work with hyper-finite factors of type II_1 (HFFs) combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter [K21]

. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. These variants of imbedding space are characterized by discrete subgroups of SU(2) acting in M^4 and CP_2 degrees of freedom as either symmetry groups or homotopy groups of covering. Among other things this picture implies a general model of fractional quantum Hall effect.

This framework also leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of a generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. The outcome is an explicit formula for the Dirac determinant consistent with the vacuum degeneracy of Kähler action and its finiteness and algebraic number property required by p-adicization requiring number theoretic universality. The zeta function associated with the eigenvalues (rather than Riemann Zeta as believed originally) in turn defines the super-symplectic conformal weights as its zeros so that a highly coherent picture result.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of the partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. It is clear that the hierarchy of Planck constants has become an essential part of the construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

(a) There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞. The groups G should relate closely to finite groups defining inclusions of HFFs.

(b) HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit and $M^4 \times CP_2$ can be interpreted as a structure related to hyper-octonions which is a subspace of complexified octonions for which metric has naturally Minkowski signature. This would mean that TGD could be seen also as a generalized number theory. This conjecture predicts the existence of two dual formulations of TGD based on the identification space-times as 4-surfaces in hyper-octonionic space M^8 *resp.* $M^4 \times CP_2$.

(c) The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

Could quantum ergodicity allow to calculate correlation functions and coupling constant evolution from the statistical properties of preferred extremals?

How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Generalized Feynman diagrams provide a powerful vision which however does not help in practical calculations. Some big idea has been lacking.

The latest (means the end of year 2012) and perhaps the most powerful idea hitherto is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. All preferred extremals in the superposition would have same correlation functions coding for S-matrix. This principle would be a quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This symmetry principle analogous to holography might allow to fix S-matrix uniquely even in the case that the hermitian square root of the density matrix appearing in the M-matrix would lead to a breaking of quantum ergodicity as also 4-D spin glass degeneracy suggests.

This principle would allow to deduce correlation functions from the statistical properties of single preferred extremal alone using just classical intuition. Also coupling constant evolution would be coded by the statistical properties of preferred extremals. Quantum ergodicity would mean an enormous simplification since one could avoid the horrible conceptual complexities involved with the functional integrals over WCW.

15.1.2 Identification of symplectic and Kac-Moody symmetries

The basic symmetries are isometries of "world of classical worlds" (WCW) proposed to be realized as symplectic transformations of the boundaries of causal diamonds (CD) locally identifiable as $\delta M^4_\pm \times CP_2$. These symplectic symmetries contains as algebra symplectic isometries which are expected to be of special importance. These transformations are expected to have continuation to deformations of the entire preferred extremal. They cannot be symmetries of Kähler action.

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating configuration space degrees of freedom contributing to the metric. These symmetries would act as gauge symmetries and related to quantum criticality due to the non-determinism of Kähler action in turn giving rise to the hierarchy of Planck constants explaining dark matter. The recent vision looks like follows.

(a) The recent interpretation is that these gauge symmetries are due to the non-determinism of Kähler action and transform to each other preferred extremals with same space-like surfaces as their ends at the boundaries of causal diamond. These space-time surfaces have same Kähler action and possess same conserved quantities.

(b) The sub-algebra of conformal symmetries acts as gauge transformations of these infinite set of degenerate preferred extremals and there is finite number n of gauge equivalence classes. n corresponds to the effective (or real depending on interpretation) value of Planck constant $h_{eff} = n \times h$. The further conjecture is that the sub-algebra of conformal algebra for which conformal weights are integers divisible by n act as genuine gauge symmetries. If Kähler action reduces to a sum of 3-D Chern-Simons terms for preferred extremals, it is enough to consider the action on light-like 3-surfaces. For gauge part of algebra the algebra acts trivially at space-like 3-surfaces.

(c) A good guess is that the Kac-Moody type algebra corresponds to the sub-algebra of symplectic isometries of $\delta M^4_\pm \times CP_2$ acting on light-like 3-surfaces and having continuation to the interior.

A stronger assumption is that isometries are in question. For CP_2 nothing would change but light-cone boundary $\delta M^4_\pm = S^2 \times R_+$ has conformal transformations of S^2 as isometries. The conformal scaling is compensated by S^2-local scaling of the light like radial coordinate of R_+.

(d) This super-conformal algebra realized in terms of spinor modes and second quantized induced spinor fields would define the Super Kac-Moody algebra. The generators of this Kac-Moody type algebra have continuation from the light-like boundaries to deformations of preferred extremals and at least the generators of sub-algebra act trivially at space-like 3-surfaces.

15.1.3 The construction of M-matrix

15.1.4 The construction of M-matrix

The construction of S-matrix involves several ideas that have emerged during last years and involve symmetries in an essential manner.

Zero energy ontology

Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing conserved net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as a space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. Obviously a profound modification of existing views about realization of symmetries is in question.

S-matrix and density matrix are unified to the notion of M-matrix defining time-like entanglement and expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

Quantum TGD as almost topological QFT

Light-likeness of the basic fundamental objects suggests that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. The original proposal that Chern-Simons action for light-like 3-surfaces defined by the regions at which the signature of the induced metric changes its sign however failed and one must use Kähler action and corresponding modified Dirac action with measurement term to define the fundamental theory. At the limit when the momenta of particles vanish, the theory reduces to topological QFT defined by Kähler action and corresponding modified Dirac action. The imaginary exponent of the instanton term associated with the induced Kähler form defines the counterpart of Chern-Simons action as a phase of the vacuum functional and contributes also to modified Dirac equation.

M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory. Super-conformal symmetries implied by the light-likeness pose very strong constraints on both state construction and on M-matrix and U-matrix. The notions of n-category and n-groupoid which represents a generalization of the notion of group could be very relevant to this view about M-matrix.

Quantum measurement theory with finite measurement resolution

The notion of measurement resolution represented in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This means that complex rays of state space are effectively replaced with $\mathcal{N}$ rays. The condition that the action of $\mathcal{N}$ commutes with the M-matrix is a powerful symmetry and implies that the time-like entanglement characterized by M-matrix is consistent with Connes tensor product. This does not fix the M-matrix as was the original belief but only realizes mathematically the notion of finite measurement resolution. Together with super-conformal symmetries this constraint should fix possible M-matrices to a very high degree if one assumes the existence of universal M-matrix from which M-matrices with finite measurement resolution are obtained.

The notion of number theoretical braid realizes the notion of finite measurement resolution at space-time level and gives a direct connection to topological QFTs describing braids. The connection with quantum groups is highly suggestive since already the inclusions of HFFs involve these groups. Effective non-commutative geometry for the quantum critical sub-manifolds $M^2 \subset M^4$ and $S^2 \subset CP_2$ might provide an alternative notion for the reduction of stringy anti-commutation relations for induced spinor fields to anti-commutations at the points of braids.

Generalization of Feynman diagrams

An essential difference between TGD and string models is the replacement of stringy diagrams with generalized Feynman diagrams obtained by gluing 3-D light-like surfaces (instead of lines) together at their ends represented as partonic 2-surfaces. This makes the construction of vertices very simple. The notion of number theoretic braid in turn implies discretization having also interpretation in terms of non-commutativity due to finite measurement resolution replacing anti-commutativity along stringy curves with anti-commutativity at points of braids. Braids can replicate at vertices which suggests an interpretation in terms of topological quantum computation combined with non-faithful copying and communication of information. The analogs of stringy diagrams have quite different interpretation in TGD for instance, photons traveling via two different paths in double slit experiment are represented in terms of stringy branching of the photonic 2-surface.

Symplectic variant of QFT as basic building block of construction

The latest discovery related to the construction of M-matrix was the realization that a symplectic variant of conformal field theories might be a further key element in the concrete construction of n-point functions and M-matrix in zero energy ontology. Although I have known super-symplectic (super-symplectic) symmetries to be fundamental symmetries of quantum TGD for almost two decades, I failed for some reason to realize the existence of symplectic QFT, and discovered it while trying to understand quite different problem - the fluctuations of cosmic microwave background! The symplectic contribution to the n-point function satisfies fusion rules and involves only variables which are symplectic invariants constructed using geodesic polygons assignable to the sub-polygons of n-polygon defined by the arguments of n-point function. Fusion rules lead to a concrete recursive formula for n-point functions and M-matrix in contrast to the iterative construction of n-point functions used in perturbative QFT.

Bosonic emergence, extended space-time supersymmetry, and generalized twistors

During year 2009 several new ideas emerged and give hopes about a concrete construction of M-matrix.

(a) The notion of bosonic emergence [K50] follows from the fact that gauge bosons are identifiable as pairs of fermion and anti-fermion at opposite light-like throats of wormhole contact. As a consequence, bosonic propagators and vertices are generated radiatively from a fundamental action for fermions and their super partners. At QFT limit without super-symmetry this means that Dirac action coupled to gauge bosons is the fundamental action and the counterpart of YM action is generated radiatively. All coupling constants follow as predictions as they indeed must do on basis of the general structure of quantum TGD.

(b) Whether the counterparts of space-time supersymmetries are possible in TGD Universe has remained a long-standing open question and my cautious belief has been that the super partners do not exist. The resolution of the problem came with the increased understanading of the dynamics of the Kähler-Dirac action [K22, K23]. In particular, the localization of the electroweakly charged modes at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces- meant an enormous simplification since the solutions of the Kähler-Dirac equation are conformal spinor modes. Chern-Simons-Dirac term associated with with boundaries of string world sheet allows to construct Dirac propagator and together with its bosonic counterpart it explains the breaking of CP and T symmetries.

The oscillator operators associated with the modes of the induced spinor field satisfy the anti-commutation relations defining the generalization of space-time super-symmetry algebra and these oscillator operators serve as the building blocks of various super-conformal algebras. The number of super-symmetry generators is very large, perhaps even infinite. This forces a generalization of the standard super field concept. The action for chiral super-fields emerges as a generalization of the Dirac action to include all possible super-partners. The huge super-symmetry gives excellent hopes about cancelation of UV divergences. The counterpart of super-symmetric YM action emerges radiatively. This formalism works at the QFT limit. The generalization of the formalism to quantum TGD proper is yet to be carried out.

Covariantly constant right-handed neutrino defines a very special solution of Kähler-Dirac equation in that it is need not be localized to 2-D string world sheets. The reason it that it does not have electroweak couplings. This mode is excellent candidate for defining $\mathcal{N}=2$ SUSY, which however differs from the standard SUSY. SUSY breaking would be simple. p-Adic thermodynamics would predict same mass formulas for particles and their superpartners but the p-adic prime characterizing correspond space-time sheets could be different and thus also the p-adic mass scale.

(c) Twistor program has become one of the most promising approaches to gauge theories. This inspired the question whether TGD could allow twistorialization [K78] . Massive states -both real and virtual- are the basic problem of twistor approach. In TGD framework the obvious idea is that massive on mass shell states can be interpreted as massless states in 8-D sense. Massive off-mass shell states in turn could be regarded as pairs of positive and negative on mass shell states. This means opening of the black box of virtual state attempted already in the model for bosonic propagators inspired by the bosonic emergence [K50] , and one can even hope that individual loop integrals are finite and that Wick rotation is not needed. The third observation is that 8-dimensional gamma matrices allow a representation in terms of octonions (matrices are not in question anymore). If the modified gamma "matrices" associated with space-time surface define a quaternionic sub-algebra of the complexified octonion algebra, they allow a matrix representation defined by octonionic structure constants. This holds true for are hyper-quaternionic space-time surfaces so that a connection with number theoretic vision emerges. This would more or less reduce the notion of twistor to its 4-dimensional counterpart.

15.1.5 Could correlation functions, S-matrix, and coupling constant evolution be coded the statistical properties of preferred extremals?

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

(a) The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

(b) The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

(c) The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

    Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root " of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

(d) Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix having M-matrices as its orthonormal rows.

(e) In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

(a) General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of

M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

(b) The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \to \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

(c) What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

(d) What about Higgs field? TGD in principle allows scalar and pseudo-scalars which could be called Higgs like states. These states are however not necessary for particle massivation although they can represent particle massivation and must do so if one assumes that QFT limit exist. p-Adic thermodynamics however describes particle massivation microscopically.

The problem is that Higgs like field does not seem to have any obvious space-time correlate. The trace of the second fundamental form is the obvious candidate but vanishes for preferred extremals which are both minimal surfaces and solutions of Einstein Maxwell equations with cosmological constant. If the string world sheets at which all spinor components except right handed neutrino are localized for the general solution ansatz of the modified Dirac equation, the corresponding second fundamental form at the level of imbedding space defines a candidate for classical Higgs field. A natural expectation is that string world sheets are minimal surfaces of space-time surface. In general they are however not minimal surfaces of the imbedding space so that one might achieve a microscopic definition of classical Higgs field and its vacuum expectation value as an average of one point correlation function over the string world sheet.

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

15.1.6 Vision about coupling constant evolution

The following summarizes the basic vision about coupling constant evolution.

p-Adic evolution in phase resolution and the spectrum of values for Planck constants

The quantization of Planck constant has been the basic theme of TGD for about five years now. The basic idea is that the the different values of Planck constant correspond to evolution in angular resolution in p-adic context characterized by quantum phase $q = exp(i\pi/n)$ characterizing Jones inclusion is. The higher the value of n, the better the angular resolution since the number of different complex phases in extension of p-adic numbers increases with n.

The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

The most recent view about coupling constant evolution

In classical TGD only Kähler coupling constant appears explicitly but does not affect the classical dynamics. Other gauge couplings do not appear at all in classical dynamics since the the definition of classical fields absorbs them as normalization constants. Hence the notion of continuous coupling constant evolution at space-time level is not needed nor makes sense in quantum TGD proper and emerges only at the QFT limit when space-time is replaced with general relativistic effective space-time.

Discrete p-adic coupling constant evolution replacing in TGD the ordinary continuous coupling constant evolution emerges only when space-time sheets are lumped together to define GRT space-time. This evolution would have as parameters the p-adic length scale characterizing the causal diamond (CD) associated with particle and the phase factors characterizing the algebraic extension of p-adic numbers involved.

The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CDs whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This "prediction" is of course subject to criticism.

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination might make possible a concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules.

Equivalence Principle and evolution of gravitational constant

The views about Equivalence Princple (EP) and GRT limit of TGD have changed quite a lot since 2007 and here the updated view is summarized. Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Gravitational constant, cosmological constant, and various gauge couplings emerge as predictions and characterized the GRT-QFT limit of TGD. Planck length should be related to CP_2 size by a dimensionless numerical factor predicted by the theory. These constants need not be universal constants: cosmological constant is certainly very large for the Euclidian variant of GRT space-time. These constants could also depend on p-adic length scale. p-Adic coupling constant evolution suggests itself as a discretized variant of coupling constant evolution and p-adic scales would relate naturally to the size scales of causal diamonds: perhaps the integer n characterizing the multiple of CP_2 scale giving the distance between the tips of CD has p-adic prime p or its power as a divisor.

A warning for the reader is in order: the attempts to understand coupling constant evolution discussed in this chapter have been made for more than decade ago and do not not fully correspond fully the above summarized overall view which have emerged during 2014. In particular, these attempts do not assume the vision about how many-sheeted space-time leads to YM-Einstein type description as an effective description.

Quantitative estimates for the values of coupling constants

All quantitative statements about coupling constants are bound to be guesswork as long as explicit formulas for M-matrix elements are lacking. p-Adic length scale hypothesis provides one guideline for the guesses. Second guideline is provided by number theoretical universality. Third guideline is general physical intuition. What is done can be however seen as exercises perhaps giving some familiarity with the basic notions.

The latest proposal for understanding of p-adic coupling constant evolution comes from a formula for Kähler coupling strength α_K in terms of Dirac determinant of the modified Dirac operator.

The formula for α_K fixes its number theoretic anatomy and also that of other coupling strengths. The assumption that simple rationals (p-adicization) are involved can be combined with the input from p-adic mass calculations and with an old conjecture for the formula of gravitational constant allowing to express it in terms of CP_2 length scale and Kähler action of topologically condensed CP_2 type vacuum extremal. The prediction is that α_K is renormalization group invariant and equals to the value of fine structure constant at electron length scale characterized by M_{127}. Newton's constant is proportional to p-adic length scale squared and ordinary gravitons correspond to M_{127}. The number theoretic anatomy of R^2/G allows to consider two options. For the first one only M_{127} gravitons are possible number theoretically. For the second option gravitons corresponding to $p \simeq 2^k$ are possible.

A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

p-Adic length scale evolution of gauge couplings

Understanding the dependence of gauge couplings constants on p-adic prime is one of the basic challenges of quantum TGD. The problem has been poorly understood even at the conceptual level to say nothing about concrete calculations. The generalization of the motion of S-matrix to that of M-matrix changed however the situation [K14] . M-matrix is always defined with respect to measurement resolution characterized in terms of an inclusion of von Neumann algebra. Coupling constant evolution reduces to a discrete evolution involving only octaves of $T(k) = 2^k T_0$ of the fundamental time scale $T_0 = R$, where R CP_2 scale. p-Adic length scale $L(k)$ is related to $T(k)$ by $L^2(k) = T(k)T_0$. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, is automatic prediction of the theory. There is also a close connection with the description of coupling constant evolution in terms of radiative corrections.

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level and why certain p-adic primes are favored.

(a) Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges invariant: this seems to be the case for known extremals which form scaling invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

(b) Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at http://www.tgdtheory.fi/cmaphtml.html [L12]. Pdf representation of same files serving as a kind of glossary can be found at http://www.tgdtheory.fi/tgdglossary.pdf [L13]. The topics relevant to this chapter are given by the following list.

- TGD as infinite-dimensional geometry [L44]
- Geometry of WCW [L21]
- p-Adic physics [L29]
- Geometrization of fields [L20]
- Generalized Feynman diagrams [L19]
- p-Adic length scale hypothesis [L30]

15.2 General vision about real and p-adic coupling constant evolution

The unification of super-symplectic and Super Kac-Moody symmetries allows new view about p-adic aspects of the theory forcing a considerable modification and refinement of the almost decade old first picture about color coupling constant evolution.

Perhaps the most important questions about coupling constant evolution relate to the basic hypothesis about preferred role of primes $p \simeq 2^k$, k an integer. Why integer values of k are favored, why prime values are even more preferred, and why Mersenne primes $M_n = 2^n - 1$ and Gaussian Mersennes seem to be at the top of the hierarchy?

Second bundle of questions relates to the color coupling constant evolution. Do Mersenne primes really define a hierarchy of fixed points of color coupling constant evolution for a hierarchy of asymptotically non-free QCD type theories both in quark and lepton sector of the theory? How the transitions $M_n \rightarrow M_{n(next)}$ occur? What are the space-time correlates for the coupling constant evolution and for for these transitions and how space-time description relates to the usual description in terms of parton loops? How the condition that p-adic coupling constant evolution reflects the real coupling constant evolution can be satisfied and how strong conditions it poses on the coupling constant evolution?

15.2.1 A general view about coupling constant evolution

Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events.

Einstein's equations, Equivalence Principle, and GRT and QFT limits of TGD

Coupling constant evolution makes sense in quantum field theory defined in fixed background space-time, say Minkowski space-time. In TGD framework imbedding space replaces this fixed space-time and in ZEO the hierarchy of causal diamonds replaces imbedding space. It is not at all clear whether at the level of basic TGD coupling constant evolution makes sense at all whereas it should make sense at QFT limit of TGD. This requires understanding of QFT and GRT limits of TGD including also Equivalence Principle.

At quantum level Equivalence Principle (EP) can be reduced to quantum classical correspondence: the conserved four-momentum associated with Kähler action equals to the eigenvalue of conserved quantal four-momentum assignable to Kähler-Dirac equation [K87]. This quantal four-momentum in turn can be associated with string world sheets which emerge naturally from Kähler-Doirac equation.

Einstein's equation give a purely local meaning for EP. How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD, whose resolution came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book). .

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Similar description applies to induced electroweak gauge potentials and color gauge potentials: the sum of these gauge potentials over space-time sheets should define the classical gauge fields of QFT limit of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

What coupling constant evolution could mean in TGD framework? Kähler action and Kähler-Dirac action do not contain any fundamental couplings affecting to the dynamics. Kähler coupling strength does not affect classical dynamics and is analogous to critical temperature, and therefore invariant under renormalization group if defined in TGD framework. This suggests that the analog of renormalization group equations at space-time level does not look feasible. Continuous coupling constant evolution might be useful notion only at the QFT limit.

The natural length scale hierarchy associated with coupling constant evolution would be the hierarchy of length scales assignable to CDs. The minimal sizes of CDs assumed to be equal to secondary p-adic length scales in the case of elementary particles. More generally, number theoretical arguments suggest that the scales of CDs come as integer multiples of CP_2 radius. What is new that coupling constant evolution would be discretized, being labelled by integers. Primes and primes near powers of 2 could correspond to physically favored minimal size scales for CDs: kind of survivors in fight for survival. Discrete coupling constant evolution as evolution of various M-matrix elements as function of the size-scale of CD would look like a reasonable TGD counterpart of coupling constant evolution. For single CD one might say that system is quantum critical, and coupling constants do not evolve.

Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [K14] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [A58] known as hyperfinite factor of type II_1 (HFF) [K14, K79, K21] . HFF [A55, A71] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [A2] . The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [A94] , anyons [D5] , quantum groups and conformal field theories [A72] , and knots and topological quantum field theories [A86, A99] .

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [K14] . The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. The original hope was that Connes tensor product realizing mathematical the finite measurement resolution could fix M-matrix to high degree turned out be too optimistic.

How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

Zero energy ontology in which zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of future and past directed light-cones are power of 2 multiples of fundamental time scale ($T_n = 2^nT_0$) implies in a natural manner coupling constant evolution. One must however emphasize that also the weaker condition $T_p = pT_0$, p prime, is possible, and would assign all p-adic time scales to the size scale hierarchy of CDs.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^nT_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

(a) The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^nT_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^kT_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

(b) p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have secondary Compton length Electron's secondary Compton time $T_e(127) = \sqrt{5}T_2(127) = .1$ seconds defines a fundamental biological rhythm. A deep connection between elementary particle physics and biology becomes highly suggestive.

(c) In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

15.2.2 Both symplectic and conformal field theories are needed in TGD framework

Before one can say anything quantitative about coupling constant evolution, one must have a formulation for its TGD counterpart and thus also a more detailed formulation for how to calculate M-matrix elements. There is also the question about infinities. By very general arguments infinities of quantum field theories are predicted to cancel in TGD Universe - basically by the non-locality of Kähler function as a functional of 3-surface and by the general properties of the vacuum functional identified as the exponent of Kähler function. The precise mechanism leading to the cancellation of infinities of local quantum field theories has remained unspecified. Only the realization that the symplectic invariance of quantum TGD provides a mechanism regulating the short distance behavior of N-point functions changed the situation in this respect. This also leads to concrete view about the generalized Feynman diagrams giving M-matrix elements and rather close resemblance with ordinary Feynman diagrammatics.

Symplectic invariance

Symplectic symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light-cone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [K15] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [K49] . In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

(a) In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

(b) For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

(c) Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s \quad . \tag{15.2.1}$$

Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

(d) The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl}f(A(s_1,s_2,s))Idd\mu_s$ so that one has

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl}f(A(s_1,s_2,s))d\mu_s \quad . \tag{15.2.2}$$

Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n=1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

(a) The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1,s_2,s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1,s_2,s_3) \quad \equiv \quad A(s_1,s_2,s_3,N) - A(s_1,s_2,s_3,S) \tag{15.2.3}$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

(b) The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\begin{aligned}\langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle &= c_{kl}^r \int f(\Delta A(s_1,s_2,s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s \\ &= \end{aligned} \tag{15.2.4}$$

$$c_{kl}^r c_{rm}\int f(\Delta A(s_1,s_2,s)) f(\Delta A(s,s_3,t)) d\mu_s d\mu_t \ . \tag{15.2.5}$$

Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

(c) 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1,s_2,s))d\mu_s \tag{15.2.6}$$

(d) There is a clear difference between $n>3$ and $n=3$ cases: for $n>3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n=4$ theory is certainly well-defined, but one can argue that so are also $n>4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

(e) To sum up, the general predictions are following. Quite generally, for $f(0)=0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

Generalization to quantum TGD

(Number theoretic) braids are identifiable as boundaries of string world sheets at which the modes of induced spinor fields are localized in the generic case in Minkowskian space-time regions. Fundamental fermions can be assigned to these lines. Braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

(a) This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

(b) Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

(c) The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

(a) It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n) + N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

(b) What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of WCW. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

(c) The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

(d) This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [K14] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

(a) *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [K67] .

(b) Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

(c) It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time.

(d) One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and WCW degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

(e) Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of WCW Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

(f) Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. The coupling constant evolution is based on the same mechanism as in QFT and symplectic invariance replaces ad hoc UV cutoff with a genuine dynamical regulation mechanism. Causal diamond itself defines the physical IR cutoff. p-Adic and real coupling constant evolutions reflect the underlying evolution in powers of two for the temporal distance between the tips of the light-cones of the causal diamond and the association of macroscopic time scale as secondary p-adic time scale to elementary particles (.1 seconds for electron) serves as a first test for the picture. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules could be treated independently.

More detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities could follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.*

renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

UV finiteness is suggested also by the generalized Feynman rules providing a phenomenological view about what TGD predicts. According to these rules fundamental fermions propagate like massless particles. In twistor Grassmann approach residue integration is expected to reduce internal fermion lines to on mass shell propagation with non-physical helicity. The fundamental 4-fermion interaction is assignable to wormhole contact and corresponds to stringy exchange of four-momentum with propagator being defined by the inverse of super-conformal scaling generator $1/L_0$. Wormhole contacts carrying fermion and antifermion at their throats behave like fundamental bosons. Stringy propagators at wormhole contacts make TGD rules a hybrid of Feynmann and stringy rules. Stringy propagators are necessary in order to avoid logarithmic divergences. Higher mass excitations crucial for finiteness belong to the representations of super-conformal algebra and can be regarded as bound states of massless fermions. Massivation of external particles allows to avoid infrared divergences. Not only physical bosons but also physical fermions emerge from fundamental massless fermions.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-symplectic conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [K15] .

(a) The state construction utilizes both super-symplectic and super Kac-Moody algebras. super-symplectic algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

(b) The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-symplectic algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-symplectic operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

(c) Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO - H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic 4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

(d) The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the CH Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-symplectic super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the WCW super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to WCW gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^{\dagger}/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

(a) Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^{d}{}^{agger}$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

(b) One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k \gamma_k^{\dagger}$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since ordinary fermionic propagator and boson-emission vertices at the ends of the line containing WCW gamma matrix and its conjugate give compensating fermion numbers [K58].

(c) If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

(d) In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

(e) The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

In this framework coupling constant evolution would correspond evolution as a function of the scale of CD. It might have interpretation also in terms of addition of intermediate zero energy states corresponding to the generalized Feynman diagrams obtained by the insertion of causal diamonds with a new shorter time scale $T = T_{prev}/2$ to the previous Feynman diagram as the size of CD is increased. p-Adic length scale hypothesis follows naturally. A very close correspondence with ordinary Feynman diagrammatics arises and and ordinary vision about coupling constant evolutions arises. The absence of infinities follows from the symplectic invariance which is genuinely new element. p-Adic and real coupling constant evolutions can be seen as completions of coupling constant evolutions for physics based on rationals and their algebraic extensions.

15.3 Quantitative guesses for the values of coupling constants

This focus of attention in this section is in quantitative for the p-adic evolution of couplings constants obtained by combining information coming from p-adic mass calculations with number theoretic constraints and general formula for gravitational constant inspired by a simple physical picture. It must be emphasized that only educated guesses are in question since real understanding of coupling constant evolution has begun to emerge only rather recently (2014) when the relationship between TGD and GRT and QFT was finally clarified.

15.3.1 A revised view about coupling constant evolution

The development of the ideas related to number theoretic aspects has been rather tortuous and based on guess work since basic theory has been lacking.

(a) The original hypothesis was that Kähler coupling strength is invariant under p-adic coupling constant evolution. Later I gave up this hypothesis and replaced it with the invariance of gravitational coupling since otherwise the prediction would have been that gravitational coupling strength is proportional to p-adic length scale squared. Second first guess was that Kähler coupling strength equals to the value of fine structure constant at electron length scale corresponding to Mersenne prime M_{127}. Later I replaced fine structure constant with electro-weak U(1) coupling strength at this length scale. The recent discussion returns back to the roots in both aspects.

(b) The recent discussion relies on the progress made in the understanding of quantum TGD at partonic level [K10] . What comes out is an explicit formula for Kähler couplings strength in terms of Dirac determinant involving only a finite number of eigenvalues of the modified Dirac operator. This formula dictates the number theoretical anatomy of g_K^2 and also of other coupling constants: the most general option is that α_K is a root of rational. The requirement that the rationals involved are simple combined with simple experimental inputs leads to very powerful predictions for the coupling parameters.

(c) A further simplification is due to the discreteness of p-adic coupling constant evolution allowing to consider only length scales coming as powers of $\sqrt{2}$. This kind of discretization is necessary also number theoretically since logarithms can be replaced with 2-adic logarithms for powers of 2 giving integers. This raises the question whether $p \simeq 2^k$ should be replaced with 2^k in all formulas as the recent view about quantum TGD suggests.

(d) The prediction is that Kähler coupling strength α_K is invariant under p-adic coupling constant evolution and from the constraint coming from electron and top quark masses very near to fine structure constant so that the identification as fine structure constant is natural. Gravitational constant is predicted to be proportional to p-adic length scale squared and corresponds to the largest Mersenne prime (M_{127}), which does not correspond to a completely super-astronomical p-adic length scale. For the parameter R^2/G p-adicization program allows to consider two options: either this constant is of form e^q or 2^q: in both cases q is rational number. $R^2/G = exp(q)$ allows only M_{127} gravitons if number theory is taken completely seriously. $R^2/G = 2^q$ allows all p-adic length scales for gravitons and thus both strong and weak variants of ordinary gravitation.

(e) A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

It seems fair to conclude that the attempts to understand the implications of p-adicization for coupling constant evolution have begun to bear fruits.

General formula for the Kähler coupling strength

The identification of exponent of Kähler function as Dirac determinant leads to a formula relating Kähler action for the preferred extremal to the Dirac determinant. The eigenvalues are proportional to $1/\alpha_K$ since the matrices $\hat{\Gamma}^\alpha$ have this proportionality. This gives the formula

$$exp(\frac{S_{K,R}(X^4(X^3))}{2g_K^2}) = \prod_i \lambda_i = \frac{\prod_i \lambda_{0,i}}{(g_K)^{2N}} \quad . \tag{15.3.1}$$

Here $\lambda_{0,i}$ by definition corresponds to $g_K^2 = 4\pi\alpha_K = 1$. $S_{K,R} = \int J^* J$ is the reduced Kähler action.

For $S_{K,R} = 0$, which might correspond to so called massless extremals [K4] one obtains the formula

$$g_K^2 = (\prod_i \lambda_{0,i})^{1/N} \quad . \tag{15.3.2}$$

Thus for $S_{K,R} = 0$ extremals one has an explicit formula for g_K^2 having interpretation as the geometric mean of the eigenvalues $\lambda_{0,i}$. Several values of α_K are in principle possible.

p-Adicization suggests that $\lambda_{0,i}$ are rational or at most algebraic numbers. This would mean that g_K^2 is N:th root of this kind of number. $S_{K,R}$ in turn would be

$$S_{K,R} = 2g_K^2 log(\frac{\prod_i \lambda_{0,i}}{g_K^{2N}}) \quad . \tag{15.3.3}$$

so that the reduced Kähler action $S_{K,R}$ would be expressible as a product N:th root of rational, and logarithm of rational. This result would provide a general answer to the question about number theoretical anatomy of Kähler coupling strength and S_K.

For CP_2 type vacuum extremal one would have $S_{K,R} = \frac{\pi^2}{2}$ in apparent conflict with the above result. The conflict is of course only apparent since topological condensation of CP_2 type vacuum extremal generates a hole in CP_2 having light-like wormhole throat as boundary so that the value of the action is modified.

Identifications of Kähler coupling strength and gravitational coupling strength

To construct an expression for gravitational constant one can use the following ingredients.

(a) The exponent $exp(S_K(CP_2))$ defining vacuum functional and thus the value of Kähler function in terms of the Kähler action $S_K(CP_2)$ of CP_2 type extremal representing elementary particle expressible as

$$S_K(CP_2) = \frac{S_{K,R}(CP_2)}{8\pi\alpha_K} = \frac{\pi}{8\alpha_K} \ . \tag{15.3.4}$$

Since CP_2 type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \rightarrow a \times S_K(CP_2) \ . \tag{15.3.5}$$

$a < 1$ conforms with the idea that a piece of CP_2 type extremal defining a wormhole contact is in question. One must however keep mind open in this respect.

(b) The p-adic length scale L_p assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime $M_{127} = 2^{127}-1$ defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that M_{127} characterizes these space-time sheets.

1. The formula for the gravitational constant

A long standing basic conjecture has been that gravitational constant satisfies the following formula

$$\begin{aligned} \hbar G &\equiv r\hbar_0 G = L_p^2 \times exp(-2aS_K(CP_2)) \ , \\ L_p &= \sqrt{p}R \ . \end{aligned} \tag{15.3.6}$$

Here R is CP_2 radius defined by the length $2\pi R$ of the geodesic circle. What was noticed before is that this relationship allows even constant value of G if a has appropriate dependence on p.

This formula seems to be correct but the argument leading to it was based on two erratic assumptions compensating each other.

(a) I assumed that modulus squared for vacuum functional is in question: hence the factor $2a$ in the exponent. The interpretation of zero energy state as a generalized Feynman diagram requires the use of vacuum functional so that the replacement $2a \rightarrow a$ is necessary.

(b) Second wrong assumption was that graviton corresponds to CP_2 type vacuum extremal-that is wormhole contact in the recent picture. This does allow graviton to have spin 2. Rather, two wormhole contacts represented by CP_2 vacuum extremals and connected by fluxes associated with various charges at their throats are needed so that graviton is string like object. This saves the factor $2a$ in the exponent.

The highly non-trivial implication to be discussed later is that ordinary coupling constant strengths should be proportional to $exp(-aS_K(CP_2))$.

The basic constraint to the coupling constant evolution comes for the invariance of g_K^2 in p-adic coupling constant evolution:

$$\begin{aligned} g_K^2 &= \frac{a(p,r)\pi^2}{log(pK)} \ , \\ K &= \frac{R^2}{\hbar G(p)} = \frac{1}{r}\frac{R^2}{\hbar_0 G(p)} \equiv \frac{K_0(p)}{r} \ . \end{aligned} \tag{15.3.7}$$

2. How to guarantee that g_K^2 is RG invariant and N:th root of rational?

Suppose that g_K^2 is N:th root of rational number and invariant under p-adic coupling constant evolution.

(a) The most general manner to guarantee the expressibility of g_K^2 as N:th root of rational is guaranteed for both options by the condition

$$a(p,r) = \frac{g_K^2}{\pi^2}log(\frac{pK_0}{r}) \ . \tag{15.3.8}$$

That a would depend logarithmically on p and $r = \hbar/\hbar_0$ looks rather natural. Even the invariance of G under p-adic coupling constant evolution can be considered.

(b) The condition

$$\frac{r}{p} < K_0(p) \ . \tag{15.3.9}$$

must hold true to guarantee the condition $a > 0$. Since the value of gravitational Planck constant is very large, also the value of corresponding p-adic prime must very large to guarantee this condition. The condition $a < 1$ is guaranteed by the condition

$$\frac{r}{p} > exp(-\frac{\pi^2}{g_K^2}) \times K_0(p) \ . \tag{15.3.10}$$

The condition implies that for very large values of p the value of Planck constant must be larger than $\hbar_0$.

(c) The two conditions are summarized by the formula

$$K_0(p) \times exp(-\frac{\pi^2}{g_K^2}) < \frac{r}{p} < K_0(p) \tag{15.3.11}$$

characterizing the allowed interval for r/p. If G does not depend on p, the minimum value for r/p is constant. The factor $exp(-\frac{\pi^2}{g_K^2})$ equals to 1.8×10^{-47} for $\alpha_K = \alpha_{em}$ so that $r > 1$ is required for $p \geq 4.2 \times 10^{-40}$. $M_{127} \sim 10^{38}$ is near the upper bound for p allowing $r = 1$. The constraint on r would be roughly $r \geq 2^{k-131}$ and $p \simeq 2^{131}$ is the first p-adic prime for which $\hbar > 1$ is necessarily. The corresponding p-adic length scale is .1 Angstroms.

This conclusion need not apply to elementary particles such as neutrinos but only to the space-time sheets mediating gravitational interaction so that in the minimal scenario it would be gravitons which must become dark above this scale. This would bring a new aspect to vision about the role of gravitation in quantum biology and consciousness.

The upper bound for r behaves roughly as $r < 2.3 \times 10^7 p$. This condition becomes relevant for gravitational Planck constant GM_1M_2/v_0 having gigantic values. For Earth-Sun system and for $v_0 = 2^{-11}$ the condition gives the rough estimate $p > 6 \times 10^{63}$. The corresponding p-adic length scale would be of around $L(215) \sim 40$ meters.

(d) p-Adic mass calculations predict the mass of electron as $m_e^2 = (5 + Y_e)2^{-127}/R^2$ where $Y_e \in [0, 1)$ parameterizes the not completely known second order contribution. Top quark mass favors a small value of Y_e (the original experimental estimates for m_t were above the range allowed by TGD but the recent estimates are consistent with small value Y_e [K46]). The range $[0, 1)$ for Y_e restricts $K_0 = R^2/\hbar_0 G$ to the range $[2.3683, 2.5262] \times 10^7$.

(e) The best value for the inverse of the fine structure constant is $1/\alpha_{em} = 137.035999070(98)$ and would correspond to $1/g_K^2 = 10.9050$ and to the range $(0.9757, 0.9763)$ for a for $\hbar = \hbar_0$ and $p = M_{127}$. Hence one can seriously consider the possibility that $\alpha_K = \alpha_{em}(M_{127}$ holds true. As a matter fact, this was the original hypothesis but was replaced later with the hypothesis that α_K corresponds to electro-weak $U(1)$ coupling strength in this length scale. The fact that M_{127} defines the largest Mersenne prime, which does not correspond to super-astrophysical length scale might relate to this co-incidence.

To sum up, the recent view about coupling constant evolution differs strongly from previous much more speculative scenarios. It implies that g_K^2 is root of rational number, possibly even rational, and can be assumed to be equal to e^2. Also $R^2/\hbar G$ could be rational. The new element is that G need not be proportional to p and can be even invariant under coupling constant evolution since the the parameter a can depend on both p and r. An unexpected constraint relating p and r for space-time sheets mediating gravitation emerges.

Are the color and electromagnetic coupling constant evolutions related?

Classical theory should be also able to say something non-trivial about color coupling strength α_s too at the general level. The basic observations are following.

(a) Both classical color YM action and electro-weak U(1) action reduce to Kähler action.

(b) Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that α_K is a strict RG invariant. One can consider two options.

(a) The original idea was that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for $\alpha_{U(1)} = \alpha_s = 2\alpha_K$. Asymptotically the couplings would approach to a fixed point defined by $2\alpha_K$ rather than to zero as in asymptotically free gauge theories.

Thus one would have

$$\frac{1}{\alpha_{U(1)}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \tag{15.3.12}$$

The relationship between $U(1)$ and em coupling strengths is

$$\begin{aligned} \alpha_{U(1)} &= \frac{\alpha_{em}}{cos^2(\theta_W)} \simeq \frac{1}{104.1867} \ , \\ sin^2(\theta_W)_{|10\ MeV} &\simeq 0.2397(13) \ , \\ \alpha_{em}(M_{127}) &= 0.00729735253327 \ . \end{aligned} \tag{15.3.13}$$

Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value $sin^2(\theta_W) = 0.2397(13)$ corresponding to 10 MeV mass scale [E4] is used. Note however that the previous argument implying $\alpha_K = \alpha_{em}(M_{127})$ excludes $\alpha = \alpha_{U(1)}(M_{127})$ option.

(b) Second option is obtained by replacing $U(1)$ with electromagnetic gauge $U(1)_{em}$.

$$\frac{1}{\alpha_{em}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \tag{15.3.14}$$

Possible justifications for this assumption are following. The notion of induced gauge field makes it possible to characterize the dynamics of classical electro-weak gauge fields using only the Kähler part of electro-weak action, and the induced Kähler form appears only in the electromagnetic part of the induced classical gauge field. A further justification is that em and color interactions correspond to unbroken gauge symmetries.

The following arguments are consistent with this conclusion.

(a) In TGD framework coupling constant is discrete and comes as powers of $\sqrt{2}$ corresponding to p-adic primes $p \simeq 2^k$. Number theoretic considerations suggest that coupling constants g_i^2 are algebraic or perhaps even rational numbers, and that the logarithm of mass scale appearing as argument of the renormalized coupling constant is replaced with 2-based logarithm of the p-adic length scale so that one would have $g_i^2 = g_i^2(k)$. g_K^2 is predicted to be N:th root of rational but could also reduce to a rational. This would allow rational values for other coupling strengths too. This is possible if $sin(\theta_W)$ and $cos(\theta_W)$ are rational numbers which would mean that Weinberg angle corresponds to a Pythagorean triangle as proposed already earlier. This would mean the formulas $sin(\theta_W) = (r^2 - s^2)/(r^2 + s^2)$ and $cos(\theta_W) = 2rs(r^2 + s^2)$.

(b) A very strong prediction is that the beta functions for color and $U(1)$ degrees of freedom are apart from sign identical and the increase of $U(1)$ coupling compensates the decrease of the color coupling. This allows to predict the hard-to-calculate evolution of QCD coupling constant strength completely.

(c) $\alpha(M_{127}) = \alpha_K$ implies that M_{127} defines the confinement length scale in which the sign of α_s becomes negative. TGD predicts that also M_{127} copy of QCD should exist and that M_{127} quarks should play a key role in nuclear physics [K63, L2] , [L2] . Hence one can argue that color coupling strength indeed diverges at M_{127} (the largest not completely super-astrophysical Mersenne prime) so that one would have $\alpha_K = \alpha(M_{127})$. Therefore the precise knowledge of $\alpha(M_{127})$ in principle fixes the value of parameter $K = R^2/G$ and thus also the second order contribution to the mass of electron.

(d) $\alpha_s(M_{89})$ is predicted to be $1/\alpha_s(M_{89}) = 1/\alpha_K - 1/\alpha(M_{89})$. $sin^2(\theta_W) = .23120$, $\alpha_{em}(M_{89}) \simeq 1/127$, and $\alpha_{U(1)} = \alpha_{em}/cos^2(\theta_W)$ give $1/\alpha_{U(1)}(M_{89}) = 97.6374$. $\alpha = \alpha_{em}$ option gives $1/\alpha_s(M_{89}) \simeq 10$, which is consistent with experimental facts. $\alpha = \alpha_{U(1)}$ option gives $\alpha_s(M_{89}) = 0.1572$, which is larger than QCD value. Hence $\alpha = \alpha_{em}$ option is favored.

To sum up, the proposed formula would dictate the evolution of α_s from the evolution of the electro-weak parameters without any need for perturbative computations. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

Can one deduce formulae for gauge couplings?

The improved physical picture behind gravitational constant allows also to consider a general formula for gauge couplings.

(a) The natural guess for the general formula would be as

$$g^2(p,r) = kg_K^2 \times exp[-a_g(p,r) \times S_K(CP_2)] \quad . \tag{15.3.15}$$

here k is a numerical constant.

(b) The condition
$g_K^2 = e^2(M_{127})$ fixes the value of k if it's value does not depend on the character of gauge interaction:

$$k = exp[a_{gr}(M_{127}, r = 1) \times S_K(CP_2)] \quad . \tag{15.3.16}$$

Hence the general formula reads as

$$g^2(p,r) = g_K^2 \times exp[(-a_g(p,r) + a_{gr}(M_{127}), r = 1)) \times S_K(CP_2)] \quad . \tag{15.3.17}$$

The value of $a(M_{127}, r = 1)$ is near to its maximum value so that the exponential factor tends to increase the value of g^2 from e^2. The formula can reproduce α_s and various electro-weak couplings although it is quite possibile that Weinberg angle corresponds to a group theoretic factor not representable in terms of $a_g(p,r)$. The volume of the CP_2 type vacuum extremal would characterize gauge bosons. Analogous formula should apply also in the case of Higgs.

(c) α_{em} in very long length scales would correspond to

$$e^2(p \to \infty, r = 1) = e^2 \times exp[(-1 + a(M_{127}), r = 1)) \times S_K(CP_2)] = e^2 x \quad , \tag{15.3.18}$$

where x is in the range $[0.6549, 0.6609]$.

Formula relating v_0 to α_K and $R^2/\hbar G$

The parameter $v_0 = 2^{-11}$ plays a key role in the formula for gravitational Planck constant and can be also seen as a fundamental constant in TGD framework. As a matter, factor v_0 has interpretation as velocity parameter and is dimensionless when $c = 1$ is used.

If v_0 is identified as the rotation velocity of distant stars in galactic plane, one can use the Newtonian model for the motion of mass in the gravitational field of long straight string giving $v_0 = \sqrt{TG}$. String tension T can be expressed in terms of Kähler coupling strength as

$$T = \frac{b}{2\alpha_K R^2} \quad ,$$

where R is the radius of geodesic circle. The factor $b \leq 1$ would explain reduction of string tension in topological condensation caused by the fact that not entire geodesic sphere contributes to the action.

This gives

$$\begin{aligned} v_0 &= \frac{b}{2\sqrt{\alpha_K K}} \ , \\ \alpha_K(p) &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{\hbar G} \ . \end{aligned} \tag{15.3.19}$$

The condition that α_K has the desired value for $p = M_{127} = 2^{127} - 1$ defining the p-adic length scale of electron fixes the value of b for given value of a. The value of b should be smaller than 1 corresponding to the reduction of string tension in topological condensation.

The condition 15.3.19 for $v_0 = 2^{-m}$, say $m = 11$, allows to deduce the value of a/b as

$$\frac{a}{b} = \frac{4 * log(pK)}{\pi}\frac{2^{2m-1}}{K} \ . \tag{15.3.20}$$

For both $K = e^q$ with $q = 17$ and $K = 2^q$ option with $q = 24 + 1/2$ $m = 10$ is the smallest integer giving $b < 1$. $K = e^q$ option gives $b = .3302$ $(.0826)$ and $K = 2^q$ option gives $b = .3362$ $(.0841)$ for $m = 10$ $(m = 11)$.

$m = 10$ corresponds to one third of the action of free cosmic string. $m = 11$ corresponds to much smaller action smaller by a factor rather near $1/12$. The interpretation would be that as m increases the action of the topologically condensed cosmic string decreases. This would correspond to a gradual transformation of the cosmic string to a magnetic flux tube.

To sum up, the resulting overall vision seems to be internally consistent and is consistent with generalized Feynman graphics, predicts exactly the spectrum of α_K, suggests the identification of the inverse of p-adic temperature with k, allows to understand the differences between fermionic and bosonic massivation. One might hope that the additional objections (to be found sooner or later!) could allow to develop a more detailed picture.

15.3.2 Why gravitation is so weak as compared to gauge interactions?

The weakness of gravitational interaction in contrast to other gauge interactions is definitely a fundamental test for the proposed picture. The heuristic argument allowing to understand the value of gravitational constant is based on the assumption that graviton exchange corresponds to the exchange of CP_2 type extremal for which vacuum functional implies huge reduction of the gravitational constant from the value $\sim L_p^2$ implied by dimensional considerations based on p-adic length scale hypothesis to a value $G = exp(-2S_K)L_p^2$ which for $p = M_{127}$ gives gravitational constant for $\alpha_K = \pi a/log(M_{127} \times K)$, where a is near unity and $K = 2 \times 3 \times 5 .... \times 23$ is a choice motivated by number theoretical arguments. The value of K is fixed rather precisely from electron mass scale and the proposed scenario for coupling constant evolution fixes both α_K and K completely in terms of electron mass (using p-adic mass calculations) and electro-magnetic coupling at electron length scale $L_{M_{127}}$ by the formula $\alpha_K = \alpha_{em}$ [K21] . The interpretation would be that gravitational masses are measured using p-adic mass scale $M_p = \pi/L_p$ as a natural unit.

Why gravitational interaction is weak?

The first problem is that CP_2 type extremal cannot represent the lowest order contribution to the interaction since otherwise the normalization of WCW vacuum functional would give $exp[-2S_K(CP_2)]$ factor cancelling the exponential in the propagator so that one would have $G = L_p^2$. The following observations allow to understand the solution of the problem.

(a) As already found, the key feature of CP_2 type vacuum extremals distinguishing them from other 3-surfaces is their non-deterministic behavior allowing them to carry off mass shell four-momenta. Other 3-surfaces can give rise only to scattering involving exchange of on mass shell particles and for space-like momentum exchanges there is no contribution.

(b) All possible light-like 3-surfaces must be allowed as propagator portions of surfaces X^3_V but in absence of non-determinism they can give rise to massless exchanges which are typically non-allowed.

(c) The contributions of CP_2 type vacuum extremals are suppressed by $exp[-2NS_K(CP_2)]$ factor in presence of N CP_2 type extremals with maximal action. CP_2 type extremals are vacuum extremals and interact with surrounding world only via the topological condensation generating 3-D CP_2 projection near the throat of the wormhole contact. This motivates the assumption that the sector of the WCW containing N CP_2 type extremals has the approximate structure $CH(N) = CH(0) \times CP^N$, where $CH(0)$ corresponds to the situation without CP_2 type extremals and CP to the degrees of freedom associated with single CP_2 type extremal. With this assumption the functional integral gives a result of form $X \times exp(-2NS_K(CP_2)$ for N CP_2 type extremals. This factorization allows to forget all the complexities of the world of classical worlds which on the first sight seem to destroy all hopes about calculating something and the normalization factor is in lowest order equal to $X(0)$ whereas single CP_2 type extremal gives $exp[-2S_K(CP_2)]$ factor. This argument generalizes also to the case when CP_2 type extremals are allowed to have varying value of action (the distance travelled by the virtual particle can vary).

Massless extremals (MEs) define a natural candidate for the lowest order contribution since for them Kähler action vanishes. MEs describes a dispersion free on-mass shell propagation of massless modes of both induced gauge fields and metric. Hence they can describe only on mass shell massless exchanges of bosons and gravitons which typically vanishes for kinematical reasons except for collinear scattering in the case of massless particles so that CP_2 type extremals would give the leading contribution to the S-matrix element.

There are however exceptional situations in which exchange of ordinary CP_2 type extremals makes kinematically possible the emission of MEs as brehmstrahlung in turn giving rise to exchange of light-like momentum. Since MEs carry also classical gravitational fields, one can wonder whether this kind of exchanges could make possible strong on mass shell gravitation made kinematically possible by ordinary gauge boson exchanges inside interacting systems.

If one takes absolutely seriously the number theoretic argument based on $R^2/G = exp(q)$ ansatz then M_{127} is selected uniquely as the space-time sheet of gravitons and the predicted gravitational coupling strength is indeed weak.

What differentiates between gravitons and gauge bosons?

The simplest explanation for the difference between gauge bosons and gravitons is that for virtual gauge bosons the volume of CP_2 type extremals is reduced dramatically from its maximal value so that $exp(-2S_K)$ brings in only a small reduction factor. The reason would be that for virtual gauge bosons the length of a typical CP_2 type extremal is far from the value giving rise to the saturation of the Kähler action. For gravitational interactions in astrophysical length scales CP_2 type extremals must indeed be very long.

Gravitational interaction should become strong sufficiently below the saturation length scale with gravitational constant approaching its stringy value L_p^2. According to the argument discussed in [K21] , this length scale corresponds to the Mersenne prime M_{127} characterizing gravitonic space-time sheets so that gravitation should become strong below electron's Compton length. This suggests a connection with stringy description of graviton. M_{127} quarks connected by the corresponding strings are indeed a basic element of TGD based model of nuclei [K63] . TGD suggests also the existence of lepto-hadrons as bound state of color excited leptons in length scale M_{127} [K71] . Also gravitons corresponding to smaller

Mersenne primes are possible but corresponding forces are much weaker than ordinary gravitation. On the other hand, M_{127} is the largest Mersenne prime which does not give rise to super-astronomical p-adic length scale so that stronger gravitational forces are not be predicted in experimentally accessible length scales.

More generally, the saturation length scale should relate very closely to the p-adic length scale L_p characterizing the particle. The amount of zitterbewegung determines the amount dS_K/dl of Kähler action per unit length along the orbit of virtual particle. L_p would naturally define the length scale below which the particle moves in a good approximation along M^4 geodesic. The shorter this length scale is, the larger the value of dS_K/dl is.

If the Kähler action of CP_2 type extremal increases linearly with the distance (in a statistical sense at least), an exponential Yukawa screening results at distances much shorter than saturation length. Therefore CP_2 extremals would provide a fundamental description of particle massivation at space-time level. p-Adic thermodynamics would characterize what happens for a topologically condensed CP_2 type extremal carrying given quantum numbers at the resulting light-like CD. Besides p-adic length scale also the quantized value $T_p = 1/n$ of the p-adic temperature would be decisive. For weak bosons Mersenne prime M_{89} would define the saturation length scale. For photons the p-adic length scale defining the Yukawa screening should be rather long. An n-ary p-adic length scale $L_{M_{89}}(n) = p^{(n-1)/2}L_{M_{89}}$ would most naturally be in question so that the p-adic temperature associated with photon would be $T_p = 1/n$, $n > 1$ [K37] . In the case of gluons confinement length scale should be much shorter than the scale at which the Yukawa screening becomes visible. If also gluons correspond to $n > 1$ this is certainly the case.

All gauge interactions would give rise to ultra-weak long ranged interactions, which are extremely weak compared to the gravitational interaction: the ratio for the strengths of these interactions would be of order $\alpha Q_1 Q_2 m_e^2/M_1 M_2$ and very small for particles whose masses are above electron mass. Note however that MEs give rise to arbitrarily unscreened long ranged weak and color interactions restricted to light-like momentum transfers and these interactions play a key role in the TGD based model of living matter [K17, K18] . This prediction is in principle testable.

15.4 p-Adic coupling constant evolution

p-Adic coupling constant evolution is one of the genuinely new elements of quantum TGD. In the following some aspects of the evolution will be discussed. The discussion is a little bit obsolete as far as the role of canonical identification is considered. The most recent view about p-adic coupling constant evolution is discussed at the end of the section.

15.4.1 General considerations

One of the basic challenges of quantum TGD is to understand whether the notion of p-adic coupling constant evolution is something related to the basic TGD or whether it emerges at GRT and QFT limits only.

(a) Since neither classical field equations for Kähler action nor Kähler-Dirac action depend on coupling constants except as overall multiplicative normalization factor, one expects that at the level of TGD space-time the notion of coupling constant evolution is not well-defined or at least fails to be a fundamental notion. Coupling constant evolution would characterize GRT and QFT limits of TGD and since causal diamond (CD) is the basic unit, the scale of CD would serve as a fundamental scale.

What would give rise to coupling constant evolution at long length scales, would be the replacement of many-sheeted space-time with GRT space-time containing gauge potentials which are sums of induced gauge potentials associated with various space-time sheets. The increase in the size of CD would induce the scaling of the size of the space-time sheet. Hence the geometric correlate for coupling constant evolution would be the scaling of CD size. The original belief was that it would be scaling of the size of the space-time sheet.

(b) This evolution is discrete by p-adic length scale hypothesis justified by zero energy ontology, where CD sizes are assumed to come as integer multiples of CP_2 mass: the discretization is for number theoretical reasons and gives hopes of number theoretical universality. The most general option is that the CD sizes come as rational multiples of CP_2 size. Discreteness means that continuous mass scale is replaced by mass scales coming as half octaves of CP_2 mass. Kähler coupling strength α_K or gravitational coupling constant is assumed to remain invariant under p-adic coupling constant evolution. The basic problem is to understand the value of α_K and here p-adic mass calculations give strong constraints.

(c) An attractive hypothesis is that Dirac determinant reduces to the vacuum functional identifiable as exponent of Kähler action S_K for a preferred extremal. The contribution from Euclidian regions corresponds to Kähler function and that from Minkowskian regions serves as analog of Minkowskian action defining Morse function at the level of WCW.

There is no hope of reducing Kähler action to Dirac action since Kähler action and Kähler-Dirac action are in completely democratic position since Kähler-Dirac gamma matrices are defined in terms of the canonical momentumd densities for Kähler action. The value of Kähler coupling strength is however expected to follow from the condition that Dirac determinant equals to vacuum functional.

(d) The realization that well-definedness of em charge requires the localization of the modes of induced spinor field to string world sheets or partonic 2-surfaces was an important step in process trying to make the notion of Dirac determinant more concrete [K87]. Dirac determinants reduce to those assignable to string world sheets and possibly also partonic 2-surfaces and would naturally correspond to square roots of determinants defined by the products of the eigenvalues of the mass squared operator for incoming on mass shell states and given by stringy mass formula. Zeta function regularization should allow to defined these determinants and one can hope that it reduces to the exponent of Kähler action for preferred extremal. Thus coupling constant evolution might allow a reduction to string model type description.

(e) If weak form of electric magnetic duality and $j \cdot A = 0$ condition for Kähler current and gauge potential in the interior of space-time sheets are satisfied, Kähler action reduces to Chern-Simons terms at light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface. Induced metric would apparently disappear from the action in accordance with the idea about TGD as almost topological QFT.

(f) The boundary conditions for Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits can be expressed as generalized eigenvalue equation $D_{C-S})\Psi = p^k\gamma_k\Psi$ with p^k interpreted as virtual momentum of the fermion propagating along the boundary of string world sheets at which it is localized by the well-definedness of em charge. C-S-D term gives rise to massless fermion propagation. Also the external fermions are massless unless there are measurement interaction terms defined as Lagrangian multiplier terms forcing classical charges in Cartan algebra to be equal to their quantum counterparts for the space-time surfaces in the quantum superposition. This implies localization in WCW analogous to state function collapse and by quantum classical correspondence could accompany state function reduction. It would be very natural if this localization would happen in zero modes so that classical charges parametrize zero modes quantum charges correspond to wave function in quantum fluctuating modes.

15.4.2 p-Adic evolution in angular resolution and dynamical Planck constant

Quantum phases $q = exp(i\pi/n)$ characterized Jones inclusions which have turned out to play key role in the understanding of macroscopic quantum phases in TGD framework. The basic idea is that the different values of Planck constant correspond to evolution in angular resolution in p-adic context characterized by quantum phase $q = exp(i\pi/n)$ characterizing Jones inclusion is. The higher the value of n, the better the angular resolution since the number of different complex phases in extension of p-adic numbers increases with n.

The quantization of Planck constant has been the basic them of TGD for more than one and half years. The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

Jones inclusions and quantization of Planck constants

Jones inclusions combined with simple anyonic arguments turned out to be the key to the unification of existing heuristic ideas about the quantization of Planck constant.

(a) The new view allows to understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: their is no landscape.

(b) In ordinary phase Planck constants of M^4 and CP_2 are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of the scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups: $\hbar(M^4) = n_a$ and $\hbar(CP_2) = n_b$ whereas scaling factors of M^4 and CP_2 metrics are n_b^2 and n_a^2 respectively. At the level of Schrödinger equation this means that Planck constant $\hbar$ corresponds to the effective Planck consant $\hbar_{eff} = (\hbar(M^4)/\hbar(CP_2))\hbar_0 = (n_a/n_b)\hbar_0$, which thus can have all possible positive rational values. For some time I believed on the scaling of metrics of M^4 *resp.* CP_2 as n_b^2 *resp.* n_a^2: this would imply invariance of Schrödinger equation under the scalings but would not be consistent with the explanation of the quantization of radii of planetary orbits requiring huge Planck constant [K59] . Poincare invariance is however achieved in the sense that mass spectrum is invariant under the scalings of Planck constants. That the ratio n_a/n_b defines effective Planck constant conforms with the fact that the value of Kähler action involves only this ratio (quantum-classical correspondence). Also the value of gravitational constant is invariant under the scalings of Planck constant since one has $G \propto g_K^2 R^2$, R radius of CP_2 for $n_a = 1$.

(c) This predicts automatically arbitrarily large values of effective Planck constant n_a/n_b and they correspond to coverings of CP_2 points by large number of M^4 points which can have large distance and have precisely correlated behavior due to the G_a symmetry. One can assign preferred values of Planck constant to quantum phases $q = exp(i\pi/n)$ expressible in terms of iterated square roots of rationals: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter seem to correspond to these special values of Planck constants. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

(d) These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index.

The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_{2n}, E_6, E_8 are possible so that TGD seems to be able to mimic these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

The values of gravitational Planck constant

The understanding of large Planck constants led to the detailed interpretation of what is involved with the emergence of gigantic gravitational Planck constant.

Gravitational Planck constant $\hbar_{gr}$ can be interpreted as effective Planck constant $\hbar_{eff} = (n_a/n_b)\hbar_0$ so that the Planck constant associated with M^4 degrees of freedom (rather than CP_2 degrees of freedom as in the original wrong picture) must be very large in this kind of situation.

The detailed spectrum for Planck constants gives very strong constraints to the values of $\hbar_{gr} = GMm/v_0$ if ones assumes that favored values of Planck constant correspond to the Jones inclusions for which quantum phase corresponds to a simple algebraic number expressible in terms of square roots of rationals. These phases correspond to n-polygons with n equal to a product of power of two and Fermat primes, which are all different. The ratios of planetary masses obey the predictions with an accuracy of 10 percent and GMm/v_0 for Sun-Earth system is consistent with $v_0 = 2^{-11}$ if the fraction of visible matter of all matter is about 3 per cent in solar system to be compared with the accepted cosmological value of 4 per cent [K59] .

If so, its huge value implies that also the von Neumann inclusions associated with M^4 degrees of freedom are involved meaning that dark matter cosmology has quantal lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M^4_+ and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

There is however strong objection based on the observation that the radius of CP_2 would become gigantic. Surprisingly, this need not have any dramatic implications as will be found. It is also quite possible that the biomolecules subgroups of rotation group as symmetries could correspond to $n_a > 1$. For instance, the tetrahedral and icosahedral molecular structures appearing in water would correspond to E_6 with $n_a = 3$ and E_8 with $n_a = 5$. Note that $n_a = 5$ is minimal value of n_a allowing universal topological quantum computation.

15.4.3 Large values of Planck constant and electro-weak and strong coupling constant evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

15.4.4 Super-symplectic gluons and non-perturbative aspects of hadron physics

What happens mathematically in the transition to non-perturbative QCD has remained more or less a mystery. The number theoretical considerations of [K77] inspired the idea that Planck constant is dynamical and has a spectrum given as $\hbar(n) = n\hbar_0$, where n characterizes the quantum phase $q = exp(i2\pi/n)$ associated with Jones inclusion. The strange finding that the orbits of planets seem to obey Bohr quantization rules with a gigantic value of Planck constant inspired the hypothesis that the increase of Planck constant provides a unique mechanism allowing strongly interacting system to stay in perturbative phase [K59, K21] . The resulting model allows to understand dark matter as a macroscopic quantum phase in astrophysical length and time scales, and strongly suggest a connection with dark matter and biology.

The phase transition increasing Planck constant could provide a model for the transition to confining phase in QCD. When combined with the recent ideas about value spectrum of Kähler coupling strength one ends up with a rather explicit model about non-perturbative aspects of hadron physics already successfully applied in hadron mass calculations [K46] .

According to the model of hadron masses [K46] , in the case of light pseudo-scalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudo-scalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of WCW degrees of freedom ("world of classical worlds") in which super-symplectic algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges.

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-symplectic gluons. It turns out the model assuming same topological mixing of super-symplectic bosons identical to that experienced by U type quarks leads to excellent understanding of hadron masses assuming that hadron spin correlates with the super-symplectic particle content of the hadronic space-time sheet.

According to the argument already discussed, at the hadronic $k = 107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s = \alpha_K = \frac{1}{4} \quad .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

This α_s would correspond to the interaction via super-symplectic colored gluons and would lead to the failure of perturbation theory. By the general criterion stating that the failure of perturbation theory leads to a phase transition increasing the value of Planck constant one expects that the value of $\hbar$ increases [K21] . The value leaving the value of α_K invariant would be $\hbar \rightarrow 26\hbar$ and would mean that p-adic length scale L_{107} is replaced with length scale $26L_{107} = 46$ fm, the size of large nucleus so that also the basic length scale nuclear physics would be implicitly coded into the structure of hadrons.

15.4.5 Why Mersenne primes should label a fractal hierarchy of physics?

There are motivations for the working hypothesis stating that there is fractal hierarchy of copies of standard model physics, and that Mersenne primes label both hadronic space-time sheets and gauge bosons. The reason for this is not yet well understood and I have considered several speculative explanations.

First picture

The first thing to come in mind is that Mersenne primes correspond to fixed points of the discrete p-adic coupling constant evolution, most naturally to the maxima of the color coupling constant strength. This would mean that gluons are emitted with higher probability than in other p-adic length scales.

There is however an objection against this idea. If one accepts the new vision about non-perturbative aspects of QCD, it would seem that super-symplectic bosons or the interaction between super-symplectic bosons and quarks for some reason favors Mersenne primes. However, if color coupling strength corresponds to $\alpha_K = \alpha_s = 1/4$ scaled down by the increase of the Planck constant, the evolution of super-symplectic color coupling strength does not seem to play any role. What becomes large should be a geometric "form factor", when the boson in the vertex corresponds to Mersenne prime rather than "bare" coupling.

The resolution of the problem could be that boson emission vertices $g(p_1, p_2, p_3)$ are functions of p-adic primes labeling the particles of the vertices so that actually three p-adic length scales are involved instead of single length scale as in the ordinary coupling constant evolution. Hence one can imagine that the interaction between particles corresponding to primes near powers of 2 and Mersenne primes is especially strong and analogous to a resonant interaction. The geometric resonance due to the fact that the length scales involved are related by a fractal scaling by a power of 2 would make the form factors $F(p_1 \simeq 2^{k_1}, p_2 \simeq 2^{k_2}, M_n)$ large. The selection of primes near powers of two and Mersenne bosons would be analogous to evolutionary selection of a population consisting of species able to interact strongly.

Since $k = 113$ quarks are possible for $k = 107$ hadron physics, it seems that quarks can have join along boundaries bonds directed to M_n space-times with $n < k$. This suggests that neighboring Mersenne primes compete for join along boundaries bonds of quarks. For instance, when the p-adic length scale characterizing quark of M_{107} hadron physics begins to approach M_{89} quarks tend to feed their gauge flux to M_{89} space-time sheet and M_{89} hadron physics takes over and color coupling strength begins to increase. This would be the space-time correlate for the loss of asymptotic freedom.

Second picture

Preferred values of Planck constants could play a key role in the selection of Mersenne primes. Ruler-and-compass hypothesis predicts that Planck constants, which correspond to ratios of ruler and compass integers proportional to a product of distinct Fermat primes (four of them are known) and any power of two are favored. As a special case one obtains ruler and compass integers. As a consequence, p-adic length scales have satellites obtained by multiplying them with ruler-and-compass integers, and entire fractal hierarchy of power-of-two multiples of a given p-adic length scale results.

Mersenne length scales would be special since their satellites would form a subset of satellites of shorter Mersenne length scales. The copies of standard model physics associated with Mersenne primes would define a kind of resonating subset of physics since corresponding wavelengths and frequencies would coincide. This would also explain why fermions labeled by primes near power of two couple strongly with Mersenne primes.

15.4.6 The formula for the hadronic string tension

It is far from clear whether the strong gravitational coupling constant has same relation to the parameter $M_0^2 = 16m_0^2 = 1/\alpha' = 2\pi T$ as it would have in string model.

(a) One could estimate the strong gravitational constant from the fundamental formula for the gravitational constant expressed in terms of exponent of Kähler action in the case that one has $\alpha_K = 1/4$. The formula reads as

$$\frac{L_p^2}{G_p} = exp(2aS_K(CP_2)) = exp(\pi/4\alpha_K) = e^{\pi} \ . \tag{15.4.1}$$

a is a parameter telling which fraction the action of wormhole contact is about the full action for CP_2 type vacuum extremal and $a \sim 1/2$ holds true. The presence of a can take care that the exponent is rational number. For $a = 1$ The number at the right hand side is Gelfond constant and one obtains

$$G_p = exp(-\pi) \times L_p^2 \ . \tag{15.4.2}$$

(b) One could relate the value of the strong gravitational constant to the parameter $M_0^2(k) = 16m(k)^2$, $p \simeq 2^k$ also assuming that string model formula generalizes as such. The basic formulas can be written in terms of gravitational constant G, string tension T, and $M_0^2(k)$ as

$$\frac{1}{8\pi G(k)} = \frac{1}{\alpha'} = 2\pi T(k) = \frac{1}{M_0^2(k)} = \frac{1}{16m(k)^2} \ . \tag{15.4.3}$$

This allows to express G in terms of the hadronic length scale $L(k) = 2\pi/m(k)$ as

$$G(k) = \frac{1}{16^2\pi^2}L(k)^2 \simeq 3.9 \times 10^{-4}L(k)^2 \ . \tag{15.4.4}$$

The value of gravitational coupling would be by two orders of magnitude smaller than for the first option.

15.4.7 How p-adic and real coupling constant evolutions are related to each other?

The relationship between p-adic and real coupling constant evolutions more or less trivializes since S-matrix elements in the approach based on number theoretical braids are algebraic numbers and thus make sense in any number field. The real and p-adic coupling constants are thus identical algebraic numbers.

One can pose many questions about p-adic coupling constant evolution. How do p-adic and corresponding real coupling constant evolution relate to each other? Why Mersenne primes and primes near prime (integer) powers of two seem to be in a special position physically? Could one say something about phase transition between perturbative and non-perturbative phases of QCD?

How p-adic amplitudes are mapped to real ones?

Before the realization that p-adic and real amplitudes could be algebraic numbers the question of the title was very relevant. If the recent picture is correct, the following considerations are to some degree obsolete.

The real and p-adic coupling constant evolutions should be consistent with each other. This means that the coupling constants $g(p_1,p_2,p_3)$ as functions of p-adic primes characterizing particles of the vertex should have the same qualitative behavior as real and p-adic functions. Hence the p-adic norms of complex rational valued (or those in algebraic extension) amplitudes must give a good estimate for the behavior of the real vertex. Hence a restriction of a continuous correspondence between p-adics and reals to rationals is highly suggestive. The restriction of the canonical identification to rationals would define this kind of correspondence but this correspondence respects neither symmetries nor unitarity in its basic form. Some kind of compromize between correspondence via common rationals and canonical identification should be found.

The compromise might be achieved by using a modification of canonical identification $I_{R_p\rightarrow R}$. Generalized numbers would be regarded in this picture as a generalized manifold obtained by gluing different number fields together along rationals. Instead of a direct identification of real and p-adic rationals, the p-adic rationals in R_p are mapped to real rationals (or vice versa) using a variant of the canonical identification $I_{R\rightarrow R_p}$ in which the expansion of rational number $q=r/s=\sum r_np^n/\sum s_np^n$ is replaced with the rational number $q_1 = r_1/s_1=\sum r_np^{-n}/\sum s_np^{-n}$ interpreted as a p-adic number:

$$q = \frac{r}{s}=\frac{\sum_n r_np^n}{\sum_m s_np^n}\rightarrow q_1=\frac{\sum_n r_np^{-n}}{\sum_m s_np^{-n}} \quad (15.4.5)$$

This variant of canonical identification is not equivalent with the original one using the infinite expansion of q in powers of p since canonical identification does not commute with product and division. The variant is however unique in the recent context when r and s in $q=r/s$ have no common factors. For integers $n<p$ it reduces to direct correspondence. R_{p1} and R_{p_2} are glued together along common rationals by an the composite map $I_{R\rightarrow R_{p_2}}I_{R_{p_1}\rightarrow R}$.

Instead of a re-interpretation of the p-adic number $g(p_1,p_2,p_3)$ as a real number or vice versa would be continued by using this variant of canonical identification. The nice feature of the map would be that continuity would be respected to high degree and something which is small in real sense would be small also in p-adic sense.

How to achieve consistency with the unitarity of topological mixing matrices and of CKM matrix?

It is easy to invent an objection against the proposed relationship between p-adic and real coupling constants. Topological mixing matrices U, D and CKM matrix $V=U^\dagger D$ define an important part of the electro-weak coupling constant structure and appear also in coupling constants. The problem is that canonical identification does not respect unitarity and does not commute with the matrix multiplication in the general case unlike gluing along common rationals. Even if matrices U and D which contain only ratios of integers smaller than p are constructed, the construction of V might be problematic since the products of two rationals can give a rational $q = r/s$ for which r or s or both are larger than p.

One might hope that the objection could be circumvented if the ratios of the integers of the algebraic extension defining the matrix elements of CKM matrix are such that the integer components of algebraic integers are smaller than p in U and D and even the products of integers in $U^\dagger D$ satisfy this condition so that modulo p arithmetics is avoided.

In the standard parameterization all matrix elements of the unitarity matrix can be expressed in terms of real and imaginary parts of complex phases ($p \ mod \ 4 = 3$ guarantees that $\sqrt{-1}$ is not an ordinary p-adic number involving infinite expansion in powers of p). These phases are expressible as products of Pythagorean phases and phases in some algebraic extension of rationals.

(a) Pythagorean phases defined as complex rationals $[r^2 - s^2 + i2rs]/(r^2 + s^2)$ are an obvious source of potential trouble. However, if the products of complex integers appearing in the numerators and denominators of the phases have real and imaginary parts smaller than p it seems to be possible to avoid difficulties in the definition of $V = U^\dagger D$.

(b) Pythagorean phases are not periodic phases. Algebraic extensions allow to introduce periodic phases of type $exp(i\pi m/n)$ expressible in terms of p-adic numbers in a finite-dimensional algebraic extension involving various roots of rationals. Also in this case the product $U^\dagger D$ poses conditions on the size of integers appearing in the numerators and denominators of the rationals involved.

If the expectation that topological mixing matrices and CKM matrix characterize the dynamics at the level $p \simeq 2^k$, $k = 107$, is correct, number theoretical constraints are not expected to bring much new to what is already predicted. Situation changes if these matrices appear already at the level k. For $k = 89$ hadron physics the restrictions would be even stronger and might force much simpler U, D and CKM matrices.

k-adicity constraint would have even stronger implications for S-matrix and could give very powerful constraints to the S-matrix of color interactions. Quite generally, the constraints would imply a p-adic hierarchy of increasingly complex S-matrices: kind of a physical realization for number theoretic emergence. The work with CKM matrix has shown how powerful the number theoretical constraints are, and there are no reasons to doubt that this could not be the case also more generally since in the lowest order the construction would be carried out in finite (Galois) fields $G(p, k)$.

How generally the hybrid of canonical identification and identification via common rationals can apply?

The proposed gluing procedure, if applied universally, has non-trivial implications which need not be consistent with all previous ideas.

(a) The basic objection against the new kind of identification is that it does not commute with symmetries. Therefore its application at imbedding space and space-time level is questionable.

(b) The mapping of p-adic probabilities by canonical identification to their real counterparts requires a separate normalization of the resulting probabilities. Also the new variant of canonical identification requires this since it does not commute with the sum.

(c) The direct correspondence of reals and p-adics by common rationals at space-time level implies that the intersections of cognitive space-time sheets with real space-time sheet have literally infinite size (p-adically infinitesimal corresponds to infinite in real sense for rational) and consist of discrete points in general. If the new gluing procedure is adopted also at space-time level, it would considerably de-dramatize the radical idea that the size for the space-time correlates of cognition is literally infinite and cognition is a literally cosmic phenomenon.

Of course, the new kind of correspondence could be also seen as a manner to construct cognitive representations by mapping rational points to rational points in the real sense and thus as a formation of cognitive representations at space-time level mapping points close to each other in real sense to points close to each other p-adically but arbitrarily far away in real sense. The image would be a completely chaotic looking set of points in the wrong topology and would realize the idea of Bohm about hidden order in a very concrete manner. This kind of mapping might be used to code visual information using the value of p as a part of the code key.

(d) In p-adic thermodynamics p-adic particle mass squared is mapped to its real counterpart by canonical identification. The objection against the use of the new variant of canonical identification is that the predictions of p-adic thermodynamics for mass squared are not rational numbers but infinite power series. p-Adic thermodynamics itself however defines a unique representation of probabilities as ratios of generalized Boltzmann weights and partition function and thus the variant of canonical identification indeed generalizes and at the same time raises worries about the fate of the earlier predictions of the p-adic thermodynamics.

Quite generally, the thermodynamical contribution to the particle mass squared is in the lowest p-adic order of form rp/s, where r is the number of excitations with conformal weight 1 and s the number of massless excitations with vanishing conformal weight. The real counterpart of mass squared for the ordinary canonical identification is of order CP_2 mass by $r/s = R + r_1p + ...$ with $R < p$ near to p. Hence the states for which massless state is degenerate become ultra heavy if r is not divisible by s. For the new variant of canonical identification these states would be light. It is not actually clear how many states of this kind the generalized construction unifying super-symplectic and super Kac-Moody algebras predicts.

A less dramatic implication would be that the second order contribution to the mass squared from p-adic thermodynamics is always very small unless the integer characterizing it is a considerable fraction of p. When ordinary canonical identification is used, the second order term of form rp^2/s can give term of form Rp^2, $R < p$ of order p. This occurs only in the case of left handed neutrinos.

The assumption that the second order term to the mass squared coming from other than thermodynamical sources gives a significant contribution is made in the most recent calculations of leptonic masses [K37] . It poses constraints on CP_2 mass which in turn are used as a guideline in the construction of a model for hadrons [K46] . This kind of contribution is possible also now and corresponds to a contribution Rp^2, $R < p$ near p.

The new variant of the canonical correspondence resolves the long standing problems related to the calculation of Z and W masses. The mass squared for intermediate gauge bosons is smaller than one unit when m_0^2 is used as a fundamental mass squared unit. The standard form of the canonical identification requires $M^2 = (m/n)p^2$ whereas in the new approach $M^2 = (m/n)p$ is allowed. Second difficult problem has been the p-adic description of the group theoretical model for m_W^2/m_Z^2 ratio. In the new framework this is not a problem anymore [K37] since canonical identification respects the ratios of small integers.

On the other hand, the basic assumption of the successful model for topological mixing of quarks [K46] is that the modular contribution to the masses is of form np. This assumption loses its original justification for this option and some other justification is needed. The first guess is that the conditions on mass squared plus probability conservation might not be consistent with unitarity unless the modular contribution to the mass squared remains integer valued in the mixing (note that all integer values are not possible [K46]). Direct numerical experimentation however shows that that this is not the case.

15.4.8 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [K14] . M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results. Recall that also the weaker condition $T_p = pT_0$, p prime, would assign secondary p-adic time scales to the size scale hierarchy of CDs: $p \simeq 2^n$ would result as an outcome of some kind of "natural selection" for this option. The highly satisfactory feature would be that p-adic time scales would reflect directly the geometry of imbedding space and WCW.

p-Adic coupling constant evolution

An attractive conjecture is that the coupling constant evolution associated with CDs in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induces p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there seems to be a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

(a) The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

(b) p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via

$T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

(c) In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For the weaker condition would be $T_p = pT_0$, p prime, $p \simeq 2^n$ could be seen as an outcome of some kind of "natural selection". In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

(d) The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \quad ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

Appendix: Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B37] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \qquad (15.4.6)$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{15.4.7}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ , \end{aligned} \tag{15.4.8}$$

and

$$B = 2re^3 \ , \tag{15.4.9}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{15.4.10}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{aligned} \tag{15.4.11}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{15.4.12}$$

where $W^\pm$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 \ , \\ Y &= \frac{e^3}{r} \ , \end{aligned} \tag{15.4.13}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \tag{15.4.14}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{15.4.15}$$

Identifying Σ_{12} and $\Sigma_{03} = 1\times\gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{15.4.16}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{15.4.17}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{15.4.18}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{15.4.19}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} , \quad (15.4.20)$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) , \quad (15.4.21)$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0\wedge e^3 + e^1\wedge e^2) , \\ R_{12} &= 2(e^0\wedge e^3 + 2e^1\wedge e^2) , \\ J &= 2(e^0\wedge e^3 + e^1\wedge e^2) , \end{aligned} \quad (15.4.22)$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) . \quad (15.4.23)$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} , \\ Z^0 &= 2R_{03} . \end{aligned} \quad (15.4.24)$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) . \quad (15.4.25)$$

Chapter 1

Appendix

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of imbedding space and related spaces are discussed and the relationship of CP_2 to standard model is summarized. The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure. Hierarchy of Planck constants can be now understood in terms of the non-determinism of Kähler action and the recent vision about connections to other key ideas is summarized.

A-1 Imbedding space $M^4 \times CP_2$ and related notions

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that imbedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Imbedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://www.tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) causal diamond (CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://www.tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://www.tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrates that it codes for the symmetries of standard models via its isometries and holonomies.

A-2.1 CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A91] will be used and their relation to the complex coordinates is given by

$$\begin{aligned}
\xi^1 &= z + it \ , \\
\xi^2 &= x + iy \ .
\end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned}
\xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\
\xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ .
\end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. http://www.tgdtheory.fi/appfigures/cp2.jpg

A-2.2 Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i \bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1 + r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r, \phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{array}{lllllll} e^0 & = & \frac{dr}{F} & , & e^1 & = & \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 & = & \frac{r\sigma_2}{\sqrt{F}} & , & e^3 & = & \frac{r\sigma_3}{F} \ . \end{array} \quad \text{(A-2.10)}$$

The explicit representations of vierbein vectors are given by

$$\begin{array}{lllllll} e^0 & = & \frac{dr}{F} \ , & & e^1 & = & \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 & = & \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} & , & e^3 & = & \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{array} \quad \text{(A-2.11)}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \quad \text{(A-2.12)}$$

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \quad \text{(A-2.13)}$$

is given by

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} & , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} & , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 & , & V_{12} & = & (2r + \frac{1}{r})e^3 \ . \end{array} \quad \text{(A-2.14)}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0 \wedge e^1 - e^2 \wedge e^3 & , & R_{23} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} & = & e^0 \wedge e^2 - e^3 \wedge e^1 & , & R_{31} & = & -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} & = & 4e^0 \wedge e^3 + 2e^1 \wedge e^2 & , & R_{12} & = & 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \quad \text{(A-2.15)}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -ig_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \quad \text{(A-2.16)}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r} J^{rl} = -s^{kl} \ . \quad \text{(A-2.17)}$$

The form J is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling). Locally one has therefore

$$J = dB \ , \tag{A-2.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

It should be noticed that the magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2} dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F} sin\Theta d\Theta d\Phi \ . \end{aligned} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1,1).

Useful coordinates for CP_2 are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= \frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

A-2.3 Spinors in CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A80] . However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1- factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

A-2.4 Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A70] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-2.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-3 CP_2 geometry and standard model symmetries

A-3.1 Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B37] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-3.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-3.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ , \end{aligned} \tag{A-3.3}$$

and

$$B = 2re^3 \ , \tag{A-3.4}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-3.5}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{aligned} \tag{A-3.6}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-3.7}$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 \ , \\ Y &= \frac{e^3}{r} \ , \end{aligned} \tag{A-3.8}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \tag{A-3.9}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{A-3.10}$$

Identifying Σ_{12} and $\Sigma_{03} = 1\times\gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{A-3.11}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-3.12}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+ 1_+ + n_- 1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{A-3.13}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{A-3.14}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-3.15}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+ 1_+ + n_- 1_-) \ , \tag{A-3.16}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{A-3.17}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-3.18}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-3.19}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-3.20}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-3.21}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \tag{A-3.22}$$

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18 + 2n_i^2)sin^2\theta_W}{9}\right] \ , \tag{A-3.23}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18 + n_i^2))} \ . \tag{A-3.24}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-3.25}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [B59] .

A-3.2 Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

(a) Symmetries must be realized as purely geometric transformations.

(b) Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B24] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-3.26}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-3.27}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-3.28}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-4 The relationship of TGD to QFT and string models

TGD could be seen as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://www.tgdtheory.fi/appfigures/particletgd.jpg`

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in completely unique position as far as symmetries are considered.

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of TGD Universe and inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situtation in Minkowskian regions of space-time surface. `http://www.tgdtheory.fi/appfigures/fermistring.jpg`

TGD based view about elementary particles has two aspects.

(a) The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidian signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

(b) Fermion number is carried by the modes of the induced spinor field. In Minkowskian space-time regions the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle corresponds 4-D generalization of world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidian signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacs carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompnied by fermionic strings connecting the throats at same sheet: the strings do not extend inside the wormhole contacts. `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

(a) The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having hadronic string as physical counterpart. Long strings should be distinguished from wormhole contacts which due to their super-conformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

(b) Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats beheave like virtual bosons so that one has BFF type vertices typically.

(c) In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particle along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://www.tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

Induction procedure for gauge fields

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has imbedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as base space the imbedded manifold. In the recent case the imbedding of space-time surface to imbedding space defines the induction procedure. The induce gauge potentials and gauge fields are projections of the spinor connection of the imbedding space to the space-time surface. Induction procedure makes sense also for the spinor fields of imbedding space and one obtains geometrization of both electroweak gauge potentials and of spinors.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://www.tgdtheory.fi/appfigures/induct.jpg`

Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP_2 projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \quad .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

A-5.1 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four imbedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of imbedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)) as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://www.tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-5.2 Imbedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

(a) Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

(b) Spinor harmonics of imbedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of imbedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

(a) Although the imbedding space spinor connection carries W gauge potentials one can say that the imbedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

(b) The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

(c) Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

(d) One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

(e) This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-5.3 Space-time surfaces with vanishing em, Z^0, or Kähler fields

In the following the induced gauge fields are studied for general space-time surface without assuming the extremal property. In fact, extremal property reduces the study to the study of vacuum extremals and surfaces having geodesic sphere as a CP_2 projection and in this sense the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + \frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-5.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + (3+p)\frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-5.2}$$

where Θ_W denotes Weinberg angle.

(a) The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k+cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \quad \text{(A-5.3)}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{(k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k+cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \quad \text{(A-5.4)}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r=0$ would correspond to $X=0$ giving $u=-k$ achieved only for $|k| \leq 1$ and $r=\infty$ to $X=1$ giving $|u+k| = [(1+r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.
The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p}Xdu \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p}J \ . \end{aligned} \quad \text{(A-5.5)}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

(b) The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F}du \wedge d\Phi$ is useful.

(c) The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u}du \wedge d\Phi = (k+u)du \wedge d\Phi \ , \\ r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\ \gamma &= -\frac{p}{2}Z^0 \ . \end{aligned} \quad \text{(A-5.6)}$$

For a vanishing value of Weinberg angle ($p=0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$
\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \qquad \text{(A-5.7)}
\end{aligned}
$$

and is useful in the construction of vacuum imbedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$
\begin{aligned}
\Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\
\Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \qquad \text{(A-5.8)}
\end{aligned}
$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \tag{A-5.9}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-6 p-Adic numbers and TGD

A-6.1 p-Adic number fields

p-Adic numbers (p is prime: 2,3,5,...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A48] . p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k, \ x(k) = 0, ..., p-1 \ . \tag{A-6.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{A-6.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \tag{A-6.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x, y) = |x - y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x, z) \leq max\{d(x, y), d(y, z)\} \ . \tag{A-6.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x, y) \leq D \ . \tag{A-6.5}$$

This division of the metric space into classes has following properties:

(a) Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

(b) Distances of points x and y inside single class are smaller than distances between different classes.

(c) Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B50] . The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

A-6.2 Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

Basic form of canonical identification

There exists a natural continuous map $I : R_p \to R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \to x = \sum_{k<N} y_k p^{-k} \quad , \\ y_k &\in \{0,1,..,p-1\} \quad . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique $(1 = 0.999...)$ for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \quad , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \quad . \end{aligned} \tag{A-6.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^k \quad , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \quad , \end{aligned} \tag{A-6.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see Fig. **??**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://www.tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p |y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \tag{A-6.9}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p |y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \tag{A-6.10}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-6.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-6.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

Generalization of number concept and notion of imbedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic imbedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real imbedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod\ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://www.tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real imbedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

A-6.3 The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles" with reverse map interpreted as a transformation of intention to action and would be realized in terms of canonical identification or some of its variants.

Fig. 16. The basic idea between p-adic manifold. `http://www.tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

(a) Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

(b) Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

(c) Canonical identification vreaks general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic imbedding space with chart maps to real imbedding space and assuming preferred coordinates made possible by isometries of imbedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-7 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the imbedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of imbedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of imbedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://www.tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. http://www.tgdtheory.fi/appfigures/reconnect1.jpg

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. http://www.tgdtheory.fi/appfigures/reconect2.jpg

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. http://www.tgdtheory.fi/appfigures/fluxtubedynamics.jpg

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. http://www.tgdtheory.fi/appfigures/cat.jpg

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred imbedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. http://www.tgdtheory.fi/appfigures/padictoreal.jpg

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. http://www.tgdtheory.fi/appfigures/sharing.jpg

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see fig. http://www.tgdtheory.fi/appfigures/timemirror.jpg or fig. 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. http://www.tgdtheory.fi/appfigures/timemirror.jpg

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Index

TOWARDS M-MATRIX

Topological Geometrodynamics (TGD) is a modification of general relativity inspired by the problems related to the definition of inertial and gravitational energies in general relativity. TGD is also a generalization of super string models. Physical space-times are seen as four-dimensional surfaces in certain 8-dimensional space H. The choice of H is fixed by symmetries of standard model and leads to a geometrization of known classical fields and elementary particle numbers. In fermionic sector strings indeed emerge.

Many-sheeted space-time replaces Einsteinian space-time, which follows as a long length scale approximation in which sheets of the many-sheeted space-time are lumped together. The extension of number concept based on the fusion of real numbers and p-adic number fields implies a further generalisation of the space-time concept allowing to identify space-time correlates of cognition and intentionality.

Zero energy ontology forces an extension of quantum measurement theory to a theory of consciousness and a hierarchy of phases identified as dark matter is predicted with far reaching implications for the understanding of consciousness and living systems. This all implies an elegant theoretical projection of our reality honoring the work by renowned scientists (such as Wheeler, Feynman, Penrose, Einstein, Josephson to name a few) and creating a solid foundation for modeling our Universe in terms of geometry.

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Matti Pitkänen started to work with the basic idea of TGD at 1977, published his thesis work about TGD at 1982, and has since then worked to transform the basic vision to a consistent predictive mathematical framework, to solve various interpretational issues, and understand the relationship of TGD with existing theories.

TGD Web Pages: http://www.tgdtheory.com
TGD Diary and Blog: http://matpitka.blogspot.com