P-ADIC LENGTH SCALE HYPOTHESIS AND DARK MATTER HIERARCHY

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0.1 Background

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [16]. The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last twenty-three years for the realization of this dream and this has resulted in seven online books [TGDview, TGDgeom, TGDquant, TGDnumber, TGDclass, TGDpad, TGDfree] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [TGDconsc, TGDselforg, TGDware, TGDholo, TGDgeme, TGDeeg, TGDmagn, 15].

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be.

The fifth thread came with the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and certainly possible in TGD framework. The identification of hierarchy of Planck constants whose values TGD "predicts" in terms of dark matter hierarchy would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The seven online books [TGDview, TGDgeom, TGDquant, TGDnumber, TGDclass, TGDpad, TGDfree] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [TGDconsc, TGDselforg, TGDware, TGDholo, TGDgeme, TGDeeg, TGDmagn, 15] are warmly recommended to the interested reader.

0.2 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD is as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

0.2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_+ \times CP_2$, where M^4_+ denotes the interior of the future light cone of the Minkowski space (to be referred as light cone in the sequel) and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [2, 18, 19, 5]. The identification of the space-time as a submanifold [21, 22] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity [Misner-Thorne-Wheeler, Logunov *et al*]. The actual choice $H = M^4_+ \times CP_2$ implies the breaking of the Poincare invariance in the cosmological scales but only at the quantum level. It soon however turned out that submanifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

0.2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

0.2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there is "vapor phase" that is a "gas" of particle like 3-surfaces (counterpart of the "baby universies" of GRT) and the nonconservation of energy in GRT corresponds to the transfer of energy between the topological condensate and vapor phase.

0.3 The five threads in the development of quantum TGD

The development of TGD has involved four strongly interacting threads: physics as infinite-dimensional geometry; p-adic physics; TGD inspired theory of consciousness and TGD as a generalized number theory. In the following these five threads are briefly described.

0.3.1 Quantum TGD as configuration space spinor geometry

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and are the following ones:

a) Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [23, 24, 25]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

b) Configuration space is endowed with the metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory.

0.3.2 p-Adic TGD

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzless and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics growed steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

0.3.3 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [TGDconsc, TGDselforg, TGDware, TGDholo, TGDgeme, TGDeeg, TGDmagn, 15].

Quantum jump as a moment of consciousness

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \quad ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

The notion of self

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions' U. Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves S_i are not experienced as such but represent kind of averages $\langle S_{ij}\rangle$ of sub-subselves S_{ij}. Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

Relationship to quantum measurement theory

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

a) The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

b) Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m-M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

Selves self-organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [I1]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

Classical non-determinism of Kähler action

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

p-Adic physics as physics of cognition and intentionality

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, ...$ p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive pinary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [E1]. The application this notion at the level of the imbedding space implies that imbedding space has a book like structure with various variants of the imbedding space glued together along common rationals (algebraics). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the projection to the real imbedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an explanation for p-adic length scale hypothesis stating that the primes $p \simeq 2^k$, k integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic pinary digits a p-valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the $p = 2^k - n$ pinary digits represent a Boolean logic B^k with k elementary statements (the points of the k-element set in the set theoretic realization) with n taboos which are constrained to be identically true.

0.3.4 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

The role of classical number fields

The vision about the physical role of the classical number fields relies on the notion of number theoretic compactifiction stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$.

As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [E2] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

One can fairly say, that number theoretical compactification is responsible for most of the understanding of quantum TGD that has emerged during last years. This includes the realization of Equivalence Principle at space-time level, dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of configuration space metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing automatically the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of configuration space represents an example.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

0.3.5 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale [40] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [D7].

Already before learning about Nottale's paper I had proposed the possibility that Planck constant is quantized [E9] and the spectrum is given in terms of logarithms of Beraha numbers: the lowest Beraha number B_3 is completely exceptional in that it predicts infinite value of Planck constant. The inverse of the gravitational Planck constant could correspond a gravitational perturbation of this as $1/\hbar_{gr} = v_0/GMm$. The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character and at the transition $n = 4 \rightarrow 3$ only the small perturbative correction to $1/\hbar(3) = 0$ remains. This would apply to QCD and to atoms with $Z > 137$ as well.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. Also a model for the formation of planetary system as a condensation of ordinary matter around quantum coherent dark matter emerges [D7].

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

p-Adic and dark matter hierarchies and hierarchy of moments of consciousness

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

a) Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as $\hbar$). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.

b) The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [M3].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [M3]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, ...$ [M3]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy and number theoretical arguments suggest a general formula for the allowed values of Planck constant λ depending logarithmically on p-adic prime [A8]. Also the value of $\hbar_0$ has spectrum characterized by Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$, varying by a factor in the range $n > 3$ [A8]. It must be however emphasized that the relation of this picture to the model of quantized gravitational Planck constant h_{gr} appearing in Nottale's model is not yet completely understood.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [M3]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [L2, M3]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [M3].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [J6, M3]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like $\hbar$. Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. The quantum parallel dissipation at the lower levels would give rise to the experience of flow of time. For instance, hadron as a macro-temporal quantum system in the characteristic time scale of hadron is a dissipating system at quark and gluon level corresponding to shorter p-adic time scales. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

3. The time span of long term memories as signature for the level of dark matter hierarchy

The simplest dimensional estimate gives for the average increment τ of geometric time in quantum jump $\tau \sim 10^4$ CP_2 times so that $2^{127}-1 \sim 10^{38}$ quantum jumps are experienced during secondary p-adic time scale $T_2(k=127) \simeq 0.1$ seconds which is the duration of physiological moment and predicted to be fundamental time scale of human consciousness [L1]. A more refined guess is that $\tau_p = \sqrt{p}\tau$ gives the dependence of the duration of quantum jump on p-adic prime p. By multi-p-fractality predicted by TGD and explaining p-adic length scale hypothesis, one expects that at least $p=2$-adic level is also always present. For the higher levels of dark matter hierarchy τ_p is scaled up by $\hbar/\hbar_0$. One can understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined τ [L2].

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. For instance, EEG time scales corresponds to $k=4$ level of hierarchy and a time scale of .1 seconds [J6], and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question.

The level would determine also the time span of long term memories as discussed in [M3]. $k=7$ would correspond to a duration of moment of conscious of order human lifetime which suggests that $k=7$ corresponds to the highest dark matter level relevant to our consciousness whereas higher levels would in general correspond to transpersonal consciousness. $k=5$ would correspond to time scale of short term memories measured in minutes and $k=6$ to a time scale of memories measured in days.

The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [L2, M3]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

0.4 Bird's eye of view about the topics of the book

The book is devoted to the applications of p-adic length scale hypothesis and dark matter hierarchy.

1. p-Adic length scale hypothesis states that primes $p \simeq 2^k$, k integer, in particular prime, define preferred p-adic length scales. Physical arguments supporting this hypothesis are based on the generalization of Hawking's area law for blackhole entropy so that it applies in case of elementary particles.

2. A much deeper number theory based justification for this hypothesis is based on the generalization of the number concept fusing real number fields and p-adic number fields among common rationals or numbers in their non-trivial algebraic extensions. This approach also justifies the notion of multi-p-fractality and allows to understand scaling law in terms of simultaneous $p \simeq 2^k$- and 2-fractality.

3. Certain anomalous empirical findings inspire in TGD framework the hypothesis about the existence of entire hierarchy of phases of matter identifiable as dark matter. The levels of dark matter hierarchy are labeled by the values of dynamical quantized Planck constant. The justification for the hypothesis provided by quantum classical correspondence and the fact the sizes of space-time sheets identifiable as quantum coherence regions can be arbitrarily large.

The organization of the book is following.

1. The first part of the book is devoted to the description of elementary particle massivation in terms of p-adic thermodynamics. In the first two chapters general theory is represented and the

remaining three chapters are devoted to the detailed calculation of masses of elementary particles and hadrons, and to various new physics suggested or predicted by the resulting scenario.

2. The second part of the book is devoted to the application of p-adic length scale hypothesis above elementary particle length scales. The notions of topological condensation and evaporation are formulated. The so called leptohadron physics, originally developed on basis of experimental anomalies, is discussed as a particular instance of an infinite fractal hierarchy of copies of standard model physics, predicted by TGD and consistent with what is known about ordinary elementary particle physics.

   TGD based view about nuclear physics involves light exotic quarks as a essential element, and dark nuclear physics could have implications also at the level of condensed matter physics and biology. Quite surprisingly, the model for dark 3-quarks states consisting of u and d quarks leads to the identification of quantum states of three-quark system as counterparts of 64 DNA and RNA codons and 20 amino-acids and of the analog of genetic code identical with the vertebrate genetic code. This suggests that dark nuclear physics with scaled up sizes of nucleon of order atomic size could play key role in living matter and provide the realization of genetc code at deeper level. Water memory would be one application of this vision.

   TGD based view about high T_c superconductors involves also in an essential manner dark matter and is summarized in the closing chapter.

The seven online books about TGD [TGDview, TGDgeom, TGDquant, TGDnumber, TGDclass, TGDpad, TGDfree] and eight online books about TGD inspired theory of consciousness and quantum biology [TGDconsc, TGDselforg, TGDware, TGDholo, TGDgeme, TGDeeg, TGDmagn, 15] are warmly recommended for the reader willing to get overall view about what is involved.

0.5 The contents of the book

0.5.1 ParT I: p-Adic description of particle massivation

In this part of the book a p-adic description of particle massivation using p-adic thermodynamics and TGD variant of Higgs mechanism is developed.

Elementary particle vacuum functionals

Genus-generation correspondence is one of the basic ideas of TGD approach. In order to answer various questions concerning the plausibility of the idea, one should know something about the dependence of the elementary particle vacuum functionals on the vibrational degrees of freedom for the boundary component. The construction of the elementary particle vacuum functionals based on Diff invariance, 2-dimensional conformal symmetry, modular invariance plus natural stability requirements indeed leads to an essentially unique form of the vacuum functionals and one can understand why $g > 2$ bosonic families are experimentally absent and why lepton numbers are conserved separately.

An argument suggesting that the number of the light fermion families is three, is developed. The argument goes as follows. Elementary particle vacuum functionals represent bound states of g handles and vanish identically for hyper-elliptic surfaces having $g > 2$. Since all $g \leq 2$ surfaces are hyper-elliptic, $g \leq 2$ and $g > 2$ elementary particles cannot appear in same non-vanishing vertex and therefore decouple. The $g > 2$ vacuum functionals not vanishing for hyper-elliptic surfaces represent many particle states of $g \leq 2$ elementary particle states being thus unstable against the decay to $g \leq 2$ states. The failure of Z_2 conformal symmetry for $g > 2$ elementary particle vacuum functionals would in turn explain why they are heavy: this however not absolutely necessary since these particles would behave like dark matter in any case.

Massless states and particle massivation

In this chapter the goal is to summarize the recent theoretical understanding of the spectrum of massless particles and particle massivation in TGD framework. After a summary of the recent phenomenological picture behind particle massivation the notions of number theoretical compactification and number theoretical braid are introduced and the construction of quantum TGD at parton level in terms of second quantization of modified Dirac action is described. The recent understanding of super-conformal symmetries are analyzed in detail. TGD color differs in several respect from QCD color and a detailed analysis of color partial waves associated with quark and lepton chiralities of imbedding space spinors fields is carried out with a special emphasis given to the contribution of color partial wave to mass squared of the fermion. The last sections are devoted to p-adic thermodynamics and to a model providing a formula for the modular contribution to mass squared.

Although the basic predictions of p-adic mass calculations were known almost 15 years ago, the justification of the basic assumptions from basic principles of TGD (and also the discovery of these principles!) has taken a considerable time. Particle massivation can be regarded as a generation of thermal conformal weight identified as mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

1. The original observation was that the pieces of CP_2 type vacuum extremals representing elementary particles have random light-like curve as an M^4 projection so that the average motion correspond to that of massive particle. Light-like randomness gives rise to classical Virasoro conditions. This picture generalizes since the basic dynamical objects are light-like but otherwise random 3-surfaces. Fermions are identified as light-like 3-surfaces at which the signature of induced metric of deformed CP_2 type extremals changes from Euclidian to the Minkowskian signature of the background space-time sheet. Gauge bosons and Higgs correspond to wormhole contacts with light-like throats carrying fermion and antifermion quantum numbers. Gravitons correspond to pairs of wormhole contacts bound to string like object by the fluxes connecting the wormhole contacts. The randomness of the light-like 3-surfaces and associated super-conformal symmetries justify the use of thermodynamics and the question remains why this thermodynamics can be taken to be p-adic. The proposed identification of bosons means enormous simplification in thermodynamical description since all calculations reduced to the calculations to fermion level.

2. The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces. Dynamics is constrained by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. As a matter fact, the basic theory relies on the modified Dirac action associated with Chern-Simons action and Kähler action in the sense that the generalizes eigenmodes of C-S Dirac operator correspond to the zero modes of Kähler action localized to the light-like 3-surfaces representing partons. In this manner the data about the dynamics of Kähler action is feeded to the eigenvalue spectrum. Eigenvalues are interpreted as square roots of ground state conformal weights.

3. The symmetries respecting light-likeness property give rise to Kac-Moody type algebra and super-symplectic symmetries emerge also naturally as well as $N = 4$ character of super-conformal invariance. The coset construction for super-symplectic Virasoro algebra and Super Kac-Moody algebra identified in physical sense as sub-algebra of former implies that the four-momenta assignable to the two algebras are identical. The interpretation is in terms of the identity of gravitational inertial masses and generalization of Equivalence Principle.

4. Instead of energy, the Super Kac-Moody Virasoro (or equivalently super-symplectic) generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics (and also in its real version assuming it exists). The fact that mass squared is thermal expectation of conformal weight guarantees Lorentz invariance. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

5. By Equivalence Principle the thermal average of mass squared can be calculated either in terms of thermodynamics for either super-symplectic of Super Kac-Moody Virasoro algebra and p-adic thermodynamics is consistent with conformal invariance.

6. A long standing problem has been whether coupling to Higgs boson is needed to explain gauge boson masses. It has turned out that p-adic thermodynamics is enough. From the beginning it was clear that is that ground state conformal weight is negative. Only quite recently it became clear that the ground state conformal weight need not be a negative integer. The deviation Δh of the total ground state conformal weight from negative integer gives rise to Higgs type contribution to the thermal mass squared and dominates in case of gauge bosons for which p-adic temperature is small. In the case of fermions this contribution to the mass squared is small. Higgs vacuum expectation is naturally proportional to Δh so that the coupling to Higgs apparently causes gauge boson massivation. The interpretation is that the effective metric defined by the modified gamma matrices associated with Kähler action has Euclidian signature. This implies that the eigenvalues of the modified Dirac operator are purely imaginary and analogous to cyclotron energies so that in the first approximation smallest conformal weights are of form $h = -n - 1/2$ and for $n = 0$ one obtains the ground state conformal weight $h = -1/2$ conjectured earlier. One cannot exclude the possibility of complex eigenvalues of D_{C-S}.

7. There is also modular contribution to the mass squared which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

An important question concerns the justification of p-adic thermodynamics.

1. The underlying philosophy is that real number based TGD can be algebraically continued to various p-adic number fields. This gives justification for the use of p-adic thermodynamics although the mapping of p-adic thermal expectations to real counterparts is not completely unique. The physical justification for p-adic thermodynamics is effective p-adic topology characterizing the 3-surface: this is the case if real variant of light-like 3-surface has large number of common algebraic points with its p-adic counterpart obeying same algebraic equations but in different number field.

2. The most natural option is that the descriptions in terms of both real and p-adic thermodynamics make sense and are consistent. This option indeed makes since the number of generalized eigen modes of modified Dirac operator is finite. The finite number of fermionic oscillator operators implies an effective cutoff in the number conformal weights so that conformal algebras reduce to finite-dimensional algebras. The first guess would be that integer label for oscillator operators becomes a number in finite field for some prime. This means that one can calculate mass squared also by using real thermodynamics but the consistency with p-adic thermodynamics gives extremely strong number theoretical constraints on mass scale. This consistency condition allows also to solve the problem how to map a negative ground state conformal weight to its p-adic counterpart. Negative conformal weight is divided into a negative half odd integer part plus positive part Δh, and negative part corresponds as such to p-adic integer whereas positive part is mapped to p-adic number by canonical identification.

p-Adic thermodynamics is what gives to this approach its predictive power.

1. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.

2. p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5}\sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

The predictions of the general theory are consistent with the earliest mass calculations, and the earlier ad hoc parameters disappear. In particular, optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles.

p-Adic particle massivation: elementary particle masses

The calculation of elementary fermion and boson masses using p-adic thermodynamics is carried out. Leptons and quarks are obey almost identical mass formulas. Charged lepton mass ratios are predicted with relative errors of order one cent and QED renormalization corrections provide a plausible explanation for the discrepancies. Neutrino masses and neutrino mixing matrix can be predicted highly uniquely if the existing experimental inputs are taken seriously: the best fit of the mass squared differences requires $k = 13^2 = 169$ so that extended form of the p-adic length scale hypothesis is needed.

The prediction or quark masses is more difficult since even the deduction of even the p-adic length scale determining the masses of u, d, and s is a non-trivial task. Second difficulty is related to the topological mixing of quarks. Somewhat surprisingly, the model for U and D matrices constructed for a decade ago predicts realistic quark mass spectrum although the new mass formula is based on different assumptions and different identification of p-adic mass scales. Current quark masses and constituent quark masses can be understood if the p-adic length scale of quark is different for free and bound quarks. The analog of Gell-Mann-Okubo type mass formula results if the p-adic length scale depends on hadron. The Higgs contribution to the fermionic mass is of second order and can be even vanishing and there is an argument implying that Higgs field cannot develop vacuum expectation at fermionic space-time sheets. Top quark mass fixes highly uniquely the CP_2 mass scale since second order correction to electron mass must be very small in order to reproduce the top quark mass in the allowed range of values. Also top quark can correspond to several p-adic mass scales and there is direct experimental evidence for this in mass distribution of top quark.

p-Adic thermodynamics cannot explain Z^0 and W boson masses: thermal masses are completely negligible for the p-adic temperature $T = 1/2$ whereas for $T = 1$ they are 20-30 per cent too high. There is a general argument implying that $T = 1/26$ holds true for bosons so that the masses would be completely negligible. TGD allows a candidate for a Higgs field with the same quantum numbers as its standard model counterpart and having wormhole contacts as space-time correlates just as ordinary gauge bosons have. Thus p-adic thermodynamics *resp.* Higgs mechanism would predict in excellent accuracy fermion *resp.* boson masses and allow the Higgs production rate to be about one per cent of the rate predicted by the standard model (the dominating fermionic couplings are now small).

The possibility of exotic states poses a serious problem for the proposed scenario. If elementary particles correspond to CP_2 type extremals, all exotic massless particles can be constructed using colored generators and by color confinement cannot induce macroscopic long range interactions. The essential assumption is that the fermionic quantization for the space-time sheets having CP_2 projection of dimension $D(CP_2) < 4$ is non-conventional. This has also direct relevance for the understanding of the matter antimatter asymmetry.

p-Adic particle massivation: hadron masses

In this chapter the results of the calculation of elementary particle masses will be used to construct a model predicting hadron masses.

1. *Topological mixing of quarks*

In TGD framework CKM mixing is induced by topological mixing of quarks (that is 2-dimensional topologies characterized by genus). Number theoretical constraints on topological mixing can be realized by assuming that topological mixing leads to a thermodynamical equilibrium. This gives an upper bound of 1200 for the number of different U and D matrices and the input from top quark mass and $\pi^+ - \pi^0$ mass difference implies that physical U and D matrices can be constructed as small perturbations of matrices expressible as direct sum of essentially unique 2×2 and 1×1 matrices. The maximally entropic solutions can be found numerically by using the fact that only the probabilities p_{11} and p_{21} can be varied freely. The solutions are unique in the accuracy used, which suggests that the system allows only single thermodynamical phase.

The matrices U and D associated with the probability matrices can be deduced straightforwardly in the standard gauge. The U and D matrices derived from the probabilities determined by the entropy maximization turn out to be unitary for most values of n_1 and n_2. This is a highly non-trivial result and means that mass and probability constraints together with entropy maximization define a sub-manifold of $SU(3)$ regarded as a sub-manifold in 9-D complex space. The choice $(n(u), n(c)) = (4, n)$,

$n < 9$, does not allow unitary U whereas $(n(u), n(c)) = (5, 6)$ does. This choice is still consistent with top quark mass and together with $n(d) = n(s) = 5$ it leads to a rather reasonable CKM matrix with a value of CP breaking invariant within experimental limits. The elements V_{i3} and V_{3i}, $i = 1, 2$ are however roughly twice larger than their experimental values deduced assuming standard model. V_{31} is too large by a factor 1.6. The possibility of scaled up variants of light quarks could lead to too small experimental estimates for these matrix elements. The whole parameter space has not been scanned so that better candidates for CKM matrices might well exist.

2. Higgs contribution to fermion masses is negligible

There are good reasons to believe that Higgs expectation for the fermionic space-time sheets is vanishing although fermions couple to Higgs. Thus p-adic thermodynamics would explain fermion masses completely. This together with the fact that the prediction of the model for the top quark mass is consistent with the most recent limits on it, fixes the CP_2 mass scale with a high accuracy to the maximal one obtained if second order contribution to electron's p-adic mass squared vanishes. This is very strong constraint on the model.

3. The p-adic length scale of quark is dynamical

The assumption about the presence of scaled up variants of light quarks in light hadrons leads to a surprisingly successful model for pseudo scalar meson masses using only quark masses and the assumption mass squared is additive for quarks with same p-adic length scale and mass for quarks labelled by different primes p. This conforms with the idea that pseudo scalar mesons are Goldstone bosons in the sense that color Coulombic and magnetic contributions to the mass cancel each other. Also the mass differences between hadrons containing different numbers of strange and heavy quarks can be understood if s, b and c quarks appear as several scaled up versions.

This hypothesis yields surprisingly good fit for meson masses but for some mesons the predicted mass is slightly too high. The reduction of CP_2 mass scale to cure the situation is not possible since top quark mass would become too low. In case of diagonal mesons for which quarks correspond to same p-adic prime, quark contribution to mass squared can be reduced by ordinary color interactions and in the case of non-diagonal mesons one can require that quark contribution is not larger than meson mass.

4. Super-canonical bosons at hadronic space-time sheet can explain the constant contribution to baryonic masses

Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD.

A possible identification of this contribution is in terms of super-canonical gluons predicted by TGD. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-canonical gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent. Super-canonical gluons also provide a possible solution to the spin puzzle of proton.

Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-canonical quanta. If the topological mixing for super-canonical bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-canonical quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight.

In the case of mesons pion could contain super-canonical boson of first generation preventing the large negative contribution of the color magnetic spin-spin interaction to make pion a tachyon. For heavier bosons super-canonical boson need not to be assumed. The preferred role of pion would relate to the fact that its mass scale is below QCD Λ.

5. Description of color magnetic spin-spin splitting in terms of conformal weight

What remains to be understood are the contributions of color Coulombic and magnetic interactions to the mass squared. There are contributions coming from both ordinary gluons and super-canonical gluons and the latter is expected to dominate by the large value of color coupling strength.

Conformal weight replaces energy as the basic variable but group theoretical structure of color magnetic contribution to the conformal weight associated with hadronic space-time sheet ($k = 107$) is same as in case of energy. The predictions for the masses of mesons are not so good than for baryons, and one might criticize the application of the format of perturbative QCD in an essentially non-perturbative situation.

The comparison of the super-canonical conformal weights associated with spin 0 and spin 1 states and spin 1/2 and spin 3/2 states shows that the different masses of these states could be understood in terms of the super-canonical particle contents of the state correlating with the total quark spin. The resulting model allows excellent predictions also for the meson masses and implies that only pion and kaon can be regarded as Goldstone boson like states. The model based on spin-spin splittings is consistent with the model.

To sum up, the model provides an excellent understanding of baryon and meson masses. This success is highly non-trivial since the fit involves only the integers characterizing the p-adic length scales of quarks and the integers characterizing color magnetic spin-spin splitting plus p-adic thermodynamics and topological mixing for super-canonical gluons. The next challenge would be to predict the correlation of hadron spin with super-canonical particle content in the case of long-lived hadrons.

p-Adic Particle Massivation: New Physics

TGD certainly predicts a lot of new physics, actually infinite hierarchies of fractal copies of standard model physics, but the precise characterization of predictions has varied as the interpretation of the theory has evolved during years. No attempt to discuss systematically the spectrum of various exotic bosons and fermions, basically due to the ground states created by color super-canonical and Kac-Moody generators, will be made. Rather, the attempt is to summarize the new physics expected on basis of recent interpretation of quantum TGD.

1. Basic new physics predictions

Concerning new physics the basic predictions are following. TGD predicts a rich spectrum of massless states for which ground states of negative super-canonical conformal weight are created by colored super-generators. By color confinement these states do not however give rise to macroscopic long range forces. A hierarchy color and weak physics is predicted. Also dark matter hierarchy corresponding to a hierarchy of Planck constants brings in a hierarchy of variants of standard model physics labelled by the values of Planck constant. Thus in TGD the question is not about about predicting some exotic particle but entire fractal hierarchies of copies of standard model physics.

The family replication for fermions correspond in case of gauge bosons prediction of bosons labelled by genera of the two lightlike wormhole throats associated with the wormhole contact representing boson. There are very general arguments predicting that the number of fermionic genera is three and this means that gauge bosons can be arranged into genus-SU(3) singlet and octet. Octet corresponds to exotic gauge bosons and its members should develop Higgs expectation value. Completely symmetric coupling between Higgs octet and boson octet allows also the bosons with vanishing genus-SU(3) quantum numbers to develop mass.

Higgs field is predicted and its vacuum expectation value explains boson masses. By a general argument p-adic temperature for bosons is low and this means that Higgs contribution to the gauge boson mass dominates. Only p-adic thermodynamics is needed to explain fermion masses and the masses of super-canonical bosons and their super counterparts. There is an argument suggesting that vacuum expectation value of Higgs at fermion space-time sheets is not possible. Almost universality of the topological mixing inducing also CKM mixing allows to predict mass spectrum of these states.

2. A general vision about coupling constant evolution

The vision about coupling constant evolution has developed slowly and especially important developments have occurred during last few years. Therefore an overall view about recent understanding is in order.

Also QCD coupling constant evolution is discussed and it is found that asymptotic freedom could be lost making possible existence of several scaled up versions of QCD existing only in a finite length scale range. The basic counter arguments against lepto-hadron hypothesis are considered and it is found that the loss of asymptotic freedom could allow lepto-hadron physics. One can also consider the possibility that the copies of say electro-weak characterized by Mersenne primes do not couple directly to each other so that the objections are circumvented.

The discovery of dark matter hierarchy about fifteen years after these argument were developed resolves the problems in much more elegant manner. TGD predicts an infinite hierarchy of electro-weak and color physics physics for which particles couple directly only via gravitons. De-coherence phase transitions can however induce processes allowing the decay of particles of a given physics to particles of another physics.

3. Summary of new physics effects

Various new physics effects are discussed.

1. There is a brief discussion of family replication phenomenon in the case of gauge bosons based on the identification of gauge bosons as wormhole contacts. Also an argument forcing the identification of partonic vertices as branchings of partonic 2-surfaces is developed.

2. ALEPH anomaly is interpreted in terms of a fractal copy of b-quark corresponding to k=197.

3. The possible signatures of M_{89} hadron physics in e^+e^- annihilation experiments are discussed using a naive scaling of ordinary hadron physics.

4. It is found that the newly born concept of Pomeron of Regge theory could be identified as the sea of perturbative QCD.

5. In p-adic context exotic representations of Super Virasoro with $M^2 \propto p^k$, $k = 1, 2, ..$m are possible. For $k = 1$ the states of these representations have same mass scale as elementary particles although in real context the masses would be gigantic. This inspires the question whether non-perturbative aspects of hadron physics could be assigned to the presence of these representations. The prospects for this are promising. Pion mass is almost exactly equal to the mass of lowest state of the exotic representation for $k = 107$ and Regge slope for rotational excitations of hadrons is predicted with three per cent accuracy assuming that they correspond to the states of $k = 101$ exotic Super Virasoro representations. This leads to the idea that hadronization and fragmentation correspond to phase transitions between ordinary and exotic Super Virasoro representations and that there is entire fractal hierarchy of hadrons inside hadrons and QCD:s inside QCD:s corresponding to p-adic length scales $L(k)$, $k = 107, 103, 101, 97, ...$

4. Cosmic primes and Mersenne primes

p-Adic length scale hypothesis suggests the existence of a scaled up copy of hadron physics associated with each Mersenne prime $M_n = 2^n - 1$,n prime: M_{107} corresponds to ordinary hadron physics.

There is some evidence for exotic hadrons. Centauro events and the peculiar events associated with $E > 10^5$ GeV radiation from Cygnus X-3 could be understood as due to the decay of gamma rays to M_{89} hadron pair in the atmosphere. The decay $\pi_n \rightarrow \gamma\gamma$ produces a peak in the spectrum of the cosmic gamma rays at energy $\frac{m(\pi_n)}{2}$ and there is evidence for the peaks at energies $E_{89} \simeq 34$ GeV and $E_{31} \simeq 3.5 \cdot 10^{10}$ GeV. The absence of the peak at $E_{61} \simeq 1.5 \cdot 10^6$ GeV can be understood as due to the strong absorption caused by the e^+e^- pair creation with photons of the cosmic microwave background. Cosmic string decays *cosmic string* $\rightarrow M_2$ *hadrons* $\rightarrow M_3$ *hadrons* ..$\rightarrow M_{107}$ *hadrons* is a new source of cosmic rays. The mechanism could explain the change of the slope in the hadronic cosmic ray spectrum at M_{61} pion rest energy $3 \cdot 10^6$ GeV. The cosmic ray radiation at energies near 10^9 GeV apparently consisting of protons and nuclei not lighter than Fe might be actually dominated by gamma rays: at these energies γ and p induced showers have same muon content and the decays of gamma rays to M_{89} and M_{61} hadrons in the atmosphere can mimic the presence of heavy nuclei in the cosmic radiation.

5. Anomalously large direct CP breaking in $K - \overline{K}$ system and exotic gluons

The recently observed anomalously large direct CP breaking in $K_L \rightarrow \pi\pi$ decays is explained in terms of loop corrections due to the predicted 2 exotic gluons having masses around 33.6 GeV. It will be also found that the TGD version of the chiral field theory believed to provide a phenomenological low energy description of QCD differs from its standard model version in that quark masses are replaced in TGD framework with shifts of quark masses induced by the vacuum expectation values of the scalar meson fields. This conforms with the TGD view about Higgs mechanism as causing only small mass shifts. It must be however emphasized that there is an argument suggesting that the vacuum expectation value of Higgs in fermionic case does not even make sense.

0.5.2 Part II: Applications of p-adic length scale hypothesis and dark matter hierarchy

Coupling constant evolution in Quantum TGD

This chapter summarizes the recent views about p-adic coupling constant evolution.

1. The most recent view about coupling constant evolution

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-canonical and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination makes possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules.

2. Equivalence Principle and evolution of gravitational constant

Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

1. The coset construction for super-symplectic and super Kac-Moody algebras implies Equivalence Principle in the sense that four-momenta assignable to the Super Virasoro generators of the two algebras are identical. The challenge is to understand this result in more concrete terms.

2. The progress made in the understanding of number theoretical compactification led to a dramatic progress in the construction of configuration space geometry and spinor structure in terms of the modified Dirac operator associated with light-like 3-surfaces appearing in the slicing of the preferred extremal $X^(X_l^3)$ of Kähler action to light-like 3-surfaces Y_l^3 "parallel" to X_l^3. Even more the M^4 projection is predicted to have a slicing into 2-dimensional stringy worldsheets having $M^2(x) \subset M^4$ as a tangent space at point x.

3. By dimensional reduction one can assign to any stringy slice Y^2 a stringy action obtained by integrating Kähler action over the transversal degrees of freedom labeling the copies of Y^2. One can assign length scale evolution to the string tension $T(x)$, which in principle can depend on the point of the string world sheet and thus evolves. $T(x)$ is not identifiable as inverse of gravitational constant but by general arguments proportional to $1/L_p^2$, where L_p is p-adic length scale.

4. Gravitational constant can be understood as a product of L_p^2 with the exponential of the Kähler action for the two pieces of CP_2 type vacuum extremals representing wormhole contacts assignable to graviton connected by the string world sheets. The volume of the typical CP_2 type extremal associated with the graviton increases with L_p so that the exponential factor decreases reducing the growth due to the increase of L_p. Hence G could be RG invariant in p-adic coupling constant evolution. It does not make sense to formulate evolution of gravitational constant at space-time level and gravitational constant characterizes given CD.

5. Gravitational mass is assigned to the stringy world sheet and should be identical with the inertial mass identified as Noether charge assignable to the preferred extremal. By construction there are good hopes that for a proper choice of G gravitational and inertial masses are identical.

3. The RG invariance of gauge couplings inside causal diamond

Quantum classical correspondence suggests that the notion of p-adic coupling constant evolution should have space-time correlate. Zero energy ontology suggests that this counterpart is realized in terms of CDs in the sense that coupling constant evolution has formulation at space-time level inside CD of given size scale and that RG invariance holds true for this evolution. Number theoretic compactification forces to conclude that space-time surfaces has slicing into light-like 3-surfaces Y_l^3: this prediction is consistent with that is known about the extremals. General Coordinate Invariance requires that basic theory can be formulated by replacing the light-like 3-surface X_l^3 associated with wormhole throats with any surface Y_l^3 appearing in the associated slicing.

The natural identification for the renormalization group parameter is as the light-like coordinate labeling different light-like slices. The light-likeness of the RG parameter suggests RG invariance. Quantum classical correspondence requires that the classical gauge fluxes to X_l^3 selected by stationary phase approximation correspond to the expectation values of gQ_g, where g is coupling constant and Q_g the expectation (eigen) value of corresponding charge matrix in the state in question. If the gauge currents are light-like and in direction of Y_l^3 as they are for known extremals under proper selection of X_l^3, RG invariance follows because Abelian gauge fluxes are conserved due to the absence of the component of vacuum current in the direction of slicing.

In principle TGD predicts the values of all coupling constants including also the value of Kähler coupling strength which follows from the identification of Kähler action of the preferred extremal $X^4(X_l^3)$ of Kähler action as Dirac determinant associated with modified Dirac action. Hence Kähler coupling strength could have several values. Quantum criticality in the strongest form however motivates the hypothesis that g_K^2 is invariant under p-adic coupling constant evolution and evolution under evolution associated with the hierarchy of Planck constants.

4. Quantitative predictions for the values of coupling constants

The latest progress in the understanding of p-adic coupling constant evolution comes from a formula for Kähler coupling strength α_K in terms of Dirac determinant of the modified Dirac operator associated with $C-S$ action. The progress came from the realization about how that data about preferred extremal of Kähler action is feeded into the eigenvalue spectrum, which - due to the almost topological character of $C-S$ action - is otherwise far from fixed.

The formula for α_K fixes its number theoretic anatomy and also that of other coupling strengths. The assumption that simple rationals (p-adicization) are involved can be combined with the input from p-adic mass calculations and with an old conjecture for the formula of gravitational constant allowing to express it in terms of CP_2 length scale and Kähler action of topologically condensed CP_2 type vacuum extremal. The prediction is that α_K is renormalization group invariant and equals to the value of fine structure constant at electron length scale characterized by M_{127}. Newton's constant is proportional to p-adic length scale squared and ordinary gravitons correspond to M_{127}. The number theoretic anatomy of R^2/G allows to consider two options. For the first one only M_{127} gravitons are possible number theoretically. For the second option gravitons corresponding to $p \simeq 2^k$ are possible.

A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

5. p-Adic length scale evolution of gauge couplings

Understanding the dependence of gauge couplings constants on p-adic prime is one of the basic challenges of quantum TGD. The problem has been poorly understood even at the conceptual level to say nothing about concrete calculations. The generalization of the motion of S-matrix to that of M-matrix changed however the situation. M-matrix is always defined with respect to measurement resolution characterized in terms of an inclusion of von Neumann algebra. Coupling constant evolution reduces to a discrete evolution involving only octaves of $T(k) = 2^k T_0$ of the fundamental time scale $T_0 = R$, where R CP_2 scale. p-Adic length scale $L(k)$ is related to $T(k)$ by $L^2(k) = T(k)T_0$. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, is automatic prediction of the theory. There is also a close connection with the description of coupling constant evolution in terms of radiative corrections.

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level and why certain p-adic primes are favored.

1. Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges invariant: this seems to be the case for known extremals which form scaling invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

2. Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

Recent status of leptohadron hypothesis

TGD suggests strongly the existence of leptohadron physics. Leptohadrons are bound states of color excited leptons and the anomalous production of e^+e^- pairs in heavy ion collisions finds a nice explanation as resulting from the decays of leptohadrons with basic condensate level $k = 127$ and having typical mass scale of one MeV. The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of ortopositronium give further support for the leptohadron hypothesis. There is also evidence for anomalous production of low energy photons and e^+e^- pairs in hadronic collisions.

The identification of leptohadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the leptohadron hypothesis (Z^0 decay width and production of colored lepton jets in e^+e^- annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the leptopion to the axial vector current as a free parameter. The prediction for e^+e^- production cross section is of correct order of magnitude only provided one assumes that leptopions decay to leptonucleon pair $e^+_{ex}e^-_{ex}$ first and that leptonucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing $n > 2$ particle and the experimental demonstration that leptonucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored μ has emerged. Towards the end of 2008 CDF anomaly gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

TGD and Nuclear Physics

This chapter is devoted to the possible implications of TGD for nuclear physics. In the original version of the chapter the focus was in the attempt to resolve the problems caused by the incorrect interpretation of the predicted long ranged weak gauge fields. What seems to be a breakthrough in this respect came only quite recently (2005), more than a decade after the first version of this chapter, and is based on TGD based view about dark matter inspired by the developments in the mathematical understanding of quantum TGD. In this approach condensed matter nuclei can be either ordinary, that is behave essentially like standard model nuclei, or be in dark matter phase in which case they generate long ranged dark weak gauge fields responsible for the large parity breaking effects in living matter. This approach resolves trivially the objections against long range classical weak fields.

The basic criterion for the transition to dark matter phase having by definition large value of $\hbar$ is that the condition $\alpha Q_1 Q_2 \simeq 1$ for appropriate gauge interactions expressing the fact that the perturbation series does not converge. The increase of $\hbar$ makes perturbation series converging since the value of α is reduced but leaves lowest order classical predictions invariant.

This criterion can be applied to color force and inspires the hypothesis that valence quarks inside nucleons correspond to large $\hbar$ phase whereas sea quark space-time sheets correspond to the ordinary value of $\hbar$. This hypothesis is combined with the earlier model of strong nuclear force based on the assumption that long color bonds with p-adically scaled down quarks with mass of order MeV at their ends are responsible for the nuclear strong force.

1. *Is strong force due to color bonds between exotic quark pairs?*

The basic assumptions are following.

1. Valence quarks correspond to large $\hbar$ phase with p-adic length scale $L(k_{eff} = 129) = L(107)/v_0 \simeq 2^{11}L(107) \simeq 5 \times 10^{-12}$ m whereas sea quarks correspond to ordinary $\hbar$ and define the standard size of nucleons.

2. Color bonds with length of order $L(127) \simeq\simeq 2.5 \times 10^{-12}$ m and having quarks with ordinary $\hbar$ and p-adically scaled down masses $m_q(dark) \simeq v_0 m_q$ at their ends define kind of rubber bands connecting nucleons. The p-adic length scale of exotic quarks differs by a factor 2 from that of dark valence quarks so that the length scales in question can couple naturally. This large length scale as also other p-adic length scales correspond to the size of the topologically quantized field body associated with system, be it quark, nucleon, or nucleus.

   Valence quarks and even exotic quarks can be dark with respect to both color and weak interactions but not with respect to electromagnetic interactions. The model for binding energies suggests darkness with respect to weak interactions with weak boson masses scaled down by a factor v_0. Weak interactions remain still weak. Quarks and nucleons as defined by their $k = 107$ sea quark portions condense at scaled up weak space-time sheet with $k_{eff} = 111$ having p-adic size 10^{-14} meters. The estimate for the atomic number of the heaviest possible nucleus comes out correctly.

   The wave functions of the nucleons fix the boundary values of the wave functionals of the color magnetic flux tubes idealizable as strings. In the terminology of M-theory nucleons correspond to small branes and color magnetic flux tubes to strings connecting them.

2. General features of strong interactions

This picture allows to understand the general features of strong interactions.

1. Quantum classical correspondence and the assumption that the relevant space-time surfaces have 2-dimensional CP_2 projection implies Abelianization. Strong isospin group can be identified as the SU(2) subgroup of color group acting as isotropies of space-time surfaces. and the U(1) holonomy of color gauge potential defines a preferred direction of strong isospin. Dark color isospin corresponds to strong isospin. The correlation of dark color with weak isospin of the nucleon is strongly suggested by quantum classical correspondence.

2. Both color singlet spin 0 pion type bonds and colored spin 1 bonds are allowed and the color magnetic spin-spin interaction between the exotic quark and anti-quark is negative in this case. p-p and n-n bonds correspond to oppositely colored spin 1 bonds and p-n bonds to colorless spin 0 bonds for which the binding energy is free times higher. The presence of colored bonds forces the presence of neutralizing dark gluon condensate favoring states with $N - P > 0$.

3. Shell model based on harmonic oscillator potential follows naturally from this picture in which the magnetic flux tubes connecting nucleons take the role of springs. Spin-orbit interaction can be understood in terms of the color force in the same way as it is understood in atomic physics.

3. Nuclear binding energies

1. The binding energies per nucleon for $A \leq 4$ nuclei can be understood if they form closed string like structures, nuclear strings, so that only two color bonds per nucleon are possible. This could be understood if ordinary quarks and exotic quarks possessing much smaller mass behave as if they were identical fermions. p-Adic mass calculations support this assumption. Also the average behavior of binding energy for heavier nuclei is predicted correctly.

2. For nuclei with $P = N$ all color bonds can be pion type bonds and have thus largest color magnetic spin-spin interaction energy. The increase of color Coulombic binding energy between colored exotic quark pairs and dark gluons however favors $N > P$ and explains also the formation of neutron halo outside $k = 111$ space-time sheet.

3. Spin-orbit interaction provides the standard explanation for magic numbers. If the maximum of the binding energy per nucleon is taken as a criterion for magic, also Z=N=4,6,12 are magic. The alternative TGD based explanation for magic numbers $Z = N = 4, 6, 8, 12, 20$ would be in terms of regular Platonic solids. Experimentally also other magic numbers are known for neutrons. The linking of nuclear strings provides a possible mechanism producing new magic nuclei from lighter magic nuclei.

4. Stringy description of nuclear reactions

The view about nucleus as a collection of linked nuclear strings suggests stringy description of nuclear reactions. Microscopically the nuclear reactions would correspond to re-distribution of exotic quarks between the nucleons in reacting nuclei.

5. Anomalies and new nuclear physics

The TGD based explanation of neutron halo has been already mentioned. The recently observed tetra-neutron states are difficult to understand in the standard nuclear physics framework since Fermi statistics does not allow this kind of state. The identification of tetra-neutron as an alpha particle containing two negatively charged color bonds allows to circumvent the problem. A large variety of exotic nuclei containing charged color bonds is predicted.

The proposed model explains the anomaly associated with the tritium beta decay. What has been observed is that the spectrum intensity of electrons has a narrow bump near the endpoint energy. Also the maximum energy E_0 of electrons is shifted downwards. I have considered two explanations for the anomaly. The original models are based on TGD variants of original models involving belt of dark neutrinos or antineutrinos along the orbit of Earth. Only recently (towards the end of year 2008) I realized that nuclear string model provides much more elegant explanation of the anomaly and has also the potential to explain much more general anomalies.

Cold fusion has not been taken seriously by the physics community but the situation has begun to change gradually. There is an increasing evidence for the occurrence of nuclear transmutations of heavier elements besides the production of ^4He and ^3H whereas the production rate of ^3He and neutrons is very low. These characteristics are not consistent with the standard nuclear physics predictions. Also Coulomb wall and the absence of gamma rays and the lack of a mechanism transferring nuclear energy to the electrolyte have been used as an argument against cold fusion. TGD based model relying on the notion of charged color bonds explains the anomalous characteristics of cold fusion.

Nuclear String Hypothesis

Nuclear string hypothesis is one of the most dramatic almost-predictions of TGD. The hypothesis in its original form assumes that nucleons inside nucleus form closed nuclear strings with neighboring nuclei of the string connected by exotic meson bonds consisting of color magnetic flux tube with quark and anti-quark at its ends. The lengths of flux tubes correspond to the p-adic length scale of electron and therefore the mass scale of the exotic mesons is around 1 MeV in accordance with the general scale of nuclear binding energies. The long lengths of em flux tubes increase the distance between nucleons and reduce Coulomb repulsion. A fractally scaled up variant of ordinary QCD with respect to p-adic length scale would be in question and the usual wisdom about ordinary pions and other mesons as the origin of nuclear force would be simply wrong in TGD framework as the large mass scale of ordinary pion indeed suggests.

1. $A > 4$ nuclei as nuclear strings consisting of $A \leq 4$ nuclei

In this article a more refined version of nuclear string hypothesis is developed.

1. It is assumed ^4He nuclei and $A < 4$ nuclei and possibly also nucleons appear as basic building blocks of nuclear strings. $A \leq 4$ nuclei in turn can be regarded as strings of nucleons. Large number of stable lightest isotopes of form $A = 4n$ supports the hypothesis that the number of ^4He nuclei is maximal. Even the weak decay characteristics might be reduced to those for $A < 4$ nuclei using this hypothesis.

2. One can understand the behavior of nuclear binding energies surprisingly well from the assumptions that total *strong* binding energy associated with $A \leq 4$ building blocks is *additive* for nuclear strings.

3. In TGD framework tetra-neutron is interpreted as a variant of alpha particle obtained by replacing two meson-like stringy bonds connecting neighboring nucleons of the nuclear string with their negatively charged variants. For heavier nuclei tetra-neutron is needed as an additional building brick.

2. Bose-Einstein condensation of color bonds as a mechanism of nuclear binding

The attempt to understand the variation of the nuclear binding energy and its maximum for Fe leads to a quantitative model of nuclei lighter than Fe as color bound Bose-Einstein condensates of pion like colored states associated with color flux tubes connecting ^4He nuclei. The color contribution to the total binding energy is proportional to n^2, where n is the number of color bonds. Fermi statistics explains the reduction of E_B for the nuclei heavier than Fe. Detailed estimate favors harmonic oscillator model over free nucleon model with oscillator strength having interpretation in terms of string tension.

Fractal scaling argument allows to understand ^4He and lighter nuclei as strings of nucleons with nucleons bound together by color bonds. Three fractally scaled variants of QCD corresponding $A > 4$, $A = 4$, and $A < 4$ nuclei are involved. The binding energies of also $A \leq 4$ are predicted surprisingly accurately by applying simple p-adic scaling to the model of binding energies of heavier nuclei.

3. Giant dipole resonance as de-coherence of Bose-Einstein condensate of color bonds

Giant resonances and so called pygmy resonances are interpreted in terms of de-coherence of the Bose-Einstein condensates associated with $A \leq 4$ nuclei and with the nuclear string formed from $A \leq 4$ nuclei. The splitting of the Bose-Einstein condensate to pieces costs a precisely defined energy. For ^4He de-coherence the model predicts singlet line at 12.74 MeV and triplet at ~ 27 MeV spanning 4 MeV wide range.

The de-coherence at the level of nuclear string predicts 1 MeV wide bands 1.4 MeV above the basic lines. Bands decompose to lines with precisely predicted energies. Also these contribute to the width. The predictions are in rather good agreement with experimental values. The so called pygmy resonance appearing in neutron rich nuclei can be understood as a de-coherence for $A = 3$ nuclei. A doublet at ~ 8 MeV and MeV spacing is predicted. The prediction for the position is correct.

4. Dark nuclear strings as analogs of as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code

A speculative picture proposing a connection between homeopathy, water memory, and phantom DNA effect is discussed and on basis of this connection a vision about how the tqc hardware represented by the genome is actively developed by subjecting it to evolutionary pressures represented by a virtual world representation of the physical environment. The speculation inspired by this vision is that genetic code as well as DNA-, RNA- and amino-acid sequences should have representation in terms of nuclear strings. The model for dark baryons indeed leads to an identification of these analogs and the basic numbers of genetic code including also the numbers of aminoacids coded by a given number of codons are predicted correctly. Hence it seems that genetic code is universal rather than being an accidental outcome of the biological evolution.

Dark Nuclear Physics and Condensed Matter

The unavoidable presence of classical long ranged weak (and also color) gauge fields in TGD Universe has been a continual source of worries for more than two decades. The basic question has been whether Z^0 charges of elementary particles are screened in electro-weak length scale or not. For a long time the hypothesis was that the charges are feeded to larger space-time sheets in this length scale rather than screened by vacuum charges so that an effective screening results in electro-weak length scale.

A more promising approach inspired by the TGD based view about dark matter assumes that weak charges are indeed screened for ordinary matter in electro-weak length scale but that dark electro-weak bosons correspond to much longer symmetry breaking length scale.

1. What darkness means?

It is not at all obvious what darkness means and one can consider two variants.

1. The weak form of darkness states that only some field bodies of the particle consisting of flux quanta mediating bound state interactions between particles become dark. One can assign to each interaction a field body (em, Z^0, W, gluonic, gravitational) and p-adic prime and the value of Planck constant characterize the size of the particular field body. One might even think that particle mass can be assigned with its em field body and that Compton length of particle corresponds to the size scale of em field body.

2. The strong form of the hypothesis states that particle space-time sheet is distinguishable from em field body and can become dark. The space-time sheet of the particle would be associated with the covering $H = M^4 \times CP_2 \rightarrow H/G_a \times G_b$, where G_a and G_b are subgroups of $SU(2)$ characterizing Jones inclusions, and would be analogous to a many-sheeted Riemann surface. The large value of $\hbar$ in dark matter phase would mean that Compton lengths and -times are scaled up. A model of dark atom based on this view about darkness leads to the notion of N-atom (each sheet of the multiple covering can carry electron so that Fermi statistics apparently fails).

Nuclear string model suggests that the sizes of color flux tubes and weak flux quanta associated with nuclei can become dark in this sense and have size of order atomic radius so that dark nuclear physics would have a direct relevance for condensed matter physics. If this happens, it becomes impossible to make a reductionistic separation between nuclear physics and condensed matter physics and chemistry anymore.

2. What dark nucleons are?

The basic hypothesis is that nuclei can make a phase transition to dark phase in which the size of both quarks and nuclei is measured in Angstroms. For the less radical option this transition could happen only for the color, weak, and em field bodies. Proton connected by dark color bonds super-nuclei with inter-nucleon distance of order atomic radius might be crucial for understanding the properties of water and perhaps even the properties of ordinary condensed matter. Large $\hbar$ phase for weak field body of D and Pd nuclei with size scale of atom would explain selection rules of cold fusion.

3. Anomalous properties of water and dark nuclear physics

A direct support for partial darkness of water comes from the $H_{1.5}O$ chemical formula supported by neutron and electron diffraction in attosecond time scale. The explanation would be that one fourth of protons combine to form super-nuclei with protons connected by color bonds and having distance sufficiently larger than atomic radius.

The crucial property of water is the presence of molecular clusters. Tedrahedral clusters allow an interpretation in terms of magic Z=8 protonic dark nuclei. The icosahedral clusters consisting of 20 tedrahedral clusters in turn have interpretation as magic dark dark nuclei: the presence of the dark dark matter explains large portion of the anomalies associated with water and explains the unique role of water in biology. In living matter also higher levels of dark matter hierarchy are predicted to be present. The observed nuclear transmutation suggest that also light weak bosons are present.

4. Implications of the partial darkness of condensed matter

The model for partially dark condensed matter inspired by nuclear string model and the model of cold fusion inspired by it allows to understand the low compressibility of the condensed matter as being due to the repulsive weak force between exotic quarks, explains large parity breaking effects in living matter, and suggests a profound modification of the notion of chemical bond having most important implications for bio-chemistry and understanding of bio-chemical evolution.

Super-Conductivity in Many-Sheeted Space-Time

In this chapter a model for high T_c super-conductivity as quantum critical phenomenon is developed.

1. Quantum criticality, hierarchy of dark matters, and dynamical $\hbar$

Quantum criticality is the basic characteristic of TGD Universe and quantum critical superconductors provide an excellent test bed to develop the ideas related to quantum criticality into a more concrete form.

The hypothesis that Planck constants in M^4 and CP_2 degrees of freedom are dynamical possessing quantized spectrum given as integer multiples of minimum value of Planck constant adds further content to the notion of quantum criticality. Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and seems to be especially favored in living matter.

Phases with different values of M^4 and CP_2 Planck constants behave like dark matter with respect to each other in the sense that they do not have direct interactions except at criticality corresponding to a leakage between different sectors of imbedding space glued together along M^4 or CP_2 factors. In large $\hbar(M^4)$ phases various quantum time and length scales are scaled up which means macroscopic and macro-temporal quantum coherence.

The only coupling constant strength of theory is Kähler coupling constant g_K^2 which appears in the definition of the Kähler function K characterizing the geometry of the configuration space of 3-surfaces (the "world of classical worlds"). The exponent of K defines vacuum functional analogous to the exponent of Hamiltonian in thermodynamics. The allowed value(s) of g_K^2, which is (are) analogous to critical temperature(s), is (are) determined by quantum criticality requirement. Contrary to the original hypothesis inspired by the requirement that gravitational coupling is renormalization group invariant, α_K does not seem to depend on p-adic prime whereas gravitational constant is proportional to L_p^2. The situation is saved by the assumption that gravitons correspond to the largest non-super-astrophysical Mersenne prime M_{127} so that gravitational coupling is effectively RG invariant in p-adic coupling constant evolution.

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anticommutation relations of various super-conformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

A further great idea is that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces gauge coupling strength α so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

TGD actually predicts an infinite hierarchy of phases behaving like dark or partially dark matter with respect to the ordinary matter and the value of $\hbar$ is only one characterizer of these phases. These phases, especially so large $\hbar$ phase, seem to be essential for the understanding of even ordinary hadronic, nuclear and condensed matter physics. This strengthens the motivations for finding whether dark matter might be involved with quantum critical super-conductivity.

Cusp catastrophe serves as a metaphor for criticality. In the recent case temperature and doping are control variables and the tip of cusp is at maximum value of T_c. Critical region correspond to the cusp catastrophe. Quantum criticality suggests the generalization of the cusp to a fractal cusp. Inside the critical lines of cusp there are further cusps which corresponds to higher levels in the hierarchy of dark matters labelled by increasing values of $\hbar$ and they correspond to a hierarchy of subtle quantum coherent dark matter phases in increasing length scales. The proposed model for high T_c super-conductivity involves only single value of Planck constant but it might be that the full description involves very many values of them.

2. Many-sheeted space-time concept and ideas about macroscopic quantum phases

Many-sheeted space-time leads to obvious ideas concerning the realization of macroscopic quantum phases.

1. The dropping of particles to larger space-time sheets is a highly attractive mechanism of super-conductivity. If space-time sheets are thermally isolated, the larger space-time sheets could be at extremely low temperature and super-conducting.

2. The possibility of large $\hbar$ phases allows to give up the assumption that space-time sheets characterized by different p-adic length scales are thermally isolated. The scaled up versions of a given space-time sheet corresponding to a hierarchy of values of $\hbar$ are possible such that the scale of kinetic energy and magnetic interaction energy remain same for all these space-time sheets. For instance, for scaled up variants of space-time sheet having size scale characterized by $L(151) = 10$ nm (cell membrane thickness) the critical temperature for superconductivity could be higher than room temperature.

3. The existence of wormhole contacts have been one of the most exotic predictions of TGD. The realization that wormhole contacts can be regarded as parton-antiparton pairs with parton and antiparton assignable to the light-like causal horizons accompanying wormhole contacts, and that Higgs particle corresponds to wormhole contact, opens the doors for more concrete models of also super-conductivity involving massivation of photons.

   The formation of a coherent state of wormhole contacts would be the counterpart for the vacuum expectation value of Higgs. The notions of coherent states of Cooper pairs and of charged Higgs challenge the conservation of electromagnetic charge. The following argument however suggests that coherent states of wormhole contacts form only a part of the description of ordinary super-conductivity. The basic observation is that wormhole contacts with vanishing fermion number define space-time correlates for Higgs type particle with fermion and antifermion numbers at light-like throats of the contact.

   The ideas that a genuine Higgs type photon massivation is involved with super-conductivity and that coherent states of Cooper pairs really make sense are somewhat questionable since the conservation of charge and fermion number is lost. A further questionable feature is that a quantum superposition of many-particle states with widely different masses would be in question. The interpretational problems could be resolved elegantly in zero energy ontology in which the total conserved quantum numbers of quantum state are vanishing. In this picture the energy, fermion number, and total charge of any positive energy state are compensated by opposite quantum numbers of the negative energy state in geometric future. This makes possible to speak about superpositions of Cooper pairs and charged Higgs bosons separately in positive energy sector.

   Rather remarkably, if this picture is taken seriously, super-conductivity can be seen as providing a direct support for both the hierarchy of scaled variants of standard model physics and for the zero energy ontology.

4. Quantum classical correspondence has turned out be a very powerful idea generator. For instance, one can ask what are the space-time correlates for various notions of condensed matter such as phonons, BCS Cooper pairs, holes, etc... For instance, TGD predicts the existence of negative energy space-time sheets so that ordinary particles can and must exist in negative energy states (in cosmological scales the density of inertial energy is predicted to vanish. The question is whether holes could have quite concrete representation as negative energy space-time sheets carrying negative energy particles and whether the notion of Cooper pair of holes could have this kind of space-time correlate.

3. Model for high T_c superconductivity

The model for high T_c super-conductivity relies on the notions of quantum criticality, dynamical Planck constant, and many-sheeted space-time.

These ideas lead to a concrete model for high T_c superconductors as quantum critical superconductors allowing to understand the characteristic spectral lines as characteristics of interior and boundary Cooper pairs bound together by phonon and color interaction respectively. The model for quantum critical electronic Cooper pairs generalizes to Cooper pairs of fermionic ions and for sufficiently large $\hbar$ stability criteria, in particular thermal stability conditions, can be satisfied in a given length scale. Also high T_c superfluidity based on dropping of bosonic atoms to Cooper pair space-time sheets where they form Bose-Einstein condensate is possible.

At qualitative level the model explains various strange features of high T_c superconductors. One can understand the high value of T_c and ambivalent character of high T_c super conductors suggesting both BCS type Cooper pairs and exotic Cooper pairs with non-vanishing spin, the existence of pseudogap and scalings laws for observables above T_c, the role of stripes and doping and the existence of a critical doping, etc... An unexpected prediction is that coherence length is actually $\hbar/\hbar_0 = 2^{11}$ times longer than the coherence length predicted by conventional theory so that type I super-conductor would be in question with stripes serving as duals for the defects of type I super-conductor in nearly critical magnetic field replaced now by ferromagnetic phase.

At quantitative level the model predicts correctly the four poorly understood photon absorption lines and the critical doping ratio from basic principles. The current carrying structures have structure locally similar to that of axon including the double layered structure of cell membrane and also the size scales are predicted to be same so that the idea that axons are high T_c superconductors is highly suggestive.

Quantum Hall effect and Hierarchy of Planck Constants

I have already earlier proposed the explanation of FQHE, anyons, and fractionization of quantum numbers in terms of hierarchy of Planck constants realized as a generalization of the imbedding space $H = M^4 \times CP_2$ to a book like structure. The book like structure applies separately to CP_2 and to causal diamonds ($CD \subset M^4$) defined as intersections of future and past directed light-cones. The pages of the Big Book correspond to singular coverings and factor spaces of CD (CP_2) glued along 2-D subspace of CD (CP_2) and are labeled by the values of Planck constants assignable to CD and CP_2 and appearing in Lie algebra commutation relations. The observed Planck constant $\hbar$, whose square defines the scale of M^4 metric corresponds to the ratio of these Planck constants. The key observation is that fractional filling factor results if $\hbar$ is scaled up by a rational number.

In this chapter I try to formulate more precisely this idea. The outcome is a rather detailed view about anyons on one hand, and about the Kähler structure of the generalized imbedding space on the other hand.

1. The key idea in the formulation of quantum TGD in terms of modified Dirac equation associated with Kähler action is that the Dirac determinant defined by the generalized eigenvalues assignable to the Dirac operator D_K equals to the vacuum functional defined as the exponent of Kähler function in turn identifiable as Kähler action for a preferred extremal for which second variation of Kähler action vanishes at least for the variations responsible for dynamical symmetries. The interpretation is in terms of quantum criticality. This representation generalizes. One can add imaginary instanton term to the Kähler function and corresponding modified Dirac operator: the hypothesis is that the resulting Dirac determinant equals the exponent of Kähler action and imaginary instanton term. The instanton term does not contribute to configuration space metric but provides a first level description for CP breaking and anyonic effects.

2. Fundamental role is played by the assumption that the Kähler gauge potential of CP_2 contains a gauge part with no physical implications in the context of gauge theories but contributing to physics in TGD framework since $U(1)$ gauge transformations are representations of symplectic transformations of CP_2. Also in the case of CD it makes also sense to speak about Kähler gauge potential. The gauge part codes for Planck constants of CD and CP_2 and leads to the identification of anyons as states associated with partonic 2-surfaces surrounding the tip of CD and fractionization of quantum numbers. Explicit formulas relating fractionized charges to the coefficients characterizing the gauge parts of Kähler gauge potentials of CD and CP_2 are proposed based on some empirical input.

3. One important implication is that Poincare and Lorentz invariance are broken inside given CD although they remain exact symmetries at the level of the geometry of world of classical worlds (WCW). The interpretation is as a breaking of symmetries forced by the selection of quantization axis.

4. Anyons would basically correspond to matter at 2-dimensional "partonic" surfaces of macroscopic size surrounding the tip of the light-cone boundary of CD and could be regarded as gigantic elementary particle states with very large quantum numbers and by charge fractionization confined around the tip of CD. Charge fractionization and anyons would be basic characteristic of dark matter (dark only in relative sense). Hence it is not surprising that anyons would have applications going far beyond condensed matter physics. Anyonic dark matter concentrated at 2-dimensional surfaces would play key key role in the the physics of stars and black holes, and also in the formation of planetary system via the condensation of the ordinary matter around dark matter. This assumption was the basic starting point leading to the discovery of the hierarchy of Planck constants. In living matter membrane like structures would represent a key example of anyonic systems as the model of DNA as topological quantum computer indeed assumes.

5. One of the basic questions has been whether TGD forces the hierarchy of Planck constants realized in terms of generalized imbedding space or not. The condition that the choice of quantization axes has a geometric correlate at the imbedding space level motivated by quantum classical correspondence of course forces the hierarchy: this has been clear from the beginning. It is now clear that first principle description of anyons requires the hierarchy in TGD Universe. The hierarchy reveals also new light to the huge vacuum degeneracy of TGD and reduces it dramatically at pages for which CD corresponds to a non-trivial covering or factor space, which suggests that mathematical existence of the theory necessitates the hierarchy of Planck constants.

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Part I

P-ADIC DESCRIPTION OF PARTICLE MASSIVATION

Chapter 1

Elementary Particle Vacuum Functionals

1.1 Introduction

One of the basic ideas of TGD approach is genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies. A more general hypothesis is that the 2-surfaces in question sections of 3-D lightlike causal determinants, say those associated with wormhole contacts carrying parton quantum numbers

1.1.1 First series of questions

The most attractive feature of this idea is universality: if the generalized string model vertices are identified as particle vertices, different particle families are predicted to behave identically with respect to the known interactions in accordance with observational facts.

Before one can accept this identification, one should however answer several questions:

1. Also elementary bosons are predicted to possess family degeneracy: why the higher boson families have not been observed? Why only $g = 0$, "spherical", bosons seem to be the bosons produced in particle accelerators? Are $g > 0$ bosons very massive or are their couplings to fermions very small?

2. Topological reactions changing the genus of boundary component are possible (some of the handles of 2-surfaces suffers pinch or new handle is created): why however different lepton numbers are conserved in such a good approximation?

3. Why the number of the observed elementary particle families seems to be three [27]?

1.1.2 Second series of questions

The questions above are obvious if one accepts string model picture about particle vertices. 25 years with TGD however leads to question the string model based interpretation of particle vertices and stimulates a slightly different series of questions.

1. What really happens in particle vertices? Is the generalization of string model diagrams the proper description of particle reactions in TGD framework? Or should one assume that vertices are direct generalizations of ordinary Feynmann diagrams so that the Feynmann diagrams correspond to singular 4-manifolds and vertices to non-singular 3-manifolds at which the ends of space-time sheets representing particles meet? The elegant treatment of fermion number and other conserved quantum numbers in the vertices and construction of the vertices themselves provides a considerable support for this view. In this framework string model type vertices would be interpreted in terms of a propagation of the particle through several paths simultaneously as in double-slit experiment.

2. The new picture about vertices predicts a profound difference between fermions and bosons: the lowest bosonic vacuum wave functionals must be completely delocalized with respect to the genus to guarantee that the gauge couplings to the fermions are universal. Why this delocalization does not occur for fermions as the successful calculation of elementary particle masses strongly suggests [TGDpad]? Why would bosonic families correspond to a hierarchy of delocalized states having $g < 3$ with a phase phase factor $expi2\pi ng/3$, $n = 0, 1, 2$ characterizing the particle family. Why would fermions correspond to states localized to $g \leq 2$? What makes bells ringing is that for topologically delocalized bosons the finiteness of the vertices would require an effective reduction of the number of particle families to a finite number N. For instance, one can consider a decomposition of the lattice $\{g \geq 0\}$ to disjoint sublattices with a complete bosonic delocalization inside each lattice.

3. Why the number of genera is just three? $g \leq 2$ Riemann surfaces are always hyper-elliptic (have global Z_2 conformal symmetry) unlike $g > 2$ surfaces. Why the complete bosonic de-localization of the light families should be restricted inside the hyper-elliptic sector? Could the reason be that $g > 2$ elementary particle vacuum functionals vanish for hyper-elliptic surfaces so that states localized to $g \leq 2$ surfaces are not transformed to $g > 2$ surfaces? Does the Z_2 symmetry make these states light?

4. There is also a second intriguing observation. Configuration space Clifford algebra is a direct integral over von Neumann algebras known as hyperfinite factors of type II_1 [21, A8]. The hierarchy of Jones inclusions for von Neumann algebras is characterized by a quantum phase $q = exp(i\pi/N)$, $N \geq 3$. $N = 3$ corresponds to the simplest algebraic extension of rationals and is TGD framework physically completely unique as compared to $N > 3$ since the value of the inverse of $\hbar$ vanishes for $N = 3$ apart from small gravitational corrections [A8]. The huge value of Planck constant means maximal quantum coherence time natural for elementary particles.

   Is the number of light particle families three because elementary particles correspond to the lowest level in the hierarchy of Jones inclusions and to the maximally quantal situation perhaps correlating with the hyper-elliptic symmetry? Could the lattice $\{g \geq 0\}$ decompose into a union of disjoint de-localization sub-lattices with $n = 3, 4, 5...$ elements corresponding to $q = exp(i\pi/n)$?

1.1.3 The notion of elementary particle vacuum functional

In order to provide answers to either series of questions one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals, is made. Irrespective of what identification of interaction vertices is adopted, the arguments involved with the construction involve only the string model type vertices so that the previous discussion seems to apply more or less as such.

The basic assumptions underlying the construction are the following ones:

1. Elementary particle vacuum functionals depend on the geometric properties of the two-surface X^2 representing elementary particle.

2. Vacuum functionals possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting preferred extremal of the Kähler action as a generalized Bohr orbit [B1]) and determined in $Diff^3$ invariant manner.

3. Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.

4. Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.

5. Elementary particle vacuum functionals are stable against the two-particle decay $g \rightarrow g_1 + g_2$ and one particle decay $g \rightarrow g-1$.

In the following the construction will be described in more detail.

1. Some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces are introduced and the concept of hyper-ellipticity is introduced. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are discussed.

2. After these preliminaries the construction of elementary particle vacuum functionals is carried out.

3. Possible explanations for the experimental absence of the higher fermion families are considered.

1.2 Identification of elementary particles

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 [C2, C3] suggest dramatic simplifications of the general picture discussed in the earlier version of this chapter. p-Adic mass calculations [F3, F4, F5] leave a lot of freedom concerning the detailed identification of elementary particles.

1.2.1 Elementary fermions and bosons

The basic open question is whether the *theory is on some sense free at parton level* as suggested by the recent view about the construction of S-matrix (actually its generalization M-matrix) and by the almost topological QFT property of quantum TGD at parton level [C3]. If partonic 2-surfaces at elementary particle level carry only free many-fermion states, no bi-local composites of second quantized induced spinor field would be needed in the construction of the quantum states and this would simplify the theory enormously.

If this is the case, the basic conclusion would be that light-like 3-surfaces - in particular the ones at which the signature of induced metric changes from Minkowskian to Euclidian - are carriers of fermionic quantum numbers. These regions are associated naturally with CP_2 type vacuum extremals identifiable as correlates for elementary fermions if only fermion number ± 1 is allowed for the stable states. The question however arises about the identification of elementary bosons.

Wormhole contacts with two light-like wormhole throats carrying fermion and anti-fermion quantum numbers are the first thing that comes in mind. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number. In this case a delicate question is whether the space-time sheets connected by wormhole contacts have opposite time orientations or not. If this the case the two fermions would correspond to positive and negative energy particles.

I considered first the identification of only Higgs as a wormhole contact but there is no reason why this identification should not apply also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions.

This picture allows to understand the difference between fermions and gauge bosons and Higgs particle. For fermions topological explanation of family replication predicts three fermionic generations [F1] corresponding to handle numbers $g = 0, 1, 2$ for the partonic 2-surface. In the case of gauge bosons and Higgs this replication is not visible. This could be due to the fact that gauge bosons form singlet and octet representation of the dynamical $SU(3)$ group associated with the handle number $g = 0, 1, 2$ since bosons correspond to pairs of handles. If octet representation is heavy the experimental absence of family replication for bosons can be understood.

1.2.2 Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The development of the understanding of gravitational coupling has had many twists and it is perhaps to summarize the basic misunderstandings.

1. CP_2 length scale R, which is roughly $10^{3.5}$ times larger than Planck length $l_P = \sqrt{\hbar G}$, defines a fundamental length scale in TGD. The challenge is to predict the value of Planck length $\sqrt{\hbar G}$. The outcome was an identification of a formula for $R^2/\hbar G$ predicting that the magnitude of Kähler coupling strength α_K is near to fine structure constant in electron length scale (for ordinary value of Planck constant should be added here).

2. The emergence of the parton level formulation of TGD finally demonstrated that G actually appears in the fundamental parton level formulation of TGD as a fundamental constant characterizing the M^4 part of CP_2 Kähler gauge potential [B4, F12]. This part is pure gauge in the sense of standard gauge theory but necessary to guarantee that the theory does not reduce to topological QFT. Quantum criticality requires that G remains invariant under p-adic coupling constant evolution and is therefore predictable in principle at least.

3. The TGD view about coupling constant evolution [C4] predicts the proportionality $G \propto L_p^2$, where L_p is p-adic length scale. Together with input from p-adic mass calculations one ends up to two conclusions. The correct conclusion was that Kähler coupling strength is equal to the fine structure constant in the p-adic length scale associated with Mersenne prime $p = M_{127} = 2^{127} - 1$ assignable to electron [C4]. I have considered also the possibility that α_K would be equal to electro-weak $U(1)$ coupling in this scale.

4. The additional - wrong- conclusion was that gravitons must always correspond to the p-adic prime M_{127} since G would otherwise vary as function of p-adic length scale. As a matter fact, the question was for years whether it is G or g_K^2 which remains invariant under p-adic coupling constant evolution. I found both options unsatisfactory until I realized that RG invariance is possible for both g_K^2 and G! The point is that the exponent of the Kähler action associated with the piece of CP_2 type vacuum extremal assignable with the elementary particle is exponentially sensitive to the volume of this piece and logarithmic dependence on the volume fraction is enough to compensate the $L_p^2 \propto p$ proportionality of G and thus guarantee the constancy of G.

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naive estimate $G \sim L_p^2$.

1.2.3 Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving 8+8=16 states altogether. Taking into account phase conjugates gives 16+16=32 states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2+1) \times (3+1) = 12$ states plus phase conjugates giving 12+12=24 states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes 48+16+16=80 states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

1.3 Basic facts about Riemann surfaces

In the following some basic aspects about Riemann surfaces will be summarized. The basic topological concepts, in particular the concept of the mapping class group, are introduced, and the Teichmueller parameters are defined as conformal invariants of the Riemann surface, which in fact specify the conformal equivalence class of the Riemann surface completely.

1.3.1 Mapping class group

The first homology group $H_1(X^2)$ of a Riemann surface of genus g contains $2g$ generators [17, 19, 18]: this is easy to understand geometrically since each handle contributes two homology generators. The so called canonical homology basis can be identified as in Fig. 1.3.1.

One can define the so called intersection number $J(a,b)$ for two elements a and b of the homology group as the number of intersection points for the curves a and b counting the orientation. Since $J(a,b)$ depends on the homology classes of a and b only, it defines an antisymmetric quadratic form in $H_1(X^2)$. In the canonical homology basis the non-vanishing elements of the intersection matrix are:

$$J(a_i,b_j) = -J(b_j,a_i) = \delta_{i,j} \ . \tag{1.3.1}$$

J clearly defines symplectic structure in the homology group.

The dual to the canonical homology basis consists of the harmonic one-forms $\alpha_i, \beta_i, i = 1,..,g$ on X^2. These 1-forms satisfy the defining conditions

$$\begin{array}{ll} \int_{a_i} \alpha_j = \delta_{i,j} & \int_{b_i} \alpha_j = 0 \ , \\ \int_{a_i} \beta_j = 0 & \int_{b_i} \beta_j = \delta_{i,j} \ . \end{array} \tag{1.3.2}$$

The following identity helps to understand the basic properties of the Teichmueller parameters

$$\int_{X^2} \theta \wedge \eta = \sum_{i=1,..,g} [\int_{a_i} \theta \int_{b_i} \eta - \int_{b_i} \theta \int_{a_i} \eta] \ . \tag{1.3.3}$$

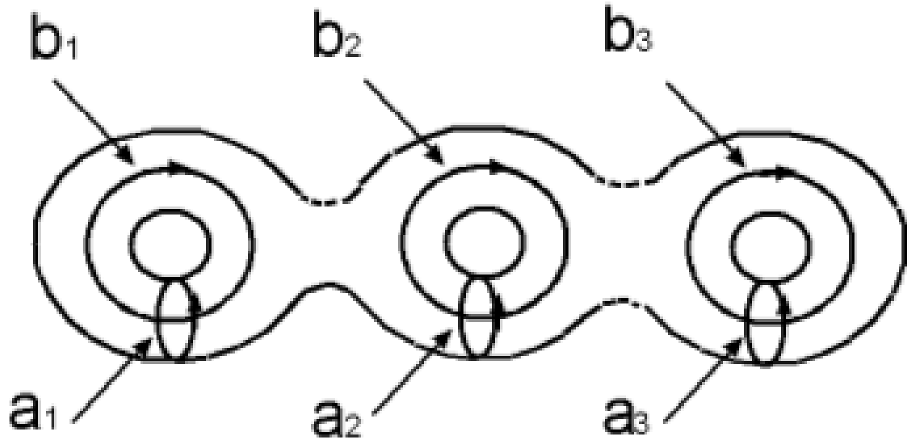

Figure 1.1: Definition of the canonical homology basis

The existence of topologically nontrivial diffeomorphisms, when X^2 has genus $g > 0$, plays an important role in the sequel. Denoting by $Diff$ the group of the diffeomorphisms of X^2 and by $Diff_0$ the normal subgroup of the diffeomorphisms homotopic to identity, one can define the mapping class group M as the coset group

$$M = Diff/Diff_0 \ . \tag{1.3.4}$$

The generators of M are so called Dehn twists along closed curves a of X^2. Dehn twist is defined by excising a small tubular neighborhood of a, twisting one boundary of the resulting tube by 2π and gluing the tube back into the surface: see Fig. 9.6.2.

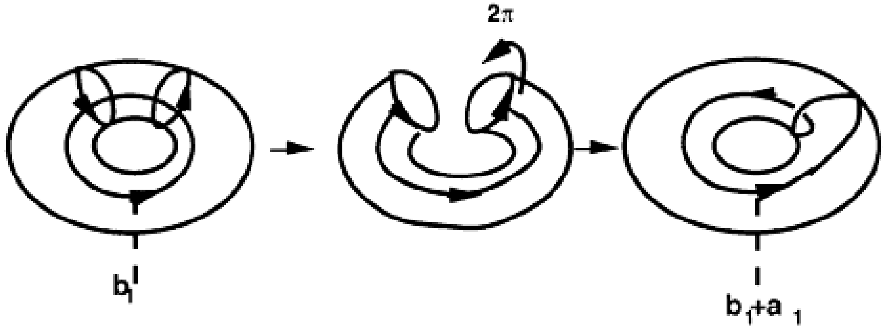

Figure 1.2: Definition of the Dehn twist

It can be shown that a minimal set of generators is defined by the following curves

$$a_1, b_1, a_1^{-1}a_2^{-1}, a_2, b_2, a_2^{-1}a_3^{-11}, ..., a_g, b_g \quad . \tag{1.3.5}$$

The action of these transformations in the homology group can be regarded as a symplectic linear transformation preserving the symplectic form defined by the intersection matrix. Therefore the matrix representing the action of $Diff$ on $H_1(X^2)$ is $2g \times 2g$ matrix M with integer entries leaving J invariant: $MJM^T = J$. Mapping class group is often referred also as a symplectic modular group and denoted by $Sp(2g, Z)$. The matrix representing the action of M in the canonical homology basis decomposes into four $g \times g$ blocks A, B, C and D

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad , \tag{1.3.6}$$

where A and D operate in the subspaces spanned by the homology generators a_i and b_i respectively and C and D map these spaces to each other. The notation $D = [A, B; C, D]$ will be used in the sequel: in this notation the representation of the symplectic form J is $J = [0, 1; -1, 0]$.

1.3.2 Teichmueller parameters

The induced metric on the two-surface X^2 defines a unique complex structure. Locally the metric can always be written in the form

$$ds^2 = e^{2\phi} dz d\bar{z} \quad . \tag{1.3.7}$$

where z is local complex coordinate. When one covers X^2 by coordinate patches, where the line element has the above described form, the transition functions between coordinate patches are holomorphic and therefore define a complex structure.

The conformal transformations ξ of X^2 are defined as the transformations leaving invariant the angles between the vectors of X^2 tangent space invariant: the angle between the vectors X and Y at point x is same as the angle between the images of the vectors under Jacobian map at the image point $\xi(x)$. These transformations need not be globally defined and in each coordinate patch they correspond to holomorphic (anti-holomorphic) mappings as is clear from the diagonal form of the metric in the local complex coordinates. A distinction should be made between local conformal transformations and globally defined conformal transformations, which will be referred to as conformal symmetries: for instance, for hyper-elliptic surfaces the group of the conformal symmetries contains two-element group Z_2.

Using the complex structure one can decompose one-forms to linear combinations of one-forms of type $(1,0)$ $(f(z,\bar{z})dz)$ and $(0,1)$ $(f(z,\bar{z})d\bar{z})$. $(1,0)$ form ω is holomorphic if the function f is holomorphic: $\omega = f(z)dz$ on each coordinate patch.

There are g independent holomorphic one forms ω_i known also as Abelian differentials of the first kind [17, 19, 18] and one can fix their normalization by the condition

$$\int_{a_i} \omega_j = \delta_{ij} \quad . \tag{1.3.8}$$

This condition completely specifies ω_i.

Teichmueller parameters Ω_{ij} are defined as the values of the forms ω_i for the homology generators b_j

$$\Omega_{ij} = \int_{b_j} \omega_i \quad . \tag{1.3.9}$$

The basic properties of Teichmueller parameters are the following:
i) The $g \times g$ matrix Ω is symmetric: this is seen by applying the formula (1.3.3) for $\theta = \omega_i$ and $\eta = \omega_j$.
ii) The imaginary part of Ω is positive: $Im(\Omega) > 0$. This is seen by the application of the same

formula for $\theta = \eta$. The space of the matrices satisfying these conditions is known as Siegel upper half plane.
iii) The space of Teichmueller parameters can be regarded as a coset space $Sp(2g,R)/U(g)$ [19]: the action of $Sp(2g,R)$ is of the same form as the action of $Sp(2g,Z)$ and $U(g) \subset Sp(2g,R)$ is the isotropy group of a given point of Teichmueller space.
iv) Teichmueller parameters are conformal invariants as is clear from the holomorphy of the defining one-forms.
v) Teichmueller parameters specify completely the conformal structure of Riemann surface [18].

Although Teichmueller parameters fix the conformal structure of the 2-surface completely, they are not in one-to-one correspondence with the conformal equivalence classes of the two-surfaces:
i) The dimension for the space of the conformal equivalence classes is $D = 3g-3$, when $g > 1$ and smaller than the dimension of Teichmueller space given by $d = (g \times g + g)/2$ for $g > 3$: all Teichmueller matrices do not correspond to a Riemann surface. In TGD approach this does not produce any problems as will be found later.
ii) The action of the topologically nontrivial diffeomorphisms on Teichmueller parameters is nontrivial and can be deduced from the action of the diffeomorphisms on the homology ($Sp(2g,Z)$ transformation) and from the defining condition $\int_{a_i} \omega_j = \delta_{i,j}$: diffeomorphisms correspond to elements $[A,B;C,D]$ of $Sp(2g,Z)$ and act as generalized Möbius transformations

$$\Omega \rightarrow (A\Omega + B)(C\Omega + D)^{-1} \ . \tag{1.3.10}$$

All Teichmueller parameters related by $Sp(2g,Z)$ transformations correspond to the same Riemann surface.
iii) The definition of the Teichmueller parameters is not unique since the definition of the canonical homology basis involves an arbitrary numbering of the homology basis. The permutation S of the handles is represented by same $g \times g$ orthogonal matrix both in the basis $\{a_i\}$ and $\{b_i\}$ and induces a similarity transformation in the space of the Teichmueller parameters

$$\Omega \rightarrow S\Omega S^{-1} \ . \tag{1.3.11}$$

Clearly, the Teichmueller matrices related by a similarity transformations correspond to the same conformal equivalence class. It is easy to show that handle permutations in fact correspond to $Sp(2g,Z)$ transformations.

1.3.3 Hyper-ellipticity

The motivation for considering hyper-elliptic surfaces comes from the fact, that $g > 2$ elementary particle vacuum functionals turn out to be vanishing for hyper-elliptic surfaces and this in turn will be later used to provide a possible explanation the non-observability of $g > 2$ particles.

Hyper-elliptic surface X can be defined abstractly as two-fold branched cover of the sphere having the group Z_2 as the group of conformal symmetries (see [19, 16, 18]. Thus there exists a map $\pi : X \rightarrow S^2$ so that the inverse image $\pi^{-1}(z)$ for a given point z of S^2 contains two points except at a finite number (say p) of points z_i (branch points) for which the inverse image contains only one point. Z_2 acts as conformal symmetries permuting the two points in $\pi^{-1}(z)$ and branch points are fixed points of the involution.

The concept can be generalized [16]: g-hyper-elliptic surface can be defined as a 2-fold covering of genus g surface with a finite number of branch points. One can consider also p-fold coverings instead of 2-fold coverings: a common feature of these Riemann surfaces is the existence of a discrete group of conformal symmetries.

A concrete representation for the hyper-elliptic surfaces [19] is obtained by studying the surface of C^2 determined by the algebraic equation

$$w^2 - P_n(z) = 0 \ , \tag{1.3.12}$$

where w and z are complex variables and $P_n(z)$ is a complex polynomial. One can solve w from the above equation

$$w_{\pm} \quad = \quad \pm\sqrt{P_n(z)} \ , \tag{1.3.13}$$

where the square root is determined so that it has a cut along the positive real axis. What happens that w has in general two roots (two-fold covering property), which coincide at the roots z_i of $P_n(z)$ and if n is odd, also at $z = \infty$: these points correspond to branch points of the hyper-elliptic surface and their number r is always even: $r = 2k$. w is discontinuous at the cuts associated with the square root in general joining two roots of $P_n(z)$ or if n is odd, also some root of P_n and the point $z = \infty$. The representation of the hyper-elliptic surface is obtained by identifying the two branches of w along the cuts. From the construction it is clear that the surface obtained in this manner has genus $k-1$. Also it is clear that Z_2 permutes the different roots $w_{\pm}$ with each other and that $r = 2k$ branch points correspond to fixed points of the involution.

The following facts about the hyper-elliptic surfaces [19, 18] turn out to be important in the sequel:
i) All $g < 3$ surfaces are hyper-elliptic.
ii) $g \geq 3$ hyper-elliptic surfaces are not in general hyper-elliptic and form a set of codimension 2 in the space of the conformal equivalence classes [19].

1.3.4 Theta functions

An extensive and detailed account of the theta functions and their applications can be found in the book of Mumford [19]. Theta functions appear also in the loop calculations of string model [17]. In the following the so called Riemann theta function and theta functions with half integer characteristics will be defined as sections (not strictly speaking functions) of the so called Jacobian variety.

For a given Teichmueller matrix Ω, Jacobian variety is defined as the $2g$-dimensional torus obtained by identifying the points z of C^g (vectors with g complex components) under the equivalence

$$z \sim z + \Omega m + n \ , \tag{1.3.14}$$

where m and n are points of Z^g (vectors with g integer valued components) and Ω acts in Z^g by matrix multiplication.

The definition of Riemann theta function reads as

$$\Theta(z|\Omega) \quad = \quad \sum_n exp(i\pi n \cdot \Omega \cdot n + i2\pi n \cdot z) \ . \tag{1.3.15}$$

Here $\cdot$ denotes standard inner product in C^g. Theta functions with half integer characteristics are defined in the following manner. Let a and b denote vectors of C^g with half integer components (component either vanishes or equals to 1/2). Theta function with characteristics $[a,b]$ is defined through the following formula

$$\Theta[a,b](z|\Omega) \quad = \quad \sum_n exp\left[i\pi(n+a)\cdot\Omega\cdot(n+a) + i2\pi(n+a)\cdot(z+b)\right] \ . \tag{1.3.16}$$

A brief calculation shows that the following identity is satisfied

$$\Theta[a,b](z|\Omega) \quad = \quad exp(i\pi a\cdot\Omega\cdot a + i2\pi a\cdot b)\times\Theta(z+\Omega a+b|\Omega) \tag{1.3.17}$$

Theta functions are not strictly speaking functions in the Jacobian variety but rather sections in an appropriate bundle as can be seen from the identities

$$\begin{aligned}
\Theta[a,b](z+m|\Omega) &= exp(i2\pi a\cdot m)\Theta[a,b](z\Omega) \ , \\
\Theta[a,b](z+\Omega m|\Omega) &= exp(\alpha)\Theta[a,b](z|\Omega) \ , \\
exp(\alpha) &= exp(-i2\pi b\cdot m)exp(-i\pi m\cdot\Omega\cdot m - 2\pi m\cdot z) \ .
\end{aligned} \tag{1.3.18}$$

The number of theta functions is 2^{2g} and same as the number of nonequivalent spinor structures defined on two-surfaces. This is not an accident [17]: theta functions with given characteristics turn out to be in a close relation to the functional determinants associated with the Dirac operators defined on the two-surface. It is useful to divide the theta functions to even and odd theta functions according to whether the inner product $4a\cdot b$ is even or odd integer. The numbers of even and odd theta functions are $2^{g-1}(2^g+1)$ and $2^{g-1}(2^g-1)$ respectively.

The values of the theta functions at the origin of the Jacobian variety understood as functions of Teichmueller parameters turn out to be of special interest in the following and the following notation will be used:

$$\Theta[a,b](\Omega) \equiv \Theta[a,b](0|\Omega) \ , \tag{1.3.19}$$

$\Theta[a,b](\Omega)$ will be referred to as theta functions in the sequel. From the defining properties of odd theta functions it can be found that they are odd functions of z and therefore vanish at the origin of the Jacobian variety so that only even theta functions will be of interest in the sequel.

An important result is that also some *even* theta functions vanish for $g>2$ hyper-elliptic surfaces : in fact one can characterize $g>2$ hyper-elliptic surfaces by the vanishing properties of the theta functions [19, 18]. The vanishing property derives from conformal symmetry (Z_2 in the case of hyper-elliptic surfaces) and the vanishing phenomenon is rather general [16]: theta functions tend to vanish for Riemann surfaces possessing discrete conformal symmetries. It is not clear (to the author) whether the presence of a conformal symmetry is in fact equivalent with the vanishing of some theta functions. As already noticed, spinor structures and the theta functions with half integer characteristics are in one-to-one correspondence and the vanishing of theta function with given half integer characteristics is equivalent with the vanishing of the Dirac determinant associated with the corresponding spinor structure or equivalently: with the existence of a zero mode for the Dirac operator [17]. For odd characteristics zero mode exists always: for even characteristics zero modes exist, when the surface is hyper-elliptic or possesses more general conformal symmetries.

1.4 Elementary particle vacuum functionals

The basic assumption is that elementary particle families correspond to various elementary particle vacuum functionals associated with the 2-dimensional boundary components of the 3-surface. These functionals need not be localized to a single boundary topology. Neither need their dependence on the boundary component be local. An important role in the following considerations is played by the fact that the minimization requirement of the Kähler action associates a unique 3-surface to each boundary component, the "Bohr orbit" of the boundary and this surface provides a considerable (and necessarily needed) flexibility in the definition of the elementary particle vacuum functionals. There are several natural constraints to be satisfied by elementary particle vacuum functionals.

1.4.1 Extended Diff invariance and Lorentz invariance

Extended Diff invariance is completely analogous to the extension of 3-dimensional Diff invariance to four-dimensional Diff invariance in the interior of the 3-surface. Vacuum functional must be invariant not only under diffeomorphisms of the boundary component but also under the diffeomorphisms of the 3- dimensional "orbit" Y^3 of the boundary component. In other words: the value of the vacuum functional must be same for any time slice on the orbit the boundary component. This is guaranteed if vacuum functional is functional of some two-surface Y^2 belonging to the orbit and defined in $Diff^3$ invariant manner.

An additional natural requirement is Poincare invariance. In the original formulation of the theory only Lorentz transformations of the light cone were exact symmetries of the theory. In this framework the definition of Y^2 as the intersection of the orbit with the hyperboloid $\sqrt{m_{kl}m^km^l} = a$ is $Diff^3$ and Lorentz invariant.

Interaction vertices as generalization of stringy vertices

For stringy diagrams Poincare invariance of conformal equivalence class and general coordinate invariance are far from being a trivial issues. Vertices are now not completely unique since there is an infinite number of singular 3-manifolds which can be identified as vertices even if one assumes space-likeness. One should be able to select a unique singular 3-manifold to fix the conformal equivalence class.

One might hope that Lorentz invariant invariant and general coordinate invariant definition of Y^2 results by introducing light cone proper time a as a height function specifying uniquely the point at which 3-surface is singular (stringy diagrams help to visualize what is involved), and by restricting the singular 3-surface to be the intersection of $a = constant$ hyperboloid of M^4 containing the singular point with the space-time surface. There would be non-uniqueness of the conformal equivalence class due to the choice of the origin of the light cone but the decomposition of the configuration space of 3-surfaces to a union of configuration spaces characterized by unions of future and past light cones could resolve this difficulty.

Interaction vertices as generalization of ordinary ones

If the interaction vertices are identified as intersections for the ends of space-time sheets representing particles, the conformal equivalence class is naturally identified as the one associated with the intersection of the boundary component or light like causal determinant with the vertex. Poincare invariance of the conformal equivalence class and generalized general coordinate invariance follow trivially in this case.

1.4.2 Conformal invariance

Conformal invariance implies that vacuum functionals depend on the conformal equivalence class of the surface Y^2 only. What makes this idea so attractive is that for a given genus g configuration space becomes effectively finite-dimensional. A second nice feature is that instead of trying to find coordinates for the space of the conformal equivalence classes one can construct vacuum functionals as functions of the Teichmueller parameters.

That one can construct this kind of functions as suitable functions of the Teichmueller parameters is not trivial. The essential point is that the boundary components can be regarded as submanifolds of $M^4_+ \times CP_2$: as a consequence vacuum functional can be regarded as a composite function:

2-surface $\rightarrow$ Teichmueller matrix Ω determined by the induced metric $\rightarrow \Omega_{vac}(\Omega)$

Therefore the fact that there are Teichmueller parameters which do not correspond to any Riemann surface, doesn't produce any trouble. It should be noticed that the situation differs from that in the Polyakov formulation of string models, where one doesn't assume that the metric of the two-surface is induced metric (although classical equations of motion imply this).

1.4.3 Diff invariance

Since several values of the Teichmueller parameters correspond to the same conformal equivalence class, one must pose additional conditions on the functions of the Teichmueller parameters in order to obtain single valued functions of the conformal equivalence class.

The first requirement of this kind is the invariance under topologically nontrivial Diff transformations inducing $Sp(2g, Z)$ transformation $(A, B; C, D)$ in the homology basis. The action of these transformations on Teichmueller parameters is deduced by requiring that holomorphic one-forms satisfy the defining conditions in the transformed homology basis. It turns out that the action of the topologically nontrivial diffeomorphism on Teichmueller parameters can be regarded as a generalized Möbius transformation:

$$\Omega \rightarrow (A\Omega + B)(C\Omega + D)^{-1} \ . \tag{1.4.1}$$

Vacuum functional must be invariant under these transformations. It should be noticed that the situation differs from that encountered in the string models. In TGD the integration measure over the configuration space is Diff invariant: in string models the integration measure is the integration measure of the Teichmueller space and this is not invariant under $Sp(2g, Z)$ but transforms like a density: as a consequence the counterpart of the vacuum functional must be also modular covariant since it is the product of vacuum functional and integration measure, which must be modular invariant.

It is possible to show that the quantities

$$(\Theta[a, b]/\Theta[c, d])^4 \ . \tag{1.4.2}$$

and their complex conjugates are $Sp(2g, Z)$ invariants [19] and therefore can be regarded as basic building blocks of the vacuum functionals.

Teichmueller parameters are not uniquely determined since one can always perform a permutation of the g handles of the Riemann surface inducing a redefinition of the canonical homology basis (permutation of g generators). These transformations act as similarities of the Teichmueller matrix:

$$\Omega \rightarrow S\Omega S^{-1} \ , \tag{1.4.3}$$

where S is the $g \times g$ matrix representing the permutation of the homology generators understood as orthonormal vectors in the g- dimensional vector space. Therefore the Teichmueller parameters related by these similarity transformations correspond to the same conformal equivalence class of the Riemann surfaces and vacuum functionals must be invariant under these similarities.

It is easy to find out that these similarities permute the components of the theta characteristics: $[a, b] \rightarrow [S(a), S(b)]$. Therefore the invariance requirement states that the handles of the Riemann surface behave like bosons: the vacuum functional constructed from the theta functions is invariant under the permutations of the theta characteristics. In fact, this requirement brings in nothing new. Handle permutations can be regarded as $Sp(2g, Z)$ transformations so that the modular invariance alone guarantees invariance under handle permutations.

1.4.4 Cluster decomposition property

Consider next the behavior of the vacuum functional in the limit, when boundary component with genus g splits to two separate boundary components of genera g_1 and g_2 respectively. The splitting into two separate boundary components corresponds to the reduction of the Teichmueller matrix Ω^g to a direct sum of $g_1 \times g_1$ and $g_2 \times g_2$ matrices ($g_1 + g_2 = g$):

$$\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2} \ , \tag{1.4.4}$$

when a suitable definition of the Teichmueller parameters is adopted. The splitting can also take place without a reduction to a direct sum: the Teichmueller parameters obtained via $Sp(2g, Z)$ transformation from $\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2}$ do not possess direct sum property in general.

The physical interpretation is obvious: the non-diagonal elements of the Teichmueller matrix describe the geometric interaction between handles and at this limit the interaction between the handles belonging to the separate surfaces vanishes. On the physical grounds it is natural to require that vacuum functionals satisfy cluster decomposition property at this limit: that is they reduce to the product of appropriate vacuum functionals associated with the composite surfaces.

Theta functions satisfy cluster decomposition property [19, 17]. Theta characteristics reduce to the direct sums of the theta characteristics associated with g_1 and g_2 ($a = a_1 \oplus a_2$, $b = b_1 \oplus b_2$) and the dependence on the Teichmueller parameters is essentially exponential so that the cluster decomposition property indeed results:

$$\Theta[a, b](\Omega^g) = \Theta[a_1, b_1](\Omega^{g_1})\Theta[a_2, b_2](\Omega^{g_2}) \ . \tag{1.4.5}$$

Cluster decomposition property holds also true for the products of theta functions. This property is also satisfied by suitable homogenous polynomials of thetas. In particular, the following quantity playing central role in the construction of the vacuum functional obeys this property

$$Q_0 = \sum_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 \ , \qquad (1.4.6)$$

where the summation is over all even theta characteristics (recall that odd theta functions vanish at the origin of C^g).

Together with the $Sp(2g,Z)$ invariance the requirement of cluster decomposition property implies that the vacuum functional must be representable in the form

$$\Omega_{vac} = P_{M,N}(\Theta^4, \bar{\Theta}^4)/Q_{MN}(\Theta^4, \bar{\Theta}^4) \qquad (1.4.7)$$

where the homogenous polynomials $P_{M,N}$ and $Q_{M,N}$ have same degrees (M and N as polynomials of $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$.

1.4.5 Finiteness requirement

Vacuum functional should be finite. Finiteness requirement is satisfied provided the numerator $Q_{M,N}$ of the vacuum functional is real and positive definite. The simplest quantity of this type is the quantity Q_0 defined previously and its various powers. $Sp(2g,Z)$ invariance and finiteness requirement are satisfied provided vacuum functionals are of the following general form

$$\Omega_{vac} = \frac{P_{N,N}(\Theta^4, \bar{\Theta}^4)}{Q_0^N} \ , \qquad (1.4.8)$$

where $P_{N,N}$ is homogenous polynomial of degree N with respect to $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$. In addition $P_{N,N}$ is invariant under the permutations of the theta characteristics and satisfies cluster decomposition property.

1.4.6 Stability against the decay $g \rightarrow g_1 + g_2$

Elementary particle vacuum functionals must be stable against the genus conserving decays $g \rightarrow g_1 + g_2$. This decay corresponds to the limit at which Teichmueller matrix reduces to a direct sum of the matrices associated with g_1 and g_2 (note however the presence of $Sp(2g,Z)$ degeneracy). In accordance with the topological description of the particle reactions one expects that this decay doesn't occur if the vacuum functional in question vanishes at this limit.

In general the theta functions are non-vanishing at this limit and vanish provided the theta characteristics reduce to a direct sum of the odd theta characteristics. For $g < 2$ surfaces this condition is trivial and gives no constraints on the form of the vacuum functional. For $g = 2$ surfaces the theta function $\Theta(a,b)$, with $a = b = (1/2, 1/2)$ satisfies the stability criterion identically (odd theta functions vanish identically), when Teichmueller parameters separate into a direct sum. One can however perform $Sp(2g,Z)$ transformations giving new points of Teichmueller space describing the decay. Since these transformations transform theta characteristics in a nontrivial manner to each other and since all even theta characteristics belong to same $Sp(2g,Z)$ orbit [19, 17], the conclusion is that stability condition is satisfied provided $g = 2$ vacuum functional is proportional to the product of fourth powers of all even theta functions multiplied by its complex conjugate.

If $g > 2$ there always exists some theta functions, which vanish at this limit and the minimal vacuum functional satisfying this stability condition is of the same form as in $g = 2$ case, that is proportional to the product of the fourth powers of all even Theta functions multiplied by its complex conjugate:

$$\Omega_{vac} = \prod_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 / Q_0^N \ , \qquad (1.4.9)$$

where N is the number of even theta functions. The results obtained imply that genus-generation correspondence is one to one for $g > 1$ for the minimal vacuum functionals. Of course, the multiplication of the minimal vacuum functionals with functionals satisfying all criteria except stability criterion gives new elementary particle vacuum functionals: a possible physical identification of these vacuum functionals is most naturally as some kind of excited states.

One of the questions posed in the beginning was related to the experimental absence of $g > 0$, possibly massless, elementary bosons. The proposed stability criterion suggests a nice explanation. The point is that elementary particles are stable against decays $g \to g_1 + g_2$ but not with respect to the decay $g \to g + \ sphere$. As a consequence the direct emission of $g > 0$ gauge bosons is impossible unlike the emission of $g = 0$ bosons: for instance the decay muon $\to$ electron $+(g = 1)$ photon is forbidden.

1.4.7 Stability against the decay $g \to g - 1$

This stability criterion states that the vacuum functional is stable against single particle decay $g \to g-1$ and, if satisfied, implies that vacuum functional vanishes, when the genus of the surface is smaller than g. In stringy framework this criterion is equivalent to a separate conservation of various lepton numbers: for instance, the spontaneous transformation of muon to electron is forbidden. Notice that this condition doesn't imply that that the vacuum functional is localized to a single genus: rather the vacuum functional of genus g vanishes for all surfaces with genus smaller than g. This hierarchical structure should have a close relationship to Cabibbo-Kobayashi-Maskawa mixing of the quarks.

The stability criterion implies that the vacuum functional must vanish at the limit, when one of the handles of the Riemann surface suffers a pinch. To deduce the behavior of the theta functions at this limit, one must find the behavior of Teichmueller parameters, when i:th handle suffers a pinch. Pinch implies that a suitable representative of the homology generator a_i or b_i contracts to a point.

Consider first the case, when a_i contracts to a point. The normalization of the holomorphic one-form ω_i must be preserved so that that ω_i must behaves as $1/z$, where z is the complex coordinate vanishing at pinch. Since the homology generator b_i goes through the pinch it seems obvious that the imaginary part of the Teichmueller parameter $\Omega_{ii} = \int_{b_i} \omega_i$ diverges at this limit (this conclusion is made also in [19]): $Im(\Omega_{ii}) \to \infty$.

Of course, this criterion doesn't cover all possible manners the pinch can occur: pinch might take place also, when the components of the Teichmueller matrix remain finite. In the case of torus topology one finds that $Sp(2g, Z)$ element $(A, B; C, D)$ takes $Im(\Omega) = \infty$ to the point C/D of real axis. This suggests that pinch occurs always at the boundary of the Teichmueller space: the imaginary part of Ω_{ij} either vanishes or some matrix element of $Im(\Omega)$ diverges.

Consider next the situation, when b_i contracts to a point. From the definition of the Teichmueller parameters it is clear that the matrix elements Ω_{kl}, with $k, l \neq i$ suffer no change. The matrix element Ω_{ki} obviously vanishes at this limit. The conclusion is that i:th row of Teichmueller matrix vanishes at this limit. This result is obtained also by deriving the $Sp(2g, Z)$ transformation permuting a_i and b_i with each other: in case of torus this transformation reads $\Omega \to -1/\Omega$.

Consider now the behavior of the theta functions, when pinch occurs. Consider first the limit, when $Im(\Omega_{ii})$ diverges. Using the general definition of $\Theta[a, b]$ it is easy to find out that all theta functions for which the i:th component a_i of the theta characteristic is non-vanishing (that is $a_i = 1/2$) are proportional to the exponent $exp(-\pi\Omega_{ii}/4)$ and therefore vanish at the limit. The theta functions with $a_i = 0$ reduce to $g-1$ dimensional theta functions with theta characteristic obtained by dropping i:th components of a_i and b_i and replacing Teichmueller matrix with Teichmueller matrix obtained by dropping i:th row and column. The conclusion is that all theta functions of type $\Theta(a, b)$ with $a = (1/2, 1/2, ..., 1/2)$ satisfy the stability criterion in this case.

What happens for the $Sp(2g, Z)$ transformed points on the real axis? The transformation formula for theta function is given by [19, 17]

$$\Theta[a, b]((A\Omega + B)(C\Omega + D)^{-1}) \quad = \quad exp(i\phi)det(C\Omega + D)^{1/2}\Theta[c, d](\Omega) \ , \tag{1.4.10}$$

where

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \left(\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} (CD^T)_d/2 \\ (AB^T)_d/2 \end{pmatrix} \right) . \tag{1.4.11}$$

Here ϕ is a phase factor irrelevant for the recent purposes and the index d refers to the diagonal part of the matrix in question.

The first thing to notice is the appearance of the diverging square root factor, which however disappears from the vacuum functionals (P and Q have same degree with respect to thetas). The essential point is that theta characteristics transform to each other: as already noticed all even theta characteristics belong to the same $Sp(2g, Z)$ orbit. Therefore the theta functions vanishing at $Im(\Omega_{ii}) = \infty$ do not vanish at the transformed points. It is however clear that for a given Teichmueller parametrization of pinch some theta functions vanish always.

Similar considerations in the case $\Omega_{ik} = 0$, i fixed, show that all theta functions with $b = (1/2, ..., 1/2)$ vanish identically at the pinch. Also it is clear that for $Sp(2g, Z)$ transformed points one can always find some vanishing theta functions. The overall conclusion is that the elementary particle vacuum functionals obtained by using $g \rightarrow g_1 + g_2$ stability criterion satisfy also $g \rightarrow g - 1$ stability criterion since they are proportional to the product of all even theta functions. Therefore the only nontrivial consequence of $g \rightarrow g - 1$ criterion is that also $g = 1$ vacuum functionals are of the same general form as $g > 1$ vacuum functionals.

A second manner to deduce the same result is by restricting the consideration to the hyper-elliptic surfaces and using the representation of the theta functions in terms of the roots of the polynomial appearing in the definition of the hyper-elliptic surface [19]. When the genus of the surface is smaller than three (the interesting case), this representation is all what is needed since all surfaces of genus $g < 3$ are hyper-elliptic.

Since hyper-elliptic surfaces can be regarded as surfaces obtained by gluing two compactified complex planes along the cuts connecting various roots of the defining polynomial it is obvious that the process $g \rightarrow g - 1$ corresponds to the limit, when two roots of the defining polynomial coincide. This limit corresponds either to disappearance of a cut or the fusion of two cuts to a single cut. Theta functions are expressible as the products of differences of various roots (Thomae's formula [19])

$$\Theta[a, b]^4 \propto \prod_{i<j \in T} (z_i - z_j) \prod_{k<l \in CT} (z_k - z_l) \ , \tag{1.4.12}$$

where T denotes some subset of $\{1, 2, ..., 2g\}$ containing $g+1$ elements and CT its complement. Hence the product of all even theta functions vanishes, when two roots coincide. Furthermore, stability criterion is satisfied only by the product of the theta functions.

Lowest dimensional vacuum functionals are worth of more detailed consideration.

i) $g = 0$ particle family corresponds to a constant vacuum functional: by continuity this vacuum functional is constant for all topologies.

ii) For $g = 1$ the degree of P and Q as polynomials of the theta functions is 24: the critical number of transversal degrees of freedom in bosonic string model! Probably this result is not an accident.

ii) For $g = 2$ the corresponding degree is 80 since there are 10 even genus 2 theta functions.

There are large numbers of vacuum functionals satisfying the relevant criteria, which do not satisfy the proposed stability criteria. These vacuum functionals correspond either to many particle states or to unstable single particle states.

1.4.8 Continuation of the vacuum functionals to higher genus topologies

From continuity it follows that vacuum functionals cannot be localized to single boundary topology. Besides continuity and the requirements listed above, a natural requirement is that the continuation of the vacuum functional from the sector g to the sector $g + k$ reduces to the product of the original vacuum functional associated with genus g and $g = 0$ vacuum functional at the limit when the surface with genus $g + k$ decays to surfaces with genus g and k: this requirement should guarantee the conservation of separate lepton numbers although different boundary topologies suffer mixing in the vacuum functional. These requirements are satisfied provided the continuation is constructed using the following rule:

Perform the replacement

$$\Theta[a,b]^4 \quad \rightarrow \quad \sum_{c,d} \Theta[a \oplus c, b \oplus d]^4 \tag{1.4.13}$$

for each fourth power of the theta function. Here c and d are Theta characteristics associated with a surface with genus k. The same replacement is performed for the complex conjugates of the theta function. It is straightforward to check that the continuations of elementary particle vacuum functionals indeed satisfy the cluster decomposition property and are continuous.

To summarize, the construction has provided hoped for answers to some questions stated in the beginning: stability requirements explain the separate conservation of lepton numbers and the experimental absence of $g > 0$ elementary bosons. What has not not been explained is the experimental absence of $g > 2$ fermion families. The vanishing of the $g > 2$ elementary particle vacuum functionals for the hyper-elliptic surfaces however suggest a possible explanation: under some conditions on the surface X^2 the surfaces Y^2 are hyper-elliptic or possess some conformal symmetry so that elementary particle vacuum functionals vanish for them. This conjecture indeed might make sense since the surfaces Y^2 are determined by the asymptotic dynamics and one might hope that the surfaces Y^2 are analogous to the final states of a dissipative system.

1.5 Explanations for the absence of the $g > 2$ elementary particles from spectrum

The decay properties of the intermediate gauge bosons [27] are consistent with the assumption that the number of the light neutrinos is $N = 3$. Also cosmological considerations pose upper bounds on the number of the light neutrino families and $N = 3$ seems to be favored [28]. It must be however emphasized that p-adic considerations [F5] encourage the consideration the existence of higher genera with neutrino masses such that they are not produced in the laboratory at present energies. In any case, for TGD approach the finite number of light fermion families is a potential difficulty since genus-generation correspondence suggests that the number of the fermion (and possibly also boson) families is infinite. Therefore one had better to find a good argument showing that the number of the observed neutrino families, or more generally, of the observed elementary particle families, is small also in the world described by TGD.

It will be later found that also TGD inspired cosmology requires that the number of the effectively massless fermion families must be small after Planck time. This suggests that boundary topologies with handle number $g > 2$ are unstable and/or very massive so that they, if present in the spectrum, disappear from it after Planck time, which correspond to the value of the light cone proper time $a \simeq 10^{-11}$ seconds.

In accordance with the spirit of TGD approach it is natural to wonder whether some geometric property differentiating between $g > 2$ and $g < 3$ boundary topologies might explain why only $g < 3$ boundary components are observable. One can indeed find a good candidate for this kind of property: namely hyper-ellipticity, which states that Riemann surface is a two-fold branched covering of sphere possessing two-element group Z_2 as conformal automorphisms. All $g < 3$ Riemann surfaces are hyper-elliptic unlike $g > 2$ Riemann surfaces, which in general do not posses this property. Thus it is natural to consider the possibility that hyper-ellipticity or more general conformal symmetries might explain why only $g < 2$ topologies correspond to the observed elementary particles.

As regards to the present problem the crucial observation is that some even theta functions vanish for the hyper-elliptic surfaces with genus $g > 2$ [19]. What is essential is that these surfaces have the group Z_2 as conformal symmetries. Indeed, the vanishing phenomenon is more general. Theta functions tend to vanish for $g > 2$ two-surfaces possessing discrete group of conformal symmetries [16]: for instance, instead of sphere one can consider branched coverings of higher genus surfaces.

From the general expression of the elementary particle vacuum functional it is clear that elementary particle vacuum functionals vanish, when Y^2 is hyper-elliptic surface with genus $g > 2$ and one might hope that this is enough to explain why the number of elementary particle families is three.

1.5.1 Hyper-ellipticity implies the separation of $g \leq 2$ and $g > 2$ sectors to separate worlds

If the vertices are defined as intersections of space-time sheets of elementary particles and if elementary particle vacuum functionals are required to have Z_2 symmetry, the localization of elementary particle vacuum functionals to $g \leq 2$ topologies occurs automatically. Even if one allows as limiting case vertices for which 2-manifolds are pinched to topologies intermediate between $g > 2$ and $g \leq 2$ topologies, Z_2 symmetry present for both topological interpretations implies the vanishing of this kind of vertices. This applies also in the case of stringy vertices so that also particle propagation would respect the effective number of particle families. $g > 2$ and $g \leq 2$ topologies would behave much like their own worlds in this approach. This is enough to explain the experimental findings if one can understand why the $g > 2$ particle families are absent as incoming and outgoing states or are very heavy.

1.5.2 What about $g > 2$ vacuum functionals which do not vanish for hyper-elliptic surfaces?

The vanishing of all $g \geq 2$ vacuum functionals for hyper-elliptic surfaces cannot hold true generally. There must exist vacuum functionals which do satisfy this condition. This suggest that elementary particle vacuum functionals for $g > 2$ states have interpretation as bound states of g handles and that the more general states which do not vanish for hyper-elliptic surfaces correspond to many-particle states composed of bound states $g \leq 2$ handles and cannot thus appear as incoming and outgoing states. Thus $g > 2$ elementary particles would decouple from $g \leq 2$ states.

1.5.3 Should higher elementary particle families be heavy?

TGD predicts an entire hierarchy of scaled up variants of standard model physics for which particles do not appear in the vertices containing the known elementary particles and thus behave like dark matter [A1, A8]. Also $g > 2$ elementary particles would behave like dark matter and in principle there is no absolute need for them to be heavy.

The safest option would be that $g > 2$ elementary particles are heavy and the breaking of Z_2 symmetry for $g \geq 2$ states could guarantee this. p-Adic considerations lead to a general mass formula for elementary particles such that the mass of the particle is proportional to $\frac{1}{\sqrt{p}}$ [TGDpad]. Also the dependence of the mass on particle genus is completely fixed by this formula. What remains however open is what determines the p-adic prime associated with a particle with given quantum numbers. Of course, it could quite well occur that p is much smaller for $g > 2$ genera than for $g \leq 2$ genera.

1.6 Elementary particle vacuum functionals for dark matter

One of the open questions is how dark matter hierarchy reflects itself in the properties of the elementary particles. The basic questions are how the quantum phase $q = ep(2i\pi/n)$ makes itself visible in the solution spectrum of the modified Dirac operator D and how elementary particle vacuum functionals depend on q. Considerable understanding of these questions emerged recently. One can generalize modular invariance to fractional modular invariance for Riemann surfaces possessing Z_n symmetry and perform a similar generalization for theta functions and elementary particle vacuum functionals. In particular, without any further assumptions $n = 2$ dark fermions have only three families. The existence of space-time correlate for fermionic 2-valuedness suggests that fermions indeed correspond to $n = 2$, or more generally to even values of n, so that this result would hold quite generally. Elementary bosons (actually exotic particles) would correspond to $n = 1$, and more generally odd values of n, and could have also higher families.

1.6.1 Connection between Hurwitz zetas, quantum groups, and hierarchy of Planck constants?

The action of modular group SL(2,Z) on Riemann zeta [23] is induced by its action on theta function [24]. The action of the generator $\tau \rightarrow -1/\tau$ on theta function is essential in providing the functional equation for Riemann Zeta. Usually the action of the generator $\tau \rightarrow \tau + 1$ on Zeta is not considered explicitly. The surprise was that the action of the generator $\tau \rightarrow \tau + 1$ on Riemann Zeta does not give back Riemann zeta but a more general function known as Hurwitz zeta $\zeta(s,z)$ for $z = 1/2$. One finds that Hurwitz zetas for certain rational values of argument define in a well defined sense representations of fractional modular group to which quantum group can be assigned naturally. This could allow to code the value of the quantum phase $q = exp(i2\pi/n)$ to the solution spectrum of the modified Dirac operator D.

Hurwitz zetas

Hurwitz zeta is obtained by replacing integers m with $m + z$ in the defining sum formula for Riemann Zeta:

$$\zeta(s,z) = \sum_m (m+z)^{-s} \quad . \tag{1.6.1}$$

Riemann zeta results for $z = n$.
Hurwitz zeta obeys the following functional equation for rational $z = m/n$ of the second argument [22]:

$$\zeta(1-s, \frac{m}{n}) = \frac{2\Gamma(s)}{2\pi n}^s \sum_{k=1}^{n} cos(\frac{\pi s}{2} - \frac{2\pi k m}{n})\zeta(s, \frac{k}{n}) \quad . \tag{1.6.2}$$

The representation of Hurwitz zeta in terms of θ [22] is given by the equation

$$\int_0^\infty \left[\theta(z,it) - 1\right] t^{s/2} \frac{dt}{t} = \pi^{(1-s)/2}\Gamma(\frac{1-s}{2}) \left[\zeta(1-s,z) + \zeta(1-s,1-z)\right] \quad . \tag{1.6.3}$$

By the periodicity of theta function this gives for $z = n$ Riemann zeta.

The action of $\tau \rightarrow \tau + 1$ transforms $\zeta(s, 0)$ to $\zeta(s, 1/2)$

The action of the transformations $\tau \rightarrow \tau + 1$ on the integral representation of Riemann Zeta [23] in terms of θ function [24]

$$\theta(z;\tau) - 1 = 2\sum_{n=1}^{\infty} [exp(i\pi\tau)]^{n^2} cos(2\pi n z) \tag{1.6.4}$$

is given by

$$\pi^{-s/2}\Gamma(\frac{s}{2})\zeta(s) = \int_0^\infty [\theta(0;it) - 1] t^{s/2} \frac{dt}{t} \quad . \tag{1.6.5}$$

Using the first formula one finds that the shift $\tau = it \rightarrow \tau + 1$ in the argument θ induces the shift $\theta(0;\tau) \rightarrow \theta(1/2;\tau)$. Hence the result is Hurwitz zeta $\zeta(s, 1/2)$. For $\tau \rightarrow \tau + 2$ one obtains Riemann Zeta.

Thus $\zeta(s, 0)$ and $\zeta(s, 1/2)$ behave like a doublet under modular transformations. Under the subgroup of modular group obtained by replacing $\tau \rightarrow \tau + 1$ with $\tau \rightarrow \tau + 2$ Riemann Zeta forms a singlet. The functional equation for Hurwitz zeta relates $\zeta(1-s, 1/2)$ to $\zeta(s, 1/2)$ and $\zeta(s, 1) = \zeta(s, 0)$ so that also now one obtains a doublet, which is not surprising since the functional equations directly reflects the modular transformation properties of theta functions. This doublet might be the proper object to study instead of singlet if one considers full modular invariance.

Hurwitz zetas form n-plets closed under the action of fractional modular group

The inspection of the functional equation for Hurwitz zeta given above demonstrates that $\zeta(s, m/n)$, $m = 0, 1, ..., n$, form in a well-defined sense an n-plet under fractional modular transformations obtained by using generators $\tau \rightarrow -1/\tau$ and $\tau \rightarrow \tau + 2/n$. The latter corresponds to the unimodular matrix $(a, b; c, d) = (1, 2/n; 0, 1)$. These matrices obviously form a group. Note that Riemann zeta is always one member of the multiplet containing n Hurwitz zetas.

These observations bring in mind fractionization of quantum numbers, quantum groups corresponding to the quantum phase $q = exp(i2\pi/n)$, and the inclusions for hyper-finite factors of type II_1 partially characterized by these quantum phases. Fractional modular group obtained using generator $\tau \rightarrow \tau + 2/n$ and Hurwitz zetas $\zeta(s, k/n)$ could very naturally relate to these and related structures.

1.6.2 Could Hurwitz zetas relate to dark matter?

These observations suggest a speculative application to quantum TGD.

Basic vision about dark matter

1. In TGD framework inclusions of HFFs of type II_1 are directly related to the hierarchy of Planck constants involving a generalization of the notion of imbedding space obtained by gluing together copies of 8-D $H = M^4 \times CP_2$ with a discrete bundle structure $H \rightarrow H/Z_{n_a} \times Z_{n_b}$ together along the 4-D intersections of the associated base spaces [A9]. A book like structure results and various levels of dark matter correspond to the pages of this book. One can say that elementary particles proper are maximally quantum critical and live in the 4-D intersection of these imbedding spaces whereas their "field bodies" reside at the pages of the Big Book. Note that analogous book like structures results when real and various p-adic variants of the imbedding space are glued together along common algebraic points.

2. The integers n_a and n_b give Planck constant as $\hbar/\hbar_0 = n_a/n_b$, whose most general value is a rational number. In Platonic spirit one can argue that number theoretically simple integers involving only powers of 2 and Fermat primes are favored physically. Phase transitions between different matters occur at the intersection.

3. The inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs relate also to quantum measurement theory with finite measurement resolution with $\mathcal{N}$ defining the measurement resolution so that N-rays replace complex rays in the projection postulate and quantum space $\mathcal{M}/\mathcal{N}$ having fractional dimension effectively replaces $\mathcal{M}$.

4. Geometrically the fractional modular invariance would naturally relate to the fact that Riemann surface (partonic 2-surface) can be seen as an $n_a \times n_b$-fold covering of its projection to the base space of H: fractional modular transformations corresponding to n_a and n_b would relate points at different sheets of the covering of M^4 and CP_2. This means $Z_{n_a n_b} = Z_{n_a} \times Z_{n_b}$ conformal symmetry. This suggests that the fractionization could be a completely general phenomenon happening also for more general zeta functions.

What about exceptional cases $n = 1$ and $n = 2$?

Also $n = 1$ and $n = 2$ are present in the hierarchy of Hurwitz zetas (singlet and doublet). They do not correspond to allowed Jones inclusion since one has $n > 2$ for them. What could this mean?

1. It would seem that the fractionization of modular group relates to Jones inclusions ($n > 2$) giving rise to fractional statistics. $n = 2$ corresponding to the full modular group Sl(2,Z) could relate to the very special role of 2-valued logic, to the degeneracy of $n = 2$ polygon in plane, to the very special role played by 2-component spinors playing exceptional role in Riemann geometry with spinor structure, and to the canonical representation of HFFs of type II_1 as fermionic Fock space (spinors in the world of classical worlds). Note also that SU(2) defines the building block of compact non-commutative Lie groups and one can obtain Lie-algebra generators of Lie groups from n copies of SU(2) triplets and posing relations which distinguish the resulting algebra from a direct sum of SU(2) algebras.

2. Also $n = 2$-fold coverings $M^4 \rightarrow M^4/Z_2$ and $CP_2 \rightarrow CP_2/Z_2$ seem to make sense. One can argue that by quantum classical correspondence the spin half property of imbedding space spinors should have space-time correlate. Could $n = 2$ coverings allow to define the space-time correlates for particles having half odd integer spin or weak isospin? If so, bosons would correspond to $n = 1$ and fermions to $n = 2$. One could of course counter argue that induced spinor fields already represent fermions at space-time level and there is no need for the doubling of the representation.

   The trivial group Z_1 and Z_2 are exceptional since Z_1 does not define any quantization axis and Z_2 allows any quantization axis orthogonal to the line connecting two points. For $n \geq 3$ Z_n fixes the direction of quantization axis uniquely. This obviously correlates with $n \geq 3$ for Jones inclusions.

Dark elementary particle functionals

One might wonder what might be the dark counterparts of elementary particle vacuum functionals. Theta functions $\theta_{[a,b]}(z,\Omega)$ with characteristic $[a,b]$ for Riemann surface of genus g as functions of z and Teichmueller parameters Ω are the basic building blocks of modular invariant vacuum functionals defined in the finite-dimensional moduli space whose points characterize the conformal equivalence class of the induced metric of the partonic 2-surface. Obviously, kind of spinorial variants of theta functions are in question with $g+g$ spinor indices for genus g.

The recent case corresponds to $g = 1$ Riemann surface (torus) so that a and b are $g = 1$-component vectors having values 0 or 1/2 and Hurwitz zeta corresponds to $\theta_{[0,1/2]}$. The four Jacobi theta functions listed in Wikipedia [24] correspond to these thetas for torus. The values for a and b are 0 and 1 for them but this is a mere convention.

The extensions of modular group to fractional modular groups obtained by replacing integers with integers shifted by multiples of $1/n$ suggest the existence of new kind of q-theta functions with characteristics $[a,b]$ with a and b being g-component vectors having fractional values k/n, $k = 0,1...n-1$. There exists also a definition of q-theta functions working for $0 \leq |q| < 1$ but not for roots of unity [25]. The q-theta functions assigned to roots of unity would be associated with Riemann surfaces with additional Z_n conformal symmetry but not with generic Riemann surfaces and obtained by simply replacing the value range of characteristics $[a,b]$ with the new value range in the defining formula

$$\Theta[a,b](z|\Omega) = \sum_n exp\left[i\pi(n+a)\cdot\Omega\cdot(n+a) + i2\pi(n+a)\cdot(z+b)\right] . \tag{1.6.6}$$

for theta functions. If Z_n conformal symmetry is relevant for the definition of fractional thetas it is probably so because it would make the generalized theta functions sections in a bundle with a finite fiber having Z_n action.

This hierarchy would correspond to the hierarchy of quantum groups for roots of unity and Jones inclusions and one could probably define also corresponding zeta function multiplets. These theta functions would be building blocks of the elementary particle vacuum functionals for dark variants of elementary particles invariant under fractional modular group. They would also define a hierarchy of fractal variants of number theoretic functions: it would be interesting to see what this means from the point of view of Langlands program [26] discussed also in TGD framework [E11] involving ordinary modular invariance in an essential manner.

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Hierarchy of Planck constants defines a hierarchy of quantum critical systems

Dark matter hierarchy corresponds to a hierarchy of conformal symmetries Z_n of partonic 2-surfaces with genus $g \geq 1$ such that factors of n define subgroups of conformal symmetries of Z_n. By the decomposition $Z_n = \prod_{p|n} Z_p$, where $p|n$ tells that p divides n, this hierarchy corresponds to an hierarchy of increasingly quantum critical systems in modular degrees of freedom. For a given prime p one has a sub-hierarchy Z_p, $Z_{p^2} = Z_p \times Z_p$, etc... such that the moduli at n+1:th level are contained by n:th level. In the similar manner the moduli of Z_n are sub-moduli for each prime factor of n. This mapping of integers to quantum critical systems conforms nicely with the general vision that biological evolution corresponds to the increase of quantum criticality as Planck constant increases.

The group of conformal symmetries could be also non-commutative discrete group having Z_n as a subgroup. This inspires a very short-lived conjecture that only the discrete subgroups of SU(2) allowed by Jones inclusions are possible as conformal symmetries of Riemann surfaces having $g \geq 1$. Besides Z_n one could have tedrahedral and icosahedral groups plus cyclic group Z_{2n} with reflection added but not Z_{2n+1} nor the symmetry group of cube. The conjecture is wrong. Consider the orbit of the subgroup of rotational group on standard sphere of E^3, put a handle at one of the orbits such that it is invariant under rotations around the axis going through the point, and apply the elements of subgroup. You obtain a Riemann surface having the subgroup as its isometries. Hence all discrete subgroups of SU(2) can act even as isometries for some value of g.

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself.

Spherical topology is exceptional since in this case the space of conformal moduli is trivial and conformal symmetries correspond to the entire $SL(2,C)$. This would suggest that only the fermions of lowest generation corresponding to the spherical topology are maximally quantum critical. This brings in mind Jones inclusions for which the defining subgroup equals to $SU(2)$ and Jones index equals to $\mathcal{M}/\mathcal{N} = 4$. In this case all discrete subgroups of $SU(2)$ label the inclusions. These inclusions would correspond to fiber space $CP_2 \rightarrow CP_2/U(2)$ consisting of geodesic spheres of CP_2. In this case the discrete subgroup might correspond to a selection of a subgroup of $SU(2) \subset SU(3)$ acting non-trivially on the geodesic sphere. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ having geodesic spheres of CP_2 as their ends could correspond to this phase dominating the very early cosmology.

Fermions in TGD Universe allow only three families

What is nice that if fermions correspond to $n = 2$ dark matter with Z_2 conformal symmetry as strong quantum classical correspondence suggests, the number of ordinary fermion families is three without any further assumptions. To see this suppose that also the sectors corresponding to $M^4 \rightarrow M^4/Z_2$ and $CP_2 \rightarrow CP_2/Z_2$ coverings are possible. Z_2 conformal symmetry implies that partonic Riemann surfaces are hyper-elliptic. For genera $g > 2$ this means that some theta functions of $\theta_{[a,b]}$ appearing in the product of theta functions defining the vacuum functional vanish. Hence fermionic elementary particle vacuum functionals would vanish for $g > 2$ and only 3 fermion families would be possible for $n = 2$ dark matter.

This results can be strengthened. The existence of space-time correlate for the fermionic 2-valuedness suggests that fermions quite generally to even values of n, so that this result would hold for all fermions. Elementary bosons (actually exotic particles belonging to Kac-Moody type representations) would correspond to odd values of n, and could possess also higher families. There is a nice argument supporting this hypothesis. n-fold discretization provided by covering associated with H corresponds to discretization for angular momentum eigenstates. Minimal discretization for $2j+1$ states corresponds to $n = 2j+1$. $j = 1/2$ requires $n = 2$ at least, $j = 1$ requires $n = 3$ at least, and so on. $n = 2j+1$ allows spins $j \leq n - 1/2$. This spin-quantum phase connection at the level of space-time correlates has counterpart for the representations of quantum SU(2).

These rules would hold only for genuinely elementary particles corresponding to single partonic component and all bosonic particles of this kind are exotics (excitations in only "vibrational" degrees of freedom of partonic 2-surface with modular invariance eliminating quite a number of them.

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Chapter 2

Massless States and Particle Massivation

2.1 Introduction

This chapter tries to represent the most recent view about particle massivation. The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

Zero energy ontology providing a detailed geometric view about bosons and fermions, the generalization of S-matrix to what I call M-matrix, the notion of finite measurement resolution characterized in terms of inclusions of von Neumann algebras, the derivation of p-adic coupling constant evolution and p-adic length scale hypothesis from the first principles, and understanding of Higgs mechanism in terms of the generalized eigenvalues of the modified Dirac operator: these are the most important steps of progress during last years with a direct relevance for the understanding of particle spectrum and massivation although the predictions of p-adic thermodynamics are not affected. What is frustrating is that the joy by every great step of progress is shadowed by the realization that it creates a lot of mammoth bones generating internal inconsistencies (there are fifteen books about TGD so that I have to fight fiercely to avoid total chaos!), and I feel that my first task before continuing is to represent apologies for not being able to identify all of them. Therefore it is better to take these chapters as lab note books about work in progress rather than final summaries.

2.1.1 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD?

What p-adic coupling constant evolution really means has remained for a long time more or less open and detailed attempts to model the situation has suffered from this. The progress made in the understanding of the S-matrix of the theory [C3] has however changed the situation dramatically.

M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [C3]. M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude. One important implication is justification for p-adic thermodynamics used to calculate particle masses: it is related to genuine quantum description of elementary particles rather than to a description of a fictive thermal ensemble.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm $log(2)$ dependence disappears so that rational number results. A weaker condition for the scale hierarchy of CDs would be $T_p = pT_0$, p prime, and would assign all p-adic time scales to the size scale hierarchy of CDs.

p-Adic coupling constant evolution

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ (or $T_p = pT_0$) induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This idea looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime $p \simeq 2^k$ would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of configuration space.

4. The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \ ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

2.1.2 Physical states as representations of super-symplectic and Super Kac-Moody algebras

Physical states belong to the representation of super-symplectic algebra and Super Kac-Moody algebra assignable $SO(2) \times SU(3) \times SU(2)_{rot} \times U(2)_{ew}$ associated with the 2-D surfaces X^2 defined by the intersections of 3-D light like causal determinants with $\delta M^4_\pm \times CP_2$. These 2-surfaces have interpretation as partons.

It has taken considerable effort to understand the relationship between super-symplectic and super Kac-Moody algebras and there are still many uncertainties involved. What looks like the most plausible option relies on the generalization of a coset construction proposed already for years ago but given up because of the lacking understanding of how SKM and SC algebras could be lifted to the level of imbedding space. The progress in the *Physics as generalized number theory* program provided finally a justification for the coset construction.

1. Assume a generalization of the coset construction in the sense that the differences of super Kac-Moody Virasoro generators (SKMV) and super-symplectic Virasoro generators (SSV) annihilate the physical states. The interpretation is in terms of TGD counterpart for Einstein's equations realizing Equivalence Principle. Mass squared is identified as the p-adic thermal expectation value of either $SKMV$ or SSV conformal weight (gravitational or inertial mass) in a superposition of states with $SKMV$ (SSV) conformal weight $n \geq 0$ annihilated by $SKMV - SSV$.

2. Construct first ground states with negative conformal weight annihilated by $SKMV$ and SSV generators G_n, L_n, $n < 0$. Apply to these states generators of tensor factors of Super Viraroso algebras to obtain states with vanishing SSV and $SKMV$ conformal weights. After this construct thermal states as superpositions of states obtained by applying $SKMV$ generators and corresponding SSV generators G_n,L_n, $n > 0$. Assume that these states are annihilated by SSV and $SKMV$ generators G_n, L_n,$n > 0$ and by the differences of all SSV and $SKMV$ generators.

3. Super-symplectic algebra represents a completely new element and in the case of hadrons the non-perturbative contribution to the mass spectrum is easiest to understand in terms of super-symplectic thermal excitations contributing roughly 70 per cent to the p-adic thermal mass of the hadron. It must be however emphasized that by SKMV-SSV duality one can regard these contributions equivalently as SKM or SC contributions.

2.1.3 Particle massivation

Particle massivation can be regarded as a generation of thermal conformal weight identified as mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

1. The original observation was that the pieces of CP_2 type vacuum extremals representing elementary particles have random light-like curve as an M^4 projection so that the average motion correspond to that of massive particle. Light-like randomness gives rise to classical Virasoro conditions. This picture generalizes since the basic dynamical objects are light-like but otherwise random 3-surfaces. Fermions are identified as light-like 3-surfaces at which the signature of induced metric of deformed CP_2 type extremals changes from Euclidian to the Minkowskian

signature of the background space-time sheet. Gauge bosons and Higgs correspond to wormhole contacts with light-like throats carrying fermion and antifermion quantum numbers. Gravitons correspond to pairs of wormhole contacts bound to string like object by the fluxes connecting the wormhole contacts. The randomness of the light-like 3-surfaces and associated super-conformal symmetries justify the use of thermodynamics and the question remains why this thermodynamics can be taken to be p-adic. The proposed identification of bosons means enormous simplification in thermodynamical description since all calculations reduced to the calculations to fermion level.

2. The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces. Dynamics is constrained by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. As a matter fact, the basic theory relies on the modified Dirac action associated with Chern-Simons action and Kähler action in the sense that the generalizes eigenmodes of C-S Dirac operator correspond to the zero modes of Kähler action localized to the light-like 3-surfaces representing partons. In this manner the data about the dynamics of Kähler action is feeded to the eigenvalue spectrum. Eigenvalues are interpreted as square roots of ground state conformal weights.

3. The symmetries respecting light-likeness property give rise to Kac-Moody type algebra and super-symplectic symmetries emerge also naturally as well as $N = 4$ character of super-conformal invariance. The coset construction for super-symplectic Virasoro algebra and Super Kac-Moody algebra identified in physical sense as sub-algebra of former implies that the four-momenta assignable to the two algebras are identical. The interpretation is in terms of the identity of gravitational inertial masses and generalization of Equivalence Principle.

4. Instead of energy, the Super Kac-Moody Virasoro (or equivalently super-symplectic) generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics (and also in its real version assuming it exists). The fact that mass squared is thermal expectation of conformal weight guarantees Lorentz invariance. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

5. By Equivalence Principle the thermal average of mass squared can be calculated either in terms of thermodynamics for either super-symplectic of Super Kac-Moody Virasoro algebra and p-adic thermodynamics is consistent with conformal invariance.

6. A long standing problem has been whether coupling to Higgs boson is needed to explain gauge boson masses. It has turned out that p-adic thermodynamics is enough. From the beginning it was clear that is that ground state conformal weight is negative. Only quite recently it became clear that the ground state conformal weight need not be a negative integer. The deviation Δh of the total ground state conformal weight from negative integer gives rise to Higgs type contribution to the thermal mass squared and dominates in case of gauge bosons for which p-adic temperature is small. In the case of fermions this contribution to the mass squared is small. Higgs vacuum expectation is naturally proportional to Δh so that the coupling to Higgs seems to cause gauge boson massivation. The interpretation is that the effective metric defined by the modified gamma matrices associated with Kähler action has Euclidian signature. This implies that the eigenvalues of the modified Dirac operator are purely imaginary and analogous to cyclotron energies so that in the first approximation smallest conformal weights are of form $h = -n-1/2$ and for $n = 0$ one obtains the ground state conformal weight $h = -1/2$ conjectured earlier. One cannot exclude the possibility of complex eigenvalues of D_{C-S}.

7. There is also modular contribution to the mass squared which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

An important question concerns the justification of p-adic thermodynamics.

1. The underlying philosophy is that real number based TGD can be algebraically continued to various p-adic number fields. This gives justification for the use of p-adic thermodynamics although the mapping of p-adic thermal expectations to real counterparts is not completely unique. The physical justification for p-adic thermodynamics is effective p-adic topology characterizing the 3-surface: this is the case if real variant of light-like 3-surface has large number of common algebraic points with its p-adic counterpart obeying same algebraic equations but in different number field.

2. The most natural option is that the descriptions in terms of both real and p-adic thermodynamics make sense and are consistent. This option indeed makes since the number of generalized eigen modes of modified Dirac operator is finite. The finite number of fermionic oscillator operators implies an effective cutoff in the number conformal weights so that conformal algebras reduce to finite-dimensional algebras. The first guess would be that integer label for oscillator operators becomes a number in finite field for some prime. This means that one can calculate mass squared also by using real thermodynamics but the consistency with p-adic thermodynamics gives extremely strong number theoretical constraints on mass scale. This consistency condition allows also to solve the problem how to map a negative ground state conformal weight to its p-adic counterpart. Negative conformal weight is divided into a negative half odd integer part plus positive part Δh, and negative part corresponds as such to p-adic integer whereas positive part is mapped to p-adic number by canonical identification.

p-Adic thermodynamics is what gives to this approach its predictive power.

1. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.

2. p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5}\sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

The predictions of the general theory are consistent with the earliest mass calculations, and the earlier ad hoc parameters disappear. In particular, optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles.

2.1.4 Topics of the chapter

In this chapter the goal is to summarize the recent theoretical understanding behind particle massivation. After a summary of the recent phenomenological picture behind particle massivation the notions of number theoretical compactification and number theoretical braid are introduced and the construction of quantum TGD at parton level in terms of second quantization of modified Dirac action is described. The recent understanding of super-conformal symmetries are analyzed in detail. TGD color differs in several respect from QCD color and a detailed analysis of color partial waves associated with quark and lepton chiralities of imbedding space spinors fields is carried out with a special emphasis given to the contribution of color partial wave to mass squared of the fermion. The last sections are devoted to p-adic thermodynamics and to a model providing a formula for the modular contribution to mass squared.

2.2 Identification of elementary particles and the role of Higgs in particle massivation

The development of the recent view about the identification of elementary particles and particle massivation has taken fifteen years since the discovery of p-adic thermodynamics around 1993. p-Adic thermodynamics worked excellently from the beginning for fermions. Only the understanding of gauge boson masses turned out to be problematic and group theoretical arguments led to the proposal that Higgs boson should be present and give the dominating contribution to the masses of gauge bosons whereas the contribution to fermion masses should be small and even negligible. The detailed understanding of quantum TGD at partonic level eventually led to the realization that the coupling to Higgs is not needed after all. The deviation Δh of the ground state conformal weight from negative integer has interpretation as effective Higgs contribution since Higgs vacuum expectation is naturally proportional to Δh but the coupling to Higgs does not cause massivation. In the following I summarize the basic identification of elementary particles and massivation. A more detailed discussion can be found in [F6].

2.2.1 Identification of elementary particles

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 [C2, C3] suggest dramatic simplifications of the general picture discussed in the earlier version of this chapter. p-Adic mass calculations [F3, F4, F5] leave a lot of freedom concerning the detailed identification of elementary particles.

Elementary fermions and bosons

The basic open question is whether the *theory is on some sense free at parton level* as suggested by the recent view about the construction of S-matrix (actually its generalization M-matrix) and by the almost topological QFT property of quantum TGD at parton level [C3]. If partonic 2-surfaces at elementary particle level carry only free many-fermion states, no bi-local composites of second quantized induced spinor field would be needed in the construction of the quantum states and this would simplify the theory enormously.

If this is the case, the basic conclusion would be that light-like 3-surfaces - in particular the ones at which the signature of induced metric changes from Minkowskian to Euclidian - are carriers of fermionic quantum numbers. These regions are associated naturally with CP_2 type vacuum extremals identifiable as correlates for elementary fermions if only fermion number ± 1 is allowed for the stable states. The question however arises about the identification of elementary bosons.

Wormhole contacts with two light-like wormhole throats carrying fermion and anti-fermion quantum numbers are the first thing that comes in mind. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number. In this case a delicate question is whether the space-time sheets connected by wormhole contacts have opposite time orientations or not. If this the case the two fermions would correspond to positive and negative energy particles.

I considered first the identification of only Higgs as a wormhole contact but there is no reason why this identification should not apply also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions.

This picture allows to understand the difference between fermions and gauge bosons and Higgs particle. For fermions topological explanation of family replication predicts three fermionic generations [F1] corresponding to handle numbers $g = 0, 1, 2$ for the partonic 2-surface. In the case of gauge bosons and Higgs this replication is not visible. This could be due to the fact that gauge bosons form singlet and octet representation of the dynamical $SU(3)$ group associated with the handle number $g = 0, 1, 2$ since bosons correspond to pairs of handles. If octet representation is heavy the experimental absence of family replication for bosons can be understood.

Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The development of the understanding of gravitational coupling has had many twists and it is perhaps to summarize the basic misunderstandings.

1. CP_2 length scale R, which is roughly $10^{3.5}$ times larger than Planck length $l_P = \sqrt{\hbar G}$, defines a fundamental length scale in TGD. The challenge is to predict the value of Planck length $\sqrt{\hbar G}$. The outcome was an identification of a formula for $R^2/\hbar G$ predicting that the magnitude of Kähler coupling strength α_K is near to fine structure constant in electron length scale (for ordinary value of Planck constant should be added here).

2. The emergence of the parton level formulation of TGD finally demonstrated that G actually appears in the fundamental parton level formulation of TGD as a fundamental constant characterizing the M^4 part of CP_2 Kähler gauge potential [B4, F12]. This part is pure gauge in the sense of standard gauge theory but necessary to guarantee that the theory does not reduce to topological QFT. Quantum criticality requires that G remains invariant under p-adic coupling constant evolution and is therefore predictable in principle at least.

3. The TGD view about coupling constant evolution [C4] predicts the proportionality $G \propto L_p^2$, where L_p is p-adic length scale. Together with input from p-adic mass calculations one ends up to two conclusions. The correct conclusion was that Kähler coupling strength is equal to the fine structure constant in the p-adic length scale associated with Mersenne prime $p = M_{127} = 2^{127}-1$ assignable to electron [C4]. I have considered also the possibility that α_K would be equal to electro-weak $U(1)$ coupling in this scale.

4. The additional - wrong- conclusion was that gravitons must always correspond to the p-adic prime M_{127} since G would otherwise vary as function of p-adic length scale. As a matter fact, the question was for years whether it is G or g_K^2 which remains invariant under p-adic coupling constant evolution. I found both options unsatisfactory until I realized that RG invariance is possible for both g_K^2 and G! The point is that the exponent of the Kähler action associated with the piece of CP_2 type vacuum extremal assignable with the elementary particle is exponentially sensitive to the volume of this piece and logarithmic dependence on the volume fraction is enough to compensate the $L_p^2 \propto p$ proportionality of G and thus guarantee the constancy of G.

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naive estimate $G \sim L_p^2$.

Gravitons are obviously not the only stringy states. For instance, one obtains spin 1 states when the ends of string correspond to gauge boson and Higgs. Also non-vanishing electro-weak and color quantum numbers are possible and stringy states couple to elementary partons via standard couplings in this case. TGD based model for nuclei as nuclear strings having length of order $L(127)$ [F8] suggests that the strings with light M_{127} quark and anti-quark at their ends identifiable as companions of the ordinary graviton are responsible for the strong nuclear force instead of exchanges of ordinary mesons or color van der Waals forces.

Also the TGD based model of high T_c super-conductivity involves stringy states connecting the space-time sheets associated with the electrons of the exotic Cooper pair [J1, J2]. Thus stringy states would play a key role in nuclear and condensed matter physics, which means a profound departure from stringy wisdom, and breakdown of the standard reductionistic picture.

Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving 8+8=16 states altogether. Taking into account phase conjugates gives 16+16=32 states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2 + 1) \times (3 + 1) = 12$ states plus phase conjugates giving 12+12=24 states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes 48+16+16=80 states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

What about light-like boundaries and macroscopic wormhole contacts?

Light-like boundaries of the space-time sheet as also wormhole throats can have macroscopic size and can carry free many-fermion states but not elementary bosons. Number theoretic braids and anyons might be assignable to these structures. Deformations of cosmic strings to magnetic flux tubes with a light-like outer boundary are especially interesting in this respect.

If the ends of a string like object move with light velocity as implied by the usual stringy boundary conditions they indeed define light-like 3-surfaces. Many-fermion states could be assigned at the ends of string. One could also connect in pairwise manner the ends of two time-like strings having opposite time orientation using two space-like strings so that the analog of boson state consisting of two wormhole contacts and analogous to graviton would result. "Wormhole throats" could have arbitrarily long distance in M^4.

Wormhole contacts can be regarded as slightly deformed CP_2 type extremals only if the size of M^4 projection is not larger than CP_2 size. The natural question is whether one can construct macroscopic wormhole contacts at all.

1. The throats of wormhole contacts cannot belong to vacuum extremals. One might however hope that small deformations of macrosopic vacuum extremals could yield non-vacuum wormhole contacts of macroscopic size.

2. A large class of macroscopic wormhole contacts which are vacuum extremals consists of surfaces of form $X_1^2 \times X_2^2 \subset (M^1 \times Y^2) \times E^3$, where Y^2 is Lagrangian manifold of CP_2 (induced Kähler form vanishes) and $M^4 = M^1 \times E^3$ represents decomposition of M^1 to time-like and space-like sub-spaces. X_2^2 is a stationary surface of E^3. Both $X_1^2 \subset M^1 \times CP_2$ and X_2^2 have an Euclidian signature of metric except at light-like boundaries $X_a^1 \times X_2^2$ and $X_b^1 \times X_2^2$ defined by ends of X_1^2 defining the throats of the wormhole contact.

3. This kind of vacuum extremals could define an extremely general class of macroscopic wormhole contacts as their deformations. These wormhole contacts describe an interaction of wormhole throats regarded as closed strings as is clear from the fact that X^2 can be visualized as an analog of closed string world sheet X_1^2 in $M^1 \times Y^2$ describing a reaction leading from a state with a given number of incoming closed strings to a state with a given number of outgoing closed strings which correspond to wormhole throats at the two space-time sheets involved.

If one accepts the hierarchy of Planck constants [A9] leading to the generalization of the notion of imbedding space, the identification of anyonic phases in terms of macroscopic light-like surfaces emerges naturally. In this kind of states large fermion numbers are possible. Dark matter would correspond to this kind of phases and "partonic" 2-surfaces could have even astrophysical size. Also black holes can be identified as dark matter at light-like 3-surfaces analogous to black hole horizons and possessing gigantic value of Planck constant [F12].

2.2.2 New view about the role of Higgs boson in massivation

The proposed identifications challenge the standard model view about particle massivation.

1. The standard model inspired interpretation would be that Higgs vacuum expectation associated with the coherent state of neutral Higgs wormhole contacts generates gauge boson mass. Higgs could not however contribute to fermion mass since Higgs condensate cannot accompany fermionic space-time sheets. Fermionic mass would be solely to p-adic thermodynamics. This assumption is consistent with experimental facts but means asymmetry between fermions and bosons.

2. The alternative interpretation inspired by p-adic thermodynamics. Besides the thermodynamical contribution to the particle mass there can be a small contribution from the ground state conformal weight unless this weight is not negative integer. Gauge boson mass would correspond to the ground state conformal weight present in both fermionic and bosonic states and in the case of gauge bosons this contribution would dominate due to the small value of p-adic temperature. For fermions p-adic thermodynamics for super Virasoro algebra would give the dominating contribution to the mass. Higgs vacuum expectation value would be proportional to the square root of ground state conformal weight for the simple reason that it is the only natural dimensional parameter available. Therefore the causal relation between Higgs and massivation would have been misunderstood in standard model inspired framework. As will be found, the generalized eigen values of the modified Dirac operator having dimension of mass have a natural interpretation as square roots of ground state conformal weight and eigenvalues reflect directly the dynamics of Kähler action.

3. The remaining problem is to understand how the negative value of the ground state conformal weight emerges. This negative conformal weight compensated by the action of Super Virasoro generators is necessary for the success of p-adic mass calculations. Also this problem finds a natural solution. The generalized eigenvalues of the modified Dirac operator are purely imaginary if the effective metric associated with the modified Dirac operator has Euclidian signature. Ground state conformal would be negative and if it is not integer, an effective Higgs contribution to the mass squared is implied. For fermions the deviation from negative integer would be small. Hence p-adic thermodynamics is able to describe the massivation without the introduction of coupling to Higgs, which in TGD framework would be necessarily only a phenomenological description.

2.2.3 General mass formulas

In the following general view about p-adic mass formulas and related problems is discussed.

Mass squared as a thermal expectation of super Kac-Moody conformal weight

The general view about particle massivation is based on the generalized coset construction allowing to understand the p-adic thermal contribution to mass squared as a thermal expectation value of the conformal weight for super Kac-Moody Virasoro algebra ($SKMV$) or equivalently super-symplectic

Virasoro algebra (SSV). Conformal invariance holds true only for the generators of the differences of $SKMV$ and SSV generators. In the case of SSV and $SKMV$ only the generators L_n, $n > 0$, annihilate the physical states. Obviously the actions of super-symplectic Virasoro (SSV) generators and Super Kac-Moody Virasoro generators on physical states are identical. The interpretation is in terms of Equivalence Principle. p-Adic mass expectation value is same irrespective of whether it is calculated for the excitations created by SSV or $KKMV$ generators and p-adic mass calculations are consisted with super-conformal invariance.

1. Super-Kac Moody conformal weights must be negative for elementary fermions and this can be understood if the ground state conformal weight corresponds to the square of the imaginary eigenvalue of the modified Dirac operator having dimensions of mass. If the value of ground state conformal weight is not negative integer, a contribution to mass squared analogous to Higgs expectation is obtained.

2. Massless state is thermalized with respect to SKMV (or SSV) with thermal excitations created by generators L_n, $n > 0$.

Under what conditions conformal weight is additive

The question whether four- momentum or conformal weight is additive in p-adic mass calculations becomes acute in hadronic mass calculations. Only the detailed understanding of quantum TGD at partonic level allowed to understand the situation. One can consider three options.

1. Conformal weight and thus mass squared is additive only inside the regions of X^3_l, which correspond to non-vanishing of induced Kähler magnetic field since these behave effectively as separate 3-surfaces as far as eigenmodes of the modified Dirac operator are considered. The spectrum of the ground state conformal weights is indeed different for these regions in the general case. The four-momenta associated with different regions would be additive. This makes sense since the tangent space of $X^4(X^3_l)$ contains at each point of X^3_l a subspace $M^2(x)) \subset M^4$ defining the plane of non-physical polarizations and the natural interpretation is that four-momentum is in this plane. Hence the problem of original mass calculations forcing to assign all partonic four-momenta to a fixed plane M^2 is avoided.

2. If assigns independent translational degrees of freedom only to disjoint partonic 2-surfaces, a separate mass formula for each X^2_i would result and four-momenta would be additive:

$$M_i^2 = \sum_i L_{0i}(SKM) \ . \tag{2.2.1}$$

   Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known. Also vacuum conformal weight is included.

3. At the other extreme one has the option is based on the assignment of the mass squared with the total cm. This option looked the only reasonable one for 15 years ago. This would give

$$M^2 = (\sum_i p_i)^2 = \sum_i M_i^2 + 2\sum_{i\neq j} p_i \cdot p_j = -\sum_i L_{0i}(SKM) \ . \tag{2.2.2}$$

   The additivity of mass squared is strong condition and p-adic mass calculations for hadrons suggest that it holds true for quarks of low lying hadrons. For this option the decomposition of the net four momentum to a sum of individual momenta can be regarded as subjective unless there is a manner to measure the individual masses.

Mass formula for bound states of partons

The coefficient of proportionality between mass squared and conformal weight can be deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \quad (2.2.3)$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \quad (2.2.4)$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

2.3 Number theoretic compactification and $M^8 - H$ duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to number theory. In strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as surfaces of H or as surfaces of M^8 composed of hyper-quaternionic and co-hyper-quaternionic regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric.

2.3.1 Basic idea behind $M^8 - M^4 \times CP_2$ duality

The hopes of giving $M^4 \times CP_2$ hyper-octonionic structure are meager. This circumstance forces to ask whether four-surfaces $X^4 \subset M^8$ could under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly so that the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. The following arguments suggest that this is indeed the case.

The hard mathematical fact behind number theoretical compactification is that the quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space) are parameterized by CP_2 just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group.

1. The space of complex structures of the octonion space is parameterized by S^6. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Hyper-quaternions can be identified as $U(2)$ Lie-algebra but it is obvious that hyper-octonions do not allow an identification as $SU(3)$ Lie algebra. Rather, octonions decompose as $1 \oplus 1 \oplus 3 \oplus \overline{3}$ to the irreducible representations of $SU(3)$.

2. Geometrically the choice of a preferred complex (quaternionic) structure means fixing of complex (quaternionic) sub-space of octonions. The fixing of a hyper-quaternionic structure of hyper-octonionic M^8 means a selection of a fixed hyper-quaternionic sub-space $M^4 \subset M^8$ implying the decomposition $M^8 = M^4 \times E^4$. If M^8 is identified as the tangent space of $H = M^4 \times CP_2$, this decomposition results naturally. It is also possible to select a fixed hyper-complex structure, which means a further decomposition $M^4 = M^2 \times E^2$.

3. The basic result behind number theoretic compactification and $M^8 - H$ duality is that hyper-quaternionic sub-spaces $M^4 \subset M^8$ containing a fixed hyper-complex sub-space $M^2 \subset M^4$ or its light-like line $M_\pm$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

4. Space-time surface $X^4 \subset M^8$ is by definition hyper-quaternionic if the tangent spaces of X^4 are hyper-quaternionic planes. Co-hyper-quaternionictity means the same for normal spaces. The presence of fixed hyper-complex structure means at space-time level that the tangent space of X^4 contains fixed M^2 at each point. Under this assumption one can map the points $(m, e) \in M^8$ to points $(m, s) \in H$ by assigning to the point (m, e) of X^4 the point (m, s), where $s \in CP_2$ characterize $T(X^4)$ as hyper-quaternionic plane.

5. The choice of M^2 can be made also local in the sense that one has $T(X^4) \supset M^2(x) \subset M^4 \subset H$. It turns out that strong form of number theoretic compactification requires this kind of generalization. In this case one must be able to fix the convention how the point of CP_2 is assigned to a hyper-quaternionic plane so that it applies to all possible choices of $M^2 \subset M^4$. Since $SO(3)$ hyper-quaternionic rotation relates the hyper-quaternionic planes to each other, the natural assumption is hyper-quaternionic planes related by $SO(3)$ rotation correspond to the same point of CP_2. Under this assumption it is possible to map hyper-quaternionic surfaces of M^8 for which $M^2 \subset M^4$ depends on point of X^4 to H.

2.3.2 Minimal form of $M^8 - H$ duality

The basic problem in the construction of quantum TGD has been the identification of the preferred extremals of Kähler action playing a key role in the definition of the theory. The most elegant manner to do this is by fixing the 4-D tangent space $T(X^4(X_l^3))$ of $X^4(X_l^3)$ at each point of X_l^3 so that the boundary value problem is well defined. What I called number theoretical compactification allows to achieve just this although I did not fully realize this in the original vision. The minimal picture is following.

1. The basic observations are following. Let M^8 be endowed with hyper-octonionic structure. For hyper-quaternionic space-time surfaces in M^8 tangent spaces are by definition hyper-quaternionic. If they contain a preferred plane $M^2 \subset M^4 \subset M^8$ in their tangent space, they can be mapped to 4-surfaces in $M^4 \times CP_2$. The reason is that the hyper-quaternionic planes containing preferred the hyper-complex plane M^2 of $M_\pm \subset M^2$ are parameterized by points of CP_2. The map is simply $(m, e) \rightarrow (m, s(m, e))$, where m is point of M^4, e is point of E^4, and $s(m, 2)$ is point of CP_2 representing the hyperquaternionic tangent plane. The inverse map assigns to each point (m, s) in $M^4 \times CP_2$ point m of M^4, undetermined point e of E^4 and 4-D plane. The requirement that the distribution of planes containing the preferred M^2 or $M_\pm$ corresponds to a distribution of planes for 4-D surface is expected to fix the points e. The physical interpretation of M^2 is in terms of plane of non-physical polarizations so that gauge conditions have purely number theoretical interpretation.

2. In principle, the condition that $T(X^4)$ contains M^2 can be replaced with a weaker condition that either of the two light-like vectors of M^2 is contained in it since already this condition assigns to $T(X^4)$ M^2 and the map $H \rightarrow M^8$ becomes possible. Only this weaker form applies in the case of massless extremals [D1] as will be found.

3. The original idea was that hyper-quaternionic 4-surfaces in M^8 containing $M^2 \subset M^4$ in their tangent space could correspond to preferred extremals of Kähler action. This condition does not seem to be consistent with what is known about the extremals of Kähler action. The weaker form of the hypothesis is that hyper-quaternionicity holds only for 4-D tangent spaces of $X_l^3 \subset H = M^4 \times CP_2$ identified as wormhole throats or boundary components lifted to 3-surfaces in 8-D tangent space M^8 of H. The minimal hypothesis would be that only $T(X^4(X_l^3))$ at X_l^3 is associative that is hyper-quaternionic for fixed M^2. $X_l^3 \subset M^8$ and $T(X^4(X_l^3))$ at X_l^3 can be mapped to $X_l^3 \subset H$ if tangent space contains also $M_\pm \subset M^2$ or $M^2 \subset M^4 \subset M^8$ itself having interpretation as preferred hyper-complex plane. This condition is not satisfied by all surfaces X_l^3 as is clear from the fact that the inverse map involves local E^4 translation. The requirements that the distribution of hyper-quaternionic planes containing M^2 corresponds to a distribution of 4-D tangent planes should fix the E^4 translation to a high degree.

4. A natural requirement is that the image of $X_l^3 \subset H$ in M^8 is light-like. The condition that the determinant of induced metric vanishes gives an additional condition reducing the number of free parameters by one. This condition cannot be formulated as a condition on CP_2 coordinate characterizing the hyper-quaternionic tangent plane. Since M^4 projections are same for the two representations, this condition is satisfied if the contributions from CP_2 and E^4 and projections to the induced metric are identical: $s_{kl}\partial_\alpha s^k \partial_\beta s^l = e_{kl}\partial_\alpha e^k \partial_\beta e^l$. This condition means that only a subset of light-like surfaces of M^8 are realized physically. One might argue that this is as it must be since the volume of E^4 is infinite and that of CP_2 finite: only an infinitesimal portion of all possible light-like 3-surfaces in M^8 can can have H counterparts. The conclusion would be that number theoretical compactification is 4-D isometry between $X^4 \subset H$ and $X^4 \subset M^8$ at X_l^3. This unproven conjecture is unavoidable.

5. $M^2 \subset T(X^4(X_l^3))$ condition fixes $T(X^4(X_l^3))$ in the generic case by extending the tangent space of X_l^3, and the construction of configuration space spinor structure fixes boundary conditions completely by additional conditions necessary when X_l^3 corresponds to a light-like 3 surfaces defining wormhole throat at which the signature of induced metric changes. What is especially beautiful that only the data in $T(X^4(X_l^3))$ at X_l^3 is needed to calculate the vacuum functional of the theory as Dirac determinant: the only remaining conjecture (strictly speaking un-necessary but realistic looking) is that this determinant gives exponent of Kähler action for the preferred extremal and there are excellent hopes for this by the structure of the basic construction.

The basic criticism relates to the condition that light-like 3-surfaces are mapped to light-like 3-surfaces guaranteed by the condition that $M^8 - H$ duality is isometry at X_l^3.

2.3.3 Strong form of $M^8 - H$ duality

The proposed picture is the minimal one. One can of course ask whether the original much stronger conjecture that the preferred extrema of Kähler action correspond to hyper-quaternionic surfaces could make sense in some form. One can also wonder whether one could allow the choice of the plane M^2 of non-physical polarization to be local so that one would have $M^2(x) \subset M^4 \subset M^4 \times E^4$, where M^4 is fixed hyper-quaternionic sub-space of M^8 and identifiable as M^4 factor of H.

1. If M^2 is same for all points of X_l^3, the inverse map $X_l^3 \subset H \to X_l^3 \subset M^8$ is fixed apart from possible non-uniquencess related to the local translation in E^4 from the condition that hyper-quaternionic planes represent light-like tangent 4-planes of light-like 3-surfaces. The question is whether not only X_l^3 but entire four-surface $X^4(X_l^3)$ could be mapped to the tangent space of M^8. By selecting suitably the local E^4 translation one might hope of achieving the achieving this. The conjecture would be that the preferred extrema of Kähler action are those for which the distribution integrates to a distribution of tangent planes.

2. There is however a problem. What is known about extremals of Kähler action is not consistent with the assumption that fixed M^2 of $M_\pm \subset M^2$is contained in the tangent space of X^4. This suggests that one should relax the condition that $M^2 \subset M^4 \subset M^8$ is a fixed hyper-complex plane associated with the tangent space or normal space X^4 and allow M^2 to vary from point to point so that one would have $M^2 = M^2(x)$. In $M^8 \to H$ direction the justification comes from the

observation (to be discussed below) that it is possible to uniquely fix the convention assigning CP_2 point to a hyper-quaternionic plane containing varying hyper-complex plane $M^2(x) \subset M^4$.

Number theoretic compactification fixes naturally $M^4 \subset M^8$ so that it applies to any $M^2(x) \subset M^4$. Under this condition the selection is parameterized by an element of $SO(3)/SO(2) = S^2$. Note that M^4 projection of X^4 would be at least 2-dimensional in hyper-quaternionic case. In co-hyper-quaternionic case E^4 projection would be at least 2-D. $SO(2)$ would act as a number theoretic gauge symmetry and the $SO(3)$ valued chiral field would approach to constant at X_l^3 invariant under global $SO(2)$ in the case that one keeps the assumption that M^2 is fixed ad X_l^3.

3. This picture requires a generalization of the map assigning to hyper-quaternionic plane a point of CP_2 so that this map is defined for all possible choices of $M^2 \subset M^4$. Since the $SO(3)$ rotation of the hyper-quaternionic unit defining M^2 rotates different choices parameterized by S^2 to each other, a natural assumption is that the hyper-quaternionic planes related by $SO(3)$ rotation correspond to the same point of CP_2. Denoting by M^2 the standard representative of M^2, this means that for the map $M^8 \rightarrow H$ one must perform $SO(3)$ rotation of hyper-quaternionic plane taking $M^2(x)$ to M^2 and map the rotated tangent plane to CP_2 point. In $M^8 \rightarrow H$ case one must first map the point of CP_2 to hyper-quaternionic plane and rotate this plane by a rotation taking $M^2(x)$ to M^2.

4. In this framework local M^2 can vary also at the surfaces X_l^3, which considerably relaxes the boundary conditions at wormhole throats and light-like boundaries and allows much more general variety of light-like 3-surfaces since the basic requirement is that M^4 projection is at least 1-dimensional. The physical interpretation would be that a local choice of the plane of non-physical polarizations is possible everywhere in $X^4(X_l^3)$. This does not seem to be in any obvious conflict with physical intuition.

These observation provide support for the conjecture that (classical) $S^2 = SO(3)/SO(2)$ conformal field theory might be relevant for (classical) TGD.

1. General coordinate invariance suggests that the theory should allow a formulation using any light-like 3-surface X^3 inside $X^4(X_l^3)$ besides X_l^3 identified as union of wormhole throats and boundary components. For these surfaces the element $g(x) \in SO(3)$ would vary also at partonic 2-surfaces X^2 defined as intersections of $\delta CD \times CP_2$ and X^3 (here CD denotes causal diamond defined as intersection of future and past directed light-cones). Hence one could have $S^2 = SO(3)/SO(2)$ conformal field theory at X^2 (regarded as quantum fluctuating so that also $g(x)$ varies) generalizing to WZW model for light-like surfaces X^3.

2. The presence of E^4 factor would extend this theory to a classical $E^4 \times S^2$ WZW model bringing in mind string model with 6-D Euclidian target space extended to a model of light-like 3-surfaces. A further extension to X^4 would be needed to integrate the WZW models associated with 3-surfaces to a full 4-D description. General Coordinate Invariance however suggests that X_l^3 description is enough for practical purposes.

3. The choices of $M^2(x)$ in the interior of X_l^3 is dictated by dynamics and the first optimistic conjecture is that a classical solution of $SO(3)/SO(2)$ Wess-Zumino-Witten model obtained by coupling $SO(3)$ valued field to a covariantly constant $SO(2)$ gauge potential characterizes the choice of $M^2(x)$ in the interior of $M^8 \supset X^4(X_l^3) \subset H$ and thus also partially the structure of the preferred extremal. Second optimistic conjecture is that the Kähler action involving also E^4 degrees of freedom allows to assign light-like 3-surface to light-like 3-surface.

4. The best that one can hope is that $M^8 - H$ duality could allow to transform the extremely non-linear classical dynamics of TGD to a generalization of WZW-type model. The basic problem is to understand how to characterize the dynamics of CP_2 projection at each point.

In H picture there are two basic types of vacuum extremals: CP_2 type extremals representing elementary particles and vacuum extremals having CP_2 projection which is at most 2-dimensional Lagrange manifold and representing say hadron. Vacuum extremals can appear only as limiting cases of preferred extremals which are non-vacuum extremals. Since vacuum extremals have so decisive role in TGD, it is natural to requires that this notion makes sense also in M^8 picture. In particular, the notion of vacuum extremal makes sense in M^8.

This requires that Kähler form exist in M^8. E^4 indeed allows full S^2 of covariantly constant Kähler forms representing quaternionic imaginary units so that one can identify Kähler form and construct Kähler action. The obvious conjecture is that hyper-quaternionic space-time surface is extremal of this Kähler action and that the values of Kähler actions in M^8 and H are identical. The elegant manner to achieve this, as well as the mapping of vacuum extremals to vacuum extremals and the mapping of light-like 3-surfaces to light-like 3-surfaces is to assume that $M^8 - H$ duality is Kähler isometry so that induced Kähler forms are identical.

This picture contains many speculative elements and some words of warning are in order.

1. Light-likeness conjecture would boil down to the hypothesis that $M^8 - H$ correspondence is Kähler isometry so that the metric and Kähler form of X^4 induced from M^8 and H would be identical. This would guarantee also that Kähler actions for the preferred extremal are identical. This conjecture is beautiful but strong.

2. The slicing of $X^4(X^3_l)$ by light-like 3-surfaces is very strong condition on the classical dynamics of Kähler action and does not make sense for pieces of CP_2 type vacuum extremals.

Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative

The 8-dimensionality of M^8 allows to consider both associativity (hyper-quaternionicity) of the tangent space and associativity of the normal space- let us call this co-assosiativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \rightarrow k$ duality.

Are the known extremals of Kähler action consistent with the strong form of $M^8 - H$ duality

It is interesting to check whether the known extremals of Kähler action [D1] are consistent with strong form of $M^8 - H$ duality assuming that M^2 or its light-like ray is contained in $T(X^4)$ or normal space.

1. CP_2 type vacuum extremals correspond cannot be hyper-quaternionic surfaces but co-hyper-quaternionicity is natural for them. In the same manner canonically imbedded M^4 can be only hyper-quaternionic.

2. String like objects are associative since tangent space obviously contains $M^2(x)$. Objects of form $M^1 \times X^3 \subset M^4 \times CP_2$ do not have M^2 either in their tangent space or normal space in H. So that the map from $H \rightarrow M^8$ is not well defined. There are no known extremals of Kähler action of this type. The replacement of M^1 random light-like curve however gives vacuum extremal with vanishing volume, which need not mean physical triviality since fundamental objects of the theory are light-like 3-surfaces.

3. For canonically imbedded CP_2 the assignment of $M^2(x)$ to normal space is possible but the choice of $M^2(x) \subset N(CP_2)$ is completely arbitrary. For a generic CP_2 type vacuum extremals M^4 projection is a random light-like curve in $M^4 = M^1 \times E^3$ and $M^2(x)$ can be defined uniquely by the normal vector $n \in E^3$ for the local plane defined by the tangent vector dx^μ/dt and acceleration vector d^2x^μ/dt^2 assignable to the orbit.

4. Consider next massless extremals. Let us fix the coordinates of X^4 as $(t,z,x,y) = (m^0,m^2,m^1,m^2)$. For simplest massless extremals CP_2 coordinates are arbitrary functions of variables $u = k\cdot m = t-z$ and $v = \epsilon\cdot m = x$, where $k = (1,1,0,0)$ is light-like vector of M^4 and $\epsilon = (0,0,1,0)$ a polarization vector orthogonal to it. Obviously, the extremals defines a decomposition $M^4 = M^2\times E^2$. Tangent space is spanned by the four H-vectors $\nabla_\alpha h^k$ with M^4 part given by $\nabla_\alpha m^k = \delta^k_\alpha$ and CP_2 part by $\nabla_\alpha s^k = \partial_u s^k k_\alpha + \partial_v s^k \epsilon_\alpha$.

   The normal space cannot contain M^4 vectors since the M^4 projection of the extremal is M^4. To realize hyper-quaternionic representation one should be able to from these vector two vectors of M^2, which means linear combinations of tangent vectors for which CP_2 part vanishes. The vector $\partial_t h^k - \partial_z h^k$ has vanishing CP_2 part and corresponds to M^4 vector $(1,-1,0,0)$ fix assigns to each point the plane M^2. To obtain M^2 one would need $(1,1,0,0)$ too but this is not possible. The vector $\partial_y h^k$ is M^4 vector orthogonal to ϵ but M^2 would require also $(1,0,0,0)$. The proposed generalization of massless extremals allows the light-like line $M_\pm$ to depend on point of M^4 [D1], and leads to the introduction of Hamilton-Jacobi coordinates involving a local decomposition of M^4 to $M^2(x)$ and its orthogonal complement with light-like coordinate lines having interpretation as curved light rays. $M^2(x)\subset T(X^4)$ assumption fails fails also for vacuum extremals of form $X^1\times X^3\subset M^4\times CP_2$, where X^1 is light-like random curve. In the latter case, vacuum property follows from the vanishing of the determinant of the induced metric.

5. The deformations of string like objects to magnetic flux quanta are basic conjectural extremals of Kähler action and the proposed picture supports this conjecture. In hyper-quaternionic case the assumption that local 4-D tangent plane of X^3 contains $M^2(x)$ but that $T(X^3)$ does not contain it, is very strong. It states that $T(X^4)$ at each point can be regarded as a product $M^2(x)\times T^2$, $T^2\subset T(CP_2)$, so that hyper-quaternionic X^4 would be a collection of Cartesian products of infinitesimal 2-D planes $M^2(x)\subset M^4$ and $T^2(x)\subset CP_2$. The extremals in question could be seen as local variants of string like objects $X^2\times Y^2\subset M^4\times CP_2$, where X^2 is minimal surface and Y^2 holomorphic surface of CP_2. One can say that X^2 is replaced by a collection of infinitesimal pieces of $M^2(x)$ and Y^2 with similar pieces of homologically non-trivial geodesic sphere $S^2(x)$ of CP_2, and the Cartesian products of these pieces are glued together to form a continuous surface defining an extremal of Kähler action. Field equations would pose conditions on how $M^2(x)$ and $S^2(x)$ can depend on x. This description applies to magnetic flux quanta, which are the most important must-be extremals of Kähler action.

Geometric interpretation of strong M^8-H duality

In the proposed framework M^8-H duality would have a purely geometric meaning and there would nothing magical in it.

1. $X^4(X^3_l)\subset H$ could be seen a curve representing the orbit of a light-like 3-surface defining a 4-D surface. The question is how to determine the notion of tangent vector for the orbit of X^3_l. Intuitively tangent vector is a one-dimensional arrow tangential to the curve at point X^3_l. The identification of the hyper-quaternionic surface $X^4(X^3_l)\subset M^8$ as tangent vector conforms with this intuition.

2. One could argue that M^8 representation of space-time surface is kind of chart of the real space-time surface obtained by replacing real curve by its tangent line. If so, one cannot avoid the question under which conditions this kind of chart is faithful. An alternative interpretation is that a representation making possible to realize number theoretical universality is in question.

3. An interesting question is whether $X^4(X^3_l)$ as orbit of light-like 3-surface is analogous to a geodesic line -possibly light-like- so that its tangent vector would be parallel translated in the sense that $X^4(X^3)$ for any light-like surface at the orbit is same as $X^4(X^3_l)$. This would give justification for the possibility to interpret space-time surfaces as a geodesic of configuration space: this is one of the first -and practically forgotten- speculations inspired by the construction of configuration space geometry. The light-likeness of the geodesic could correspond at the level of X^4 the possibility to decompose the tangent space to a direct sum of two light-like spaces and 2-D transversal space producing the foliation of X^4 to light-like 3-surfaces X^3_l along light-like curves.

4. M^8-H duality would assign to X_l^3 classical orbit and its tangent vector at X_l^3 as a generalization of Bohr orbit. This picture differs from the wave particle duality of wave mechanics stating that once the position of particle is known its momentum is completely unknown. The outcome is however the same: for X_l^3 corresponding to wormhole throats and light-like boundaries of X^4, canonical momentum densities in the normal direction vanish identically by conservation laws and one can say that the the analog of (q,p) phase space as the space carrying wave functions is replaced with the analog of subspace consisting of points $(q,0)$. The dual description in M^8 would not be analogous to wave functions in momentum space space but to those in the space of unique tangents of curves at their initial points.

The Kähler and spinor structures of M^8

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action. As found, this leads to the conclusion that theM^8-H duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units.

2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The free Kähler forms could thus allow to produce M^8 counterparts of these gauge potentials possessing same couplings as their H counterparts. This picture would produce parity breaking in M^8 picture correctly.

4. Only the charged parts of classical electro-weak gauge fields would be absent. This would conform with the standard thinking that charged classical fields are not important. The predicted classical W fields is one of the basic distinctions between TGD and standard model and in this framework. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

5. Also super-symmetries of quantum TGD crucial for the construction of configuration space geometry force this picture. In the absence of coupling to Kähler gauge potential all constant spinor fields and their conjugates would generate super-symmetries so that M^8 would allow $N=8$ super-symmetry. The introduction of the coupling to Kähler gauge potential in turn means that all covariantly constant spinor fields are lost. Only the representation of all three neutral parts of electro-weak gauge potentials in terms of three independent Kähler gauge potentials allows right-handed neutrino as the only super-symmetry generator as in the case of H.

6. The $SO(3)$ element characterizing $M^2(x)$ is fixed apart from a local $SO(2)$ transformation, which suggests an additional $U(1)$ gauge field associated with $SO(2)$ gauge invariance and representable

as Kähler form corresponding to a quaternionic unit of E^4. A possible identification of this gauge field would be as a part of electro-weak gauge field.

M^8 dual of configuration space geometry and spinor structure?

If one introduces M^8 spinor structure and preferred extremals of M^8 Kähler action, one cannot avoid the question whether it is possible or useful to formulate the notion of configuration space geometry and spinor structure for light-like 3-surfaces in M^8 using the exponent of Kähler action as vacuum functional.

1. The isometries of the configuration space in M^8 and H formulations would correspond to symplectic transformation of $\delta M^4_\pm \times E^4$ and $\delta M^4_\pm \times CP_2$ and the Hamiltonians involved would belong to the representations of $SO(4)$ and $SU(3)$ with 2-dimensional Cartan sub-algebras. In H picture color group would be the familiar $SU(3)$ but in M^8 picture it would be $SO(4)$. Color confinement in both $SU(3)$ and $SO(4)$ sense could allow these two pictures without any inconsistency.

2. For $M^4 \times CP_2$ the two spin states of covariantly constant right handed neutrino and antineutrino spinors generate super-symmetries. This super-symmetry plays an important role in the proposed construction of configuration space geometry. As found, this symmetry would be present also in M^8 formulation so that the construction of M^8 geometry should reduce more or less to the replacement of CP_2 Hamiltonians in representations of $SU(3)$ with E^4 Hamiltonians in representations of $SO(4)$. These Hamiltonians can be taken to be proportional to functions of E^4 radius which is $SO(4)$ invariant and these functions bring in additional degree of freedom.

3. The construction of Dirac determinant identified as a vacuum functional can be done also in M^8 picture and the conjecture is that the result is same as in the case of H. In this framework the construction is much simpler due to the flatness of E^4. In particular, the generalized eigen modes of the Chern-Simons Dirac operator D_{C-S} identified as zero modes of 4-D Dirac operator D_K restricted to the X^3_l correspond to a situation in which one has fermion in induced Maxwell field mimicking the neutral part of electro-weak gauge field in H as far as couplings are considered. Induced Kähler field would be same as in H. Eigen modes are localized to regions inside which the Kähler magnetic field is non-vanishing and apart from the fact that the metric is the effective metric defined in terms of canonical momentum densities via the formula $\hat{\Gamma}^\alpha = \partial L_K/\partial h^k_\alpha \Gamma_k$ for effective gamma matrices. This in fact, forces the localization of modes implying that their number is finite so that Dirac determinant is a product over finite number eigenvalues. It is clear that M^8 picture could dramatically simplify the construction of configuration space geometry.

4. The eigenvalue spectra of the transversal parts of D_K operators in M^8 and H should identical. This motivates the question whether it is possible to achieve a complete correspondence between H and M^8 pictures also at the level of spinor fields at X^3 by performing a gauge transformation eliminating the classical W gauge boson field altogether at X^3_l and whether this allows to transform the modified Dirac equation in H to that in M^8 when restricted to X^3_l. That something like this might be achieved is supported by the fact that in Coulombic gauge the component of gauge potential in the light-like direction vanishes so that the situation is effectively 2-dimensional and holonomy group is Abelian.

Why $M^8 - H$ duality is useful?

Skeptic could of course argue that $M^8 - H$ duality produces only an inflation of unproven conjectures. There are however strong reasons for $M^8 - H$ duality: both theoretical and physical.

1. The map of $X^3_l \subset H \rightarrow X^3_l \subset M^8$ and corresponding map of space-time surfaces would allow to realize number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as natural coordinates in which one can say what it means that the point of imbedding space is rational/algebraic. The point of $X^4 \subset H$ is algebraic if it is mapped to an algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactication could in fact be motivated by the number theoretical universality.

2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action since the Kähler form in E^4 has constant components. If the spinor connection in E^4 is combination of the three Kähler forms mimicking neutral part of electro-weak gauge potential, the eigenvalue spectrum for the modified Dirac operator would correspond to that for a fermion in $U(1)$ magnetic field defined by an Abelian magnetic field whereas in $M^4 \times CP_2$ picture $U(2)_{ew}$ magnetic fields would be present.

3. $M^8 - H$ duality provides insights to low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

2.3.4 $M^8 - H$ duality and low energy hadron physics

The description of $M^8 - H$ at the configuration space level can be applied to gain a view about color confinement and its dual for electro-weak interactions at short distance limit. The basic idea is that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quark color using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since configuration space degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass

calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [F4].

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

2.3.5 The notion of number theoretical braid

The notion of number theoretic braid is essential for the view about quantum TGD as almost topological quantum field theory. It also realization discretization as a space-time correlate for the finite measurement resolution. Number theoretical universality leads to this notion also and requires that the points in the intersection of the number theoretic braid with partonic 2-surface correspond to rational or at most algebraic points of H in preferred coordinates fixed by symmetry considerations. The challenge has been to find a unique identification of the number theoretic braid. Number theoretic vision indeed makes this possible.

The core element of number theoretic vision is that the laws of physics could be reduced to associativity conditions. One realization for associativity conditions is the level of M^8 endowed with hyper-octonionic structure as a condition that the points sets possible as arguments of N-point function in X^4 are associative and thus belong to hyper-quaternionic subspace $M^4 \subset M^8$. This decomposition must be consistent with the $M^4 \times E^4$ decomposition implied by $M^4 \times CP_2$ decomposition of H. What comes first in mind is that partonic 2-surfaces X^2 belong to $\delta M^4_\pm \subset M^8$ defining the ends of the causal diamond and are thus associative. This boundary condition however freezes E^4 degrees of freedom completely so that M^8 configuration space geometry trivializes.

Are the points of number theoretic braid commutative?

One can also consider the commutativity condition by requiring that arguments belong to a preferred commutative hyper-complex sub-space M^2 of M^8 which can be regarded as a curve in complex plane. Fixing preferred real and imaginary units means a choice of M^2 interpreted as a partial choice of quantization axes at the level of M^8. One must distinguish this choice from the hyper-quaternionicity of space-time surfaces and from the condition that each tangent space of X^4 contains $M^2(x) \subset M^4$ in its tangent space or normal space. Commutativity condition indeed implies the notion of number theoretic braid and fixes it uniquely once a global selection of $M^2 \subset M^8$ is made. There is also an alternative identification of number theoretic braid based on the assumption that braids are light-like curves with tangent vector in $M^2(x)$.

1. The strong form of commutativity condition would require that the arguments of the n-point function at partonic 2-surface belong to the intersection $X^2 \cap M_\pm$. This however allows quite too few points since an intersection of 2-D and 1-D objects in 7-D space would be in question. Associativity condition would reduce cure the problem but would trivialize configuration space geometry.

2. The weaker condition that only $\delta M^4_\pm$ projections for the points of X^2 commute is however sensible since the intersection of 1-D and 2-D surfaces of 3-D space results. This condition is also invariant under number theoretical duality. In the generic case this gives a discrete set of points as intersection of light-like radial geodesic and the projection $P_{\delta M^4_\pm}(X^2)$. This set is naturally identifiable in terms of points in the intersection of number theoretic braids with $\delta CD \times E^4$. One should show that this set of points consists of rational or at most algebraic points. Here the possibility to choose X^2 to some degree could be essential. Any radial light ray from the tip of light-cone allows commutativity and one can consider the possibility of integrating over n-point functions with arguments at light ray to obtain maximal information.

3. For the pre-images of light-like 3-surfaces commutativity of the points in $\delta M^4_\pm$ projection would allow the projections to be one-dimensional curves of M^2 having thus interpretation as braid strands. M^2 would play exactly the same role as the plane into which braid strands are projected in the construction of braid invariants. Therefore the plane of non-physical polarizations in gauge theories corresponds to the plane to which braids and knots are projected in braid and knot theories. A further constraint is that the braid strand connects algebraic points of M^8 to

algebraic points of M^8. It seems that this can be guaranteed only by posing some additional conditions to the light-like 3-surfaces themselves which is of course possible since they are in the role of fundamental dynamical objects.

4. An alternative identification of the number theoretic braid would give up commutativity condition for M^4 projection and assume braid strand to be as a light-like curve having light-like tangent belonging to the local hyper-complex tangent sub-space $M^2(x)$ at point x. This definition would apply both in $X^3 \subset \delta M^4_\pm \times CP_2$ and in X^3_l. Also now one would have a continuous distribution of number theoretic braids, with one braid assignable to each light-like curve with tangent $\delta M^4_+ \supset M_+(x) \subset M^2(x)$. In this case each light-like curve at δM^4_+ with tangent in $M_+(x)$ would define a number theoretic braid so that the only difference would be the replacement of light-like ray with a more general light-like curve.

Are number theoretic braids light-like curves with tangent in $M^2(x)$?

There are reasons why the identification of the number theoretic braid strand as a curve having hyper-complex light-like tangent looks more attractive.

1. The preferred plane $M^2(x)$ can be interpreted as the local plane of non-physical polarizations so that the interpretation as a number theoretic analog of gauge conditions posed in both quantum field theories and string models is possible. In TGD framework this would mean that super-conformal degrees of freedom are restricted to the orthogonal complement of $M^2(x)$ and $M^2(x)$ does not contribute to the configuration space metric. In Hamilton-Jacobi coordinates the pairs of light-like curves associated with coordinate lines can be interpreted as curved light rays. Hence the partonic planes $M^2(x_i)$ associated with the points of the number theoretic braid could be also regarded as carriers four-momenta of fermions associated with the braid strands so that the standard gauge conditions $\epsilon \cdot p = 0$ for polarization vector and four-momentum would be realized geometrically. The possibility of M^2 to depend on point of X^3_l would be essential to have non-collinear momenta and for a classical description of interactions between braid strands.

2. One could also define analogs of string world sheets as sub-manifolds of $P_{M^4_+}(X^4)$ having $M^2(x) \subset M^4$ as their tangent space or being assignable to their tangent containing $M_+(x)$ in the case that the distribution defined by the planes $M^2(x)$ exists and is integrable. It must be emphasized that in the case of massless extremals one can assign only $M_+(x) \subset M^4$ to $T(X^4(x))$ so that only a foliation of X^4 by light-like curves in M^4 is possible. For $P_{M^4_+}(X^4)$ however a foliation by 2-D stringy surfaces is obtained. Integrability of this distribution and thus the duality with stringy description has been suggested to be a basic feature of the preferred extremals and is equivalent with the existence of Hamilton-Jacobi coordinates for a large class of extremals of Kähler action [D1].

3. The possibility of dual descriptions based on integrable distribution of planes $M^2(x)$ allowing identification as 2-dimensional stringy sub-manifolds of $X^4(X^3)$ and the flexibility provided by the hyper-complex conformal invariance raise the hopes of achieving the lifting of super-symplectic algebra SS and super Kac-Moody algebra SKM to H. At the light-cone boundary the light-like radial coordinate could be lifted to a hyper-complex coordinate defining coordinate for M^2. At X^3_l one could fix the light-like coordinate varying along the braid strands and it can can be lifted to a light-like hyper-complex coordinate in M^4 by requiring that the tangent to the coordinate curve is light-like line of $M^2(x)$ at point x. The total four-momenta and color quantum numbers assignable to SS and SKM degrees of freedom are naturally identical since they can be identified as the four-momentum of the partonic 2-surface $X^2 \subset X^3 \cap \delta M^4_\pm \times CP_2$. Equivalence Principle would emerge as an identity.

Are also CP_2 duals of number theoretic braids possible?

This picture is probably not enough. From the beginning the idea that also the CP_2 projections of points of X^2 define number theoretic braids has been present. The dual role of the braids defined by M^2 and CP_2 projections of X^2 is suggested both by the construction of the symplectic fusion algebras [C4] and by the model of anyons [F12]. M^2 and the geodesic sphere $S^2_i \subset CP_2$, where one has either $i = I$ or $i = II$, where $i = I/II$ corresponds to homologically trivial/non-trivial geodesic sphere, are in a key role in the geometric realization of the hierarchy of Planck constants in terms of the book like structure of the generalized imbedding space. The fact that S^2_I corresponds to vacuum extremals would suggest that only the intersection $S^2_{II} \cap P_{CP_2}(X^2)$ can define CP_2 counterpart of the number theoretic braid. M^4 braid could be the proper description in the associative case (Minkowskian signature of induced metric) and CP_2 braid in the co-associative case (Euclidian signature of induced metric). The duality of these descriptions would be reflected also by the fact that the physical Planck constant is given by $\hbar = r\hbar_0$, $r = \hbar(M^4)/\hbar(CP_2)$, so that only the ratio of the two Planck constants matters in commutation relations.

What about symplectic contribution to number theoretic braids?

Also the symplectically invariant degrees of freedom representing zero modes must be treated and this leads to the notion of symplectic QFT. The explicit construction of symplectic fusion rules has been discussed in [C4]. These rules make sense only as a discretized version. Discreteness can be understood also as a manifestation of finite measurement resolution: at this time it is associated with the impossibility to know the induced Kähler form at each point of partonic 2-surface. What one can measure is the Kähler flux associated with a triangle and the density of triangulation determines the measurement accuracy. The discrete set of points associated with the symplectic algebra characterizes the measurement resolution and there is an infinite hierarchy of symplectic fusion algebras corresponding to gradually increasing measurement resolution in classical sense [C4].

Second interesting question is whether the symplectic triangulation could be used to represent a hierarchy of cutoffs of super conformal algebras by introducing additional fermionic oscillators at the points of the triangulation. The M^4 coordinates at the points of symplectic triangulation of S^2_i, $i = I, II$ projection and CP_2 coordinates at the points of symplectic triangulation of S^2 could define discrete version of quantized conformal fields. The functional integral over symplectic group would mean integral over symplectic triangulations. Note that M^2 number theoretic braid is trivial as symplectic triangulation since the points are along light-like geodesic of $\delta M^4_\pm$.

In the original variant of symplectic triangulation [C4] the exact form of triangulation was left free. It would be however nice if symplectic triangulation could be fixed purely physically by the properties of the induced Kähler form since also the number of fermionic oscillator modes and number theoretical braids is fixed by the dynamics of Kähler action.

1. A symplectically invariant manner to fix the nodes of the triangulation could be in terms of extrema of the symplectic invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ (the dependence on metric is only apparent). Here the Kähler forms of both S^2 and CP_2 can be considered. The maxima for the magnitude of Kähler magnetic field are indeed natural observables as also the areas of projections of X^2 to S^2. The nodes are completely fixed by dynamics and the contribution to number theoretic braid involves no ad hoc elements. Physical intuition suggests that this is not enough: magnetic flux quantization is what strongly suggests itself as additional source of braid points.

2. $J = constant$ curves define the analogs of height curves surrounding the extrema of J. Inside each region where J has definite sign, the quantization of the Kähler magnetic flux defines a collection of height curves bounding disks for which Kähler magnetic flux is given by $Flux = \int_{J<J_q} JdS = q2\pi r$, where $r = \hbar/\hbar_0$ and q are rational.

3. Symplectic and Kac-Moody algebras [B2] algebras are local with respect to X^2 but the dependence is only through J. Hence the analogy with conformal field theory would suggest that the quantization of the fermionic oscillator operators should treat $J = constant$ curve more or less as a single point or at most as a disrete point set. Hence the addition of height curves would give additional "points" to the number theoretic braid.

4. Could one reduce the set of symplectic height curves to a discrete point set? The canonically conjugate coordinate Φ for J (analogous to canonical momentum) defined with respect to the symplectic form $\epsilon^{\mu\nu}$ of X^2 and by the condition $\{\Phi, J\} = 1$ defines an angle variable varying in the range $(0, 2\pi)$. The flux would be given in these coordinates simply as $Flux = \int_{J_q} Jd\Phi = 2\pi J = q \times 2\pi r$ so that $J = qr$ would be rational valued for rational values of magnetic flux. Rational values $\Phi = m2\pi/n$ would divide symplectic disks with quantized flux to quadrangles

with quantized flux reduced by factor $1/n$. Symplectic transformations of $\delta M^4_\pm \times CP_2$ and of X^2 would leave the fluxes invariant. A discrete point set could be selected as the intersection of the coordinate curves associated with J and Φ and would define number theoretic braid, which can be used in the second quantization of the induced spinor fields.

5. If the precise specification of the edges of the triangulation [C4] has any physical meaning, this meaning must come from the quantization of magnetic fluxes for symplectic triangles and from their unique specification. A possible definition of symplectic triangulation satisfying these criteria relies on the observation that $J = constant$ and $\Phi = constant$ coordinate curves divide the region surrounding given extremum of J to quadrangles. By connecting the vertices of quadrangles by straight lines in linear coordinates defined by J and Φ, one obtains unique symplectic triangulation with rationally quantized fluxes. Also sub-triangulations with the same property can be constructed.

To sum up, the symplectic contribution to all three types of number theoretic braids could be present and would differ from the above described contribution in that the points of the braid are not critical with respect to phase transitions changing Planck constant.

What makes braids number theoretic?

Are braids always number theoretic or are they number theoretic only under special conditions which might be said to characterize number theoretic criticality. To answer these questions one must define precisely what one means with number theoretic universality, which has been one of the basic guide lines in the construction of quantum TGD. There are two forms of the principle.

1. The strong form of number theoretical universality states that physics for any system should effectively reduce to a physics in algebraic extension of rational numbers at the level of M-matrix so that an interpretation in both real and p-adic sense (allowing a suitable algebraic extension of p-adics) is possible. One can however worry whether this principle only means that physics is algebraic so that there would be no need to talk about real and p-adic physics at the level of M-matrix elements. It is not possible to get rid of real and p-adic numbers at the level of classical physics since calculus is a prerequisite for the basic variational principles used to formulate the theory. For this option the possibility of completion is what poses conditions on M-matrix.

2. The weak form of principle requires only that both real and p-adic variants of physics make sense and that the intersection of these physics consist of physics associated with various algebraic extensions of rational numbers. In this rational physics would be like rational numbers allowing infinite number of algebraic extensions and real numbers and p-adic number fields as its completions. Real and p-adic physics would be completions of rational physics. In this framework criticality with respect to phase transitions changing number field becomes a viable concept. This form of principle allows also purely p-adic phenomena such as p-adic pseudo non-determinism assigned to imagination and cognition. Genuinely p-adic physics does not however allow definition of notions like conserved quantities since the notion of definite integral is lacking and only the purely local form of real physics allows p-adic counterpart.

Experience has taught that it is better to avoid too strong statements and perhaps the weak form of the principle is enough. It is however clear that number theoretical criticality could provide important insights to quantum TGD: p-adic thermodynamics is excellent example of this. In consciousness theory the transitions transforming intentions to actions and actions to cognitions would be key applications and number theoretic criticality would be almost defining feature of living matter. Needless to say, zero energy ontology is absolutely essential: otherwise this kind of transitions would not make sense.

Number theoretical criticality (or number theoretical universality in strong sense) requires that M-matrix elements are algebraic numbers. This is achieved naturally if the definition of M-matrix elements involves only the data associated with the number theoretic braid with the property that the coordinates for the points of imbedding space in question are algebraic numbers and that possible other data are also algebraic. This point has been discussed in more detail in [C3].

2.3.6 Connection with string model and Equivalence Principle at space-time level

Coset construction allows to generalize Equivalence Principle and understand it at quantum level. This is however not quite enough: a precise understanding of Equivalence Principle is required also at the classical level. Also the mechanism selecting via stationary phase approximation a preferred extremal of Kähler action providing a correlation between quantum numbers of the particle and geometry of the preferred extremals is still poorly understood.

Is stringy action principle coded by the geometry of preferred extremals?

It seems very difficult to deduce Equivalence Principle as an identity of gravitational and inertial masses identified as Noether charges associated with corresponding action principles. Since string model is an excellent theory of quantum gravitation, one can consider a less direct approach in which one tries to deduce a connection between classical TGD and string model and hope that the bridge from string model to General Relativity is easier to build. Number theoretical compactification gives good hopes that this kind of connection exists.

1. Number theoretic compactification implies that the preferred extremals of Kähler action have the property that one can assign to each point of M^4 projection $P_{M^4}(X^4(X^3_l))$ of the preferred extremal $M^2(x)$ identified as the plane of non-physical polarizations and also as the plane in which local massless four-momentum lies.

2. If the distribution of the planes $M^2(x)$ is integrable, one can slice $P_{M^4}(X^4(X^3_l))$ to string world-sheets. The intersection of string world sheets with $X^3 \subset \delta M^4_\pm \times CP_2$ corresponds to a light-like curve having tangent in local tangent space $M^2(x)$ at light-cone boundary. This is the first candidate for the definition of number theoretic braid. Second definition assumes M^2 to be fixed at δCD: in this case the slicing is parameterized by the sphere S^2 defined by the light rays of $\delta M^4_\pm$.

3. One can assign to the string world sheet -call it Y^2 - the standard area action

$$S_G(Y^2) = \int_{Y^2} T\sqrt{g_2}d^2y \ , \tag{2.3.1}$$

where g_2 is either the induced metric or only its M^4 part. The latter option looks more natural since M^4 projection is considered. T is string tension.

4. The naivest guess would be $T = 1/\hbar G$ apart from some numerical constant but one must be very cautious here since $T = 1/L_p^2$ apart from a numerical constant is also a good candidate if one accepts the basic argument identifying G in terms of p-adic length L_p and Kähler action for two pieces of CP_2 type vacuum extremals representing propagating graviton. The formula reads $G = L_p^2 exp(-2aS_K(CP_2))$, $a \leq 1$ [A9, C4]. The interaction strength which would be L_p^2 without the presence of CP_2 type vacuum extremals is reduced by the exponential factor coming from the exponent of Kähler function of configuration space.

5. One would have string model in either $CD \times CP_2$ or $CD \subset M^4$ with the constraint that stringy world sheet belongs to $X^4(X^3_l)$. For the extremals of $S_G(Y^2)$ gravitational four-momentum defined as Noether charge is conserved. The extremal property of string world sheet need not however be consistent with the preferred extremal property. This constraint might bring in coupling of gravitons to matter. The natural guess is that graviton corresponds to a string connecting wormhole contacts. The strings could also represent formation of gravitational bound states when they connect wormhole contacts separated by a large distance. The energy of the string is roughly $E \sim \hbar TL$ and for $T = 1/\hbar G$ gives $E \sim L/G$. Macroscopic strings are not allowed except as models of black holes. The identification $T \sim 1/L_p^2$ gives $E \sim \hbar L/L_p^2$, which does not favor long strings for large values of $\hbar$. The identification $G_p = L_p^2/\hbar_0$ gives $T = 1/\hbar G_p$ and $E \sim \hbar_0 L/L_p^2$, which makes sense and allows strings with length not much longer than p-adic length scale. Quantization - that is the presence of configuration space degrees of freedom-

would bring in massless gravitons as deformations of string whereas strings would carry the gravitational mass.

6. The exponent $exp(iS_G)$ can appear as a phase factor in the definition of quantum states for preferred extremals. S_G is not however enough. One can assign also to the points of number theoretic braid action describing the interaction of a point like current Qdx^μ/ds with induced gauge potentials A_μ. The corresponding contribution to the action is

$$S_{braid} = \int_{braid} iTr(Q\frac{dx^\mu}{ds}A_\mu)dx \quad . \tag{2.3.2}$$

In stationary phase approximation subject to the additional constraint that a preferred extremal of Kähler action is in question one obtains the desired correlation between the geometry of preferred extremal and the quantum numbers of elementary particle. This interaction term carries information only about the charges of elementary particle. It is quite possible that the interaction term is more complex: for instance, it could contain spin dependent terms (Stern-Gerlach experiment).

7. The constraint coming from preferred extremal property of Kähler action can be expressed in terms of Lagrange multipliers

$$S_c = \int_{Y^2} \lambda^k D_\alpha(\frac{\partial L_K}{\partial_\alpha h^k})\sqrt{g_2}d^2y \quad . \tag{2.3.3}$$

8. The action exponential reads as

$$exp(iS_G + S_{braid} + S_c) \quad . \tag{2.3.4}$$

The resulting field equations couple stringy M^4 degrees of freedom to the second variation of Kähler action with respect to M^4 coordinates and involve third derivatives of M^4 coordinates at the right hand side. If the second variation of Kähler action with respect to M^4 coordinates vanishes, free string results. This is trivially the case if a vacuum extremal of Kähler action is in question.

9. An interesting question is whether the preferred extremal property boils down to the condition that the second variation of Kähler action with respect to M^4 coordinates or actually all coordinates vanishes so that gravitonic string is free. As a matter fact, the stronger condition is required that the Noether currents associated with the modified Dirac action are conserved. The physical interpretation would be in terms of quantum criticality which is the basic conjecture about the dynamics of quantum TGD. This is clear from the fact that in 1-D system criticality means that the potential $V(x) = ax + bx^2 + ..$ has $b = 0$. In field theory criticality corresponds to the vanishing of the term $m^2\phi^2/2$ so that massless situation corresponds to massless theory and criticality and long range correlations.

What does the equality of gravitational and inertial masses mean?

Consider next the question in what form Equivalence Principle could be realized in this framework.

1. Coset construction inspires the conjecture that gravitational and inertial four-momenta are identical. Also some milder form of it would make sense. What is clear is that the construction of preferred extremal involving the distribution of $M^2(x)$ implies that conserved four-momentum associated with Kähler action can be expressed formally as stringy four-momentum. The integral of the conserved inertial momentum current over X^3 indeed reduces to an integral over the curve

defining string as one integrates over other two degrees of freedom. It would not be surprising if a stringy expression for four-momentum would result but with string tension depending on the point of string and possibly also on the component of four-momentum. If the dependence of string tension on the point of string and on the choice of the stringy world sheet is slow, the interpretation could be in terms of coupling constant evolution associated with the stringy coordinates. An alternative interpretation is that string tension corresponds to a scalar field. A quite reasonable option is that for given X^3_l T defines a scalar field and that the observed T corresponds to the average value of T over deformations of X^3_l.

2. The minimum option is that Kähler mass is equal to the sum gravitational masses assignable to strings connecting points of wormhole throat or two different wormhole throats. This hypothesis makes sense even for wormhole contacts having size of order Planck length.

3. The condition that gravitational mass equals to the inertial mass (rest energy) assigned to Kähler action is the most obvious condition that one can imagine. The breaking of Poincare invariance to Lorentz invariance with respect to the tip of CD supports this form of Equivalence Principle. This would predict the value of the ratio of the parameter R^2T and p-adic length scale hypothesis would allow only discrete values for this parameter. $p \simeq 2^k$ following from the quantization of the temporal distance $T(n)$ between the tips of CD as $T(n) = 2^nT_0$ would suggest string tension $T_n = 2^nR^2$ apart from a numerical factor. $G_p \propto 2^nR^2/\hbar_0$ would emerge as a prediction of the theory. G can be seen either as a prediction or RG invariant input parameter fixed by quantum criticality. The arguments related to p-adic coupling constant evolution suggest $R^2/\hbar_0 G = 3\times 2^{23}$ [A9, C4].

4. The scalar field property of string tension should be consistent with the vacuum degeneracy of Kähler action. For instance, for the vacuum extremals of Kähler action stringy action is non-vanishing. The simplest possibility is that one includes the integral of the scalar $J^{\mu\nu}J_{\mu\nu}$ over the degrees transversal to M^2 to the stringy action so that string tension vanishes for vacuum extremals. This would be nothing but dimensional reduction of 4-D theory to a 2-D theory using the slicing of $X^4(X^3_l)$ to partonic 2-surfaces and stringy word sheets. For cosmic strings Kähler action reduces to stringy action with string tension $T \propto 1/g_K^2R^2$ apart from a numerical constant. If one wants consistency with $T \propto 1/L_p^2$, one must have $T \propto 1/g_K^2 2^nR^2$ for the cosmic strings deformed to Kähler magnetic flux tubes. This looks rather plausible if the thickness of deformed string in M^4 degrees of freedom is given by p-adic length scale.

2.4 Does the modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

There are two choices: either the 3-D Chern-Simons Dirac action or 4-D Kähler action. The first was suggested by the vision that the almost-topological QFT defined by $C-S$ action codes the exponent of Kähler action for the preferred extremal as Dirac determinant and also by the fact that it was difficult to imagine how to assign to D_K eigenvalue spectrum. It however turned out that D_K is needed to code for the data about the preferred extremal to the spectrum of D_{C-S}, and after that it did not take long time to realized that D_K is the correct choice. In the Appendix also $C-S$ Dirac action is analyzed in order to see its failures.

2.4.1 Modified Dirac equation

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced.

Problems associated with the ordinary Dirac action

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [B2, E2].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the configuration space geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of the configuration space geometry so that there is internal inconsistency.

Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k_{\ \alpha}} L_K \ . \end{aligned} \tag{2.4.1}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{2.4.2}$$

having a vanishing covariant divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \tag{2.4.3}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \tag{2.4.4}$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned}\hat{\Gamma}^{\alpha} D_{\alpha}\Psi &= 0 \ , \\ \hat{\Gamma}^{\alpha} &= T^{\alpha}_{l}\Gamma^{l} \ . \end{aligned} \quad (2.4.5)$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi}\hat{\Gamma}^{\alpha} D_{\alpha}\Psi \ . \quad (2.4.6)$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_{\mu}\hat{\Gamma}^{\mu} = 0 \quad (2.4.7)$$

guaranteing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_{\mu}$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_{\mu},\hat{\Gamma}_{\nu}\} = 2T^{k}_{\mu}T_{\nu k} \ . \quad (2.4.8)$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [E2]. In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X^3_l, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the symplectic currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action, Kähler coupling strength would be determined completely by the anti-commutation relations of the super-symplectic algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

Which Dirac action?

Which modified Dirac action should one choose? The four-dimensional modified Dirac action associated with Kähler action or 3-D Dirac action associated with $C-S$ action? Or something else?

1. The first guess inspired by TGD as almost-TQFT was that $C-S$ action is enough. The problems are encountered when one tries to define Dirac determinant. The eigenvalues of the modified Dirac equation are functions rather than constants and this leads to difficulties in the definition of the Dirac determinant. The proposal was that Dirac determinant could be defined as product of the the values of generalized eigenvalues in the set of points defined by the number theoretic braid. This kind of definition is however questionable since it does not have obvious connection with the standard definition.

2. Second guess was that also 4-D modified Dirac action is needed. The physical picture would be that the induced spinor fields restricted to the light-like 3-surfaces are singular solutions of 4-D Dirac operator. Since the modified Dirac equation can be written as a conservation law for super current this idea translates to the condition that the "normal" component of the super current vanishes at X^43_l and tangential component satisfies current conservation meaning that 3-D variant of modified Dirac equation results. There is a unique function of the light-like coordinate r defining the time coordinate and eigenmodes of transversal part of modified Dirac operator define the spectrum of also the modified Dirac operator associated with $C-S$ action naturally. The system is 2-dimensional and if the modes of spinor fields are localized in regions of strong induced electro-weak magnetic field, their number is finite and the Dirac determinant defined in the standard manner is finite. A close connection with anyonic systems emerges. One can indeed define the action of D_K also at the limit when the light-like 3-surface associated with a wormhole throat is approached. This limit is singular since $det(g_4)=0$ and $det(g_3)=0$ hold true at this limit. As a consequence the normal component of Kähler electric field typically diverges in accordance with the idea that at short distances $U(1)$ gauge charges approach to infinity. Also the modified Gamma matrices diverge like $1/det(g_4)^3$. One of the problems is that only light-like 3-surfaces with 2-D CP_2 projection are allowed since D_{C-S} reduces to 1-D operator only for these.

3. The third guess inspired by the results relating to the number theoretic compactification was that D_{C-S} is not needed at all! Number theoretical compactification strongly suggets dual slicings of X^4 to string word sheets Y^2 and partonic 2-surfaces X^2, and the generalized eigenvalues can be identified as those associated with the longitudinal part $D_K(Y^2)$ or transverse part $D_K(X^2)$ of the modified Dirac operator D_K. The outcome is exactly the same as for D_{C-S} except that one avoids the problems associated with it. There is also an additional symmetry: the eigenvalue spectra associated with transversal slices must be such that Kähler action gives rise to the same Kähler metric.

4. The fourth guess was the inclusion of instanton term to the action meaning complexification of Kähler action. This does not affect configuration space metric at all but brings in CP breaking and also makes possible construction of generalized Feynman diagrammatics.

2.4.2 How to define Dirac determinant?

The basic challenge is to define Dirac determinant expected to give rise to the exponent of Kähler action associated with the preferred extremal. Modified Dirac operator as such does not carry information about the preferred extremal. If D_K is to be useful, the generalized eigenvalues must carry information about the tangent space $T(X^4(X_l^3))$ at X_l^3 and also about the extremals of Kähler action with boundary conditions defined by this tangent space. Number theoretic compactification results if $T(X^4(X_l^3))$ contains the plane $M^2 \subset M^4$ in its tangent space for all points of X_l^3. As a consequence the tangent space of $X^4(X^3)$ is fixed at each point and one can in principle solve the field equations defining the extrema of Kähler action as a limiting case. Thus one can start from the assumption that it is possible to assign to X_l^3 a unique preferred extremal and that this extremal has the properties implied by number theoretical compactification.

Could the generalized eigen modes of D_{C-S} define Dirac determinant?

The basic idea is that the spinor field at X_l^3 can be regarded as a singular spinor field in $X^4(X^3)_l$ located to X_l^3 in the sense that the conserved super current associated with Ψ has vanishing normal component X_l^3 and 4-D modified Dirac equation for Kähler action reduces to the conservation of this super current. If the conditions $g_{ui} = 0$ and $J_{ui} = 0$ for the induced metric and Kähler form hold true in some coordinates at X_l^3, it is possible to realize this picture very elegantly. The conditions state the decoupling of tangential and normal dynamics of the induced metric and Kähler form.

This would suggest the identification of the generalized eigenmodes of D_{C-S} with zero modes of D_K restricted to X_l^3. To achieve this the modified Dirac operator must reduce to 1-D form for this identification to make sense. This happens only for the extremals of $C - S$ action having 2-D CP_2 projection. The value of $C-S$ action also vanishes for the extremals. These are not desirable features. One would like to have arbitrary light-like 3-surfaces as surfaces at which the signature of the induced metric changes. Thus the question is whether one could get rid of D_{C-S} completely. One can also ask whether D_{C-S} is really needed if D_K codes for all the relevant information.

Or is D_K enough?

The difficulties of D_{C-S} approach force to ask whether one could get rid of D_{C-S} inspired by the TGD as almost-topological QFT vision and replace it with D_K since D_K in any case would code the needed information. As a matter fact, the original idea was that Kähler action corresponds to a Dirac determinant assignable to D_K.

1. Quantum holography is possible if the spectrum of D_K is such that it gives the same Dirac determinant for every choice Y_l^3 in the slicing of X^4 by light-like 3-surfaces parallel to X_l^3. This requires 4-D analogs of spinor shock waves. A good guess is as 4-D modes but having no dependence on the second light-like coordinate u labeling the slices Y_l^3.

2. The generalized eigenmodes involve in an essential manner the decomposition to light-like curve and partonic 2-surface X^2. Number theoretical compactification indeed implies the slicing of X^4 by stringy 2-surfaces Y^2 and their partonic duals X^2 fundamental for the understanding of Equivalence Principle at space-time level. Therefore the natural identification of generalized eigenvalues is as those associated with the Y^2 or X^2 in accordance with parton-string duality. The situation would be exactly similar to that achieved by considering zero modes localized to Y_l^3. From the point of view of Higgs mechanism it is essential that the effective metric defined by the modified gamma matrices for Y_l^3 has Euclidian signature. By constancy in u-direction condition is the same as it would be for $C - S$ Dirac operator.

3. The super-conformal gauge symmetries assignable with the zero modes of D_{C-S} and physical intuition suggests that they should be recovered somehow. Physically they should correspond to the infinite number of eigen modes in Y^2, which correspond to the same eigenvalue and fixed eigen mode in X^2. In terms of the notion of finite measurement resolution these stringy modes would be below measurement resolution. The natural proposal is that all all modes which are not constant with respect to light-like u coordinate are zero modes.

4. The introduction of D_{C-S} is thus by no means necessary. This allows also to get rid of the condition that CP_2 projection of X_l^3 is 2-dimensional and other problems plaguing D_{C-S}.

5. What about the phase defined by the exponent of Chern-Simons action which was lost in the approach based on D_{C-S}? Could $C-S$ action emerge somehow from the theory? I have already earlier proposed that $C-S$ action could emerge as a phase of the Dirac determinant. Chern-Simons action is also associated with anyonic phases and in TGD framework anyons are possible for phases with non-standard value of Planck constant assignable to the pages of the book like structures associated with CD and CP_2. Since singular coverings and factor spaces are in question, it is possible to add to the Kähler gauge potential an anomalous gauge term. This modifies the transverse eigenvalue spectrum for the modified Dirac equation for D_K. This could bring to the Dirac determinant a phase factor. This factor could come from both CD and CP_2 degrees of freedom since one can assign gauge part of Kähler gauge potential also in CD. M^2 selects global quantization axes of angular momentum and pure gauge part $A_\phi = constant$ would induce fractionization of angular momentum. Also if A has component in M^2, a shift of the eigenvalue spectrum results. Perhaps a more plausible modification $A_\eta = constant$, where η corresponds to hyper-bolic angle of M^2 since this corresponds to Lorentz transformation in the direction of quantization axis of spin. The latter two modifications are however somewhat questionable since only the transverse part of the stringy slicing allows symplectic structure.

General vision about how the eigenmodes of D_K can code information about preferred extremal

Before doing anything practical it would be a good idea to formulate a general vision about how the eigenmodes of D_K can code information about the preferred extremal of Kähler action. In practice one is of course never able to follow this good practice, and the following arguments rely strongly on the experience gained with the erratic approach based on the identification of the spectrum of D_{C-S} with that of D_K.

1. The original vision was that almost-topological QFT should be defined by Chern-Simons action and its fermionic counterpart. This seemed to be the only possibility since the vanishing of determinant of 3-metric does not allow any other action principle. There is however hole in this argument that I should have become aware long time ago. The modified gamma matrices appearing in the modified Dirac operator define effective metric for X^3_l and this effective metric need not be degenerate even if the genuine 3-metric is. Just the fact that D_K allows only finite number of eigenmodes effectively restricted to X^3_l would realize the attribute "almost-topological". In the correct approach Kähler action would be the hen and $C-S$ action the egg rather than vice versa. $C-S$ action would emerge naturally in sectors of H with non-standard value of Planck constant and implying anyonization and charge fractionization if the Kähler gauge potential for non-standard values of Planck constant has singular pure gauge parts not possible to transform away by a gauge transformation. Here one must of course notice that the gauge transformations are realized as symplectic tranformation of H and that these transformations are symmetries of only vacuum extremals.

2. The basic implication of number theoretic compactification is the slicing of M^4 projection of X^4 by string world sheets Y^2 and their partonic duals X^2. This string-parton duality should become manifest in the properties of D_K eigenmodes. Hamilton Jacobi coordinates $(u, v, w, \overline{w})$ for M^4 express string-parton duality concretely. (u, v) are light-like coordinates for stringy world sheet Y^2 and w complex coordinate for the partonic 2-surface X^2. I discovered this decomposition for a very general family of extremals of Kähler for years ago [D1] but failed to realize its implications. Additional information about the nature of this slicing can be deduced by requiring that the already known general picture about eigen value spectrum follows from it.

3. One implication of string-parton duality is that by the $D_K\Psi = (D_K(Y^2) + D_K(X^2))\Psi = 0$ condition the generalized eigenvalue spectrum assignable either with string world sheet part $D_K(Y^2)$ or with partonic part $D_K(X^2)$ defines the Dirac determinant and hence Kähler action if the basic conjecture is correct. For light-like 3-surfaces decomposing into regions with non-vanishing induced Kähler form the number of eigenmodes of $D_K(X^2)$ of D_K is finite. This realizes the almost TQFT property concretely and implies also that Dirac determinant is finite and algebraic number if eigenvalues are algebraic numbers. This is important for number theoretic universality.

4. Quantum holography in the sense that the slices Y_l^3 represent holographic copies of the dynamics at X_l^3 is assumed. The most stringent condition is that the eigenvalue spectrum is independent of Y_l^3. The weakest condition is that only Kähler metric of configuration space defined by the Dirac determinant associated with Y_l^3 is independent of Y_l^3. This is guaranteed if the eigenvalues are scaled by functionals of CH which depend only on the real part of a holomorphic function of configuration space (WCW) coordinates.

5. Spinorial shock waves are replaced with spinor modes which are either constant with respect to coordinate u or can be taken to be such. The latter option requires super-conformal gauge symmetry meaning that u-coordinate becomes non-dynamical. This symmetry would have mirror counterpart in X^2 degrees of freedom. These symmetries are standard super-conformal symmetries of Dirac operator. If $\hat{\Gamma}^u$ is light-like then "hyper-holomorphic" spinor modes proportional to $\hat{\Gamma}^u u^n$ are annihilated by $\hat{\Gamma}^u D_u$ and D_K reduces effectively to 3-D modified Dirac operator $D_K(Y^1)+D_K(X^2)$. Ψ can thus have any dependence on u which corresponds to super-conformal gauge symmetry realization quantum holography. Also the square of $D_K(Y^2)$ should reduced to the square of $D_K(Y^1)$ and this takes place always if the component $\hat{g}^{uv}$ of effective metric vanishes. If not, then the condition $D_u\Psi = 0$ is required.

6. p-Adic thermodynamics requires that the ground state conformal weights identified squares of the generalized eigenvalues must be non-positive. This is the case if the effective metric of Y_l^3 has Euclidian signature. This is true if the square of $\hat{\Gamma}^v$ is non-positive. Positivity is would one might naively expect. These two conditions pose strong constraint on the preferred extremal and mean also asymmetry between u and v directions, which is of course expected since classical conserved currents should flow along Y_l^3.

7. If the effective metric defined by the anticommutators of the modified gamma matrices of X^2 is Kähler metric in the sense that it has only $\hat{g}_{w\overline{w}}$ as a non-vanishing component, the eigenmodes of $D_K(X^2)$ have similar gauge invariance locally and if one selects them to be proportional to $\hat{\Gamma}^w$ they can have arbitrary dependence on w. It is however difficult to obtain eigenmodes in this manner and these modes are also un-bounded. Hence the more reasonable view about situation is as fermion in Kähler magnetic field providing analogy with cyclotron states restricted to the regions were induced Kähler form is non-vanishing. This allows only a finite number of eigenmodes.

8. In the approach based on D_{C-S} the non-conservation of gauge charges posed the basic problem and led to the introduction of the gauge part A_a of Kähler gauge potential (see Appendix).

   (a) In the recent case modified Dirac action provides excellent candidates for Noether charges. They are however conserved only if D_K is stationary with respect to the variations of X^4 so that the only contribution to Noether current comes from the variation of Ψ. Since the first variation of S_K defines the modified gamma matrices, this means that second variation of S_K or at least the second variations defining the conserved currents must vanish. The second variations in question should respect boundary conditions, in particular X_l^3 and perhaps also the basic string-parton decomposition of X^4. This corresponds also to the vanishing of the first variation of the modified Dirac action with respect to H coordinates as is seen by the explicit calculation of the second variation of modified Dirac action and by the transformation of the terms containing derivatives of Ψ and $\overline{\psi}$ to give a total divergence plus the term $\overline{\Psi} D_\alpha D_\beta(\partial^2 L_K/\partial h^k_\alpha \partial h^l_\beta)\Gamma^k\Psi$ proportional to the second variation of Kähler action. It is essential that modified Dirac equation holds true so that modified Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of induced metric.

   (b) The vanishing of second variation for some deformations means that the system is critical, in the present case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. For some mysterious reason I failed to realized that quantum criticality realized as the vanishing of the second variation makes possible a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality. Both the supersymmetry of D_K and conservation Dirac Noether currents for modified Dirac action have thus a connection with quantum criticality.

i. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations, which are of third order in imbedding space coordinates.

ii. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$. Recall that the formulation of Equivalence Principle in string picture demonstrated that the reduction of stringy dynamics to that for free strings requires that second variation with respect to M^4 coordinates vanish. This condition would guarantee the conservation of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers but not those for gravitational color quantum numbers. Encouragingly, the action of CP_2 type vacuum extremals having random light-like curve as M^4 projection have vanishing second variation with respect to M^4 coordinates (this follows from the vanishing of Kähler energy momentum tensor, second fundamental form, and Kähler gauge current). In this case however the momentum is vanishing.

iii. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. Conserved Noether charges characterize quantum criticality. The notion of finite quantum measurement based on the hierarchy of Jones inclusions suggests also the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [A9] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom. Since second variation is purely local in the sense that $\delta^2 S_K/\delta h^k(x)\delta h^l(y)$ is proportional to $\delta^4(x,y)$ one might hope that infinite-dimensional criticality reduces to a finite-dimensional criticality in the sense that at given point one has finite-dimensional criticality.

iv. Does this criticality have anything to do with the criticality against the phase transitions changing the value of Planck constant? If the geodesic sphere S^2_I for which induced Kähler form vanishes corresponds to the back of the CP_2 book (as one expects), this could be the case. The homologically non-trivial geodesic sphere $S^1 2_{II}$ is as far as possible from vacuum extremals. If it corresponds to the back of CP_2 book, cosmic strings would be quantum critical with respect to phase transition changing Planck constant. They cannot however correspond to preferred extremals.

(c) Besides basic conservation laws associated with isometries the Noether currents assignable to super-symplectic and super Kac-Moody symmetries realized as transformations respecting the light-likeness of X^3_l give rise to conserved fermionic Noether currents so that quantum criticality would be realized at least in this sense. Effective 2-dimensionality of space-like 3-surface X^3 would suggest that the charges can be expressed as an integrals over the partonic 2-surface X^2.

Cosmic strings and massless extremals provide simplest test beds for this vision. Both have 2-D CP_2 projection and if the hierarchy of Planck constants involves also homologically non-trivial geodesic spheres S^2_{II} of CP_2 as critical manifolds representing the back of CP_2 book, cosmic strings and MEs with CP_2 projection in S^2_{II} cannot correspond to preferred extremals because the value of Planck constant would be ill-defined for them. Also the absence of Euclidian space-like regions identified as generalized Feynman diagrams supports this conclusion. Encouragingly, also the proposed scenario excludes these extremals as preferred ones.

1. Cosmic strings have automatically $Y^2 \times X^2$ decomposition, where Y^2 denotes now string orbit and X^2 is a complex surface of CP_2. For cosmic strings the effective metric in Y^2 is proportional

to the metric of Y^2 so that $\hat{\Gamma}^u$ is light-like but $\hat{\Gamma}^v$ fails to be space-like and also $\hat{g}^{uv}$ is non-vanishing. Effective 3-dimensionality of D_K requires $D_u\Psi = 0$. This in turn implies that the only allowed eigenvalue of $D_K(X^2)$ is zero and corresponds to covariantly constant right-handed neutrino. Ground state conformal weight would vanish and Higgs like contribution to the fermion mass would vanish. Dirac determinant would be equal to unity and cannot therefore correspond to the exponent of Kähler action for cosmic string. The deformation induced by the topological condensation of CP_2 type vacuum extremals making CP_2 projection 3-D should make $\hat{\Gamma}^v$ space-like. If one gives up the condition about effective 3-dimensionality, the eigenvalue spectrum of the square of $D_K(X^2)$ defines ground state conformal weights. This spectrum is unbounded so that Dirac determinant would be infinite.

2. For MEs the decomposition corresponds to Hamilton-Jacobi coordinates such that the canonical momentum currents in Y^2 are in v-direction. Light-likeness implies that the square of $D_K(Y^2)$ vanishes and $\lambda = 0$ is the only eigenvalue allowed for $D_K(X^2)$. Covariantly constant right handed neutrino belongs to the spectrum but represents a pure gauge degree of freedom. In this case the exponent of Kähler function as product of non-vanishing eigenvalues is predicted to be equal to one which conforms with the fact that Kähler action vanishes.

Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X_l^3)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ wth the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the configuration space metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables.

This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

2.4.3 Dirac determinant as a product of eigenvalues for transverse part of D_K

The previous considerations led to the conclusion that the zero modes of D_K define generalized eigenmodes $D_K(X^2)$ and this in turn led to the realization that thanks to the string-parton duality one can express Dirac determinant in terms of generalized eigenvalues assignable to either partonic or stringy Dirac operator.

Generalized eigenmodes of $D_K(X^2)$ at X^3_l

The general description of generalized eigenmodes of D_K assuming the slicing by string world sheets Y^2 and partonic 2-surfaces X^2 has been already discussed. In the following the situation is studied in more detail.

1. Modified Dirac equation can be written as a conservation condition for the super current

$$J^{\alpha} = \frac{\partial L_K}{\partial h^k_{\ \alpha}} \Gamma^k \Psi \ . \tag{2.4.9}$$

The reduction of D_K to effectively 3-D Dirac operator requires the takes place if the conditions that super current flows along slices Y^3_l:

$$J^u = 0 \ . \tag{2.4.10}$$

The physical interpretation is that fermionic currents flow along Y^3_l.

2. An equivalent formulation is in terms of the conditions on the effective metric

$$\hat{g}^{uu} = 0 \ , \ \hat{g}^{uv} = 0 \ , \hat{g}^{vv} \leq 0 \ . \tag{2.4.11}$$

The third condition states that the effective metric of Y^3_l has Euclidian or light-like signature and is required by the p-adic particle massivation.

3. If the condition $\hat{g}^{uv} = 0$ is dropped one must posed the condition

$$D_u \Psi = 0 \tag{2.4.12}$$

so that the modes are covariantly constant in u-direction. Otherwise u-direction corresponds to a pure gauge degree of freedom.

4. Light-likeness of $\hat{\Gamma}^u$ can be replaced with a stronger condition

$$\hat{\Gamma}^u = 0 \qquad (2.4.13)$$

equivalent with the conditions

$$\frac{\partial L_K}{\partial h^k{}_u} = 0 \qquad (2.4.14)$$

guaranteing the vanishing of the components of various conserved currents assignable to L_K in u-direction. This is natural if effective 3-dimensionality holds true classically. In this case the multiplication of the spinor modes by $\hat{\Gamma}^u$ is neither needed nor allowed. This condition implies that Noether currents do not flow through $det(g_4) = 0$ throats so that Euclidian regions represent elementary particle like units which they indeed are as generalized Feynman diagrams. Only exchanges of particles between particles can mediate exchange of quantum numbers.

5. If the conditions

$$g_{ui} = 0 \ , \ J_{ui} = 0 \qquad (2.4.15)$$

are satisfied, the decoupling of metric in normal and tangential directions implies $T^{u\alpha} = 0$ and $\hat{\Gamma}^u = 0$. Also additional consistency conditions might be required. $J_{ui} = 0$ does not imply the vanishing of Kähler gauge charge at the limit $Y_l^3 \rightarrow X_l^3$ since $J^{ui}\sqrt{g_4}$ can be non-vanishing at this limit. The vanishing of the component j_K^u of Kähler current implies that gauge flux is conserved. The properties of known extremals of Kähler action support the view that all gauge currents flow along Y_l^3 so that gauge fluxes through partonic 2-surfaces at Y_l^3 are conserved. This has interpretation as a justification for p-adic coupling constant evolution [C4].

6. $D_K\Psi = 0$ with the decomposition $D_K = D_K(X^2) + D_K(Y^2)$ allows to identify generalized eigenvalues as those assignable to $D_K(X^2)$ or $D_K(Y^2)$. The square of the modified Dirac operator gives eigenvalue equations familiar from the separation of variables. One can hope that Dirac determinant defined as the product of eigenvalues gives the exponent of Kähler action. One expects that the solutions of D_K to be of form $exp(i\lambda t)$ for a suitable choice of the light-like radial coordinate r. As a matter fact, λ is imaginary if the effective metric of Y^2 has Euclidian signature. $(\hat{\Gamma}^v)^2 \equiv \hat{g}^{vv}$ acts like a component of contravariant metric, which suggests that the time coordinate is analogous to the proper time coordinate so that one hat $dt = dr/\sqrt{\hat{g}^{vv}}$. This condition gives eigenvalue equation for the square of the transverse Dirac operator in the form

$$D^2(X^2)\Psi = \lambda^2\Psi \ . \qquad (2.4.16)$$

The eigen modes can be solved and the system in question describes fermion in electro-weak magnetic field defined by the induced spinor connection. Of course, the replacement of the contravariant metric with effective metric $T^{\alpha k}T^{\beta l}h_{kl}$ means also considerable differences.

The vanishing of $det(g_4)$ makes the limit $Y_l^3 \rightarrow X_l^3$ somewhat delicate. Quantum holography allows in principle to avoid these difficulties (one can of course worry about whether preferred extremals for which Y_l^3 approaching X_l^3 exist!) but it is interesting to see what happens. Assume $\hat{\Gamma}^u = 0$.

1. The non-vanishing components of $g^4_{\alpha\beta}$ at X_l^3 can be written as $g_{u\alpha}$ and g^3_{ij} such that the conditions $det(g_3) = 0$ and $det(g_4) = 0$ hold true. The components of the scaled effective metric $\hat{g}^{\alpha\beta} = det(g)g^{\alpha\beta}$ are finite and $\hat{g}^{nn} = 0$ holds true.

2. By studying the general structure of $T_K^{\alpha k} = \partial L_K/\partial h^k_{\ \alpha}$ appearing in the definition of the modified gamma matrix, one finds that the contribution $T^{\alpha\beta}\partial_\beta h^k$ can be written as a sum of three terms proportional to $1/det(g_4)^n$, $n = 1, 2, 3$. In particular, the contribution from $J^{\alpha\beta} J_{l\beta} h^{kl}$ contains terms proportional to $1/det(g_4)^2$ and $1/det(g_4)$. The strongest form for $J^u = 0$ states that the coefficients of J_i^u in the decomposition

$$J^u = \frac{J_1^u}{det(g_4)} + \frac{J_2^u}{det(g_4)^2} + \frac{J_3^u}{det(g_4)^3} \tag{2.4.17}$$

vanish separately. The weakest form for $J^u = 0$ states that only J_3^u vanishes. The fact that a limiting case is in question would in turn suggest the proportionality $J_i^u \propto det(g_4)^i$ so that one cannot treat these contributions as independent ones.

3. For the modified Dirac equation one obtains similar decomposotion

$$\sum_{n=0,1} \frac{\hat{\Gamma}_n^i}{det(g_4)^n} D_i\Psi = 0 \ . \tag{2.4.18}$$

The terms associated with the powers of $det(g)$ must be proportional to each other in order to have complete internal consistency.

Definition of Dirac determinant

The standard manner to define Dirac determinant would be as the product of the eigenvalues λ.

1. This definition usually leads to divergence difficulties and the definition works only if the number of generalized eigenvalues is finite. D_T describes 2-D fermion in a varying electro-weak gauge field. It is somewhat matter of taste whether one wants to speak about electric and magnetic fields or only magnetic field. This suggests that the analogs of bound state solutions are localized in regions, where the induced electro-weak magnetic field is strong. Partonic 2-surface decomposes into regions, where the sign of $B_K = \epsilon^{\alpha\beta} J_{\alpha\beta}$ is fixed. These regions are typically separated by curves, where $B_K = 0$ holds true. These regions provide 2-D representation of the spin glass degeneracy assigned to Kähler action naturally define the loci for the localized cyclotron states. The localization in a finite region is possible for a finite number of cyclotron states only. If only this kind of localized states are allowed, one can hope that the number of states is finite so that also Dirac determinant is finite.

2. A more precise argument goes as follows. The effective metric appearing in the modified Dirac operator corresponds to

$$\hat{g}^{\alpha\beta} = \frac{\partial L_K}{\partial h^k_\alpha}\frac{\partial L_K}{\partial h^l_\beta} h_{kl} \ ,$$

and vanishes at the boundaries of regions carrying non-vanishing Kähler magnetic field. Hence the spinorial shock waves must be localized to regions $X^3_{l,i}$ containing a non-vanishing Kähler magnetic field. Assume that it is induced Kähler magnetic field B_K that matters. The vanishing of the effective contravariant metric near the boundary of $X^3_{l,i}$ corresponds to an infinite effective mass for a massive particle in constant magnetic field so that the counterpart for the cyclotron frequency scale eB/m reduces to zero. The radius of the cyclotron orbit is proportional to $1/\sqrt{eB}$ and approaches to infinity. Hence the required localization is not possible only for cyclotron states for which the cyclotron radius is below that the transversal size scale of $X^3_{l,i}$.

3. Dirac determinant decomposes into a product over these regions just as the exponent of Kähler action for preferred extremal is expected to do. In the case of elementary particles one expects that both the size and the number of these regions is small so that also the number of cyclotron states is small. When the size of the partonic 2-surface is macroscopic - as in the case of anyonic states in TGD based model of QHE [F12] - both the number and sizes of the regions can be large so that larger number of cyclotron states is possible.

4. The eigenvalues are analogous to cyclotron energies and in the first approximation proportional to $n+1/2$ up to some maximum integer $n_{max,i}$ in the i:th region carrying strong electro-weak magnetic field. In this approximation and in absence of degenerate eigen values the zeta function associated with the system would be a sum of zeta functions associated with these regions and correspond to cutoffs of Riemann zeta defined as $\zeta_{1/2}(s, n_{max}) = \sum_{n=11}^{n_{max}} (n+1/2)^{-s}$. Thus something resembling Riemann Zeta indeed emerges as speculated for long time ago. By using the integral representation completely analogous to that for Riemann Zeta whether it should be easy to see whether this zeta function has zeros at line $Re(s) = 1/2$.

5. A nice property of this definition is that vacuum functional is defined also in p-adic context if the eigenvalues λ are algebraic numbers and their number is finite. Hence it is possible to speak about exponent of Kähler function also in p-adic context although the Kähler action is not well-defined as integral.

Formula for the Kähler coupling strength

The identification of exponent of Kähler function as Dirac determinant leads to a formula relating Kähler action for the preferred extremal to the Dirac determinant. The eigenvalues are proportional to $1/\alpha_K$ since the matrices $\hat{\Gamma}^\alpha$ have this proportionality. This gives the formula

$$exp(\frac{S_K(X^4(X^3))}{8\pi\alpha_K}) = \prod_i \lambda_i = \frac{\prod_i \lambda_{0,i}}{\alpha_K^N} . \quad (2.4.19)$$

Here $\lambda_{0,i}$ corresponds to $\alpha_K = 1$. $S_K = \int J^* J$ is the reduced Kähler action.

For $S_K = 0$, which might correspond to so called massless extremals [D1] one obtains the formula

$$\alpha_K = (\prod_i \lambda_{0,i})^{1/N} . \quad (2.4.20)$$

Thus for $S_K = 0$ extremals one has an explicit formula for α_K having interpretation as the geometric mean of the eigenvalues $\lambda_{0,i}$. Several values of α_K are in principle possible.

p-Adicization suggests that $\lambda_{0,i}$ are rational or at most algebraic numbers. This would mean that α_K is N:th root of this kind of number. S_K in turn would be

$$S_K = 8\pi\alpha_K log(\frac{\prod_i \lambda_{0,i}}{\alpha_K^N}) . \quad (2.4.21)$$

so that S_K would be expressible as a product of the transcendental π, N:th root of rational, and logarithm of rational. This result would provide a general answer to the question about number theoretical anatomy of Kähler coupling strength and S_K. Note that S_K makes sense p-adically only if one adds π and its all powers to the extension of p-adic numbers. The exponent of Kähler function however makes sense also p-adically.

Eigenvalues of D_K as vacuum expectations of Higgs field?

The interpretation inspired by p-adic mass calculations is that the squares λ_i^2 of the eigenvalues of $D_K(X^2)$ correspond to the conformal weights of ground states. Another natural physical interpretation of λ is as an analog of the Higgs vacuum expectation. The instability of the Higgs=0 phase would corresponds to the fact that $\lambda = 0$ mode is not localized to any region in which ew magnetic field or induced Kähler field is non-vanishing. A good guess is that induced Kähler magnetic field B_K dictates the magnitude of the eigenvalues which is thus of order $h_0 = \sqrt{B_K R}$, R CP_2 radius. The first guess is that eigenvalues in the first approximation come as $(n+1/2)h_0$. Each region where induced Kähler field is non-vanishing would correspond to different scale mass scale h_0.

1. The vacuum expectation value of Higgs is only proportional to an eigenvalue λ, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and antifermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this).

2. Physical considerations suggest that the vacuum expectation of Higgs field corresponds to a particular eigenvalue λ_i of modified Dirac operator so that the eigenvalues λ_i would define TGD counterparts for the minima of Higgs potential. Since the vacuum expectation of Higgs corresponds to a condensate of wormhole contacts giving rise to a coherent state, the vacuum expectation cannot be present for topologically condensed CP_2 type vacuum extremals representing fermions since only single wormhole throat is involved. This raises a hen-egg question about whether Higgs contributes to the mass or whether Higgs is only a correlate for massivation having description using more profound concepts. From TGD point of view the most elegant option is that Higgs does not give rise to mass but Higgs vacuum expectation value accompanies bosonic states and is naturally proportional to λ_i. With this interpretation λ_i could give a contribution to both fermionic and bosonic masses.

3. If the coset construction for super-symplectic and super Kac-Moody algebra implying Equivalence Principle is accepted, one encounters what looks like a problem. p-Adic mass calculations require negative ground state conformal weight compensated by Super Virasoro generators in order to obtain massless states. The tachyonicity of the ground states would mean a close analogy with both string models and Higgs mechanism. λ_i^2 is very natural candidate for the ground state conformal weights identified but would have wrong sign if the effective metric of X_l^3 defined by the inner products $T_K^{k\alpha}T_K^{l\beta}h_{kl}$ of the Kähler energy momentum tensor $T^{k\alpha} = h^{kl}\partial L_K/\partial h^l_\alpha$ and appearing in the modified Dirac operator D_K has Minkowskian signature.

   The situation changes if the effective metric has Euclidian signature. This seems to be the case for the light-like surfaces assignable to the known extremals such as MEs and cosmic strings. In this kind of situation light-like coordinate possesses Euclidian signature and real eigenvalue spectrum is replaced with a purely imaginary one. Since Dirac operator is in question both signs for eigenvalues are possible and one obtains both exponentially increasing and decreasing solutions. This is essential for having solutions extending from the past end of X_l^3 to its future end. Non-unitary time evolution is possible because X_l^3 does not strictly speaking represent the time evolution of 2-D dynamical object but actual dynamical objects (by light-likeness both interpretation as dynamical evolution and dynamical object are present). The Euclidian signature of the effective metric would be a direct analog for the tachyonicity of the Higgs in unstable minimum and the generation of Higgs vacuum expectation would correspond to the compensation of ground state conformal weight by conformal weights of Super Virasoro generators.

4. In accordance with this λ_i^2 would give constant contribution to the ground state conformal weight. What contributes to the thermal mass squared is the deviation of the ground state conformal weight from half-odd integer since the negative integer part of the total conformal weight can be compensated by applying Virasoro generators to the ground state. The first guess motivated by cyclotron energy analogy is that the lowest conformal weights are of form $h_c = \lambda_i^2 = -1/2 - n + \Delta h_c$ so that lowest ground state conformal weight would be $h_c = -1/2$ in the first approximation. The negative integer part of the net conformal weight can be canceled using Super Virasoro generators but Δh_c would give to mass squared a contribution analogous to Higgs contribution. The mapping of the real ground state conformal weight to a p-adic number by canonical identification involves some delicacies.

5. p-Adic mass calculations are consistent with the assumption that Higgs type contribution is vanishing (that is small) for fermions and dominates for gauge bosons. This requires that the deviation of λ_i^2 with smallest magnitude from half-odd integer value in the case of fermions is considerably smaller than in the case of gauge bosons in the scale defined by p-adic mass scale

$1/L(k)$ in question. Somehow this difference could relate to the fact that bosons correspond to pairs of wormhole throats.

Connection between Jones inclusions, hierarchy of Planck constants, and finite number of spinor modes

The original generalization of the imbedding space to accommodate the hierarchy of Planck constants was based on the idea that the singular coverings and factor spaces associated with the causal diamond CD and CP_2, which appears as factors of $CD \times CP_2$ correspond somehow to Jones inclusions, and that the integers n_a and n_b characterizing the orders of maximal cyclic groups of groups G_a and G_b associated with the two Cartesian factors correspond to quantum phases $q = exp(i2\pi/n_i)$ in such a manner that singular factor spaces correspond to Jones inclusions with index $\mathcal{M} : \mathcal{N} \leq 4$ and coverings to those with index $\mathcal{M} : \mathcal{N} \geq 4$.

Since Jones inclusions are interpreted in terms of finite measurement resolution, the mathematical realization of this heuristic picture should rely on the same concept realized also by the fact that the number of non-zero modes for induced spinor fields is finite. This allows to consider two possible interpretations.

1. The finite number of modes defines an approximation to the hyper-finite factor of type II_1 defined by configuration space Clifford algebra.
2. The Clifford algebra spanned by fermionic oscillator operators is quantum Clifford algebra and corresponds to the somewhat nebulous object $\mathcal{N}/\mathcal{M}$ associated with the inclusion $\mathcal{M} \subset \mathcal{N}$ and coding the finite measurement resolution to a finite quantum dimension of the Clifford algebra. The fact that quantum dimension is smaller than the actual dimension would reflect correlations between spinor components so that they are not completely independent.

If the latter interpretation is correct then second quantized induced spinor fields should obey quantum variant of anticommutation relations reducing to ordinary anticommutation relations only for $n_a = n_b = 0$ (no singular coverings nor factor spaces). This would give the desired connection between inclusions and hierarchy of Planck constants. It is possible to have infinite number of quantum group like structure for $\hbar = \hbar_0$ [**?**].

There are two quantum phases q and one should understand what is the phase that appears in the quantum variant of anti-commutation relations. A possible resolution of the problem relies on the observation that there are two kinds of number theoretic braids. The first kind of number theoretic braid is defined as the intersection of M_+ (or light-like curve of δM^4_+ in more general case) and of δM^4_+ projection of X^2. Second of braid is defined as the intersection of CP_2 projection of X^2 of homologically non-trivial sphere S^2_{II} of CP_2. The intuitive expectation is that these dual descriptions apply for light-like 3-surfaces associated *resp.* co-associative regions of space-time surface and that both descriptions apply at wormhole throats. The duality of these descriptions is guaranteed also at wormhole throats if physical Planck constant is given by $\hbar = r\hbar_0$, $r = \hbar(M^4)/\hbar(CP_2)$, so that only the ratio of the two Planck constants matters in commutation relations. This would suggest that it is $q = exp(i2\pi/r)$, which appears in quantum variant of anti-commutation relations of the induced spinor fields.

Does the zeta function defined by eigenvalues of D_K have physical meaning?

There is a considerable amount of evidence that the zeros of Riemann zeta relate to critical quantum systems in the sense that the energy spectrum seems to obey a distribution similar to that associated with the zeros of Riemann ζ. This led to a rather ad hoc idea that Riemann Zeta might play a key role in quantum TGD. One of the speculations was that super-symplectic conformal weights correspond to the zeros of Riemann Zeta. Combining this idea with p-adicization led to a handful of number theoretic conjectures and many of them turned out to be wrong: one of them was that zeros of ζ are algebraic numbers.

As I realized the role of the eigenvalues of the modified Dirac operator D_K, Riemann Zeta was naturally replaced with the zeta function determined by the generalized eigenvalues of D_K:

$$\zeta_D(s) = \sum_i \lambda_i^{-s} \quad . \tag{2.4.22}$$

Dirac Zeta might be the appropriate nickname for this zeta. The analogy between λ_i and cyclotron energies of a fermion in electro-weak magnetic field suggests that the eigenvalue spectrum apart from possible degeneracies is in a reasonable approximation integer or half integer valued but has cutoff for some integer n_{max} so that cutoff variant of Riemann Zeta might not be too bad an approximation to $\zeta_D(s)$.

Configuration space Kähler function can be regarded as the derivative of Dirac Zeta $\zeta_D(s)$ at origin:

$$K = \sum_i log(\lambda_i) = \frac{d\zeta_D}{ds}_{|s=0} \quad . \tag{2.4.23}$$

The derivatives $\partial_k \partial_{\bar{l}} K$ and thus Dirac Zeta would give the configuration space metric in complex coordinates.

$\zeta_D(s)$ also codes the information about eigenvalues of D_K. From the asymptotic behavior at the limit $s \rightarrow \infty$ one can deduce the eigenvalue with the smallest magnitude, by subtracting the corresponding contribution the next eigenvalue, and so on. One might hope that some general conditions could allow to deduce information about the behavior of ζ_D as a functional of X^3_l as well as a function of general light-like 3-surface Y^3_l in the slicing of $X^4(X^3_l)$ by light-like 3-surfaces.

It would be however disappointing if Dirac Zeta had no other role in quantum TGD. This kind of role indeed emerges naturally from thermodynamics for the eigenvalues of D_K.

1. In zero energy ontology zero energy states code in their structure also thermodynamics since M-matrix is product of real square root of density matrix and unitary S-matrix: quantum theory could be regarded as a square root of thermodynamics and the square roots of thermal states would be genuine quantum states.

2. In this framework a square root of thermodynamical ensemble defined for the modes of D_K could be associated naturally with zero energy states. ζ_D with complex argument is perfectly acceptable candidate for defining the square root of partition function with partition function identified as its modulus squared. $\zeta_D(s)$ would define a square root of thermodynamics for $log(\lambda_i)$ instead of λ_i. The argument s of ζ_D would define the analog of inverse temperature and the counterpart of the thermal energy would be thermal average over the eigenvalues $log(\lambda_i)$.

3. What looks problematic that instead of λ_i with dimensions of mass, $log(\lambda_i/\lambda_0)$ takes the role of energy. The first possibility is that fractal thermodynamics based fixing the thermal average of logarithm of energy rather than average of energy is in question. Second possibility is that the energies are indeed proportional to $log(\lambda_i)$ defining fundamental parameters of the system. Since Dirac determinant is the product for Dirac determinants for sub-systems identified as regions in which induced Kähler form is non-vanishing the spectra of $D_K(X^2)$ are multiplied in the formation of many particle systems. The energy spectra of subsystems are however additive, which suggests that if energies are expressible in terms of λ_i, they must be proportional to $log(\lambda_i)$.

4. It is not clear under what conditions on the spectrum of $D_K(X^2)$ Dirac Zeta satisfies Riemann hypothesis or whether it has any physically interesting zeros. The first idea is that a finite cutoff for Riemann Zeta making sense also in the region $Re(s) < 1$ should be able to give approximate expressions for the zeros of Riemann Zeta but as such this idea does not look good. One possibility is that zero energy state is proportional to the product $Z(s) = \zeta_D(s)\zeta_D(\bar{s})$, of zeta functions corresponding to the systems at temperatures $T = 1/s$ and its complex conjugate. For real values of $log(\lambda_i)$ $Z(s)$ is real and zeros are possible for some values of $Re(s)$. In physically interesting situation number theoretical constraints pose conditions on the values of s and very probably only approximate zeros are in question. Therefore complex critical temperatures would correspond to approximate zeros of $Z(s)$. In ordinary thermodynamics the zeros of the partition function are identified in terms of quantum criticality so that quantum criticality could be regarded as the square root of ordinary criticality. One might hope that the zeros of similar partition function $Z(1/2 + iy)$ associated with a cutoff of Riemann Zeta could give a reasonable approximation for the zeros of Riemann Zeta too.

5. Quite generally, this kind of situation is achieved if one has systems with energy spectra $E_i = log(\lambda_i)$ and $E_i = \pm log(\lambda_i)$ at temperatures $1/T = s = x + iy$ and $1/T = \pm\overline{s} = \pm(x - iy)$. The interpretation in terms of positive and negative energy parts of a zero energy state is suggestive. The counterpart of thermal energy would be the thermal average $\langle log(\lambda_i)\rangle$. Since the vanishing of $Z(s)$ is expected to be only approximate, partition function remains non-vanishing and the thermal average remains well defined albeit large reflecting the presence of large quantum fluctuations.

For critical quantum systems the distribution of energy eigenvalues is similar to the distribution of zeros of Riemann Zeta and it is interesting to see whether this could be understood in the proposed framework.

1. Suppose that the energies correspond to the thermal expectations $\langle log(\lambda_i)\rangle$ at criticality and therefore to approximate zeros of $Z(s)$. A further requirement is that energies are proportional to $Im(1/T) = Im(s) = y$.

2. Criticality is expected to correspond to a situation in which Kähler function vanishes as the extremum of Kähler action. This gives the condition $d\zeta_D(s)/ds = 0$ for $s = 0$. This implies in the first order approximation the condition

$$\frac{d\zeta_D(s)}{ds} = a \times s \ , \tag{2.4.24}$$

where a is complex number.

3. The general thermodynamical formula

$$\langle log(\lambda_i)\rangle = -\frac{d\zeta_D(s)/ds}{\zeta_D(s)} - \frac{d\zeta_D(\overline{s})/d\overline{s}}{\zeta_D(\overline{s})} \tag{2.4.25}$$

gives in the lowest order approximation

$$\langle log(\lambda_i)\rangle = -a \times s - \overline{a} \times \overline{s} = 2Re(as) \ . \tag{2.4.26}$$

If a is purely imaginary one obtains $\langle log(\lambda_i)\rangle \simeq 2Im(a)y$ so that in this approximation the energy expectation is indeed proportional to the imaginary part of the approximate zero of ζ_D at criticality as required. If the linear approximation for K is good for all values of y and the spectrum of zeros for ζ_D resembles that for Riemann Zeta to a sufficiently high degree, the experimental results can be understood.

Should one introduce induced spinor fields at string world sheets?

In the previous section it was found that TGD should allow also dimensionally reduced descriptions in terms of either string world sheets or partonic 2-surfaces. This raises the question whether it makes sense to introduce induced spinor fields at string world sheets. This is indeed the case. The modified Dirac action would in this case correspond to the Dirac operator for the dimensionally reduced Kähler action. The effective minimal surface property of Y^2 would guarantee the conservation of the super current. The realization of the effective 3-dimensionality in turn means that the stringy coordinate u corresponds to a gauge degree of freedom or to the condition $D_u\Psi = 0$. There would no spinor waves propagating along this direction of string and only the deformations of string represented by symplectic and Kac-Moody algebras present also in the dynamics of Kähler action responsible for the p-adic thermodynamics would be present. Besides this there would be the fermionic excitations associated with the ends of the string and correspond to the eigenmodes of $D_K(X^2)$ or equivalently with $D_K(Y^1)$ so that the Dirac determinant would be the same as obtained for D_K. For the description in terms of partonic 2-surfaces the Dirac operator would be just $D_K(X^2)$ and also now the equivalence with the 4-D description follows trivially.

2.4.4 Generalization of the representation of Kähler function in terms of Dirac determinant to include instanton term

In 4-D gauge theories one cannot avoid instanton term inducing breaking of CP symmetry and also in TGD framework this term is possible. It would give an imaginary contribution to Kähler action and Kähler function but by its topological nature would not affect configuration space metric. This suggests also a generalization of modified Dirac equation and this leads to a nice picture about how CP breaking takes place at the fundamental level.

Addition of imaginary instanton term to Kähler function

Chern-Simons action relates to the description of anyonic phases and one expects that this the case also in TGD framework. The phase factor defined by the imaginary exponent of Chern-Simons action defined by Lagrangian density $L_{C-S} = (k/4\pi)A\wedge J$, k integer, for the modified Kähler gauge potential should be present in the quantum state and somehow relates to the space-time representations for the fractionization. Note that the absence of $\hbar$ is dictated by the dimensionless character of Kähler potential distinguishing it from quantized gauge potentials. Also it should be noticed that in Kähler action the inverse of $\alpha_{K,0} = g_K^2/4\pi\hbar_0$ appears as a scaling factor. This is the only possible choice since the over-all scaling of the $1/\hbar$ factor of the modified Dirac action induces to Kähler function only an additive constant term rather than a scaling of $1/\alpha_K$ factor.

1. If one assumes that the quantum state is proportional to the imaginary exponent of the "instanton density" $(k/4\pi)J\wedge J$, which is total divergence locally, this kind of exponent results automatically and is associated with the light-like 3-surfaces Y_l^3 and possible boundaries of X^4. Chern-Simons term should would be present also for $n_a = n_b = 1$ but no charge fractionization would result at the level of modified Dirac action. A good guess for the value of k is $k = 1$ since in this case Kac-Moody currents allow a representation as fermionic bilinears.

2. The addition of imaginary part to Kähler action is what is done effectively when instanton term is introduced. If instanton term is not just a phase factor assigned to the quantum state but reflects directly the structure of fundamental theory, one must consider the possibility of generalizing the exponent of Kähler function by adding to it the phase factor from 4-dimensional $J\wedge J$ term giving rise to a term analogous to Chern-Simons action at X_l^3. This would add to Kähler action density an imaginary part proportional to $J\wedge J$. Since the second variation of $J\wedge J$ term vanishes identically, the corresponding term in Kähler function does not contribute to the Kähler metric of the configuration space. Also the modified Dirac operator would receive a non-vanishing imaginary part since $J\wedge J$ implies the replacement

$$\hat{\Gamma}^{\alpha} \rightarrow \hat{\Gamma}^{\alpha} + \frac{2k}{\pi}\epsilon^{\alpha\beta\gamma\delta}J_{k\beta}J_{\gamma\delta} \ . \qquad (2.4.27)$$

   The imaginary part of the super current $\hat{\Gamma}^{\alpha}\Psi$ would be identically conserved so that no additional conditions on preferred extremals are posed.

3. The imaginary part of $\hat{\Gamma}(X^2)$ from $J\wedge J$ to the modified Dirac operator and hence also to $D_K(X^2)$ is non-vanishing if J restricted to Y_l^3 has non-vanishing components of form J_{vi} and J_{ui}. This is the case when the CP_2 projection of $X^3(X_l^3)$ is 4-dimensional: $D_{CP_2} = 4$. Since the contribution of $J\wedge J$ to $\hat{\Gamma}^u$ is non-vanishing unlike that from Kähler action with the assumptions made about slicing, the condition $D_u\Psi = 0$ is necessary to achieve the effective 3-dimensionality. This makes the eigenvalues λ complex but in the square $\overline{D}_K D_K$ only $|\lambda|^2$ appears and ground state conformal weights are real. The Dirac determinant becomes complex and the outcome should be Kähler function with imaginary part from instanton density.

Connection with CP breaking

There is also a connection to CP breaking, in particular matter anti-matter asymmetry, which still remains one of the mysteries of fundamental physics.

1. CP breaking appears in CKM mixing of quarks and I have proposed a TGD based model for CKM matrix [F4] based on the identification of fermion families in terms of the genus of partonic 2-surface X^2 assigned with fermion. CKM matrix is induced by topological mixing in the sense that quantum state corresponds to superposition of partonic two-surfaces with different genera and the mixings are different for different charge states of fermion.

2. Matter antimatter asymmetry means that antimatter is effectively absent in the Universe. I have considered several explanations for this. A small breaking of matter symmetry during the primordial cosmic evolution should induce small asymmetry in densities of matter and antimatter. After the annihilation of matter and antimatter this asymmetry would be visible as a non-vanishing density of matter in that part of Universe that we can observe. Antimatter could reside in the interior of cosmic strings, at different p-adic space-time sheets, or at dark space-time sheets with different value of $\hbar$.

3. The open question is how CP breaking emerges in the fundamental formulation of quantum TGD. This breaking could also mean that the arrow of geometric time correlating with the arrow of experienced [MPb] is same everywhere.

4. The introduction of the imaginary instanton term to the exponent of Kähler function could provide the long sought solution to the problem since it means breaking of CP and thus T. CP breaking is present only when the CP_2 projection is 4-dimensional and its dependence on $\hbar$ comes only through the breaking of scale invariance.

2.4.5 Does CP breaking term imply infinite number of conformal excitations?

The above picture looks rather nice but one must remain critical. Is the proposed picture based on finite number of spinor modes really realistic and really consistent with stringy picture? The proposed picture suggests the realization of $\mathcal{M}/\mathcal{N}$ but what about $\mathcal{M}$? Shouldn't it be there also? Could $\mathcal{N}$ be identified in terms of conformal excitations labeled by conformal weight? In the following this is proposed. In [C3] a detailed proposal for how the TGD counterpart of stringy perturbation theory emerges from the proposed picture.

Could super-conformal symmetry in the direction of slicing help to have stringy picture?

The only possibility which comes in mind relates to the super-conformal invariance associated with the coordinate u labeling the slices Y_l^3 in the slicing of $X^4(X_l^3)$ and implying effective 3-dimensionality of $X^4(X_l^3)$.

1. Suppose that virtual states break the effective 3-dimensionality and therefore are not annihilated by $D_K(X^3)$). One would have virtual states in 3-D sense but on mass shells states in 4-D sense, that is solutions of D_K. This would be the fermionic counterpart for the preferred extremal of Kähler action property.One can imagine also a detailed scenario based on CP breaking term in $D(X^4) = D_K(X^4) + iD_I(X^4)$.

2. The presence of instanton term for which the contribution to $\hat{\Gamma}^u$ is non-vanishing when CP_2 projection has dimension $D_{CP_2} \geq 3$ allows to keep the assumption that the contribution to $\hat{\Gamma}^u$ from Kähler action is light-like and the contribution to $\hat{g}^{uv}$ vanishes. If this is the case, then effective 3-dimensionality would be broken only by the instanton term. Effective 3-dimensionality indeed requires $D_u\Psi = O$ in the presence of instanton term.

3. The reduction

$$\begin{aligned} D(X^4) &= D_K(X^4) + iD_I(X^4) \rightarrow D(Y_l^3) + iD_I(Y^1) \ , \\ D(Y_l^3) &= D_K(Y_l^3) + iD_I(Y_l^3) \end{aligned}$$

would still take place. The operator

$$D^2(X^2) \equiv \{D(X^4), D^\dagger(X^4)\}/2 = D_K^2(X^4) + D_I^2(X^4) = D_K^2(Y_l^3) + D_I^2(X^4)$$

annihilates the spinor modes. Suppose that this operator decomposes as

$$D^2(Y_l^3) + D_I^2(Y^1) \ .$$

If $D_I^2(Y^1)\Psi = \pm n\Psi$ holds true, $1/D_I^2(Y^1)$ behaves as the bosonic propagator $1/L_0$. Kind of topological modes would be in question. The propagator would have pole for $n = 0$ in accordance with idea that these states are the only on mass shell states. The generalized eigenvalue equation for $D_I(Y^1)$ reduces to the condition

$$D_u\Psi = \pm i\sqrt{n}\Psi \qquad (2.4.28)$$

giving

$$iD(Y^1)\Psi = - \mp \sqrt{n}\hat{\Gamma}^u\Psi \ .$$

The corresponding equations in Y_l^3 are

$$D(Y_l^3)\Psi = \pm\sqrt{n}\hat{\Gamma^u}\Psi \ , \quad D(X^1)\Psi = (\pm\sqrt{n} - h_c)\hat{\Gamma^u}\Psi \ , \quad D(X^2)\Psi = h_c\hat{\Gamma^u}\Psi \ .$$

4. The topological character of the instanton term raises the hope that the spectrum of the operator $iD_I(Y^1)$ is universal and $\sqrt{n}$ valued. The most plausible option is that $\sqrt{n}$ valued spectrum codes for conformal invariance and in turn poses conditions on the spectrum of $D(Y_l^3)$. This works if boundary conditions do not pose any restrictions of the spectrum. For $D_{CP_2} < 3$ the instanton term is vanishing so that no conformal excitations are possible. This conforms with the vision that $D_{CP_2} > 2$ holds true for the preferred extremals always.

5. Do $iD_I(Y^1)$ and the super-generator G used in p-adic mass calculations correspond to different representations of the same super-conformal symmetry? One might argue that $D_K(X^3)$ corresponds to super-canonical algebra and $iD(Y^1)$ to super Kac-Moody algebra. Does this mean that $iD(Y^1)$, which is also non-Hermitian because of the presence of instanton term in D_K corresponds to the fundamental representation of super-conformal symmetry and that the super-conformal algebras creating particle states and defining configuration space Dirac operator correspond to second quantized non-Hermitian representations of this super-conformal symmetry? This question is pondered in [C3]. Here it is enough to notice that $1/iD_I(Y^1)$ as such does not give rise to stringy propagator since the only pole corresponds to $n = 0$. In [C3] a mechanism transforming $1/iD(Y^1)$ to stringy propagator is considered and the cautious conclusion is that CP breaking term is absolutely essential for the emergence of M propagation in stringy sense. This mechanism would also provide the answer to the question how to modify basic QFT picture to avoid self energy divergences: the answer is that kinetic and mass terms in M^4 degrees are not present from the beginning but emerge purely dynamically (this corresponds to the vanishing of renomalization constants relating bare fields to renormalized ones).

Could conformal modes correspond to fermionic oscillator operators?

Conformal modes correspond to genuine physical degrees of freedom at least in the sense that conformal algebra realized in terms of fermionic oscillators operators corresponding to ground states can generate a finite number of conformal excitations required by the p-adic mass calculations and p-adic thermodynamics would be due to the presence of instanton term responsible also for CP breaking and matter antimatter asymmetry. One can of course ask whether the interpretation in terms of virtual states really correct. Should one quantize also the modes of D with higher values of conformal weight so that an infinite number of fermionic oscillators would result and the effective finite-dimensionality of the configuration space Clifford algebra would be lost?

1. The propagator interpretation would suggest that the introduction of an infinite number of oscillator operators is not necessary since the propagator expressible in standard manner in terms of the eigen modes of $iD(Y^1)$ is annihilated by $iD(Y^1)$ inside Y^1 whereas at the ends delta function singularity appears. Therefore the conformal weights appearing in the propagator

would be analogous to the virtual momenta of particles appearing in ordinary Feynman graphs. The objection is that in p-adic calculations the super-conformal excitations are assumed to be physical. One might however argue that since interactions cause p-adic particle massivation and since physical states correspond to Feynman diagrams virtual states must be allowed.

2. The interpretation as genuine oscillator modes finds support from the notion of finite measurement resolution realized in terms of Jones inclusions. The Clifford algebra generated by all modes would define the Clifford algebra of the entire configuration space and finite-dimensional Clifford algebra generated by $n=0$ oscillator operators would define quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ for quantum variant of configuration space. On mass shell property characterizes the finite measurement resolution and leave only $n=0$ states as on mass shell states. One would obtain automatically genuine representations of conformal algebras and their quantum variants by dropping away the oscillator operators having $n>0$.

3. The generalization of the notion of number theoretic braid to include also the points corresponding to extrema of $J=\epsilon\mu\nu J_{\mu\nu}$ for both CP_2 and $\delta M^4_{\pm}$ symplectic forms could allow to realize maximal conformal cutoff n_{max}, and at the limit when the number of extrema approaches infinite, entire $\mathcal{M}$ could be represented.

4. One could argue that the allowance of a infinite number of eigenvalues of D leads to problems with the definition of Kähler function as a Dirac determinant unless zeta function regularization works. Below it will be found that due the special form of the spectrum of conformal weights zeta function regularization reduces to the standard zeta function regularization.

What about the definition of Dirac determinant?

If one allows the conformal modes in the oscillator algebra, the definition of the Dirac determinant and Kähler function might become problematic since the product of the eigenvalues diverges without regularization.

1. Zeta function regularization would Kähler function as

$$K = -\frac{\zeta_D(s)}{ds}_{|s=0} = \sum_{k,n} log(\lambda_k+\sqrt{n}) \tag{2.4.29}$$

Dirac Zeta is defined as

$$\begin{aligned}\zeta_D(s) &= \sum_{\lambda_k}\zeta_D(s,\lambda_k) \ , \\ \zeta(s,\lambda_k) &= \sum_n(\lambda_k+\sqrt{n})^{-s}\end{aligned} \tag{2.4.30}$$

$\zeta(s,\lambda)$ differs from Riemann zeta by the replacement $n\rightarrow\lambda+n$.

2. For Riemann Zeta the analytic continuation to a well defined function is possible and one might hope that the shift $n\rightarrow\lambda+n$ does not spoil this property. In fact, one can expand $\zeta(s,\lambda)$ by using the generalized binomial formula for $(\lambda+n)^s$ in terms of gamma functions and Riemann zetas with shifted arguments

$$\begin{aligned}\zeta(s,\lambda) &= \sum_n(\lambda+\sqrt{n})^{-s} \\ &= \sum_{k\geq 0}\begin{pmatrix} -s \\ k\end{pmatrix}\lambda^k\sum_n n^{(-s-k)/2} \\ &= \sum_{k\geq 0}\frac{\Gamma(-s+1)}{\Gamma(-s-k+1)k!}\lambda^k\zeta(\frac{s+k}{2}) \ .\end{aligned} \tag{2.4.31}$$

3. This expansion might provide the desired analytic continuation allowing to calculate Kähler function. At $s = 0$ the binomial coefficients vanish for $k > 0$ since $\Gamma(-k+1)$ develops a pole. The derivative of the binomial coefficient equals to $1/Res(\Gamma(-k+1)) = (-1)^{k-1}(k+1)!$ (at pole one has by definition $f(z) \simeq Res(f(z_0))/(z-z_0))$ for $k > 0$ and vanishes for $k = 0$. Hence the derivative of $\zeta(s,\lambda)$ at origin is given by

$$-\frac{d\zeta(s,\lambda)}{ds}_{|s=0} = -\frac{1}{2}\times\frac{d\zeta(s)}{ds}_{|s=0} + \sum_{k>0}(-1)^k(k+1)\zeta(\frac{k}{2})\lambda^k \ . \quad (2.4.32)$$

This expression is be well-defined as an alternative series and by the fact that ζ approaches zero for large values of real argument. For $\lambda = n$ one has

$$\zeta(s,n) = \zeta(s) - \sum_{k=1}^{n} k^{-s} \ , \quad (2.4.33)$$

so that the analytic continuation is well-defined also now and Kähler function is calculable.

4. Kähler function would be sum over the contributions corresponding to different values of λ_k.

$$\begin{aligned} K &= -\frac{d\zeta_D(s)}{ds}_{|s=0} = -\sum_k \frac{d\zeta(s,\lambda_k)}{ds}_{|s=0} \\ &= -\frac{N}{2}\times\frac{d\zeta(s)}{ds}_{|s=0} + \sum_{n>0}(-1)^n(n+1)\zeta(\frac{n}{2})\sum_k \lambda_k^n \\ &\equiv \sum_{n>0}(-1)^n(n+1)\zeta(\frac{n}{2})\sum_k \lambda_k^n \ . \end{aligned} \quad (2.4.34)$$

Here N is the number of ground state conformal weights. As a matter fact, the contribution of the constant term does not affect configuration space metric so that it can be dropped.

5. The zeros of $\zeta_D(s)$ are not expected to reside at the critical line $Re(s) = 1/2$ (as the case $\lambda = m \in Z$ shows). The nice feature of this option is that the long standing conjecture that Riemann zeta is a fundamental element of quantum TGD would not be completely wrong. In particular, the often held belief that Zeta functions accompany critical systems would conform with the quantum criticality of TGD.

6. One can always make also optimistic conjectures about mathematical miracles. There are two Dirac determinants since both $D(X^2)$ with eigenvalue spectrum λ_k and $D(X^1)$ with eigenvalue spectrum $-\lambda_k + \sqrt{n}$ define Dirac determinants. Which option should one choose? Perhaps there is no need to choose! Maybe the Dirac determinants defined by the two candidates for ζ_D give identical expressions for the configuration space metric. This does not require identical expressions for Kähler function which can contain arbitrary additive part given by a real part of holomorphic function of configuration space coordinates. This would give physical justification for zeta function regularization. The identity of the two zeta functions -which is by no means necessary- is not supported by the observation that the additive contribution of λ_k to Kähler function equals to $log(\lambda_k)$ for finite-dimensional Dirac determinant whereas in infinite-dimensional case the contribution analytic function of λ_k.

2.4.6 Some comments about super-conformal symmetries

Here only a brief summary of super-conformal symmetries of the modified Dirac action is given with more detailed discussion left to a separate section.

1. The topological character of the solution spectrum extends the super-conformal symmetries in X^2 degrees of freedom by replacing holomorphic functions with arbitrary functions so that conformal symmetries become pure gauge symmetries. Arbitrary diffeomorphisms of CP_2, including local SU(3) and its holomorphic counterpart, act as symmetries of the non-vacuum solutions. Also the symplectic transformations of CP_2 inducing a U(1) gauge transformation are symmetries. More generally, the symplectic transformations of $\delta M^4_\pm \times CP_2$ define configuration space symmetries.

2. Diffeomorphisms of M^4 respecting the light-likeness condition of X^3_l and the condition $M^2(x) \subset T(X^4(X^3_l))$ or the weaker condition replacing $M^2(x)$ with light-like $M_+(x) \subset M^2(x)$ define Kac-Moody symmetries. In particular, holomorphic deformations of X^3_l defined in E^2 factor of $M^2 \times E^2$ compensated by a hyper-complex deformation in M^2 degrees taking care that light-likeness is not lost, act as symmetry transformations. This requires that M^2 and E^2 contributions of the deformation to the induced metric compensate each other.

3. The radial conformal symmetries associated with both $\delta M^4_\pm \times CP_2$ and X^3_l in light-like direction generalize the dynamical conformal symmetries characterizing CP_2 type vacuum extremals and could be regarded as dynamical conformal symmetries defining the spectrum of super-symplectic conformal weights assigned originally to the radial light-like coordinate of $\delta M^4_\pm$. It deserves to be emphasized that the topological QFT character of TGD at fundamental level broken only by the light-likeness of X^3_l carrying information about H metric makes possible these symmetries.

4. The fact that the modified Dirac equation reduces to a one-dimensional Dirac equation allows the action of Kac-Moody algebra as a symmetry algebra of spinor fields and the fermionic conserved currents can be deduced as Noether currents. In M^4 degrees of freedom X^2-local SL(2,C) acts as super-conformal symmetries and extends the SU(2) Kac-Moody algebra of $N = 4$ super-conformal algebra to $SL(2, C)$. The reduction to SU(2) occurs naturally. These symmetries act on all spinor components rather than on the second spinor chirality or right handed neutrinos only.

5. The situation with electro-weak $U(2)$ is somewhat problematic. Since color symmetries are realized in terms of super-symplectic generators one could argue that the $U(2)_{ew}$ can be identified as $U(2)$ subgroup of color $SU(3)$ as far as Kac-Moody symmetries at space-time level are considered. If so, then the Noether charges associated with color isospin and hyper-charges would define as their linear combinations electro-weak charges and realized electro-weak gauge symmetries as genuine gauge symmetries in transversal space X^2 and as Kac-Moody symmetries in light-like direction. Hence all the desired Kac-Moody symmetries would be realized.

6. The action of Super Kac-Moody symmetries corresponds to the addition of a linear combination of zero modes of D_K to Ψ whose non-gauge part is identified as the superposition of the generalized eigen modes having coefficients in the fermionic oscillator algebra. The coefficients of gauge parts are assumed to be ordinary numbers. These transformations can be written as X^2 local super-Hamiltonian transformations $\delta\Psi = \epsilon\Phi(x) j^k_A \Gamma_k \Psi_0$, where j^k_A is the vector field of symplectic transformation and Ψ_0 is arbitrary c-number spinor. Super-generators anti-commute to X^2-local Hamiltonians so that genuine super symmetries are in question.

7. $N = 4$ super-conformal symmetry corresponding to the maximal representation with the group $SU(2) \times SU(2) \times U(1)$ acting as rotations and electro-weak symmetries on imbedding space spinors is in question. This symmetry is broken for light-like 3-surfaces not satisfying field equations. It seems that rotational SU(2) can be extended to the full Lorentz group.

The picture about symplectic symmetries remained unchanged for a long time. The discovery that Kac-Moody algebra consisting of X^2 local symmetries generated by Hamiltonians of isometry sub-algebra of symplectic algebra forced to challenge this picture and ask whether also X^2-local transformations of symplectic group could be involved.

1. The basic condition is that the X^2 local transformation leaves induced Kähler form invariant apart from diffeomorphism. Denote the infinitesimal generator of X^2 local symplectomorphism by $\Phi_A(x)j^{Ak}$, where A labels Hamiltonians in the sum and by j^α the generator of X^2 diffeomorphism.

2. The invariance of $J \equiv \epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ modulo diffeomorphism under the infinitesimal symplectic transformation gives

$$\{H^A, \Phi_A\} \equiv \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi_A = \partial_\alpha J j^\alpha \ . \tag{2.4.35}$$

3. Note that here the Poisson bracket is not defined by $J^\alpha\beta$ but $\epsilon^{\alpha\beta}$ defined by the induced metric. Left hand side reflects the failure of symplectomorphism property due to the dependence of $\Phi_A(x)$ on X^2 coordinate which and comes from the gradients of $\delta M^4 \times CP_2$ coordinates in the expression of the induced Kähler form. Right hand side corresponds to the action of infinitesimal diffeomorphism.

4. Let us assume that one can restrict the consideration to single Hamiltonian so that the transformation is generated by $\Phi(x)H_A$ and that to each $\Phi(x)$ there corresponds a diffeomorphism of X^2, which is a symplectic transformation of X^2 with respect to symplectic form $\epsilon^{\alpha\beta}$ and generated by Hamiltonian $\Psi(x)$. This transforms the invariance condition to

$$\{H^A, \Phi\} \equiv \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi = \partial_\alpha J \epsilon^{\alpha\beta}\partial_\beta \Psi_A = \{J, \Psi_A\} \ . \tag{2.4.36}$$

   This condition can be solved identically by assuming that Φ_A and Ψ are proportional to arbitrary smooth function of J:

$$\Phi = f(J) \ , \quad \Psi_A = -f(J)H_A \ . \tag{2.4.37}$$

   Therefore the X^2 local symplectomorphisms of H reduce to symplectic transformations of X^2 with Hamiltonians depending on single coordinate J of X^2. The analogy with conformal invariance for which transformations depend on single coordinate z is obvious. As far as the anti-commutation relations for induced spinor fields are considered this means that $J = consant$ curves behave as points points. For extrema of J appearing as candidates for points of number theoretic braids $J = constant$ curves reduce to points.

5. From the structure of the conditions it is easy to see that the transformations generate a Lie-algebra. For the transformations $\Phi^1_A H^A$ $\Phi^2_A H^A$ the commutator is

$$\Phi_A^{[1,2]} = f_A^{\ BC}\Phi_B\Phi_C \ , \tag{2.4.38}$$

   where $f_A^{\ BC}$ are the structure constants for the symplectic algebra of $\delta M^4_\pm \times CP_2$. From this form it is easy to check that Jacobi identifies are satisfied. The commutator has same form as the commutator of gauge algebra generators. BRST gauge symmetry is perhaps the nearest analog of this symmetry. In the case of isometries these transforms realized local color gauge symmetry in TGD sense.

6. If space-time surface allows a slicing to light-like 3-surfaces Y^3_l parallel to X^3_l, these conditions make sense also for the partonic 2-surfaces defined by the intersections of Y^3_l with $\delta M^4_\pm \times CP_2$ and "parallel" to X^2. The local symplectic transformations also generalize to their local variants in X^3_l. Light-likeness of X^3_l means effective metric 2-dimensionality so that 2-D Kähler metric and symplectic form as well as the invariant $J = \epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ exist. A straightforward calculation shows that the the notion of local symplectic transformation makes sense also now and formulas are exactly the same as above.

2.4.7 Number theoretic braids and global view about anti-commutations of induced spinor fields

The anti-commutations of induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-surfaces taking which would thus seem to take the role of fundamental dynamical objects. Conformal invariance in turn seems to make the 2-D partons 1-D objects and number theoretical braids in turn discretizes strings.

Somehow these apparently contradictory views should be unifiable in a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales. The notions of measurement resolution and number theoretic braid indeed provide the needed insights in this respect.

Second quantization of induced spinor fields

Second quantized induced spinor field Ψ can be expressed as sum of the eigen modes in standard manner with coefficients of eigen modes identified as oscillator operators whose anti-commutations should give rise to the counterpart of the standard anti-commutation relations. According to the basic argument the number of modes for induced spinor fields is finite since each region X^2_i in which induced Kähler field is non-vanishing corresponds only finite number of states analogous cyclotron states. This means that it is not possible to realize the standard anti-commutation relations for Ψ and $\Psi^\dagger$ except in a finite subset of points of X^2.

1. The standard canonical anti-commutation relations for the induced the spinor fields would given by

$$\{\overline{\Psi}\hat{\Gamma}^0(x), \Psi(y)\} = \delta^2(x,y) \ . \tag{2.4.39}$$

   The factor that $\hat{\Gamma}^0(x) = (\partial L/\partial h^k_0)\Gamma^k$ corresponds to the canonical momentum density associated with Kähler action so that the coefficients are components of generalized energy momentum tensor.

2. These relations must be replaced by their discrete variant given by

$$\{\overline{\Psi}\hat{\Gamma}^0(x_i), \Psi(x_j)\} = \delta_{i,j} \ . \tag{2.4.40}$$

   where x_i and x_j label the points of the number theoretic braid B.

3. B is defined the set of points of X^2, whose $\delta M^4_\pm$ projections commute.The points of belong to an intersection $M_\pm \cap P_{M^4}(X^2)$ where $M_\pm \subset M^2 \subset M^4 \subset M^8$ is light-like radial ray and P_{M^4} denotes the projection map. This definition is completely symmetric with respect to M^8 and H and the representations of B in H and M^8 are related by $M^8 - H$ duality. Not that the outcome is a fractal hierarchy of state basis corresponding to the hierarchy of CDs.

The finiteness of the number of eigen modes and the decomposition of X^2 into regions inside which induce Kähler field is non-vanishing bring in some delicacies.

1. There is no obvious reason why the number N_m of eigen modes for a given region should be same as the number of points of number theoretic braid N_B. For $N_m > N_B$ the solution to the conditions is not unique and for $N_m < N_B$ it seems necessary to drop some commutative points from consideration in order to obtained discrete version of purely local anti-commutation relations essential for the realization of the super-symplectic algebra (Poisson bracket must result as an outcome).

2. Effectively the symplectic algebra decomposes into a direct sum corresponding to the regions inside which induced Kähler form is non-vanishing and each region corresponds effectively to a Cartesian factor of the configuration space possessing dimension defined by the number N_m of eigen-modes. The dimension of the Clifford algebra for given fermionic chirality (leptonic or quark like) of configuration space reduces to 2^{N_m} corresponding to the pairs formed by spinor modes and their conjugates. There are $8N_B^2$ anti-commutativity conditions and one must have $8N_B^2 \leq N_m$ in order to have solutions to the conditions ($D = 8$ is the number spinor components). If this not the case one must drop some points from the number theoretic braid.

3. Symplectic fusion algebra [C4] might also be important element in quantization. The relationship between symplectic fusion algebra and its conjugate has not been characterized and one can consider the possibility that the algebra generators satisfy the conditions $e_m\overline{e}_n = \delta_{m,n}$. If induced spinor field at points of number theoretic braid defining the symplectic fusion algebra is multiplied by e_m then the anti-commutation relations reduce automatically to a form in which anti-commutators at same point are involved. This would reduce the number of conditions to $8N_B$ from $8N_B^2$. The notion of finite measurement resolution could be used to defend this option.

Interesting questions relate to the non-uniqueness of the number theoretic braid.

1. Since the commutativity conditions are only relative conditions, very large number of points sets satisfying the defining conditions of number theoretic braid exists. The fixing of the commutative sub-manifold to $M_\pm$ however reduces this degeneracy to a high degree.

2. Second question relates to the choice of $M_\pm$. It would seem natural to allow a kind of direct integral over all possible choices of this ray in order to a gain maximum amount of information about X^2. This would mean that the sphere describing the possible choices of $M_\pm$ defines a moduli space.

The decomposition into 3-D patches and QFT description of particle reactions at the level of number theoretic braids

What is the physical meaning of the decomposition of 3-D light-like surface to patches? It would be very desirable to keep the picture in which number theoretic braid connects the incoming positive/negative energy state to the partonic 2-surfaces defining reaction vertices. This is not obvious if X_l^3 decomposes into causally independent patches. One can however argue that although each patch can define its own fermion state it has a vanishing net quantum numbers in zero energy ontology, and can be interpreted as an intermediate virtual state for the evolution of incoming/outgoing partonic state.

This picture conforms with zero energy ontology in which hierarchy of causal diamonds (CDs) within CDs gives rise to a hierarchy of generalized Feynman diagrams and geometric description of the radiative corrections. Each sub-CD gives also rise to to zero energy states and thus particle reactions in its own time scale so that improvement of the time resolution brings in also new physics as it does also in reality.

Another problem - actually only apparent problem - has been whether it is possible to have a generalization of the braid dynamics able to describe particle reactions in terms of the fusion and decay of braid strands. This kind of description is un-necessary if one accepts the notion of generalized Feynman diagram.

One could also worry about whether the generalized Feynman diagrams give rise to non-trivial vertices. This is the case. The point is that the ends of incoming and outgoing braid strands at partonic 2-surfaces but their ends do not co-incide in general. Therefore reactions changing particle numbers become possible if the N-point functions do not reduce to those free conformal field theory. Although the incoming (outgoing) fermionic oscillator operators anti-commute the anticommutators between incoming and outgoing oscillator operators are non-vanishing so that the N-point functions defining the vertices are non-trivial. Hence the finiteness of the number of eigenmodes and the notion of number theoretic braid are absolutely essential for non-triviality of the theory.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

How generalized braid diagrams relate to the perturbation theory?

The association of generalized braid diagrams to incoming and outgoing partonic legs and internal lines of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams. The basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancelation occurs only for critical values of Kähler coupling strength α_K: for general values of α_K cancellation would require separate vanishing of each term in the sum and does not occur.

This would mean following.

1. One would not have perturbation theory around a given maximum of Kähler function but as a sum over increasingly complex maxima of Kähler function. Radiative corrections in the sense of perturbative functional integral around a given maximum would vanish (so that the expansion in terms of braid topologies would not make sense around single maximum). Radiative corrections would not vanish in the sense of a sum over 3-topologies obtained by adding radiative corrections as zero energy states in shorter time scale.

2. Connes tensor product with a given measurement resolution would correspond to a restriction on the number of maxima of Kähler function labeled by the braid diagrams. For zero energy states in a given time scale the maxima of Kähler function could be assigned to braids of minimal complexity with braid vertices interpreted in terms of an addition of radiative corrections. Hence a connection with QFT type Feyman diagram expansion would be obtained and the Connes tensor product would have a practical computational realization.

3. The cutoff in the number of topologies (maxima of Kähler function contributing in a given resolution defining Connes tensor product) would be always finite in accordance with the algebraic universality.

4. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

There are still some questions. Radiative corrections around given 3-topology vanish. Could radiative corrections sum up to zero in an ideal measurement resolution also in 2-D sense so that the initial and final partonic 2-surfaces associated with a partonic 3-surface of minimal complexity would determine the outcome completely? Could the 3-surface of minimal complexity correspond to a trivial diagram so that free theory would result in accordance with asymptotic freedom as measurement resolution becomes ideal?

The answer to these questions seems to be 'No'. In the p-adic sense the ideal limit would correspond to the limit $p \rightarrow 0$ and since only $p \rightarrow 2$ is possible in the discrete length scale evolution defined by primes, the limit is not a free theory. This conforms with the view that CP_2 length scale defines the ultimate UV cutoff.

2.5 Super-conformal symmetries at space-time and configuration space level

The physical interpretation and detailed mathematical understanding of super-conformal symmetries has developed rather slowly and has involved several side tracks. In the following I try to summarize the basic picture with minimal amount of formulas with the understanding that the statement "Noether charge associated with geometrically realized Kac-Moody symmetry" is enough for the reader to write down the needed formula explicitly.

2.5.1 Configuration space as a union of symmetric spaces

In finite-dimensional context globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. The hope is that same holds true in infinite-dimensional context. The most one can hope of obtaining is the decomposition $C(H) = \cup_i G/H_i$ over orbits of G. One could allow also symmetry breaking in the sense that G and H depend on the orbit: $C(H) = \cup_i G_i/H_i$ but it seems that G can be chosen to be same for all orbits. What is essential is that these groups are infinite-dimensional. The basic properties of the coset space decomposition give very strong constraints on the group H, which certainly contains the subgroup of G, whose action reduces to diffeomorphisms of X^3.

Consequences of the decomposition

If the decomposition to a union of coset spaces indeed occurs, the consequences for the calculability of the theory are enormous since it suffices to find metric and curvature tensor for single representative 3-surface on a given orbit (contravariant form of metric gives propagator in perturbative calculation of matrix elements as functional integrals over the configuration space). The representative surface can be chosen to correspond to the maximum of Kähler function on a given orbit and one obtains perturbation theory around this maximum (Kähler function is not isometry invariant).

The task is to identify the infinite-dimensional groups G and H and to understand the zero mode structure of the configuration space. Almost twenty (seven according to long held belief!) years after the discovery of the candidate for the Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy as well as special properties of the Kähler action.

The guess (not the first one!) would be following. G corresponds to the symplectic transformations of $\delta M^4_\pm \times CP_2$ leaving the induced Kähler form invariant. If G acts as isometries the values of Kähler form at partonic 2-surfaces (remember effective 2-dimensionality) are zero modes and configuration space allows slicing to symplectic orbits of the partonic 2-surface with fixed induced Kähler form. Quantum fluctuating degrees of freedom would correspond to symplectic group and to the fluctuations of the induced metric. The group H dividing G would in turn correspond to the Kac-Moody symmetries respecting light-likeness of X^3_l and acting in X^3_l but trivially at the partonic 2-surface X^2. This coset structure was originally discovered via coset construction for super Virasoro algebras of super-symplectic and super Kac-Moody algebras and realizes Equivalence Principle at quantum level.

Configuration space isometries as a subgroup of $Diff(\delta M^4_+ \times CP_2)$

The reduction to light cone boundary leads to the identification of the isometry group as some subgroup of for the group G for the diffeomorphisms of $\delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the the space of 3-surfaces in $\delta M^4_+ \times CP_2$. Configuration space is expected to decompose to a union of the coset spaces G/H_i, where H_i corresponds to some subgroup of G containing the transformations of G acting as diffeomorphisms for given X^3. Geometrically the vector fields acting as diffeomorphisms of X^3 are tangential to the 3-surface. H_i could depend on the topology of X^3 and since G does not change the topology of 3-surface each 3-topology defines separate orbit of G. Therefore, the union involves sum over all topologies of X^3 plus possibly other 'zero modes'. Different topologies are naturally glued together since singular 3-surfaces intermediate between two 3-topologies correspond to points common to the two sectors with different topologies.

2.5.2 Isometries of configuration space geometry as symplectic transformations of $\delta M^4_+ \times CP_2$

During last decade I have considered several candidates for the group G of isometries of the configuration space as the sub-algebra of the subalgebra of $Diff(\delta M^4_+ \times CP_2)$. To begin with let us write the general decomposition of $diff(\delta M^4_+ \times CP_2)$:

$$diff(\delta M^4_+ \times CP_2) \quad = \quad S(CP_2) \times diff(\delta M^4_+) \oplus S(\delta M^4_+) \times diff(CP_2) \ . \tag{2.5.1}$$

Here $S(X)$ denotes the scalar function basis of space X. This Lie-algebra is the direct sum of light cone diffeomorphisms made local with respect to CP_2 and CP_2 diffeomorphisms made local with respect to light cone boundary.

The idea that entire diffeomorphism group would act as isometries looks unrealistic since the theory should be more or less equivalent with topological field theory in this case. Consider now the various candidates for G.

1. The fact that symplectic transformations of CP_2 and M^4_+ diffeomorphisms are dynamical symmetries of the vacuum extremals suggests the possibility that the diffeomorphisms of the light cone boundary and symplectic transformations of CP_2 could leave Kähler function invariant and thus correspond to zero modes. The symplectic transformations of CP_2 localized with respect to light cone boundary acting as symplectic transformations of CP_2 have interpretation as local color transformations and are a good candidate for the isometries. The fact that local color transformations are not even approximate symmetries of Kähler action is not a problem: if they were exact symmetries, Kähler function would be invariant and zero modes would be in question.

2. CP_2 local conformal transformations of the light cone boundary act as isometries of δM^4_+. Besides this there is a huge group of the symplectic symmetries of $\delta M^4_+ \times CP_2$ if light cone boundary is provided with the symplectic structure. Both groups must be considered as candidates for groups of isometries. $\delta M^4_+ \times CP_2$ option exploits fully the special properties of $\delta M^4_+ \times CP_2$, and one can develop simple argument demonstrating that $\delta M^4_+ \times CP_2$ symplectic invariance is the correct option. Also the construction of configuration space gamma matrices as super-symplectic charges supports $\delta M^4_+ \times CP_2$ option.

This picture remained same for a long time. The discovery that Kac-Moody algebra consisting of X^2 local symmetries generated by Hamiltonians of isometry sub-algebra of symplectic algebra forced to challenge this picture and ask whether also X^2-local transformations of symplectic group could be involved.

1. The basic condition is that the X^2 local transformation acts leaves induced Kähler form invariant apart from diffeomorphism. Denote the infinitesimal generator of X^2 local symplecto morphism by $\Phi_A(x)j^{Ak}$, where A labels Hamiltonians in the sum and by j^α the generator of X^2 diffeomorphism.

2. The invariance of $J = \epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ modulo diffeomorphism under the infinitesimal symplectic transformation gives

$$\{H^A, \Phi_A\} \quad \equiv \quad \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi_A = \partial_\alpha J j^\alpha \ . \tag{2.5.2}$$

3. Note that here the Poisson bracket is not defined by $J^\alpha\beta$ but $\epsilon^{\alpha\beta}$ defined by the induced metric. Left hand side reflects the failure of symplectomorphism property due to the dependence of $\Phi_A(x)$ on X^2 coordinate which and comes from the gradients of $\delta M^4 \times CP_2$ coordinates in the expression of the induced Kähler form. Right hand side corresponds to the action of infinitesimal diffeomorphism.

4. Let us assume that one can restrict the consideration to single Hamiltonian so that the transformation is generated by $\Phi(x)H_A$ and that to each $\Phi(x)$ there corresponds a diffeomorphism of X^2, which is a symplectic transformation of X^2 with respect to symplectic form $\epsilon^{\alpha\beta}$ and generated by Hamiltonian $\Psi(x)$. This transforms the invariance condition to

$$\{H^A,\Phi\} \equiv \partial_\alpha H^A \epsilon^{\alpha\beta}\partial_\beta \Phi = \partial_\alpha J \epsilon^{\alpha\beta}\partial_\beta \Psi_A = \{J,\Psi_A\} \ . \tag{2.5.3}$$

This condition can be solved identically by assuming that Φ_A and Ψ are proportional to arbitrary smooth function of J:

$$\Phi = f(J) \ , \quad \Psi_A = -f(J)H_A \ . \tag{2.5.4}$$

Therefore the X^2 local symplectomorphisms of H reduce to symplectic transformations of X^2 with Hamiltonians depending on single coordinate J of X^2. The analogy with conformal invariance for which transformations depend on single coordinate z is obvious. As far as the anti-commutation relations for induced spinor fields are considered this means that $J = consant$ curves behave as points points. For extrema of J appearing as candidates for points of number theoretic braids $J = constant$ curves reduce to points.

5. From the structure of the conditions it is easy to see that the transformations generate a Lie-algebra. For the transformations $\Phi^1_A H^A$ $\Phi^2_A H^A$ the commutator is

$$\Phi_A^{[1,2]} = f_A^{\ BC}\Phi_B\Phi_C \ , \tag{2.5.5}$$

where $f_A^{\ BC}$ are the structure constants for the symplectic algebra of $\delta M^4_\pm \times CP_2$. From this form it is easy to check that Jacobi identifies are satisfied. The commutator has same form as the commutator of gauge algebra generators. BRST gauge symmetry is perhaps the nearest analog of this symmetry. In the case of isometries these transforms realized local color gauge symmetry in TGD sense.

6. If space-time surface allows a slicing to light-like 3-surfaces Y^3_l parallel to X^3_l, these conditions make sense also for the partonic 2-surfaces defined by the intersections of Y^3_l with $\delta M^4_\pm \times CP_2$ and "parallel" to X^2. The local symplectic transformations also generalize to their local variants in X^3_l. Light-likeness of X^3_l means effective metric 2-dimensionality so that 2-D Kähler metric and symplectic form as well as the invariant $J = \epsilon^{\alpha\beta}J_{\alpha\beta}$ exist. A straightforward calculation shows that the the notion of local symplectic transformation makes sense also now and formulas are exactly the same as above.

2.5.3 Identification of Kac-Moody symmetries

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating configuration space degrees of freedom contributing to the metric.

Identification of Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{2.5.6}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \rightarrow x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \tag{2.5.7}$$

Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \ . \tag{2.5.8}$$

This gives

$$\begin{aligned} c_A(x) & \left[D_k j^A_l + D_l j^A_k\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l \\ = & \ \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \end{aligned} \tag{2.5.9}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j^A_l + D_l j^A_k = 2h_{kl} \ . \tag{2.5.10}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \tag{2.5.11}$$

A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k = 0 \ . \tag{2.5.12}$$

The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r) d_A$ by a one obtains

$$d_A h_{kl} j^{Ak} \partial_r h^k = 0 \ . \tag{2.5.13}$$

so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3- local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i \quad = \quad \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \quad . \tag{2.5.14}$$

The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj} \partial_i \xi^k + g_{ki} \partial_j \xi^k \quad = \quad \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \quad . \tag{2.5.15}$$

These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] \quad = \quad c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \quad , \tag{2.5.16}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{aligned} \left[D, K^k\right] &= -K^k \ , & \left[D, P^k\right] &= P^k \ , \\ \left[K^k, K^l\right] &= 0 \ , & \left[K^k, P^l\right] &= m^{kl}D - M^{kl} \ . \end{aligned} \quad (2.5.17)$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak}h_{kl}\partial_r h^l$$

   is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labeled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$. Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3) \times E^3$ respecting the radial conformal weights would reduce to $SO(2) \times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

Hamiltonians

The action of these transformations on Kähler action is well-defined and one can deduce the conserved quantities having identification as configuration space Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_\pm \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

The action of Kac-Moody algebra on spinors and fermionic representations of Kac-Moody algebra

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

1. The basic goal is to deduce the fermionic Noether charge associated with the bosonic Kac-Moody symmetry and this can be done by a standard recipe. The first contribution to the charge comes from the transformation of modified gamma matrices appearing in the modified Dirac action associated with fermions. Second contribution comes from spinor rotation.

2. Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors.

How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-symplectic algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

About the interpretation of super Kac-Moody symmetries

Also the light like 3-surfaces X_l^3 of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to the configuration space metric. In this case the symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X_l^3 so that Kac-Moody type symmetry results. Also the condition $\sqrt{g_3} = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction.

If is enough to localize only the H-isometries with respect to X_l^3, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation need not be equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them.

The following arguments support one possible identification.

1. The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

2. Since X_l^3-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

3. The fact that only quarks appear in the gamma matrices of the configuration space supports the view that action of the generators of X_l^3-local color transformations on configuration space spinor fields represents local color transformations. If the action of X_l^3-local $SU(3)$ transformations on configuration space spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N = 4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only righthanded neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of configuration space gamma matrices forming a subalgebra of the full super-algebra.

$N = 2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N = 2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N = 2$ super-conformal representations are modular invariant, and that $N = 2$ algebra defines so called chiral ring defining a topological quantum field theory [22], lend a further support for the belief that $N = 2$ super-conformal algebra acts in super-symplectic degrees of freedom.

The values of c and conformal weights for $N = 2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \quad l = 0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \quad m = -l,-l+2,....,l-2,l \ . \end{aligned} \tag{2.5.18}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k = 1$ one would have $q = 0, 1/3, -1/3$, which brings in mind anyons. $\Delta_{l=0,m=0} = 0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [22].

For Ramond representation $L_0 - c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta = c/24$ giving the condition $k = 2\left[l(l+2) - m^2\right]$ (note that k must be even and that $(k, l, m) = (4, 1, 1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac} = \pm c/12 = \pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N = 4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on configuration space Hamiltonians expressible in terms of Hamiltonians of $X^3_l \times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h = 1$ whereas T and G have conformal weights $h = 2$ and $h = 3/2$.

The experience with $N = 4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h = 1/2$ and their super-partners with $h = 0$ and realized as fermion-antifermion bilinears. Since G and Ψ are labeled by 2×4 spinor indices, super-partners would correspond to $2 \times (3 + 1) = 8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

2.5.4 Coset space structure for configuration space as a symmetric space

The key ingredient in the theory of symmetric spaces is that the Lie-algebra of G has the following decomposition

$$
\begin{aligned}
& g = h + t \ , \\
& [h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ .
\end{aligned}
$$

In present case this has highly nontrivial consequences. The commutator of *any* two infinitesimal generators generating nontrivial deformation of 3-surface belongs to h and thus vanishing norm in the configuration space metric at the point which is left invariant by H. In fact, this same condition follows from Ricci flatness requirement and guarantees also that G acts as isometries of the configuration space. This generalization is supported by the properties of the unitary representations of Lorentz group at the light cone boundary and by number theoretical considerations.

The algebras suggesting themselves as candidates are symplectic algebra of $\delta M^{\pm} \times CP_2$ and Kac-Moody algebra mapping light-like 3-surfaces to light-like 3-surfaces to be discussed in the next section.

The identification of the precise form of the coset space structure is however somewhat delicate.

1. The essential point is that both symplectic and Kac-Moody algebras allow representation in terms of X^3_l-local Hamiltonians. The general expression for the Hamilton of Kac-Moody algebra is

$$
H \ = \ \sum \Phi_A(x) H^A \ . \tag{2.5.19}
$$

   Here H^A are Hamiltonians of $SO(3) \times SU(3)$ acting in $\delta X^3_l \times CP_2$. For symplectic algebra any Hamiltonian is allowed. If x corresponds to any point of X^3_l, one must assume a slicing of the causal diamond CD by translates of $\delta M^4_{\pm}$.

2. For symplectic generators the dependence of form on r^{Δ} on light-like coordinate of $\delta X^3_l \times CP_2$ is allowed. Δ is complex parameter whose modulus squared is interpreted as conformal weight. Δ is identified as analogous quantum number labeling the modes of induced spinor field.

3. One can wonder whether the choices of the $r_M = constant$ sphere S^2 is the only choice. The Hamiltonin-Jacobi coordinate for $X^4_{X^3_l}$ suggest an alternative choice as E^2 in the decomposition of $M^4 = M^2(x) \times E^2(x)$ required by number theoretical compactification and present for known extremals of Kähler action with Minkowskian signature of induced metric. In this case $SO(3)$ would be replaced with $SO(2)$. It however seems that the radial light-like coordinate u of $X^4(X^3_l)$ would remain the same since any other curve along light-like boundary would be space-like.

4. The vector fields for representing Kac-Moody algebra must vanish at the partonic 2-surface $X^2 \subset \delta M^4_{\pm} \times CP_2$. The corresponding vector field must vanish at each point of X^2:

$$
j^k \ = \ \sum \Phi_A(x) J^{kl} H^A_l = 0 \ . \tag{2.5.20}
$$

   This means that the vector field corresponds to $SO(2) \times U(2)$ defining the isotropy group of the point of $S^2 \times CP_2$.

   This expression could be deduced from the idea that the surfaces X^2 are analogous to origin of CP_2 at which $U(2)$ vector fields vanish. Configuration space at X^2 could be also regarded as the analog of the origin of local $S^2 \times CP_2$. This interpretation is in accordance with the original idea which however was given up in the lack of proper realization. The same picture can be deduced from braiding in which case the Kac-Moody algebra corresponds to local $SO(2) \times U(2)$ for each point of the braid at X^2. The condition that Kac-Moody generators with positive conformal weight annihilate physical states could be interpreted by stating effective 2-dimensionality in the sense that the deformations of X^3_l preserving its light-likeness do not affect the physics. Note however that Kac-Moody type Virasoro generators do not annihilate physical states.

5. Kac-Moody algebra generator must leave induced Kähler form invariant at X^2. This is of course trivial since the action leaves each point invariant. The conditions of Cartan decomposition are satisfied. The commutators of the Kac-Moody vector fields with symplectic generators are non-vanishing since the action of symplectic generator on Kac-Moody generator restricted to X^2 gives a non-vanishing result belonging to the symplectic algebra. Also the commutators of Kac-Moody generators are Kac-Moody generators.

2.5.5 Comparison of TGD and stringy views about super-conformal symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super-symmetries in TGD framework differs from that in string models in several fundamental aspects.

1. In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of symplectic transformations rather than vector fields generating them. This kind of representation applies also in Kac-Moody sector since the local transversal isometries localized in X^3_l and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

2. Super-symmetry generators can be identified as configuration space gamma matrices carrying quark and lepton numbers and the notion of super-space is not needed at all. Therefore no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an 1/2 disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore). This means that the interpretation of λ_i^2 (λ_i is generalized eigenvalue of $D_K(X^2)$) as ground state conformal weight does not lead to difficulties.

3. Kac-Moody and symplectic algebras generate larger algebra obtained by making symplectic algebra X^2 local. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom).

4. The finite number of spinor modes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework and the notion of number theoretic braid indeed implies this. The physical interpretation is in terms of finite measurement resolution.

Basic super-conformal symmetries

The identification of explicit representations of super conformal algebras was for a long time plagued by the lack of appropriate formalism. The modified Dirac operator D_K associated with Kähler action resolves this problem if one accepts the implications of number theoretic compactification supported by what is known about preferred extremals of Kähler action and one can identify the charges associated with symplectic and Kac-Moody algebra as Noether charges. Fermionic generators can in turn be identified from the condition that they anticommute to X^2 local Hamiltonians of corresponding bosonic transformations. In case of Super Virasoro algebra Sugaware construction allows to construct super generators G.

1. Covariantly constant right handed neutrino is the fundamental generator of dynamical super conformal symmetries and appears in both leptonic and quark-like realizations of gamma matrices. Γ matrices have also Super Kac-Moody counterparts and reduce in special case to symplectic ones. Also super currents whose anti-commutators give products of corresponding Hamiltonians can be defined so that both ordinary product and Poisson bracket give rise to quark and lepton like realizations of super-symmetries. Besides this there are also electric and magnetic representations of the gamma matrices.

2. The zero modes of $D_K(X^2)$ which do not depend on the light-like radial coordinate of X_l^3 define super conformal symmetries for which any c-number spinor field generates super conformal symmetry. These symmetries are pure gauge symmetries but also them can be parameterized by Hamiltonians and by functions depending only on the coordinates of the transverse section X^2 so that one obtains also now both function algebra and symplectic algebra localized with respect to X^2. Similar picture applies in both super-symplectic and super Kac-Moody sector. In particular, one can deduce canonical expressions for the super currents associated with these super symmetries. Since all charge states are possible for the generators of these super symmetries, these super symmetries naturally correspond to those assignable to electro-weak degrees of freedom.

3. The notion of X^2 local super-symmetry makes sense if the choice of coordinates x for X^2 is specified by the inherent properties of X^2 so that same coordinates x apply for all surfaces obtained as deformations of X^2. The regions, where induced Kähler form is non-vanishing define good candidates for coordinate patches. The Hamilton-Jacobi coordinates associated with the decomposition of M^4 are a natural choice. Also geodesic coordinates can be considered. The redundancy related to rotations of coordinate axis around origin can be reduced by choosing second axis so that it connects the origin to nearest point of the number theoretic braid.

4. The diffeomorphisms of light-like coordinate of $\delta M^4_\pm$ and X_l^3 playing the role of conformal transformations. One can construct fermionic representations of as Noether charges associated with modified Dirac action. The problem is however that that super-generators cannot be derived in this manner so that these transformations cannot be regarded as symplectic transformations. The manner to circumvent the difficulty is to construct fermionic super charges Γ_A as gamma matrices for both super symplectic and super Kac-Moody algebras in terms of generators $j^{Ak}\Gamma_k$ and corresponding Kac-Moody algebra elements T^A as fermionic super charges. From these operators super generators G can be constructed by the standard Sugawara construction allowing to interpret operators $G = T^A\Gamma_A$ as Dirac operators at the level of configuration space. By coset construction the actions of super-symplectic and super Kac-Moody Dirac operators are identical. Internal consistency requires that the Virasoro generators obtained as anticommutator $L = \{G, G^\dagger\}$ are equal to the Virasoro generators derived as fermionic Noether charges.

Finite measurement resolution and cutoff in the spectrum of conformal weights

The basic properties of Kähler action imply that the number generalized eigenvalues λ_i of $D_K(X^2)$ is finite. The interpretation is that the notion of finite measurement resolution is coded by Kähler action to space-time dynamics. This has also implications for the representations of super-conformal algebras.

1. The fermionic representations of various super-algebras involve only finite number of oscillator operators. Hence some kind of cutoff in the number of states reflecting the finiteness of the measurement resolution is unavoidable. A cutoff reduce integers as labels of the generators of super-conformal algebras to a finite number of integers. Finite field $G(p,1)$ for some prime p would be a natural candidate. Since p-adic integers modulo p are in question the cutoff could relate closely to effective p-adicity and p-adic length scale-hypothesis.

2. The interpretation of the eigenvalues of the modified Dirac operator as ground state conformal weights raises the question how to represent states with conformal weights $n + \lambda_i^2$, $n > 0$. The notion of number theoretic braid allows to circumvent the difficulty. Since canonical anti-commutation relations fail, one must replace the integral representations of super-conformal generators with discrete sums over the points of number theoretic braid, the resulting representations of super-conformal algebras must reduce to representation of finite-dimensional algebras. The cutoff on conformal weight must result from the fact that the higher Virasoro generators are expressible in terms of lower ones. The cutoff is not a problem since $n < 3$ cutoff for conformal weights gives an excellent accuracy in p-adic mass calculations. A not-very-educated guess but the only one that one can imagine is that for $p \simeq 2^k$, $n_{max} = k$ defines the cutoff on allowed conformal weights.

What are the counter parts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

1. Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [B2]. Thus the real variable J replaces complex coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

2. The slicing of X^2 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate.

3. An further identification for TGD parts of conformal fields is inspired by $M^8 - H$ duality. Conformal fields would be fields in configuration space. The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of configuration space Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

Generalized coset representation

X^2 local super-symplectic algebra as super Kac-Moody algebra as sub-algebra. Since X^2 locality corresponds to a full 2-D gauge invariance, one can conclude that SKM is in well defined sense sub-algebra of super-symplectic algebra so that generalized coset construction makes sense and generalizes Equivalence Principle in the sense that not only four-momenta but all analogous quantum numbers associated with SKM and SS algebras are identical.

1. In this framework the ground state conformal weights associated with both super-symplectic and super Kac-Moody algebras can be identified as squares of the eigenvalues λ_i of $D_K(X^2)$. This identification together with p-adic mass thermodynamics predicts that λ_i^2 gives to mass squared a contribution analogous to the square of Higgs vacuum expectation. This identification would resolve the long-standing problem of identifying the values of these ground state conformal weights for super-conformal algebras and give a direct connection with Higgs mechanism.

2. The identification of SKM as a sub-algebra of super-symplectic algebra becomes more convincing if the light-like coordinate r allows lifting to a light-like coordinate of H. This is achieved if r is identified as coordinate associated with a light-like curve whose tangent at point $x \in X_l^3$ is light-like vector in $M^2(x) \subset T(X^4(X^3)$. With this interpretation of SKM algebra as sub-algebra of super-symplectic algebra becomes natural.

3. The existence of a lifting of SS and SKM algebras to entire H would solve the problems. The lifting problem is obviously non-trivial only in M^4 degrees of freedom. Suppose that the existence of an integrable distribution of planes $M^2(x)$ and their orthogonal complements $E^2(x)$ belonging to the tangent space of M^4 projection $P_{M^4}(X^4(X^3))$ characterizes the preferred extremals with Minkowskian signature of induced metric. In this case the lifting of the super-symplectic and super Kac-Moody algebras to entire H is possible. The local degrees of freedom contributing to the configuration space metric would belong to the integrable distribution of orthogonal complements $E^2(x)$ of $M^2(x)$ having physical interpretation as planes of physical polarizations.

2.6 Trying to understand $N = 4$ super-conformal symmetry

The original idea was that $N = 4$ super-conformal symmetry is a symmetry generated by the solutions of the modified Dirac equation for the second quantized induced spinor fields. Later I was ended up with this symmetry by considering the general structure of these algebras interpreted in TGD framework. In the following the latter approach is discussed in detail.

Needless to say, a lot remains to be understood. One of the problems is that my understanding of $N = 4$ super-conformal symmetry at technical level is rather modest. There are also profound differences between these two kinds of super conformal symmetries. In TGD framework super generators carry quark or lepton number, super-symplectic and super Kac-Moody generators are identified as Hamiltonians rather than vector fields, and symplectic group is infinite-dimensional whereas the Lie groups associated with Kac-Moody algebras are finite-dimensional. On the other hand, finite measurement resolution implies discretization and cutoff in conformal weight. Therefore the naive attempt to re-interpret results of standard super-conformal symmetry to TGD framework might lead to erratic conclusions.

$N > 0$ super-conformal algebras contain besides super Virasoro generators also other types of generators and this raises the question whether it might be possible to find an algebra coding the basic quantum numbers of the induced spinor fields.

There are several variants of $N = 4$ SCAs and they correspond to the Kac-Moody algebras $SU(2)$ (small SCA), $SU(2)\times SU(2)\times U(1)$ (large SCA) and $SU(2)\times U(1)^4$. Rasmussen has found also a fourth variant based on $SU(2)\times U(1)$ Kac-Moody algebra [26]. It seems that only minimal and maximal $N = 4$ SCAs can represent realistic options. The reduction to almost topological string theory in critical phase is probably lost for other than minimal SCA but could result as an appropriate limit for other variants.

2.6.1 Large $N = 4$ SCA

Large $N = 4$ SCA is described in the following in detail since it might be a natural algebra in TGD framework.

The structure of large $N = 4$ SCA algebra

Large $N = 4$ super-conformal symmetry with $SU(2)_+\times SU(2)_-\times U(1)$ inherent Kac-Moody symmetry correspond to a fundamental partonic super-conformal symmetry in TGD framework.

A concise discussion of this symmetry with explicit expressions of commutation and anticommutation relations can be found in [26]. The representations of SCA are characterized by three central extension parameters for Kac-Moody algebras but only two of them are independent and given by

$$\begin{aligned} k_\pm &\equiv k(SU(2)_\pm) \ , \\ k_1 &\equiv k(U(1)) = k_+ + k_- \ . \end{aligned} \tag{2.6.1}$$

The central extension parameter c is given as

$$c = \frac{6k_+k_-}{k_+ + k_-} \ . \tag{2.6.2}$$

and is rational valued as required.

A much studied $N = 4$ SCA corresponds to the special case

$$\begin{aligned} k_- &= 1 \ , \ k_+ = k+1 \ , \ k_1 = k+2 \ , \\ c &= \frac{6(k+1)}{k+2} \ . \end{aligned} \tag{2.6.3}$$

$c = 0$ would correspond to $k_+ = 0, k_- = 1, k_1 = 1$. For $k_+ > 0$ one has $k_1 = k_+ + k_- \neq k_+$.

About unitary representations of large $N = 4$ SCA

The unitary representations of large $N = 4$ SCA are briefly discussed in [28]. The representations are labeled by the ground state conformal weigh h, SU(2) spins l_+, l_-, and U(1) charge u. Besides the inherent Kac-Moody algebra there is also "external" Kac-Moody group G involved and could correspond in TGD framework to the symplectic algebra associated with $\delta H_\pm = \delta M^4_\pm \times CP_2$ or to Kac-Moody group respecting light-likeness of light-like 3-surfaces.

Unitarity constraints apply completely generally irrespective of G so that one can apply them also in TGD framework. There are two kinds of unitary representations.

1. Generic/long/massive representations which are ge generated from vacuum state as usual. In this case there are no null vectors.

2. Short or massless representations have a null vector. The expression for the conformal weigt h_{short} of the null vector reads in terms of l_+, l_- and k_+, k_- as

$$h_{short} = \frac{1}{k_+ + k_-}(k_- l_+ + k_+ l_- + (l_+ - l_-)^2 + u^2) \ . \tag{2.6.4}$$

   Unitarity demands that both short and long representations lie at or above $h \geq h_{short}$ and that spins lie in the range $l_\pm = 0, 1/2, ..., (k_\pm - 1)/2$.

   Interesting examples of $N = 4$ SCA are provided by WZW coset models $\mathcal{W} \times U(1)$, where $\mathcal{W}$ is WZW model associated wto a quaternionic (Wolf) space. Examples based on classical groups are $\mathcal{W} = G/H = SU(n)/SU(n-1) \times U(1)$, $SO(n)/SO(n-4) \times SU(2)$, and $Sp(2n)/Sp(2n-2)$. For $n = 3$ first series gives CP_2 whereas second series gives for $n = 4$ $SO(4)/SU(2) = SU(2)$. In this case one has $k_+ = \kappa + 1$, and $k_- = \hat{c}_G$, where κ is the level of the bosonic current algebra for G and $\hat{c}_G$ is its dual Coxeter number.

2.6.2 Overall view about how different $N = 4$ SCAs could emerge in TGD framework

The basic idea is simple $N = 4$ fermion states obtained as different combinations of spin and isospin for given H-chirality of imbedding space spinor correspond to $N = 4$ multiplet. In case of leptons the holonomy group of $S^2 \times CP_2$ for given spinor chirality is $SU(2)_R \times SU(2)_R$ or $SU(2)_L \times SU(2)_R$ depending on M^4 chirality of the spinor. In case of quark one has $SU(2)_L \times SU(2)_L$ or $SU(2)_R \times SU(2)_R$. The coupling to Kähler gauge potential adds to the group $U(1)$ factor so that large $N = 4$ SCA is obtained. For covariantly constant right handed neutrino electro-weak part of holonomy group drops away as also $U(1)$ factor so that one obtains $SU(2)_L$ or $SU(2)_R$ and small $N = 4$ SCA.

How maximal $N = 4$ SCA could emerge in TGD framework?

Consider the Kac-Moody algebra $SU(2) \times SU(2) \times U(1)$ associated with the maximal $N = 4$ SCA. Besides Kac-Moody currents it contains 4 spin 1/2 fermions having an identification as quantum counterparts of leptonic spinor fields. The interpretation of the first $SU(2)$ is as rotations as rotations leaving invariant the sphere $S^2 \subset \delta M^4_\pm$. $U(2)$ has interpretation as electro-weak gauge group and as maximal linearly realized subgroup of $SU(3)$. This algebra acts naturally as symmetries of the 8-component spinors representing super partners of quaternions.

The algebra involves the integer value central extension parameters k_+ and k_- associated with the two SU(2) algebras as parameters. The value of $U(1)$ central extension parameter k is given by $k = k_+ + k_-$. The value of central extension parameter c is given by

$$c = 6k_- \frac{x}{1+x} < 6k_+ \ , \ x = \frac{k_+}{k_-} \ .$$

c can have all non-negative rational values m/n for positive values of $k_\pm$ given by $k_+ = rm, k_- = (6nr-1)m$. Unitarity might pose further restrictions on the values of c. At the limit $k_- = k$, $k_+ \to \infty$ the algebra reduces to the minimal $N = 4$ SCA with $c = 6k$ since the contributions from the second $SU(2)$ and $U(1)$ to super Virasoro currents vanish at this limit.

How small $N = 4$ SCA could emerge in TGD framework?

Consider the TGD based interpretation of the small $N = 4$ SCA.

1. The group $SU(2)$ associated with the small $N = 4$ SCA and acting as rotations of covariantly constant right-handed neutrino spinors allows also an interpretation as a group $SO(3)$ leaving invariant the sphere S^2 of the light-cone boundary identified as $r_M = m^0$=constant surface defining generalized Kähler and symplectic structures in $\delta M^4_\pm$. Electro-weak degrees of freedom are obviously completely frozen so that $SU(2)_- \times U1$ factor indeed drops out.

2. The choice of the preferred coordinate system should have a physical justification. The interpretation of $SO(3)$ as the isotropy group of the rest system defined by the total four-momentum assignable to the 3-surface containing partonic 2-surfaces is supported by the quantum classical correspondence. The subgroup $U(1)$ of $SU(2)$ acts naturally as rotations around the axis defined by the light ray from the tip of $M^4_\pm$ orthogonal to S^2. For $c = 0, k = 0$ case these groups define local gauge symmetries. In the more general case local gauge invariance is broken whereas global invariance remains as it should.

   In $M^2 \times E^2$ decomposition E^2 corresponds to the tangent space of S^2 at a given point and M^2 to the plane orthogonal to it. The natural assumption is that the right handed neutrino spinor is annihilated by the momentum space Dirac operator corresponding to the light-like momentum defining $M^2 \times E^2$ decomposition.

3. For covariantly constant right handed neutrinos the dynamics would be essentially that defined by a topological quantum field theory and this kind of almost trivial dynamics is indeed associated with small $N = 4$ SCA.

1. Why $N = 4$ super-conformal symmetry would be so nice?

$N = 2$ super-conformal invariance has been claimed to imply the vanishing of all amplitudes with more than 3 external legs for closed critical $N = 2$ strings having $c = 6, k = 1$ which is proposed to correspond to $n \to \infty$ limit and $q = 1$ for Jones inclusions [20, 21]. Only the partition function and $2 \leq N \leq 3$ scattering amplitudes would be non-vanishing. The argument of [20] relies on the imbedding of $N = 2$ super-conformal field theory to $N = 4$ topological string theory whereas in [21] the Ward identities for additional unbroken symmetries associated with the chiral ring accompanying $N = 2$ super-symmetry [22] are utilized. In fact, $N = 4$ topological string theory allows also imbeddings of $N = 1$ super strings [20].

The properties of $c = 6$ critical theory allowing only integral valued $U(1)$ charges and fermion numbers would conform nicely with what we know about the perturbative electro-weak physics of leptons and gauge bosons. $c = 1, k = 1$ sector with $N = 2$ super-conformal symmetry would involve genuinely stringy physics since all N-point functions would be non-vanishing and the earlier hypothesis that strong interactions can be identified as electro-weak interactions which have become strong inspired by HO-H duality [E2] could find a concrete realization.

In $c = 6$ phase $N = 2$-vertices the loop corrections coming from the presence of higher lepton genera in amplitude could be interpreted as topological mixing forced by unitarity implying in turn leptonic CKM mixing for leptons. The non-triviality of 3-point amplitudes would in turn be enough to have a stringy description of particle number changing reactions, such as single photon brehmstrahlung. The amplitude for the emission of more than one brehmstrahlung photons from a given lepton would vanish. Obviously the connection with quantum field theory picture would be extremely tight and imbeddability to a topological $N = 4$ quantum field theory could make the theory to a high degree exactly solvable.

2. Objections

There are also several reasons for why one must take the idea about the usefulness of $c = 6$ super-conformal strings from the point of view of TGD with an extreme caution.

1. Stringy diagrams have quite different interpretation in TGD framework. The target space for these theories has dimension four and metric signature (2,2) or (0,4) and the vanishing theorems hold only for $(2,2)$ signature. In lepton sector one might regard the covariantly constant complex right-handed neutrino spinors as generators of $N = 2$ real super-symmetries but in quark sector there are no super-symmetries.

2. The spectrum looks unrealistic: all degrees of freedom are eliminated by symmetries except single massless scalar field so that one can wonder what is achieved by introducing the extremely heavy computational machinery of string theories. This argument relies on the assumption that time-like modes correspond to negative norm so that the target space reduces effectively to a 2-dimensional Euclidian sub-space E^2 so that only the vibrations in directions orthogonal to the string in E^2 remain. The situation changes if one assigns negative conformal weights and negative energies to the time like excitations. In the generalized coset representation used to construct physical states this is indeed assumed.

3. The central charge has only values $c = 6k$, where k is the central extension parameter of SU(2) algebra [27] so that it seems impossible to realize the genuinely rational values of c which should correspond to the series of Jones inclusions. One manner to circumvent the problem would be the reduction to $N = 2$ super-conformal symmetry.

4. SU(2) Kac-Moody algebra allows to introduce only 2-component spinors naturally whereas super-quaternions allow quantum counterparts of 8-component spinors.

The $N = 2$ super-conformal algebra automatically extends to the so called small $N = 4$ algebra with four super-generators $G_{\pm}$ and their conjugates [20]. In TGD framework $G_{\pm}$ degeneracy corresponds to the two spin directions of the covariantly constant right handed neutrinos and the conjugate of $G_{\pm}$ is obtained by charge conjugation of right handed neutrino. From these generators one can build up a right-handed SU(2) algebra.

Hence the $SU(2)$ Kac-Moody of the small $N = 4$ algebra corresponds to the three imaginary quaternionic units and the $U(1)$ of $N = 2$ algebra to ordinary imaginary unit. Energy momentum tensor T and SU(2) generators would correspond to quaternionic units. $G_{\pm}$ to their super counterparts and their conjugates would define their "square roots".

What about $N = 4$ SCA with $SU(2) \times U(1)$ Kac-Moody algebra?

Rasmussen [26] has discovered an $N = 4$ super-conformal algebra containing besides Virasoro generators and 4 Super-Virasoro generators $SU(2) \times U(1)$ Kac-Moody algebra and two spin 1/2 fermions and a scalar.

The first identification of $SU(2) \times U(1)$ is as electro-weak algebra for a given spin state. Second and more natural identification is as the algebra defined by rotation group and electromagnetic or Kähler charge acting on given charge state of fermion and naturally resulting in electro-weak symmetry breaking. Scalar might relate to Higgs field which is M^4 scalar but CP_2 vector.

There are actually two versions about Rasmussen's article [26]: in the first version the author talks about $SU(2) \times U(1)$ Kac-Moody algebra and in the second one about $SL(2) \times U(1)$ Kac-Moody algebra.

These variants could correspond in TGD framework to two different inclusions of hyper-finite factors of type II_1.

1. The first inclusion could be defined by $G = SL(2, R) \subset SO(3, 1)$ acting on M^4 part of H-spinors (or alternatively, as Lorentz group inducing motions in the plane E^2 orthogonal to a light-like ray from the origin of light-cone M^4_+). Physically the inclusion would mean that Lorentz degrees of

freedom are frozen in the physical measurement. This leaves electro-weak group $SU(2)_L \times U(1)$ as the group acting on H-spinors.

2. The second inclusion would be defined by the electro-weak group $SU(2)_L$ so that Kac-Moody algebra $SL(2,R) \times U(1)$ remains dynamical.

2.6.3 How large $N = 4$ SCA could emerge in quantum TGD?

The discovery of the formulation of TGD as a $N = 4$ almost topological super-conformal QFT with light-like partonic 3-surfaces identified as basic dynamical objects increased considerably the understanding of super-conformal symmetries and their breaking in TGD framework. $N = 4$ super-conformal algebra corresponds to the maximal algebra with $SU(2) \times U(2)$ Kac-Moody algebra as inherent fermionic Kac-Moody algebra.

Concerning the interpretation the first guess would be that $SU(2)_+$ and $SU(2)_-$ correspond to vectorial spinor rotations in M^4 and CP_2 and $U(1)$ to Kähler charge or electromagnetic charge. For given imbedding space chirality (lepton/quark) and M^4 chirality $SU(2)$ groups are completely fixed.

Identification of super generators

Consider first the fermionic generators of the super Kac-Moody algebra.

1. Assume that the modified Dirac operator decomposition $D = D(Y^2) + D(X^2) = D(Y^1) + D(X^1) + D(X^2)$ reflecting the dual slicings of space-time surfaces to string world sheets Y^2 and partonic 2-surfaces X^2.

2. Y^1 represents light-like direction and also string connecting braid strands at same component of X^3_l or at two different components of X^3_l. Modified Dirac equation implies that the charges

$$\int_{X^3_l} \overline{\Psi}_{\lambda_k,n}\hat{\Gamma}^v\Psi \tag{2.6.5}$$

define conserved super charges in time direction associated with Y^1 and carrying quark or lepton number. Here $\Psi_{\lambda_k,n}$ corresponds to n:th conformal excitation of Ψ_{λ_k} and λ_k is is a generalized eigenvalue of $D(X^2)$, whose modulus squared has interpretation as ground state conformal weight. In the case of ordinary Dirac equation essentially fermionic oscillator operators would be in question.

3. The zero modes of $D(X^2)$ define a sub-algebra which represents super gauge symmetries. In particular, covariantly constant right handed neutrinos define this kind of super gauge supersymmetries. $N = 2$ super-conformal symmetry would correspond in TGD framework to covariantly constant complex right handed neutrino spinors with two spin directions forming a right handed doublet and would be exact and act only in the leptonic sector relating configuration space Hamiltonians and super-Hamiltonians. This algebra extends to the so called small $N = 4$ algebra if one introduces the conjugates of the right handed neutrino spinors. This symmetry is exact if only leptonic chirality is present in theory or if free quarks carry leptonic charges.

Identification of Kac-Moody generators

Consider next the generators of inherent Kac-Moody algebras for $SU(2) \times SU(L) \times U(1)$ and freely chosen group G.

1. Generators of Kac-Moody algebra associated with isometries correspond Noether currents associated with the infinitesimal action of Kac-Moody algebra to the induced spinor fields. Local $SO(3) \times SU(3)$ algebra is in question and excitations should have dependence on the coordinate u in direction of Y^1. The most natural guess is that this algebra corresponds to the Kac-Moody algebra for group G.

2. The natural candidate for the inherent Kac-Moody algebra is the holonomy algebra associated with $S^2 \times CP_2$. This algebra should correspond to a broken symmetry. The generalized eigen modes of $D(X^2)$ labeled by λ_k should from the representation space in this case. If Kac-Moody symmetry were not broken these representations would correspond a degeneracy associated with given value of λ_k. Electro-weak symmetry breaking is however present and coded already into the geometry of CP_2. Also $SO(3)$ symmetry is broken due to the presence of classical electro-weak magnetic fields. The broken symmetries could be formulated in terms of initial values of generalized eigen modes at X^2 defining either end of X^3_l. One can rotate these initial values by spinor rotations. Symmetry breaking would mean that the modes obtained by a rotation by angle $\phi = \pi$ from a mode with fixed eigenvalue λ_k have different eigenvalues. Four states would be obtained for a given imbedding space chirality (quark or lepton). One expects that an analog of cyclotron spectrum with cutoff results with each cyclotron state split to four states with different eigenvalues λ_k. Kac-Moody generators could be expressed as matrices acting in the space spanned by the eigen modes.

Consistency with p-adic mass calculations

The consistency with p-adic mass calculations provides a strong guide line in attempts to interpret $N = 4$ SCA. The basis ideas of p-adic mass calculations are following.

1. Fermionic partons move in color partial waves in their cm degrees of freedom. This gives to conformal weight a vacuum contribution equal to the CP_2 contribution to mass squared. The contribution depends on electro-weak isospin and equals $h_c(U) = 2$ and $h_c(D) = 3$ for quarks and one has $h_c(\nu) = 1$ and $h_c(L) = 2$.

2. The ground state can correspond also to non-negative value of L_0 for SKMV algebra which gives rise to a thermal degeneracy of massless states. p-Adic mass calculations require $(h_{gr}(D), h_{gr}(U)) = (0, -1,)$ and $(h_{gr}(L), h_{gr}(\nu)) = (-1, -2)$ so that the super-symplectic operator O_c screening the anomalous color charge has conformal weight $h_c = -3$ for all fermions.

The simplest interpretation is that the free parameter h appearing in the representations of the SCA corresponds to the conformal weight due to the color partial wave so that the correlation with electromagnetic charge would indeed emerge but from the correlation of color partial waves and electro-weak quantum numbers.

The requirement that ground states are null states with respect to the SCV associated with the radial light-like coordinate of $\delta M^4_\pm$ gives an additional consistency condition and $h_c = -3$ should satisfy this condition. p-Adic mass calculations do not pose non-trivial conditions on h for option 1) if one makes the identification $u = Q_{em}$ since one has $h_{short} < 1$ for all values of $k_+ + k_-$. Therefore both options 1) and 2) can be considered.

About symmetry breaking for large $N = 4$ SCA

Partonic formulation predicts that large $N = 4$ SCA is a broken symmetry, and the first guess is that breaking occurs via several steps. First a "small" $N = 4$ SCA with Kac-Moody group $SU(2)_+ \times U(1)$, where $SU(2)_+$ corresponds to ordinary rotations on spinor with fixed helicity, would result in electro-weak symmetry breaking. The next step break spin symmetry would lead to $N = 2$ SCA and the final step to $N = 0$ SCA. Several symmetry breaking scenarios are possible.

1. The interpretation of $SU(2)_+$ in terms of right- or left- handed spin rotations and $U(1)$ as electromagnetic gauge group conforms with the general vision about electro-weak symmetry breaking in non-stringy phase. The interpretation certainly makes sense for covariantly constant right handed neutrinos for which spin direction is free. For left handed charged electro-weak bosons the action of right-handed spinor rotations is trivial so that the interpretation would make sense also now.

2. The next step in the symmetry breaking sequence would be $N = 2$ SCA with electromagnetic Kac-Moody algebra as inherent Kac-Moody algebra $U(1)$.

2.6.4 The interpretation of the critical dimension $D = 4$ and the objection related to the signature of the space-time metric

The first task is to show that $D = 4$ ($D = 8$) as critical dimension of target space for $N = 2$ ($N = 4$) super-conformal symmetry makes sense in TGD framework and that the signature (2,2) ($(4,4)$ of the metric of the target space is not a fatal flaw. One must also remember that super-conformal symmetry in TGD sense differs from that in the standard sense so that one must be very cautious with comparisons at this level.

Space-time as a target space for partonic string world sheets?

Since partonic 2-surfaces are sub-manifolds of 4-D space-time surface, it would be natural to interpret space-time surface as the target space for $N = 2$ super-conformal string theory so that space-time dimension would find a natural explanation. Different Bohr orbit like solutions of the classical field equations could be the TGD counterpart for the dynamic target space metric of M-theory. Since partonic two-surfaces belong to 3-surface X^3_V, the correlations caused by the vacuum functional would imply non-trivial scattering amplitudes with CP_2 type extremals as pieces of X^3_V providing the correlate for virtual particles. Hence the theory could be physically realistic in TGD framework and would conform with perturbative character for the interactions of leptons. $N = 2$ super-conformal theory would of course not describe everything. This algebra seems to be still too small and the question remains how the functional integral over the configuration space degrees of freedom is carried out. It will be found that $N = 4$ super-conformal algebra results neatly when super Kac-Moody and super-symplectic degrees of freedom are combined.

The interpretation of the critical signature

The basic problem with this interpretation is that the signature of the induced metric cannot be (2,2) which is essential for obtaining the cancellation for $N = 2$ SCA imbedded to $N = 4$ SCA with critical dimension $D = 8$ and signature (4,4). When super-generators carry fermion number and do not reduce to ordinary gamma matrices for vanishing conformal weights, there is no need to pose the condition of the metric signature. The (4,4) signature of the target space metric is not so serious limitation as it looks if one is ready to consider the target space appearing in the calculation of N-point functions as a fictive notion.

The resolution of the problems relies on two observations.

1. The super Kac-Moody and super-symplectic Cartan algebras have dimension $D = 2$ in both M^4 and CP_2 degrees of freedom giving total effective dimension $D = 8$.

2. The generalized coset construction to be discussed in the sequel allows to assign opposite signatures of metric to super Kac-Moody Cartan algebra and corresponding super-symplectic Cartan algebra so that the desired signature (4,4) results. Altogether one has 8-D effective target space with signature (4,4) characterizing $N = 4$ super-conformal topological strings. Hence the number of physical degrees of freedom is $D_{phys} = 8$ as in super-string theory. Including the non-physical M^2 degrees of freedom, one has critical dimension $D = 10$. If also the radial degree of freedom associated with $\delta M^4_\pm$ is taken into account, one obtains $D = 11$ as in M-theory.

The connection between super-conformal algebras and classical division algebras

There are well-known connections with classical number fields and super-conformal algebras.

1. There exists two proposals for a simple super-affinization of the octonionic algebra realized in terms of spin 1/2 super fields obeying expected octonionic anticommutation relations in the fermionic sector. Otherwise the fields behave like like octonionic units. These constructions are discussed in [22, 23].

2. It is known that only $N \leq 4$ super-conformal algebras allow Sugawara construction [22]. For $N = 8$ super-affine octonionic algebra the Sugawara construction does not give a closed algebraic structure except at the limit $k \to \infty$ for the Kac-Moody central charge [23]: this algebra is the non-associative SCA discovered first by Englert *et al* [24]. This limit could be interpreted in terms

of a critical conformal field theory. The minimal super-affine quaternionic sub-algebra reduces to a small $N = 4$ SCA and allows Sugawara construction [22]. This limit would correspond to $n \rightarrow \infty$ limit for the Jones inclusion and critical value of c corresponding to the almost-topologization of $N = 2$ n-point functions. The problem is that the representations do not exist for finite values of k which are also needed.

The number theoretical vision supports the view that only quaternionic SCA can be used in the construction of physical states. A stronger conclusion would be that only the quaternionic SCA is possible so that quarks would be fractionally charged leptons in $k = 1$ phase. The topologication of $N = 4$ n-point functions in the critical phase could be consistent with the possibility to describe electro-weak interactions perturbatively since partonic 2-surfaces would still interact classically and these interactions would correspond to exchanges of virtual particles represented by CP_2 type extremals.

Small $N = 4$ SCA as sub-algebra of $N = 8$ SCA in TGD framework?

A possible interpretation of the small $N = 4$ super-conformal algebra would be quaternionic sub-SCA of the non-associative octonionic SCA. The $N = 4$ algebra associated with a fixed fermionic chirality would represent the fermionic counterpart for the restriction to the hyper-quaternionic submanifold of HO and $N = 2$ algebra in the further restriction to commutative sub-manifold of HO so that this algebra would naturally appear at the parton level. Super-affine version of the quaternion algebra can be constructed straightforwardly as a special case of corresponding octonionic algebra [23]. The construction implies 4 fermion spin doublets corresponding and unit quaternion naturally corresponds to right handed neutrino spin doublet. The interpretation is as leptonic spinor fields appearing in Sugawara representation of Super Virasoro algebra.

A possible octonionic generalization of Super Virasoro algebra would involve 4 doublets $G^{i)}_{\pm}$, $i = 1, ..., 4$ of super-generators and their conjugates having interpretation as SO(8) spinor and its its conjugate. $G^{i)}_{\pm}$ and their conjugates $\overline{G}^{i)}_{\pm}$ would anti-commute to SO(8) vector octet having an interpretation as a super-affine algebra defined by the octonionic units: this would conform nicely with SO(8) triality.

One could say that the energy momentum tensor T extends to an octonionic energy momentum tensor T as real component and affine generators as imaginary components: the real part would have conformal weight $h = 2$ and imaginary parts conformal weight $h = 1$ in the proposed constructions reflecting the special role of real numbers. The ordinary gamma matrices appearing in the expression of G in Sugawara construction should be represented by units of complexified octonions to achieve non-associativity. This construction would differ from that of [23] in that G fields would define an SO(8) octet in the proposed construction: HO-H duality would however suggest that these constructions are equivalent.

One can consider two possible interpretations for $G^{i)}_{\pm}$ and corresponding analogs of super Kac-Moody generators in TGD framework.

1. Leptonic right handed neutrino spinors correspond to $G^{i)}_{\pm}$ generating quaternionic units and quark like left-handed neutrino spinors with leptonic charges to the remaining non-associative octonionic units. The interpretation in terms of so called mirror symmetry would be natural. What is is clear the direct sum of $N = 4$ SCAs corresponding to the Kac-Moody group $SU(2) \times SU(2)$ would be exact symmetry if free quarks and leptons carry integer charges. One might however hope of getting also $N = 8$ super-conformal algebra. The problem with this interpretation is that SO(8) transformations would in general mix states with different fermion numbers. The only way out would be the allowance of mixtures of right-handed neutrinos of both chiralities and also of their conjugates which looks an ugly option.

   In any case, the well-definedness of the fermion number would require the restriction to $N = 4$ algebra. Obviously this restriction would be a super-symmetric version for the restriction to 4-D quaternionic- or co-quaternionic sub-manifold of H.

2. One can ask whether $G^{i)}_{\pm}$ and their conjugates could be interpreted as components of leptonic H-spinor field. This would give 4 doublets plus their conjugates and mean $N = 16$ super-symmetry by generalizing the interpretation of $N = 4$ super-symmetry. In this case fermion number

conservation would not forbid the realization of $SO(8)$ rotations. Super-conformal variant of complexified octonionic algebra obtained by adding a commuting imaginary unit would result. This option cannot be excluded since in TGD framework complexified octonions and quaternions play a key role. The fact that only right handed neutrinos generate associative super-symmetries would mean that the remaining components $G_{\pm}^{i)}$ and their conjugates could be used to construct physical states. $N = 8$ super-symmetry would thus break down to small $N = 4$ symmetry for purely number theoretic reasons and the geometry of CP_2 would reflect this breaking.

The objection is that the remaining fermion doublets do not allow covariantly constant modes at the level of imbedding space. They could however allow these modes as induced H-spinors in some special cases which is however not enough and this option can be considered only if one accepts breaking of the super-conformal symmetry from beginning. The conclusion is that the $N = 8$ or even $N = 16$ algebra might appear as a spectrum generating algebra allowing elegant coding of the primary fermionic fields of the theory.

2.7 Color degrees of freedom

The ground states for the Super Virasoro representations correspond to spinor harmonics in $M^4 \times CP_2$ characterized by momentum and color quantum numbers. The correlation between color and electro-weak quantum numbers is wrong for the spinor harmonics and these states would be also hyper-massive. The super-symplectic generators allow to build color triplet states having negative vacuum conformal weights, and their values are such that p-adic massivation is consistent with the predictions of the earlier model differing from the recent one in the quark sector. In the following the construction and the properties of the color partial waves for fermions and bosons are considered. The discussion follows closely to the discussion of [5].

2.7.1 SKM algebra and counterpart of Super Virasoro conditions

The geometric part of SKM algebra is defined as an algebra respecting the light-likeness of the partonic 3-surface. It consists of X^3-local conformal transformations of $M^4_{\pm}$ and SU(3)-local SU(3) rotations. The requirement that generators have well defined radial conformal weight with respect to the lightlike coordinate r of X^3 restricts M^4 conformal transformations to the group $SO(3) \times E^3$. This involves choice of preferred time coordinate. If the preferred M^4 coordinate is chosen to correspond to a preferred light-like direction in $\delta M^4_{\pm}$ characterizing the theory, a reduction to $SO(2) \times E^2$ more familiar from string models occurs. The algebra decomposes into a direct sum of sub-algebras mapped to themselves by the Kac-Moody algebra generated by functions depending on r only. SKM algebra contains also $U(2)_{ew}$ Kac-Moody algebra acting as holonomies of CP_2 and having no bosonic counterpart.

p-Adic mass calculations require $N = 5$ sectors of super-conformal algebra. These sectors correspond to the 5 tensor factors for the $SO(3) \times E^3 \times SU(3) \times U(2)_{ew}$ (or $SO(2) \times E^2 \times SU(3) \times U(2)_{ew}$) decomposition of the SKM algebra to gauge symmetries of gravitation, color and electro-weak interactions. These symmetries act on the intersections $X^2 = X^3_l \cap X^7$ of 3-D light like causal determinants (CDs) X^3_l and 7-D light like CDs $X^7 = \delta M^4_+ \times CP_2$. This constraint leaves only the 2 transversal M^4 degrees of freedom since the translations in light like directions associated with X^3_l and δM^4_+ are eliminated.

The algebra differs from the standard one in that super generators $G(z)$ carry lepton and quark numbers are not Hermitian as in super-string models (Majorana conditions are not satisfied). The counterparts of Ramond representations correspond to zero modes of a second quantized spinor field with vanishing radial conformal weight. Non-zero modes with generalized eigenvalues $\lambda = 1/2 + iy$, $y = \sum_k n_k y_k$, $n_k \geq 0$, of the modified Dirac operator with $s_k = 1/2 + iy_k$ a zero or Rieman Zeta, define ground states of N-S type super Virasoro representations.

What is new is the imaginary part of conformal weight which means that the arrow of geometric time manifests itself via the sign of the imaginary part y already at elementary particle level. More concretely, positive energy particle propagating to the geometric future is not equivalent with negative energy particle propagating to the geometric past. The strange properties of the phase conjugate provide concrete physical demonstration of this difference. p-Adic mass calculations suggest the interpretation of y in terms of a decay width of the particle.

The Ramond or N-S type Virasoro conditions satisfied by the physical states in string model approach are replaced by the formulas expressing mass squared as a conformal weight. The condition is not equivalent with super Virasoro conditions since four-momentum does not appear in super Virasoro generators. It seems possible to assume that the commutator algebra $[SKM, SC]$ and the commutator of $[SKMV, SSV]$ of corresponding Super Virasoro algebras annihilate physical states. This would give rise to the analog of Super Virasoro conditions which could be seen as a Dirac equation in the world of classical worlds.

CP_2 CM degrees of freedom

Important element in the discussion are center of mass degrees of freedom parameterized by imbedding space coordinates. By the effective 2-dimensionality it is indeed possible to assign to partons momenta and color partial waves and they behave effectively as free particles. In fact, the technical problem of the earlier scenario was that it was not possible to assign symmetry transformations acting only on on the boundary components of 3-surface.

One can assign to each eigen state of color quantum numbers a color partial wave in CP_2 degrees of freedom. Thus color quantum numbers are not spin like quantum numbers in TGD framework except effectively in the length scales much longer than CP_2 length scale. The correlation between color partial waves and electro-weak quantum numbers is not physical in general: only the covariantly constant right handed neutrino has vanishing color.

Mass formula, and condition determining the effective string tension

Mass squared eigenvalues are given by

$$M^2 = m^2_{CP_2} + kL_0 \ . \tag{2.7.1}$$

The contribution of CP_2 spinor Laplacian to the mass squared operator is in general not integer valued.

The requirement that mass squared spectrum is integer valued for color partial waves possibly representing light states fixes the possible values of k determining the effective string tension modulo integer. The value $k = 1$ is the only possible choice. The earlier choice $k_L = 1$ and $k_q = 2/3$, $k_B = 1$ gave integer conformal weights for the lowest possible color partial waves. The assumption that the total vacuum weight h_{vac} is conserved in particle vertices implied $k_B = 1$.

2.7.2 General construction of solutions of Dirac operator of H

The construction of the solutions of massless spinor and other d'Alembertians in $M^4_+ \times CP_2$ is based on the following observations.

1. d'Alembertian corresponds to a massless wave equation $M^4 \times CP_2$ and thus Kaluza-Klein picture applies, that is M^4_+ mass is generated from the momentum in CP_2 degrees of freedom. This implies mass quantization:

$$M^2 = M^2_n \ ,$$

   where M^2_n are eigenvalues of CP_2 Laplacian. Here of course, ordinary field theory is considered. In TGD the vacuum weight changes mass squared spectrum.

2. In order to get a respectable spinor structure in CP_2 one must couple CP_2 spinors to an odd integer multiple of the Kähler potential. Leptons and quarks correspond to $n = 3$ and $n = 1$ couplings respectively. The spectrum of the electromagnetic charge comes out correctly for leptons and quarks.

3. Right handed neutrino is covariantly constant solution of CP_2 Laplacian for $n = 3$ coupling to Kähler potential whereas right handed 'electron' corresponds to the covariantly constant solution for $n = -3$. From the covariant constancy it follows that all solutions of the spinor Laplacian are obtained from these two basic solutions by multiplying with an appropriate solution of the

scalar Laplacian coupled to Kähler potential with such a coupling that a correct total Kähler charge results. Left handed solutions of spinor Laplacian are obtained simply by multiplying right handed solutions with CP_2 Dirac operator: in this operation the eigenvalues of the mass squared operator are obviously preserved.

4. The remaining task is to solve scalar Laplacian coupled to an arbitrary integer multiple of Kähler potential. This can be achieved by noticing that the solutions of the massive CP_2 Laplacian can be regarded as solutions of S^5 scalar Laplacian. S^5 can indeed be regarded as a circle bundle over CP_2 and massive solutions of CP_2 Laplacian correspond to the solutions of S^5 Laplacian with $exp(is\tau)$ dependence on S^1 coordinate such that s corresponds to the coupling to the Kähler potential:

$$s = n/2 \ .$$

Thus one obtains

$$D_5^2 = (D_\mu - iA_\mu\partial_\tau)(D^\mu - iA^\mu\partial_\tau) + \partial_\tau^2 \qquad (2.7.2)$$

so that the eigen values of CP_2 scalar Laplacian are

$$m^2(s) = m_5^2 + s^2 \qquad (2.7.3)$$

for the assumed dependence on τ.

5. What remains to do, is to find the spectrum of S^5 Laplacian and this is an easy task. All solutions of S^5 Laplacian can be written as homogenous polynomial functions of C^3 complex coordinates Z^k and their complex conjugates and have a decomposition into the representations of $SU(3)$ acting in natural manner in C^3.

6. The solutions of the scalar Laplacian belong to the representations $(p, p+s)$ for $s \geq 0$ and to the representations $(p+|s|, p)$ of $SU(3)$ for $s \leq 0$. The eigenvalues $m^2(s)$ and degeneracies d are

$$\begin{aligned} m^2(s) &= \frac{2\Lambda}{3}[p^2 + (|s|+2)p + |s|] \ , \ p > 0 \ , \\ d &= \frac{1}{2}(p+1)(p+|s|+1)(2p+|s|+2) \ . \end{aligned} \qquad (2.7.4)$$

Λ denotes the 'cosmological constant' of CP_2 ($R_{ij} = \Lambda s_{ij}$).

2.7.3 Solutions of the leptonic spinor Laplacian

Right handed solutions of the leptonic spinor Laplacian are obtained from the asatz of form

$$\nu_R = \Phi_{s=0}\nu_R^0 \ ,$$

where u_R is covariantly constant right handed neutrino and Φ scalar with vanishing Kähler charge. Right handed 'electron' is obtained from the ansats

$$e_R = \Phi_{s=3}e_R^0 \ ,$$

where e_R^0 is covariantly constant for $n = -3$ coupling to Kähler potential so that scalar function must have Kähler coupling $s = n/2 = 3$ a in order to get a correct Kähler charge. The d'Alembert equation reduces to

$$(D_\mu D^\mu - (1-\epsilon)\Lambda)\Phi = -m^2\Phi \ , \quad \epsilon(\nu) = 1 \ , \quad \epsilon(e) = -1 \ . \tag{2.7.5}$$

The two additional terms correspond to the curvature scalar term and $J_{kl}\Sigma^{kl}$ terms in spinor Laplacian. The latter term is proportional to Kähler coupling and of different sign for ν and e, which explains the presence of the sign factor ϵ in the formula.

Right handed neutrinos correspond to (p,p) states with $p \geq 0$ with mass spectrum

$$\begin{aligned} m^2(\nu) &= \frac{m_1^2}{3}\left[p^2+2p\right] \ , \ p \geq 0 \ , \\ m_1^2 &\equiv 2\Lambda \ . \end{aligned} \tag{2.7.6}$$

Right handed 'electrons' correspond to $(p,p+3)$ states with mass spectrum

$$m^2(e) = \frac{m_1^2}{3}\left[p^2+5p+6\right] \ , \ p \geq 0 \ . \tag{2.7.7}$$

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions and have the same mass spectrum and representational content as right handed leptons with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino ($(p=0, p=0)$ state) annihilates it.

2.7.4 Quark spectrum

Quarks correspond to the second conserved H-chirality of H-spinors. The construction of the color partial waves for quarks proceeds along similar lines as for leptons. The Kähler coupling corresponds to $n=1$ (and $s=1/2$) and right handed U type quark corresponds to a right handed neutrino. U quark type solutions are constructed as solutions of form

$$U_R = u_R\Phi_{s==1} \ ,$$

where u_R possesses the quantum numbers of covariantly constant right handed neutrino with Kähler charge $n=3$ ($s=3/2$). Hence Φ_s has $s=-1$. For D_R one has

$$D_R = d_r\Phi_{s=2} \ .$$

d_R has $s=-3/2$ so that one must have $s=2$. For U_R the representations $(p+1,p)$ with triality one are obtained and $p=0$ corresponds to color triplet. For D_R the representations $(p,p+2)$ are obtained and color triplet is missing from the spectrum ($p=0$ corresponds to $\bar{6}$).

The CP_2 contributions to masses are given by the formula

$$\begin{aligned} m^2(U,p) &= \frac{m_1^2}{3}\left[p^2+3p+2\right] \ , \ p \geq 0 \ , \\ m^2(D,p) &= \frac{m_1^2}{3}\left[p^2+4p+4\right] \ , \ p \geq 0 \ . \end{aligned} \tag{2.7.8}$$

Left handed quarks are obtained by applying Dirac operator to right handed quark states and mass formulas and color partial wave spectrum are the same as for right handed quarks.

The color contributions to p-adic mass squared are integer valued if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since canonical identification does not commute with a division by integer. More precisely, the images of number xp in canonical identification has a value of order 1 when x is a non-trivial rational whereas for $x=np$ the value is n/p and extremely is small for physically interesting primes. This choice does not however affect the spectrum of massless states but can affect the spectrum of light states in case of electro-weak gauge bosons.

2.8 Exotic states

The possibility of exotic states poses a serious problem. The assumption that only free many fermion states are possible eliminates a huge number of exotics and only the degrees of freedom associated with ground states remain. Coset construction implying duality between SSV and $SKMV$ algebras removes a huge number of exotic states but genuinely SC contributions with a vanishing conformal weight are possible. Also other kinds of exotic states are predicted.

2.8.1 What kind of exotic states one expects

The physical consequences of the exotic light leptons, quarks, and bosons are considered in the chapter devoted to the New Physics [F5]. Here it only suffices to make a short summary. Consider first what kind of exotic particles extended conformal symmetries predict.

1. Massless states are expected to become massive by p-adic thermodynamics meaning that one has superposition of states with Super Kac-Moody conformal weight equal to Super Virasoro conformal weight and annihilated by SKMV and SSV generators G_n,L_n, $n > 0$. This condition allows degeneracy since there are many manners to create a ground state with a given angular momentum and color quantum numbers and conformal weight n and annihilated by L_n, $n < 0$, by using super-symplectic generators. The combinations of super-symplectic generators which do not belong to SKM algebra and create singlets in color and rotational degrees of freedom would be responsible for this degeneracy. The condition that the states in the superposition are annihilated by G_n, L_n, $n > 0$, reduces the number of the massless states.

2. The original expectation that the spectrum has $N = 1$ space-time super-symmetry seems to be wrong. The understanding of the super-conformal symmetries as at parton level allowed to identify partonic super-conformal symmetries in terms of a generalization of large $N = 4$ SCA with Kac-Moody group extended to contain also symplectic transformations of $\delta H_\pm$. Thus an immense generalization of string model conformal symmetries is in question. This allows to conclude that sparticles in the sense of super Poincare symmetry are certainly absent. This does not affect the mass calculations in any manner and dramatically reduces the number of exotic states.

3. If elementary particles correspond to CP_2 type extremals, one can argue that all massless exotic massless particles can be constructed using colored generators and by color confinement cannot induce macroscopic long range interactions.

4. The possibility that conformal weights have imaginary part expressible as linear combination of imaginary parts of zeros of ζ function associated with the modified Dirac operator satisfying Riemann hypothesis brings in additional richness of structure. A possible interpretation is that the non-vanishing imaginary part allows to distinguish between positive energy particle propagating into geometric future and negative energy propagating to the geometric past. Phase conjugate photons for which dissipation occurs in time reversed direction would be basic examples of this. Dissipation would be visible already in the mathematical description of partons. The imaginary part of the conformal weight might relate directly to the decay rate of the particle or to the length of the time interval separating positive energy particle and corresponding negative energy particle in zero energy ontology where all physical states have vanishing net quantum numbers [C3].

These exotic particles relate to the extended conformal symmetries. There are also other kinds of exotic particles.

1. The existence of fermionic families suggests the existence of higher bosonic families too. If gauge bosons correspond to wormhole contacts, three families would mean that bosons are labelled by pairs (g_i, g_j) of genera associated with wormhole contacts and $U(3)$ dynamical gauge symmetry emerges naturally. The observed gauge bosons would correspond to SU(3) singlets which do not induced genus changing transitions. The new view about particle decay as a branching of partonic 2-surface is consistent with this picture but not the earlier stringy view. Only three fermion families are predicted if $g > 2$ topologies for partonic 2-surfaces correspond to free

many-handle states rather than bound states as for $g < 3$ topologies: who this could happen is discussed in [F1].

2. Also p-adically scaled up copies of various particles are possible as well as scaled-up/scaled-down versions of QCD associated with both quarks [F8] and colored leptons [F7]. There is now quite a lot of evidence that neutrino masses depend on environment [44]: this dependence could have an explanation in terms of topological condensation occurring in several p-adic length scales.

3. Dark matter hierarchy based on the spectrum of Planck constants [A9] infinite number of zoomed up copies of ordinary elementary particles with same mass spectrum.

4. Electro-weak doublet Higgs particle would be present in the spectrum and be identifiable as wormhole contact, contrary to the long held beliefs. Also $q - \overline{q}$ bound states of M_{89} hadron physics such that quark and anti-quark have parallel spins and relative angular momentum $L = 1$ could mimic scalar mesons. The effective couplings of these states to leptons and quarks could mimic the couplings of Higgs boson to some degree. Scalar bound states of heavy quarks are also present in ordinary hadron physics.

2.8.2 Are S^2 degrees frozen for elementary particles?

As the system approaches CP_2 type extremal, radial waves in $\delta M^4_\pm$ for 2-D partonic surface having 0-dimensional $\delta M^4_\pm$ projection become constant. Hence one might argue that the radial conformal weights vanish for SC. This would however lead to a contradiction since radial conformal weights are absolutely essential for p-adic mass calculations. Parton picture allows to understand what really happens. artons correspond to light-like 3-surfaces correspond to wormhole throats resulting when CP_2 type extremal is glued to the space-time sheet with Minkowskian signature of induced metric so that M^4 projection is necessarily 3-dimensional although metrically 2-D.

One can however consider the possibility that the S^2 degrees of freedom associated with δM^4_+ are essentially frozen at elementary particle level with graviton forming a possible exception. The reason would be simply the extremely small size of wormhole contacts implying that the super-symplectic generators are essentially constant in S^2 degrees of freedom. Only color Hamiltonians would generate tachyonic ground states as null states.

2.8.3 More detailed considerations

The exotic states can emerge both from super-symplectic and super Kac-Moody sectors. The tachyonic ground states correspond to null states of super-symplectic Super Virasoro representations having negative conformal weight $h < 0$ and satisfying the condition $L_n|h\rangle = 0$, $n < 0$. Massless state is obtained by applying super Hamiltonians and SKM generators to this state. Null state conditions certainly reduce dramatically the number of ground states since this kind of states are possible only for special values of c and h. For instance, in $N = 2$ super-conformal theories only very special rational values of c and h are possible and the number of null states is finite.

First vision

If one assumes that elementary particles correspond to CP_2 type extremals, and that $SO(3)$ Hamiltonians with vanishing conformal weight are "frozen" to a constant at this limit, the predicted exotic massless states would be generated by color Hamiltonians only. This justifies the hope that new macroscopic long range forces are absent in TGD Universe. It will be found that this assumption is not necessary and fails at hadronic space-time sheets.

1. Super-symplectic sector. In super-symplectic sector S^2 generators are frozen to constant and fermionic generators vanish so that infinite number of generators otherwise giving rise to degeneracy of massless states is eliminated. Color generators appear as pairs of Hamiltonian and its super-partner with an "anomalous" conformal weight determined by the color representation, and due to the breaking of conformal symmetry induced by CP_2 geometry reflecting itself as a massivation of spinor harmonics. Poisson bracket action does not conserve color conformal weights. This can be understood in terms of the breaking of conformal invariance. The ground states with negative conformal weight would be generated by color Hamiltonians and

their spartners having same conformal weights. Color confinement suggests that the massless particles generated from these ground states cannot give rise to macroscopic long range forces.

2. SKM generators in NS representation.
   N-S sector gives rise to super generators with conformal weight $n+1/2$,$n \geq 0$ since $h=-1/2$ generators are not allowed by the representation used. Therefore the dangerous $n=0$ operators are absent.

3. Ramond sector of SKM algebra corresponding to $SO(3) \times SU(2)_L \times U(1)$ holonomies.
   $n=0$ generators are absent in holonomy degrees of freedom. That the right handed neutrino is covariantly constant, is annihilated by charge matrices, and is orthogonal with $\lambda \neq 0$ modes of the modified Dirac operator D, implies that $n=0$ fermionic generators vanish. Also the covariant constancy of em charge matrix and the anomalous conformal weight $h_c=2$ of the left-handed electro-weak charge matrices is of importance. Hence no spartners are predicted in $SO(3) \times SU(2)_L \times U(1)$ degrees of freedom.

4. Ramond sector of SKM algebra corresponding to $SO3) \times SU(3)$ isometries.
   i) $n=0$ bosonic $SO(3) \times SU(3)$ SKM generators act directly as operators $j^{Ar}D_r$ on the Hamiltonians of X^7 appearing in the definitions of configuration space Hamiltonians. In the same manner $j^{Ar}D_r$ transforms $j^{Bk}\Gamma_k$ to $j^{[A,B]k}\Gamma_k$ and does not affect the representation of H_B. Hence the KM algebra corresponding to isometries does not increase the "particle" number defined as the number of X^2 non-local operators in the state nor change the representation of $SO(3) \times SU(3)$.
   ii) Fermionic $SO(3)$ generators have $h_c=0$ but for $n=0$ they vanish by the orthogonality of ν_R and $\lambda>0$ eigen modes of D. Fermionic $SU(3)$ SKM generators have an anomalous conformal weight $h_c=1$.

The cautious conclusion would be that massless exotics are all created by color Hamiltonians and their spartners subject to the condition that tachyonic ground state is annihilated by SSV and SKMV generators G_n, L_n, $n<0$. This might be enough to achieve consistency with the experimental facts since color confinement would restrict the new long range interactions to a finite range.

Improved vision

An objection against the effective absence of rotational degrees of freedom came from the realization that super-symplectic degrees of freedom are absolutely essential for the understanding of the hadron mass spectrum [F4, F5].

1. Hadronic space-time sheet labelled $k=107$ would be a carrier of many-particle state of super-symplectic bosons carrying both spin and color quantum numbers. The additivity of the conformal weight implies string mass formula and gives a connection with the hadronic string model. String tension is predicted correctly and the states of the Regge trajectories correspond to many particle states for super-symplectic bosons. Hadron masses are predicted with an accuracy better than one per cent.

2. The super-symplectic part of the hadron is dark matter in a strict sense of the word and highly analogous to a black hole. This leads a model explaining RHIC events, where black-hole like states would be created in the collisions of heavy Gold nuclei by the fusion of the hadronic space-time sheets involving also the materialization of collision energy to super-symplectic matter [45, 35]. The model also explains the re-incarnated Pomeron [71]. The strange cosmic ray events as well as the observation of cosmic rays with energy larger than the limiting energy 5×10^{10} GeV could be understood as resulting when extremely energetic proton has lost its valence quarks (Pomeron) and propagates as a mini black-hole without interactions with microwave background. LHC gives a possibility to test this picture.

3. The realization that neutron star can be regarded as a gigantic hadron leads to a microscopic description of black-holes as super-symplectic black-holes and the requirement that horizon radius equals to Compton length fixes the Planck constant to $\hbar_{gr}=2GM^2$. This form is a generalization of the gravitational Planck constant appearing in the Bohr quantization of planetary orbits [D7].

To sum up, it seems that all basic ingredients of TGD Universe are present already at the level of the standard physics.

2.9 Particle massivation

In TGD framework p-adic thermodynamics provides a microscopic theory of particle massivation. The idea is very simple. The mass of the particle results from a thermal mixing of the massless states with CP_2 mass excitations of super-conformal algebra. In p-adic thermodynamics the Boltzmann weight $exp(-E/T)$ does not exist in general and must be replaced with p^{L_0/T_p} which exists for Virasoro generator L_0 if the inverse of the p-adic temperature is integer valued $T_p = 1/n$. The expansion in powers of p converges extremely rapidly for physical values of p, which are rather large. Therefore the three lowest terms in expansion give practically exact results. Thermal massivation does not not necessarily lead to light states and this drops a large number of exotic states from the spectrum of light particles. The partition functions of N-S and Ramond type representations are not changed in TGD framework despite the fact that fermionic super generators carry fermion numbers and are not Hermitian. Thus the practical calculations are relatively straightforward.

In free fermion picture the p-adic thermodynamics in the boson sector is for fermion-antifermion states associated with the two throats of the bosonic wormhole. The question is whether the thermodynamical mass squared is just the sum of the two independent fermionic contributions for Ramond representations or should one use N-S type representation resulting as a tensor product of Ramond representations.

The overall conclusion about p-adic mass calculations is that fermionic mass spectrum is predicted in an excellent accuracy but that the thermal masses of the intermediate gauge bosons come 20-30 per cent to large for $T_p = 1$ and are completely negligible for $T_p = 1/2$. This forces to consider very seriously the possibility that thermal contribution to the bosonic mass is negligible and that TGD can, contrary to the original expectations, provide dynamical Higgs field as a fundamental field. The identification of Higgs as wormhole contact would provide this field. The bound state character of the boson states could be responsible for $T_p < 1$. For this option the Higgs contribution to fermion masses would be negligible.

The correct option is based on the identification of the Higgs like contribution in terms of the deviation of the ground state conformal weight from negative integer. The negative ground state conformal weights in turn correspond to the squares of the generalized eigenvalues of the modified Dirac operator determined by the dynamics of Kähler action for preferred extremals. For this option Higgs vacuum expectation in bosonic sector would be proportional to the generalized eigenvalue simply because no other natural parameter with dimensions of mass is available. The space-time correlate of tachyonicity would be the Euclidian signature of effective metric defined by the modified Dirac operator associated with Kähler action.

2.9.1 Partition functions are not changed

One must write Super Virasoro conditions for L_n and *both* G_n and $G_n^\dagger$ rather than for L_n and G_n as in the case of the ordinary Super Virasoro algebra, and it is a priori not at all clear whether the partition functions for the Super Virasoro representations remain unchanged. This requirement is however crucial for the construction to work at all in the fermionic sector, since even the slightest changes for the degeneracies of the excited states can change light state to a state with mass of order m_0 in the p-adic thermodynamics.

Super conformal algebra

Super Virasoro algebra is generated by the bosonic the generators L_n (n is an integer valued index) and by the fermionic generators G_r, where r can be either integer (Ramond) or half odd integer (NS). G_r creates quark/lepton for $r > 0$ and antiquark/antilepton for $r < 0$. For $r = 0$, G_0 creates lepton and its Hermitian conjugate anti-lepton. The defining commutation and anti-commutation relations are the following:

$$
\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{2}m(m^2-1)\delta_{m,-n} \ , \\
[L_m, G_r] &= (\frac{m}{2} - r)G_{m+r} \ , \\
\left[L_m, G_r^\dagger\right] &= (\frac{m}{2} - r)G_{m+r}^\dagger \ , \\
\{G_r, G_s^\dagger\} &= 2L_{r+s} + \frac{c}{3}(r^2 - \frac{1}{4})\delta_{m,-n} \ , \\
\{G_r, G_s\} &= 0 \ , \\
\{G_r^\dagger, G_s^\dagger\} &= 0 \ .
\end{aligned} \tag{2.9.1}
$$

By the inspection of these relations one finds some results of a great practical importance.

1. For the Ramond algebra G_0, G_1 and their Hermitian conjugates generate the $r \geq 0, n \geq 0$ part of the algebra via anti-commutations and commutations. Therefore all what is needed is to assume that Super Virasoro conditions are satisfied for these generators in case that G_0 and $G_0^\dagger$ annihilate the ground state. Situation changes if the states are *not* annihilated by G_0 and $G_0^\dagger$ since then one must assume the gauge conditions for both L_1, G_1 and $G_1^\dagger$ besides the mass shell conditions associated with G_0 and $G_0^\dagger$, which however do not affect the number of the Super Virasoro excitations but give mass shell condition and constraints on the state in the cm spin degrees of freedom. This will be assumed in the following. Note that for the ordinary Super Virasoro only the gauge conditions for L_1 and G_1 are needed.

2. NS algebra is generated by $G_{1/2}$ and $G_{3/2}$ and their Hermitian conjugates (note that $G_{3/2}$ cannot be expressed as the commutator of L_1 and $G_{1/2}$) so that only the gauge conditions associated with these generators are needed. For the ordinary Super Virasoro only the conditions for $G_{1/2}$ and $G_{3/2}$ are needed.

Conditions guaranteing that partition functions are not changed

The conditions guaranteing the invariance of the partition functions in the transition to the modified algebra must be such that they reduce the number of the excitations and gauge conditions for a given conformal weight to the same number as in the case of the ordinary Super Virasoro.

1. The requirement that physical states are invariant under $G \leftrightarrow G^\dagger$ corresponds to the charge conjugation symmetry and is very natural. As a consequence, the gauge conditions for G and $G^\dagger$ are not independent and their number reduces by a factor of one half and is the same as in the case of the ordinary Super Virasoro.

2. As far as the number of the thermal excitations for a given conformal weight is considered, the only remaining problem are the operators $G_nG_n^\dagger$, which for the ordinary Super Virasoro reduce to $G_nG_n = L_{2n}$ and do not therefore correspond to independent degrees of freedom. In present case this situation is achieved only if one requires

$$
(G_nG_n^\dagger - G_n^\dagger G_n)|phys\rangle = 0 \ . \tag{2.9.2}
$$

It is not clear whether this condition must be posed separately or whether it actually follows from the representation of the Super Virasoro algebra automatically.

Partition function for Ramond algebra

Under the assumptions just stated, the partition function for the Ramond states not satisfying any gauge conditions

$$
Z(t) = 1 + 2t + 4t^2 + 8t^3 + 14t^4 + ... \ , \tag{2.9.3}
$$

which is identical to that associated with the ordinary Ramond type Super Virasoro.

For a Super Virasoro representation with $N = 5$ sectors, of main interest in TGD, one has

$$\begin{aligned} Z_N(t) &= Z^{N=5}(t) = \sum D(n)t^n \\ &= 1 + 10t + 60t^2 + 280t^3 + ... \ . \end{aligned} \tag{2.9.4}$$

The degeneracies for the states satisfying gauge conditions are given by

$$d(n) = D(n) - 2D(n-1) \ . \tag{2.9.5}$$

corresponding to the gauge conditions for L_1 and G_1. Applying this formula one obtains for $N = 5$ sectors

$$d(0) = 1 \ , \quad d(1) = 8 \ , \quad d(2) = 40 \ , \quad d(3) = 160 \ . \tag{2.9.6}$$

The lowest order contribution to the p-adic mass squared is determined by the ratio

$$r(n) = \frac{D(n+1)}{D(n)} \ ,$$

where the value of n depends on the effective vacuum weight of the ground state fermion. Light state is obtained only provided the ratio is integer. The remarkable result is that for lowest lying states the ratio is integer and given by

$$r(1) = 8 \ , \quad r(2) = 5 \ , \quad r(3) = 4 \ . \tag{2.9.7}$$

It turns out that $r(2) = 5$ gives the best possible lowest order prediction for the charged lepton masses and in this manner one ends up with the condition $h_{vac} = -3$ for the tachyonic vacuum weight of Super Virasoro.

Partition function for NS algebra

For NS representations the calculation of the degeneracies of the physical states reduces to the calculation of the partition function for a single particle Super Virasoro

$$Z_{NS}(t) = \sum_n z(n/2)t^{n/2} \ . \tag{2.9.8}$$

Here $z(n/2)$ gives the number of Super Virasoro generators having conformal weight $n/2$. For a state with N active sectors (the sectors with a non-vanishing weight for a given ground state) the degeneracies can be read from the N-particle partition function expressible as

$$Z_N(t) = Z^N(t) \ . \tag{2.9.9}$$

Single particle partition function is given by the expression

$$Z(t) = 1 + t^{1/2} + t + 2t^{3/2} + 3t^2 + 4t^{5/2} + 5t^3 + ... \ . \tag{2.9.10}$$

Using this representation it is an easy task to calculate the degeneracies for the operators of conformal weight Δ acting on a state having N active sectors.

One can also derive explicit formulas for the degeneracies and calculation gives

$$\begin{array}{ll} D(0,N)=1\ , & D(1/2,N)=N\ , \\ D(1,N)=\frac{N(N+1)}{2}\ , & D(3/2,N)=\frac{N}{6}(N^2+3N+8)\ , \\ D(2,N)=\frac{N}{2}(N^2+2N+3)\ , & D(5/2,N)=9N(N-1)\ , \\ D(3,N)=12N(N-1)+2N(N-1) & . \end{array} \qquad (2.9.11)$$

as a function of the conformal weight $\Delta = 0, 1/2, ..., 3$.

The number of states satisfying Super Virasoro gauge conditions created by the operators of a conformal weight Δ, when the number of the active sectors is N, is given by

$$d(\Delta,N) = D(\Delta,N) - D(\Delta-1/2,N) - D(\Delta-3/2,N)\ . \qquad (2.9.12)$$

The expression derives from the observation that the physical states satisfying gauge conditions for $G^{1/2}$, $G^{3/2}$ satisfy the conditions for all Super Virasoro generators. For $T_p = 1$ light bosons correspond to the integer values of $d(\Delta+1,N)/d(\Delta,N)$ in case that massless states correspond to thermal excitations of conformal weight Δ: they are obtained for $\Delta = 0$ only (massless ground state). This is what is required since the thermal degeneracy of the light boson ground state would imply a corresponding factor in the energy density of the black body radiation at very high temperatures. For the physically most interesting nontrivial case with $N = 2$ two active sectors the degeneracies are

$$d(0,2)=1\ ,\quad d(1,2)=1\ ,\quad d(2,2)=3\ ,\quad d(3,2)=4\ . \qquad (2.9.13)$$

N,Δ	0	1/2	1	3/2	2	5/2	3
2	1	1	1	3	3	4	4
3	1	2	3	9	11		
4	1	3	5	19	26		
5	1	4	10	24	150		

Table 3. Degeneracies $d(\Delta,N)$ of the operators satisfying NS type gauge conditions as a function of the number N of the active sectors and of the conformal weight Δ of the operator. Only those degeneracies, which are needed in the mass calculation for bosons assuming that they correspond to N-S representations are listed.

2.9.2 Fundamental length and mass scales

The basic difference between quantum TGD and super-string models is that the size of CP_2 is not of order Planck length but much larger: of order $10^{3.5}$ Planck lengths. This conclusion is forced by several consistency arguments, the mass scale of electron, and by the cosmological data allowing to fix the string tension of the cosmic strings which are basic structures in TGD inspired cosmology.

The relationship between CP_2 radius and fundamental p-adic length scale

One can relate CP_2 'cosmological constant' to the p-adic mass scale: for $k_L = 1$ one has

$$m_0^2 = \frac{m_1^2}{k_L} = m_1^2 = 2\Lambda\ . \qquad (2.9.14)$$

$k_L = 1$ results also by requiring that p-adic thermodynamics leaves charged leptons light and leads to optimal lowest order prediction for the charged lepton masses. Λ denotes the 'cosmological constant' of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$ with cosmological term).

The real counterpart of the p-adic thermal expectation for the mass squared is sensitive to the choice of the unit of p-adic mass squared which is by definition mapped as such to the real unit in canonical identification. Thus an important factor in the p-adic mass calculations is the correct identification of the p-adic mass squared scale, which corresponds to the mass squared unit and hence to the unit of the p-adic numbers. This choice does not affect the spectrum of massless states but can affect the spectrum of light states in case of intermediate gauge bosons.

1. For the choice

$$M^2 = m_0^2 \leftrightarrow 1 \quad (2.9.15)$$

   the spectrum of L_0 is integer valued.

2. The requirement that all sufficiently small mass squared values for the color partial waves are mapped to real integers, would fix the value of p-adic mass squared unit to

$$M^2 = \frac{m_0^2}{3} \leftrightarrow 1 \ . \quad (2.9.16)$$

   For this choice the spectrum of L_0 comes in multiples of 3 and it is possible to have a first order contribution to the mass which cannot be of thermal origin (say $m^2 = p$). This indeed seems to happen for electro-weak gauge bosons.

p-Adic mass calculations [F3] allow to relate m_0 to electron mass and to Planck mass by the formula

$$\begin{aligned} \frac{m_0}{m_{Pl}} &= \frac{1}{\sqrt{5+Y_e}} \times 2^{127/2} \times \frac{m_e}{m_{Pl}} \ , \\ m_{Pl} &= \frac{1}{\sqrt{\hbar G}} \ . \end{aligned} \quad (2.9.17)$$

For $Y_e = 0$ this gives $m_0 = .2437 \times 10^{-3} m_{Pl}$.

This means that CP_2 radius R defined by the length $L = 2\pi R$ of CP_2 geodesic is roughly $10^{3.5}$ times the Planck length. More precisely, using the relationship

$$\Lambda = \frac{3}{2R^2} = M^2 = m_0^2 \ ,$$

one obtains for

$$L = 2\pi R = 2\pi \sqrt{\frac{3}{2}} \frac{1}{m_0} \simeq 3.1167 \times 10^4 \sqrt{\hbar G} \ \text{for} \ Y_e = 0 \ . \quad (2.9.18)$$

The result came as a surprise: the first belief was that CP_2 radius is of order Planck length. It has however turned out that the new identification solved elegantly some long standing problems of TGD.

Y_e	0	.5	.7798
$(m_0/m_{Pl})10^3$	.2437	.2323	.2266
$K_R \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{\hbar G}) \times 10^{-4}$	3.1580	3.3122	3.3954
$K \times 10^{-7}$	2.4606	2.4606	2.4606
$(L/\sqrt{\hbar G}) \times 10^{-4}$	3.1167	3.1167	3.1167
K_R/K	1.0267	1.1293	1.1868

Table 1. Table gives the values of the ratio $K_R = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$. Also the ratio of K_R/K, where $K = 2\times3\times5\times7\times11\times13\times17\times19\times23\times2^{-3} * (15/17)$ is rational number producing R^2/G approximately is given.

The value of top quark mass favors $Y_e = 0$ and $Y_e = .5$ is largest value of Y_e marginally consistent with the limits on the value of top quark mass.

CP_2 radius as the fundamental p-adic length scale

The identification of CP_2 radius as the fundamental p-adic length scale is forced by the Super Virasoro invariance. The pleasant surprise was that the identification of the CP_2 size as the fundamental p-adic length scale rather than Planck length solved many long standing problems of older TGD.

1. The earliest formulation predicted cosmic strings with a string tension larger than the critical value giving the angle deficit 2π in Einstein's equations and thus excluded by General Relativity. The corrected value of CP_2 radius predicts the value k/G for the cosmic string tension with k in the range $10^{-7} - 10^{-6}$ as required by the TGD inspired model for the galaxy formation solving the galactic dark matter problem.

2. In the earlier formulation there was no idea as how to derive the p-adic length scale $L \sim 10^{3.5}\sqrt{\hbar G}$ from the basic theory. Now this problem becomes trivial and one has to predict gravitational constant in terms of the p-adic length scale. This follows in principle as a prediction of quantum TGD. In fact, one can deduce G in terms of the p-adic length scale and the action exponential associated with the CP_2 extremal and gets a correct value if α_K approaches fine structure constant at electron length scale (due to the fact that electromagnetic field equals to the Kähler field if Z^0 field vanishes).

   Besides this, one obtains a precise prediction for the dependence of the Kähler coupling strength on the p-adic length scale by requiring that the gravitational coupling does not depend on the p-adic length scale. p-Adic prime p in turn has a nice physical interpretation: the critical value of α_K is same for the zero modes with given p. As already found, the construction of graviton state allows to understand the small value of the gravitational constant in terms of a de-coherence caused by multi-p fractality reducing the value of the gravitational constant from L_p^2 to G.

3. p-Adic length scale is also the length scale at which super-symmetry should be restored in standard super-symmetric theories. In TGD this scale corresponds to the transition to Euclidian field theory for CP_2 type extremals. There are strong reasons to believe that sparticles are however absent and that super-symmetry is present only in the sense that super-generators have complex conformal weights with $Re(h) = \pm 1/2$ rather than $h = 0$. The action of this super-symmetry changes the mass of the state by an amount of order CP_2 mass.

2.9.3 Spectrum of elementary particles

The assumption that $k = 1$ holds true for all particles forces to modify the earlier construction of quark states. This turns out to be possible without affecting the p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (which can be negative).

Leptonic spectrum

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \ mod\ 3 \neq 2 \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p, p) representations with $p \geq 1$ whereas charged leptons correspond to $(p, p+3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2 + 2p)/3$, $p \geq 1$, for neutrinos and $p = 1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2 + 5p + 6)/3$ gives $h_c = 2$ for $p = 0$ (decuplet). In both cases super-symplectic operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-symplectic conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-symplectic conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-symplectic conformal weights contains both octet and decuplet so that singlets are obtained. What strengthens the hopes that the construction is not adhoc is that the same operator appears in the construction of quark states too.

Right handed neutrino remains essentially massless. $p = 0$ right handed neutrino does not however generate $N = 1$ space-time (or rather, imbedding space) super symmetry so that no sparticles are predicted. The breaking of the electro-weak symmetry at the level of the masses comes out basically from the anomalous color electro-weak correlation for the Kaluza-Klein partial waves implying that the weights for the ground states of the fermions depend on the electromagnetic charge of the fermion. Interestingly, TGD predicts leptohadron physics based on color excitations of leptons and color bound states of these excitations could correspond topologically condensed on string like objects but not fundamental string like objects.

Spectrum of quarks

Earlier arguments [F4] related to a model of CKM matrix as a rational unitary matrix suggested that the string tension parameter k is different for quarks, leptons, and bosons. The basic mass formula read as

$$M^2 = m^2_{CP_2} + kL_0 \quad .$$

The values of k were $k_q = 2/3$ and $k_L = k_B = 1$. The general theory however predicts that $k = 1$ for all particles.

1. By earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as unit, color conformal weight $h_c \equiv m^2_{CP_2}$ is integer for $p \ mod \ = \pm 1$ for U type quark belonging to $(p+1, p)$ type representation and obeying $h_c(U) = (p^2 + 3p + 2)/3$ and for $p \ mod \ 3 = 1$ for D type quark belonging $(p, p+2)$ type representation and obeying $h_c(D) = (p^2 + 4p + 4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

2. In the recent case $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This forces the super-symplectic operator O to compensate the anomalous color to have a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-symplectic operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SSV: this would also explain why h_{sc} would be same for all fermions.

3. It would seem that the tensor product of the spinor harmonic of quarks (as also leptons) with Hamiltonians gives rise to a large number of exotic colored states which have same thermodynamical mass as ordinary quarks (and leptons). Why these states have smaller values of p-adic prime that ordinary quarks and leptons, remains a challenge for the theory. Note that the decay widths of intermediate gauge bosons pose strong restrictions on the possible color excitations of quarks. On the other hand, the large number of fermionic color exotics can spoil the asymptotic freedom, and it is possible to have and entire p-adic length scale hierarchy of QCDs existing only in a finite length scale range without affecting the decay widths of gauge bosons.

The following table summarizes the color conformal weights and super-symplectic vacuum conformal weights for the elementary particles.

	L	ν_L	U	D	W	γ, G, g
h_{vac}	-3	-3	-3	-3	-2	0
h_c	2	1	2	3	2	0

Table 2. The values of the parameters h_{vac} and h_c assuming that $k = 1$. The value of $h_{vac} \leq -h_c$ is determined from the requirement that p-adic mass calculations give best possible fit to the mass spectrum.

Photon, graviton and gluon

For photon, gluon and graviton the conformal weight of the $p = 0$ ground state is $h_{gr} = h_{vac} = 0$. The crucial condition is that $h = 0$ ground state is non-degenerate: otherwise one would obtain several physically more or less identical photons and this would be seen in the spectrum of black-body radiation. This occurs if one can construct several ground states not expressible in terms of the action of the Super Virasoro generators.

Masslessness or approximate masslessness requires low enough temperature $T_p = 1/n$, $n > 1$ at least and small enough value of the possible contribution coming from the ground state conformal weight.

In NS thermodynamics the only possibility to get exactly massless states in thermal sense is to have $\Delta = 0$ state with one active sector so that NS thermodynamics becomes trivial due to the absence of the thermodynamical excitations satisfying the gauge conditions. For neutral gauge bosons this is indeed achieved. For $T_p = 1/2$, which is required by the mass spectrum of intermediate gauge bosons, the thermal contribution to the mass squared is however extremely small even for W boson.

2.9.4 Can p-adic thermodynamics explain the masses of intermediate gauge bosons?

The requirement that the electron-intermediate gauge boson mass ratios are sensible, serves as a stringent test for the hypothesis that intermediate gauge boson masses result from the p-adic thermodynamics. It seems that the only possible option is that the parameter k has same value for both bosons, leptons, and quarks:

$$k_B = k_L = k_q = 1 \ .$$

In this case all gauge bosons have $D(0) = 1$ and there are good changes to obtain boson masses correctly. $k = 1$ together with $T_p = 1$ implies that the thermal masses of very many boson states are extremely heavy so that the spectrum of the boson exotics is reduced drastically. For $T_p = 1/2$ the thermal contribution to the mass squared is completely negligible.

Contrary to the original optimistic beliefs based on calculational error, it turned out impossible to predict W/e and Z/e mass ratios correctly in the original p-adic thermodynamics scenario. Although the errors are of order 20-30 percent, they seemed to exclude the explanation for the massivation of gauge bosons using p-adic thermodynamics.

1. The thermal mass squared for a boson state with N active sectors (non-vanishing vacuum weight) is determined by the partition function for the tensor product of N NS type Super Virasoro algebras. The degeneracies of the excited states as a function of N and the weight Δ of the operator creating the massless state are given in the table below.

2. Both W and Z must correspond to $N = 2$ active Super Virasoro sectors for which $D(1) = 1$ and $D(2) = 3$ so that (using the formulas of p-adic thermodynamics [TGDpad, F3]) the thermal mass squared is $m^2 = k_B(p + 5p^2)$ for $T_p = 1$. The second order contribution to the thermal mass squared is extremely small so that Weinberg angle vanishes in the thermal approximation. $k_B = 1$ gives Z/e mass-ratio which is about 22 per cent too high. For $T_p = 1/2$ thermal masses are completely negligible.

3. The thermal prediction for W-boson mass is the same as for Z^0 mass and thus even worse since the two masses are related $M_W^2 = M_Z^2 cos^2(\theta_W)$.

It seem that the Achilles's heel of the p-adic thermodynamics is bosonic sector whereas the weak point of the standard model is fermionic sector. The first natural reaction -before realizing that ground state conformal weights need not be and very probably are not exactly equal to -1/2- was that it might be possible to combine these two approaches. $T_p = 1/2$ is certainly the only possible p-adic temperature for intermediate gauge bosons so that gauge boson masses should result by a TGD variant of the Higgs mechanism.

1. It is indeed possible to identify a candidate for Higgs boson with correct quantum numbers also in TGD framework. The simplest identification of Higgs boson is as a wormhole contact carrying

appropriate fermion and anti-fermion numbers at the two light-like 3-surfaces defined by the wormhole throats. The coherent state associated with this kind of Higgs can contribute only to gauge boson masses. It must be emphasized that this Higgs behaves as scale in M^4 degrees of freedom but like vector with respect to CP_2. This means obvious difference to standard model.

2. The minimum p-adic mass squared is the p-adic mass squared unit $m_0^2/3$. This corresponds in a reasonable approximation to the mass of W boson so that the mass scale would be predicted correctly. The calculation of leptonic masses however requires the use of m_0^2 as a mass squared unit for which intermediate gauge boson masses are smaller than one unit. The way out of the difficulty could be based on the use of a variant of the canonical identification I acting as $I_1(r/s) = I(r)/I(s)$. This map respects under certain additional conditions various symmetries and is the only sensible possibility at the level of scattering amplitudes. This variant predicts that the real counterpart of $m^2 = (m/n)p$ is $(m/n)/p$ rather than of order CP_2 squared so that intermediate gauge boson masses can be smaller than one unit even if $O(p)$ p-adically, and allows an elegant group theoretic description of m_W/m_Z mass ratio in terms of Weinberg angle. This point is discussed in [F4, F5].

3. After the realization that the generalized eigenvalues of the modified Dirac operator play a key role in quantum TGD the identification of Higgs field as the generalized eigenvalue emerged naturally. Since C-S action defines almost topological QFT, the generalized eigenvalues had dependence on the coordinates transversal to the light-like coordinate of X_l^3. The interpretation was as Higgs field and Higgs vacuum expectation was assigned with the values of this field at points of number theoretic braid. The unsatisfactory feature of this interpretation was the asymmetry between bosons and fermions and unability to predict the vacuum expectation value of Higgs.

Only after the discovery how the information about preferred extremal of Kähler action can be feeded to the spectrum of modified Dirac operator (see the discussion about modified Dirac action), a real understanding of the situation emerged (at least this is my belief!).

1. The generalized eigenvalues are simply square roots of ground state conformal weights and by analogy with cyclotron energies the conformal weights are in reasonable approximation given by $h = -n - 1/2$ giving the desired $h \simeq -1/2$ for lowest state plus finite number of additional ground states. The deviation Δh of h from half odd integer value cannot be compensated by the action of Virasoro generators and it is this contribution which has interpretation as Higgs contribution to mass squared. Δh is present for both fermions and bosons, should be small for fermions and dominate for gauge bosons. The vacuum expectation of Higgs is indeed naturally proportional to Δh but the presence of Higgs condensate does not cause the massivation.

2. Before one can buy a bottle of champaign, one must understand the relationship $M_W^2 = M_Z^2 cos^2(\theta_W)$ requiring $\Delta h(W)/\Delta h(Z) = cos^2(\theta_W)$. Essentially, one should understand the dependence of the quantum averaged the spectrum of modified Dirac operator on the quantum numbers of elementary particle over configuration space degrees of freedom. Suppose that the zero energy state describing particle is proportional to a phase factor depending on electro-weak and color quantum numbers of the particle. This phase factor would be simply $exp[i \int Tr(gQA_\mu)(dx^\mu/ds)ds]$ assignable to the strand of the number theoretic braid: gQ is the diagonal charge matrix characterizing the particle and A_μ represents gauge potential: in the electro-weak case components of the induced spinor connection and the case of color interactions the space-time projection of Killing forms j_k^A of color isometries. Stationary phase approximation selects a preferred light-like 3-surface X_l^3 for given quantum numbers and boundary conditions assign to this preferred extremal of Kähler action defining the exponent of Kähler function so that also Δh depends on quantum numbers of the particle.

Second challenge is to understand how the mixing of neutral gauge bosons B_3 and B_0 relates to the group theoretic factor $cos^2(\theta_W)$. The condition that the Higgs expectation value for gauge boson B is proportional to $\Delta h(B)$ and that the coherent state of Higgs couples gauge bosons regarded as fermion anti-fermion pairs should explain the mixing.

1. If gauge bosons and Higgs correspond to wormhole contacts, the discussion reduces to one-fermion level. The value of Δh should be different for different charge states $F_{\pm 1/2}$ of elementary fermion (in the following I will drop from discussion delicacies due to the fact that both quarks and leptons and fermion families are involved). The values of λ of fermion and anti-fermion assignable to gauge boson are naturally identical

$$\Delta\lambda(F_{\pm 1/2} = \Delta\lambda(\overline{F}_{\pm 1/2}) \equiv x_{\pm 1/2} \ . \tag{2.9.19}$$

This implies

$$\begin{aligned} \Delta h(Z,W) &\equiv \Delta h(Z) - \Delta h(W) = m_Z^2 - m_W^2 = m_Z^2 sin^2(\theta) \ , \\ \Delta h(Z) &= 1/2\sum_{\pm}(\Delta\lambda(F_{\pm 1/2}) + \Delta\lambda(\overline{F}_{\mp 1/2})^2 = 2\sum_{\pm} x_{\pm 1/2}^2 \ , \\ \Delta h(W) &= 1/2\sum_{\pm}(\Delta\lambda(F_{\pm 1/2}) + \Delta\lambda(\overline{F}_{\pm 1/2})^2 = (x_{1/2} + x_{-1/2})^2 \ . \end{aligned} \tag{2.9.20}$$

This gives

$$\Delta h(Z,W) = (x_{1/2} - x_{-1/2})^2 \tag{2.9.21}$$

giving the condition

$$(x_{1/2} - x_{-1/2})^2 = (x_{1/2} + x_{-1/2})^2 sin^2(\theta_W) \ . \tag{2.9.22}$$

The interpretation is as breaking of electro-weak $SU(2)_L$ symmetry coded by the geometry of CP_2 in the structure of spinor connection so that the symmetry breaking is expected to take place. One can *define* the value of Weinberg angle from the formula

$$sin(\theta_W) \equiv \pm \frac{x_{1/2} - x_{-1/2}}{x_{1/2} + x_{-1/2}} \ . \tag{2.9.23}$$

2. This definition of Weinberg angle should be consistent with the identification of Weinberg angle coming from the couplings of Z^0 and photon to fermions. Also here the reduction of couplings to one-fermion level might help to understand the symmetry breaking. Z^0 and γ decompose as $Z_0 = cos(\theta_W)B_3 + sin(\theta_W)B_0$ and $\gamma = -sin(\theta_W)B_3 + cos(\theta_W)B_0$, where B_3 corresponds to the gauge potential in $SU(2)_L$ triplet and B_0 the gauge potential in $SU(2)_L$ singlet. Why this mixing should be induced by the splitting of the conformal weights? What induces the mixing of electro-weak triplet with singlet?

3. Could it be the coherent state of Higgs field which transforms left handed and right handed fermions to each other and hence also B_3 to B_0 and vice versa? If the Higgs expectation value associated with the coherent state is proportional to Δh, it would not be too surprising if the mixing between B_3 and B_0 caused by the coherent Higgs state were proportional to $(x_{1/2} - x_{-1/2})/(x_{1/2} + x_{-1/2})$. The reason would be that B_3 is antisymmetric with respect to the exchange of weak isospins whereas B_0 is symmetric. Therefore also the mixing amplitude should be antisymmetric with respect to the exchange of isospins and proportional to $(x_{1/2} - x_{-1/2})$. The presence of the numerator is needed to make the amplitude dimensionless. Under this assumption the two identifications of the Weinberg angle are equivalent.

This - admittedly oversimplified - picture obviously changes considerably what-causes-what's in the description of gauge boson massivation and the basic argument should be developed into a more precise form.

2.9.5 Some probabilistic considerations

There are uniqueness problems related to the mapping of p-adic probabilities to real ones. These problems find a nice resolution from the requirement that the map respects probability conservation. The implied modification of the original mapping does not change measurably the predictions for the masses of light particles.

How unique the map of p-adic probabilities and mass squared values are mapped to real numbers is?

The mapping of p-adic thermodynamical probabilities and mass squared values to real numbers is not completely unique.

1. Canonical identification $I : \sum x_n p^n \rightarrow \sum x_n p^{-n}$ takes care of this mapping but does not respect the sum of probabilities so that the real images $I(p_n)$ of the probabilities must be normalized. This is a somewhat alarming feature.

2. The modification of the canonical identification mapping rationals by the formula $I(r/s) = I(r)/I(s)$ has appeared naturally in various applications, in particular because it respects unitarity of unitary matrices with rational elements with $r < p, s < p$. In the case of p-adic thermodynamic the formula $I(g(n)p^n/Z) \rightarrow I(g(n)p^n)/I(Z)$ would be very natural although Z need not be rational anymore. For $g(n) < p$ the real counterparts of the p-adic probabilities would sum up to one automatically for this option. One cannot deny that this option is more convincing than the original one. The generalization of this formula to map p-adic mass squared to a real one is obvious.

3. Options 1) and 2) differ dramatically when the $n = 0$ massless ground state has ground state degeneracy $D > 1$. For option 1) the real mass is predicted to be of order CP_2 mass whereas for option 2) it would be by a factor $1/D$ smaller than the minimum mass predicted by the option a). Thus option 2) would predict a large number of additional exotic states. For those states which are light for option 1), the two options make identical predictions as far as the significant two lowest order terms are considered. Hence this interpretation would not change the predictions of the p-adic mass calculations in this respect. Option 2) is definitely more in accord with the real physics based intuitions and the main role of p-adic thermodynamics would be to guarantee the quantization of the temperature and fix practically uniquely the spectrum of the "Hamiltonian".

Under what conditions the mapping of p-adic ensemble probabilities to real probabilities respects probability conservation?

One can consider also a more general situation. Assume that one has an ensemble consisting of independent elementary events such that the number of events of type i is N_i. The probabilities are given by $p_i = N_i/N$ and $N = \sum N_i$ is the total number of elementary events. Even in the case that N is infinite as a real number it is natural to map the p-adic probabilities to their real counterparts using the rational canonical identification $I(p_i) = I(N_i)/I(N)$. Of course, N_i and N exist as well defined p-adic numbers under very stringent conditions only.

The question is under what conditions this map respects probability conservation. The answer becomes obvious by looking at the pinary expansions of N_i and N. If the integers N_i (possibly infinite as real integers) have pinary expansions having no common pinary digits, the sum of probabilities is conserved in the map. Note that this condition can assign also to a finite ensemble with finite number of a unique value of p.

This means that the selection of a basis for independent events corresponds to a decomposition of the set of integers labelling pinary digits to disjoint sets and brings in mind the selection of orthonormalized basis of quantum states in quantum theory. What is physically highly non-trivial that this "orthogonalization" alone puts strong constraints on probabilities of the allowed elementary events. One can say that the probabilities define distributions of pinary digits analogous to non-negative probability amplitudes in the space of integers labelling pinary digits, and the probabilities of independent events must be orthogonal with respect to the inner product defined by point-wise multiplication in the space of pinary digits.

p-Adic thermodynamics for which Boltzman weights $g(E)exp(-E/T)$ are replaced by $g(E)p^{E/T}$ such that one has $g(E) < p$ and E/T is integer valued, satisfies this constraint. The quantization of E/T to integer values implies quantization of both T and "energy" spectrum and forces so called super conformal invariance in TGD applications, which is indeed a basic symmetry of the theory.

There are infinitely many ways to choose the elementary events and each choice corresponds to a decomposition of the infinite set of integers n labelling the powers of p to disjoint subsets. These subsets can be also infinite. One can assign to this kind of decomposition a resolution which is the poorer the larger the subsets involved are. p-Adic thermodynamics would represent the situation in which the resolution is maximal since each set contains only single pinary digit. Note the analogy with the basis of completely localized wave functions in a lattice.

2.10 Modular contribution to the mass squared

The success of the p-adic mass calculations gives convincing support for the generation-genus correspondence. The basic physical picture is following.

1. Fermionic mass squared is dominated by partonic contribution, which is sum of cm and modular contributions: $M^2 = M^2(cm) + M^2(mod)$. Here 'cm' refers to the thermal contribution. Modular contribution can be assumed to depend on the genus of the boundary component only.

2. If Higgs contribution for diagonal (g, g) bosons (singlets with respect to "topological" $SU(3)$) dominates, the genus dependent contribution can be assumed to be negligible. This should be due to the bound state character of the wormhole contacts reducing thermal motion and thus the p-adic temperature.

3. Modular contribution to the mass squared can be estimated apart from an overall proportionality constant. The mass scale of the contribution is fixed by the p-adic length scale hypothesis. Elementary particle vacuum functionals are proportional to a product of all even theta functions and their conjugates, the number of even theta functions and their conjugates being $2N(g) = 2^g(2^g + 1)$. Also the thermal partition function must also be proportional to $2N(g)$:th power of some elementary partition function. This implies that thermal/ quantum expectation $M^2(mod)$ must be proportional to $2N(g)$. Since single handle behaves effectively as particle, the contribution must be proportional to genus g also. The success of the resulting mass formula encourages the belief that the argument is essentially correct.

The challenge is to construct theoretical framework reproducing the modular contribution to mass squared. There are two alternative manners to understand the origin modular contribution.

1. The realization that super-symplectic algebra is relevant for elementary particle physics leads to the idea that two thermodynamics are involved with the calculation of the vacuum conformal weight as a thermal expectation. The first thermodynamics corresponds to Super Kac-Moody algebra and second thermodynamics to super-symplectic algebra. This approach allows a first principle understanding of the origin and general form of the modular contribution without any need to introduce additional structures in modular degrees of freedom. The very fact that super-symplectic algebra does not commute with the modular degrees of freedom explains the dependence of the super-symplectic contribution on moduli.

2. The earlier approach was based on the idea that he modular contribution could be regarded as a quantum mechanical expectation value of the Virasoro generator L_0 for the elementary particle vacuum functional. Quantum treatment would require generalization the concepts of the moduli space and theta function to the p-adic context and finding an acceptable definition of the Virasoro generator L_0 in modular degrees of freedom. The problem with this interpretation is that it forces to introduce, not only Virasoro generator L_0, but the entire super Virasoro

algebra in modular degrees of freedom. One could also consider of interpreting the contribution of modular degrees of freedom to vacuum conformal weight as being analogous to that of CP_2 Laplacian but also this would raise the challenge of constructing corresponding Dirac operator. Obviously this approach has become obsolete.

The thermodynamical treatment taking into account the constraints from that p-adicization is possible might go along following lines.

1. In the real case the basic quantity is the thermal expectation value $h(M)$ of the conformal weight as a function of moduli. The average value of the deviation $\Delta h(M) = h(M) - h(M_0)$ over moduli space $\mathcal{M}$ must be calculated using elementary particle vacuum functional as a modular invariant partition function. Modular invariance is achieved if this function is proportional to the logarithm of elementary particle vacuum functional: this reproduces the qualitative features basic formula for the modular contribution to the conformal weight. p-Adicization leads to a slight modification of this formula.

2. The challenge of algebraically continuing this calculation to the p-adic context involves several sub-tasks. The notions of moduli space $\mathcal{M}_p$ and theta function must be defined in the p-adic context. An appropriately defined logarithm of the p-adic elementary particle vacuum functional should determine $\Delta h(M)$. The average of $\Delta h(M)$ requires an integration over $\mathcal{M}_p$. The problems related to the definition of this integral could be circumvented if the integral in the real case could be reduced to an algebraic expression, or if the moduli space is discrete in which case integral could be replaced by a sum.

3. The number theoretic existence of the p-adic Θ function leads to the quantization of the moduli so that the p-adic moduli space is discretized. Accepting the sharpened form of Riemann hypothesis [E8], the quantization means that the imaginary *resp.* real parts of the moduli are proportional to integers *resp.* combinations of imaginary parts of zeros of Riemann Zeta. This quantization could occur also for the real moduli for the maxima of Kähler function. This reduces the problematic p-adic integration to a sum and the resulting sum defining $\langle \Delta h \rangle$ converges extremely rapidly for physically interesting primes so that only the few lowest terms are needed.

2.10.1 Conformal symmetries and modular invariance

The full SKM invariance means that the super-conformal fields depend only on the conformal moduli of 2-surface characterizing the conformal equivalence class of the 2-surface. This means that all induced metrics differing by a mere Weyl scaling have same moduli. This symmetry is extremely powerful since the space of moduli is finite-dimensional and means that the entire infinite-dimensional space of deformations of parton 2-surface X^2 degenerates to a finite-dimensional moduli spaces under conformal equivalence. Obviously, the configurations of given parton correspond to a fiber space having moduli space as a base space. Super-symplectic degrees of freedom could break conformal invariance in some appropriate sense.

Conformal and SKM symmetries leave moduli invariant

Conformal transformations and super Kac Moody symmetries must leave the moduli invariant. This means that they induce a mere Weyl scaling of the induced metric of X^2 and thus preserve its non-diagonal character $ds^2 = g_{z\overline{z}}dzd\overline{z}$. This is indeed true if

1. the Super Kac Moody symmetries are holomorphic isometries of $X^7 = \delta M^4_\pm \times CP_2$ made local with respect to the complex coordinate z of X^2, and

2. the complex coordinates of X^7 are holomorphic functions of z.

Using complex coordinates for X^7 the infinitesimal generators can be written in the form

$$J^{An} = z^n j^{Ak} D_k + \overline{z}^n j^{A\overline{k}} D_{\overline{k}} \ . \tag{2.10.1}$$

The intuitive picture is that it should be possible to choose X^2 freely. It is however not always possible to choose the coordinate z of X^2 in such a manner that X^7 coordinates are holomorphic functions of z since a consistency of inherent complex structure of X^2 with that induced from X^7 is required. Geometrically this is like meeting of two points in the space of moduli.

Lorentz boosts produce new inequivalent choices of S^2 with their own complex coordinate: this set of complex structures is parameterized by the hyperboloid of future light cone (Lobatchevski space or mass shell), but even this is not enough. The most plausible manner to circumvent the problem is that only the maxima of Kähler function correspond to the holomorphic situation so that super-symplectic algebra representing quantum fluctuations would induce conformal anomaly.

The isometries of δM^4_+ are in one-one correspondence with conformal transformations

For CP_2 factor the isometries reduce to $SU(3)$ group acting also as symplectic transformations. For $\delta M^4_+ = S^2 \times R_+$ one might expect that isometries reduce to Lorentz group containing rotation group of $SO(3)$ as conformal isometries. If r_M corresponds to a macroscopic length scale, then X^2 has a finite sized S^2 projection which spans a rather small solid angle so that group $SO(3)$ reduces in a good approximation to the group $E^2 \times SO(2)$ of translations and rotations of plane.

This expectation is however wrong! The light-likeness of δM^4_+ allows a dramatic generalization of the notion of isometry. The point is that the conformal transformations of S^2 induce a conformal factor $|df/dw|^2$ to the metric of δM^4_+ and the local radial scaling $r_M \rightarrow r_M/|df/dw|$ compensates it. Hence the group of conformal isometries consists of conformal transformations of S^2 with compensating radial scalings. This compensation of two kinds of conformal transformations is the deep geometric phenomenon which translates to the condition $L_{SC} - L_{SKM} = 0$ in the sub-space of physical states. Note that an analogous phenomenon occurs also for the light-like CDs X^3_l with respect to the metrically 2-dimensional induced metric.

The X^2-local radial scalings $r_M \rightarrow r_M(z,\overline{z})$ respect the conditions $g_{zz} = g_{\overline{z}\overline{z}} = 0$ so that a mere Weyl scaling leaving moduli invariant results. By multiplying the conformal isometries of δM^4_+ by z^n (z is used as a complex coordinate for X^2 and w as a complex coordinate for S^2) a conformal localization of conformal isometries would result. Kind of double conformal transformations would be in question. Note however that this requires that X^7 coordinates are holomorphic functions of X^2 coordinate. These transformations deform X^2 unlike the conformal transformations of X^2. For X^3_l similar local scalings of the light like coordinate leave the moduli invariant but lead out of X^7.

Canonical transformations break the conformal invariance

In general, infinitesimal symplectic transformations induce non-vanishing components $g_{zz}, g_{\overline{z}\overline{z}}$ of the induced metric and can thus change the moduli of X^2. Thus the quantum fluctuations represented by super-symplectic algebra and contributing to the configuration space metric are in general moduli changing. It would be interesting to know explicitly the conditions (the number of which is the dimension of moduli space for a given genus), which guarantee that the infinitesimal symplectic transformation is moduli preserving.

2.10.2 The physical origin of the genus dependent contribution to the mass squared

Different p-adic length scales are not enough to explain the charged lepton mass ratios and an additional genus dependent contribution in the fermionic mass formula is required. The general form of this contribution can be guessed by regarding elementary particle vacuum functionals in the modular degrees of freedom as an analog of partition function and the modular contribution to the conformal weight as an analog of thermal energy obtained by averaging over moduli. p-Adic length scale hypothesis determines the overall scale of the contribution.

The exact physical origin of this contribution has remained mysterious but super-symplectic degrees of freedom represent a good candidate for the physical origin of this contribution. This would mean a sigh of relief since there would be no need to assign conformal weights, super-algebra, Dirac operators, Laplacians, etc.. with these degrees of freedom.

Thermodynamics in super-symplectic degrees of freedom as the origin of the modular contribution to the mass squared

The following general picture is the simplest found hitherto.

1. Elementary particle vacuum functionals are defined in the space of moduli of surfaces X^2 corresponding to the maxima of Kähler function. There some restrictions on X^2. In particular, p-adic length scale poses restrictions on the size of X^2. There is an infinite hierarchy of elementary particle vacuum functionals satisfying the general constraints but only the lowest elementary particle vacuum functionals are assumed to contribute significantly to the vacuum expectation value of conformal weight determining the mass squared value.

2. The contribution of Super-Kac Moody thermodynamics to the vacuum conformal weight h coming from Virasoro excitations of the $h = 0$ massless state is estimated in the previous calculations and does not depend on moduli. The new element is that for a partonic 2-surface X^2 with given moduli, Virasoro thermodynamics is present also in super-symplectic degrees of freedom.

   Super-symplectic thermodynamics means that, besides the ground state with $h_{gr} = -h_{SC}$ with minimal value of super-symplectic conformal weight h_{SC}, also thermal excitations of this state by super-symplectic Virasoro algebra having $h_{gr} = -h_{SC} - n$ are possible. For these ground states the SKM Virasoro generators creating states with net conformal weight $h = h_{SKM} - h_{SC} - n \geq 0$ have larger conformal weight so that the SKM thermal average h depends on n. It depends also on the moduli M of X^2 since the Beltrami differentials representing a tangent space basis for the moduli space $\mathcal{M}$ do not commute with the super-symplectic algebra. Hence the thermally averaged SKM conformal weight h_{SKM} for given values of moduli satisfies

$$h_{SKM} \quad = \quad h(n, M) \ . \tag{2.10.2}$$

3. The average conformal weight induced by this double thermodynamics can be expressed as a super-symplectic thermal average $\langle \cdot \rangle_{SC}$ of the SKM thermal average $h(n, M)$:

$$h(M) \quad = \quad \langle h(n, M) \rangle_{SC} = \sum p_n(M) h(n) \ , \tag{2.10.3}$$

   where the moduli dependent probability $p_n(M)$ of the super-symplectic Virasoro excitation with conformal weight n should be consistent with the p-adic thermodynamics. It is convenient to write $h(M)$ as

$$h(M) \quad = \quad h_0 + \Delta h(M) \ , \tag{2.10.4}$$

   where h_0 is the minimum value of $h(M)$ in the space of moduli. The form of the elementary particle vacuum functionals suggest that h_0 corresponds to moduli with $Im(\Omega_{ij}) = 0$ and thus to singular configurations for which handles degenerate to one-dimensional lines attached to a sphere.

4. There is a further averaging of $\Delta h(M)$ over the moduli space $\mathcal{M}$ by using the modulus squared of elementary particle vacuum functional so that one has

$$h \quad = \quad h_0 + \langle \Delta h(M) \rangle_{\mathcal{M}} \ . \tag{2.10.5}$$

   Modular invariance allows to pose very strong conditions on the functional form of $\Delta h(M)$. The simplest assumption guaranteing this and thermodynamical interpretation is that $\Delta h(M)$ is proportional to the logarithm of the vacuum functional Ω:

$$\Delta h(M) \quad \propto \quad -log(\frac{\Omega(M)}{\Omega_{max}}) \ . \tag{2.10.6}$$

Here Ω_{max} corresponds to the maximum of Ω for which $\Delta h(M)$ vanishes.

Justification for the general form of the mass formula

The proposed general ansatz for $\Delta h(M)$ provides a justification for the general form of the mass formula deduced by intuitive arguments.

1. The factorization of the elementary particle vacuum functional Ω into a product of $2N(g) = 2^g(2^g+1)$ terms and the logarithmic expression for $\Delta h(M)$ imply that the thermal expectation values is a sum over thermal expectation values over $2N(g)$ terms associated with various even characteristics (a,b), where a and b are g-dimensional vectors with components equal to 1/2 or 0 and the inner product $4a\cdot b$ is an even integer. If each term gives the same result in the averaging using Ω_{vac} as a partition function, the proportionality to $2N_g$ follows.

2. For genus $g\geq 2$ the partition function defines an average in $3g-3$ complex-dimensional space of moduli. The analogy of $\langle\Delta h\rangle$ and thermal energy suggests that the contribution is proportional to the complex dimension $3g-3$ of this space. For $g\leq 1$ the contribution the complex dimension of moduli space is g and the contribution would be proportional to g.

$$\begin{aligned}
\langle\Delta h\rangle &\propto g\times X(g) \ \text{ for } \ g\leq 1 \ , \\
\langle\Delta h\rangle &\propto (3g-3)\times X(g) \ \text{ for } \ g\geq 2 \ , \\
X(g) &= 2^g(2^g+1) \ .
\end{aligned} \tag{2.10.7}$$

   If X^2 is hyper-elliptic for the maxima of Kähler function, this expression makes sense only for $g\leq 2$ since vacuum functionals vanish for hyper-elliptic surfaces.

3. The earlier argument, inspired by the interpretation of elementary particle vacuum functional as a partition function, was that each factor of the elementary particle vacuum functional gives the same contribution to $\langle\Delta h\rangle$, and that this contribution is proportional to g since each handle behaves like a particle:

$$\langle\Delta h\rangle \quad \propto \quad g\times X(g) \ . \tag{2.10.8}$$

   The prediction following from the previous differs by a factor $(3g-3)/g$ for $g\geq 2$. This would scale up the dominant modular contribution to the masses of the third $g=2$ fermionic generation by a factor $\sqrt{3/2}\simeq 1.22$. One must of course remember, that these rough arguments allow $g-$ dependent numerical factors of order one so that it is not possible to exclude either argument.

2.10.3 Generalization of Θ functions and quantization of p-adic moduli

The task is to find p-adic counterparts for theta functions and elementary particle vacuum functionals. The constraints come from the p-adic existence of the exponentials appearing as the summands of the theta functions and from the convergence of the sum. The exponentials must be proportional to powers of p just as the Boltzmann weights defining the p-adic partition function. The outcome is a quantization of moduli so that integration can be replaced with a summation and the average of $\Delta h(M)$ over moduli is well defined.

It is instructive to study the problem for torus in parallel with the general case. The ordinary moduli space of torus is parameterized by single complex number τ. The points related by $SL(2,Z)$ are equivalent, which means that the transformation $\tau\rightarrow (A\tau+B)/(C\tau+D)$ produces a point equivalent with τ. These transformations are generated by the shift $\tau \rightarrow \tau + 1$ and $\tau \rightarrow -1/\tau$. One can choose the fundamental domain of moduli space to be the intersection of the slice $Re(\tau) \in [-1/2, 1/2]$ with the exterior of unit circle $|\tau| = 1$. The idea is to start directly from physics and to look whether one might some define p-adic version of elementary particle vacuum functionals in the p-adic counter part of this set or in some modular invariant subset of this set.

Elementary particle vacuum functionals are expressible in terms of theta functions using the functions $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ as a building block. The general expression for the theta function reads as

$$\Theta[a,b](\Omega) = \sum_n exp(i\pi(n+a)\cdot\Omega\cdot(n+a))exp(2i\pi(n+a)\cdot b) \ . \tag{2.10.9}$$

The latter exponential phase gives only a factor $\pm i$ or ± 1 since $4a \cdot b$ is integer. For $p \ mod \ 4 = 3$ imaginary unit exists in an algebraic extension of p-adic numbers. In the case of torus (a,b) has the values $(0,0)$, $(1/2,0)$ and $(0,1/2)$ for torus since only even characteristics are allowed.

Concerning the p-adicization of the first exponential appearing in the summands in Eq. 2.10.9, the obvious problem is that π does not exists p-adically. The introduction of the scaled variable $\hat{\tau} = \pi\tau$ resolves this problem. The second modification is the replacement of the factors $exp(X)$ with $p^{X/log(p)}$ in order to achieve a rapid p-adic convergence of the sum defining the theta function. This requires a further scaling so that one has $\Omega_p = \pi\Omega/log(p)$ is the appropriate variable and the terms in the sum are apart from the phase factor of form $p^{i(n+a)\cdot\Omega_p\cdot(n+a)}$.

If the exponents

$$p^{i(n+a)\cdot Im(\Omega_{ij,p})\cdot(n+a)} = p^{-a\cdot Im(\Omega_{ij,p})\cdot a} \times p^{-2a\cdot Im(\Omega_{ij,p})n} \times p^{-n\cdot Im(\Omega_{ij,p})\cdot n}$$

are integer powers of p, $\Theta_{[a,b]}$ exist in R_p. A milder condition is that only the building blocks $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ exist in R_p. The problematic factor is the first exponent since the components of the vector a can have values $1/2$ and 0 and its existence implies a quantization of $Im(\Omega_{ij,p})$ as

$$Im(\Omega_{ij,p}) = -Kn_{ij} \ , \ n_{ij} \in Z \ , \ n_{ij} \geq 1 \ , \tag{2.10.10}$$

$K = 4$ guarantees the existence of Θ functions and $K = 1$ the existence of elementary particle vacuum functionals. Obviously the sum defining Θ converges rapidly with respect to the p-adic norm.

The problem is that the condition $Im(\Omega_{ij,p}) > 0$ is not satisfied. There is however no reason why the p-adic theta function could not be defined by changing the sign of the exponents so that one would have

$$\begin{aligned} \Theta[a,b](\Omega)_p &= \sum_n p^{-i(n+a)\cdot\Omega_p\cdot(n+a)} \times exp\left[2i\pi(n+a)\cdot b\right] \ , \\ Im(\Omega_{ij,p}) &= Kn_{ij} \ , \ n_{ij} \geq 1 \ . \end{aligned} \tag{2.10.11}$$

$K = 4$ guarantees the existence of Θ functions in R_p and $K = 1$ the existence of elementary vacuum functional in R_p: in this case $\Theta_{[a,b]}$ exists in appropriate algebraic extension of R_p. Note that a similar change of sign must be performed in p-adic thermodynamics for powers of p to map p-adic probabilities to real ones.

A further requirement is that the phases $p^{-iRe(\Omega_{ij,p})/4}$ exist p-adically. A weaker condition that only the phases $p^{-iRe(\Omega_{ij,p})}$ exist p-adically guarantees that elementary particle vacuum functionals exist p-adically. The condition that p^{iy} exists for certain preferred values of y for all values of prime p is encountered repeatedly in the algebraic continuation of quantum TGD to p-adic context. The sharpening of the Riemann Hypothesis [E8] stating that the partition functions $1/(1-p^z)$ appearing in the product expansion of Rieman Zeta in various p-adic number fields exist for the zeros $z = 1/2 + iy$ of Riemann Zeta, is number theoretically highly attractive.

This conjecture implies that p^{iy} is in general a product of a phase factor $exp(i2\pi m/n)$ in some algebraic extension of p-adic numbers and of a Pythagorean phase $(k+il)/\sqrt{k^2+l^2}$, $k^2 + l^2 = n^2$. A potential problem is that this phase factor does not possess unit p-adic norm in the general case.

The explicit form for the allowed (k,l) and (l,k) pairs is given by

$$\begin{aligned} k &= 2rs \ , \\ l &= r^2 - s^2 \ , \\ n &= r^2 + s^2 \ . \end{aligned} \tag{2.10.12}$$

where r and s are relatively prime integers, not both odd. Note that (l,k) is also an allowed solution. An important point to be noticed is that the p-adic norm of Pythagorean phase is not larger than one for physically most interesting primes satisfying $p \ mod \ 4 = 3$ since $n \ mod \ 4 = 1$ holds true as a simple calculation shows. This guarantees that the phase factors of the Θ function cannot spoil the p-adic convergence of the sum defining the p-adic theta function.

The sharpening of the Riemann hypothesis, when combined with the requirement that the logarithmic radial waves $(r_M/r_0)^{iz}$ exists in some finite-dimensional extension of any p-adic number fields when r_M/r_0 is rational valued, implies that the radial conformal weights z of the super-symplectic algebra correspond to the zeros of Zeta and their appropriate combinations. The quantization condition is

$$Re(\Omega_{ij,p}) = K\sum n_k y_k \ , \tag{2.10.13}$$

where y_k correspond to zeros of Zeta. $K=4$ guarantees that Θ functions exist p-adically. $K=1$ is enough to guarantee the existence of elementary particle vacuum functionals.

In the real context the quantization of moduli of torus would correspond to

$$\begin{aligned} \tau &= K(\sum n_k y_k + in) \times \frac{log(p)}{\pi} \ , \\ |\tau| &= K\sqrt{n^2 + (\sum_k n_k y_k)^2} \ , \\ \Phi &= atan(\frac{n}{\sum_k n_k y_k}) \ . \end{aligned} \tag{2.10.14}$$

$K=1$ guarantees the existence of elementary particle vacuum functionals and $K=4$ the existence of Theta functions. The ratio for the complex vectors defining the sides of the plane parallelogram defining torus via the identification of the parallel sides is quantized. In other words, the angles Φ between the sides and the ratios of the sides given by $|\tau|$ have quantized values.

The quantization rules for the moduli of the higher genera read as

$$\Omega_{ij} = K\left[\sum n_k(i,j)y_k + in(i,j)\right] \times \frac{log(p)}{\pi} \ , \tag{2.10.15}$$

If the quantization rules hold true also for the maxima of Kähler function in the real context, there are good hopes that the p-adicized expression for Δh is obtained by a simple algebraic continuation of the real formula. Thus p-adic length scale characterizes partonic surface X^2 rather than the light like causal determinant X_l^3 containing X^2. Therefore the idea that various p-adic primes label various X_l^3 connecting fixed partonic surfaces X_i^2 would not be correct.

The set of the moduli allowed by the quantization rules is not invariant under modular transformations. For instance, in the case of torus the $SL(2,Z)$ Möbius transformations $\Omega \rightarrow \Omega + n$ and $\Omega \rightarrow 1/\Omega$ lead out of the allowed moduli space. This is not however a problem if there ar no modular transformations relating quantized moduli so that they can be thought of as forming single fundamental domain containing possibly non-equivalent moduli from several fundamental domains in the conventional sense of the word.

Quite generally, the quantization of moduli means that the allowed 2-dimensional shapes form a lattice and are thus additive. It also means that the maxima of Kähler function obey a linear superposition in an extreme abstract sense. The proposed number theoretical quantization is expected to apply for any complex space allowing some preferred complex coordinates. In particular, configuration space of 2-surfaces could allow this kind of quantization in the complex coordinates naturally associated with isometries and this could allow to define configuration space integration, at least the counterpart of integration in zero mode degrees of freedom, as a summation.

2.10.4 The calculation of the modular contribution $\langle \Delta h \rangle$ to the conformal weight

The quantization of the moduli implies that the integral over moduli can be defined as a sum over moduli. The theta function $\Theta[a,b](\Omega)_p(\tau_p)$ is proportional to $p^{a\cdot a Im(\Omega_{ij,p})} = p^{K n_{ij} m(a)/4}$ for $a \cdot a = m(a)/4$, where $K = 1$ *resp.* $K = 4$ corresponds to the existence existence of elementary particle vacuum functionals *resp.* theta functions in R_p. These powers of p can be extracted from the thetas defining the vacuum functional. The numerator of the vacuum functional gives $(p^n)^{2K\sum_{a,b} m(a)}$. The numerator gives $(p^n)^{2K\sum_{a,b} m(a_0)}$, where a_0 corresponds to the minimum value of $m(a)$. $a_0 = (0,0,..,0)$ is allowed and gives $m(a_0) = 0$ so that the p-adic norm of the denominator equals to one. Hence one has

$$|\Omega_{vac}(\Omega_p)|_p \quad = \quad p^{-2nK\sum_{a,b} m(a)} \qquad (2.10.16)$$

The sum converges extremely rapidly for large values of p as function of n so that in practice only few moduli contribute.

The definition of $log(\Omega_{vac})$ poses however problems since in $log(p)$ does not exist as a p-adic number in any p-adic number field. The argument of the logarithm should have a unit p-adic norm. The simplest manner to circumvent the difficulty is to use the fact that the p-adic norm $|\Omega_p|_p$ is also a modular invariant, and assume that the contribution to conformal weight depends on moduli as

$$\Delta h_p(\Omega_p) \quad \propto \quad log(\frac{\Omega_{vac}}{|\Omega_{vac}|_p}) \ . \qquad (2.10.17)$$

The sum defining $\langle \Delta h_p \rangle$ converges extremely rapidly and gives a result of order $O(p)$ p-adically as required.

The p-adic expression for $\langle \Delta h_p \rangle$ should result from the corresponding real expression by an algebraic continuation. This encourages the conjecture that the allowed moduli are quantized for the maxima of Kähler function, so that the integral over the moduli space is replaced with a sum also in the real case, and that Δh given by the double thermodynamics as a function of moduli can be defined as in the p-adic case. The positive power of p multiplying the numerator could be interpreted as a degeneracy factor. In fact, the moduli are not primary dynamical variables in the case of the induced metric, and there must be a modular invariant weight factor telling how many 2-surfaces correspond to given values of moduli. The power of p could correspond to this factor.

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Chapter 3

p-Adic Particle Massivation: Elementary Particle Masses

3.1 Introduction

In this chapter the detailed predictions of the p-adic description of particle massivation are studied. The theoretical background is represented in detail in [F2].

3.1.1 Particle massivation

Particle massivation can be regarded as a generation of thermal conformal weight identified as mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

1. The original observation was that the pieces of CP_2 type vacuum extremals representing elementary particles have random light-like curve as an M^4 projection so that the average motion correspond to that of massive particle. Light-like randomness gives rise to classical Virasoro conditions. This picture generalizes since the basic dynamical objects are light-like but otherwise random 3-surfaces. Fermions are identified as light-like 3-surfaces at which the signature of induced metric of deformed CP_2 type extremals changes from Euclidian to the Minkowskian signature of the background space-time sheet. Gauge bosons and Higgs correspond to wormhole contacts with light-like throats carrying fermion and antifermion quantum numbers. Gravitons correspond to pairs of wormhole contacts bound to string like object by the fluxes connecting the wormhole contacts. The randomness of the light-like 3-surfaces and associated super-conformal symmetries justify the use of thermodynamics and the question remains why this thermodynamics can be taken to be p-adic. The proposed identification of bosons means enormous simplification in thermodynamical description since all calculations reduced to the calculations to fermion level.

2. The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces. Dynamics is constrained by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. As a matter fact, the basic theory relies on the modified Dirac action associated with Chern-Simons action and Kähler action in the sense that the generalizes eigenmodes of C-S Dirac operator correspond to the zero modes of Kähler action localized to the light-like 3-surfaces representing partons. In this manner the data about the dynamics of Kähler action is feeded to the eigenvalue spectrum. Eigenvalues are interpreted as square roots of ground state conformal weights.

3. The symmetries respecting light-likeness property give rise to Kac-Moody type algebra and super-symplectic symmetries emerge also naturally as well as $N = 4$ character of super-conformal

invariance. The coset construction for super-symplectic Virasoro algebra and Super Kac-Moody algebra identified in physical sense as sub-algebra of former implies that the four-momenta assignable to the two algebras are identical. The interpretation is in terms of the identity of gravitational inertial masses and generalization of Equivalence Principle.

4. Instead of energy, the Super Kac-Moody Virasoro (or equivalently super-symplectic) generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics (and also in its real version assuming it exists). The fact that mass squared is thermal expectation of conformal weight guarantees Lorentz invariance. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

5. By Equivalence Principle the thermal average of mass squared can be calculated either in terms of thermodynamics for either super-symplectic of Super Kac-Moody Virasoro algebra and p-adic thermodynamics is consistent with conformal invariance.

6. A long standing problem has been whether coupling to Higgs boson is needed to explain gauge boson masses. It has turned out that p-adic thermodynamics is enough. From the beginning it was clear that is that ground state conformal weight is negative. Only quite recently it became clear that the ground state conformal weight need not be a negative integer. The deviation Δh of the total ground state conformal weight from negative integer gives rise to Higgs type contribution to the thermal mass squared and dominates in case of gauge bosons for which p-adic temperature is small. In the case of fermions this contribution to the mass squared is small. Higgs vacuum expectation is naturally proportional to Δh so that the coupling to Higgs seems to cause gauge boson massivation. The interpretation is that the effective metric defined by the modified gamma matrices associated with Kähler action has Euclidian signature. This implies that the eigenvalues of the modified Dirac operator are purely imaginary and analogous to cyclotron energies so that in the first approximation smallest conformal weights are of form $h = -n-1/2$ and for $n = 0$ one obtains the ground state conformal weight $h = -1/2$ conjectured earlier. One cannot exclude the possibility of complex eigenvalues of D_{C-S}.

7. There is also modular contribution to the mass squared which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

3.1.2 Basic contributions to the particle mass squared

The formula for the p-adic mass squared contains three additive contributions. The first contribution is proportional to the thermal expectation of the Virasoro generator L_0 in in Super Virasoro degrees of freedom. The miracle is that this contribution is small for the particles with the quantum numbers of the observed light particles, when Super Virasoro has $N = 5$ sectors as it does in TGD approach. These sectors correspond to the 5 tensor factors for the $M^4 \times SU(3) \times U(2)_{ew}$ decomposition of the super Kac Moody algebra to gauge symmetries of gravitation, color and electro-weak interactions. These symmetries act on the intersections $X^2 = X^3_l \cap X^7$ of 3-D light like causal determinants (CDs) X^3_l and 7-D light like CDs $X^7 = \delta M^4_+ \times CP_2$. This constraint leaves only the 2 transversal degrees M^4 degrees of freedom since the translations in light like directions associated with X^3_l and δM^4_+ are eliminated.

Second contribution comes from the modular degrees of freedom associated with the boundary component of the particle like 3-surface and the interpretation as a thermal contribution is possible but not necessary.

Third contribution comes from the deviation of ground state conformal weight from negative half odd integer since it cannot be compensated by conformal weight of Super Virasoro generators.

The fourth contribution consists corresponds to the interactions between partonic 2-surfaces and are important inside hadrons.

The lowest order contributions to the charged lepton masses are predicted correctly and information about the yet non-calculable second order corrections is obtained by requiring that the mass ratios are reproduced exactly. Since the contribution from modular degrees of freedom dominates for higher generations the successful predictions for fermion masses give strong support for the topological explanation of the family replication phenomenon.

The prediction or quark masses is more difficult since even the deduction of even the p-adic length scale determining the masses of u, d, and s is a non-trivial task. Second difficulty is related to the topological mixing of quarks. Somewhat surprisingly, the model for U and D matrices constructed for a decade ago predicts realistic quark mass spectrum although the new mass formula is based on different assumptions and different identification of p-adic mass scales. Top quark mass is completely exceptional since it can be deduced reliably from experiment. The recent experimental value of top quark mass agrees with TGD prediction.

Graviton, photon and gluons are predicted to be exactly massless. Contrary to the long held beliefs, intermediate boson mass scale is predicted to be 20-30 per cent too high if p-adic thermodynamics with temperature $T_p = 1$ is solely responsible for gauge boson masses. $T_p = 1/2$ predicts completely negligible masses. The resolution of difficulty is the contribution from ground state conformal weight. Its precise calculation is in principle possible but require the knowledge of the lowest eigenvalue of the modified Dirac operator.

3.1.3 Exotic states

The physical consequences of the exotic light leptons, quarks, and bosons are considered in the chapter devoted to the New Physics [F5]. Here it only suffices to make a short summary.

The possibility of exotic states poses a serious problem. The assumption that only free many fermion states are possible eliminates a huge number of exotics and only the degrees of freedom associated with ground states remain. Coset construction implying duality between SCV and $SKMV$ algebras removes a huge number of exotic states. The strongest form of the duality holds is that one can use either SC or SKM to construct states. In this case the situation reduces more or less to that in super string models in algebraic sense. Also other kinds of exotic states are predicted.

3.2 Various contributions to the particle masses

In the sequel various contributions to the mass squared are discussed.

3.2.1 General mass squared formula

The thermal independence of Super Virasoro and modular degrees of freedom implies that mass squared for elementary particle is the sum of Super Virasoro, modular and renormalization correction contributions:

$$M^2 = M^2(color) + M^2(SV) + M^2(mod) + M^2(ren) \ . \tag{3.2.1}$$

At this stage the small second order renormalization correction is not yet calculable but can be deduced from the known particle masses by comparing them to the predictions of the theory.

3.2.2 Color contribution to the mass squared

The mass squared contains a non-thermal color contribution to the ground state conformal weight coming from the mass squared of CP_2 spinor harmonic. The color contribution is an integer multiple of $m_0^2/3$, where $m_0^2 = 2\Lambda$ denotes the 'cosmological constant' of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$).

The color contribution to the p-adic mass squared is integer valued only if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since the simplest form of the canonical identification does not commute with a division by integer. More precisely, the image of number xp in canonical identification has a value of order 1 when x is a non-trivial rational number whereas for $x = np$ the value is n/p and extremely is small for physically interesting primes.

The choice of the p-adic mass squared unit are no effects on zeroth order contribution which must vanish for light states: this requirement eliminates quark and lepton states for which the CP_2 contribution to the mass squared is not integer valued using m_0^2 as a unit. There can be a dramatic effect on the first order contribution. The mass squared $m^2 = p/3$ using $m_0^2/3$ means that the particle is light. The mass squared becomes $m^2 = p/3$ when m_0^2 is used as a unit and the particle has mass of order 10^{-4} Planck masses. In the case of W and Z^0 bosons this problem is actually encountered. For light states using $m_0^2/3$ as a unit only the second order contribution to the mass squared is affected by this choice.

3.2.3 Modular contribution to the mass of elementary particle

The general form of the modular contribution is derivable from p-adic partition function for conformally invariant degrees of freedom associated with the boundary components. The general form of the vacuum functionals as modular invariant functions of Teichmuller parameters was derived in [F1] and the square of the elementary particle vacuum functional can be identified as a partition function. Even theta functions serve as basic building blocks and the functionals are proportional to the product of all even theta functions and their complex conjugates. The number of theta functions for genus $g > 0$ is given by

$$N(g) = 2^{g-1}(2^g + 1) . \tag{3.2.2}$$

One has $N(1) = 3$ for muon and $N(2) = 10$ for τ.

1. Single theta function is analogous to a partition function. This implies that the modular contribution to the mass squared must be proportional to $2N(g)$. The factor two follows from the presence of both theta functions and their conjugates in the partition function.

2. The factorization properties of the vacuum functionals imply that handles behave effectively as particles. For example, at the limit, when the surface splits into two pieces with g_1 and $g - g_1$ handles, the partition function reduces to a product of g_1 and $g - g_1$ partition functions. This implies that the contribution to the mass squared is proportional to the genus of the surface. Altogether one has

$$\begin{aligned} M^2(mod, g) &= 2k(mod)N(g)g\frac{m_0^2}{p} , \\ k(mod) &= 1 . \end{aligned} \tag{3.2.3}$$

   Here $k(mod)$ is some integer valued constant (in order to avoid ultra heavy mass) to be determined. $k(mod) = 1$ turns out to be the correct choice for this parameter.

Summarizing, the real counterpart of the modular contribution to the mass of a particle belonging to $g + 1$:th generation reads as

$$\begin{aligned} M^2(mod) &= 0 \; for \; e, \nu_e, u, d , \\ M^2(mod) &= 9\frac{m_0^2}{p(X))} \; for \; X = \mu, \nu_\mu, c, s , \\ M^2(mod) &= 60\frac{m_0^2}{p(X)} \; for \; X = \tau, \nu_\tau, t, b . \end{aligned} \tag{3.2.4}$$

The requirement that hadronic mass spectrum and CKM matrix are sensible however forces the modular contribution to be the same for quarks, leptons and bosons. The higher order modular contributions to the mass squared are completely negligible if the degeneracy of massless state is $D(0, mod, g) = 1$ in the modular degrees of freedom as is in fact required by $k(mod) = 1$.

3.2.4 Thermal contribution to the mass squared

One can deduce the value of the thermal mass squared in order $O(p^2)$ (an excellent approximation) using the general mass formula given by p-adic thermodynamics. Assuming maximal p-adic temperature $T_p = 1$ one has

$$\begin{aligned} M^2 &= k(sp + Xp^2 + O(p^3)) \ , \\ s_\Delta &= \frac{D(\Delta+1)}{D(\Delta)} \ , \\ X_\Delta &= 2\frac{D(\Delta+2)}{D(\Delta)} - \frac{D^2(\Delta+1)}{D^2(\Delta)} \ , \\ k &= 1 \ . \end{aligned} \quad (3.2.5)$$

Δ is the conformal weight of the operator creating massless state from the ground state.

The ratios $r_n = D(n+1)/D(n)$ allowing to deduce the values of s and X have been deduced from p-adic thermodynamics in [F2]. Light state is obtained only provided $r(\Delta)$ is an integer. The remarkable result is that for lowest lying states this is the case. For instance, for Ramond representations the values of r_n are given by

$$(r_0, r_1, r_2, r_3) = (8, 5, 4, \frac{55}{16}) \ . \quad (3.2.6)$$

The values of s and X are

$$\begin{aligned} (s_0, s_1, s_2) &= (8, 5, 4) \ , \\ (X_0, X_1, X_2) &= (16, 15, 11 + 1/2)) \ . \end{aligned} \quad (3.2.7)$$

The result means that second order contribution is extremely small for quarks and charged leptons having $\Delta < 2$. For neutrinos having $\Delta = 2$ the second order contribution is non-vanishing.

3.2.5 The contribution from the deviation of ground state conformal weight from negative integer

The interpretation inspired by p-adic mass calculations is that the squares λ_i^2 of the eigenvalues of the modified Dirac operator D_{C-S} correspond to the conformal weights of ground states. Another natural physical interpretation of λ is as an analog of the Higgs vacuum expectation. The instability of the Higgs=0 phase would corresponds to the fact that $\lambda = 0$ mode is not localized to any region in which ew magnetic field or induced Kähler field is non-vanishing. A good guess is that induced Kähler magnetic field B_K dictates the magnitude of the eigenvalues which is thus of order $h_0 = \sqrt{B_K}R$, R CP_2 radius. The first guess is that eigenvalues in the first approximation come as $(n + 1/2)h_0$. Each region where induced Kähler field is non-vanishing would correspond to different scale mass scale h_0.

1. The vacuum expectation value of Higgs is only proportional to an eigenvalue λ, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and antifermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this).

2. Physical considerations suggest that the vacuum expectation of Higgs field corresponds to a particular eigenvalue λ_i of modified Dirac operator so that the eigenvalues λ_i would define TGD counterparts for the minima of Higgs potential. Since the vacuum expectation of Higgs corresponds to a condensate of wormhole contacts giving rise to a coherent state, the vacuum expectation cannot be present for topologically condensed CP_2 type vacuum extremals representing fermions since only single wormhole throat is involved. This raises a hen-egg question

about whether Higgs contributes to the mass or whether Higgs is only a correlate for massivation having description using more profound concepts. From TGD point of view the most elegant option is that Higgs does not give rise to mass but Higgs vacuum expectation value accompanies bosonic states and is naturally proportional to λ_i. With this interpretation λ_i could give a contribution to both fermionic and bosonic masses.

3. If the coset construction for super-symplectic and super Kac-Moody algebra implying Equivalence Principle is accepted, one encounters what looks like a problem. p-Adic mass calculations require negative ground state conformal weight compensated by Super Virasoro generators in order to obtain massless states. The tachyonicity of the ground states would mean a close analogy with both string models and Higgs mechanism. λ_i^2 is very natural candidate for the ground state conformal weights identified but would have wrong sign if the effective metric of X_l^3 defined by the inner products $T_K^{k\alpha}T_K^{l\beta}h_{kl}$ of the Kähler energy momentum tensor $T^{k\alpha} = h^{kl}\partial L_K/\partial h_\alpha^l$ and appearing in the modified Dirac operator D_K has Minkowskian signature.

   The situation changes if the effective metric has Euclidian signature. This seems to be the case for the light-like surfaces assignable to the known extremals such as MEs and cosmic strings. In this kind of situation light-like coordinate possesses Euclidian signature and real eigenvalue spectrum is replaced with a purely imaginary one. Since Dirac operator is in question both signs for eigenvalues are possible and one obtains both exponentially increasing and decreasing solutions. This is essential for having solutions extending from the past end of X_l^3 to its future end. Non-unitary time evolution is possible because X_l^3 does not strictly speaking represent the time evolution of 2-D dynamical object but actual dynamical objects (by light-likeness both interpretation as dynamical evolution and dynamical object are present). The Euclidian signature of the effective metric would be a direct analog for the tachyonicity of the Higgs in unstable minimum and the generation of Higgs vacuum expectation would correspond to the compensation of ground state conformal weight by conformal weights of Super Virasoro generators.

4. In accordance with this λ_i^2 would give constant contribution to the ground state conformal weight. What contributes to the thermal mass squared is the deviation of the ground state conformal weight from half-odd integer since the negative integer part of the total conformal weight can be compensated by applying Virasoro generators to the ground state. The first guess motivated by cyclotron energy analogy is that the lowest conformal weights are of form $h_c = \lambda_i^2 = -1/2 - n + \Delta h_c$ so that lowest ground state conformal weight would be $h_c = -1/2$ in the first approximation. The negative integer part of the net conformal weight can be canceled using Super Virasoro generators but Δh_c would give to mass squared a contribution analogous to Higgs contribution. The mapping of the real ground state conformal weight to a p-adic number by canonical identification involves some delicacies.

5. p-Adic mass calculations are consistent with the assumption that Higgs type contribution is vanishing (that is small) for fermions and dominates for gauge bosons. This requires that the deviation of λ_i^2 with smallest magnitude from half-odd integer value in the case of fermions is considerably smaller than in the case of gauge bosons in the scale defined by p-adic mass scale $1/L(k)$ in question. Somehow this difference could relate to the fact that bosons correspond to pairs of wormhole throats.

3.2.6 General mass formula for Ramond representations

By taking the modular contribution from the boundaries into account the general p-adic mass formulas for the Ramond type states read for states for which the color contribution to the conformal weight is integer valued as

$$
\begin{aligned}
\frac{m^2(\Delta = 0)}{m_0^2} &= (8 + n(g))p + Yp^2 \ , \\
\frac{m^2(\Delta = 1)}{m_0^2} &= (5 + n(g)p + Yp^2 \ , \\
\frac{m^2(\Delta = 2)}{m_0^2} &= (4 + n(g))p + (Y + \frac{23}{2})p^2 \ , \\
n(g) &= 3g \cdot 2^{g-1}(2^g + 1) \ . \qquad (3.2.8)
\end{aligned}
$$

Here Δ denotes the conformal weight of the operators creating massless states from the ground state and g denotes the genus of the boundary component. The values of $n(g)$ for the three lowest generations are $n(0) = 0$, $n(1) = 9$ and $n(2) = 60$. The value of second order thermal contribution is nontrivial for neutrinos only. The value of the rational number Y can, which corresponds to the renormalization correction to the mass, can be determined using experimental inputs.

Using m_0^2 as a unit, the expression for the mass of a Ramond type state reads in terms of the electron mass as

$$
\begin{aligned}
M(\Delta, g, p)_R &= K(\Delta, g, p)\sqrt{\frac{M_{127}}{p}}m_e \\
K(0, g, p) &= \sqrt{\frac{n(g) + 8 + Y_R}{X}} \\
K(1, g, p) &= \sqrt{\frac{n(g) + 5 + Y_R}{X}} \\
K(2, g, p) &= \sqrt{\frac{n(g) + 4 + Y_R}{X}} \ , \\
X &= \sqrt{5 + Y(e)_R} \ . \qquad (3.2.9)
\end{aligned}
$$

Y can be assumed to depend on the electromagnetic charge and color representation of the state and is therefore same for all fermion families. Mathematica provides modules for calculating the real counterpart of the second order contribution and for finding realistic values of Y.

3.2.7 General mass formulas for NS representations

Using $m_0^2/3$ as a unit, the expression for the mass of a light NS type state for $T_p = 1$ ad $k_B = 1$ reads in terms of the electron mass as

$$
\begin{aligned}
M(\Delta, g, p, N)_R &= K(\Delta, g, p, N)\sqrt{\frac{M_{127}}{p}}m_e \\
K(0, g, p, 1) &= \sqrt{\frac{n(g) + Y_R}{X}} \ , \\
K(0, g, p, 2) &= \sqrt{\frac{n(g) + 1 + Y_R}{X}} \ , \\
K(1, g, p, 3) &= \sqrt{\frac{n(g) + 3 + Y_R}{X}} \ , \\
K(2, g, p, 4) &= \sqrt{\frac{n(g) + 5 + Y_R}{X}} \ , \\
K(2, g, p, 5) &= \sqrt{\frac{n(g) + 10 + Y_R}{X}} \ , \\
X &= \sqrt{5 + Y(e)_R} \ . \qquad (3.2.10)
\end{aligned}
$$

Here N is the number of the 'active' NS sectors (sectors for which the conformal weight of the massless state is non-vanishing). Y denotes the renormalization correction to the boson mass and in general depends on the electro-weak and color quantum numbers of the boson.

The thermal contribution to the mass of W boson is too large by roughly a factor $\sqrt{3}$ for $T_p = 1$. Hence $T_p = 1/2$ must hold true for gauge bosons and their masses must have a non-thermal origin perhaps analogous to Higgs mechanism. Alternatively, the non-covariant constancy of charge matrices could induce the boson mass [F2].

It is interesting to notice that the minimum mass squared for gauge boson corresponds to the p-adic mass unit $M^2 = m_0^2 p/3$ and this just what is needed in the case of W boson. This forces to ask whether $m_0^2/3$ is the correct choice for the mass squared unit so that non-thermally induced W mass would be the minimal $m_W^2 = p$ in the lowest order. This choice would mean the replacement

$$Y_R \rightarrow \frac{(3Y)_R}{3}$$

in the preceding formulas and would affect only neutrino mass in the fermionic sector. $m_0^2/3$ option is excluded by charged lepton mass calculation. This point will be discussed later.

3.2.8 Primary condensation levels from p-adic length scale hypothesis

p-Adic length scale hypothesis states that the primary condensation levels correspond to primes near prime powers of two $p \simeq 2^k$, k integer with prime values preferred. Black hole-elementary particle analogy [E5] suggests a generalization of this hypothesis by allowing k to be a power of prime. The general number theoretical vision discussed in [E1] provides a first principle justification for p-adic length scale hypothesis in its most general form. The best fit for the neutrino mass squared differences is obtained for $k = 13^2 = 169$ so that the generalization of the hypothesis might be necessary.

A particle primarily condensed on the level k can suffer secondary condensation on a level with the same value of k: for instance, electron ($k = 127$) suffers secondary condensation on $k = 127$ level. u, d, s quarks ($k = 107$) suffer secondary condensation on nuclear space-time sheet having $k = 113$). All quarks feed their color gauge fluxes at $k = 107$ space-time sheet. There is no deep reason forbidding the condensation of p on p. Primary and secondary condensation levels could also correspond to different but nearly identical values of p with the same value of k.

3.3 Fermion masses

In the earlier model the coefficient of $M^2 = kL_0$ had to be assumed to be different for various particle states. $k = 1$ was assumed for bosons and leptons and $k = 2/3$ for quarks. The fact that $k = 1$ holds true for all particles in the model including also super-canonical invariance forces to modify the earlier construction of quark states. This turns out to be possible without affecting the earlier p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (h_{gr} can be negative). The structure of lepton and quark states in color degrees of freedom was discussed in [F2].

3.3.1 Charged lepton mass ratios

The overall mass scale for lepton and quark masses is determined by the condensation level given by prime $p \simeq 2^k$, k prime by length scale hypothesis. For charged leptons k must correspond to $k = 127$ for electron, $k = 113$ for muon and $k = 107$ for τ. For muon $p = 2^{113} - 1 - 4 * 378$ is assumed (smallest prime below 2^{113} allowing $\sqrt{2}$ but not $\sqrt{3}$). So called Gaussian primes are to complex integers what primes are for the ordinary integers and the Gaussian counterparts of the Mersenne primes are Gaussian primes of form $(1 \pm i)^k - 1$. Rather interestingly, $k = 113$ corresponds to a Gaussian Mersenne so that all charged leptons correspond to generalized Mersenne primes.

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \ mod \ 3 \neq 2 \ \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p, p) representations with $p \geq 1$ whereas charged leptons correspond to $(p, p + 3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2+2p)/3$, $p \geq 1$, for neutrinos and $p = 1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2 + 5p + 6)/3$ gives $h_c = 2$ for $p = 0$ (decuplet). In both cases super-canonical operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-canonical conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-canonical conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-canonical conformal weights contains both octet and decuplet so that singlets are obtained. What strengthens the hopes that the construction is not adhoc is that the same operator appears in the construction of quark states too.

Using CP_2 mass scale m_0^2 [F2] as a p-adic unit, the mass formulas for the charged leptons read as

$$\begin{aligned}
M^2(L) &= A(\nu)\frac{m_0^2}{p(L)} \ , \\
A(e) &= 5 + X(p(e)) \ , \\
A(\mu) &= 14 + X(p(\mu)) \ , \\
A(\tau) &= 65 + X(p(\tau)) \ .
\end{aligned} \tag{3.3.1}$$

$X(\cdot)$ corresponds to the yet unknown second order corrections to the mass squared.

The following table gives the basic parameters as determined from the mass of electron for some values of Y_e. The mass of top quark favors as maximal value of CP_2 mass which corresponds to $Y_e = 0$.

Y_e	0	.5	.7798
$(m_0/m_{Pl}) \times 10^3$	.2437	.2323	.2266
$K \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G}) \times 10^{-4}$	3.1580	3.3122	3.3954

Table 1. Table gives the values of CP_2 mass m_0 using Planck mass $m_{Pl} = 1/\sqrt{G}$ as unit, the ratio $K = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$.

The following table lists the lower and upper bounds for the charged lepton mass ratios obtained by taking second order contribution to zero or allowing it to have maximum possible value. The values of lepton masses are $m_e = .510999$ MeV, $m_\mu = 105.76583$ MeV, $m_\tau = 1775$ MeV.

$$\begin{aligned}
\frac{m(\mu)_+}{m(\mu)} &= \sqrt{\frac{15}{5}2^7\frac{m_e}{(\mu)}} \simeq 1.0722 \ , \\
\frac{m(\mu)_-}{m(\mu)} &= \sqrt{\frac{14}{6}2^7\frac{m_e}{m(\mu)}} \simeq 0.9456 \ , \\
\frac{m(\tau)_+}{m(\tau)} &= \sqrt{\frac{66}{5}2^{10}\frac{m_e}{m(\tau)}} \simeq 1.0710 \ , \\
\frac{m(\tau)_-}{m(\tau)} &= \sqrt{\frac{65}{6}2^{10}\frac{m_e}{m(\tau)}} \simeq .9703 \ .
\end{aligned} \tag{3.3.2}$$

For the maximal value of CP_2 mass the predictions for the mass ratio are systematically too large by a few per cent. From the formulas above it is clear that the second order corrections to mass squared can be such that correct masses result.

τ mass is least sensitive to $X(p(e)) \equiv Y_e$ and the maximum value of $Y_e \equiv Y_{e,max}$ consistent with τ mass corresponds to $Y_{e,max} = .7357$ and $Y_\tau = 1$. This means that the CP_2 mass is at least a fraction .9337 of its maximal value. If Y_L is same for all charged leptons and has the maximal value $Y_{e,max} = .7357$, the predictions for the mass ratios are

$$\begin{aligned} \frac{m(\mu)_{pr}}{m(\mu)} &= \sqrt{\frac{14+Y_{e,max}}{5+Y_{e,max}}} \times 2^7 \frac{m_e}{m(\mu)} \simeq .9922 \ , \\ \frac{m(\tau)_{pr}}{m(\tau)} &= \sqrt{\frac{65+Y_{e,max}}{5+Y_e(max}} \times 2^{10} \frac{m_e}{m(\tau)} \simeq .9980 \ . \end{aligned} \tag{3.3.3}$$

The error is .8 per cent *resp.* .2 per cent for muon *resp.* τ.

The argument leading to estimate for the modular contribution to the mass squared [F2] leaves two options for the coefficient of the modular contribution for $g = 2$ fermions: the value of coefficient is either $X = g$ for $g \leq 1$, $X = 3g - 3$ for $g \geq 2$ or $X = g$ always. For $g = 2$ the predictions are $X = 2$ and $X = 3$ in the two cases. The option $X = 3$ allows slightly larger maximal value of Y_e equal to $Y^{1)}_{e,max} = Y_{e,max} + (5 + Y_{e,max})/66$.

3.3.2 Neutrino masses

The estimation of neutrino masses is difficult at this stage since the prediction of the primary condensation level is not yet possible and neutrino mixing cannot yet be predicted from the basic principles. The cosmological bounds for neutrino masses however help to put upper bounds on the masses. If one takes seriously the LSND data on neutrino mass measurement of [16, 17] and the explanation of the atmospheric ν-deficit in terms of $\nu_\mu - \nu_\tau$ mixing [32, 33] one can deduce that the most plausible condensation level of μ and τ neutrinos is $k = 167$ or $k = 13^2 = 169$ allowed by the more general form of the p-adic length scale hypothesis suggested by the blackhole-elementary particle analogy. One can also deduce information about the mixing matrix associated with the neutrinos so that mass predictions become rather precise. In particular, the mass splitting of μ and τ neutrinos is predicted correctly if one assumes that the mixing matrix is a rational unitary matrix.

Super Virasoro contribution

Using $m_0^2/3$ as a p-adic unit, the expression for the Super Virasoro contribution to the mass squared of neutrinos is given by the formula

$$\begin{aligned} M^2(SV) &= (s + (3Yp)_R/3)\frac{m_0^2}{p} \ , \\ s &= 4 \text{ or } 5 \ , \\ Y &= \frac{23}{2} + Y_1 \ , \end{aligned} \tag{3.3.4}$$

where m_0^2 is universal mass scale. One can consider two possible identifications of neutrinos corresponding to $s(\nu) = 4$ with $\Delta = 2$ and $s(\nu) = 5$ with $\Delta = 1$. The requirement that CKM matrix is sensible forces the asymmetric scenario in which quarks and, by symmetry, also leptons correspond to lowest possible excitation so that one must have $s(\nu) = 4$. Y_1 represents second order contribution to the neutrino mass coming from renormalization effects coming from self energy diagrams involving intermediate gauge bosons. Physical intuition suggest that this contribution is very small so that the precise measurement of the neutrino masses should give an excellent test for the theory.

With the above described assumptions and for $s = 4$, one has the following mass formula for neutrinos

$$\begin{aligned} M^2(\nu) &= A(\nu)\frac{m_0^2}{p(\nu))} \ , \\ A(\nu_e) &= 4+\frac{(3Y(p(\nu_e)))_R}{3} \ , \\ A(\nu_\mu) &= 13+\frac{(3Y(p(\nu_\mu)))_R}{3} \ , \\ A(\nu_\tau) &= 64+\frac{(3Y(p(\nu_\tau)))_R}{3} \ , \\ 3Y &\simeq \frac{1}{2} \ . \end{aligned} \tag{3.3.5}$$

The predictions must be consistent with the recent upper bounds [75] of order 10 eV, 270 keV and 0.3 MeV for ν_e, ν_μ and ν_τ respectively. The recently reported results of LSND measurement [17] for $\nu_e -> \nu_\mu$ mixing gives string limits for $\Delta m^2(\nu_e,\nu_\mu)$ and the parameter $sin^2(2\theta)$ characterizing the mixing: the limits are given in the figure 30 of [17]. The results suggests that the masses of both electron and muon neutrinos are below 5 eV and that mass squared difference $\Delta m^2 = m^2(\nu_\mu) - m^2(\nu_e)$ is between $.25 - 25\ eV^2$. The simplest possibility is that ν_μ and ν_e have common condensation level (in analogy with d and s quarks). There are three candidates for the primary condensation level: namely $k = 163, 167$ and $k = 169$. The p-adic prime associated with the primary condensation level is assumed to be the nearest prime below 2^k allowing p-adic $\sqrt{2}$ but not $\sqrt{3}$ and satisfying $p\ mod\ 4 = 3$. The following table gives the values of various parameters and unmixed neutrino masses in various cases of interest.

k	p	$(3Y)_R/3$	$m(\nu_e)/eV$	$m(\nu_\mu)/eV$	$m(\nu_\tau)/eV$
163	$2^{163} - 4*144 - 1$	1.36	1.78	3.16	6.98
167	$2^{167} - 4*144 - 1$	.34	.45	.79	1.75
169	$2^{169} - 4*210 - 1$	.17	.22	.40	.87

Could neutrino topologically condense also in other p-adic length scales than $k = 169$?

One must keep mind open for the possibility that there are several p-adic length scales at which neutrinos can condense topologically. In fact, the quantum model for hearing [M6] requires that both $k = 169$ and $k = 151$ correspond to p-adic length scales at which neutrinos can condense topologically. Rather interestingly, the ratio for the mass scales of $k = 151$ and $k = 169$ neutrinos equals to 512 and is same as the ratio of the mass scales of the ordinary $k = 107$ hadron physics and $k = 89$ hadronic physics predicted by TGD.

In fact, all intermediate p-adic length scales $k = 151, 157, 163, 167$ could correspond to metastable neutrino states. The point is that these p-adic lengths scales are number theoretically completely exceptional in the sense that there exist Gaussian Mersenne $2^k \pm i$ (prime in the ring of complex integers) for all these values of k. Since charged leptons, atomic nuclei ($k = 113$) , hadrons and intermediate gauge bosons correspond to ordinary or Gaussian Mersennes, it would not be surprising if the biologically important Gaussian Mersennes would correspond to length scales giving rise to metastable neutrino states. Of course, one can keep mind open for the possibility that $k = 167$ rather than $k = 13^2 = 169$ is the length scale defining the stable neutrino physics.

Neutrino mixing

Consider next the neutrino mixing. A quite general form of the neutrino mixing matrix D given by the table below will be considered.

	ν_e	ν_μ	ν_τ
ν_e	c_1	s_1c_3	s_1s_3
ν_μ	$-s_1c_2$	$c_1c_2c_3 - s_2s_3exp(i\delta)$	$c_1c_2s_3 + s_2c_3exp(i\delta)$
ν_τ	$-s_1s_2$	$c_1s_2c_3 + c_2s_3exp(i\delta)$	$c_1s_2s_3 - c_2c_3exp(i\delta)$

Physical intuition suggests that the angle δ related to CP breaking is small and will be assumed to be vanishing. Topological mixing is active only in modular degrees of freedom and one obtains for the first order terms of mixed masses the expressions

$$\begin{aligned} s(\nu_e) &= 4+9|U_{12}|^2+60|U_{13}|^2 = 4+n_1 \ , \\ s(\nu_\mu) &= 4+9|U_{22}|^2+60|U_{23}|^2 = 4+n_2 \ , \\ s(\nu_\tau) &= 4+9|U_{32}|^2+60|U_{33}|^2 = 4+n_3 \ . \end{aligned} \tag{3.3.6}$$

The requirement that resulting masses are not ultraheavy implies that $s(\nu)$ must be small integers. The condition $n_1+n_2+n_3=69$ follows from unitarity. The simplest possibility is that the mixing matrix is a rational unitary matrix. The same ansatz was used successfully to deduce information about the mixing matrices of quarks. If neutrinos are condensed on the same condensation level, rationality implies that $\nu_\mu-\nu_\tau$ mass squared difference must come from the first order contribution to the mass squared and is therefore quantized and bounded from below.

The first piece of information is the atmospheric ν_μ/ν_e ratio, which is roughly by a factor 2 smaller than predicted by standard model [32]. A possible explanation is the CKM mixing of muon neutrino with τ-neutrino, whereas the mixing with electron neutrino is excluded as an explanation. The latest results from Kamiokande [32] are in accordance with the mixing $m^2(\nu_\tau)-m^2(\nu_\mu)\simeq 1.6\cdot 10^{-2}\ eV^2$ and mixing angle $sin^2(2\theta)=1.0$: also the zenith angle dependence of the ratio is in accordance with the mixing interpretation. If mixing matrix is assumed to be rational then only $k=169$ condensation level is allowed for ν_μ and ν_τ. For this level $\nu_\mu-\nu_\tau$ mass squared difference turns out to be $\Delta m^2\simeq 10^{-2}\ eV^2$ for $\Delta s\equiv s(\nu_\tau)-s(\nu_\mu)=1$, which is the only acceptable possibility and predicts $\nu_\mu-\nu_\tau$ mass squared difference correctly within experimental uncertainties! The fact that the predictions for mass squared differences are practically exact, provides a precision test for the rationality assumption.

What is measured in LSND experiment is the probability $P(t,E)$ that ν_μ transforms to ν_e in time t after its production in muon decay as a function of energy E of ν_μ. In the limit that ν_τ and ν_μ masses are identical, the expression of $P(t,E)$ is given by

$$\begin{aligned} P(t,E) &= sin^2(2\theta)sin^2(\frac{\Delta E t}{2}) \ , \\ sin^2(2\theta) &= 4c_1^2s_1^2c_2^2 \ , \end{aligned} \tag{3.3.7}$$

where ΔE is energy difference of ν_μ and ν_e neutrinos and t denotes time. LSND experiment gives stringent conditions on the value of $\sin^2(2\theta)$ as the figure 30 of [17] shows. In particular, it seems that $sin^2(2\theta)$ must be considerably below 10^{-1} and this implies that s_1^2 must be small enough.

The study of the mass formulas shows that the only possibility to satisfy the constraints for the mass squared and $sin^2(2\theta)$ given by LSND experiment is to assume that the mixing of the electron neutrino with the tau neutrino is much larger than its mixing with the muon neutrino. This means that s_3 is quite near to unity. At the limit $s_3=1$ one obtains the following (nonrational) solution of the mass squared conditions for $n_3=n_2+1$ (forced by the atmospheric neutrino data)

$$\begin{aligned} s_1^2 &= \frac{69-2n_2-1}{60} \ , \\ c_2^2 &= \frac{n_2-9}{2n_2-17} \ , \\ sin^2(2\theta) &= \frac{4(n_2-9)}{51}\frac{(34-n_2)(n_2-4)}{30^2} \ , \\ s(\nu_\mu)-s(\nu_e) &= 3n_2-68 \ . \end{aligned} \tag{3.3.8}$$

The study of the LSND data shows that there is only one acceptable solution to the conditions obtained by assuming maximal mass squared difference for ν_e and ν_μ

$$
\begin{aligned}
n_1 &= 2 \;\; n_2 = 33 \;\; n_3 = 34 \;\; , \\
s_1^2 &= \frac{1}{30} \;\; c_2^2 = \frac{24}{49} \;\; , \\
sin^2(2\theta) &= \frac{24}{49}\frac{2}{15}\frac{29}{30} \simeq .0631 \;\; , \\
s(\nu_\mu) - s(\nu_e)) &= 31 \leftrightarrow .32 \; eV^2 \;\; .
\end{aligned}
\tag{3.3.9}
$$

That c_2^2 is near 1/2 is not surprise taking into account the almost mass degeneracy of ν_{mu} and ν_τ. From the figure 30 of [17] it is clear that this solution belongs to 90 per cent likelihood region of LSND experiment but $sin^2(2\theta)$ is about two times larger than the value allowed by Bugey reactor experiment. The study of various constraints given in [17] shows that the solution is consistent with bounds from all other experiments. If one assumes that $k > 169$ for ν_e $\nu_\mu - \nu_e$ mass difference increases, implying slightly poorer consistency with LSND data.

There are reasons to hope that the actual rational solution can be regarded as a small deformation of this solution obtained by assuming that c_3 is non-vanishing. $s_1^2 = \frac{69-2n_2-1}{60-51c_3^2}$ increases in the deformation by $O(c_3^2)$ term but if c_3 is positive the value of $c_2^2 \simeq \frac{24-102c_1^0c_2^0s_2^0c_3}{49} \sim \frac{24-61c_3}{49}$ decreases by $O(c_3)$ term so that it should be possible to reduce the value of $sin^2(2\theta)$. Consistency with Bugey reactor experiment requires $.030 \leq sin^2(2\theta) < .033$. $sin^2(2\theta) = .032$ is achieved for $s_1^2 \simeq .035$,$s_2^2 \simeq .51$ and $c_3^2 \simeq .068$. The construction of U and D matrices for quarks shows that very stringent number theoretic conditions are obtained and as in case of quarks it might be necessary to allow complex CP breaking phase in the mixing matrix. One might even hope that the solution to the conditions is unique.

For the minimal rational mixing one has $s(\nu_e) = 5$, $s(\nu_\mu) = 36$ and $s(\nu_\tau) = 37$ if unmixed ν_e corresponds to $s = 4$. For $s = 5$ first order contributions are shifted by one unit. The masses ($s = 4$ case) and mass squared differences are given by the following table.

k	$m(\nu_e)$	$m(\nu_\mu)$	$m(\nu_\tau)$	$\Delta m^2(\nu_\mu - \nu_e)$	$\Delta m^2(\nu_\tau - \nu_\mu)$
169	.27 eV	.66 eV	.67 eV	.32 eV^2	.01 eV^2

Predictions for neutrino masses and mass squared splittings for $k = 169$ case.

Evidence for the dynamical mass scale of neutrinos

In recent years (I am writing this towards the end of year 2004 and much later than previous lines) a great progress has been made in the understanding of neutrino masses and neutrino mixing. The pleasant news from TGD perspective is that there is a strong evidence that neutrino masses depend on environment [44]. In TGD framework this translates to the statement that neutrinos can suffer topological condensation in several p-adic length scales. Not only in the p-adic length scales suggested by the number theoretical considerations but also in longer length scales, as will be found.

The experiments giving information about mass squared differences can be divided into three categories [44].

i) There along baseline experiments, which include solar neutrino experiments [24], KamLAND [27], K2K [26], and SuperK [25] as well as earlier studies of solar neutrinos. These experiments see evidence for the neutrino mixing and involve significant propagation through dense matter. For the solar neutrinos and KamLAND the mass splittings are estimated to be of order $O(8\times 10^{-5})$ eV^2 or more cautiously 8×10^{-5} eV^2 $< \delta m^2 < 2\times 10^{-3}$ eV^2. For K2K and atmospheric neutrinos the mass splittings are of order $O(2\times 10^{-3})eV^2$ or more cautiously $\delta m^2 > 10^{-3}$eV^2. Thus the scale of mass splitting seems to be smaller for neutrinos in matter than in air, which would suggest that neutrinos able to propagate through a dense matter travel at space-time sheets corresponding to a larger p-adic length scale than in air.

ii) There are null short baseline experiments including CHOOZ, Bugey, and Palo Verde reactor experiments, and the higher energy CDHS, JARME, CHORUS, and NOMAD experiments, which involve muonic neutrinos (for references see [44]. No evidence for neutrino oscillations have been seen in these experiments.

iii) The results of LSND experiment[17] are consistent with oscillations with a mass splitting greater than $3 \times 10^{-2} eV^2$. LSND has been generally been interpreted as necessitating a mixing with sterile neutrino. If neutrino mass scale is dynamical, situation however changes.

If one assumes that the p-adic length scale for the space-time sheets at which neutrinos can propagate is different for matter and air, the situation changes. According to [44] a mass 3×10^{-2} eV in air could explain the atmospheric results whereas mass of of order .1 eV and $.07 eV^2 < \delta m^2 < .26 eV^2$ would explain the LSND result. These limits are of the same order as the order of magnitude predicted by $k = 169$ topological condensation.

Assuming that the scale of the mass splitting is proportional to the p-adic mass scale squared, one can consider candidates for the topological condensation levels involved.

1. Suppose that $k = 169 = 13^2$ is indeed the condensation level for LSND neutrinos. $k = 173$ would predict $m_{\nu_e} \sim 7 \times 10^{-2}$ eV and $\delta m^2 \sim .02$ eV^2. This could correspond to the masses of neutrinos propagating through air. For $k = 179$ one has $m_{\nu_e} \sim .8 \times 10^{-2}$ eV and $\delta m^2 \sim 3 \times 10^{-4}$ eV^2 which could be associated with solar neutrinos and KamLAND neutrinos.

2. The primes $k = 157, 163, 167$ associated with Gaussian Mersennes would give $\delta m^2(157) = 2^6 \delta m^2(163) = 2^{10} \delta m^2(167) = 2^{12} \delta m^2(169)$ and mass scales $m(157) \sim 22.8$ eV, $m(163) \sim 3.6$ eV, $m(167) \sim .54$ eV. These mass scales are unrealistic or propagating neutrinos. The interpretation consistent with TGD inspired model of condensed matter in which neutrinos screen the classical Z^0 force generated by nucleons would be that condensed matter neutrinos are confined inside these space-time sheets whereas the neutrinos able to propagate through condensed matter travel along $k > 167$ space-time sheets.

The results of MiniBooNE group as a support for the energy dependence of p-adic mass scale of neutrino

The basic prediction of TGD is that neutrino mass scale can depend on neutrino energy and the experimental determinations of neutrino mixing parameters support this prediction. The newest results (11 April 2007) about neutrino oscillations come from MiniBooNE group which has published its first findings [19] concerning neutrino oscillations in the mass range studied in LSND experiments [18].

1. The motivation for MiniBooNE

Neutrino oscillations are not well-understood. Three experiments LSND, atmospheric neutrinos, and solar neutrinos show oscillations but in widely different mass regions (1 eV^2 , 3×10^{-3} eV^2, and 8×10^{-5} eV^2).

In TGD framework the explanation would be that neutrinos can appear in several p-adically scaled up variants with different mass scales and therefore different scales for the differences Δm^2 for neutrino masses so that one should not try to try to explain the results of these experiments using single neutrino mass scale. In single-sheeted space-time it is very difficult to imagine that neutrino mass scale would depend on neutrino energy since neutrinos interact so extremely weakly with matter. The best known attempt to assign single mass to all neutrinos has been based on the use of so called sterile neutrinos which do not have electro-weak couplings. This approach is an ad hoc trick and rather ugly mathematically and excluded by the results of MiniBooNE experiments.

2. The result of MiniBooNE experiment

The purpose of the MiniBooNE experiment was to check whether LSND result $\Delta m^2 = 1 eV^2$ is genuine. The group used muon neutrino beam and looked whether the transformations of muonic neutrinos to electron neutrinos occur in the mass squared region $\Delta m^2 \simeq 1$ eV^2. No such transitions were found but there was evidence for transformations at low neutrino energies.

What looks first as an over-diplomatic formulation of the result was *MiniBooNE researchers showed conclusively that the LSND results could not be due to simple neutrino oscillation, a phenomenon in which one type of neutrino transforms into another type and back again.* rather than direct refutation of LSND results.

3. LSND and MiniBooNE are consistent in TGD Universe

The habitant of the many-sheeted space-time would not regard the previous statement as a mere diplomatic use of language. It is quite possible that neutrinos studied in MiniBooNE have suffered topological condensation at different space-time sheet than those in LSND if they are in different energy range (the preferred rest system fixed by the space-time sheet of the laboratory or Earth). To see whether this is the case let us look more carefully the experimental arrangements.

1. In LSND experiment 800 MeV proton beam entering in water target and the muon neutrinos resulted in the decay of produced pions. Muonic neutrinos had energies in 60-200 MeV range [18].

2. In MiniBooNE experiment [19] 8 GeV muon beam entered Beryllium target and muon neutrinos resulted in the decay of resulting pions and kaons. The resulting muonic neutrinos had energies the range 300-1500 GeV to be compared with 60-200 MeV.

Let us try to make this more explicit.

1. Neutrino energy ranges are quite different so that the experiments need not be directly comparable. The mixing obeys the analog of Schrödinger equation for free particle with energy replaced with $\Delta m^2/E$, where E is neutrino energy. The mixing probability as a function of distance L from the source of muon neutrinos is in 2-component model given by

$$P = sin^2(\theta)sin^2(1.27\Delta m^2 L/E) \ .$$

   The characteristic length scale for mixing is $L = E/\Delta m^2$. If L is sufficiently small, the mixing is fifty-fifty already before the muon neutrinos enter the system, where the measurement is carried out and no mixing is detected. If L is considerably longer than the size of the measuring system, no mixing is observed either. Therefore the result can be understood if Δm^2 is much larger or much smaller than E/L, where L is the size of the measuring system and E is the typical neutrino energy.

2. MiniBooNE experiment found evidence for the appearance of electron neutrinos at low neutrino energies (below 500 MeV) which means direct support for the LSND findings and for the dependence of neutron mass scale on its energy relative to the rest system defined by the space-time sheet of laboratory.

3. Uncertainty Principle inspires the guess $L_p \propto 1/E$ implying $m_p \propto E$. Here E is the energy of the neutrino with respect to the rest system defined by the space-time sheet of the laboratory. Solar neutrinos indeed have the lowest energy (below 20 MeV) and the lowest value of Δm^2. However, atmospheric neutrinos have energies starting from few hundreds of MeV and $\Delta; m^2$ is by a factor of order 10 higher. This suggests that the the growth of Δm^2 with E^2 is slower than linear. It is perhaps not the energy alone which matters but the space-time sheet at which neutrinos topologically condense. For instance, MiniBooNE neutrinos above 500 MeV would topologically condense at space-time sheets for which the p-adic mass scale is higher than in LSND experiments and one would have $\Delta m^2 >> 1$ eV^2 implying maximal mixing in length scale much shorter than the size of experimental apparatus.

4. One could also argue that topological condensation occurs in condensed matter and that no topological condensation occurs for high enough neutrino energies so that neutrinos remain massless. One can even consider the possibility that the p-adic length scale L_p is proportional to E/m_0^2, where m_0 is proportional to the mass scale associated with non-relativistic neutrinos. The p-adic mass scale would obey $m_p \propto m_0^2/E$ so that the characteristic mixing length would be by a factor of order 100 longer in MiniBooNE experiment than in LSND.

Comments

Some comments on the proposed scenario are in order: some of the are written much later than the previous text.

1. Mass predictions are consistent with the bound $\Delta m(\nu_\mu, \nu_e) < 2\, eV^2$ coming from the requirement that neutrino mixing does not spoil the so called r-process producing heavy elements in Super Novae [28].

2. TGD neutrinos cannot solve the dark matter problem: the total neutrino mass required by the cold+hot dark matter models would be about 5 eV. In [D5] a model of galaxies based on string like objects of galaxy size and providing a more exotic source of dark matter, is discussed.

3. One could also consider the explanation of LSND data in terms of the interaction of ν_μ and nucleon via the exchange of $g=1$ W boson. The fraction of the reactions $\bar{\nu}_\mu + p \rightarrow e^+ + n$ is at low neutrino energies $P \sim \frac{m^4_W(g=0)}{m^4_W(g=1)} sin^2(\theta_c)$, where θ_c denotes Cabibbo angle. Even if the condensation level of $W(g=1)$ is $k=89$, the ratio is by a factor of order .05 too small to explain the average $\nu_\mu \rightarrow \nu_e$ transformation probability $P \simeq .003$ extracted from LSND data.

4. The predicted masses exclude MSW and vacuum oscillation solutions to the solar neutrino problem unless one assumes that several condensation levels and thus mass scales are possible for neutrinos. This is indeed suggested by the previous considerations.

3.3.3 Quark masses

The prediction or quark masses is more difficult due the facts that the deduction of even the p-adic length scale determining the masses of these quarks is a non-trivial task, and the original identification was indeed wrong. Second difficulty is related to the topological mixing of quarks. The new scenario leads to a unique identification of masses with top quark mass as an empirical input and the thermodynamical model of topological mixing as a new theoretical input. Also CKM matrix is predicted highly uniquely.

Basic mass formulas

By the earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U)=-1$ and $h_{gr}(D)=0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as a unit, color conformal weight $h_c \equiv m^2_{CP_2}$ is integer for $p \ mod \ = \pm 1$ for U type quark belonging to $(p+1,p)$ type representation and obeying $h_c(U)=(p^2+3p+2)/3$ and for $p \ mod \ 3 = 1$ for D type quark belonging $(p,p+2)$ type representation and obeying $h_c(D)=(p^2+4p+4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

In the recent case the minimal $p=1$ states correspond to $h_c(U)=2$ and $h_c(D)=3$. $h_{gr}(U)=-1$ and $h_{gr}(D)=0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This requires super-canonical operators O with a net conformal weight $h_{sc}=-3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-canonical operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc}=-3$ defines null state for SCV: this would also explain why h_{sc} would be same for all fermions.

Consider now the mass squared values for quarks. For $h(D)=0$ and $h(U)=-1$ and using $m_0^2/3$ as a unit the expression for the thermal contribution to the mass squared of quark is given by the formula

$$\begin{aligned} M^2 &= (s+X)\frac{m_0^2}{p} \ , \\ s(U) &= 5 \ , \ s(D)=8 \ , \\ X &\equiv \frac{(3Yp)_R}{3} \ , \end{aligned} \tag{3.3.10}$$

where the second order contribution Y corresponds to renormalization effects coming and depending on the isospin of the quark. When m_0^2 is used as a unit X is replaced by $X=(Y_p)_R$.

With the above described assumptions one has the following mass formula for quarks

$$M^2(q) = A(q)\frac{m_0^2}{p(q)} \ ,$$

$$\begin{array}{lll} A(u) = 5 + X_U(p(u) \ , & A(c) = 14 + X_U(p(c)) \ , & A(t) = 65 + X_U(p(t)) \ , \\ A(d) = 8 + X_D(p(d)) \ , & A(s) = 17 + X_D(p(s)) \ , & A(b) = 68 + X_D(p(b)) \ . \end{array} \quad (3.3.11)$$

p-Adic length scale hypothesis allows to identify the p-adic primes labelling quarks whereas topological mixing of U and D quarks allows to deduce topological mixing matrices U and D and CKM matrix V and precise values of the masses apart from effects like color magnetic spin orbit splitting, color Coulombic energy, etc..

Integers n_{q_i} satisfying $\sum_i n(U_i) = \sum_i n(D_i) = 69$ characterize the masses of the quarks and also the topological mixing to high degree. The reason that modular contributions remain integers is that in the p-adic context non-trivial rationals would give CP_2 mass scale for the real counterpart of the mass squared. In the absence of mixing the values of integers are $n_d = n_u = 0$, $n_s = n_c = 9$, $n_b = n_t = 60$.

The fact that CKM matrix V expressible as a product $V = U^\dagger D$ of topological mixing matrices is near to a direct sum of 2×2 unit matrix and 1×1 unit matrix motivates the approximation $n_b \simeq n_t$. The large masses of top quark and of $t\bar{t}$ meson encourage to consider a scenario in which $n_t = n_b = n \leq 60$ holds true.

The model for topological mixing matrices and CKM matrix predicts U and D matrices highly uniquely and allows to understand quark and hadron masses in surprisingly detailed level.

1. $n_d = n_u = 60$ is not allowed by number theoretical conditions for U and D matrices and by the basic facts about CKM matrix but $n_t = n_b = 59$ allows almost maximal masses for b and t. This is not yet a complete hit. The unitarity of the mixing matrices and the construction of CKM matrix to be discussed in the next section forces the assignments

$$(n_d, n_s, n_b) = (5, 5, 59) \ , \quad (n_u, n_c, n_t) = (5, 6, 58) \ . \quad (3.3.12)$$

   fixing completely the quark masses apart possible Higgs contribution [F4]. Note that top quark mass is still rather near to its maximal value.

2. The constraint that valence quark contribution to pion mass does not exceed pion mass implies the constraint $n(d) \leq 6$ and $n(u) \leq 6$ in accordance with the predictions of the model of topological mixing. $u - d$ mass difference does not affect $\pi^+ - \pi^0$ mass difference and the quark contribution to $m(\pi)$ is predicted to be $\sqrt{(n_d + n_u + 13)/24} \times 136.9$ MeV for the maximal value of CP_2 mass (second order p-adic contribution to electron mass squared vanishes).

The p-adic length scales associated with quarks and quark masses

The identification of p-adic length scales associated with the quarks has turned to be a highly non-trivial problem. The reasons are that for light quarks it is difficult to deduce information about quark masses for hadron masses and that the unknown details of the topological mixing (unknown until the advent of the thermodynamical model [F4]) made possible several p-adic length scales for quarks. It has also become clear that the p-adic length scale can be different form free quark and bound quark and that bound quark p-adic scale can depend on hadron.

Two natural constraints have however emerged from the recent work.

1. Quark contribution to the hadron mass cannot be larger than color contribution and for quarks having $k_q \neq 107$ quark contribution to mass is added to color contribution to the mass. For quarks with same value of k conformal weight rather than mass is additive whereas for quarks with different value of k masses are additive. An important implication is that for diagonal mesons $M = q\bar{q}$ having $k(q) \neq 107$ the condition $m(M) \geq \sqrt{2}m_q$ must hold true. This gives strong constraints on quark masses.

2. The realization that scaled up variants of quarks explain elegantly the masses of light hadrons allows to understand large mass splittings of light hadrons without the introduction of strong isospin-isospin interaction.

The new model for quark masses is based on the following identifications of the p-adic length scales.

1. The nuclear p-adic length scale $L(k)$, $k = 113$, corresponds to the p-adic length scale determining the masses of u, d, and s quarks. Note that $k = 113$ corresponds to a so called Gaussian Mersenne. The interpretation is that quark massivation occurs at nuclear space-time sheet at which quarks feed their em fluxes. At $k = 107$ space-time sheet, where quarks feed their color gauge fluxes, the quark masses are vanishing in the first p-adic order. This could be due to the fact that the p-adic temperature is $T_p = 1/2$ at this space-time sheet so that the thermal contribution to the mass squared is negligible. This would reflect the fact that color interactions do not involve any counterpart of Higgs mechanism.

   p-Adic mass calculations turn out to work remarkably well for massive quarks. The reason could be that M_{107} hadron physics means that *all* quarks feed their color gauge fluxes to $k = 107$ space-time sheets so that color contribution to the masses becomes negligible for heavy quarks as compared to Super-Kac Moody and modular contributions corresponding to em gauge flux feeded to $k > 107$ space-time sheets in case of heavy quarks. Note that Z^0 gauge flux is feeded to space-time sheets at which neutrinos reside and screen the flux and their size corresponds to the neutrino mass scale. This picture might throw some light to the question of whether and how it might be possible to demonstrate the existence of M_{89} hadron physics.

   One might argue that $k = 107$ is not allowed as a condensation level in accordance with the idea that color and electro-weak gauge fluxes cannot be feeded at the space-time space time sheet since the classical color and electro-weak fields are functionally independent. The identification of η' meson as a bound state of scaled up $k = 107$ quarks is not however consistent with this idea unless one assumes that $k = 107$ space-time sheets in question are separate.

2. The requirement that the masses of diagonal pseudoscalar mesons of type $M = q\overline{q}$ are larger but as near as possible to the quark contribution $\sqrt{2}m_q$ to the valence quark mass, fixes the p-adic primes $p \simeq 2^k$ associated with c, b quarks but not t since toponium does not exist. These values of k are "nominal" since k seems to be dynamical. c quark corresponds to the p-adic length scale $k(c) = 104 = 2^3 \times 13$. b quark corresponds to $k(b) = 103$ for $n(b) = 5$. Direct determination of p-adic scale from top quark mass gives $k(t) = 94 = 2 \times 47$ so that secondary p-adic length scale is in question.

   Top quark mass tends to be slightly too low as compared to the most recent experimental value of $m(t) = 169.1$ GeV with the allowed range being [164.7, 175.5] GeV [59]. The optimal situation corresponds to $Y_e = 0$ and $Y_t = 1$ and happens to give top mass exactly equal to the most probable experimental value. It must be emphasized that top quark is experimentally in a unique position since toponium does not exist and top quark mass is that of free top.

In the case of light quarks there are good reasons to believe that the p-adic mass scale of quark is different for free quark and bound state quark and that in case of bound quark it can also depend on hadron. This would explain the notions of valence (constituent) quark and current quark mass as masses of bound state quark and free quark and leads also to a TGD counterpart of Gell-Mann-Okubo mass formula [F4].

1. Constituent quark masses

Constituent quark masses correspond to masses derived assuming that they are bound to hadrons. If the value of k is assumed to depend on hadron one obtains nice mass formula for light hadrons as will be found later. The following table summarizes constituent quark masses as predicted by this model.

q	d	u	s	c	b	t
n_q	4	5	6	6	59	58
s_q	12	10	14	11	67	63
$k(q)$	113	113	113	104	103	94
$m(q)/GeV$	.105	.092	.105	2.191	7.647	167.8

Table 2. Constituent quark masses predicted for diagonal mesons assuming $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$, maximal CP_2 mass scale($Y_e = 0$), and vanishing of second order contributions.

2. Current quark masses

Current quark masses would correspond to masses of free quarks which tend to be lower than valence quark masses. Hence k could be larger in the case of light quarks. The table of quark masses in Wikipedia [61] gives the value ranges for current quark masses depicted in the table below together with TGD predictions for the spectrum of current quark masses.

q	d	u	s
$m(q)_{exp}/MeV$	4-8	1.5-4	80-130
$k(q)$	(122,121,120)	(125,124,123,122)	(114,113,112)
$m(q)/MeV$	(4.5,6.6,9.3)	(1.4,2.0,2.9,4.1)	(74,105,149)
q	c	b	t
$m(q)_{exp}/MeV$	1150-1350	4100-4400	1691
$k(q)$	(106,105)	(105,104)	92
$m(q)/MeV$	(1045,1477)	(3823,5407)	167.8×10^3

Table 3. The experimental value ranges for current quark masses [61] and TGD predictions for their values assuming $(n_d, n_s, n_b) = (5, 5, 59)$, $(n_u, n_c, n_t) = (5, 6, 58)$, and $Y_e = 0$. For top quark $Y_t = 0$ is assumed. $Y_t = 1$ would give 169.2 GeV.

Some comments are in order.

1. The long p-adic length associated with light quarks seem to be in conflict with the idea that quarks have sizes smaller than hadron size. The paradox disappears when one realized that $k(q)$ characterizes the electromagnetic "field body" of quark having much larger size than hadron.

2. u and d current quarks correspond to a mass scale not much higher than that of electron and the ranges for mass estimates suggest that u could correspond to scales $k(u) \in (125, 124, 123, 122) = (5^3, 4\times 31, 3\times 41, 2\times 61)$, whereas d would correspond to $k(d) \in (122, 121, 120) = (2\times 61, 11^2, 3\times 5 \times 8)$.

3. The TGD based model for nuclei based on the notion of nuclear string leads to the conclusion that exotic copies of $k = 113$ quarks having $k = 127$ are present in nuclei and are responsible for the color binding of nuclei [F8, F9].

4. The predicted values for c and b masses are slightly too low for $(k(c), k(b)) = (106, 105) = (2 \times 53, 3 \times 5 \times 7)$. Second order Higgs contribution could increase the c mass into the range given in [61] but not that of b.

5. The mass of top quark has been slightly below the experimental estimate for long time. The experimental value has been coming down slowly and the most recent value obtained by CDF [60] is $m_t = 165.1 \pm 3.3 \pm 3.1$ GeV and consistent with the TGD prediction for $Y_e = Y_t = 0$.

Can Higgs field develop a vacuum expectation in fermionic sector at all?

An important conclusion following from the calculation of lepton and quark masses is that if Higgs contribution is present, it can be of second order p-adically and even negligible, perhaps even vanishing. There is indeed an argument forcing to consider this possibility seriously. The recent view about elementary particles is following.

1. Fermions correspond to CP_2 type vacuum extremals topologically condensed at positive/negative energy space-time sheets carrying quantum numbers at light-like wormhole throat. Higgs and gauge bosons correspond to wormhole contacts connecting positive and negative energy space-time sheets and carrying fermion and anti-fermion quantum numbers at the two light-like wormhole throats.

2. If the values of p-adic temperature are $T_p = 1$ and $T_p = 1/n$, $n > 1$f or fermions and bosons the thermodynamical contribution to the gauge boson mass is negligible.

3. Different p-adic temperatures and Kähler coupling strengths for fermions and bosons make sense if bosonic and fermionic partonic 3-surfaces meet only along their ends at the vertices of generalized Feynman diagrams but have no other common points [C3]. This forces to consider the possibility that fermions cannot develop Higgs vacuum expectation value although they can couple to Higgs. This is not in contradiction with the modification of sigma model of hadrons based on the assumption that vacuum expectation of σ field gives a small contribution to hadron mass [F5] since this field can be assigned to some bosonic space-time sheet pair associated with hadron.

4. Perhaps the most elegant interpretation is that ground state conformal is equal to the square of the eigenvalue of the modified Dirac operator. The ground state conformal weight is negative and its deviation from half odd integer value gives contribution to both fermion and boson masses. The Higgs expectation associated with coherent state of Higgs like wormhole contacts is naturally proportional to this parameter since no other parameter with dimensions of mass is present. Higgs vacuum expectation determines gauge boson masses only apparently if this interpretation is correct. The contribution of the ground state conformal weight to fermion mass square is near to zero. This means that λ is very near to negative half odd integer and therefore no significant difference between fermions and gauge bosons is implied.

3.4 Boson masses

The original model for boson masses was based on N-S representations and predicted slightly too high masses for intermediate gauge bosons. The identification of gauge bosons and Higgs as wormhole contacts leads to much simpler model using Ramond representations for fermions. Ground state conformal weight is assumed to be given by the square of the sum of eigenvalues of the modified Dirac operator for fermion and antifermion and p-adic thermodynamics is applied independently to fermion and antifermion.

3.4.1 Are bosons pairs of positive energy fermion and negative energy antifermion?

The identification of gauge bosons is not quite straightforward. There seems to be no obvious reason for not allowing bosons to be bound states of positive energy fermion and negative energy anti-fermion or vice versa. The following arguments indeed support the identification of bosons as bound states of positive and negative energy fermion.

1. The space-like polarization can be realized only if either the massless fermion or anti-fermion has non-physical polarization or equivalently, opposite time orientation.

2. If the energies of fermion and anti-fermion are of opposite sign, $\lambda_F = -\lambda_{\overline{F}}$ looks natural. If the state is symmetric under the exchange of fermion and anti-fermion, the average value of λ_B vanishes. If conformal weight is given by $\langle \lambda_B \rangle^2$, the contribution to ground state conformal weight vanishes. Hence strong CP breaking seems to be necessary and one must consider seriously the old idea that the particles of antimatter have negative energy and that the matter antimatter asymmetry of the Universe is due to this asymmetry. An alternative model assumes that antimatter is inside "big" cosmic strings inside large voids [D5].

3. The charge matrix Q characterizing the fermionic couplings of boson appears in the bilinear of fermion and anti-fermion oscillator operators. If Q has definite parity, the eigenvalue spectra of

D_{C-S} are identical for fermion and anti-fermion and $\lambda = 0$ for boson. This is true for neutral vector bosons, that is for photon, gluons, and graviton. Since Virasoro generators L_n, $n > 0$ annihilate the ground state, thermal excitations are not possible and the state remains massless.

4. If the sign of energy is same for both fermion and anti-fermion, the analogy with cyclotron states suggests $\lambda \simeq 1$ so that strictly massless states are not possible. Arbitrarily small masses are however possible for low enough p-adic temperatures.

5. Critical reader probably asks whether gauge bosons and gravitons can appear at all in initial and final states if this interpretation is correct. If all fermions and anti-fermions of the state are required to have positive/negative energy, the answer is indeed 'No'. If it is the sign of net energy which matters, the answer is 'Yes'. To my opinion, the latter option is the only sensible one.

If these arguments are accepted, one can understand the basic pattern of gauge boson masses. The understanding of the details behind p-adic mass calculations has however involved many wrong guesses so that one must keep mind open for alternative interpretations.

3.4.2 Photon, graviton and gluon

The only possibility to get massless states is to have $\Delta = 0$ state with one active sector so that NS thermodynamics becomes trivial due to the absence of the thermodynamical excitations satisfying the gauge conditions. The model for the Weinberg mixing suggests that a second order non-thermal contribution to the mass squared is present in case of photon. Also in case of the gluon this contribution is expected to be present. For graviton the contribution should be extremely small or even higher order in p.

3.4.3 Can p-adic thermodynamics explain the masses of intermediate gauge bosons?

The requirement that the electron-intermediate gauge boson mass ratios are sensible, serves as a stringent test for the hypothesis that intermediate gauge boson masses result from the p-adic thermodynamics. It seems that the only possible option is that the parameter k has same value for both bosons, leptons, and quarks:

$$k_B = k_L = k_q = 1 \ .$$

In this case all gauge bosons have $D(0) = 1$ and there are good changes to obtain boson masses correctly. $k = 1$ together with $T_p = 1$ implies that the thermal masses of very many boson states are extremely heavy so that the spectrum of the boson exotics is reduced drastically. For $T_p = 1/2$ the thermal contribution to the mass squared is completely negligible.

Simplest for of p-adic thermodynamics fails to explain gauge boson masses

Contrary to the original optimistic beliefs based on calculational error, it turned out impossible to predict W/e and Z/e mass ratios correctly in the original p-adic thermodynamics scenario. Although the errors are of order 20-30 percent, they seemed to exclude the explanation for the massivation of gauge bosons using p-adic thermodynamics.

1. The thermal mass squared for a boson state with N active sectors (non-vanishing vacuum weight) is determined by the partition function for the tensor product of N NS type Super Virasoro algebras. The degeneracies of the excited states as a function of N and the weight Δ of the operator creating the massless state are given in the table below.

2. Both W and Z must correspond to $N = 2$ active Super Virasoro sectors for which $D(1) = 1$ and $D(2) = 3$ so that (using the formulas of p-adic thermodynamics [TGDpad]) the thermal mass squared is $m^2 = k_B(p + 5p^2)$ for $T_p = 1$. The second order contribution to the thermal mass squared is extremely small so that Weinberg angle vanishes in the thermal approximation. $k_B = 1$ gives Z/e mass-ratio which is about 22 per cent too high. For $T_p = 1/2$ thermal masses are completely negligible.

3. The thermal prediction for W-boson mass is the same as for Z^0 mass and thus even worse since the two masses are related $M_W^2 = M_Z^2 cos^2(\theta_W)$.

Could coupling to Higgs contribute to gauge boson masses

It seem that the Achilles's heel of the p-adic thermodynamics is bosonic sector whereas the weak point of the standard model is fermionic sector. The first natural reaction -before realizing that ground state conformal weights need not be and very probably are not exactly equal to -1/2- was that it might be possible to combine these two approaches. $T_p = 1/2$ is certainly the only possible p-adic temperature for intermediate gauge bosons so that gauge boson masses should result by a TGD variant of the Higgs mechanism.

1. It is indeed possible to identify a candidate for Higgs boson with correct quantum numbers also in TGD framework. The simplest identification of Higgs boson is as a wormhole contact carrying appropriate fermion and anti-fermion numbers at the two light-like 3-surfaces defined by the wormhole throats. The coherent state associated with this kind of Higgs can contribute only to gauge boson masses. It must be emphasized that this Higgs behaves as scale in M^4 degrees of freedom but like vector with respect to CP_2. This means obvious difference to standard model.

2. The minimum p-adic mass squared is the p-adic mass squared unit $m_0^2/3$. This corresponds in a reasonable approximation to the mass of W boson so that the mass scale would be predicted correctly. The calculation of leptonic masses however requires the use of m_0^2 as a mass squared unit for which intermediate gauge boson masses are smaller than one unit. The way out of the difficulty could be based on the use of a variant of the canonical identification I acting as $I_1(r/s) = I(r)/I(s)$. This map respects under certain additional conditions various symmetries and is the only sensible possibility at the level of scattering amplitudes. This variant predicts that the real counterpart of $m^2 = (m/n)p$ is $(m/n)/p$ rather than of order CP_2 squared so that intermediate gauge boson masses can be smaller than one unit even if $O(p)$ p-adically, and allows an elegant group theoretic description of m_W/m_Z mass ratio in terms of Weinberg angle. This point is discussed in [F4, F5].

3. After the realization that the generalized eigenvalues of the modified Dirac operator play a key role in quantum TGD the identification of Higgs field as the generalized eigenvalue emerged naturally. Since C-S action defines almost topological QFT, the generalized eigenvalues had dependence on the coordinates transversal to the light-like coordinate of X_l^3. The interpretation was as Higgs field and Higgs vacuum expectation was assigned with the values of this field at points of number theoretic braid. The unsatisfactory feature of this interpretation was the asymmetry between bosons and fermions and unability to predict the vacuum expectation value of Higgs.

Higgs contributes only apparently to elementary particle masses

Only after the discovery how the information about preferred extremal of Kähler action can be feeded to the spectrum of modified Dirac operator (see the discussion about modified Dirac action), a real understanding of the situation emerged (at least this is my belief!).

1. The generalized eigenvalues are simply square roots of ground state conformal weights and by analogy with cyclotron energies the conformal weights are in reasonable approximation given by $h = -n - 1/2$ giving the desired $h \simeq -1/2$ for lowest state plus finite number of additional ground states. The deviation Δh of h from half odd integer value cannot be compensated by the action of Virasoro generators and it is this contribution which has interpretation as Higgs contribution to mass squared. Δh is present for both fermions and bosons, should be small for fermions and dominate for gauge bosons. The vacuum expectation of Higgs is indeed naturally proportional to Δh but the presence of Higgs condensate does not cause the massivation.

2. Before one can buy a bottle of champaign, one must understand the relationship $M_W^2 = M_Z^2 cos^2(\theta_W)$ requiring $\Delta h(W)/\Delta h(Z) = cos^2(\theta_W)$. Essentially, one should understand the dependence of the quantum averaged the spectrum of modified Dirac operator on the quantum numbers of elementary particle over configuration space degrees of freedom. Suppose that the

zero energy state describing particle is proportional to a phase factor analogous to Chern-Simons action depending on electro-weak quantum numbers of the particle. Stationary phase approximation selects a preferred light-like 3-surface and boundary conditions assign to this preferred extremal of Kähler action defining the exponent of Kähler function so that also Δh depends on quantum numbers of the particle. The discretization implied by the notion of number theoretic braid should simplify this problem considerably.

Why the two definitions of Weinberg angle should be identical?

Second challenge is to understand how the mixing of neutral gauge bosons B_3 and B_0 relates to the group theoretic factor $cos^2(\theta_W)$. The condition that the Higgs expectation value for gauge boson B is proportional to $\Delta h(B)$ and that the coherent state of Higgs couples gauge bosons regarded as fermion anti-fermion pairs should explain the mixing.

1. If gauge bosons and Higgs correspond to wormhole contacts, the discussion reduces to one-fermion level. The value of Δh should be different for different charge states $F_{\pm 1/2}$ of elementary fermion (in the following I will drop from discussion delicacies due to the fact that both quarks and leptons and fermion families are involved). The values of λ of fermion and anti-fermion assignable to gauge boson are naturally identical

$$\Delta\lambda(F_{\pm 1/2} = \Delta\lambda(\overline{F}_{\pm 1/2}) \equiv x_{\pm 1/2} \ . \tag{3.4.1}$$

This implies

$$\begin{aligned} \Delta h(Z,W) &\equiv \Delta h(Z) - \Delta h(W) = m_Z^2 - m_W^2 = m_Z^2 sin^2(\theta) \ , \\ \Delta h(Z) &= 1/2\sum_{\pm}(\Delta\lambda(F_{\pm 1/2}) + \Delta\lambda(\overline{F}_{\mp 1/2})^2 = 2\sum_{\pm} x_{\pm 1/2}^2 \ , \\ \Delta h(W) &= 1/2\sum_{\pm}(\Delta\lambda(F_{\pm 1/2}) + \Delta\lambda(\overline{F}_{\pm 1/2})^2 = (x_{1/2} + x_{-1/2})^2 \ . \end{aligned} \tag{3.4.2}$$

This gives

$$\Delta h(Z,W) = (x_{1/2} - x_{-1/2})^2 \tag{3.4.3}$$

giving the condition

$$(x_{1/2} - x_{-1/2})^2 = (x_{1/2} + x_{-1/2})^2 sin^2(\theta_W) \ . \tag{3.4.4}$$

The interpretation is as breaking of electro-weak $SU(2)_L$ symmetry coded by the geometry of CP_2 in the structure of spinor connection so that the symmetry breaking is expected to take place. One can *define* the value of Weinberg angle from the formula

$$sin(\theta_W) \equiv \pm\frac{x_{1/2} - x_{-1/2}}{x_{1/2} + x_{-1/2}} \ . \tag{3.4.5}$$

2. This definition of Weinberg angle should be consistent with the identification of Weinberg angle coming from the couplings of Z^0 and photon to fermions. Also here the reduction of couplings to one-fermion level might help to understand the symmetry breaking. Z^0 and γ decompose as $Z_0 = cos(\theta_W)B_3 + sin(\theta_W)B_0$ and $\gamma = -sin(\theta_W)B_3 + cos(\theta_W)B_0$, where B_3 corresponds to the gauge potential in $SU(2)_L$ triplet and B_0 the gauge potential in $SU(2)_L$ singlet. Why this mixing should be induced by the splitting of the conformal weights? What induces the mixing of electro-weak triplet with singlet?

3. Could it be the coherent state of Higgs field which transforms left handed and right handed fermions to each other and hence also B_3 to B_0 and vice versa? If the Higgs expectation value associated with the coherent state is proportional to Δh, it would not be too surprising if the mixing between B_3 and B_0 caused by the coherent Higgs state were proportional to $(x_{1/2}-x_{-1/2})/(x_{1/2}+x_{-1/2})$. The reason would be that B_3 is antisymmetric with respect to the exchange of weak isospins whereas B_0 is symmetric. Therefore also the mixing amplitude should be antisymmetric with respect to the exchange of isospins and proportional to $(x_{1/2} - x_{-1/2})$. The presence of the numerator is needed to make the amplitude dimensionless. Under this assumption the two identifications of the Weinberg angle are equivalent.

This - admittedly oversimplified - picture obviously changes considerably what-causes-what's in the description of gauge boson massivation and the basic argument should be developed into a more precise form.

Is Higgs really needed and does it exist?

The mass range containing the Higgs mass is becoming narrower and narrower[55, 56], and one cannot avoid the question whether the Higgs really exists. This issue remains far from decided also in TGD framework where also the question whether Higgs is needed at all to explain the massivation of gauge bosons must be raised.

1. My long-held belief was that Higgs does not exist. One motivation for this belief was that there is no really nice space-time correlate for the Higgs field. Higgs should correspond to M^4 scalar and CP_2 vector but one cannot identify any natural candidate for Higgs field in the geometry of CP_2. The trace of CP_2 part of the second fundamental form could be considered as a candidate but depends on second derivatives of the imbedding space coordinates. Its counterpart for Kähler action would be the covariant divergence of the vector defined by modified gamma matrices and this vanishes identically.

2. For a long time I believed that p-adic thermodynamics is not able to describe realistically gauge boson massivation and the group theoretical expression for the mass ratio of W and Z gauge bosons led to the cautious conclusion that Higgs is needed and generates a coherent state and that the ordinary Higgs mechanism has TGD counterpart. This field theoretic description is of course purely phenomenological in TGD framework and whether it extends to a microscopic description is far from clear.

3. The identification of bosons in terms of wormhole contacts having fermion and antifermion at their light-like throats allowed a construction of also Higgs like particle. One can estimate its mass by p-adic thermodynamics using the existing bounds to determine the p-adic length scale in question: $p \simeq 2^k$, $k = 94$, is the best guess and gives $m_H = 129$ GeV, which is consistent with the experimental constraints. Higgs expectation cannot however contribute to fermion masses if fermions are identified as CP_2 type vacuum extremals topologically condensed to single space-time sheet so that there can be only one wormhole throat present. This would mean that Higgs condensate -whatever it means in precise sense- is topologically impossible in fermionic sector. p-Adic thermodynamics for fermions allows only a very small Higgs contribution to the mass so that this is not a problem.

4. The next step was the realization that the deviation of the ground state conformal weights from half integer values could give rise to Higgs type contribution to both fermion and boson mass. Furthermore, the contribution to the ground state conformal weight corresponds to the modulus squared for the generalized eigenvalue λ of the modified Dirac operator D. This picture suggests

a microscopic description of gauge boson masses and the Weinberg angle determining W/Z mass ratio can be expressed in terms of the generalized eigenvalues of D. Higgs could be still present and if it generates vacuum expectation (characterizing coherent state) its value should be expressible in terms of the generalized eigenvalues of modified Dirac operator. The causal relation between Higgs and massivation would not however be what it is generally believed to be.

The massivation of Z^0 and generation of longitudinal polarizations are the problems, which should be understood in detail before one can take seriously in TGD inspired microscopic description.

1. The presence of an axial part in the decomposition of gauge bosons to fermion-antifermion pairs located at the throats of the wormhole contact should explain the massivation of intermediate gauge bosons and the absence of it the masslessness of photon, gluon, and gravitons.
2. One can understand the massivation of W bosons in terms of the differences of the generalized eigenvalues of the modified Dirac operator. In the case of W bosons fermions have different charges so that the generalized eigenvalues of the modified Dirac operator differ and their difference gives rise to a non-vanishing mass. Both transverse and longitudinal polarizations are in the same position as they should be.
3. The problem is how Z^0 boson can generate mass. For Z^0 the fermions for transverse polarizations should have in a good approximation same spectrum generalized eigenvalues so that the mass would vanish *unless* fermion and anti-fermion correspond to different eigenvalues for some reason for Z^0. The requirement that the photon and Z^0 states are orthogonal to each other might require different eigen values. If fermion and antifermion in both Z^0 and γ correspond to the same eigen mode of the modified Dirac operator, their inner product is proportional to the trace of the charge matrices given by $Tr(Q_{em}(I_L^3 + sin^2(\theta_W)Q_{em})$, which is non-vanishing in general. For different eigenmodes in the case of Z^0 the states would be trivially orthogonal.
4. Gauge bosons must allow also longitudinal polarization states. The fact that the modes associated with wormhole throats are different in the case of Z^0 could allow also longitudinal polarizations. The state would have the structure $\overline{\Psi}_-(D_{\rightarrow} - D_{\leftarrow})Q_Z\Psi_+$, $D = p^k\gamma_k$. This state does not vanish for intermediate gauge bosons since the action of $p^k\gamma_k$ to the two modes of the induced spinor field is different. For photon and gluons the state would vanish.
5. In the standard approach the gradient of Higgs field is transformed to a longitudinal polarization of massive gauge bosons. It is not clear whether this kind of idea makes sense at all microscopically in TGD framework. The point is that Higgs as a particle corresponds to a superposition of fermion-antifermion pairs with opposite M^4 chiralities whereas the longitudinal part corresponds to pairs with same M^4 chiralities. Hence the idea about the gradient of Higgs field transforming to the longitudinal part of gauge boson need not make sense in TGD framework although Higgs can quite well exist.

To sum up, these arguments could be seen as a support for the possibility that Higgs is not needed at all in particle massivation in TGD Universe but leave open the question whether Higgs exists as particle and possibly develops coherent state.

3.4.4 Recent situation in Higgs search

Why Higgs is not detected?

Higgs has not yet been detected yet. Various contributions to Higgs production are discussed in [29, 30]. The basic contributions to the production of Higgs bosons in p-p collisions at LHC corresponds to gluon fusion, associated production, and vector boson fusion. Various production cross sections for $p - p$ collisions at cm energy of $\sqrt{s} = 14$ TeV are given in [29], see also the figures of [30]. The dominating contribution corresponds to the triangle diagram $gg \rightarrow q\overline{q} \rightarrow H$. Since the coupling of quarks to Higgs in TGD can be much smaller than in standard model, this contribution can be very small in TGD framework. Also the rates for direct annihilations $q\overline{q} \rightarrow H$ are small for the same reason.

The rates for the vector boson fusion and associated production are in the lowest order same in TGD as in the standard model. Vector boson fusion corresponds to the scattering of quarks via the exchange of W/Z boson coupling to Higgs ($q\overline{q} \rightarrow q\overline{q}H$). The rate for this process is roughly one hundred times lower than for the associated production. Associated production corresponds to the diagram $q\overline{q} \rightarrow W \rightarrow W + H$. The rate is below the rate of the vector boson fusion if Higgs mass is above $\sim$ 100 GeV: on basis recent searches the Higgs mass is known to be in the range $114.4 - 237$ GeV [30].

It seems safe to conclude that TGD predicts Higgs particle. The fact that the rate of Higgs production can be about 100 times lower than in standard model and even this could easily explain the unsuccessful search for Higgs.

Higgs mass determination from high precision electro-weak observables

Higgs mass can be estimated from the measured values of electro-weak high precision electro-weak observables. The values of these observables can be deduced from fermion-antifermions scattering at Z^0 resonance [45]. The dependence on Higgs mass comes from radiative corrections involving the coupling of Higgs to the fundamental fermions and gauge bosons. The radiative corrections affect the couplings of gauge bosons to fundamental fermions and introduce renormalization corrections to gauge boson masses and decay widths. Hence one can deduce Higgs mass in several independent manners and at the same time test the internal consistency of the theory. The variation of the values of observables is surprisingly wide: roughly an order of magnitude variation appears [46].

The dependence of the loop corrections on Higgs mass is logarithmic and this together with experimental uncertainties could explain the great variation. One could also ask whether this finding could be seen as an evidence for small couplings of fundamental fermions to Higgs so that $h - f - \overline{f}$ contributions to radiative corrections would effectively vanish and only boson-Higgs couplings contribute significantly. This is indeed allowed by TGD where fermionic masses come from p-adic thermodynamics rather than coupling to Higgs vacuum expectation.

Unfortunately this idea does not work as the detailed discussion of high precision electro-weak observables in fermion-antifermion scattering at Z^0 resonance pole can be found in [45] shows. The point is that already in standard model fermion-Higgs type contributions to radiative corrections are very small except for top quark since the contribution of $hf\overline{f}$ vertex in the loop is proportional to the fermion mass. Hence the radiative corrections from the couplings of gauge bosons to Higgs appearing in the boson propagators dominate. For $t\overline{t}$ scattering left-right asymmetry due to $\gamma - Z^0$ interference and forward-backward asymmetry involve sizable contributions from Higgs exchange and in principle could be used to distinguish between TGD and standard model. In practice this is not possible.

Constraints on Higgs mass from the evolution of Higgs self coupling

The constraints on the coupling constant evolution of λ give constraints on the Higgs mass. There are two competing effects. Quartic self coupling tends to increase λ and if it dominates it gives rise to a logarithmic behavior leading to large values of λ in the ultraviolet [43]. This situation prevails provided some critical value of λ can be reached since other couplings tend to slow down the growth of λ. An alterative option is that the Yukawa coupling to top quark wins and λ becomes very small and even changes sign. Coupling constant evolution can also induce the change of minimum of Higgs potential to a maximum.

1. Upper bound on Higgs mass from perturbative unitarity

An upper bound on Higgs mass comes from the requirement that perturbative unitarity is not lost in the energy range considered characterized by the value Λ of UV cutoff. The loss of perturbative unitary would have interpretation in terms of new physics above Λ. This requires that the initial value of λ cannot be too high.

1. The upper bound for $\lambda(t_Z)$ at intermediate gauge boson mass using the basic formula in terms of vacuum expectation value and λ: $m_H^2 = 2\lambda v^2$. Here $v = \sqrt{\sqrt{2}G_F} = 247$ GeV is fixed from the intermediate boson mass scale and therefore genuine upper bound results. $\lambda = .2098$ for $m_H = 160$ GeV makes sense at this energy.

2. If the initial value of λ, or equivalently Higgs boson mass, is too large, λ starts to grow leading to a loss of perturbative unitarity at some energy. The requirement that this does not occur below Λ defining the mass scale scale for the new physics gives an upper bound on Higgs mass. For instance, if the new physics is not allowed below GUT mass scale 10^{16} GeV, one obtains the upper bound $m_H < 153$ GeV [43]. The counterpart for GUT length scale is CP_2 size and corresponds to energy of $M \sim 10^{-4} M_{Planck} \sim 10^{15}$ GeV.

2. Standard model lower bound on Higgs mass from coupling constant evolution of λ

The natural conditions are that λ stays positive and that the extremum of the effective potential $V(H(t))$ does not transform from minimum to a maximum. The large coupling to top quark tends to reduce λ and the latter condition gives a lower bound on the low energy value of λ and thus to Higgs mass. For instance, according to some estimates $\Lambda_{GUT} \simeq 10^{16}$ GeV restricts the Higgs mass in the range 130-190 GeV, which does not have overlap with the mass range allowed by the range allowed by the best fit using high precision estimates for electro-weak parameters.

An estimate for Higgs mass

In standard model Higgs and W boson masses are given by

$$\begin{aligned} m_H^2 &= 2v^2\lambda = \mu^2\lambda^3 \ , \\ m_W^2 &= \frac{g^2v^2}{4} = \frac{e^2}{8sin^2(\theta_W)}\mu^2\lambda^2 \ . \end{aligned} \tag{3.4.6}$$

This gives

$$\lambda = \frac{4\pi}{8\alpha_{em} sin^2(\theta_W)}(\frac{m_H}{m_W})^2 \ . \tag{3.4.7}$$

In standard model one cannot predict the value of m_H.

In TGD framework one can try to understand Higgs mass from p-adic thermodynamics as resulting via the same mechanism as fermion masses so that the value of the parameter λ would follow as a prediction.

One must assume that p-adic temperature equals to $T_p = 1$. The natural assumption is that Higgs can be regarded as superposition of pairs of fermion and anti-fermion at opposite throats of wormhole contact. With these assumptions the thermal expectation of the Higgs conformal weight is just the sum of contributions from both throats and *two* times the average of the conformal weight over that for quarks and leptons (one might question the presence of factor 2):

$$\begin{aligned} s_H &= 2 \times \langle s \rangle = 2 \times \frac{\left[\sum_q s_q + \sum_L s_L\right]}{N_q + N_L} \\ &= 2\frac{\sum_{g=0}^{2} s_{mod}(g)}{3} + \frac{(s_L + s_{\nu_L} + s_U + s_D)}{2} \\ &= 26 + \frac{5+4+5+8}{2} = 37 \ . \end{aligned} \tag{3.4.8}$$

A couple of comments about the formula are in order.

1. The first term - two times the average of the genus dependent modular contribution to the conformal weight - equals to 26, and comes from modular degrees of freedom and does not depend on the charge of fermion.

2. The contribution of p-adic thermodynamics for super-conformal generators gives same contribution for all fermion families and depends on the em charge of fermion. The values of thermal conformal weights deduced earlier have been used. Note that only the value $s_{\nu_L} = 4$ (also $s_{\nu_L} = 5$ could be considered. This is possible if one requires that the conformal weight is integer. If the

standard form of the canonical identification mapping p-adics to reals is used, this must be the case since otherwise real mass would be super-heavy.

The first guess would be that the p-adic length scale associated with Higgs boson is M_{89}. Second option is $p \simeq 2^k$, $k = 97$ (restricting k to be prime). If one allows k to be non-prime (these values of k are also realized. One can consider also $k = 91 = 7 \times 13$. By scaling from the expression for the electron mass, one obtains the estimates

$$\begin{aligned} m_H(89) &\simeq \sqrt{\frac{37}{5}} \times 2^{21} m_e \simeq 727.3 \ \ GeV \ \ , \\ m_H(91) &\simeq \sqrt{\frac{37}{5}} \times 2^{18} m_e \simeq 363.5 GeV \ \ , \\ m_H(97) &\simeq \sqrt{\frac{37}{5}} \times 2^{15} m_e \simeq 45.5 \ \ GeV \ \ . \end{aligned} \tag{3.4.9}$$

A couple of comments are in order.

1. From [46] one learns that the latest estimates for Higgs mass give two widely different values, namely $m_H = 31^{33}_{-19}$ GeV and $m_H = 420^{+420}_{-190}$ GeV. Since the p-adic mass scale of both neutrinos and quarks and possibly even electron can vary in TGD framework, one cannot avoid the question whether - depending on experimental situation- Higgs could appear in two different mass scales corresponding to $k = 91$ and 97.

2. The low value of $m_H(97)$ might be consistent with experimental facts since the couplings of fermions to Higgs can in TGD framework be weaker than in standard model.

The value of λ is given in the three cases given by

$$\lambda(89) \simeq 4.41 \ , \ \lambda(91) = 1.10 \ \ , \ \lambda(97) = .2757 \ . \tag{3.4.10}$$

Unitarity would thus favor $k = 97$ and $k = 91$ also favored by the high precision data and $k = 91$ is just at the unitarity bound $\lambda = 1$ (here I am perhaps naive!). A possible interpretation is that for M_{89} Higgs mass forces λ to break unitarity bound and that this corresponds to the emergence of M_{89} copy of hadron physics.

In August 2008 some fresh information about Higgs mass emerged.

1. A press release from Tevatron [52] excluded the possibility that the mass is in a narrow interval around 170 GeV, roughly the average of the above mentioned mass values. Ironically, this mass value corresponds exactly to the Higgs mass predicted by the non-commutative variant of standard model of Alain Connes [53].

2. The second piece of information [54] discussed in detail in Tommaso Dorigo's blog [55] gives much stronger limits on Higgs mass. The first plot discussed in Tommaso's blog is obtained by combining enormous amount of information except that coming from LEPII and Tevatron and at 1 sigma limit bounds Higgs mass to the interval 57-100 GeV with favored value around 80 GeV. At 2 sigma the interval is 39-156 GeV. In TGD framework $k = 96$ would predict mass 91 GeV which is near the upper bound of the 1 sigma range 57-100 GeV. $k = 97$ would predict mass 45.5 GeV belonging to the lower boundary of the 2 sigma range.

3. If one includes also the information from LEPII and Tevatron the mass range 115-135 GeV [54]. TGD would predict mass 129 GeV for k=94 which is near the upper end of the allowed interval 115-135 GeV. If these limits are taken absolutely seriously, one can say that TGD is able to predict correctly also Higgs mass. Recalling that the prediction is exponentially sensitive to the value of the integer k, this could be regarded as as triumph of TGD. The reported results are however consistent with the proposal that Higgs appears with at least two different mass values. All these mass values and even others could be there depending on experimental conditions.

3.4.5 Has Higgs been detected?

In the beginning of year 2007 there were cautious claims [37, 47, 49] about the possible detection of first Higgs events. Before the end of the year the indications about Higgs events had suffered the usual fate of a statistical fluke.

These speculations however inspired more precise considerations of the experimental signatures of TGD counterpart of Higgs. This kind of theorizing is of course speculative and remains on general qualitative level only since no calculational formalism exists and one must assume that gauge field theory provides an approximate description of the situation. I leave it for the reader to decided whether to skip over this subsection.

Indications for Higgs

The indications for Higgs comes from two sources [37, 47, 49]. In both cases Higgs would have been produced as gluons decay to two $b\overline{b}$ pairs and virtual $b\overline{b}$ pair fuses to Higgs, which then decays either to τ lepton pair or b quark pair.

John Conway, the leader of CDF team analyzing data from Tevatron, has reported about a slight indication for Higgs with mass $m_H = 160$ GeV as a small excess of events in the large bump produced by the decays of Z^0 bosons with mass of $m_Z \simeq 94$ GeV to $\tau\overline{\tau}$ pairs in the blog Cosmic Variance [47]. These events have 2σ significance level meaning that the probability that they are statistical fluctuations is about 2 per cent.

The interpretation suggested by Conway is as Higgs of minimal super-symmetric extension of standard model (MSSM) [42]. In MSSM there are two complex Higgs doublets and this predicts three neutral Higgs particles denoted by h, H, and A. If A is light then the rate for the production of Higgs bosons is proportional to the parameter $tan(\beta)$ define as the ratio of vacuum expectation values of the two doublets. The rate for Higgs production is by a factor $tan(\beta)^2$ higher than in standard model and this has been taken as a justification for the identification as MSSM Higgs [47] (the proposed value is $tan(\beta) \sim 50$ [49]). If the identification is correct, about recorded 100 Higgs candidates should already exist [37] so that this interpretation can be checked.

Also Tommaso Dorigo, the blogging member of second team analyzing CDF results, has reported in his blog [49] a slight evidence for an excess of $b\overline{b}$ pairs in $Z^0 \to b\overline{b}$ decays at the same mass $m_H = 160$ GeV [49]. The confidence level is around 2 sigma. The excess could result from the decays of Higgs to $b\overline{b}$ pair associated with $b\overline{b}$ production.

What forces to take these reports with some seriousness is that the value of m_H is same in both cases. John Conway has however noticed [48] that if both signals correspond to Higgs then it is possible to deduce estimate for the number of excess events in $Z^0 \to b\overline{b}$ peak from the excess in $\tau\overline{\tau}$ peak. The predicted excess is considerably larger than the real excess. Therefore a statistical fluke could be in question, or staying in an optimistic mood, there is some new particle there but it is not Higgs.

$m_H = 160$ GeV is not consistent with the standard model estimate by D0 collaboration for the mass of standard model Higgs boson mass based on high precision measurement of electro-weak parameters $sin(\theta_W)$, α, α_s , m_t and m_Z depending on $log(m_H)$ via the radiative corrections. The best fit is in the range $96 - 117$ GeV [38]. The upper bound from the same analysis for Higgs mass is 251 GeV with 95 per cent confidence level. The estimate $m_t = 178.0 \pm 4.3$ GeV for the mass of top quark is used. The range for the best estimate is not consistent with the lower bound of 114 GeV on m_H coming from the consistency conditions on the renormalization group evolution of the effective potential $V(H)$ for Higgs [37], see Fig. 3.4.5. Here one must of course remember that the estimates vary considerably.

TGD picture about Higgs briefly

Since TGD cannot yet be coded to precise Feynman rules, the comparison of TGD to standard model is not possible without some additional assumptions. It is assumed that p-adic coupling constant evolution reduces in a reasonable approximation to the coupling constant evolution predicted by a gauge theory so that one can apply at qualitative level the basic wisdom about the effects of various couplings of Higgs to the coupling constant evolution of the self coupling λ of Higgs giving upper and lower bounds for the Higgs mass. This makes also possible to judge the determinations of Higgs mass from high precision measurements of electro-weak parameters in TGD framework.

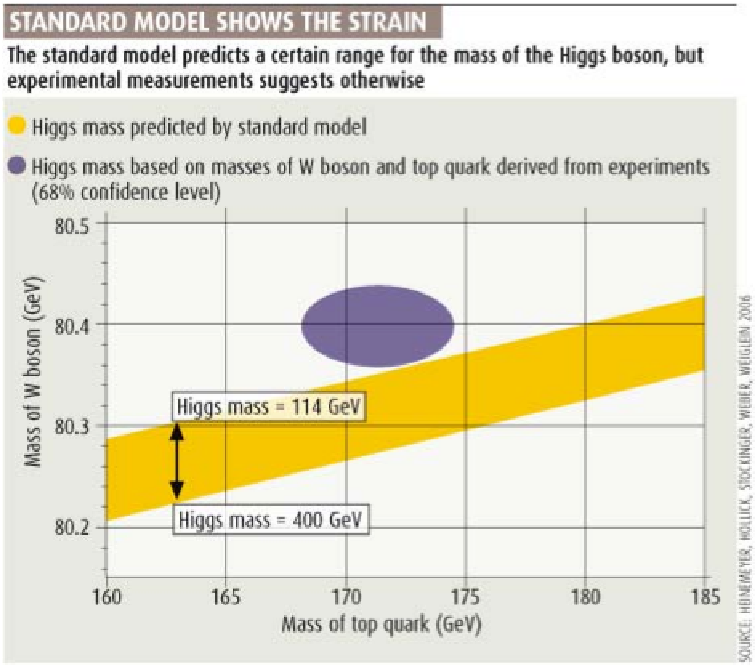

Figure 3.1: The regions of parameter space allowed by high precision measurements of top and W boson masses and the bounds on Higgs mass coming from the evolution of Higgs self coupling λ do not overlap.

In TGD framework the Yukawa coupling of Higgs to fermions can be much weaker than in standard model. This has several implications.

1. The rate for the production of Higgs via channels involving fermions is much lower. This could explain why Higgs has not been observed even if it had mass around 100 GeV.

2. In standard model the large Yukawa coupling of Higgs to top, call it h, tends to reduce the quartic self coupling constant λ for Higgs in ultraviolet. The condition that the minimum for Higgs potential is not transformed to a maximum gives a lower bound on the initial value of λ and thus to the value of m_H. In TGD framework the weakness of fermionic couplings implies that there is no lower bound to Higgs mass.

3. The weakness of Yukawa couplings means that self coupling of Higgs tends to increase λ faster than in standard model. Note also that when Yukawa coupling h_t to top is small ($h_t^2 < \lambda$, see [39]), its contribution tends to increase the value of β_λ. Thus the upper bound from perturbative unitarity to the scalar coupling λ (and m_H) is reduced. This would force the value of Higgs mass to be even lower than in standard model. The coupling constant evolution using $\beta_\lambda = (3/4\pi^2)\lambda^2$ obtained taking into account only the contribution of Higgs would give $\lambda(t) = \lambda_0/(1-k\lambda_0 log(t/t_0))$, $k = 3/4\pi^2$. For $\lambda(M_Z^2) = .2$ the value $\lambda(t) = 1$ would be achieved for $M/M_Z \simeq 5 \times 10^5$.

   In TGD framework new physics can however emerge in the length scales corresponding to Mersenne primes $M_n = 2^n - 1$. Ordinary QCD corresponds to M_{107} and one cannot exclude even M_{89} copy of QCD corresponding to mass scale $M \sim 128$ GeV. M_{61} corresponding to the mass scale $M \sim 2 \times 10^6$ GeV would define the next candidate. The quarks of M_{89} QCD would give to the beta function β_λ a negative contribution tending to reduce the value λ so that unitary bound would not be violated. If this new physics is accepted $m_H = 160$ GeV can be considered.

Can one then identify the Higgs candidate with $m_H = 160$ with the TGD variant of standard model Higgs? This is far from clear.

1. Even in standard model the rate for the production of Higgs is low. In TGD the rate for the production of the counterpart of standard model Higgs is reduced since the coupling of quarks to Higgs is expected to be much smaller than in standard model. This might exclude the interpretation as Higgs.

2. The slow rate for the production of Higgs could also allow the presence of Higgs at much lower mass and explain why Higgs has not been detected in the mass range $m_H < 114$ GeV. Interestingly, around 1990 a 2σ evidence for Higgs with mass about 100 GeV was reported and one might wonder whether there might be genuine Higgs there after all.

3. M_{89} hadron physics might be required in TGD framework by the requirement of perturbative unitarity. By a very naive scaling by factor $2^{(107-89)/2} = 2^9$ the mass of the pion of M_{89} physics would be about 70 GeV. This estimate is not reliable since color spin-spin splittings distinguishing between pion and ρ mass do not scale naively. For M_{89} mesons this splitting should be very small since color magnetic moments are very small. The mass of pion in absence of splitting would be around 297 MeV and 512-fold scaling gives $M(\pi_{89}) = 152$ GeV which is not too far from 160 GeV. Could the decays of this exotic pion give rise to the excess of fermion pairs? This interpretation might also allow to understand why b-pair and t-pair excesses are not consistent. Monochromatic photon pairs with photon energy around 76 GeV would be the probably easily testable experimental signature of this option.

Could the claimed inconsistency of $Z^0 \rightarrow \tau\overline{\tau}$ and $Z^0 \rightarrow b\overline{b}$ excesses be understood in TGD framework?

According to simple argument of John Conway [48] based on branching ratios of Z^0 and standard model Higgs to $\tau\overline{\tau}$ and $b\overline{b}$, $Z^0 \rightarrow \tau\overline{\tau}$ excess predicts that the ratio of Higgs events to Z^0 events $Z^0 \rightarrow b\overline{b}$ is related by a scaling factor

$$\frac{B(H \rightarrow b\overline{b})}{B(H \rightarrow \tau\overline{\tau})} / \frac{B(Z^0 \rightarrow b\overline{b})}{B(Z^0 \rightarrow \tau\overline{\tau})} \simeq \frac{10}{5.6} = 1.8$$

to that in $Z^0 \rightarrow \tau\overline{\tau}$ case. The prediction seems to be too high.

In a shamelessly optimistic mood and forgetting that mere statistical fluctuations might be in question, one might ask whether the inconsistency of $\tau\overline{\tau}$ and $b\overline{b}$ excesses could be understood in TGD framework.

1. The couplings of Higgs to fermions need not scale as mass in TGD framework. Rather, the simplest guess is that the Yukawa couplings scale like p-adic mass scale $m(k) = 1/L(k)$, where $L(k)$ is the p-adic length scale of fermion. Fermionic masses can be written as $m(F) = x(F)/L(k)$, where the numerical factor $x(F) > 1$ depends on electro-weak quantum numbers and is different for quarks and leptons. If the leading contribution to the fermion mass comes from p-adic thermodynamics, Yukawa couplings in TGD framework can be written as $h(F) = \varepsilon(F)m(F)/x(F)$, $\varepsilon << 1$. The parameter ε should be same for all quarks *resp.* leptons but need not be same for leptons and quarks so that that one can write $\varepsilon(quark) = \varepsilon_Q$ and $\varepsilon(lepton) = \varepsilon_L$. This is obviously an important feature distinguishing between Higgs decays in TGD and standard model.

2. The dominating contribution to the mass of highest generation fermion which in absence of topological mixing correspond to genus $g = 2$ partonic 2-surface comes from the modular degrees of freedom and is same for quarks and leptons and does not depend on electro-weak quantum numbers at all (p-adic length scale is what matters only). Topological mixing inducing CKM mixing affects $x(F)$ and tends to reduce $x(\tau), x(b)$, and $x(t)$ but the reduction is very small [F4].

3. In TGD framework the details of the dynamics leading to the final states involving Z^0 bosons and Higgs bosons are different since one expects that it fermion-Higgs vertices suppressed to the degree that weak-boson-Higgs vertices could dominate in the production of Higgs. Since these details should not be relevant for the experimental determination of $Z^0 \rightarrow \tau\overline{\tau}$ and $Z^0 \rightarrow b\overline{b}$ distributions, then the above argument can be modified in a straightforward manner by looking how the branching ratio $R(b\overline{b})/R(\tau\overline{\tau})$ is affected by the modification of Yukawa couplings for b and τ. What happens is following:

$$\frac{B(H \to b\bar{b})}{B(H \to \tau\bar{\tau})} = \frac{m_b^2}{m_\tau^2} \to \frac{B(H \to b\bar{b})}{B(H \to \tau\bar{\tau})} X \ , \ X = \frac{\varepsilon^2(q)}{\varepsilon^2(L)} \frac{x_\tau^2}{x_b^2} \ .$$

Generalizing the simple argument of Conway one therefore has

$$\frac{H}{Z^0}(b\bar{b}) = 1.8 \frac{\varepsilon_Q^2}{\varepsilon_L^2} \frac{x_\tau^2}{x_b^2} \times \frac{H}{Z^0}(\tau\bar{\tau}) \ .$$

Since the topological mixing of both charged leptons and quarks of genus 2 with lower genera is predicted to be very small, $x_\tau/x_b \simeq 1$ is expected to hold true. Hence the situation is not improved unless one has $r = \frac{\varepsilon_Q}{\varepsilon_L} < 1$ meaning that the coupling of Higgs to the p-adic mass scale would be weaker for quarks than for leptons.

Can one then guess then value of r and perhaps even Yukawa coupling from general arguments?

1. The actual value of r should relate to electro-weak physics at very fundamental level. The ratio $r = 1/3$ of Kähler couplings of quarks and leptons is certainly this kind of number. This would reduce the prediction for $\frac{H}{Z^0}(b\bar{b})$ by a factor of 1/9.

Of course, it might turn out that fake Higgs is in question. What is however important is that the deviation of the Yukawa coupling allowed by TGD for Higgs from those predicted by standard model could manifest itself in the ratio of $Z_0 \to b\bar{b}$ and $Z^0 \to \tau\bar{\tau}$ excesses.

3.5 Exotic states

The physical consequences of the exotic light leptons, quarks, and bosons are considered in the chapter devoted to the New Physics [F5]. Here it only suffices to make a short summary. Consider first what kind of exotic particles extended conformal symmetries predict.

1. Massless states are expected to become massive by p-adic thermodynamics meaning that one has superposition of states with Super Kac-Moody conformal weight equal to Super Virasoro conformal weight and annihilated by SKMV and SCV generators G_n,L_n, $n > 0$. This condition allows degeneracy since there are many manners to create a ground state with a given angular momentum and color quantum numbers and conformal weight n and annihilated by L_n, $n < 0$, by using super-canonical generators. The combinations of super-canonical generators which do not belong to super Kac-Moody algebra and create singlets in color and rotational degrees of freedom would be responsible for this degeneracy. The condition that the states in the superposition are annihilated by G_n, L_n, $n > 0$, reduces the number of the massless states.

2. The original expectation that the spectrum has $N = 1$ space-time super-symmetry seems to be wrong. The understanding of the super-conformal symmetries as at parton level allowed to identify partonic super-conformal symmetries in terms of a generalization of large $N = 4$ SCA with Kac-Moody group extended to contain also canonical transformations of $\delta H_\pm$. Thus an immense generalization of string model conformal symmetries is in question. This allows to conclude that sparticles in the sense of super Poincare symmetry are certainly absent. This does not affect the mass calculations in any manner and dramatically reduces the number of exotic states.

3. If elementary particles correspond to CP_2 type vacuum extremals, one can argue that all massless exotic massless particles can be constructed using colored generators and by color confinement cannot induce macroscopic long range interactions.

These exotic particles relate to the extended conformal symmetries. There are also other kinds of exotic particles.

1. The existence of fermionic families suggests the existence of higher bosonic families too. If gauge bosons correspond to wormhole contacts, three families would mean that bosons are labelled by pairs (g_i, g_j) of genera associated with wormhole contacts and $U(3)$ dynamical gauge symmetry emerges naturally. The observed gauge bosons would correspond to SU(3) singlets which do not induced genus changing transitions. The new view about particle decay as a branching of partonic 2-surface is consistent with this picture but not the earlier stringy view. Only three fermion families are predicted if $g > 2$ topologies for partonic 2-surfaces correspond to free many-handle states rather than bound states as for $g < 3$ topologies: how this could happen is discussed in [F1].

2. If ground state conformal weights are identified as squares of the generalized eigenvalues of modified Dirac operator D_{C-S} [F2] analogous to cyclotron energies in Kähler magnetic field. This suggests that the spectrum of ground state conformal weights for a given region of light-like 3-surface X_l^3 inside which the induced Kähler form is non-vanishing, is of form $\lambda_n = -n-1/2+x$, $n \geq 0$, x small. Standard model fermions would correspond to $n = 0$ and there would be a finite number of higher excitations of fermions corresponding to $n > 0$. The mass spectrum of these is in principle calculable by p-adic thermodynamics.

3. Also p-adically scaled up copies of various particles are possible as well as scaled-up/scaled-down versions of QCD associated with both quarks [F8] and colored leptons: there is now evidence for color variants of all lepton families [F7]. There is also evidence that neutrino masses depend on environment [44]: this dependence could have an explanation in terms of topological condensation occurring in several p-adic length scales.

4. Dark matter hierarchy based on the spectrum of Planck constants [A9] infinite number of zoomed up copies of ordinary elementary particles with same mass spectrum.

5. Electro-weak doublet Higgs particle would be present in the spectrum contrary to the long held beliefs. Also $q - \overline{q}$ bound states of M_{89} hadron physics such that quark and anti-quark have parallel spins and relative angular momentum $L = 1$ could mimic scalar mesons. The effective couplings of these states to leptons and quarks could mimic the couplings of Higgs boson to some degree. Scalar bound states of heavy quarks are also present in ordinary hadron physics.

To sum up, the results of the calculations provide a considerable support for TGD and the notion p-adicization. What remains still to be understood are the values of some integer valued small parameters fixed completely by the empirical constraints. The detailed analysis and application of the results to derive information about hadron masses is left to the next chapter.

3.6 Appendix

The appendix has become somewhat obsolete because of the dramatic simplifications in the construction of states. I have however decided to still keep it.

3.6.1 Gauge invariant states in color sector

The construction of states satisfying Super Virasoro and Kac Moody gauge conditions for various values of conformal weight is essential ingredient in the calculation of degeneracies for various values of mass squared operator in order to estimate thermal mass expectation value. If one has obtained the multiplicities of various representations with weight n then it is easy to calculate the multiplicities for the states satisfying Super Virasoro conditions and Kac Moody conditions. Kac Moody conditions are implied by Super Virasoro conditions since $T^{a,n} \propto [L^n, T^{a,0}]$ holds true so that only Super Virasoro conditions need to be taken into account. If the gauge conditions associated with $G^{1/2}$ and $G^{3/2}$ in N-S representation induce surjective maps to the levels $n-1/2$,$n-1$ and $n-2$ then the multiplicity of gauge invariant representation is given by $m = m(n) - m(n-1/2) - m(n-3/2)$. In Ramond sector the gauge conditions for L^1 and G^1 guarantee the remaining gauge conditions and one has $m = m(n) - 2m(n-1)$ under similar assumptions.

The construction of the gauge invariant states relies on the following observations.

1. The states at each level n (conformal weight) of color Kac Moody algebra can be classified into irreducible representations of color group. The states are created by the monomials $O(F)$ of the 'fermionic' generators $F^{A,k}$, which can be regarded as an element of Grassmann algebra generated by $F^{A,k}$. The monomials of $F^{A,1/2}$ satisfy the gauge conditions of the bosonic Kac Moody identically.

2. The operators $O(F)$ creating nonzero norm sates can be classified into irreducible representations of the color group. The basic building blocks are the representations defined by N:th order monomials of generators F^{Ak} with k fixed. These representations are completely antisymmetrized tensor products of $N = 0, 1, ..., 8$ octets and representation content is same for all values of k. The representation content can be coded into multiplicity vector $m(N; k)$, $k = 1, 8, 10, ...$

3. Once the representation contents for antisymmetrized tensor products are known in terms of multiplicity vectors, the representation contents for tensor products of N_1, k_1 and N_2, k_2 can be determined by standard tensor product construction since anticommutativity does not produce no effects for $k_1 \neq k_2$. One can express the multiplicity vector for the tensor product $(N_1, k_1) \otimes (N_2, k_2)$ in terms of the multiplicity vector $D(k_1, k_2, k_3)$ for the tensor product of irreducible representations $k_1, k_2 = 1, 8, 10, ...$

$$m((N_1, k_1) \otimes (N_2, k_2; k) = m(N_1; k_1)D(k_1, k_2, k_3)m(N_2; k_2) \ . \tag{3.6.1}$$

4. It is useful to calculate total multiplicity vector $m(n; k)$ for each conformal weight n by considering all possible states having this conformal weight. The multiplicity vector is just the sum of multiplicity vectors of various tensor products satisfying $\sum N_i k_i = N$:

$$\begin{aligned} m(n; k) &= \sum_{S=N} m((N_1, k_1) \otimes .... \otimes (N_r, k_r; k)) \ , \\ S &\equiv \sum N_i k_i \ . \end{aligned} \tag{3.6.2}$$

   The multiplicity vectors $m(n; k)$ are basic objects in the systematic construction of tensor products of several Super Virasoro algebras.

Multiplicity vectors for antisymmetric tensor products

Consider first the construction of N-fold antisymmetric tensor products of octets F^{Ak}, k fixed. The tensor products are obviously analogous to the antisymmetric tensors of 8-dimensional space. The completely antisymmetric 8-dimensional permutation symbol $\epsilon_{A_1,....,A_8}$ transforms as color singlet and induces duality operation in the set of antisymmetric representations: the antisymmetric representations N are mapped to representations $8 - N$. This implies that the representation contents are same for $N = 0$ and 8, $N = 1$ and 7, $N = 2$ and $N = 6$, $N = 3$ and $N = 5$ respectively. $N = 4$ is self dual. It is relatively easy to determine the representation content of the lowest completely antisymmetric representations and the results can be summarized conveniently as multiplicity vectors defined as

$$\begin{aligned} \bar{m} &\equiv (m(1), m(8), m(10), m(\bar{10}), m(27), m(28), m(\bar{28}), \\ & m(64), m(81), m(\bar{81}), m(125), ...) \end{aligned} \tag{3.6.3}$$

The multiplicity vectors are given by the following formulas

$$\begin{aligned} \bar{m}(F) = \bar{m}(F^7) &= (0, 1) \ , \\ \bar{m}(F^2) = \bar{m}(F^6) &= (0, 1, 1, 1) \ , \\ \bar{m}(F^3) = \bar{m}(F^5) &= (1, 1, 1, 1, 1) \ , \\ \bar{m}(F^4) &= (0, 2, 0, 0, 2) \ , \end{aligned} \tag{3.6.4}$$

where F^N denotes N:th tensor power of F^{Ak}.

Multiplicity vectors for general states

The next task is to calculate multiplicity vectors for various conformal weights. The task is straightforward application of Young Tableaux. The representation contents for various conformal weights for N-S algebra are given by

$$
\begin{aligned}
n &= 0 : 1 \\
n &= 1/2 : 1/2 \\
n &= 1 : (1/2)^2) \\
n &= 3/2 : 3/2 \oplus (1/2)^3 \\
n &= 2 : (3/2) \otimes (1/2) \oplus (1/2)^4 \\
n &= 5/2 : 5/2 \oplus (3/2) \otimes (1/2)^2 \oplus (1/2)^5 \\
n &= 3 : (5/2) \otimes (1/2) \oplus (3/2)^2 \oplus (3/2) \otimes (1/2)^3 \oplus (1/2)^6 \\
n &= 7/2 : 7/2 \oplus (5/2) \otimes (1/2)^2 \oplus (3/2)^2 \otimes (1/2) \oplus 3/2 \otimes (1/2)^4 \oplus (1/2)^7 \\
n &= 4 : (7/2) \otimes (1/2) \oplus (5/2) \otimes (3/2) \oplus (5/2) \otimes (1/2)^3 \oplus (3/2)^2 \otimes (1/2)^2 ... \\
&\oplus (3/2) \otimes (1/2)^5 \oplus (1/2)^8 \\
n &= 9/2 : 9/2 \oplus (7/2) \otimes (1/2)^2 ... \\
&\oplus (5/2) \otimes (3/2) \otimes (1/2) \oplus 5/2 \otimes (1/2)^4 \oplus (3/2)^3 .... \\
&\oplus (3/2)^2 \otimes (1/2)^3 \oplus (3/2) \otimes (1/2)^6
\end{aligned}
\tag{3.6.5}
$$

Multiplicity vectors obtained as sums of multiplicity vectors associated with summands in the direct sum composition and are given by the following table

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	81	$\overline{81}$
0	1											
1/2		1										
1		1	1	1								
3/2	1	2	1	1	1							
2	1	4	1	1	3							
5/2	2	6	3	3	4							
3	2	10	6	6	6			2	2	1		
7/2	4	16	8	8	12			4	4	2		
4	8	24	12	12	21	1	1	7	7	4		
9/2	10	36	21	21	32	1	1	12	12	8	1	1

Table 5. Multiplicity vectors for various conformal weights for N-S type Super Virasoro algebra.

Similar arguments can be used to deduce the multiplicity vectors in case of Ramond type Super Virasoro algebra.

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	80	$\overline{80}$	81	$\overline{81}$	125
0	1														
1		1													
2		1	1	1											
3	2	4	2	2	2										
4	2	10	4	4	6			1	1						
5	6	20	10	10	14			4	4	1					
6	12	40	22	22	32	1	1	10	10	6					
7	17	68	36	36	55	1	1	20	20	11			1	1	
8	33	124	70	70	113	5	5	44	44	29			5	5	1
9	70	276	170	170	276	16	16	122	122	94	1	1	22	22	6

Table 6. Multiplicity vectors for various conformal weights for Ramond type Super Virasoro algebra.

Multiplicity vectors for conformally invariant states

Multiplicity vectors for gauge invariant states are obtained from the formulas $m(n) \rightarrow m(n) - m(n - 1/2) - m(n-3/2)$ and $m(n) \rightarrow m(n) - 2m(n-1)$. The inspection of the above tables gives following tables for the multiplicity vectors of gauge invariant states needed in the mass calculations to find the possible ground states.

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	81	$\overline{81}$
0	1											
1/2		1										
1		1	1	1								
3/2		1		2	1							
2	1	1		1	2							
5/2	1	1	1	1	1							
3	1	2	2	1	1			2	2	1		
7/2		2	1	3	2			2	2	1		
4	2	2	1	3	2	1	1	3	3	2		
9/2		2	3	5	1			3	3	3	1	1

Table 7. Multiplicity vectors for the conformal weights of gauge invariant states for N-S type Super Virasoro algebra.

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	80	$\overline{80}$	81	$\overline{81}$	125
0	1														
1		1													
2			1	1											
3	2	2			2										
4		2			2			1	1						
5	2		2	2	2			2	2	1					
6			2	2	4	1	1	2	2	4					
7													1	1	
8					3	3	3	4	4	7			3	3	1
9	4	28	30	30	50	6	6	34	34	36	1	1	12	12	4

Table 8. Multiplicity vectors for various conformal weights of gauge invariant states of Ramond type Super Virasoro algebra.

3.6.2 Number theoretic auxiliary results

The ground state degeneracies for fermions and bosons need not to be identical to their ideal values $D = 64$ and $D = 16$ and it is of interest to find under what conditions the degeneracy can be said to be near to its ideal value. This amounts to calculating the p-adic inverse of the D in general case. Mathematica provides means for calculating modular inverses as well as modular powers (also fractional assuming that they exists). Despite this it is useful show how the real counterpart of a fractional p-adic number can be deduced.

The calculation of the modular inverse goes as follows.

1. The problem is to find the lowest order term in p-adic expansion of the inverse y of p-adic number $x \in 1, ...p-1$. The remaining terms in expansion in powers of p can be found iteratively. The equation to be solved is

$$yx = 1 \; mod \; p \; , \quad (3.6.6)$$

for a given value of x, which gives $y = mp + 1$.

2. One can express p in the form

$$p = Nx + r \ . \tag{3.6.7}$$

The evaluation of N and $r \in \{1, .., x-1\}$ is a straightforward exercise in modulo arithmetics. The defining equation for y can be written as

$$yx = m(Nx + r) + 1 = mNx + mr + 1 \ . \tag{3.6.8}$$

From this one must have

$$mr + 1 = kx \ , \tag{3.6.9}$$

and any pair (m, k) satisfying this condition gives solution to y:

$$y = mN + k \ . \tag{3.6.10}$$

y must be chosen to be the smallest possible one.

Consider as examples two practical cases.

1. $p = M_n = 2^n - 1$ and $x = 15 = 2^4 - 1$. One obtains r by substituting repeatedly $2^4 = 1 \ mod \ x$ to the expression of M_n. M_n can be written in the form $M_n = 15(2^{n-4} + 2^{n-8} + ...) + r$ and the previous condition reads $mr + 1 = 15k$.
   i) For M_{89} one has $r = 1$ and $(m, k) = (14, 1)$ giving $y = 14(2^{n-4} + 2^{n-8} + ...) + 1$. For the real counterpart of $Xp^2/2D$ one has the approximate expression $(7X \ mod \ 16)/15$ and approximately N-S mass formula for small quantum numbers results.
   ii) For M_{127} and M_{107} one has $r = 7$ and $7m + 1 = 15k$ gives $(m, k) = 2, 1)$ and $y = 2(2^{n-4} + ...) + 1$. For $Xp^2/2D$ one has $Xmod16/15$: the factor 7 is absent.

2. $p = M^n$ and $x = 63 = 64 - 1$. One obtains r by substituting repeatedly $2^6 - 1 \ mod \ x$ to the expression of M_n. One has $r = 1$ for $n = 127$, $r = 31$ for $n = 107$ and $n = 89$. For the real counterpart R of Xp^2/D one has $R = (62X \ mod \ 64)/(63M_n)$ and $y = (60X \ mod \ 64)/(63M_n)$ for $n = 127$ and $107, 89$ respectively so that mass formulas change somewhat and in n-dependent manner if one has $D = 63$ instead of $D = 64$.

3. 1/5 factor appears in mass formulas for leptons and the previous argument leads to the expression $p^2/5 = (2^{126} - 2^{124} + 2^{122} - ..)p^2$. From this formula the real counterpart of, say 1/5, is in a good approximation 4/5. It must be emphasized that Mathematica provides the number theoretical modules for calculating the real counterparts for numbers of form rp, r rational number.

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Chapter 4

p-Adic Particle Massivation: Hadron Masses

4.1 Introduction

In this chapter the results of the calculation of elementary particle masses will be used to construct a model predicting hadron masses. The new elements are a revised identification for the p-adic length scales of quarks and the realization that number theoretical constraints on topological mixing can be realized by assuming that topological mixing leads to a thermodynamical equilibrium. This gives an upper bound of 1200 for the number of different U and D matrices and the input from top quark mass and $\pi^+ - \pi^0$ mass difference implies that physical U and D matrices can be constructed as small perturbations of matrices expressible as a direct sum of essentially unique 2×2 and 1×1 matrices.

The assumption about the presence of scaled up variants of light quarks in light hadrons leads to a surprisingly successful model for pseudo scalar meson masses in terms of only quark masses. This conforms with the idea that at least light pseudo scalar mesons are Goldstone bosons in the sense that color Coulombic and magnetic contributions to the mass cancel each other. Also the mass differences between baryons containing different numbers of strange quarks can be understood if s quark appears as three scaled up versions. The earlier model for the purely hadronic contributions to hadron masses simplifies dramatically and only the color Coulombic and magnetic contributions to color conformal weight are needed.

4.1.1 Construction of U and D matrices

The basic constraint on the topological mixing that the modular contributions to the conformal weight defining the mass squared remain integer valued in the proper units: if this condition does not hold true, the order of magnitude for the real counterpart of the p-adic mass squared corresponds to 10^{-4} Planck masses.

Number theory gives strong constraints on CKM matrix. p-Adicization requires that U and D matrix elements are algebraic numbers. A strong constraint would be that the mixing probabilities are rational numbers implying that matrices defined by the moduli of U and D involve only square roots of rationals. The phases of matrix elements should belong to a finite extension of complex rationals.

Little can be said about the details of the dynamics of topological mixing. Nothing however prevents for constructing a thermodynamical model for the mixing. A thermodynamical model for U and D matrices maximizing the entropy defined by the mixing probabilities subject to the constraints fixing the values of n_{q_i} and the sums of row/column probabilities to one gives a thermodynamical ensemble with two quantized temperatures and two quantized chemical potentials. The resulting polynomial equations allow at most 1200 different solutions so that the number of U and D matrices is relatively small. The fact that matrix elements are algebraic numbers guarantees that the matrices are continuable to p-adic number fields as required.

The detailed study of quark mass spectrum leads to a tentative identification $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$ of the modular contributions of conformal weights of quarks: note that in absence of mixing the contributions would be $(0, 9, 60)$ for both U and D type quarks. That b and t quark masses are nearly maximal and thus mix very little with lighter quarks is forced by the masses of t quark and $t\bar{t}$ meson. The values of n_{q_i} for light quarks follow by considering $\pi^+ - \pi^0$ mass difference.

One might consider the possibility that n_{q_i} for slightly dynamical and can vary in light mesons in order to guarantee that $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ give identical modular contributions to the conformal weight in states which are linear combinations of quark pairs. It turns out that unitarity does not allow the choices $(n_1 = 4, n_2 < 9)$, and that the choice $(n_d, n_s) = (5, 5)$, $(n_u, n_c) = (5, 6)$ is the unique choice producing a realistic CKM matrix. The requirement that quark contribution to pseudo scalar meson mass is smaller than meson mass is possible to satisfy and gives a constraint on CP_2 mass scale consistent with the prediction of leptonic masses when second order p-adic contribution to lepton mass is allowed to be non-vanishing.

The small mixing with b and t quarks is natural since the modular conformal weight of unmixed state having spectrum $\{0, 9, 60\}$ is analogous to energy so that Boltzmann weight for $n(g = 3)$ thermal excitation is small for $g = 1, 2$ ground states.

The maximally entropic solutions can be found numerically by using the fact that only the probabilities p_{11} and p_{21} can be varied freely. The solutions are unique in the accuracy used, which suggests that the system allows only single thermodynamical phase.

The matrices U and D associated with the probability matrices can be deduced straightforwardly in the standard gauge. The U and D matrices derived from the probabilities determined by the entropy maximization turn out to be unitary for most values of n_1 and n_2. This is a highly non-trivial result and means that mass and probability constraints together with entropy maximization define a sub-manifold of $SU(3)$ regarded as a sub-manifold in 9-D complex space. The choice $(n_u, n_c) = (4, n)$, $n < 9$, does not allow unitary U whereas $(n_u, n_c) = (5, 6)$ does. This choice is still consistent with top quark mass and together with $n_d = n_s = 5$ it leads to a rather reasonable CKM matrix with a value of CP breaking invariant within experimental limits. The elements V_{i3} and V_{3i}, $i = 1, 2$ are however roughly twice larger than their experimental values deduced assuming standard model. V_{31} is too large by a factor 1.6. The possibility of scaled up variants of light quarks could lead to too small experimental estimates for these matrix elements. The whole parameter space has not been scanned so that better candidates for CKM matrices might well exist.

4.1.2 Observations crucial for the model of hadron masses

The evolution of the model for hadron masses involves several key observations made during the more decade that I have been working with p-adic mass calculations.

The p-adic mass scales of quarks are dynamical

The existence of scaled up variants of quarks is suggested by various anomalies such as Aleph anomaly [50] and the strange bumpy structure of the distribution of the mass of the top quark candidate. This leads to the idea that the the integer $k(q)$ characterizing the p-adic mass scale of quark is different for free quarks and bound quarks and that $k(q)$ can depend on hadron. Hence one can understand not only the notions of current quark mass and constituent quark mass but reproduce also the p-adic counterpart of Gell-Mann-Okubo mass formula. Indeed, the assumption about scaled up variants of u, d, s, and even c quarks in light hadrons leads to an excellent fit of meson masses with quark contribution explaining almost all of meson mass.

Quarks give dominating contribution to the masses of pseudoscalar mesons

The interpretation is that color Coulombic and color magnetic interaction conformal weights (rather than interaction energies) cancel each other in a approximation for pseudoscalar mesons in accordance with the idea that pseudo scalar mesons are massless as far as color interactions are considered. In the case of baryons the assumption that s quark appears in three different scaled up versions (which are Λ, $\{\Sigma, \Xi\}$, and Ω) allows to understand the mass differences between baryons with different s quark content. The dominating contribution to baryon mass has however remained hitherto unidentified.

What it means that Higgs like contribution to fermion masses is negligible?

The failure of the simplest form of p-adic thermodynamics for intermediate gauge bosons led to the unsatisfactory conclusion that p-adic thermodynamics is not enough and the coupling to Higgs bosons contributes to the gauge boson masses. This option had its own problems.

1. There are good, purely topological - reasons to believe that Higgs expectation for the fermionic space-time sheets is vanishing although fermions couple to Higgs. p-Adic thermodynamics would explain fermion masses completely: this indeed turns out to be the case within experimental uncertainties. The absence of Higgs contribution to fermion masses would however mean asymmetry between fermions and bosons.

2. After the understanding of the spectrum of the modified Dirac operator it became clear that ground state conformal weight is proportional to the square of the eigenvalue of the Dirac operator, and that it is the deviation of the ground state conformal weight from negative half odd integer which is responsible for the Higgs type contribution. This contribution to the mass squared is present for both fermions and bosons but the contribution must be small for fermions and dominate for gauge bosons.

3. In the case of gauge bosons Higgs vacuum expectation is proportional to this deviation for the simple reason that there is no other fundamental parameters with dimensions of mass available. Hence the role of Higgs boson would be misunderstood in standard model framework.

The fact that the prediction of the model for the top quark mass is consistent with the most recent limits on it [58], fixes the CP_2 mass scale with a high accuracy to the maximal one obtained if second order contribution to electron's p-adic mass squared vanishes. This is very valuable constraint on the model.

Conformal weights are additive for quarks with same p-adic prime

An essential element of the new understanding is that conformal weight (mass squared is additive) for quarks with the same p-adic length scale whereas mass is additive for quarks with different values of p. For instance, the masses of heavy $q\overline{q}$ mesons are equal to $\sqrt{2} \times m(q)$ rather than $2m(q)$. Since $k = 107$ for hadronic space-time sheet, for quarks with $k(q) \neq 107$, additivity holds true for the quark and color contributions for mass rather than mass squared.

This hypothesis yields surprisingly good fit for meson masses but for some mesons the predicted mass is slightly too high. The reduction of CP_2 mass scale to cure the situation is not possible since top quark mass would become too low. In case of diagonal mesons for which quarks correspond to same p-adic prime, quark contribution to mass squared can be reduced by ordinary color interactions and in case of non-diagonal mesons one can require that quark contribution is not larger than meson mass.

A remark about terminology

Before continuing a remark about terminology is in order.

1. In the generalized coset construction the symplectic algebra of $\delta M^4_\pm \times CP_2$ and Super-Kac Moody algebras at light-like partonic surfaces X^3 are lifted to hyper-complex algebras inside the causal diamond of $M^4 \times CP_2$ carrying the zero energy states. SKM is identified as a sub-algebra of SC and the differences of SC and SKM Super-Virasoro generators annihilate the physical states. All purely geometric contributions and their super-counterparts can be regarded as SC contributions. The fermionic contributions in electro-weak and spin degrees of freedom responsible also for color partial waves are trivially one and same. One could say that there is no other contribution than SC which can be however divided into a contribution from imbedded SKM subalgebra and a genuine SC contribution.

2. In the coset construction a tachyonic ground state of negative SC conformal weight from which SKM generators create massless states must have a negative conformal weight also in SKM sense. Therefore the earlier idea that genuine SC generators create the ground states with a negative conformal weight assignable to elementary particles does not work anymore: the

negative conformal weight must be due to SKM generators with conformal weight which is most naturally of form $h = -1/2 + iy$.

3. Super-canonical contribution with a positive conformal weight can be regarded also as a product of genuine SC contribution with a vanishing conformal weight and a contribution having also interpretation as SKM contribution. What motivates the term "super-canonical bosons" used in the sequel is that in a non-perturbative situation this contribution is most naturally calculated by regarding it as a super-canonical contribution. This contribution is highly constrained since it comes solely from generators which are color octets and singlets have spin one or spin zero. Genuine SC contribution with a zero conformal weight comes from the products of super-Hamiltonians in higher representations of $SU(3) \times SO(3)$ containing both positive and negative conformal weights compensating each other. This contribution must have vanishing color quantum numbers and spin since otherwise Dirac operators of H in SKM and SC degrees of freedom could not act on it in the same manner. Note that gluons do not correspond to SKM generators but to pairs of quark and antiquark at throats of a wormhole contact.

Super-canonical bosons at hadronic space-time sheet can explain the constant contribution to baryonic masses

Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD.

A possible identification of this contribution is in terms of super-canonical gluons predicted by TGD. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-canonical gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent. Super-canonical gluons also provide a possible solution to the spin puzzle of proton.

Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-canonical quanta. If the topological mixing for super-canonical bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-canonical quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight.

In the case of mesons pion could contain super-canonical boson of first generation preventing the large negative contribution of the color magnetic spin-spin interaction to make pion a tachyon. For heavier bosons super-canonical boson is not absolutely necessary but a very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-canonical particle content.

Color magnetic spin-spin splitting formulated in terms of conformal weight

What remains to be understood are the contributions of color Coulombic and magnetic interactions to the mass squared. There are several delicate points to be taken into account.

1. The QCD based formula for the color magnetic interaction energy fails completely since the dependence of color magnetic spin-spin splittings on quark mass scale is nearer to logarithmic dependence on p-adic length scale than being of form $1/m(q_i)m(q_j) \propto L(k_i)L(k_j)$. This finding supports the decade old idea that the proper notion is not color interaction energy but color conformal weight. A model based on this assumption is constructed assuming that all pseudoscalars are Goldstone boson like states. The predictions for the masses of mesons are not so good than for baryons, and one might criticize the application of the format of perturbative QCD in an essentially non-perturbative situation.

2. The comparison of the super-canonical conformal weights associated with spin 0 and spin 1 states and spin 1/2 and spin 3/2 states shows that the different masses of these states could be understood in terms of the super-canonical particle contents of the state correlating with the total quark spin. The resulting model allows excellent predictions also for the meson masses and implies that only pion and kaon can be regarded as Goldstone boson like states. The model based on spin-spin splittings is consistent with the model.

To sum up, the model provides an excellent understanding of baryon and meson masses. This success is highly non-trivial since the fit involves only the integers characterizing the p-adic length scales of quarks and the integers characterizing color magnetic spin-spin splitting plus p-adic thermodynamics and topological mixing for super-canonical gluons. The next challenge would be to predict the correlation of hadron spin with super-canonical particle content in the case of long-lived hadrons.

4.1.3 A possible model for hadron

These findings suggest that the following model for hadrons deserves a testing. Hadron can be characterized in terms of $k \geq 113$ partonic 2-surfaces $X^2(q_i)$ connected by join along boundaries bonds (JABs, flux tubes) to $k = 107$ 2-surface $X^2(H)$ corresponding to hadron. These flux tubes which for $k = 113$ have size much larger than hadron can be regarded as "field bodies" of quarks which themselves have sub-hadronic size. Color flux tubes between quarks are replaced with pairs of flux tubes from $X^2(q_1)) \rightarrow X^2(H) \rightarrow X^2(q_2)$ mediating color Coulombic and magnetic interactions between quarks. In contrast to the standard model, mesons are characterized by two flux tubes rather than only one flux tube. Certainly this model gives nice predictions for hadron masses and even the large color Coulombic contribution to baryon masses can be deduced from $\rho - \pi$ mass splitting in a good approximation.

4.2 Quark masses

The prediction or quark masses is more difficult due the facts that the deduction of even the p-adic length scale determining the masses of these quarks is a non-trivial task, and the original identification was indeed wrong. Second difficulty is related to the topological mixing of quarks. The new scenario leads to a unique identification of masses with top quark mass as an empirical input and the thermodynamical model of topological mixing as a new theoretical input. Also CKM matrix is predicted highly uniquely.

4.2.1 Basic mass formulas

By the earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as a unit, color conformal weight $h_c \equiv m_{CP_2}^2$ is integer for $p\ mod\ = \pm 1$ for U type quark belonging to $(p+1, p)$ type representation and obeying $h_c(U) = (p^2+3p+2)/3$ and for $p\ mod\ 3 = 1$ for D type quark belonging $(p, p+2)$ type representation and obeying $h_c(D) = (p^2+4p+4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

In the recent case the minimal $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This requires super-canonical operators O with a net conformal weight $h_{sc} = -3$ to compensate the anomalous color just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-canonical operator is same for both quarks and leptons suggest that the construction is not had hoc.

Consider now the mass squared values for quarks. For $h(D) = 0$ and $h(U) = -1$ and using $m_0^2/3$ as a unit the expression for the thermal contribution to the mass squared of quark is given by the formula

$$\begin{aligned} M^2 &= (s+X)\frac{m_0^2}{p} \ , \\ s(U) &= 5 \ , \ s(D) = 8 \ , \\ X &\equiv \frac{(3Yp)_R}{3} \ , \end{aligned} \tag{4.2.1}$$

where the second order contribution Y corresponds to renormalization effects coming and depending on the isospin of the quark.

With the above described assumptions one has the following mass formula for quarks

$$
M^2(q) = A(q)\frac{m_0^2}{p(q)} \ ,
$$

$$
\begin{array}{lll}
A(u) = 5 + X_U(p(u) \ , & A(c) = 14 + X_U(p(c)) \ , & A(t) = 65 + X_U(p(t)) \ , \\
A(d) = 8 + X_D(p(d)) \ , & A(s) = 17 + X_D(p(s)) \ , & A(b) = 68 + X_D(p(b)) \ .
\end{array}
\tag{4.2.2}
$$

p-Adic length scale hypothesis allows to identify the p-adic primes labelling quarks whereas topological mixing of U and D quarks allows to deduce topological mixing matrices U and D and CKM matrix V and precise values of the masses apart from effects like color magnetic spin orbit splitting, color Coulombic energy, etc..

Integers n_{q_i} satisfying $\sum_i n(U_i) = \sum_i n(D_i) = 69$ characterize the masses of the quarks and also the topological mixing to high degree. The reason that modular contributions remain integers is that in the p-adic context non-trivial rationals would give CP_2 mass scale for the real counterpart of the mass squared. In the absence of mixing the values of integers are $n_d = n_u = 0$, $n_s = n_c = 9$, $n_b = n_t = 60$.

The fact that CKM matrix V expressible as a product $V = U^\dagger D$ of topological mixing matrices is near to a direct sum of 2×2 unit matrix and 1×1 unit matrix motivates the approximation $n_b \simeq n_t$.

The model for topological mixing matrices and CKM matrix predicts U and D matrices highly uniquely and allows to understand quark and hadron masses in surprisingly detailed level.

The large masses of top quark and of $t\bar{t}$ meson encourage to consider a scenario in which $n_t = n_b = n \leq 60$ holds true.

1. $n_d = n_u = 60$ is not allowed by number theoretical conditions for U and D matrices and by the basic facts about CKM matrix but $n_t = n_b = 59$ allows almost maximal masses for b and t. This is not yet a complete hit. The unitarity of the mixing matrices and the construction of CKM matrix to be discussed in the next section forces the assignments

$$
(n_d, n_s, n_b) = (5, 5, 59) \ , \quad (n_u, n_c, n_t) = (5, 6, 58) \ . \tag{4.2.3}
$$

fixing completely the quark masses apart from a possible few per cent renormalization effects of hadronic mass scale in topological condensation which seem to be present and will be discussed later [1]. Note that top quark mass is still rather near to its maximal value.

2. The constraint that quark contribution to pion mass does not exceed pion mass implies the constraint $n(d) \leq 6$ and $n(u) \leq 6$ in accordance with the predictions of the model of topological mixing. It is important to notices that $u - d$ mass difference does not affect $\pi^+ - \pi^0$ mass difference and the quark contribution to $m(\pi)$ is predicted to be $\sqrt{(n_d + n_u + 13)/24} \times 136.9$ MeV for the maximal value of CP_2 mass (second order p-adic contribution to electron mass squared vanishes).

4.2.2 The p-adic length scales associated with quarks and quark masses

The identification of p-adic length scales associated with the quarks has turned to be a highly non-trivial problem. The reasons are that for light quarks it is difficult to deduce information about quark masses for hadron masses and that the unknown details of the topological mixing (unknown until the advent of the thermodynamical model) made possible several p-adic length scales for quarks. It has also become clear that the p-adic length scale can be different form free quark and bound quark and that bound quark p-adic scale can depend on hadron.

Two natural constraints have however emerged from the recent work.

[1] As this was written I had not realized that there is also a Higgs contribution which tends to increase top quark mass

1. Quark contribution to the hadron mass cannot be larger than color contribution and for quarks having $k_q \neq 107$ quark contribution to mass is added to color contribution to the mass. For quarks with same value of k conformal weight rather than mass is additive whereas for quarks with different value of k masses are additive. An important implication is that for diagonal mesons $M = q\overline{q}$ having $k(q) \neq 107$ the condition $m(M) \geq \sqrt{2}m_q$ must hold true. This gives strong constraints on quark masses.

2. The realization that scaled up variants of quarks explain elegantly the masses of light hadrons allows to understand large mass splittings of light hadrons without the introduction of strong isospin-isospin interaction.

The new model for quark masses is based on the following identifications of the p-adic length scales.

1. The nuclear p-adic length scale $L(k)$, $k = 113$, corresponds to the p-adic length scale determining the masses of u, d, and s quarks. Note that $k = 113$ corresponds to a so called Gaussian Mersenne. The interpretation is that quark massivation occurs at nuclear space-time sheet at which quarks feed their em fluxes. At $k = 107$ space-time sheet, where quarks feed their color gauge fluxes, the quark masses are vanishing in the first p-adic order. This could be due to the fact that the p-adic temperature is $T_p = 1/2$ at this space-time sheet so that the thermal contribution to the mass squared is negligible. This would reflect the fact that color interactions do not involve any counterpart of Higgs mechanism.

   p-Adic mass calculations turn out to work remarkably well for massive quarks. The reason could be that M_{107} hadron physics means that *all* quarks feed their color gauge fluxes to $k = 107$ space-time sheets so that color contribution to the masses becomes negligible for heavy quarks as compared to Super-Kac Moody and modular contributions corresponding to em gauge flux feeded to $k > 107$ space-time sheets in case of heavy quarks. Note that Z^0 gauge flux is feeded to space-time sheets at which neutrinos reside and screen the flux and their size corresponds to the neutrino mass scale. This picture might throw some light to the question of whether and how it might be possible to demonstrate the existence of M_{89} hadron physics.

   One might argue that $k = 107$ is not allowed as a condensation level in accordance with the idea that color and electro-weak gauge fluxes cannot be feeded at the space-time space time sheet since the classical color and electro-weak fields are functionally independent. The identification of η' meson as a bound state of scaled up $k = 107$ quarks is not however consistent with this idea unless one assumes that $k = 107$ space-time sheets in question are separate.

2. The requirement that the masses of diagonal pseudoscalar mesons of type $M = q\overline{q}$ are larger but as near as possible to the quark contribution $\sqrt{2}m_q$ to the valence quark mass, fixes the p-adic primes $p \simeq 2^k$ associated with c, b quarks but not t since toponium does not exist. These values of k are "nominal" since k seems to be dynamical. c quark corresponds to the p-adic length scale $k(c) = 104 = 2^3 \times 13$. b quark corresponds to $k(b) = 103$ for $n(b) = 5$. Direct determination of p-adic scale from top quark mass gives $k(t) = 94 = 2 \times 47$ so that secondary p-adic length scale is in question.

3. Top quark is experimentally in a unique position since toponium does not exist and top quark mass is that of free top. The prediction for top quark mass (see Table 1 below) is 167.8 GeV for $Y_t = Y_e = 0$ (second order contributions to mass vanish) and 169.1 GeV for $Y_t = 1$ and $Y_e = 0$ (maximal possible mass for top). The experimental estimate for m_t remained for a long time somewhat higher than the prediction but the estimates have gradually reduced. The previous experimental average value was $m(t) = 169.1$ GeV with the allowed range being [164.7, 175.5] GeV [58, 61]. The fine tuning $Y_e = 0, Y_t = 1$ giving 169.1 GeV is somewhat un-natural. The most recent value obtained by CDF and discussed in detail by Tommaso Dorigo [60] is $m_t = 165.1 \pm 3.3 \pm 3.1$ GeV. This is value is consistent with the lower bound predicted by TGD for $Y_e = Y_t = 0$ and increase of Y_t increases the value of the predicted mass. Clearly, TGD passes the stringent test posed by top quark.

4. There are good reasons to believe that the p-adic mass scale of quark is different for free quark and bound state quark and that in case of bound quark it can also depend on hadron. This

would explain the notions of valence (constituent) quark and current quark mass as masses of bound state quark and free quark and leads also to a TGD counterpart of Gell-Mann-Okubo mass formula.

1. Constituent quark masses

Constituent quark masses correspond to masses derived assuming that they are bound to hadrons. If the value of k is assumed to depend on hadron one obtains nice mass formula for light hadrons as will be found later. The following table summarizes constituent quark masses labelled by k_q deduced from the masses of diagonal mesons.

q	d	u	s	c	b	t
n_q	4	5	6	6	59	58
s_q	12	10	14	11	67	63
$k(q)$	113	113	113	104	103	94
$m(q)/GeV$	.105	.0923	.105	2.191	7.647	167.8

Table 1. Constituent quark masses predicted for diagonal mesons assuming $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$, maximal CP_2 mass scale($Y_e = 0$), and vanishing of second order contributions.

2. Current quark masses

Current quark masses would correspond to masses of free quarks which tend to be lower than valence quark masses. Hence k could be larger in the case of light quarks. The table of quark masses in Wikipedia [61] gives the value ranges for current quark masses depicted in the table below together with TGD predictions for the spectrum of current quark masses.

q	d	u	s
$m(q)_{exp}/MeV$	4-8	1.5-4	80-130
$k(q)$	(122,121,120)	(125,124,123,122)	(114,113,112)
$m(q)/MeV$	(4.5,6.6,9.3)	(1.4,2.0,2.9,4.1)	(74,105,149)
q	c	b	t
$m(q)_{exp}/MeV$	1150-1350	4100-4400	1691
$k(q)$	(106,105)	(105,104)	92
$m(q)/MeV$	(1045,1477)	(3823,5407)	167.8

Table 2. The experimental value ranges for current quark masses [61] and TGD predictions for their values assuming $(n_d, n_s, n_b) = (5, 5, 59)$, $(n_u, n_c, n_t) = (5, 6, 58)$, $Y_e = 0$, and vanishing of second order contributions.

Some comments are in order.

1. The long p-adic length associated with light quarks seem to be in conflict with the idea that quarks have sizes smaller than hadron size. The paradox disappears when one realized that $k(q)$ characterizes the electromagnetic "field body" of quark having much larger size than hadron.

2. u and d current quarks correspond to a mass scale not much higher than that of electron and the ranges for mass estimates suggest that u could correspond to scales $k(u) \in (125, 124, 123, 122) = (5^3, 4\times 31, 3\times 41, 2\times 61)$, whereas d would correspond to $k(d) \in (122, 121, 120) = (2\times 61, 11^2, 3\times 5 \times 8)$.

3. The TGD based model for nuclei based on the notion of nuclear string leads to the conclusion that exotic copies of $k = 113$ quarks having $k = 127$ are present in nuclei and are responsible for the color binding of nuclei [F8, F9].

4. The predicted values for c and b masses are slightly too low for $(k(c), k(b)) = (106, 105) = (2 \times 53, 3 \times 5 \times 7)$. Second order Higgs contribution could increase the c mass into the range given in [61] but not that of b.

4.2.3 Are scaled up variants of quarks also there?

The following arguments suggest that p-adically scaled up variants of quarks might appear not only at very high energies but even in low energy hadron physics.

Aleph anomaly and scaled up copy of b quark

The prediction for the b quark mass is consistent with the explanation of the Aleph anomaly [50] inspired by the finding that neutrinos seem to condense at several p-adic length scales [44]. If b quark condenses at $k(b) = 97$ level, the predicted mass is $m(b, 97) = 52.3$ GeV for $n_b = 59$ for the maximal CP_2 mass consistent with η' mass. If the the mass of the particle candidate is defined experimentally as one half of the mass of resonance, b quark mass is actually by a factor $\sqrt{2}$ higher and scaled up b corresponds to $k(b) = 96 = 2^5 \times 3$. The prediction is consistent with the estimate 55 GeV for the mass of the Aleph particle and gives additional support for the model of topological mixing. Also the decay characteristics of Aleph particle are consistent with the interpretation as a scaled up b quark.

Scaled variants of top quark

Tony Smith has emphasized the fact that the distribution for the mass of the top quark candidate has a clear structure suggesting the existence of several states, which he interprets as excited states of top quark [53]. According to the figures 4.2.3 and 4.2.3 representing published FermiLab data, this structure is indeed clearly visible.

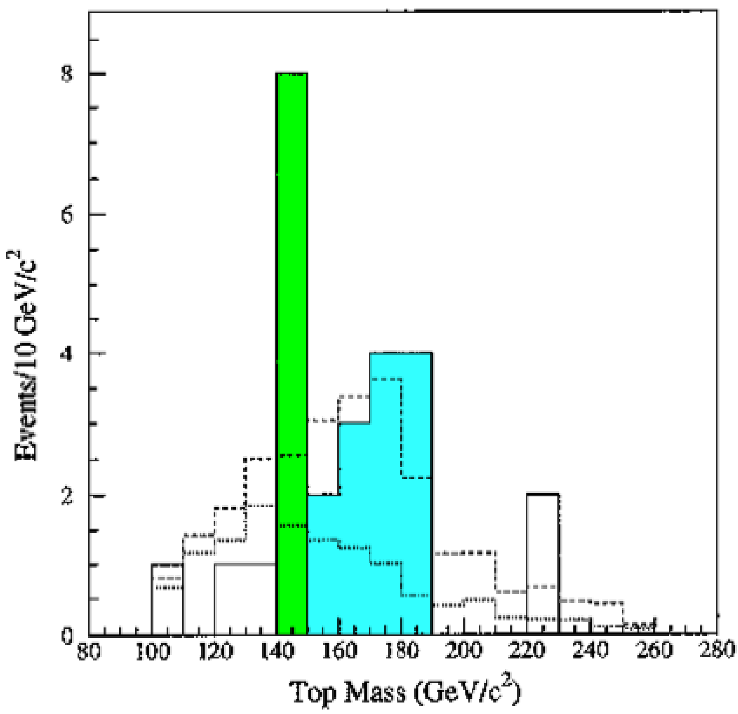

Figure 4.1: Fermilab semileptonic histogram for the distribution of the mass of top quark candidate (FERMILAB-PUB-94/097-E).

There is evidence for a sharp peak in the mass distribution of the top quark in 140-150 GeV range (Fig. 4.2.3). There is also a peak slightly below 120 GeV, which could correspond to a p-adically scaled down variant t quark with $k = 95$ having mass 119.6 GeV for $(Y_e = 0, Y_t = 1)$ There is also a small peak also around 265 GeV which could relate to $m(t(93)) = 240.4$ GeV. There top could appear at least for the p-adic scales $k = 93, 94, 95$ as also u and d quarks seem to appear as current quarks.

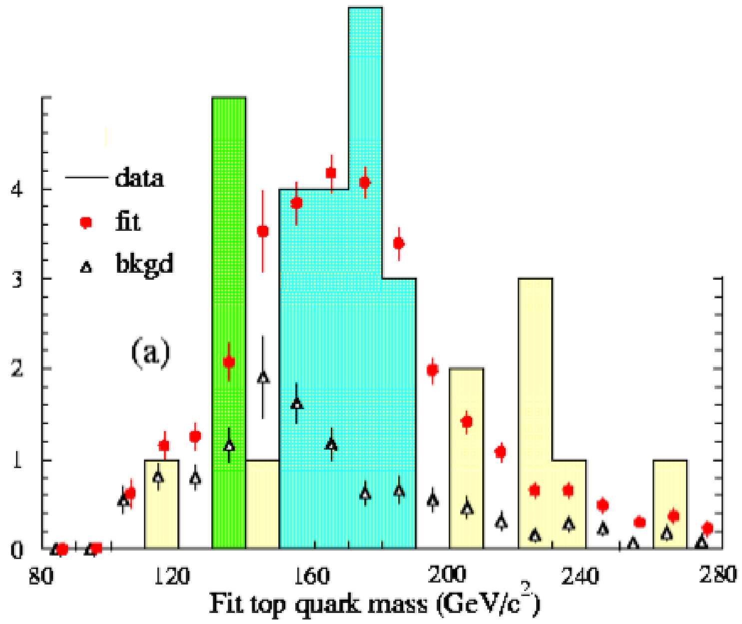

Figure 4.2: Fermilab D0 semileptonic histogram for the distribution of the mass of top quark candidate (hep-ex/9703008, April 26, 1994

Scaled up variants of d, s, u, c in top quark mass scale

The fact that all neutrinos seem to appear as scaled up versions in several scales, encourages to look whether also u, d, s, and c could appear as scaled up variants transforming to the more stable variants by a stepwise increase of the size scale involving the emission of electro-weak gauge bosons. In the following the scenario in which t and b quarks mix minimally is considered.

q	$m(92)/GeV$	$m(91)/GeV$	$m(90)/GeV$
u	134	189	267
d	152	216	304
c	140	198	280
s	152	216	304

Table 3. The masses of $k = 92, 91$ and $k = 90$ scaled up variants of u,d,c,s quarks assuming same integers n_{q_i} as for ordinary quarks in the scenario $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$ and maximal CP_2 mass consistent with the η' mass.

1. For $k = 92$, the masses would be $m(q, 92)$ =134,140,152,152 GeV in the order q= u,c,d,s so that all these quarks might appear in the critical region where the top quark mass has been wandering.

2. For $k = 91$ copies would have masses $m(q, 91)$ =189, 198, 256, 256 GeV in the order q= u,c,d,s. The masses of u and c are somewhat above the value of latest estimate 170 GeV for top quark mass [58].

Note that it is possible to distinguish between scaled up quarks of M_{107} hadron physics and the quarks of M_{89} hadron physics since the unique signature of M_{89} hadron physics would be the increase of the scale of color Coulombic and magnetic energies by a factor of 512. As will be found, this allows to estimate the masses of corresponding mesons and baryons by a direct scaling. For instance, M_{89} pion and nucleon would have masses 71.7 GeV and 481 GeV.

It must be added that the detailed identifications are sensitive to the exact value of the CP_2 mass scale. The possibility of at most 2.5 per cent downward scaling of masses occurs is allowed by the recent value range for top quark mass.

Fractally scaled up copies of light quarks and low mass hadrons?

One can of course ask, whether the fractally scaled up quarks could appear also in low lying hadrons. The arguments to be developed in detail later suggest that u, d, and s quark masses could be dynamical in the sense that several fractally scaled up copies can appear in low mass hadrons and explain the mass differences between hadrons.

In this picture the mass splittings of low lying hadrons with different flavors would result from fractally scaled up excitations of s and also u and d quarks in case of mesons. This notion would also throw light into the paradoxical presence of two kinds of quark masses: constituent quark masses and current quark masses having much smaller values than constituent quarks masses. That color spin-spin splittings are of same order of magnitude for all mesons supports the view that color gauge fluxes are feeded to $k = 107$ space-time sheet.

The alert reader has probably already asked whether also proton mass could be understood in terms of scaled up copies of u and d quarks. This does not seem to be the case, and an argument predicting with 23 per cent error proton mass scale from $\rho - \pi$ and $\Delta - N$ color magnetic splittings emerges.

To sum up, it seems quite possible that the scaled up quarks predicted by TGD have been observed for decade ago in FermiLab about that the prevailing dogmas has led to their neglect as statistical fluctuations. Even more, scaled up variants of s quarks might have been in front of our eyes for half century! Phenomenon is an existing phenomenon only if it is an understood phenomenon.

The mystery of two Ω_b baryons

Tommaso Dorigo has three interesting postings [64] about the discovery of Ω_b baryon containing two strange quarks and one bottom quark. Ω_b has been discovered -even twice. This is not a problem. The problem is that the masses of these Ω_bs differ quite too much. D0 collaboration discovered Ω_b with a significance of 5.4 sigma and a mass of 6165 ± 16.4 MeV [65]. Later CDF collaboration announced the discovery of the same particle with a significance of 5.5 sigma and a mass of 6054.4 ± 6.9 MeV. Both D0 and CDF agree that the particle is there at better than 5 sigma significance and also that the other collaboration is wrong. They cant both be right Or could they? In some other Universe that that of standard model and all its standard generalizations, maybe in some less theoretically respected Universe, say TGD Universe?

The mass difference between the two Ω_b candidates is 111 MeV, which represents the mass scale of strange quark. TDG inspired model for quark masses relies on p-adic thermodynamics and predicts that quarks can appear in several p-adic mass scales forming a hierarchy of half octaves - in other words mass scales comes as powers of square root of two. This property is absolutely essential for the TGD based model for masses of even low lying baryons and mesons where strange quarks indeed appear with several different p-adic mass scales. It also explains the large difference of the mass scales assigned to current quarks and constituent quarks. Light variants of quarks appear also in nuclear string model where nucleons are connected by color bonds containing light quark and antiquark at their ends.

Ω_b contains two strange quarks and the mass difference between the two candidates is of order of mass of strange quark. Could it be that both Ω_b s are real and the discrepancy provides additional support for p-adic length scale hypothesis? The prediction of p-adic mass calculations for the mass of s quark is 105 MeV (see Table 1) so that the mass difference can be understood if the second s-quark in Ω_b has mass which is twice the "standard" value. Therefore the strange finding about Ω_b could give additional support for quantum TGD. Before buying a bottle of champaigne, one should however understand why D0 and CDF collaborations only one Ω_b instead of both of them.

4.3 Topological mixing of quarks

The requirement that hadronic mass spectrum is physical requires mixing of U and D type boundary topologies. In this section quark masses and the mixing of the boundary topologies are considered on the general level and CKM matrix is derived using the existing empirical information plus the constraints on the quark masses to be derived from the hadronic mass spectrum in the later sections.

4.3.1 Mixing of the boundary topologies

In TGD the different mixings of the boundary topologies for U and D type quarks provide the fundamental mechanism for CKM mixing and also CP breaking. In the determination of CKM matrix one can use following conditions.

1. Mass squared expectation values in order $O(p)$ for the topologically mixed states must be integers and the study of the hadron mass spectrum leads to very stringent conditions on the values of these integers. Physical values for these integers imply essentially correct value for Cabibbo angle provided U and D matrices differ only slightly from the mixing matrices mixing only the two lowest generations.

2. The matrices U and D describing the mixing of U and D type boundary topologies are unitary in the p-adic sense. The requirement that the moduli squared of the matrix elements are rational numbers, is very attractive since it suggests equivalence of p-adic and real probability concepts and therefore could solve some conceptual problems related to the transition from the p-adic to real regime. It must be however immediately added that rationality assumption for the probabilities defined by S-matrix turns out to be non-physical. It turns out that the mixing scenario reproducing a physical CKM matrix is consistent with the rationality of the moduli squared of the matrix elements of U and D matrices but not with the rationality of the matrix elements themselves. The phase angles appearing in U and D matrix can be rational and in this case they correspond to Pythagorean triangles. In principle the rationality of the CKM matrix is possible.

3. The requirements that Cabibbo angle has correct value and that the elements $V(t,d)$ and $V(u,b)$ of the CKM matrix have small values not larger than 10^{-2} fixes the integers n_i characterizing quark masses to a very high degree and in a good approximation one can estimate the angle parameters analytically. remains open at this stage. The requirement of a realistic CKM matrix leads to a scenario for the values of n_i, which seems to be essentially unique.

The mass squared constraints give for the D matrix the following conditions

$$\begin{aligned}
9|D_{12}|^2+60|D_{13}|^2 &= n_1(D)\equiv n_d \ , \\
9|D_{22}|^2+60|D_{23}|^2 &= n_2(D)\equiv n_s \ , \\
9|D_{32}|^2+60|D_{33}|^2 &= n_3(D)\equiv n_b = 69-n_2(D)-n_1(D) \ .
\end{aligned} \tag{4.3.1}$$

The third condition is not independent since the sum of the conditions is identically true by unitarity.

For U matrix one has similar conditions:

$$\begin{aligned}
9|U_{12}|^2+60|U_{13}|^2 &= n_1(U)\equiv n_u \ , \\
9|U_{22}|^2+60|U_{23}|^2 &= n_2(U)\equiv n_c \ , \\
9|U_{32}|^2+60|U_{33}|^2 &= n_3(U)\equiv n_t = 69-n_2(U)-n_1(U) \ .
\end{aligned} \tag{4.3.2}$$

The integers n_d, n_s and n_u, n_c characterize the masses of the physical quarks and the task is to derive the values of these integers by studying the spectrum of the hadronic masses. The second task is to find unitary mixing matrices satisfying these conditions.

The general form of U and D matrices can be deduced from the standard parametrization of the CKM matrix given by

$$V = \begin{bmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3exp(i\delta_{CP}) & c_1c_2s_3 + s_2c_3exp(i\delta_{CP}) \\ -s_1s_2 & c_1s_2c_3 + c_2s_3exp(i\delta_{CP}) & c_1s_2s_3 - c_2c_3exp(i\delta_{CP}) \end{bmatrix} \quad (4.3.3)$$

This form of the CKM matrix is always possible to achieve by multiplying each U and D type quark fields with a suitable phase factor: this induces a multiplication U and D from left by a diagonal phase factor matrix inducing the multiplication of the columns of U and D by phase factors:

$$\begin{aligned} &U \rightarrow U \times d(\phi_1, \phi_2, \phi_3) \ , \\ &D \rightarrow D \times d(\chi_1, \chi_2, \chi_3) \ , \\ &d(\phi_1, \phi_2, \phi_3) \equiv diag(exp(i\phi_1), exp(i\phi_2), exp(i\phi_3)) \ . \end{aligned}$$

The multiplication of the columns by the phase factors affects CKM matrix defined as

$$V \ = \ U^\dagger D \rightarrow d(-\phi_1, -\phi_2, -\phi_3)Vd(\chi_1, \chi_2), \chi_3) \ . \quad (4.3.4)$$

By a suitable choice of the phases, the first row and column of V can be made real. The multiplication of the rows of U and D from the left by the same phase factors does not affect the elements of V. One can always choose D to be of the same general form as the CKM matrix but must allow U to have nontrivial phase overall factors on the second and third row so that the most general U matrix is parameterized by six parameters.

Mass squared conditions give two independent conditions on the values of the moduli of the matrix elements of U and D. This eliminates two coordinates so that the most general D matrix can be chosen to depend on 2 parameters, which can be taken to be $r_{11} \equiv |D_{11}|$ and $r_{21} \equiv |D_{21}|$. U matrix contains also the overall phase angles associated with the second and third row and hence depends on four parameters altogether.

4.3.2 The constraints on U and D matrices from quark masses

The new view about quark masses allows a surprisingly simple model for U and D matrices predicting in the lowest order approximation that the probabilities defined by these matrices are identical and that the integers characterizing the masses of U and D type quarks are identical.

The constraints on $|U|$ and $|D|$ matrices from quark masses

The understanding of quark masses pose strong constraints on U and D matrices. The constraints are identical in the approximation that V-matrix is identity matrix and read in the case of D-matrix as

$$\begin{aligned} n_d \ &= \ 13 = P^D_{12} \times 9 + P^D_{13} \times 60 \ , \\ n_s \ &= \ 31 = P^D_{22} \times 9 + P^D_{23} \times 60 \ . \end{aligned} \quad (4.3.5)$$

The conditions for b quark give nothing new. The extreme cases when only $g = 1$ or $g = 2$ contributes to n_q gives the bounds

$$\begin{aligned} \frac{15}{36} \ &\leq \ P^D_{13} \leq \frac{15}{60}, \\ \frac{22}{60} \ &\leq \ P^D_{23} \leq \frac{31}{60} \ . \end{aligned} \quad (4.3.6)$$

Unitarity conditions

The condition $D = VU$ and the fact that V is in not too far from unit matrix being in a good approximation a direct sum of 2×2 matrix and 1×1 identity matrix, imply together that U an D cannot differ much from each other. At least the probabilities defined by the moduli squared of matrix elements are near to each other.

1. Instead of trying numerically to solve U and D matrices by a direct numerical search, it is more appropriate to try to deduce estimates for the probabilities $P_{ij}^U = |U_{ij}|^2$ and $P_{ij}^D = |D_{ij}|^2$ determined by the moduli squared of the matrix elements and satisfying the unitarity conditions $\sum_j P_{ij}^X = 1$ and $\sum_i P_{ij}^X = 1$.
2. The formula $D = UV$ using the fact that V_{i3} is small for $i = 1, 2$ implies $|D_{i3}| \simeq |U_{i3}|$. By probability conservation also the condition $|D_{33}| \simeq |U_{33}|$ must hold true so that the third columns of U and D are same in a reasonable approximation.

1. Parametrization of $|U|$ and $|D|$ matrices

The following parametrization is natural for the matrices P_{ij}^X.

$$\begin{aligned} P_{12}^D &= \tfrac{k_D}{9} \ , & P_{13}^D &= \tfrac{n_d-k_D}{9} \ , \\ P_{22}^D &= \tfrac{l_D}{9} \ , & P_{23}^D &= \tfrac{n_s-l_D}{60} \ , \\ P_{32}^D &= \tfrac{9-k_D-l_D}{9} \ , & P_{33}^D &= \tfrac{60-n_s-n_d-k_D-l_D}{60} \ . \end{aligned} \tag{4.3.7}$$

A similar parametrization holds true for P_{ij}^U but with $n_d = n_u$ and $n_s = n_c$ but possibly different values of k_U and l_U. Since $l_D \ll n_s$ is expected to hold true, P_{23}^D is in a good approximation equal to $P_{23}^D = n_s/60 = 31/60$. Same applies to P_{23}^U.

$k_X = 2$ (k_X need not be an integer) gives a good first estimate for mixing probabilities of u and d quark. Thus only the parameter l_X remains free if $k_D = 2$ is accepted.

The approximation $P_{i3}^U = P_{i3}^D$ motivated by the near unit matrix property of V, gives the parametrization

$$P_{12}^D = P_{12}^U = \frac{k}{9} \ , \ P_{13}^D = P_{13}^U \frac{n_d - k}{60} \ . \tag{4.3.8}$$

2. Constraints from CKM matrix in $|U| = |D|$ approximation

The condition $D_{12} = (UV)_{12}$ when feeded to the condition

$$P_{12}^U = P_{12}^D \tag{4.3.9}$$

using the approximation $k_D = k_U = k$ $l_D = l_U = l$ gives

$$|U_{i2}|^2 - |U_{i1}V_{12} + U_{i2}V_{22} + U_{i3}V_{32}|^2 = 0 \ . \tag{4.3.10}$$

$i = 1, 2, 3$ In the approximation that the small V_{32} term does not contribute, this gives

$$|U_{i1}V_{12} + U_{i2}V_{22})|^2 = |U_{i2}|^2 \ . \tag{4.3.11}$$

By dividing with $|U_{i1}|^2|V_{22}|^2$ and using the approximation $|V_{22}|^2 = 1$ this gives

$$\begin{aligned} v_i^2 &+ 2u_iv_i \times cos(\Psi_i) = 0 \ , \\ \Psi_i &= arg(V_{i2}) - arg(V_{32}) + arg(U_{i1}) - arg(U_{i2}) \ . \\ u_i &= |\frac{U_{i2}}{U_{i1}}| \ , \ v_i = |\frac{V_{i2}}{V_{22}}| \ . \end{aligned} \tag{4.3.12}$$

This gives

$$
\begin{aligned}
cos(\Psi_i) &= -\frac{v_i}{2u_i} = -\frac{v_i}{2}\sqrt{\frac{9x_i}{k_i}} \ , \\
x_i &= P^D_{ii} = 1 - \frac{k_i}{9} - \frac{n(i)-k(i)}{60} \ , \\
k(1) &= k \ , \ k(2) = l \ , \ n(1) = n_d, \ n(2) \equiv n_s \ . \qquad (4.3.13)
\end{aligned}
$$

The condition $|cos(\Psi)| \leq 1$ is trivially satisfied. For $n_d = 13$ and $k = 2$ the condition gives $x = .59$ and $cos(\Psi_1) = .185$. $k = 1.45$ gives $x = .65$ and $cos(\Psi) = .226$, which is rather near to V_{12}.

4.3.3 Constraints from CKM matrix

Besides the constraints from hadron masses, there are constraints from CKM matrix $V = U^\dagger D$ on U and D matrices.

1. The fact that CKM matrix is near unit matrix implies that U and D matrix are near to each other and the assumption $n(U_i) = n(D_i)$ predicting quark masses correctly is consistent with this.
2. Cabibbo angle allows to derive the estimate for the difference $|U_{11}| - |D_{11}|$. Together with other conditions this difference fixes the scenario essentially uniquely.
3. The requirement that CP breaking invariant J has a correct order of magnitude gives a very strong constraint on D matrix. The smallness of J implies that V is nearly orthogonal matrix and same assumption can be made about U and D matrices.
4. The requirement that the moduli the first row (column) of CKM matrix are predicted correctly makes it possible to deduce for given D (U) U (D) matrix essentially uniquely. Unitarity requirement poses very strong additional constraints. It must be emphasized that the constraints from the moduli of the CKM alone are sufficient to determine U and D matrices and hence also quark masses and hadron masses to very high degree.

1. Bounds on CKM matrix elements

The most recent experimental information [36] concerning CKM matrix elements is summarized in table below

$\|V_{13}\| \equiv \|V_{ub}\| = (0.087 \pm 0.075)V_{cb} : 0.42 \cdot 10^{-3} < \|V_{ub}\| < 6.98 \cdot 10^{-3}$
$\|V_{23}\| \equiv \|V_{cb}\| = (41.2 \pm 4.5) \cdot 10^{-3}$
$\|V_{31}\| \equiv \|V_{td}\| = (9.6 \pm 0.9) \cdot 10^{-3}$
$\|V_{32}\| \equiv \|V_{ts}\| = (40.2 \pm 4.4) \cdot 10^{-3}$
$s_{Cab} = 0.226 \pm 0.002$

Table 4. The experimental constraints on the absolute values of the CKM matrix elements.

$$
\begin{aligned}
s_1 &= .226 \pm .002 \ , \\
s_1 s_2 &= V_{31} = (9.6 \pm .9) \cdot 10^{-3} \ , \\
s_1 s_3 &= V_{13} = (.087 \pm .075) \cdot V_{23} \ , \\
V_{23} &= (40.2 \pm 4.4) \cdot 10^{-3} \ . \qquad (4.3.14)
\end{aligned}
$$

The remaining parameter is $sin(\delta)$ or equivalently the CP breaking parameter J:

$$
J = Im(V_{11}V_{22}\overline{V}_{12}\overline{V}_{21}) = c_1c_2c_3s_2s_3s_1^2 sin(\delta) \ , \qquad (4.3.15)
$$

where the upper bound is for $sin(\delta)=1$ and the previous average values of the parameters s_i, c_i (note that the poor knowledge of s_3 affects on the upper bound for J considerably). Unitary triangle [66] gives for the CP breaking parameter the limits

$$1.0\times 10^{-4}\leq J\leq 1.7\times 10^{-4} \quad . \qquad (4.3.16)$$

2. CP breaking in $M-\overline{M}$ systems as a source of information about CP breaking phase

Information about the value of $sin(\delta)$ as well as on the range of possible top quark masses comes from CP breaking in $K-\bar{K}$ and $B-\bar{B}$ systems.

The observables in $K_L\rightarrow 2\pi$ system [112]

$$\begin{aligned}
\eta_{+-} &= \frac{A(K_L\rightarrow \pi^+\pi^-)}{A(K_S\rightarrow \pi^+\pi^-)}=\epsilon+\frac{\epsilon'}{1+\omega/\sqrt{2}} \ , \\
\eta_{00} &= \frac{A(K_L\rightarrow \pi^0\pi^0)}{A(K_S\rightarrow \pi^0\pi^0)}=\epsilon-2\frac{\epsilon'}{1-\sqrt{2}\omega} \ , \\
\omega &\sim \frac{1}{20} \ , \\
\epsilon &= (2.27\pm .02)\cdot 10^{-3}\cdot exp(i43.7^o) \ , \\
|\frac{\epsilon'}{\epsilon}| &= (3.3\pm 1.1)\cdot 10^{-3} \ . \qquad (4.3.17)
\end{aligned}$$

The phases of ϵ and ϵ' are in good approximation identical. CP breaking in $K-\bar{K}$ mass matrix comes from the CP breaking imaginary part of $\bar{s}d\rightarrow s\bar{d}$ amplitude M_{12} (via the decay to intermediate W^+W^- pair) whereas $K^0\bar{K}^0$ mass difference Δm_K comes from the real part of this amplitude: the calculation of the real part cannot be done reliably for kaon since perturbative QCD does not work in the energy region in question. On can however relate the real part to the known mass difference between K_L and K_S: $2Re(M_{12})=\Delta m_K$.

Using the results of [112]) one can express ϵ and ϵ'/ϵ in the following numerical form

$$\begin{aligned}
|\epsilon| &= \frac{1}{\sqrt{2}}\frac{Im(M_{12}^{sd})}{\Delta m_K}-.05\cdot|\frac{\epsilon'}{\epsilon}|=2J(22.2B_K\cdot X(m_t)-.28B_K') \ , \\
|\frac{\epsilon'}{\epsilon}| &= C\cdot J\cdot B_K' \ , \\
X(m_t) &= \frac{H(m_t)}{H(m_t=60\ GeV)} \ , \\
H(m_t) &= -\eta_1F(x_c)+\eta_2F(x_t)K+\eta_3G(x_c,x_t) \ , \\
x_q &= \frac{m(q)^2}{m_W^2} \ , \\
K &= s_2^2+s_2s_3cos(\delta) \ . \qquad (4.3.13)
\end{aligned}$$

Here the values of QCD parameters η_i depend on top mass slightly. B_K' and B_K are strong interaction matrix elements and vary between 1/3 and 1. The functions F and G [112] are given by

$$\begin{aligned}
F(x) &= x\left[\frac{1}{4}+\frac{9}{4}\frac{1}{1-x}-\frac{3}{2}\frac{1}{(1-x)^2}\right]+\frac{3}{2}(\frac{x}{x-1})^3log(x) \ , \\
G(x,y) &= xy\left[\frac{1}{x-y}\left[\frac{1}{4}+\frac{3}{2}\frac{1}{1-x}-\frac{3}{4}\frac{1}{(1-x)^2}\right]log(x)+(y\rightarrow x)-\frac{3}{4}\frac{1}{(1-x)(1-y)}\right] \ . \qquad (4.3.12)
\end{aligned}$$

One can solve parameter B_K' by requiring that the value of ϵ'/ϵ corresponds to the experimental mean value:

$$B'_K = \frac{1}{C \times J} \frac{\epsilon'}{\epsilon} . \quad (4.3.13)$$

The most recent measurements by KTeV collaboration in Fermi Lab [38] give for the ratio $|\epsilon'/\epsilon|$ the value $|\epsilon'/\epsilon| = (28 \pm 1) \times 10^{-4}$. The proposed standard model explanation for the large value of B'_K is that s-quark has running mass about $m_s(m_c) \simeq .1$ GeV at m_c [85]. The explanation is marginally consistent with the TGD prediction $m(s) = 127$ MeV for the mass of s quark. Also the effects caused by the predicted higher gluon generations having masses around 33 GeV can increase the value of ϵ'/ϵ by a factor 3 in the lowest approximation since the corrections involve sum over three different one-gluon loop diagrams with gluon mass small respect to intermediate boson mass scale [F5].

A second source of information comes from $B - \bar{B}$ mass difference. At the energies in question perturbative QCD is expected to be applicable for the calculation of the mass difference and mass difference is predicted correctly if the mass of the top quark is essentially the mass of the observed top candidate [32].

3. U and D matrices could be nearly orthogonal matrices

The smallness of the CP breaking phase angle δ_{CP} means that V is very near to an orthogonal matrix. This raises the hope that in a suitable gauge also U and D are nearly orthogonal matrices and would be thus almost determined by single angle parameter θ_X, $X = U, D$. Cabibbo angle $s_c = sin(\theta_c) = .226$ which is not too far from $sin^2(\theta_W) \simeq .23$ and appears in V matrix rotating the rows of U to those of D. In very vague sense this angle would characterize between the difference of angle parameters characterizing U and D matrices. If U is orthogonal matrix then the decomposition

$$V = V_1 V_2 = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 c_2 & c_1 c_2 & s_2 exp(i\delta_{CP}) \\ -s_1 s_2 & c_1 s_2 & -c_2 exp(i\delta_{CP}) \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{bmatrix} \quad (4.3.14)$$

suggests that CP breaking can be visualized as a process in which first s and b quarks are slightly mixed to s' and b' by V_2 ($s_3 \simeq 1.4 \times 10^{-2}$) after which V_1 induces a slightly CP-breaking mixing of d and s' with b' ($s_2 \simeq .04$).

4. How the large mixing between u and c results

The prediction that u quark spends roughly 1/3 of time in $g = 0$ state looks bizarre and it is desirable to understand this from basic principles. The basic observations are following.

1. V matrix is in good approximation direct sum of 2×2 matrix inducing relatively large rotation with $sin(\theta_c) \simeq .23$ and unit matrix. In particular, V_{i3} are very small for $i = 1, 2$. Using the formula $D = UV$ one finds that $|U_{i3}| = |D_{i3}|$ in a good approximation for $i = 1, 2$ and by unitarity also for $I = 3$. Thus the third columns of U and D are identical in a good approximation.

2. Assume that also U_{i3} and D_{i3} are small for $i = 1, 2$. A stronger assumption is that even the contribution of D_{13} and U_{13} are so small that they do not affect u and d masses. This implies

$$\begin{aligned} n_d &= 9|D_{12}|^2 + 60|D_{13}|^2 \simeq 9|D_{12}|^2 , \\ n_u &\simeq 9|U_{12}|^2 . \end{aligned} \quad (4.3.14)$$

Unitarity implies in this approximation

$$\begin{aligned} &|U_{11}|^2 \leq 1 - \frac{n_u}{9} = \frac{1}{3} , \\ &|D_{11}|^2 \leq 1 - \frac{n_d}{9} = \frac{5}{9} . \end{aligned} \quad (4.3.14)$$

3. It might be that there are also solutions for which mixing of u resp. d quark is mostly with t resp. b quarks but numerical experimentation does not favor this idea since CP breaking becomes extremely small. Since mixing presumably involves topology change, it seems obvious that topological mixing involving a creation or annihilation of two handles is improbable.

4.4 Construction of U, D, and CKM matrices

In this section it will be found that various mathematical and experimental constraints on U and D matrices determine them essentially uniquely.

4.4.1 The constraints from CKM matrix and number theoretical conditions

The requirement that U, D and V allow an algebraic continuation to finite-dimensional extensions of various p-adic number fields provides a very strong additional constraints. The mathematical problem is to understand how many unitary V matrices acting on U as $U \rightarrow D = UV$ respect the number theoretic constraints plus the constraints $n_u = n_d + 2$ and $n_c = n_d - 2$.

It is instructive to what happens in much simpler 2-dimensional case. In this case the conditions boil down to the conditions on $n(i)$ imply $|U| = |D|$ and this condition is equivalent with (say) the condition $|U_{11}| = D_{11}$. U and D can be parameterized as

$$U = \begin{pmatrix} cos(\theta)exp(i(\psi) & sin(\theta)exp(i\phi) \\ -sin(\theta)exp(-i\phi) & cos(\theta)exp(-i\psi) \end{pmatrix} .$$

If $cos(\theta)^2$ and $sin(\theta)^2$ are rational numbers, $exp(i\theta)$ is associated with a Gaussian integer. A more general requirement is that $exp(i\theta)$ belongs to a finite-dimensional extension of rational numbers and thus corresponds to a products of a phase associated with Gaussian integer and a phase in a finite-dimensional algebraic extension of rational numbers.

Eliminating the trivial multiplicative phases gives a set of matrices U identifiable as a double coset space $X^2 = SU(2)/U(1)_R \times U(1)_L$. The value of $cos(\theta) = |U_{11}|$ serving as a coordinate for X^2 is respected by the right multiplication with V. Eliminating trivial $U(1)_R$ phase multiplication, the space of V:s reduces to $S^2 = SU(2)/U(1)_R$. The condition that $cos(\theta)$ is not changed leaves one parameter set of allowed matrices V.

The translation of these results to 3-dimensional case is rather straightforward. In the 3-dimensional case the probabilities P_{i2}, P_{i3}, $i = 1, 2$ characterize a general matrix $|U|$, and V can affect these probabilities subject to constraints on $n(I)$. When trivial phases affecting the probabilities are eliminated, the matrices U correspond naturally to points of the 4-dimensional double coset space $X^4 = SU(3)/(U(1) \times U(1))_R \times U(1) \times U(1))_L$ having dimension $D = 4$.

The two constraints on the probabilities mean that allowed solutions for given values of $n(I)$ define a 2-dimensional surface X^2 in X^4. The allowed unitary transformations V must be such that they move U along this surface. Certainly they exist since X^2 can be regarded as a local section in $SU(3) \rightarrow X^2$ bundle obtained as a restriction of $SU(3) \rightarrow X^4$ bundle. The action of V on rows of U is ordinary unitary transformation plus a 2-dimensional unitary transformation preserving the Hermitian degenerate lengths $L_i = 9|U_{i2}|^2 + 60|U_{i3}|^2 = n_i$ defining the sub-bundle $SU(3) \rightarrow X^2$. Note for $L_1 = 0$ ($L_2 = 0$) the situation becomes 2-dimensional and solutions correspond to points in S^2. Thus these points seem to represent a conical singularity of X^2.

The 2-dimensionality of the solution space means that two moduli (probabilities) of any row or column of U or D matrix characterize the matrix apart from the non-uniqueness due to the gauge choice allowing $U(1)_L \times U(1)_R$ transformation of U. Of course, discrete sign degeneracy might be present.

A highly non-trivial problem is whether the set X^2 contains rational points and what is the number of these points. For instance, Fermat's theorem says that no rational solutions to the equation $x^n + y^n - z^n = 0$ exist for $n > 2$. The fact that the degenerate situation allows infinite number of rational solutions suggest that they exist also in the general case. Note also that the additional conditions are second order polynomial equations with rational coefficients so that $SU(3, Q)$ should contain non-trivial solutions to the equations.

It is possible to write $|U|$ in a form containing minimal number of square roots:

$$\begin{array}{lll} |U_{11}| = \sqrt{n_u}\frac{p_1}{N_1} \ , & |U_{12}| = \sqrt{\frac{n_u}{9}}\frac{r_1}{N_1} \ , & |U_{13}| = \sqrt{\frac{n_u}{60}}\frac{s_1}{N_1} \ , \\ |U_{21}| = \sqrt{n_c}\frac{p_2}{N_2} \ , & |U_{22}| = \sqrt{\frac{n_c}{9}}\frac{r_2}{N_2} \ , & |U_{23}| = \sqrt{\frac{n_c}{60}}\frac{s_2}{N_2} \ , \\ |U_{31}| = \sqrt{n_t}\frac{p_3}{N_3} \ , & |U_{32}| = \sqrt{\frac{n_t}{9}}\frac{r_3}{N_3} \ , & |U_{23}| = \sqrt{\frac{n_t}{60}}\frac{s_3}{N_3} \ . \end{array} \quad (4.4.1)$$

Completely analogous expression holds true for D. r_i, s_i and N_i are integers, and the defining equations reduce in both cases to equations generalizing those satisfied by Pythagorean triangles

$$\begin{aligned} r_1^2 + s_1^2 &= N_1^2 \ , \\ r_2^2 + s_2^2 &= N_2^2 \ , \\ r_3^2 + s_3^2 &= N_3^2 \ . \end{aligned} \quad (4.4.0)$$

The square roots of n_i are also eliminated from the unitarity conditions which become equations with rational coefficients for the phases appearing in U and D. Hence there are good hopes that even rational solutions to the conditions exist.

4.4.2 Number theoretic conditions on U and D matrices

The most stringent requirement would be that U and D matrices are rational unitary matrices. A less stringent condition is that only the moduli squared of U and D are rational numbers. p-Adicization allows also matrices for which various phases are products of Pythagorean phases with phases in an an extension of rational numbers defining a finite-dimensional extension of p-adic numbers. The number theoretic conditions following from the rational unitarity on the moduli of the U and D matrices are not completely independent of the parametrization used. The reason is that the products of the parameters in some algebraic extension of the rationals can combine to give a rational number. The safest parametrization to use is the one based on the moduli of the U and D matrix.

Assuming rationality for the mixing matrix all moduli can be written in the form

$$|D_{ij}| = \frac{n_{ij}}{N} \ . \quad (4.4.1)$$

If only moduli squared are required to be rational, the condition is replaced with a milder one:

$$|D_{ij}| = \frac{n_{ij}}{\sqrt{N}} \ . \quad (4.4.2)$$

Here $\sqrt{N}$ belongs to square root allowing algebraic extension of the p-adic numbers but is not an integer itself. An even milder condition is

$$|D_{ij}| = \sqrt{\frac{n_{ij}}{N}} \ . \quad (4.4.3)$$

The following arguments show that only this option is allowed. This option is also natural in light or preceding general considerations.

1. *Unitary and mass conditions modulo 8*

For $p_{ij} = (\sqrt{\frac{n_{ij}}{N}})^k$, $k = 1$ or 2, the requirement that the rows are unit vectors implies

$$\begin{aligned} \sum_j n_{i,j}^k &= N^k \ , \\ k &= 1 \text{ or } 2 \ . \end{aligned} \quad (4.4.3)$$

The problem of finding vectors with integer valued components and with a given integer valued length squared m ($k = 2$ case) is a well known and well understood problem of the number theory [22]. The basic idea is to write the conditions modulo 8 and use the fact that the square of odd (even) integer is 1 (0 or 4) modulo 8. The result is that one must have

$$m \in \{1,2,3,5,6\} \ , \tag{4.4.4}$$

for the conditions to possess nontrivial solutions. For $m = N$ case this is the only condition needed. In $m = N^2$ case the condition implies that N must be odd.

Using this result one can write the mass squared conditions modulo 8 for $k = 2$ as

$$\begin{aligned} 3n_{i,2}^2 + 4n_{i,3}^2 &= n_iX \ , \\ X &= 1 \text{ for } m = N^2 \ , \\ X &\in \{1,2,3,5,6\} \text{ for } m = N \ . \end{aligned} \tag{4.4.3}$$

Here modulo 8 arithmetics is understood. In $m = N^2$ case one must have $n_i \in \{0,3,4\}$ modulo 8. These conditions are not satisfied in general. For $m = N$ conditions allow considerably more general set of solutions. By summing the equations and using probability conservation one however obtains $7N = 5N$ implying $2N = 0$ so that the non-allowed value $N = 4$ or 0 results.

For $k = 1$ no obvious conditions result on the values of n_i and only this option is allowed by mass conditions for the physical masses.

2. Rational unitarity cannot hold true for U and D matrices separately

The mixing scenario is not consistent with the assumption that the matrix elements of U and D matrix are complex rational numbers. If this were the case then matrix elements had to be proportional to a common denominator $1/N$ such that N is odd integer (otherwise the conditions stating that the unit vector property of the rows is not satisfied). The conditions

$$\begin{aligned} \sum_j r_{ij} &= 1 \ , \\ 9r_{12} + 60r_{13} &= n_d \ , \\ 9r_{22} + 60r_{23} &= n_s \ , \\ 9r_{32} + 60r_{33} &= n_b \ , \\ r_{ij} &= \frac{n_{ij}}{N_i} \ , \end{aligned} \tag{4.4.-1}$$

can be written modulo 8 as

$$\begin{aligned} \sum_j n_{ij}^k &= N^k \ , \\ n_{12}^k + 4n_{13}^k &= n_dN^k \ , \\ n_{22}^k + 4n_{23}^k &= n_sN^k \ , \\ n_{32}^k + 4n_{33}^k &= n_bN^k \ , \\ r_{ij} &= (\frac{n_{ij}}{N})^{k/2} \ , \quad k = 1 \text{ or } 2 \ . \end{aligned} \tag{4.4.-5}$$

1. Consider first the case $k = 2$. For odd n $n^2 = 1$ holds true and for even n $n^2 = 4$ or 0 holds true. It is easy to see that the conditions can be satisfied only of all integers are proportional to 4 but this cannot be possible since it would be possible since n_{ij} an N cannot contain common factors. Thus at least an extension allowing square roots is needed. Quite generally from $N^2 = 1 \ mod \ 8$ the above equations give

   $$n_{q_i} \ mod \ 8 \in \{0,3,4,7\} \ .$$

   This condition fails to be satisfied by in the general case.

2. For the option $k = 1$ for which only the probabilities are rational the sum of all three equations gives $5N = 5N$ so that equations are consistent.

3. Phase factors

The phase factors associated with the rows of the mixing matrix are rational provided the corresponding angles correspond to Pythagorean triangles. Combining this property with the orthogonality conditions for the rows of the U matrix, one obtains highly nontrivial conditions relating the integers characterizing the sides of the Pythagorean triangle to the integers n_{ij}. The requirement that the imaginary parts of the inner product vanish, gives the conditions

$$\frac{s_{i,2}}{s_{i3}} = \frac{n_{13}n_{i3}}{n_{12}n_{22}} \ , \ i = 2,3 \ . \tag{4.4.-4}$$

Combining this conditions with the general representation for the sines of the Pythagorean triangle

$$sin(\phi) = \frac{2rs}{r^2+s^2} \text{ or } \frac{r^2-s^2}{r^2+s^2} \ , \tag{4.4.-3}$$

one obtains conditions relating the integers appearing characterizing the triangle to the integers on the right hand side.

An interesting possibility is that the lengths of the hypothenusae of the triangles associated with $s(i,2)$ $((r(i),s(i)))$ and s_{i3} $((r_1(i),s_1(i)))$ are the same and sines correspond to the products $2rs$:

$$\begin{aligned} r^2(i)+s^2(i) &= r_1^2(i)+s_1^2(i) \ , \\ s_{i,2} &= 2r(i)s(i)/(r^2(i)+s^2(i)) \ , \\ s_{i,3} &= 2r_1(i)s_1(i)/(r_1^2(i)+s_1^2(i)) \ . \end{aligned} \tag{4.4.-4}$$

In this case the conditions give

$$\frac{r(i)s(i)}{r_1(i)s_1(i)} = \frac{n_{13}n_{i3}}{n_{12}n_{22}} \ . \tag{4.4.-3}$$

The conditions are satisfied if one has

$$\begin{aligned} r(i)s(i) &= n_{13}n_{i3} \ , \\ r_1(i)s_1(i) &= n_{12}n_{22} \ . \end{aligned} \tag{4.4.-3}$$

This implies that $r(i)$ and $s(i)$ are products of the factors contained in the product $n_{13}n_{i3}$. Analogous conclusion applies to $r_1(i)$ and $s_1(i)$.

Additional number theoretic conditions are obtained from the requirement that the real parts of the inner products between first row and second and third rows vanish:

$$n_{11}n_{i1} + c_{i,2}n_{12}n_{i2} + c_{i,3}n_{13}n_{i3} = 0 \ , \ i = 2,3 \ . \tag{4.4.-2}$$

4.4.3 The parametrization suggested by the mass squared conditions

To understand the consequences of the mass squared conditions, it is useful to use a parametrization, which is more natural for the treatment of the mass squared conditions than the standard parametrization:

$$U = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21}x_2 & r_{22}x_2exp(i\phi_{22}) & r_{23}x_2exp(i\phi_{23}) \\ r_{31}x_3 & r_{32}x_3exp(i\phi_{32}) & r_{33}x_3exp(i\phi_{33}) \end{bmatrix} \tag{4.4.-1}$$

$$x_2 = exp(i\phi_2) \ ,$$
$$x_3 = exp(i\phi_3) \ .$$

In case of D matrix, the phase factors x_2 and x_3 can be chosen to be trivial. As far as the treatment of the mass conditions and unitarity conditions for the rows is considered, one can restrict the consideration to the case, when the overall phase factors are trivial. The remaining parameters are not independent and one on can deduce the formulas relating the moduli r_{ij} as well as the phase angles ϕ_{ij} to the parameters r_{11} and r_{12}. In general, the resulting parameters are not real and unitarity is broken.

Mass squared conditions and the requirement that the rows are unit vectors:

$$\begin{aligned} 9r_{i2}^2 + 60r_{i3}^2 &= n_i \ , \ i = 1,2 \ , \\ \sum_k r_{ik}^2 &= 1 \ , \end{aligned} \tag{4.4.-1}$$

allows one to express r_{i2} and r_{i3} in terms of r_{i1}

$$\begin{aligned} r_{i2} &= \sqrt{[-\frac{n_i}{51} + \frac{20}{17}(1 - r_{i1}^2)]} \ , \\ r_{i3} &= \sqrt{[\frac{n_i}{51} - \frac{3}{17}(1 - r_{i1}^2)]} \ . \end{aligned} \tag{4.4.-1}$$

The requirement that the rows are orthogonal to each other, relates the phase angles ϕ_{ij} in terms to r_{11} and r_{21}. Using the notations $sin(\phi_{ij}) = s_{ij}$ and $cos(\phi_{ij}) = c_{ij}$, one has

$$\begin{array}{ll} c_{i2} = \frac{a_i}{b_i} \ , & c_{i3} = -\frac{(A_{1i}+c_{i2}A_{2i})}{A_{3i}} \ , \\ s_{i2} = \epsilon(i)\sqrt{1 - c_{i2}^2} \ , & s_{i3} = -\frac{A_{2i}}{A_{3i}}s_{i2} \ , \\ A_{1i} = r_{11}r_{i1} \ , & A_{2i} = r_{12}r_{i2} \\ A_{3i} = r_{13}r_{i3} \ , & \epsilon(i) = \pm 1 \ . \\ a_i = A_{3i}^2 - A_{1i}^2 - A_{2i}^2 \ , & b_i = 2A_{1i}A_{2i} \ , \end{array} \tag{4.4.0}$$

The sign factors $\epsilon(i)$ are not completely free and must be chosen so that the second and third row are orthogonal.

The mass conditions imply the following bounds for the parameters r_{i1}

$$\begin{aligned} m_i &\leq r_{i1} \leq M_i \ , \\ m_i &= \sqrt{1 - \frac{n_i}{9}} \text{ for } n_i \leq 9 \ , \\ m_i &= 0 \text{ for } n_i \geq 9 \ , \\ M_i &= \sqrt{1 - \frac{n_i}{60}} \ . \end{aligned} \tag{4.4.-2}$$

The boundaries for the regions of the solution manifold in (r_{11}, r_{21}) plane can be understood as follows. For given values of r_{11} and r_{21} there are in general two solutions corresponding to the sign factor $\epsilon(i)$ appearing in the equations defining the solutions of the mass squared conditions. This means just that complex conjugation gives a new solution from a given one. These two branches become degenerate, when the phase factors become ± 1 so that (s_{i2}, s_{i3}) vanishes for $i = 2$ or $i = 3$. Thus the curves at which one has $(s_{i2} = 0, s_{i3} = 0)$ define the boundaries of the projection of the solution manifold to (r_{11}, r_{21}) plane. At the boundaries the orthogonality conditions reduce to the form

$$\begin{aligned} & r_{11}r_{i1} + \epsilon(i,2)r_{12}r_{i2} + \epsilon_{i3}r_{13}r_{i3} = 0 \ , \ \ i = 2 \text{ or } 3 \ , \\ & \epsilon_{22} = \epsilon_{32} \ , \\ & \epsilon_{23} = -\epsilon_{33} \end{aligned} \quad (4.4.\text{-}1)$$

where ϵ_{ij} corresponds to the value of the cosine of the phase angle in question. Consistency requires that either second or third row becomes real on the boundary of the unitarity region and that the matrices reduce to orthogonal matrices at the dip of the region allowed by unitarity.

4.4.4 Thermodynamical model for the topological mixing

What would be needed is a physical model for the topological mixing allowing to deduce U and D matrices from first principles. The physical mechanism behind the mixing is change of the topology of X^2 in the dynamical evolution defined by the light like 2-surface X_l^3 defining parton orbit. This suggests that the topology changes $g \rightarrow g \pm 1$ dominate the dynamics so that matrix elements U_{13} and D_{13} should be indeed small so that the weird looking result $P_{11}^U \simeq 1/3$ follows from the requirement $n_u = 6$. This model however suggests that the matrix elements U_{23} and D_{23} could be large unlike in the original model for U and D matrices.

Solution of thermodynamical model

A possible approach to the construction of mixing matrices is based on the idea that the interactions causing the mixing lead to a thermal equilibrium so that the entropies for the ensemble defined by the probabilities p_{ij}^U and p_{ij}^D matrix is maximized (the subscripts U and D are dropped in the sequel).

1. The elements in the three rows of the mixing matrix represent probabilities for three states of the system with energies $(E_{i1}, E_{i2}, E_{i3}) = (0, 9, 60)$ and average energy is fixed to $\langle E \rangle = 69$.
2. There are usual constraints from probability conservation for each row plus two independent constraints from columns. The latter constraints can be regarded as a constraint on a second quantity equal to 1 for each column and brings in variable analogous to chemical potential besides temperature.

The constraint from mass squared for the third row follows from these constraints. The independent constraints can be chosen to be the following ones

$$\begin{aligned} & \sum_j p_{ij} - 1 = 0 \ , \ \ i = 1,2,3 \qquad \sum_i p_{ij} - 1 = 0, \ \ j = 1,2 \ , \\ & 9p_{i2} + 60p_{i3} - n_{q_i} = 0 \ , \ \ i = 1,2 \ . \end{aligned} \quad (4.4.0)$$

The obvious notations $(q_1, q_2) = (d, s)$ and $(q_1, 2_2) = (u, c)$ are introduced. The conditions on mass squared are completely analogous to the conditions fixing the energy of the ensemble and thus its temperature, and thermodynamical intuition suggests that the probabilities p_{ij} decrease exponentially as function of E_j in the absence of additional constraints coming from the probability conservation for the columns and meaning presence of chemical potential.

The variational principle maximizing entropy in presence of these constraints can be expressed as

$$\begin{aligned} L &= S + S_c \\ S &= \sum_{i,j} p_{ij} \times log(p_{ij}) \\ S_c &= \sum_i \lambda_i (\sum_j p_{ij} - 1) + \sum_{j=1,2} \mu_j (\sum_i p_{ij} - 1) + \sum_{i=1,2} \sigma_i (9p_{i2} + 60p_{i3} - n_{q_i}) \ . \end{aligned} \quad (4.4.\text{-}2)$$

The variational equation is

$$\partial_{p_{ij}} L = 0 \ , \tag{4.4.-1}$$

and gives the probabilities as

$$\begin{array}{lll} p_{11} = \frac{1}{Z_1} \ , & p_{12} = \frac{xx_1^3}{Z_1} & p_{13} = \frac{yx_1^{20}}{Z_1} \ , \\ p_{21} = \frac{1}{Z_2} \ , & p_{22} = \frac{xx_2^3}{Z_2} & p_{13} = \frac{yx_2^{20}}{Z_2} \ , \\ p_{31} = \frac{1}{Z_3} \ , & p_{32} = \frac{x}{Z_3} & p_{33} = \frac{y}{Z_3} \ , \end{array} \tag{4.4.-1}$$

Here the parameters x, y, x_1, x_2 are defined as

$$\begin{array}{ll} x = exp(-\mu_2) \ , & y = exp(-\mu_3) \ , \\ x_1 = exp(-3\sigma_1) \ , & x_2 = exp(-3\sigma_2) \ . \end{array} \tag{4.4.-1}$$

whereas the row partition functions Z_i are defined as

$$Z_1 = 1 + xx_1^3 + yx_1^{20} \ , \quad Z_2 = 1 + xx_2^3 + yx_2^{20} \ , \quad Z_3 = 1 + x + y \ . \tag{4.4.0}$$

Note that the parameters λ_i have been eliminated. There are four parameters $\mu_2, \mu_3, \sigma_2, \sigma_3$ and 2 conditions from columns and 2 mass conditions so that the number of solutions is discrete and only finite number of U and D matrices are possible in the thermodynamical approximation.

Mass squared conditions

The mass squared conditions read as

$$9xx_1^3 + 60yx_1^{20} = n(q_1)Z_1 \ , \quad 9xx_2^3 + 60yx_2^{20} = n(q_2)Z_2 \ . \tag{4.4.1}$$

These equations allow to solve y as a simple linear function of x

$$y = \frac{n(q_1) - xx_1^3(9 - n(q_1))}{(60 - n(q_1))x_1^{20}} \equiv kx + l \ , \quad y = \frac{n(q_2) - xx_2^3(9 - n(q_2))}{(60 - n(q_2)x_2^{20}} \ . \tag{4.4.2}$$

The identification of the two expressions for y allows to solve x_1 in terms of x_2 using equation of form $x_1^{20} - bx_1^3 + c = 0$:

$$\begin{array}{ll} & \left[60 - n(q_2)x_2^{20}\right] \left[n(q_1) - xx_1^3(9 - n(q_1))\right] \\ = & \left[60 - n(q_1))x_1^{20}\right] \left[n(q_2) - xx_2^3(9 - n(q_2))\right] \ . \end{array} \tag{4.4.2}$$

In the most general case the equation allows 20 roots $x_1 = x_2(x_1)$.

Probability conservation

Probability conditions give additional information. By solving $1/Z_3$ from the first column gives

$$Z_1Z_2Z_3 - Z_1Z_2 - Z_2Z_3 - Z_1Z_3 = 0 \ , \tag{4.4.3}$$

(4.4.4)

This equation is a polynomial equation for in x_1 and x_2 with degree 20 and together with Eq. 4.4.2 having same degree determines and (x_1, x_2) the possible values of x_1 and x_2 as function of x. The number of real positive roots is at most $20^2 = 400$.

Probability conservation for the second column gives

$$x\left[(1-x_1^3)Z_2 + (1-x_2^3)Z_1\right] + (1-x)Z_1Z_2 = 0 \ . \qquad (4.4.5)$$

The row partition functions Z_i are linear functions of x and y and mass squared conditions give $y = kx + l$ (see Eq. 4.4.2) so that a third order polynomial equation for x results and gives the roots as functions of control parameters x_1 and x_2. Either 1 or 3 real roots are obtained for x. The values of x_1 and x_2 are determined by the probability constraint Eq. 4.4.4 for the first column and Eq. 4.4.2 relating x_1 and x_2.

The analogy with spontaneous magnetization

Physically the situation is analogous to a spontaneous symmetry breaking with y representing the external magnetizing field and x linear magnetization or vice versa. x_1 and x_2 are control parameters characterizing the interaction between spins. For single real root for x no spontaneous magnetization occurs but for 3 real roots there are two directions of spontaneous magnetization plus unstable state. In the recent case the roots must be positive. Since the maximal number of roots for (x_1, x_2) is 400, the maximal number of real roots is 1200. The trivial solution to the conditions is $p_{11} = 1, p_{22} = 1$, $p_{33} = 1$ with $x = y = 0$ represents corresponds to the absence of external magnetizing field and of magnetization.

Catastrophe theoretic description of the system

In the catastrophe theoretic approach one can see that situation as a cusp catastrophe with x as a behavior variable and x_1, x_2 in the role of control variables. In the standard parametrization of the cusp catastrophe [18] the conditions correspond to the equation

$$x^3 - a - bx \ = \ 0 \ , \qquad (4.4.5)$$

In the recent case a more general polynomial $P_3(x)$ easily transformable to the standard form is in question. The coefficients of the polynomial $P_3(x) = Dx^3 + Cx^2 + Bx + A$ are

$$\begin{aligned}
A &= Q(x_1)Q(x_2) \ , \\
B &= P(x_1)Q(x_2) + P(x_2)Q(x_1) + R(x_2) + R(x_1) \ , \\
C &= P(x_1)R(x_2) + P(x_2)R(x_1) - R(x_1)Q(x_2) - R(x_2)Q(x_1) \ , \\
D &= R(x_1)R(x_2) \ , \\
P(u) &= 1 - u^3 \ , \ Q(u) = 1 + lu^{20} \ , \ R(u) = u^3 + ku^{20} \ .
\end{aligned} \qquad (4.4.2)$$

The trivial scaling transformation $A \rightarrow A/D = \hat{A}$, $B \rightarrow B/D = \hat{B}$, $C \rightarrow C/D = \hat{C}$ and the shift $x \rightarrow x + \hat{C}/3$ casts the equation in the standard form and gives

$$\begin{aligned}
a &= -\hat{A} + \frac{\hat{C}^3}{9} \ , \\
b &= -\hat{B} + \frac{\hat{C}^2}{3} \ .
\end{aligned} \qquad (4.4.1)$$

The curve

$$a = \pm 2(\frac{b}{3})^{3/2} \ , b \geq 0 \tag{4.4.2}$$

represents the bifurcation set for the solutions. For $b \geq 0$, $|a| \leq (\frac{b}{3})^{3/2}$ three roots are obtained for x. $a = b = 0$ corresponds to the dip of the cusp. Three solutions result under the conditions

$$\begin{aligned}
\frac{\hat{C}^2}{3} &\geq 3\hat{B} \ , \\
(-\hat{B}+\frac{\hat{C}^2}{3})^3 &\leq \frac{(-\hat{A}+\frac{\hat{C}^3}{9})^2}{4} \ , \\
\hat{A} &= \frac{Q(x_1)Q(x_2)}{R(x_1)R(x_2)} \ , \\
\hat{B} &= \frac{P(x_1)Q(x_2)+P(x_2)Q(x_1)+R(x_2)+R(x_1)}{R(x_1)R(x_2)} \ , \\
\hat{C} &= \frac{P(x_1)R(x_2)+P(x_2)R(x_1)-R(x_1)Q(x_2)-R(x_2)Q(x_1)}{R(x_1)R(x_2)} \ , \\
P(u) &= 1-u^3 \ , \ Q(u) = 1+lu^{20} \ , \ R(u) = u^3+ku^{20} \ .
\end{aligned} \tag{4.4.-2}$$

The boundaries of the regions are defined by polynomial equations for x_1 and x_2. . The two mass squared conditions and the probability conservation for the first row select a discrete set of parameter combinations.

One might ask whether U and D matrices could correspond to different solutions of these equations for same values of n_{q_i}. This cannot be the case since this would predict too large $u-d$ mass difference. Orthogonalization conditions for the rows should determine the phases more or less uniquely and could force CP breaking. The requirement that probabilities are rational valued implies that x_1, x_2, x and y are rational and poses very strong additional conditions to the solutions. The roots should correspond to very special solutions possessing symmetries so that the solutions of polynomial equations give probabilities as rational numbers. Note however that the solutions of polynomial equations with integer coefficients are in question and the solutions are algebraic numbers: this is enough as far as the p-adicization of the theory is considered.

Maximization of entropy solving constraint equations explicitly

The mass squared conditions allow to express the probabilities p_{ij} in terms of p_{11} and p_{21} (for instance) and this allows a rather concise representation for the solution to the maximization the entropy of topological mixing. The key formulas are following.

$$\begin{aligned}
p_{31} &= 1-p_{11}-p_{12} \ , \\
p_{i2} &= -\frac{n_i}{51}+\frac{20}{17}(1-p_{i1}) \ , \ i=1,2 \ , \\
p_{i3} &= \frac{n_i}{51}-\frac{3}{17}(1-p_{i1}) \ , \ i=1,2 \ .
\end{aligned} \tag{4.4.-3}$$

Expressing entropy directly in terms of p_{11} and p_{21}, the conditions for the maximization of entropy imply the equations

$$log(p_{ij})X^{ij} = 0 \ , \ log(p_{ij})Y^{ij} = 0 \ , \tag{4.4.-2}$$

where a summation over repeated indices is carried out. The matrices $X^{ij} = \partial_{p_{11}} p_{ij}$ and $Y^{ij} = \partial_{p_{21}} p_{ij}$ are given by

$$X = \begin{pmatrix} 1 & -\frac{20}{17} & \frac{3}{17} \\ 0 & 0 & 0 \\ -1 & \frac{20}{17} & -\frac{3}{17} \end{pmatrix}$$
$$Y = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -\frac{20}{17} & \frac{3}{17} \\ -1 & \frac{20}{17} & -\frac{3}{17} \end{pmatrix}$$

(4.4.-3)

The equations can be transformed into the form

$$\prod_{ij} p_{ij}^{X_{ij}} = 1 \ , \quad \prod_{ij} p_{ij}^{Y_{ij}} = 1 \ . \qquad (4.4.\text{-}2)$$

When written explicitly, these equations read as

$$\frac{p_{11}}{1-p_{11}-p_{21}} \times \left(\frac{-n_1+60(1-p_{11})}{-n_3+60(p_{11}+p_{21})}\right)^{-20/17} \times \left(\frac{n_1-9(1-p_{11})}{n_3-9(p_{11}+p_{21})}\right)^{3/17} = 1 \ ,$$
$$\frac{p_{21}}{1-p_{11}-p_{21}} \times \left(\frac{-n_2+60(1-p_{21})}{-n_3+60(p_{11}+p_{21})}\right)^{-20/17} \times \left(\frac{n_2-9(1-p_{21})}{n_3-9(p_{11}+p_{21})}\right)^{3/17} = 1 \ .$$

(4.4.-2)

The equations can be cast into polynomial equations in p_{11} and p_{21} by taking 17:th power of both equations. This gives polynomial equations of degree $d = 17 + 20 + 3 = 40$. The total number of solutions to the equations is at most $40 \times 40 = 1600$. The previous estimate gave upper bound $3 \times 20 \times 20 = 1200$ for the number of solution. It might be that some symmetry is involved and reduces the upper bound by a factor 3/4.

The solutions can be sought using gradient dynamics in which system in (p_{11}, p_{21}) plane drifts in the force field defined by the gradient ∇S of the entropy $S = -\sum_{ij} p_{ij} log(p_i j)$ and ends up to the maximum of S, $S = -\sum_{ij} p_{ij} log(p_i j)$.

$$\frac{dp_{11}}{dt} = \partial_{p_{11}} S = -X^{ij} log(p_{ij}) \ ,$$
$$\frac{dp_{21}}{dt} = \partial_{p_{21}} S = -Y^{ij} log(p_{ij}) \ ,$$

(4.4.-2)

The conditions that the probabilities are positive give the constraints

$$\begin{aligned} 1 - \frac{n_1}{9} &\leq p_{11} \leq 1 - \frac{n_1}{60} \ , \\ 1 - \frac{n_2}{9} &\leq p_{21} \leq 1 - \frac{n_2}{60} \ , \\ 0 &\leq p_{21} \leq 1 - p_{11} \ , \\ \frac{69 - n_1 - n_2}{60} - p_{11} &\leq p_{21} \leq \frac{69 - n_1 - n_2}{9} - p_{11} \end{aligned}$$

(4.4.-5)

on the region containing the solutions.

4.4.5 U and D matrices from the knowledge of top quark mass alone?

As already found, a possible resolution to the problems created by top quark is based on the additivity of mass squared so that top quark mass would be about 230 GeV, which indeed corresponds to a peak in mass distribution of top candidate, whereas $t\bar{t}$ meson mass would be 163 GeV. This requires that top quark mass changes very little in topological mixing. It is easy to see that the mass constraints imply that for $n_t = n_b = 60$ the smallness of V_{i3} and $V(3i)$ matrix elements implies that both U and D must be direct sums of 2×2 matrix and 1×1 unit matrix and that V matrix would have also similar decomposition. Therefore $n_b = n_t = 59$ seems to be the only number theoretically acceptable option. The comparison with the predictions with pion mass led to a unique identification $(n_d, n_b, n_b) = (5, 5, 59)$,$(n_u, n_c, n_t) = (4, 6, 59)$.

U and D matrices as perturbations of matrices mixing only the first two genera

This picture suggests that U and D matrices could be seen as small perturbations of very simple U and D matrices satisfying $|U| = |D|$ corresponding to $n = 60$ and having $(n_d, n_b, n_b) = (4, 5, 60)$, $(n_u, n_c, n_t) = (4, 5, 60)$ predicting V matrix characterized by Cabibbo angle alone. For instance, CP breaking parameter would characterize this perturbation. The perturbed matrices should obey thermodynamical constraints and it could be possible to linearize the thermodynamical conditions and in this manner to predict realistic mixing matrices from first principles. The existence of small perturbations yielding acceptable matrices implies also that these matrices be near a point at which two different matrices resulting as a solution to the thermodynamical conditions coincide.

D matrix can be deduced from U matrix since $9|D_{12}|^2 \simeq n_d$ fixes the value of the relative phase of the two terms in the expression of D_{12}.

$$\begin{aligned}
|D_{12}|^2 &= |U_{11}V_{12} + U_{12}V_{22}|^2 \\
&= |U_{11}|^2|V_{12}|^2 + |U_{12}|^2|V_{22}|^2 \\
&+ 2|U_{11}||V_{12}||U_{12}||V_{22}|cos(\Psi) = \frac{n_d}{9} \ , \\
\Psi &= arg(U_{11}) + arg(V_{12}) - arg(U_{12}) - arg(V_{22}) \ .
\end{aligned} \tag{4.4.-8}$$

Using the values of the moduli of U_{ij} and the approximation $|V_{22}| = 1$ this gives for $cos(\Psi)$

$$\begin{aligned}
\cos(\Psi) &= \frac{A}{B} \ , \\
A &= \frac{n_d - n_u}{9} - \frac{9 - n_u}{9}|V_{12}|^2 \ , \\
B &= \frac{2}{9|V_{12}|}\sqrt{n_u(9 - n_u)} \ .
\end{aligned} \tag{4.4.-9}$$

The experimentation with different values of n_d and n_u shows that $n_u = 6, n_d = 4$ gives $cos(\Psi) = -1.123$. Of course, $n_u = 6, n_d = 4$ option is not even allowed by $n_t = 60$. For $n_d = 4, n_u = 5$ one has $cos(\Psi) = -0.5958$. $n_d = 5, n_u = 6$ corresponding to the perturbed solution gives $cos(\Psi) = -0.6014$.

Hence the initial situation could be $(n_u = 5, n_s = 4, n_b = 60)$, $(n_d = 4, n_s = 5, n_t = 60)$ and the physical U and D matrices result from U and D matrices by a small perturbation as one unit of t (b) mass squared is transferred to u (s) quark and produces symmetry breaking as $(n_d = 5, n_s = 5, n_b = 59)$, $(n_u = 6, n_c = 4, n_t = 59)$.

The unperturbed matrices $|U|$ and $|D|$ would be identical with $|U|$ given by

$$|U_{11}| = |U_{22}| = \frac{2}{3} \ , \quad |U_{12}| = |U_{21}| = \frac{\sqrt{5}}{3} \ , \tag{4.4.-8}$$

The thermodynamical model allows solutions reducing to a direct sum of 2×2 and 1×1 matrices, and since $|U|$ matrix is fixed completely by the mass constraints, it is trivially consistent with the thermodynamical model.

Direct search of U and D matrices

The general formulas for p^U and p^D in terms of the probabilities p_{11} and p_{21} allow straightforward search for the probability matrices having maximum entropy just by scanning the (p_{11}, p_{21}) plane constrained by the conditions that all probabilities are positive and smaller than 1. In the physically interesting case the solution is sought near a solution for which the non-vanishing probabilities are $p_{11} = p_{22} = (9 - n_1)/9$, $p_{12} = p_{21} = n_1/9$, $p_{33} = 1$, $n_1 = 4$ or 5. The inequalities allow to consider only the values $p_{11} \geq (9 - n_1)/9$.

1. *Probability matrices p^U and p^D*

The direct search leads to maximally entropic p^D matrix with $(n_d, n_s) = (5, 5)$:

$$p^D = \begin{pmatrix} 0.4982 & 0.4923 & 0.0095 \\ 0.4981 & 0.4924 & 0.0095 \\ 0.0037 & 0.0153 & 0.9810 \end{pmatrix} \ , \ p_0^D = \begin{pmatrix} 0.5556 & 0.4444 & 0 \\ 0.4444 & 0.5556 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ . \tag{4.4.-8}$$

p_0^D represents the unperturbed matrix p_0^D with $n(d=4), n_s = 5$ and is included for the purpose of comparison. The entropy $S(p^D) = 1.5603$ is larger than the entropy $S(p_0^D) = 1.3739$. A possible interpretation is in terms of the spontaneous symmetry breaking induced by entropy maximization in presence of constraints.

A maximally entropic p^U matrix with $(n_u, n_c) = (5, 6)$ is given by

$$p^U = \begin{pmatrix} 0.5137 & 0.4741 & 0.0122 \\ 0.4775 & 0.4970 & 0.0254 \\ 0.0088 & 0.0289 & 0.9623 \end{pmatrix} \tag{4.4.-8}$$

The value of entropy is $S(p^U) = 1.7246$. There could be also other maxima of entropy but in the range covering almost completely the allowed range of the parameters and in the accuracy used only single maximum appears.

The probabilities p_{ii}^D *resp.* p_{ii}^U satisfy the constraint $p(i,i) \geq .492$ *resp.* $p_{ii} \geq .497$ so that the earlier proposal for the solution of proton spin crisis must be given up and the solution discussed in [D2] remains the proposal in TGD framework.

2. Near orthogonality of U and D matrices

An interesting question whether U and D matrices can be transformed to approximately orthogonal matrices by a suitable $(U(1) \times U(1))_L \times (U(1) \times U(1))_R$ transformation and whether CP breaking phase appearing in CKM matrix could reflect the small breaking of orthogonality. If this expectation is correct, it should be possible to construct from $|U|$ ($|D|$) an approximately orthogonal matrix by multiplying the matrix elements $|U_{ij}|$, $i,j \in \{2,3\}$ by appropriate sign factors. A convenient manner to achieve this is to multiply $|U|$ ($|D|$) in an element wise manner ($(A \circ B)_{ij} = A_{ij}B_{ij}$) by a sign factor matrix S.

1. In the case of $|U|$ the matrix $U = S \circ |U|$, $S(2,2) = S(2,3) = S(3,2) = -1$, $S_{ij} = 1$ otherwise, is approximately orthogonal as the fact that the matrix U^TU given by

$$U^TU = \begin{pmatrix} 1.0000 & 0.0006 & -0.0075 \\ 0.0006 & 1.0000 & -0.0038 \\ -0.0075 & -0.0038 & 1.0000 \end{pmatrix}$$

   is near unit matrix, demonstrates.

2. For D matrix there are two nearly orthogonal variants. For $D = S \circ |D|$, $S(2,2) = S(2,3) = S(3,2) = -1$, $S_{ij} = 1$ otherwise, one has

$$D^TD = \begin{pmatrix} 1.0000 & -0.0075 & 0.0604 \\ -0.0075 & 1.0000 & 0.0143 \\ 0.0604 & 0.0143 & 1.0000 \end{pmatrix} \ .$$

   The choice $D = S \circ D$, $S(2,2) = S(2,3) = S(3,3) = -1$, $S_{ij} = 1$ otherwise, is slightly better

$$D^TD = \begin{pmatrix} 1.0000 & -0.0075 & 0.0604 \\ -0.0075 & 1.0000 & 0.0143 \\ 0.0601 & 0.0143 & 1.0000 \end{pmatrix} \ .$$

3. The matrices U and D in the standard gauge

Entropy maximization indeed yields probability matrices associated with unitary matrices. 8 phase factors are possible for the matrix elements but only 4 are relevant as far as the unitarity conditions are considered. The vanishing of the inner products between row vectors, gives 6 conditions altogether so that the system seems to be over-determined. The values of the parameters s_1, s_2, s_3 and phase angle δ in the "standard gauge" can be solved in terms of r_{11} and r_{21}.

The requirement that the norms of the parameters c_i are not larger than unity poses non-trivial constraints on the probability matrices. This should should be the case since the number of unitarity conditions is 9 whereas probability conservation for columns and rows gives only 5 conditions so that not every probability matrix can define unitary matrix. It would seem that that the constraints are satisfied only if the the 2 mass squared conditions and 2 conditions from the entropy maximization are equivalent with 4 unitarity conditions so that the number of conditions becomes 5+4=9. Therefore entropy maximization and mass squared conditions would force the points of complex 9-dimensional space defined by 3×3 matrices to a 9-dimensional surface representing group $U(3)$ so that these conditions would have a group theoretic meaning.

The formulas

$$\begin{aligned} r_{i2} &= \sqrt{[-\frac{n_i}{51} + \frac{20}{17}(1 - r_{i1}^2)]} \ , \\ r_{i3} &= \sqrt{[\frac{n_i}{51} - \frac{3}{17}(1 - r_{i1}^2)]} \ . \end{aligned} \tag{4.4.-8}$$

and

$$U = \left[\begin{array}{ccc} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3exp(i\delta) & c_1c_2s_3 + s_2c_3exp(i\delta) \\ -s_1s_2 & c_1s_2c_3 + c_2s_3exp(i\delta) & c_1s_2s_3 - c_2c_3exp(i\delta) \end{array} \right] \tag{4.4.-7}$$

give

$$\begin{aligned} c_1 = r_{11} \ , \quad & c_2 = \frac{r_{21}}{\sqrt{1-r_{11}^2}} \ , \\ s_3 = \frac{r_{13}}{\sqrt{1-r_{11}^2}} \ , \quad & cos(\delta) = \frac{c_1^2c_2^2c_3^2 + s_2^2s_3^2 - r_{22}^2}{2c_1c_2c_3s_2s_3} \ . \end{aligned} \tag{4.4.-6}$$

Preliminary calculations show that for $n_1 = n_2 = 5$ case the matrix of moduli allows a continuation to a unitary matrix but that for $n_1 = 4, n_2 = 6$ the value of $cos(\delta)$ is larger than one. This would suggest that unitarity indeed gives additional constraints on the integers n_i. The unitary (in the numerical accuracy used) $(n_d, n_s) = (5, 5)$ D matrix is given by

$$D = \left(\begin{array}{ccc} 0.7059 & 0.7016 & 0.0975 \\ -0.7057 & 0.7017 - 0.0106i & 0.0599 + 0.0766i \\ -0.0608 & 0.0005 + 0.1235i & 0.4366 - 0.8890i \end{array} \right) \ .$$

The unitarity of this matrix supports the view that for certain integers n_i the mass squared conditions and entropy maximization reduce to group theoretic conditions. The numerical experimentation shows that the necessary condition for the unitarity is $n_1 > 4$ for $n_2 < 9$ whereas for $n_2 \geq 9$ the unitarity is achieved also for $n_1 = 4$.

Direct search for CKM matrices

The standard gauge in which the first row and first column of unitary matrix are real provides a convenient representation for the topological mixing matrices: it is convenient to refer to these representations as U_0 and D_0. The possibility to multiply the rows of U_0 and D_0 by phase factors ($(U(1) \times U(1))_R$ transformations) provides 2 independent phases affecting the values of $|V|$. The phases $exp(i\phi_j)$, $j = 2, 3$ multiplying the second and third row of D_0 can be estimated from the matrix elements of $|V|$, say from the elements $|V_{11}| = cos(\theta_c) \equiv v_{11}$, $sin\theta_c = .226 \pm .002$ and $|V_{31}| =$

$(9.6 \pm .9) \cdot 10^{-3} \equiv v_{31}$. Hence the model would predict two parameters of the CKM matrix, say s_3 and δ_{CP}, in its standard representation.

The fact that the existing empirical bounds on the matrix elements of V are based on the standard model physics raises the question about how seriously they should be taken. The possible existence of fractally scaled up versions of light quarks could effectively reduce the matrix elements for the electro-weak decays $b \rightarrow c + W$, $b \rightarrow u + W$ *resp.* $t \rightarrow s + W$, $t \rightarrow d + W$ since the decays involving scaled up versions of light quarks can be counted as decays $W \rightarrow bc$ *resp.* $W \rightarrow tb$. This would favor too small experimental estimates for the matrix elements V_{i3} and V_{3i}, $i = 1, 2$. In particular, the matrix element $V_{31} = V_{td}$ could be larger than the accepted value.

Various constraints do not leave much freedom to choose the parameters n_{q_i}. The preliminary numerical experimentation shows that the choice $(n_d, n_s) = (5, 5)$ and $(n_u, n_c) = (5, 6)$ yields realistic U and D matrices. In particular, the conditions $|U(1,1)| > .7$ and $|D(1,1)| > .7$ hold true and mean that the original proposal for the solution of spin puzzle of proton must be given up. In [D2] an alternative proposal based on more recent findings is discussed. Only for this choice reasonably realistic CKM matrices have been found.

1. The requirement that the parameters $|V_{11}|$ (or equivalently, Cabibbo angle and $|V_{31}|$ are produced correctly, yields CKM matrices for which CP breaking parameter J is roughly one half of its accepted value. The matrix elements $V_{23} \equiv V_{cb}$, $V_{32} \equiv V_{tc}$, and $V_{13} \equiv V_{ub}$ are roughly twice their accepted value. This suggests that the condition on V_{31} should be loosened.

2. The following tables summarize the results of the search requiring that
   i) the value of the Cabibbo angle s_{Cab} is within the experimental limits $s_{Cab} = .223 \pm .002$,
   ii) $V_{31} = (9.6 \pm .9) \cdot 10^{-3}$, is allowed to have value at most twice its upper bound,
   iii) V_{13} whose upper bound is determined by probability conservation, is within the experimental limits $.42 \cdot 10^{-3} < |V_{ub}| < 6.98 \cdot 10^{-3}$ whereas $V_{23} \simeq 4 \times 10^{-3}$ should come out as a prediction,
   iv) the CP breaking parameter satisfies the condition $|(J - J_0)/J_0| < .6$, where $J_0 = 10^{-4}$ represents the lower bound for J (the experimental bounds for J are $J \times 10^4 \in (1 - 1.7)$).

The pairs of the phase angles (ϕ_1, ϕ_2) defining the phases $(exp(i\phi_1), exp(i\phi_2))$ are listed below

$$
\begin{array}{llllll}
class\ 1: & \phi_1 & 0.1005 & 0.1005 & 4.8129 & 4.8129 \\
 & \phi_2 & 0.0754 & 1.4828 & 4.7878 & 6.1952 \\
class\ 2: & \phi_1 & 0.1005 & 0.1005 & 4.8129 & 4.8129 \\
 & \phi_2 & 2.3122 & 5.5292 & 0.7414 & 3.9584
\end{array}
\quad (4.4.\text{-}6)
$$

The phase angle pairs correspond to two different classes of U, D, and V matrices. The U, D and V matrices inside each class are identical at least up to 11 digits(!). Very probably the phase angle pairs are related by some kind of symmetry.

The values of the fitted parameters for the two classes are given by

	$\lvert V_{11}\rvert$	$\lvert V_{31}\rvert$	$\lvert V_{13}\rvert$	$J/10^{-4}$
class 1	0.9740	0.0157	0.0069	.93953
class 2	0.9740	0.0164	0.0067	1.0267

V_{31} is predicted to be about 1.6 times larger than the experimental upper bound and for both classes V_{23} and V_{32} are roughly too times too large. Otherwise the fit is consistent with the experimental limits for class 2. For class 1 the CP breaking parameter is 7 per cent below the experimental lower bound. In fact, the value of J is fixed already by the constraints on V_{31} and V_{11} and reduces by a factor of one half if V_{31} is required to be within its experimental limits.

U, D and $|V|$ matrices for class 1 are given by

$$U = \begin{bmatrix} 0.7167 & 0.6885 & 0.1105 \\ -0.6910 & 0.7047-0.0210i & 0.0909+0.1310i \\ -0.0938 & 0.0696+0.1550i & 0.1747-0.9653i \end{bmatrix}$$
$$D = \begin{bmatrix} 0.7059 & 0.7016 & 0.0975 \\ -0.6347-0.3085i & 0.6358+0.2972i & 0.0203+0.0951i \\ -0.0587-0.0159i & -0.0317+0.1194i & 0.6534-0.7444i \end{bmatrix}$$
$$|V| = \begin{bmatrix} 0.9740 & 0.2265 & 0.0069 \\ 0.2261 & 0.9703 & 0.0862 \\ 0.0157 & 0.0850 & 0.9963 \end{bmatrix} \tag{4.4.-8}$$

U, D and $|V|$ matrices for class 2 are given by

$$U = \begin{bmatrix} 0.7167 & 0.6885 & 0.1105 \\ -0.6910 & 0.7047-0.0210i & 0.0909+0.1310i \\ -0.0938 & 0.0696+0.1550i & 0.1747-0.9653i \end{bmatrix}$$
$$D = \begin{bmatrix} 0.7059 & 0.7016 & 0.0975 \\ -0.6347-0.3085i & 0.6358+0.2972i & 0.0203+0.0951i \\ -0.0589-0.0151i & -0.0302+0.1198i & 0.6440-0.7525i \end{bmatrix}$$
$$|V| = \begin{bmatrix} 0.9740 & 0.2265 & 0.0067 \\ 0.2260 & 0.9704 & 0.0851 \\ 0.0164 & 0.0838 & 0.9963 \end{bmatrix} \tag{4.4.-10}$$

What raises worries is that the values of $|V_{23}| = |V_{cb}|$ and $|V_{32}| = |V_{ts}|$ are roughly twice their experimental estimates. This, as well as the discrepancy related to V_{31}, might be understood in terms of the electro-weak decays of b and t to scaled up quarks causing a reduction of the branching ratios $b \rightarrow c + W$, $t \rightarrow s + W$ and $t \rightarrow t + d$. The attempts to find more successful integer combinations n_i has failed hitherto. The model for pseudoscalar meson masses, the predicted relatively small masses of light quarks, and the explanation for $t\bar{t}$ meson mass supports this mixing scenario.

4.5 Hadron masses

Besides the quark contributions already discussed, hadron mass squared can contain several other contributions and the task is to find a model allowing to identify and estimate these contributions. There are several guidelines for the numerical experimentation.

1. Conformal weight, that is mass squared, is assumed to be additive for quarks corresponding to the same p-adic prime. For instance, in case of $q\bar{q}$ mesons the mass would be $\sqrt{2}m(q)$ and the contribution of $k = 113$ u, d, s quarks to nucleon mass would be $< \sqrt{3} \times 100$ MeV and thus surprisingly small. For cd meson quark masses would be additive.

2. Old fashioned quark model explains reasonably well hadron masses in terms of constituent quark masses. Effective 2-dimensionality of partons suggests an interpretation for the constituent quark as a composite structure formed by the current quark identified as a partonic 2-surface X^2 characterized by $k(q)$ and by join along boundaries bond, kind of a gluonic "rubber band" characterized by $k = 107$ and connecting X^2 to the $k = 107$ hadronic 2-surface $X^2(H)$ representing hadron. $X^2(q_i)$ could be perhaps regarded as a hole in $k = k(q)$ 3-surface. The 2-dimensional visualization for a 3-dimensional topological condensation would become much more than a mere visualization. This view about hadrons brings in mind unavoidably the surreal 2-dimensional structures formed by organs like retina. Of course, effective 2-dimensionality allows to characterize the entire Universe as an extremely complex fractal 2-surface.

The large mass of the constituent quark would be due to the color Coulombic and spin-spin interaction conformal weights of join along boundaries bond. Quark mass and the mass due to the color interaction conformal weight would be additive unless $k = 107$ for the quark (it seems that for η' this is indeed the case!). Classical color gauge fluxes would flow between $k = 107$ and $k \neq 107$ space-time sheets along the bonds. Color dynamics would take place at $k = 107$ space-time sheet in the sense that color gauge flux between quarks q_1 and q_2 flows first from $X^2((k(q_1)$ to the hadronic 2-surface $X^2(k = 107)$ and then back to $X^2(k(q_2))$. The induced Kähler field is always accompanied by a classical color gauge field and the classical color gauge flux would represent non-perturbative aspects of color interactions at space-time level.

3. A crucial observation is that the mass of η meson is rather precisely 4 times the pion mass whereas the mass of its spin excited companion ω is very nearly the same as the mass of ρ meson. This suggests that u, d quarks correspond to $k = 109$ inside η but to $k = 113$ inside ω. This inspires the idea that the p-adic mass scale of quarks is dynamical and sensitive to small perturbations as the fact that for CP_2 type extremals the operators corresponding to different p-adic primes reduce to one and same operator forces to suspect. If k characterizes the length scale associated with the elementary particle horizon as $\sqrt{k}$ multiple of CP_2 length scale, quark mass would be characterized by the size of elementary particle horizon sensitive to the dynamics in hadronic mass scale.

   The physical states would result as small perturbations of this degenerate ground state and the value of $k(q)$ would be sensitive to the perturbation. A rather nice fit for meson and baryon masses results by assuming that the p-adic length scale of the quark is dynamical.

4. In the case of pseudoscalar mesons the scaled up versions of light quarks identifiable as constituent quarks, turn out to explain almost all of the pseudo scalar meson mass, and this inspires a new formulation for the old vision about pseudoscalar mesons as Goldstone bosons. At least light pseudoscalar mesons are Goldstone bosons in the sense that the color Coulombic and spin-spin interaction energies cancel in a good approximation so that quarks at $k \neq 107$ space-time sheets are responsible for most of the meson mass. The assumption that only $k(s)$ is dynamical for light baryons is enough to understand the mass differences between baryons having different numbers of strange quarks.

5. Color magnetic spin-spin interaction energies are indeed surprisingly constant among baryons. Also for mesons spin-spin interaction energies vary much less than the scaling of quark masses would predict on basis of QCD formula. This motivates the replacement of the interaction energy with interaction conformal weight in the case of color interactions. The interaction conformal weight is assignable to $k = 107$ space-time sheet, and the fact that spin-spin splittings of also heavy hadrons can be measured in few hundred MeVs, supports this identification. The mild dependence of color Coulombic conformal weight and spin-spin interaction conformal weight on hadron would be due to their dependence on the primes $k(q_i)$ and $k = 107$ characterizing space-time sheets connected by the the color bonds $q_i \rightarrow 107$ and $107 \rightarrow q_j$.

6. The values for the parameters s^c_{ij} and S_{ij} characterizing color Coulombic and color magnetic interaction conformal weights can be deduced from the mass squared differences for hadrons and assuming definite values for the parameters $k(q_i)$ characterizing quark masses. It seems that no other sources to meson mass (or at least pion mass) are needed.

7. In the case of nucleons the understanding of nucleon mass requires a large additional contribution about 780 MeV since quarks contribute only about 160 MeV to the mass of nucleon. This contribution can be assumed to be same for all baryons as the possibility to understand baryon mass differences in terms of quark masses demonstrates. The most plausible identification of this contribution is in terms of 2- or 3-particle state formed by super-canonical gluons assignable to $k = 107$ hadronic space-time sheet and having conformal weight $s = 16$ corresponding to mass 934.2 MeV (rather near to nucleon mass and η' mass). This leads to a vision about non-perturbative aspects of color interactions and allows to understand baryon masses with accuracy better than one per cent. Also a connection with hadronic string model emerges and hadronic string tension is predicted correctly.

4.5.1 The definition of the model for hadron masses

The defining assumptions of the model of hadron masses are following. CP_2 mass defines the overall elementary particle mass scale. Electron mass determines this mass only in certain limits.

Model for hadronic quarks

The numerical construction of U and D matrices using the thermodynamical model for the topological mixing justifies the assumptions $n_d = n_s = 5, n_b = 59$ and $n_u = 5, n_c = 6, n_t = 58$.

Quarks can appear both as free quarks and bound state quarks and the value of $k(q)$ is in general different for free and bound state quarks and can depend on hadron in case of bound state quarks. This allows to understand satisfactorily the masses of low lying hadrons.

Quark mass contribution to the mass of the hadron

Quark mass squared is p-adically additive for quarks with same value of p-adic prime. In the case of meson one has

$$m_M^2(p_1 = p_2) = m_{q_1}^2 + m_{q_2}^2 \ . \tag{4.5.1}$$

m_q denotes constituent quark mass which is larger than current quark mass due to the smaller value of k.

Masses are additive for different values of p.

$$m_M(p_1 \neq p_2) = m_{q_1} + m_{q_2} \ . \tag{4.5.2}$$

The generalization of these formulas to the cse of baryons is trivial.

Super-canonical gluons and non-perturbative aspects of hadron physics

At least in the case of light pseudoscalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudoscalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudoscalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of configuration space degrees of freedom ("world of classical worlds") in which super-canonical algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges [F5].

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-canonical gluons. For $g = 0$ these gluons are massless and in absence of topological mixing could form a contribution analogous to sea or Bose-Einstein condensate. For $g = 1$ their mass can be calculated. It turns out the model assuming same topological mixing as in case of U quarks leads to excellent understanding of baryon masses assuming that hadron spin correlates with the super-canonical particle content of the hadronic space-time sheet.

Top quark mass as a fundamental constraint

CP_2 mass is an important parameter of the model. The vanishing second order contribution to electron mass gives an upper bound for CP_2 mass. The bound $Y_e \leq .7357$ can be derived from the requirement that it is possible to reproduce τ mass in p-adic thermodynamics. Maximal second order contribution corresponds to a minimal CP_2 mass reduced by a factor $\sqrt{5/6} = .9129$ from its maximal value. There is a natural mechanism making second order contribution negligible. Leptonic masses tend to be predicted to be few per cent too high [F3] if the second order contribution from p-adic thermodynamics to the electron mass vanishes, which suggests that second order contribution might be there.

For $Y_e = 0$ and $Y_t = 1$ the most recent experimental best estimate 169.1 GeV [58] for top quark mass is reproduced exactly. Even $Y_t = 0$ allows a prediction in the allowed range. For too large Y_e top quark mass is predicted to be too small unless one allows first order Higgs contribution to the top quark mass. This means that CP_2 mass can be scaled down from its maximal value at most 2.5 per cent. This translates to the condition $Y_e < .26$. It is possible to understand quark masses satisfactorily by assuming that Higgs contribution is second order p-adically and even negligible. In fact, there are good arguments suggesting that Higgs does not develop vacuum expectation at fermionic space-time sheets [F3]. If this is the case, top quark mass gives a very strong constraint to the model.

The super-canonical color interactions associated with $k = 107$ space-time sheet give rise to the dominant reduction of the color conformal weight having interpretation in terms of color magnetic and electric conformal weights. Canonical correspondence implies that this contribution is always non-negative. Therefore the simple additive formula can lead to a situation in which the contribution of quarks to the meson mass can be slightly larger than meson mass and it is not obvious whether it is possible to reduce this contribution by any means since the reduction of CP_2 mass scale makes top quark mass too small.

For diagonal mesons for which quarks have the same value of p-adic prime, ordinary color interaction between quarks can contribute negative conformal weight reducing the contribution to the mass squared. In the case of non-diagonal mesons it is not clear whether this kind of color interaction exists. This kind of gluons would correspond to pairs of light-like partonic 3-surfaces for which throats correspond to different values of p-adic prime p. These are in principle possible but could couple weakly to matter. It seems that the parameters of the model, essentially CP_2 mass scale strongly constrained by the top quark mass, allow the quark contributions of non-diagonal mesons to be below the mass of the meson.

The fact that standard QCD model for color binding energies works rather well for heavy mesons suggests that the notion of negative color binding energy might make sense and could explain the discrepancy. The mixing of real and p-adic physics descriptions is however aesthetically very unappealing but might be the only way out of the problem. The p-adic counterpart of this description in case of heavy diagonal mesons would be based on the introduction of a negative color Coulombic contribution to the the conformal weight of quark pair.

Smallness of isospin splittings

The smallness of isospin splittings inside $I_s = 1/2$ doublets poses an further constraint. $d_{113} - u_{113}$ mass difference is about $\Delta m_{d-u} = 13$ MeV and larger than typical isospin splitting. The repulsive Coulomb interaction between quarks typically tends to reduce the mass differences due to Δm_{d-u} and the the sign of Δm_{d-u} explains the "wrong" sign of n-p mass difference equal to $\Delta m_{n-p} = 1.3$ MeV. Non-diagonal hadrons containing scaled up u and d quarks would have anomalously large isospin splittings. On the other hand, for a diagonal meson containing b quark and scaled up u and d quark isospin splitting is proportional to $(m_d^2 - m_u^2)/m_b$ and small. B meson corresponds to this kind of situation.

4.5.2 The anatomy of hadronic space-time sheet

Although the presence of the hadronic space-time sheet having $k = 107$ has been obvious from the beginning, the questions about its anatomy emerged only quite recently after the vision about the spectrum of Kähler coupling strength had emerged [C4, F5].

In the case of pseudoscalar mesons quarks give the dominating contribution to the meson mass. This is not true for spin 1/2 baryons and the dominating contribution must have some other origin.

TGD allows to identify this contribution in terms of states created by purely bosonic generators of super-canonical algebra and having as a space-time correlate CP_2 type vacuum extremals topologically condensed at $k = 107$ hadronic space-time sheet (or having this space-time sheet as field body). Proton and neutron masses are predicted with .5 per cent accuracy and $\Delta - N$ mass splitting with .6 per cent accuracy. A further outcome is a possible solution to the spin puzzle of proton proposed already earlier [F5].

Quark contribution cannot dominate light baryon mass

The first guess would be that the masses give dominating contribution to the mass of baryon. Since mass squared is additive, this would require rather large quark masses for proton and neutron. $k(d) = k(u) = k(s) = 108$ would give $(m(d), m(u), m(s)) = (571.3, 520.4, 616.6)$ MeV and $(m(n), m(p)) = (961.1, 931.7)$ MeV to be compared with the actual masses $(m(n), m(p) = (939.6, 938.3)$ MeV. The difference looks too large to be explainable in terms of Coulombic self-interaction energy. $\lambda - n$ mass splitting would be 27.6 MeV for $k(s) = 108$ which is much smaller than the real mass splitting 176.0 MeV. For $k(s) = 110$ one would have 120.0 MeV.

Does $k = 107$ hadronic space-time sheet give the large contribution to baryon mass?

In the sigma model for baryons the dominating contribution to the mass of baryon results as a vacuum expectation value of scalar field and light pseudoscalar mesons are analogous to Goldstone bosons whose masses are basically due to the masses of light quarks.

This would suggest that $k = 107$ gluonic/hadronic space-time sheet gives a large contribution to the mass squared of baryon. p-Adic thermodynamics allows to expect that the contribution to the mass squared is in a good approximation of form

$$\Delta m^2 = nm^2(107) \ ,$$

where $m^2(107)$ is the minimum possible p-adic mass mass squared and n a positive integer. One has $m(107) = 2^{10}m(127) = 2^{10}m_e/\sqrt(5) = 233.55$ MeV for $Y_e = 0$ favored by the top quark mass.

1. $n = 11$ predicts $(m(n), m(p)) = (944.5, 939.3)$ MeV for $k = 113$ quarks: the actual masses are $(m(n), m(p) = (939.6, 938.3)$ MeV. Coulombic repulsion between u quarks could reduce the p-n difference to a realistic value.

2. $\lambda - n$ mass splitting would be 184.7 MeV for $k(s) = 111$ to be compared with the real difference which is 176.0 MeV. Note however that color magnetic spin-spin splitting requires that the ground state mass squared is larger than $11m_0^2(107)$.

What is responsible for the large ground state mass of the baryon?

The observations made above do not leave much room for alternative models. The basic problem is the identification of the large contribution to the mass squared coming from the hadronic space-time sheet with $k = 107$. This contribution could have the energy of classical color field as a space-time correlate.

1. The assignment of the energy to the vacuum expectation value of sigma boson does not look very promising since the very existence sigma boson is questionable and it does not relate naturally to classical color gauge fields. More generally, since no gauge symmetry breaking is involved, the counterpart of Higgs mechanism as a development of a coherent state of scalar bosons does not look a plausible idea.

2. One can however consider the possibility of a Bose-Einstein condensate or of a more general many-particle state of massive bosons possibly carrying color quantum numbers. A many-boson state of exotic bosons at $k = 107$ space-time sheet having net mass squared

$$m^2 = nm_0^2(107) \ , \ \ n = \sum_i n_i$$

could explain the baryonic ground state mass. Note that the possible values of n_i are predicted by p-adic thermodynamics with $T_p = 1$.

Glueballs cannot be in question

Glueballs [62, 63] define the first candidate for the exotic boson in question. There are however several objections against this idea.

1. QCD predicts that lightest glue-balls consisting of two gluons have $J^{PC} = 0^{++}$ and 2^{++} and have mass 1650 MeV [63]. If one takes QCD seriously, one must exclude this option. One can also argue that light glue balls should have been observed long ago and wonder why their Bose-Einstein condensate is not associated with mesons.

2. There are also theoretical objections in TGD framework.

   i) Can one really apply p-adic thermodynamics to the bound states of gluons? Even if this is possible, can one assume the p-adic temperature $T_p = 1$ for them if $T_p < 1$ holds true for gauge bosons consisting of fermion-antifermion pairs [F5, C4].

   ii) Baryons are fermions and one can argue that they must correspond to single space-time sheet rather than a pair of positive and negative energy space-time sheets required by the glueball Bose-Einstein condensate realized as wormhole contacts connecting these space-time sheets. This argument should be taken with a big grain of salt.

Do exotic colored bosons give rise to the ground state mass of baryon?

The objections listed above lead to an identification of bosons responsible for the ground state mass, which looks much more promising.

1. Super-canonical gluons

TGD predicts exotic bosons and their super-conformal partners. The bosons created by the purely bosonic part of super-canonical algebra [B2, B3, B4], whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. The super-partners of the super-canonical bosons have quantum numbers of a right handed neutrino and have no electro-weak couplings. Recall that super-canonical algebra is crucial for the construction of configuration space Kähler geometry.

Exotic bosons are single-sheeted structures meaning that they correspond to a single wormhole throat associated with a CP_2 type vacuum extremal. The assignment of these bosons to hadronic space-time is an attractive idea. The only contribution to the mass would come from the genus and $g = 0$ state would be massless in absence of topological mixing. In this case $g = 0$ bosons could condense on the ground state and define the analog of gluonic contribution to the parton sea. If they mix situation changes.

$g = 1$ unmixed super-canonical boson would have mass squared $9m_0^2(k)$ (mass would be 700.7 MeV). For a ground state containing two $g = 1$ exotic bosons, one would have ground state mass squared $M_0^2 = 18m_0^2$ corresponding to $(m(n), m(p)) = (1160.8, 1155.6)$ MeV. Negative color Coulombic conformal and color magnetic spin-spin splitting can reduce the mass of system as well as. Electromagnetic Coulomb interaction energy can reduce the p-n mass splitting to a realistic value.

1. Color magnetic spin-spin splitting for baryons gives a test for this hypothesis. The splitting of the conformal weight is by group theoretic arguments of the same general form as that of color magnetic energy and given by $(m^2(N), m^2(\Delta)) = (18m_0^2 - X, 18m_0^2 + X)$ in absence of topological mixing. $n = 11$ for nucleon mass implies $X = 7$ and $m(\Delta) = 5m_0(107) = 1338$ MeV to be compared with the actual mass $m(\Delta) = 1232$ MeV. The prediction is too large by about 8.6 per cent.

   If one allows negative color Coulombic conformal weight $\Delta s = -2$ the mass squared reduces by 2 units. The alternative is topological mixing one can have $m^2 = 8m_0^2$ instead of $9m_0^2$. This gives $m(\Delta) = 1240$ MeV so that the error is only .6 per cent. The mass of topologically mixed exotic boson would be 660.6 MeV and equals to m_{104}. Amusingly, $k = 104$ happens to corresponds to the inverse of α_K for gauge bosons.

2. One must consider also the possibility that super-canonical gluons suffer topological mixing identical with that suffered by say U type quarks in which the conformal weights would be

(5,6,58) for the three lowest generations. The ground state of baryon could consist of 2 gluons of lowest generation and one gluon of second generation ($5+5+6=16$). If mixing is same as for D type quarks with weights (5,5,59), one can have only $s=15$ state. It turns out that this option allows to predict hadron masses with amazing precision if one assumes correlation between hadron spin and its super-canonical particle content.

3. The conclusion is that a many-particle state of super-canonical bosons could be responsible for the ground state mass of baryon. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.

2. The value of α_s in super-canonical phase

If one takes seriously the reduction of the spectrum of α_K and p-adic temperature to that for the Chern-Simons coupling k [F5, C4], one ends up with the conclusion that particles which correspond to single light-like wormhole throat (ordinary fermions and super-canonical bosons and fermions) must correspond to $k=1$ implying $T_p=1$ and $\alpha_K=1/4$. Ordinary gauge bosons would correspond to pairs of light-like wormhole throats (wormhole contacts).

The large value of the Kähler coupling strength $\alpha_K=1/4$ would characterize the hadronic space-time sheet as opposed to $\alpha_K=1/104$ assignable to the gauge bosons. Hence the color gauge coupling characterizing their interactions would be strong. This would provide a precise articulation for what the generation of the hadronic space-time sheet in the phase transition to a non-perturbative or confining phase of QCD really means.

One can even guess the value of α_s in the non-perturbative phase. The fact that Kähler action is proportional to both electro-weak U(1) action and classical color YM action led already earlier to the formula

$$\frac{1}{\alpha_s}+\frac{1}{\alpha_{U(1)}}=\frac{1}{\alpha_K} \ .$$

holding true for ordinary gauge interactions. This formula leads to a prediction of α_K if one assumes that α_s diverges at electron length scale $p=M_{127}$ so that one has

$$\frac{1}{\alpha_K}=\frac{1}{\alpha_{U(1)}(M_{127})} \ .$$

From the experimental value of $\alpha_{U(1)}(M_{127})$ and formula $\alpha_K=1/4k$ one can deduce $k=26$.

At the hadronic $k=107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s=\alpha_K=\frac{1}{4} \ .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

The large value of α_K suggests that the criterion for a phase transition increasing the value of Planck constant [A9] and leading to a phase, where $\alpha_K \propto 1/hbar$ is reduced, could occur. This would mean that super-canonical bosons would represent dark matter in a well-defined sense. Note however that the fact that super-canonical bosons have no electro-weak interactions, could imply their dark matter character even for the ordinary value of Planck constant.

An interesting side question is what the attribute 'non-perturbative' could mean in p-adic context. Could it mean that the contribution of color interactions to the conformal weight of system consisting of quarks labelled by same prime p is of first order in p? This definition would allow also ordinary color interactions to be non-perturbative in case of diagonal mesons.

3. A connection with hadronic string model

Hadronic string model provides a phenomenological description of the non-perturbative aspects of hadron physics, and TGD was born also as a generalization of the hadronic string model. Hence one can ask whether something resembling hadronic string model might emerge from the super-canonical sector. TGD allows string like objects but the fundamental string tension is gigantic, roughly a factor 10^{-8} of that defined by Planck constant. The hypothesis motivated by the p-adic length scale hypothesis is that vacuum extremals deformed to non-vacuum extremals give to a hierarchy of string like structures with string tension $T \propto 1/L_p^2$, L_p the p-adic length scale. The challenge has been the identification of quantum counterpart of this picture.

The fundamental mass formula of the string model relates mass squared and angular momentum of the stringy state. It has the form

$$M^2 = kJ \ , \ k \simeq .9 \ GeV^2 \ . \qquad (4.5.3)$$

A more general formula is $M^2 = kn$.

This kind of formula results from the additivity of the conformal weight (and thus mass squared) if one constructs a many particle state from $g = 1$ super-canonical bosons with a thermal mass squared $M^2 = M_0^2 n$, $M_0^2 = n_0 m_{107}^2$. The angular momentum of the building blocks has some spectrum fixed by Virasoro conditions. If the basic building block has angular momentum J_0 and mass squared M_0^2, one obtains $M^2 = M_0^2 J$, $k = M_0^2$, $J = nJ_0$. The values of n are even in old fashioned string model for a Regge trajectory with a fixed parity. $J_0 = 2$ implies the same result so that basic unit might be called "strong graviton".

One can consider several candidates for the values of n_0. In the absence of topological mixing one has $n_0 = 9$ for super-canonical gluons. The bound state of two super-canonical $g = 1$ bosons with mass squared $M_0^2 = 16m_{107}^2$ (two units of color binding conformal weight) could be responsible for the ground state mass of baryons. If topological mixing occurs and is same as for U type quarks then also a bound state of 2 gluons of first generation and 1 gluon of second generation gives $M_0^2 = 16m_{107}^2$.

The table below summarizes the prediction for the string tension in various cases. The identification of the basic excitations as many-particle states from from bound states of super-canonical gluons with $M_0^2 = 16m_{107}^2$ predicts the nominal value of the .9 GeV with 3 per cent accuracy.

n_0	5	9	16	18
M_0^2/GeV^2	.273	.490	0.872	0.982

Table 6. The prediction for the hadronic string tension for some values of the mass squared of super-canonical particle used to construct hadronic excitations.

Pomeron [70] represented an anomaly of the hadronic string model as a hadron like particle which was not accompanied by a Regge trajectory. A natural interpretation would be as a space-time sheet containing valence quarks as a structure connected by color flux tubes to single structure. There is recent quite direct experimental evidence for the existence of Pomeron [71, 72, 74] in proton photon collisions: Pomeron seems to leave the hadronic space-time sheet for a moment and collide with photon after which it topologically condenses back to the hadronic space-time sheet. For a more detailed discussion see [F5].

This picture allows also to consider a possible mechanism explaining spin puzzle of proton and I have already earlier considered an explanation in terms of super-canonical spin [F5] assuming that state is superposition of ordinary $(J = 0, J_q = 1/2)$ state and $(J = 2, J_q = 3/2)$ state in which super-canonical bound state has spin 2.

4. Some implications

If one accepts this picture, it becomes possible to derive general mass formulas also for the baryons of scaled up copies of QCD possibly associated with various Mersenne primes and Gaussian Mersennes. In particular, the mass formulas for leptobaryons, in particular "electro-baryons", can be deduced [F7]. Good estimates for the masses of the mesons and baryons of M_{89} hadron physics are also obtained by simple scaling of the ordinary hadron masses by factor 512. Scaled up isospin splittings would be one signature of M_{89} hadron physics: for instance, n-p splitting of 1.3 MeV would scale up to 665.6 MeV.

What about mesons?

The original short-lived belief was that only baryons are accompanied by a pair of super-canonical bosons condensed at hadronic $k = 107$ space-time sheet. By noticing that color magnetic spin-spin splitting requires an additional contribution to the mass conformal weight of meson cancelled by spin-spin splitting conformal weight in the case of pseudoscalar mesons to first order in p, one ends up with the conclusion that also mesons could possess the hadronic space-time sheet.

It however turns out that the contribution of super-canonical massive boson is necessarily only in the case of $\pi - \rho$ system and produces mere nuisance in the case of heavier mesons. The special role of $\pi - \rho$ system could be understood in terms of color confinement which would make pion $k = 107$ tachyon without the presence of additional mass squared.

The super-canonical contribution must correspond to a conformal weight of 5 units in the case of pion and thus to *single* super-canonical boson with $m^2 = 5m_{107}^2$ instead of $9m_{107}^2$ as for $g = 1$ super-canonical bosons. A possible interpretation is in terms of $g = 0$ boson which has suffered a topological mixing. That 5 units of conformal weight result also in the topological mixing of u and d quarks supports this option and forces to ask whether also super-canonical topological mixing is same inside baryons and mesons. If it is same for U type quarks and super-canonical bosons one has $(s_1, s_2, s_3) = (5, 6, 58)$ for the super-canonical gluons. As noticed, $S_{SC} = 16$ for baryons is obtained if one has a bound state of 2 bosons of first generation and one boson of second generation giving $s_{SC} = 5 + 5 + 6 = 16$. One can wonder how tightly the super-canonical gluons are associated with baryonic valence quarks.

4.5.3 Pseudoscalar meson masses

The requirement that all contributions to the meson masses have p-adic origin allows to fix the model uniquely and also constraints on the value of the parameter Y_e emerge. In the following only pseudoscalar mesons will be considered.

Light pseudoscalar mesons as analogs of Goldstone bosons

Fractally scaled up versions of light quarks allow a rather simple model for hadron masses. In the old fashioned $SU(3)$ based quark model η meson is regarded as a combination $u\overline{u} + d\overline{d} - 2s\overline{s}$. The basic observation is that η mass is rather precisely 4 times the mass of π whereas the mass of ω is very near to ρ mass. This suggests that η results by a fractal scaling of quark masses obtained by the replacement $k(q) = 113 \rightarrow 109$ for the quarks appearing in η. This inspires the idea that mesonic quarks are scaled up variants of light quarks and at least light pseudoscalar mesons are almost Goldstone bosons in the sense that quark contribution to the mass is as large as possible but smaller than meson mass. This idea must of course be taken as an interesting ansatz and in the end of the chapter it will be found that this idea might work only in the case of pion and kaon systems.

Quark contributions to meson masses

The following table summarizes the predictions for quark contributions to the masses of mesons assuming $k(q)$ depending on meson and assuming $Y_e = 0$ guaranteing maximum value of top quark mass.

$Meson$	$scaled\ quarks$	$m_q(M)/MeV$	m_{exp}/MeV
π^0	d_{113}, u_{113}	140.0	135.0
π^+	d_{113}, u_{113}	140.0	139.6
K^0	d_{114}, s_{109}	495.5	497.7
K^+	u_{114}, s_{109}	486.3	493.7
η	$u_{109}, d_{109}, s_{109}$	522.2	548.9
η'	$u_{107}, d_{107}, s_{107}, c_{107}$	1144.2	957.6
$\eta' = B_{SC} + \sum_i q_i\overline{q}_i$	q_{118}	959.2	957.6
η_c	c_{104}	3098	2980
D^0	c_{105}, u_{113}	1642	1865
D^+	c_{105}, d_{113}	1654	1870
Υ	b_{103}	10814	9460
B	$b_{104}, d_{104}, u_{104}$	5909	5270

Table 7. Summary of the model for contribution of quarks to the masses of mesons containing

scaled up u,d, and s quarks. The model assumes the maximal value of CP_2 mass allowed by η' mass and the condition $Y_e = 0$ favored by top quark mass.

1. The quark contribution to pion mass is predicted to be 140 MeV, which is by few percent above the pion mass. Ordinary color interactions between pionic quarks can however reduce the conformal weight of pion by one unit. The reduction of CP_2 mass scaled cannot be considered since it would reduce top quark mass to 163.3 GeV which is slightly below the favored range of values [58].

2. The success of the fit requires that spin-spin splitting cancels the mass of super-canonical boson in a good approximation for pseudoscalar mesons. This would be in accordance with the Goldstone boson interpretation of pseudoscalar mesons in the sense that color contribution to the mass from $k = 107$ space-time sheet vanishes in the lowest p-adic order.

3. In the case of η *resp.* η' meson it has been assumed that the states have form $(u\overline{u}+\overline{d}-2s\overline{s})/\sqrt{6}$ *resp.* $(u\overline{u}+\overline{d}+s\overline{s})/\sqrt{3}$.

4. B mesons have anomalously large coupling to $\eta' K$ and $\eta' X$ [55], which indicates an anomalously large coupling of η' to gluons [56]. The interpretation has been in terms of a considerable mixing η' with gluon-gluon bound state.

   η' mass is only 2.5 per cent higher than the mass $4m_{107}$ of super-canonical boson B_{SC} associated with the hadronic space-time sheet of hadron. Large mixing scenario is however not consistent with the existence of Φ with nearly the same mass. This encourages to consider the possibility that η' corresponds to a super-canonical boson B_{SC} plus quark pair with $k(d) = k(u) = k(s) = k(c) = 118$ with maximal mixing. In this case the contribution of quarks to the mass would be 25.1 MeV and one would have $m(\eta') = 959.2$ MeV which coincides with the actual mass with 1 per mille accuracy. Note that this model predicts identical couplings to various quark pairs as does also the model assuming that $\eta' - \Phi$ system is singlet with respect to flavor $SU(3)$ (having no fundamental status in TGD).

It is clear from the above table that the quark contributions to the masses of π, η' and B are slightly above the meson masses. In the case of B the discrepancy is largest and about 12 per cent. If one assumes that all contributions to the mass have p-adic origin, they are necessarily non-negative.

1. In the case of diagonal mesons the ordinary color interactions can reduce the contribution of quark masses to the mass of the meson. In the case of η' baruyonic super-canonical gluon B_{SC} could resolve the problem.

2. In the case of non-diagonal mesons the only possible solution of the problem is that $Y_e > 0$ holds true so that mass scale is reduced by a factor $1 - P = \sqrt{5/(5+Y_e)}$ giving $Y_e \simeq .056$. The prediction for top quark mass is reduced by 1.1 per cent to 167.2 GeV which belongs to the allowed range [58].

3. In the case of B meson one is forced to assume $k_b = k_d = k_u = 104$ although it would be possible to achieve smaller quark contribution by an alternative choice. This choice explains the observed very small isospin splitting and diagonality allows the ordinary color interaction to reduce the quark contribution to the B meson mass.

4. At the end of the chapter an alternative scenario in which quark masses give in good approximation only the masses of pion and kaon will be considered.

An example about how the mesonic mass formula works

The mass of the B_c meson (bound state of b and c quark) has been measured with a precision by CDF (see the blog posting by Tommaso Dorigo [59]) and is found to be $M(B_c) = 6276.5 \pm 4.8$ MeV. Dorigo notices that there is a strange "crackpottian" co-incidence involved. Take the masses of the fundamental mesons made of $c\overline{c}$ (Ψ) and $b\overline{b}$ (Υ), add them, and divide by two. The value of mass turns out to be 6278.6 MeV, less than one part per mille away from the B_c mass!

The general p-adic mass formulas and the dependence of k_q on hadron explain the co-incidence. The mass of B_c is given as $m(B_c) = m(c, k_c(B_c)) + m(b, k_b(B_c))$, whereas the masses of Ψ and Υ are given by $m(\Psi) = \sqrt{2}m(c, k_\Psi)$ and $m(\Upsilon) = \sqrt{2}m(b, k_\Upsilon)$. Assuming $k_c(B_c) = k_c(\Psi)$ and $k_b(B_c) = k_b(\Upsilon)$ would give $m(B_c) = [m(\Psi) + m(\Upsilon)]/\sqrt{2}$ which is by a factor $\sqrt{2}$ higher than the prediction of the "crackpot" formula. $k_c(B_c) = k_c(\Psi) + 1$ and $k_b(B_c) = k_b(\Upsilon) + 1$ however gives the correct result.

As such the formula makes sense but the one part per mille accuracy must be an accident in TGD framework.

1. The predictions for Ψ and Υ masses are too small by 2 *resp.* 5 per cent in the model assuming no effective scaling down of CP_2 mass.

2. The formula makes sense if the quarks are effectively free inside hadrons and the only effect of the binding is the change of the mass scale of the quark. This makes sense if the contribution of the color interactions, in particular color magnetic spin-spin splitting, to the heavy meson masses are small enough. Ψ and η_c correspond to spin 1 and spin 0 states and their masses by 3.7 per cent ($m(\eta_c) = 2980$ MeV and $m(\Psi) = 3096.9$) so that color magnetic spin-spin splitting is measured using per cent as natural unit.

4.5.4 Baryonic mass differences as a source of information

The first step in the development of the model for the baryon masses was the observations that $B - n$ mass differences can be understood if baryons are assumed to contain scaled versions of strange and heavy quarks. The deduction of precise values of $k(q)$ is however not quite straightforward since the color magnetic contribution to the mass affects the situation. Thus a working hypothesis worth of studying is that ground state contribution is same for all baryons and that for spin 1/2 baryons quark contribution to the mass added to this contribution is near as possible to the real mass but smaller than it.

The purpose of the following explicit is to to convince the reader that baryon mass difference can be indeed understood in terms of quark mass differences.

1. $\Lambda - n$ mass difference is 176 MeV and $(k(s) = 111, k(d) = 114)$ for λ would predict the mass difference $m(\lambda) - m(n) = m_q(\lambda) - m_q(n)$, where one has $m_q(\lambda) = m(s_{111}) + \sqrt{2}m(d_{114}) - m(n)$, $m_q(n) = \sqrt{m(u_{113})^2 + 2m(d_{113})^2)}$. The prediction equals to 141 MeV. It is possible to achieve smaller discrepancy but more precise considerations support this identification. Note that the spin-spin interaction energy is same if u and d quark form the paired quark system which is in $J = 0$ or $J = 1$ state so that the mass difference indeed can be regarded as quark mass difference.

2. $\Sigma - n$ mass difference is 257 MeV. If sigma contains s_{111}, u_{114} and d_{114}, the mass difference is predicted to be $m_q(\Sigma) - m_q(n)$, $m_q(\Sigma) = m(s_{111}) + \sqrt{2}m(d_{114})$ and comes out as 228 MeV.

3. If Ξ contains two s_{110} quarks and u_{113} (d_{113}), he mass difference comes out as 351 MeV to be compared with the experimental value 381 MeV.

4. Even single hadron, such as Ω, could contain several scaled up variants of s quark. $s_{108} + 2s_{111}$ decomposition would give mass difference 718 MeV to be compared with the real mass difference 734 MeV.

5. For Λ_c the mass is 2282 MeV. For $k(c) = 105$ instead of $k(c) = 104$ the predicted $\Lambda_c - n$ mass difference is 1341 MeV whereas the experimental difference is 1344 MeV.

6. For Λ_b the mass is 5425 MeV. For $k(b) = 104$ instead of $k(b) = 103$ the predicted $\Lambda_b - n$ mass difference is 4403 MeV. The experimental difference is 4485 MeV.

Baryon	*s content*	$\Delta m/MeV$	$\Delta m_{exp}/MeV$
Λ	s_{111}	141	176
Σ	s_{110}	228	257
Ξ	$s_{110} + s_{111}$	351	381
Ω	$s_{108} + 2s_{110}$	718	734
Λ_c	$c_{105}, d_{112}, u_{112}$	1341	1344
Λ_b	$b_{105}, u_{106}, d_{106}$	4403	4485

Table 8. Summary of the model for the quark contribution to the masses of baryons containing strange quarks deduced from mass differences and neglecting second order contributions to the mass. Δm denotes the predicted $B-n$ mass difference $m(B)-m(n)$. The subscript 'exp' refer to experimental value of the quantity in question.

4.5.5 Color magnetic spin-spin splitting

Color magnetic hyperfine splitting makes it possible to understand the $\rho-\pi$, $K^{\star}-K$, $\Delta-N$, etc. mass differences [35]. That the order of magnitude for the splittings remains same over the entire spectrum of hadrons serves as a support for the idea that color fluxes are feeded to $k=107$ space-time sheet. This would suggest that color coupling strength does not run for the physical states and runs only for the intermediate states appearing in parton description of the hadron reactions. A possible manner to see the situation in terms of intermediate states feeding color gauge flux to space-time sheets with $k>107$ so that the additive color Coulombic interaction conformal weights $s(q_i,q_j)$ would depend only on the integers $k(q_i),k(q_j)$. It will be found that the dependence is roughly of form $1/(k(q_i)+k(q_j))$, which brings in mind a logarithmic dependence of α_s on p-adic length scales involved.

There are two approaches to the problem of estimating spin-spin splitting: the first one is based on spin-spin interaction energy and the second one on spin-spin interaction conformal weight. The latter one turns out to be the only working one.

The model based on spin-spin interaction energy fails

Classical model would apply real number based physics to estimate the splittings and calculate color magnetic interaction energies. Standard QCD approach predicts that the color magnetic interaction energy is of form

$$\Delta E = S\sum_{pairs}\frac{\bar{s}_i\cdot\bar{s}_j}{m_im_jr_{ij}^3} . \tag{4.5.4}$$

The mass differences for hadrons allow to deduce information about the nature of color magnetic interaction and make some conclusions about the applicability this model.

1. For mesons the spin-spin splitting various from 630 MeV for $\rho-\pi$ system to 120 MeV $\Psi-\eta_c$ excludes the classical model predicting that the splitting should be proportional to $1/m(q_1)m(q_2)$ (variation by a factor $2^{113-106}=128$ instead of 5 would be predicted if the size of the hadron remains same). Also the predicted ratio of K^*-K splitting to $\rho-\pi$ splitting would be 1/4 rather than .63. The ratio of $\eta-\omega$ splitting to $\rho-\pi$ splitting would be 1/16 rather than .34. The ratio of $\Phi-\eta\prime$ splitting to $\rho-\pi$ splitting would be $1/32\simeq .03$ instead of .11.

   The inspection of the spin-spin interaction energies would suggest that the interaction energy scales $E(i,j)$ obey roughly the formula

   $$E(i,j)\sim \frac{5}{2}\times\frac{1}{(\Delta k(q_1)+\Delta k(q_2))} =$$

   $$5\times\frac{1}{log_2\{[L(113)/L(k(q_1)]\times[L(113)/L(k(q_2)]\}}$$

   $$\Delta k(q) = 113-k(q)$$

   rather than being proportional to $2^{-k(q_1)-k(q_1)}$. The hypothesis that p-adic length scale $L(k)$ of order CP_2 length scale range corresponds to the size of elementary particle horizon associated with wormhole contacts feeding gauge fluxes of the CP_2 type extremal representing particle to the larger space-time sheet with $p\simeq 2^k$ might allow to understand this dependence.

2. $\Delta-N$, $\Sigma^*-\Sigma$, and $\Xi^*-\Xi$ mass differences are 291 MeV, 194 MeV, 220 MeV. If strange quark inside Σ corresponds to $k=110$, the ratio of $\Sigma^*-\Sigma$ splitting to $\Delta-N$ splitting is predicted to be by a factor 1.17 larger than experimental ratio. $\Xi^*-\Xi$ splitting assuming $k(s)=109$ the ratio would be .19 and quite too small. Assuming that s,u,d quarks have more or less same mass, the model would predict reasonably well the ratios of the splittings. Either the idea about scaled

up variants of s is wrong or the notion of interaction energy must be replaced with interaction conformal weight in order to calculate the effects of color interactions to hadron masses.

The modelling of color magnetic spin-spin interaction in terms of conformal weight

The model based on the notion of interaction conformal weight generalizes the formula for color magnetic interaction energy to the p-adic context so that color magnetic interaction contributes directly to the conformal weight rather than rest mass. The effect is so large that it must be p-adically first order (the maximal contribution in second order to hadron mass would be however only 224 MeV) and the generalization of the mass splitting formula is rather obvious:

$$\Delta s = \sum_{pairs} S_{ij}\bar{s}_i \cdot \bar{s}_j \ . \tag{4.5.5}$$

The coefficients S_{ij} depend must be such that integer valued Δs results and CP_2 masses are avoided: this makes the model highly predictive. Coefficients can depend both on quark pair and on hadron since the size of hadron need not be constant. In any case, very limited range of possibilities remains for the coefficients.

This might be understood if the color flux carrying JAB connecting quark to $k = 107$ hadronic space-time sheet is also characterized by a value of $k \geq 113$. This fixes practically completely the model in the case of mesons. If the interaction strengths $s_c(i,j)$ characterizing color Coulombic interaction conformal weight between two quarks depends only on the flux tube pair connecting the quarks via $k = 107$ space-time sheet via the integers $k(q_i)$, the model contains only very few parameters.

The modelling of color magnetic- spin-spin splitting in terms of super-canonical boson content

The recent variant for the model of the color magnetic spin-spin splitting replacing energy with conformal weight is considerably simpler than the earlier one. Still one can argue that a model using perturbative QCD as a format is not the optimal one in a genuinely non-perturbative situation.

The explicit comparison of the super-canonical conformal weights associated with spin 0 and spin 1 states on one hand and spin 1/2 and spin 3/2 states on the other hand is carried out at the end of the chapter. The comparison demonstrates that the difference between these states could be understood in terms of super-canonical particle contents of the states by introducing only single additional negative conformal weight s_c describing color Coulombic binding . s_c is constant for baryons($s_c = -4$) and in the case of mesons non-vanishing only for pions ($s_c = -5$) and kaons ($s_c = -12$). This leads to an excellent prediction for the masses also in the meson sector since pseudoscalar mesons heavier than kaon are not Golstone boson like states in this model.

The correlation of the spin of quark-system with the particle content of the super-canonical sector increases dramatically the predictive power of the model since the allowed conformal weights of super-canonical bosons are (5,6,58). One can even consider the possibility that also exotic hadrons with different super-canonical particle content exist: this means a natural generalization of the notion of Regge trajectories. This description will be summarized at the end of the chapter.

4.5.6 Color magnetic spin-spin interaction and super-canonical contribution to the mass of hadron

Since $k = 107$ contribution to hadron mass is always non-negative, spin-spin interaction conformal weight and also color Coulombic conformal weight must be subtracted from some additional contribution both in the case of pseudo-scalars and spin 1/2 baryons.

Baryonic case

In the case of baryons the additional contribution could be identified as a 2-particle state of super-canonical bosons with mass squared $9m_{107}^2$ in case of baryons. The net mass is $s_{CS} = 18m_{107}^2$. The study of $N - \Delta$ system shows that color Coulombic interaction energy for single super-canonical structural unit corresponds to $\Delta s_{SC} = -2$ in the case of nucleon system so that one has $s_{SC} = 18 \rightarrow$

16. If topological mixing for super-canonical bosons is same as for U type quarks with conformal weights (5,6,58), the already discussed three-particle state of would give $s_{SC} = 5+5+6 = 16$.

The basic requirement is that the the sum of spin-spin interaction conformal weight and s_{CS} reduces to the conformal weight corresponding to the difference of nucleon mass and quark contribution to 774.6 MeV and correspond to $s = 11$.

One might hope that the situation could be the same for all baryons but it is safer to introduce an additional color Coulombic conformal weight $s_c(B)$ which vanishes for $N-\Delta$ system and hope that it is small as suggested by the fact that quark contributions explain quite satisfactorily the mass differences of baryons. This conformal weight could be assigned to the interaction of quarks via super-canonical gluons and would represent a correction to the simplest model. Strictly speaking, the term "*color Coulombic*" should be taken as a mere convenient letter sequence.

Pseudo scalars

In the case of pseudoscalars the situation is not so simple. What is clear that quark masses determine the meson mass in good accuracy.

In this case s_{CS} can be determined uniquely from the requirement that in case of pion it is cancelled the conformal weight characterizing $\rho-\pi$ color magnetic spin-spin splitting:

$$s_{SC} = |\Delta s(\pi, spin-spin)| \ . \tag{4.5.6}$$

This gives $s_{SC} = 21/4 \simeq 5$.

The interpretation as a bound state of super-canonical $g=1$ and $g=0$ gluon would require binding conformal weight by 4 units which looks somewhat strange. The masslessness of $g=0$ gluond does not support the formation of this kind of bound state. An alternative option is in terms of topological mixing in which case $g=0$ boson should receive 5 units of conformal weight.

Explicit calculations demonstrate that for mesons heavier than pion the role of s_c is to compensate s_{SC}. This suggests that the boson of lowest generation is present only inside $\pi-\rho$ system and prevents the large and negative color magnetic spin-spin interaction conformal weight to make pion a tachyon. The special role of pion could be understood in terms of a phase transition to color confining phase. Note however that the mass of η' could be understood by assuming baryonic super-canonical boson of conformal weight $s_{SC} = 16$ and fully mixed $k = 118$ quarks.

Formulas for $s_c(H)$ for mesons

For option I one has $s_{SC} = 5$ for all mesons. For option II s_{CS} vanishes for all mesons except π and ρ. For option I one obtains the formula

$$s_c(M) = -s_{SC} - \Delta s(M_0, spin-spin) = -5 + |\Delta s(M_0, spin-spin)| \ . \tag{4.5.6}$$

For option II one has

$$s_c(M) = -5 + |\Delta s(M_0, spin-spin)| \ , \quad M = \pi, \rho \ , \tag{4.5.6}$$

$$s_c(M) = |\Delta s(M_0, spin-spin)| \ , \quad M \neq \pi, \rho \ . \tag{4.5.7}$$

M_0 refers to the pseudoscalar.

Formulas for $s_c(H)$ for baryons

In the case of spin 1/2 baryons the requirement that the sum of color Coulombic and color magnetic conformal weights is same as for nucleons fixes the values of $s_c(B)$:

$$
\begin{aligned}
s_c(B) &= s_0 - s_{SC} - \Delta s(B_{1/2}, spin-spin) = -5 + |\Delta s(B_{1/2}, spin-spin)| \ , \\
s_{SC} &= 16 \ , \\
s_0 &= S(m(n) - m_q(n), 107) \ , \\
m_q(n) &= \sqrt{2m_d^2 + m_u^2} \ , \\
S(x, 107) &\equiv [(\frac{x}{m_{107}})^2] \ . \qquad (4.5.4)
\end{aligned}
$$

$s_0 = 11$ corresponds to the contribution difference of (say) neutron mass and quark contribution to the nucleon mass. s_{CS} corresponds to the conformal weight due to super-canonical bosons. In the defining formula for $S(x, 107)$ $[x]$ denotes the integer closest to x.

The conformal weights associated with spin-spin splitting

The general formula for the spin-spin splitting allowing to determine the parameters S_{ij} from the masses of a pair $H^\star - H$ of hadrons (say $\rho - \pi$ or $\Delta - N$). The parameters can be deduced from the observation that the mass difference $m(M^*) - m(M)$ for mesons corresponds to the difference of spin-splitting contributions to the mass:

$$
\Delta s(M^*) - \Delta s(M) = S(m(M^*) - m(M), 107) \ . \qquad (4.5.5)
$$

For baryons one has

$$
\begin{aligned}
\Delta s(B^*) - \Delta s(B) &= X_1 - X_0 \ , \\
X_1 &= S(m(B^*) - m_q(B), 107) = \ , \\
X_0 &= S(m(B) - m_q(B), 107) \ . \qquad (4.5.4)
\end{aligned}
$$

Here $m_q(B) = m_q(B^*)$ denotes the quark contribution to the nucleon mass. The possibility to understand the mass differences of spin 1/2 baryons in terms of differences for$m_q(B)$ inspires the hypothesis that X_0 is constant also for baryons (it vanishes for mesons). If so X_0 can be determined from neutron mass as

$$
\begin{aligned}
X_0 &= S(m(n) - m_q(n), 107) \ , \\
m_q(n) &= \sqrt{2m_d^2 + m_u^2)} \ . \qquad (4.5.4)
\end{aligned}
$$

Here $m_q(n)$ is the contribution of quarks to neutron mass.

These formulas are *not* identical with those used in the earlier calculations and the difference is due to the fact that $k = 107$ contributions and quark contributions are calculated separately unless quarks correspond $k = 107$. The formula allows to calculate second order contribution to the mass splitting.

p-Adicization brings in additional constraints. The requirement that the predicted mass of spin 1 boson and spin 3/2 fermion is not larger than than the experimental mass can pose strong constraints the scaling factor $\sqrt{5/(5 + Y_e)}$ in the case of non-diagonal hadrons unless one is willing to modify the model for spin-spin splittings. It was already found that in case of $\rho - \pi$ system this implies that top quark mass is at the lower limit of the allowed mass interval. One cannot take these constraints so seriously as the constraints that quark mass contribution is lower than meson mass in the case the quarks do not correspond to $k = 107$.

General mass formula

The general formula for the mass of hadron can be written as a sum of perturbative and non-perturbative contributions as

$$m(H) = m_P + m_{NP} \quad . \tag{4.5.5}$$

Preceding considerations lead to a simple formula for the non-perturbative contribution m_{NP} to the masses of spin 0 and spin 1 doublet of mesons:

$$\begin{aligned} m_{NP}(M) &= \sqrt{s_{NP}(M)} \times m_{107} \ , \\ s_{NP}(M_0) &= 0 \ , \\ s_{NP}(M_1) &= S(m(M^*) - m(M), 107) \ . \end{aligned} \tag{4.5.4}$$

For spin 1/2 and 3/2 doublet of baryons one has

$$\begin{aligned} m_{NP}(B) &= \sqrt{s_{NP}(B)} \times m_{107} \ , \\ s_{NP}(B_{1/2}) &= S(m_n - \sqrt{2m_d^2 + m_u^2}, 107) \ , \\ s_{NP}(B_{3/2}) &= S(m(B^*) - m_q(B), 107) \ . \end{aligned} \tag{4.5.3}$$

Perturbative contribution m_P contains in the lowest order approximation only the contribution of quark masses. In the case of diagonal mesons also a contribution from the ordinary color-Coulombic force and color magnetic spin-spin splitting can be present. For heavy mesons this contribution seems necessary since pure quark contribution is exceeds by few per cent the mass of meson.

Spin-spin interaction conformal weights for baryons

Consider now the determination of S_{ij} in the case of baryons. The general splitting pattern for baryons resulting from color Coulombic, and spin-spin interactions is given by the following table. The following equations summarize spin-spin splittings for baryons in a form of a table.

baryon	J	J_{12}	Δs^{spin}
N	$\frac{1}{2}$	0	$-\frac{3}{4}S_{d,d}$
Δ	$\frac{3}{2}$	1	$\frac{3}{4}S_{d,d}$
Λ	$\frac{1}{2}$	0	$-\frac{3}{4}S_{d,d}$
Σ	$\frac{1}{2}$	0	$-\frac{3}{4}S_{d,d}$
Σ^*	$\frac{1}{2}$	0	$\frac{1}{4}S_{d,d}$ $+\frac{1}{2}S_{d,s}$
Ξ	$\frac{1}{2}$	0	$-\frac{3}{4}S_{s,s}$
Ξ^*	$\frac{1}{2}$	0	$\frac{1}{4}S_{s,s}$ $+\frac{1}{2}S_{d,s}$
Ω	$\frac{3}{2}$	1	$\frac{3}{4}S_{s,s}$

(4.5.3)

Spin-spin splittings are deduced from the formulas

$$\begin{aligned} \Delta s^{spin} &= S_{q_1,q_)}(\frac{J_{12}(J_{12}+1)}{2} - \frac{3}{4}) \ , \\ &+ \frac{1}{4}(S_{q_1,q_3} + S_{q_2,q_3})(J(J+1) - J_{12}(J_{12}+1) - \frac{3}{4}) \ , \end{aligned} \tag{4.5.2}$$

where J_{12} is the angular momentum eigenvalue of the 'first two quarks', whose value is fixed by the requirement that magnetic moments are of correct sign.

The masses determine the values of the parameters uniquely if one assumes that color binding energy is constant as indeed suggested by the very notion of M_{107} hadron physics. The requirement is that the mass difference squared for $\Delta - N$, $\Sigma^* - \Sigma$, and $\Xi^* - \Xi$ come out correctly.

Consider now the values of S_{ij} for the models assuming $k = 113$ light quarks and dynamical $k(s)$.

1. For $N - \Delta$ system the equation is

$$S_{d_{113},d_{113}} = \frac{1}{3}S(m(\Delta) - m_q(N), 107) - S(m(N) - m_q(N), 107) \ .$$

   Here $m_q(N)$ refers to the quark contribution to the baryon mass.

2. For $\Sigma^* - \Sigma$ system the basic equation can be written as

$$S_{d_{114},s_{110}} = 2[S(m(\Sigma^*) - m_q(\Sigma), 107) - S(m(\Sigma) - m_q(\Sigma), 107) - S(d_{114}, d_{114})] \ .$$

   One must make some assumption in order to find a unique solution. The simplest assumption is that $S_{d_{114},d_{114}} = S_{d_{114},s_{110}}$. This implies

$$S_{d_{114},d_{114}} = \frac{1}{3}[S(m(\Sigma^*) - m_q(\Sigma), 107) - S(m(\Sigma) - m_q(\Sigma), 107)] \ .$$

3. In the case of $\Xi^* - \Xi$ system the equation is

$$S_{s_{110},s_{110}} = -\frac{1}{2}S_{d_{113},s_{110}} + [S(m(\Xi^*) - m_q(\Xi), 107) - S(m(\Xi) - m_q(\Xi), 107)] \ .$$

   If one assumes $S_{s_{110},s_{110}} = S_{d_{113},s_{110}}$ one obtains

$$S_{s_{110},s_{110}} = \frac{1}{3}[S(m(\Xi^*) - m_q(\Xi), 107) - S(m(\Xi) - m_q(\Xi), 107)] \ .$$

The resulting values of the parameters characterizing baryonic spin-spin splittings are in the table below. The parameters rela

$S_{d_{113},d_{113}}$	$S_{d_{114},d_{114}}$	$S_{d_{114},s_{110}}$	$S_{s_{110},s_{110}}$	$S_{d_{113},s_{110}}$
7	6	6	$\frac{8}{3}$	$\frac{8}{3}$

(4.5.3)

The mass squared unit used is m_0^2 and $k = 107$ defines the p-adic length scale used. The elements of $S_{i,j}$ between different p-adic primes are assumed to be vanishing. The matrix elements are quite near to each other which raises the hope that the model indeed makes sense.

Color Coulombic binding conformal weights are given by the expression $s_c = -5 + |\Delta s(B_{1/2}, spin-spin)|$. The weights are given in the following table

baryon	N	Σ	Ξ
s_c	$\frac{1}{4}$	$-\frac{1}{2}$	-3

(4.5.3)

Some remarks are in order.

1. A good sign is that the values of s_c are small as compared to the value of $s_{CS} = 18$ in all baryons so that only a small correction is in question.

2. The magnitude of s_c increases with the mass of baryon which does not conform with the expectations raised by ordinary QCD evolution. Could this mean that asymptotic freedom means that the color interaction between quarks occurs increasingly via super-canonical gluons? For $N - \Delta$ system the actual value of s_c should vanish.

3. One might worry about the fact the color binding conformal weights are not integral valued. The total conformal weights determining the mass squared are however integers.

Spin-spin interaction conformal weights for mesons

The values of mesonic interaction strengths $S_{i,j}$ can in principle deduced from the observed mass splittings. The following equations summarize the spin-spin splitting pattern for mesons in a form of table.

$meson$	Δs^{spin}
π	$-\frac{3}{4}S_{d,d}$
ρ	$\frac{1}{4}S_{d,d}$
η	$-\frac{3}{4}S_{d,d}$
ω	$\frac{1}{4}S_{d,d}$
$K^{\pm}, K^0(CP=1)$	$-\frac{3}{4}S_{d,s}$
$K^0(CP=-1)$	$-\frac{3}{4}S_{d,s}$
$K^{*,\pm}, K^{*,0}(CP=1)$	$\frac{1}{4}S_{d,s}$
$K^{*,0}(CP=-1)$	$\frac{1}{4}S_{d,s}$
η'	$-\frac{3}{4}S_{s,s}$
Φ	$\frac{1}{4}S_{s,s}$
η_c	$-\frac{3}{4}S_{c,c}$
Ψ	$\frac{1}{4}S_{c,c}$
$D^{\pm}, D^0(CP=1)$	$-\frac{3}{4}S_{d,c}$
$D^0(CP=-1)$	$-\frac{3}{4}S_{d,c}$
$D^{*,\pm}, D^{*0}(CP=1)$	$\frac{1}{4}S_{d,c}$
$D^{*0}(CP=-1)$	$\frac{1}{4}S_{d,c}$

(4.5.3)

Consider the spin-spin interaction for mesons.

1. For $\rho-\pi$ system one has

$$S_{d_{113},d_{113}} = s(m(\rho)-m_q(\pi)) \quad .$$

   Using $s(\rho)=14$ and $s(\pi)=0$ gives $S(d_{113},d_{113})=13$.

2. $\omega-\eta$ system one obtains

$$S_{q_{109},q_{109}} = S(m(\omega)-m_q(\eta),107)$$

3. $K^{\star}-K$-splitting gives $S_{d_{114},s_{109}} = S(m(K^*)-m_q(K),107)$.
4. $\Phi-\eta'$ splitting gives $S_{q_{107},q_{107}} = S(m(\Phi)-m_q(\eta'),107)$.
5. D^*-D mass splitting gives $S_{d_{113},c_{105}} = S(m(D^*)-m_q(D),107)$.
6. $\Psi-\eta_c$ mass difference gives $S_{c_{104},c_{104}} = s(m(\Psi)-m_q(\eta_c),107)$.

The results for the spin-spin interaction strengths S_{ij} are summarized in the table below. q_{109} refers to u, d, and s quarks.

$S_{d_{113},d_{113}}$	$S_{q_{109},q_{109}}$	$S_{q_{107},q_{107}}$	$S_{d_{114},s_{109}}$	$S_{d_{113},c_{105}}$	$S_{c_{104},c_{104}}$
7	1	0	3	2	0

(4.5.3)

Note that interaction strengths tend to be weaker for mesons than for baryons. For scaled up quarks the value of interaction strength tends to decrease and is smaller for non-diagonal than diagonal interactions. Since the values of $k(q_i)$ maximize the quark contribution to hadron masses, the interaction strength produce a satisfactory mass fit for hadrons with errors of not larger than about five cent.

The color Coulombic binding conformal weights for meson states are given in the following table:

$meson$	π	K	η	η'	D	ψ
$s_c(I)$	$+1/4$	$-4-1/4$	-6	$-3-1/4$	$-4-1/2$	-5
$s_c(II)$	$1/4$	$3/4$	1	$1+3/4$	$1/2$	0

(4.5.3)

For option I $g=1$ boson is present in all mesons. The magnitude of s_c increases with the mass of the meson and more or less compensates $s_{CS}=5$. This forces to consider the possibility that only pion contains the super-canonical boson compensating the large and negative spin-spin interaction conformal weight making the state tachyon otherwise. For option II s_c is relatively small and positive for this option.

4.5.7 Summary about the predictions for hadron masses

The following tables summarize the predictions for baryon masses following from the proposed model with optimal choices of the integers $k(q)$ characterizing the mass scales of quarks and requiring that the predicted isospin splittings are of same order than the observed splittings. This condition is non-trivial: for instance, in case of B meson the smallness of splitting forces the condition $k(b)=k(d)=k(u)=104$ so that mass squared is additive and the large contribution of b quark minimizes the isospin splitting.

Meson masses assuming that all pseudoscalars are Goldstone bosons

$Meson$	$quarks$	$m_{pr}(M)/MeV$	m_{exp}/MeV
π^0	$B_{SC,1}+d_{113},u_{113}$	140.0	135.0
π^+	d_{113},u_{113}	140.0	139.6
ρ^0	d_{113},u_{113}	756	772
ρ^+	d_{113},u_{113}	756	770
K^0	d_{114},s_{109}	496	498
K^+	u_{114},s_{109}	486	494
K^{0*}	d_{114},s_{109}	896	900
K^{+*}	u_{114},s_{109}	892	891
η	u_{109},d_{109},s_{109}	522	549
ω^0	u_{109},d_{109},s_{109}	817	783
η'	$u_{107},d_{107},s_{107},c_{107}$	1144	958
Φ	$u_{107},d_{107},s_{107},c_{107}$	1144	1019
η_c	c_{104}	3098	2980
D^0	c_{105},u_{113}	1642	1865
D^+	c_{105},d_{113}	1655	1870
D^{*0}	c_{105},u_{114}	1971	2007
D^{*+}	c_{105},d_{114}	1985	2010
F	$c_{105},s(106)$	1954	2021
Υ	b_{103}	10814	9460
B	b_{104},d_{104},u_{104}	5909	5270

Table 9. The prediction of meson masses. The model assumes the maximal value of CP_2 mass allowed by η' mass and the condition $Y_e=0$ favored by top quark mass.

In the case of meson masses the predictions for masses are not so good as for baryons. Errors are at worst about 5 per cent. For non-diagonal mesons the predicted masses are smaller than actual masses and in the case of kaons excellent. Also the prediction of pion mass is good but about 5 per cent too large. In the case of diagonal mesons ordinary color interactions could reduce the predicted masses in case that they are larger than actual ones.

Meson masses assuming that only pion and kaon are Goldstone bosons

The Golstone boson interpretation does not seem completely satisfactory. In order to make progress one can check whether the masses associated with super-canonical bosons could serve as basic units for pseudoscalar and vector boson masses. A more general fit would be based on the use of fictive boson B_{107} with mass m_{107} as a basic unit in $k = 107$ contribution to the mass. The table below gives very accurate formulas for the meson masses in terms of the scale m_{107} and quark contribution to the masses.

$Meson$	$quarks$	$m_{pr}(M)/MeV$	m_{exp}/MeV
π^0	d_{113}, u_{113}	140.0	135.0
π^+	d_{113}, u_{113}	140.0	139.6
ρ^0	$6B_{107} + d_{113}, u_{113}$	758	772
ρ^+	$6B_{107} + d_{113}, u_{113}$	758	770
K^0	d_{114}, s_{109}	496	498
K^+	u_{114}, s_{109}	486	494
K^{0*}	$3B_{107} + d_{114}, s_{109}$	901	900
K^{+*}	$3B_{107} + u_{114}, s_{109}$	891	891
η	$B_{SC,1} + u_{118}, d_{118}, s_{118}$	548	549
ω^0	$2B_{SC,1} + u_{118}, d_{118}, s_{118}$	803	783
η'	$2B_{SC,1} + B_{SC,2} + q_{118}$	959	958
Φ	$2B_{SC,1} + B_{SC,2} + q_{118}$	959	1019
η_c	$2B_{SC,1} + c_{105}$	2929	2980
Ψ	$3B_{SC,1} + c_{105}$	3098	3100
D^0	$2m_{SC,1} + c_{106}, u_{118}$	1853	1865
D^+	$2m_{SC,1} + c_{106}, d_{118}$	1850	1870
D^{*0}	$3m_{SC,1} + c_{106}, u_{118}$	2019	2007
D^{*+}	$3m_{SC,1} + c_{106}, d_{118}$	2016	2010
F	$3m_{SC,2} + c_{105}, s(113)$	2010	2021
Υ	$B_{SC,3} + b_{104}$	9441	9460
$B^{\pm}$	$3B_{SC,2} + b_{105}, d_{105}, u_{105}$	5169	5270

Table 10. Table demonstrates that scalar and vector meson masses can be effectively regarded as expressible in terms of quark contribution and contribution coming from many particle states of super-canonical bosons $B_{SC,k}$, $k = 1, 2, 3$, with conformal weights (5,6,28) associated also with U type quarks. B_{107} denotes effective super-canonical boson with mass conformal weight 1 and mass m_{107}. $Y_e = 0$ favored by top quark mass is assumed.

The table demonstrates following.

1. For mesons heavier than kaons, the masses are effective sums of masses for quarks and many-particle state formed by super-canonical bosons allowed by the topological mixing of U type quarks.

2. For $\pi - \rho$ *resp.* $K - K^*$ systems the masses can be expressed using effective $7B_{107}$ state state *resp.* $3B_{107}$ state. Second order contribution to the conformal weight from the super-canonical color interaction can explain the too small mass of ρ and too large mass of π if it interferes with the corresponding quark mass contribution.

3. For pseudo-scalars heavier than kaon the mass of the super-canonical meson is not completely compensated by spin-spin splitting for the pseudoscalar state so that Goldstone boson interpretation does not make sense anymore. In the case of heavy mesons the predicted masses of pseudoscalars are slightly below the actual mass.

4. The predicted masses are not larger than actual masses (ω_0 is the troublemaker) if one assumes 2.5 per cent reduction of CP_2 mass scale for which top quark mass is at the lower bound of the allowed mass range.

5. Color magnetic spin-spin splitting parameters can be deduced from the differences of super-canonical conformal weights for pseudoscalar and spin one boson. There is however no absolute need for this perturbative construct.

6. One can consider the possibility that the super-canonical boson content is actual and correlates with the spin of quark-antiquark system for mesons heavier than kaons. The point would be that the representability in terms of super-canonical bosons would make the model for the color magnetic spin-spin splittings highly predictive. This interpretation makes sense in the case of $\pi-\rho$ and $K-K^*$ systems only if one introduces negative color Coulombic conformal weight s_c. For heavier mesons only this contribution would be second order in p which is more or less consistent with the view about color coupling evolution. $\pi-\rho$ would correspond to B_1 ($s=5$) and $2B_2$ ($s=12$) ground states with color Coulombic conformal weight $s_c=-12$. $K-K^*$ would correspond to $2B_2$ ($s=12$) and $3B_1$ with $s_c=-12$. The presence of ground state bosons saves π and K from becoming tachyons.

Whatever the correct physical interpretation of the mass formulas represented by the table above is, it is clear that m_{107} defines a fundamental mass scale also for meson systems.

Baryon masses

One can ask whether the representability of spin-spin splitting in terms of super-canonical conformal boson content is possible also in the case of baryons so that perturbative formulas altogether would not be necessary. The physical interpretation would be that the total spin of baryonic quarks correlates with the content of super-canonical bosons. The existence of this kind of representation would be one step towards understanding of also spin-spin splitting from first principles.

This is indeed the case if one accepts negative color Coulombic conformal weight $s_c=-4$. Spin 1/2 ground states would correspond to $3B_1$ with conformal weight $s=15$, one B_1 for each valence quark. Spin 3/2 states would correspond to $5B_1$ with $s=25$ in the case of Δ, to $2B_1+B_2$ in the case of Σ^* with $s=23$, and to B_1+3B_2 with $s=24$ in case of Ξ^*.

$Baryon$	$quarks$	$m_{pr}(B)/MeV$	m_{exp}/MeV
p	$3B_1+u_{113},d_{113}$	942.3	938.3
n	$3B_1+u_{113},d_{113}$	949.8	939.6
Δ^{++}	$5B_1+u_{113}$	1230	1231
Δ^{+}	$5B_1+u_{113},d_{113}$	1238	1235
Δ^{0}	$5B_1+u_{113},d_{113}$	1245	1237
Δ^{-}	$5B_1+d_{113}$	1253	≤ 1238
Λ	$3B_1+u_{114},d_{114},s_{111}$	1090	1116
Σ^{+}	$3B_1+u_{114},s_{110}$	1165	1189
Σ^{0}	$3B_1+u_{114},d_{113},s_{110}$	1171	1192
Σ^{-}	$3B_1+d_{114},s_{110}$	1178	1197
Σ^{*+}	$2B_1+2B_2+u_{114},s_{110}$	1381	1385
Ξ^{0}	$2B_1+2B_2+u_{113},s_{110},s_{111}$	1301	1315
Ξ^{-}	$3B_1+d_{113},s_{110}$	1288	1321
Ξ^{*0}	$B_1+3B_2+u_{113},s_{110}$	1531	1532
Ξ^{*-}	$B_1+3B_2+d_{113},s_{110}$	1505	1535
Ω^{-}	$3B_1+s_{108},s_{111}$	1667	1672
Λ_c	$3B_1+d_{110},u_{110},c_{106}$	2261	2282
Λ_b	$3B_1+d_{108},u_{108},b_{105}$	5390	5425

Table11. The predictions for baryon masses assuming $Y_e=0$.

From the table for the predicted baryon masses one finds that the predicted masses are slightly below the experimental masses for all baryons except for some baryons in $N-\Delta$ multiplet and for Ω. The reduction of the CP_2 mass scale by a factor of order per cent consistent with what is known about top quark mass cures this problem (also ordinary color interactions could take of the problem).

In principle the quark contribution to the hadron mass is measurable. Suppose that color binding conformal weight can be assigned to the color interaction in super-canonical degrees of freedom alone. Above the "ionization" energy, which corresponds to the contribution of quarks to the mass of hadron, valence quark space-time sheet can separate from the hadronic space-time sheet in the collisions of hadrons. This threshold might be visible in the collision cross sections for say nucleon-nucleon collisions. For nucleons this energy corresponds to 170 MeV.

4.5.8 Some critical comments

The number theoretical model for quark masses and topological mixing matrices and CKM matrix as well as the simple model for hadron masses give strong support for the belief that the general vision is correct. One must bear in mind that the scenario need not be final so that the basic objections deserve an explicit articulation.

Is the canonical identification the only manner to map mass squared values to their real counterparts

In p-adic thermodynamics p-adic particle mass squared is mapped to its real counterpart by the canonical identification. If the $O(p)$ contribution corresponds to non-trivial rational number, the real mass is of order CP_2 mass. This allows to eliminate a large number of exotics. In particular, it implies that the modular contribution to the mass squared must be of form np rather than $(r/s)p$. This assumption is absolutely crucial in the model of topological mixing matrices and CKM matrix.

One can however question the use of the standard form of the canonical identification to map p-adic mass squared to its real counterpart. The requirement that p-adic and real S-matrix elements (in particular coupling constants) are related in a realistic manner, forces a modification of the canonical identification. Instead of a direct identification of real and p-adic rationals, the p-adic rationals in R_p are mapped to real rationals (or vice versa) using a variant of the canonical identification $I_{R\rightarrow R_p}$ in which the expansion of rational number $q = r/s = \sum r_n p^n / \sum s_n p^n$ is replaced with the rational number $q_1 = r_1/s_1 = \sum r_n p^{-n} / \sum s_n p^{-n}$ interpreted as a p-adic number:

$$q = \frac{r}{s} = \frac{\sum_n r_n p^n}{\sum_m s_n p^n} \rightarrow q_1 = \frac{\sum_n r_n p^{-n}}{\sum_m s_n p^{-n}} = \frac{I(r)}{I(s)} \ . \tag{4.5.4}$$

The nice feature of this variant of the canonical identification is that it respects quantitative behavior of amplitudes, respects symmetries, and maps unitary matrices to unitary matrices if the matrix elements correspond to rationals (or generalized rationals in algebraic extension of rationals) if the p-adic integers involved are smaller than p. At the limit of infinitely large p this is always satisfied.

Quite generally, the thermodynamical contribution to the particle mass squared is in the lowest p-adic order of form rp/s, where r is the number of excitations with conformal weight 1 and s the number of massless excitations with vanishing conformal weight. The real counterpart of mass squared for the ordinary canonical identification is of order CP_2 mass by $r/s = R + r_1 p + ...$ with $R < p$ near to p. Hence the states for which massless state is degenerate become ultra heavy if r is not divisible by s. For the new variant of canonical identification these states would be light.

Even worse, the new form does not require the modular contribution to the p-adic mass squared to be of form np. Some other justification for this assumption would be needed. The first guess is that the conditions on mass squared plus probability conservation might not be consistent with unitarity unless the modular contribution to the mass squared remains integer valued in the mixing (note that all integer values are not possible). Direct numerical experimentation however shows that that this is not the case.

The predicted integer valued contributions to the mass squared are minimal in the case of u and d quarks and very nearly maximal in the case t and b quarks. This suggests a possible way out of the difficulty. Perhaps the rational valued p-adic mass squared of u and d quarks are minimal and those of b and t quarks maximal or nearly maximal. This might also allow to improve the prediction for the CKM matrix.

The objection against the use of the new variant of canonical identification is that the predictions of p-adic thermodynamics for mass squared are not rational numbers but infinite power series. p-Adic thermodynamics itself however defines a unique representation of probabilities as ratios of generalized

Boltzmann weights and partition function and thus the variant of canonical identification might indeed generalize. If this representation generalizes to the sum of modular and Virasoro contributions, then the new form of canonical identification becomes very attractive. Also an elegant model for the masses of intermediate gauge bosons results if $O(p)$ contribution to mass squared is allowed to be a rational number.

Uncertainties related to the CP_2 length scale

The uncertainties related to the CP_2 length scale mean that one cannot take the detailed model for hadron masses too literally unless one takes the recent value of top quark mass at face value and requiring $(Y_e = 0, Y_t = 1)$ in rather high accuracy. This constraint allows at most 2.5 per cent reduction of the fundamental mass scale and baryonic masses suggest a 1 per cent reduction. The accurate knowledge of top quark mass is therefore of fundamental importance from the point of view of TGD.

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Chapter 5

p-Adic Particle Massivation: New Physics

5.1 Introduction

TGD certainly predicts a lot of new physics, actually infinite hierarchies of fractal copies of standard model physics, but the precise characterization of predictions has varied as the interpretation of the theory has evolved during years.

5.1.1 Basic new physics predictions

Concerning new physics the basic predictions are following. TGD predicts a rich spectrum of massless states for which ground states of negative super-canonical conformal weight are created by colored super-generators. By color confinement these states do not however give rise to macroscopic long range forces. A hierarchy color and weak physics is predicted. Also dark matter hierarchy corresponding to a hierarchy of Planck constants brings in a hierarchy of variants of standard model physics labelled by the values of Planck constant. Thus in TGD the question is not about about predicting some exotic particles but an entire fractal hierarchies of copies of standard model physics.

The family replication for fermions correspond in case of gauge bosons prediction of bosons labelled by genera of the two light-like wormhole throats associated with the wormhole contact representing boson. There are very general arguments predicting that the number of fermionic genera is three and this means that gauge bosons can be arranged into genus-SU(3) singlet and octet. Octet corresponds to exotic gauge bosons and its members should develop Higgs expectation value. Completely symmetric coupling between Higgs octet and boson octet allows also the bosons with vanishing genus-SU(3) quantum numbers to develop mass.

Higgs field is predicted and its vacuum expectation value explains boson masses. By a general argument p-adic temperature for bosons is low and this means that Higgs contribution to the gauge boson mass dominates. Only p-adic thermodynamics is needed to explain fermion masses and the masses of super-canonical bosons and their super counterparts. There is an argument suggesting that vacuum expectation value of Higgs at fermion space-time sheets is not possible. Almost universality of the topological mixing inducing also CKM mixing allows to predict mass spectrum of these states.

5.1.2 Outline of the topics of the chapter

A general vision about coupling constant evolution

The vision about coupling constant evolution has developed slowly and especially important developments have occurred during last few years. Therefore an overall view about recent understanding is in order.

Also QCD coupling constant evolution is discussed and it is found that asymptotic freedom could be lost making possible existence of several scaled up versions of QCD existing only in a finite length scale range. The basic counter arguments against lepto-hadron hypothesis are considered and it is found that the loss of asymptotic freedom could allow lepto-hadron physics. One can also consider the possibility that the copies of say electro-weak characterized by Mersenne primes do not couple directly to each other so that the objections are circumvented.

The discovery of dark matter hierarchy about fifteen years after these argument were developed resolves the problems in much more elegant manner. TGD predicts an infinite hierarchy of electro-weak and color physics physics for which particles couple directly only via gravitons. De-coherence phase transitions can however induce processes allowing the decay of particles of a given physics to particles of another physics.

Summary of new physics effects

Various new physics effects related to the predicted exotic particles are discussed. No attempt to discuss systematically the spectrum of various exotic bosons and fermions, basically due to the ground states created by color super-canonical and Kac-Moody generators, will be made. Rather, the attempt is to summarize the new physics expected on basis of recent interpretation of quantum TGD.

1. There is a brief discussion of family replication phenomenon in the case of gauge bosons based on the identification of gauge bosons as wormhole contacts. Also an argument forcing the identification of partonic vertices as branchings of partonic 2-surfaces is developed.

2. Fractal copies of quarks is basic prediction and now a key part of the model for hadron masses. ALEPH anomaly is interpreted in terms of a fractal copy of b-quark corresponding to k=197.

3. The possible signatures of M_{89} hadron physics in e^+e^- annihilation experiments are discussed using a naive scaling of ordinary hadron physics.

4. The possibility that the newly born concept of Pomeron of Regge theory might be identified as the sea of perturbative QCD is considered.

5. In p-adic context exotic representations of Super Virasoro with $M^2 \propto p^k$, $k = 1, 2, ..$m are possible. For $k = 1$ the states of these representations have same mass scale as elementary particles although in real context the masses would be gigantic. This inspires the question whether non-perturbative aspects of hadron physics could be assigned to the presence of these representations. The prospects for this are promising. Pion mass is almost exactly equal to the mass of lowest state of the exotic representation for $k = 107$ and Regge slope for rotational excitations of hadrons is predicted with three per cent accuracy assuming that they correspond to the states of $k = 101$ exotic Super Virasoro representations. This leads to the idea that hadronization and fragmentation correspond to phase transitions between ordinary and exotic Super Virasoro representations and that there is entire fractal hierarchy of hadrons inside hadrons and QCD:s inside QCD:s corresponding to p-adic length scales $L(k)$, $k = 107, 103, 101, 97, ...$

Cosmic primes and Mersenne primes

p-Adic length scale hypothesis suggests the existence of a scaled up copy of hadron physics associated with each Mersenne prime $M_n = 2^n - 1$,n prime: M_{107} corresponds to ordinary hadron physics. There is some evidence for exotic hadrons. Centauro events and the peculiar events associated with $E > 10^5\ GeV$ radiation from Cygnus X-3 could be understood as due to the decay of gamma rays to M_{89} hadron pair in the atmosphere. The decay $\pi_n \rightarrow \gamma\gamma$ produces a peak in the spectrum of the cosmic gamma rays at energy $\frac{m(\pi_n)}{2}$ and there is evidence for the peaks at energies $E_{89} \simeq 34\ GeV$ and $E_{31} \simeq 3.5 \cdot 10^{10}\ GeV$. The absence of the peak at $E_{61} \simeq 1.5 \cdot 10^6\ GeV$ can be understood as due to the strong absorption caused by the e^+e^- pair creation with photons of the cosmic microwave background. Cosmic string decays *cosmic string* $\rightarrow M_2$ *hadrons* $\rightarrow M_3$ *hadrons* $.. \rightarrow M_{107}$ *hadrons* is a new source of cosmic rays. The mechanism could explain the change of the slope in the hadronic cosmic ray spectrum at M_{61} pion rest energy $3 \cdot 10^6\ GeV$. The cosmic ray radiation at energies near $10^9\ GeV$ apparently consisting of protons and nuclei not lighter than Fe might be actually dominated by gamma rays: at these energies γ and p induced showers have same muon content and the decays of gamma rays to M_{89} and M_{61} hadrons in the atmosphere can mimic the presence of heavy nuclei in the cosmic radiation.

Anomalously large direct CP breaking in $K - \overline{K}$ system and exotic gluons

The recently observed anomalously large direct CP breaking in $K_L \rightarrow \pi\pi$ decays is explained in terms of loop corrections due to the predicted 2 exotic gluons having masses around 33.6 GeV. It will be also found that the TGD version of the chiral field theory believed to provide a phenomenological low energy description of QCD differs from its standard model version in that quark masses are replaced in TGD framework with shifts of quark masses induced by the vacuum expectation values of the scalar meson fields. This conforms with the TGD view about Higgs mechanism as causing only small mass shifts. It must be however emphasized that there is an argument suggesting that the vacuum expectation value of Higgs in fermionic case does not even make sense.

5.2 General vision about real and p-adic coupling constant evolution

The unification of super-canonical and Super Kac-Moody symmetries allows new view about p-adic aspects of the theory forcing a considerable modification and refinement of the almost decade old first picture about color coupling constant evolution.

Perhaps the most important questions about coupling constant evolution relate to the basic hypothesis about preferred role of primes $p \simeq 2^k$, k an integer. Why integer values of k are favored, why prime values are even more preferred, and why Mersenne primes $M_n = 2^n - 1$ and Gaussian Mersennes seem to be at the top of the hierarchy?

Second bundle of questions relates to the color coupling constant evolution. Do Mersenne primes really define a hierarchy of fixed points of color coupling constant evolution for a hierarchy of asymptotically non-free QCD type theories both in quark and lepton sector of the theory? How the transitions $M_n \rightarrow M_{n(next)}$ occur? What are the space-time correlates for the coupling constant evolution and for for these transitions and how space-time description relates to the usual description in terms of parton loops? How the condition that p-adic coupling constant evolution reflects the real coupling constant evolution can be satisfied and how strong conditions it poses on the coupling constant evolution?

5.2.1 A general view about coupling constant evolution

Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state. Equivalence Principle is expected to hold true for elementary particles and their composites but not for the quantum states defined around non-vacuum extremals.

Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [C3] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [19] known as hyperfinite factor of type II_1 (HFF) [A9, A8, C3]. HFF [20, 26] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [27]. The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [28], anyons [30], quantum groups and conformal field theories[21, 22], and knots and topological quantum field theories [26, 27].

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [C3]. The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. M-matrix is identifiable in terms of Connes tensor product [26] and therefore exists and is almost unique. Connes tensor product implies that the Hermitian elements of the included algebra commute with M-matrix and hence act like infinitesimal symmetries. A connection with integrable quantum field theories is suggestive. The remaining challenge is the calculation of M-matrix and the needed machinery might already exist.

The tension is present also now. The connection with visions should come from the discretization in terms of number theoretic braids providing space-time correlate for the finite measurement resolution and making p-adicization in terms of number theoretic braids possible. Number theoretic braids give a connection with the construction of configuration space geometry in terms of Dirac determinant and with TGD as almost TQFT and with conformal field theory approach. The mathematics for the inclusions of hyper-finite factors of type II_1 is also closely related to that for conformal field theories including quantum groups relating closely to Connes tensor product and non-commutativity.

How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

Zero energy ontology in which zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of future and past directed light-cones are power of 2 multiples of fundamental time ($T_n = 2^nT_0$) scale implies in a natural manner coupling constant evolution. A weaker condition would be $T_p = pT_0$, p prime, and would assign all p-adic time scales to the size scale hierarchy of CDs.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^nT_0$ (or $T_p = pT_0$) induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$

suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of configuration space.

5.2.2 Both symplectic and conformal field theories are needed in TGD framework

Before one can say anything quantitative about coupling constant evolution, one must have a formulation for its TGD counterpart and thus also a more detailed formulation for how to calculate M-matrix elements. There is also the question about infinities. By very general arguments infinities of quantum field theories are predicted to cancel in TGD Universe - basically by the non-locality of Kähler function as a functional of 3-surface and by the general properties of the vacuum functional identified as the exponent of Kähler function. The precise mechanism leading to the cancellation of infinities of local quantum field theories has remained unspecified. Only the realization that the symplectic invariance of quantum TGD provides a mechanism regulating the short distance behavior of N-point functions changed the situation in this respect. This also leads to concrete view about the generalized Feynman diagrams giving M-matrix elements and rather close resemblance with ordinary Feynman diagrammatics.

Symplectic invariance

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of lightcone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [C2] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [D8]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c^m_{kl}\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c^m_{kl} f(A(s_1, s_2, s_3))\Phi_m(s)d\mu_s \quad . \tag{5.2.1}$$

   Here the coefficients c^m_{kl} are constants and $A(s_1, s_2, s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1, s_2, s))Id d\mu_s$ so that one has

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1, s_2, s))d\mu_s \quad . \tag{5.2.2}$$

   Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n =$ 1- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1, s_2, s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1,s_2,s_3) \equiv A(s_1,s_2,s_3,N) - A(s_1,s_2,s_3,S) \quad (5.2.3)$$

   of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c^r_{kl}\int f(\Delta A(s_1,s_2,s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \quad (5.2.3)$$
$$c^r_{kl}c_{rm}\int f(\Delta A(s_1,s_2,s))f(\Delta A(s,s_3,t))d\mu_s d\mu_t \ . \quad (5.2.4)$$

   Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1,s_2,s))d\mu_s \quad (5.2.5)$$

4. There is a clear difference between $n>3$ and $n=3$ cases: for $n>3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n=4$ theory is certainly well-defined, but one can argue that so are also $n>4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0)=0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

Generalization to quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments coincide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n)+N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

   The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of configuration space. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions

could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [C3] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [E2].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time.

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and configuration space degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of the configuration space Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. The coupling constant evolution is based on the same mechanism as in QFT and symplectic invariance replaces ad hoc UV cutoff with a genuine dynamical regulation mechanism. Causal diamond itself defines the physical IR cutoff. p-Adic and real coupling constant evolutions reflect the underlying evolution in powers of two for the temporal distance between the tips of the light-cones of the causal diamond and the association of macroscopic time scale as secondary p-adic time scale to elementary particles (.1 seconds for electron) serves as a first test for the picture. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules could be treated independently.

More detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities would follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-canonical conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [C2].

1. The state construction utilizes both super-canonical and super Kac-Moody algebras. Super-canonical algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-canonical algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-canonical operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO-H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic 4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the CH Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-canonical super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the CH super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to CH gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^\dagger/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^{dagger}$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k\gamma_k^\dagger$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since fermionic propagator and boson-emission vertices give compensating fermion numbers.

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

In this framework coupling constant evolution would have interpretation in terms of addition of intermediate zero energy states corresponding to the generalized Feynman diagrams obtained by the insertion of causal diamonds with a new shorter time scale $T = T_{prev}/2$ to the previous Feynman diagram. p-Adic length scale hypothesis follows naturally. A very close correspondence with ordinary Feynman diagrammatics arises and and ordinary vision about coupling constant evolutions arises. The absence of infinities follows from the symplectic invariance which is genuinely new element. p-Adic and real coupling constant evolutions can be seen as completions of coupling constant evolutions for physics based on rationals and their algebraic extensions.

5.2.3 How p-adic and real coupling constant evolutions are related to each other?

The real and p-adic coupling constant evolutions should be consistent with each other. This means that the coupling constants $g(p_1, p_2, p_3)$ as functions of p-adic primes characterizing particles of the vertex should have the same qualitative behavior as real and p-adic functions. Hence the p-adic norms of complex rational valued (or those in algebraic extension) amplitudes must give a good estimate for the behavior of the real vertex. Hence a restriction of a continuous correspondence between p-adics and reals to rationals is highly suggestive. The restriction of the canonical identification to rationals would define this kind of correspondence but this correspondence respects neither symmetries nor unitarity in its basic form. Some kind of compromize between correspondence via common rationals and canonical identification should be found.

The compromize might be achieved by using a modification of canonical identification $I_{R_p \rightarrow R}$. Generalized numbers would be regarded in this picture as a generalized manifold obtained by gluing different number fields together along rationals. Instead of a direct identification of real and p-adic rationals, the p-adic rationals in R_p are mapped to real rationals (or vice versa) using a variant of the canonical identification $I_{R \rightarrow R_p}$ in which the expansion of rational number $q = r/s = \sum r_n p^n / \sum s_n p^n$ is replaced with the rational number $q_1 = r_1/s_1 = \sum r_n p^{-n} / \sum s_n p^{-n}$ interpreted as a p-adic number:

$$q = \frac{r}{s} = \frac{\sum_n r_n p^n}{\sum_m s_n p^n} \rightarrow q_1 = \frac{\sum_n r_n p^{-n}}{\sum_m s_n p^{-n}} \tag{5.2.6}$$

This variant of canonical identification is not equivalent with the original one using the infinite expansion of q in powers of p since canonical identification does not commute with product and division. The variant is however unique in the recent context when r and s in $q = r/s$ have no common factors. For integers $n < p$ it reduces to direct correspondence. R_{p1} and R_{p_2} are glued together along common rationals by an the composite map $I_{R \rightarrow R_{p_2}} I_{R_{p_1} \rightarrow R}$.

Instead of a re-interpretation of the p-adic number $g(p_1, p_2, p_3)$ as a real number or vice versa would be continued by using this variant of canonical identification. The nice feature of the map would be that continuity would be respected to high degree and something which is small in real sense would be small also in p-adic sense.

How to achieve consistency with the unitarity of topological mixing matrices and of CKM matrix?

It is easy to invent an objection against the proposed relationship between p-adic and real coupling constants. Topological mixing matrices U, D and CKM matrix $V = U^\dagger D$ define an important part of the electro-weak coupling constant structure and appear also in coupling constants. The problem is that canonical identification does not respect unitarity and does not commute with the matrix multiplication in the general case unlike gluing along common rationals. Even if matrices U and D which contain only ratios of integers smaller than p are constructed, the construction of V might be problematic since the products of two rationals can give a rational $q = r/s$ for which r or s or both are larger than p.

One might hope that the objection could be circumvented if the ratios of the integers of the algebraic extension defining the matrix elements of CKM matrix are such that the integer components of algebraic integers are smaller than p in U and D and even the products of integers in $U^\dagger D$ satisfy this condition so that modulo p arithmetics is avoided.

In the standard parametrization all matrix elements of the unitarity matrix can be expressed in terms of real and imaginary parts of complex phases ($p \ mod \ 4 = 3$ guarantees that $\sqrt{-1}$ is not an ordinary p-adic number involving infinite expansion in powers of p). These phases are expressible as products of Pythagorean phases and phases in some algebraic extension of rationals.

i) Pythagorean phases defined as complex rationals $[r^2 - s^2 + i2rs]/(r^2 + s^2)$ are an obvious source of potential trouble. However, if the products of complex integers appearing in the numerators and denominators of the phases have real and imaginary parts smaller than p it seems to be possible to avoid difficulties in the definition of $V = U^\dagger D$.

ii) Pythagorean phases are not periodic phases. Algebraic extensions allow to introduce periodic phases of type $exp(i\pi m/n)$ expressible in terms of p-adic numbers in a finite-dimensional algebraic extension involving various roots of rationals. Also in this case the product $U^\dagger D$ poses conditions on the size of integers appearing in the numerators and denominators of the rationals involved.

If the expectation that topological mixing matrices and CKM matrix characterize the dynamics at the level $p \simeq 2^k$, $k = 107$, is correct, number theoretical constraints are not expected to bring much new to what is already predicted. Situation changes if these matrices appear already at the level k. For $k = 89$ hadron physics the restrictions would be even stronger and might force much simpler U, D and CKM matrices.

k-adicity constraint would have even stronger implications for S-matrix and could give very powerful constraints to the S-matrix of color interactions. Quite generally, the constraints would imply a p-adic hierarchy of increasingly complex S-matrices: kind of a physical realization for number theoretic emergence. The work with CKM matrix has shown how powerful the number theoretical constraints are, and there are no reasons to doubt that this could not be the case also more generally since in the lowest order the construction would be carried out in finite (Galois) fields $G(p, k)$.

How generally the hybrid of canonical identification and identification via common rationals can apply?

The proposed gluing procedure, if applied universally, has non-trivial implications which need not be consistent with all previous ideas.

1. The basic objection against the new kind of identification is that it does not commute with symmetries. Therefore its application at imbedding space and space-time level is questionable.

2. The mapping of p-adic probabilities by canonical identification to their real counterparts requires a separate normalization of the resulting probabilities. Also the new variant of canonical identification requires this since it does not commute with the sum.

3. The direct correspondence of reals and p-adics by common rationals at space-time level implies that the intersections of cognitive space-time sheets with real space-time sheet have literally infinite size (p-adically infinitesimal corresponds to infinite in real sense for rational) and consist of discrete points in general. If the new gluing procedure is adopted also at space-time level, it would considerably de-dramatize the radical idea that the size for the space-time correlates of cognition is literally infinite and cognition is a literally cosmic phenomenon.

   Of course, the new kind of correspondence could be also seen as a manner to construct cognitive representations by mapping rational points to rational points in the real sense and thus as a formation of cognitive representations at space-time level mapping points close to each other in real sense to points close to each other p-adically but arbitrarily far away in real sense. The image would be a completely chaotic looking set of points in the wrong topology and would realize the idea of Bohm about hidden order in a very concrete manner. This kind of mapping might be used to code visual information using the value of p as a part of the code key.

4. In p-adic thermodynamics p-adic particle mass squared is mapped to its real counterpart by canonical identification. The objection against the use of the new variant of canonical identification is that the predictions of p-adic thermodynamics for mass squared are not rational numbers

but infinite power series. p-Adic thermodynamics itself however defines a unique representation of probabilities as ratios of generalized Boltzmann weights and partition function and thus the variant of canonical identification indeed generalizes and at the same time raises worries about the fate of the earlier predictions of the p-adic thermodynamics.

Quite generally, the thermodynamical contribution to the particle mass squared is in the lowest p-adic order of form rp/s, where r is the number of excitations with conformal weight 1 and s the number of massless excitations with vanishing conformal weight. The real counterpart of mass squared for the ordinary canonical identification is of order CP_2 mass by $r/s = R + r_1p + ...$ with $R < p$ near to p. Hence the states for which massless state is degenerate become ultra heavy if r is not divisible by s. For the new variant of canonical identification these states would be light. It is not actually clear how many states of this kind the generalized construction unifying super-canonical and super Kac-Moody algebras predicts.

A less dramatic implication would be that the second order contribution to the mass squared from p-adic thermodynamics is always very small unless the integer characterizing it is a considerable fraction of p. When ordinary canonical identification is used, the second order term of form rp^2/s can give term of form Rp^2, $R < p$ of order p. This occurs only in the case of left handed neutrinos.

The assumption that the second order term to the mass squared coming from other than thermodynamical sources gives a significant contribution is made in the most recent calculations of leptonic masses [F3]. It poses constraints on CP_2 mass which in turn are used as a guideline in the construction of a model for hadrons [F4]. This kind of contribution is possible also now and corresponds to a contribution Rp^2, $R < p$ near p.

The new variant of the canonical correspondence resolves the long standing problems related to the calculation of Z and W masses. The mass squared for intermediate gauge bosons is smaller than one unit when m_0^2 is used as a fundamental mass squared unit. The standard form of the canonical identification requires $M^2 = (m/n)p^2$ whereas in the new approach $M^2 = (m/n)p$ is allowed. Second difficult problem has been the p-adic description of the group theoretical model for m_W^2/m_Z^2 ratio. In the new framework this is not a problem anymore [F3] since canonical identification respects the ratios of small integers.

On the other hand, the basic assumption of the successful model for topological mixing of quarks [F4] is that the modular contribution to the masses is of form np. This assumption loses its original justification for this option and some other justification is needed. The first guess is that the conditions on mass squared plus probability conservation might not be consistent with unitarity unless the modular contribution to the mass squared remains integer valued in the mixing (note that all integer values are not possible [F4]). Direct numerical experimentation however shows that that this is not the case.

5.2.4 A revised view about the interpretation and evolution of Kähler coupling strength

The original hypothesis was that Kähler coupling strength is invariant under p-adic coupling constant evolution. Later I gave up this hypothesis and replaced it with the invariance of gravitational coupling since otherwise the prediction would have been that gravitational coupling strength is proportional to p-adic length scale squared. The recent view means return to the roots: Kähler coupling strength is invariant under p-adic coupling constant evolution and has value spectrum dictated by the Chern-Simons coupling k defining the theory at the parton level. Gravitational coupling constant corresponds in this framework to the largest Mersenne prime M_{127} which does not correspond to a completely super-astronomical p-adic length scale.

Formula for Kähler coupling constant

To construct expression for gravitational constant one can use the following ingredients.

1. The exponent $exp(2S_K(CP_2))$ defining the value of Kähler function in terms of the Kähler action $S_K(CP_2)$ of CP_2 type extremal representing elementary particle expressible as

$$S_K(CP_2) = \frac{\pi}{8\alpha_K} . \tag{5.2.7}$$

Since CP_2 type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \rightarrow a \times S_K(CP_2) \ . \tag{5.2.8}$$

Naively one would expect reduction of the action so that one would have $a \leq 1$. One must however keep mind open in this respect.

2. The p-adic length scale L_p assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime $M_{127} = 2^{127} - 1$ defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that M_{127} characterizes these space-time sheets.

The formula for gravitational constant would read as

$$\begin{aligned} G &= L_p^2 \times exp(-2aS_K(CP_2)) \ . \\ L_p &= \sqrt{p}R \ . \end{aligned} \tag{5.2.8}$$

Here R is CP_2 radius defined by the length $2\pi R$ of the geodesic circle. The relationship boils down to

$$\begin{aligned} \alpha_K &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{G} \ . \end{aligned} \tag{5.2.8}$$

The value of K is fixed by the requirement that electron mass scale comes out correctly in the p-adic mass calculations and minimal value of K is factor. The uncertainties related to second order contributions however leave the precise value open.

The earlier calculations contained two errors. First error was related to the value of the parameter $K = R^2/G$ believed to be in good approximation given by the product of primes smaller than 26. Second error was that $1/\alpha_K$ was by a factor 2 too large for $a = 1$. This led first to a conclusion that α_K is very near to fine structure constant and perhaps equal to it. The physically more plausible option turned out to corresponds to $1/\alpha_K = 104$, which corresponds in good approximation to the value of electro-weak U(1) coupling at electron length scale but gave $a > 1$ whereas $a < 1$ would be natural since the action for a wormhole contact formed by a piece of CP_2 type vacuum extremal is expected to be smaller than the full action of CP_2 type vacuum extremal.

The correct calculation gives $a < 1$ for $\alpha_K = 1/104$. From the table one finds that if the parameter a equals to $a = 1/2$ the value of α_K is about 133. It would require $a = .6432$ for $Y_e = 0$ favored by the value of top quark mass. This value of a conforms with the idea that a piece of CP_2 type extremal defining a wormhole contact is in question. Note that a proper choices of value of a can make $K = R^2/G$ rational. The table gives values of various quantities assuming

$$K = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23 \times 2^{-3} * (15/17) \ . \tag{5.2.9}$$

giving simplest approximation as a rational for K producing K_R for $Y_e = 0$ with error of 2.7 per cent which is still marginally consistent with the mass of top quark. This approximation should not be taken too seriously.

Y_e	0	.5	.7798
$(m_0/m_{Pl})10^3$	.2437	.2323	.2266
$K_R \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G}) \times 10^{-4}$	3.1580	3.3122	3.3954
$1/\alpha_K$	133.7850	133.9064	133.9696
a_{104}	0.6432	0.6438	0.6441
a_α	0.4881	0.4886	0.4888
$K \times 10^{-7}$	2.4606	2.4606	2.4606
$(L/\sqrt{G}) \times 10^{-4}$	3.1167	3.1167	3.1167
$1/\alpha_K$	133.9158	133.9158	133.9158
a_{104}	0.6438	0.6438	0.6438
a_α	.4886	0.4886	0.4886
K_R/K	1.0267	1.1293	1.1868

Table 1. Table gives the values of the ratio $K_R = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$. Also the ratio of K_R/K, where $K = 2\times3\times5\times7\times11\times13\times17\times19\times23\times2^{-3}*(15/17)$ is rational number producing R^2/G approximately is given*[1]. The values of α_K deduced using the formula above are given for $a = 1/2$ and the values of $a = a_{104}$ giving $\alpha_K = 1/104$ are given. Also the values of $a = a_\alpha$ for which α_K equals to the fine structure constant $1/\alpha_{em} = 137.0360$ are given.

If one assumes that α_K is of order fine structure constant in electron length scale, the value of the parameter a is slightly below 1/2 cannot be far from unity. Symmetry principles do not favor the identification. Later it will be found that rather general arguments predict integer spectrum for $1/\alpha_K$ given by $1/\alpha_K = 4k$. For this option $\alpha_K = 1/137$ is not allowed whereas the $1/\alpha_K = 104 = 4 \times 26$ is.

Formula relating v_0 to α_K and R^2/G

If v_0 is identified as the rotation velocity of distant stars in galactic plane, one can use the Newtonian model for the motion of mass in the gravitational field of long straight string giving $v_0 = \sqrt{TG}$. String tension T can be expressed in terms of Kähler coupling strength as

$$T = \frac{b}{2\alpha_K R^2} \ ,$$

where R is the radius of geodesic circle. The factor $b \leq 1$ would explain reduction of string tension in topological condensation caused by the fact that not entire geodesic sphere contributes to the action.

This gives

$$\begin{aligned} v_0 &= \frac{b}{2\sqrt{\alpha_K K}} \ , \\ \alpha_K(p) &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{G} \ . \end{aligned} \tag{5.2.8}$$

The condition that α_K has the desired value for $p = M_{127} = 2^{127} - 1$ defining the p-adic length scale of electron fixes the value of b for given value of a. The value of b should be smaller than 1 corresponding to the reduction of string tension in topological condensation.

The condition 6.8.11 for $v_0 = 2^{-m}$, say $m = 11$, allows to deduce the value of a/b as

$$\frac{a}{b} = \frac{4 * log(pK)}{\pi}\frac{2^{2m-1}}{K} \ . \tag{5.2.9}$$

The table gives the values of b calculated assuming $a = a_{104}$ producing $\alpha_K = 1/104$ for various values of Y_e.

[1]The earlier calculations giving $K = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23$ in a good approximation contained an error

Y_e	0	.5	.7798
$b_{9,104}$	0.9266	1.0193	1.0711
$b_{11,104}$	0.0579	0.0637	0.0669
$b_{9,\alpha}$	0.7032	0.7736	0.81291
$b_{11,\alpha}$	0.0440	0.0483	0.050

Table 2. Table gives the values of b for $Y_e \in \{0, .5, .7798\}$ assuming the values $a = a_{104}$ guaranteing $\alpha_K = 1/104$ and $\alpha_K = \alpha_{em}$. $b_{9,...}$ corresponds to $m = 9$ and $b_{11,...}$ corresponds to $m = 11$.

From the table one finds that for $\alpha_K = 1/104$ $m = 9$ corresponds to the almost full action for topological condensed cosmic string. $m = 11$ corresponds to much smaller action smaller by a factor of order 1/16. The interpretation would be that as m increases the action of the topologically condensed cosmic string decreases. This would correspond to a gradual transformation of the cosmic string to a magnetic flux tube.

Does α_K correspond α_{em} or weak coupling strength $\alpha_{U(1)}$ at electron length scale?

The preceding arguments allow the original identification $\alpha_K \simeq 1/137$. On the other hand, group theoretical arguments encourage the identification of α_K as weak $U(1)$ coupling strength $\alpha_{U(1)}$:

$$\begin{aligned} \alpha_K &= \alpha_{U(1)} = \frac{\alpha_{em}}{cos^2(\theta_W)} \simeq \frac{1}{104.1867} \ , \\ sin^2(\theta_W)_{|10\ MeV} &\simeq 0.2397(13) \ , \\ \alpha_{em}(M_{127}) &= 0.00729735253327 \ . \end{aligned} \tag{5.2.8}$$

Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value $sin^2(\theta_W) = 0.2397(13)$ corresponding to 10 MeV mass scale [33] is used.

Later it will be found that general argument predicts that $1/\alpha_K$ is integer valued: $1/\alpha_K = 4k$. This option excludes identification as $\alpha_{em}(127)$ but encourages strongly the identification as $\alpha_{U(1)}(127)$.

Is gravitational constant an approximate RG invariant?

The original model for the p-adic evolution of α_K was based on the p-adic renormalization group invariance of gravitational constant. The starting point was the observation that on purely dimension analytic basis one can write $G = exp(-2S_K(CP_2))L_p^2$. If α_K is p-adic RG invariant, G scales like L_p^2 which looked completely non-sensible at that time so that the identification $\alpha_K = \alpha_{U(1)}$ was given up. Discrete p-adic evolution for α_K is consistent with RG invariance and quantum criticality at a given p-adic space-time sheet.

This view however leads to problems with the identification $\alpha_K = \alpha_{U(1)}$ since the evolution of α_K dictated by RG invariance of G is much faster than that of $\alpha_{U(1)}$. The condition

$$cos^2(\theta_W)(89) = \frac{log(M_{127}K)}{log(M_{89}K)} \times \frac{\alpha_{em}(M_{127})}{\alpha_{em}(M_{89})} \times cos^2(\theta_W)(127) \ . \tag{5.2.9}$$

together with the experimental value $1/\alpha_{em}(M_{89}) \simeq 128$ as predicted by standard model, gives $sin^2(\theta_W)(89) = .0474$ to be compared with the measured valued .23120(15) at intermediate boson mass scale [33]!

Furthermore, if α_K evolves with p then v_0 is predicted to evolve too but $v_0 = 2^{-11}$ is consistent with experimental facts (apart from possible presence of sub-harmonics which can be however understood in TGD framework).

Or is α_K RG invariant?

One is forced to ask whether one must give up the existing scenario for the p-adic evolution of α_K and identify it with the evolution of $\alpha_{U(1)}$ or perhaps even p-adic RG invariance of α_K. The predicted very fast evolution $G \propto L_p^2$ in the approximation that α_K is RG invariant makes sense only if L_p

characterizes the space-time sheets carrying gravitational interaction or even to gravitons and if these space-time sheet corresponds to $p = M_{127}$ under normal conditions.

If bosons correspond to Mersenne primes, this would be naturally the case since the Mersenne prime next to M_{127} corresponds to a completely super-astrophysical length scale. In this case p-adic length scale hypothesis would predict $v_0^{-2}(L(k)) = 2^{-22}2^{-k+127}$ if α_K is RG invariant so that it would behave as a power of 2. $\hbar_{gr}$ would scale as 2^{-k+127} and approach rapidly to zero as $L(k)$ increases whereas gravitational force would become strong.

If same p_0 characterizes all ordinary gauge bosons with their dark variants included, one would have $p_0 = M_{89} = 2^{89} - 1$. In this case however the gravitational coupling strength would be weaker by a factor 2^{-38}. M_{127} also defines a dark length scale in TGD inspired quantum model of living matter [F9, J6].

A further nice feature of this identification is that one can also allow the scaling of CP_2 metric and thus R^2 by $\lambda^2 = (\hbar/\hbar_0)^2$ inducing $K \to \lambda^2 K$. $1/v_0 \to \lambda/v_0$ implies that $\hbar_{gr}$ scales in the same manner as $\hbar$. Hence it would seem that $\hbar$ corresponds to M^4- and $\hbar_{gr}$ to CP_2 degrees of freedom and the huge value of $\hbar_{gr}$ would mean that there is that cosmology has quantal lattice like structure in cosmological length scales with H_a/G, $G \subset SL(2,C)$, serving as a basic lattice cell (here H_a denotes $a = constant$ hyperboloid of M^4_+). The observed sub-harmonics of v_0 could thus be understood in terms of scalings of CP_2 gravitational constant. This structure is supported also by the quantization of cosmological red shifts [32].

The huge value of h_{gr} assignable to color algebra does not mean that colored states would have huge values of color charges since fractionization of color quantum numbers occurs. It however means that dark color charges are de-localized in huge length scales and cosmological color could be seen as responsible for a macroscopic quantum coherence in astrophysical length scales.

What about color coupling strength?

Classical theory should be also able to say something non-trivial about color coupling strength α_s too at the general level. The basic observations are following.

1. Both classical color YM action and electro-weak U(1) action reduce to Kähler action.
2. Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that α_K is a strict RG invariant. The first idea would be that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for $\alpha_{U(1)} = \alpha_s = 2\alpha_K$. Asymptotically the couplings approach to a fixed point defined by $2\alpha_K$ rather than to zero as in asymptotically free gauge theories.

Thus one would have

$$\frac{1}{\alpha_{U(1)}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \tag{5.2.10}$$

The formula implies that the beta functions for color and $U(1)$ degrees of freedom are apart from sign identical and the increase of $U(1)$ coupling compensates the decrease of the color coupling. This gives the formula

$$\alpha_s = \frac{1}{\frac{1}{\alpha_K} - \frac{1}{\alpha_{U(1)}}} . \tag{5.2.11}$$

At least formally $\alpha_s(QCD)$ could become negative below the confinement length scale so that $\alpha_K < \alpha_{U(1)}$ for M_{127} is consistent with this formula. For M_{89} $\alpha_{em} \simeq 1/127$ gives $1/\alpha_{U(1)}(89) = 1/97.6374$.

1. $\alpha_K = \alpha_{em}(127)$ option does not work. Confinement length scale corresponds to the point at which one has $\alpha_{U(1)} = \alpha_K$ and in principle can be predicted precisely using standard model. In the case that $\alpha_s(107)$ diverges, one has

$$\alpha_{em}(107) = cos^2(\theta_W)\alpha_{U(1)} = cos^2(\theta_W)\alpha_K = \frac{cos^2(\theta_W)}{136} \quad .$$

The resulting value of α_{em} is too small and the situation worsens for $k > 107$ since $\alpha_{U(1)}$ decreases. Hence this option is excluded.

2. TGD predicts that also M_{127} copy of QCD should exist and that M_{127} quarks should play a key role in nuclear physics [F8]. Hence one could argue that color coupling strength diverges at M_{127} (the largest not completely super-astrophysical Mersenne prime) so that one would have $\alpha_K = \alpha_{U(1)}(M_{127})$. Therefore the precise knowledge of $\alpha_{U(1)}(M_{127})$ in principle fixes the value of parameter $K = R^2/G$ and thus also the second order contribution to the mass of electron. On ther other hand, quite a general argument predicts $\alpha_K = 1/104$ so that an exact prediction for $U(1)$ coupling follows.

The predicted value of $\alpha_s(M_{89})$ follows from $sin^2(\theta_W) = .23120$ and $\alpha_{em} \simeq 1/127$ at intermediate boson mass scale using $\alpha_{U(1)} = \alpha_{em}/cos^2(\theta_W)$ and $1/\alpha_s = 1/\alpha_K - 1/\alpha_{U(1)}$. The predicted value $\alpha_s(89) = 0.1572$ is quite reasonable although somewhat larger than QCD value. For $1/\alpha_K = 108 = 4 \times 27$ one would have $\alpha_s(89) = 0.0965$.

The new vision about the value spectrum of Kähler coupling strength and hadronic space-time sheet suggests $\alpha_K = \alpha_s = \alpha_s = 1/4$ at hadronic space-time sheet labelled by M_{107}. α_s here refers however to super-canonical gluons which do not consist of quark-antiquark pairs. If the two values of α_s are identical at $k = 107$ (ordinary gluons might be perhaps mix strongly with super-canonical ones at this length scale), one has $\alpha_{U(1)}(107) = 1/100$. Using $sint^2(\theta_W) = 2397$ at 10 MeV this predicts $\alpha_{em}(107) = 1/131.53$.

To sum up, the proposed formula would dictate the evolution of α_s from the evolution of the electro-weak parameters without any need for perturbative computations and number theoretical prediction for U(1) coupling at electron length scale would be exact. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

5.2.5 Does the quantization of Kähler coupling strength reduce to the quantization of Chern-Simons coupling at partonic level?

Kähler coupling strength associated with Kähler action (Maxwell action for the induced Kähler form) is the only coupling constant parameter in quantum TGD, and its value (or values) is in principle fixed by the condition of quantum criticality since Kähler coupling strength is completely analogous to critical temperature. The quantum TGD at parton level reduces to almost topological QFT for light-like 3-surfaces. This almost TQFT involves Abelian Chern-Simons action for the induced Kähler form.

This raises the question whether the integer valued quantization of the Chern-Simons coupling k could predict the values of the Kähler coupling strength. I considered this kind of possibility already for more than 15 years ago but only the reading of the introduction of the [30] about his new approach to 3-D quantum gravity led to the discovery of a childishly simple argument that the inverse of Kähler coupling strength could indeed be proportional to the integer valued Chern-Simons coupling k: $1/\alpha_K = 4k$. $k = 26$ is forced by the comparison with some physical input. Also p-adic temperature could be identified as $T_p = 1/k$.

Quantization of Chern-Simons coupling strength

For Chern-Simons action the quantization of the coupling constant guaranteing so called holomorphic factorization is implied by the integer valuedness of the Chern-Simons coupling strength k. As Witten explains, this follows from the quantization of the first Chern-Simons class for closed 4-manifolds plus the requirement that the phase defined by Chern-Simons action equals to 1 for a boundaryless 4-manifold obtained by gluing together two 4-manifolds along their boundaries. As explained by Witten in his paper, one can consider also "anyonic" situation in which k has spectrum Z/n^2 for n-fold covering of the gauge group and in dark matter sector one can consider this kind of quantization.

Formula for the Kähler coupling strength

The quantization argument for k seems to generalize to the case of TGD. What is clear that this quantization should closely relate to the quantization of the Kähler coupling strength appearing in the 4-D Kähler action defining Kähler function for the world of classical worlds and conjectured to result as a Dirac determinant. The conjecture has been that g_K^2 has only single value. With some physical input one can make educated guesses about this value. The connection with the quantization of Chern-Simons coupling would however suggest a spectrum of values. This spectrum is easy to guess.

1. Wick rotation argument

The U(1) counterpart of Chern-Simons action is obtained as the analog of the "instanton" density obtained from Maxwell action by replacing $J\wedge *J$ with $J\wedge J$. This looks natural since for self dual J associated with CP_2 type vacuum extremals Maxwell action reduces to instanton density and therefore to Chern-Simons term. Also the interpretation as Chern-Simons action associated with the classical SU(3) color gauge field defined by Killing vector fields of CP_2 and having Abelian holonomy is possible. Note however that *instanton density is multiplied by imaginary unit in the action exponential of path integral*. One should find justification for this "Wick rotation" not changing the value of coupling strength and later this kind of justification will be proposed.

Wick rotation argument suggests the correspondence $k/4\pi = 1/4g_K^2$ between Chern-Simons coupling strength and the Kähler coupling strength g_K appearing in 4-D Kähler action. This would give

$$g_K^2 = \frac{\pi}{k} \ , \frac{1}{\alpha_K} = 4k \ . \tag{5.2.12}$$

The spectrum of $1/\alpha_K$would be integer valued. The result is very nice from the point of number theoretic vision since the powers of α_K appearing in perturbative expansions would be rational numbers (ironically, radiative corrections should vanish by number theoretic universality but this might happen only for these rational values ofα_K!).

2. Are more general values of k possible

Note however that if k is allowed to have values in Z/n^2, the strongest possible coupling strength is scaled to $n^2/4$ unless $\hbar$ is not scaled: already for $n=2$ the resulting perturbative expansion might fail to converge. In the scalings of $\hbar$ associated with M^4 degrees of freedom $\hbar$ however scales as $1/n^2$ so that the spectrum of α_K would remain invariant.

3. Experimental constraints on α_K

It is interesting to compare the prediction with the experimental constraints on the value of $1/\alpha_K$. As already found, there are two options to consider.

1. $\alpha_K=\alpha_{em}$ option suggests $1/\alpha_K=137$ inconsistent with $1/\alpha_K=4k$ condition. $1/\alpha_K=136=4\times 34$ combined with the formula $1/\alpha_s+1/\alpha_{U(1)}=1/\alpha_K$ leads to nonsensical predictions.

2. For $1/\alpha_s+1/\alpha_{U(1)} = 1/\alpha_K = 104$ option option the basic empirical input is that electro-weak $U(1)$ coupling strength reduces to Kähler coupling at electron length scale. This gives $\alpha_K=\alpha_{U(1)}(M_{127})\simeq 104.1867$, which corresponds to $k=26.0467$. $k=26$ would give $\alpha_K=104$: the difference would be only .2 per cent and one would obtain exact prediction for $\alpha_{U(1)}(M_{127})$. Together with electro-weak coupling constant evolution this would also explain why the inverse of the fine structure constant is so near to 137 but not quite. Amusingly, $k=26$ is the critical space-time dimension of the bosonic string model. Also the conjectured formula for the gravitational constant in terms of α_K and p-adic prime p involves all primes smaller than 26.

Justification for Wick rotation

It is not too difficult to believe to the formula $1/\alpha_K=qk$, q some rational. $q=4$ however requires a justification for the Wick rotation bringing the imaginary unit to Chern-Simons action exponential lacking from Kähler function exponential.

In this kind of situation one might hope that an additional symmetry might come in rescue. The guess is that number theoretic vision could justify this symmetry.

1. To see what this symmetry might be consider the generalization of the [31] obtained by combining theta angle and gauge coupling to single complex number via the formula

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} \ . \tag{5.2.13}$$

   What this means in the recent case that for CP_2 type vacuum extremals [D1] Kähler action and instanton term reduce by self duality to Kähler action obtained by the replacement g^2 with $-i\tau/4\pi$. The first duality $\tau \rightarrow \tau + 1$ corresponds to the periodicity of the theta angle. Second duality $\tau \rightarrow -1/\tau$ corresponds to the generalization of Montonen-Olive duality $\alpha \rightarrow 1/\alpha$. These dualities are definitely not symmetries of the theory in the recent case.

2. Despite the failure of dualities, it is interesting to write the formula for τ in the case of Chern-Simons theory assuming $g_K^2 = \pi/k$ with $k > 0$ holding true for Kac-Moody representations. What one obtains is

$$\tau = 4k(1-i) \ . \tag{5.2.14}$$

   The allowed values of τ are integer spaced along a line whose direction angle corresponds to the phase $exp(i2\pi/n)$, $n = 4$. The transformations $\tau \rightarrow \tau + 4(1-i)$ generate a dynamical symmetry and as Lorentz transformations define a subgroup of the group E^2 leaving invariant light-like momentum (this brings in mind quantum criticality!). One should understand why this line is so special.

3. This formula conforms with the number theoretic vision suggesting that the allowed values of τ belong to an integer spaced lattice. Indeed, if one requires that the phase angles are proportional to vectors with rational components then only phase angles associated with orthogonal triangles with short sides having integer valued lengths m and n are possible. The additional condition that the phase angles correspond to *roots of unity*! This leaves only $m = n$ and $m = -n > 0$ into consideration so that one would have $\tau = n(1-i)$ from $k > 0$.

4. Notice that theta angle is a multiple of $8k\pi$ so that a trivial strong CP breaking results and no QCD axion is needed (this of one takes seriously the equivalence of Kähler action to the classical color YM action).

Is p-adicization needed and possible only in 3-D sense?

The action of CP_2 type extremal is given as $S = \pi/8\alpha_K = k\pi/2$. Therefore the exponent of Kähler action appearing in the vacuum functional would be $exp(k\pi)$ - Gelfond's constant - known to be a transcendental number [17]. Also its powers are transcendental. If one wants to p-adicize also in 4-D sense, this raises a problem.

Before considering this problem, consider first the 4-D p-adicization more generally.

1. The definition of Kähler action and Kähler function in p-adic case can be obtained only by algebraic continuation from the real case since no satisfactory definition of p-adic definite integral exists. These difficulties are even more serious at the level of configuration space unless algebraic continuation allows to reduce everything to real context. If TGD is integrable theory in the sense that functional integral over 3-surfaces reduces to calculable functional integrals around the maxima of Kähler function, one might dream of achieving the algebraic continuation of real formulas. Note however that for light-like 3-surface the restriction to a category of algebraic surfaces essential for the re-interpretation of real equations of 3-surface as p-adic equations. It is far from clear whether also preferred extremals of Kähler action have this property.

2. Is 4-D p-adicization the really needed? The extension of light-like partonic 3-surfaces to 4-D space-time surfaces brings in classical dynamical variables necessary for quantum measurement theory. p-Adic physics defines correlates for cognition and intentionality. One can argue that these are not quantum measured in the conventional sense so that 4-D p-adic space-time sheets would not be needed at all. The p-adic variant for the exponent of Chern-Simons action can make sense using a finite-D algebraic extension defined by $q = exp(i2\pi/n)$ and restricting the allowed light-like partonic 3-surfaces so that the exponent of Chern-Simons form belongs to this extension of p-adic numbers. This restriction is very natural from the point of view of dark matter hierarchy involving extensions of p-adics by quantum phase q.

If one remains optimistic and wants to p-adicize also in 4-D sense, the transcendental value of the vacuum functional for CP_2 type vacuum extremals poses a problem (not the only one since the p-adic norm of the exponent of Kähler action can become completely unpredictable).

1. One can also consider extending p-adic numbers by introducing $exp(\pi)$ and its powers and possibly also π. This would make the extension of p-adics infinite-dimensional which does not conform with the basic ideas about cognition. Note that e^p is not p-adic transcendental so that extension of p-adics by powers e is finite-dimensional and if p-adics are first extended by powers of π then further extension by $exp(\pi)$ is p-dimensional.

2. A more tricky manner to overcome the problem posed by the CP_2 extremals is to notice CP_2 type extremals are necessarily deformed and contain a hole corresponding to the light-like 3-surface or several of them. This would reduce the value of Kähler action and one could argue that the allowed p-adic deformations are such that the exponent of Kähler action is a p-adic number in a finite extension of p-adics. This option does not look promising.

5.2.6 What could happen in the transition to non-perturbative QCD?

What happens mathematically in the transition to non-perturbative QCD has remained more or less a mystery. The number theoretical considerations of [E9] inspired the idea that Planck constant is dynamical and has a spectrum given as $\hbar(n) = n\hbar_0$, where n characterizes the quantum phase $q = exp(i2\pi/n)$ associated with Jones inclusion. The strange finding that the orbits of planets seem to obey Bohr quantization rules with a gigantic value of Planck constant inspired the hypothesis that the increase of Planck constant provides a unique mechanism allowing strongly interacting system to stay in perturbative phase [A9, D7]. The resulting model allows to understand dark matter as a macroscopic quantum phase in astrophysical length and time scales, and strongly suggest a connection with dark matter and biology.

The phase transition increasing Planck constant could provide a model for the transition to confining phase in QCD. When combined with the recent ideas about value spectrum of Kähler coupling strength one ends up with a rather explicit model about non-perturbative aspects of hadron physics already successfully applied in hadron mass calculations [F4]. Mersenne primes seem to define the p-adic length scales of gauge bosons and of hadronic space-time sheets. The quantization of Planck constant provides additional insight to p-adic length scales hypothesis and to the preferred role of Mersenne primes.

Super-canonical gluons and non-perturbative aspects of hadron physics

According to the model of hadron masses [F4], in the case of light pseudoscalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudoscalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of configuration space degrees of freedom ("world of classical worlds") in which super-canonical algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges.

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-canonical gluons. It turns out the model assuming same topological mixing of super-canonical bosons identical to that experienced by U type quarks leads to excellent understanding of hadron masses assuming that hadron spin correlates with the super-canonical particle content of the hadronic space-time sheet.

According to the argument already discussed, at the hadronic $k = 107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s = \alpha_K = \frac{1}{4} \ .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

This α_s would correspond to the interaction via super-canonical colored gluons and would lead to the failure of perturbation theory. By the general criterion stating that the failure of perturbation theory leads to a phase transition increasing the value of Planck constant one expects that the value of $\hbar$ increases [A9]. The value leaving the value of α_K invariant would be $\hbar \rightarrow 26\hbar$ and would mean that p-adic length scale L_{107} is replaced with length scale $26L_{107} = 46$ fm, the size of large nucleus so that also the basic length scale nuclear physics would be implicitly coded into the structure of hadrons.

Why Mersenne primes should label a fractal hierarchy of physics?

There are motivations for the working hypothesis stating that there is fractal hierarchy of copies of standard model physics, and that Mersenne primes label both hadronic space-time sheets and gauge bosons. The reason for this is not yet well understood and I have considered several speculative explanations.

1. *First picture*

The first thing to come in mind is that Mersenne primes correspond to fixed points of the discrete p-adic coupling constant evolution, most naturally to the maxima of the color coupling constant strength. This would mean that gluons are emitted with higher probability than in other p-adic length scales.

There is however an objection againt this idea. If one accepts the new vision about non-perturbative aspects of QCD, it would seem that super-canonical bosons or the interaction between super-canonical bosons and quarks for some reason favors Mersenne primes. However, if color coupling strength corresponds to $\alpha_K = \alpha_s = 1/4$ scaled down by the increase of the Planck constant, the evolution of super-canonical color coupling strength does not seem to play any role. What becomes large should be a geometric "form factor", when the boson in the vertex corresponds to Mersenne prime rather than "bare" coupling.

The resolution of the problem could be that boson emission vertices $g(p_1, p_2, p_3)$ are functions of p-adic primes labelling the particles of the vertices so that actually three p-adic length scales are involved instead of single length scale as in the ordinary coupling constant evolution. Hence one can imagine that the interaction between particles corresponding to primes near powers of 2 and Mersenne primes is especially strong and analogous to a resonant interaction. The geometric resonance due to the fact that the length scales involved are related by a fractal scaling by a power of 2 would make the form factors $F(p_1 \simeq 2^{k_1}, p_2 \simeq 2^{k_2}, M_n)$ large. The selection of primes near powers of two and Mersenne bosons would be analogous to evolutionary selection of a population consisting of species able to interact strongly.

Since $k = 113$ quarks are possible for $k = 107$ hadron physics, it seems that quarks can have join along boundaries bonds directed to M_n space-times with $n < k$. This suggests that neighboring Mersenne primes compete for join along boundaries bonds of quarks. For instance, when the p-adic length scale characterizing quark of M_{107} hadron physics begins to approach M_{89} quarks tend to feed their gauge flux to M_{89} space-time sheet and M_{89} hadron physics takes over and color coupling strength begins to increase. This would be the space-time correlate for the loss of asymptotic freedom.

2. Second picture

Preferred values of Planck constants could play a key role in the selection of Mersenne primes. Ruler-and-compass hypothesis predicts that Planck constants, which correspond to ratios of ruler and compass integers proportional to a product of distinct Fermat primes (four of them are known) and any power of two are favored. As a special case one obtains ruler and compass integers. As a consequence, p-adic length scales have satellites obtained by multiplying them with ruler-and-compass integers, and entire fractal hierarchy of power-of-two multiples of a given p-adic length scale results.

Mersenne length scales would be special since their satellites would form a subset of satellites of shorter Mersenne length scales. The copies of standard model physics associated with Mersenne primes would define a kind of resonating subset of physics since corresponding wavelengths and frequencies would coincide. This would also explain why fermions labelled by primes near power of two couple strongly with Mersenne primes.

5.2.7 The formula for the hadronic string tension

It is far from clear whether the strong gravitational coupling constant has same relation to the parameter $M_0^2 = 16m_0^2 = 1/\alpha' = 2\pi T$ as it would have in string model.

1. One could estimate the strong gravitational constant from the fundamental formula for the gravitational constant expressed in terms of exponent of Kähler action in the case that one has $\alpha_K = 1/4$. The formula reads as

$$\frac{L_p^2}{G_p} = exp(2aS_K(CP_2)) = exp(\pi/4\alpha_K) = e^{\pi} \quad . \tag{5.2.15}$$

   a is a parameter telling which fraction the action of wormhole contact is about the full action for CP_2 type vacuum extremal and $a \sim 1/2$ holds true. The presence of a can take care that the exponent is rational number. For $a = 1$ The number at the right hand side is Gelfond constant and one obtains

$$G_p = exp(-\pi) \times L_p^2 \quad . \tag{5.2.16}$$

2. One could relate the value of the strong gravitational constant to the parameter $M_0^2(k) = 16m(k)^2$, $p \simeq 2^k$ also assuming that string model formula generalizes as such. The basic formulas can be written in terms of gravitational constant G, string tension T, and $M_0^2(k)$ as

$$\frac{1}{8\pi G(k)} = \frac{1}{\alpha'} = 2\pi T(k) = \frac{1}{M_0^2(k)} = \frac{1}{16m(k)^2} \quad . \tag{5.2.16}$$

   This allows to express G in terms of the hadronic length scale $L(k) = 2\pi/m(k)$ as

$$G(k) = \frac{1}{16^2\pi^2}L(k)^2 \simeq 3.9 \times 10^{-4}L(k)^2 \quad . \tag{5.2.17}$$

   The value of gravitational coupling would be by two orders of magnitude smaller than for the first option.

5.3 Exotic particles predicted by TGD

Besides lepto-hadrons and M_{89} hadrons TGD suggests also other new physics effects such as higher generations for bosons and fractally scaled up versions of quarks. The basic challenge is to decide on experimental grounds whether partonic vertices correspond to fusions or branchings and the physics of $M\overline{M}$ systems allows to do this. More exotic effects are related to the new concept of space time: for example the concept of topological evaporation (formation of Baby Universies in elementary particle length scale) suggests an explanation for the Pomeron. Also exotic p-adic Super Virasoro representations for which the CP_2 mass scale is replaced effectively divided by a power of p can be considered as possible associated with non-perturbative aspects of hadronic physics.

5.3.1 Higher gauge boson families

TGD predicts that also gauge bosons, with gravitons included, should be characterized by family replication phenomenon but not quite in the expected manner. The first expectation was that these gauge bosons would have at least 3 light generations just like quarks and leptons.

Only within last two years it has become clear that there is a deep difference between fermions and gauge bosons. Elementary fermions and particles super-conformally related to elementary fermions correspond to single throat of a wormhole contact assignable to a topologically condensed CP_2 type vacuum extremal whereas gauge bosons would correspond to a wormhole throat pair assignable to wormhole contact connecting two space-time sheets. Wormhole throats correspond to light-like partonic 3-surfaces at which the signature of the induced metric changes.

In the case of 3 generations gauge bosons can be arranged to octet and singlet representations of a dynamical SU(3) and octet bosons for which wormhole throats have different genus could be massive and effectively absent from the spectrum.

Exotic gauge boson octet would induce particle reactions in which conserved handle number would be exchanged between incoming particles such that total handle number of boson would be difference of the handle numbers of positive and negative energy throat. These gauge bosons would induce flavor changing but genus conserving neutral current. There is no evidence for this kind of currents at low energies which suggests that octet mesons are heavy. Typical reaction would be $\mu + e \rightarrow e + \mu$ scattering by exchange of $\Delta g = 1$ exotic photon.

New view about interaction vertices and bosons

There are two options for the identification of particle vertices as topological vertices.

1. *Option a)*

The original assumption was that one can assign also to bosons a partonic 2-surface X^2 with more or less well defined genus g. The hypothesis is consistent with the view that particle reactions are described by smooth 4-surfaces with vertices being singular 3-surfaces intermediate between two three-topologies. The basic objection against this option is that it can induce too high rates for flavor changing currents. In particular $g > 0$ gluons could induce these currents. Second counter argument is that stable $n > 4$-particle vertices are not possible.

2. *Option b)*

According to the new vision (option 2)), particle decays correspond to branchings of the partonic 2-surfaces in the same sense as the vertices of the ordinary Feynman diagrams do correspond to branchings of lines. The basic mathematical justification for this vision is the enormous simplification caused by the fact that vertices correspond to non-singular 2-manifolds. This option allows also $n > 3$-vertices as stable vertices.

A consistency with the experimental facts is achieved if the observed gauge bosons have each value of $g(X^2)$ with the same probability. Hence the general boson state would correspond to a phase $exp(in2\pi g/3)$, $n = 0, 1, 2$, in the discrete space of 3 lowest topologies $g = 0, 1, 2$. The observed bosons would correspond to $n = 0$ state and exotic higher states to $n = 1, 2$.

The nice feature of this option is that no flavor changing neutral electro-weak or color currents are predicted. This conforms with the fact that CKM mixing can be understood as electro-weak phenomenon described most naturally by causal determinants X_l^3 (appearing as lines of generalized Feynman diagram) connecting fermionic 2-surfaces of different genus.

Consider now objections against this scenario.

1. Since the modular contribution does not depend on the gradient of the elementary particle vacuum functional but only on its logarithm, all three boson states should have mass squared which is the average of the mass squared values $M^2(g)$ associated with three generations. The fact that modular contribution to the mass squared is due to the super-canonical thermodynamics allows to circumvent this objection. If the super-canonical p-adic temperature is small, say $T_p = 1/2$, then the modular contribution to the mass squared is completely negligible also for $g > 0$ and photon, graviton, and gluons could remain massless. The wiggling of the elementary particle vacuum functionals at the boundaries of the moduli spaces $\mathcal{M}_g$ corresponding to 2-surfaces intermediate between different 2-topologies (say pinched torus and self-touching sphere) caused by the change of overall phase might relate to the higher p-adic temperature T_p for exotic bosons.

2. If photon states had a 3-fold degeneracy, the energy density of black body radiation would be three times higher than it is. This problem is avoided if the the super-canonical temperature for $n = 1, 2$ states is higher than for $n = 0$ states, and same as for fermions, say $T_p = 1$. In this case two mass degenerate bosons would be predicted with mass squared being the average over the three genera. In this kind of situation the factor $1/3$ could make the real mass squared very large, or order CP_2 mass squared, unless the sum of the modular contributions to the mass squared values $M^2_{mod}(g) \propto n(g)$ is divisible by 3. This would make also photon, graviton, and gluons massive. Fortunately, $n(g)$ is divisible by 3 as is clear form $n(0) = 0$, $n(1) = 9$, $n(2) = 60$.

Masses of genus-octet bosons

For option 1) ordinary bosons are accompanied by $g > 0$ massive partners. For option 2) both ordinary gauge bosons and their exotic partners have suffered maximal topological mixing in the case that they are singlets with respect to the dynamical SU(3). There are good reasons to expect that Higgs mechanism for ordinary gauge bosons generalizes as such and that $1/T_p > 1$ means that the contribution of p-adic thermodynamics to the mass is negligible. The scale of Higgs boson expectation would be given by p-adic length scale and mass degeneracy of octet is expected. A good guess is obtained by scaling the masses of electro-weak bosons by the factor $2^{(k-89)/2}$. Also the masses of genus-octet of gluons and photon should be non-vanishing and induced by a vacuum expectation of Higgs particle which is electro-weak singlet but genus-octet.

5.3.2 The physics of $M - \overline{M}$ systems forces the identification of vertices as branchings of partonic 2-surfaces

For option 2) gluons are superpositions of $g = 0, 1, 2$ states with identical probabilities and vertices correspond to branchings of partonic 2-surfaces. Exotic gluons do not induce mixing of quark families and genus changing transitions correspond to light like 3-surfaces connecting partonic 2-surfaces with different genera. CKM mixing is induced by this topological mixing. The basic testable predictions relate to the physics of $M\overline{M}$ systems and are due to the contribution of exotic gluons and large direct CP breaking effects in $K - \overline{K}$ favor this option.

For option 1) vertices correspond to fusions rather than branchings of the partonic 2-surfaces. The prediction that quarks can exchange handle number by exchanging $g > 0$ gluons (to be denoted by G_g in the sequel) could be in conflict with the experimental facts.

1. CP breaking in $K - \bar{K}$ as a basic test

CP breaking physics in kaon-antikaon and other neutral pseudoscalar meson systems is very sensitive to the new physics. What makes the situation especially interesting, is the recently reported high precision value for the parameter ϵ'/ϵ describing direct CP breaking in kaon-antikaon system [105]. The value is almost by an order of magnitude larger than the standard model expectation. $K - \bar{K}$ mass difference predicted by perturbative standard model is 30 per cent smaller than the the experimental value and one cannot exclude the possibility that new physics instead of/besides non-perturbative QCD might be involved.

In standard model the low energy effective action is determined by box and penguin diagrams. $\Delta S = 2$ piece of the effective weak Lagrangian, which describes processes like $s\bar{d} \rightarrow d\bar{s}$, determines the value of the $K - \bar{K}$ mass difference Δm_K and since this piece determines $K \rightarrow \bar{K}$ amplitude it also contributes to the parameter ϵ characterizing indirect CP breaking. $\Delta S = 2$ part of the weak effective action corresponds to box diagrams involving two W boson exchanges.

2. Δm_K kills option a

For option 1) box diagrams involving Z and $g > 0$ exchanges are allowed provided exchanges correspond to exchange of both Z and $g > 0$ gluon. The most obvious objection is that the exchanges of $g > 0$ gluons make strong $\Delta S > 0$ decays of mesons possible: $K_S \rightarrow \pi\pi$ is a good example of this kind of decay. The enhancement of the decay rate would be of order $(\alpha_s(g = 1)/\alpha_{em})^2(m_W/m_G(g = 1)^2 \sim 10^3$. Also other $\Delta S = 1$ decay rates would be enhanced by this factor. The real killer prediction is a gigantic value of Δm_K for kaon-antikaon system resulting from the possibility of $\overline{s}d \rightarrow \overline{d}s$ decay by single $g = 1$ gluon exchange. This prediction alone excludes option 1).

3. Option 2) could explain direct CP breaking

For option 2) box diagrams are not affected in the lowest order by exotic gluons. The standard model contributions to Δm_K and indirect CP breaking are correct for the observed value of the top quark mass which results if top corresponds to a secondary padic length scale $L(2, k)$ associated with $k = 47$ (Appendix). Higher order gluonic contribution could increase the value of Δm_K predicted to be about 30 per cent too small by the standard model.

In standard model penguin diagrams contribute to $\Delta S = 1$ piece of the weak Lagrangian, which determines the direct CP breaking characterized by the parameter ϵ'/ϵ. Penguin diagrams, which describe processes like $s\bar{d} \rightarrow d\bar{d}$, are characterized by effective vertices dsB, where B denotes photon, gluon or Z boson. dsB vertices give the dominant contribution to direct CP breaking in standard model. The new penguin diagrams are obtained from ordinary penguin diagrams by replacing ordinary gluons with exotic gluons.

For option 2) the contributions predicted by the standard model are multiplied by a factor 3 in the approximation that exotic gluon mass is negligible in the mass scale of intermediate gauge boson. These diagrams affect the value of the parameter ϵ'/ϵ characterizing direct CP breaking in $K - \bar{K}$ system found experimentally to be almost order of magnitude larger than standard model expectation [105].

5.3.3 Super-canonical bosons

TGD predicts also exotic bosons which are analogous to fermion in the sense that they correspond to single wormhole throat associated with CP_2 type vacuum extremal whereas ordinary gauge bosons corresponds to a pair of wormhole contacts assignable to wormhole contact connecting positive and negative energy space-time sheets. These bosons have super-conformal partners with quantum numbers of right handed neutrino and thus having no electro-weak couplings. The bosons are created by the purely bosonic part of super-canonical algebra [B2, B3, B4], whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. Recall that super-canonical algebra is crucial for the construction of configuration space Kähler geometry. If one assumes that super-canonical gluons suffer topological mixing identical with that suffered by say U type quarks, the conformal weights would be (5,6,58) for the three lowest generations. The application of super-canonical bosons in TGD based model of hadron masses is discussed in [F4] and here only a brief summary is given.

As explained in [F4], the assignment of these bosons to hadronic space-time sheet is an attractive idea.

1. Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD. A possible identification of this contribution is in terms of super-canonical gluons. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-canonical gluons with net conformal weight of 16 units. This leads

to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent.

2. Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-canonical quanta. If the topological mixing for super-canonical bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-canonical quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight. A very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-canonical particle content.

3. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.

4. Super-canonical bosons suggest a solution to several other anomalies related to hadron physics. The events observed for a couple of years ago in RHIC [119] suggest a creation of a black-hole like state in the collision of heavy nuclei and inspire the notion of color glass condensate of gluons, whose natural identification in TGD framework would be in terms of a fusion of hadronic space-time sheets containing super-canonical matter materialized also from the collision energy. In the collision, valence quarks connected together by color bonds to form separate units would evaporate from their hadronic space-time sheets in the collision, and would define TGD counterpart of Pomeron, which experienced a reincarnation for few years ago [71]. The strange features of the events related to the collisions of high energy cosmic rays with hadrons of atmosphere (the particles in question are hadron like but the penetration length is anomalously long and the rate for the production of hadrons increases as one approaches surface of Earth) could be also understood in terms of the same general mechanism.

5.3.4 A new twist in the spin puzzle of proton

The so called proton spin crisis or spin puzzle of proton was an outcome of the experimental finding that the quarks contribute only 13-17 per cent of proton spin [113, 114] whereas the simplest valence quark model predicts that quarks contribute about 75 per cent to the spin of proton with the remaining 25 per cent being due to the orbital motion of quarks. Besides the orbital motion of valence quarks also gluons could contribute to the spin of proton. Also polarized sea quarks can be considered as a source of proton spin.

Quite recently, the spin crisis got a new twist [115]. One of the few absolute predictions of perturbative QCD (pQCD) is that at the limit, when the momentum fraction of quark approaches unity, quark spin should be parallel to the proton spin. This is due to the helicity conservation predicted by pQCD in the lowest order. The findings are consistent with this expectation in the case of protonic u quarks but not in the case of protonic d quark. The discovery is of a special interest from the point of view of TGD since it might have an explanation involving the notions of many-sheeted space-time, of color-magnetic flux tubes, the predicted super-canonical "vacuum" spin, and also the concept of quantum parallel dissipation.

The experimental findings

In the experiment performed in Jefferson Lab [115] neutron spin asymmetries A_1^n and polarized structure functions $g_{1,2}^n$ were deduced for three kinematic configurations in the deep inelastic region from e-^{3}He scattering using 5.7 GeV longitudinally polarized electron beam and a polarized ^3He target. A_1^n and $g_{1,2}^n$ were deduced for $x = .33, .47$, and $.60$ and $Q^2 = 2.7, 3.5$ and 4.8 (GeV/c)^2. A_1^n and g_1^n at $x = .33$ are consistent with the world data. At $x = .47$ A_1^n crosses zero and is significantly positive at $x = 0.60$. This finding agrees with the next-to-leading order QCD analysis of previous world data without the helicity conservation constraint. The trend of the data agrees with the predictions of the constituent quark model but disagrees with the leading order pQCD assuming hadron helicity conservation.

By isospin symmetry one can translate the result to the case of proton by the replacement $u \leftrightarrow d$. By using world proton data, the polarized quark distribution functions were deduced for proton using isospin symmetry between neuron and proton. It was found that $\Delta u/u$ agrees with the predictions of various models while $\Delta d/d$ disagrees with the leading-order pQCD.

Let us denote by $q(x) = q^{\uparrow} + q^{\downarrow}(x)$ the spin independent quark distribution function. The difference $\Delta q(x) = q^{\uparrow} - q^{\downarrow}(x)$ measures the contribution of quark q to the spin of hadron. The measurement allowed to deduce estimates for the ratios $(\Delta q(x) + \Delta\overline{q}(x))/(q(x) + \overline{q}(x))$.

The conclusion of [115] is that for proton one has

$$\frac{\Delta u(x) + \Delta\overline{u}(x)}{u(x) + \overline{u}(x)} \simeq .737 \pm .007 \ \ , \ \ \text{for} \ \ x = .6 \ \ .$$

This is consistent with the pQCD prediction. For d quark the experiment gives

$$\frac{\Delta d(x) + \Delta\overline{d}(x)}{d(x) + \overline{d}(x)} \simeq -.324 \pm .083 \ \ \text{for} \ \ x = .6 \ \ .$$

The interpretation is that d quark with momentum fraction $x > .6$ in proton spends a considerable fraction of time in a state in which its spin is opposite to the spin of proton so that the helicity conservation predicted by first order pQCD fails. This prediction is of special importance as one of the few absolute predictions of pQCD.

The finding is consistent with the relativistic $SU(6)$ symmetry broken by spin-spin interaction and the QCD based model interpolated from data but giving up helicity conservation [115]. $SU(6)$ is however not a fundamental symmetry so that its success is probably accidental.

It has been also proposed that the spin crisis might be illusory [116] and due to the fact that the vector sum of quark spins is not a Lorentz invariant quantity so that the sum of quark spins in infinite-momentum frame where quark distribution functions are defined is not same as, and could thus be smaller than, the spin sum in the rest frame. The correction due to the transverse momentum of the quark brings in a non-negative numerical correction factor which is in the range $(0, 1)$. The negative sign of $\Delta d/d$ is not consistent with this proposal.

TGD based model for the findings

The TGD based explanation for the finding involves the following elements.

1. TGD predicts the possibility of vacuum spin due to the super-canonical symmetry. Valence quarks can be modelled as a star like formation of magnetic flux tubes emanating from a vertex with the conservation of color magnetic flux forcing the valence quarks to form a single coherent structure. A good guess is that the super-canonical spin corresponds classically to the rotation of the the star like structure.

2. By parity conservation only even values of super-canonical spin J are allowed and the simplest assumption is that the valence quark state is a superposition of ordinary $J = 0$ states predicted by pQCD and $J = 2$ state in which all quarks have spin which is in a direction opposite to the direction of the proton spin. The state of $J = 1/2$ baryon is thus replaced by a new one:

$$\begin{aligned}
|B, \frac{1}{2}, \uparrow\rangle &= a|B, 1/2, \frac{1}{2}\rangle|J = J_z = 0\rangle + b|B, \frac{3}{2}, -\frac{3}{2}\rangle|J = J_z = 2\rangle \ \ , \\
|B, 1/2, \frac{1}{2}\rangle &= \sum_{q_1,q_2,q_3} c_{q_1,q_2,q_3} q_1^{\uparrow} q_2^{\uparrow} q_3^{\downarrow} \ \ , \\
|B, \frac{3}{2}, -\frac{3}{2}\rangle &= d_{q_1,q_2,q_3} q_1^{\downarrow} q_2^{\downarrow} q_3^{\downarrow} \ \ . \qquad (5.3.-1)
\end{aligned}$$

$|B, 1/2, \frac{1}{2}\rangle$ is in a good approximation the baryon state as predicted by pQCD. The coefficients c_{q_1,q_2,q_3} and d_{q_1,q_2,q_3} depend on momentum fractions of quarks and the states are normalized so that $|a|^2 + |b|^2 = 1$ is satisfied: the notation $p = |a|^2$ will be used in the sequel. The quark parts of $J = 0$ and $J = 2$ have quantum numbers of proton and Δ resonance. $J = 2$ part need not however have the quark distribution functions of Δ.

3. The introduction of $J=0$ and $J=2$ ground states with a simultaneous use of quark distribution functions makes sense if one allows quantum parallel dissipation. Although the system is coherent in the super-canonical degrees of freedom which correspond to the hadron size scale, there is a de-coherence in quark degrees of freedom which correspond to a shorter p-adic length scale and smaller space-time sheets.

4. Consider now the detailed structure of the $J=2$ state in the case of proton. If the d quark is at the rotation axis, the rotating part of the triangular flux tube structure resembles a string containing u-quarks at its ends and forming a di-quark like structure. Di-quark structure is taken to mean correlations between u-quarks in the sense that they have nearly the same value of x so that $x<1/2$ holds true for them whereas the d-quark behaving more like a free quark can have $x>1/2$.

   A stronger assumption is that di-quark behaves like a single colored hadron with a small value of x and only the d-quark behaves as a free quark able to have large values of x. Certainly this would be achieved if u quarks reside at their own string like space-time sheet having $J=2$.

From these assumptions it follows that if u quark has $x>1/2$, the state effectively reduces to a state predicted by pQCD and $u(x)\rightarrow 1$ for $x\rightarrow 1$ is predicted. For the d quark the situation is different and introducing distribution functions $q^{J)}(x)$ for $J=0,2$ separately, one can write the spin asymmetry at the limit $x\rightarrow 1$ as

$$\begin{aligned} A_d &\equiv \frac{\Delta d(x)+\Delta\overline{d}(x)}{d(x)+\overline{d}(x)} = \frac{p(\Delta d_0+\Delta\overline{d}_0)+(1-p)(\Delta d_2+\Delta\overline{d}_2)}{p(d_0+\overline{d}_0)+(1-p)(d_2+\overline{d}_2)} \ , \\ p &= |a|^2 \ . \end{aligned} \tag{5.3.-1}$$

Helicity conservation gives $\Delta d_0/d_0\rightarrow 1$ at the limit $x\rightarrow 1$ and one has trivially $\Delta d_2/d_2=-1$. Taking the ratio

$$y=\frac{d_2}{d_0}$$

as a parameter, one can write

$$A_d \rightarrow \frac{p-(1-p)y}{p+(1-p)y} \tag{5.3.0}$$

at the limit $x\rightarrow 1$. This allows to deduce the value of the parameter y once the value of p is known:

$$y = \frac{p}{1-p}\times\frac{1-A_d}{1+A_d} \ . \tag{5.3.1}$$

From the requirement that quarks contribute a fraction $\Sigma=\sum_q \Delta q\in(13,17)$ per cent to proton spin, one can deduce the value of p using

$$\frac{p\times\frac{1}{2}-(1-p)\times\frac{3}{2}}{\frac{1}{2}} = \Sigma \tag{5.3.2}$$

giving $p=(3+\Sigma)/4\simeq .75$.

Eq. 5.3.1 allows estimate the value of y. In the range $\Sigma\in(.13,.30)$ defined by the lower and upper bounds for the contribution of quarks to the proton spin, $A_d=-.32$ gives $y\in(6.98,9.15)$. $d_2(x)$ would be more strongly concentrated at high values of x than $d_0(x)$. This conforms with the assumption that u quarks tend to carry a small fraction of proton momentum in $J=2$ state for which uu can be regarded as a string like di-quark state.

A further input to the model comes from the ratio of neutron and proton F_2 structure functions expressible in terms of quark distribution functions of proton as

$$R^{np} \equiv \frac{F_2^n}{F_2p} = \frac{u(x)+4d(x)}{4u(x)+d(x)} \quad . \tag{5.3.3}$$

According to [115] $R^{np}(x)$ is a straight line starting with $R^{np}(x \rightarrow 0) \simeq 1$ and dropping below 1/2 as $x \rightarrow 1$. The behavior for small x can be understood in terms of sea quark dominance. The pQCD prediction for R^{np} is $R^{np} \rightarrow 3/7$ for $x \rightarrow 1$, which corresponds to $d/u \rightarrow z = 1/5$. TGD prediction for R^{np} for $x \rightarrow 1$

$$\begin{aligned} R^{np} &\equiv \frac{F_2^n}{F_2^p} = \frac{pu_0+4(pd_0+(1-p)d_2)}{4pu_0+pd_0+(1-p)d_2} \\ &= \frac{p+4z(p+(1-p)y}{4p+z(p+(1-p)y)} \quad . \end{aligned} \tag{5.3.3}$$

In the range $\Sigma \in (.13, .30)$ which corresponds to $y \in (6.98, 9.15)$ for $A_d = -.32$ $R^{np} = 1/2$ gives $z \simeq .1$, which is 20 per cent of pQCD prediction. 80 percent of d-quarks with large x predicted to be in $J=0$ state by pQCD would be in $J=2$ state.

5.3.5 Fractally scaled up versions of quarks

The strange anomalies of neutrino oscillations [44] suggesting that neutrino mass scale depends on environment can be understood if neutrinos can suffer topological condensation in several p-adic length scales [F3]. The obvious question whether this could occur also in the case of quarks led to a very fruitful developments leading to the understanding of hadronic mass spectrum in terms of scaled up variants of quarks. Also the mass distribution of top quark candidate exhibits structure which could be interpreted in terms of heavy variants of light quarks. The ALEPH anomaly [49], which I first erratically explained in terms of a light top quark has a nice explanation in terms of b quark condensed at $k = 97$ level and having mass ~ 55 GeV. These points are discussed in detail in [F4].

The emergence of ALEPH results [49] meant a an important twist in the development of ideas related to the identification of top quark. In the LEP 1.5 run with $E_{cm} = 130-140\ GeV$, ALEPH found 14 e^+e^- annihilation events, which pass their 4-jet criteria whereas 7.1 events are expected from standard model physics. Pairs of dijets with vanishing mass difference are in question and dijets could result from the decay of a new particle with mass about 55 GeV.

The data do not allow to conclude whether the new particle candidate is a fermion or boson. Top quark pairs produced in e^+e^- annihilation could produce 4-jets via gluon emission but this mechanism does not lead to an enhancement of 4-jet fraction. No $b\bar{b}b\bar{b}$ jets have been observed and only one event containing b has been identified so that the interpretation in terms of top quark is not possible unless there exists some new decay channel, which dominates in decays and leads to hadronic jets not initiated by b quarks. For option 2), which seems to be the only sensible option, this kind of decay channels are absent.

Super symmetrized standard model suggests the interpretation in terms of super partners of quarks or/and gauge bosons [79]. It seems now safe to conclude that TGD does not predict sparticles. If the exotic particles are gluons their presence does not affect Z^0 and W decay widths. If the condensation level of gluons is $k = 97$ and mixing is absent the gluon masses are given by $m_g(0) = 0$, $m_g(1) = 19.2\ GeV$ and $m_g(2) = 49.5\ GeV$ for option 1) and assuming $k = 97$ and hadronic mass renormalization. It is however very difficult to understand how a pair of $g = 2$ gluons could be created in e^+e^- annihilation. Moreover, for option 2), which seems to be the only sensible option, the gluon masses are $m_g(0) = 0$, $m_g(1) = m_g(2) = 30.6\ GeV$ for $k = 97$. In this case also other values of k are possible since strong decays of quarks are not possible.

The strong variations in the order of magnitude of mass squared differences between neutrino families [44] can be understood if they can suffer a topological condensation in several p-adic length scales. One can ask whether also t and b quark could do the same. In absence of mixing effects the masses of $k = 97$ t and b quarks would be given by $m_t \simeq 48.7$ GeV and $m_b \simeq 52.3$ GeV taking into account the hadronic mass renormalization. Topological mixing reduces the masses somewhat. The fact that b quarks are not observed in the final state leaves only $b(97)$ as a realistic option. Since Z^0 boson mass is ~ 94 GeV, $b(97)$ does not appreciably affect Z^0 boson decay width. The observed anomalies concentrate at cm energy about 105 GeV. This energy is 15 percent smaller than the total mass of top pair. The discrepancy could be understood as resulting from the binding energy of the $b(97)\bar{b}(97)$ bound states. Binding energy should be a fraction of order $\alpha_s \simeq .1$ of the total energy and about ten per cent so that consistency is achieved.

5.3.6 What M_{89} Hadron Physics would look like?

TGD suggests the existence of the scaled up copies of hadron physics corresponding to the Mersenne primes $M_n = 89, 61, 31, ..$ at least in the sense that α_s has maximum at these length scales. The assumption of QCD:s decoupling completely from each other seems more unrealistic.

The requirement of unitarity forces the existence of Higgs particle in gauge theories. The failure of the p-adic mass calculations to predict intermediate gauge boson masses correctly forces to give up the idea that boson masses are of purely thermodynamical origin. A possible TGD counterpart for the Higgs fields is as the fields defined by the Kac-Moody generators associated with the complement of the $u(2)$ algebra of $su(3)$ associated with the conserved charges Q_J defined by the variation of the modified Dirac action with respect to the induced Kähler form. As found in the [F3], the small coupling of the Higgs to fermionic masses resolves the paradoxical situation created by the failure to detect Higgs boson. Also the fact that left handed electro-weak charge matrices are not covariantly constant could explain Higgs vacuum expectation value without an introduction of an elementary scalar field.

One could of course, consider also other explanations for Higgs. The only scalar mesons with masses in intermediate boson mass scale allowed by TGD are bound states of quark and antiquark of M_{89} hadron physics such that quark and antiquark have parallel spins and relative angular momentum $L = 1$. The effective couplings of these states to leptons and quarks could mimic the couplings of Higgs boson to some degree. Scalar bound states of heavy quarks are also present in ordinary hadron physics. The coherent states formed by these particles could mimic the effects caused by a fundamental Higgs field.

M_{89} would be obtained in the first approximation by scaling the ordinary hadron physics by the ratio $\sqrt{\frac{M_{89}}{M_{107}}}$. This implies that QCD Λ, string tension, etc. get scaled by the appropriate power of this factor. If one estimates the u_{89} mass as $m(u_{89}) = m(\rho_{89})/2$ one obtains the TGD prediction for its mass as $m(u_{89}) = 512m(\rho_{107})/2 \simeq 197\ GeV$. Defining $u(89)$ mass by scaling the mass of ordinary u quark defined as one third of proton mass one obtains u_{89} mass about 160 GeV. This estimate for u_{89} mass happened to be within experimental uncertainties equal to the mass of the top candidate discovered just when the mass calculations were carried out and led to a tentative identification of the top candidate as u_{89}.

The fact that top candidate turned out to have production and decay characteristics of the real top forced to give up this hypothesis. Also the study of CKM matrix led to the cautious conclusion that only the mass of the experimental top candidate is consistent with CP breaking observed in $K - \bar{K}$ and $B - \bar{B}$ system (Appendix). Even more, the direct calculation of the u_{89} mass from p-adic thermodynamics gives $m(u_{89}) \simeq 262$ GeV and demonstrates the the idea about identifying top quark as u_{89} quark was a result of sloppy order of magnitude thinking. The relatively high mass however leaves open the possibility that M_{89} physics exists.

M_{89} physics means the emergence of a new condensate level in the hadronic physics. One can visualize M_{89} hadrons as very tiny objects possibly condensed on the quarks and gluons of M_{107} hadron physics. The New Physics begins to reveal itself, when the collision energy is so high that M_{89} hadrons inside quarks and gluons can exist as on mass shell particles (M_{89} hadron inside M_{107} hadron is comparable to a bee of size of one cm in a room of size about 5 meters!).

The new Physics at the energies not much above the energy scale of top is essentially the counterpart of ordinary hadron physics at cm energies of the order of ρ/ω meson mass. Therefore M_{89} meson resonances and their interactions described rather satisfactorily by the old fashioned string model with string tension scaled by factor 2^{18} should describe the situation. The electro-weak interactions should be in turn describable using generalization of current algebra ideas, such as PCAC and vector dominance model. If M_{89} hadrons condense on quarks and gluons this physics must be convoluted with the distribution functions of M_{89} hadrons inside quarks and gluons. The resonance structures are partially smeared out by the convolution process.

M_{89} vector mesons should be observed as resonances in e^+e^- annihilation and charged M_{89} pion should be pair produced at e^+e^- collision energies achievable in near future at LEP. Gamma pairs form unique signature of neutral leptopion. The following table gives the naive scaling estimate for the masses of lowest lying M_{89} hadrons.

meson	m/GeV	baryon	m/GeV
π^0	69.1	p	480.4
π^+	71.5	n	481.0
K^+	252.8	Λ	571.2
K^0	254.8	Σ^+	609.0
η	281.0	Σ^0	610.4
η'	490.5	Σ^-	610.5
ρ	394.2	Ξ^0	673.2
ω	400.9	Ξ^-	676.5
K^*	456.7	Ω^-	856.2
Φ	522		

Table 3. Masses of low lying hadrons for M_{89} hadron physics obtained by scaling ordinary hadron masses by a factor of 512.

Consider next the estimation of the production and decay rates for $\rho(89)$ /$\omega(89)$ and more generally M_{89} mesons. In e^+e^- annihilation vector boson resonances are produced via the decay of virtual photon or Z^0. Since low energies are in question at M_{89} level the scaled up version of vector dominance model described in the nice book of Feynman [36] should give a satisfactory description for the production of M_{89} mesons via resonance mechanism. The idea is to introduce direct coupling $F_V = m_V^2/g_V$ of photon (or gauge boson) to vector boson (ρ, ω, ϕ). The diagrams describing the production of mesons via decay of vector boson contain vector boson propagator $\frac{1}{p^2-m_V^2+im_V\Delta}$ and the production rate is enhanced by a factor $R = 4\pi m_V^2/(\Delta^2 g_V^2)$ in the resonance: the factor should be same in M_{89} physics as in ordinary hadron physics. The ratio $r = \alpha_{em}R/\alpha_s$ gives a rough measure for the ratio of the rates of production for $u(89)$ and ordinary top quark. A rough estimate for what is to be expected is obtained by scaling the results of ordinary hadron physics. The table below gives the estimates for the quantity r and one has $r = 15.1$ for ω.

meson	$m/512\ MeV$	$\Delta/512\ MeV$	$g_V^2/4\pi$	r
ρ	770	150	2.27	0.52
ω	783	10	18.3	15.1
Φ	1019	4.2	13.3	230.8

Table 4. Scaled up resonance production parameters for ρ, ω and Φ. The last column of the table gives the value of the quantity $r = \alpha_{em}R/\alpha_s$, which should give a measure for the ratio of production rate of $u({}_89)$ and of the production of ordinary top quark pair.

Centauro type events [97] might find nice explanation in terms of M_{89} hadron physics. If electro-weak decay channels dominate over hadronic decay channels for M_{89} mesons this might lead to anomalously small abundance of ordinary pions in Centauro events. In particular, neutral M_{89} pions are expected to decay dominantly to photon pairs and since monoenergetic gamma pairs are used as a signature of pions the observed abundance of ordinary pions becomes small. Evidence for M_{89} pions comes from anomalous gamma pairs detected in the decays of Z^0 bosons[106] with total energy of about 60 GeV. The pairs might be related to the decay of M_{89} exotic pion predicted to have mass $m_{89} \simeq 2^9 m_\pi \simeq 67.5\ GeV$.

The resonance production of M_{89} vector mesons via the graph $q\bar{q} \rightarrow \gamma(virt) \rightarrow M(M_{89})$ and their decay to dijets gives small contribution to dijet production rate.

At high enough cm energies, presumably of order $\sqrt{s} \sim 10\ TeV$ in $p\bar{p}$ collisions the jets of M_{89} hadron physics should begin to manifest themselves. The unique signature of M_{89} jets is that the p_T spectrum for the hadrons of the jet, which is of form $exp(-kp_T^2/\Lambda(89))$, is by factor 512 wider than the p_T spectrum of hadrons for ordinary jets.

Following list gives some of the unique signatures of New Physics.

1. At higher energies exotic pions are produced abundantly and might be detectable via annihilation

to monoenergetic photon pair. π^0 of the New Physics should have mass 69.1 GeV and $\gamma\gamma$ annihilation width $512 \cdot 7.63\ eV = 3.9\ MeV$ (obtained by scaling from that for ordinary pion). The width for the decay by W emission from either quark of $\pi^0(89)$ (the second is assumed to act as spectator) is of order $G_F^2 m(u(89))^5/(192\pi^3)$ and of order 2.5 MeV.

2. The scaling of mass splittings inside isopin multiplets with the scale factor 512 as compared to ordinary hadron physics is a unique signature of M_{89} hadrons.

3. The scaled up versions of ρ and ω meson should be found at nearby energies. Kaon (and s quark) of the New Physics should be seen as a decay product of $\Phi(522\ GeV) \rightarrow K + \bar{K}$: from table 5.3.6 one finds that that Φ should have rather small hadronic width $\Delta \simeq 2.2\ GeV$ so that the parameter measuring its production rate to the production rate of ordinary quark is as high as $r \simeq 230.8$ at resonance.

5.3.7 Topological evaporation and the concept of Pomeron

Topological evaporation provides an explanation for the mysterious concept of Pomeron originally introduced to describe hadronic diffractive scattering as the exchange of Pomeron Regge trajectory [70]. No hadrons belonging to Pomeron trajectory were however found and via the advent of QCD Pomeron was almost forgotten. Pomeron has recently experienced reincarnation [71, 72, 74]. In Hera [71] $e-p$ collisions, where proton scatters essentially elastically whereas jets in the direction of incoming virtual photon emitted by electron are observed. These events can be understood by assuming that proton emits color singlet particle carrying small fraction of proton's momentum. This particle in turn collides with virtual photon (antiproton) whereas proton scatters essentially elastically.

The identification of the color singlet particle as Pomeron looks natural since Pomeron emission describes nicely diffractive scattering of hadrons. Analogous hard diffractive scattering events in pX diffractive scattering with $X = \bar{p}$ [72] or $X = p$ [74] have also been observed. What happens is that proton scatters essentially elastically and emitted Pomeron collides with X and suffers hard scattering so that large rapidity gap jets in the direction of X are observed. These results suggest that Pomeron is real and consists of ordinary partons.

TGD framework leads to two alternative identifications of Pomeron relying on same geometric picture in which Pomeron corresponds to a space-time sheet separating from hadronic space-time sheet and colliding with photon.

Earlier model

The earlier model is based on the assumption that baryonic quarks carry the entire four-momentum of baryon. p-Adic mass calculations have shown that this assumption is wrong. The modification of the model requires however to change only wordings so that I will represent the earlier model first.

The TGD based identification of Pomeron is very economical: Pomeron corresponds to sea partons, when valence quarks are in vapor phase. In TGD inspired phenomenology events involving Pomeron correspond to pX collisions, where incoming X collides with proton, when valence quarks have suffered coherent simultaneous (by color confinement) evaporation into vapor phase. System X sees only the sea left behind in evaporation and scatters from it whereas valence quarks continue without noticing X and condense later to form quasi-elastically scattered proton. If X suffers hard scattering from the sea the peculiar hard diffractive scattering events are observed. The fraction of these events is equal to the fraction f of time spent by valence quarks in vapor phase.

Dimensional argument can be used to derive a rough order of magnitude estimate for f as $f \sim 1/\alpha = 1/137 \sim 10^{-2}$ for f: f is of same order of magnitude as the fraction (about 5 per cent) of peculiar events from all deep inelastic scattering events in Hera. The time spent in condensate is by dimensional arguments of the order of the p-adic legth scale $L(M_{107})$, not far from proton Compton length. Time dilation effects at high collision energies guarantee that valence quarks indeed stay in vapor phase during the collision. The identification of Pomeron as sea explains also why Pomeron Regge trajectory does not correspond to actual on mass shell particles.

The existing detailed knowledge about the properties of sea structure functions provides a stringent test for the TGD scenario. According to [72] Pomeron structure function seems to consist of soft $((1-x)^5\ )$, hard $((1-x)\ )$ and super-hard component (delta function like component at $x=1$).

The peculiar super hard component finds explanation in TGD based picture. The structure function $q_P(x,z)$ of parton in Pomeron contains the longitudinal momentum fraction z of the Pomeron as a parameter and $q_P(x,z)$ is obtained by scaling from the sea structure function $q(x)$ for proton $q_P(x,z) = q(zx)$. The value of structure function at $x = 1$ is non-vanishing: $q_P(x = 1, z) = q(z)$ and this explains the necessity to introduce super hard delta function component in the fit of [72].

Updated model

The recent developments in the understanding of hadron mass spectrum involve the realization that hadronic $k = 107$ space-time sheet is a carrier of super-canonical bosons (and possibly their super-counterparts with quantum numbers of right handed neutrino) [F4]. The model leads to amazingly simple and accurate mass formulas for hadrons. Most of the baryonic momentum is carried by super-canonical quanta: valence quarks correspond in proton to a relatively small fraction of total mass: about 170 MeV. The counterparts of string excitations correspond to super-canonical many-particle states and the additivity of conformal weight proportional to mass squared implies stringy mass formula and generalization of Regge trajectory picture. Hadronic string tension is predicted correctly. Model also provides a solution to the proton spin puzzle.

In this framework valence quarks would naturally correspond to a color singlet state formed by space-time sheets connected by color flux tubes having no Regge trajectories and carrying a relatively small fraction of baryonic momentum. In the collisions discussed valence quarks would leave the hadronic space-time sheet and suffer a collision with photon. The lightness of Pomeron and and electro-weak neutrality of Pomeron support the view that photon stripes valence quarks from Pomeron, which continues its flight more or less unperturbed. Instead of an actual topological evaporation the bonds connecting valence quarks to the hadronic space-time sheet could be stretched during the collision with photon.

The large value of $\alpha_K = 1/4$ for super-canonical matter suggests that the criterion for a phase transition increasing the value of Planck constant [A9] and leading to a phase, where $\alpha_K \propto 1/hbar$ is reduced, could occur. For α_K to remain invariant, $\hbar_0 \rightarrow 26\hbar_0$ would be required. In this case, the size of hadronic space-time sheet, "color field body of the hadron", would be $26 \times L(107) = 46$ fm, roughly the size of the heaviest nuclei. Hence a natural expectation is that the dark side of nuclei plays a role in the formation of atomic nuclei. Note that the sizes of electromagnetic field bodies of current quarks u and d with masses of order few MeV is not much smaller than the Compton length of electron. This would mean that super-canonical bosons would represent dark matter in a well-defined sense and Pomeron exchange would represent temporary separation of ordinary and dark matter.

Note however that the fact that super-canonical bosons have no electro-weak interactions, implies their dark matter character even for the ordinary value of Planck constant: this could be taken as an objection against dark matter hierarchy. My own interpretation is that super-canonical matter is dark matter in the strongest sense of the world whereas ordinary matter in the large hbar phase is only apparently dark matter because standard interactions do not reveal themselves in the expected manner.

Astrophysical counterpart of Pomeron events

Pomeron events have direct analogy in astrophysical length scales. In the collision of two galaxies dark and visible matter parts of the colliding galaxies have been found to separate by Chandra X-ray Observatory [117].

Imagine a collision between two galaxies. The ordinary matter in them collides and gets interlocked due to the mutual gravitational attraction. Dark matter, however, just keeps its momentum and keeps going on leaving behind the colliding galaxies. This kind of event has been detected by the Chandra X-Ray Observatory by using an ingenious manner to detect dark matter. Collisions of ordinary matter produces a lot of X-rays and the dark matter outside the galaxies acts as a gravitational lens.

5.3.8 Wild speculations about non-perturbative aspects of hadron physics and exotic Super Virasoro representations

If the canonical correspondence mapping the p-adic mass squared values to real numbers is taken completely seriously, then TGD predicts infinite hierarchy of exotic light representations of Super

Virasoro. These exotic states are created by sub-algebras of Super Kac-Moody and SKM algebras whose generators have conformal weights divisible by p^n, $n = 1, 2, ...$ Ordinary representations would correspond to $n = 0$.

For the exotic representations the p-adic mass squared of the particle is proportional to Virasoro p^n. When the value of the p-adic mass squared is power of p: $M^2 \propto p^n$, $n = 1, 2, ..,$ the real counterpart of the mass squared in canonical identification is extremely small since it is proportional to $1/p^n$ in this case. It is of course not at all clear whether these representations have have any real counterpart and if even this the case the could be thermally unstable in an environment with higher p-adic temperature.

Also ordinary low temperature ($T_p = 1/n$) Super Virasoro representations allow extremely light states but in this case there is no subalgebra generating these states. If these representations exist they could correspond to low energy-long length scale fractal copies of elementary particles. Due to the state degeneracy providing an enormous information storage capacity associated with these states these representations, if realized in nature, might have biological relevance [H2, J4].

There is however an objection against this idea: these representations are possible also in elementary particle length scales and for $M^2 \propto L_0 = npm_0^2$ the representations have same mass scale as ordinary elementary particles. These representations couple to ordinary elementary particles via classical gauge fields and could therefore be present also in elementary particle physics. For reasons which become clear below, exotic Super Virasoro representations might provide a model for low energy hadron physics.

1. The formula

$$M_R^2 = \frac{nm_0^2}{p}$$

is generalization of the mass formula of hadronic string models and reduces to it when the angular momentum

$$J = \alpha' M^2$$

of the hadronic state satisfies $J = n$. From this Regge slope α' and string tension T are given by

$$T = \frac{1}{2\pi\alpha'} \quad , \quad \frac{1}{\alpha'} = \frac{m_0^2}{p} \quad .$$

The observed value of the Regge slope is $\alpha' = .9/GeV^2$.

2. The value of the predicted string tension is easily found. The prediction of TGD based mass calculations for the value of the p-adic pion mass squared is

$$m_\pi^2 = pm_0^2 + O(p^2) \simeq pm_0^2 \quad , \quad p = M_{107} \quad .$$

$m_\pi \geq m_0/\sqrt{M_{107}}$ and $m_\pi = 134$ MeV gives upper bound for m_0 which is consistent with the prediction for the mass of electron. For $k = 107$ the value of α' would be roughly 64 times too large as simple calculation shows. For $k = 101$ one has

$$\alpha' = \frac{.87}{GeV^2} \quad ,$$

which deviates from the value $\alpha' = .9/GeV^2$ determined from ρ Regge trajectory only by three per cent.

3. This would suggest that excited states of ordinary hadrons contain $k = 101$ space-time sheets with p-adic length scale of .3 fm condensed on $k = 107$ hadronic space-time sheet with 8 times larger p-adic length scale and that the angular momentum of these excitations is not assignable to the ordinary quarks but to the states of $k = 101$ exotic Super Virasoro representation. The slight deviation from $.9/GeV^2$ could be explained if the contribution of quarks and gluons to the

mass squared decreases as a function of J so that the effective value of α' increases and effective string tension increases. This might be due to the transformation of parton mass squared to the mass squared associated with $k = 101$ exotic Super Virasoro states. Note that $n = 1$ excitation of $k = 101$ Super Virasoro has mass $m_1 = 1.07$ GeV, which is larger than proton mass: therefore the spin of these excitations cannot resolve the spin crisis of proton.

4. For $k = 103$ the predicted value of string tension is by a factor $1/4$ smaller. An interesting question is whether $k = 107$ and $k = 103$ excitations might be observable in low energy hadron physics.

The second thought provoking observation is that pion mass squared corresponds in excellent approximation to that for $n = 1$ state of exotic Super Virasoro representation for $k = 107$. This suggests that in case of pion quark masses are compensated apart from $O(p^2)$ contributions completely by various interaction energy and the energy associated with exotic Super Virasoro representation contributes to the mass squared. This would be p-adic articulation for the statement that pion is massless Goldstone boson. Since pion represents essentially non-perturbative aspects of QCD, this raises the possibility that exotic Super Virasoro representations could provide the long sought first principle theory of low energy hadronic physics.

1. In this theory hadrons would correspond to exotic Super Virasoro representations whereas quark-gluon plasma would correspond to ordinary p-adic Super Virasoro representations. In color confined phase p-adic α_c would have increased to the critical value $\alpha_c = p + O(p^2)$ implying dramatic drop of the real counterpart of α_c to $\alpha_c^R \simeq 1/p$ so that color interactions would disappear effectively and only electro-weak interactions and the geometric interactions between the space-time sheets would remain. What is important is that these phases can exist inside hadron for several values of p. This suggests a fractal hierarchy of hadrons inside hadrons and QCD:s inside QCD:s with the values of $\Lambda(k) \propto 1/L^2(k)$, $k = 107, 103, 101, ...$ In particular, rotational excitations would mean generation of $k = 101$ hadrons inside $k = 107$ hadrons.

2. Hadronization and fragmentation are semi-phenomenological aspects of QCD and would correspond at fundamental level to the phase transitions between the exotic Super Virasoro representations and ordinary Super Virasoro representations. Also the concepts of sea and Pomeron could be reduced the states of exotic Super Virasoro representations associated with $k = 107, 103, 101, 97, ...$

In light of the successes of the hadron model based on super-canonical many-particle states assigned to hadrons [F4] the exotic Super Virasoro representations do not look attractive from the point of view of ordinary hadron physics. Also the thermal instability is a good objection against them.

5.4 Simulating Big Bang in laboratory

Ultra-high energy collisions of heavy nuclei at Relativistic Heavy Ion Collider (RHIC) can create so high temperatures that there are hopes of simulating Big Bang in laboratory. The experiment with PHOBOS detector [118] probed the nature of the strong nuclear force by smashing two Gold atoms together at ultrahigh energies. The analysis of the experimental data has been carried out by Prof. Manly and his collaborators at RHIC in Brookhaven, NY [119]. The surprise was that the hydrodynamical flow for non-head-on collisions did not possess the expected longitudinal boost invariance.

This finding stimulates in TGD framework the idea that something much deeper might be involved.

1. The quantum criticality of the TGD inspired very early cosmology predicts the flatness of 3-space as do also inflationary cosmologies. The TGD inspired cosmology is 'silent whisper amplified to big bang' since the matter gradually topologically condenses from decaying cosmic string to the space-time sheet representing the cosmology. This suggests that one could model also the evolution of the quark-gluon plasma in an analogous manner. Now the matter condensing to the quark-gluon plasma space-time sheet would flow from other space-time sheets. The evolution of the quark-gluon plasma would very literally look like the very early critical cosmology.

2. What is so remarkable is that critical cosmology is not a small perturbation of the empty cosmology represented by the future light cone. By perturbing this cosmology so that the spherical symmetry is broken, it might possible to understand qualitatively the findings of [119]. Even more, the breaking of the spherical symmetry in the collision could be understood as a strong gravitational effect on distances transforming the spherical shape of the plasma ball to a non–spherical shape without affecting the spherical shape of its M^4_+ projection.

3. The model seems to work and predicts strong gravitational effects in elementary particle length scales so that TGD based gravitational physics would differ dramatically from that predicted by the competing theories. Standard cosmology cannot produce these effects without a large breaking of the cherished Lorentz and rotational symmetries forming the basis of elementary particle physics. Thus the the PHOBOS experiment gives direct support for the view that Poincare symmetry is symmetry of the imbedding space rather than that of the space-time.

4. This picture was completed a couple of years later by the progress made in hadronic mass calculations [F4]. It has already earlier been clear that quarks are responsible only for a small part of the mass of baryons (170 GeV in case of nucleons). The assumption that hadronic $k = 107$ space-time sheet carries a many-particle state of super-canonical particles with vanishing electro-weak quantum numbers (meaning darkness in the strongest sense of the wor

5. allows a model of hadrons predicting their masses with accuracy better than one per cent. The large value of Kähler coupling strength $\alpha_K = \alpha_s = 1/4$ for ordinary value of Planck constant motivates the hypothesis that a transition to large $\hbar$ phase occurs: $\hbar = 26 \times \hbar_0$ would leave the value of α_K for gauge boson field bodies ($\alpha_K = 1/104$) invariant [C4]. $J = 2$ excitations have identification as strong gravitons. In this framework color glass condensate can be identified as a state formed when the hadronic space-time sheets of colliding hadrons fuse to single long stringy object and collision energy is transformed to super-canonical hadrons.

5.4.1 Experimental arrangement and findings

Heuristic description of the findings

In the experiments using PHOBOS detector ultrahigh energy Au+Au collisions at center of mass energy for which nucleon-nucleon center of mass energy is $\sqrt{s_{NN}} = 130$ GeV, were studied [118].

1. In the analyzed collisions the Au nuclei did not collide quite head-on. In classical picture the collision region, where quark gluon plasma is created, can be modelled as the intersection of two colliding balls, and its intersection with plane orthogonal to the colliding beams going through the center of mass of the system is defined by two pieces of circles, whose intersection points are sharp tips. Thus rotational symmetry is broken for the initial state in this picture.

2. The particles in quark-gluon plasma can be compared to a persons in a crowded room trying to get out. The particles collide many times with the particles of the quark gluon plasma before reaching the surface of the plasma. The distance $d(z, \phi)$ from the point $(z, 0)$ at the beam axis to the point $(0, \phi)$ at the plasma surface depends on ϕ. Obviously, the distance is longest to the tips $\phi = \pm\pi/2$ and shortest to the points $\phi = 0, \phi = \phi$ of the surface at the sides of the collision region. The time $\tau(z, \phi)$ spent by a particle to the travel to the plasma surface should be a monotonically increasing function $f(d)$ of d:

$$\tau(z, \phi) = f(d(z, \phi)) \ .$$

   For instance, for diffusion one would have $\tau \propto d^2$ and $\tau \propto d$ for a pure drift.

3. What was observed that for $z = 0$ the difference

$$\Delta\tau = \tau(z = 0, \pi/2) - \tau(z = 0, 0)$$

   was indeed non-vanishing but that for larger values of z the difference tended to zero. Since the variation of z correspond that for the rapidity variable y for a given particle energy, this means that particle distributions depend on rapidity which means a breaking of the longitudinal boost invariance assumed in hydrodynamical models of the plasma. It was also found that the difference vanishes for large values of y: this finding is also important for what follows.

A more detailed description

Consider now the situation in a more quantitative manner.

1. Let z-axis be in the direction of the beam and ϕ the angle coordinate in the plane E^2 orthogonal to the beam. The kinematical variables are the rapidity of the detected particle defined as $y = log[E + p_z)/(E - p_z)]/2$ (E and p_z denote energy and longitudinal momentum), Feynman scaling variable $x_F \simeq 2E/\sqrt{s}$, and transversal momentum p_T.

2. By quantum-classical correspondence, one can translate the components of momentum to space-time coordinates since classically one has $x^\mu = p^\mu a/m$. Here a is proper time for a future light cone, whose tip defines the point where the quark gluon plasma begins to be generated, and $v^\mu = p^\mu/m$ is the four-velocity of the particle. Momentum space is thus mapped to an $a = constant$ hyperboloid of the future light cone for each value of a.

   In this correspondence the rapidity variable y is mapped to $y = log[(t + z)/(t - z)]$, $|z| \leq t$ and non–vanishing values for y correspond to particles which emerge, not from the collision point defining the origin of the plane E^2, but from a point above or below E^2. $|z| \leq t$ tells the coordinate along the beam direction for the vertex, where the particle was created. The limit $y \to 0$ corresponds to the limit $a \to \infty$ and the limit $y \to \pm\infty$ to $a \to 0$ (light cone boundary).

3. Quark-parton models predict at low energies an exponential cutoff in transverse momentum p_T; Feynman scaling $dN/dx_F = f(x_F)$ independent of s; and longitudinal boost invariance, that is rapidity plateau meaning that the distributions of particles do not depend on y. In the space-time picture this means that the space-time is effectively two-dimensional and that particle distributions are Lorentz invariant: string like space-time sheets provide a possible geometric description of this situation.

4. In the case of an ideal quark-gluon plasma, the system completely forgets that it was created in a collision and particle distributions do not contain any information about the beam direction. In a head-on collision there is a full rotational symmetry and even Lorentz invariance so that transverse momentum cutoff disappears. Rapidity plateau is predicted in all directions.

5. The collisions studied were not quite head-on collisions and were characterized by an impact parameter vector with length b and direction angle ψ_2 in the plane E^2. The particle distribution at the boundary of the plane E^2 was studied as a function of the angle coordinate $\phi - \psi_2$ and rapidity y which corresponds for given energy distance to a definite point of beam axis.

The hydrodynamical view about the situation looks like follows.

1. The particle distributions $N(p^\mu)$ as function of momentum components are mapped to space-time distributions $N(x^\mu, a)$ of particles. This leads to the idea that one could model the situation using Robertson-Walker type cosmology. Co-moving Lorentz invariant particle currents depending on the cosmic time only would correspond in this picture to Lorentz invariant momentum distributions.

2. Hydrodynamical models assign to the particle distribution $d^2N/dyd\phi$ a hydrodynamical flow characterized by four-velocity $v^\mu(y, \phi)$ for each value of the rapidity variable y. Longitudinal boost invariance predicting rapidity plateau states that the hydrodynamical flow does not depend on y at all. Because of the breaking of the rotational symmetry in the plane orthogonal to the beam, the hydrodynamical flow v depends on the angle coordinate $\phi - \psi_2$. It is possible to Fourier analyze this dependence and the second Fourier coefficient v_2 of $cos(2(\phi - \psi_2)$ in the expansion

$$\frac{dN}{d\phi} \simeq 1 + \sum_n v_n cos(n(\phi - \psi_2)) \quad (5.4.1)$$

was analyzed in [119].

3. It was found that the Fourier component v_2 depends on rapidity y, which means a breaking of the longitudinal boost invariance. v_2 also vanishes for large values of y. If this is true for all Fourier coefficients v_n, the situation becomes effectively Lorentz invariant for large values of y since one has $v(y, \phi) \rightarrow 1$.

   Large values of y correspond to small values of a and to the initial moment of big bang in cosmological analogy. Hence the finding could be interpreted as a cosmological Lorentz invariance inside the light cone cosmology emerging from the collision point. Small values of y in turn correspond to large values of a so that the breaking of the spherical symmetry of the cosmology should be manifest only at $a \rightarrow \infty$ limit. These observations suggest a radical re-consideration of what happens in the collision: the breaking of the spherical symmetry would not be a property of the initial state but of the final state.

5.4.2 TGD based model for the quark-gluon plasma

Consider now the general assumptions the TGD based model for the quark gluon plasma region in the approximation that spherical symmetry is not broken.

1. Quantum-classical correspondence supports the mapping of the momentum space of a particle to a hyperboloid of future light cone. Thus the symmetries of the particle distributions with respect to momentum variables correspond directly to space-time symmetries.

2. The M^4_+ projection of a Robertson-Walker cosmology imbedded to $H = M^4_+ \times CP_2$ is future light cone. Hence it is natural to model the hydrodynamical flow as a mini-cosmology. Even more, one can assume that the collision quite literally creates a space-time sheet which locally obeys Robertson-Walker type cosmology. This assumption is sensible in many-sheeted space-time and conforms with the fractality of TGD inspired cosmology (cosmologies inside cosmologies).

3. If the space-time sheet containing the quark-gluon plasma is gradually filled with matter, one can quite well consider the possibility that the breaking of the spherical symmetry develops gradually, as suggested by the finding $v_2 \rightarrow 1$ for large values of $|y|$ (small values of a). To achieve Lorentz invariance at the limit $a \rightarrow 0$, one must assume that the expanding region corresponds to $r = constant$ "coordinate ball" in Robertson-Walker cosmology, and that the breaking of the spherical symmetry for the induced metric leads for large values of a to a situation described as a "not head-on collision".

4. Critical cosmology is by definition unstable, and one can model the Au+Au collision as a perturbation of the critical cosmology breaking the spherical symmetry. The shape of $r = constant$ sphere defined by the induced metric is changed by strong gravitational interactions such that it corresponds to the shape for the intersection of the colliding nuclei. One can view the collision as a spontaneous symmetry breaking process in which a critical quark-gluon plasma cosmology develops a quantum fluctuation leading to a situation described in terms of impact parameter. This kind of modelling is not natural for a hyperbolic cosmology, which is a small perturbation of the empty M^4_+ cosmology.

The imbedding of the critical cosmology

Any Robertson-Walker cosmology can be imbedded as a space-time sheet, whose M^4_+ projection is future light cone. The line element is

$$ds^2 = f(a)da^2 - a^2(K(r)dr^2 + r^2 d\Omega^2) \ . \tag{5.4.2}$$

Here a is the scaling factor of the cosmology and for the imbedding as surface corresponds to the future light cone proper time.

This light cone has its tip at the point, where the formation of quark gluon plasma starts. (θ, ϕ) are the spherical coordinates and appear in $d\Omega^2$ defining the line element of the unit sphere. a and r are related to the spherical Minkowski coordinates (m^0, r_M, θ, ϕ) by $(a = \sqrt{(m^0)^2 - r_M^2}, r = r_M/a)$.

If hyperbolic cosmology is in question, the function $K(r)$ is given by $K(r) = 1/(1+r^2)$. For the critical cosmology 3-space is flat and one has $K(r) = 1$.

1. The critical cosmologies imbeddable to $H = M^4_+ \times CP_2$ are unique apart from a single parameter defining the duration of this cosmology. Eventually the critical cosmology must transform to a hyperbolic cosmology. Critical cosmology breaks Lorentz symmetry at space-time level since Lorentz group is replaced by the group of rotations and translations acting as symmetries of the flat Euclidian space.

2. Critical cosmology replaces Big Bang with a silent whisper amplified to a big but not infinitely big bang. The silent whisper aspect makes the cosmology ideal for the space-time sheet associated with the quark gluon plasma: the interpretation is that the quark gluon plasma is gradually transferred to the plasma space-time sheet from the other space-time sheets. In the real cosmology the condensing matter corresponds to the decay products of cosmic string in 'vapor phase'. The density of the quark gluon plasma cannot increase without limit and after some critical period the transition to a hyperbolic cosmology occurs. This transition could, but need not, correspond to the hadronization.

3. The imbedding of the critical cosmology to $M^4_+ \times S^2$ is given by

$$\begin{aligned} sin(\Theta) &= \frac{a}{a_m} \ , \\ \Phi &= g(r) \ . \end{aligned} \tag{5.4.2}$$

Here Θ and Φ denote the spherical coordinates of the geodesic sphere S^2 of CP_2. One has

$$\begin{aligned} f(a) &= 1 - \frac{R^2k^2}{(1-(a/a_m)^2)} \ , \\ (\partial_r\Phi)^2 &= \frac{a_m^2}{R^2} \times \frac{r^2}{1+r^2} \ . \end{aligned} \tag{5.4.2}$$

Here R denotes the radius of S^2. From the expression for the gradient of Φ it is clear that gravitational effects are very strong. The imbedding becomes singular for $a = a_m$. The transition to a hyperbolic cosmology must occur before this.

This model for the quark-gluon plasma would predict Lorentz symmetry and $v = 1$ (and $v_n = 0$) corresponding to head-on collision so that it is not yet a realistic model.

TGD based model for the quark-gluon plasma without breaking of spherical symmetry

There is a highly unique deformation of the critical cosmology transforming metric spheres to highly non–spherical structures purely gravitationally. The deformation can be characterized by the following formula

$$sin^2(\Theta) = (\frac{a}{a_m})^2 \times (1 + \Delta(a,\theta,\phi)^2) \ . \tag{5.4.3}$$

1. This induces deformation of the g_{rr} component of the induced metric given by

$$g_{rr} = -a^2 \left[1 + \Delta^2(a,\theta,\phi)\frac{r^2}{1+r^2} \right] \ . \tag{5.4.4}$$

Remarkably, g_{rr} does not depend at all on CP_2 size and the parameter a_m determining the duration of the critical cosmology. The disappearance of the dimensional parameters can be understood to reflect the criticality. Thus a strong gravitational effect independent of the gravitational constant (proportional to R^2) results. This implies that the expanding plasma space-time sheet having sphere as M^4_+ projection differs radically from sphere in the induced metric for large values of a. Thus one can understand why the parameter v_2 is non-vanishing for small values of the rapidity y.

2. The line element contains also the components g_{ij}, $i,j \in \{a,\theta,\phi\}$. These components are proportional to the factor

$$\frac{1}{1-(a/a_m)^2(1+\Delta^2)} \ , \tag{5.4.5}$$

which diverges for

$$a_m(\theta,\phi) = \frac{a_m}{\sqrt{1+\Delta^2}} \ . \tag{5.4.6}$$

Presumably quark-gluon plasma phase begins to hadronize first at the points of the plasma surface for which $\Delta(\theta,\phi)$ is maximum, that is at the tips of the intersection region of the colliding nuclei. A phase transition producing string like objects is one possible space-time description of the process.

5.4.3 Further experimental findings and theoretical ideas

The interaction between experiment and theory is pure magic. Although experimenter and theorist are often working without any direct interaction (as in case of TGD), I have the strong feeling that this disjointness is only apparent and there is higher organizing intellect behind this coherence. Again and again it has turned out that just few experimental findings allow to organize separate and loosely related physical ideas to a consistent scheme. The physics done in RHIC has played completely unique role in this respect.

Super-canonical matter as the TGD counterpart of CGC?

The model discussed above explained the strange breaking of longitudinal Lorentz invariance in terms of a hadronic mini bang cosmology. The next twist in the story was the shocking finding, compared to Columbus's discovery of America, was that, rather than behaving as a dilute gas, the plasma behaved like a liquid with strong correlations between partons, and having density 30-50 times higher than predicted by QCD calculations [17]. When I learned about these findings towards the end of 2004, I proposed how TGD might explain them in terms of what I called conformal confinement [F2]. This idea - although not wrong for any obvious reason - did not however have any obvious implications. After the progress made in p-adic mass calculations of hadrons leading to highly successful model for both hadron and meson masses [F4], the idea was replaced with the hypothesis that the condensate in question is Bose-Einstein condensate like state of super-canonical particles formed when the hadronic space-time sheets of colliding nucleons fuse together to form a long string like object.

Fireballs behaving like black hole like objects

The latest discovery in RHIC is that fireball, which lasts a mere 10^{-23} seconds, can be detected because it absorbs jets of particles produced by the collision [45]. The association with the notion black hole is unavoidable and there indeed exists a rather esoteric M-theory inspired model "The RHIC fireball as a dual black hole" by Hortiu Nastase [123] for the strange findings.

The Physics Today article [35] "What Have We Learned From the Relativistic Heavy Ion Collider?" gives a nice account about experimental findings. Extremely high collision energies are in question: Gold nuclei contain energy of about 100 GeV per nucleon: 100 times proton mass. The expectation was that a large volume of thermalized Quark-Gluon Plasma (QCP) is formed in which partons lose rapidly their transverse momentum. The great surprise was the suppression of high transverse momentum collisions suggesting that in this phase strong collective interactions are present. This has inspired the proposal that quark gluon plasma is preceded by liquid like phase which has been christened as Color Glass Condensate (CGC) thought to contain Bose-Einstein condensate of gluons.

The theoretical ideas relating CGC to gravitational interactions

Color glass condensate relates naturally to several gravitation related theoretical ideas discovered during the last year.

1. Classical gravitation and color confinement

Just some time ago it became clear that strong classical gravitation might play a key role in the understanding of color confinement [E2]. Whether the situation looks confinement or asymptotic freedom would be in the eyes of beholder: one example of dualities filling TGD Universe. If one looks the situation at the hadronic space-time sheet one has asymptotic freedom, particles move essentially like free massless particles. But, and this is absolutely essential, in the induced metric of hadronic space-time sheet. This metric represents classical gravitational field becoming extremely strong near hadronic boundary. From the point of view of outsider, the motion of quarks slows down to rest when they approach hadronic boundary: confinement. The distance to hadron surface is infinite or at least very large since the induced metric becomes singular at the light-like boundary! Also hadronic time ceases to run near the boundary and finite hadronic time corresponds to infinite time of observer. When you look from outside you find that this light-like 3-surface is just static surface like a black hole horizon which is also a light-like 3-surface. Hence confinement.

2. Dark matter in TGD

The evidence for hadronic black hole like structures is especially fascinating. In TGD Universe dark matter can be (not always) ordinary matter at larger space-time sheets in particular magnetic flux tubes. The mere fact that the particles are at larger space-time sheets might make them more or less invisible.

Matter can be however dark in much stronger sense, should I use the word "black"! The findings suggesting that planetary orbits obey Bohr rules with a gigantic Planck constant [40, D7] would suggest quantum coherence of dark matter even in astrophysical length scales and this raises the fascinating possibility that Planck constant is dynamical so that fine structure constant for these charged coherent states would be proportional to $1/\hbar_{gr}$ and extremely small: hence darkness. This quantization saves from black hole collapse just as the quantization of hydrogen atom saves from the infrared catastrophe.

The obvious questions are following. Could black hole like objects/magnetic flux tubes/cosmic strings consist of quantum coherent dark matter? Does this dark matter consist dominantly from hadronic space-time sheets which have fused together and contain super-canonical bosons and their super-partners (with quantum numbers of right handed neutrino) having therefore no electro-weak interactions.

Since $\alpha_K = \alpha_s = 1/4$ would indeed justify large value of Planck constant, $\hbar = 26\hbar_0$ would leave α_K unchanged and predicts that the size of the hadronic space-time sheet is that of a large nucleus. The hadronic string tension would be predicted correctly and strong gravitation would correspond to the exchange of super-canonical $J = 2$ quanta.

This overall view would be of enormous importance even for the understanding of living matter since dark matter at magnetic flux tubes would be responsible for the quantum control of the ordinary matter. Note however that TGD based quantum model for living matter involves also dark variants of ordinary elementary particles.

From outside non-stringy TGD analogs of black holes would look just like ordinary black holes but the interior metric would be of course different from the usual one since matter would not be collapsed to a point.

Dark matter option cannot be realized in a purely hadronic system at RHIC energies since the product GM_1M_2 characterizing the interaction strength of two masses must be larger than unity ($\hbar = c = 1$) for the phase transition increasing Planck constant to occur. Hence the collision energy should be above Planck mass for the phase transition to occur if gravitational interactions are responsible for the transition.

The hypothesis is however much more general and states that the system does its best to stay perturbative by increasing its Planck constant in discrete steps and applies thus also in the case of color interactions and governs the phase transition to the TGD counterpart of non-perturbative QCD. Criterion would be roughly $\alpha_s Q_s^2 > 1$ for two color charges of opposite sign. Hadronic string picture would suggests that the criterion is equivalent to the generalization of the gravitational criterion to its strong gravity analog $nL_p^2 M^2 > 1$, where L_p is the p-adic length scale characterizing color magnetic energy density (hadronic string tension) and M is the mass of the color magnetic flux tube and n is a numerical constant. Presumably L_p, $p = M_107 = 2^{107} - 1$, is the p-adic length scale since Mersenne prime M_{107} labels the space-time sheet at which partons feed their color gauge fluxes. The temperature during this phase could correspond to Hagedorn temperature (for the history and various interpretations of Hagedorn temperature see the CERN Courier article [124]) for strings and is determined by string tension and would naturally correspond also to the temperature during the critical phase determined by its duration as well as corresponding black-hole temperature. This temperature is expected to be somewhat higher than hadronization temperature found to be about $\simeq 176$ MeV. The density of inertial mass would be maximal during this phase as also the density of gravitational mass during the critical phase.

Lepto-hadron physics [F7], one of the predictions of TGD, is one instance of a similar situation. In this case electromagnetic interaction strength defined in an analogous manner becomes larger than unity in heavy ion collisions just above the Coulomb wall and leads to the appearance of mysterious states having a natural interpretation in terms of lepto-pion condensate. Lepto-pions are pairs of color octet excitations of electron and positron.

One can ask whether the Bose-Einstein condensed gluons at color magnetic flux tubes possess complex super-canonical conformal weights and whether conformal confinement could be responsible for the particle like behavior of CGC. An equally interesting question is whether ordinary liquid flow could involve Bose-Einstein condensates of particles which are not "conformal singlets".

3. Description of collisions using analogy with black holes

The following view about RHIC events represents my immediate reaction to the latest RHIC news in terms of black-hole physics instead of notions related to big bang. Since black hole collapse is roughly time reversal of big bang, the description is complementary to the earliest one.

In TGD context one can ask whether the fireballs possibly detected in RHIC are produced when a portion of quark-gluon plasma in the collision region formed by to Gold nuclei separates from hadronic space-time sheets which in turn fuse to form a larger space-time sheet separated from the remaining collision region by a light-like 3-D surface (I have used to speak about light-like causal determinants) mathematically completely analogous to a black hole horizon. This larger space-time sheet would contain color glass condensate of super-canonical gluons formed from the collision energy. A formation of an analog of black hole would indeed be in question.

The valence quarks forming structures connected by color bonds would in the first step of the collision separate from their hadronic space-time sheets which fuse together to form color glass condensate. Similar process has been observed experimentally in the collisions demonstrating the experimental reality of Pomeron, a color singlet state having no Regge trajectory [71] and identifiable as a structure formed by valence quarks connected by color bonds. In the collision it temporarily separates from the hadronic space-time sheet. Later the Pomeron and the new mesonic and baryonic Pomerons created in the collision suffer a topological condensation to the color glass condensate: this process would be analogous to a process in which black hole sucks matter from environment.

Of course, the relationship between mass and radius would be completely different with gravitational constant presumably replacement by the the square of appropriate p-adic length scale presumably of order pion Compton length: this is very natural if TGD counterparts of black-holes are formed by color magnetic flux tubes. This gravitational constant expressible in terms of hadronic string tension of .9 GeV^2 predicted correctly by super-canonical picture would characterize the strong gravitational interaction assignable to super-canonical $J = 2$ gravitons. I have long time ago in the context of p-adic mass calculations formulated quantitatively the notion of elementary particle black hole analogy making the notion of elementary particle horizon and generalization of Hawking-Bekenstein law [E5].

The size L of the "hadronic black hole" would be relatively large using protonic Compton radius as a unit of length. For $hbar = 26\hbar_0$ the size would be $26 \times L(107) = 46$ fm,and correspond to a size of a heavy nucleus. This large size would fit nicely with the idea about nuclear sized color glass condensate.

The density of partons (possibly gluons) would be very high and large fraction of them would have been materialized from the brehmstrahlung produced by the de-accelerating nuclei. Partons would be gravitationally confined inside this region. The interactions of partons or conformal confinement would lead to a generation of a liquid like dense phase and a rapid thermalization would occur. The collisions of partons producing high transverse momentum partons occurring inside this region would yield no detectable high p_T jets since the matter coming out from this region would be somewhat like a thermal radiation from an evaporating black hole identified as a highly entangled hadronic string in Hagedorn temperature. This space-time sheet would expand and cool down to QQP and crystallize into hadrons.

4. Quantitative comparison with experimental data

Consider now a quantitative comparison of the model with experimental data. The estimated freeze-out temperature of quark gluon plasma is $T_f \simeq 175.76$ MeV [35, 123], not far from the total contribution of quarks to the mass of nucleon, which is 170 MeV [F4]. Hagedorn temperature identified as black-hole temperature should be higher than this temperature. The experimental estimate for the hadronic Hagedorn temperature from the transversal momentum distribution of baryons is $\simeq 160$ MeV. On the other hand, according to the estimates of hep-ph/0006020 the values of Hagedorn temperatures for mesons and baryons are $T_H(M) = 195$ MeV and $T_H(B) = 141$ MeV respectively.

D-dimensional bosonic string model for hadrons gives for the mesonic Hagedorn temperature the expression [124]

$$T_H = \frac{\sqrt{6}}{2\pi(D-2)\alpha'} , \tag{5.4.7}$$

For a string in $D = 4$-dimensional space-time and for the value $\alpha' \sim 1$ GeV^{-2} of Regge slope, this would give $T_H = 195$ MeV, which is slightly larger than the freezing out temperature as it indeed should be, and in an excellent agreement with the experimental value of [125]. It deserves to be noticed that in the model for fireball as a dual 10-D black-hole the rough estimate for the temperature of color glass condensate becomes too low by a factor 1/8 [123]. In light of this I would not yet rush to conclude that the fireball is actually a 10-dimensional black hole.

Note that the baryonic Hagedorn temperature is smaller than mesonic one by a factor of about $\sqrt{2}$. According to [125] this could be qualitatively understood from the fact that the number of degrees of freedom is larger so that the effective value of D in the mesonic formula is larger. $D_{eff} = 6$ would give $T_H = 138$ MeV to be compared with $T_H(B) = 141$ MeV. On the other hand, TGD based model for hadronic masses [F4] assumes that quarks feed their color fluxes to $k = 107$ space-time sheets. For mesons there are two color flux tubes and for baryons three. Using the same logic as in [125], one would have $D_{eff}(B)/D_{eff}(M) = 3/2$. This predicts $T_H(B) = 159$ MeV to be compared with 160 MeV deduced from the distribution of transversal momenta in p-p collisions.

5.4.4 Are ordinary black-holes replaced with super-canonical black-holes in TGD Universe?

Some variants of super string model predict the production of small black-holes at LHC. I have never taken this idea seriously but in a well-defined sense TGD predicts black-holes associated with super-canonical gravitons with strong gravitational constant defined by the hadronic string tension. The proposal is that super-canonical black-holes have been already seen in Hera, RHIC, and the strange cosmic ray events.

Baryonic super-canonical black-holes of the ordinary M_{107} hadron physics would have mass 934.2 MeV, very near to proton mass. The mass of their M_{89} counterparts would be 512 times higher, about 478 GeV if quark massses scale also by this factor. This need not be the case: if one has $k = 113 \rightarrow 103$ instead of 105 one has 434 GeV mass. "Ionization energy" for Pomeron, the structure formed by valence quarks connected by color bonds separating from the space-time sheet of super-canonical black-hole in the production process, corresponds to the total quark mass and is about 170 MeV for ordinary proton and 87 GeV for M_{89} proton. This kind of picture about black-hole formation expected to occur in LHC differs from the stringy picture since a fusion of the hadronic mini black-holes to a larger black-hole is in question.

An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are baryons, which have lost their valence quarks in a collision with hadron and therefore have no interactions with the microwave background so that they are able to propagate through long distances.

In neutron stars the hadronic space-time sheets could form a gigantic super-canonical black-hole and ordinary black-holes would be naturally replaced with super-canonical black-holes in TGD framework (only a small part of black-hole interior metric is representable as an induced metric). This obviously means a profound difference between TGD and string models.

1. Hawking-Bekenstein black-hole entropy would be replaced with its p-adic counterpart given by

$$S_p = (\frac{M}{m(CP_2)})^2 \times log(p) \ , \tag{5.4.8}$$

where $m(CP_2)$ is CP_2 mass, which is roughly 10^{-4} times Planck mass. M is the contribution of p-adic thermodynamics to the mass. This contribution is extremely small for gauge bosons but for fermions and super-canonical particles it gives the entire mass.

2. If p-adic length scale hypothesis $p \simeq 2^k$ holds true, one obtains

$$S_p = klog(2) \times (\frac{M}{m(CP_2)})^2, \tag{5.4.9}$$

$m(CP_2) = \hbar/R$, R the "radius" of CP_2, corresponds to the standard value of $\hbar_0$ for all values of $\hbar$.

3. Hawking-Bekenstein area law gives in the case of Schwartschild black-hole

$$S = \frac{A}{4G} \times \hbar = \pi G M^2 \times \hbar \ . \tag{5.4.10}$$

For the p-adic variant of the law Planck mass is replaced with CP_2 mass and $klog(2) \simeq log(p)$ appears as an additional factor. Area law is obtained in the case of elementary particles if k is prime and wormhole throats have M^4 radius given by p-adic length scale $L_k = \sqrt{k}R$ which is exponentially smaller than L_p. For macroscopic super-canonical black-holes modified area law results if the radius of the large wormhole throat equals to Schwartschild radius. Schwartschild radius is indeed natural: in [D4] I have shown that a simple deformation of the Schwartschild exterior metric to a metric representing rotating star transforms Schwartschild horizon to a light-like 3-surface at which the signature of the induced metric is transformed from Minkowskian to Euclidian.

4. The formula for the gravitational Planck constant appearing in the Bohr quantization of planetary orbits and characterizing the gravitational field body mediating gravitational interaction between masses M and m [D7] reads as

$$\hbar_{gr} = \frac{GMm}{v_0}\hbar_0 \ .$$

$v_0 = 2^{-11}$ is the preferred value of v_0. One could argue that the value of gravitational Planck constant is such that the Compton length $\hbar_{gr}/M$ of the black-hole equals to its Schwartshild radius. This would give

$$\hbar_{gr} = \frac{GM^2}{v_0}\hbar_0 \ , \ \ v_0 = 1/2 \ . \tag{5.4.11}$$

The requirement that $\hbar_{gr}$ is a ratio of ruler-and-compass integers expressible as a product of distinct Fermat primes (only four of them are known) and power of 2 would quantize the mass spectrum of black hole [D7]. Even without this constraint M^2 is integer valued using p-adic mass squared unit and if p-adic length scale hypothesis holds true this unit is in an excellent approximation power of two.

5. The gravitational collapse of a star would correspond to a process in which the initial value of v_0 , say $v_0 = 2^{-11}$, increases in a stepwise manner to some value $v_0 \leq 1/2$. For a supernova with solar mass with radius of 9 km the final value of v_0 would be $v_0 = 1/6$. The star could have an onion like structure with largest values of v_0 at the core as suggested by the model of planetary system. Powers of two would be favored values of v_0. If the formula holds true also for Sun one obtains $1/v_0 = 3 \times 17 \times 2^{13}$ with 10 per cent error.

6. Black-hole evaporation could be seen as means for the super-canonical black-hole to get rid of its electro-weak charges and fermion numbers (except right handed neutrino number) as the antiparticles of the emitted particles annihilate with the particles inside super-canonical black-hole. This kind of minimally interacting state is a natural final state of star. Ideal super-canonical black-hole would have only angular momentum and right handed neutrino number.

7. In TGD light-like partonic 3-surfaces are the fundamental objects and space-time interior defines only the classical correlates of quantum physics. The space-time sheet containing the highly entangled cosmic string might be separated from environment by a wormhole contact with size of black-hole horizon.

This looks the most plausible option but one can of course ask whether the large partonic 3-surface defining the horizon of the black-hole actually contains all super-canonical particles so that super-canonical black-hole would be single gigantic super-canonical parton. The interior of super-canonical black-hole would be a space-like region of space-time, perhaps resulting as a large deformation of CP_2 type vacuum extremal. Black-hole sized wormhole contact would define a gauge boson like variant of the black-hole connecting two space-time sheets and getting its mass through Higgs mechanism. A good guess is that these states are extremely light.

5.4.5 Conclusions

The model for quark-gluon plasma in terms of valence quark space-time sheets separated from hadronic space-time sheets forming a color glass condensate relies on quantum criticality and implies gravitation like effects due to the presence of super-canonical strong gravitons. At space-time level the change of the distances due to strong gravitation affects the metric so that the breaking of spherical symmetry is caused by gravitational interaction. TGD encourages to think that this mechanism is quite generally at work in the collisions of nuclei. One must take seriously the possibility that strong gravitation is present also in longer length scales (say biological), in particular in processes in which new space-time sheets are generated. Critical cosmology might provide a universal model for the emergence of a new space-time sheet.

The model supports TGD based early cosmology and quantum criticality. In standard physics framework the cosmology in question is not sensible since it would predict a large breaking of the Lorentz invariance, and would mean the breakdown of the entire conceptual framework underlying elementary particle physics. In TGD framework Lorentz invariance is not lost at the level of imbedding space, and the experiments provide support for the view about space-time as a surface and for the notion of many-sheeted space-time.

The attempts to understand later strange events reported by RHIC have led to a dramatic increase of understanding of TGD and allow to fuse together separate threads of TGD.

1. The description of RHIC events in terms of the formation of hadronic black hole and its evaporation seems to be also possible and essentially identical with description as a mini bang.

2. It took some time to realize that scaled down TGD inspired cosmology as a model for quark gluon plasma predicts a new phase identifiable as color glass condensate and still a couple of years to realize the proper interpretation of it in terms of super-canonical bosons having no counterpart in QCD framework.

3. Also dark matter could be identified as a macroscopic quantum phase in which individual particles have complex conformal weights. This phase could be even responsible for the properties of living matter. There is also a connection with the dramatic findings suggesting that Planck constant for dark matter has a gigantic value.

4. Black holes and their scaled counterparts would not be merciless information destroyers in TGD Universe. The entanglement of particles possessing different conformal weights to give states with a vanishing net conformal weight and having particle like integrity would make black hole like states ideal candidates for quantum computer like systems. One could even imagine that the galactic black hole is a highly tangled cosmic string in Hagedorn temperature performing quantum computations the complexity of which is totally out of reach of human intellect! Indeed, TGD inspired consciousness predicts that evolution leads to the increase of information and intelligence, and the evolution of stars should not form exception to this. Also the interpretation of black hole as consisting of dark matter follows from this picture.

Summarizing, it seems that thanks to some crucial experimental inputs the new physics predicted by TGD is becoming testable in laboratory.

5.5 Cosmic rays and Mersenne Primes

TGD suggests the existence of a scaled up copy of hadron physics associated with each Mersenne prime $M_n = 2^n - 1$, n prime: M_{107} corresponds to ordinary hadron physics. There is some evidence for exotic hadrons. Also Gaussian Mersennes $(1+i)^k - 1$, could correspond to hadron physics. Four of them ($k = 151, 157, 163, 167$) are in the biologically interesting length scale range between cell membrane thickness and the size of cell nucleus.

Centauro events and the peculiar events associated with $E > 10^5\ GeV$ radiation from Cygnus X-3 could be understood as due to the decay of gamma rays to M_{89} hadron pair in the atmosphere. The decay $\pi_n \rightarrow \gamma\gamma$ produces a peak in the spectrum of the cosmic gamma rays at energy $\frac{m(\pi_n)}{2}$ and there is evidence for the peaks at energies $E_{89} \simeq 34\ GeV$ and $E_{31} \simeq 3.5 \cdot 10^{10}\ GeV$. The absence of the peak at $E_{61} \simeq 1.5 \cdot 10^6\ GeV$ can be understood as due to the strong absorption caused by the e^+e^- pair creation with photons of the cosmic microwave background.

Cosmic string decays *cosmic string* $\rightarrow$ M_2 *hadrons* $\rightarrow$ M_3 *hadrons* ..$\rightarrow$ M_{107} *hadrons* is a new source of cosmic rays. The mechanism could explain the change of the slope in the hadronic cosmic ray spectrum at $3 \cdot 10^6\ GeV$ which is not far from M_{61} pion rest energy $1.2 \cdot 10^6\ GeV$.

The cosmic ray radiation at energies near $10^9\ GeV$ apparently consisting of protons and nuclei not lighter than Fe might be actually dominated by gamma rays: at these energies γ and p induced showers have same muon content and the decays of gamma rays to M_{89} and M_{61} hadrons in the atmosphere can mimic the presence of heavy nuclei in the cosmic radiation.

The identification of the hadronic space-time sheet as a super-canonical mini black-hole [F4] suggests that part of ultra-high energy cosmic rays could be protons which have lost their valence quarks. These particles would have essentially same mass as proton and would behave like mini black-holes consisting of dark matter. They could even give a dominating contribution to the dark matter. Since electro-weak interactions are absent, the scattering from microwave background is absent, and they could propagate over much longer distances than ordinary particles. An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are super-canonical mini black-holes associated with M_{107} hadron physics or some other copy of hadron physics.

5.5.1 Mersenne primes and mass scales

p-Adic mass calculations lead to quite detailed predictions for elementary particle masses. In particular, there are reasons to believe that the most importabnt fundamental elementary particle mass scales correspond to Mersenne primes $M_n = 2^n - 1$, $n = 2, 3, 7, 13, 17, 19, ...$

$$m_n^2 = \frac{m_0^2}{M_n} ,$$
$$m_0 \simeq 1.41 \cdot \frac{10^{-4}}{\sqrt{G}} , \quad (5.5.0)$$

where $\sqrt{G}$ is Planck length. The known elementary particle mass scales were identified as mass scales associated identified with Mersenne primes $M_{127} \simeq 10^{38}$ (leptons), M_{107} (hadrons) and M_{89} (intermediate gauge bosons). Of course, also other p-adic length scales are possible and it is quite possible that not all Mersenne primes are realized.

Theory predicts also some higher mass scales corresponding to the Mersenne primes M_n for $n = 89, 61, 31, 19, 17, 13, 7, 3$ and suggests the existence of a scaled up copy of hadron physics with each of these mass scales. In particular, masses should be related by simple scalings to the masses of the ordinary hadrons.

An attractive hypothesis is that the color interactions of the particles of level M_n can be described using the ordinary QCD scaled up to the level M_n so that that masses and the confinement mass scale Λ is scaled up by the factor $\sqrt{M_n/M_{107}}$.

$$\Lambda_n = \sqrt{\frac{M_n}{M_{107}}}\Lambda . \quad (5.5.1)$$

In particular, the masses of the exotic pions associated with M_n are given by

$$m(\pi_n) = \sqrt{\frac{M_n}{M_{107}}}m_\pi . \quad (5.5.2)$$

Here $m_\pi \simeq 135\ MeV$ is the mass of the ordinary pion.

The interactions between the different level hadrons are mediated by the emission of electro-weak gauge bosons and by gluons with cm energies larger than the energy defined by the confinement scale of level with smaller p. The decay of the exotic hadrons at level M_{n_k} to exotic hadrons at level $M_{n_{k+1}}$ must take place by a transition sequence leading from the effective M_{n_k}-adic space-time topology to effective $M_{n_{k+1}}$-adic topology. All intermediate p-adic topologies might be involved.

5.5.2 Cosmic strings and cosmic rays

Cosmic strings are fundamental objects in quantum TGD and dominated during early cosmology.

Cosmic strings

Cosmic strings (not quite the same thing in TGD as in GUTs) are basic objects in TGD inspired cosmology [D6, D5].

1. In TGD inspired galaxy model galaxies are regarded as mass concentrations around cosmic strings and the energy of the string corresponds to the dark energy whereas the particles condensed at cosmic strings and magnetic flux tubes resulting from them during cosmic expansion correspond to dark matter [D6, D5]. The galactic nuclei, often regarded as candidates for black holes, are the most probable seats for decaying highly entangled cosmic strings.

2. Galaxies are known to organize to form larger linear structures. This can be understood if the highly entangled galactic strings organize around long strings like pearls in necklace. Long strings could correspond to galactic jets and their gravitational field could explain the constant velocity spectrum of distant stars in the galactic halo.

3. In [D6, D5, D7] it is suggested that decaying cosmic strings might provide a common explanation for the energy production of quasars, galactic jets and gamma ray bursters and that the visible matter in galaxies could be regarded as decay products of cosmic strings. The magnetic and Z^0 magnetic flux tubes resulting during the cosmic expansion from cosmic strings allow to assign at

least part of gamma ray bursts to neutron stars. Hot spots (with temperature even as high as $T \sim \frac{10^{-3,5}}{\sqrt{G}}$) in the cosmic string emitting ultra high energy cosmic rays might be created under the violent conditions prevailing in the galactic nucleus.

The decay of the cosmic strings provides a possible mechanism for the production of the exotic hadrons and in particular, exotic pions. In [86] the idea that cosmic strings might produce gamma rays by decaying first into 'X' particles with mass of order 10^{15} GeV and then to gamma rays, was proposed. As authors notice this model has some potential difficulties resulting from the direct production of gamma rays in the source region and the presence of intensive electromagnetic fields near the source. These difficulties are overcome if cosmic strings decay first into exotic hadrons of type M_{n_0}, $n_0 \geq 3$ of energy of order $2^{-n_0+2}10^{25}$ GeV , which in turn decay to exotic hadrons corresponding to M_k, $k > n_0$ via ordinary color interaction, and so on so that a sequence of M_k:s starting some value of n_0 in $n = 2, 3, 7, 13, 17, 19, 31, 61, 89, 107$ is obtained. The value of n remains open at this stage and depends on the temperature of the hot spot and much smaller temperatures than the $T \sim m_0$ are possible: favored temperatures are the temperatures $T_n \sim m_n$ at which M_n hadrons become unstable against thermal decay.

Decays of cosmic strings as producer of high energy cosmic gamma rays

In [87] the gamma ray signatures from ordinary cosmic strings were considered and a dynamical QCD based model for the decay of cosmic string was developed. In this model the final state particles were assumed to be ordinary hadrons and final state interactions were neglected. In present case the string decays first to M_{n_0} hadrons and the time scale of for color interaction between M_{n_0} hadrons is extremely short (given by the length scale defined by the inverse of π_{n_0} mass) as compared to the time time scale in case of ordinary hadrons. Therefore the interactions between the final state particles must be taken into account and there are good reasons to expect that thermal equilibrium sets on and much simpler thermodynamic description of the process becomes possible.

A possible description for the decaying part of the highly tangled cosmic string is as a 'fireball' containing various M_{n_0} ($n \geq 3$) partons in thermal equilibrium at Hagedorn temperature T_{n_0} of order $T_{n_0} \sim m_{n_0} = 2^{-2+n_0}\frac{10^{-4}}{k\sqrt{G}}$, $k \simeq 1.288$. The experimental discoveries made in RHIC suggest [35] that high energy nuclear collisions create instead of quark gluon plasma a liquid like phase involving gluonic BE condensate christened as color glass condensate. Also black hole like behavior is suggested by the experiments.

RHIC findings inspire a TGD based model for this phase as a macroscopic quantum phase condensed on a highly tangled color magnetic string at Hagedorn temperature. The model relies also on the notion of dynamical but quantized $\hbar$ [J6] and its recent form to the realization that super-canonical many-particle states at hadronic space-time sheets give dominating contribution to the baryonic mass and explain hadronic masses with an excellent accuracy.

This phase has no direct gauge interactions with ordinary matter and is identified in TGD framework as a particular instance of dark matter. Quite generally, quantum coherent dark matter would reside at magnetic flux tubes idealizable as string like objects with string tension determined by the p-adic length scale and thus outside the "ordinary" space-time. This suggests that color glass condensate forms when hadronic space-time sheets fuse to single long string like object containing large number of super-canonical bosons.

Color glass condensate has black-hole like properties by its electro-weak darkness and there are excellent reasons to believe that also ordinary black holes could by their large density correspond to states in which super-canonical matter would form single connected string like structure (if Planck constant is larger for super-canonical hadrons, this fusion is even more probable).

This inspires the following mechanism for the decay of exotic boson.

1. The tangled cosmic string begins to cool down and when the temperature becomes smaller than $m(\pi_{n_0})$ mass it has decayed to M_{n_1} matter which in turn continues to decay to M_{n_2} matter. The decay to M_{n_1} matter could occur via a sequence $n_0 \rightarrow n_0 - 1 \rightarrow ...n_1$ of phase transitions corresponding to the intermediate p-adic length scales $p \simeq 2^k$, $n_1 \geq k > n_0$. Of course, all intermediate p-adic length scales are in principle possible so that the process would be practically continuous and analogous to p-adic length scale evolution with $p \simeq 2^k$ representing more stable intermediate states.

2. The first possibility is that virtual hadrons decay to virtual hadrons in the transition $k \rightarrow k-1$. The alternative option is that the density of final state hadrons is so high that they fuse to form a single highly entangled hadronic string at Hagedorn temperature T_{k-1} so that the process would resemble an evaporation of a hadronic black hole staying in quark plasma phase without freezing to hadrons in the intermediate states. This entangled string would contain partons as "color glass condensate".

3. The process continues until all particles have decayed to ordinary hadrons. Part of the M_n low energy thermal pions decay to gamma ray pairs and produce a characteristic peak in cosmic gamma ray spectrum at energies $E_n = \frac{m(\pi_n)}{2}$ (possibly red-shifted by the expansion of the Universe). The decay of the cosmic string generates also ultra high energy hadronic cosmic rays, say protons. Since the creation of ordinary hadron with ultra high energy is certainly a rare process there are good hopes of avoiding the problems related to the direct production of protons by cosmic strings (these protons produce two high flux of low energy gamma rays, when interacting with cosmic microwave background [86]).

Topologically condensed cosmic strings as analogs super-canonical black-holes?

Super-canonical matter has very stringy character. For instance, it obeys stringy mass formula due the additivity and quantization of mass squared as multiples of p-adic mass scale squared [F4]. The ensuing additivity of mass squared defines a universal formula for binding energy having no independence on interaction mechanism. Highly entangled strings carrying super-canonical dark matter are indeed excellent candidates for TGD variants of black-holes. The space-time sheet containing the highly entangled cosmic string is separated from environment by a wormhole contact with a radius of black-hole horizon. Schwartschild radius has also interpretation as Compton length with Planck constant equal to gravitational Planck constant $\hbar/\hbar_0 = 2GM^2$. In this framework the proposed decay of cosmic strings would represent nothing but the TGD counterpart of Hawking radiation. Presumably the value of p-adic prime in primordial stage was as small as possible, even $p = 2$ can be considered.

Exotic cosmic ray events and exotic hadrons

One signature of the exotic hadrons is related to the interaction of the ultra high energy gamma rays with the atmosphere. What can happen is that gamma rays in the presence of an atmospheric nucleus decay to virtual exotic quark pair associated with M_{n_k}, which in turn produces a cascade of exotic hadrons associated with M_{n_k} through the ordinary scaled up color interaction. These hadrons in turn decay $M_{n_{k+1}}$ type hadrons via mechanisms to be discussed later. At the last step ordinary hadrons are produced. The collision creates in the atmospheric nucleus the analog of quark gluon plasma which forms a second kind of fireball decaying to ordinary hadrons. RHIC experiments have already discovered these fireballs and identified them as color glass condensates [35]. It must be emphasized that it is far from clear whether QCD really predicts this phase.

These showers differ from ordinary gamma ray showers in several respects.

1. Exotic hadrons can have small momenta and the decay products can have isotropic angular distribution so that the shower created by gamma rays looks like that created by a massive particle.

2. The muon content is expected to be similar to that of a typical hadronic shower generated by proton and larger than the muon content of ordinary gamma ray shower [88].

3. Due to the kinematics of the reactions of type $\gamma + p \rightarrow H_{M_n} + ... + p$ the only possibility at the available gamma ray energies is that M_{89} hadrons are produced at gamma ray energies above 10 TeV. The masses of these hadrons are predicted to be above 70 GeV and this suggests that these hadrons might be identified incorrectly as heavy nuclei (heavier than ^{56}Fe). These signatures will be discussed in more detail in the sequel in relation to Centauro type events, Cygnus X-3 events and other exotic cosmic ray events. For a good review for these events and models form them see the review article [103].

Some cosmic ray events [89, 90] have total laboratory energy as high as 3000 TeV which suggests that the shower contains hadron like particles, which are more penetrating than ordinary hadrons.

1. One might argue that exotic hadrons corresponding M_k, $k > 107$with interact only electro-weakly (color is confined in the length scale associated with M_n) with the atmosphere one might argue that they are more penetrating than the ordinary hadrons.

2. The observed highly penetrating fireballs could also correspond super-canonical dark matter part of incoming, possibly exotic, hadron fused with that for a hadron of atmosphere. Both hadrons would have lost their valence quarks in the collision just as in the case of Pomeron events. Large fraction of the collision energy would be transformed to super-canonical quanta in the process and give rise to a large color spin glass condensate. These condensates would have no direct electro-weak interactions with ordinary matter which would explain their long penetration lengths in the atmosphere. Sooner or later the color glass condensate would decay to hadrons by the analog of blackhole evaporation. This process is different from QCD type hadronization process occurring in hadronic collisions and this might allow to understand the anomalously low production of neutral pions.

Exotic mesons can also decay to lepton pairs and neutral exotic pions produce gamma pairs. These gamma pairs in principle provide a signature for the presence of exotic pions in the cosmic ray shower. If M_{89} proton is sufficiently long-lived enough they might be detectable.The properties of Centauro type events however suggest that M_{89} protons are short lived.

5.5.3 Peaks in cosmic gamma ray spectrum

The decay of the M_n pions at rest to two gamma rays produces gamma rays with energy $E_n = \frac{m(\pi_n)}{2}$. Therefore the cosmic gamma ray spectrum might show detectable signatures at these energies.

There is indeed some evidence for this kind of signatures in cosmic gamma ray background.

1. There are indications that the energy density of the cosmic gamma ray spectrum has peak at energy near 33.5 GeV ([91], see Fig. 5.8). A possible identification is as gamma rays produced by the decay of M_{89} pions. The energy distribution would be induced from the non-relativistic thermal distribution with temperature near $m(\pi_{89})$.

2. M_{61} corresponds to gamma ray threshold energy of $1.7 \cdot 10^6$ GeV. There is no visible signature at this energy but there is a good explanation for this. The e^+e^- pair production of the gamma rays with energy in certain energy range above 10^6 GeV with the photons of the cosmic microwave background implies strong reduction of the gamma ray flux by 2-3 orders of magnitude [87]. According to [87] the cutoff red-shift is of order $z-1 \simeq e^{-5}$ at this energy and corresponds to an upper bound of order 10^8 light years (the size of the large voids) for the distance of the source to be observable. The energy of the gamma rays coming from M_{61} pions happens to belong to the region with strongest absorption.

3. M_{31} corresponds to energy of the order of $1.7 \cdot 10^{10}$ GeV and jump in cosmic ray energy density is expected. As figure 5.8 shows, the cosmic ray spectrum contains indeed an bump at this energy [92, 86, 87]: the energy flux has a peak in short energy interval above $1.7 \cdot 10^{10}$ GeV. The simplest possibility is that the bump results from the decay of thermal M_{31} pions created in the decay of cosmic string. The effect is partially masked by the annihilation of gamma rays and photons of the cosmic radio wave background to e^+e^- pairs above the energy $5 \cdot 10^9$ GeV and the greatest effect comes at $3 \cdot 10^{10}$ GeV [87] (the mass of the exotic pion!).

   An alternative explanation for the bump is based on the assumption that cosmic rays are predominantly protons at these energies [93]. The proton component of the cosmic ray spectrum is predicted to effectively terminate at energy about $7 \cdot 10^{10}$ GeV due to pion production from cosmic microwave background. The experimental situation is unclear at this moment. Haverah Park detector claims the detection of 4 events with energies above 10^{11} GeV whereas Fly's Eye detector reports no events [94].

4. The theory predicts further peaks at $m_{19} = 64 \cdot m_{31} \simeq 2 \cdot 10^{12}$ GeV, $m_{17} = 2m_{19}$,... It might well be possible in not so far future to verify whether cosmic gamma ray flux contains these peaks.

5.5.4 Centauro type events, Cygnus X-3 and M_{89} hadrons

The results reported by Brazil-Japan Emulsion Chamber Collaboration [89, 95] on multiple production of hadrons induced by cosmic rays with energies $E_{lab} > 10^5$ GeV provide evidence for new Physics. The distributions for the transverse momentum p_T and longitudinal momentum fraction x for pions were found to differ from the distributions extrapolated from lower energies. The widening of the transversal momentum distributions has also been observed at accelerator energies (ISR above $\sqrt{s} = 63$ GeV and CERN SPS-p$\bar{p}$ Collider at $\sqrt{s} = 540$ GeV). Furthermore, exotic events called Geminion, Centauro, Chiron with emission of $n_B \leq 100$ hundred baryons but practically no pions were detected. There are also peculiar events associated with the radiation coming from Cygnus X-3. A recent summary about peculiar events is given in the review article [103].

Mirim, Acu and Quacu

The exotic cosmic ray events are described in the review article of [89]. In [89] the multiple production of pions is classified into 3 jet types called Mirim, Acu and Quacu. Although the transverse momentum distributions for pions observed at low energies are universal, Acu and Quacu jets are characterized by wider transverse momentum distributions with larger value of average transverse momentum p_T than in low energy pionization: this widening is in accordance with accelerator results. The distributions for the longitudinal momentum fraction x scale but differ from the low energy situation for Acu and Quacu jets.

In [89, 96, 97] a description of these events in terms of 'fireballs' decaying into ordinary hadrons were considered. The p_T distribution associated with Mirim is just the ordinary low energy transverse momentum distribution whereas the distributions associated with Acu and Quacu are wider. The masses of the fireballs were assumed to be discrete and were found to be $M_0 \sim 2-3$ GeV (Mirim), $M_1 \sim 15-30$ GeV (Acu) , $M_2 \sim 100-300$ GeV (Quacu). It should be noticed that the upper bounds for the masses associated with Acu and Quacu fireballs are roughly by a factor of two smaller than the masses 481 GeV associated with M_{89} pion and M_{89} proton. The temperatures were found to be in range $0.4-10$ GeV for Acu and Quacu fireball and to be substantially larger than the ordinary Hagedorn temperature $T_H \simeq 0.16$ GeV.

Chirons, Centauros, anti-Centauros, and Geminions

For the second class of events consisting of Chirons, Centauros and Geminions observed at laboratory energies $100-1000$ TeV pion production is strongly suppressed (gamma pairs resulting from the decay of neutral pions are almost absent) [89]. The primary event takes place few hundred meters above the detector and decay products are known to be hadrons and mostly baryons: about 15 (100) for Mini-Centauros (Centauros). This excludes the possibility that exotic hadrons decay in emulsion chamber and implies also that the decay mechanism of the primary particle is such that very few mesons are produced.

The fireball hypothesis has been applied also to Centauro type events assuming that fireballs corresponds to a different phase than in the case of Mirim, Acu and Quacu [89, 97]. The fireball masses associated with Mini-Centauro and Centauro are according to the estimate of [89] $M_{mini} = 35$ GeV and $M_{Centauro} = 23$ GeV. These masses are almost exactly one half of the masses of the M_{89} pion (70 GeV) and proton (470 GeV) respectively!

$$\begin{aligned} M_{Mini} &\simeq \frac{m(\pi_{89})}{2} , \\ M_{Centauro} &\simeq \frac{m(p_{89})}{2} . \end{aligned} \tag{5.5.2}$$

This suggests that the decay of cosmic gamma ray to M_{89} quark pair which in turn hadronizes to (possibly virtual) M_{89} hadrons induced by the interaction with the nucleon of atmosphere is the origin of Mini-Centauro/Centauro events.

The basic difference between the decaying fireballs in Acu/Quacu events and Centauro type events is that Acu/Quacu decays produce neutral pions unlike Centauros.

The appearance of the factor of 1/2 in the mass estimates needs an explanation. One explanation is systematic error in the evaluation of hadronic energy: for instance, the gamma inelasticity k_γ telling which fraction of hadronic energy is transformed to electromagnetic energy might be actually smaller than believed by a factor of order two. An alternative explanation is related to the decay mechanism of M_{89} particle: if the decay takes place via a decay to two off mass shell M_{89} hadrons decaying in turn to hadrons then the average rest energy of the fireball is indeed one half of the mass of the decaying on mass shell particle. The reason for the necessity of off mass shell intermediate states is perhaps the stability of the on mass shell exotic hadrons against the direct decay to ordinary hadrons.

Anti-Centauros are much like Centauros except that neutral pions are over-abundant [103]. The speculative model [104] relies on the notion of chiral condensates consisting of neutral pions in the case of Centauros and charged pions in the case of anti-Centauros.

The case of Cygnus X-3

There are peculiar events associated with the cosmic rays coming from Cygnus X-3 at gamma ray energies above 10^5 GeV [98]. The primary particle must be massless particle and is most probably ordinary gamma ray. The structure of the shower however suggests that the decaying particle is very massive! Furthermore, the muon content of the shower is larger than that associated with gamma ray shower. A possible explanation is that the gamma rays coming from Cygnus X-3 with energy above the threshold 10^4 GeV produce M_{89} hadrons, which in turn create the cosmic ray shower through the decay to M_{89} hadrons and the decay of these to the ordinary M_{107} hadrons: this indeed means that the gamma rays behave like a massive particles in the atmosphere.

5.5.5 TGD based explanation of the exotic events

The TGD based model for exotic events involve p-adic length scale hierarchy, many-sheeted space-time, and TGD inspired view about dark matter. A decisive empirical input comes from RHIC events suggesting that quark gluon plasma is actually a liquid like "macroscopic" quantum phase identifiable as a particular instance of dark matter.

General considerations

The mass estimates for the fireballs and the absence of neutral pions suggest that Mini-Centauro/Centauro type events correspond to the decay of M_{89} hadrons (pion/proton) to ordinary hadrons. The general model for the exotic events would be following.

1. Cosmic gamma ray decays first into M_{89} quark pair via electromagnetic interaction with the nucleon of the atmosphere. Pairs of Centauros/anti-Centauros and quark-gluon-plasma blobs explaing Mirim/Qcu/Quacu events would be naturally created in these collisions.

2. The quark pair in turn hadronizes to M_{89} hadrons decaying to virtual $k > 89$ hadrons which in turn end up via a sequential decay process to ordinary hadrons. This process is kinematically possible if the condition $E_{tot} > 2M^2/m_p$, is satisfied (M is the mass of the exotic hadron). For example, the energy of the gamma ray must be larger than 500 TeV for exotic proton pair production. For the exotic pion the corresponding lower bound is about 10 TeV. The energies of the exotic events are indeed above 100 TeV in accordance with these bounds. The average total energy is about $E_{tot} = 1740$ TeV for Centauros and $E_{tot} \simeq 903$ TeV for Mini-Centauros [97]. The mechanism implies that two M_{89} fireballs are produced. 'Binocular' events (Geminions) consisting of two widely separated fireballs have indeed been observed [89].

3. If anti-Centauros result via the same mechanism there must be a mechanism explaining why the production of neural pions varies from event to event. One proposal is that the difference is due to a formation of pion condensates consisting of neural *resp.* charged pions in the two situations [104]. This hypothesis would unify Centauro events with anti-Centauro events in which the production of neutral pions is abnormally high [103].

4. Mirim/Acu/Quacu events could correspond to the decay of a high temperature quark-gluon plasma blob, or rather color glass condensate, to hadrons (recall that the estimated plasma temperatures are much lower than for Centauros). The collision of M_{89} hadron possibly generated

in the interaction of the cosmic gamma ray with ordinary nucleon could induce both the decay of M_{89} hadron to virtual hadrons and generate quark-gluon plasma blob in the atmospheric target nucleus. Hagedorn temperature $T(k)$, $89 < k \leq 107$ is a good guess for the temperature of this plasma blob. RHIC findings [35] suggest that the blob corresponds to highly tangled hadronic string containing super-canonical dark matter and decaying by de-coherence to ordinary hadrons [J6].

Connection with TGD based model for RHIC events

The counterparts of Centauros and other exotic events have not been observed in accelerator experiments. More than a decade after writing the first version of the model for Centauros came however data from RHIC experiment [35], which seems to provide a connection between laboratory and cosmic ray data. In RHIC collisions of very energetic Gold nuclei are studied. The collisions were expected to create a quark gluon plasma freezing to ordinary hadrons. The surprise was that the resulting state behaves like an ideal liquid and has also black hole like properties [35].

Recall that the TGD based model [D7, J6] for RHIC findings is following.

1. The state in question corresponds to a highly entangled hadronic string at Hagedorn temperature defining the analog of black hole and decaying by evaporation. The gravitational constant defined by Planck length is effectively replaced by a hadronic gravitational constant defined by the hadronic length scale. p-Adic length scale hypothesis predicts entire hierarchy of Hagedorn temperatures.

2. Bose-Einstein condensate of gluons referred to as color glass condensate has been proposed as an explanation for the liquid like behavior of the quark-gluon phase. TGD based explanation for the liquid like state is that that the state in question corresponds to a large Bose-Einstein condensate like state of super-canonical particles resulting as hadronic space-time sheets fuse. Super-canonical bosons have vanishing electro-weak quantum numbers since super-canonical generators are either purely bosonic or possess quantum numbers of right handed neutrino. Dark matter is in question.

3. The large value of $\alpha_K = \alpha_s = 1/4$ for super-canonical bosons for ordinary value of $\hbar$ motivates the assumption is that the super-canonical many-particle state corresponds to a large value of $\hbar$ increasing the length and time scales of quantum coherence since typical length and time scales are proportional to $\hbar$. In the lowest order in $\hbar$ (classical limit) the physics does not change but higher order corrections are reduced since gauge coupling strengths are reduced. For the situation involving non-perturbative effects (typically binding energies) the change of $\hbar$ induces more dramatic effects.

A more precise model for exotic events

A more detailed formulation necessitates a rough model for the transformation of M_{89} hadrons to M_{107} hadrons.

1. On mass shell exotic hadrons can be assumed to be stable against direct decay to ordinary hadrons so that their decay must take place via a sequential decay to off mass shell exotic hadrons characterized by $107 > k > 89$, which eventually decay to ordinary hadrons. The simplest decay mode is the decay to two virtual exotic hadrons with average mass, which is one half of the mass of the decaying exotic hadron in accordance with observations.

2. M_{89} hadron decays to virtual hadrons with $p \simeq 2^k > M_{89}$ dominate over electro-weak decays since the characteristic time scale is defined by $\Lambda(QCD, M_{89}) = 512\Lambda(QCD, 107)$. This means that most of the energy in the process goes to virtual $k > 89$ virtual mesons. Neutral $k > 89$ virtual pions, if created, can decay to gamma pairs so that the problem of understanding the absence of neutral pions remains.

3. M_{89} hadronic space-time sheet suffers a topological phase transition to M_{107} hadronic space-time sheet via several steps $k = 89 \rightarrow k_1 > 89.. \rightarrow k_n = 107$. In the process the size of hadronic surface suffers a $2^9 = 512$-fold expansion meaning the increase of volume by a factor

for $2^{27} \sim 10^9/8$ so that a small scale Big Bang is really in question! The expansion brings in mind liquid-vapor phase transition but the freezing to hadrons (due to the properties of color coupling constant evolution) makes the transition more like a liquid-solid phase transition.

As noticed, all p-adic length scales in the range involved could be present but $p \simeq 2^k$ would define more stable intermediate states. A possible experimental signature for the sequence of the phase transitions labelled by $89 \leq k \leq 107$ is a bumpy structure of the detected hadronic cascades with a maximum of 17 maxima. This kind of structure with a constant distance between maxima and 11 maxima has been indeed observed for some cascades (see Fig. 8 of [103]).

A good guess for the critical temperature of the Big Bang like phase transition to occur is $T_{cr}(89) = km_{89}$, where k is some numerical factor. TGD inspired model for the early cosmology provides a universal hydrodynamics model for this period as a mini Big Bang, or rather "a soft whisper amplified to a relatively big bang", containing the duration of the period as the only parameter [D6].

4. If the decay process is fast enough, the density of virtual hadrons in the final state becomes so high that they form single highly tangled cosmic string in Hagedorn temperature $T(k)$. An entire sequence of $T(k) = km_k$, $107 > k > 89$ of phase transition temperatures could be involved without intermediate freezing to hadrons. Since the transformation of $k = 89$ hadrons to $k = 107$ hadrons would be essentially a decay process, the distribution of decay products is isotropic in the center of mass frame of $k = 89$ hadron (Centauros/anti-Centauros). The same conclusion holds true for the decay of quark gluon plasma (Mirim/Qcu/Quacu).

How to understand the anomalous production of pions?

One can imagine two different explanations for the varying number of pions in the events.

1. Restoration of electro-weak symmetry?

The anomalous production of pions might relate to the restoration of electro-weak symmetry in case of M_{89} hadrons. For M_{89} hadrons the restoration of the electro-weak symmetry would be natural since in TGD framework classical induce fields are massless for known non-vacuum extremals below the p-adic length scale $L(89)$ defining the fundamental electro-weak length scale. The finite size of the space-time sheet carrying these fields brings in the length scale determining the boson mass when the space-time sheet in question looks point like in the length scale resolution used.

Both Centauros and anti-Centauros can be understood if the transformation of M_{89} hadrons to ordinary hadrons generates "mis-aligned" pionic BE condensates. $U(2)_{ew}$ symmetry is restored for M_{89} hadrons and there is no preferred isospin direction for the order parameter of M_{89} pionic BE condensate. This BE condensate is however excluded by energetic considerations. The sequence of phase transitions leading to M_{107} hadrons involving intermediate p-adic length scales could however generate this kind of BE condensate.

If an overcooling occurs in the sense that electro-weak symmetry is not lost, the first intermediate pion condensate can correspond to π_+,π_- or π_0. Charged π condensates would be created in pairs with opposite charges. In this kind of situation the number of gamma rays produced in the decay to ordinary hadrons would vary from event to event.

The presence of pionic BE condensates favors the decay to M_{107} hadrons via hadronic intermediate states rather than via the cooling of partonic phase condensed on single tangled string whose length grows. This and the idea that $U(2)_{ew}$ symmetry could be exact for the dark matter phase, encourages to consider also the possibility that M_{89} hadron decays to a state consisting of dark M_{107} hadrons forming a BE condensate like state behaving like single coherent unit and interacting with the ordinary matter only via emission of dark gauge boson BE condensates de-cohering to ordinary gauge bosons.

Dark pionic BE condensates with various charges could be present. These dark π condensates would decay coherently to pairs of dark ew boson "laser beams", which can interact with the ordinary matter only after they have de-cohered to ordinary ew gauge bosons and remain undetected if the de-coherence time for dark bosons is long enough, probably not so. Dark hadron option could thus explain also the abnormally long penetration lengths.

2. Is long range charge entanglement involved?

The variation for the number of pions could involve electromagnetic charge entanglement between particles produced in the event and ordinary matter. This would guarantee strict charge conservation when the quantization axis for weak isospin for the resulting hadrons differs from that for the ordinary matter. The decay of the pion to gamma pair becomes possible only after the entanglement is reduced and if de-coherence time is long enough it is possible to understand the variation.

5.5.6 Cosmic ray spectrum and exotic hadrons

The hierarchy of M_n hadron physics provides also a mechanism producing ultra high energy cosmic gamma rays and hadrons.

Do gamma rays dominate the spectrum at ultrahigh energies?

A possible piece of evidence for M_{89} hadrons is related to the analysis [100] of the cosmic ray composition near 10^9 GeV. The analysis was based on the assumption that the spectrum consists of nuclei. The assumptions and conclusions of the analysis can be criticized:

1. There is argument [101], which states that the interaction of protons having energy above 10^9 GeV with the cosmic microwave background implies pion pair creation and a rapid loss of proton energy so that the contribution of protons should be strongly suppressed in the cosmic ray spectrum above $E = 7 \cdot 10^{10}$ GeV. If protons dominate, cosmic ray spectrum should effectively terminate at energy of order $7 \cdot 10^{10}$ GeV: some events above $E = 10^{11}$ GeV have been however detected [94].

2. It is not obvious whether one can distinguish between protons and gamma rays at these energies since the muon content of the photon and proton showers are near to each other at these energies [86]. Therefore the particles identified as protons might well be gamma rays.

3. The spectrum can be fitted assuming that cosmic ray spectrum has two components. Light component ('protons') can be identified as protons and He nuclei. The heavy component ('Fe') corresponds to Fe and heavier nuclei. The nuclei between He and Fe seem to be peculiarly absent. Furthermore, there are also indications that spectrum contains only light nuclei in the range $3 \cdot 10^7 - 10^{11}$ GeV [102].

An alternative interpretation suggested also in [86] is that cosmic ray flux is dominated by gamma rays at these energies. 'Protons' correspond to gamma rays interacting ordinarily with matter. 'Fe nuclei' correspond to the fraction of gamma rays decaying first into M_{89} exotic quark pair producing corresponding exotic hadrons, which then decay to ordinary hadrons and produce showers resembling ordinary heavy nucleus shower. Super-canonical vision allows to consider the possibility that 'protons' correspond to super-canonical part of proton having essentially the same mass.

Hadronic component of the cosmic ray spectrum

The properties of the hadronic cosmic ray spectrum above $4 \cdot 10^5$ GeV are not well understood.

1. It has turned out difficult to invent acceleration mechanisms producing hadronic cosmic rays having energies above 10^5 GeV [100].

2. The spectrum contains a 'knee' (power $E^{-2.7}$ changes to about E^{-3} at the knee), which is at the energy $3 \cdot 10^6$ GeV [100] and equals to the mass of M_{61} pion. It is difficult to understand how the knee is generated although several explanations have been proposed (these are reviewed shortly in [100]).

A possible solution of the problems is that part of the hadronic cosmic rays are generated in the decay of string like objects rather than by some acceleration mechanism. Assume that M_{n_k} hadron is created in the decay cascade. Since $M_{n_{k+m}}$, $m = 1, 2, ..$ hadrons can have rest masses above M_{n_k} threshold mass, one can consider the possibility that M_{n_k} hadron decays sequentially to ordinary M_{107} hadron with arbitrary large rest mass (even larger than M_{n_k} pion mass) and that this ordinary hadron in turn produces some very energetic low mass hadrons, say proton and antiproton, identifiable as cosmic rays. The most efficient producers of hadrons are M_{n_k} pions since these are produced most abundantly in the decay of $M_{n_{k+1}}$ hadrons. M_{n_k} pion at rest cannot however decay to ordinary hadrons with energy above M_{n_k} pion mass. Therefore the slope of the cosmic ray energy flux should become steeper above M_{n_k}, in particular M_{61}, threshold.

The problem of relic quarks and hierarchy of QCD:s

Baryon and lepton numbers are conserved separately in TGD and one of the basic problems of the gauge theories with conserved baryon number is the problem of relic quarks. Hadronization starts in temperature of the order of quark mass and since hadronization is basically many quark process it continues until the expansion rate of the Universe becomes larger than the rate of the hadronization. As a consequence the number density of relic quarks is much larger than the upper bound $n_{relic} < \rho_B/m_q = 10^{-9}n_\gamma m_p/m_q$ obtained from the requirement that the contribution of relic quarks to mass density is smaller than the baryonic mass density. There is also an experimental upper bound $n_{relic} < 10^{-28}n_\gamma$.

The assumption about the existence of QCD:s with a hierarchy of increasing scales $\Lambda_{QCD}(M_n)$ implies that the length scale $L(n) \sim 1/\sqrt{\Lambda_{QCD}(M_n)}$ below which quarks are free, decreases with increasing cosmic temperature and therefore the problem of the relic quarks disappears.

5.5.7 Ultrahigh energy cosmic rays as super-canonical quanta?

Near the end of year 2007 Pierre Auger Collaboration made a very important announcement relating to ultrahigh energy cosmic rays. I glue below a popular summary of the findings [128].

Scientists of the Pierre Auger Collaboration announced today (8 Nov. 2007) that active galactic nuclei are the most likely candidate for the source of the highest-energy cosmic rays that hit Earth. Using the Pierre Auger Observatory in Argentina, the largest cosmic-ray observatory in the world, a team of scientists from 17 countries found that the sources of the highest-energy particles are not distributed uniformly across the sky. Instead, the Auger results link the origins of these mysterious particles to the locations of nearby galaxies that have active nuclei in their centers. The results appear in the Nov. 9 issue of the journal Science.

Active Galactic Nuclei (AGN) are thought to be powered by supermassive black holes that are devouring large amounts of matter. They have long been considered sites where high-energy particle production might take place. They swallow gas, dust and other matter from their host galaxies and spew out particles and energy. While most galaxies have black holes at their center, only a fraction of all galaxies have an AGN. The exact mechanism of how AGNs can accelerate particles to energies 100 million times higher than the most powerful particle accelerator on Earth is still a mystery.

What has been found?

About million cosmic ray events have been recorded and 80 of them correspond to particles with energy above the so called GKZ bound, which is $.54 \times 10^{11}$ GeV. Electromagnetically interacting particles with these energies from distant galaxies should not be able to reach Earth. This would be due to the scattering from the photons of the microwave background. About 20 particles of this kind however comes from the direction of distant active galactic nuclei and the probability that this is an accident is about 1 per cent. Particles having only strong interactions would be in question. The problem is that this kind of particles are not predicted by the standard model (gluons are confined).

What does TGD say about the finding?

TGD provides an explanation for the new kind of particles.

1. The original TGD based model for the galactic nucleus is as a highly tangled cosmic string (in TGD sense of course [D5]. Much later it became clear that also TGD based model for black-hole is as this kind of string like object near Hagedorn temperature [D5]. Ultrahigh energy particles could result as decay products of a decaying split cosmic string as an extremely energetic galactic jet. Kind of cosmic fire cracker would be in question. Originally I proposed this decay as an explanation for the gamma ray bursts. It seems that gamma ray bursts however come from thickened cosmic strings having weaker magnetic field and much lower energy density [D7].

2. TGD predicts particles having only strong interactions [F2]. I have christened these particles super-canonical quanta. These particles correspond to the vibrational degrees of freedom of partonic 2-surface and are not visible at the quantum field theory limit for which partonic 2-surfaces become points.

What super-canonical quanta are?

Super-canonical quanta are created by the elements of super-canonical algebra, which creates quantum states besides the super Kac-Moody algebra present also in super string model. Both algebras relate closely to the conformal invariance of light-like 3-surfaces.

1. The elements of super-canonical algebra are in one-one correspondence with the Hamiltonians generating symplectic transformations of $\delta M^4_+ \times CP_2$. Note that the 3-D light-cone boundary is metrically 2-dimensional and possesses degenerate symplectic and Kähler structures so that one can indeed speak about symplectic (canonical) transformations.

2. This algebra is the analog of Kac-Moody algebra with finite-dimensional Lie group replaced with the infinite-dimensional group of symplectic transformations [B3]. This should give an idea about how gigantic a symmetry is in question. This is as it should be since these symmetries act as the largest possible symmetry group for the Kähler geometry of the world of classical worlds (WCW) consisting of light-like 3-surfaces in 8-D imbedding space for given values of zero modes (labelling the spaces in the union of infinite-dimensional symmetric spaces). This implies that for the given values of zero modes all points of WCW are metrically equivalent: a generalization of the perfect cosmological principle making theory calculable and guaranteing that WCW metric exists mathematically. Super-canonical generators correspond to gamma matrices of WCW and have the quantum numbers of right handed neutrino (no electro-weak interactions). Note that a geometrization of fermionic statistics is achieved.

3. The Hamiltonians and super-Hamiltonians have only color and angular momentum quantum numbers and no electro-weak quantum numbers so that electro-weak interactions are absent. Super-canonical quanta however interact strongly.

Also hadrons contain super-canonical quanta

One can say that TGD based model for hadron is at space-time level kind of combination of QCD and old fashioned string model forgotten when QCD came in fashion and then transformed to the highly unsuccessful but equally fashionable theory of everything.

1. At quantum level the energy corresponding to string tension explaining about 70 per cent of proton mass corresponds to super-canonical quanta [F4]. Supercanonical quanta allow to understand hadron masses with a precision better than 1 per cent.

2. Super-canonical degrees of freedom allow also to solve spin puzzle of the proton: the average quark spin would be zero since same net angular momentum of hadron can be obtained by coupling quarks of opposite spin with angular momentum eigen states with different projection to the direction of quantization axis.

3. If one considers proton without valence quarks and gluons, one obtains a boson with mass very nearly equal to that of proton (for proton super-canonical binding energy compensates quark masses with high precision). These kind of pseudo protons might be created in high energy collisions when the space-time sheets carrying valence quarks and super-canonical space-time sheet separate from each other. Super-canonical quanta might be produced in accelerators in this manner and there is actually experimental support for this from Hera.

4. The exotic particles could correspond to some p-adic copy of hadron physics predicted by TGD and have very large mass smaller however than the energy. Mersenne primes $M_n = 2^n - 1$ define excellent candidates for these copies. Ordinary hadrons correspond to M_{107}. The protons of M_{31} hadron physics would have the mass of proton scaled up by a factor $2^{(107-31)/2} = 2^{38} \simeq 2.6\times 10^{11}$. Energy should be above 2.6×10^{11} GeV and is above $.54 \times 10^{11}$ GeV for the particles above the GKZ limit. Even super-canonical quanta associated with proton of this kind could be in question. Note that CP_2 mass corresponds roughly to about 10^{14} proton masses.

5. Ideal blackholes would be very long highly tangled string like objects, scaled up hadrons, containing only super-canonical quanta. Hence it would not be surprising if they would emit super-canonical quanta. The transformation of supernovas to neutron stars and possibly blackholes would involve the fusion of hadronic strings to longer strings and eventual annihilation and evaporation of the ordinary matter so that only super-canonical matter would remain eventually. A wide variety of intermediate states with different values of string tension would be possible and the ultimate blackhole would correspond to highly tangled cosmic string. Dark matter would be in question in the sense that Planck constant could be very large.

5.6 TGD based explanation for the anomalously large direct CP violation in $K \to 2\pi$ decay

KTeV collaboration in Fermilab [105] has measured the parameter $|\epsilon'/\epsilon|$ characterizing the size of the direct CP violation in the decays of kaons to two pions. The value of the parameter was found to be $|\epsilon'/\epsilon| = (2.8 \pm .1)10^{-3}$ and is almost by an order of magnitude larger than the naive standard model expectations based on the hypothesis that direct CP breaking is induced by CKM matrix. In [85] it was shown that the value of the parameter could be understood without introducing any new physics if the value of running strange quark mass at m_c is about $m_s(m_c) = .1$ GeV and $m_d << m_s$ holds true.

5.6.1 How to solve the problems in TGD framework

Problems

Also in TGD framework the situation looks confusing.

1. The TGD based prediction for the value of the CP breaking parameter for CKM matrices satisfying the constraints coming from p-adicity is within the experimental constraints $1.0 \times 10^{-4} \leq J \leq 1.7 \times 10^{-4}$ coming from the standard model so that J produces no problems (see [F4] or Appendix for the CKM matrix as predicted by TGD).

2. The dominating contributions of the chiral field theory to $Re(\epsilon'/\epsilon)$ are proportional to $1/(m_s + m_d)^2$. The predictions of p-adic thermodynamics for s and d quark masses for $k(d) = k(s) = 113$ are $m_d(113) = m_s(113) = 90$ MeV and if this mass can be interpreted as $m_s(m_c) \simeq 0.1$ GeV, the prediction is too small by a factor 1/4. Even worse, if m_s corresponds to the scaled up mass $m_s(109) \simeq 360$ MeV of the s quark inside kaon, the situation changes completely and ϵ'/ϵ is too small by a factor $\sim 1/4.5^2 \simeq .05$.

3. TGD predicts that family replication phenomenon has also a bosonic counterpart. In the original scenario gauge bosons had single light-like wormhole throat as space-time correlate just like fermions and two exotic gluons were predicted corresponding to $g = 1$ and $g = 2$. The assumption that fermions at partonic space-time sheets are free fermions however forces to identify gauge bosons as wormhole contacts such that the two light-like wormhole throats carry quantum numbers of fermion and antifermion. Gauge bosons can be arranged into SU(3) singlet corresponding to ordinary gauge bosons and octet, where SU(3) states correspond to pairs (g_1, g_2) of handle numbers $0 \leq g_i \leq 2$.

   The experimental non-existence of flavor changing currents suggest strongly that the masses of octet gauge bosons are high. This requires that they correspond to $L(89)$ or even shorter p-adic length scale. Hence these gauge bosons are not interesting from the point of view of CP breaking.

4. The recent breakthrough in p-adic mass calculations for hadrons [F4] led also the understanding of non-perturbative aspects of hadron physics in terms of super-canonical bosons which correspond to single light-like wormhole throat so that they couplings to quarks in the sense of generalized Feynman diagrams do not imply flavor changing neutral currents.

The basic prediction is that topologically mixed super-canonical bosons are responsible for the most of the mass of proton and that it is possible to deduce the super-canonical content of hadrons from their masses if their topological mixing is assumed to be same as for U type quarks. The masses of these bosons correspond to p-adic length scale $L(107)$ and are small in length scale $L(89)$ relevant for CP breaking. These observations suggest that higher gluon genera of the earlier model should be replaced with super-canonical gluons.

In the standard diagrammatic expression for the CP breaking parameter gluon propagators are replaced by a sum of ordinary massless and two exotic gluon massive gluon propagators. The fact that the matrix elements relevant for the estimation of the CP breaking parameter are estimated at momentum transfer of order $\mu = m_W$, implies that gluon masses do not significantly change the contribution of the super-canonical gluons to the amplitude apart from the change in value of color coupling strength. Hence the penguin amplitudes are simply multiplied by some factor X determined by the number of super-canonical gluons light in length scale $L(89)$ and by the coupling constants of these gluons and the ratio ϵ'/ϵ is multiplied by a factor X. Unfortunately, it is not possible to calculate this factor at this stage.

The model based on exotic gluons and current quarks

It is essential that exotic gluons correspond to single light-like wormhole throat and thus have family replication phenomenon analogous to that of fermions. Two models can be considered.

1. The original model based on the assumption that ordinary gauge bosons correspond to single wormhole throat. There are good reasons to believe that this interpretation is wrong.

2. The new model based on super-canonical exotic gluons whose number is not known but is multiple of 3. The couplings to quarks are not known. Also color single super-canonical bosons could be also present.

1. The difficulty of the original model

The problem of this model is that assuming exotic gluons in sense 1) ϵ'/ϵ would be still by a factor .15 too small for $m_s(109)$ relevant for kaons.

The basic observation is that the gluon contribution is proportional to $1/(m_s + md)^2$ and for $m_s(113)$ instead of $m_s(119)$ ϵ'/ϵ would be a fraction $(16+1)/2 = 8.5$ large and by a factor 1.275 larger than the experimental value since $m_d = m_s$ rather than $m_d << m_s$ holds true.

This observation stimulated the idea that a transition $s_{109} \rightarrow s_{113}$ occurs before electro-weak process and would have an interpretation as a transformation of a constituent quark to current quark. This requires that the amplitudes for the transition $s(109) \rightarrow s(113)$ and its reversal are near to unity.

The question is why $s(109) \rightarrow s(113)$ constituent-current transformation should occur in electro-weak interactions and why it occurs with amplitude $A \sim 1$. Of course it could also be that also d quark is transformed to a very low mass variant with mass about 4 MeV predicted by chiral field theory. This would correspond to $k = 125$. As a result the amplitude would be multiplied by a factor 4 and $A = 1/2$ would become possible.

For some reason the join along boundaries bonds feeding em gauge flux of s quark to $k = 109$ space-time sheet would be transferred to nuclear space-time sheets with $k = 113$ before the electro-weak scattering process responsible for the CP breaking. Note that the value of strange quark mass about 176 MeV deduced from τ lepton decay rate corresponds to $m_s(111)$ in a good approximation. Also this indicates that various scaled up variants of quark masses can appear in the electro-weak dynamics as intermediate states.

The assumption for the proportionality $\epsilon'/\epsilon \propto 1/(m_s + m_d)^2$ derivable from chiral field theory can be criticized. Finding a justification for this assumption seems to be a non-trivial challenge since it is not at all clear that chiral field theory based on $SU(3)$ flavor symmetry makes sense in TGD context.

2. Super-canonical variant of the original model

For super-canonical gluons one can predict only that the relevant gluon exchange amplitude increases by a factor

$$X = \sum_{i,j} \alpha_s(B_{i,j})) \quad ,$$

where $\alpha_s(B_{i,j})$ is the color coupling strength to j:th generation of the super-canonical gluon B_i. In principle also color neutral super-canonical bosons having spin might contribute.

For $\alpha_s(B_{i,j}) = \alpha_s(B_i)$ one would have

$$X = 3\sum_i \alpha_s(B_i) \ .$$

If the number of light super-canonical gluons large enough, it is possible to have a large enough value of X to compensate for the factor .14 so that the assumption about the transformation $s(109) \to s(113)$ from constituent quark to current quark would become un-necessary. $X \sim 8$ would be needed.

Recall that super-canonical algebra is analogous to Kac-Moody algebra in the sense that finite-dimensional Lie-group is replaced with symplectic group. Super-canonical gluons correspond to states created by super-algebra generators, which are in one-one correspondence Hamiltonians of $\delta M^4_+ \times CP_2$ subject to some additional conditions making subset of states zero norm states. Therefore more than single octet of super-canonical bosons and also higher dimensional representations might be possible.

All depends on which of these super-canonical states correspond to light particles. This in turn depends on the details of super-canonical representations (they correspond to the states of negative conformal weight annihilated by Virasoro generators L_n, $n < 0$ [C2]). Here the help of a mathematician specialized to the representations of super-conformal algebras would be needed.

At this moment it is not possible to know whether the transformation to current quark is needed or even possible and this motivates the following discussion of the basic notions and chiral field theory approach in more detail in order to clarify what is involved.

5.6.2 Basic notations and concepts

Until 1963 CP symmetry was believed to be an exact symmetry of Nature. In this year it was however observed by Christensen, Cronin, Fitch and Turlay that CP symmetry is violated in hadronic decays of neutral kaons. In order to interpret the experimental evidence one must consider the strong Hamiltonian eigen states K^0 and its CP conjugate $\bar{K}^0$ as a mixture of physical short lived K_S decaying predominantly to two pions and long-lived K_L decaying mostly semi-leptonically and into 3 pion states. Two- and three pion final states have odd and even CP parity. In absence of CP breaking one would identify K_S and K_L as the CP even and CP odd states

$$\begin{aligned} K_1 &= (K^0 + \bar{K}^0)/\sqrt{2} \ , \\ K_2 &= (K^0 - \bar{K}^0)/\sqrt{2} \ . \end{aligned} \tag{5.6.0}$$

What was observed in 1963 was that K_L decays also to two-pion final states.

There are two mechanisms of CP violation. The indirect mechanism involves a slight mixing of K^1 and K^2 characterized by a complex mixing parameter $\bar{\epsilon}$

$$\begin{aligned} K_S &= \frac{K_1 + \bar{\epsilon}K_2}{1+|\bar{\epsilon}|^2} \ , \\ K_L &= \frac{K_2 + \bar{\epsilon}K_1}{1+|\bar{\epsilon}|^2} \ . \end{aligned} \tag{5.6.0}$$

Direct mechanism involves the direct decay of K_2 to two pion state and is induced by the weak interaction Lagrangian L_W directly. Both mechanisms can be parameterized in terms of the small ratios

$$\begin{aligned} \eta_{00} &= \frac{\langle \pi^0\pi^0|L_W|K_L\rangle}{\langle \pi^0\pi^0|L_W|K_S\rangle} \ , \\ \eta_{+-} &= \frac{\langle \pi^+\pi^-|L_W|K_L\rangle}{\langle \pi^+\pi^-|L_W|K_S\rangle} \ . \end{aligned} \tag{5.6.-1}$$

Here L_W represents the $\Delta S = 1$ part of the weak Lagrangian. The equations for η parameters can be also written as

$$\begin{aligned} \eta_{00} &= \epsilon - \frac{2\epsilon'}{1-\omega\sqrt{2}} \simeq \epsilon - 2\epsilon' \ , \\ \eta_{+-} &= \epsilon - \frac{2\epsilon'}{1+\omega/\sqrt{2}} \simeq \epsilon + \epsilon' \ . \end{aligned} \quad (5.6.-1)$$

Parameter $\bar{\epsilon}$ is simply related to ϵ. The parameter ω measures the ratio

$$|\omega| = \frac{|\langle(\pi\pi)_{I=2}|L_W|K_S\rangle|}{|\langle(\pi\pi)_{I=0}|L_W|K_S\rangle|} \simeq 1/22.2 \ . \quad (5.6.0)$$

$I = 0$ and $I = 2$ denote the isospin states of final state pions.

The CP violating parameters are expressible in terms of $K_{S,L}$ decay amplitudes as

$$\begin{aligned} \epsilon &= \frac{\langle(\pi\pi)_{I=0}|L_W|K_L\rangle}{\langle(\pi\pi)_{I=0}|L_W|K_S\rangle} \ , \\ \epsilon' &= \frac{\epsilon}{\sqrt{2}}\left[\frac{\langle(\pi\pi)_{I=2}|L_W|K_L\rangle}{\langle(\pi\pi)_{I=0}|L_W|K_L\rangle} - \frac{\langle(\pi\pi)_{I=2}|L_W|K_S\rangle}{\langle(\pi\pi)_{I=0}|L_W|K_S\rangle}\right] \ . \end{aligned} \quad (5.6.0)$$

By Watson's theorem one can write the generic amplitudes for K^0 and $\bar{K}^0$ decay as

$$\begin{aligned} \langle(\pi\pi)_I|L_W|K^0\rangle &= -iA_I exp(i\delta_I) \ , \\ \langle(\pi\pi)_I|L_W|\bar{K}^0\rangle &= -iA_I^* exp(i\delta_I) \ , \end{aligned} \quad (5.6.0)$$

where the phases δ_I arise from the pion finals state interactions. In good approximation ($|\bar{\epsilon} ImA_0| \ll |ReA_0|$, $|\bar{\epsilon}|^2 \ll 1$) one can write

$$\begin{aligned} \epsilon' &= exp(i(\pi/2+\delta_2-\delta_1)) \times \frac{\omega}{\sqrt{2}} \times (\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}) \ , \\ \omega &= \frac{ReA_2}{ReA_0} \ . \end{aligned} \quad (5.6.0)$$

With the approximations used one obtains a relationship

$$\epsilon' = \bar{\epsilon} + i\frac{ImA_0}{ReA_0} \ . \quad (5.6.1)$$

One can find a more detailed representation of the subject in various review articles [112, 63]. The standard model of CP breaking is based on the presence of complex phases in CKM matrix.

The value of the parameter ϵ describing indirect CP violation is well established and given by

$$|\epsilon| = (2.266 \pm .017) \times 10^{-3} \ .$$

The phases of ϵ and ϵ' are in good approximation identical so that their signs are same. The value of $Re(\epsilon'/\epsilon)$ was finally established by KTeV collaboration at Fermi Lab to be

$$Re(\frac{\epsilon'}{\epsilon}) = (2.8 \pm .01) \times 10^{-3} \ .$$

The measurement is consistent with the result of the CERN experiment NA31, which has also found a non-vanishing value for this parameter.

There are several theories of CP violation. The so called milliweak theory predicts vanishing value of ϵ'. The model based on the presence of CP breaking phases in three-generation CKM matrix predicts non-vanishing value for the parameter. Also Higgs particles can effect the value of the parameter in standard model. Standard model predicts this parameter to be nonzero but the expectation has been that the value is roughly ten times smaller than the measured value.

A possible explanation of the effect which does not introduce new physics is based on the hypothesis that the mass of s quark is smaller than the mass of d quark: the running mass $m_s(2\ GeV) \simeq .1$ GeV is needed to explain the anomaly if CP breaking parameter J is taken to be in the range $(1-1.7)\times 10^{-4}$ claimed in [66] to follow from unitarity. There is however experimental evidence from τ decays for $m_s(m(\tau)) = (172 \pm 31)\ MeV$. This suggests that some new short length scale physics is involved.

Standard model prediction for $Re(\epsilon'/\epsilon)$ [85] can be summarized in a handy formula

$$\begin{aligned} Re(\frac{\epsilon'}{\epsilon}) &= J \times \left[-1.35 + R_s\left(A_6 B_6^{1/2} + A_8 B_8^{3/2}\right)\right] \ , \\ A_6 &= 1.1|r_Z^8| \ , \\ A_8 &= 1.0 - .67|r_Z^8| \ . \end{aligned} \tag{5.6.0}$$

The subscript Z refers to renormalization mass m_Z. The parameter R_s is given by

$$R_s \simeq \left[\frac{150\ MeV}{m_s(m_c) + m_d(m_c)}\right]^2 \ . \tag{5.6.1}$$

The dominating contributions to $Re(\frac{\epsilon'}{\epsilon})$ come from second (A_6) and third terms (A_8). These terms correspond to gluonic and electro-weak penguin diagram contributions to the CP breaking decays and of opposite sign. Clearly, the sum of the two terms is roughly one third of the gluonic term alone.

5.6.3 Separation of short and long distance physics using operator product expansion

The calculation of CP breaking parameters involves physics in very wide energy scale. The strategy is to derive low energy effective action by functionally integrating over the short distance effects coming from energies larger than m_c. This leads to Wilson expansion for the low energy electro-weak effective Lagrangian

$$L_{low,W} = -\sum_i C_i(\mu, m_c, m_b, m_t, m_W, ...)Q_i(\mu) \ . \tag{5.6.2}$$

The coefficients C_i of the operators Q_i in the low energy effective action for light quarks (u, d, s) are functionals of various parameters characterizing short distance physics. The coefficients $C_i(\mu)$ in Wilson expansion of electro-weak effective action can be written as

$$C_i(\mu) = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \left[x_i(\mu) + \tau y_i(\mu)\right] \ . \tag{5.6.3}$$

Here x_i and y_i are Wilson coefficients. V_{ij} denotes CKM matrix and τ is defined as $\tau = V_{td}V_{ts}^*/V_{ud}V_{us}^*$. $V_{td}V_{ts}^*$ is identical with CP breaking invariant J in standard parametrization. Coefficients $y_i(\mu)$ summarize short distance CP breaking physics and in order to determine CP breaking one needs to consider only the coefficients y_i.

Long distance physics is the difficult part of the calculation since it involves calculation of matrix elements of the quark operators Q_i between initial kaon state and final two-pion state. There are several approaches to the problem. Chiral field theory [82] is phenomenological approach and relies on the idea that low energy effective action for quarks can be expressed in a good approximation using meson fields. Lattice QCD is believed to provide a more fundamental direct method for the calculation of the correlation functions of Q_i.

Short distance physics

In present initial states are kaons and μ denotes the momentum exchange for a typical diagram associated with the scattering of $d\bar{s}$ quark to final state consisting of of light quarks. μ is taken to be of order m_W and by using renormalization group equations one can deduce the values of the coefficients $C_i(\mu)$ at energy scales, typically of order 1 GeV.

The basic standard diagrams contributing to the $\Delta S = 1$ and $\Delta S = 2$ processes are given by the figure below.

The quark operators Q_i appearing in the expansion can be classified. In present case the list of relevant operators correspond to various terms possible in four-fermion Fermi interaction and are given by the following list.

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A} \ , \tag{5.6.3}$$

$$Q_2 = (\bar{s}u)_{V-A}(\bar{u}d)_{V-A} \ , \tag{5.6.3}$$

$$\begin{aligned} Q_{3,5} &= (\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V\mp A} \ , \\ Q_{4,6} &= (\bar{s}_\alpha d_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V\mp A} \ , \\ Q_{7,9} &= \frac{3}{2}(\bar{s}d)_{V-A}\sum_q \hat{e}_q(\bar{q}q)_{V\pm A} \ , \\ Q_{8,10} &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A}\sum_q \hat{e}_q(\bar{q}_\beta q_\alpha)_{V\pm A} \ . \end{aligned} \tag{5.6.1}$$

α, β denote color indices and $\hat{e}_q$ denote quark charges. $V \pm A$ refers to the Dirac structure $\gamma_\mu(1 \pm \gamma_5)$. Q_2 is induced by mere W exchange whereas gluonic loop corrections to Q_2 induce Q_1. QCD through penguin loop induces the penguin operators Q_{3-6}. Electro-weak loops, in which penguin gluon is replaced with electro-weak gauge boson, induce $Q_{7,9}$ and part of Q_3. The operators $Q_{8,10}$ are induced by the QCD renormalization of the electro-weak loop operators $Q_{7,9}$.

As far as the calculation of ϵ'/ϵ is considered, the dominating contributions come from the penguin diagrams, which are proportional to the vertices $s\bar{d}V$, where V is either gluon or electro-weak gauge boson and to the propagator denominator of V with momentum squared equal to momentum exchange between initial state quarks, which equals to $(p_i - p_j)^2 = \mu^2$. For option 2) the standard gluon contribution is replaced with a sum over contributions of ordinary and exotic gluons. For option 1) situation is more complicated since $g > 0$ gluons can change the genus of the fermion.

The operators Q_6 and Q_8 give the dominating contributions to ϵ'/ϵ and these contributions are competing. Q_6 and Q_8 differ only by the fact that in Q_8 penguin gluon is replaced with penguin electro-weak boson γ or Z^0. For neutral kaon initial state electro-weak penguin diagram is proportional to the product $e_q e_{\bar{q}} = -e_q^2$ of the virtual quark whereas in case of gluons the factor $Tr(T^aT^a) > 0$ appears. Therefore the contributions associated with Q_6 and Q_8 are of opposite sign and mutually competing.

Detailed calculations lead to the formula already described:

$$\begin{aligned} Re(\frac{\epsilon'}{\epsilon}) &= J \times \left[-1.35 + R_s\left(A_6 B_6^{1/2} + A_8 B_8^{3/2}\right)\right] \ , \\ A_6 &= 1.1|r_Z^8| \ , \\ A_8 &= 1.0 - .67|r_Z^8| \ . \end{aligned} \tag{5.6.0}$$

for $Re(\epsilon'/\epsilon)$. The coefficients B_6 and B_8 code the long distance physics and their values do not differ too much from $B_6 = B_8 = 1$. Clearly, the sum of Q_6 and Q_8 contributions is roughly one third of the Q_6 contribution alone. From the general structure of Feynman diagrams it ss clear that for option 2) the effect caused by the introduction of exotic gluons is in a good approximation a simple scaling of the Q_6 contribution by a factor 3 in the approximation that gluon masses are negligible as compared to W mass, and that this new contribution can enhance direct CP breaking dramatically.

Chiral field theory approach

The basic problem is to calculate electro-weak matrix elements of the quark effective action between hadronic states. These matrix elements reduce to vacuum expectation values of various quark bi-linears appearing in four-fermion Fermi interaction Lagrangian. This problem is very difficult since non-perturbative QCD is involved in an essential manner. An attempt to circumvent this problem [82] is based on the hypothesis that low energy effective action for quarks is essentially equivalent with the low energy effective action, where pseudoscalar meson fields as dynamical fields and scalar, vector and axial vector meson fields occur as external fields not subject to variations. Quark masses are identified as vacuum expectation values of the external scalar meson field. The approximate symmetry of the chiral field theory is flavor $SU(3)_L \times SU(3)_R$ which is exact symmetry at the limit of massless quarks. This symmetry can be realized if mesons are represented by an element U of $SU(3)$ regarded as a dynamical field: the two $SU(3)$:s act on U from left and right respectively. For small perturbations around ground state mesons correspond to various Lie-algebra generators of $SU(3)$. Chiral field develops vacuum expectation value. If vacuum expectation is not proportional to unit matrix it corresponds to the presence of coherent states associated with the neutral components of the pseudo scalar meson field.

The basic formulation of the chiral field theory approach is described in [82] whereas its application to the calculation of ϵ'/ϵ is described in [63]. The strong part of the chiral action [82] is given by the formula

$$\begin{aligned} L_S &= \frac{f^2}{4}\left[Tr\{D_\mu U^\dagger D^\mu U\} + 2B_0 Tr\{(s-ip)U\} + 2B_0^* Tr\{(s+ip)U^\dagger\}\right] \\ &+ \frac{1}{12} H_0 D_\mu \theta D^\mu \theta \ . \end{aligned} \quad (5.6.0)$$

D_μ denotes the covariant derivative defined by the couplings to the left and right handed gauge bosons L_μ and R_μ defined as superpositions $R_\mu = v_\mu + a_\mu$ and $L_\mu = v_\mu - a_\mu$ of the vector and axial vector mesons fields v and a. Action contains three coupling constant parameters: f, B_0 and H_0, which is present because the presence of color instantons can lead to a non-vanishing value of the θ parameter in QCD. In lowest order f is pion decay constant f_π and B_0 sets the scale in the formula $M_M^2 = B_0(\sum_i m(q_i))$ inspired by broken $SU(3)$ symmetry and resulting as a prediction of the model. The components for the non-vanishing vacuum expectation value for the external scalar field are identified as quark masses. The generation of vacuum expectation value of s implies that quark condensates are developed:

$$\begin{aligned} \langle \bar{q}_i q_j \rangle &= B_0 f^2 \delta_{i,j} \ , \\ B_0 f^2 &= \frac{f_\pi^2 m_\pi^2}{(m_u + m_d)} = \frac{f_K^2 m_K^2}{(m_s + m_d)} \ . \end{aligned} \quad (5.6.0)$$

Note that the strong part of the chiral Lagrangian is invariant under the overall scaling of quark masses.

The weak part of the chiral action corresponds to the sigma model counterpart of the most general electro-weak four-fermion action. The recipe for constructing this action is described in more detail in [63] and can be summarized as rules associating with various fermionic bi-linears appearing in the generalized Fermi action corresponding terms of the weak part of the chiral action. In particular, the following rules hold true:

$$\begin{aligned} \bar{q}_L^j \gamma^\mu q_L^i &\rightarrow -i f_\pi^2 (U^\dagger D_\mu U)_{ij} \ , \\ \bar{q}_R^j \gamma^\mu q_R^i &\rightarrow -i f_\pi^2 (U D_\mu U^\dagger)_{ij} \ , \\ \bar{q}_L^j \gamma^\mu q_R^i &\rightarrow -2B_0 \left[\frac{f^2}{4} U + \text{higher order terms}\right]_{ij} \ , \\ \bar{q}_R^j \gamma^\mu q_L^i &\rightarrow -2B_0 \left[\frac{f^2}{4} U^\dagger + \text{higher order terms}\right]_{ij} \ . \end{aligned} \quad (5.6.\text{-}2)$$

The chiral counterparts of the left and right handed currents are proportional to BM and depend on the ratios of quark masses only. The terms giving dominating contribution to the $\Delta S = 1$ part of the weak effective action involve the chiral counterparts of terms $\bar{q}_L^j q_R^i$ breaking chiral invariance. The chiral counterparts of these terms are proportional to B and, in accordance with expectations, fail to be invariant under the overall scaling of quark masses. The higher order contributions to these terms are important for the calculations of direct CP breaking effects but are not written explicitly here because they are not needed in the estimate for how the predictions of the standard model are modified in TGD framework. The terms breaking chiral symmetry give rise to ϵ'/ϵ a contribution, which is proportional to $1/(m_s + m_d)^2$.

The $\Delta S = 2$ part of effective quark action is involved with $K^0 \rightarrow \bar{K}^0$ transitions and the corresponding quark operator is given by

$$Q_{S2} = (\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma^\mu d_L) \ . \qquad (5.6.-1)$$

The chiral counterpart of this operator is obviously invariant under overall scaling of quark masses.

Does chiral theory approach make sense in TGD framework?

The TGD based model for the large direct CP breaking based on exotic gluons and on the transformation of s_{109} to s_{113} has been already discussed. The open question is whether the $1/(m_s + m_d)^2$ proportionality of the CP breaking amplitude can be justified in TGD context where it is not at all clear that chiral theory approach makes sense.

In standard model framework chiral field theory provides a phenomenological description of the low energy hadron physics and makes possible the calculation of various hadronic matrix elements needed to derive the predictions for CP breaking effect.

Chiral field theory limit however involves some questionable assumptions about the relationship between QCD and low energy hadron physics.

1. $SU(3)$ symmetry is assumed and allows description of light mesons in terms of $SU(3)$ valued chiral field U possessing $SU(3)_R \times SU(3)_R$ symmetry broken only by quark mass matrix. In TGD framework $SU(3)$ symmetry is purely phenomenological symmetry since the fundamental gauge group is the gauge group of the standard model.

2. The generation of quark masses is described as effective spontaneous symmetry breaking caused by the vacuum expectation value of $SU(3)$ Lie-algebra valued external scalar field s. Quark masses are identified as the components of the diagonal vacuum expectation value of this field. Physically the scalar field corresponds to scalar meson field so that quark masses would result from the coupling of the quarks to coherent states of scalar mesons. This cannot be a correct physical description in TGD framework, where p-adic thermodynamics gives rise to quark masses. Of course, the presence of the scalar field can give rise to a small shift in the values of the quark masses. Also Higgs field could be in question.

3. The coupling of the field s to chiral field U implies in the standard model context that the mass squared values of mesons are proportional to the sums of masses of the mesonic quarks: for instance, $M_\pi^2 = B_0(m_u + m_d)$ and $M_K^2 = B_0(m_s + m_d)$, where B_0 is one of the basic coupling constants of the chiral field theory. This formula is not consistent with the p-adic mass calculations, where quark mass squared is additive for quarks with the same value of k_q and quark mass for different values of k_q. Indeed, the formulas $M_\pi^2 = m_u^2 + m_d^2$ and $M_K^2 = (m_s + m_d)^2$ are true. The chiral field formula predicts $m_s/m_d \simeq 24$ requiring $m_u = m_d \simeq 13$ MeV ($k = 121$) for $m_s(113) = 320$ MeV whereas TGD predicts $m_s(109)/m_d(107) = 4$. For $m_s \simeq 100$ MeV the prediction is $m_d \simeq 4.2$ MeV. This looks suspiciously small.

To sum up, although the basic assumptions of chiral field theory limit look too specific in TGD framework, its predictions for low energy hadron physics are well-tested and TGD could be consistent with them. If this the case, the assumption about $s_{109} \rightarrow s_{107}$ transition allows a correct prediction of direct CP breaking amplitude using chiral field theory limit.

5.7 Appendix

5.7.1 Effective Feynman rules and the effect of top quark mass on the effective action

The effective low energy field theory relevant for $K - \bar{K}$ systems is in the standard model context summarized elegantly using the Feynman rules of effective field theory deriving from box and penguin diagrams. The rules in t'Hooft-Feynman gauge are summarized in excellent review article of Buras and Fleischer [64]. For box diagrams the rules are following:

$$\begin{aligned}
Box(\Delta S = 2) &= \lambda_i^2 \frac{G_F^2}{16\pi^2} M_W^2 S_0(x_i)(\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \ , \\
Box(T_3 = -1/2) &= \lambda_i \frac{G_F}{\sqrt{2}} \frac{\alpha}{sin^2(\theta_W)} B_0(x_i)(\bar{s}d)_{V-A}(\bar{\mu}\mu)_{V-A} \ , \\
Box(T_3 = 1/2) &= \lambda_i \frac{G_F}{\sqrt{2}} \frac{\alpha}{sin^2(\theta_W)} \left[-4B_0(x_i)\right](\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} \ , \\
\lambda_i &= V_{is}^* V_{id} \ . \qquad (5.7.\text{-}2)
\end{aligned}$$

The box vertices listed here describe the decays $K_0 \rightarrow \bar{K}_0$ and contribute to $K_0 \rightarrow \bar{\mu}\mu$ and $K_0 \rightarrow \bar{\nu}\nu$ decays. $(\bar{q}_1 q_2)_{V-A}$ is shorthand notation for the left handed weak current involving gamma matrices and the products of fermionic bi-linears actually involve contraction of the gamma matrix indices.

Penguin diagrams can be characterized by the effective vertices $\bar{s}dB$, where B is photon, Z boson or gluon, which is treated as usual in effective field theory

$$\begin{aligned}
\bar{s}Zd &= i\lambda_i \frac{G_F}{\sqrt{2}} \frac{g_Z}{2\pi^2} M_Z^2 g_Z C_0(x_i)\bar{s}\gamma^\mu(1-\gamma_5)d \ , \\
\bar{s}\gamma d &= -i\lambda_i \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} D_0(x_i)\bar{s}(q^2\gamma^\mu - q^\mu q^\nu \gamma_\nu)(1-\gamma_5)d \ , \\
\bar{s}G^a d &= -i\lambda_i \frac{G_F}{\sqrt{2}} \frac{g_s}{8\pi^2} E_0(x_i)\bar{s}(q^2\gamma^\mu - q^\mu q^\nu \gamma_\nu)(1-\gamma_5)T^a d \ . \qquad (5.7.\text{-}3)
\end{aligned}$$

The vertices above correspond to the exchange of Z, photon and gluon between the quarks. Boson propagator and second vertex is constructed using the standard Feynman rules. The counterparts of the sdB vertices are easily constructed for $g > 0$ gluons. The orthogonality of single hadron states requires that flavor is conserved for $g > 0$ exchanges.

The functions $B_0, C_0, ...$ characterize the low energy effective action at mass scale $\mu = m_W$. The subscript '0' refers to the values of these functions without QCD corrections, which are taken into account using renormalization group equations to deduced the functions at mass scale of order 1 GeV. The functions are listed below:

$$\begin{aligned}
B_0(x_t) &= \frac{1}{4}\left[\frac{x_t}{y_t} + \frac{x_t log(x_t)}{y_t^2}\right] \ , \\
C_0(x_t) &= \frac{x_t}{8}\left[-\frac{x_t - 6}{y_t} + \frac{3x_t + 2}{y_t^2} log(x_t)\right] \ , \\
D_0(x_t) &= -\frac{4}{9} log(x_t) - \frac{25x_t^2 - 19x_t^3}{36y_t^3} + \frac{x_t^2(-6 - 2x_t + 5x_t^2)}{18y_t^3} log(x_t) \ , \\
E_0(x_t) &= -\frac{2}{3} log(x_t) + \frac{x_t^2(15 - 16x_t - 4x_t^2)}{6y_t^4} log(x_t) + \frac{x_t(18 - 11x_t - x_t^2)}{12y_t^3} \ , \\
S_0(x_t) &= \frac{4x_t - 11x_t^2 + x_t^3}{4y_t^2} - \frac{3x_t^2 log(x_t)}{2y_t^3} \ , \\
S_0(x_c, x_t) &= x_c\left[log(\frac{x_t}{x_c}) - \frac{3x_t}{4y_t} - \frac{3x_t^2 log(x_t)}{4y_t^2}\right] \ , \\
& x_c = (\tfrac{m_c}{m_W})^2 \quad x_t = (\tfrac{m_t}{m_W})^2 \ , \quad y_t = 1 - x_t \ . \qquad (5.7.\text{-}8)
\end{aligned}$$

Although x_t, being the interesting parameter, appears as the only argument of these functions, also the contributions coming from light quarks propagating in the loops are included. For comparison purposes it is useful to give the explicit relations between electro-weak coupling parameters and G_F.

$$\begin{aligned} \frac{G_F}{\sqrt{2}} &= \frac{g_W^2}{8m_W^2} , \\ g_W &= \frac{e}{sin(\theta_W)} , \\ g_Z &= \frac{e}{sin(\theta_W)cost(\theta_W)} . \end{aligned} \quad (5.7.-9)$$

The following table summarizes the effect of the change of the top quark mass on the functions $B_0, C_0, ...$ What is given are the ratios $r(f) = f(55)/f(175)$ of the functions $B_0, C_0, ..$ evaluated for top quark masses 55 GeV and 175 GeV respectively.

f	$B_0(x_t)$	$C_0(x_t)$	$D_0(x_t)$	$E_0(x_t)$	$S_0(x_t)$	$S_0(x_c, x_t)$
r	.51	.09	$-.70$	3.44	.15	.81

(5.7.-8)

These results leave allow only the identification of the experimental candidate as a realistic candidate for top quark.

1. The function B_0 is reduced only by a factor of 1/2 and there are no new physics contributions to B_0 in the lowest order. The function C_0 characterizing Z penguin diagrams is reduced by an order of magnitude. The coefficient $C_0(x_t) - 4B_0(x_t)$ characterizes the dominating contribution to $K \rightarrow \mu^+\mu^-$ decay in standard model and the decay amplitude is reduced by a factor .27 so that this decay would provide a stringent test selecting between 55 GeV top quark and 175 GeV top quark. Unfortunately, the predicted $K \rightarrow \mu^+\mu^-$ rate is still by several orders of magnitude below the experimental upper bound.

2. The function $S_0(x_t)$ characterizing $B - \bar{B}$ and $K - \bar{K}$ mass differences is reduced almost by an order of magnitude. Note that in case of Δm_K the ratio $r(tt/ct)$ of the WW box diagram amplitudes with two top quarks and c and t in internal fermion lines is $r(tt/ct) \sim 738$ for $m_t = 175$ GeV and $r(tt/ct) \sim 138$ for $m_t = 55$ GeV (the moduli of the factors coming from CKM matrix are taken into account). Thus $m_t = 175$ GeV is the only sensible choice.

5.7.2 U and D matrices from the knowledge of top quark mass alone?

As already found, a possible resolution to the problems created by top quark is based on the additivity of mass squared so that top quark mass would be about 230 GeV, which indeed corresponds to a peak in mass distribution of top candidate, whereas $t\bar{t}$ meson mass would be 163 GeV. This requires that top quark mass changes very little in topological mixing. It is easy to see that the mass constraints imply that for $n_t = n_b = 60$ the smallness of V_{i3} and $V(3i)$ matrix elements implies that both U and D must be direct sums of 2×2 matrix and 1×1 unit matrix and that V matrix would have also similar decomposition. Therefore $n_b = n_t = 59$ seems to be the only number theoretically acceptable option. The comparison with the predictions with pion mass led to a unique identification $(n_d, n_b, n_b) = (5, 5, 59)$,$(n_u, n_c, n_t) = (4, 6, 59)$.

U and D matrices as perturbations of matrices mixing only the first two genera

This picture suggests that U and D matrices could be seen as small perturbations of very simple U and D matrices satisfying $|U| = |D|$ corresponding to $n = 60$ and having$(n_d, n_b, n_b) = (4, 5, 60)$, $(n_u, n_c, n_t) = (4, 5, 60)$ predicting V matrix characterized by Cabibbo angle alone. For instance, CP breaking parameter would characterize this perturbation. The perturbed matrices should obey thermodynamical constraints and it could be possible to linearize the thermodynamical conditions and in this manner to predict realistic mixing matrices from first principles. The existence of small perturbations yielding acceptable matrices implies also that these matrices be near a point at which two different matrices resulting as a solution to the thermodynamical conditions coincide.

D matrix can be deduced from U matrix since $9|D_{12}|^2 \simeq n_d$ fixes the value of the relative phase of the two terms in the expression of D_{12}.

$$\begin{aligned}
|D_{12}|^2 &= |U_{11}V_{12} + U_{12}V_{22}|^2 \\
&= |U_{11}|^2|V_{12}|^2 + |U_{12}|^2|V_{22}|^2 \\
&+ 2|U_{11}||V_{12}||U_{12}||V_{22}|cos(\Psi) = \frac{n_d}{9} \ , \\
\Psi &= arg(U_{11}) + arg(V_{12}) - arg(U_{12}) - arg(V_{22}) \ .
\end{aligned} \tag{5.7.-11}$$

Using the values of the moduli of U_{ij} and the approximation $|V_{22}| = 1$ this gives for $cos(\Psi)$

$$\begin{aligned}
\cos(\Psi) &= \frac{A}{B} \ , \\
A &= \frac{n_d - n_u}{9} - \frac{9 - n_u}{9}|V_{12}|^2 \ , \\
B &= \frac{2}{9|V_{12}|}\sqrt{n_u(9 - n_u)} \ .
\end{aligned} \tag{5.7.-12}$$

The experimentation with different values of n_d and n_u shows that $n_u = 6, n_d = 4$ gives $cos(\Psi) = -1.123$. Of course, $n_u = 6, n_d = 4$ option is not even allowed by $n_t = 60$. For $n_d = 4, n_u = 5$ one has $cos(\Psi) = -0.5958$. $n_d = 5, n_u = 6$ corresponding to the perturbed solution gives $cos(\Psi) = -0.6014$.

Hence the initial situation could be $(n_u = 5, n_s = 4, n_b = 60)$, $(n_d = 4, n_s = 5, n_t = 60)$ and the physical U and D matrices result from U and D matrices by a small perturbation as one unit of t (b) mass squared is transferred to u (s) quark and produces symmetry breaking as $(n_d = 5, n_s = t, n_b = 59)$, $(n_u = 6, n_c = 4, n_t = 59)$.

The unperturbed matrices $|U|$ and$|D|$ would be identical with $|U|$ given by

$$|U_{11}| = |U_{22}| = \tfrac{2}{3} \ , \quad |U_{12}| = |U_{21}| = \tfrac{\sqrt{5}}{3} \ , \tag{5.7.-11}$$

The thermodynamical model allows solutions reducing to a direct sum of 2×2 and 1×1 matrices, and since $|U|$ matrix is fixed completely by the mass constraints, it is trivially consistent with the thermodynamical model.

Direct search of U and D matrices

The general formulas for p^U and p^D in terms of the probabilities p_{11} and p_{21} allow straightforward search for the probability matrices having maximum entropy just by scanning the (p_{11}, p_{21}) plane constrained by the conditions that all probabilities are positive and smaller than 1. In the physically interesting case the solution is sought near a solution for which the non-vanishing probabilities are $p_{11} = p_{22} = (9 - n_1)/9$, $p_{12} = p_{21} = n_1/9$, $p_{33} = 1$, $n_1 = 4$ or 5. The inequalities allow to consider only the values $p_{11} \geq (9 - n_1)/9$.

1. *Probability matrices p^U and p^D*

The direct search leads to maximally entropic p^D matrix with $(n_d, n_s) = (5, 5)$:

$$p^D = \begin{pmatrix} 0.4982 & 0.4923 & 0.0095 \\ 0.4981 & 0.4924 & 0.0095 \\ 0.0037 & 0.0153 & 0.9810 \end{pmatrix} \ , \ p_0^D = \begin{pmatrix} 0.5556 & 0.4444 & 0 \\ 0.4444 & 0.5556 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ . \tag{5.7.-11}$$

p_0^D represents the unperturbed matrix p_0^D with $n(d = 4), n_s = 5$ and is included for the purpose of comparison. The entropy $S(p^D) = 1.5603$ is larger than the entropy $S(p_0^D) = 1.3739$. A possible interpretation is in terms of the spontaneous symmetry breaking induced by entropy maximization in presence of constraints.

A maximally entropic p^U matrix with $(n_u, n_c) = (5, 6)$ is given by

$$p^U = \begin{pmatrix} 0.5137 & 0.4741 & 0.0122 \\ 0.4775 & 0.4970 & 0.0254 \\ 0.0088 & 0.0289 & 0.9623 \end{pmatrix} \tag{5.7.-11}$$

The value of entropy is $S(p^U) = 1.7246$. There could be also other maxima of entropy but in the range covering almost completely the allowed range of the parameters and in the accuracy used only single maximum appears.

The probabilities p^D_{ii} *resp.* p^U_{ii} satisfy the constraint $p(i,i) \geq .492$ *resp.* $p_{ii} \geq .497$ so that the earlier proposal for the solution of proton spin crisis must be given up and the solution discussed in [D2] remains the proposal in TGD framework.

2. Near orthogonality of U and D matrices

An interesting question whether U and D matrices can be transformed to approximately orthogonal matrices by a suitable $(U(1) \times U(1))_L \times (U(1) \times U(1))_R$ transformation and whether CP breaking phase appearing in CKM matrix could reflect the small breaking of orthogonality. If this expectation is correct, it should be possible to construct from $|U|$ ($|D|$) an approximately orthogonal matrix by multiplying the matrix elements $|U_{ij}|$, $i, j \in \{2, 3\}$ by appropriate sign factors. A convenient manner to achieve this is to multiply $|U|$ ($|D|$) in an element wise manner ($(A \circ B)_{ij} = A_{ij}B_{ij}$) by a sign factor matrix S.

1. In the case of $|U|$ the matrix $U = S \circ |U|$, $S(2,2) = S(2,3) = S(3,2) = -1$, $S_{ij} = 1$ otherwise, is approximately orthogonal as the fact that the matrix U^TU given by

$$U^TU = \begin{pmatrix} 1.0000 & 0.0006 & -0.0075 \\ 0.0006 & 1.0000 & -0.0038 \\ -0.0075 & -0.0038 & 1.0000 \end{pmatrix}$$

is near unit matrix, demonstrates.

2. For D matrix there are two nearly orthogonal variants. For $D = S \circ |D|$, $S(2,2) = S(2,3) = S(3,2) = -1$, $S_{ij} = 1$ otherwise, one has

$$D^TD = \begin{pmatrix} 1.0000 & -0.0075 & 0.0604 \\ -0.0075 & 1.0000 & 0.0143 \\ 0.0604 & 0.0143 & 1.0000 \end{pmatrix} .$$

The choice $D = S \circ D$, $S(2,2) = S(2,3) = S(3,3) = -1$, $S_{ij} = 1$ otherwise, is slightly better

$$D^TD = \begin{pmatrix} 1.0000 & -0.0075 & 0.0604 \\ -0.0075 & 1.0000 & 0.0143 \\ 0.0601 & 0.0143 & 1.0000 \end{pmatrix} .$$

3. The matrices U and D in the standard gauge

Entropy maximization indeed yields probability matrices associated with unitary matrices. 8 phase factors are possible for the matrix elements but only 4 are relevant as far as the unitarity conditions are considered. The vanishing of the inner products between row vectors, gives 6 conditions altogether so that the system seems to be over-determined. The values of the parameters s_1, s_2, s_3 and phase angle δ in the "standard gauge" can be solved in terms of r_{11} and r_{21}.

The requirement that the norms of the parameters c_i are not larger than unity poses non-trivial constraints on the probability matrices. This should should be the case since the number of unitarity conditions is 9 whereas probability conservation for columns and rows gives only 5 conditions so that not every probability matrix can define unitary matrix. It would seem that that the constraints are satisfied only if the the 2 mass squared conditions and 2 conditions from the entropy maximization are equivalent with 4 unitarity conditions so that the number of conditions becomes 5+4=9. Therefore entropy maximization and mass squared conditions would force the points of complex 9-dimensional space defined by 3×3 matrices to a 9-dimensional surface representing group $U(3)$ so that these conditions would have a group theoretic meaning.

The formulas

$$\begin{aligned} r_{i2} &= \sqrt{[-\frac{n_i}{51} + \frac{20}{17}(1 - r_{i1}^2)]} \ , \\ r_{i3} &= \sqrt{[\frac{n_i}{51} - \frac{3}{17}(1 - r_{i1}^2)]} \ . \end{aligned} \tag{5.7.-11}$$

and

$$U = \left[\begin{array}{ccc} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3exp(i\delta) & c_1c_2s_3 + s_2c_3exp(i\delta) \\ -s_1s_2 & c_1s_2c_3 + c_2s_3exp(i\delta) & c_1s_2s_3 - c_2c_3exp(i\delta) \end{array} \right] \tag{5.7.-10}$$

give

$$\begin{aligned} c_1 = r_{11} \ , \quad & c_2 = \frac{r_{21}}{\sqrt{1-r_{11}^2}} \ , \\ s_3 = \frac{r_{13}}{\sqrt{1-r_{11}^2}} \ , \quad & cos(\delta) = \frac{c_1^2c_2^2c_3^2 + s_2^2s_3^2 - r_{22}^2}{2c_1c_2c_3s_2s_3} \ . \end{aligned} \tag{5.7.-9}$$

Preliminary calculations show that for $n_1 = n_2 = 5$ case the matrix of moduli allows a continuation to a unitary matrix but that for $n_1 = 4, n_2 = 6$ the value of $cos(\delta)$ is larger than one. This would suggest that unitarity indeed gives additional constraints on the integers n_i. The unitary (in the numerical accuracy used) $(n_d, n_s) = (5,5)$ D matrix is given by

$$D = \left(\begin{array}{lll} 0.7059 & 0.7016 & 0.0975 \\ -0.7057 & 0.7017 - 0.0106i & 0.0599 + 0.0766i \\ -0.0608 & 0.0005 + 0.1235i & 0.4366 - 0.8890i \end{array} \right) \ .$$

The unitarity of this matrix supports the view that for certain integers n_i the mass squared conditions and entropy maximization reduce to group theoretic conditions. The numerical experimentation shows that the necessary condition for the unitarity is $n_1 > 4$ for $n_2 < 9$ whereas for $n_2 \geq 9$ the unitarity is achieved also for $n_1 = 4$.

Direct search for CKM matrices

The standard gauge in which the first row and first column of unitary matrix are real provides a convenient representation for the topological mixing matrices: it is convenient to refer to these representations as U_0 and D_0. The possibility to multiply the rows of U_0 and D_0 by phase factors ($(U(1) \times U(1))_R$ transformations) provides 2 independent phases affecting the values of $|V|$. The phases $exp(i\phi_j)$, $j = 2,3$ multiplying the second and third row of D_0 can be estimated from the matrix elements of $|V|$, say from the elements $|V_{11}| = cos(\theta_c) \equiv v_{11}$, $sin\theta_c = .226 \pm .002$ and $|V_{31}| = (9.6 \pm .9) \cdot 10^{-3} \equiv v_{31}$. Hence the model would predict two parameters of the CKM matrix, say s_3 and δ_{CP}, in its standard representation.

The fact that the existing empirical bounds on the matrix elements of V are based on the standard model physics raises the question about how seriously they should be taken. The possible existence of fractally scaled up versions of light quarks could effectively reduce the matrix elements for the electro-weak decays $b \rightarrow c + W$, $b \rightarrow u + W$ *resp.* $t \rightarrow s + W$, $t \rightarrow d + W$ since the decays involving scaled up versions of light quarks can be counted as decays $W \rightarrow bc$ *resp.* $W \rightarrow tb$. This would favor too small experimental estimates for the matrix elements V_{i3} and V_{3i}, $i = 1,2$. In particular, the matrix element $V_{31} = V_{td}$ could be larger than the accepted value.

Various constraints do not leave much freedom to choose the parameters n_{q_i}. The preliminary numerical experimentation shows that the choice $(n_d, n_s) = (5,5)$ and $(n_u, n_c) = (5,6)$ yields realistic

U and D matrices. In particular, the conditions $|U(1,1)| > .7$ and $|D(1,1)| > .7$ hold true and mean that the original proposal for the solution of spin puzzle of proton must be given up. In [D2] an alternative proposal based on more recent findings is discussed. Only for this choice reasonably realistic CKM matrices have been found. For $n_t = 58$ the mass of $t\bar{t}$ meson mass is reduced by one percent from 2×163 GeV for $n(5) = 59$ so that $n_t = 58$ is still acceptable if the additivity of conformal weight rather than mass is accepted for diagonal mesons.

1. The requirement that the parameters $|V_{11}|$ (or equivalently, Cabibbo angle) and $|V_{31}|$ are produced correctly, yields CKM matrices for which CP breaking parameter J is roughly one half of its accepted value. The matrix elements $V_{23} \equiv V_{cb}$, $V_{32} \equiv V_{tc}$, and $V_{13} \equiv V_{ub}$ are roughly twice their accepted value. This suggests that the condition on V_{31} should be loosened.

2. The following tables summarize the results of the search requiring that
   i) the value of the Cabibbo angle s_{Cab} is within the experimental limits $s_{Cab} = .223 \pm .002$,
   ii) $V_{31} = (9.6 \pm .9)\cdot 10^{-3}$, is allowed to have value at most twice its upper bound,
   iii) V_{13} whose upper bound is determined by probability conservation, is within the experimental limits $.42\cdot 10^{-3} < |V_{ub}| < 6.98\cdot 10^{-3}$ whereas $V_{23} \simeq 4\times 10^{-3}$ should come out as a prediction,
   iv) the CP breaking parameter satisfies the condition $|(J - J_0)/J_0| < .6$, where $J_0 = 10^{-4}$ represents the lower bound for J (the experimental bounds for J are $J \times 10^4 \in (1 - 1.7)$).

The pairs of the phase angles (ϕ_1, ϕ_2) defining the phases $(exp(i\phi_1), exp(i\phi_2))$ are listed below

$$\begin{array}{llllll} class\ 1: & \phi_1 & 0.1005 & 0.1005 & 4.8129 & 4.8129 \\ & \phi_2 & 0.0754 & 1.4828 & 4.7878 & 6.1952 \\ class\ 2: & \phi_1 & 0.1005 & 0.1005 & 4.8129 & 4.8129 \\ & \phi_2 & 2.3122 & 5.5292 & 0.7414 & 3.9584 \end{array} \qquad (5.7.-9)$$

The phase angle pairs correspond to two different classes of U, D, and V matrices. The U, D and V matrices inside each class are identical at least up to 11 digits(!). Very probably the phase angle pairs are related by some kind of symmetry.

The values of the fitted parameters for the two classes are given by

	$\lvert V_{11}\rvert$	$\lvert V_{31}\rvert$	$\lvert V_{13}\rvert$	$J/10^{-4}$
class 1	0.9740	0.0157	0.0069	.93953
class 2	0.9740	0.0164	0.0067	1.0267

V_{31} is predicted to be about 1.6 times larger than the experimental upper bound and for both classes V_{23} and V_{32} are roughly too times too large. Otherwise the fit is consistent with the experimental limits for class 2. For class 1 the CP breaking parameter is 7 per cent below the experimental lower bound. In fact, the value of J is fixed already by the constraints on V_{31} and V_{11} and reduces by a factor of one half if V_{31} is required to be within its experimental limits.

U, D and $|V|$ matrices for class 1 are given by

$$U = \begin{bmatrix} 0.7167 & 0.6885 & 0.1105 \\ -0.6910 & 0.7047 - 0.0210i & 0.0909 + 0.1310i \\ -0.0938 & 0.0696 + 0.1550i & 0.1747 - 0.9653i \end{bmatrix}$$

$$D = \begin{bmatrix} 0.7059 & 0.7016 & 0.0975 \\ -0.6347 - 0.3085i & 0.6358 + 0.2972i & 0.0203 + 0.0951i \\ -0.0587 - 0.0159i & -0.0317 + 0.1194i & 0.6534 - 0.7444i \end{bmatrix}$$

$$|V| = \begin{bmatrix} 0.9740 & 0.2265 & 0.0069 \\ 0.2261 & 0.9703 & 0.0862 \\ 0.0157 & 0.0850 & 0.9963 \end{bmatrix} \qquad (5.7.-11)$$

U, D and $|V|$ matrices for class 2 are given by

$$U = \begin{bmatrix} 0.7167 & 0.6885 & 0.1105 \\ -0.6910 & 0.7047 - 0.0210i & 0.0909 + 0.1310i \\ -0.0938 & 0.0696 + 0.1550i & 0.1747 - 0.9653i \end{bmatrix}$$

$$D = \begin{bmatrix} 0.7059 & 0.7016 & 0.0975 \\ -0.6347 - 0.3085i & 0.6358 + 0.2972i & 0.0203 + 0.0951i \\ -0.0589 - 0.0151i & -0.0302 + 0.1198i & 0.6440 - 0.7525i \end{bmatrix}$$

$$|V| = \begin{bmatrix} 0.9740 & 0.2265 & 0.0067 \\ 0.2260 & 0.9704 & 0.0851 \\ 0.0164 & 0.0838 & 0.9963 \end{bmatrix} \tag{5.7.-13}$$

What raises worries is that the values of $|V_{23}| = |V_{cb}|$ and $|V_{32}| = |V_{ts}|$ are roughly twice their experimental estimates. This, as well as the discrepancy related to V_{31}, might be understood in terms of the electro-weak decays of b and t to scaled up quarks causing a reduction of the branching ratios $b \rightarrow c + W$, $t \rightarrow s + W$ and $t \rightarrow t + d$. The attempts to find more successful integer combinations n_i has failed hitherto. The model for pseudoscalar meson masses, the predicted relatively small masses of light quarks, and the explanation for $t\bar{t}$ meson mass supports this mixing scenario.

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5.8 Figures and Illustrations

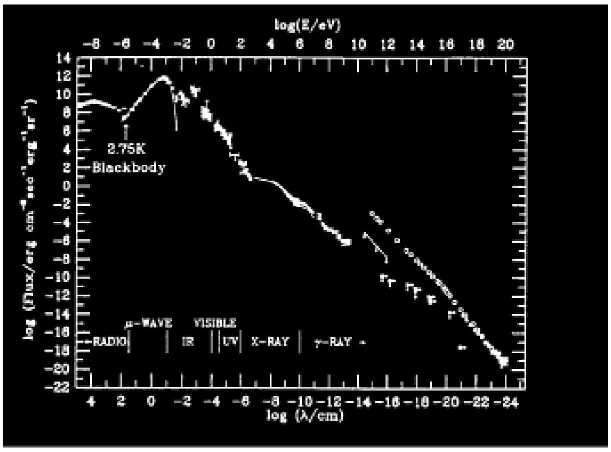

Figure 5.1: There are some indications that cosmic gamma ray flux contains a peak in the energy interval $10^{10} - 10^{11}$ eV. Figure is taken from [91].

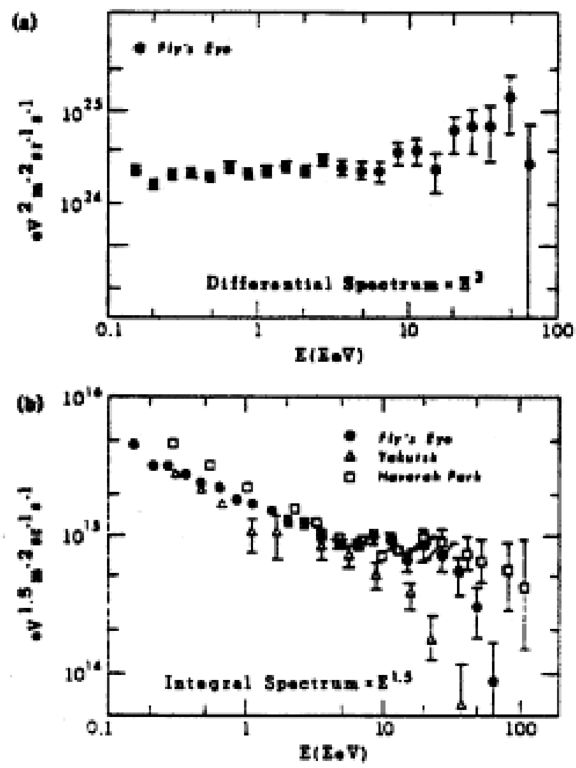

FIG. 2. (a) Differential spectrum $j(E)$ plotted as $E^3j(E)$. A power-law best fit of the form $j(E) = aE^{-\gamma}$ yields $a = 109.6 \pm 2.2$ EeV^{-1} km^{-2} sr^{-1} yr^{-1} and $\gamma = 2.94 \pm 0.02$ for events at $E < 10$ EeV. Between 10 and 50 EeV we obtain $a = 34 \pm 17$ EeV^{-1} km^{-2} sr^{-1} yr^{-1} and $\gamma = 2.42 \pm 0.27$. The lack of events above 50 EeV indicates that the flattened slope does not continue. (b) Integral spectrum $I(>E)$ plotted as $E^{1.5}I(>E)$. Data from both Haverah Park and Yakutsk (Refs. 10, 12, and 13) experiments are also shown.

Figure 5.2:

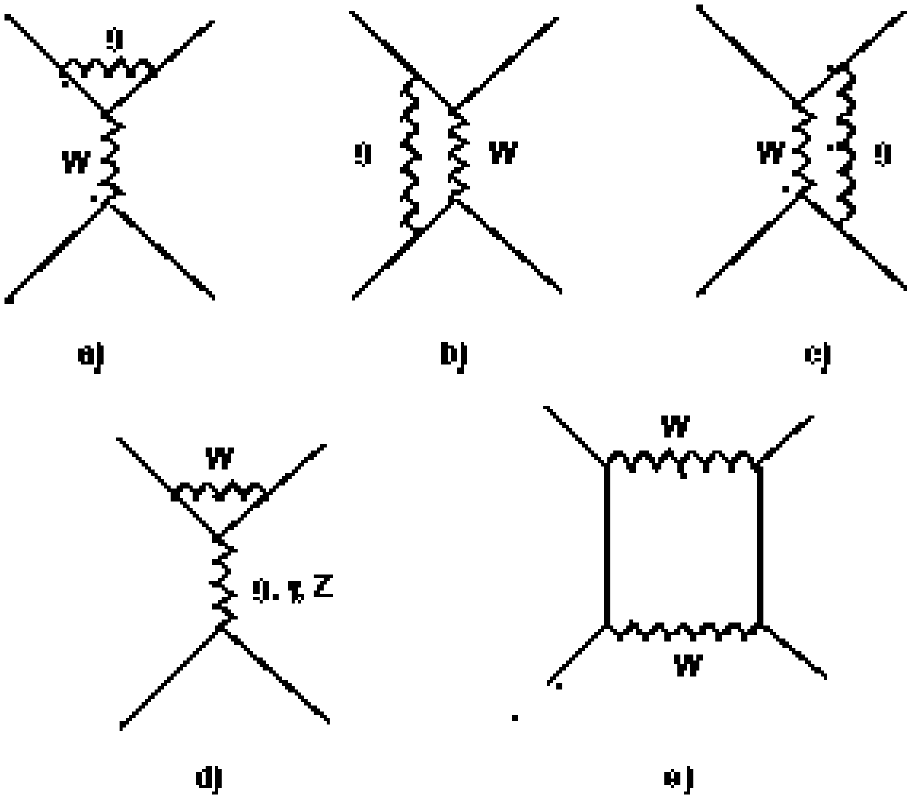

Figure 5.3: Standard model contributions to the matching of the quark operators in the effective flavor-changing Lagrangian

Part II

P-ADIC LENGTH SCALE HYPOTHESIS AND DARK MATTER HIERARCHY

Chapter 6

Coupling Constant Evolution in Quantum TGD

6.1 Introduction

In quantum TGD two kinds of discrete coupling constant evolutions emerge. p-Adic coupling constant evolution is with respect to the discrete hierarchy of p-adic length scales and p-adic length scale hypothesis suggests that only the length scales coming as half octaves of a fundamental length scale are relevant here. Second coupling constant evolution corresponds to hierarchy of Planck constants requiring a generalization of the notion of imbedding space. One can assign this evolution with angle resolution in number theoretic approach.

The notion of zero energy ontology allows to justify p-adic length scale hypothesis and formulate the discrete coupling constant evolution at fundamental level. Configuration space would consists of sectors which correspond to causal diamonds (CDs) identified as intersections of future and past directed light-cones. If the sizes of CDs come in powers of 2^n, p-adic length scale hypothesis emerges, and coupling constant evolution is discrete provided RG invariance holds true inside CDs for space-time evolution of coupling constants defined in some sense to be defined. In this chapter arguments supporting this conclusion are given by starting from a detailed vision about the basic properties of preferred extremals of Kähler action.

The chapter decomposes into two parts. In the first part basic notions are introduced and a general vision about p-adic coupling constant evolution is introduced. After that a general formulation of coupling constant evolution at space-time level and related interpretational issues are considered. In the second sections quantitative predictions involving some not completely rigorous arguments, which I however dare to take rather seriously, are discussed.

6.1.1 Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of

TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of the configuration space forms what I have used to called super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

   Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. The assumption that the commutator algebra of these super-symplectic and super Kac-Moody algebras annihilates physical states gives rise to Super Virasoro conditions which could be regarded as analogs of configuration space Dirac equation.

   Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

3. Configuration space spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

p-Adic physics as physics of cognition and intentionality

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labelled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of configuration space spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [E3]. It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The ideas is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points ((hyper-)quaternions, (hyper-)octonions) become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

Hierarchy of Planck constants and dark matter hierarchy

The work with hyper-finite factors of type II_1 (HFFs) combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter [A9]. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. These variants of imbedding space are characterized by discrete subgroups of SU(2) acting in M^4 and CP_2 degrees of freedom as either symmetry groups or homotopy groups of covering. Among other things this picture implies a general model of fractional quantum Hall effect.

This framework also leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of a generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. The outcome is an explicit formula for the Dirac determinant consistent with the vacuum degeneracy of Kähler action and its finiteness and algebraic number property required by p-adicization requiring number theoretic universality. The zeta function associated with the eigenvalues (rather than Riemann Zeta as believed originally) in turn defines the super-symplectic conformal weights as its zeros so that a highly coherent picture result.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of the partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. It is clear that the hierarchy of Planck constants has become an essential part of the construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups

as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞. The groups G should relate closely to finite groups defining inclusions of HFFs.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit and $M^4 \times CP_2$ can be interpreted as a structure related to hyper-octonions which is a subspace of complexified octonions for which metric has naturally Minkowski signature. This would mean that TGD could be seen also as a generalized number theory. This conjecture predicts the existence of two dual formulations of TGD based on the identification space-times as 4-surfaces in hyper-octonionic space M^8 *resp.* $M^4 \times CP_2$.

3. The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

6.1.2 The construction of S-matrix

The construction of S-matrix involves several ideas that have emerged during last years and involve symmetries in an essential manner.

Zero energy ontology

Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing conserved net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as a space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. Obviously a profound modification of existing views about realization of symmetries is in question.

S-matrix and density matrix are unified to the notion of M-matrix defining time-like entanglement and expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

Quantum TGD as almost topological QFT

Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory. Super-conformal symmetries implied by the light-likeness pose very strong constraints on both state construction and on M-matrix and U-matrix. The notions of n-category and n-groupoid which represents a generalization of the notion of group could be very relevant to this view about M-matrix.

Quantum measurement theory with finite measurement resolution

The notion of measurement resolution represented in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This means that complex rays of state space are effectively replaced with $\mathcal{N}$ rays. The condition that the action of

$\mathcal{N}$ commutes with the M-matrix is a powerful symmetry and implies that the time-like entanglement characterized by M-matrix corresponds to Connes tensor product. Together with super-conformal symmetries this symmetry should fix possible M-matrices to a very high degree.

The notion of number theoretical braid realizes the notion of finite measurement resolution at space-time level and gives a direct connection to topological QFTs describing braids. The connection with quantum groups [?] is highly suggestive since already the inclusions of HFFs involve these groups. Effective non-commutative geometry for the quantum critical sub-manifolds $M^2 \subset M^4$ and $S^2 \subset CP_2$ might provide an alternative notion for the reduction of stringy anti-commutation relations for induced spinor fields to anti-commutations at the points of braids.

Generalization of Feynman diagrams

An essential difference between TGD and string models is the replacement of stringy diagrams with generalized Feynman diagrams obtained by gluing 3-D light-like surfaces (instead of lines) together at their ends represented as partonic 2-surfaces. This makes the construction of vertices very simple. The notion of number theoretic braid in turn implies discretization having also interpretation in terms of non-commutativity due to finite measurement resolution replacing anti-commutativity along stringy curves with anti-commutativity at points of braids. Braids can replicate at vertices which suggests an interpretation in terms of topological quantum computation combined with non-faithful copying and communication of information. The analogs of stringy diagrams have quite different interpretation in TGD: for instance, photons travelling via two different paths in double slit experiment are represented in terms of stringy branching of the photonic 2-surface.

Symplectic variant of QFT as basic building block of construction

The latest discovery related to the construction of M-matrix was the realization that a symplectic variant of conformal field theories might be a further key element in the concrete construction of n-point functions and M-matrix in zero energy ontology. Although I have known super-symplectic (super-symplectic) symmetries to be fundamental symmetries of quantum TGD for almost two decades, I failed for some reason to realize the existence of symplectic QFT, and discovered it while trying to understand quite different problem - the fluctuations of cosmic microwave background! The symplectic contribution to the n-point function satisfies fusion rules and involves only variables which are symplectic invariants constructed using geodesic polygons assignable to the sub-polygons of n-polygon defined by the arguments of n-point function. Fusion rules lead to a concrete recursive formula for n-point functions and M-matrix in contrast to the iterative construction of n-point functions used in perturbative QFT.

6.1.3 Vision about coupling constant evolution

The following summarizes the basic vision about coupling constant evolution.

p-Adic evolution in phase resolution and the spectrum of values for Planck constants

The quantization of Planck constant has been the basic theme of TGD for about five years now. The basic idea is that the the different values of Planck constant correspond to evolution in angular resolution in p-adic context characterized by quantum phase $q = exp(i\pi/n)$ characterizing Jones inclusion is. The higher the value of n, the better the angular resolution since the number of different complex phases in extension of p-adic numbers increases with n.

The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

The most recent view about coupling constant evolution

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-canonical and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination makes possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules.

Equivalence Principle and evolution of gravitational constant

Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

1. The coset construction for super-symplectic and super Kac-Moody algebras implies Equivalence Principle in the sense that four-momenta assignable to the Super Virasoro generators of the two algebras are identical. The challenge is to understand this result in more concrete terms.

2. The progress made in the understanding of number theoretical compactification led to a dramatic progress in the construction of configuration space geometry and spinor structure in terms of the modified Dirac operator associated with light-like 3-surfaces appearing in the slicing of the preferred extremal $X^(X^3_l)$ of Kähler action to light-like 3-surfaces Y^3_l "parallel" to X^3_l. Even more the M^4 projection is predicted to have a slicing into 2-dimensional stringy worldsheets having $M^2(x) \subset M^4$ as a tangent space at point x.

3. By dimensional reduction one can assign to any stringy slice Y^2 a stringy action obtained by integrating Kähler action over the transversal degrees of freedom labeling the copies of Y^2. One can assign length scale evolution to the string tension $T(x)$, which in principle can depend on the point of the string world sheet and thus evolves. $T(x)$ is not identifiable as inverse of gravitational constant but by general arguments proportional to $1/L_p^2$, where L_p is p-adic length scale.

4. Gravitational constant can be understood as a product of L_p^2 with the exponential of the Kähler action for the two pieces of CP_2 type vacuum extremals representing wormhole contacts assignable to graviton connected by the string world sheets. The volume of the typical CP_2 type extremal associated with the graviton increases with L_p so that the exponential factor decreases reducing the growth due to the increase of L_p. Hence G could be RG invariant in p-adic coupling constant evolution. It does not make sense to formulate evolution of gravitational constant at space-time level and gravitational constant characterizes given CD.

5. Gravitational mass is assigned to the stringy world sheet and should be identical with the inertial mass identified as Noether charge assignable to the preferred extremal. By construction there are good hopes that for a proper choice of G gravitational and inertial masses are identical.

The RG invariance of gauge couplings inside causal diamond

Quantum classical correspondence suggests that the notion of p-adic coupling constant evolution should have space-time correlate. Zero energy ontology suggests that this counterpart is realized in terms of CDs in the sense that coupling constant evolution has formulation at space-time level inside CD of given size scale and that RG invariance holds true for this evolution. Number theoretic compactification forces to conclude that space-time surfaces has slicing into light-like 3-surfaces Y^3_l: this prediction is consistent with that is known about the extremals. General Coordinate Invariance requires that basic theory can be formulated by replacing the light-like 3-surface X^3_l associated with wormhole throats with any surface Y^3_l appearing in the associated slicing.

The natural identification for the renormalization group parameter is as the light-like coordinate labeling different light-like slices. The light-likeness of the RG parameter suggests RG invariance. Quantum classical correspondence requires that the classical gauge fluxes to X^3_l selected by stationary phase approximation correspond to the expectation values of gQ_g, where g is coupling constant and Q_g the expectation (eigen) value of corresponding charge matrix in the state in question. If the gauge currents are light-like and in direction of Y^3_l as they are for known extremals under proper selection of X_l^3, RG invariance follows because Abelian gauge fluxes are conserved due to the absence of the component of vacuum current in the direction of slicing.

In principle TGD predicts the values of all coupling constants including also the value of Kähler coupling strength which follows from the identification of Kähler action of the preferred extremal $X^4(X_l^3)$ of Kähler action as Dirac determinant associated with modified Dirac action. Hence Kähler coupling strength could have several values. Quantum criticality in the strongest form however motivates the hypothesis that g_K^2 is invariant under p-adic coupling constant evolution and evolution under evolution associated with the hierarchy of Planck constants.

Quantitative predictions for the values of coupling constants

The latest progress in the understanding of p-adic coupling constant evolution comes from a formula for Kähler coupling strength α_K in terms of Dirac determinant of the modified Dirac operator.

The formula for α_K fixes its number theoretic anatomy and also that of other coupling strengths. The assumption that simple rationals (p-adicization) are involved can be combined with the input from p-adic mass calculations and with an old conjecture for the formula of gravitational constant allowing to express it in terms of CP_2 length scale and Kähler action of topologically condensed CP_2 type vacuum extremal. The prediction is that α_K is renormalization group invariant and equals to the value of fine structure constant at electron length scale characterized by M_{127}. Newton's constant is proportional to p-adic length scale squared and ordinary gravitons correspond to M_{127}. The number theoretic anatomy of R^2/G allows to consider two options. For the first one only M_{127} gravitons are possible number theoretically. For the second option gravitons corresponding to $p \simeq 2^k$ are possible.

A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

p-Adic length scale evolution of gauge couplings

Understanding the dependence of gauge couplings constants on p-adic prime is one of the basic challenges of quantum TGD. The problem has been poorly understood even at the conceptual level to say nothing about concrete calculations. The generalization of the motion of S-matrix to that of M-matrix changed however the situation [C3]. M-matrix is always defined with respect to measurement resolution characterized in terms of an inclusion of von Neumann algebra. Coupling constant evolution reduces to a discrete evolution involving only octaves of $T(k) = 2^k T_0$ of the fundamental time scale $T_0 = R$, where R CP_2 scale. p-Adic length scale $L(k)$ is related to $T(k)$ by $L^2(k) = T(k)T_0$. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, is automatic prediction of the theory. There is also a close connection with the description of coupling constant evolution in terms of radiative corrections.

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level and why certain p-adic primes are favored.

1. Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges invariant: this seems to be the case for known extremals which form scaling invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

2. Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

6.2 Basic conceptual framework

The notions of topological condensate and p-adic length scale hierarchy are in a central role in TGD and for a long time it seemed that the physical interpretation of these notions is relatively straightforward.

The evolution of number theoretical ideas however forced to suspect that the implications for physics might be much deeper and involve not only a solution to the mysteries of dark matter but also force to bring basic notions of TGD inspired theory of consciousness. At this moment the proper interpretation of the mathematical structures involving typically infinite hierarchies generalizing considerably the mathematical framework of standard physics is far from established so that it is better to represent just questions with some plausible looking answers.

6.2.1 Basic concepts

It is good to discuss the basic notions before discussing the definition of gauge charges and gauge fluxes.

CP_2 type vacuum extremals

CP_2 type extremals behave like elementary particles (in particular, light-likeness of M^4 projection gives rise to Virasoro conditions). CP_2 type vacuum extremals have however vanishing four-momentum although they carry classical color charges. This raises the question how they can gain elementary particle quantum numbers.

In topological condensation of CP_2 type vacuum extremal a light-like causal horizon is created. Number theoretical considerations strongly suggest that the horizon carries elementary particle numbers and can be identified as a parton. The quantum numbers or parton would serve as sources of the classical gauge fields created by the causal horizon.

In topological evaporation CP_2 type vacuum extremal carrying only classical color charges is created. This would suggest that the scattering of CP_2 type vacuum extremals defines a topological quantum field theory resulting as a limit of quantum gravitation (CP_2 is gravitational instanton) and that CP_2 type extremals define the counterparts of vacuum lines appearing in the formulation of generalized Feynman diagrams.

contacts as parton pairs

The earlier view about # contacts as passive mediators of classical gauge and gravitational fluxes is not quite correct. The basic modification is due to the fact that one can assign parton or parton pair to the # contact so that it becomes a particle like entity. This means that an entire p-adic hierarchy of new physics is predicted.

1. Formally # contact can be constructed by drilling small spherical holes S^2 in the 3-surfaces involved and connecting the spherical boundaries by a tube $S^2 \times D^1$. For instance, CP_2 type extremal can be glued to space-time sheet with Minkowskian signature or space-time sheets with Minkowskian signature can be connected by # contact having Euclidian signature of the induced metric. Also more general contacts are possible since S^2 can be replaced with a 2-surface of arbitrary genus and family replication phenomenon can be interpreted in terms of the genus.

   The # contact connecting two space-time sheets with Minkowskian signature of metric is accompanied by two "elementary particle horizons", which are light-like 3-surfaces at which the induced 4-metric becomes degenerate. Since these surfaces are causal horizons, it is not clear whether # contacts can mediate classical gauge interactions. If there is an electric gauge flux associated with elementary particle horizon it tends to be either infinite by the degeneracy of the induced metric. It is not clear whether boundary conditions allow to have finite gauge fluxes of electric type. A similar difficulty is encountered when one tries to assign gravitational flux to the # contact: in this case even the existence of flux in non-singular case is far from obvious. Hence the naive extrapolation of Newtonian picture might not be quite correct.

2. Number theoretical considerations suggests that the two light-like horizons associated with # contacts connecting space-time sheets act as dynamical units analogous to shock waves or light fronts carrying quantum numbers so that the identification as partons is natural. Quantum holography would suggest itself in the sense that the quantum numbers associated with causal horizons would determine the long range fields inside space-time sheets involved.

3. # contacts can be modelled in terms of CP_2 type extremals topologically condensed simultaneously to the two space-time sheets involved. The topological condensation of CP_2 type extremal creates only single parton and this encourages the interpretation as elementary particle. The gauge currents for CP_2 type vacuum extremals have a vanishing covariant divergence so that there are no conserved charges besides Kähler charge. Hence electro-weak gauge charges are not conserved classically in the region between causal horizons whereas color gauge charges are. This could explain the vacuum screening of electro-weak charges at space-time level. This is required since for the known solutions of field equations other than CP_2 type extremals vacuum screening does not occur.

4. In the special case space-time sheets have opposite time orientations and the causal horizons carry opposite quantum numbers (with four-momentum included) the # contact would serve the passive role of flux mediator and one could assign to the contact generalized gauge fluxes as quantum numbers associated with the causal horizons. This is the case if the contact is created from vacuum in topological condensation so that the quantum numbers associated with the horizons define naturally generalized gauge fluxes. Kind of generalized quantum dipoles living in two space-times simultaneously would be in question. # contacts in the ground state for space-time sheets with opposite time orientation can be also seen as zero energy parton-antiparton pairs bound together by a piece of CP_2 type extremal.

5. When space-time sheets have same time orientation, the two-parton state associated with the # contact has non-vanishing energy and it is not clear whether it can be stable.

$\#_B$ contacts as bound parton pairs

Besides # contacts also join along boundaries bonds (JABs, $\#_B$ contacts) are possible. They can connect outer boundaries of space-time sheets or the boundaries of small holes associated with the interiors of two space-time sheets which can have Minkowskian signature of metric and can mediate classical gauge fluxes and are excellent candidates for mediators of gauge interactions between space-time sheet glued to a larger space-time sheet by topological sum contacts and join along boundaries contacts. The size scale of the causal horizons associated with parton pairs can be arbitrary whereas the size scale of # contacts is given by CP_2 radius.

The existence of the holes for real space-time surfaces is a natural consequence of the induced gauge field concept: for sufficiently strong gauge fields the imbeddability of gauge field as an induced gauge field fails and hole in space-time appears as a consequence. The holes connected by $\#_B$ contacts obey field equations, and a good guess is that they are light-like 3-surfaces and carry parton quantum numbers. This would mean that both # and $\#_B$ contacts allow a fundamental description in terms of pair of partons.

Magnetic flux tubes provide a representative example of $\#_B$ contact. Instead of $\#_B$ contact also more descriptive terms such as join along boundaries bond (JAB), color bond, and magnetic flux tube are used. $\#_B$ contacts serve also as a space-time correlate for bound state formation and one can even consider the possibility that entanglement might have braiding of bonds defined by # contacts as a space-time correlate [E9].

It seems difficult to exclude join along boundaries contacts between between holes associated with the two space-time sheets at different levels of p-adic hierarchy. If these contacts are possible, a transfer of conserved gauge fluxes would be possible between the two space-time sheets and one could speak about interaction in conventional sense.

Topological condensation and evaporation

Topological condensation corresponds to a formation of # or $\#_B$ contacts between space-time sheets. Topological evaporation means the splitting of # or $\#_B$ contacts. In the case of elementary particles the process changes almost nothing since the causal horizon carrying parton quantum numbers does not disappear. The evaporated CP_2 type vacuum extremal having interpretation as a gravitational instanton can carry only color quantum numbers.

As # contact splits partons are created at the two space-time sheets involved. This process can obviously generate from vacuum space-time sheets carrying particles with opposite signs of energies and other quantum numbers. Positive energy matter and negative energy anti-matter could be thus created by the formation of # contacts with zero net quantum numbers which then split to produce pair of positive and negative energy particles at different space-time sheets having opposite time orientations. This mechanism would allow a creation of positive energy matter and negative energy antimatter with an automatic separation of matter and antimatter at space-time sheets having different time orientation. This might resolve elegantly the puzzle posed by matter-antimatter asymmetry.

The creation of # contact leads to an appearance of radial gauge field in condensate and this seems to be impossible at the limit of infinitely large space-time sheet since it involves a radical instantaneous change in field line topology. The finite size of the space-time sheet can however resolve the difficulty.

If all quantum numbers of elementary particle are expressible as gauge fluxes, the quantum numbers of topologically evaporated particles should vanish. In the case of color quantum numbers and Poincare quantum numbers there is no obvious reason why this should be the case. Despite this the cancellation of the interior quantum numbers by those at boundaries or light-like causal determinants could occur and would conform with the effective 2-dimensionality stating that quantum states are characterized by partonic boundary states associated with causal determinants. This could be also seen as a holographic duality of interior and boundary degrees of freedom [A3].

6.2.2 Gauge charges and gauge fluxes

The concepts of mass and gauge charge in TGD has been a source of a chronic headache. There are several questions waiting for a definite answer. How to define gauge charge? What is the microscopic physics behind the gauge charges necessarily accompanying long range gravitational fields? Are these gauge charges quantized in elementary particle level? Can one associate to elementary particles classical electro-weak gauge charges equal to its quantized value or are all electro-weak charges screened at intermediate boson length scale? Is the generation of the vacuum gauge charges, allowed in principle by the induced gauge field concept, possible in macroscopic length scales? What happens to the gauge charges in topological evaporation? Should Equivalence Principle be modified in order to understand the fact that Robertson-Walker metrics are inertial but not gravitational vacua.

How to define the notion of gauge charge?

In TGD gauge fields are not primary dynamical variables but induced from the spinor connection of CP_2. There are two manners to define gauge charges.

1. In purely group theoretical approach one can associate non-vanishing gauge charge to a 3-surface of finite size and quantization of the gauge charge follows automatically. This definition should work at Planck length scales, when particles are described as 3-surfaces of CP_2 size and classical space-time mediating long range interactions make no sense. Gauge interactions are mediated by gauge boson exchange, which in TGD has topological description in terms of CP_2 type vacuum extremals [D1].

2. Second definition of gauge charge is as a gauge flux over a closed surface. In this case quantization is not obvious nor perhaps even possible at classical level except perhaps for Abelian charges. For a closed 3-surface gauge charge vanishes and one might well argue that this is the case for finite 3-surface with boundary since the boundary conditions might well generate gauge charge near the boundary cancelling the gauge charge created by particles condensed on 3-surface. This would mean that at low energies (photon wavelength large than size of the 3-surfaces) the 3-surfaces in vapor phase look like neutral particles. Only at high energies the evaporated particles would behave as ordinary elementary particles. Furthermore, particle leaves in topological evaporation its gauge charge in the condensate.

The alternative possibility that the long range $\frac{1}{r^2}$ gauge field associated with particle disappears in the evaporation, looks topologically impossible at the limit when larger space-time sheet has infinite size: only the simultaneous evaporation of opposite gauge charges might be possible in this manner at this limit. Topological evaporation provides a possible mechanism for the generation of vacuum gauge charges, which is one basic difference between TGD and standard gauge theories.

There is a strong temptation to draw a definite conclusion but it is better to be satisfied with a simplifying working hypothesis that gauge charges are in long length scales definable as gauge fluxes and vanish for macroscopic 3-surfaces of finite size in vapor phase. This would mean that the topological evaporation of say electron as an electromagnetically charged particle would not be possible except at CP_2 length scale: in the evaporation from secondary condensation level electron would leave its gauge charges in the condensate. Vapor phase particle still looks electromagnetically charged in length scales smaller than the size of the particle surface if the neutralizing charge density is near (or at) the boundary of the surface and gauge and gravitational interactions are mediated by the exchange of CP_2 type extremals.

In what sense could # contacts feed gauge fluxes?

One can associate with the # throats magnetic gauge charges $\pm Q_i$ defined as gauge flux running to or from the throat. The magnetic charges are of opposite sign and equal magnitude on the two space-time sheets involved. For Kähler form the value of magnetic flux is quantized and non-vanishing only if the the $t = constant$ section of causal horizon corresponds to a non-trivial homology equivalence class in CP_2 so that # contact can be regarded as a homological magnetic monopole. In this case # contacts can be regarded as extremely small magnetic dipoles formed by tightly bound # throats possessing opposite magnetic gauge charges. # contacts couple to the difference of the classical gauge fields associated with the two space-time sheets and matter-# contact and # contact-# contact interaction energies are in general non-vanishing.

Electric gauge fluxes through # throat evaluated at the light-like elementary particle horizon X_l^3 tend to be either zero or infinite. The reason is that without appropriate boundary conditions the normal component of electric $F^{tn}\sqrt{(g_4)}/g^2$ either diverges or is infinite since g^{tt} diverges by the effective three-dimensionality of the induced metric at X_l^3. In the gravitational case an additional difficulty is caused by the fact that it is not at all clear whether the notion of gravitational flux is well defined. It is however possible to assign gravitational mass to a given space-time sheets as will be found in the section about space-time description of charge renormalization.

The simplest conclusion would be that the notions of gauge and gravitational fluxes through # contacts do not make sense and that # contacts mediate interactions in a more subtle manner. For instance, for a space-time sheet topologically condensed at a larger space-time sheet the larger space-time sheet would characterize the basic coupling constants appearing in the S-matrix associated with the topologically condensed space-time sheets. In particular, the value of $\hbar$ would characterize the relation between the two space-time sheets. A stronger hypothesis would be that the value of $\hbar$ is coded partially by the Jones inclusion between the state spaces involved. The larger space-time sheet would correspond to dark matter from the point of view of smaller space-time sheet [A8, F9].

One can however try to find loopholes in the argument.

1. It might be possible to pose the finiteness of $F^{tn}\sqrt{g_4}/g^2$ as a boundary condition. The variation principle determining space-time surfaces implies that space-time surfaces are analogous to Bohr orbits so that there are also hopes that gauge fluxes are quantized.

2. Another way out of this difficulty could be based on the basic idea behind renormalization in TGD framework. Gauge coupling strengths are allowed to depend on space-time point so that the gauge currents are conserved. Gauge coupling strengths $g^2/4\pi$ could become infinite at causal horizon. The infinite values of gauge couplings at causal horizons might be a TGD counterpart for the infinite values of bare gauge couplings in quantum field theories. There are however several objections against this idea. The values of coupling constants should depend on space-time sheet only so that the situation is not improved by this trick in CP_2 length scale. Dependence of g^2 on space-time point means also that in the general case the definition of gauge charge as gauge flux is lost so that gauge charges do not reduce to fluxes.

It seems that the notion of a finite electric gauge flux through the causal horizon need not make sense as such. Same applies to the notion of gravitational gauge flux. The notion of gauge flux seems however to have a natural quantal generalization. The creation of a # contact between two space-time sheets creates two causal horizons identifiable as partons and carrying conserved charges assignable with the states created using the fermionic oscillator operators associated with the second quantized induced spinor field. These charges must be of opposite sign so that electric gauge fluxes through causal horizons are replaced by quantal gauge charges. For opposite time orientations also four-momenta cancel each other. The particle states can of course transform by interactions with matter at the two-space-time sheets so that the resulting contact is not a zero energy state always.

This suggests that for gauge fluxes at the horizon are identifiable as opposite quantized gauge charges of the partons involved. If the the net gauge charges of # contact do not vanish, it can be said to possess net gauge charge and does not serve as a passive flux mediator anymore. The possibly screened classical gauge fields in the region faraway from the contact define the classical correlates for gauge fluxes. A similar treatment applies to gravitational flux in the case that the time orientations are opposite and gravitational flux is identifiable as gravitational mass at the causal horizon.

Internal consistency would mildly suggest that # contacts are possible only between space-time sheets of opposite time orientation so that gauge fluxes between space-time sheets of same time orientation would flow along $\#_B$ bonds.

Are the gauge fluxes through # and $\#_B$ contacts quantized?

There are good reasons (the absolute minimization of the Kähler action plus maximization of the Kähler function) to expect that the gauge fluxes through # (if well-defined) and $\#_B$ contacts are quantized. The most natural guess would be that the unit of electric electromagnetic flux for $\#_B$ contact is 1/3 since this makes it possible for the electromagnetic gauge flux of quarks to flow to larger space-time sheets. Anyons could however mean more general quantization rules [E9]. The quantization of electromagnetic gauge flux could serve as a unique experimental signature for # and $\#_B$ contacts and their currents. The contacts can carry also magnetic fluxes. In the case of $\#_B$ contacts the flux quantization would be dynamical and analogous to that appearing in super conductors.

Hierarchy of gauge and gravitational interactions

The observed elementary particles are identified as CP_2 type extremals topologically condensed at space-time sheets with Minkowski signature of induced metric with elementary particle horizon being responsible for the parton aspect. This suggests that at CP_2 length scale gauge and gravitational interactions correspond to the exchanges of CP_2 type extremals with light-like M^4 projection with branching of CP_2 type extremal serving as the basic vertex as discussed first in the earliest attempt to construct [C6] and years later in terms of generalized Feynman diagrams. The gravitational and gauge interactions between the partons assignable to the two causal horizons associated with # contact would be mediated by the # contact, which can be regarded as a gravitational instanton and the interaction with other particles at space-time sheets via classical gravitational fields.

Gauge fluxes flowing through the $\#_B$ contacts would mediate higher level gauge and interactions between space-time sheets rather than directly between CP_2 type extremals. The hierarchy of flux tubes defining string like objects strongly suggests a p-adic hierarchy of "strong gravities" with gravitational constant of order $G \sim L_p^2$, and these strong gravities might correspond to gravitational fluxes mediated by the flux tubes.

6.2.3 Can one regard # *resp.* $\#_B$ contacts as particles *resp.* string like objects?

#-contacts have obvious particle like aspects identifiable as either partons or parton pairs. $\#_B$ contacts in turn behave like string like objects. Using the terminology of M-theory, $\#_B$ contacts connecting the boundaries of space-time sheets could be also seen as string like objects connecting two branes. Again the ends holes at the ends of $\#_B$ contacts carry well defined gauge charges.

contacts as particles and $\#_B$ contacts as string like objects?

The fact that # contacts correspond to parton pairs raises the hope that it is possible to apply p-adic thermodynamics to calculate the masses of # contact and perhaps even the masses of the partons. If this the case, one has an order of magnitude estimate for the first order contribution to the mass of the parton as $m \sim 1/L(p_i)$, $i = 1, 2$. It can of course happen that the first order contribution vanishes: in this case an additional factor $1/\sqrt{p_i}$ appears in the estimate and makes the mass extremely small.

For # contacts connecting space-time sheets with opposite time orientations the vanishing of the net four-momentum requires $p_1 = p_2$. According to the number theoretic considerations below it is possible to assign several p-adic primes to a given space-time sheet and the largest among them, call it p_{max}, determines the p-adic mass scale. The milder condition is that p_{max} is same for the two space-time sheets.

There are some motivations for the working hypothesis that # contacts and the ends of $\#_B$ contacts feeding the gauge fluxes to the lower condensate levels or vice versa tend to be located near the boundaries of space-time sheets. For gauge charges which are not screened by vacuum charges (em and color charges) the imbedding of the gauge fields created by the interior gauge charges becomes impossible near the boundaries and the only possible manner to satisfy boundary conditions is that gauge fluxes flow to the larger space-time sheet and space-time surface becomes a vacuum extremal of the Kähler action near the boundary.

For gauge bosons the density of boundary $\#_B$ contacts should be very small in length scales, where matter is essentially neutral. For gravitational $\#_B$ contacts the situation is different. One might well argue that there is some upper bound for the gravitational flux associated with single # or $\#_B$ contact (or equivalently the gravitational mass associated with causal horizon) given by Planck mass or CP_2 mass so that the number of gravitational contacts is proportional to the mass of the system.

Could # and $\#_B$ contacts form macroscopic quantum phases?

The description as # contact as a parton pair suggests that it is possible to assign to # contacts inertial mass, say of order $1/L(p)$, they should be describable using d'Alembert type equation for a scalar field. # contacts couple dynamically to the geometry of the space-time since the induced metric defines the d'Alembertian. There is a mass gap and hence # contacts could form a Bose-Einstein (BE) condensate to the ground state. If # contacts are located near the boundary of the space-time surface, the d'Alembert equation would be 3-dimensional. One can also ask whether # contacts define a particular form of dark matter having only gravitational interactions with the ordinary matter.

Also coherent states of # contacts are possible and as will be found, Higgs mechanism could be understood as a generation of coherent state of neutral Higgs particles identified as wormhole contacts having quantum numbers of left (right) handed fermion and right (left) handed antifermion.

Also the probability amplitudes for the positions of the ends of $\#_B$ contacts located at the boundary of the space-time sheet could be described using an order parameter satisfying d'Alembert equation with some mass parameter and whether the notion of Bose-Einstein condensate makes sense also now. The model for atomic nucleus assigns to the ends of the $\#_B$ contact realized as a color magnetic flux tube having at its ends quark and anti-quark with mass scale given by $k = 127$ (MeV scale) [F8].

6.2.4 TGD based description of external fields

The description of a system in external field provides a nontrivial challenge for TGD since the system corresponds now to a p-adic space-time sheet k_1 condensed on background 3-surface $k_2 > k_1$. The problem is to understand how external fields penetrate into the smaller space-time sheet and also how the gauge fluxes inside the smaller space-time sheet flow to the external space-time sheet. One should also understand how the penetrating magnetic or electric field manages to preserve its value (if it does so). A good example is provided by the description of system, such as atom or nucleus, in external magnetic or electric field. There are several mechanisms of field penetration:

Induction mechanism

In the case of induction fields are mediated from level k_1 to levels $k_2 \neq k_1$. The external field at given level k_1 acts on # and $\#_B$ throats (both accompanied by a pair of partons) connecting levels k_2 and k_1. The motion of # and $\#_B$ contacts, induced by the gauge and gravitational couplings of partons involved to classical gauge and gravitational fields, creates gauge currents serving as sources of classical gauge field at the space-time sheets involved. This mechanism involves "dark" partons not predicted by standard model.

A good example is provided by the rotation of charged # throats induced by a constant magnetic field, which in turn creates constant magnetic field inside a cylindrical condensate space-time sheet. A second example is the polarization of the charge density associated with the # throats in the external electric field, which in turn creates a constant electric field inside the smaller space-time sheet.

One can in principle formulate general field equations governing the penetration of a classical gauge field from a given condensate level to other levels. The simplified description is based on the introduction of series of fields associated with various condensate levels as analogs of H and B and D and E fields in the ordinary description of the external fields. The simplest assumption is that the fields are linearly related. A general conclusion is that due to the delicacies of the induced field concept, the fields on higher levels appear in the form of flux quanta and typically the field strengths at the higher condensate levels are stronger so that the penetration of field from lower levels to the higher ones means a decomposition into separate flux tubes.

The description of magnetization in terms of the effective field theory of Weiss introduces effective field H, which is un-physically strong: a possible explanation as a field consisting of flux quanta at higher condensate levels. A general order of magnitude estimate for field strength of magnetic flux quantum at condensate level k is as $1/L^2(k)$.

Penetration of magnetic fluxes via # contacts

At least magnetic gauge flux can flow from level p_1 to level p_2 via # contacts. These surfaces are of the form $X^2 \times D^1$, where X^2 is a closed 2-surface. The simplest topology for X^2 is that of sphere S^2. This leads to the first nontrivial result. If a nontrivial magnetic flux flows through the contact, it is quantized. The reason is that magnetic flux is necessarily over a closed surface.

The concept of induced gauge field implies that magnetic flux is nontrivial only if the surface X^2 is homologically nontrivial: CP_2 indeed allows homologically nontrivial sphere. Ordinary magnetic field can be decomposed into co-homologically trivial term plus a term proportional to Kähler form and the flux of ordinary magnetic field comes only from the part of the magnetic field proportional to the Kähler form and the magnetic flux is an integer multiple of some basic flux.

The proposed mechanism predicts that magnetic flux can change only in multiples of basic flux quantum. In super conductors this kind of behavior has been observed. Dipole magnetic fields can be transported via several # contacts: the minimum is one for ingoing and one for return flux so that magnetic dipoles are actual finite sized dipoles on the condensed surface. Also the transfer of magnetic dipole field of, say neutron inside nucleus, to lower condensate level leads to similar magnetic dipole structure on condensate level. For this mechanism the topological condensation of elementary particle, say charged lepton space-time sheet, would involve at least two # contacts and the magnetic moment is proportional to the distance between these contacts. The requirement that the magnetic dipole formed by the # contacts gives the magnetic moment of the particle gives an estimate for the distance d between # throats: by flux quantization the general order of magnitude is given by $d \sim \frac{\alpha_{em} 2\pi}{m}$.

Penetration of electric gauge fluxes via # contacts

For # contact for the opposite gauge charges of partons define the value of generalized gauge electric flux between the two space-time sheets. In this case it is also possible to interpret the partons as sources of the fields at the two space-time sheets. If the # contacts are near the boundary of the smaller space-time sheet the interpretation as a flow of gauge flux to a larger space-time sheet is perfectly sensible. The partons near the boundary can be also seen as generators of a gauge field compensating the gauge fluxes from interior.

The distance between partons can be much larger than p-adic cutoff length $L(k)$ and a proper spatial distribution guarantees homogeneity of the magnetic or electric field in the interior. The distances of the magnetic monopoles are however large in this kind of situation and it is an open problem whether this kind of mechanism is consistent with experimental facts.

An estimate for the electric gauge flux Q_{em} flowing through the # contact is obtained as $n \sim \frac{E}{QL(k)}$: $Q \sim EL^2(k)$, which is of same order of magnitude as electric gauge flux over surface of are $L^2(k)$. In magnetic case the estimate gives $Q_M \sim BL^2(k)$: the quantization of Q_M is consistent with homogeneity requirement only provided the condition $B > \frac{\Phi_0}{L^2(k)}$, where Φ_0 is elementary flux quantum, holds true. This means that flux quantization effects cannot be avoided in weak magnetic fields. The second consequence is that too weak magnetic field cannot penetrate at all to the condensed surface: this is certainly the case if the total magnetic flux is smaller than elementary flux quantum. A good example is provided by the penetration of magnetic field into cylindrical super conductor through the end of the cylinder. Unless the field is strong enough the penetrating magnetic field decomposes into vortex like flux tubes or does not penetrate at all.

The penetration of flux via dipoles formed by # contacts from level to a second level in the interior of condensed surface implies phenomena analogous to the generation of polarization (magnetization) in dielectric (magnetic) materials. The conventional description in terms of fields H, B, M and D, E, P has nice topological interpretation (which does not mean that the mechanism is actually at work in condensed matter length scales). Magnetization M (polarization P) can be regarded as the density of fictitious magnetic (electric) dipoles in the conventional theory: the proposed topological picture suggests that these quantities essentially as densities for # contact pairs. The densities of M and P are of opposite sign on the condensed surface and condensate. $B = H - M$ corresponds to the magnetic field at condensing surface level reduced by the density $-M$ of # contact dipoles in the interior. H denotes the external field at condensate level outside the condensing surface, M $(-M)$ is the magnetic field created by the # contact dipoles at condensate (condensed) level. Similar interpretation can be given for D, E, P fields. The penetrating field is homogenous only above length scales larger than the distance between # throats of dipoles: p-adic cutoff scale $L(k)$ gives natural upper bound for this distance: if this is the case and the density of the contacts is at least of order $n \sim \frac{1}{L^3(k)}$ the penetrating field can be said to be constant also inside the condensed surface.

In condensed matter systems the generation of ordinary polarization and magnetization fields might be related to the permanent # contacts of atomic surfaces with, say, $k = 139$ level. The field created by the neutral atom contains only dipole and higher multipoles components and therefore at least two # contacts per atom is necessary in gas phase, where join along boundaries contacts between atoms are absent. In the absence of external field these dipoles tend to have random directions. In external field # throats behave like opposite charges and their motion in external field generates dipole field. The expression of the polarization field is proportional to the density of these static dipole pairs in static limit. # contacts make possible the penetration of external field to atom, where it generates atomic transitions and leads to the emission of dipole type radiation field, which gives rise to the frequency dependent part of dielectric constant.

Penetration via $\#_B$ contacts

The field can also through $\#_B$ contacts through the boundary of the condensed surface or through the small holes in its interior. The quantization of electric charge quantization would reduce to the quantization of electric gauge flux in $\#_B$ contacts. If there are partons at the ends of contact they affect the gauge gauge flux.

The penetration via $\#_B$ contacts necessitates the existence of join along boundaries bonds starting from the boundary of the condensed system and ending to the boundary component of a hole in the background surface. The field flux flows simply along the 3-dimensional stripe $X^2 \times D^1$: since X^2 has boundary no flux quantization is necessary. This mechanism guarantees automatically the homogeneity of the penetrating field inside the condensed system.

An important application for the theory of external fields is provided by bio-systems in which the penetration of classical electromagnetic fields between different space-time sheets should play central role: what makes the situation so interesting is that the order parameter describing the # and $\#_B$ Bose-Einstein condensates carries also phase information besides the information about the strength of the normal component of the penetrating field.

6.2.5 Number theoretical considerations

Number theoretical considerations allow to develop more quantitative vision about the how p-adic length scale hypothesis relates to the ideas just described.

How to define the notion of elementary particle?

p-Adic length scale hierarchy forces to reconsider carefully also the notion of elementary particle. p-Adic mass calculations led to the idea that particle can be characterized uniquely by single p-adic prime characterizing its mass squared. It however turned out that the situation is probably not so simple.

The work with modelling dark matter suggests that particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions. It would also seem that only the space-time sheets containing common primes in this collection can interact. This leads to the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The p-adic prime characterizing the mass of the particle would perhaps correspond to the largest p-adic prime associated with the particle. Graviton which corresponds to infinitely long ranged interactions, could correspond to the same p-adic prime or collection of them common to all particles. This might apply also to photons. Infinite range might mean that the join along boundaries bonds mediating these interactions can be arbitrarily long but their transversal sizes are characterized by the p-adic length scale in question.

The natural question is what this collection of p-adic primes characterizing particle means? The hint about the correct answer comes from the number theoretical vision, which suggests that at fundamental level the branching of boundary components or more generally wormhole throats to two or more components, completely analogous to the branching of line in Feynman diagram, defines vertices [C6, C4].

1. If space-time sheets correspond holographically to multi-p p-adic topology such that largest p determines the mass scale, the description of particle reactions in terms of branchings indeed makes sense. This picture allows also to understand the existence of different scaled up copies of QCD and weak physics. Multi-p p-adicity could number theoretically correspond to q-adic topology for $q = m/n$ a rational number consistent with p-adic topologies associated with prime factors of m and n ($1/p$-adic topology is homeomorphic with p-adic topology).

2. One could also imagine that different p-adic primes in the collection correspond to different space-time sheets condensed at a larger space-time sheet or boundary components of a given space-time sheet. If the boundary topologies for gauge bosons are completely mixed, as the model of hadrons forces to conclude, this picture is consistent with the topological explanation of the family replication phenomenon and the fact that only charged weak currents involve mixing of quark families. The problem is how to understand the existence of different copies of say QCD. The second difficult question is why the branching leads always to an emission of gauge boson characterized by a particular p-adic prime, say M_{89}, if this p-adic prime does not somehow characterize also the particle itself.

What effective p-adic topology really means?

The need to characterize elementary particle p-adically leads to the question what p-adic effective topology really means. p-Adic mass calculations leave actually a lot of room concerning the answer to this question.

1. The naivest option is that each space-time sheet corresponds to single p-adic prime. A more general possibility is that the boundary components of space-time sheet correspond to different p-adic primes. This view is not favored by the view that each particle corresponds to a collection of p-adic primes each characterizing one particular interaction that the particle in question participates.

2. A more abstract possibility is that a given space-time sheet or boundary component can correspond to several p-adic primes. Indeed, a power series in powers of given integer n gives rise to a well-defined power series with respect to all prime factors of n and effective multi-p-adicity could emerge at the level of field equations in this manner.

One could say that space-time sheet or boundary component corresponds to several p-adic primes through its effective p-adic topology in a hologram like manner. This option is the most flexible one as far as physical interpretation is considered. It is also supported by the number theoretical considerations predicting the value of gravitational coupling constant [E3].

An attractive hypothesis is that only space-time sheets characterized by integers n_i having common prime factors can be connected by join along boundaries bonds and can interact by particle exchanges and that each prime p in the decomposition corresponds to a particular interaction mediated by an elementary boson characterized by this prime.

The physics of quarks and hadrons provides an immediate test for this interpretation. The surprising and poorly understood conclusion from the p-adic mass calculations was that the p-adic primes characterizing light quarks u,d,s satisfy $k_q < 107$, where $k = 107$ characterizes hadronic space-time sheet [F4].

1. The interpretation of $k = 107$ space-time sheet as a hadronic space-time sheet implies that quarks topologically condense at this space-time sheet so that $k = 107$ cannot belong to the collection of primes characterizing quark.

2. Quark space-time sheets must satisfy $k_q < 107$ unless $\hbar$ is large for the hadronic space-time sheet so that one has $k_{eff} = 107 + 22 = 129$. This predicts two kinds of hadrons. Low energy hadrons consists of u, d, and s quarks with $k_q < 107$ so that hadronic space-time sheet must correspond to $k_{eff} = 129$ and large value of $\hbar$. One can speak of confined phase. This allows also $k = 127$ light variants of quarks appearing in the model of atomic nucleus [F8]. The hadrons consisting of c,t,b and the p-adically scaled up variants of u,d,s having $k_q > 107$, $\hbar$ has its ordinary value in accordance with the idea about asymptotic freedom and the view that the states in question correspond to short-lived resonances.

6.3 Identification of elementary particles and the role of Higgs in particle massivation

The development of the recent view about the identification of elementary particles and particle massivation has taken fifteen years since the discovery of p-adic thermodynamics around 1993. p-Adic thermodynamics worked excellently from the beginning for fermions. Only the understanding of gauge boson masses turned out to be problematic and group theoretical arguments led to the proposal that Higgs boson should be present and give the dominating contribution to the masses of gauge bosons whereas the contribution to fermion masses should be small and even negligible. The detailed understanding of quantum TGD at partonic level eventually led to the realization that the coupling to Higgs is not needed after all. The deviation Δh of the ground state conformal weight from negative integer has interpretation as effective Higgs contribution since Higgs vacuum expectation is naturally proportional to Δh but the coupling to Higgs does not cause massivation. In the following I summarize the basic identification of elementary particles and massivation. A more detailed discussion can be found in [F6].

6.3.1 Identification of elementary particles

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 [C2, C3] suggest dramatic simplifications of the general picture discussed in the earlier version of this chapter. p-Adic mass calculations [F3, F4, F5] leave a lot of freedom concerning the detailed identification of elementary particles.

Elementary fermions and bosons

The basic open question is whether the *theory is on some sense free at parton level* as suggested by the recent view about the construction of S-matrix (actually its generalization M-matrix) and by the almost topological QFT property of quantum TGD at parton level [C3]. If partonic 2-surfaces at elementary particle level carry only free many-fermion states, no bi-local composites of second quantized induced spinor field would be needed in the construction of the quantum states and this would simplify the theory enormously.

If this is the case, the basic conclusion would be that light-like 3-surfaces - in particular the ones at which the signature of induced metric changes from Minkowskian to Euclidian - are carriers of fermionic quantum numbers. These regions are associated naturally with CP_2 type vacuum extremals identifiable as correlates for elementary fermions if only fermion number ± 1 is allowed for the stable states. The question however arises about the identification of elementary bosons.

Wormhole contacts with two light-like wormhole throats carrying fermion and anti-fermion quantum numbers are the first thing that comes in mind. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number. In this case a delicate question is whether the space-time sheets connected by wormhole contacts have opposite time orientations or not. If this the case the two fermions would correspond to positive and negative energy particles.

I considered first the identification of only Higgs as a wormhole contact but there is no reason why this identification should not apply also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions.

This picture allows to understand the difference between fermions and gauge bosons and Higgs particle. For fermions topological explanation of family replication predicts three fermionic generations [F1] corresponding to handle numbers $g = 0, 1, 2$ for the partonic 2-surface. In the case of gauge bosons and Higgs this replication is not visible. This could be due to the fact that gauge bosons form singlet and octet representation of the dynamical $SU(3)$ group associated with the handle number $g = 0, 1, 2$ since bosons correspond to pairs of handles. If octet representation is heavy the experimental absence of family replication for bosons can be understood.

Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The development of the understanding of gravitational coupling has had many twists and it is perhaps to summarize the basic misunderstandings.

1. CP_2 length scale R, which is roughly $10^{3.5}$ times larger than Planck length $l_P = \sqrt{\hbar G}$, defines a fundamental length scale in TGD. The challenge is to predict the value of Planck length $\sqrt{\hbar G}$. The outcome was an identification of a formula for $R^2/\hbar G$ predicting that the magnitude of Kähler coupling strength α_K is near to fine structure constant in electron length scale (for ordinary value of Planck constant should be added here).

2. The emergence of the parton level formulation of TGD finally demonstrated that G actually appears in the fundamental parton level formulation of TGD as a fundamental constant characterizing the M^4 part of CP_2 Kähler gauge potential [B4, F12]. This part is pure gauge in the sense of standard gauge theory but necessary to guarantee that the theory does not reduce to topological QFT. Quantum criticality requires that G remains invariant under p-adic coupling constant evolution and is therefore predictable in principle at least.

3. The TGD view about coupling constant evolution predicts the proportionality $G \propto L_p^2$, where L_p is p-adic length scale. Together with input from p-adic mass calculations one ends up to two conclusions. The correct conclusion was that Kähler coupling strength is equal to the fine structure constant in the p-adic length scale associated with Mersenne prime $p = M_{127} = 2^{127}-1$ assignable to electron. I have considered also the possibility that α_K would be equal to electro-weak $U(1)$ coupling in this scale.

4. The additional - wrong- conclusion was that gravitons must always correspond to the p-adic prime M_{127} since G would otherwise vary as function of p-adic length scale. As a matter fact,

the question was for years whether it is G or g_K^2 which remains invariant under p-adic coupling constant evolution. I found both options unsatisfactory until I realized that RG invariance is possible for both g_K^2 and G! The point is that the exponent of the Kähler action associated with the piece of CP_2 type vacuum extremal assignable with the elementary particle is exponentially sensitive to the volume of this piece and logarithmic dependence on the volume fraction is enough to compensate the $L_p^2 \propto p$ proportionality of G and thus guarantee the constancy of G.

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naive estimate $G \sim L_p^2$.

Gravitons are obviously not the only stringy states. For instance, one obtains spin 1 states when the ends of string correspond to gauge boson and Higgs. Also non-vanishing electro-weak and color quantum numbers are possible and stringy states couple to elementary partons via standard couplings in this case. TGD based model for nuclei as nuclear strings having length of order $L(127)$ [F8] suggests that the strings with light M_{127} quark and anti-quark at their ends identifiable as companions of the ordinary graviton are responsible for the strong nuclear force instead of exchanges of ordinary mesons or color van der Waals forces.

Also the TGD based model of high T_c super-conductivity involves stringy states connecting the space-time sheets associated with the electrons of the exotic Cooper pair [J1, J2]. Thus stringy states would play a key role in nuclear and condensed matter physics, which means a profound departure from stringy wisdom, and breakdown of the standard reductionistic picture.

Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving 8+8=16 states altogether. Taking into account phase conjugates gives 16+16=32 states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2+1) \times (3+1) = 12$ states plus phase conjugates giving 12+12=24 states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes 48+16+16=80 states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

What about light-like boundaries and macroscopic wormhole contacts?

Light-like boundaries of the space-time sheet as also wormhole throats can have macroscopic size and can carry free many-fermion states but not elementary bosons. Number theoretic braids and anyons might be assignable to these structures. Deformations of cosmic strings to magnetic flux tubes with a light-like outer boundary are especially interesting in this respect.

If the ends of a string like object move with light velocity as implied by the usual stringy boundary conditions they indeed define light-like 3-surfaces. Many-fermion states could be assigned at the ends of string. One could also connect in pairwise manner the ends of two time-like strings having opposite time orientation using two space-like strings so that the analog of boson state consisting of two wormhole contacts and analogous to graviton would result. "Wormhole throats" could have arbitrarily long distance in M^4.

Wormhole contacts can be regarded as slightly deformed CP_2 type extremals only if the size of M^4 projection is not larger than CP_2 size. The natural question is whether one can construct macroscopic wormhole contacts at all.

1. The throats of wormhole contacts cannot belong to vacuum extremals. One might however hope that small deformations of macrosopic vacuum extremals could yield non-vacuum wormhole contacts of macroscopic size.

2. A large class of macroscopic wormhole contacts which are vacuum extremals consists of surfaces of form $X_1^2 \times X_2^2 \subset (M^1 \times Y^2) \times E^3$, where Y^2 is Lagrangian manifold of CP_2 (induced Kähler form vanishes) and $M^4 = M^1 \times E^3$ represents decomposition of M^1 to time-like and space-like sub-spaces. X_2^2 is a stationary surface of E^3. Both $X_1^2 \subset M^1 \times CP_2$ and X_2^2 have an Euclidian signature of metric except at light-like boundaries $X_a^1 \times X_2^2$ and $X_b^1 \times X_2^2$ defined by ends of X_1^2 defining the throats of the wormhole contact.

3. This kind of vacuum extremals could define an extremely general class of macroscopic wormhole contacts as their deformations. These wormhole contacts describe an interaction of wormhole throats regarded as closed strings as is clear from the fact that X^2 can be visualized as an analog of closed string world sheet X_1^2 in $M^1 \times Y^2$ describing a reaction leading from a state with a given number of incoming closed strings to a state with a given number of outgoing closed strings which correspond to wormhole throats at the two space-time sheets involved.

If one accepts the hierarchy of Planck constants [A9] leading to the generalization of the notion of imbedding space, the identification of anyonic phases in terms of macroscopic light-like surfaces emerges naturally. In this kind of states large fermion numbers are possible. Dark matter would correspond to this kind of phases and "partonic" 2-surfaces could have even astrophysical size. Also black holes can be identified as dark matter at light-like 3-surfaces analogous to black hole horizons and possessing gigantic value of Planck constant [F12].

6.3.2 New view about the role of Higgs boson in massivation

The proposed identifications challenge the standard model view about particle massivation.

1. The standard model inspired interpretation would be that Higgs vacuum expectation associated with the coherent state of neutral Higgs wormhole contacts generates gauge boson mass. Higgs could not however contribute to fermion mass since Higgs condensate cannot accompany fermionic space-time sheets. Fermionic mass would be solely to p-adic thermodynamics. This assumption is consistent with experimental facts but means asymmetry between fermions and bosons.

2. The alternative interpretation inspired by p-adic thermodynamics. Besides the thermodynamical contribution to the particle mass there can be a small contribution from the ground state conformal weight unless this weight is not negative integer. Gauge boson mass would correspond to the ground state conformal weight present in both fermionic and bosonic states and in the case of gauge bosons this contribution would dominate due to the small value of p-adic temperature. For fermions p-adic thermodynamics for super Virasoro algebra would give the dominating contribution to the mass. Higgs vacuum expectation value would be proportional to

the square root of ground state conformal weight for the simple reason that it is the only natural dimensional parameter available. Therefore the causal relation between Higgs and massivation would have been misunderstood in standard model inspired framework. As will be found, the generalized eigen values of the modified Dirac operator having dimension of mass have a natural interpretation as square roots of ground state conformal weight and eigenvalues reflect directly the dynamics of Kähler action.

3. The remaining problem is to understand how the negative value of the ground state conformal weight emerges. This negative conformal weight compensated by the action of Super Virasoro generators is necessary for the success of p-adic mass calculations. Also this problem finds a natural solution. The generalized eigenvalues of the modified Dirac operator are purely imaginary if the effective metric associated with the modified Dirac operator has Euclidian signature. Ground state conformal would be negative and if it is not integer, an effective Higgs contribution to the mass squared is implied. For fermions the deviation from negative integer would be small. Hence p-adic thermodynamics is able to describe the massivation without the introduction of coupling to Higgs, which in TGD framework would be necessarily only a phenomenological description.

6.3.3 Microscopic identification of Weinberg angle

Only after the discovery how the information about preferred extremal of Kähler action can be feeded to the spectrum of modified Dirac operator (see the discussion about modified Dirac action), a real understanding of TGD invariant of Higgs mechanism emerged.

1. The generalized eigenvalues of the transversal part $D_K(X^2)$ of the modified Dirac operator D_K associated with Kähler action are simply square roots of ground state conformal weights and by analogy with cyclotron energies the conformal weights are in reasonable approximation given by $h = -n - 1/2$ giving the desired $h \simeq -1/2$ for lowest state plus finite number of additional ground states. The deviation Δh of h from half odd integer value cannot be compensated by the action of Virasoro generators and it is this contribution which has interpretation as Higgs contribution to mass squared. Higgs zero phase thus corresponds to integer value for h which is highly improbable since the induced ew magnetic field at X^3_l does not correspond exactly to constant magnetic field. Δh is present for both fermions and bosons, should be small for fermions and dominate for gauge bosons. The vacuum expectation of Higgs is indeed naturally proportional to Δh but the presence of Higgs condensate does not cause the massivation.

2. One must also understand the relationship $M^2_W = M^2_Z cos^2(\theta_W)$ requiring $\Delta h(W)/\Delta h(Z) = cos^2(\theta_W)$. Essentially, one should understand the dependence of the quantum averaged the spectrum of modified Dirac operator on the quantum numbers of elementary particle over configuration space degrees of freedom. Suppose that the zero energy state describing particle is proportional to a phase factor depending on electro-weak and color quantum numbers of the particle. This phase factor would be simply $exp[i\int Tr(gQA_\mu)(dx^\mu/ds)ds]$ assignable to the strand of the number theoretic braid: gQ is the diagonal charge matrix characterizing the particle and A_μ represents gauge potential: in the electro-weak case components of the induced spinor connection and the case of color interactions the space-time projection of Killing forms j^A_k of color isometries. Stationary phase approximation selects a preferred light-like 3-surface X^3_l for given quantum numbers and boundary conditions assign to this preferred extremal of Kähler action defining the exponent of Kähler function so that also Δh depends on quantum numbers of the particle.

Second challenge is to understand how the mixing of neutral gauge bosons B_3 and B_0 relates to the group theoretic factor $cos^2(\theta_W)$. The condition that the Higgs expectation value for gauge boson B is proportional to $\Delta h(B)$ and that the coherent state of Higgs couples gauge bosons regarded as fermion anti-fermion pairs should explain the mixing.

1. If gauge bosons and Higgs correspond to wormhole contacts, the discussion reduces to one-fermion level. The value of Δh should be different for different charge states $F_{\pm 1/2}$ of elementary fermion (in the following I will drop from discussion delicacies due to the fact that both quarks and leptons and fermion families are involved). The values of λ of fermion and anti-fermion assignable to gauge boson are naturally identical

$$\Delta\lambda(F_{\pm 1/2} = \Delta\lambda(\overline{F}_{\pm 1/2}) \equiv x_{\pm 1/2} \ . \tag{6.3.0}$$

This implies

$$\begin{aligned} \Delta h(Z,W) &\equiv \Delta h(Z) - \Delta h(W) = m_Z^2 - m_W^2 = m_Z^2 sin^2(\theta) \ , \\ \Delta h(Z) &= 1/2\sum_{\pm}(\Delta\lambda(F_{\pm 1/2}) + \Delta\lambda(\overline{F}_{\mp 1/2})^2 = 2\sum_{\pm} x_{\pm 1/2}^2 \ , \\ \Delta h(W) &= 1/2\sum_{\pm}(\Delta\lambda(F_{\pm 1/2}) + \Delta\lambda(\overline{F}_{\pm 1/2})^2 = (x_{1/2} + x_{-1/2})^2 \ . \end{aligned} \tag{6.3.-2}$$

This gives

$$\Delta h(Z,W) = (x_{1/2} - x_{-1/2})^2 \tag{6.3.-2}$$

giving the condition

$$(x_{1/2} - x_{-1/2})^2 = (x_{1/2} + x_{-1/2})^2 sin^2(\theta_W) \ . \tag{6.3.-1}$$

The interpretation is as breaking of electro-weak $SU(2)_L$ symmetry coded by the geometry of CP_2 in the structure of spinor connection so that the symmetry breaking is expected to take place. One can *define* the value of Weinberg angle from the formula

$$sin(\theta_W) \equiv \pm\frac{x_{1/2} - x_{-1/2}}{x_{1/2} + x_{-1/2}} \ . \tag{6.3.0}$$

2. This definition of Weinberg angle should be consistent with the identification of Weinberg angle coming from the couplings of Z^0 and photon to fermions. Also here the reduction of couplings to one-fermion level might help to understand the symmetry breaking. Z^0 and γ decompose as $Z_0 = cos(\theta_W)B_3 + sin(\theta_W)B_0$ and $\gamma = -sin(\theta_W)B_3 + cos(\theta_W)B_0$, where B_3 corresponds to the gauge potential in $SU(2)_L$ triplet and B_0 the gauge potential in $SU(2)_L$ singlet. Why this mixing should be induced by the splitting of the conformal weights? What induces the mixing of electro-weak triplet with singlet?

3. Could it be the coherent state of Higgs field which transforms left handed and right handed fermions to each other and hence also B_3 to B_0 and vice versa? If the Higgs expectation value associated with the coherent state is proportional to Δh, it would not be too surprising if the mixing between B_3 and B_0 caused by the coherent Higgs state were proportional to $(x_{1/2}-x_{-1/2})/(x_{1/2}+x_{-1/2})$. The reason would be that B_3 is antisymmetric with respect to the exchange of weak isospins whereas B_0 is symmetric. Therefore also the mixing amplitude should be antisymmetric with respect to the exchange of isospins and proportional to $(x_{1/2} - x_{-1/2})$. The presence of the numerator is needed to make the amplitude dimensionless. Under this assumption the two identifications of the Weinberg angle are equivalent.

4. It is important to notice that Weinberg angle is a quantity assignable operationally to the wormhole contacts at the light-like boundaries of $CD \times CP_2$ but not to the generalized light-like 3-surfaces Y_l^3 parallel X_l^3. This suggest that Weinberg angle is necessarily constant for given CD and its evolution reduces to discrete p-adic coupling constant evolution labelled by the scales of CDs coming as powers of 2.

This - admittedly oversimplified - picture obviously changes considerably what-causes-what's in the description of gauge boson massivation and the basic argument should be developed into a more precise form.

6.4 Number theoretic compactification and $M^8 - H$ duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to number theory. In strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as surfaces of H or as surfaces of M^8 composed of hyper-quaternionic and co-hyper-quaternionic regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric.

6.4.1 Basic idea behind $M^8 - M^4 \times CP_2$ duality

The hopes of giving $M^4 \times CP_2$ hyper-octonionic structure are meager. This circumstance forces to ask whether four-surfaces $X^4 \subset M^8$ could under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly so that the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. The following arguments suggest that this is indeed the case.

The hard mathematical fact behind number theoretical compactification is that the quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space) are parameterized by CP_2 just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group.

1. The space of complex structures of the octonion space is parameterized by S^6. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Hyper-quaternions can be identified as $U(2)$ Lie-algebra but it is obvious that hyper-octonions do not allow an identification as $SU(3)$ Lie algebra. Rather, octonions decompose as $1 \oplus 1 \oplus 3 \oplus \overline{3}$ to the irreducible representations of $SU(3)$.

2. Geometrically the choice of a preferred complex (quaternionic) structure means fixing of complex (quaternionic) sub-space of octonions. The fixing of a hyper-quaternionic structure of hyper-octonionic M^8 means a selection of a fixed hyper-quaternionic sub-space $M^4 \subset M^8$ implying the decomposition $M^8 = M^4 \times E^4$. If M^8 is identified as the tangent space of $H = M^4 \times CP_2$, this decomposition results naturally. It is also possible to select a fixed hyper-complex structure, which means a further decomposition $M^4 = M^2 \times E^2$.

3. The basic result behind number theoretic compactification and $M^8 - H$ duality is that hyper-quaternionic sub-spaces $M^4 \subset M^8$ containing a fixed hyper-complex sub-space $M^2 \subset M^4$ or its light-like line $M_\pm$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

4. Space-time surface $X^4 \subset M^8$ is by definition hyper-quaternionic if the tangent spaces of X^4 are hyper-quaternionic planes. Co-hyper-quaternionictity means the same for normal spaces. The presence of fixed hyper-complex structure means at space-time level that the tangent space of X^4 contains fixed M^2 at each point. Under this assumption one can map the points $(m, e) \in M^8$

to points $(m,s) \in H$ by assigning to the point (m,e) of X^4 the point (m,s), where $s \in CP_2$ characterize $T(X^4)$ as hyper-quaternionic plane.

5. The choice of M^2 can be made also local in the sense that one has $T(X^4) \supset M^2(x) \subset M^4 \subset H$. It turns out that strong form of number theoretic compactification requires this kind of generalization. In this case one must be able to fix the convention how the point of CP_2 is assigned to a hyper-quaternionic plane so that it applies to all possible choices of $M^2 \subset M^4$. Since $SO(3)$ hyper-quaternionic rotation relates the hyper-quaternionic planes to each other, the natural assumption is hyper-quaternionic planes related by $SO(3)$ rotation correspond to the same point of CP_2. Under this assumption it is possible to map hyper-quaternionic surfaces of M^8 for which $M^2 \subset M^4$ depends on point of X^4 to H.

6.4.2 Minimal form of $M^8 - H$ duality

The basic problem in the construction of quantum TGD has been the identification of the preferred extremals of Kähler action playing a key role in the definition of the theory. The most elegant manner to do this is by fixing the 4-D tangent space $T(X^4(X_l^3))$ of $X^4(X_l^3)$ at each point of X_l^3 so that the boundary value problem is well defined. What I called number theoretical compactification allows to achieve just this although I did not fully realize this in the original vision. The minimal picture is following.

1. The basic observations are following. Let M^8 be endowed with hyper-octonionic structure. For hyper-quaternionic space-time surfaces in M^8 tangent spaces are by definition hyper-quaternionic. If they contain a preferred plane $M^2 \subset M^4 \subset M^8$ in their tangent space, they can be mapped to 4-surfaces in $M^4 \times CP_2$. The reason is that the hyper-quaternionic planes containing preferred the hyper-complex plane M^2 of $M_\pm \subset M^2$ are parameterized by points of CP_2. The map is simply $(m,e) \rightarrow (m, s(m,e))$, where m is point of M^4, e is point of E^4, and $s(m,2)$ is point of CP_2 representing the hyperquaternionic tangent plane. The inverse map assigns to each point (m,s) in $M^4 \times CP_2$ point m of M^4, undetermined point e of E^4 and 4-D plane. The requirement that the distribution of planes containing the preferred M^2 or $M_\pm$ corresponds to a distribution of planes for 4-D surface is expected to fix the points e. The physical interpretation of M^2 is in terms of plane of non-physical polarizations so that gauge conditions have purely number theoretical interpretation.

2. In principle, the condition that $T(X^4)$ contains M^2 can be replaced with a weaker condition that either of the two light-like vectors of M^2 is contained in it since already this condition assigns to $T(X^4)$ M^2 and the map $H \rightarrow M^8$ becomes possible. Only this weaker form applies in the case of massless extremals [D1] as will be found.

3. The original idea was that hyper-quaternionic 4-surfaces in M^8 containing $M^2 \subset M^4$ in their tangent space could correspond to preferred extremals of Kähler action. This condition does not seem to be consistent with what is known about the extremals of Kähler action. The weaker form of the hypothesis is that hyper-quaternionicity holds only for 4-D tangent spaces of $X_l^3 \subset H = M^4 \times CP_2$ identified as wormhole throats or boundary components lifted to 3-surfaces in 8-D tangent space M^8 of H. The minimal hypothesis would be that only $T(X^4(X_l^3))$ at X_l^3 is associative that is hyper-quaternionic for fixed M^2. $X_l^3 \subset M^8$ and $T(X^4(X_l^3))$ at X_l^3 can be mapped to $X_l^3 \subset H$ if tangent space contains also $M_\pm \subset M^2$ or $M^2 \subset M^4 \subset M^8$ itself having interpretation as preferred hyper-complex plane. This condition is not satisfied by all surfaces X_l^3 as is clear from the fact that the inverse map involves local E^4 translation. The requirements that the distribution of hyper-quaternionic planes containing M^2 corresponds to a distribution of 4-D tangent planes should fix the E^4 translation to a high degree.

4. A natural requirement is that the image of $X_l^3 \subset H$ in M^8 is light-like. The condition that the determinant of induced metric vanishes gives an additional condition reducing the number of free parameters by one. This condition cannot be formulated as a condition on CP_2 coordinate characterizing the hyper-quaternionic tangent plane. Since M^4 projections are same for the two representations, this condition is satisfied if the contributions from CP_2 and E^4 and projections to the induced metric are identical: $s_{kl}\partial_\alpha s^k \partial_\beta s^l = e_{kl}\partial_\alpha e^k \partial_\beta e^l$. This condition means that only a subset of light-like surfaces of M^8 are realized physically. One might argue that this is as it

must be since the volume of E^4 is infinite and that of CP_2 finite: only an infinitesimal portion of all possible light-like 3-surfaces in M^8 can can have H counterparts. The conclusion would be that number theoretical compactification is 4-D isometry between $X^4 \subset H$ and $X^4 \subset M^8$ at X_l^3. This unproven conjecture is unavoidable.

5. $M^2 \subset T(X^4(X_l^3))$ condition fixes $T(X^4(X_l^3))$ in the generic case by extending the tangent space of X_l^3, and the construction of configuration space spinor structure fixes boundary conditions completely by additional conditions necessary when X_l^3 corresponds to a light-like 3 surfaces defining wormhole throat at which the signature of induced metric changes. What is especially beautiful that only the data in $T(X^4(X_l^3))$ at X_l^3 is needed to calculate the vacuum functional of the theory as Dirac determinant: the only remaining conjecture (strictly speaking un-necessary but realistic looking) is that this determinant gives exponent of Kähler action for the preferred extremal and there are excellent hopes for this by the structure of the basic construction.

The basic criticism relates to the condition that light-like 3-surfaces are mapped to light-like 3-surfaces guaranteed by the condition that $M^8 - H$ duality is isometry at X_l^3.

6.4.3 Strong form of $M^8 - H$ duality

The proposed picture is the minimal one. One can of course ask whether the original much stronger conjecture that the preferred extrema of Kähler action correspond to hyper-quaternionic surfaces could make sense in some form. One can also wonder whether one could allow the choice of the plane M^2 of non-physical polarization to be local so that one would have $M^2(x) \subset M^4 \subset M^4 \times E^4$, where M^4 is fixed hyper-quaternionic sub-space of M^8 and identifiable as M^4 factor of H.

1. If M^2 is same for all points of X_l^3, the inverse map $X_l^3 \subset H \rightarrow X_l^3 \subset M^8$ is fixed apart from possible non-uniquencess related to the local translation in E^4 from the condition that hyper-quaternionic planes represent light-like tangent 4-planes of light-like 3-surfaces. The question is whether not only X_l^3 but entire four-surface $X^4(X_l^3)$ could be mapped to the tangent space of M^8. By selecting suitably the local E^4 translation one might hope of achieving the achieving this. The conjecture would be that the preferred extrema of Kähler action are those for which the distribution integrates to a distribution of tangent planes.

2. There is however a problem. What is known about extremals of Kähler action is not consistent with the assumption that fixed M^2 of $M_\pm \subset M^2$is contained in the tangent space of X^4. This suggests that one should relax the condition that $M^2 \subset M^4 \subset M^8$ is a fixed hyper-complex plane associated with the tangent space or normal space X^4 and allow M^2 to vary from point to point so that one would have $M^2 = M^2(x)$. In $M^8 \rightarrow H$ direction the justification comes from the observation (to be discussed below) that it is possible to uniquely fix the convention assigning CP_2 point to a hyper-quaternionic plane containing varying hyper-complex plane $M^2(x) \subset M^4$.

   Number theoretic compactification fixes naturally $M^4 \subset M^8$ so that it applies to any $M^2(x) \subset M^4$. Under this condition the selection is parameterized by an element of $SO(3)/SO(2) = S^2$. Note that M^4 projection of X^4 would be at least 2-dimensional in hyper-quaternionic case. In co-hyper-quaternionic case E^4 projection would be at least 2-D. $SO(2)$ would act as a number theoretic gauge symmetry and the $SO(3)$ valued chiral field would approach to constant at X_l^3 invariant under global $SO(2)$ in the case that one keeps the assumption that M^2 is fixed ad X_l^3.

3. This picture requires a generalization of the map assigning to hyper-quaternionic plane a point of CP_2 so that this map is defined for all possible choices of $M^2 \subset M^4$. Since the $SO(3)$ rotation of the hyper-quaternionic unit defining M^2 rotates different choices parameterized by S^2 to each other, a natural assumption is that the hyper-quaternionic planes related by $SO(3)$ rotation correspond to the same point of CP_2. Denoting by M^2 the standard representative of M^2, this means that for the map $M^8 \rightarrow H$ one must perform $SO(3)$ rotation of hyper-quaternionic plane taking $M^2(x)$ to M^2 and map the rotated tangent plane to CP_2 point. In $M^8 \rightarrow H$ case one must first map the point of CP_2 to hyper-quaternionic plane and rotate this plane by a rotation taking $M^2(x)$ to M^2.

4. In this framework local M^2 can vary also at the surfaces X_l^3, which considerably relaxes the boundary conditions at wormhole throats and light-like boundaries and allows much more general

variety of light-like 3-surfaces since the basic requirement is that M^4 projection is at least 1-dimensional. The physical interpretation would be that a local choice of the plane of non-physical polarizations is possible everywhere in $X^4(X^3_l)$. This does not seem to be in any obvious conflict with physical intuition.

These observation provide support for the conjecture that (classical) $S^2 = SO(3)/SO(2)$ conformal field theory might be relevant for (classical) TGD.

1. General coordinate invariance suggests that the theory should allow a formulation using any light-like 3-surface X^3 inside $X^4(X^3_l)$ besides X^3_l identified as union of wormhole throats and boundary components. For these surfaces the element $g(x) \in SO(3)$ would vary also at partonic 2-surfaces X^2 defined as intersections of $\delta CD \times CP_2$ and X^3 (here CD denotes causal diamond defined as intersection of future and past directed light-cones). Hence one could have $S^2 = SO(3)/SO(2)$ conformal field theory at X^2 (regarded as quantum fluctuating so that also $g(x)$ varies) generalizing to WZW model for light-like surfaces X^3.

2. The presence of E^4 factor would extend this theory to a classical $E^4 \times S^2$ WZW model bringing in mind string model with 6-D Euclidian target space extended to a model of light-like 3-surfaces. A further extension to X^4 would be needed to integrate the WZW models associated with 3-surfaces to a full 4-D description. General Coordinate Invariance however suggests that X^3_l description is enough for practical purposes.

3. The choices of $M^2(x)$ in the interior of X^3_l is dictated by dynamics and the first optimistic conjecture is that a classical solution of $SO(3)/SO(2)$ Wess-Zumino-Witten model obtained by coupling $SO(3)$ valued field to a covariantly constant $SO(2)$ gauge potential characterizes the choice of $M^2(x)$ in the interior of $M^8 \supset X^4(X^3_l) \subset H$ and thus also partially the structure of the preferred extremal. Second optimistic conjecture is that the Kähler action involving also E^4 degrees of freedom allows to assign light-like 3-surface to light-like 3-surface.

4. The best that one can hope is that $M^8 - H$ duality could allow to transform the extremely non-linear classical dynamics of TGD to a generalization of WZW-type model. The basic problem is to understand how to characterize the dynamics of CP_2 projection at each point.

In H picture there are two basic types of vacuum extremals: CP_2 type extremals representing elementary particles and vacuum extremals having CP_2 projection which is at most 2-dimensional Lagrange manifold and representing say hadron. Vacuum extremals can appear only as limiting cases of preferred extremals which are non-vacuum extremals. Since vacuum extremals have so decisive role in TGD, it is natural to requires that this notion makes sense also in M^8 picture. In particular, the notion of vacuum extremal makes sense in M^8.

This requires that Kähler form exist in M^8. E^4 indeed allows full S^2 of covariantly constant Kähler forms representing quaternionic imaginary units so that one can identify Kähler form and construct Kähler action. The obvious conjecture is that hyper-quaternionic space-time surface is extremal of this Kähler action and that the values of Kähler actions in M^8 and H are identical. The elegant manner to achieve this, as well as the mapping of vacuum extremals to vacuum extremals and the mapping of light-like 3-surfaces to light-like 3-surfaces is to assume that $M^8 - H$ duality is Kähler isometry so that induced Kähler forms are identical.

This picture contains many speculative elements and some words of warning are in order.

1. Light-likeness conjecture would boil down to the hypothesis that $M^8 - H$ correspondence is Kähler isometry so that the metric and Kähler form of X^4 induced from M^8 and H would be identical. This would guarantee also that Kähler actions for the preferred extremal are identical. This conjecture is beautiful but strong.

2. The slicing of $X^4(X^3_l)$ by light-like 3-surfaces is very strong condition on the classical dynamics of Kähler action and does not make sense for pieces of CP_2 type vacuum extremals.

Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative

The 8-dimensionality of M^8 allows to consider both associativity (hyper-quaternionicity) of the tangent space and associativity of the normal space- let us call this co-assosiativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \rightarrow k$ duality.

Are the known extremals of Kähler action consistent with the strong form of $M^8 - H$ duality

It is interesting to check whether the known extremals of Kähler action [D1] are consistent with strong form of $M^8 - H$ duality assuming that M^2 or its light-like ray is contained in $T(X^4)$ or normal space.

1. CP_2 type vacuum extremals correspond cannot be hyper-quaternionic surfaces but co-hyper-quaternionicity is natural for them. In the same manner canonically imbedded M^4 can be only hyper-quaternionic.

2. String like objects are associative since tangent space obviously contains $M^2(x)$. Objects of form $M^1 \times X^3 \subset M^4 \times CP_2$ do not have M^2 either in their tangent space or normal space in H. So that the map from $H \rightarrow M^8$ is not well defined. There are no known extremals of Kähler action of this type. The replacement of M^1 random light-like curve however gives vacuum extremal with vanishing volume, which need not mean physical triviality since fundamental objects of the theory are light-like 3-surfaces.

3. For canonically imbedded CP_2 the assignment of $M^2(x)$ to normal space is possible but the choice of $M^2(x) \subset N(CP_2)$ is completely arbitrary. For a generic CP_2 type vacuum extremals M^4 projection is a random light-like curve in $M^4 = M^1 \times E^3$ and $M^2(x)$ can be defined uniquely by the normal vector $n \in E^3$ for the local plane defined by the tangent vector dx^μ/dt and acceleration vector d^2x^μ/dt^2 assignable to the orbit.

4. Consider next massless extremals. Let us fix the coordinates of X^4 as $(t, z, x, y) = (m^0, m^2, m^1, m^2)$. For simplest massless extremals CP_2 coordinates are arbitrary functions of variables $u = k \cdot m = t - z$ and $v = \epsilon \cdot m = x$, where $k = (1, 1, 0, 0)$ is light-like vector of M^4 and $\epsilon = (0, 0, 1, 0)$ a polarization vector orthogonal to it. Obviously, the extremals defines a decomposition $M^4 = M^2 \times E^2$. Tangent space is spanned by the four H-vectors $\nabla_\alpha h^k$ with M^4 part given by $\nabla_\alpha m^k = \delta^k_\alpha$ and CP_2 part by $\nabla_\alpha s^k = \partial_u s^k k_\alpha + \partial_v s^k \epsilon_\alpha$.

   The normal space cannot contain M^4 vectors since the M^4 projection of the extremal is M^4. To realize hyper-quaternionic representation one should be able to from these vector two vectors of M^2, which means linear combinations of tangent vectors for which CP_2 part vanishes. The vector $\partial_t h^k - \partial_z h^k$ has vanishing CP_2 part and corresponds to M^4 vector $(1, -1, 0, 0)$ fix assigns to each point the plane M^2. To obtain M^2 one would need $(1, 1, 0, 0)$ too but this is not possible. The vector $\partial_y h^k$ is M^4 vector orthogonal to ϵ but M^2 would require also $(1, 0, 0, 0)$. The proposed generalization of massless extremals allows the light-like line $M_\pm$ to depend on point of M^4 [D1], and leads to the introduction of Hamilton-Jacobi coordinates involving a local decomposition of M^4 to $M^2(x)$ and its orthogonal complement with light-like coordinate lines having interpretation as curved light rays. $M^2(x) \subset T(X^4)$ assumption fails fails also for vacuum extremals of form $X^1 \times X^3 \subset M^4 \times CP_2$, where X^1 is light-like random curve. In the latter case, vacuum property follows from the vanishing of the determinant of the induced metric.

5. The deformations of string like objects to magnetic flux quanta are basic conjectural extremals of Kähler action and the proposed picture supports this conjecture. In hyper-quaternionic case the assumption that local 4-D tangent plane of X^3 contains $M^2(x)$ but that $T(X^3)$ does not contain it, is very strong. It states that $T(X^4)$ at each point can be regarded as a product $M^2(x) \times T^2$, $T^2 \subset T(CP_2)$, so that hyper-quaternionic X^4 would be a collection of Cartesian products of infinitesimal 2-D planes $M^2(x) \subset M^4$ and $T^2(x) \subset CP_2$. The extremals in question could be seen as local variants of string like objects $X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 holomorphic surface of CP_2. One can say that X^2 is replaced by a collection of infinitesimal pieces of $M^2(x)$ and Y^2 with similar pieces of homologically non-trivial geodesic sphere $S^2(x)$ of CP_2, and the Cartesian products of these pieces are glued together to form a continuous surface defining an extremal of Kähler action. Field equations would pose conditions on how $M^2(x)$ and $S^2(x)$ can depend on x. This description applies to magnetic flux quanta, which are the most important must-be extremals of Kähler action.

Geometric interpretation of strong $M^8 - H$ duality

In the proposed framework $M^8 - H$ duality would have a purely geometric meaning and there would nothing magical in it.

1. $X^4(X^3_l) \subset H$ could be seen a curve representing the orbit of a light-like 3-surface defining a 4-D surface. The question is how to determine the notion of tangent vector for the orbit of X^3_l. Intuitively tangent vector is a one-dimensional arrow tangential to the curve at point X^3_l. The identification of the hyper-quaternionic surface $X^4(X^3_l) \subset M^8$ as tangent vector conforms with this intuition.

2. One could argue that M^8 representation of space-time surface is kind of chart of the real space-time surface obtained by replacing real curve by its tangent line. If so, one cannot avoid the question under which conditions this kind of chart is faithful. An alternative interpretation is that a representation making possible to realize number theoretical universality is in question.

3. An interesting question is whether $X^4(X^3_l)$ as orbit of light-like 3-surface is analogous to a geodesic line -possibly light-like- so that its tangent vector would be parallel translated in the sense that $X^4(X^3)$ for any light-like surface at the orbit is same as $X^4(X^3_l)$. This would give justification for the possibility to interpret space-time surfaces as a geodesic of configuration space: this is one of the first -and practically forgotten- speculations inspired by the construction of configuration space geometry. The light-likeness of the geodesic could correspond at the level of X^4 the possibility to decompose the tangent space to a direct sum of two light-like spaces and 2-D transversal space producing the foliation of X^4 to light-like 3-surfaces X^3_l along light-like curves.

4. $M^8 - H$ duality would assign to X^3_l classical orbit and its tangent vector at X^3_l as a generalization of Bohr orbit. This picture differs from the wave particle duality of wave mechanics stating that once the position of particle is known its momentum is completely unknown. The outcome is however the same: for X^3_l corresponding to wormhole throats and light-like boundaries of X^4, canonical momentum densities in the normal direction vanish identically by conservation laws and one can say that the the analog of (q, p) phase space as the space carrying wave functions is replaced with the analog of subspace consisting of points $(q, 0)$. The dual description in M^8 would not be analogous to wave functions in momentum space space but to those in the space of unique tangents of curves at their initial points.

The Kähler and spinor structures of M^8

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action. As found, this leads to the conclusion that the$M^8 - H$ duality is

Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units.

2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.

3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The free Kähler forms could thus allow to produce M^8 counterparts of these gauge potentials possessing same couplings as their H counterparts. This picture would produce parity breaking in M^8 picture correctly.

4. Only the charged parts of classical electro-weak gauge fields would be absent. This would conform with the standard thinking that charged classical fields are not important. The predicted classical W fields is one of the basic distinctions between TGD and standard model and in this framework. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

5. Also super-symmetries of quantum TGD crucial for the construction of configuration space geometry force this picture. In the absence of coupling to Kähler gauge potential all constant spinor fields and their conjugates would generate super-symmetries so that M^8 would allow $N = 8$ super-symmetry. The introduction of the coupling to Kähler gauge potential in turn means that all covariantly constant spinor fields are lost. Only the representation of all three neutral parts of electro-weak gauge potentials in terms of three independent Kähler gauge potentials allows right-handed neutrino as the only super-symmetry generator as in the case of H.

6. The $SO(3)$ element characterizing $M^2(x)$ is fixed apart from a local $SO(2)$ transformation, which suggests an additional $U(1)$ gauge field associated with $SO(2)$ gauge invariance and representable as Kähler form corresponding to a quaternionic unit of E^4. A possible identification of this gauge field would be as a part of electro-weak gauge field.

M^8 dual of configuration space geometry and spinor structure?

If one introduces M^8 spinor structure and preferred extremals of M^8 Kähler action, one cannot avoid the question whether it is possible or useful to formulate the notion of configuration space geometry and spinor structure for light-like 3-surfaces in M^8 using the exponent of Kähler action as vacuum functional.

1. The isometries of the configuration space in M^8 and H formulations would correspond to symplectic transformation of $\delta M^4_\pm \times E^4$ and $\delta M^4_\pm \times CP_2$ and the Hamiltonians involved would belong to the representations of $SO(4)$ and $SU(3)$ with 2-dimensional Cartan sub-algebras. In H picture color group would be the familiar $SU(3)$ but in M^8 picture it would be $SO(4)$. Color confinement in both $SU(3)$ and $SO(4)$ sense could allow these two pictures without any inconsistency.

2. For $M^4 \times CP_2$ the two spin states of covariantly constant right handed neutrino and antineutrino spinors generate super-symmetries. This super-symmetry plays an important role in the proposed construction of configuration space geometry. As found, this symmetry would be present also in M^8 formulation so that the construction of M^8 geometry should reduce more or less to the replacement of CP_2 Hamiltonians in representations of $SU(3)$ with E^4 Hamiltonians in representations of $SO(4)$. These Hamiltonians can be taken to be proportional to functions of E^4 radius which is $SO(4)$ invariant and these functions bring in additional degree of freedom.

3. The construction of Dirac determinant identified as a vacuum functional can be done also in M^8 picture and the conjecture is that the result is same as in the case of H. In this framework the construction is much simpler due to the flatness of E^4. In particular, the generalized eigen modes of $D_K(X^2)$ restricted X_l^3 correspond to a situation in which one has fermion in induced Maxwell field mimicking the neutral part of electro-weak gauge field in H as far as couplings are considered. Induced Kähler field would be same as in H. Eigen modes are localized to regions inside which the Kähler magnetic field is non-vanishing and apart from the fact that the metric is the effective metric defined in terms of canonical momentum densities via the formula $\hat{\Gamma}^\alpha = \partial L_K/\partial h^k_\alpha \Gamma_k$ for effective gamma matrices. This in fact, forces the localization of modes implying that their number is finite so that Dirac determinant is a product over finite number eigenvalues. It is clear that M^8 picture could dramatically simplify the construction of configuration space geometry.

4. The eigenvalue spectra of the transversal parts of D_K operators in M^8 and H should identical. This motivates the question whether it is possible to achieve a complete correspondence between H and M^8 pictures also at the level of spinor fields at X^3 by performing a gauge transformation eliminating the classical W gauge boson field altogether at X_l^3 and whether this allows to transform the modified Dirac equation in H to that in M^8 when restricted to X_l^3. That something like this might be achieved is supported by the fact that in Coulombic gauge the component of gauge potential in the light-like direction vanishes so that the situation is effectively 2-dimensional and holonomy group is Abelian.

Why $M^8 - H$ duality is useful?

Skeptic could of course argue that $M^8 - H$ duality produces only an inflation of unproven conjectures. There are however strong reasons for $M^8 - H$ duality: both theoretical and physical.

1. The map of $X_l^3 \subset H \to X_l^3 \subset M^8$ and corresponding map of space-time surfaces would allow to realize number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as natural coordinates in which one can say what it means that the point of imbedding space is rational/algebraic. The point of $X^4 \subset H$ is algebraic if it is mapped to an algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactication could in fact be motivated by the number theoretical universality.

2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action since the Kähler form in E^4 has constant components. If the spinor connection in E^4 is combination of the three Kähler forms mimicking neutral part of electro-weak gauge potential, the eigenvalue spectrum for the modified Dirac operator would correspond to that for a fermion in $U(1)$ magnetic field defined by an Abelian magnetic field whereas in $M^4 \times CP_2$ picture $U(2)_{ew}$ magnetic fields would be present.

3. $M^8 - H$ duality provides insights to low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong

$SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

6.4.4 The notion of number theoretical braid

The notion of number theoretic braid is essential for the view about quantum TGD as almost topological quantum field theory. It also realization discretization as a space-time correlate for the finite measurement resolution. Number theoretical universality leads to this notion also and requires that the points in the intersection of the number theoretic braid with partonic 2-surface correspond to rational or at most algebraic points of H in preferred coordinates fixed by symmetry considerations. The challenge has been to find a unique identification of the number theoretic braid. Number theoretic vision indeed makes this possible.

The core element of number theoretic vision is that the laws of physics could be reduced to associativity conditions. One realization for associativity conditions is the level of M^8 endowed with hyper-octonionic structure as a condition that the points sets possible as arguments of N-point function in X^4 are associative and thus belong to hyper-quaternionic subspace $M^4 \subset M^8$. This decomposition must be consistent with the $M^4 \times E^4$ decomposition implied by $M^4 \times CP_2$ decomposition of H. What comes first in mind is that partonic 2-surfaces X^2 belong to $\delta M^4_\pm \subset M^8$ defining the ends of the causal diamond and are thus associative. This boundary condition however freezes E^4 degrees of freedom completely so that M^8 configuration space geometry trivializes.

Are the points of number theoretic braid commutative?

One can also consider the commutativity condition by requiring that arguments belong to a preferred commutative hyper-complex sub-space M^2 of M^8 which can be regarded as a curve in complex plane. Fixing preferred real and imaginary units means a choice of M^2 interpreted as a partial choice of quantization axes at the level of M^8. One must distinguish this choice from the hyper-quaternionicity of space-time surfaces and from the condition that each tangent space of X^4 contains $M^2(x) \subset M^4$ in its tangent space or normal space. Commutativity condition indeed implies the notion of number theoretic braid and fixes it uniquely once a global selection of $M^2 \subset M^8$ is made. There is also an alternative identification of number theoretic braid based on the assumption that braids are light-like curves with tangent vector in $M^2(x)$.

1. The strong form of commutativity condition would require that the arguments of the n-point function at partonic 2-surface belong to the intersection $X^2 \cap M_\pm$. This however allows quite too few points since an intersection of 2-D and 1-D objects in 7-D space would be in question. Associativity condition would reduce cure the problem but would trivialize configuration space geometry.

2. The weaker condition that only $\delta M^4_\pm$ projections for the points of X^2 commute is however sensible since the intersection of 1-D and 2-D surfaces of 3-D space results. This condition is also invariant under number theoretical duality. In the generic case this gives a discrete set of points as intersection of light-like radial geodesic and the projection $P_{\delta M^4_\pm}(X^2)$. This set is naturally identifiable in terms of points in the intersection of number theoretic braids with $\delta CD \times E^4$. One should show that this set of points consists of rational or at most algebraic points. Here the possibility to choose X^2 to some degree could be essential. Any radial light ray from the tip of light-cone allows commutativity and one can consider the possibility of integrating over n-point functions with arguments at light ray to obtain maximal information.

3. For the pre-images of light-like 3-surfaces commutativity of the points in $\delta M^4_\pm$ projection would allow the projections to be one-dimensional curves of M^2 having thus interpretation as braid strands. M^2 would play exactly the same role as the plane into which braid strands are projected in the construction of braid invariants. Therefore the plane of non-physical polarizations in gauge theories corresponds to the plane to which braids and knots are projected in braid and knot theories. A further constraint is that the braid strand connects algebraic points of M^8 to

algebraic points of M^8. It seems that this can be guaranteed only by posing some additional conditions to the light-like 3-surfaces themselves which is of course possible since they are in the role of fundamental dynamical objects.

Are number theoretic braids light-like curves with tangent in $M^2(x)$?

There are reasons why the identification of the number theoretic braid strand as a curve having hyper-complex light-like tangent looks more attractive.

1. An alternative identification of the number theoretic braid would give up commutativity condition for M^4 projection and assume braid strand to be as a light-like curve having light-like tangent belonging to the local hyper-complex tangent sub-space $M^2(x)$ at point x. This definition would apply both in $X^3 \subset \delta M^4_\pm \times CP_2$ and in X^3_l. Also now one would have a continuous distribution of number theoretic braids, with one braid assignable to each light-like curve with tangent $\delta M^4_+ \supset M_+(x) \subset M^2(x)$. In this case each light-like curve at δM^4_+ with tangent in $M_+(x)$ would define a number theoretic braid so that the only difference would be the replacement of light-like ray with a more general light-like curve.

2. The preferred plane $M^2(x)$ can be interpreted as the local plane of non-physical polarizations so that the interpretation as a number theoretic analog of gauge conditions posed in both quantum field theories and string models is possible. In TGD framework this would mean that super-conformal degrees of freedom are restricted to the orthogonal complement of $M^2(x)$ and $M^2(x)$ does not contribute to the configuration space metric. In Hamilton-Jacobi coordinates the pairs of light-like curves associated with coordinate lines can be interpreted as curved light rays. Hence the partonic planes $M^2(x_i)$ associated with the points of the number theoretic braid could be also regarded as carriers four-momenta of fermions associated with the braid strands so that the standard gauge conditions $\epsilon \cdot p = 0$ for polarization vector and four-momentum would be realized geometrically. The possibility of M^2 to depend on point of X^3_l would be essential to have non-collinear momenta and for a classical description of interactions between braid strands.

3. One could also define analogs of string world sheets as sub-manifolds of $P_{M^4_+}(X^4)$ having $M^2(x) \subset M^4$ as their tangent space or being assignable to their tangent containing $M_+(x)$ in the case that the distribution defined by the planes $M^2(x)$ exists and is integrable. It must be emphasized that in the case of massless extremals one can assign only $M_+(x) \subset M^4$ to $T(X^4(x))$ so that only a foliation of X^4 by light-like curves in M^4 is possible. For $P_{M^4_+}(X^4)$ however a foliation by 2-D stringy surfaces is obtained. Integrability of this distribution and thus the duality with stringy description has been suggested to be a basic feature of the preferred extremals and is equivalent with the existence of Hamilton-Jacobi coordinates for a large class of extremals of Kähler action [D1].

4. The possibility of dual descriptions based on integrable distribution of planes $M^2(x)$ allowing identification as 2-dimensional stringy sub-manifolds of $X^4(X^3)$ and the flexibility provided by the hyper-complex conformal invariance raise the hopes of achieving the lifting of super-symplectic algebra SS and super Kac-Moody algebra SKM to H. At the light-cone boundary the light-like radial coordinate could be lifted to a hyper-complex coordinate defining coordinate for M^2. At X^3_l one could fix the light-like coordinate varying along the braid strands and it can can be lifted to a light-like hyper-complex coordinate in M^4 by requiring that the tangent to the coordinate curve is light-like line of $M^2(x)$ at point x. The total four-momenta and color quantum numbers assignable to SS and SKM degrees of freedom are naturally identical since they can be identified as the four-momentum of the partonic 2-surface $X^2 \subset X^3 \cap \delta M^4_\pm \times CP_2$. Equivalence Principle would emerge as an identity.

Are also CP_2 duals of number theoretic braids possible?

This picture is probably not enough. From the beginning the idea that also the CP_2 projections of points of X^2 define number theoretic braids has been present. The dual role of the braids defined by M^2 and CP_2 projections of X^2 is suggested both by the construction of the symplectic fusion algebras [C4] and by the model of anyons [F12]. M^2 and the geodesic sphere $S^2_i \subset CP_2$, where one has either $i = I$ or $i = II$, where $i = I/II$ corresponds to homologically trivial/non-trivial geodesic sphere, are in a key role in the geometric realization of the hierarchy of Planck constants in terms of the book like structure of the generalized imbedding space. The fact that S^2_I corresponds to vacuum extremals would suggest that only the intersection $S^2_{II} \cap P_{CP_2}(X^2)$ can define CP_2 counterpart of the number theoretic braid. M^4 braid could be the proper description in the associative case (Minkowskian signature of induced metric) and CP_2 braid in the co-associative case (Euclidian signature of induced metric). The duality of these descriptions would be reflected also by the fact that the physical Planck constant is given by $\hbar = r\hbar_0$, $r = \hbar(M^4)/\hbar(CP_2)$, so that only the ratio of the two Planck constants matters in commutation relations.

6.5 General vision about real and p-adic coupling constant evolution

The unification of super-canonical and Super Kac-Moody symmetries allows new view about p-adic aspects of the theory forcing a considerable modification and refinement of the almost decade old first picture about color coupling constant evolution.

Perhaps the most important questions about coupling constant evolution relate to the basic hypothesis about preferred role of primes $p \simeq 2^k$, k an integer. Why integer values of k are favored, why prime values are even more preferred, and why Mersenne primes $M_n = 2^n - 1$ and Gaussian Mersennes seem to be at the top of the hierarchy?

Second bundle of questions relates to the color coupling constant evolution. Do Mersenne primes really define a hierarchy of fixed points of color coupling constant evolution for a hierarchy of asymptotically non-free QCD type theories both in quark and lepton sector of the theory? How the transitions $M_n \rightarrow M_{n(next)}$ occur? What are the space-time correlates for the coupling constant evolution and for for these transitions and how space-time description relates to the usual description in terms of parton loops? How the condition that p-adic coupling constant evolution reflects the real coupling constant evolution can be satisfied and how strong conditions it poses on the coupling constant evolution?

6.5.1 A general view about coupling constant evolution

Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state. Equivalence Principle is expected to hold true for elementary particles and their composites but not for the quantum states defined around non-vacuum extremals.

Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [C3] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [19] known as hyperfinite factor of type II_1 (HFF) [A9, A8, C3]. HFF [20, 26] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [27]. The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [28], anyons [30], quantum groups and conformal field theories[21, 22], and knots and topological quantum field theories [26, 27].

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [C3]. The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. M-matrix is identifiable in terms of Connes tensor product [26] and therefore exists and is almost unique. Connes tensor product implies that the Hermitian elements of the included algebra commute with M-matrix and hence act like infinitesimal symmetries. A connection with integrable quantum field theories is suggestive. The remaining challenge is the calculation of M-matrix and the needed machinery might already exist.

The tension is present also now. The connection with visions should come from the discretization in terms of number theoretic braids providing space-time correlate for the finite measurement resolution and making p-adicization in terms of number theoretic braids possible. Number theoretic braids give a connection with the construction of configuration space geometry in terms of Dirac determinant and with TGD as almost TQFT and with conformal field theory approach. The mathematics for the inclusions of hyper-finite factors of type II_1 is also closely related to that for conformal field theories including quantum groups relating closely to Connes tensor product and non-commutativity.

How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

Zero energy ontology in which zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of future and past directed light-cones are power of 2 multiples of fundamental time scale ($T_n = 2^nT_0$) implies in a natural manner coupling constant evolution. One must however emphasize that also the weaker condition $T_p = pT_0$, p prime, is possible, and would assign all p-adic time scales to the size scale hierarchy of CDs.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^nT_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$

suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

6.5.2 Both symplectic and conformal field theories are needed in TGD framework

Before one can say anything quantitative about coupling constant evolution, one must have a formulation for its TGD counterpart and thus also a more detailed formulation for how to calculate M-matrix elements. There is also the question about infinities. By very general arguments infinities of quantum field theories are predicted to cancel in TGD Universe - basically by the non-locality of Kähler function as a functional of 3-surface and by the general properties of the vacuum functional identified as the exponent of Kähler function. The precise mechanism leading to the cancellation of infinities of local quantum field theories has remained unspecified. Only the realization that the symplectic invariance of quantum TGD provides a mechanism regulating the short distance behavior of N-point functions changed the situation in this respect. This also leads to concrete view about the generalized Feynman diagrams giving M-matrix elements and rather close resemblance with ordinary Feynman diagrammatics.

Symplectic invariance

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of lightcone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [C2] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [D8]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s \quad . \tag{6.5.1}$$

   Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1,s_2,s))Idd\mu_s$ so that one has

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1,s_2,s))d\mu_s \quad . \tag{6.5.2}$$

   Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n = 1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1,s_2,s_3) \equiv A(s_1,s_2,s_3,N) - A(s_1,s_2,s_3,S) \quad (6.5.3)$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c^r_{kl}\int f(\Delta A(s_1,s_2,s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \quad (6.5.3)$$
$$c^r_{kl}c_{rm}\int f(\Delta A(s_1,s_2,s))f(\Delta A(s,s_3,t))d\mu_s d\mu_t \ . \quad (6.5.4)$$

Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1,s_2,s))d\mu_s \quad (6.5.5)$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

Generalization to quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments coincide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n{+}1$-simplices for $n{+}1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n{+}1)$ are given by $N(k,n{+}1) = N(k{-}1,n){+}N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

   The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of configuration space. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions

could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [C3] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [E2].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time.

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and configuration space degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of the configuration space Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. The coupling constant evolution is based on the same mechanism as in QFT and symplectic invariance replaces ad hoc UV cutoff with a genuine dynamical regulation mechanism. Causal diamond itself defines the physical IR cutoff. p-Adic and real coupling constant evolutions reflect the underlying evolution in powers of two for the temporal distance between the tips of the light-cones of the causal diamond and the association of macroscopic time scale as secondary p-adic time scale to elementary particles (.1 seconds for electron) serves as a first test for the picture. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules could be treated independently.

More detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities would follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-canonical conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [C2].

1. The state construction utilizes both super-canonical and super Kac-Moody algebras. Super-canonical algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-canonical algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-canonical operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO - H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic 4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the CH Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-canonical super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the CH super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to CH gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^\dagger/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^{dagger}$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k\gamma_k^\dagger$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since fermionic propagator and boson-emission vertices give compensating fermion numbers.

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

In this framework coupling constant evolution would have interpretation in terms of addition of intermediate zero energy states corresponding to the generalized Feynman diagrams obtained by the insertion of causal diamonds with a new shorter time scale $T = T_{prev}/2$ to the previous Feynman diagram. p-Adic length scale hypothesis follows naturally. A very close correspondence with ordinary Feynman diagrammatics arises and and ordinary vision about coupling constant evolutions arises. The absence of infinities follows from the symplectic invariance which is genuinely new element. p-Adic and real coupling constant evolutions can be seen as completions of coupling constant evolutions for physics based on rationals and their algebraic extensions.

6.6 Does the evolution of gravitational coupling make sense at space-time level?

Coset construction for super-symplectic and super Kac-Moody algebras discussed in [B4, F2, C2] allows to generalize Equivalence Principle and understand it at quantum level. This is however not quite enough: a precise understanding of Equivalence Principle is required also at the classical level. In the following the notion of gravitational mass and its equivalence with inertial mass is discussed first. The strategy is to deduce connection with string model type description rather than trying to show that General Relativity emerges from TGD. This connection emerges trough dimensional reduction of the dynamics defined by Kähler action to stringy dynamics. If one believes that string model description implies General Relativity in long scales, the situation is settled. The determination of gravitational mass as flux does not apply generally so that one cannot identify GM as a gravitational flux assignable to a wormhole throat. Hence one cannot formulate the evolution of G at space-time level as evolution of gravitational fluxes and it seems that only p-adic coupling constant evolution makes sense for G.

6.6.1 Is stringy action principle coded by the geometry of preferred extremals?

It seems very difficult to deduce Equivalence Principle as an identity of gravitational and inertial masses identified as Noether charges associated with corresponding action principles. Since string model is an excellent theory of quantum gravitation, one can consider a less direct approach in which one tries to deduce a connection between classical TGD and string model and hope that the bridge from string model to General Relativity is easier to build. Number theoretical compactification gives good hopes that this kind of connection exists.

1. Number theoretic compactification implies that the preferred extremals of Kähler action have the property that one can assign to each point of M^4 projection $P_{M^4}(X^4(X^3_l))$ of the preferred extremal $M^2(x)$ identified as the plane of non-physical polarizations and also as the plane in which local massless four-momentum lies.

2. If the distribution of the planes $M^2(x)$ is integrable, one can slice $P_{M^4}(X^4(X^3_l))$ to string world-sheets. The intersection of string world sheets with $X^3 \subset \delta M^4_\pm \times CP_2$ corresponds to a light-like curve having tangent in local tangent space $M^2(x)$ at light-cone boundary. This is the first candidate for the definition of number theoretic braid. Second definition assumes M^2 to be fixed at δCD: in this case the slicing is parameterized by the sphere S^2 defined by the light rays of $\delta M^4_\pm$.

3. One can assign to the string world sheet -call it Y^2 - the standard area action

$$S_G(Y^2) = \int_{Y^2} T\sqrt{g_2}d^2y \ , \tag{6.6.1}$$

where g_2 is either the induced metric or only its M^4 part. The latter option looks more natural since M^4 projection is considered. T is string tension.

4. The naivest guess would be $T = 1/\hbar G$ apart from some numerical constant but one must be very cautious here since $T = 1/L_p^2$ apart from a numerical constant is also a good candidate if one accepts the basic argument identifying G in terms of p-adic length L_p and Kähler action for two pieces of CP_2 type vacuum extremals representing propagating graviton. The formula reads $G = L_p^2 exp(-2aS_K(CP_2))$, $a \leq 1$. The interaction strength which would be L_p^2 without the presence of CP_2 type vacuum extremals is reduced by the exponential factor coming from the exponent of Kähler function of configuration space.

5. One would have string model in either $CD \times CP_2$ or $CD \subset M^4$ with the constraint that stringy world sheet belongs to $X^4(X_l^3)$. For the extremals of $S_G(Y^2)$ gravitational four-momentum defined as Noether charge is conserved. The extremal property of string world sheet need not however be consistent with the preferred extremal property. This constraint might bring in coupling of gravitons to matter. The natural guess is that graviton corresponds to a string connecting wormhole contacts. The strings could also represent formation of gravitational bound states when they connect wormhole contacts separated by a large distance. The energy of the string is roughly $E \sim \hbar TL$ and for $T = 1/\hbar G$ gives $E \sim L/G$. Macroscopic strings are not allowed except as models of black holes. The identification $T \sim 1/L_p^2$ gives $E \sim \hbar L/L_p^2$, which does not favor long strings for large values of $\hbar$. The identification $G_p = L_p^2/\hbar_0$ gives $T = 1/\hbar G_p$ and $E \sim \hbar_0 L/L_p^2$, which makes sense and allows strings with length not much longer than p-adic length scale. Quantization - that is the presence of configuration space degrees of freedom- would bring in massless gravitons as deformations of string whereas strings would carry the gravitational mass.

6. The exponent $exp(iS_G)$ can appear as a phase factor in the definition of quantum states for preferred extremals. S_G is not however enough. One can assign also to the points of number theoretic braid action describing the interaction of a point like current Qdx^μ/ds with induced gauge potentials A_μ. The corresponding contribution to the action is

$$S_{braid} = \int_{braid} iTr(Q\frac{dx^\mu}{ds}A_\mu)dx \ . \tag{6.6.2}$$

In stationary phase approximation subject to the additional constraint that a preferred extremal of Kähler action is in question one obtains the desired correlation between the geometry of preferred extremal and the quantum numbers of elementary particle. This interaction term carries information only about the charges of elementary particle. It is quite possible that the interaction term is more complex: for instance, it could contain spin dependent terms (Stern-Gerlach experiment).

7. The constraint coming from preferred extremal property of Kähler action can be expressed in terms of Lagrange multipliers

$$S_c = \int_{Y^2} \lambda^k D_\alpha(\frac{\partial L_K}{\partial_\alpha h^k})\sqrt{g_2}d^2y \ . \tag{6.6.3}$$

8. The action exponential reads as

$$exp(iS_G + S_{braid} + S_c) \ . \tag{6.6.4}$$

The resulting field equations couple stringy M^4 degrees of freedom to the second variation of Kähler action with respect to M^4 coordinates and involve third derivatives of M^4 coordinates at the right hand side. If the second variation of Kähler action with respect to M^4 coordinates vanishes, free string results. This is trivially the case if a vacuum extremal of Kähler action is in question.

9. An interesting question is whether the preferred extremal property boils down to the condition that the second variation of Kähler action with respect to M^4 coordinates vanishes so that gravitonic string is free. The physical interpretation would be in terms of quantum criticality which is the basic conjecture about the dynamics of quantum TGD. This is clear from the fact that in 1-D system criticality means that the potential $V(x) = ax + bx^2 + ..$ has $b = 0$. In field theory criticality corresponds to the vanishing of the term $m^2\phi^2/2$ so that massless situation corresponds to massless theory and criticality and long range correlations.

6.6.2 What does the equality of gravitational and inertial masses mean?

Consider next the question in what form Equivalence Principle could be realized in this framework.

1. Coset construction inspires the conjecture that gravitational and inertial four-momenta are identical. Also some milder form of it would make sense. What is clear is that the construction of preferred extremal involving the distribution of $M^2(x)$ implies that conserved four-momentum associated with Kähler action can be expressed formally as stringy four-momentum. The integral of the conserved inertial momentum current over X^3 indeed reduces to an integral over the curve defining string as one integrates over other two degrees of freedom. It would not be surprising if a stringy expression for four-momentum would result but with string tension depending on the point of string and possibly also on the component of four-momentum. If the dependence of string tension on the point of string and on the choice of the stringy world sheet is slow, the interpretation could be in terms of coupling constant evolution associated with the stringy coordinates. An alternative interpretation is that string tension corresponds to a scalar field. A quite reasonable option is that for given X^3_l T defines a scalar field and that the observed T corresponds to the average value of T over deformations of X^3_l.

2. The minimum option is that Kähler mass is equal to the sum gravitational masses assignable to strings connecting points of wormhole throat or two different wormhole throats. This hypothesis makes sense even for wormhole contacts having size of order Planck length.

3. The condition that gravitational mass equals to the inertial mass (rest energy) assigned to Kähler action is the most obvious condition that one can imagine. The breaking of Poincare invariance to Lorentz invariance with respect to the tip of CD supports this form of Equivalence Principle. This would predict the value of the ratio of the parameter R^2T and p-adic length scale hypothesis would allow only discrete values for this parameter. $p \simeq 2^k$ following from the quantization of the temporal distance $T(n)$ between the tips of CD as $T(n) = 2^nT_0$ would suggest string tension $T_n = 2^nR^2$ apart from a numerical factor. $G_p \propto 2^nR^2/\hbar_0$ would emerge as a prediction of the theory. G could be seen as a prediction or RG invariant input parameter fixed by quantum criticality. The arguments related to p-adic coupling constant evolution suggest $R^2/\hbar_0G = 3 \times 2^{23}$ [A9].

4. The scalar field property of string tension should be consistent with the vacuum degeneracy of Kähler action. For instance, for the vacuum extremals of Kähler action stringy action is non-vanishing. The simplest possibility is that one includes the integral of the scalar $J^{\mu\nu}J_{\mu\nu}$ over the degrees transversal to M^2 to the stringy action so that string tension vanishes for vacuum extremals. This would be nothing but dimensional reduction of 4-D theory to a 2-D theory using the slicing of $X^4(X^3_l)$ to partonic 2-surfaces and stringy word sheets. For cosmic strings Kähler action reduces to stringy action with string tension $T \propto 1/g_K^2R^2$ apart from a numerical constant. If one wants consistency with $T \propto 1/L_p^2$, one must have $T \propto 1/g_K^2 2^nR^2$ for the cosmic strings deformed to Kähler magnetic flux tubes. This looks rather plausible if the thickness of deformed string in M^4 degrees of freedom is given by p-adic length scale.

Should one introduce induced spinor fields at string world sheets?

In the previous section it was found that TGD should allow also dimensionally reduced descriptions in terms of either string world sheets or partonic 2-surfaces. This raises the question whether it makes sense to introduce induced spinor fields at string world sheets. This is indeed the case. The modified Dirac action would in this case correspond to the Dirac operator for the dimensionally reduced Kähler action. The effective minimal surface property of Y^2 would guarantee the conservation of the super current. The realization of the effective 3-dimensionality in turn means that the stringy coordinate u corresponds to a gauge degree of freedom or to the condition $D_u\Psi = 0$. There would no spinor waves propagating along this direction of string and only the deformations of string represented by symplectic and Kac-Moody algebras present also in the dynamics of Kähler action responsible for the p-adic thermodynamics would be present. Besides this there would be the fermionic excitations associated with the ends of the string and correspond to the eigenmodes of $D_K(X^2)$ or equivalently with $D_K(Y^1)$ so that the Dirac determinant would be the same as obtained for D_K. For the description in terms of partonic 2-surfaces the Dirac operator would be just $D_K(X^2)$ and also now the equivalence with the 4-D description follows trivially.

6.6.3 What is the connection with General Relativity?

The connection with the stringy description makes it easier to believe that General Relativity gives a reasonable approximate description of gravitational interactions in long length scales also in TGD framework. The vacuum degeneracy of Kähler action is in key role here. The topological condensation of CP_2 type vacuum extremals representing fermions and pieces of CP_2 type extremals (wormhole contacts) identified as gauge bosons deforms the vacuum extremals to non-vacuum extremals, and the resulting density of inertial momentum equals to the density of gravitational momentum in stringy sense. If stringy gravitational energy momentum density is proportional to $1/L_p^2$ and if G relates to L_p^2 in the proposed manner, the natural hypothesis is that Einstein tensor provides a good approximation for the density of gravitational four-momentum as non-conserved Noether currents for the curvature scalar action associated with the induced metric. In zero energy ontology the non-conservation of the density of gravitational momentum does not lead to a contradiction with the conservation of inertial four-momentum since inertial four-momentum is defined only for CD in given scale so that conservation laws hold also only in this scale and in finite measurement resolution.

6.6.4 What does one mean with the evolution of gravitational constant?

From above it is clear that although it is possible to speak about the evolution of string tension $T(x)$ for string space-time sheets inside given CD, it does not makes sense to speak about evolution of G inside CDs because the relationship between T and G is not so simple as one might naively expect. One can of course consider the possibility that $T(x)$ is RG invariant and thus constant for the preferred extremals of Kähler action. This could hold module finite measurement resolution for M^4 coordinates defined by the size of the sub-CDs of a given CD. Hence string model description would be exact under quantum criticality assumption in the sense that the second variation of Kähler action with respect to M^4 coordinates vanishes.

As found, gravitational constant can be understood as a product of L_p^2 with the exponential of Kähler action for the two pieces of CP_2 type vacuum extremals representing wormhole contacts assignable to graviton connected by string world sheet. The volume of the typical CP_2 type extremals associated with the graviton increases with L_p so that the exponential factor decreases reducing the growth due to the increase of L_p. Hence G could be RG invariant in p-adic coupling constant evolution: this requires that volume depends on logarithmically on L_p. This point will be discussed in more detail later.

6.7 RG invariance of gauge couplings inside CD

The first question is whether the RG evolution of all gauge couplings could have interpretation as a flow at space-time level and what the flow in question could be. Second question is how the p-adic coupling constant evolution suggesting that coupling constants are piece constant functions of length scale is realized at space-time level. The obvious guess would be that RG invariance holds true for given CD. This would conform with the fact that partonic wormhole throats associated with the light-like boundaries of CDs can be regarded as carriers of quantum numbers in zero energy ontology.

6.7.1 Are all gauge couplings RG invariants within given CD?

No extremals for which the gauge currents would have non-vanishing ordinary divergence are known at this moment (gauge currents are light-like always). Therefore one cannot exclude the possibility that all gauge coupling constants are renormalization group invariants within given CD , so that the hypothesis that RG evolution reduces to a discrete p-adic coupling constant evolution would be correct.

This requires that also Weinberg angle, being determined by the ratio of $SU(2)$ and $U(1)$ couplings, is constant inside a given space-time sheet. Its value in this case is determined most naturally by the requirement that the net vacuum em charge of the space-time sheet vanishes.

A further hypothesis is Kähler coupling strength is invariant also under p-adic coupling constant evolution. Kähler coupling strength is in principle prediction of the theory if Dirac determinant gives Kähler action so that this hypothesis can in principle be checked.

6.7.2 Slicing of space-time surface by light-like 3-surfaces

The basic question concerns the identification of the geometric parameter identifiable as the space-time counterpart of the scale associated with RG evolution. Number theoretical compactification gives clues concerning the identification of this kind of parameter.

1. Number theoretic compactification implies that the preferred extremals of Kähler action have the property that one can assign to each point of M^4 projection $P_{M^4}(X^4(X_l^3))$ of the preferred extremal $M^2(x)$ identified as the plane of non-physical polarizations and also as the plane in which local massless four-momentum lies.

2. If the distribution of the planes $M^2(x)$ is integrable, one can slice $P_{M^4}(X^4(X_l^3))$ to string world-sheets. The intersection of string world sheets with $X^3 \subset \delta M^4_\pm \times CP_2$ corresponds to a light-like curve having tangent in local tangent space $M^2(x)$ at light-cone boundary. This is the first candidate for the definition of number theoretic braid. Second definition assumes M^2 to be fixed at δCD: in this case the slicing is parameterized by the sphere S^2 defined by the light rays of $\delta M^4_\pm$.

3. Another slicing is based on the use of light-like 3-surfaces for which second light-like coordinate associated with M^2 - call it u - is constant for a given slice. By general coordinate invariance it should be a matter of taste whether deduced the predictions of the theory using any of these light-like 3-surfaces. In particular the value of Kähler function remains invariant. The conditions guaranteing under what conditions this is true are discussed in [C2, B4]

The natural identification of the RG group parameter would be as the light-like coordinate u of M^4. This parameter corresponds roughly to radial motion away from wormhole throat and in this sense scaling. Light-likeness however means that M^4 length along this coordinate line is zero so that the length of RG parameter does not increase during RG evolution. Hence RG invariance looks natural.

6.7.3 Coupling constant evolution as evolution of classical gauge fluxes

Wormhole throats are in special role in the evolution as fixed points which is obvious from the fact that the determinant of induced metric approaches to zero. At the wormhole throats one must pose the conditions $g_{ui} = 0$ and $J_{ui} = 0$ in order to guarantee that the normal components of conserved currents vanish. This guarantees standard conservation laws for space-like 3-surfaces and is also required by zero energy ontology. The condition J_{ui} does not imply that the flux of Kähler electric field associated with 2-surface at wormhole throat vanishes. The point is that J^{uv} diverges whereas $\sqrt{g_4}$ vanishes at this limit and the limiting value of the flux defined by $J^{ub}\sqrt{g_4}$ can be finite and should be so unless there is Kähler charge density associated with vacuum, or more precisely, $j^u = D_i J^{ui}$ is non-vanishing. j^v can be non-vanishing and would mean that there are light-like currents along the light-like 3-surfaces Y_l^3 associated with the slicing. This of course conforms with the idea that any light-like 3-surface can be regarded as a carrier of quantum numbers. The only known extremals of Kähler action for which gauge currents are non-vanishing are indeed those for which they are light-like. If $j^v = 0$ holds for Kähler current it holds true for all gauge currents and it would not be surprising that the gauge fluxes vanish.

Consider first electro-weak coupling constant evolution.

1. It is natural to restrict the coupling constant evolution to the neutral part F_{nc} of the electro-weak gauge field consisting of γ and Z^0, whose expressions in terms of Kähler form and R_{03} component of spinor curvature are given by

$$\begin{aligned} F_{nc} &= \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ , \\ \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{6.7.-1}$$

These expressions are discussed in more detail in Appendix.

2. One must find a gauge field which is Abelian in order that the notion of gauge flux is well-defined. If one restricts the consideration to right-handed parts Z^0 and γ this is achieved since W has only left handed part. The fluxes are determined by γ and Z^0 and the charge matrices multiplied with $(1-\gamma_5)$ and I_L^3 is dropped from the charge matrix of Z^0.

3. Quantum classical correspondence suggests a quantization of classical gauge charged. This can be understood as resulting from the presence of phase factors of form $exp(i(dx^{mu}/dv)Tr(QA_\mu)$ associated with braid strands at X_l^3. In stationary phase approximation an extremal with classical charges equal to those associated with positive (negative) energy part of zero energy state is selected. This extremal should have the property that classical gauge flux equals to the appropriate diagonal element of charge matrix multiplied by the corresponding coupling constant. This boils down to the conditions

$$\begin{aligned} e\langle Q_{em}\rangle &= \int \gamma da = (3J - sin^2(\theta_W)R_{03})da \ , \\ -g_Z sin^2(\theta_W)\langle Q_{em}\rangle &= \int Z_0 da = 2\int R_{03} da \ . \end{aligned} \tag{6.7.-2}$$

The most natural possibility is that the diagonal charge matrix element is between positive and negative energy parts of the zero energy state associated with CD. A stronger form of quantum classical correspondence would require that similar equations hold true also for the fluxes of W bosons. The expectation values would be vanishing in charge eigen states so that also the classical fluxes should vanish.

4. Since the fluxes of J and R_{03} remain constant by the previous assumption e and g_Z are RG invariants if $sin^2(\theta_W)$ is RG invariant. There is no natural manner for $sin^2(\theta)$ to evolve since it is determined in terms of quantities associated with the throats associated with gauge boson wormhole contact.

The RG evolution of α_s inside CD can be discussed along similar lines.

1. Color gauge field is given by $G^A = kH^AJ_{\alpha\beta}$, where k is numerical constant and H^A is a Hamiltonian of color isometry. Color gauge field has Abelian holonomy, which suggests that one can reduce the situation to Abelian one by performing a local gauge rotation rotating the color gauge field to a fixed direction. This is however somewhat tricky point since strictly local color rotations are not symmetries of Kähler action. The naive guess would be

$$g_s\langle T^A\rangle = k\int H^AJda \ , \tag{6.7.-1}$$

Also no expectation would be naturally between positive and negative energy parts of zero energy state. Only the fluxes associated with I_3 and Y_A would be non-vanishing so that additional conditions of color fluxes would be obtained.

2. The formula

$$\frac{1}{\alpha_{em}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K}$$

proposed in previous section would fix the p-adic evolution of color coupling and imply the RG invariance of α_s within given CD.

6.7.4 Questions related to the physical interpretation

This picture raises several interesting questions related to the physical interpretation.

1. What is the TGD counterpart of Higgs=0 phase? The dimension of CP_2 projection is is analogous to temperature and one can argue that massivation is analogous to a loss of correlations due to the increase of D bringing in additional degrees of freedom. Massless extremals having $D = 2$ all induced gauge fields are massless so that they are excellent candidates for Higgs=0 phase. Indeed, the construction of S-matrix leads to the interpretation that MEs allow massless particle exchanges with arbitrary long range but the very fact that the scattering is limited to massless momentum exchanges it is difficult to detect. Note that this scattering is not possible in two-particle system. Does the result mean that already $D = 3$ space-time sheets correspond to a massive phase?

2. Why electro-weak length scale corresponding to Mersenne prime M_{89} is preferred [F3]? Are there also other length scales in which electro-weak massivation occurs and thus scaled copies of electro-weak bosons? These questions reduce to the questions about the stability of the proposed bifurcations.

3. The basic problem of TGD based model of condensed matter is to explain why classical long range gauge fields do not give rise to large parity breaking effects in atomic length scale but do so in cell length length scale at least in the case of living matter (bio-catalysis). The proposal has been that particles feed electro-weak and em gauge fluxes to different space-time sheets. Could it be that blocks of bio-matter with size larger than cell the space-time sheets at which em and weak charges are feeded can be in Higgs=0 phase whereas for smaller blocks screening occurs already at quark and lepton level.

   This would be consistent with the fact that the dimension D of CP_2 projection tends to decrease with the size of the space-time sheet: the larger the space-time sheet, the nearer it is to a vacuum extremal. Robertson-Walker cosmologies are exact vacuum extremals carrying however non-vanishing gravitational 4-momentum densities. By previous argument W and Z masses are identical in this kind of phase if the vanishing of vacuum em field is used to fix p. The weakening of correlations caused by classical non-determinism might imply massivation.

4. Do long ranged non-screened vacuum Z^0 and W gauge fields have some quantum counterparts as quantum-classical correspondence would suggest? Does dark matter identified as a phase with large value of $\hbar$ [J6] correspond to a phase in which electro-weak symmetry breaking is absent in the bosonic sector?

   This phase would differ from the ordinary one in that the weak charges of dark counterparts of leptons and quarks are not screened in electro-weak length scale but that their masses are very nearly the same as in Higgs=0 phase since the dominant contribution to the masses of elementary fermions is not given by a coupling to Higgs type particle but determined by p-adic thermodynamics [F2, F3]. According to the TGD based model of condensed matter developed in [F9], em charges would be feeded to space-time sheets of order atomic size in this phase.

   Does bio-matter involve this kind of phase at larger space-time sheets as chirality selection suggests [F9]? Does this phase of condensed matter emerge only above length scale defined by the cell size or cell membrane thickness?

   The possibility to assign separate spectrum of values of M^4 and CP_2 Planck constants means also spectrum of scale factors of metric for both M^4 and CP_2 with scaling of covariant metric given by the square of integer n characterizing the quantum phase. If gravitational Planck constant can be identified as CP_2 Planck constant, gigantic values of CP_2 radius are possible in the sectors of the imbedding space corresponding to the dark matter.

Even if one does not accept this identification, the conclusion would seem to be that CP_2 radius can be very large in these phases. Obviously the ranges of weak and color interactions in this kind of phases would be macroscopic and even astrophysical. Second implication would be the presence of precise quantal lattice like structure involving strict quantum correlations in macroscopic length scales. The unavoidable question is whether the extremely tiny size of CP_2 could be scaled up to a macroscopic length scale even at the level of living matter and whether even the science fictive notion of hyper-space travel (which I have never liked!) might make sense after all.

5. An interesting question relates to the predicted presence of long ranged classical color gauge fields in all length scales suggesting a hierarchy of QCD type physics if quantum classical correspondence is taken seriously. The possibility to define the color Hamiltonians apart from an additive constant in principle makes possible to have vanishing classical color isospin and hyper charges at a given space-time sheet without affecting the color transformation properties of Hamiltonians. It is however far from clear whether this trick is enough. A more natural approach is to take seriously the prediction of infinite p-adic hierarchy of QCD type physics and look what the implications are.

6.8 Quantitative predictions for the values of coupling constants

This focus of attention in this section is in quantitative for the p-adic evolution of couplings constants obtained by combining information coming from p-adic mass calculations with number theoretic constraints and general formula for gravitational constant inspired by simple physical picture.

6.8.1 A revised view about coupling constant evolution

The development of the ideas related to number theoretic aspects has been rather tortuous and based on guess work since basic theory has been lacking.

1. The original hypothesis was that Kähler coupling strength is invariant under p-adic coupling constant evolution. Later I gave up this hypothesis and replaced it with the invariance of gravitational coupling since otherwise the prediction would have been that gravitational coupling strength is proportional to p-adic length scale squared. Second first guess was that Kähler coupling strength equals to the value of fine structure constant at electron length scale corresponding to Mersenne prime M_{127}. Later I replaced fine structure constant with electro-weak U(1) coupling strength at this length scale. The recent discussion returns back to the roots in both aspects.

2. The recent discussion relies on the progress made in the understanding of quantum TGD at partonic level [B4]. What comes out is an explicit formula for Kähler couplings strength in terms of Dirac determinant involving only a finite number of eigenvalues of the modified Dirac operator. This formula dictates the number theoretical anatomy of g_K^2 and also of other coupling constants: the most general option is that α_K is a root of rational. The requirement that the rationals involved are simple combined with simple experimental inputs leads to very powerful predictions for the coupling parameters.

3. A further simplification is due to the discreteness of p-adic coupling constant evolution allowing to consider only length scales coming as powers of $\sqrt{2}$. This kind of discretization is necessary also number theoretically since logarithms can be replaced with 2-adic logarithms for powers of 2 giving integers. This raises the question whether $p \simeq 2^k$ should be replaced with 2^k in all formulas as the recent view about quantum TGD suggests.

4. The prediction is that Kähler coupling strength α_K is invariant under p-adic coupling constant evolution and from the constraint coming from electron and top quark masses very near to fine

structure constant so that the identification as fine structure constant is natural. Gravitational constant is predicted to be proportional to p-adic length scale squared and corresponds to the largest Mersenne prime (M_{127}), which does not correspond to a completely super-astronomical p-adic length scale. For the parameter R^2/G p-adicization program allows to consider two options: either this constant is of form e^q or 2^q: in both cases q is rational number. $R^2/G = exp(q)$ allows only M_{127} gravitons if number theory is taken completely seriously. $R^2/G = 2^q$ allows all p-adic length scales for gravitons and thus both strong and weak variants of ordinary gravitation.

5. A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

It seems fair to conclude that the attempts to understand the implications of p-adicization for coupling constant evolution have begun to bear fruits.

General formula for the Kähler coupling strength

The identification of exponent of Kähler function as Dirac determinant leads to a formula relating Kähler action for the preferred extremal to the Dirac determinant. The eigenvalues are proportional to $1/\alpha_K$ since the matrices $\hat{\Gamma}^\alpha$ have this proportionality. This gives the formula

$$exp(\frac{S_{K,R}(X^4(X^3))}{2g_K^2}) = \prod_i \lambda_i = \frac{\prod_i \lambda_{0,i}}{(g_K)^{2N}} \ . \qquad (6.8.1)$$

Here $\lambda_{0,i}$ by definition corresponds to $g_K^2 = 4\pi\alpha_K = 1$. $S_{K,R} = \int J^* J$ is the reduced Kähler action.

For $S_{K,R} = 0$, which might correspond to so called massless extremals [D1] one obtains the formula

$$g_K^2 = (\prod_i \lambda_{0,i})^{1/N} \ . \qquad (6.8.2)$$

Thus for $S_{K,R} = 0$ extremals one has an explicit formula for g_K^2 having interpretation as the geometric mean of the eigenvalues $\lambda_{0,i}$. Several values of α_K are in principle possible.

p-Adicization suggests that $\lambda_{0,i}$ are rational or at most algebraic numbers. This would mean that g_K^2 is N:th root of this kind of number. $S_{K,R}$ in turn would be

$$S_{K,R} = 2g_K^2 log(\frac{\prod_i \lambda_{0,i}}{g_K^{2N}}) \ . \qquad (6.8.3)$$

so that the reduced Kähler action $S_{K,R}$ would be expressible as a product N:th root of rational, and logarithm of rational. This result would provide a general answer to the question about number theoretical anatomy of Kähler coupling strength and S_K.

For CP_2 type vacuum extremal one would have $S_{K,R} = \frac{\pi^2}{2}$ in apparent conflict with the above result. The conflict is of course only apparent since topological condensation of CP_2 type vacuum extremal generates a hole in CP_2 having light-like wormhole throat as boundary so that the value of the action is modified.

Identifications of Kähler coupling strength and gravitational coupling strength

To construct an expression for gravitational constant one can use the following ingredients.

1. The exponent $exp(S_K(CP_2))$ defining vacuum functional and thus the value of Kähler function in terms of the Kähler action $S_K(CP_2)$ of CP_2 type extremal representing elementary particle expressible as

$$S_K(CP_2) = \frac{S_{K,R}(CP_2)}{8\pi\alpha_K} = \frac{\pi}{8\alpha_K} \ . \qquad (6.8.4)$$

Since CP_2 type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \rightarrow a \times S_K(CP_2) \ . \tag{6.8.5}$$

$a < 1$ conforms with the idea that a piece of CP_2 type extremal defining a wormhole contact is in question. One must however keep mind open in this respect.

2. The p-adic length scale L_p assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime $M_{127} = 2^{127} - 1$ defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that M_{127} characterizes these space-time sheets.

1. The formula for the gravitational constant

A long standing basic conjecture has been that gravitational constant satisfies the following formula

$$\begin{aligned} \hbar G &\equiv r\hbar_0 G = L_p^2 \times exp(-2aS_K(CP_2)) \ , \\ L_p &= \sqrt{p}R \ . \end{aligned} \tag{6.8.5}$$

Here R is CP_2 radius defined by the length $2\pi R$ of the geodesic circle. What was noticed before is that this relationship allows even constant value of G if a has appropriate dependence on p.

This formula seems to be correct but the argument leading to it was based on two erratic assumptions compensating each other.

1. I assumed that modulus squared for vacuum functional is in question: hence the factor $2a$ in the exponent. The interpretation of zero energy state as a generalized Feynman diagram requires the use of vacuum functional so that the replacement $2a \rightarrow a$ is necessary.

2. Second wrong assumption was that graviton corresponds to CP_2 type vacuum extremal- that is wormhole contact in the recent picture. This does allow graviton to have spin 2. Rather, two wormhole contacts represented by CP_2 vacuum extremals and connected by fluxes associated with various charges at their throats are needed so that graviton is string like object. This saves the factor $2a$ in the exponent.

The highly non-trivial implication to be discussed later is that ordinary coupling constant strengths should be proportional to $exp(-aS_K(CP_2))$.

The basic constraint to the coupling constant evolution comes for the invariance of g_K^2 in p-adic coupling constant evolution:

$$\begin{aligned} g_K^2 &= \frac{a(p,r)\pi^2}{log(pK)} \ , \\ K &= \frac{R^2}{\hbar G(p)} = \frac{1}{r}\frac{R^2}{\hbar_0 G(p)} \equiv \frac{K_0(p)}{r} \ . \end{aligned} \tag{6.8.5}$$

2. How to guarantee that g_K^2 is RG invariant and N:th root of rational?

Suppose that g_K^2 is N:th root of rational number and invariant under p-adic coupling constant evolution.

1. The most general manner to guarantee the expressibility of g_K^2 as N:th root of rational is guaranteed for both options by the condition

$$a(p,r) = \frac{g_K^2}{\pi^2} log(\frac{pK_0}{r}) \ . \tag{6.8.6}$$

That a would depend logarithmically on p and $r = \hbar/\hbar_0$ looks rather natural. Even the invariance of G under p-adic coupling constant evolution can be considered.

2. The condition

$$\frac{r}{p} < K_0(p) \ . \tag{6.8.7}$$

must hold true to guarantee the condition $a > 0$. Since the value of gravitational Planck constant is very large, also the value of corresponding p-adic prime must very large to guarantee this condition. The condition $a < 1$ is guaranteed by the condition

$$\frac{r}{p} > exp(-\frac{\pi^2}{g_K^2}) \times K_0(p) \ . \tag{6.8.8}$$

The condition implies that for very large values of p the value of Planck constant must be larger than $\hbar_0$.

3. The two conditions are summarized by the formula

$$K_0(p) \times exp(-\frac{\pi^2}{g_K^2}) < \frac{r}{p} < K_0(p) \tag{6.8.9}$$

characterizing the allowed interval for r/p. If G does not depend on p, the minimum value for r/p is constant. The factor $exp(-\frac{\pi^2}{g_K^2})$ equals to 1.8×10^{-47} for $\alpha_K = \alpha_{em}$ so that $r > 1$ is required for $p \geq 4.2 \times 10^{-40}$. $M_{127} \sim 10^{38}$ is near the upper bound for p allowing $r = 1$. The constraint on r would be roughly $r \geq 2^{k-131}$ and $p \simeq 2^{131}$ is the first p-adic prime for which $\hbar > 1$ is necessarily. The corresponding p-adic length scale is .1 Angstroms.

This conclusion need not apply to elementary particles such as neutrinos but only to the space-time sheets mediating gravitational interaction so that in the minimal scenario it would be gravitons which must become dark above this scale. This would bring a new aspect to vision about the role of gravitation in quantum biology and consciousness.

The upper bound for r behaves roughly as $r < 2.3 \times 10^7 p$. This condition becomes relevant for gravitational Planck constant GM_1M_2/v_0 having gigantic values. For Earth-Sun system and for $v_0 = 2^{-11}$ the condition gives the rough estimate $p > 6 \times 10^{63}$. The corresponding p-adic length scale would be of around $L(215) \sim 40$ meters.

4. p-Adic mass calculations predict the mass of electron as $m_e^2 = (5+Y_e)2^{-127}/R^2$ where $Y_e \in [0, 1)$ parameterizes the not completely known second order contribution. Top quark mass favors a small value of Y_e (the original experimental estimates for m_t were above the range allowed by TGD but the recent estimates are consistent with small value Y_e [F4]). The range $[0, 1)$ for Y_e restricts $K_0 = R^2/\hbar_0 G$ to the range $[2.3683, 2.5262] \times 10^7$.

5. The best value for the inverse of the fine structure constant is $1/\alpha_{em} = 137.035999070(98)$ and would correspond to $1/g_K^2 = 10.9050$ and to the range $(0.9757, 0.9763)$ for a for $\hbar = \hbar_0$ and $p = M_{127}$. Hence one can seriously consider the possibility that $\alpha_K = \alpha_{em}(M_{127}$ holds true. As a matter fact, this was the original hypothesis but was replaced later with the hypothesis that α_K corresponds to electro-weak $U(1)$ coupling strength in this length scale. The fact that M_{127} defines the largest Mersenne prime, which does not correspond to super-astrophysical length scale might relate to this co-incidence.

To sum up, the recent view about coupling constant evolution differs strongly from previous much more speculative scenarios. It implies that g_K^2 is root of rational number, possibly even rational, and can be assumed to be equal to e^2. Also $R^2/\hbar G$ could be rational. The new element is that G need not be proportional to p and can be even invariant under coupling constant evolution since the the parameter a can depend on both p and r. An unexpected constraint relating p and r for space-time sheets mediating gravitation emerges.

Are the color and electromagnetic coupling constant evolutions related?

Classical theory should be also able to say something non-trivial about color coupling strength α_s too at the general level. The basic observations are following.

1. Both classical color YM action and electro-weak U(1) action reduce to Kähler action.
2. Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that α_K is a strict RG invariant. One can consider two options.

1. The original idea was that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for $\alpha_{U(1)} = \alpha_s = 2\alpha_K$. Asymptotically the couplings would approach to a fixed point defined by $2\alpha_K$ rather than to zero as in asymptotically free gauge theories.

   Thus one would have

$$\frac{1}{\alpha_{U(1)}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} \ . \tag{6.8.10}$$

   The relationship between $U(1)$ and em coupling strengths is

$$\begin{aligned} \alpha_{U(1)} &= \frac{\alpha_{em}}{cos^2(\theta_W)} \simeq \frac{1}{104.1867} \ , \\ sin^2(\theta_W)_{|10\ MeV} &\simeq 0.2397(13) \ , \\ \alpha_{em}(M_{127}) &= 0.00729735253327 \ . \end{aligned} \tag{6.8.9}$$

   Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value $sin^2(\theta_W) = 0.2397(13)$ corresponding to 10 MeV mass scale [33] is used. Note however that the previous argument implying $\alpha_K = \alpha_{em}(M_{127})$ excludes $\alpha = \alpha_{U(1)}(M_{127})$ option.

2. Second option is obtained by replacing $U(1)$ with electromagnetic gauge $U(1)_{em}$.

$$\frac{1}{\alpha_{em}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} . \tag{6.8.10}$$

   Possible justifications for this assumption are following. The notion of induced gauge field makes it possible to characterize the dynamics of classical electro-weak gauge fields using only the Kähler part of electro-weak action, and the induced Kähler form appears only in the electromagnetic part of the induced classical gauge field. A further justification is that em and color interactions correspond to unbroken gauge symmetries.

The following arguments are consistent with this conclusion.

1. In TGD framework coupling constant is discrete and comes as powers of $\sqrt{2}$ corresponding to p-adic primes $p \simeq 2^k$. Number theoretic considerations suggest that coupling constants g_i^2 are algebraic or perhaps even rational numbers, and that the logarithm of mass scale appearing as argument of the renormalized coupling constant is replaced with 2-based logarithm of the p-adic length scale so that one would have $g_i^2 = g_i^2(k)$. g_K^2 is predicted to be N:th root of rational but could also reduce to a rational. This would allow rational values for other coupling strengths too. This is possible if $sin(\theta_W)$ and $cos(\theta_W)$ are rational numbers which would mean that Weinberg angle corresponds to a Pythagorean triangle as proposed already earlier. This would mean the formulas $sin(\theta_W) = (r^2 - s^2)/(r^2 + s^2)$ and $cos(\theta_W) = 2rs(r^2 + s^2)$.

2. A very strong prediction is that the beta functions for color and $U(1)$ degrees of freedom are apart from sign identical and the increase of $U(1)$ coupling compensates the decrease of the color coupling. This allows to predict the hard-to-calculate evolution of QCD coupling constant strength completely.

3. $\alpha(M_{127}) = \alpha_K$ implies that M_{127} defines the confinement length scale in which the sign of α_s becomes negative. TGD predicts that also M_{127} copy of QCD should exist and that M_{127} quarks should play a key role in nuclear physics [F8, F9]. Hence one can argue that color coupling strength indeed diverges at M_{127} (the largest not completely super-astrophysical Mersenne prime) so that one would have $\alpha_K = \alpha(M_{127})$. Therefore the precise knowledge of $\alpha(M_{127})$ in principle fixes the value of parameter $K = R^2/G$ and thus also the second order contribution to the mass of electron.

4. $\alpha_s(M_{89})$ is predicted to be $1/\alpha_s(M_{89}) = 1/\alpha_K - 1/\alpha(M_{89})$. $sin^2(\theta_W) = .23120$, $\alpha_{em}(M_{89}) \simeq 1/127$, and $\alpha_{U(1)} = \alpha_{em}/cos^2(\theta_W)$ give $1/\alpha_{U(1)}(M_{89}) = 97.6374$. $\alpha = \alpha_{em}$ option gives $1/\alpha_s(M_{89}) \simeq 10$, which is consistent with experimental facts. $\alpha = \alpha_{U(1)}$ option gives $\alpha_s(M_{89}) = 0.1572$, which is larger than QCD value. Hence $\alpha = \alpha_{em}$ option is favored.

To sum up, the proposed formula would dictate the evolution of α_s from the evolution of the electro-weak parameters without any need for perturbative computations. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

Can one deduce formulae for gauge couplings?

The improved physical picture behind gravitational constant allows also to consider a general formula for gauge couplings.

1. The natural guess for the general formula would be as

$$g^2(p,r) = kg_K^2 \times exp[-a_g(p,r) \times S_K(CP_2)] \ . \tag{6.8.11}$$

here k is a numerical constant.

2. The condition

$g_K^2 = e^2(M_{127})$ fixes the value of k if it's value does not depend on the character of gauge interaction:

$$k = exp[a_{gr}(M_{127}, r=1) \times S_K(CP_2)] \ . \tag{6.8.12}$$

Hence the general formula reads as

$$g^2(p,r) = g_K^2 \times exp[(-a_g(p,r) + a_{gr}(M_{127}), r=1)) \times S_K(CP_2)] \ . \tag{6.8.12}$$

The value of $a(M_{127}, r=1)$ is near to its maximum value so that the exponential factor tends to increase the value of g^2 from e^2. The formula can reproduce α_s and various electro-weak couplings although it is quite possibile that Weinberg angle corresponds to a group theoretic factor not representable in terms of $a_g(p,r)$. The volume of the CP_2 type vacuum extremal would characterize gauge bosons. Analogous formula should apply also in the case of Higgs.

3. α_{em} in very long length scales would correspond to

$$e^2(p\rightarrow \infty, r=1) \;\; = \;\; e^2\times exp[(-1+a(M_{127}),r=1))\times S_K(CP_2)] = e^2x \ , \tag{6.8.12}$$

where x is in the range $[0.6549, 0.6609]$.

Formula relating v_0 to α_K and $R^2/\hbar G$

The parameter $v_0 = 2^{-11}$ plays a key role in the formula for gravitational Planck constant and can be also seen as a fundamental constant in TGD framework. As a matter, factor v_0 has interpretation as velocity parameter and is dimensionless when $c = 1$ is used.

If v_0 is identified as the rotation velocity of distant stars in galactic plane, one can use the Newtonian model for the motion of mass in the gravitational field of long straight string giving $v_0 = \sqrt{TG}$. String tension T can be expressed in terms of Kähler coupling strength as

$$T = \frac{b}{2\alpha_K R^2} \ ,$$

where R is the radius of geodesic circle. The factor $b \leq 1$ would explain reduction of string tension in topological condensation caused by the fact that not entire geodesic sphere contributes to the action.

This gives

$$\begin{aligned} v_0 &= \frac{b}{2\sqrt{\alpha_K K}} \ , \\ \alpha_K(p) &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{\hbar G} \ . \end{aligned} \tag{6.8.11}$$

The condition that α_K has the desired value for $p = M_{127} = 2^{127}-1$ defining the p-adic length scale of electron fixes the value of b for given value of a. The value of b should be smaller than 1 corresponding to the reduction of string tension in topological condensation.

The condition 6.8.11 for $v_0 = 2^{-m}$, say $m = 11$, allows to deduce the value of a/b as

$$\frac{a}{b} = \frac{4*log(pK)}{\pi}\frac{2^{2m-1}}{K} \ . \tag{6.8.12}$$

For both $K = e^q$ with $q = 17$ and $K = 2^q$ option with $q = 24 + 1/2$ $m = 10$ is the smallest integer giving $b < 1$. $K = e^q$ option gives $b = .3302$ (.0826) and $K = 2^q$ option gives $b = .3362$ (.0841) for $m = 10$ ($m = 11$).

$m = 10$ corresponds to one third of the action of free cosmic string. $m = 11$ corresponds to much smaller action smaller by a factor rather near $1/12$. The interpretation would be that as m increases the action of the topologically condensed cosmic string decreases. This would correspond to a gradual transformation of the cosmic string to a magnetic flux tube.

To sum up, the resulting overall vision seems to be internally consistent and is consistent with generalized Feynman graphics, predicts exactly the spectrum of α_K, suggests the identification of the inverse of p-adic temperature with k, allows to understand the differences between fermionic and bosonic massivation. One might hope that the additional objections (to be found sooner or later!) could allow to develop a more detailed picture.

6.8.2 Why gravitation is so weak as compared to gauge interactions?

The weakness of gravitational interaction in contrast to other gauge interactions is definitely a fundamental test for the proposed picture. The heuristic argument allowing to understand the value of gravitational constant is based on the assumption that graviton exchange corresponds to the exchange of CP_2 type extremal for which vacuum functional implies huge reduction of the gravitational constant from the value $\sim L_p^2$ implied by dimensional considerations based on p-adic length scale hypothesis to a value $G = exp(-2S_K)L_p^2$ which for $p = M_{127}$ gives gravitational constant for $\alpha_K = \pi a/log(M_{127} \times K)$, where a is near unity and $K = 2\times3\times5....\times23$ is a choice motivated by number theoretical arguments. The value of K is fixed rather precisely from electron mass scale and the proposed scenario for coupling constant evolution fixes both α_K and K completely in terms of electron mass (using p-adic mass calculations) and electro-magnetic coupling at electron length scale $L_{M_{127}}$ by the formula $\alpha_K = \alpha_{em}$ [A9]. The interpretation would be that gravitational masses are measured using p-adic mass scale $M_p = \pi/L_p$ as a natural unit.

Why gravitational interaction is weak?

The first problem is that CP_2 type extremal cannot represent the lowest order contribution to the interaction since otherwise the normalization of the configuration space vacuum functional would give $exp[-2S_K(CP_2)]$ factor canceling the exponential in the propagator so that one would have $G = L_p^2$. The following observations allow to understand the solution of the problem.

1. As already found, the key feature of CP_2 type vacuum extremals distinguishing them from other 3-surfaces is their non-deterministic behavior allowing them to carry off mass shell four-momenta. Other 3-surfaces can give rise only to scattering involving exchange of on mass shell particles and for space-like momentum exchanges there is no contribution.

2. All possible light-like 3-surfaces must be allowed as propagator portions of surfaces X_V^3 but in absence of non-determinism they can give rise to massless exchanges which are typically non-allowed.

3. The contributions of CP_2 type vacuum extremals are suppressed by $exp[-2NS_K(CP_2)]$ factor in presence of N CP_2 type extremals with maximal action. CP_2 type extremals are vacuum extremals and interact with surrounding world only via the topological condensation generating 3-D CP_2 projection near the throat of the wormhole contact. This motivates the assumption that the sector of the configuration space containing N CP_2 type extremals has the approximate structure $CH(N) = CH(0)\times CP^N$, where $CH(0)$ corresponds to the situation without CP_2 type extremals and CP to the degrees of freedom associated with single CP_2 type extremal. With this assumption the functional integral gives a result of form $X \times exp(-2NS_K(CP_2)$ for N CP_2 type extremals. This factorization allows to forget all the complexities of the world of classical worlds which on the first sight seem to destroy all hopes about calculating something and the normalization factor is in lowest order equal to $X(0)$ whereas single CP_2 type extremal gives $exp[-2S_K(CP_2)]$ factor. This argument generalizes also to the case when CP_2 type extremals are allowed to have varying value of action (the distance travelled by the virtual particle can vary).

Massless extremals (MEs) define a natural candidate for the lowest order contribution since for them Kähler action vanishes. MEs describes a dispersion free on-mass shell propagation of massless modes of both induced gauge fields and metric. Hence they can describe only on mass shell massless exchanges of bosons and gravitons which typically vanishes for kinematical reasons except for collinear scattering in the case of massless particles so that CP_2 type extremals would give the leading contribution to the S-matrix element.

There are however exceptional situations in which exchange of ordinary CP_2 type extremals makes kinematically possible the emission of MEs as brehmstrahlung in turn giving rise to exchange of light-like momentum. Since MEs carry also classical gravitational fields, one can wonder whether this kind of exchanges could make possible strong on mass shell gravitation made kinematically possible by ordinary gauge boson exchanges inside interacting systems.

If one takes absolutely seriously the number theoretic argument based on $R^2/G = exp(q)$ ansatz then M_{127} is selected uniquely as the space-time sheet of gravitons and the predicted gravitational coupling strength is indeed weak.

What differentiates between gravitons and gauge bosons?

The simplest explanation for the difference between gauge bosons and gravitons is that for virtual gauge bosons the volume of CP_2 type extremals is reduced dramatically from its maximal value so that $exp(-2S_K)$ brings in only a small reduction factor. The reason would be that for virtual gauge bosons the length of a typical CP_2 type extremal is far from the value giving rise to the saturation of the Kähler action. For gravitational interactions in astrophysical length scales CP_2 type extremals must indeed be very long.

Gravitational interaction should become strong sufficiently below the saturation length scale with gravitational constant approaching its stringy value L_p^2. According to the argument discussed in [A9], this length scale corresponds to the Mersenne prime M_{127} characterizing gravitonic space-time sheets so that gravitation should become strong below electron's Compton length. This suggests a connection with stringy description of graviton. M_{127} quarks connected by the corresponding strings are indeed a basic element of TGD based model of nuclei [F8]. TGD suggests also the existence of lepto-hadrons as bound state of color excited leptons in length scale M_{127} [F7]. Also gravitons corresponding to smaller Mersenne primes are possible but corresponding forces are much weaker than ordinary gravitation. On the other hand, M_{127} is the largest Mersenne prime which does not give rise to super-astronomical p-adic length scale so that stronger gravitational forces are not be predicted in experimentally accessible length scales.

More generally, the saturation length scale should relate very closely to the p-adic length scale L_p characterizing the particle. The amount of zitterbewegung determines the amount dS_K/dl of Kähler action per unit length along the orbit of virtual particle. L_p would naturally define the length scale below which the particle moves in a good approximation along M^4 geodesic. The shorter this length scale is, the larger the value of dS_K/dl is.

If the Kähler action of CP_2 type extremal increases linearly with the distance (in a statistical sense at least), an exponential Yukawa screening results at distances much shorter than saturation length. Therefore CP_2 extremals would provide a fundamental description of particle massivation at space-time level. p-Adic thermodynamics would characterize what happens for a topologically condensed CP_2 type extremal carrying given quantum numbers at the resulting light-like CD. Besides p-adic length scale also the quantized value $T_p = 1/n$ of the p-adic temperature would be decisive. For weak bosons Mersenne prime M_{89} would define the saturation length scale. For photons the p-adic length scale defining the Yukawa screening should be rather long. An n-ary p-adic length scale $L_{M_{89}}(n) = p^{(n-1)/2}L_{M_{89}}$ would most naturally be in question so that the p-adic temperature associated with photon would be $T_p = 1/n$, $n > 1$ [F3]. In the case of gluons confinement length scale should be much shorter than the scale at which the Yukawa screening becomes visible. If also gluons correspond to $n > 1$ this is certainly the case.

All gauge interactions would give rise to ultra-weak long ranged interactions, which are extremely weak compared to the gravitational interaction: the ratio for the strengths of these interactions would be of order $\alpha Q_1Q_2m_e^2/M_1M_2$ and very small for particles whose masses are above electron mass. Note however that MEs give rise to arbitrarily unscreened long ranged weak and color interactions restricted to light-like momentum transfers and these interactions play a key role in the TGD based model of living matter [J6, M3]. This prediction is in principle testable.

6.9 p-Adic coupling constant evolution

p-Adic coupling constant evolution is one of the genuinely new elements of quantum TGD. In the following some aspects of the evolution will be discussed. The discussion is a little bit obsolete as far as the role of canonical identification is considered. The most recent view about p-adic coupling constant evolution is discussed at the end of the section.

6.9.1 p-Adic coupling constant evolution associated with length scale resolution at space-time level

If gauge couplings are indeed RG invariants inside a given space-time sheet, gauge couplings must be regarded as being characterized by the p-adic prime associated with the space-time sheet. The question is whether it is possible to understand also the p-adic coupling constant evolution at space-time level.

A natural view about p-adic length scale evolution is as an existence of a dynamical symmetry mapping the preferred extremal space-time sheet of Kähler action characterized by a p-adic prime p_1 to a space-time sheet characterized by p-adic prime $p_2 > p_1$ sufficiently near to p_1. The simplest guess is that the symmetry transformation corresponds to a scaling of M^4 coordinates in the intersection X^3 of the space-time surface with light-cone boundary $\delta M^4_+ \times CP_2$ by a scaling factor p_2/p_1, which in turn induces a transformation of $X^4(X^3)$, which in general does not reduce to M^4 scaling outside X^3 since scalings are not symmetries of the Kähler action.

This transformation induces a change of the vacuum gauge charges: $Q_i \rightarrow Q_i + \Delta Q_i$, and the renormalization group evolution boils down to the condition

$$\frac{Q_i + \Delta Q_i}{g_i^2 + \Delta g_i^2} = \frac{Q_i}{g_i^2} . \tag{6.9.1}$$

The problem is that this transformation has a continuous variant so that p-adic length scale evolution could reduce to continuous one.

A possible resolution of the problem is based on the observation that the values of the gauge charges depend on the initial values of the time derivatives of the imbedding space coordinates. RG invariance at space-time level suggests that small scalings leave the gauge charge and thus also coupling constant invariant. As a matter fact, this seems to be the case for all known extremals since they form scaling invariant families. The scalings by p_2/p_1 for some $p_2 > p_1$ would correspond to critical points in which bi-furcations occur in the sense that two space-time surfaces $X^4(X^3)$ satisfying the minimization conditions for Kähler action and with different gauge charges appear.

The new space-time surface emerging in the bifurcation would obey effective p_2-adic topology in some length scale range instead of p_1-adic topology. Stability considerations would dictate whether $p_1 \rightarrow p_2$ transition occurs and could also explain why primes $p \simeq 2^k$, k integer, are favored. This kind of bifurcations or even multi-furcations are certainly possible by the breaking of the classical determinism.

6.9.2 p-Adic evolution in angular resolution and dynamical Planck constant

Quantum phases $q = exp(i\pi/n)$ characterized Jones inclusions which have turned out to play key role in the understanding of macroscopic quantum phases in TGD framework. The basic idea is that the different values of Planck constant correspond to evolution in angular resolution in p-adic context characterized by quantum phase $q = exp(i\pi/n)$ characterizing Jones inclusion is. The higher the value of n, the better the angular resolution since the number of different complex phases in extension of p-adic numbers increases with n.

The quantization of Planck constant has been the basic them of TGD for more than one and half years. The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

Jones inclusions and quantization of Planck constants

Jones inclusions combined with simple anyonic arguments turned out to be the key to the unification of existing heuristic ideas about the quantization of Planck constant.

1. The new view allows to understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: their is no landscape.

2. In ordinary phase Planck constants of M^4 and CP_2 are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which

quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of the scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups: $\hbar(M^4) = n_a$ and $\hbar(CP_2) = n_b$ whereas scaling factors of M^4 and CP_2 metrics are n_b^2 and n_a^2 respectively.

At the level of Schrödinger equation this means that Planck constant $\hbar$ correspodns to the effective Planck consant $\hbar_{eff} = (\hbar(M^4)/\hbar(CP_2))\hbar_0 = (n_a/n_b)\hbar_0$, which thus can have all possible positive rational values. For some time I believed on the scaling of metrics of M^4 *resp.* CP_2 as n_b^2 *resp.* n_a^2: this would imply invariance of Schrödinger equation under the scalings but would not be consistent with the explanation of the quantization of radii of planetary orbits requiring huge Planck constant [D7]. Poincare invariance is however achieved in the sense that mass spectrum is invariant under the scalings of Planck constants. That the ratio n_a/n_b defines effective Planck constant conforms with the fact that the value of Kähler action involves only this ratio (quantum-classical correspondence). Also the value of gravitational constant is invariant under the scalings of Planck constant since one has $G \propto g_K^2 R^2$, R radius of CP_2 for $n_a = 1$.

3. This predicts automatically arbitrarily large values of effective Planck constant n_a/n_b and they correspond to coverings of CP_2 points by large number of M^4 points which can have large distance and have precisely correlated behavior due to the G_a symmetry. One can assign preferred values of Planck constant to quantum phases $q = exp(i\pi/n)$ expressible in terms of iterated square roots of rationals: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter seem to correspond to these special values of Planck constants. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

4. These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_{2n}, E_6, E_8 are possible so that TGD seems to be able to mimic these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

The values of gravitational Planck constant

The understanding of large Planck constants led to the detailed interpretation of what is involved with the emergence of gigantic gravitational Planck constant.

Gravitational Planck constant $\hbar_{gr}$ can be interpreted as effective Planck constant $\hbar_{eff} = (n_a/n_b)\hbar_0$ so that the Planck constant associated with M^4 degrees of freedom (rather than CP_2 degrees of freedom as in the original wrong picture) must be very large in this kind of situation.

The detailed spectrum for Planck constants gives very strong constraints to the values of $\hbar_{gr} = GMm/v_0$ if ones assumes that favored values of Planck constant correspond to the Jones inclusions for which quantum phase corresponds to a simple algebraic number expressible in terms of square roots of rationals. These phases correspond to n-polygons with n equal to a product of power of two and Fermat primes, which are all different. The ratios of planetary masses obey the predictions with an accuracy of 10 percent and GMm/v_0 for Sun-Earth system is consistent with $v_0 = 2^{-11}$ if the fraction of visible matter of all matter is about 3 per cent in solar system to be compared with the accepted cosmological value of 4 per cent [D7].

If so, its huge value implies that also the von Neumann inclusions associated with M^4 degrees of freedom are involved meaning that dark matter cosmology has quantal lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M_+^4 and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

There is however strong objection based on the observation that the radius of CP_2 would become gigantic. Surprisingly, this need not have any dramatic implications as will be found. It is also quite possible that the biomolecules subgroups of rotation group as symmetries could correspond to $n_a > 1$. For instance, the tedrahedral and icosahedral molecular structures appearing in water would correspond to E_6 with $n_a = 3$ and E_8 with $n_a = 5$. Note that $n_a = 5$ is minimal value of n_a allowing universal topological quantum computation.

6.9.3 Large values of Planck constant and electro-weak and strong coupling constant evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

6.9.4 Super-canonical gluons and non-perturbative aspects of hadron physics

What happens mathematically in the transition to non-perturbative QCD has remained more or less a mystery. The number theoretical considerations of [E9] inspired the idea that Planck constant is dynamical and has a spectrum given as $\hbar(n) = n\hbar_0$, where n characterizes the quantum phase $q = exp(i2\pi/n)$ associated with Jones inclusion. The strange finding that the orbits of planets seem to obey Bohr quantization rules with a gigantic value of Planck constant inspired the hypothesis that the increase of Planck constant provides a unique mechanism allowing strongly interacting system to stay in perturbative phase [A9, D7]. The resulting model allows to understand dark matter as a macroscopic quantum phase in astrophysical length and time scales, and strongly suggest a connection with dark matter and biology.

The phase transition increasing Planck constant could provide a model for the transition to confining phase in QCD. When combined with the recent ideas about value spectrum of Kähler coupling strength one ends up with a rather explicit model about non-perturbative aspects of hadron physics already successfully applied in hadron mass calculations [F4].

According to the model of hadron masses [F4], in the case of light pseudoscalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudoscalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of configuration space degrees of freedom ("world of classical worlds") in which super-canonical algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges.

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-canonical gluons. It turns out the model assuming same topological mixing of super-canonical bosons identical to that experienced by U type quarks leads to excellent understanding of hadron masses assuming that hadron spin correlates with the super-canonical particle content of the hadronic space-time sheet.

According to the argument already discussed, at the hadronic $k = 107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s = \alpha_K = \frac{1}{4} \quad .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

This α_s would correspond to the interaction via super-canonical colored gluons and would lead to the failure of perturbation theory. By the general criterion stating that the failure of perturbation theory leads to a phase transition increasing the value of Planck constant one expects that the value of $\hbar$ increases [A9]. The value leaving the value of α_K invariant would be $\hbar \rightarrow 26\hbar$ and would mean that p-adic length scale L_{107} is replaced with length scale $26L_{107} = 46$ fm, the size of large nucleus so that also the basic length scale nuclear physics would be implicitly coded into the structure of hadrons.

6.9.5 Why Mersenne primes should label a fractal hierarchy of physics?

There are motivations for the working hypothesis stating that there is fractal hierarchy of copies of standard model physics, and that Mersenne primes label both hadronic space-time sheets and gauge bosons. The reason for this is not yet well understood and I have considered several speculative explanations.

First picture

The first thing to come in mind is that Mersenne primes correspond to fixed points of the discrete p-adic coupling constant evolution, most naturally to the maxima of the color coupling constant strength. This would mean that gluons are emitted with higher probability than in other p-adic length scales.

There is however an objection againt this idea. If one accepts the new vision about non-perturbative aspects of QCD, it would seem that super-canonical bosons or the interaction between super-canonical bosons and quarks for some reason favors Mersenne primes. However, if color coupling strength corresponds to $\alpha_K = \alpha_s = 1/4$ scaled down by the increase of the Planck constant, the evolution of super-canonical color coupling strength does not seem to play any role. What becomes large should be a geometric "form factor", when the boson in the vertex corresponds to Mersenne prime rather than "bare" coupling.

The resolution of the problem could be that boson emission vertices $g(p_1, p_2, p_3)$ are functions of p-adic primes labelling the particles of the vertices so that actually three p-adic length scales are involved instead of single length scale as in the ordinary coupling constant evolution. Hence one can imagine that the interaction between particles corresponding to primes near powers of 2 and Mersenne primes is especially strong and analogous to a resonant interaction. The geometric resonance due to the fact that the length scales involved are related by a fractal scaling by a power of 2 would make the form factors $F(p_1 \simeq 2^{k_1}, p_2 \simeq 2^{k_2}, M_n)$ large. The selection of primes near powers of two and Mersenne bosons would be analogous to evolutionary selection of a population consisting of species able to interact strongly.

Since $k = 113$ quarks are possible for $k = 107$ hadron physics, it seems that quarks can have join along boundaries bonds directed to M_n space-times with $n < k$. This suggests that neighboring Mersenne primes compete for join along boundaries bonds of quarks. For instance, when the p-adic length scale characterizing quark of M_{107} hadron physics begins to approach M_{89} quarks tend to feed their gauge flux to M_{89} space-time sheet and M_{89} hadron physics takes over and color coupling strength begins to increase. This would be the space-time correlate for the loss of asymptotic freedom.

Second picture

Preferred values of Planck constants could play a key role in the selection of Mersenne primes. Ruler-and-compass hypothesis predicts that Planck constants, which correspond to ratios of ruler and compass integers proportional to a product of distinct Fermat primes (four of them are known) and any power of two are favored. As a special case one obtains ruler and compass integers. As a consequence, p-adic length scales have satellites obtained by multiplying them with ruler-and-compass integers, and entire fractal hierarchy of power-of-two multiples of a given p-adic length scale results.

Mersenne length scales would be special since their satellites would form a subset of satellites of shorter Mersenne length scales. The copies of standard model physics associated with Mersenne primes would define a kind of resonating subset of physics since corresponding wavelengths and frequencies would coincide. This would also explain why fermions labelled by primes near power of two couple strongly with Mersenne primes.

6.9.6 The formula for the hadronic string tension

It is far from clear whether the strong gravitational coupling constant has same relation to the parameter $M_0^2 = 16m_0^2 = 1/\alpha' = 2\pi T$ as it would have in string model.

1. One could estimate the strong gravitational constant from the fundamental formula for the gravitational constant expressed in terms of exponent of Kähler action in the case that one has $\alpha_K = 1/4$. The formula reads as

$$\frac{L_p^2}{G_p} = exp(2aS_K(CP_2)) = exp(\pi/4\alpha_K) = e^{\pi} \quad . \tag{6.9.2}$$

   a is a parameter telling which fraction the action of wormhole contact is about the full action for CP_2 type vacuum extremal and $a \sim 1/2$ holds true. The presence of a can take care that the exponent is rational number. For $a = 1$ The number at the right hand side is Gelfond constant and one obtains

$$G_p = exp(-\pi) \times L_p^2 \quad . \tag{6.9.3}$$

2. One could relate the value of the strong gravitational constant to the parameter $M_0^2(k) = 16m(k)^2$, $p \simeq 2^k$ also assuming that string model formula generalizes as such. The basic formulas can be written in terms of gravitational constant G, string tension T, and $M_0^2(k)$ as

$$\frac{1}{8\pi G(k)} = \frac{1}{\alpha'} = 2\pi T(k) = \frac{1}{M_0^2(k)} = \frac{1}{16m(k)^2} \quad . \tag{6.9.3}$$

   This allows to express G in terms of the hadronic length scale $L(k) = 2\pi/m(k)$ as

$$G(k) = \frac{1}{16^2\pi^2}L(k)^2 \simeq 3.9\times 10^{-4}L(k)^2 \quad . \tag{6.9.4}$$

The value of gravitational coupling would be by two orders of magnitude smaller than for the first option.

6.9.7 How p-adic and real coupling constant evolutions are related to each other?

It must be emphasized that part of this section was written before the realization that the generalized eigenvalue equation for the modified Dirac operator provides a fundamental definition of the p-adic coupling constant evolution and some of the considerations are therefore only heuristic. For instance, the relationship between p-adic and real coupling constant evolutions more or less trivializes since S-matrix elements in the approach based on number theoretical braids are algebraic numbers and thus make sense in any number field. The real and p-adic coupling constants are thus identical algebraic numbers.

Questions

One can pose many questions about p-adic coupling constant evolution. How do p-adic and corresponding real coupling constant evolution relate to each other? Why Mersenne primes and primes near prime (integer) powers of two seem to be in a special position physically? Could one say something about phase transition between perturbative and non-perturbative phases of QCD?

How p-adic amplitudes are mapped to real ones?

Before the realization that p-adic and real amplitudes could be algebraic numbers the question of the title was very relevant. If the recent picture is correct, the following considerations are to some degree obsolete.

The real and p-adic coupling constant evolutions should be consistent with each other. This means that the coupling constants $g(p_1,p_2,p_3)$ as functions of p-adic primes characterizing particles of the vertex should have the same qualitative behavior as real and p-adic functions. Hence the p-adic norms of complex rational valued (or those in algebraic extension) amplitudes must give a good estimate for the behavior of the real vertex. Hence a restriction of a continuous correspondence between p-adics and reals to rationals is highly suggestive. The restriction of the canonical identification to rationals would define this kind of correspondence but this correspondence respects neither symmetries nor unitarity in its basic form. Some kind of compromize between correspondence via common rationals and canonical identification should be found.

The compromise might be achieved by using a modification of canonical identification $I_{R_p\rightarrow R}$. Generalized numbers would be regarded in this picture as a generalized manifold obtained by gluing different number fields together along rationals. Instead of a direct identification of real and p-adic rationals, the p-adic rationals in R_p are mapped to real rationals (or vice versa) using a variant of the canonical identification $I_{R\rightarrow R_p}$ in which the expansion of rational number $q=r/s=\sum r_np^n/\sum s_np^n$ is replaced with the rational number $q_1=r_1/s_1=\sum r_np^{-n}/\sum s_np^{-n}$ interpreted as a p-adic number:

$$q = \frac{r}{s}=\frac{\sum_n r_np^n}{\sum_m s_np^n}\rightarrow q_1=\frac{\sum_n r_np^{-n}}{\sum_m s_np^{-n}} \tag{6.9.5}$$

This variant of canonical identification is not equivalent with the original one using the infinite expansion of q in powers of p since canonical identification does not commute with product and division. The variant is however unique in the recent context when r and s in $q=r/s$ have no common factors. For integers $n<p$ it reduces to direct correspondence. R_{p1} and R_{p_2} are glued together along common rationals by an the composite map $I_{R\rightarrow R_{p_2}}I_{R_{p_1}\rightarrow R}$.

Instead of a re-interpretation of the p-adic number $g(p_1,p_2,p_3)$ as a real number or vice versa would be continued by using this variant of canonical identification. The nice feature of the map would be that continuity would be respected to high degree and something which is small in real sense would be small also in p-adic sense.

How to achieve consistency with the unitarity of topological mixing matrices and of CKM matrix?

It is easy to invent an objection against the proposed relationship between p-adic and real coupling constants. Topological mixing matrices U, D and CKM matrix $V = U^\dagger D$ define an important part of the electro-weak coupling constant structure and appear also in coupling constants. The problem is that canonical identification does not respect unitarity and does not commute with the matrix multiplication in the general case unlike gluing along common rationals. Even if matrices U and D which contain only ratios of integers smaller than p are constructed, the construction of V might be problematic since the products of two rationals can give a rational $q = r/s$ for which r or s or both are larger than p.

One might hope that the objection could be circumvented if the ratios of the integers of the algebraic extension defining the matrix elements of CKM matrix are such that the integer components of algebraic integers are smaller than p in U and D and even the products of integers in $U^\dagger D$ satisfy this condition so that modulo p arithmetics is avoided.

In the standard parametrization all matrix elements of the unitarity matrix can be expressed in terms of real and imaginary parts of complex phases ($p \ mod \ 4 = 3$ guarantees that $\sqrt{-1}$ is not an ordinary p-adic number involving infinite expansion in powers of p). These phases are expressible as products of Pythagorean phases and phases in some algebraic extension of rationals.

i) Pythagorean phases defined as complex rationals $[r^2 - s^2 + i2rs]/(r^2 + s^2)$ are an obvious source of potential trouble. However, if the products of complex integers appearing in the numerators and denominators of the phases have real and imaginary parts smaller than p it seems to be possible to avoid difficulties in the definition of $V = U^\dagger D$.

ii) Pythagorean phases are not periodic phases. Algebraic extensions allow to introduce periodic phases of type $exp(i\pi m/n)$ expressible in terms of p-adic numbers in a finite-dimensional algebraic extension involving various roots of rationals. Also in this case the product $U^\dagger D$ poses conditions on the size of integers appearing in the numerators and denominators of the rationals involved.

If the expectation that topological mixing matrices and CKM matrix characterize the dynamics at the level $p \simeq 2^k$, $k = 107$, is correct, number theoretical constraints are not expected to bring much new to what is already predicted. Situation changes if these matrices appear already at the level k. For $k = 89$ hadron physics the restrictions would be even stronger and might force much simpler U, D and CKM matrices.

k-adicity constraint would have even stronger implications for S-matrix and could give very powerful constraints to the S-matrix of color interactions. Quite generally, the constraints would imply a p-adic hierarchy of increasingly complex S-matrices: kind of a physical realization for number theoretic emergence. The work with CKM matrix has shown how powerful the number theoretical constraints are, and there are no reasons to doubt that this could not be the case also more generally since in the lowest order the construction would be carried out in finite (Galois) fields $G(p, k)$.

How generally the hybrid of canonical identification and identification via common rationals can apply?

The proposed gluing procedure, if applied universally, has non-trivial implications which need not be consistent with all previous ideas.

1. The basic objection against the new kind of identification is that it does not commute with symmetries. Therefore its application at imbedding space and space-time level is questionable.

2. The mapping of p-adic probabilities by canonical identification to their real counterparts requires a separate normalization of the resulting probabilities. Also the new variant of canonical identification requires this since it does not commute with the sum.

3. The direct correspondence of reals and p-adics by common rationals at space-time level implies that the intersections of cognitive space-time sheets with real space-time sheet have literally infinite size (p-adically infinitesimal corresponds to infinite in real sense for rational) and consist of discrete points in general. If the new gluing procedure is adopted also at space-time level, it would considerably de-dramatize the radical idea that the size for the space-time correlates of cognition is literally infinite and cognition is a literally cosmic phenomenon.

Of course, the new kind of correspondence could be also seen as a manner to construct cognitive representations by mapping rational points to rational points in the real sense and thus as a formation of cognitive representations at space-time level mapping points close to each other in real sense to points close to each other p-adically but arbitrarily far away in real sense. The image would be a completely chaotic looking set of points in the wrong topology and would realize the idea of Bohm about hidden order in a very concrete manner. This kind of mapping might be used to code visual information using the value of p as a part of the code key.

4. In p-adic thermodynamics p-adic particle mass squared is mapped to its real counterpart by canonical identification. The objection against the use of the new variant of canonical identification is that the predictions of p-adic thermodynamics for mass squared are not rational numbers but infinite power series. p-Adic thermodynamics itself however defines a unique representation of probabilities as ratios of generalized Boltzmann weights and partition function and thus the variant of canonical identification indeed generalizes and at the same time raises worries about the fate of the earlier predictions of the p-adic thermodynamics.

Quite generally, the thermodynamical contribution to the particle mass squared is in the lowest p-adic order of form rp/s, where r is the number of excitations with conformal weight 1 and s the number of massless excitations with vanishing conformal weight. The real counterpart of mass squared for the ordinary canonical identification is of order CP_2 mass by $r/s = R + r_1p + ...$ with $R < p$ near to p. Hence the states for which massless state is degenerate become ultra heavy if r is not divisible by s. For the new variant of canonical identification these states would be light. It is not actually clear how many states of this kind the generalized construction unifying super-canonical and super Kac-Moody algebras predicts.

A less dramatic implication would be that the second order contribution to the mass squared from p-adic thermodynamics is always very small unless the integer characterizing it is a considerable fraction of p. When ordinary canonical identification is used, the second order term of form rp^2/s can give term of form Rp^2, $R < p$ of order p. This occurs only in the case of left handed neutrinos.

The assumption that the second order term to the mass squared coming from other than thermodynamical sources gives a significant contribution is made in the most recent calculations of leptonic masses [F3]. It poses constraints on CP_2 mass which in turn are used as a guideline in the construction of a model for hadrons [F4]. This kind of contribution is possible also now and corresponds to a contribution Rp^2, $R < p$ near p.

The new variant of the canonical correspondence resolves the long standing problems related to the calculation of Z and W masses. The mass squared for intermediate gauge bosons is smaller than one unit when m_0^2 is used as a fundamental mass squared unit. The standard form of the canonical identification requires $M^2 = (m/n)p^2$ whereas in the new approach $M^2 = (m/n)p$ is allowed. Second difficult problem has been the p-adic description of the group theoretical model for m_W^2/m_Z^2 ratio. In the new framework this is not a problem anymore [F3] since canonical identification respects the ratios of small integers.

On the other hand, the basic assumption of the successful model for topological mixing of quarks [F4] is that the modular contribution to the masses is of form np. This assumption loses its original justification for this option and some other justification is needed. The first guess is that the conditions on mass squared plus probability conservation might not be consistent with unitarity unless the modular contribution to the mass squared remains integer valued in the mixing (note that all integer values are not possible [F4]). Direct numerical experimentation however shows that that this is not the case.

6.9.8 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [C3]. M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results. Recall that also the weaker condition $T_p = pT_0$, p prime, would assign secondary p-adic time scales to the size scale hierarchy of CDs: $p \simeq 2^n$ would result as an outcome of some kind of "natural selection" for this option. The highly satisfactory feature would be that p-adic time scales would reflect directly the geometry of imbedding space and configuration space.

p-Adic coupling constant evolution

An attractive conjecture is that the coupling constant evolution associated with CDs in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induces p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there seems to be a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For the weaker condition would be $T_p = pT_0$, p prime, $p \simeq 2^n$ could be seen as an outcome of some kind of "natural selection". In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of configuration space.

4. The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \ ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

6.10 Appendix A: Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [4] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{6.10.0}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{6.10.1}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ , \end{aligned} \tag{6.10.2}$$

and

$$B = 2re^3 \ , \tag{6.10.3}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{6.10.4}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01}-\Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02}-\Sigma_{13})}{2} \ . \end{aligned} \tag{6.10.4}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{6.10.5}$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 \ , \\ Y &= \frac{e^3}{r} \ , \end{aligned} \tag{6.10.5}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \tag{6.10.5}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{6.10.4}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{6.10.5}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{6.10.6}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+ 1_+ + n_- 1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{6.10.6}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{6.10.6}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{6.10.7}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+ 1_+ + n_- 1_-) \ , \tag{6.10.8}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{6.10.7}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{6.10.8}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{6.10.8}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{6.10.9}$$

6.11 Appendix B: Some number theoretical conjectures related to p-adicization

This section is a digression from basic theme being about some number theoretical ideas related to p-adicization. The justification for its inclusion is that p-adicization poses strong constraints on coupling constants and actually led to the parton level formulation of the p-adic coupling constant evolution.

6.11.1 Fusion of p-adic and real physics to single coherent whole by algebraic continuation

The development of the TGD inspired theory of consciousness theory and the vision about physics as a generalized number theory led to a general philosophy which provides powerful conceptual tools in attempts to answer the questions stated above.

Physics as a generalized number theory

The basic ideas behind physics as a generalized number theory approach deserve a brief summary.

1. The interpretation of p-adicity

Various p-adic versions of quantum TGD are interpreted as kind of cognitive representations of the real theory. p-Adic space-time sheets appear also at space-time level. Also real space-time sheets decompose into regions obeying effective p-adic topologies with the value of p characterizing the non-determinism of Kähler action in a particular region. This explains why p-adic thermodynamics predicts particle masses. Also algebraic extensions of the number fields R_p are important.

2. The construction of real and p-adic physics by algebraic continuation of rational number based physics

Real number based quantum TGD can be algebraically continued to various number fields [E1]. It seems that the generalization of the notion of number obtained by gluing reals and various p-adic number fields along common rationals might be crucial in this respect. In the spirit of manifold theory also more general gluing maps than gluing along common rationals can be imagined and canonical identification and its variants seem to be natural as far as probabilities are considered.

The algebraic continuation applies to all mathematical structures involved. Two continuations share some set of points which can be regarded as common to the number field involved. Examples of the structures involved are real and p-adic variants of imbedding space, of configuration space of 3-surfaces, and of Hilbert space of quantum states.

The continuation from rationals to various number fields abstracts the basic facts about the relationship between physical world and theories about it. Rational points correspond to physical data or numerical predictions of mathematical theories. Physical world represents algebraic continuation to reals and various p-adic continuations a hierarchy of increasingly refined theories.

3. Number theoretical existence

Number theoretical existence requirement becomes a leading guide line in the construction of the theory. p-Adic mass calculations and the construction of topological mixing matrices U, D and CKM matrix $V = U^\dagger D$ provide an example of a successful application of the number theoretical existence requirements [F4]. Coupling constant evolution represents second obvious application yet to be developed. Here the challenges relate to the realization of non-algebraic functions like logarithm appearing typically in the formulas.

Two rather general number theoretical conjectures are inspired by the physics as a generalized number theory vision [E8, E1, E2, E3].

i) The ratios of logarithms of rationals are rationals. In particular, $log_2(q) = log(q)/log(2)$ is always rational so that pits are rational multiples of bits. Among other things this makes possible to construct rational valued iterated 2-based logarithms $log_2(...(log_2(q))...)$ expected to appear in running coupling constants.

ii) The numbers p^{iy} exist in finite dimensional extensions of rational numbers for each value of prime p and each zero $z = 1/2+iy$ of Riemann Zeta. The obvious implication is that the exponents $q^{i\sum n_k y_k}$ satisfy the same condition. The construction of p-adic variant of Teichmueller parameters and moduli space provides an application of the conjecture [F2]. The implication is that two-dimensional shapes obey linear superposition and that the maxima of Kähler function correspond to these quantized 2-dimensional shapes.

As far as coupling strength evolution is considered the question whether one should require g^2 or $\alpha = g^2/4\pi$ or some other combination to be rational or in an finite-dimensional algebraic extension of R_p is of fundamental importance. The existence of the exponent of Kähler action for CP_2 support the view that g_K^2, and presumably all coupling constants should be proportional to π^2. Unless an infinite-dimensional extension of p-adic numbers defined by powers of π (possibly making sense) is allowed, a combination of $\alpha/2\pi$ or something equivalent with it should appear in Feynman diagrams.

Is it possible to introduce infinite-dimensional extension of p-adic numbers containing π?

The assumption that only finite-dimensional extensions of p-adic numbers are possible, is only a convenient working hypothesis. π appears in the basic formulas of geometry, in particular in the geometry of CP_2. π appears also in the Feynmann rules of quantum field theories and expressions for reaction rates, and the idea about the algebraic continuation of real integrals as a manner to define various momentum space integrals p-adically is very attractive. These observations strongly encourage to consider the possibility of an infinite-dimensional extension of p-adic numbers containing both positive and negative powers of π with additional constraints coming from conditions like $exp(i\pi/2) = i$.

The definition of p-adic norm should obey the usual conditions, in particular the requirement that the norm of product is product of norms. One can imagine two alternative definitions of the p-adic norm.

1. The first definition is as $N_p(x) = |det(x)|_p$, where $det(x)$ is the determinant of the linear map of the infinite-dimensional linear space spanned by powers of π defined by $x = \sum_{k=m_x}^{n_x} x_k\pi^k$. This definition is straightforward generalization of the usual definition and guarantees that norm is indeed algebraic homomorphism respecting product.

2. The second definition is as the limit of the $N_p(x) = lim_{N\rightarrow\infty}|(det(x)_N^{1/N})|_p$, truncated to the subspace defined by basis $\{\pi^{-N}, ...\pi^N\}$. The motivation for this definition is that first definition tends to give vanishing of infinite norm. This norm does not however define an algebraic homomorphism.

The linear map is represented by a matrix for which non-vanishing elements form a band parallel to the diagonal having width $n_x - m_x + 1$ with each vertical entry in band equal to the column vector $(x_{m_x}, ..., x_{n_x})^T$ defined by the components of x. Diagonal entries are equal to x_0. The determinant is a sum over all downward direct paths along the this band with the product of components x_i along the path assigned with a given path. The paths are not allowed to visit the same horizontal point twice so that an analog of a functional integral associated with a self-avoiding random walk constrained inside the diagonal band in a discrete lattice along x-axis is in question. Quantum fluctuations are restricted to the interval $[m_x, n_x]$ surrounding the classical path.

The condition that $x_n\pi^n$ approaches zero p-adically is natural and requires that the p-adic norm of x_n approaches zero. The stronger condition

$$... < |x_{n+1}|_p < |x_n|_p < ... \ , \tag{6.11.1}$$

simplifies dramatically the calculation of the determinant since $x_{m_x}\pi^{m_x}$ can be factored out and $|det(x)|_p$ becomes a norm of the determinant of a lower triangular matrix with units at diagonal multiplied by an infinite power of $|x_{m_x}|_p$.

In case a) the norm is simply $|x_{m_x}|_p^{N\rightarrow\infty}$ quite generally and diverges for $|x_{m_x}| > 1$, vanishes for $|x_{m_x}| < 1$ and equals to one for $|x_{m_x}| = 1$. In case b) the norm is $|x_{m_x}|$.

Trigonometric functions $cos(\pi q)$ and $sin(\pi q)$ allow to test the sensibility of the proposed alternative definitions. For instance, it is possible to check whether the norm of $sin(n\pi)$ vanishes for allowed values of n.

Option a): $x = cos(\pi q)$ would correspond to a lower triangular matrix with units along the diagonal so that the norm would be equal to 1 irrespective of the value of q. This is not consistent with

$cos(\pi/2) = 0$. This is not a catastrophe, since $q = 1/2$ has p-adic norm $(1/2)_p \geq 1$ so that the series is not p-adically converging and does not satisfy the condition posed above. The minimum requirement is $q = rp$, r rational with unit p-adic norm. By the product decomposition $sin(\pi q)$ for $|q| < 1$ has a vanishing norm. Thus the condition $|x_n|_p \leq p^{-n}$ guarantees the consistency with the basic trigonometric formulas.

Option b: The p-adic norm of $cos(\pi q)$ is equal to 1 whereas the p-adic norm of $sin(\pi q)$ is equal to $|q|_p$ from product decomposition. In particular, the norm is non-vanishing for $x = np\pi$ so that an inconsistency results.

If the conjecture that $log(p) = x_p/\pi$, where x_p belongs to some finite-dimensional extension holds true, then also the powers of logarithms of rationals would belong to the extension.

What effective p-adic topology really means?

The need to characterize elementary particle p-adically leads to the question what p-adic effective topology really means. p-Adic mass calculations leave actually a lot of room concerning the answer to this question.

1. The naivest option is that each space-time sheet corresponds to single p-adic prime. A more general possibility is that the boundary components of space-time sheet correspond to different p-adic primes. This view is not favored by the view that each particle corresponds to a collection of p-adic primes each characterizing one particular interaction that the particle in question participates.

2. A more abstract possibility is that a given space-time sheet or boundary component can correspond to several p-adic primes. Indeed, a power series in powers of given integer n gives rise to a well-defined power series with respect to all prime factors of n and effective multi-p-adicity could emerge at the level of field equations in this manner.

One could say that space-time sheet or boundary component corresponds to several p-adic primes through its effective p-adic topology in a hologram like manner. This option is the most flexible one as far as physical interpretation is considered. It is also supported by the number theoretical considerations predicting the value of gravitational coupling constant [E3].

An attractive hypothesis is that only space-time sheets characterized by integers n_i having common prime factors can be connected by join along boundaries bonds and can interact by particle exchanges and that each prime p in the decomposition corresponds to a particular interaction mediated by an elementary boson characterized by this prime.

The physics of quarks and hadrons provides an immediate test for this interpretation. The surprising and poorly understood conclusion from the p-adic mass calculations was that the p-adic primes characterizing light quarks u,d,s satisfy $k_q < 107$, where $k = 107$ characterizes hadronic space-time sheet [F4].

1. The interpretation of $k = 107$ space-time sheet as a hadronic space-time sheet implies that quarks topologically condense at this space-time sheet so that $k = 107$ cannot belong to the collection of primes characterizing quark.

2. Quark space-time sheets must satisfy $k_q < 107$ unless $\hbar$ is large for the hadronic space-time sheet so that one has $k_{eff} = 107 + 22 = 129$. This predicts two kinds of hadrons. Low energy hadrons consists of u, d, and s quarks with $k_q < 107$ so that hadronic space-time sheet must correspond to $k_{eff} = 129$ and large value of $\hbar$. One can speak of confined phase. This allows also $k = 127$ light variants of quarks appearing in the model of atomic nucleus [F8]. The hadrons consisting of c,t,b and the p-adically scaled up variants of u,d,s having $k_q > 107$, $\hbar$ has its ordinary value in accordance with the idea about asymptotic freedom and the view that the states in question correspond to short-lived resonances.

Do infinite primes code for q-adic effective space-time topologies?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes [E3], hierarchy of Jones inclusions [A8], hierarchy of dark matters with increasing values of $\hbar$ [F9, J6], the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related.

1. Some facts about infinite primes

The hierarchy of infinite primes can be interpreted in terms of an infinite hierarchy of second quantized super-symmetric arithmetic quantum field theories allowing a generalization to quaternionic or perhaps even octonionic context [E3]. Infinite primes, integers, and rationals have decomposition to primes of lower level.

Infinite prime has fermionic and bosonic parts having no common primes. Fermionic part is finite and corresponds to an integer containing and bosonic part is an integer multiplying the product of all primes with fermionic prime divided away. The infinite prime at the first level of hierarchy corresponds in a well defined sense a rational number $q = m/n$ defined by bosonic and fermionic integers m and n having no common prime factors.

2. Do infinite primes code for effective q-adic space-time topologies?

The most obvious question concerns the space-time interpretation of this rational number. Also the question arises about the possible relation with the integers characterizing space-time sheets having interpretation in terms of multi-p-adicity. On can assign to any rational number $q = m/n$ so called q-adic topology. This topology is not consistent with number field property like p-adic topologies. Hence the rational number q assignable to infinite prime could correspond to an effective q-adic topology.

If this interpretation is correct, arithmetic fermion and boson numbers could be coded into effective q-adic topology of the space-time sheets characterizing the non-determinism of Kähler action in the relevant length scale range. For instance, the power series of $q > 1$ in positive powers with integer coefficients in the range $[0, q)$ define q-adically converging series, which also converges with respect to the prime factors of m and can be regarded as a p-adic power series. The power series of q in negative powers define in similar converging series with respect to the prime factors of n.

I have proposed earlier that the integers defining infinite rationals and thus also the integers m and n characterizing finite rational could correspond at space-time level to particles with positive *resp.* negative time orientation with positive *resp.* negative energies. Phase conjugate laser beams would represent one example of negative energy states. With this interpretation super-symmetry exchanging the roles of m and n and thus the role of fermionic and bosonic lower level primes would correspond to a time reversal.

1. The first interpretation is that there is single q-adic space-time sheet and that positive and negative energy states correspond to primes associated with m and n respectively. Positive (negative) energy space-time sheets would thus correspond to p-adicity ($1/p$-adicity) for the field modes describing the states.

2. Second interpretation is that particle (in extremely general sense that entire universe can be regarded as a particle) corresponds to a pair of positive and negative energy space-time sheets labelled by m and n characterizing the p-adic topologies consistent with $m-$ and n-adicities. This looks natural since Universe has necessary vanishing net quantum numbers. Unless one allows the non-uniqueness due to $m/n = mr/nr$, positive and negative energy space-time sheets can be connected only by # contacts so that positive and negative energy space-time sheets cannot interact via the formation of $\#_B$ contacts and would be therefore dark matter with respect to each other.

Positive energy particles and negative energy antiparticles would also have different mass scales. If the rate for the creation of # contacts and their CP conjugates are slightly different, say due to the presence of electric components of gauge fields, matter antimatter asymmetry could be generated primordially.

These interpretations generalize to higher levels of the hierarchy. There is a homomorphism from infinite rationals to finite rationals. One can assign to a product of infinite primes the product of the corresponding rationals at the lower level and to a sum of products of infinite primes the sum of the corresponding rationals at the lower level and continue the process until one ends up with a finite rational. Same applies to infinite rationals. The resulting rational $q = m/n$ is finite and defines q-adic effective topology, which is consistent with all the effective p-adic topologies corresponding to the primes appearing in factorizations of m and n. This homomorphism is of course not 1-1.

If this picture is correct, effective p-adic topologies would appear at all levels but would be dictated by the infinite-p p-adic topology which itself could refine infinite-P p-adic topology [E3] coding information too subtle to be catched by ordinary physical measurements.

Obviously, one could assign to each elementary particle infinite prime, integer, or even rational to this a rational number $q = m/n$. q would associate with the particle q-adic topology consistent with a collection of p-adic topologies corresponding to the prime factors of m and n and characterizing the interactions that the particle can participate directly. In a very precise sense particles would represent both infinite and finite numbers.

6.11.2 The number theoretical universality of Riemann Zeta

The conjecture about number theoretical universality of Riemann Zeta involves several conjectures.

The first conjecture emerged from the idea that at least the building blocks $1/(1-p^{-s})$ of Riemann Zeta for zeros of Riemann Zeta should make sense in all number fields provided that algebraic extensions of p-adic numbers are allowed [E8]. This kind of universality would guarantee that the radial logarithmic waves $r^{\zeta^{-1}(z)}$ assigned to the Hamiltonians of $\delta M^4_\pm \times CP_2$ exist for the rational values of the argument r as algebraic numbers for points $z = \sum_k n_k s_k$, where s_k is zero of ζ and z is the projection of partonic 2-surface to a geodesic sphere of CP_2 or to a geodesic sphere S^2 assignable to the light cone boundary. This condition emerges naturally in the p-adicization of quantum TGD using the notion of number theoretical braid [C2] defined as a subset of the intersection of real and p-adic variants of partonic 2-surface obeying same algebraic equations. The intersection consists of points in the algebraic extension of p-adic numbers used.

This form of number theoretical universality can be extended by requiring that also $\zeta(s)$ at points $s = \sum_k n_k s_k$, where n_k are non-negative numbers is an algebraic number. This version of conjecture has several forms of varying strength.

An strongest form of the conjecture requires that also the zeros s_k themselves are algebraic numbers and is motivated by the requirement that vacuum functional, which is conjectured to reduce to an exponent of Kähler function and identified as Dirac determinant, is an algebraic number for the maxima of Kähler function. The question what Dirac determinant means is far from trivial and the definition based on the notion of number theoretic braid is discussed in [C2].

To sum up, one can say that these conjectures would fit very nicely with the general spirit of TGD but it would be wrong to say that TGD is lost if these conjectures are wrong.

6.11.3 Some wrong number theoretical conjectures

Further speculative number theoretical conjectures inspired by TGD state that the ratio $log(q_1)/log(q_2)$ is rational for any pair or rational numbers and that $\pi/log(q)$ is rational for any rational q. It will be found that these conjectures are wrong. Also the conjecture stating that there is single rational q such that $\pi/log(q)$ is rational turns out to be wrong. This destroys a rather fascinating possibility to fix the basic parameters of quantum TGD.

About the action of group of rationals in the group of reals

These conjectures can be looked in a wider context by studying the orbits for the group of rationals in the group of reals. Two numbers whose ratio is rational belong to the same orbit. Rationals form trivially a normal subgroup so that the orbits form also a group. This means that the additive (or multiplicative) invariants $D(u)$ associated with orbits form an additive group so that $D(uv) = D(u) + D(v)$ and $D(u/v) = D(u) - D(v)$ hold true.

This brings in mind irreducible representations of the translation group: D(u) could be seen as a kind of number theoretic momentum. The invariant could be constructed by identifying the generators of the orbit group and picking up one representative from each generator orbit, take its logarithm, and use the additive group property to deduce the rest. These number theoretic momenta would like basic units of a momentum lattice: now however the dimension of lattice would be uncountably infinite. This construction would be very much like a gauge choice with rationals acting as a gauge group. This picture generalizes to the algebraic extensions of rationals too. The rational powers of any generator are generators and rational powers map orbits into orbits and therefore respect number theoretical "gauge invariance". Since 2^x covers all non-negative values of reals, x defines a convenient invariant labelling the orbits. At least x and $x + log(q)/log(2)$ correspond to the same orbit.

The failure of the conjectures

The rationality of $log(q_1)/log(q_2)$ would mean that the additive "gauge group" formed by logarithms of rationals responsible for the non-uniqueness of additive invariants D(u) would form single orbit itself! Logarithm would respect the orbit defining the unit element. This gives some aesthetic support for $log(q_1)/log(q_2)$ conjecture.

1. One could ask whether the ratio $log(x_1)/log(x_2)$ for numbers x_1 and x_2 at *any* orbit is rational. Besides rational powers logarithm would map orbits to orbits and respect the number theoretical "gauge invariance". The answer to this question is negative. Consider the orbit of e. The ratio of logarithms of e and $q \times e$ is $1/(1 + log(q))$ and definitely not rational.

2. Conjecture fails also in the less general case. Assume that one has $log(q_2)/log(2) = q_1 = m_1/n_1$ such that q_2 is not rational power of 2. This implies $q_2 = 2^{log(q_2)/log(2)} = 2^{q_1}$. By taking n_1:th power one obtains $q_2^{n_1} = 2^{m_1}$. Since q_2 is not a power of 2, the decomposition of q_2 to produce of powers of primes would not be unique so that the conjecture cannot hold true.

3. Also the conjecture that $log(q)/\pi$ is rational for all rationals q fails. Assume that the conjecture holds true for two rationals q_1 and q_2. Taking the ratio one obtains that $log(q_1)/log(q_2)$ is always rational which cannot hold true.

4. Even the rationality of $\pi/log(q)$ for single q leads to a contradiction since it implies that $exp(\pi)$ is an algebraic number. This would in fact look extremely nice since the algebraic character of $exp(\pi)$ would conform with the algebraic character of the phases $exp(i\pi/n)$. Unfortunately this is not the case [16]. The argument showing this is based on the representation $i^{2i} = exp(\pi)$ and to the theorem that non-rational exponent of an algebraic number is transcendental. Hence one loses the attractive possibility to fix the basic parameters of theory completely from number theory unless one is somehow able to say something highly non-trivial about the parameter a.

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Chapter 7

Recent Status of Lepto-Hadron Hypothesis

7.1 Introduction

TGD suggest strongly ('predicts' is perhaps too strong expression) the existence of color excited leptons. The mass calculations based on p-adic thermodynamics and p-adic conformal invariance lead to a rather detailed picture about color excited leptons.

1. The simplest color excited neutrinos and charged leptons belong to the color octets ν_8 and L_{10} and $L_{\bar{10}}$ decuplet representations respectively and lepto-hadrons are formed as the color singlet bound states of these and possible other representations. Electro-weak symmetry suggests strongly that the minimal representation content is octet and decuplets for both neutrinos and charged leptons.

2. The basic mass scale for lepto-hadron physics is completely fixed by p-adic length scale hypothesis. The first guess is that color excited leptons have the levels $k = 127, 113, 107, ...$ ($p \simeq 2^k$, k prime or power of prime) associated with charged leptons as primary condensation levels. p-Adic length scale hypothesis allows however also the level $k = 11^2 = 121$ in case of electronic lepto-hadrons. Thus both $k = 127$ and $k = 121$ must be considered as a candidate for the level associated with the observed lepto-hadrons. If also lepto-hadrons correspond non-perturbatively to exotic Super Virasoro representations, lepto-pion mass relates to pion mass by the scaling factor $L(107)/L(k) = k^{(107-k)/2}$. For $k = 121$ one has $m_{\pi_L} \simeq 1.057$ MeV which compares favorably with the mass $m_{\pi_L} \simeq 1.062$ MeV of the lowest observed state: thus $k = 121$ is the best candidate contrary to the earlier beliefs. The mass spectrum of lepto-hadrons is expected to have same general characteristics as hadronic mass spectrum and a satisfactory description should be based on string tension concept. Regge slope is predicted to be of order $\alpha' \simeq 1.02/MeV^2$ for $k = 121$. The masses of ground state lepto-hadrons are calculable once primary condensation levels for colored leptons and the CKM matrix describing the mixing of color excited lepton families is known.

The strongest counter arguments against color excited leptons are the following ones.

1. The decay widths of Z^0 and W boson allow only $N = 3$ light particles with neutrino quantum numbers. The introduction of new light elementary particles seems to make the decay widths of Z^0 and W untolerably large.

2. Lepto-hadrons should have been seen in e^+e^- scattering at energies above few MeV. In particular, lepto-hadronic counterparts of hadron jets should have been observed.

A possible resolution of these problems is provided by the loss of asymptotic freedom in lepto-hadron physics. Lepto-hadron physics would effectively exist in a rather limited energy range about one MeV.

The development of the ideas about dark matter hierarchy [F6, F8, F9, J6] led however to a much more elegant solution of the problem.

1. TGD predicts an infinite hierarchy of various kinds of dark matters which in particular means a hierarchy of color and electro-weak physics with weak mass scales labelled by appropriate p-adic primes different from M_{89}: the simplest option is that also ordinary photons and gluons are labelled by M_{89}.

2. There are number theoretical selection rules telling which particles can interact with each other. The assignment of a collection of primes to elementary particle as characterizer of p-adic primes characterizing the particles coupling directly to it, is inspired by the notion of infinite primes [E3], and discussed in [F6]. Only particles characterized by integers having common prime factors can interact by the exchange of elementary bosons: the p-adic length scale of boson corresponds to a common primes.

3. Also the physics characterized by different values of $\hbar$ are dark with respect to each other as far quantum coherent gauge interactions are considered. Laser beams might well correspond to photons characterized by p-adic prime different from M_{89} and de-coherence for the beam would mean decay to ordinary photons. De-coherence interaction involves scaling down of the Compton length characterizing the size of the space-time of particle implying that particles do not anymore overlap so that macroscopic quantum coherence is lost.

4. Those dark physics which are dark relative to each other can interact only via graviton exchange. If lepto-hadrons correspond to a physics for which weak bosons correspond to a p-adic prime different from M_{89}, intermediate gauge bosons cannot have direct decays to colored excitations of leptons irrespective of whether the QCD in question is asymptotically free or not. Neither are there direct interactions between the QED:s and QCD:s in question if M_{89} characterizes also ordinary photons and gluons. These ideas are discussed and applied in detail in [F6, F8, F9].

Skeptic reader might stop the reading after these counter arguments unless there were definite experimental evidence supporting the lepto-hadron hypothesis.

1. The production of anomalous e^+e^- pairs in heavy ion collisions (energies just above the Coulomb barrier) suggests the existence of pseudoscalar particles decaying to e^+e^- pairs. A natural identification is as lepto-pions that is bound states of color octet excitations of e^+ and e^-.

2. The second puzzle, Karmen anomaly, is quite recent [18]. It has been found that in charge pion decay the distribution for the number of neutrinos accompanying muon in decay $\pi \to \mu + \nu_\mu$ as a function of time seems to have a small shoulder at $t_0 \sim ms$. A possible explanation is the decay of charged pion to muon plus some new weakly interacting particle with mass of order $30\ MeV$ [19]: the production and decay of this particle would proceed via mixing with muon neutrino. TGD suggests the identification of this state as color singlet leptobaryon of, say type $L_B = f_{abc}L_8^a L_8^b \bar{L}_8^c$, having electro-weak quantum numbers of neutrino.

3. The third puzzle is the anomalously high decay rate of orto-positronium. [20]. e^+e^- annihilation to virtual photon followed by the decay to real photon plus virtual lepto-pion followed by the decay of the virtual lepto-pion to real photon pair, $\pi_L\gamma\gamma$ coupling being determined by axial anomaly, provides a possible explanation of the puzzle.

4. There exists also evidence for anomalously large production of low energy e^+e^- pairs [21, 22, 23, 24] in hadronic collisions, which might be basically due to the production of lepto-hadrons via the decay of virtual photons to colored leptons.

In this chapter a revised form of lepto-hadron hypothesis is described.

1. Sigma model realization of PCAC hypothesis allows to determine the decay widths of lepto-pion and lepto-sigma to photon pairs and e^+e^- pairs. Ortopositronium anomaly determines the value of $f(\pi_L)$ and therefore the value of lepto-pion-lepto-nucleon coupling and the decay rate of lepto-pion to two photons. Various decay widths are in accordance with the experimental data and corrections to electro-weak decay rates of neutron and muon are small.

2. One can consider several alternative interpretations for the resonances.

Option 1: For the minimal color representation content, three lepto-pions are predicted corresponding to $8, 10, \overline{10}$ representations of the color group. If the lightest lepto-nucleons e_{ex} have masses only slightly larger than electron mass, the anomalous e^+e^- could be actually $e^+_{ex} + e^-_{ex}$ pairs produced in the decays of lepto-pions. One could identify 1.062, 1.63 and 1.77 MeV states as the three lepto-pions corresponding to $8, 10, \overline{10}$ representations and also understand why the latter two resonances have nearly degenerate masses. Since d and s quarks have same primary condensation level and same weak quantum numbers as coloured e and μ, one might argue that also colored e and μ correspond to $k = 121$. From the mass ratio of the colored e and μ, as predicted by TGD, the mass of the muonic lepto-pion should be about 1.8 MeV in the absence of topological mixing. This suggests that 1.83 MeV state corresponds to the lightest $g = 1$ lepto-pion.

Option 2: If one believes sigma model (in ordinary hadron physics the existence of sigma meson is not established and its width is certainly very large if it exists), then lepto-pions are accompanied by sigma scalars. If lepto-sigmas decay dominantly to e^+e^- pairs (this might be forced by kinematics) then one could adopt the previous sceneario and could identify 1.062 state as lepto-pion and 1.63, 1.77 and 1.83 MeV states as lepto-sigmas rather than lepto-pions. The fact that muonic lepto-pion should have mass about 1.8 MeV in the absence of topological mixing, suggests that the masses of lepto-sigma and lepto-pion should be rather close to each other.

Option 3: One could also interpret the resonances as string model 'satellite states' having interpretation as radial excitations of the ground state lepto-pion and lepto-sigma. This identification is not however so plausible as the genuinely TGD based identification and will not be discussed in the sequel.

3. PCAC hypothesis and sigma model leads to a general model for lepto-hadron production in the electromagnetic fields of the colliding nuclei and production rates for lepto-pion and other lepto-hadrons are closely related to the Fourier transform of the instanton density $\bar{E} \cdot \bar{B}$ of the electromagnetic field created by nuclei. The first source of anomalous e^+e^- pairs is the production of $\sigma_L \pi_L$ pairs from vacuum followed by $\sigma_L \rightarrow e^+e^-$ decay. If $e^+_{ex}e^-_{ex}$ pairs rather than genuine e^+e^- pairs are in question, the production is production of lepto-pions from vacuum followed by lepto-pion decay to lepto-nucleon pair.

   Option 1: For the production of lepto-nucleon pairs the cross section is only slightly below the experimental upper bound for the production of the anomalous e^+e^- pairs and the decay rate of lepto-pion to lepto-nucleon pair is of correct order of magnitude.

   Option 2: The rough order of magnitude estimate for the production cross section of anomalous e^+e^- pairs via $\sigma_l \pi_l$ pair creation followed by $\sigma_L \rightarrow e^+e^-$ decay, is by a factor of order $1/\sum N_c^2$ (N_c is the total number of states for a given colour representation and sum over the representations contributing to the ortopositronium anomaly appears) smaller than the reported cross section in case of 1.8 MeV resonance. The discrepancy could be due to the neglect of the large radiative corrections (the coupling $g(\pi_L \pi_L \sigma_L) = g(\sigma_L \sigma_L \sigma_L)$ is very large) and also due to the uncertainties in the value of the measured cross section.

   Given the unclear status of sigma in hadron physics, one has a temptation to conclude that anomalous e^+e^- pairs actually correspond to lepto-nucleon pairs.

4. The vision about dark matter suggests that direct couplings between leptons and lepto-hadrons are absent in which case no new effects in the direct interactions of ordinary leptons are predicted. If colored leptons couple directly to ordinary leptons, several new physics effects such as resonances in photon-photon scattering at cm energy equal to lepto-pion masses and the production of $e_{ex}\bar{e}_{ex}$ (e_{ex} is leptobaryon with quantum numbers of electron) and $e_{ex}\bar{e}$ pairs in heavy ion collisions, are possible. Lepto-pion exchange would give dominating contribution to $\nu - e$ and $\bar{\nu} - e$ scattering at low energies. Lepto-hadron jets should be observed in e^+e^- annihilation at energies above few MeV:s unless the loss of asymptotic freedom restricts lepto-hadronic physics to a very narrow energy range and perhaps to entirely non-perturbative regime of lepto-hadronic QCD.

This chapter is a revised version of the earlier chapter [16] and still a work in progress. I apologize for the reader for possible inconvenience. The motivation for the re-writing came from the evidence for the production of τ-pions in high energy proton-antiproton collisions [59, 60]. Since the kinematics of these collisions differs dramatically from that for heavy ion collisions, a critical re-examination of the earlier model - which had admittedly somewhat ad hoc character- became necessary. As a consequence the earlier model simplified dramatically. As far as basic calculations are considered, the modification makes itself visible only at the level of coefficients. Even more remarkably, it turned out possible to calculate exactly the lepto-pion production amplitude under a very natural approximation, which can be also generalized so that the calculation of production amplitude can be made analytically in high accuracy and only the integration over lepto-pion momentum must be carried out numerically. As a consequence, a rough analytic estimate for the production cross section follows and turns out to be of correct order of magnitude. It must be however stressed that the cross section is highly sensitive to the value of the cutoff parameter (at least in this naive estimate) and only a precise calculation can settle the situation.

7.2 Lepto-hadron hypothesis

7.2.1 Anomalous e^+e^- pairs in heavy ion collisions

Heavy ion-collision experiments carried out at the Gesellschaft fur Schwerionenforschung in Darmstadt, West Germany [25, 26, 36, 37] have yielded a rather puzzling set of results. The expectation was that in heavy ion collisions in which the combined charge of the two colliding ions exceeds 173, a composite nucleus with $Z > Z_{cr}$ would form and the probability for spontaneous positron emission would become appreciable.

Indeed, narrow peaks of widths of roughly 50-70 keV and energies about 350$\pm$ 50 keV were observed in the positron spectra but it turned out that the position of the peaks seems to be a constant function of Z rather that vary as Z^{20} as expected and that peaks are generated also for Z smaller than the critical Z. The collision energies at which peaks occur lie in the neighbourhood of 5.7-6 MeV/nucleon. Also it was found that positrons are accompanied by e^-- emission. Data are consistent with the assumption that some structure at rest in cm is formed and decays subsequently to e^+e^- pair.

Various theoretical explanations for these peaks have been suggested [27, 28]. For example, lines might be created by pair conversion in the presence of heavy nuclei. In nuclear physics explanations the lines are due to some nuclear transition that occurs in the compound nucleus formed in the collision or in the fragmets formed. The Z-independence of the peaks seems however to exclude both atomic and nuclear physics explanations [27]. Elementary particle physics explanations [27, 28] seem to be excluded already by the fact that several peaks have been observed in the range $1.6 - 1.8\ MeV$ with widths of order $10^1 - 10^2\ keV$. These states decay to e^+e^- pairs. There is evidence for one narrow peak with width of order one keV at 1.062 Mev [27]: this state decays to photon-photon pairs.

Thus it seems that the structures produced might be composite, perhaps resonances in e^+e^- system. The difficulty of this explanation is that conventional QED seems to offer no natural explanation for the strong force needed to explain the energy scale of the states. One idea is that the strong electromagnetic fields create a new phase of QED [27] and that the resonances are analogous to pseudoscalar mesons appearing as resonances in strongly interacting systems.

TGD based explanation relies on the following hypothesis motivated by Topological Geometrodynamics.

1. Ordinary leptons are not point like particles and can have colored excitations, which form color singlet bound states. A natural identification for the primary condensate level is $k = 121$ so that the mass scale is of order one MeV for the states containing lowest generation colored leptons. The fact that d and s quarks, having the same weak quantum numbers as charged leptons, have same primary condensation level, suggests that both colored electron and muon condense to the same level. The expectation that lepto-hadron physics exists in a narrow energy interval only, suggests that also colored τ should condense on the same level.

2. The states in question are lepto-hadrons, that is color confined states formed from the colored excitations of e^+ and e^-. The decay rate to lepto-nucleon pairs $e^+_{ex}e^-_{ex}$ is large and turns out to give rise to correct order of magnitude for the decay width. Hence two options emerge.

   Option 1: Lepto-nucleons e_{ex} have masses only slightly above the electron mass and since they behave like electrons, anomalous e^+e^- pairs could actually correspond to lepto-nucleon pairs created in the decays of lepto-pions. 1.062, 1.63 and 1.77 MeV states can be identified as lowest generation lepto-pions correspond to octet and two decuplets. 1.83 MeV state could be identified as the second generation lepto-pion corresponding to colored muon. The small branching fraction to gamma pairs explains why the decays of the higher mass lepto-pions to gamma pairs has not been observed. $g = 0$ lepto-pion decays to lepto-nucleon pairs can be visualized as occuring via dual diagrams obeying Zweig's rule (annihilation is not allowed inside incoming or outgoing particle states). The decay of $g = 1$ colored muon pair occurs via Zweig rule violating annihilation to two gluon intermediate state, which transforms back to virtual $g = 0$ colored electron pair decaying via dual diagram: the violation of Zweig's rule suggests that the decay rate for 1.8 MeV state is smaller than for the lighter states. Quantitive model shows that this scenario is the most plausible one.

Option 2: Lepto-sigmas, which are the scalar partners of lepto-pions predicted by sigma model, are the source of anomalous (and genuine) e^+e^- pairs. In this case 1.062 state must correspond to lepto-pion whereas higher states must be identified as lepto-sigmas. Also now new lepto-pion states decaying to gamma pairs are predicted and one could hence argue that this prediction is not consistent with what has been observed. A crucial assumption is that lepto-sigmas are light and cannot decay to other lepto-mesons. Ordinary hadronic physics suggests that this need not be the case: the hadronic decay width of the ordinary sigma, if it exists, is very large.

The program of the section is following:

1. PCAC hypothesis, successful in low energy pion physics, is generalized to the case of lepto-pion. Hypothesis allows to deduce the coupling of lepto-pion to leptons and lepto-baryons in terms of leptobaryon-lepton mixing angles. Ortopositronium anomaly allows to deduce precise value of $f(\pi_L)$ characterizing the decay rate of lepto-pion so that the crucial parameters of the model are completely fixed. The decay rates of lepto-pion to photon pair and of lepto-sigma to ordinary e^+e^- pairs are within experimental bounds and corrections to muon and beta decay rates are small. New calculable resonance contributions to photon-photon scattering at cm energy equal to lepto-pion masses are predicted.

2. If anomalous e^+e^- pairs are actually lepto-nucleon pairs, only a model for the creation of lepto-pions from vacuum is needed. In an external electromagnetic field lepto-pion develops a vacuum expectation value proportional to electromagnetic anomaly term [29] so that the production amplitude for the lepto-pion is essentially the Fourier transform of the scalar product of the electric field of the stationary target nucleus with the magnetic field of the colliding nucleus.

3. If anomalous e^+e^- pairs are produced in the decays of lepto-sigmas, the starting point is sigma model providing a realization of PCAC hypothesis. Sigma model makes it possible to relate the production amplitude for $\sigma_L\pi_L$ pairs to the lepto-pion production amplitude: the key element of the model is the large value of the $\sigma\pi_L\pi_L$ coupling constant.

4. Lepto-hadron production amplitudes are proportional to lepto-pion production amplitude and this motivates a detailed study of lepto-pion production. Two models for lepto-pion production are developed: in classical model colliding nucleus is treated classically whereas in quantum model the colliding nucleus is described quantum mechanically. It turns out that classical model explains the peculiar production characteristics of lepto-pion but that production cross section is too small by several orders of magnitude. Quantum mechanical model predicts also diffractive effects: production cross section varies rapidly as a function of the scattering angle and for a fixed value of scattering angle there is a rapid variation with the collision velocity. The estimate for the total lepto-pion production cross section increases by several orders of magnitude due to the coherent summation of the contributions to the amplitude from different values of the impact parameter at the peak.

5. The production rate for lepto-nucleon pairs is only slightly smaller than the experimental upper bound but the e^+e- production rate predicted by sigma model approach is still by a factor of order $1/\sum N_c^2$ smaller than the reported maximum cross section. A possible explanation for this discrepancy is the huge value of the coupling $g(\pi_L, \pi_L, \sigma_L) = g(\sigma_L, \sigma_L, \sigma_L)$ implying that the diagram involving the exchange of virtual sigma can give the dominant contribution to the production cross section of $\sigma_L\pi_L$ pair.

7.2.2 Lepto-pions and generalized PCAC hypothesis

One can say that the PCAC hypothesis predicts the existence of pions and a connection between the pion nucleon coupling strength and the pion decay rate to leptons. In the following we give the PCAC argument and its generalization and consider various consequences.

PCAC for ordinary pions

The PCAC argument for ordinary pions goes as follows [30]:

1. Consider the contribution of the hadronic axial current to the matrix element describing lepton nucleon scattering (say $N+\nu \rightarrow P+e^-$) by weak interactions. The contribution in question reduces to the well-known current-current form

$$\begin{aligned} M &= \frac{G_F}{\sqrt{2}} g_A L_\alpha \langle P|A^\alpha|P\rangle \ , \\ L_\alpha &= \bar{e}\gamma_\alpha(1+\gamma_5)\nu \ , \\ \langle P|A^\alpha|P\rangle &= \bar{P}\gamma^\alpha N \ , \end{aligned} \tag{7.2.-1}$$

where $G_F = \frac{\pi\alpha}{2m_W^2 sin^2(\theta_W)} \simeq 10^{-5}/m_p^2$ denotes the dimensional weak interaction coupling strength and g_A is the nucleon axial form factor:$g_A \simeq 1.253$.

2. The matrix element of the hadronic axial current is not divergenceless, due to the nonvanishing nucleon mass,

$$a_\alpha \langle P|A^\alpha|P\rangle \simeq 2m_p \bar{P}\gamma_5 N \ . \tag{7.2.0}$$

Here q^α denotes the momentum transfer vector. In order to obtain divergenceless current, one can modify the expression for the matrix element of the axial current

$$\langle P|A^\alpha|N\rangle \rightarrow \langle P|A^\alpha|N\rangle - q^\alpha 2m_p \bar{P}\gamma_5 N \frac{1}{q^2} \ . \tag{7.2.1}$$

3. The modification introduces a new term to the lepton-hadron scattering amplitude identifiable as an exchange of a massless pseudoscalar particle

$$\delta T = \frac{G_F g_A}{\sqrt{2}} L_\alpha \frac{2m_p q^\alpha}{q^2} \bar{P}\gamma_5 N \ . \tag{7.2.2}$$

The amplitude is identifiable as the amplitude describing the exchange of the pion, which gets its mass via the breaking of chiral invariance and one obtains by the straightfowread replacement $q^2 \rightarrow q^2 - m_\pi^2$ the correct form of the amplitude.

4. The nontrivial point is that the interpretations as pion exhange is indeed possible since the amplitude obtained is to a good approximation identical to that obtained from the Feynman diagram describing pion exchange, where the pion nucleon coupling constant and pion decay amplitude appear

$$T_2 = \frac{G}{\sqrt{2}} f_\pi q^\alpha L_\alpha \frac{1}{q^2 - m_\pi^2} g\sqrt{2}\bar{P}\gamma_5 N \ . \tag{7.2.3}$$

The condition $\delta T \sim T_2$ gives from Goldberger-Treiman [30]

$$g_A(\simeq 1.25) = \sqrt{2}\frac{f_\pi g}{2m_p}(\simeq 1.3) \ , \tag{7.2.4}$$

satisfied in a good accuracy experimentally.

PCAC in leptonic sector

A natural question is why not generalize the previous argument to the leptonic sector and look at what one obtains. The generalization is based on following general picture.

1. There are two levels to be considered: the level of ordinary leptons and the level of leptobaryons of, say type $f_{ABC}\nu_8^A\nu_8^B\bar{L}_{10}^C$, possessing same quantum numbers as leptons. The interaction transforming these states to each other causes in mass eigenstates mixing of leptobaryons with ordinary leptons described by mixing angles. The masses of lepton and corresponding leptobaryon could be quite near to each other and in case of electron this should be the case as it turns out.

2. A counterargument against the applications of PCAC hypothesis at level of ordinary leptons is that baryons and mesons are both bound states of quarks whereas ordinary leptons are not bound states of colored leptons. The divergence of the axial current is however completely independent of the possible internal structure of leptons and microscopic emission mechanism. Ordinary lepton cannot emit lepto-pion directly but must first transform to leptobaryon with same quantum numbers: phenomenologically this process can be described using mixing angle $sin(\theta_B)$. The emission of lepto-pion proceeds as $L \rightarrow B_L$: $B_L \rightarrow B_L + \pi_L$: $B_L \rightarrow L$, where B_L denotes leptobaryon of type structure $f_{ABC}L_8^AL_8^B\bar{L}_8^C$. The transformation amplitude $L \rightarrow B_L$ is proportional to the mixing angle $sin(\theta_L)$.

Three different PCAC type identities are assumed to hold true:
PCAC1) The vertex for the emission of lepto-pion by ordinary lepton is equivalent with the graph in which lepton L transforms to leptobaryon L^{ex} with same quantum numbers, emits lepto-pion and transforms back to ordinary lepton. The assumption relates the couplings $g(L_1,L_2)$ and $g(L_1^{ex},L_2^{ex})$ (analogous to strong coupling) and mixing angles to each other

$$g(L_1,L_2) = g(L_1^{ex},L_2^{ex})sin(\theta_1)sin(\theta_2) \ . \tag{7.2.5}$$

The condition implies that in electro-weak interactions ordinary leptons do not transform to their exotic counterparts.
PCAC2) The generalization of the ordinary Goldberger-Treiman argument holds true, when ordinary baryons are replaced with leptobaryons. This gives the condition expressing the coupling $f(\pi_L)$ of the lepto-pion state to axial current defined as

$$\langle vac|A_\alpha|\pi_L\rangle = ip_\alpha f(\pi_L) \ , \tag{7.2.6}$$

in terms of the masses of leptobaryons and strong coupling g.

$$f(\pi_L) = \sqrt{2}g_A\frac{(m_{ex}(1)+m_{ex}(2))sin(\theta_1)sin(\theta_2)}{g(L_1,L_2)} \ , \tag{7.2.7}$$

where g_A is parameter characterizing the deviation of weak coupling strength associated with leptobaryon from ideal value: $g_A \sim 1$ holds true in good approximation.
PCAC3) The elimination of leptonic axial anomaly from leptonic current fixes the values of $g(L_i,L_j)$.
i) The standard contribution to the scattering of leptons by weak interactions given by the expression

$$\begin{aligned} T &= \frac{G_F}{\sqrt{2}} \langle L_1|A^\alpha|L_2\rangle\langle L_3|A_\alpha|L_4\rangle \ , \\ \langle L_i|A^\alpha|L_j\rangle &= \bar{L}_i\gamma^\alpha\gamma_5 L_j \ . \end{aligned} \tag{7.2.7}$$

ii) The elimination of the leptonic axial anomaly

$$q_\alpha\langle L_i|A^\alpha|L_j\rangle = (m(L_i)+m(L_j))\bar{L}_i\gamma_5 L_j \ , \tag{7.2.8}$$

by modifying the axial current by the anomaly term

$$\langle L_i|A^\alpha|L_j\rangle \rightarrow \langle L_i|A^\alpha|L_j\rangle - (m(L_i)+m(L_j))\frac{q^\alpha}{q^2}\bar{L}_i\gamma_5 L_j \ , \tag{7.2.9}$$

induces a new interaction term in the scattering of ordinary leptons.

iii) It is assumed that this term is equivalent with the exchange of lepto-pion. This fixes the value of the coupling constant $g(L_1,L_2)$ to

$$\begin{aligned} g(L_1,L_2) &= 2^{1/4}\sqrt{G_F}(m(L_1)+m(L_2))\xi \ , \\ \xi(charged) &= 1 \ , \\ \xi(neutral) &= cos(\theta_W) \ . \end{aligned} \tag{7.2.8}$$

Here the coefficient ξ is related to different values of masses for gauge bosons W and Z appearing in charged and neutral current interactions. An important factor 2 comes from the modification of the axial current in both matrix elements of the axial current.

Lepto-pion exchange interaction couples right and left handed leptons to each other and its strength is of the same order of magnitude as the strength of the ordinary weak interaction at energies not considerably large than the mass of the lepto-pion. At high energies this interaction is negligible and the existence of the lepto-pion predicts no corrections to the parameters of the standard model since these are determined from weak interactions at much higher energies. If lepto-pion mass is sufficiently small (as found, $m(\pi_L) < 2m_e$ is allowed by the experimental data), the interaction mediated by lepto-pion exchange can become quite strong due to the presence of the lepto-pion progator. The value of the lepton-lepto-pion coupling is $g(e,e) \equiv g \sim 5.6\cdot 10^{-6}$. It is perhaps worth noticing that the value of the coupling constant is of the same order as lepton-Higgs coupling constant and also proportional to the mass of the lepton.

PCAC identities fix the values of coupling constants apart from the values of mixing angles. If one assumes that the strong interaction mediated by lepto-pions is really strong and the coupling strength $g(L_{ex},L_{ex})$ is of same order of magnitude as the ordinary pion nucleon coupling strength $g(\pi NN) \simeq 13.5$ one obtains an estimate for the value of the mixing angle $sin(\theta_e)$ $sin^2(\theta_e) \sim \frac{g(\pi NN)}{g(L,L)} \sim 2.4\cdot 10^{-6}$. This implies the order of magnitude $f(\pi_L) \sim 10^{-6}m_W \sim 10^2\ keV$ for $f(\pi_L)$. The order of magnitude is correct as will be found. Ortopositronium decay rate anomaly $\Delta\Gamma/\Gamma \sim 10^{-3}$ and the assumption $m_{ex} \geq 1.3\ MeV$ (so that $e_{ex}\bar{e}$ decay is not possible) gives the upper bound $sin(\theta_e) \leq x\cdot\sqrt{N_c}\cdot 10^{-4}$, where the value of $x \sim 1$ depends on the number of the lepto-pion type states and on the precise value of the Op anomaly.

7.2.3 Lepto-pion decays and PCAC hypothesis

The PCAC argument makes it possible to predict the lepto-pion coupling and decay rates of the lepto-pion to various channels. Actually the orders of magnitude for the decay rates of the lepto-sigma and other lepto-mesons can be deduced also. The comparison with the experimental data is made difficult by the uncertainty of the identifications. The lightest candidate has mass 1.062 MeV and decay width of order 1 keV [27]: only photon photon decay has been observed for this state. The next lepto-meson candidates are in the mass range $1.6-1.8\ MeV$. Perhaps the best status is possessed by 'Darmstadtium' with mass 1.8 MeV. For these states decays to final states identified as

e^+e^- pairs dominate: if indeed e^+e^- pairs, these states probably correspond to the decay products of lepto-sigma. Another possibility is that pairs are actually lepto-nucleon pairs with the mass of the lepto-nucleon only slightly larger than electron mass. Hadron physics experience suggests that the decay widths of the lepto-hadrons (lepto-pion forming a possible exception) should be about 1-10 per cent of particle mass as in hadron physics. The upper bounds for the widths are indeed in the range $50 - 70\ keV$ [27].

$\Gamma(\pi_L \rightarrow \gamma\gamma)$

As in the case of the ordinary pion, anomaly considerations give the following approximate expression for the decay rate of the lepto-pion to two-photon final states [29])

$$\Gamma(\pi_L \rightarrow \gamma\gamma) = \frac{N_c^2\alpha^2 m^3(\pi_L)}{64 f(\pi_L)^2\pi^3} . \tag{7.2.9}$$

$N_c = 8, 10$ is the number of the colored lepton states coming from the axial anomaly loop. For $m(\pi_L) = 1.062\ MeV$ and $f(\pi_L) = N_c \cdot 7.9\ keV$ implied by the ortopositronium decay rate anomaly $\Delta\Gamma/\Gamma = 10^{-3}$ one has $\Gamma(\gamma\gamma) = .52\ keV$, which is consistent with the experimental estimate of order $1\ keV$ [27].

In fact, several lepto-pion states could exist (4 at least corresponding to the resonances at 1.062, 1.63, 1.77 and 1.83 MeV). Since all these lepto-pion states contribute to Op decay rate, the actual value of $f(\pi_L)$ assumed to scale as $m(\pi_L)$, is actually larger in this case: it turns out that $f(\pi_L)$ for the lightest lepto-pion increases to $f(\pi_L)(lightest) = N_c \cdot 15\ keV$ and gives $\Gamma(\gamma\gamma) \simeq .13\ keV$ in case of the lightest lepto-pion if lepto-pions are assumed to correspond the resonances. Note that the order of magnitude for $f(\pi_L)$ is same as deduced from the assumption that lepto-hadronic counterpart of $g(\pi NN)$ equals to the ordinary $g(\pi NN)$. The increase of the ortopositronium anomaly by a factor of, say 4, implies corresponding decrease in $f(\pi_L)^2$. The value of $f(\pi_L)$ is also sensitive to the precise value of the mass of the lightest lepto-pion.

Lepto-pion-lepton coupling

The value of the lepto-pion-lepton coupling can be used to predict the decay rate of lepto-pion to leptons. One obtains for the decay rate $\pi_L^0 \rightarrow e^+e^-$ the estimate

$$\begin{aligned}
\Gamma(\pi_L \rightarrow e^+e^-) &= 4\frac{g(e,e)^2\pi}{2(2\pi)^2}(1-4x^2)m(\pi_L) \\
&= 16 G m_e^2 cos^2(\theta_W)\frac{\sqrt{2}}{4\pi}(1-4x^2)m(\pi_L) \ , \\
x &= \frac{m_e}{m(\pi_L)} \ .
\end{aligned} \tag{7.2.8}$$

for the decay rate of the lepto-pion: for lepto-pion mass $m(\pi_L) \simeq 1.062\ MeV$ one obtains for the decay rate the estimate $\Gamma \sim 1/(1.3 \cdot 10^{-8}\ sec)$: the low decay rate is partly due to the phase space suppression and implies that e^+e^- decay products cannot be observed in the measurement volume. The low decay rate is in accordance with the identification of the lepto-pion as the $m = 1.062\ MeV$ lepto-pion candidate. In sigma model lepto-pion and lepto-sigma have identical lifetimes and for lepto-sigma mass of order $1.8\ MeV$ one obtains $\Gamma(\sigma_L \rightarrow e^+e^-) \simeq 1/(8.2 \cdot 10^{-10}\ sec)$: the prediction is larger than the lower limit $\sim 1/(10^{-9}\ sec)$ for the decay rate implied by the requirement that σ_L decays inside the measurement volume. The estimates of the lifetime obtained from heavy ion collisions [31] give the estimate $\tau \geq 10^{-10}\ sec$. The large value of the lifetime is in accordance with the limits for the lifetime obtained from Babbha scattering [32], which indicate that the lifetime must be longer than 10^{-12} sec.

For lepto-meson candidates with mass above $1.6\ MeV$ no experimental evidence for other decay modes than $X \rightarrow e^+e^-$ has been found and the empirical upper limit for $\gamma\gamma/e^+e^-$ branching ratio [33] is $\Gamma(\gamma\gamma)/\Gamma(e^+e^-) \leq 10^{-3}$. If the identification of the decay products as e^+e^- pairs is correct then the only possible conclusion is that these states cannot correspond to lepto-pion since lepto-pion should decay dominantly into photon photon pairs. Situation changes if pairs of lepton-ucleons $e_{ex}\bar{e}_{ex}$ of type $e_{ex} = e_8\nu_8\bar{\nu}_8$ pair are in question.

I realized that this conclusion might be questioned for more than decade after writing the above text as I developed a model for CDF anomaly suggesting the existence of τ-pions. Since colored leptons are color octets, anomalous magnetic moment type coupling of form $\overline{L}Tr(F^{\mu\nu}\Sigma_{\mu\nu}L_8)$ (the trace is over the Lie-algebra generators of $SU(3)$ and $F^{\mu\nu}$ denotes color gauge field) between ordinary lepton, colored lepton and lepto-gluon is possible. The exchange of a virtual lepto-gluon allows lepto-pion to decay by lepto-strong interactions to electron-positron pairs. The decay rate is limited by the kinematics for the lightest state very near to the final state mass and might make decay rate to in this case very small. If the rate for the decay to electron-positron pair is comparable to that for the decay to two photons the production rate for electron-positron pairs could be of the same order of magnitude as leptopion production rate. The anomalous magnetic moment of electron however poses strong limitations on this coupling and it might be that the coupling is too small. This coupling could however induce the mixing of e_{ex} with e.

$\Gamma(\pi_L \rightarrow e + \bar{\nu_e})$

The expression for the decay rate $\pi_L \rightarrow e + \bar{\nu_e}$ reads as

$$\begin{aligned} \Gamma(\pi_L^- \rightarrow e\nu_e) &= 8Gm_e^2\frac{(1-x^2)^2}{2(1+x^2)}\frac{\sqrt{2}}{(2\pi)^5}m(\pi_L) \ , \\ &= \frac{4}{cos^2(\theta_W)}\frac{(1-x^2)}{(1+x^2)(1-4x^2)}\Gamma(\pi_L^0 \rightarrow e^+e^-) \ , \end{aligned} \quad (7.2.8)$$

and gives $\Gamma(\pi_L^- \rightarrow e\nu_e) \simeq 1/(3.6\cdot 10^{-10}\ sec)$ for $m(\pi_L) = 1.062\ MeV$.

$\Gamma(\pi_L/\sigma_L \rightarrow e_{ex}\bar{e}_{ex})$ **and** $\Gamma(\pi_L/\sigma_L \rightarrow e_{ex}\bar{e})$

Sigma model predicts lepto-pion and lepto-sigma to have same coupling to lepto-nucleon e_{ex} pair so that in the sequel only lepto-pion decay rates are considered. One must consider also the possibility that lepto-pion decay products are either $e_{ex}\bar{e}_{ex}$ or $e_{ex}\bar{e}$ pairs with e_{ex} having mass of near the mass of electron so that it could be misidentified as electron. If the mass of lepto-nucleon e_{ex} with quantum numbers of electron is smaller than $m(\pi_L)/2$ it can be produced in lepto-pion annihilation. One must also assume $m(e_{ex}) > m_e$: otherwise electrons would spontaneously decay to lepto-nucleons via photon emission. The production rate to lepto-nucleon pair can be written as

$$\begin{aligned} \Gamma(\pi_L \rightarrow e_{ex}^+e_{ex}^-) &= \frac{1}{sin^4(\theta_e)}\frac{(1-4y^2)}{(1-4x^2)}\Gamma(\pi_L \rightarrow e^+e^-) \ , \\ y &= \frac{m(e_{ex})}{m(\pi_L)} \ . \end{aligned} \quad (7.2.8)$$

If $e - e_{ex}$ mass difference is sufficiently small the kinematic suppression does not differ significantly from that for e^+e^- pair. The limits from Babbha scattering give no bounds on the rate of $\pi_L \rightarrow e_{ex}^+e_{ex}^-$ decay. The decay rate $\Gamma \sim 10^{20}/sec$ implied by $sin(\theta_e) \sim 10^{-4}$ implies decay width of order .1 MeV: the order of magnitude is the naively expected one and means that the decay to $e_{ex}^+e_{ex}^-$ pairs dominates over the decay to gamma pairs except in the case of the lightest lepto-pion state for which the decay is kinematically forbidden.

The decay rate of the lepto-pion to $\bar{e}e_{ex}$ pair has sensible order of magnitude: for $sin(\theta_e) = 1.2\cdot 10^{-3}$, $m_{\sigma_L} = 1.8\ MeV$ and $m_{e_{ex}} = 1.3\ MeV$ one has $\Gamma \simeq 60\ eV$ allowed by the experimental limits. This decay is kinematically possible only provided the mass of e_{ex} is in below 1.3 MeV. These decays should dominate by a factor $1/sin^2(\theta_e)$ over e^+e^- decays if kinematically allowed.

A signature of these events, if identified erratically as electron positron pairs, is the non-vanishing value of the energy difference in the cm frame of the pair: $E(e^-) - E(e^+) \simeq (m^2(e_{ex}) - m_e^2)/2E > 160\ keV$ for $E = 1.8\ MeV$. If the decay $e_{ex} \rightarrow e + \gamma$ takes place before the detection the energy asymmetry changes its sign. Energy asymmetry [34] increasing with the rest energy of the decaying object has indeed been observed: the proposed interpretation has been that electron forms a bound state with the second nucleus so that its energy is lowered. Also a deviation from the momentum distribution implied by the decay of neutral particle to e^+e^- pair (momenta are opposite in the rest frame) results from the emission of photon. This kind of deviation has also been observed [34]: the proposed explanation is that third object is involved in the decay. A possible alternative explanation for the asymmetries is the production mechanism ($\sigma_L\pi_L$ pairs instead of single particle states).

$\Gamma(e_{ex} \rightarrow e+\gamma)$

The decay to electron and photon would be a unique signature of e_{ex}. The general feature of of fermion family mixing is that mixing takes place in charged currents. In present case mixing is of different type so that $e_{ex} \rightarrow e+\gamma$ might be allowed. If this is not the case then the decay takes place as weak decay via the emission of virtual W boson: $e_{ex} \rightarrow e+\nu_e+\bar{\nu}_e$ and is very slow due to the presence of mixing angle and kinematical supression. The energy of the emitted photon is $E_\gamma = (m_{ex}^2 - m_e^2)/2m_e$. The decay rate $\Gamma(e_{ex} \rightarrow e+\gamma)$ is given by

$$\begin{aligned}
\Gamma(e_{ex} \rightarrow e+\gamma) &= \alpha_{em} sin^2(\theta_e) X m_e \ , \\
X &= \frac{(m_1-m_e)^3(m_1+m_e)m_e}{(m_1^2+m_e^2)^2 m_1} \ .
\end{aligned} \tag{7.2.7}$$

For $m(e_{ex}) = 1.3\ MeV$ the decay of order $1/(1.4\cdot 10^{-12}\ sec)$ for $sin(\theta_e) = 1.2\cdot 10^{-3}$ so that lepto-nucleons would decay to electrons in the measurement volume. In the experiments positrons are identified via pair annihilation and since pair annihilation rate for $\bar{e}_{ex}$ is by a factor $sin^2(\theta_e)$ slower than for e^+ the particles identified as positrons must indeed be positrons. For sufficiently small mass difference $m(e_{ex}) - m_e$ the particles identified as electron could actually be e_{ex}. The decay of e_{ex} to electron plus photon before its detection seems however more reasonable alternative since it could explain the observed energy asymmetry [34].

Some implications

The results have several implications as far as the decays of on mass shell states are considered:

1. For $m(e_{ex}) > 1.3\ MeV$ the only kinematically possible decay mode is the decay to e^+e^- pair. Production mechanism might explain the asymmetries [34]. The decay rate of on mass shell π_L and σ_L (or $\eta_L, \rho_L, ..$) is above the lower limit allowed by the detection in the measurement volume.

2. If the mass of e_{ex} is larger than $.9\ MeV$ but smaller than $1.3\ MeV$ $e_{ex}\bar{e}$ decays dominate over e^+e^- decays. The decay $e_{ex} \rightarrow e+\gamma$ before detection could explain the observed energy asymmetry.

3. It will be found that the direct production of $e_{ex}\bar{e}$ pairs is also possible in the heavy ion collision but the rate is much smaller due to the smaller phase space volume in two-particle case. The annihilation rate of $\bar{e}_{ex}$ in matter is by a factor $sin^2(\theta_e)$ smaller than the annihilation rate of positron. This provides a unique signature of e_{ex} if e^+ annihilation rate in matter is larger than the decay rate of $\bar{e}_{ex}$. In lead the lifetime of positron is $\tau \sim 10^{-10}\ sec$ and indeed larger that e_{ex} lifetime.

Karmen anomaly

A brief comment on the Karmen anomaly [18] observed in the decays of π^+ is in order. The anomaly suggests the existence [19] of new weakly interacting neutral particle x, which mixes with muon neutrino. Since $g=1$ neutrino corresponds to charmed quark in hadron physics context having $k=103$ rather than $k=107$ as primary condensation level, a natural guess for its primary condensation level is $k=113$, which would mean that the mass scale would be of order muon mass: the particle candidate indeed has mass of order 30 MeV. One class of solutions to laboratory constraints, which might evade also cosmological and astrophysical constraints, corresponds to object x mixing with muon type neutrino and decaying radiatively to $\gamma+\nu_{\mu}$ via the emission of virtual W boson. The value of the mixing parameter $U(\mu,x)$ describing $\nu_{mu}-x$ mixing satisfies $|U_{\mu,x}|^4\simeq .8\cdot 10^{-10}$.

The following naive PCAC argument gives order of magnitude estimate for $|U(\mu,x)|\sim sin(\theta_{\mu})$. The value of $g(\mu,\mu)$ is by a factor $m(\mu)/m_e$ larger than $g(e,e)$. If the lepto-hadronic couplings $g(\mu_{ex},\mu_{ex})$ and $g(e_{ex},e_{ex})$ are of same order of magnitude then one has $sin(\theta_{\mu})\leq .02$ (3 lepto-pion states and Op anomaly equal to $Op=5\cdot 10^{-3}$): the lower bound is 6.5 times larger than the value .003 deduced in [19]. The actual value could be considerably smaller since e_{ex} mass could be larger than 1.3 MeV by a factor of order 10.

7.2.4 Lepto-pions and weak decays

The couplings of lepto-meson to electro-weak gauge bosons can be estimated using PCAC and CVC hypothesis [29]. The effective $m_{\pi_L}-W$ vertex is the matrix element of electro-weak axial current between vacuum and charged lepto-meson state and can be deduced using same arguments as in the case of ordinary charged pion

$$\langle 0|J_A^{\alpha}|\pi_L^-\rangle = Km(\pi_l)p^{\alpha} \ , \tag{7.2.7}$$

where K is some numerical factor and p^{α} denotes the momentum of lepto-pion. For neutral lepto-pion the same argument gives vanishing coupling to photon by the conservation of vector current. This has the important consequence that lepto-pion cannot be observed as resonance in e^+e^- annihilation in single photon channel. In two photon channel lepto-pion should appear as resonance. The effective interaction Lagrangian is the 'instanton' density of electromagnetic field giving additional contribution to the divergence of the axial current and was used to derive a model for lepto-pion production in heavy ion collisions.

Lepto-hadrons and lepton decays

The lifetime of charged lepto-pion is from PCAC estimates larger than 10^{-10} seconds by the previous PCAC estimates. Therefore lepto-pions are practically stable particles and can appear in the final states of particle reactions. In particular, lepto-pion atoms are possible and by Bose statistics have the peculiar property that ground state can contain many lepto-pions.

Lepton decays $L\rightarrow\nu_{\mu}+H_L$, $L=e,\mu,\tau$ via emission of virtual W are kinematically allowed and an anomalous resonance peak in the neutrino energy spectrum at energy

$$E(\nu_L) \ = \ \frac{m(L)}{2}-\frac{m_H^2}{2m(L)} \ , \tag{7.2.8}$$

provides a unique test for the lepto-hadron hypothesis. If lepto-pion is too light electrons would decay to charged lepto-pions and neutrinos unless the condition $m(\pi_L)>m_e$ holds true.

The existence of a new decay channel for muon is an obvious danger to the lepto-hadron scenario: large changes in muon decay rate are not allowed.

Consider first the decay $\mu\rightarrow\nu_{\mu}+\pi_L$ where π_L is on mass shell lepto-pion. Lepto-pion has energy $\sim m(\mu)/2$ in muon rest system and is highly relativistic so that in the muon rest system the lifetime of lepto-pion is of order $\frac{m(\mu)}{2m(\pi_L)}\tau(\pi_L)$ and the average length traveled by lepto-pion before decay is of order 10^8 meters! This means that lepto-pion can be treated as stable particle. The presence of a new decay channel changes the lifetime of muon although the rate for events using $e\nu_e$ pair as signature is not changed. The effective H_L-W vertex was deduced above. The rate for the decay via lepto-pion emission and its ratio to ordinary rate for muon decay are given by

$$\begin{aligned}
\Gamma(\mu\rightarrow\nu_{\mu}+H_L) \ &= \ \frac{G^2K^2}{2^5\pi}m^4(\mu)m^2(H_L)(1-\frac{m^2(H_L)}{m^2(\mu)})\frac{(m^2(\mu)-m^2(H_L))}{(m^2(\mu)+m^2(H_L))} \ ,\\
\frac{\Gamma(\mu\rightarrow\nu_{\mu}+H_L)}{\Gamma(\mu\rightarrow\nu_{\mu}+e+\bar{\nu}_e)} \ &= \ 6\cdot(2\pi^4)K^2\frac{m^2(H_L)}{m^2(\mu)}\frac{(m^2(\mu)-m^2(H_L))}{(m^2(\mu)+m^2(H_L))} \ ,
\end{aligned} \tag{7.2.7}$$

and is of order $.93K^2$ in case of lepto-pion. As far as the determination of G_F or equivalently m_W^2 from muon decay rate is considered the situation seems to be good since the change introduced to G_F is of order $\Delta G_F/G_F \simeq 0.93K^2$ so that K must be considerably smaller than one. For the physical value of K: $K \leq 10^{-2}$ the contribution to the muon decay rate is negligible.

Lepto-hadrons can appear also as virtual particles in the decay amplitude $\mu \rightarrow \nu_\mu + e\nu_e$ and this changes the value of muon decay rate. The correction is however extremely small since the decay vertex of intermediate off mass shell lepto-pion is proportional to its decay rate.

Lepto-pions and beta decay

If lepto-pions are allowed as final state particles lepto-pion emission provides a new channel $n \rightarrow p + \pi_L$ for beta decay of nuclei since the invariant mass of virtual W boson varies within the range $(m_e = 0.511\ MeV, m_n - m_p = 1.293 MeV$. The resonance peak for $m(\pi_L) \simeq 1\ MeV$ is extremely sharp due to the long lifetime of the charged lepto-pion. The energy of the lepto-pion at resonance is

$$E(\pi_L) = (m_n - m_p)\frac{(m_n + m_p)}{2m_n} + \frac{m(\pi_L)^2}{2m_n} \simeq m_n - m_p \ . \tag{7.2.8}$$

Together with long lifetime this lepto-pions escape the detector volume without decaying (the exact knowledge of the energy of charged lepto-pion might make possible its direct detection).

The contribution of lepto-pion to neutron decay rate is not negligible. Decay amplitude is proportional to superposition of divergences of axial and vector currents between proton and neutron states.

$$M = \frac{G}{\sqrt{2}} K m(\pi_L)(q^\alpha V_\alpha + q^\alpha A_\alpha) \ . \tag{7.2.9}$$

For exactly conserved vector current the contribution of vector current vanishes identically. The matrix element of the divergence of axial vector current at small momentum transfer (approximately zero) is in good approximation given by

$$\begin{aligned} \langle p|q^\alpha A_\alpha|n\rangle &= g_A(m_p + m_n)\bar{u}_p\gamma_5 u_n \ , \\ g_A &\simeq 1.253 \ . \end{aligned} \tag{7.2.9}$$

Straightforward calculation shows that the ratio for the decay rate via lepto-pion emission and ordinary beta decay rate is in good approximation given by

$$\begin{aligned} \frac{\Gamma(n \rightarrow p + \pi_L)}{\Gamma(n \rightarrow p + e + \bar{\nu}_e)} &= \frac{30\pi^2 g_A^2 K^2}{0.47 \cdot (1 + 3g_A^2)} \frac{m_{\pi_L}^2(\Delta^2 - m_{\pi_L}^2)^2}{\Delta^6} \ , \\ \Delta &= m(n) - m(p) \ . \end{aligned} \tag{7.2.9}$$

Lepto-pion contribution is smaller than ordinary contribution if the condition

$$K < \left[\frac{.47 \cdot (1 + 3g_A^2)}{30\pi^2 g_A^2} \frac{\Delta^6}{(\Delta^2 - m_{\pi_L}^2)^2 m_{\pi_L}^2}\right]^{1/2} \simeq .28 \ , \tag{7.2.10}$$

is satisfied. The upper bound $K \leq 10^{-2}$ coming from the lepto-pion decay width and Op anomaly implies that the contribution of the lepto-pion to beta decay rate is very small.

7.2.5 Ortopositronium puzzle and lepto-pion in photon photon scattering

The decay rate of ortopositronium (Op) has been found to be slightly larger than the rate predicted by QED [20, 35]: the discrepancy is of order $\Delta\Gamma/\Gamma \sim 10^{-3}$. For parapositronium no anomaly has been observed. Most of the proposed explanations [35] are based on the decay mode $Op \rightarrow X + \gamma$, where X is some exotic particle. The experimental limits on the branching ratio $\Gamma(Op \rightarrow X + \gamma)$ are below the required value of order 10^{-3}. This explanation is excluded also by the standard cosmology [35].

Lepto-pion hypothesis suggests an obvious solution of the Op-puzzle. The increase in annihilation rate is due to the additional contribution to $Op \rightarrow 3\gamma$ decay coming from the decay $Op \rightarrow \gamma_V$ (V denotes 'virtual') followed by the decay $\gamma_V \rightarrow \gamma + \pi_L^V$ followed by the decay $\pi_L^V \rightarrow \gamma + \gamma$ of the virtual lepto-pion to two photon state. $\gamma\gamma\pi_L$ vertices are induced by the axial current anomaly $\propto E \cdot B$. Also a modification of parapositronium decay rate is predicted. The first contribution comes from the decay $Op \rightarrow \pi_L^V \rightarrow \gamma + \gamma$ but the contribution is very small due the smallness of the coupling $g(e, e)$. The second contribution obtained from ortopositronium contribution by replacing one outgoing photon with a loop photon is also small. Since the production of a real lepto-pion is impossible, the mechanism is consistent with the experimental constraints.

The modification to the Op annihilation amplitude comes in a good approximation from the interference term between the ordinary e^+e^- annihilation amplitude F_{st} and lepto-pion induced annihilation amplitude F_{new}:

$$\Delta\Gamma \propto 2Re(F_{st}\bar{F}_{new}) \ , \tag{7.2.11}$$

and rough order of magnitude estimate suggests $\Delta\Gamma/\Gamma \sim K^2/e^2 = \alpha^2/4\pi \sim 10^{-3}$. It turns out that the sign and the order of magnitude of the new contribution are correct for $f(\pi_L) \sim 2\ keV$ deduced also from the anomalous e^+e^- production rate.

The new contribution to $e^+e^- \rightarrow 3\gamma$ decay amplitude is most easily derivable using for lepto-pion-photon interaction the effective action

$$\begin{aligned} L_1 &= K\pi_L F \wedge F \ , \\ K &= \frac{\alpha_{em} N_c}{8\pi f(\pi_L)} \ , \end{aligned} \tag{7.2.11}$$

where F is quantized electromagnetic field. The calculation of the lepto-pion contribution proceeds in manner described in [29], where the expression for the standard contribution and an elegant method for treating the average over e^+e^- spin triplet states and sum over photon polarizations, can be found. The contribution to the decay rate can be written as

$$\begin{aligned} \frac{\Delta\Gamma}{\Gamma} &\simeq K_1 I_0 \ , \\ K_1 &= \frac{3\alpha N_c^2}{(\pi^2-9)2^9(2\pi)^3}(\frac{m_e}{f(\pi_L)})^2 \ , \\ I_0 &= \int_0^1 \int_{-1}^{umax} \frac{f}{v+f-1-x^2} v^2(2(f-v)u+2-v-f)dvdu \ , \\ f &\equiv f(v,u) = 1-\frac{v}{2}-\sqrt{(1-\frac{v}{2})^2-\frac{1-v}{1-u}} \ , \\ u &= \bar{n}_1 \cdot \bar{n}_2 \ , \ \bar{n}_i = \frac{\bar{k}_i}{\omega_i} \ , \ umax = \frac{(\frac{v}{2})^2}{(1-\frac{v}{2})^2} \ , \\ v &= \frac{\omega_3}{m_e} \ , \ x = \frac{m_{\pi_L}}{2m_e} \ . \end{aligned} \tag{7.2.7}$$

ω_i and $\bar{k}_i$ denote the energies of photons, u denotes the cosine of the angle between first and second photon and v is the energy of the third photon using electron mass as unit. The condition $\Delta\Gamma/\Gamma = 10^{-3}$ gives for the parameter $f(\pi_L)$ the value $f(\pi_L)(1.062\ MeV) \simeq N_c \cdot 7.9\ keV$. If there are several lepto-pion states, they contribute to the decay anomaly additively. If the four known resonances correspond directly to lepto-pions decaying to lepto-nucleon pairs and $f(\pi_L)$ is assumed to scale as $N_c m_{\pi_L}$, one obtains $f(\pi_L)(1.062\ MeV) \simeq N_c \cdot 14.7\ keV$. From the PCAC relation one obtains for $sin(\theta_e)$ the upper bound $sin(\theta_e) \leq x \cdot \sqrt{N_c}10^{-4}$ assuming $m_{ex} \geq 1.3\ MeV$ (so that $e_{ex}\bar{e}$ decay is not possible), where $x = 1.2$ for single lepto-pion state and $x = 1.36$ for four lepto-pion states identified as the observed resonances.

Lepto-pion photon interaction implies also a new contribution to photon-photon scattering. Just at the threshold $E = m_{\pi_L}/2$ the creation of lepto-pion in photon photon scattering is possible and the appearance of lepto-pion as virtual particle gives resonance type behaviour to photon photon scattering near $s = m_{\pi_L}^2$. The total photon-photon cross section in zero decay width approximation is given by

$$\sigma = \frac{\alpha^4 N_c^2}{2^{14}(2\pi)^6}\frac{E^6}{f_{\pi_L}^4(E^2-\frac{m_{\pi_L}^2}{4})^2} . \quad (7.2.8)$$

N	$Op/10^{-3}$	$f(\pi_L)/(N_c keV)$	$sin(\theta_e)(m_{ex}/1.3\ MeV)^{1/2}$	$\Gamma(\pi_L)/keV$
1	1	7.9	$1.2 \cdot 10^{-4}\sqrt{N_c}$	.51
3	1	14.7	$1.7 \cdot 10^{-4}\sqrt{N_c}$	.13
3	5	6.5	$3.6 \cdot 10^{-4}\sqrt{N_c}$	.73

Table 1: The dependence of various quantities on the number of lepto-pion type states and Op anomaly, whose value is varied assuming the proportionality $f(\pi_L) \propto N_c m_{\pi_L}$. N_c refers to the number of lepto-pion states in given representation and Op denotes lepto-pion anomaly.

7.2.6 Spontaneous vacuum expectation of lepto-pion field as source of lepto-pions

The basic assumption in the model of lepto-pion and lepto-hadron production is the spontaneous generation of lepto-pion vacuum expectation value in strong nonorthogonal electric and magnetic fields. This assumption is in fact very natural in TGD [1].

1. The well known relation [29] expressing pion field as a sum of the divergence of axial vector current and anomaly term generalizes to the case of lepto-pion

$$\pi_L = \frac{1}{f(\pi_L)m^2(\pi_L)}(\nabla \cdot j^A + \frac{\alpha_{em}N_c}{2\pi}E\cdot B) . \quad (7.2.9)$$

   In the case of lepto-pion case the value of $f(\pi_L)$ has been already deduced from PCAC argument. Anomaly term gives rise to pion decay to two photons so that one obtains an estimate for the lifetime of the lepto-pion.

   This relation is taken as the basis for the model describing also the production of lepto-pion in external electromagnetic field. The idea is that the presence of external electromagnetic field gives rise to a vacuum expectation value of lepto-pion field. Vacuum expectation is obtained by assuming that the vacuum expectation value of axial vector current vanishes.

$$\langle vac \mid \pi \mid vac\rangle = KE\cdot B ,$$
$$K = \frac{\alpha_{em}N_c}{2\pi f(\pi_L)m^2(\pi_L)} . \quad (7.2.9)$$

   Some comments concerning this hypothesis are in order here:

[1] 'Instanton density' generates coherent state of lepto-pions just like classical em current generates coherent state of photons

i) The basic hypothesis making possible to avoid large parity breaking effects in atomic and molecular physics is that p-adic condensation levels with length scale $L(n) < 10^{-6}\ m$ are purely electromagnetic in the sense that nuclei feed their Z^0 charges on condensate levels with $L(n) \geq 10^{-6}\ m$. The absence of Z^0 charges does not however exclude the possibility of the classical Z^0 fields induced by the nonorthogonality of the ordinary electric and magnetic fields (if Z^0 fields vanish E and B are orthogonal in TGD.

ii) The nonvanishing vacuum expectation value of the lepto-pion field implies parity breaking in atomic length scales. This is understandable from basic principles of TGD since classical Z^0 field has parity breaking axial coupling to electrons and protons. The nonvanishing classical lepto-pion field is in fact more or less equivalent with the presence of classical Z^0 field.

2. The amplitude for the production of lepto-pion with four momentum $p = (p_0, \bar{p})$ in an external electromagnetic field can be deduced by writing lepto-pion field as sum of classical and quantum parts: $\pi_L = \pi_L(class) + \pi_L(quant)$ and by decomposing the mass term into interaction term plus c-number term and standard mass term:

$$
\begin{aligned}
\frac{m^2(\pi_L)\pi_L^2}{2} &= L_{int} + L_0 \ , \\
L_0 &= \frac{m^2(\pi_L)}{2}(\pi_L^2(class) + \pi_L^2(quant)) \ , \\
L_{int} &= m^2(\pi_L)\pi_L(class)\pi_L(quant) \ . \qquad (7.2.8)
\end{aligned}
$$

Interaction Lagrangian corresponds to L_{int} linear in lepto-pion oscillator operators. Using standard LSZ reduction formula and normalization conventions of [29] one obtains for the probability amplitude for creating lepto-pion of momentum p from vacuum the expression

$$
\begin{aligned}
A(p) &\equiv \langle a(p)\pi_L\rangle = (2\pi)^3 m^2(\pi_L)\int f_p(x)\langle vac \mid \pi \mid vac\rangle d^4x \ , \\
f_p &= e^{ip\cdot x} \ . \qquad (7.2.8)
\end{aligned}
$$

The probability for the production of lepto-pion in phase space volume element $d3p$ is obtained by multiplying with the density of states factor $d^3n = V\frac{d^3p}{(2\pi)^3}$:

$$
\begin{aligned}
dP &= A|U|^2 V d^3p \ , \\
A &= (\frac{\alpha_{em}N_c^2 m^2(\pi_L)}{2\pi f(\pi_L)})^2 \ , \\
U &= \int e^{ip\cdot x} E\cdot B d^4x \ . \qquad (7.2.7)
\end{aligned}
$$

The first conclusion that one can draw is that nonstatic electromagnetic fields are required for lepto-pion creation since in static fields energy conservation forces lepto-pion to have zero energy and thus prohibits real lepto-pion production. In particular, the spontaneous creation lepto-pion in static Coulombic and magnetic dipole fields of nucleus is impossible.

7.2.7 Sigma model and creation of lepto-hadrons in electromagnetic fields

Why sigma model approach?

For several reasons it is necessary to generalize the model for lepto-pion production to a model for lepto-hadron production.

1. Sigma model approach is necessary if one assumes that anomalous e^+e^- pairs are genuine e^+e^- pairs rather lepto-nucleon pairs produced in the decays of lepto-sigmas.

2. A model for the production of lepto-hadrons is obtained from an effective action describing the strong and electromagnetic interactions between lepto-hadrons. The simplest model is sigma model describing the interaction between lepto-nucleons, lepto-pion and a hypothetical scalar particle σ_L [29]. This model realizes lepto-pion field as a divergence of the axial current and gives the standard relation between $f(\pi_L), g$ and m_{ex}. All couplings of the model are related to the masses of e_{ex}, π_L and σ_L. The generation of lepto-pion vacuum expectation value in the proposed manner takes place via triangle anomaly diagrams in the external electromagnetic field.

3. If needed the model can be generalized to contain terms describing also other lepto-hadrons. The generalized model should contain also vector bosons ρ_L and ω_L as well as pseudoscalars η_L and η'_L and radial excitations of π_L and σ_L. An open question is whether also η and η' generate vacuum expectation value proportional to $E \cdot B$. Actually all these states appear as 3-fold degenerate for the minimal color representation content of the theory.

4. The following observations are useful for what follows.

   i) Ortopositronium decay width anomaly gives the estimate $f(\pi_L) \sim N_c \cdot 7.9\ keV$ and from this one can deduce an upper bound for lepto-pion production cross section in an external electromagnetic field. The calculation of lepto-pion production cross section shows that lepto-pion production cross section is somewhat smaller than the upper bound for the observed anomalous e^+e^- production cross section, even when one tunes the values of the various parameters. This is consistent with the idea that lepto-nucleon pairs, with lepto-nucleon mass being only slightly larger than electron mass, are in question.

   ii) Also the direct production of the lepto-nucleon pairs via the interaction term $gcos(\theta_e)\bar{e}_{ex}\gamma_5 e_{ex}\pi_L(cl)$ is possible but gives rise to continuum mass squared spectrum rather than resonant structures. The direct production of the pairs via the interaction term $gsin(\theta_e)\bar{e}\gamma_5 e_{ex}\pi_L(cl)$ from is much slower process than the production via the meson decays and does not give rise to resonant structures since Also the production via the $\bar{e}e_{ex}$ decay of virtual lepto-pion created from classical field is slow process since it involves $sin^2(\theta_e)$.

   iii) e^+e^- production can also proceed also via the creation of many particle states. The simplest candidates are $V_L + \pi_L$ states created via $\partial_\alpha \pi_L V^\alpha \pi_L(class)$ term in action and $\sigma_L + \pi_L$ states created via the the $k\sigma_L\pi_L\pi_L(class)$ term in the sigma model action. The production cross section via the decays of vector mesons is certainly very small since the production vertex involves the inner product of vector boson 3 momentum with its polarization vector and the situation is nonrelativistic.

   iv) If the strong decay of σ_L to lepto-mesons is kinematically forbidden (this is not suggested by the experience with the ordinary hadron physics), the production rate for σ_L meson is large since the coupling k turns out to be given by $k = (m^2_{\sigma_L} - m^2_{\pi_L})/2f(\pi_L)$ and is anomalously large for the value of $f(\pi_L) \geq 7.9 \cdot N_c\ keV$ derived from ortopositronium anomaly: $k \sim 336 m(\pi_L)/N_c$ for $f(\pi_L) \sim N_c \cdot 7.9\ keV$. The resulting additional factor in the production cross section compensates the reduction factor coming from two-particle phase space volume. Despite this the estimate for the production cross section of anomalous e^+e^- pairs is roughly by a factor $1/N_c^2$ smaller than the maximum experimental cross section. The radiative corrections are huge and should give the dominant contribution to the cross section. It is however questionable very the assumed small lepto-hadronic decay width and mass of σ_L is consistent with the extremely strong interactions of σ_L.

Simplest sigma model

A detailed description of the sigma model can be found in [29] and it suffices to outline only the crucial features here.

1. The action of lepto-hadronic sigma model reads as

$$\begin{aligned} L &= L_S + c\sigma_L \ , \\ L_S &= \bar{\psi}_L(i\gamma^k\partial_k + g(\sigma_L + i\pi_L\cdot\tau\gamma_5))\psi_L + \frac{1}{2}((\partial\pi_L)^2 + (\partial\sigma_L)^2) \\ &- \frac{\mu^2}{2}(\sigma_L^2 + \pi_L^2) - \frac{\lambda}{4}(\sigma_L^2 + \pi_L^2)^2 \ . \end{aligned} \tag{7.2.6}$$

π_L is isospin triplet and σ_L isospin singlet. ψ_L is isospin doublet with electro-weak quantum numbers of electron and neutrino (e_{ex} and ν_{ex}). The model allows $so(4)$ symmetry. Vector current is conserved but for $c \neq 0$ axial current generates divergence, which is proportional to pion field: $\partial^\alpha A_\alpha = -c\pi_L$.

2. The presence of the linear term implies that σ_L field generates vacuum expectation value $\langle 0|\sigma_L|0\rangle = v$. When the action is written in terms of new quantum field $\sigma_L' = \sigma_L - v$ one has

$$\begin{aligned} L &= \bar{\psi}_L(i\gamma^k\partial_k + m + g(\sigma_L' + i\pi_L\cdot\tau\gamma_5))\psi_L + \frac{1}{2}((\partial\pi_L)^2 + (\partial\sigma_L')^2) \\ &- \frac{1}{2}m_{\sigma_L}^2(\sigma_L')^2 - \frac{m_{\pi_L}^2}{2}\pi_L^2 \\ &- \lambda v\sigma_L'((\sigma_L')^2 + \pi_L^2) - \frac{\lambda}{4}((\sigma_L')^2 + \pi_L^2)^2 \ , \end{aligned} \tag{7.2.4}$$

The masses are given by

$$\begin{aligned} m_{\pi_L}^2 &= \mu^2 + \lambda v^2 \ , \\ m_{\sigma_L}^2 &= \mu^2 + 3\lambda v^2 \ , \\ m &= -gv \ . \end{aligned} \tag{7.2.3}$$

These formulas relate the parameters μ, v, g to lepto-hadrons masses.

3. The requirement that σ_L' has vanishing vacuum expectation implies in Born approximation

$$c - \mu^2 v - \lambda v^3 = 0 \ , \tag{7.2.4}$$

which implies

$$\begin{aligned} f(\pi_L) &= -v = -\frac{c}{m^2(\pi_L)} \ , \\ m_{ex} &= gf(\pi_L) \ . \end{aligned} \tag{7.2.4}$$

Note that e_{ex} and ν_{ex} are predicted to have identical masses in this approximation. The value of the strong coupling constant g of lepto-hadronic physics is indeed strong from $m_{ex} > m_e$ and $f(\pi_L) < N_c \cdot 10$ keV.

4. A new feature is the generation of the lepto-pion vacuum expectation value in an external electromagnetic field (of course, this is possible for the ordinary pion field, too!). The vacuum expectation is generated via the triangle anomaly diagram in a manner identical to the generation of a non-vanishing photon-photon decay amplitude and is proportional to the instanton density of the electromagnetic field. By redefining the pion field as a sum $\pi_L = \pi_L(cl) + \pi'_L$ one obtains effective action describing the creation of the lepto-hadrons in strong electromagnetic fields.

5. As far as the production of $\sigma_L \pi_L$ pairs is considered, the interaction term $\lambda v \sigma'_L \pi_L^2$ is especially interesting since it leads to the creation of $\sigma_L \pi_L$ pairs via the interaction term $k\lambda v \sigma'_L \pi_L(qu)\pi_L(cl)$.

   The coefficient of this term can be expressed in terms of the lepto-meson masses and $f(\pi_L)$:

$$\begin{aligned} k &\equiv 2\lambda v = \frac{m_{\sigma_L}^2 - m_{\pi_L}^2}{2f(\pi_L)} = x m_{\pi_L} \ , \\ x &= \frac{1}{2}(\frac{m_{\sigma_L}^2}{m_{\pi_L}^2} - 1)\frac{m_{\pi_L}}{f(\pi_L)} \ . \end{aligned} \tag{7.2.4}$$

   The large value of the coupling deriving from $f(\pi_L) = N_c \cdot 7.9\ keV)$ compensates the reduction of the production rate coming from the smallness of two-particle phase space volume as compared with single particle-phase space volume but fails to produce large enough production cross section. The large value of $g(\sigma_L, \sigma_L, \sigma_L) = g(\sigma_L, \pi_L, \pi_L)$ however implies that the radiative contribution to the production cross section coming from the emission of a virtual sigma in the production vertex is much larger than the lowest order production cross section and with a rather small value of the relative $\sigma_L - \pi_L$ mass difference correct order of magnitude of cross section should be possible.

7.2.8 Classical model for lepto-pion production

The nice feature of both quantum and classical model is that the production amplitudes associated with all lepto-hadron production reactions in external electromagnetic field are proportional to the lepto-pion production amplitude and apart from phase space volume factors production cross sections are expected to be given by lepto-pion production cross section. Therefore it makes sense to construct a detailed model for lepto-pion production despite the fact that lepto-pion decays probably contribute only a very small fraction to the observed e^+e^- pairs.

General considerations

Angular momentum barrier makes the production of lepto-mesons with orbital angular momentum $L > 0$ improbable. Therefore the observed resonances are expected to be $L = 0$ pseudoscalar states. Lepto-pion production has two signatures which any realistic model should reproduce.

1. Data are consistent with the assumption that states are produced at rest in cm frame.

2. The production probability has a peak in a narrow region of velocities of colliding nucleus around the velocity needed to overcome Coulomb barrier in head on collision. The relative width of the velocity peak is of order $\Delta\beta/\beta \simeq \cdot 10^{-2}$ [36]. In Th-Th system [36] two peaks at projectile energies 5.70 MeV and 5.75 MeV per nucleon have been observed. This suggests that some kind of diffraction mechanism based on the finite size of nuclei is at work.
   In this section a model treating nuclei as point like charges and nucleus-nucleus collision purely classically is developed. This model yields qualitative predictions in agreement with the signature 1) but fails to reproduce the possible diffraction behavior although one can develop argument for understanding the behavior above Coulomb wall.

The general expression for the amplitude for creation of lepto-pion in external electric and magnetic fields has been derived in Appendix. Let us now specialize to the case of heavy ion collision. We consider the situation, where the scattering angle of the colliding nucleus is measured. Treating the collision completely classically we can assume that collision occurs with a well defined value of the impact parameter in a fixed scattering plane. The coordinates are chosen so that target nucleus is at rest at the origin of the coordinates and colliding nucleus moves in z-direction in y=0 plane with velocity β. The scattering angle of the scattered nucleus is denoted by α, the velocity of the lepto-pion by v and the direction angles of lepto-pion velocity by (θ, ϕ).

The minimum value of the impact parameter for the Coulomb collision of point like charges is given by the expression

$$\begin{aligned} b &= \frac{b_0 cot(\alpha/2)}{2} \ , \\ b_0 &= \frac{2Z_1Z_2\alpha_{em}}{M_R\beta^2} \ , \end{aligned} \tag{7.2.4}$$

where b_0 is the expression for the distance of the closest approach in head on collision. M_R denotes the reduced mass of the nucleus-nucleus system.

To estimate the amplitude for lepto-pion production the following simplifying assumptions are made.

1. Nuclei can be treated as point like charges. This assumption is well motivated, when the impact parameter of the collision is larger than the critical impact parameter given by the sum of radii of the colliding nuclei:

$$b_{cr} = R_1 + R_2 \ . \tag{7.2.5}$$

   For scattering angles that are sufficiently large the values of the impact parameter do not satisfy the above condition in the region of the velocity peak. p-Adic considerations lead to the conclusion that nuclear condensation level corresponds to prime $p \sim 2^k$, $k = 113$ (k is prime). This suggest that nuclear radius should be replaced by the size $L(113)$ of the p-adic convergence cube associated with nucleus (see the chapter "TGD and Nuclear Physics": $L(113) \sim 1.7 \cdot 10^{-14}\ m$ implies that cutoff radius is $b_{cr} \sim 2L(113) \sim 3.4 \cdot 10^{-14}\ m$.

2. Since the velocities are non-relativistic (about $0.12c$) one can treat the motion of the nuclei non-relativistically and the non-retarded electromagnetic fields associated with the exactly known classical orbits can be used. The use of classical orbit doesn't take into account recoil effect caused by lepto-pion production. Since the mass ratio of lepto-pion and the reduced mass of heavy nucleus system is of order 10^{-5} the recoil effect is however negligible.

3. The model simplifies considerably, when the orbit is idealized with a straight line with impact parameter determined from the condition expressing scattering angle in terms of the impact parameter. This approximation is certainly well founded for large values of impact parameter. For small values of impact parameter the situation is quite different and an interesting problem is whether the contributions of long range radiation fields created by accelerating nuclei in head-on collision could give large contribution to lepto-pion production rate. On the line connecting the nuclei the electric part of the radiation field created by first nucleus is indeed parallel to the magnetic part of the radiation field created by second nucleus. In this approximation the instanton density in the rest frame of the target nucleus is just the scalar product of the Coulombic electric field E of the target nucleus and of the magnetic field B of the colliding nucleus obtained by boosting it from the Coulomb field of nucleus at rest.

Expression of the classical cross section

First some kinematical notations. Lepto-pion four-momentum in the rest system of target nucleus is given by the following expression

$$\begin{aligned} p &= (p_0, \bar{p}) = m\gamma_1(1, vsin(\theta)cos(\phi), vsin(\theta)sin(\phi), vcos(\theta)) \ , \\ \gamma_1 &= 1/(1-v^2)^{1/2} \ . \end{aligned} \tag{7.2.5}$$

The velocity and Lorentz boost factor of the projectile nucleus are denoted by β and $\gamma = 1/\sqrt{1-\beta^2}$.

The double differential cross section in the classical model can be written as

$$\begin{aligned}
d\sigma &= dP 2\pi b db \ , \\
dP &= K|A(b,p)|^2 d^3n \ , per d^3n = V\frac{d^3p}{(2\pi)^3} \ , \\
K &= (Z_1Z_2)^2(\alpha_{em})^4 \times N_c^2(\frac{m(\pi_L}{f(\pi_L)})^2\frac{1}{2\pi^{13}} \ , \\
A(b,p) &= N_0\frac{4\pi}{Z_1Z_2\alpha_{em}} \times U(b,p) \ , \\
U(b,p) &= \int e^{ip\cdot x} E\cdot B d^4x \ , \\
N_0 &= \frac{(2\pi)^7}{i} \ .
\end{aligned} \quad (7.2.1)$$

where b denotes impact parameter. The formula generalizes the classical formula for the cross section of Coulomb scattering. In the calculation of the total cross section one must introduce some cutoff radii and the presence of the volume factor V brings in the cutoff volume explicitly (particle in the box description for lepto-pions). Obviously the cutoff length must be longer than lepto-pion Compton length. Normalization factor N_0 has been introduced in order to extract out large powers of 2π.

From this one obtains differential cross section as

$$\begin{aligned}
d\sigma &= P 2\pi b db \ , \\
P &= \int K|A(b,p)|^2 V\frac{d^3p}{(2\pi)^3} \ , \ .
\end{aligned} \quad (7.2.1)$$

The first objection is the need to explicitly introduce the reaction volume: this obviously breaks manifest Lorentz invariance. The cross section was estimated in the earlier version of the model [16] and turned to be too small by several orders of magnitude. This inspired the idea that constructive interference for the production amplitudes for different values of impact parameter could increase the cross section.

7.2.9 Quantum model for lepto-pion production

There are good reasons for considering the quantum model. First, the lepto-pion production cross section is by several orders of magnitude too small in classical model. Secondly, in Th-Th collisions there are indications about the presence of two velocity peaks with separation $\delta\beta/\beta \sim 10^{-2}$ [36] and this suggests that quantum mechanical diffraction effects might be in question. These effects could come from the upper and/or lower length scale cutoff and from the delocalization of the wave function of incoming nucleus.

The question is what quantum model means. The most natural thing is to start from Coulomb scattering and multiply Coulomb scattering amplitude for a given impact parameter value b with the amplitude for lepto-pion production. This because the classical differential cross section given by $2\pi b db$ in Coulomb scattering equals to the quantum cross section. One might however argue that on basis of $S = 1+T$ decomposition of S-matrix the lowest order contribution to lepto-pion production in quantum situation corresponds to the absence of any scattering. The lepto-pion production amplitude is indeed non-vanishing also for the free motion of nuclei. The resolution of what looks like a paradox could come from many-sheeted space-time concept: if no scattering occurs, the space-time sheets representing colliding nuclei do not touch and all and there is no interference of em fields so that there is no lepto-pion production. It turns however that lowest order contribution indeed corresponds to the absence of scattering in the model that works.

Two possible approaches

One can imagine two approaches to the construction of the model for production amplitude in quantum case.

The first approach is based on eikonal approximation [61]. Eikonal approximation applies at high energy limit when the scattering angle is small and one can approximate the orbit of the projectile with a straight orbit.

The expression for the scattering amplitude in eikonal approximation reads as

$$\begin{aligned} f(\theta,\phi) &= \frac{k}{2\pi i}\int d^2b exp(-ik\cdot b)exp(i\xi(b))-1) \ , \\ \xi(b) &= \frac{-m}{k\hbar^2}\int_{z=-\infty}^{z=\infty} dzV(z,b) \ , \\ \frac{d\sigma}{d\Omega} &= |f^2| \ . \end{aligned} \tag{7.2.0}$$

as one expands the exponential in lowest in spherically symmetric potential order one obtains the

$$f(\theta,\phi) \simeq -\frac{m}{2\pi\hbar^2}\int J_0(k_Tb)\xi(b)bdb \ . \tag{7.2.0}$$

The challenge is to find whether it is possible to generalize this expression so that it applies to the production of lepto-pions.

1. The simplest guess is that one should multiply the eikonal amplitude with the dimensionless amplitude $A(b)$:

$$\begin{aligned} f(\theta,\phi) &\rightarrow f(\theta,\phi,p) = \frac{k}{2\pi i}\int d^2b exp(-ik\cdot b)exp(i\xi(b))-1)A(b,p) \\ &\simeq -\frac{m}{2\pi\hbar^2}\int J_0(k_Tb)\xi(b)A(b,p)bdb \ . \end{aligned} \tag{7.2.0}$$

2. Amplitude squared must give differential cross section for lepto-pion production and scattering

$$\begin{aligned} d\sigma &= |f(\theta,\phi,p)|^2 d\Omega d^3n \ , \\ d^3n &= Vd^3p \ . \end{aligned} \tag{7.2.0}$$

   This requires an explicit introduction of a volume factor V via a spatial cutoff. This cutoff is necessary for the coordinate z in the case of Coulomb potential, and would have interpretation in terms of a finite spatio-temporal volume in which the space-time sheets of the colliding particles are in contact and fields interfere.

3. There are several objections against this approach. The loss of a manifest relativistic invariance in the density of states factor for lepto-pion does not look nice. One must keep count about the scattering of the projectile which means a considerable complication from the point of view of numerical calculations. In classical picture for orbits the scattering angle in principle is fixed once impact parameter is known so that the introduction of scattering angles does not look logical.

Second approach starts from the classical picture in which each impact parameter corresponds to a definite scattering angle so that the resulting amplitude describes lepto-pion production amplitude and says nothing about the scattering of the projectile. This approach is more in spirit with TGD since classical physics is exact part of quantum TGD and classical orbit is absolutely real from the point of view of lepto-pion production amplitude.

1. The counterpart of the eikonal exponent has interpretation as the exponent of classical action associated with the Coulomb interaction

$$S(b) = \int_\gamma V ds \tag{7.2.1}$$

along the orbit γ of the particle, which can be taken also as a real classical orbit but will be approximated with rectilinear orbit in sequel.

2. The first guess for the production amplitude is

$$\begin{aligned} f(p) &= \int d^2b exp(-i\Delta k(b)\cdot b)exp[\frac{i}{\hbar}S(b)]A(b,p) \\ &= \int J_0(k_T(b)b)(1+\frac{i}{\hbar}\int_{z=-a}^{z=a} dzV(z,b)+..)A(b,p) \ . \end{aligned} \tag{7.2.1}$$

Δk is the change of the momentum in the classical scattering and in the scattering plane. The cutoff $|z|\leq a$ in the longitudinal direction corresponds to a finite imbedding space volume inside which the space-time sheets of target and projectile are in contact.

3. The production amplitude is non-trivial even if the interaction potential vanishes being given by

$$f(p) = \int d^2b exp(-ik\cdot b))A(b,p) = 2\pi int J_0(k_T(b)b)\times A(b,p)bdb \ . \tag{7.2.2}$$

This formula can be seen as a generalization of quantum formula in the sense that incoherent integral over production probabilities at various values of b is replaced by an integral over production amplitude over b so that interference effects become possible.

4. This result could be seen as a problem. On basis of $S = 1+iT$ decomposition corresponding to free motion and genuine interaction, one could argue that since the exponent of action corresponds to S, $A(p,b)$ vanishes when the space-time sheets are not in contact. The improved guess for the amplitude is

$$\begin{aligned} f(p) &= \int d^2b exp(-ik\cdot b)exp(\frac{i}{\hbar}S(b))A(b,p) \\ &= \int J_0(k_T(b)b)(\frac{i}{\hbar}\int_{z=-a}^{z=a} V(z,b)+..)A(b,p) \ . \end{aligned} \tag{7.2.2}$$

This would mean that there would be no classical limit when coherence is assumed to be lost. At this stage one must keep mind open for both options.

5. The dimension of $f(p)$ is $L^2/\hbar$

$$d\sigma = |f(p)|^2\frac{d^3p}{2E_p(2\pi)^3} \ . \tag{7.2.3}$$

has correct dimension. This model will be considered in sequel. The earlier work in [16] was however based on the first option.

Production amplitude

The Fourier transform of $E \cdot B$ can be expressed as a convolution of Fourier transforms of E and B and the resulting expression for the amplitude reduces by residue calculus (see APPENDIX) to the following general form

$$\begin{aligned} A(b,p) &\equiv N_0 \times \frac{4\pi}{Z_1 Z_2 \alpha_{em}} \times U(b,p) = 2\pi i (CUT_1 + CUT_2) \ , \\ N_0 &= \frac{(2\pi)^7}{i} \ . \end{aligned} \tag{7.2.3}$$

where nuclear charges are such that Coulomb potential is $1/r$. The motivation for the strange looking notation is to extract all powers of 2π so that the resulting amplitudes contain only factors of order unity.

The contribution of the first cut for $\phi \in [0, \pi/2]$ is given by the expression

$$\begin{aligned} CUT_1 &= D_1 \times \int_0^{\pi/2} exp(-\frac{b}{b_0} cos(\psi)) A_1 d\psi \ , \\ D_1 &= -\frac{1}{2}\frac{sin(\phi)}{sin(\theta)} \ , \ b_0 = \frac{\hbar}{m}\frac{\beta\gamma}{\gamma_1} \ , \\ A_1 &= \frac{A + iBcos(\psi)}{cos^2(\psi) + 2iCcos(\psi) + D} \ , \\ A &= sin(\theta)cos(\phi) \ , \ B = K \ , \\ C &= K\frac{cos(\phi)}{sin(\theta)} \ , \ D = -sin^2(\phi) - \frac{K^2}{sin^2(\theta)} \ , \\ K &= \beta\gamma(1 - \frac{v_{cm}}{\beta}cos(\theta)) \ , \ v_{cm} = \frac{2v}{1+v^2} \ . \end{aligned} \tag{7.2.-2}$$

The definitions of the various kinematical variables are given in previous formulas. The notation is tailored to express the facts that A_1 is rational function of $cos(\psi)$ and that integrand depends exponentially on the impact parameter.

The expression for CUT_2 reads as

$$\begin{aligned} CUT_2 &= D_2 \times \int_0^{\pi/2} exp(i\frac{b}{b_1} cos(\psi)) A_2 d\psi \ , \\ D_2 &= -\frac{sin(\frac{\phi}{2})}{usin(\theta)} \times exp(-\frac{b}{b_2}) \ , \\ b_1 &= \frac{\hbar}{m}\frac{\beta}{\gamma_1} \ , \ b_2 = \frac{\hbar}{mb}\frac{1}{\gamma_1 \times sin(\theta)cos(\phi)} \\ A_2 &= \frac{Acos(\psi) + B}{cos^2(\psi) + Ccos(\psi) + D} \ , \\ A &= sin(\theta)cos(\phi)u \ , \ B = \frac{w}{v_{cm}} + \frac{v}{\beta}sin^2(\theta)[sin^2(\phi) - cos^2(\phi)] \ , \\ C &= 2i\frac{\beta w}{uv_{cm}}\frac{cos(\phi)}{sin(\theta)} \ , \ D = -\frac{1}{u^2}(\frac{sin^2(\phi)}{\gamma^2} + \beta^2(v^2 sin^2(\theta) - \frac{2vw}{v_{cm}})cos^2(\phi)) \\ &+ \frac{w^2}{v_{cm}^2 u^2 sin^2(\theta)} + 2i\frac{\beta v}{u}sin(\theta)cos(\phi) \ , \\ u &= 1 - \beta v cos(\theta) \ , \ w = 1 - \frac{v_{cm}}{\beta}cos(\theta) \ . \qquad (7.2.\text{-}8) \end{aligned} \tag{7.2.-7}$$

The denominator X_2 has no poles and the contribution of the second cut is therefore always finite. Again the expression is tailored to make clear the functional dependence of the integrand on $cos(\psi)$

and on impact parameter. Besides this the exponential damping makes in non-relativistic situation the integrand small everywhere expect in the vicinity of $cos(\Psi) = 0$ and for small values of the impact parameter.

Using the symmetries

$$\begin{aligned} U(b, p_x, -p_y) &= -U(b, p_x, p_y) \ , \\ U(b, -p_x, -p_y) &= \bar{U}(b, p_x, p_y) \ , \end{aligned} \quad (7.2.\text{-}7)$$

of the amplitude one can calculate the amplitude for other values of ϕ.

CUT_1 gives the singular contribution to the amplitude. The reason is that the factor X_1 appearing in denominator of cut term vanishes, when the conditions

$$\begin{aligned} cos(\theta) &= \frac{\beta}{v_{cm}} \ , \\ sin(\phi) &= cos(\psi) \ , \end{aligned} \quad (7.2.\text{-}7)$$

are satisfied. In forward direction this condition tells that z- component of the lepto-pion momentum in velocity center of mass coordinate system vanishes. In laboratory this condition means that the lepto-pion moves in certain cone defined by the value of its velocity. The condition is possible to satisfy only above the threshold $v_{cm} \geq \beta$.
For $K = 0$ the integral reduces to the form

$$CUT_1 = \frac{1}{2} cos(\phi) sin(\phi) \lim_{\varepsilon \to 0} \frac{\int_0^{\pi/2} exp(-\frac{cos(\psi)}{sin(\phi_0)}) d\psi}{(sin^2(\phi) - cos^2\psi + i\varepsilon)} \ . \quad (7.2.\text{-}7)$$

One can estimate the singular part of the integral by replacing the exponent term with its value at the pole. The integral contains two parts: the first part is principal value integral and second part can be regarded as integral over a small semicircle going around the pole of integrand in upper half plane. The remaining integrations can be performed using elementary calculus and one obtains for the singular cut contribution the approximate expression

$$\begin{aligned} CUT_1 &\simeq e^{-(b/a)(sin(\phi)/sin(\phi_0))} (\frac{ln(X)}{2} + \frac{i\pi}{2}) \ , \\ X &= \frac{((1+s)^{1/2} + (1-s)^{1/2})}{((1+s)^{1/2} - (1-s)^{1/2})} \ , \\ s &= sin(\phi) \ , \\ sin(\phi_0) &= \frac{\beta\gamma}{\gamma_1 m(\pi_L) a} \ . \end{aligned} \quad (7.2.\text{-}9)$$

The principal value contribution to the amplitude diverges logarithmically for $\phi = 0$ and dominates over 'pole' contribution for small values of ϕ. For finite values of impact parameter the amplitude decreases exponentially as a function of ϕ.

If the singular term appearing in CUT_1 indeed gives the dominant contribution to the lepto-pion production one can make some conclusions concerning the properties of the production amplitude. For given lepto-pion cm velocity v_{cm} the production associated with the singular peak is predicted to occur mainly in the cone $cos(\theta) = \beta/v_{cm}$: in forward direction this corresponds to the vanishing of the z-component of the lepto-pion momentum in velocity center of mass frame. Since the values of $sin(\theta)$ are of order .1 the transversal momentum is small and production occurs almost at rest in cm frame as observed. In addition, the singular production cross section is concentrated in the production plane ($\phi = 0$) due to the exponential dependence of the singular production amplitude on the angle ϕ and impact parameter and the presence of the logarithmic singularity. The observed lepto-pion velocities are in the range $\Delta v/v \simeq 0.2$ [36] and this corresponds to the angular width $\Delta\theta \simeq 34$ degrees.

Differential cross section in the quantum model

There are two options to consider depending on whether one uses $exp(iS)$ or $exp(iS)-1$ to define the production amplitude.

1. For the $exp(iS)$ option the expression for the differential cross section reads in the lowest order as

$$
\begin{aligned}
d\sigma &= K|f_B|^2\frac{d^3p}{2E_p} \ , \\
f_B &\simeq i\int exp(-i\Delta k\cdot r)(CUT_1+CUT_2)bdbdzd\phi \ , \\
K &= (Z_1Z_2)^2\alpha_{em}^4N_c^2(\frac{m(\pi_L)}{f(\pi_L)})^2\frac{1}{(2\pi)^{15}} \ .
\end{aligned}
\tag{7.2.-10}
$$

   Here Δk is the momentum exchange in Coulomb scattering and a vector in the scattering plane so that the above described formula is obtained for the linear orbits.

2. For the $exp(iS)-1$ option the differential production cross section for lepto-pion is in the lowest non-trivial approximation for the exponent of action S given by the expression

$$
\begin{aligned}
d\sigma &= K|f_B|^2\frac{d^3p}{2E_p} \ , \\
f_B &\simeq \int exp(-i\Delta k\cdot r)V(z,b)(CUT_1+CUT_2)bdbdzd\phi \ , \\
V(z,b) &= \frac{1}{r} \ , \\
K &= (Z_1Z_2)^4\alpha_{em}^6N_c^2(\frac{m(\pi_L)}{f(\pi_L)})^2\frac{1}{(2\pi)^{15}} \ .
\end{aligned}
\tag{7.2.-12}
$$

   Effectively the Coulomb potential is replaced with the product of the Coulomb potential and lepto-pion production amplitude $A(b,p)$. Since α_{em} is assumed to correspond to relate to its standard value by a scaling $\hbar_0/\hbar$ factor.

3. Coulomb potential brings in an additional $(Z_1Z_2\alpha_{em})^2$ factor to the differential cross section, which in the case of heavy ion scattering increases the contribution to the cross section by a factor of order 3×10^3 but reduces it by a factor of order 5×10^{-5} in the case of proton-antiproton scattering. The increase of $\hbar$ expected to be forced by the requirement that perturbation theory is not lost however reduces the contribution from higher orders in V. It should be possible to distinguish between the two options on basis of these differences.

The scattering amplitude can be reduced to a simpler form by using the defining integral representation

$$J_0(x) = \frac{1}{2\pi}\int_0^{2\pi} exp(-ixsin(\phi))d\phi$$

of Bessel functions.

1. For $exp(iS)$ option this gives

$$
\begin{aligned}
f_B &= 2\pi i\int J_0(\Delta kb)(CUT_1+CUT_2)bdb \ , \\
\Delta k &= 2ksin(\frac{\alpha}{2}) \ , \ \ k = M_R\beta \ , \\
M_R &\simeq A_Rm_p \ , \quad A_R = \frac{A_1A_2}{A_1+A_2} \ ,
\end{aligned}
\tag{7.2.-13}
$$

where the length scale cutoffs in various integrations are not written explicitly. The value of α can be deduced once the value of impact parameter is known in the case of the classical Coulomb scattering.

2. For $exp(iS)-1$ option one has

$$\begin{aligned} f_B &= 2\pi i\int F(b)J_0(\Delta kb)(CUT_1+CUT_2)bdb \ , \\ F(b\geq b_{cr}) &= 2\int dz\frac{1}{\sqrt{z^2+b^2}} = ln(\frac{\sqrt{a^2-b^2}+a}{b}) \ , \\ & . \end{aligned} \tag{7.2.-14}$$

Note that the factors K appearing in the different cross section are different in these to cases.

Calculation of the lepto-pion production amplitude in the quantum model

The details related to the calculation of the production amplitude can be found in appendix and it suffices to describe only the general treatment here. The production amplitude of the quantum model contains integrations over the impact parameter and angle parameter ψ associated with the cut. The integrands appearing in the definition of the contributions CUT_1 and CUT_2 to the scattering amplitude have simple exponential dependence on impact parameter. The function F appearing in the definition of the scattering amplitude is a rather slow varying function as compared to the Bessel function, which allows trigonometric approximation and for small values of scattering angle equals to its value at origin. This motivates the division of the impact parameter range into pieces so that F can approximated with its mean value inside each piece so that integration over cutoff parameters can be performed exactly inside each piece.

In Appendix the explicit expansion in power series with respect to impact parameter is derived by assuming $J_0(k_Tb)\simeq 1$ and $F(b)=F=constant$. These formulas can be easily generalized by assuming a piecewise constancy of these two functions. This means that the only the integration over the lepto-pion phase space must be carried out numerically.

CUT_1 becomes also singular at $cos(\theta)=\beta/v_{cm}$, $cos(\psi)=\sin(\phi)$. The singular contribution of the production amplitude can be extracted by putting $cos(\psi)=sin(\phi)$ in the arguments of the exponent functions appearing in the amplitude so that one obtains a rational function of $cos(\psi)$ and $sin(\psi)$ integrable analytically. The remaining nonsingular contribution can be integrated numerically.

Formula for the production cross section

In the case of heavy ion collisions the rectilinear motion is not an excellent approximation since the anomalous events are observed near Coulomb wall and $\beta\simeq .1$ holds true. Despite this this can be taken as a first approximation.

The expression for the differential cross section for lepto-pion production in heavy ion collisions is given by

$$d\sigma = KF^2|\int(CUT_1+CUT_2)bdb|^2\frac{d^3p}{2E} \ , \tag{7.2.-14}$$

This expression and also the expressions of the integrals of CUT_1 and CUT_2 are calculated explicitly as powers series of the impact parameter in the Appendix.

1. For $exp(iS)$ option one has

$$\begin{aligned} K &= (Z_1Z_2)^2\alpha_{em}^4N_c^2[\frac{m(\pi_L)}{f(\pi_L)}]^2\frac{1}{(2\pi)^{13}} \ , \\ F &= 1 \ . \end{aligned} \tag{7.2.-14}$$

2. For $exp(iS)-1$ option one has

$$
\begin{aligned}
K &= (Z_1Z_2)^4\alpha_{em}^6 N_c^2[\frac{m(\pi_L)}{f(\pi_L)}]^2\frac{1}{(2\pi)^{13}} \ , \\
F &= 2\langle\langle ln(\frac{\sqrt{a^2-b^2}+a}{b})\rangle\rangle \ . \qquad (7.2.-14)
\end{aligned}
$$

In the approximation that F is constant the two lowest order predictions are related by a scaling factor

$$
R = (Z_1Z_2\alpha_{em})^2F^2 \ . \qquad (7.2.-13)
$$

It is interesting to get a rough order of magnitude feeling about the situation assuming that the contributions of CUT_1 and CUT_2 are of order unity. For $Z_1 = Z_2 = 92$ and $m(\pi_L)/f(\pi_L) \simeq 1.5$ -as in the case of ordinary pion- one obtains following results. It must be emphasized that these estimates are extremely sensitive to the over all scaling of f_B and to the choice of the cutoff parameter a and cannot be taken too seriously.

1. From $\beta \simeq .1$ one has $b_0 \simeq .1/m(\pi_L)$. One can argue that the impact parameter cutoff $a = xb_0$ should satisfy $a \geq 1/m_{\pi_L}$ so that $x \geq 10$ should hold true.

2. For $expi(S)-1$ option one has $K = 4.7\times 10^{-6}$. From the classical model the allowed phase space volume is of order $\frac{1}{3}\Delta v^3 \sim 10^{-4}$. By using $a = m(\pi_L)$ as a cutoff and $m(\pi_L) \simeq 2m_e$ one obtains $\sigma \sim 4$ μb, which is of same order of magnitude as the experimental estimate 5 μb.

3. For $exp(iS)$ option one has $K = 1.2\times 10^{-9}$ and the estimate for cross section is 1.1 nb for $a = 1/m(\pi_L)$. A correct order of magnitude is obtained by assuming $a = 5.5/m(\pi_L)$ and that a^4 scaling holds true. At larger values of impact parameter a^2 scaling sets on and would require $a \sim 30/m(\pi_L)$ which would correspond to .36 A and to atomic length scale. It is not possible to distinguish between the two options.

4. The singular contribution near to production plane at the cone $v_{cm}cos(\theta) = \beta$ is expected to enhance the total cross section. The strong sensitivity of the cross section to the choice of the cutoff parameter allows to reproduce the experimental findings easily and it would be important to establish strong bounds on the value of the impact parameter.

Dominating contribution to production cross section and diffractive effects

Consider now the behavior of the dominating singular contribution to the production amplitude at the cone $cos(\theta) = \beta/v_{cm}$ depending on b via the exponent factor . This amplitude factorizes into a product

$$
\begin{aligned}
f_{B,sing} &= K_0a^2B(\Delta k)A_{sing}(b,p) \ , \\
B(\Delta k) &= \int F(ax)J_0(\Delta kax)exp(-\frac{sin(\phi)}{sin(\phi_0)}x)xdx \ , \\
&\sim \sqrt{\frac{2}{\pi\Delta ka}}\int F(ax)cos(\Delta kax-\frac{\pi}{4})exp(-\frac{sin(\phi)}{sin(\phi_0)}x)\sqrt{x}dx \ , \\
x &= \frac{b}{a} \ . \qquad (7.2.-15)
\end{aligned}
$$

The factor $A_{sing}(b,p) \equiv (4\pi/(Z_1Z_2\alpha_{em})U_{sing}(b,p)$ is the analytically calculable singular and dominating part of the lepto-pion production amplitude (see appendix) with the exponential factor excluded. The factor B is responsible for diffractive effects. The contribution of the peak to the total production cross section is of same order of magnitude as the classical production cross section.

At the peak $\phi \sim 0$ the contribution the exponent of the production amplitude is constant at this limit one obtains product of the Fourier transform of Coulomb potential with cutoffs with the production amplitude. One can calculate the Fourier transform of the Coulomb potential analytically to obtain

$$\begin{aligned} f_{B,sing} &\simeq 4\pi K_0 \frac{(cos(\Delta ka) - cos(\Delta kb_{cr}))}{\Delta k^2} CUT_1 \\ \Delta k &= 2\beta sin(\frac{\alpha}{2}) \ . \end{aligned} \quad (7.2.\text{-}15)$$

One obtains oscillatory behavior as a function of the collision velocity in fixed angle scattering and the period of oscillation depends on scattering angle and varies in wide limits.

The relationship between scattering angle α and impact parameter in Coulomb scattering translates the impact parameter cutoffs to the scattering angle cutoffs

$$\begin{aligned} a &= \frac{Z_1 Z_2 \alpha_{em}}{M_R \beta^2} cot(\alpha(min)/2) \ , \\ b_{cr} &= \frac{Z_1 Z_2 \alpha_{em}}{M_R \beta^2} cot(\alpha(max)/2) \ . \end{aligned} \quad (7.2.\text{-}15)$$

This gives for the argument Δkb of the Bessel function at lower and upper cutoffs the approximate expressions

$$\begin{aligned} \Delta ka &\simeq \frac{2Z_1 Z_2 \alpha_{em}}{\beta} \sim \frac{124}{\beta} \ , \\ \Delta kb_{cr} &\simeq x_0 \frac{2Z_1 Z_2 \alpha_{em}}{\beta} \sim \frac{124 x_0}{\beta} \ . \end{aligned} \quad (7.2.\text{-}15)$$

The numerical values are for $Z_1 = Z_2 = 92$ (U-U collision). What is remarkable that the argument Δka at upper momentum cutoff does not depend at all on the value of the cutoff length. The resulting oscillation at minimum scattering angle is more rapid than allowed by the width of the observed peak: $\Delta\beta/\beta \sim 3 \cdot 10^{-3}$ instead of $\Delta\beta/\beta \sim 10^{-2}$: of course, the measured value need not correspond to minimum scattering angle. The oscillation associated with the lower cutoff comes from $cos(2M_R b_{cr} \beta sin(\alpha/2))$ and is slow for small scattering angles $\alpha < 1/A_R \sim 10^{-2}$. For $\alpha(max)$ the oscillation is rapid: $\delta\beta/\beta \sim 10^{-3}$.

In the total production cross section integrated over all scattering angles (or finite angular range) diffractive effects disappear. This might explain why the peak has not been observed in some experiments [36].

Cutoff length scales

Consider next the constraints on the upper cutoff length scale.

1. The production amplitude turns out to decrease exponentially as a function of impact parameter b unless lepto-pion is produced in scattering plane. The contribution of lepto-pions produced in scattering plane however gives divergent contribution to the total cross section integrated over all impact parameter values and upper cutoff length scale a is necessary. If one considers scattering with scattering angle between specified limits this is of course not a problem of classical model.

2. Upper cutoff length scale must be longer than the Compton length of lepto-pion.

3. Upper cutoff length scale a should be certainly smaller than the interatomic distance. For partially ionized atoms a more stringent upper bound for a is the size r of atom defined as the distance above which atom looks essentially neutral: a rough extrapolation from hydrogen atom gives $r \sim a_0/Z^{1/3} \sim 1.5 \cdot 10^{-11}$ m (a_0 is Bohr radius of hydrogen atom). Therefore cutoff scale would be between Bohr radius $a_0/Z \sim .5 \cdot 10^{-12}$ m and r. In the recent case however atoms are completely ionized so that cutoff length scale can be longer. It turns out that 10 A reproduces the empirical estimate for the cross section correctly.

Numerical estimate for the electro-pion production cross section

The numerical estimate for the electro-pion production cross section is carried out for thorium with $(Z = 90, A = 232)$. The value of the collision velocity of the incoming nucleus in the rest frame of the second nucleus is taken as $\beta = .1$. From the width $\delta v/v = .2$ of velocity distribution in the same frame the upper bound $\gamma \leq 1 + \delta$, $\delta \simeq 2 \times 10^{-3}$ for the Lorentz boost factor of electro-pion in cm system is deduced. The cutoff is necessary because energy conservation is not coded to the structure of the model.

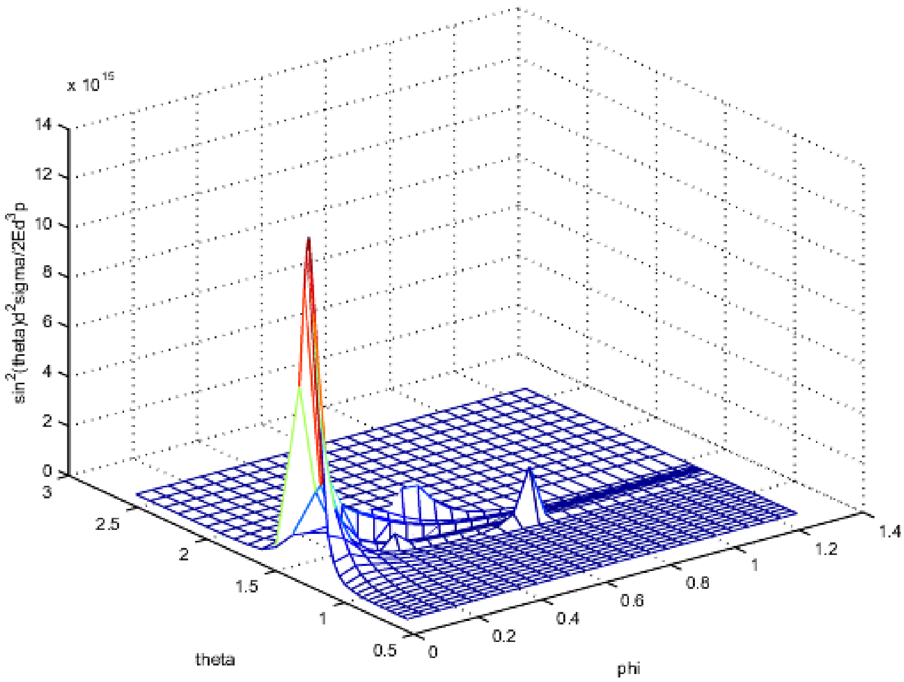

Figure 7.1: Differential cross section $sin^2(\theta) \times \frac{d^2\sigma}{2Ed^3p}$ for τ-pion production for $\gamma_1 = 1.0319 \times 10^3$ in the rest system of antiproton for $\delta = 1.5$. $m(\pi_\tau)$ defines the unit of energy and nb is the unit for cross section. The ranges of θ and ϕ are $(0, \pi)$ and $(0, \pi/2)$.

As expected, the singular contribution from the cone $v_{cm}cos(\theta) = \beta$, $v_{cm} = 2v/(1 + v^2)$ gives the dominating contribution to the cross section. This contribution is proportional to the value of b^2_{max} at the limit $\phi = 0$. Cutoff radius is taken to be $b_{max} = 150 \times \gamma_{cm}\hbar/m(\pi_e) = 1.04$ A. The numerical estimate for the cross section using the parameter values listed comes out as $\sigma = 5.6$ μb to be compared with the rough experimental estimate of about 5 μb. The interpretation would be that the space-time sheet associated with colliding nuclei during the collision has this transversal size in cm system. At this space-time sheet the electric and magnetic fields of the nuclei interfere.

From this one can cautiously conclude that lepto-pion model is consistent with both electro-pion production and τ-pion production in proton antiproton collisions. One can of course criticize the large value of impact parameter and a good justification for 1 Angstrom should be found. One could also worry about the singular character of the amplitude making the integration of total cross section somewhat risky business using the rather meager numerical facilities available. The rigorous method to calculate the contribution near the singularity relies on stepwise halving of the increment $\Delta\theta$ as one approaches the singularity. The calculation gives essentially the same result as that with constant value of $\Delta\theta$. Hence it seems that one can trust on the result of calculation.

Figure 2. gives the differential production cross section for $\gamma_1 = 1.0319$. Obviously the differential cross section is strongly concentrated at the cone due to singularity of the production amplitude for fixed b.

The important conclusion is that the same model can reproduce the value of production cross section for both electro-pions explaining the old electron-positron anomaly of heavy ion collisions and τ-pions explaining the CDF anomaly of proton-antiproton collisions at cm energy $\sqrt{s} = 1.96$ TeV (to be discussed later) with essentially same and rather reasonable assumptions (do not however forget the large maximal value of the impact parameter!).

In the case of electro-pions one must notice that depending on situation the final states are gamma pairs for the electron-pion with mass very nearly equal to electron mass. In the case of neutral tau-pion the strong decay to three p-adically scaled down versions of τ-pion proceeds faster or at least rate comparable to that for the decay to gamma pair. For higher mass variants of electro-pion for which there is evidence (for instance, one with mass 1.6 MeV) the final states are dominated by electron-positron pairs. This is true if the primary decay products are electro-baryons of form (say) $e_{ex} = e_8\nu_8\nu_{c,8}$ resulting via electro-strong decays instead of electrons and having slightly larger mass than electron. Otherwise the decay to gamma pair would dominate also the decays of higher mass states. A small magnetic moment type coupling between e, e_{ex} and electro-gluon field made possible by the color octet character of colored leptons induces the mixing of e and e_{ex} so that e_{ex} can transform to e by the emission of photon. The anomalous magnetic moment of electron poses restrictions on the color magnetic coupling.

$e^+_{ex}e^-_{ex}$ pairs from lepto-pions or e^+e^- pairs from lepto-sigmas?

If one assumes that anomalous e^+e^- pairs correspond to lepto-nucleon pairs, then lepto-pion production cross section gives a direct estimate for the production rate of e^+e^- pairs. The results of the table 3 show that in case of 1.8 MeV state, the predicted cross section is roughly by a factor 5 smaller than the experimental upper bound for the cross section. Since this lepto-pion state is rather massive, positron decay width allows smaller $f(\pi_L)$ in this case and the production cross section could be larger than the estimate used by the $1/f(\pi_L)^2$ proportionality of the cross section. Both the simplicity and predictive power of this option and the satisfactory agreement with the experimental data suggest that this option provides the most plausible explanation of the anomalous e^+e^- pairs.

N	$Op/10^{-3}$	$\Gamma(\pi_L)/keV$	$\sigma(\pi_L)/\mu b$	$\sigma(\pi_L)/\mu b$
			$a = .01$	$a = .1$
1	1	.51	.13	1.4
3	1	.13	.04	.41
3	5	.73	.19	2.1

Table 2. The table summarizes lepto-pion lifetime and the upper bounds for lepto-pion (and lepto-nucleon pair) production cross sections for the lightest lepto-pion. N refers to the number of lepto-pion states and $Op = \Delta\Gamma/\Gamma$ refers to ortopositronium decay anomaly. The values of upper cutoff length a are in units of 10^{-10} m.

If one assumes that anomalous e^+e^- pairs result from the decays of lepto-sigmas, the value of e^+e^- production cross section can be estimated as follows. e^+e^- pairs are produced from via the creation of $\sigma_L\pi_L$ pairs from vacuum and subsequent decay σ_L to e^+e^- pairs. The estimate for (or rather for the upper bound of) $\pi_L\sigma_L$ production cross section is obtained as

$$\begin{aligned} \sigma(e^+e^-) &\simeq X\sigma(\pi_L) \ , \\ X &= \frac{V_2}{V_1}(\frac{km_{\sigma_L}}{m^2_{\pi_L}})^2 \ , \\ \frac{V_2}{V_1} &= V_{rel} = \frac{v^3_{12}}{3(2\pi)^2} \sim 1.1\cdot 10^{-5} \ , \\ \frac{k}{m_{pi_L}} &= \frac{(m^2_\sigma - m^2_{\pi_L})}{2m_{\pi_L}f(\pi_L)} \ . \end{aligned} \tag{7.2.-17}$$

Here V_2/V_1 of two-particle and single particle phase space volumes. V_2 is in good approximation the product $V_1(cm)V_1(rel)$ of single particle phase space volumes associated with cm coordinate and relative coordinate and one has $V_2/V_1 \sim V_{rel} = \frac{v_{12}^3}{3(2\pi)^2)} \simeq 1.1 \cdot 10^{-5}$ if the maximum value of the relative velocity is $v_{12} \sim .1$.

Situation is partially saved by the anomalously large value of $\sigma_L \pi_L \pi_L$ coupling constant k appearing in the production vertex $k\sigma_L \pi_L \pi_L(class)$. Production cross section is very sensitive to the value of $f(\pi_L)$ and Op anomaly $\Delta\Gamma/\Gamma = 5 \cdot 10^{-3}$ gives upper bound 2 $\mu b/N_c^2$ for $a = 10^{-11}$ m, which is considerably smaller than the experimental upper bound 5 μb. The huge value of the $g(\pi_L, \pi_L, \sigma_L)$ and $g(\sigma_L, \sigma_L, \sigma_L)$, however implies that radiative corrections to the cross section given by σ exchange are much larger than the lowest order contribution to the cross section! If this is the case then lepto-sigma option might survive but perturbative approach probably would not make sense. On the other hand, one could argue that sigma model action should be regarded as an effective action giving only tree diagrams so that radiative corrections cannot save the situation. There are also purely physical counter arguments against lepto-sigma option: hadronic physics experience suggests that the mass of lepto-sigma is much larger than lepto-pion mass so that lepto-sigma becomes very wide resonance decaying strongly and having negligibly small branching ratio to e^+e^- pairs.

It must be emphasized that the estimates are very rough (the replacement of the integral over the angle α with rough upper bound, estimate for the phase space volume, the values of cutoff radii, the neglect of the velocity dependence of the production cross section, the estimate for the minimum scattering angle, ...). Also the measured production cross section is subject to considerable uncertainties (even the issue whether or not anomalous pairs are produced is not yet completely settled!).

Summary

The usefulness of the modeling lepto-pion production is that the knowledge of lepto-pion production rate makes it possible to estimate also the production rates for other lepto-hadrons and even for many particle states consisting of lepto-hadrons using some effective action describing the strong interactions between lepto-hadrons. One can consider two basic models for lepto-pion production. The models contain no free parameters unless one regards cutoff length scales as such. Classical model predicts the singular production characteristics of lepto-pion. Quantum model predicts several velocity peaks at fixed scattering angle and the distance between the peaks of the production cross section depends sensitively on the value of the scattering angle. Production cross section depends sensitively on the value of the scattering angle for a fixed collision velocity. In both models the reduction of the lepto-pion production rate above Coulomb wall could be understood as a threshold effect: for the collisions with impact parameter smaller than two times nuclear radius, the production amplitude becomes very small since $E \cdot B$ is more or less random for these collisions in the interaction region. The effect is visible for fixed sufficiently large scattering angle only. The value of the anomalous e^+e^- production cross section is of nearly the observed order of magnitude provided that e^+e^- pairs are actually lepto-nucleon pairs originating from the decays of the lepto-pions. Alternative mechanism, in which anomalous pairs originate from the creation of $\sigma_L \pi_L$ pairs from vacuum followed by the decay $\sigma_L \rightarrow e^+e^-$ gives too small production cross section by a factor of order $1/N_c^2$ in lowest order calculation. This alternative works only provided that radiative corrections give the dominant contribution to the production rate of $\pi_L \sigma_L$ pairs as is the case if $\pi_L \sigma_L$ mass difference is of order ten per cent. The existence of at least three colored leptons and family replication provide the most plausible explanation the appearance of several peaks.

The proposed models are certainly over idealizations: in particular the approximation that nuclear motion is free motion fails for those values of the impact parameter, which are most important in the classical model. To improve the models one should calculate the Fourier transform of $E \cdot B$ using the fields of nuclei for classical orbits in Coulomb field rather than free motion. The second improvement is related to the more precise modelling of the situation at length scales below b_{cr}, where nuclei do not behave like point like charges. A peculiar feature of the model from the point of view of standard physics is the appearance of the classical electromagnetic fields associated with the classical orbits of the colliding nuclei in the definition of the quantum model. This is in spirit with Quantum TGD: Quantum TGD associates a unique space-time surface (classical history) to a given 3-surface (counterpart of quantum state).

7.3 Further developments

This section represents further developments of leptohadron model which have emerged during years after the first version of the model published in International Journal of Theoretical Physics.

7.3.1 How to observe leptonic color?

The most obvious argument against lepto-hadrons is that their production via the decay of virtual photons to lepto-mesons has not been observed in hadronic collisions. The argument is wrong. Anomalously large production of low energy e^+e^- pairs [21, 22, 23, 24] in hadronic collisions has been actually observed. The most natural source for photons and e^+e^- pairs are lepto-hadrons. There are two possibilities for the basic production mechanism.

1. Colored leptons result directly from the decay of hadronic gluons. Internal consistency excludes this alternative.
2. Colored leptons result from the decay of virtual photons. This hypothesis is in accordance with the general idea that the QCD:s associated with different condensate levels of p-adic topological condensate do not communicate. More precisely, in TGD framework leptons and quarks correspond to different chiralities of configuration space spinors: this implies that baryon and lepton numbers are conserved exactly and therefore the stability of proton. In particular, leptons and quarks correspond to different Kac Moody representations: important difference as compared with typical unified theory, where leptons and quarks share common multiplets of the unifying group. The special feature of TGD is that there are several gluons since it is possible to associate to each Kac-Moody representation gluons, which are "irreducible" in the sense that they couple only to a single Kac Moody representation. It is clear that if the physical gluons are "irreducible" the world separates into different Kac Moody representations having their own color interactions and communicating only via electro-weak and gravitational interactions. In particular, no strong interactions between leptons and hadrons occur. Since colored lepton corresponds to colored ground state of Kac-Moody representations the gluonic color coupling between ordinary lepton and colored lepton vanishes.

If this picture is correct then lepto-hadrons are produced only via the ordinary electro-weak interactions: at higher energies via the decay of virtual photon to colored lepton pair and at low energies via the emission of lepto-pion by photon. Consider next various manners to observe the effects of lepton color.

1. Resonance structure in the photon-photon scattering and energy near lepto-pion mass is a unique signature of lepto-pion.
2. The production of lepto-mesons in strong classical electromagetic fields (of nuclei, for example) is one possibility. There are several important constraints for the production of lepto-pions in this kind of situation.

   i) The scalar product $E \cdot B$ must be large. Faraway from the source region this scalar product tends to vanish: consider only Coulomb field.

   ii) The region, where $E \cdot B$ has considerable size cannot be too small as compared with lepto-pion de Broglie wavelength (large when compared with the size of nuclei for example). If this condition doesn't hold true the plane wave appearing in Fourier amplitude is essentially constant spatially and since the fields are approximately static the Fourier component of $E \cdot B$ is expressible as a spatial divergence, which reduces to a surface integral over a surface faraway from the source region. Resulting amplitude is small since fields in faraway region have essentially vanishing $E \cdot B$.

   iii) If fields are exactly static, then energy conservation prohibits lepto-hadron production.
3. Also the production of $e^+_{ex}e^-_{ex}$ and $e^+e^-_{ex}$ pairs in nuclear electromagnetic fields with nonvanishing $E \cdot B$ is possible either directly or as decay products of lepto-pions. In thedirect production, the predicted cross section is small due to the presence of two-particle phase space factor. One signature of e^-_{ex} is emission line accompanying the decay $e^-_{ex} \rightarrow e^- + \gamma$. The collisions of nuclei in highly ionized (perhaps astrophysical) plasmas provide a possible source of leptobaryons.

4. The interaction of quantized em field with classical electromagnetic fields is one experimental arrangement to come into mind. The simplest arrangement consisting of linearly polarized photons with energy near lepto-pion mass plus constant classical em field does not however work. The direct production of $\pi_L - \gamma$ pairs in rapidly varying classical electromagnetic field with frequency near lepto-pion mass is perhaps a more realistic possibility . An interesting possibility is that violent collisions inside astrophysical objects could lead to gamma ray bursts via the production of pions and lepto-pions in rapidly varying classical E and B fields.

5. In the collisions of hadrons, virtual photon produced in collision can decay to lepto-hadrons, which in turn produce lepto-pions decaying to lepto-nucleon pairs. As already noticed, anomalous production of low energy e^+e^- pairs (actually lepto-nucleon pairs!) [21] in hadronic collisions has been observed.

6. $e-\nu_e$ and $e-\bar{\nu}_e$ scattering at energies below one MeV provide a unique signature of lepto-pion. In $e-\bar{\nu}_e$ scattering π_L appears as resonance.

7. If leptonic color coupling strength has sufficiently small value in the energy range at which lepto-hadronic QCD exists, e^+e^- annihilation at energies above few MeV should produce colored pairs and lepto-hadronic counterparts of the hadron jets should be observed. The fact that nothing like this has been observed, suggests that lepto-hadronic coupling constant evolution does not allow the perturbative QCD phase.

7.3.2 New experimental evidence

After writing this chapter astrophysical support for the notion of lepto-pions has appeared. There is also experimental evidence for the existence of colored muons

Could lepto-hadrons correspond to dark matter?

The proposed identification of cosmic strings (in TGD sense) as the ultimate source of both visible and dark matter discussed in [D5] does not exclude the possibility that a considerable portion of topologically condensed cosmic strings have decayed to some light particles. In particular, this could be the situation in the galactic nuclei.

The idea that lepto-hadrons might have something to do with the dark matter has popped up now and then during the last decade but for some reason I have not taken it seriously. Situation changed towards the end of the year 2003. There exist now detailed maps of the dark matter in the center of galaxy and it has been found that the density of dark matter correlates strongly with the intensity of monochromatic photons with energy equal to the rest mass of electron [38].

The only explanation for the radiation is that some yet unidentified particle of mass very nearly equal to $2m_e$ decays to an electron positron pair. Electron and positron are almost at rest and this implies a high rate for the annihilation to a pair of gamma rays. A natural identification for the particle in question would be as a lepto-pion (or rather, electro-pion). By their low mass lepto-pions, just like ordinary pions, would be produced in high abundance, in lepto-hadronic strong reactions and therefore the intensity of the monochromatic photons resulting in their decays would serve as a measure for the density of the lepto-hadronic matter. Also the presence of lepto-pionic condensates can be considered.

These findings force to take seriously the identification of the dark matter as lepto-hadrons. This is however not the only possibility. The TGD based model for tetra-neutrons discussed in [F8] is based on the hypothesis that mesons made of scaled down versions of quarks corresponding to Mersenne prime M_{127} (ordinary quarks correspond to $k = 107$) and having masses around one MeV could correspond to the color electric flux tubes binding the neutrons to form a tetra-neutron. The same force would be also relevant for the understanding of alpha particles.

There are also good theoretical arguments for why lepto-hadrons should be dark matter in the sense of having a non-standard value of Planck constant.

1. Since particles with different Planck constant correspond to different pages of the book like structure defining the generalization of the imbedding space, the decays of intermediate gauge bosons to colored excitations of leptons would not occur and would thus not contribute to their decay widths.

2. In the case of electro-pions the large value of the coupling parameter $Z_1Z_2\alpha_{em} > 1$ combined with the hypothesis that a phase transition increasing Planck constant occurs as perturbative QFT like description fails would predict that electro-pions represent dark matter. Indeed, the power series expansion of the $exp(iS)$ term might well fail to converge in this case since S is proportional to Z_1Z_2. For τ-pion production one has $Z_1 = -Z_2 = 1$ and in this case one can consider also the possibility that τ-pions are not dark in the sense of having large Planck constant. Contrary to the original expectations darkness does not affect the lowest order prediction for the production cross section of lepto-pion.

The proposed identification raises several questions.

1. Why the ratio of the lepto-hadronic mass density to the mass density of the ordinary hadrons would be so high, of order 7? Could an entire hierarchy of asymptotically non-free QCDs be responsible for the dark matter so that lepto-hadrons would explain only a small portion of the dark matter?

2. Under what conditions one can regard lepto-hadronic matter as a dark matter? Could short life-times of lepto-hadrons make them effectively dark matter in the sense that there would be no stable enough atom like structures consisting of say charged lepto-baryons bound electromagnetically to the ordinary nuclei or electrons? But what would be the mechanism producing lepto-hadrons in this case (nuclear collisions produce lepto-pions only under very special conditions)?

3. What would be the role of the many-sheeted space-time: could lepto-hadrons and atomic nuclei reside at different space-time sheets so that lepto-baryons could be long-lived? Could dark matter quite generally correspond to the matter at different space-time sheets and thus serve as a direct signature of the many-sheeted space-time topology?

Experimental evidence for colored muons

Also μ and τ should possess colored excitations. About fifteen years after this prediction was made. Direct experimental evidence for these states finally emerges (the year I am adding this comment is 2007) [16, 17]. The mass of the new particle, which is either scalar or pseudoscalar, is 214.4 MeV whereas muon mass is 105.6 MeV. The mass is about 1.5 per cent higher than two times muon mass. The proposed interpretation is as a light Higgs. I do not immediately resonate with this interpretation although p-adically scaled up variants of also Higgs bosons live happily in the fractal Universe of TGD. The most natural TGD inspired interpretation is as a pion like bound state of colored excitations of muon completely analogous to lepto-pion (or rather, electro-pion).

Scaled up variants of QCD appear also in nuclear string model [F8, F9], where scaled variant of QCD for exotic quarks in p-adic length scale of electron is responsible for the binding of ^4He nuclei to nuclear strings. One cannot exclude the possibility that the fermion and anti-fermion at the ends of color flux tubes connecting nucleons are actually colored leptons although the working hypothesis is that they are exotic quark and anti-quark. One can of course also turn around the argument: could it be that lepto-pions are "lepto-nuclei", that is bound states of ordinary leptons bound by color flux tubes for a QCD in length scale considerably shorter than the p-adic length scale of lepton.

7.3.3 Evidence for τ-hadrons

The evidence for τ-leptons came in somewhat funny but very pleasant manner. During my friday morning blog walk, the day next to my birthday October 30, I found that Peter Woit had told in his blog about a possible discovery of a new long-lived particle by CDF experiment [55] emphasizing how revolutionary finding is if it is real. There is a detailed paper [59] with title *Study of multi-muon events produced in p-pbar collisions at* $\sqrt(s) = 1.96\ TeV$ by CDF collaboration added to the ArXiv October 29 - the eve of my birthday. I got even second gift posted to arXiv the very same day and reporting an anomalously high abundance of positrons in cosmic ray radiation [48]. Both of these article give support for basic predictions of TGD differentiating between TGD and standard model and its generalizations.

The first gift

A brief summary of Peter Woit about the finding gives good idea about what is involved.

The article originates in studies designed to determine the b-bbar cross-section by looking for events, where a b-bbar pair is produced, each component of the pair decaying into a muon. The b-quark lifetime is of order a picosecond, so b-quarks travel a millimeter or so before decaying. The tracks from these decays can be reconstructed using the inner silicon detectors surrounding the beam-pipe, which has a radius of 1.5 cm. They can be characterized by their impact parameter, the closest distance between the extrapolated track and the primary interaction vertex, in the plane transverse to the beam.

If one looks at events where the b-quark vertices are directly reconstructed, fitting a secondary vertex, the cross-section for b-bbar production comes out about as expected. On the other hand, if one just tries to identify b-quarks by their semi-leptonic decays, one gets a value for the b-bbar cross-section that is too large by a factor of two. In the second case, presumably there is some background being misidentified as b-bbar production.

The new result is based on a study of this background using a sample of events containing two muons, varying the tightness of the requirements on observed tracks in the layers of the silicon detector. The background being searched for should appear as the requirements are loosened. It turns out that such events seem to contain an anomalous component with unexpected properties that disagree with those of the known possible sources of background. The number of these anomalous events is large (tens of thousands), so this cannot just be a statistical fluctuation.

One of the anomalous properties of these events is that they contain tracks with large impact parameters, of order a centimeter rather than the hundreds of microns characteristic of b-quark decays. Fitting this tail by an exponential, one gets what one would expect to see from the decay of a new, unknown particle with a lifetime of about 20 picoseconds. These events have further unusual properties, including an anomalously high number of additional muons in small angular cones about the primary ones.

The lifetime is estimated to be considerably longer than b quark life time and below the lifetime 89.5 ps of $K_{0,s}$ mesons. The fit to the tail of "ghost" muons gives the estimate of 20 picoseconds.

The second gift

In October 29 also another remarkable paper [48] had appeared in arXiv. It was titled *Observation of an anomalous positron abundance in the cosmic radiation.* PAMELA collaboration finds an excess of cosmic ray positron at energies 10 → 50 GeV. PAMELA anomaly is discussed in Resonaances blog [56]. ATIC collaboration in turn sees an excess of electrons and positrons going all the way up to energies of order 500-800 GeV [49].

Also Peter Woit refers to these cosmic ray anomalies and also to the article *LHC Signals for a SuperUnified Theory of Dark Matter* by Nima Arkadi-Hamed and Neal Weiner [50], where a model of dark matter inspired by these anomalies is proposed together with a prediction of lepton jets with invariant masses with mass scale of order GeV. The model assumes a new gauge interaction for dark matter particles with Higgs and gauge boson masses around GeV. The prediction is that LHC should detect "lepton jets" with smaller angular separations and GeV scale invariant masses.

Explanation of the CDF anomaly

Consider first the CDF anomaly. TGD predicts a fractal hierarchy of QCD type physics. In particular, colored excitations of leptons are predicted to exist. Neutral lepto-pions would have mass only slightly above two times the charged lepton mass. Also charged lepto-pions are predicts and their masses depend on what is the p-adic mass scale of neutrino and it is not clear whether it is much longer than that for charge colored lepton as in the case of ordinary leptons.

1. There exists a considerable evidence for colored electrons as already found. The anomalous production of electron positron pairs discovered in heavy ion collisions can be understood in terms of decays of electro-pions produced in the strong non-orthogonal electric and magnetic fields created in these collisions. The action determining the production rate would be proportional to the product of the lepto-pion field and highly unique "instanton" action for electromagnetic field determined by anomaly arguments so that the model is highly predictive.

2. Also the .511 MeV emission line [51, 52] from the galactic center can be understood in terms of decays of neutral electro-pions to photon pairs. Electro-pions would reside at magnetic flux tubes of strong galactic magnetic fields. It is also possible that these particles are dark in TGD sense.

3. There is also evidence for colored excitations of muon and muo-pion [16, 17]. Muo-pions could be produced by the same mechanism as electro-pions in high energy collisions of charged particles when strong non-orthogonal magnetic and electric fields are generated.

Also τ-hadrons are possible and CDF anomaly can be understood in terms of a production of higher energy τ-hadrons as the following argument demonstrates.

1. τ-QCD at high energies would produce "lepton jets" just as ordinary QCD. In particular, muon pairs with invariant energy below $2m(\tau) \sim 3.6$ GeV would be produced by the decays of neutral τ-pions. The production of monochromatic gamma ray pairs is predicted to dominate the decays. Note that the space-time sheet associated with both ordinary hadrons and τ lepton correspond to the p-adic prime $M_{107} = 2^{107} - 1$.

2. The model for the production of electro-pions in heavy ion collisions suggests that the production of τ-pions could take place in higher energy collisions of protons generating very strong non-orthogonal magnetic and electric fields. This This would reduce the model to the quantum model for electro-pion production.

3. One can imagine several options for the detailed production mechanism.

   (a) The decay of *virtual* τ-pions created in these fields to pairs of leptobaryons generates lepton jets. Since colored leptons correspond to color octets, lepto-baryons could correspond to states of form LLL or $L\overline{L}L$.

   (b) The option inspired by a blog discussion with Ervin Goldfein is that a coherent state of τ-pions is created first and is then heated to QCD plasma like state producing the lepton jets like in QCD. The linear coupling to $E \cdot B$ defined by em fields of colliding nucleons would be analogous to the coupling of harmonic oscillator to constant force and generate the coherent state.

   (c) The option inspired by CDF model [60] is that a p-adically scaled up variant of *on mass shell* neutral τ-pion having $k = 103$ and 4 times larger mass than $k = 107$ τ-pion is produced and decays to three $k = 105$ τ-pions with $k = 105$ neutral τ-pion in turn decaying to three $k = 107$ τ-pions.

4. The basic characteristics of the anomalous muon pair prediction seems to fit with what one would expect from a jet generating a cascade of τ-pions. Muons with both charges would be produced democratically from neutral τ-pions; the number of muons would be anomalously high; and the invariant masses of muon pairs would be below 3.6 GeV for neutral τ-pions and below 1.8 GeV for charged τ-pions if colored neutrinos are light.

5. The lifetime of 20 ps can be assigned with charged τ-pion decaying weakly only into muon and neutrino. This provides a killer test for the hypothesis. In absence of CKM mixing for colored neutrinos, the decay rate to lepton and its antineutrino is given by

$$\Gamma(\pi_\tau \to L + \overline{\nu}_L) = \frac{G^2 m(L)^2 f^2(\pi)(m(\pi_\tau)^2 - m(L)^2)^2}{4\pi m^3(\pi_\tau)} . \tag{7.3.1}$$

   The parameter $f(\pi_\tau)$ characterizing the coupling of pion to the axial current can be written as $f(\pi_\tau) = r(\pi_\tau)m(\pi_\tau)$. For ordinary pion one has $f(\pi) = 93$ MeV and $r(\pi) = .67$. The decay rate for charged τ-pion is obtained by simple scaling giving

$$\begin{aligned}
\Gamma(\pi_\tau \rightarrow L + \overline{\nu}_L) &= 8x^2u^2y^3(1-z^2)\frac{1}{cos^2(\theta_c)}\Gamma(\pi \rightarrow \mu + \overline{\nu_\mu}) \ , \\
x &= \frac{m(L)}{m(\mu)} \ , \ y = \frac{m(\tau)}{m(\pi)} \ , \ z = \frac{m(L)}{2m(\tau)} \ , \ u = \frac{r(\pi_\tau)}{r(\pi)} \ .
\end{aligned} \tag{7.3.0}$$

If the p-adic mass scale of the colored neutrino is same as for ordinary neutrinos, the mass of charged lepto-pion is in good approximation equal to the mass of τ and the decay rates to τ and electron are for the lack of phase space much slower than to muons so that muons are produced preferentially.

6. For $m(\tau) = 1.8$ GeV and $m(\pi) = .14$ GeV and the same value for f_π as for ordinary pion the lifetime is obtained by scaling from the lifetime of charged pion about 2.6×10^{-8} s. The prediction is 3.31×10^{-12} s to be compared with the experimental estimate about 20×10^{-12} s. $r(\pi_\tau) = .41 r_\pi$ gives a correct prediction. Hence the explanation in terms of τ-pions seems to be rather convincing unless one is willing to believe in really nasty miracles.

7. Neutral τ-pion would decay dominantly to monochromatic pairs of gamma rays. The decay rate is dictated by the product of τ-pion field and "instanton" action, essentially the inner product of electric and magnetic fields and reducing to total divergence of instanton current locally. The rate is given by

$$\begin{aligned}
\Gamma(\pi_\tau \rightarrow \gamma + \gamma) &= \frac{\alpha_{em}^2 m^3(\pi_\tau)}{64\pi^3 f(\pi_\tau)^2} = 2x^{-2}y \times \Gamma(\pi \rightarrow \gamma + \gamma) \ , \\
x &= \frac{f(\pi_\tau)}{m(\pi_\tau)} \ , \ y = \frac{m(\tau)}{m(\pi)} \ .\Gamma(\pi \rightarrow \gamma + \gamma) = 7.37 \ eV \ .
\end{aligned} \tag{7.3.-1}$$

The predicted lifetime is 1.17×10^{-17} seconds.

8. Second decay channel is to lepton pairs, with muon pair production dominating for kinematical reasons. The invariant mass of the pairs is 3.6 GeV of no other particles are produced. Whether the mass of colored neutrino is essentially the same as that of charged lepton or corresponds to the same p-adic scale as the mass of the ordinary neutrino remains an open question. If colored neutrino is light, the invariant mass of muon-neutrino pair is below 1.78 GeV.

PAMELA and ATIC anomalies

TGD predicts also a hierarchy of hadron physics assignable to Mersenne primes. The mass scale of M_{89} hadron physics is by a factor 512 higher than that of ordinary hadron physics. Therefore a very rough estimate for the nucleons of this physics is 512 GeV. This suggest that the decays of M_{89} hadrons are responsible for the anomalous positrons and electrons up to energies 500-800 GeV reported by ATIC collaboration. An equally naive scaling for the mass of pion predicts that M_{89} pion has mass 72 GeV. This could relate to the anomalous cosmic ray positrons in the energy interval 10-50 GeV reported by PAMELA collaboration. Be as it may, the prediction is that M_{89} hadron physics exists and could make itself visible in LHC.

The surprising finding is that positron fraction (the ratio of flux of positrons to the sum of electron and positron fluxes) increases above 10 GeV. If positrons emerge from secondary production during the propagation of cosmic ray-nuclei, this ratio should decrease if only standard physics is be involved with the collisions. This is taken as evidence for the production of electron-positron pairs, possibly in the decays of dark matter particles.

Leptohadron hypothesis predicts that in high energy collisions of charged nuclei with charged particles of matter it is possible to produce also charged electro-pions, which decay to electrons or positrons depending on their charge and produce the electronic counterparts of the jets discovered in CDF. This proposal - and more generally leptohadron hypothesis - could be tested by trying to find whether also electronic jets can be found in proton-proton collisions. They should be present at considerably lower energies than muon jets. I decided to check whether I have said something about this earlier and found that I have noticed years ago that there is evidence for the production of anomalous electron-positron pairs in hadronic reactions [21, 22, 23, 24]: some of it dates back to seventies.

The first guess is that the center of mass energy at which the jet formation begins to make itself visible is in a constant ratio to the mass of charged lepton. From CDF data this ratio satisfies $\sqrt{s}/m_\tau = x < 10^3$. For electro-pions the threshold energy would be around $10^{-3}x \times .5$ GeV and for muo-pions around $10^{-3}x \times 100$ GeV.

Comparison of TGD model with the model of CDF collaboration

Few days after the experimental a theoretical paper by CDF collaboration proposing a phenomenological model for the CDF anomaly appeared in the arXiv [60], and it is interesting to compare the model with TGD based model (or rather, one of them corresponding to the third option mentioned above).

The paper proposes that three new particles are involved. The masses for the particles - christened h_3, h_2, and h_1 - are assumed to be 3.6 GeV, 7.3 GeV, and 15 GeV. h_1 is assumed to be pair produced and decay to h_2 pair decaying to h_3 pair decaying to a τ pair.

h_3 is assumed to have mass 3.6 GeV and life-time of 20×10^{-12} seconds. The mass is same as the TGD based prediction for neutral τ-pion mass, whose lifetime however equals to 1.12×10^{-17} seconds ($\gamma + \gamma$ decay dominates). The correct prediction for the lifetime provides a strong support for the identification of long-lived state as charged τ-pion with mass near τ mass so that the decay to μ and its antineutrino dominates. Hence the model is not consistent with leptohadronic model.

p-Adic length scale hypothesis predicts that allowed mass scales come as powers of $\sqrt{2}$ and these masses indeed come in good approximation as powers of 2. Several p-adic scales appear in low energy hadron physics for quarks and this replaces Gell-Mann formula for low-lying hadron masses. Therefore one can ask whether the proposed masses correspond to neutral tau-pion with $p = M_k = 2^k - 1$, $k = 107$, and its p-adically scaled up variants with $p \simeq 2^k$, $k = 105$, and $k = 103$ (also prime). The prediction for masses would be 3.6 GeV, 7.2 GeV, 14.4 GeV.

This co-incidence cannot of course be taken too seriously since the powers of two in CDF model have a rather mundane origin: they follow from the assumed production mechanism producing 8 τ-leptons from h_1. One can however spend some time by looking whether it could be realized somehow allowing p-adically scaled up variants of τ-pion.

1. The proposed model for the production of muon jets is based on production of k=103 neutral τ-pion (or several of them) having 8 times larger mass than k=107 τ-pion in strong EB background of the colliding proton and antiproton and decaying via strong interactions to k=105 and k=107 τ-pions.

2. The first step would be

$$\pi_\tau^0(103) \rightarrow \pi_\tau^0(105) + \pi_\tau^+(105) + \pi_\tau^-(105) \ .$$

This step is not kinematically possible if masses are obtained by exact scaling and if $m(\pi_\tau^0) < m(pi_\tau^\pm)$ holds true as for ordinary pion. p-Adic mass formulas do not however predict exact scaling. In the case that reaction is not kinematically possible, it must be replaced with a reaction in which second charged k=105 pion is virtual and decays weakly. This option however reduces the rate of the process dramatically and might be excluded.

3. Second step would consist of a scaled variant of the first step

$$\pi_\tau^0(105) \rightarrow \pi_\tau^0(107) + \pi_\tau^+(107) + \pi_\tau^-(107) \ ,$$

where second charged pion also can be virtual and decay weakly, and the weak decays of the $\pi_\tau^\pm(105)$ with mass $2m(\tau)$ to lepton pairs. The rates for these are obtained from previous formulas by scaling.

4. The last step would involve the decays of both charged and neutral $\pi_\tau(107)$. The signature of the mechanism would be anomalous γ pairs with invariant masses $2^k \times m(\tau)$, $k = 1, 2, 3$ coming from the decays of neutral τ-pions.

5. Dimensionless four-pion coupling λ determines the decay rates for neutral τ-pions appearing in the cascade. Rates are proportional to phase space-volumes, which are rather small by kinetic reasons.

The total cross section for producing single lepto-pion can be estimated by using the quantum model for lepto-pion production. Production amplitude is essentially Coulomb scattering amplitude for a given value of the impact parameter b for colliding proton and anti-proton multiplied by the amplitude $U(b, p)$ for producing on mass shell $k = 103$ lepto-pion with given four-momentum in the fields E and B and given essentially by the Fourier transform of $E \cdot B$. The replacement of the motion with free motion should be a good approximation.

UV and IR cutoffs for the impact parameter appear in the model and are identifiable as appropriate p-adic length scales. UV cutoff could correspond to the Compton size of nucleon ($k = 107$) and IR cutoff to the size of the space-time sheets representing topologically quantized electromagnetic fields of colliding nucleons (perhaps $k = 113$ corresponding to nuclear p-adic length scale and size for color magnetic body of constituent quarks or $k = 127$ for the magnetic body of current quarks with mass scale of order MeV). If one has $\hbar/\hbar_0 = 2^7$ one could also guess that the IR cutoff corresponds to the size of dark em space-time sheet equal to $2^7 L(113) = L(127)$ (or $2^7 L(127) = L(141)$), which corresponds to electron's p-adic length scale. These are of course rough guesses.

Quantitatively the jet-likeness of muons means that the additional muons are contained in the cone $\theta < 36.8$ degrees around the initial muon direction. If the decay of $\pi^0_\tau(k)$ can occur to on mass shell $\pi^0_\tau(k+2)$, $k = 103, 105$, it is possible to understand jets as a consequence of the decay kinematics forcing the pions resulting as decay products to be almost at rest.

1. Suppose that the decays to three pions can take place as on mass shell decays so that pions are very nearly at rest. The distribution of decay products $\mu\overline{\nu}$ in the decays of $\pi^\pm(105)$ is spherically symmetric in the rest frame and the energy and momentum of the muon are given by

$$[E, p] = [m(\tau) + \frac{m^2(\mu)}{4m(\tau)}, m(\tau) - \frac{m^2(\mu)}{4m(\tau)}] \ .$$

The boost factor $\gamma = 1/\sqrt{1 - v^2}$ to the rest system of muon is $\gamma = \frac{m(\tau)}{m(\mu)} + \frac{m(\mu)}{4m(\tau)}) \sim 18$.

2. The momentum distribution for μ^+ coming from π^+_τ is spherically symmetric in the rest system of π^+ . In the rest system of μ^- the momentum distribution is non-vanishing only for when the angle θ between the direction of velocity of μ^- is below a maximum value of given by $tan(\theta_{max}) = 1$ corresponding to a situation in which the momentum μ^+ is orthogonal to the momentum of μ^- (the maximum transverse momentum equals to $m(\mu)v\gamma$ and longitudinal momentum becomes $m(\mu)v\gamma$ in the boost). This angle corresponds to 45 degrees and is not too far from 36.8 degrees.

3. At the next step the energy of muons resulting in the decays of $\pi^\pm(103)$

$$[E, p] = [\frac{m(\tau)}{2} + \frac{m^2(\mu)}{2m(\tau)}, \frac{m(\tau)}{2} - \frac{m^2(\mu)}{2m(\tau)}] \ ,$$

and the boost factor is $\gamma_1 = \frac{m(\tau)}{2m(\mu)} + \frac{m(\mu)}{2m(\tau)} \sim 9$. θ_{max} satisfies the condition $tan(\theta_{max}) = \gamma_1 v_1/\gamma v \simeq 1/2$ giving $\theta_{max} \simeq 26.6$ degrees.

If on mass shell decays are not allowed the situation changes since either of the charged pions is off mass shell. In order to obtain similar result the virtual should occur dominantly via states near to on mass shell pion. Since four-pion coupling is just constant, this option does not seem to be realized.

Quantitatively the jet-likeness of muons means that the additional muons are contained in the cone $\theta < 36.8$ degrees around the initial muon direction. If the decay of $\pi^0_\tau(k)$ can occur to on mass shell $\pi^0_\tau(k+2)$, $k = 103, 105$, it is possible to understand jets as a consequence of the decay kinematics forcing the pions resulting as decay products to be almost at rest.

1. Suppose that the decays to three pions can take place as on mass shell decays so that pions are very nearly at rest. The distribution of decay products $\mu\overline{\nu}$ in the decays of $\pi^{\pm}(105)$ is spherically symmetric in the rest frame and the energy and momentum of the muon are given by

$$[E,p] = [m(\tau) + \frac{m^2(\mu)}{4m(\tau)}, m(\tau) - \frac{m^2(\mu)}{4m(\tau)}] \ .$$

The boost factor $\gamma = 1/\sqrt{1-v^2}$ to the rest system of muon is $\gamma = \frac{m(\tau)}{m(\mu)} + \frac{m(\mu)}{4m(\tau} \sim 18$.

2. The momentum distribution for μ^+ coming from π_{τ}^+ is spherically symmetric in the rest system of π^+ . In the rest system of μ^- the momentum distribution is non-vanishing only for when the angle θ between the direction of velocity of μ^- is below a maximum value of given by $tan(\theta_{max}) = 1$ corresponding to a situation in which the momentum μ^+ is orthogonal to the momentum of μ^- (the maximum transverse momentum equals to $m(\mu)v\gamma$ and longitudinal momentum becomes $m(\mu)v\gamma$ in the boost). This angle corresponds to 45 degrees and is not too far from 36.8 degrees.

3. At the next step the energy of muons resulting in the decays of $\pi^{\pm}(103)$

$$[E,p] = [\frac{m(\tau)}{2} + \frac{m^2(\mu)}{2m(\tau)}, \frac{m(\tau)}{2} - \frac{m^2(\mu)}{2m(\tau)}] \ ,$$

and the boost factor is $\gamma_1 = \frac{m(\tau)}{2m(\mu)} + \frac{m(\mu)}{2m(\tau)} \sim 9$. θ_{max} satisfies the condition $tan(\theta_{max}) = \gamma_1 v_1/\gamma v \simeq 1/2$ giving $\theta_{max} \simeq 26.6$ degrees.

If on mass shell decays are not possible, the situation changes since either of the charged pions is off mass shell. In order to obtain similar result the virtual should occur dominantly via states near to on mass shell pion. Since four-pion coupling is just constant, this option does not seem to be realized.

Numerical estimate for the production cross section

The numerical estimate of the cross section involves some delicacies. The model has purely physical cutoffs which must be formulated in a precise manner.

1. Since energy conservation is not coded into the model, some assumption about the maximal τ-pion energy in cm system expressed as a fraction ϵ of proton's center of mass energy is necessary. Maximal fraction corresponds to the condition $m(\pi_\tau) \leq m(\pi_\tau)\gamma_1 \leq \epsilon m_p \gamma_{cm}$ in cm system giving $[m(\pi_\tau)/(m_p\gamma_{cm}) \leq \epsilon \leq 1$. γ_{cm} can be deduced from the center of mass energy of proton as $\gamma_{cm} = \sqrt{s}2m_p$, $\sqrt{s} = 1.96$ TeV. This gives $1.6 \times 10^{-2} < \epsilon < 1$ in a reasonable approximation. It is convenient to parameterize ϵ as

$$\epsilon = (1+\delta) \times \frac{m(\pi_\tau)}{m_p} \times \frac{1}{\gamma_{cm}} \ .$$

The coordinate system in which the calculations are carried out is taken to be the rest system of (say) antiproton so that one must perform a Lorentz boost to obtain upper and lower limits for the velocity of τ-pion in this system. In this system the range of γ_1 is fixed by the maximal cm velocity fixed by ϵ and the upper/lower limit of γ_1 corresponds to a direction parallel/opposite to the velocity of proton.

2. By Lorentz invariance the value of the impact parameter cutoff b_{max} should be expressible in terms τ-pion Compton length and the center of mass energy of the colliding proton and the assumption is that $b_{max} = \gamma_{cm} \times \hbar/m(\pi_\tau)$, where it is assumed $m(\pi_\tau) = 8m(\tau)$. The production cross section does not depend much on the precise choice of the impact parameter cutoff b_{max} unless it is un-physically large in which case b_{max}^2 proportionality is predicted.

The numerical estimate for the production cross section involves some delicacies.

1. The power series expansion of the integral of CUT_1 using partial fraction representation does not converge since that roots $c_\pm$ are very large in the entire integration region. Instead the approximation $A_1 \simeq iBcos(\psi)/D$ simplifying considerably the calculations can be used. Also the value of $b_1 L$ is rather small and one can use stationary phase approximation for CUT_2. It turns out that the contribution of CUT_2 is negligible as compared to that of CUT_1.

2. Since the situation is singular for $\theta = 0$ and $\phi = 0$ and $\phi = \pi/2$ (by symmetry it is enough to calculate the cross section only for this kinematical region), cutoffs

$$\theta \in [\epsilon_1, (1-\epsilon_1)] \times \pi \ , \quad \phi \in [\epsilon_1, (1-\epsilon_1)] \times \pi/2 \ , \quad \epsilon_1 = 10^{-3} \ .$$

   The result of the calculation is not very sensitive to the value of the cutoff.

3. Since the available numerical environment was rather primitive (MATLAB in personal computer), the requirement of a reasonable calculation time restricted the number of intervals in the discretization for the three kinematical variables γ, θ, ϕ to be below $N_{max} = 80$. The result of calculation did not depend appreciably on the number of intervals above $N = 40$ for γ_1 integral and for θ and ϕ integrals even $N = 10$ gave a good estimate.

The calculations were carried for the $exp(iS)$ option since in good approximation the estimate for $exp(iS) - 1$ model is obtained by a simple scaling. $exp(iS)$ model produces a correct order of magnitude for the cross section whereas $exp(iS) - 1$ variant predicts a cross section, which is by several orders of magnitude smaller by downwards α_{em}^2 scaling. As I asked Tommaso Dorigo for an estimate for the production cross section in his first blog posting [58], he mentioned that authors refer to a production cross section is 100 nb which looks to me suspiciously large (too large by three orders of magnitude), when compared with the production rate of muon pairs from b-bbar. $\delta = 1.5$ which corresponds to τ-pion energy 36 GeV gives the estimate $\sigma = 351$ nb. The energy is suspiciously high.

In fact, in the recent blog posting of Tommaso Dorigo [57] a value of order .1 nb for the production cross section was mentioned. Electro-pions in heavy ion collisions are produced almost at rest and one has $\Delta v/v \simeq .2$ giving $\delta = \Delta E/m(\pi) \simeq 2 \times 10^{-3}$. If one believes in fractal scaling, this should be at least the order of magnitude also in the case of τ-pion. This would give the estimate $\sigma = 1$ nb. For $\delta = \Delta E/m(\pi) \simeq 10^{-3}$ a cross section $\sigma = .16$ nb would result.

One must of course take the estimate cautiously but there are reasons to hope that large systematic errors are not present anymore. In any case, the model can explain also the order of magnitude of the production cross section under reasonable assumptions about cutoffs.

Does the production of lepto-pions involve a phase transition increasing Planck constant?

The critical argument of Tommaso Dorigo in his blog inspired an attempt to formulate more precisely the hypothesis $\sqrt{s}/m_\tau > x < 10^3$. This led to the realization that a phase transition increasing Planck constant might happen in the production process as also the model for the production of electro-pions requires.

Suppose that the instanton coupling gives rise to *virtual* neutral lepto-pions which ultimately produce the jets (this is first of the three models that one can imagine). E and B could be associated with the colliding proton and antiproton or quarks.

1. The amplitude for lepto-pion production is essentially Fourier transform of $E \cdot B$, where E and B are the non-orthogonal electric and magnetic fields of the colliding charges. At the level of scales one has $\tau \sim \hbar/E$, where τ is the time during which $E \cdot B$ is large enough during collision and E is the energy scale of the virtual lepto-pion giving rise to the jet.

2. In order to have jets one must have $m(\pi_\tau) << E$. If the scaling law $E \propto \sqrt{s}$ hold true, one indeed has $\sqrt{s}/m(\pi_\tau) > x < 10^3$.

3. If proton and antiproton would move freely, τ would be of the order of the time for proton to move through a distance, which is 2 times the Lorentz contracted radius of proton: $\tau_{free} = 2 \times \sqrt{1-v^2} R_p/v = 2\hbar/E_p$. This would give for the energy scale of virtual τ-pion the estimate $E = \hbar/\tau_{free} = \sqrt{s}/4$. $x = 4$ is certainly quite too small value. Actually $\tau > \tau_{free}$ holds true but one can argue that without new physics the time for the preservation of $E \cdot B$ cannot be by a factor of order 2^8 longer than for free collision.

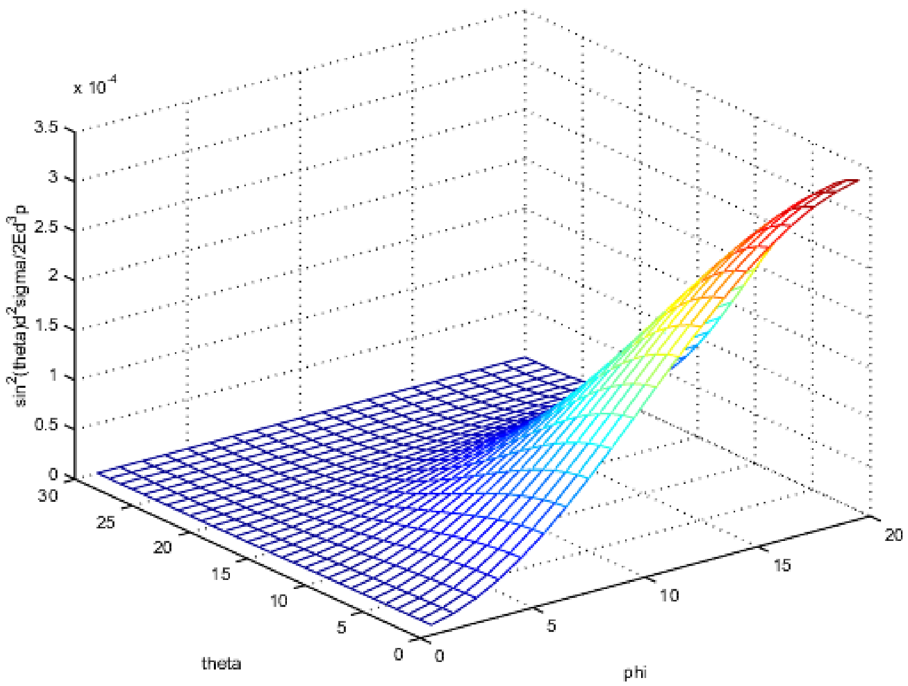

Figure 7.2: Differential cross section $sin^2(\theta) \times \frac{d^2\sigma}{2Ed^3p}$ for τ-pion production for $\gamma_1 = 1.090 \times 10^3$ in the rest system of antiproton for $\delta = 1.5$. $m(\pi_\tau)$ defines the unit of energy and nb is the unit for cross section. The ranges of θ and ϕ are $(0, \pi)$ and $(0, \pi/2)$.

4. For a colliding quark pair one would have $\tau_{free} = 4\hbar/\sqrt{s_{pair}(s)}$, where $\sqrt{s_{pair}(s)}$ would be the typical invariant energy of the pair which is exponentially smaller than $\sqrt{s}$. Somewhat paradoxically from classical physics point of view, the time scale would be much longer for the collision of quarks than that for proton and antiproton.

The possible new physics relates to the possibility that lepto-pions are dark matter in the sense that they have Planck constant larger than the standard value.

1. Suppose that the produced lepto-pions have Planck constant larger than its standard value $\hbar_0$. Originally the idea was that larger value of $\hbar$ would scale up the production cross section. It turned out that this is not the case. For $exp(iS)$ option the lowest order contribution is not affected by the scaling of $\hbar$ and for $exp(iS) - 1$ option the lowest order contribution scales down as $1/hbar^2$. The improved formulation of the model however led to a correct order of magnitude estimates for the production cross section.

2. Assume that a phase transition increasing Planck constant occurs during the collision. Hence τ is scaled up by a factor $y = \hbar/\hbar_0$. The inverse of the lepto-pion mass scale is a natural candidate for the scaled up dark time scale. $\tau(\hbar_0) \sim \tau_{free}$, one obtains $y \sim \sqrt{s_{min}}/4m(\pi_\tau) \leq 2^8$ giving for proton-antiproton option the first guess $\sqrt{s}/m(\pi_\tau) > x < 2^{10}$. If the value of y does not depend on the type of lepto-pion, the proposed estimates for muo- and electro-pion follow.

3. If the fields E and B are associated with colliding quarks, only colliding quark pairs with $\sqrt{s_{pair}(s)} > (>)m(\pi_\tau)$ contribute giving $y_q(s) = \sqrt{s_{pair}(s)/s} \times y$.

If the τ-pions produced in the magnetic field are on-mass shell τ-pions with $k = 113$, the value of $\hbar$ would satisfy $\hbar/\hbar_0 < 2^5$ and $\sqrt{s}/m(\pi_\tau) > x < 2^7$.

Could it have been otherwise?

To sum up, the probability that a correct prediction for the lifetime of the new particle using only known lepton masses and standard formulas for weak decay rates follows by accident is extremely low. Throwing billion times coin and getting the same result every time might be something comparable to this. Therefore my sincere hope is that colleagues would be finally mature to take TGD seriously. If TGD based explanation of the anomalous production of electron positron pairs in heavy ion collisions would have been taken seriously for fifteen years ago, particle physics might look quite different now.

7.3.4 Could lepto-hadrons be replaced with bound states of exotic quarks?

Can one then exclude the possibility that electron-hadrons correspond to colored quarks condensed around $k = 127$ hadronic space-time sheet: that is M_{127} hadron physics? There are several objections against this identification.

1. The recent empirical evidence for the colored counterpart of μ and τ supports the view that colored excitations of leptons are in question.
2. The octet character of color representation makes possible the mixing of leptons with lepto-baryons of form $L\nu_L\overline{\nu}_L$ by color magnetic coupling between lepto-gluons and ordinary and colored lepton. This is essential for understanding the production of electron-positron pairs.
3. In the case CDF anomaly also the assumption that colored variant of τ neutrino is very light is essential. In the case of colored quarks this assumption is not natural.

7.3.5 About the masses of lepto-hadrons

The progress made in understanding of dark matter hierarchy [A9] and non-perturbative aspects of hadron physics [F4, F5] allow to sharpen also the model of lepto-hadrons.

The model for the masses of ordinary hadrons [F4] applies also to the scaled up variants of the hadron physics. The two contributions to the hadron mass correspond to quark contribution and a contribution from super-canonical bosons. For quarks labeled identical p-adic primes mass squared is additive and for quarks labeled by different primes mass is additive. Quark contribution is calculable once the p-adic primes of quarks are fixed.

Super-canonical contribution comes from super-canonical bosons at hadronic space-time sheet labeled by Mersenne prime and is universal if one assumes that the topological mixing of the super-canonical bosons is universal. If this mixing is same as for U type quarks, hadron masses can be reproduced in an excellent approximation if the super-canonical boson content of hadron is assumed to correlate with the net spin of quarks.

In the case of baryons and pion and kaon one must assume the presence of a negative color conformal weight characterizing color binding. The value of this conformal weight is same for all baryons and super-canonical contribution dominates over quark contribution for nucleons. In the case of mesons binding conformal weight can be assumed to vanish for mesons heavier than kaon and one can regard pion and kaon as Golstone bosons in the sense that quark contribution gives the mass of the meson.

This picture generalizes to the case of lepto-hadrons.

1. By the additivity of the mass squared leptonic contribution to lepto-pion mass would be $\sqrt{2}m_e(k)$, where k characterizes the p-adic length scale of colored electron. For $k = 127$ the mass of lepto-pion would would be .702 MeV and too small. For $k = 126$ the mass would be $2m_e = 1.02$ MeV and is very near to the mass of the lepto-pion. Note that for ordinary hadrons quarks can appear in several scaled up variants inside hadrons and the value of k depends on hadron. The prediction for the mass of lepto-ρ would be $m_{\pi_L} + \sqrt{7}m_{127} \simeq 1.62$ MeV ($m_{127} = m_e/\sqrt{5}$).
2. The state consisting of three colored electrons would correspond to leptonic variant of Δ_{++} having charge $q = -3$. The quark contribution to the mass of $\Delta_L \equiv \Delta_{L,3-}$ would be by the additivity of mass squared $\sqrt{3} \times m_e(k = 126) = 1.25$ MeV. If super-canonical particle content is same as for Δ_L, super-canonical contribution would be $m_{SC} = 5 \times m_{127}$, and equal to $m_{SC} = .765$ MeV so that the mass of Δ_L would be $m_{\Delta_L} = 2.34$ MeV. If colored neutrino corresponds to the same p-adic prime as colored electron, also lepto-proton has mass in MeV scale.

7.4 APPENDIX

7.4.1 Evaluation of lepto-pion production amplitude

General form of the integral

The amplitude for lepto-pion production with four momentum

$$\begin{aligned} p &= (p_0,\bar{p}) = m\gamma_1(1, vsin(\theta)cos(\phi), vsin(\theta)sin(\phi), vcos(\theta)) \ , \\ \gamma_1 &= 1/(1-v^2)^{1/2} \ , \end{aligned} \quad (7.4.0)$$

is essentially the Fourier component of the instanton density

$$U(b,p) = \int e^{ip\cdot x} E\cdot B d^4x \quad (7.4.1)$$

associated with the electromagnetic field of the colliding nuclei.

In order to avoid cumbersome numerical factors, it is convenient to introduce the amplitude $A(b,p)$ as

$$\begin{aligned} A(b,p) &= N_0 \times \frac{4\pi}{Z_1 Z_2 \alpha_{em}} \times U(b,p) \ , \\ N_0 &= \frac{(2\pi)^7}{i} \end{aligned} \quad (7.4.1)$$

Coordinates are chosen so that target nucleus is at rest at the origin of coordinates and colliding nucleus moves along positive z direction in $y=0$ plane with velocity β. The orbit is approximated with straight line with impact parameter b.

Instanton density is just the scalar product of the static electric field E of the target nucleus and magnetic field B the magnetic field associated with the colliding nucleus, which is obtained by boosting the Coulomb field of static nucleus to velocity β. The flux lines of the magnetic field rotate around the direction of the velocity of the colliding nucleus so that instanton density is indeed non vanishing.

The Fourier transforms of E and B for nuclear charge 4π (chose for convenience) giving rise to Coulomb potential $1/r$ are given by the expressions

$$\begin{aligned} E_i(k) &= N\delta(k_0)k_i/k^2 \ , \\ B_i(k) &= N\delta(\gamma(k_0-\beta k_z))k_j\varepsilon_{ijz}e^{ik_xb}/((\frac{k_z}{\gamma})^2+k_T^2) \ , \\ N &= \frac{1}{(2\pi)^2} \ . \end{aligned} \quad (7.4.0)$$

The normalization factor corresponds to momentum space integration measure d^4p. The Fourier transform of the instanton density can be expressed as a convolution of the Fourier transforms of E and B.

$$\begin{aligned} A(b,p) &\equiv = N_0N_1\int E(p-k)\cdot B(k)d^4k \ , \\ N_1 &= \frac{1}{(2\pi)^4} \ . \end{aligned} \quad (7.4.0)$$

Where the fields correspond to charges $\pm 4\pi$. In the convolution the presence of two delta functions makes it possible to integrate over k_0 and k_z and the expression for U reduces to a two-fold integral

$$\begin{aligned}
A(b,p) &= \beta\gamma \int dk_x dk_y exp(ikxb)(k_xp_y - k_yp_x)/AB \ , \\
A &= (p_z - \frac{p_0}{\beta})^2 + p_T^2 + k_T^2 - 2k_T \cdot p_T \\
B &= k_T^2 + (\frac{p_0}{\beta\gamma})^2 \ , \\
p_T &= (p_x, p_y) \ . \qquad (7.4.\text{-}2)
\end{aligned}$$

To carry out the remaining integrations one can apply residue calculus.

1. k_y integral is expressed as a sum of two pole contributions
2. k_x integral is expressed as a sum of two pole contributions plus two cut contributions.

k_y-integration

Integration over k_y can be performed by completing the integration contour along real axis to a half circle in upper half plane (see Fig. 7.4.1).

The poles of the integrand come from the two factors A and B in denominator and are given by the expressions

$$\begin{aligned}
k_y^1 &= i(k_x^2 + (\frac{p_0}{\beta\gamma})^2)^{1/2} \ , \\
k_y^2 &= p_y + i((p_z - \frac{p_0}{\beta})^2 + p_x^2 + k_x^2 - 2p_xk_x)^{1/2} \ . \qquad (7.4.\text{-}2)
\end{aligned}$$

One obtains for the amplitude an expression as a sum of two terms

$$A(b,p) = 2\pi i \int e^{ik_xb}(U_1 + U_2)dk_x \ , \qquad (7.4.\text{-}1)$$

corresponding to two poles in upper half plane.

The explicit expression for the first term is given by

$$\begin{aligned}
U_1 &= RE_1 + iIM_1 \ , \\
RE_1 &= (k_x\frac{p_0}{\beta}y - p_xre_1/2)/(re_1^2 + im_1^2) \ , \\
IM_1 &= (-k_xp_yre_1/2K_1^{1/2} - p_xp_yK_1^{1/2})/(re_1^2 + im_1^2) \ , \\
re_1 &= (p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2 - 2p_xk_x \ , \\
im_1 &= -2K_1^{1/2}p_y \ , \\
K_1 &= k_x^2 + (\frac{p_0}{\beta\gamma})^2 \ . \qquad (7.4.\text{-}5)
\end{aligned}$$

The expression for the second term is given by

$$\begin{aligned}
U_2 &= RE_2 + iIM_2 \ , \\
RE_2 &= -((k_xp_y - p_xp_y)p_y + p_xre_2/2)/(re_2^2 + im_2^2) \ , \\
IM_2 &= (-(k_xp_y - p_xp_y)re_2/2K_2^{1/2} + p_xp_yK_2^{1/2})/(re_2^2 + im_2^2) \ , \\
re_2 &= -(p_z - \frac{p_0}{\beta})^2 + (\frac{p_0}{\beta\gamma})^2 + 2p_xk_x + \frac{p_0}{\beta}y - \frac{p_0}{\beta}x \ , \\
im_2 &= 2p_yK_2^{1/2} \ , \\
K_2 &= (p_z - \frac{p_0}{\beta})^2 + \frac{p_0}{\beta}x + k_x^2 - 2p_xk_x \ . \qquad (7.4.\text{-}9)
\end{aligned}$$

A little inspection shows that the real parts cancel each other:$RE_1 + RE_2 = 0$. A further useful result is the identity $im_1^2 + re_1^2 = re_2^2 + im_2^2$ and the identity $re_2 = -re_1 + 2p_y^2$.

k_x-integration

One cannot perform k_x-integration completely using residue calculus. The reason is that the terms IM_1 and IM_2 have cuts in complex plane. One can however reduce the integral to a sum of pole terms plus integrals over the cuts.

The poles of U_1 and U_2 come from the denominators and are in fact common for the two integrands. The explicit expressions for the pole in upper half plane, where integrand converges exponentially are given by

$$
\begin{aligned}
re_i^2 &+ im_i^2 = 0 \ , \ \ i = 1,2 \ , \\
k_x &= (-b + i(-b^2 + 4ac)^{1/2})/2a \ , \\
a &= 4p_T^2 \ , \\
b &= -4((p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2)p_x \ , \\
c &= ((p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2)^2 + 4(\frac{p_0}{\beta\gamma})^2 p_y^2 \ . \qquad (7.4.-12)
\end{aligned}
$$

A straightforward calculation using the previous identities shows that the contributions of IM_1 and IM_2 at pole have opposite signs and the contribution from poles vanishes identically!

The cuts associated with U_1 and U_2 come from the square root terms K_1 and K_2. The condition for the appearance of the cut is that K_1 (K_2) is real and positive. In case of K_1 this condition gives

$$
k_x = it, \ t \in (0, \frac{p_0}{\beta\gamma}) \ . \qquad (7.4.-11)
$$

In case of K_2 the same condition gives

$$
k_x = p_x + it, \ t \in (0, \frac{p_0}{\beta} - p_z) \ . \qquad (7.4.-10)
$$

Both cuts are in the direction of imaginary axis.

The integral over real axis can be completed to an integral over semi-circle and this integral in turn can be expressed as a sum of two terms (see Fig. 7.4.1).

$$
A(b,p) = 2\pi i(CUT_1 + CUT_2) \ . \qquad (7.4.-9)
$$

The first term corresponds to contour, which avoids the cuts and reduces to a sum of pole contributions. Second term corresponds to the addition of the cut contributions.

In the following we shall give the expressions of various terms in the region $\phi \in [0, \pi/2]$. Using the symmetries

$$
\begin{aligned}
A(b, p_x, -p_y) &= -A(b, p_x, p_y) \ , \\
A(b, -p_x, -p_y) &= \bar{A}(b, p_x, p_y) \ . \qquad (7.4.-9)
\end{aligned}
$$

of the amplitude one can calculate the amplitude for other values of ϕ.

The integration variable for cuts is the imaginary part t of complexified k_x. To get a more convenient form for cut integrals one can perform a change of the integration variable

$$
\begin{aligned}
cos(\psi) &= \frac{t}{(\frac{p_0}{\beta\gamma})} \ , \\
cos(\psi) &= \frac{t}{(\frac{p_0}{\beta} - p_z)} \ , \\
\psi &\in [0, \pi/2] \ . \qquad (7.4.-10)
\end{aligned}
$$

1. The contribution of the first cut

By a painstaking calculation one verifies that the expression for the contribution of the first cut is given by

$$
\begin{aligned}
CUT_1 &= D_1 \times \int_0^{\pi/2} exp(-\frac{b}{b_0}cos(\psi))A_1 d\psi \ , \\
D_1 &= -\frac{1}{2}\frac{sin(\phi)}{sin(\theta)} \ , \ b_0 = \frac{\hbar}{m}\frac{\beta\gamma}{\gamma_1} \ , \\
A_1 &= \frac{A + iBcos(\psi)}{cos^2(\psi) + 2iCcos(\psi) + D} \ , \\
A &= sin(\theta)cos(\phi) \ , \ B = K \ , \\
C &= K\frac{cos(\phi)}{sin(\theta)} \ , \ D = -sin^2(\phi) - \frac{K^2}{sin^2(\theta)} \ , \\
K &= \beta\gamma(1 - \frac{v_{cm}}{\beta}cos(\theta)) \ , \ v_{cm} = \frac{2v}{1+v^2} \ .
\end{aligned}
\tag{7.4.-15}
$$

The definitions of the various kinematical variables are given in previous formulas. The notation is tailored to express that A_1 is rational function of $cos(\psi)$.

1. The exponential $exp(-bcos(\psi)/b_0)$ is very small in the condition

$$
cos(\psi) \geq cos(\psi_0) \equiv \frac{\hbar}{mb}\frac{\beta\gamma}{\gamma_1 cos(\phi)} \tag{7.4.-14}
$$

   holds true. Here $\hbar = 1$ convention has been given up to make clear that the increase of the Compton length of lepto-pion due to the scaling of $\hbar$ increase the magnitude of the contribution. If the condition $cos(\psi_0) << 1$ holds true, the integral over ψ receives contributions only from narrow range of values near the upper boundary $\psi = \pi/2$ plus the contribution corresponding to the pole of X_1. The practical condition is in terms of critical parameter b_{max} above which exponential approaches zero very rapidly.

2. For $cos(\psi_0) << 1$, that is for $b > b_{max}$ and in the approximation that the function multiplying the exponent is replaced with its value for $\psi = \pi/2$, one obtains for CUT_1 the expression

$$
\begin{aligned}
CUT_1 &\simeq D_1 A_1(\psi = \pi/2)\frac{\hbar}{mb} \\
&= \frac{1}{2} \times \frac{\beta\gamma}{\gamma_1} \times \frac{\hbar}{mb} \times \frac{sin^2(\theta)cos(\phi)sin(\phi)}{sin^2(\theta)sin^2(\phi) + K^2} \ .
\end{aligned}
\tag{7.4.-14}
$$

3. For $cos(\psi_0) >> 1$ exponential factor can be replaced by unity in good approximation and the integral reduces to an integral of rational function of $cos(\psi)$ having the form

$$
D_1 \frac{A + iBcos(\psi)}{cos^2(\psi) + 2iC \times cos(\psi) + D} \ . \tag{7.4.-14}
$$

   which can be expressed in terms of the roots $c_\pm$ of the denominator as

$$D_1 \times \sum_{\pm} \frac{A \mp iBc_{\pm}}{cos(\psi) - c_{\pm}} \quad , \quad c_{\pm}) \quad = \quad -iC \pm \sqrt{-C^2 - D} \quad . \tag{7.4.-13}$$

Integral reduces to an integral of rational function over the interval $[0,1]$ by the standard substitution $tan(\psi/2) = t$, $d\psi = 2dt/(1+t^2)$, $cos(\psi) = (1-t^2)/(1+t^2)$, $sin(\psi) = 2t/(1+t^2)$.

$$I \quad = \quad 2D_1 \sum_{\pm} \int_0^1 dt \frac{A \mp iBc_{\pm}}{1 - c_{\pm} - (1+c_{\pm})t^2} \tag{7.4.-12}$$

This gives

$$I \quad = \quad 2D_1 \sum_{\pm} \frac{A \mp iBc_{\pm}}{s_{\pm}} \times arctan(\frac{1+c_{\pm}}{1-c_{\pm}}) \quad . \tag{7.4.-12}$$

$s_{\pm}$ is defined as $\sqrt{1-c_{\pm}^2}$ and one must be careful with the signs. This gives for CUT_1 the approximate expression

$$\begin{aligned} CUT_1 \quad &= \quad D_1 \sum_{\pm} \frac{sin(\theta)cos(\phi) \mp iKc_{\pm}}{s_{\pm}} \times arctan(\frac{1+c_{\pm}}{1-c_{\pm}}) \quad , \\ c_{\pm} \quad &= \quad \frac{-iKcos(\phi) \pm sin(\phi)\sqrt{sin^2(\theta)+K^2}}{sin(\theta)} \quad . \end{aligned} \tag{7.4.-12}$$

Arcus tangent function must be defined in terms of logarithm functions since the argument is complex.

4. In the intermediate region, where the exponential differs from unity one can use expansion in Taylor polynomial to sum over integrals of rational functions of $cos(\psi)$ and one obtains the expression

$$\begin{aligned} CUT_1 \quad &= \quad D_1 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (\frac{b}{b_0})^n I_n \quad , \\ I_n \quad &= \quad \sum_{\pm} (A \mp iBc_{\pm} I_n(c_{\pm}) \quad , \\ I_n(c) \quad &= \quad \int_0^{\pi/2} \frac{cos^n(\psi)}{cos(\psi) - c} \quad . \end{aligned} \tag{7.4.-14}$$

$I_n(c)$ can be calculated explicitly by expanding in the integrand $cos(\psi))^n$ to polynomial with respect to $cos(\psi)) - c$, $c \equiv c_{\pm}$

$$\frac{cos^n(\psi)}{cos(\psi) - c} = \sum_{m=0}^{n-1} \binom{n}{m} c^m (cos(\psi) - c)^{n-m-1} + \frac{c^n}{cos(\psi) - c} \quad . \tag{7.4.-14}$$

After the change of the integration variable the integral reads as

$$
\begin{aligned}
I_n(c) &= \sum_{m=0}^{n-1}\sum_{k=0}^{n-m-1} \left(\begin{array}{c} n \\ m \end{array}\right)\left(\begin{array}{c} n-m-1 \\ k \end{array}\right)(-1)^k(1-c)^{n-m-1-k}(1+c)^k c^m I(k,n-m) \\
&+ \frac{c^n}{1-c}\times log[\frac{\sqrt{1-c}+\sqrt{1+c}}{\sqrt{1-c}-\sqrt{1+c}}] \ , \\
I(k,n) &= 2\int dt \frac{t^{2k}}{(1+t^2)^n} \ . \qquad (7.4.\text{-}15)
\end{aligned}
$$

Partial integration for $I(k,n)$ gives the recursion formula

$$
I(k,n) = -\frac{2^{-n+1}}{n-1} + \frac{2k-1}{2(n-1)} \times I(k-1,n-1) \ . \qquad (7.4.\text{-}14)
$$

The lowest term in the recursion formula corresponds to $I(0,n-k)$, can be calculated by using the expression

$$
\begin{aligned}
(1+t^2)^{-n} &= \sum_{k=0}^{n} c(n,k)[(1+it)^{-k}+(1-it)^{-k}] \ , \\
c(n,k) &= \sum_{l=0}^{n-k-1} c(n-1,k+l)2^{-l-2} + c(n-1,n-1)2^{-n+k-1} \ . \qquad (7.4.\text{-}14)
\end{aligned}
$$

The formula is deducible by assuming the expression to be known for n and multiplying the expression with $(1+t^2)^{-1} = [(1+it)^{-1}+(1-it)^{-1}]/2$ and applying this identity to the resulting products of $(1+it)^{-1}$ and $(1-it)^{-1}$. This gives

$$
I(0,n) = -2i\sum_{k=2,n}\frac{c(n,k)}{(k-1)}[1+2^{(k-1)/2}sin((k-1)\pi/4)] + c(n,1)log(\frac{1+i}{1-i}) \ . \qquad (7.4.\text{-}13)
$$

This boils down to the following expression for CUT_1

$$
\begin{aligned}
CUT_1 &= D_1\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}(\frac{b}{b_0})^n I_n \ , \\
I_n &= \sum_{\pm}[A\mp iBc_{\pm}]I_n(cos(c_{\pm}) \ , \\
I_n(c) &= \sum_{m=1}^{n-1}\sum_{k=0}^{n-m-1}\left(\begin{array}{c} n \\ m \end{array}\right)\left(\begin{array}{c} n-m-1 \\ k \end{array}\right)(1-c)^{n-m-1-k}(1+c)^k c^m I(k,n-m-1) \\
&+ \frac{c^n}{1-c}\times log[\frac{\sqrt{1-c}+\sqrt{1+c}}{\sqrt{1-c}-\sqrt{1+c}}] \ , \\
I(k,n) &= -\frac{2^{-n+1}}{n-1} + \frac{2k-1}{2(n-1)}\times I(k-1,n-1) \ , \\
I(0,n) &= -2i\sum_{k=2}^{n}\frac{c(n,k)}{(k-1)}[1+2^{(k-1)/2}sin((k-1)\pi/4)] - c(n,1) \ , \\
c(n,k) &= \sum_{l=0}^{n-k-1} c(n-1,k+l)2^{-l-2} + c(n-1,n-1)2^{-n+k-1} \ . \qquad (7.4.\text{-}18)
\end{aligned}
$$

This expansion in powers of $c_\pm$ fails to converge when their values are very large. This happens in the case of τ-pion production amplitude. In this case one typically has however the situation in which the conditions $A_1 \simeq iBcos(\psi)/D$ holds true in excellent approximation and one can write

$$\begin{aligned} CUT_1 &\simeq i\frac{D_1 B}{D} \times \sum_{n=0,1,...} \frac{(-1)^n}{n!2^n}(\frac{b}{b_0})^n I_n \times \ , \\ I_n &= \int_0^{\pi/2} cos(\psi)^{n+1} d\psi = \sum_{k=0}^{n+1} \left(\begin{array}{c} n+1 \\ k \end{array} \right) \frac{i^{n-2k}-1}{n+1-2k} \ . \end{aligned} \quad (7.4.\text{-}18)$$

The denominator X_1 vanishes, when the conditions

$$\begin{aligned} cos(\theta) &= \frac{\beta}{v_{cm}} \ , \\ sin(\phi) &= cos(\psi) \end{aligned} \quad (7.4.\text{-}18)$$

hold. In forward direction the conditions express the vanishing of the z-component of the lepto-pion velocity in velocity cm frame as one can realize by noticing that condition reduces to the condition $v = \beta/2$ in non-relativistic limit. This corresponds to the production of lepto-pion with momentum in scattering plane and with direction angle $cos(\theta) = \beta/v_{cm}$.

CUT_1 diverges logarithmically for these values of kinematical variables at the limit $\phi \rightarrow 0$ as is easy to see by studying the behavior of the integral near as K approaches zero so that X_1 approaches zero at $sin(\phi) = cos(\Phi)$ and the integral over a small interval of length $\Delta\Psi$ around $cos(\Psi) = sin(\phi)$ gives a contribution proportional to $log(A + B\Delta\Psi))/B$, $A = K[K - 2isin(\theta)sin^2(\phi)]$ and $B = 2sin(\theta)cos(\phi)[sin(\theta)sin(\phi) - iKcos(\phi)]$. Both A and B vanish at the limit $\phi \rightarrow 0$, $K \rightarrow 0$. The exponential damping reduces the magnitude of the singular contribution for large values of $sin(\phi)$ as is clear form the first formula.

2. The contribution of the second cut

The expression for CUT_2 reads as

$$\begin{aligned} CUT_2 &= D_2 exp(-\frac{b}{b_2}) \times \int_0^{\pi/2} exp(i\frac{b}{b_1}cos(\psi))A_2 d\psi \ , \\ D_2 &= -\frac{sin(\frac{\phi}{2})}{usin(\theta)} \ , \\ b_1 &= \frac{\hbar}{m}\frac{\beta}{\gamma_1} \ , \ b_2 = \frac{\hbar}{mb}\frac{1}{\gamma_1 \times sin(\theta)cos(\phi)} \\ A_2 &= \frac{Acos(\psi) + B}{cos^2(\psi) + 2iCcos(\psi) + D} \ , \\ A &= sin(\theta)cos(\phi)u \ , \ B = \frac{w}{v_{cm}} + \frac{v}{\beta}sin^2(\theta)[sin^2(\phi) - cos^2(\phi)] \ , \\ C &= \frac{\beta w}{uv_{cm}}\frac{cos(\phi)}{sin(\theta)} \ , \ D = -\frac{1}{u^2}(\frac{sin^2(\phi)}{\gamma^2} + \beta^2(v^2 sin^2(\theta) - \frac{2vw}{v_{cm}})cos^2(\phi)) \\ &+ \frac{w^2}{v_{cm}^2 u^2 sin^2(\theta)} + 2i\frac{\beta v}{u}sin(\theta)cos(\phi) \ , \\ u &= 1 - \beta vcos(\theta) \ , \ w = 1 - \frac{v_{cm}}{\beta}cos(\theta) \ . \end{aligned} \quad (7.4.\text{-}24)$$

(7.4.-23)

The denominator X_2 has no poles and the contribution of the second cut is therefore always finite.

1. The factor $exp(-b/b_2)$ gives an exponential reduction and the contribution of CUT_2 is large only when the criterion

$$b < \frac{\hbar}{m} \times \frac{1}{v\gamma_1 sin(\theta)cos(\phi)}$$

for the impact parameter b is satisfied. Large values of $\hbar$ increase the range of allowed impact parameters since the Compton length of lepto-pion increases.

2. At the limit when the exponent becomes very large the variation of the phase factor implies destructive interference and one can perform stationary phase approximation around $\psi = \pi/2$. This gives

$$\begin{aligned} CUT_2 &\simeq \sqrt{\frac{2\pi b_1}{b}} \times D_2 \times exp(\frac{b}{b_2})A_2(\psi = 0) \ , \\ D_2 &= -\frac{sin(\frac{\phi}{2})}{usin(\theta)} \ , \ A_2 = \frac{A}{D} \ . \end{aligned} \quad (7.4.\text{-}23)$$

3. As for CUT_1, the integral over ψ can be expressed as a finite sum of integrals of rational functions, when the value of $(b/b_1)cos(\psi)$ is so small that $exp(i(b/b_1)cos(\psi))$ can be approximated by a Taylor polynomial. More generally, one obtains the expansion

$$\begin{aligned} CUT_2 &= D_2 exp(-\frac{b}{b_2}) \times \sum_{n=0}^{\infty} \frac{1}{n!} i^n (\frac{b}{b_1})^n I_n(A,B,C,D) \ , \\ I_n(A,B,C,D) &= \int_0^{\pi/2} cos(\psi)^n \frac{A + iBcos(\psi)}{cos^2(\psi) + Ccos(\psi) + D} \ . \end{aligned} \quad (7.4.\text{-}23)$$

The integrand of $I_n(A,B,C,D)$ is same rational function as in the case of CUT_1 but the parameters A, B, C, D given in the expression for CUT_2 are different functions of the kinematical variables. The functions appearing in the expression for integrals $I_n(c)$ correspond to the roots of the denominator of A_2 and are given by $c_\pm = -iC \pm \sqrt{-C^2 - D}$, where C and D are the function appearing in the general expression for CUT_2 in Eq. 7.4.-23.

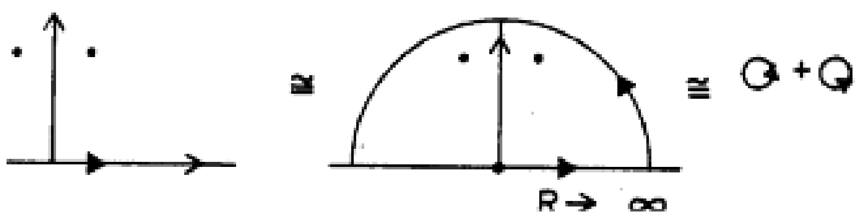

Figure 7.3: Evaluation of k_y-integral using residue calculus.

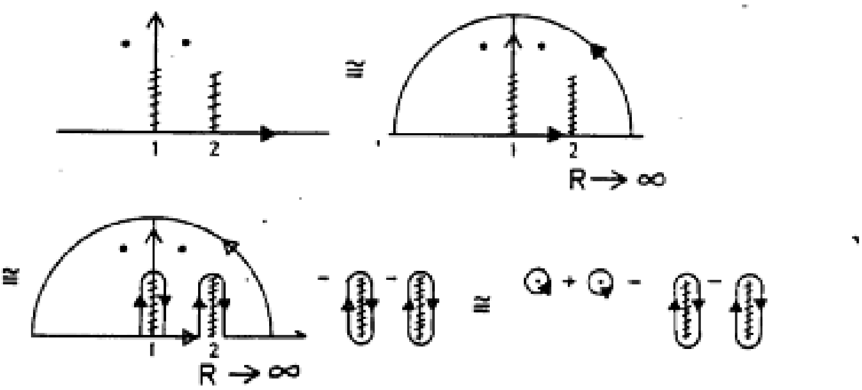

Figure 7.4: Evaluation of k_x-integral using residue calculus.

7.4.2 Production amplitude in quantum model

The previous expressions for CUT_1 and CUT_2 as such give the production amplitude for given b in the classical model and the cross section can be calculated by integrating over the values of b. The finite Taylor expansion of the amplitude in powers of b allows explicit formulas when impact parameter cutoff is assumed.

General expression of the production amplitude

In quantum model the production amplitude can be reduced to simpler form by using the defining integral representation of Bessel functions

$$\begin{aligned} f_B &= i\int F(b)J_0(\Delta kb)(CUT_1+CUT_2)bdb \ , \\ F &= 1 \quad \text{for } exp(i(S)) \text{ option} \ , \\ F(b\geq b_{cr}) &= \int dz\frac{1}{\sqrt{z^2+b^2}} = 2ln(\frac{\sqrt{a^2-b^2}+a}{b}) \ \text{ for } exp(i(S))-1 \text{ option} \ , \\ \Delta k &= 2ksin(\frac{\alpha}{2}) \ , \quad k=M_R\beta \quad . \end{aligned} \tag{7.4.-25}$$

Note that F is a rather slowly varying function of b and in good approximation can be replaced by its average value $A(b,p)$, which has been already explicitly calculated as power series in b. α_{em} corresponds to the value of α_{em} for the standard value of Planck constant.

The limit $\Delta k = 0$

The integral of the contribution of CUT_1 over the impact parameter b involves integrals of the form

$$\begin{aligned} J_{1,n} &= b_0^2\int J_0(\Delta kb)F(b)x^{n+1}dx \ , \\ x &= \frac{b}{b_0} \ . \end{aligned} \tag{7.4.-25}$$

Here a is the upper impact parameter cutoff. For CUT_2 one has integrals of the form

$$\begin{aligned} J_{2,n} &= b_1^2(\frac{b_2}{b_1})^{n+2}\int J_0(\Delta kb)F(b)exp(-x)x^{n+1}dx \ , \\ x &= \frac{b}{b_2} \ . \end{aligned} \tag{7.4.-25}$$

Using the following approximations it is possible to estimate the integrals analytically.

1. The logarithmic term is slowly varying function and can be replaced with its average value

$$F(b) \rightarrow \langle F(b)\rangle \equiv F \ . \tag{7.4.-24}$$

2. Δk is fixed once the value of the impact parameter is known. At the limit $\Delta k = 0$ making sense for very high energy collisions one can but the value of Bessel function to $J_0(0) = 1$. Hence it is advantageous to calculated the integrals of $\int CUT_ibdb$.

Consider first the integral $\int CUT_1bdb$. If exponential series converges rapidly one can use Taylor polynomial and calculate the integrals explicitly. When this is not the case one can calculate integral approximately and the total integral is sum over two contributions:

$$\int CUT_1bdb = I_a + I_b \ . \tag{7.4.-23}$$

1. The region in which Taylor expansion converges rapidly gives rise integrals

$$\begin{aligned} I_{1,n} &\simeq b_0^2 \int x^{n+1}dx = b_0^2 \frac{1}{n+2}[(\frac{b_{max}}{b_0})^{n+2} - (\frac{b_{cr}}{b_0})^{n+2}] \simeq b_0^2 \frac{1}{n+2}(\frac{b_{max}}{b_0})^{n+2} \ , \\ I_{2,n} &\simeq b_1^2 (\frac{b_2}{b_1})^{n+2} \int exp(-x)x^{n+1}dx = b_1^2(\frac{b_2}{b_1})^{n+2}(n+1)! \ . \end{aligned} \quad (7.4.\text{-}24)$$

2. For the perturbative part of CUT_1 one obtains the expression

$$\begin{aligned} I_a &= \int_0^{b_{max}} CUT_1 bdb = D_1 \times b_0^2 \times \sum_{n=0}^{\infty} \frac{1}{n!(n+2)}(\frac{b_{max}}{b_0})^{n+2} I_n(A,B,C,D) \ , \\ D_1 &= -\frac{1}{2}\frac{sin(\phi)}{sin(\theta)} \ , \ b_0 = \frac{\hbar\beta\gamma}{m\gamma_1} \ . \end{aligned} \quad (7.4.\text{-}25)$$

   There b_{max} is the largest value of b for which the series converges sufficiently rapidly.

3. The convergence of the exponential series is poor for large values of b/b_0, that is for $b > b_m$. In this case one can use the approximation in which the multiplier of exponent function in the integrand is replaced with its value at $\psi = \pi/2$ so that amplitude becomes proportional to b_0/b. In this case the integral over b gives a factor proportional to ab_0, where a is the impact parameter cutoff.

$$\begin{aligned} I_b \equiv \int_{b_m}^{a} CUT_1 bdb &\simeq b_0(a-b_m)D_1 \times A_1(\psi = \pi/2) \\ &= \frac{\beta\gamma}{\gamma_1} \times \frac{\hbar}{m} \times \frac{sin^2(\theta)cos(\phi)sin(\phi)}{sin^2(\theta)sin^2(\phi)+K^2} \ , \\ D_1 &= -\frac{1}{2}\frac{sin(\phi)}{sin(\theta)} \ , \ A_1(\psi = \pi/2) = \frac{A}{D} \ . \end{aligned} \quad (7.4.\text{-}27)$$

4. As already explained, the expansion based on partial fractions does not converge, when the roots $c_\pm$ have very large values. This indeed occurs in the case of τ-pion production cross section. In this case one has $A_1 \simeq iBcos(\psi)/D$ in excellent approximation and one can calculate CUT_1 in much easier manner. Using the formula of Eq. 7.4.-18 for CUT_1, one obtains

$$\begin{aligned} \int CUT_1 bdb &\simeq b_0^2 \frac{D_1 B}{D} \times \sum_{n=0,1,...} \frac{(-1)^n}{n!(n+2)2^n} \times \sum_{k=0}^{n+1} \left(\begin{array}{c} n+1 \\ k \end{array} \right) c_{n,k} \times (\frac{b_{max}}{b_0})^n \ , \\ c_{n,k} &= \frac{i^{n+1-2k}-1}{n+1-2k} for \ \ n \neq 2k-1 \ , \ c_{n,k} = \frac{i\pi}{2} \ for \ \ n = 2k-1 \ , \end{aligned} \quad (7.4.\text{-}27)$$

   Note that for $n = 2k+1 = k$ the coefficient diverges formally and actua

Highly analogous treatment applies to the integral of CUT_2.

1. For the perturbative contribution to $\int CUT_2 bdb$ one obtains

$$I_a = \int_0^{b_{1,max}} CUT_2 bdb = b_1^2 D_2 \sum_{n=0}^{\infty} (n+1) i^n I_n(A,B,C,D) \times (\frac{b_2}{b_1})^{n+2} ,$$
$$D_2 = -\frac{sin(\frac{\phi}{2})}{usin(\theta)} ,$$
$$b_1 = \frac{\hbar\beta}{m\gamma_1} , \quad b_2 = \frac{\hbar}{m\gamma_1}\frac{1}{sin(\theta)cos(\phi)} . \quad (7.4.\text{-}28)$$

2. Taylor series converges slowly for

$$\frac{b_1}{b_2} = \frac{sin(\theta)cos(\phi)}{\beta} \to 0 .$$

In this case one can replace $exp(-b/b_2)$ with unity or expand it as Taylor series taking only few terms. This gives the expression for the integral which is of the same general form as in the case of CUT_1

$$I_a = \int_0^{b_{max}} CUT_2 bdb = b_1^2 D_2 \sum_{n=0}^{\infty} \frac{i^n}{n!(n+2)} I_n(A,B,C,D)(\frac{b_{max}}{b_1})^{n+1} . \quad (7.4.\text{-}28)$$

3. Also when b/b_1 becomes very large, one must apply stationary phase approximation to calculate the contribution of CUT_2 which gives a result proportional to $\sqrt{b_1/b}$. Assume that $b_m >> b_1$ is the value of impact parameter above which stationary phase approximation is good. This gives for the non-perturbative contribution to the production amplitude the expression

$$I_b = \int_{b_m}^{a} CUT_2 bdb = k\sqrt{\frac{2\pi b_1}{b_2}} b_2^2 \times D_2 \times A_2(\psi = \pi/2) ,$$
$$k = \int_{x_1}^{x_2} exp(-x)x^{1/2}dx = 2\int_{\sqrt{x_1}}^{\sqrt{x_2}} exp(-u^2)u^2 du ,$$
$$x_1 = \frac{b_m}{b_2} , \quad x_2 = \frac{a}{b_2} . \quad (7.4.\text{-}29)$$

In good approximation one can take $x_2 = \infty$. $x_1 = 0$ gives the upper bound $k \leq \sqrt{\pi}$ for the integral.

Some remarks relating to the numerics are in order.

1. The contributions of both CUT_1 and CUT_2 are proportional to $1/sin(\theta)$ in the forward direction. The denominators of A_i however behave like $1/sin^2(\theta)$ at this limit so that the amplitude behaves as $sin(\theta)$ at this limit and the amplitude approaches to zero like $sin(\theta)$ Therefore the singularity is only apparent but must be taken into account in the calculation since one has $c_\pm \to i\infty$ at this limit for CUT_2 and for CUT_1 the roots approach to $c_+ = c_- = i\infty$. One must pose a cutoff θ_{min} below which the contribution of CUT_1 and CUT_2 are calculated directly using approximate he expressions for D_iA_i.

$$D_1A_1 \to -\frac{i}{K}cos(\psi) \times sin(\theta) \to 0$$
$$D_2A_2 \to -\frac{uv_{cm}}{w} \times sin(\theta) \to 0 . \quad (7.4.\text{-}29)$$

In good approximation both contributions vanish since also $sin^2(\theta)$ factor from the phase space integration reduces the contribution.

2. A second numerical problem is posed by the possible vanishing of

$$K = \beta\gamma(1 - \frac{v_c m}{\beta}cos(\theta)) \quad .$$

In this case the roots $c_\pm = \pm sin(\phi)$ are real and c_+ gives rise to a pole in the integrand.

The singularity to the amplitude comes from the logarithmic contributions in the Taylor series expansion of the amplitude. The sum of the singular contributions coming from c_+ and c_- are of form

$$\frac{c_n}{2}(\sqrt{1-sin(\phi)} + \sqrt{1+sin(\phi)}log(\frac{1+u}{1-u}) \ , \ u = \sqrt{\frac{1+sin(\phi)}{1-sin(\phi)}} \ .$$

Here c_n characterizes the $1/(cos(\psi)-c_\pm)$ term of associated with the $cos(\psi)^n$ term in the Taylor expansion. Logarithm becomes singular for the two terms in the sum at the limit $\phi \rightarrow 0$. The sum however behaves as

$$\frac{c_n}{2}sin(\phi)log(\frac{sin(\phi)}{2}) \ .$$

so that the net result vanishes at the limit $\phi \rightarrow 0$. It is essential that the logarithmic singularities corresponding to the roots c_+ and c_- cancel each other and this must be taken into account in numerics. There is also apparent singularity at $\phi = \pi/2$ canceled by $cos(\phi)$ factor in D_1. The simplest manner to get rid of the problem is to exclude small intervals $[0,\epsilon]$ and $[\pi/2-\epsilon, \pi/2]$ from the phase space volume.

Improved approximation to the production cross section

The approximation $J_0(\Delta k_T(b)b) = 1$ and $F(b) = F = constant$ allows to perform the integrations over impact parameter explicitly (for $exp(iS)$ option $F = 1$ holds true identically in the lowest order approximation). An improved approximation is obtained by diving the range of impact parameters to pieces and performing the integrals over the impact parameter ranges exactly using the average values of these functions. This requires only a straightforward generalization of the formulas derived above involving integrals of the functions x^n and $exp(-x)x^n$ over finite range. Obviously this is still numerically well-controlled procedure.

7.4.3 Evaluation of the singular parts of the amplitudes

The singular parts of the amplitudes $CUT_{1,sing}$ and $B_{1,sing}$ are rational functions of $cos(\psi)$ and the integrals over ψ can be evaluated exactly.

In the classical model the expression for $U_{1,sing}$ appearing as integrand in the expression of $CUT_{1,sing}$ reads as

$$
\begin{aligned}
A_{1,sing} &= -\frac{1}{2\sqrt{K^2+sin^2(\theta)}}(sin(\theta)cos(\phi)A_a+iKA_b) \ , \\
A_a &= I_1(\beta,\pi/2)=\int_0^{\pi/2}d\psi f_1 \ , \\
A_b &= I_2(\beta,\pi/2)=\int_0^{\pi/2}d\psi f_2 \ , \\
f_1 &= \frac{1}{(cos(\psi)-c_1)(cos(\psi)-c_2)} \ , \\
f_2 &= cos(\psi)f_1 \ , \\
c_1 &= \frac{-iKcos(\phi)+sin(\phi)\sqrt{K^2+sin^2(\theta)}}{sin(\theta)} \ , \\
c_2 &= -\bar{c}_1 \ .
\end{aligned} \tag{7.4.-34}
$$

Here c_i are the roots of the polynomial X_1 appearing in the denominator of the integrand.

In quantum model the approximate expression for the singular contribution to the production amplitude can be written as

$$
\begin{aligned}
B_{1,sing} &\simeq k_1\frac{sin(\theta)sin(\phi)}{2\sqrt{K^2+sin^2(\theta)}}\sum_n\langle F\rangle_n(I(x(n+1))-I(x(n))) \ , \\
I(x) &= exp(-\frac{sin(\phi)x}{sin(\phi_0)})(sin(\theta)cos(\phi)A_a(\Delta ka,x)+iKA_b(\Delta ka,x)) \ , \\
k_1 &= 2\pi^2M_RZ_1Z_2\alpha_{em}\frac{\sqrt{2}}{\sqrt{\Delta k\pi}}sin(\phi_0) \ .
\end{aligned} \tag{7.4.-36}
$$

The expressions for the amplitudes $A_a(k,x)$ and $A_b(k,x)$ read as

$$
\begin{aligned}
A_a(k,x) &= cos(kx)I_3(k,0,\pi/2)+isin(\phi_0)ksin(kx)I_5(k,0,\pi/2) \ , \\
A_b(k,x) &= cos(kx)I_4(k,0,\pi/2)+isin(\phi_0)ksin(kx)I_3(k,0,\pi/2) \ , \\
I_i(k,\alpha,\beta) &= \int_\alpha^\beta f_i(k)d\psi \ , \\
f_3(k) &= \frac{cos(\psi)}{(cos^2(\psi)+sin^2(\phi_0)k^2)}f_1(k) \ , \\
f_4(k) &= cos(\psi)f_3(k) \ , \\
f_5(k) &= \frac{1}{(cos^2(\psi)+sin^2(\phi_0)k^2)}f_1(k) \ .
\end{aligned} \tag{7.4.-41}
$$

The expressions for the integrals I_i as functions of the endpoints α and β can be written as

$$
\begin{aligned}
I_1(k,\alpha,\beta) &= I_0(c_1,\alpha,\beta) - I_0(c_2,\alpha,\beta) \ , \\
I_2(\alpha,\beta) &= c_1 I_0(c_1,\alpha,\beta) - c_2 I_0(c_2,\alpha,\beta) \ , \\
I_3 &= C_{34} \sum_{i=1,2,j=3,4} \frac{1}{(c_i - c_j)} (c_i I_0(c_i,\alpha,\beta) - c_j I_0(c_j,\alpha,\beta)) \ , \\
I_4 &= C_{34} \sum_{i=1,2,j=3,4} \frac{1}{(c_i - c_j)} ((c_i - c_j)(\beta - \alpha) - c_i^2 I_0(c_i,\alpha,\beta) + c_j^2 I_0(c_j,\alpha,\beta)) \ , \\
I_5 &= C_{34} \sum_{i=1,2,j=3,4} \frac{1}{(c_i - c_j)} (I_0(c_i,\alpha,\beta) - I_0(c_j,\alpha,\beta)) \ , \\
C_{34} &= \frac{1}{c_3 - c_4} = \frac{1}{2ikasin(\phi_0)} \ .
\end{aligned} \tag{7.4.-45}
$$

The parameters c_1 and c_2 are the zeros of X_1 as function of $cos(\psi)$ and c_3 and c_4 the zeros of the function $cos^2(\psi) + k^2a^2sin^2(\phi_0)$:

$$
\begin{aligned}
c_1 &= \frac{-iKcos(\phi) + sin(\phi)\sqrt{K^2 + sin^2(\theta)}}{sin(\theta)} \ , \\
c_2 &= \frac{-iKcos(\phi) - sin(\phi)\sqrt{K^2 + sin^2(\theta)}}{sin(\theta)} \ , \\
c_3 &= ikasin(\phi_0) \ , \\
c_4 &= -ikasin(\phi_0) \ .
\end{aligned} \tag{7.4.-48}
$$

The basic integral $I_0(c,\alpha,\beta)$ appearing in the formulas is given by

$$
\begin{aligned}
I_0(c,\alpha,\beta) &= \int_\alpha^\beta d\psi \frac{1}{(cos(\psi) - c)} \ , \\
&= \frac{1}{\sqrt{1-c^2}} (f(\alpha) - f(\beta)) \ , \\
f(x) &= ln(\frac{(1 + tan(x/2)t_0)}{(1 - tan(x/2)t_0)}) \ , \\
t_0 &= \sqrt{\frac{1-c}{1+c}} \ .
\end{aligned} \tag{7.4.-50}
$$

From the expression of I_0 one discovers that scattering amplitude has logarithmic singularity, when the condition $tan(\alpha/2) = 1/t_0$ or $tan(\beta/2) = 1/t_0$ is satisfied and appears, when c_1 and c_2 are real. This happens at the cone $K = 0$ ($\theta = \theta_0$), when the condition

$$
\begin{aligned}
\sqrt{\frac{(1 - sin(\phi))}{(1 + sin(\phi))}} &= tan(x/2) \ , \\
x &= \alpha \text{ or } \beta \ .
\end{aligned} \tag{7.4.-50}
$$

holds true. The condition is satisfied for $\phi \simeq x/2$. $x = 0$ is the only interesting case and gives singularity at $\phi = 0$. In the classical case this gives logarithmic singularity in production amplitude for all scattering angles.

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Chapter 8

TGD and Nuclear Physics

8.1 Introduction

Despite the immense amount of data about nuclear properties, the first principle understanding of the nuclear strong force is still lacking. The conventional meson exchange description works at qualitative level only and does not provide a viable perturbative approach to the description of the strong force. The new concept of atomic nucleus forced by TGD suggests quite different approach to the quantitative description of the strong force in terms of the notion of field body, join along boundaries bond concept, long ranged color gauge fields associated with dark hadronic matter, and p-adic length scale hierarchy.

8.1.1 p-Adic length scale hierarchy

p-Adic length scale hypothesis

The concept of the p-adic topological condensate is the corner stone of p-adic TGD. Various levels of the topological condensate obey effective p-adic topology and are assumed to form a p-adic hierarchy ($p_1 \leq p_2$ can condense on p_2). By the length scale hypothesis, the physically interesting length scales should come as square roots of powers of 2: $L(k) \simeq 2^{\frac{k}{2}} l$, $l \simeq 1.288E + 4\sqrt{G}$ and prime powers of k are especially interesting.

For nuclear physics applications the most interesting values of k are: $k = 107$ (hadronic space-time sheet at which quarks feed their color gauge fluxes), $k = 109$ (radius of light nucleus such as alpha particle[1], $k = 113$ (the space-time at which quarks feed their electromagnetic gauge fluxes), $k = k_{em} = 127$ or 131 (electronic or atomic space-time sheet receiving electromagnetic gauge fluxes of nuclei).

The so called Gaussian primes are to complex integers what primes are for the ordinary integers and the Gaussian counterparts of the Mersenne primes are Gaussian primes of form $(1 \pm i)^k - 1$. Rather interestingly, $k = 113$ corresponds to a Gaussian Mersenne. Also the primes $k = 151, 157, 163, 167$ defining biologically important length scales correspond to Gaussian Mersennes. Thus the electromagnetic p-adic length scales associated with quarks, hadrons, and nuclear physics as well as with muon are in well defined sense also Mersenne length scales. A possible interpretation for complex primes is in terms of complex conformal weights for elementary particles. If the net conformal weights of physical states are required to be real this gives rise to conformal confinement.

There are however arguments suggesting the conformal weights can be complex for particles and that the imaginary part of the conformal weight defines a new kind of conserved quantum number, "scaling momentum", whose sign distinguishes between particles and their phase conjugates which can be regarded as particles of negative energy traveling to the direction of geometric past. There would be inherent arrow of geometric time associated with particles with complex conformal weight. For instance, the strange properties of phase conjugate photons could be understood since second law of thermodynamics would hold true in a reversed direction of geometric time for them.

[1] For some mysterious reason I realized that $k = 109$ is also prime only at the third millenium: for more than half decade after writing the first version of this chapter!

Particles are characterized by a collection of p-adic primes

It seems that is not correct to speak about particle as a space-time sheet characterized by single p-adic prime. Already p-adic mass calculations suggest that there are several sizes corresponding to space-time sheets at which particle feeds its gauge charges. p-Adic length scale hypothesis provides further insight: the length scale is more like the size of field body and possibly also delocalization volume of particle determining the p-adic mass scale in p-adic thermodynamics rather than the geometric size for the elementary particle.

What one can definitely say that each particle is characterized by a collection of p-adic primes and one of them characterizes the mass scale of the particle whereas other characterize its interactions. There are two possible interpretations and both of them allow to resolve objections against p-adic hierarchies of color and electro-weak physics.

1. These primes characterize the space-time sheets at which it feeds its gauge fluxes and particles can interact only via their common space-time sheets and are otherwise dark with respect to each other.

2. Number theoretical vision supports the notion of multi-p p-adicity and the idea that elementary particles correspond to infinite primes, integers, or perhaps even rationals [E3, F6]. To infinite primes, integers, and rationals it is possible to associate a finite rational $q = m/n$ by a homomorphism. q defines an effective q-adic topology of space-time sheet consistent with p-adic topologies defined by the primes dividing m and n ($1/p$-adic topology is homeomorphic to p-adic topology). The largest prime dividing m determines the mass scale of the space-time sheet in p-adic thermodynamics. m and n are exchanged by super-symmetry and the primes dividing m (n) correspond to space-time sheets with positive (negative) time orientation. Two space-time sheets characterized by rationals having common prime factors can be connected by a $\#_B$ contact and can interact by the exchange of particles characterized by divisors of m or n.

   The nice feature of this option is that single multi-p p-adic space-time sheet rather than a collection of them characterizes elementary particle. Concerning the description of interaction vertices as generalization of vertices of Feynman graphs (vertices as branchings of 3-surfaces) this option is decisively simpler than option 1) and is consistent with earlier number theoretic argument allowing to evaluate gravitational coupling strength [E3, F6]. It is also easier to to understand why the largest prime in the collection determines the mass scale of elementary particle.

Interestingly, these two options are not necessarily mutually exclusive: single multi-p p-adic space-time sheet could correspond to many-sheeted structure with respect to real topology.

What is the proper interpretation of p-adic length scales

One of the surprises of p-adic mass calculations was that for u and d quarks electromagnetic size corresponds to $k = 113$ which corresponds to the length scale of 2×10^{-14} m. This leads to the view that also hadrons and nuclei have this size in some sense. The charge radii of even largest nuclei without neutron halo are smaller than this.

1. If electromagnetic charges of quarks inside nucleons were separately delocalized in the scale $L(113)$, also the distributions of electromagnetic charges of nuclei would be non-trivial in surprisingly long length scale. Em charges would exhibit fractionality in this length scale and Rutherford scattering cross sections would be modified. The fact that the height of the Coulomb wall at $L(113)$ is lower than the observed heights of the Coulomb wall would lead to a paradox.

   This suggests that the p-adic length scale $L(113)$ does not characterize the geometric size of neither nucleons nor nuclei but to the size, perhaps height, of the electromagnetic field body associated with quark/hadron/nucleus.

2. If protons feed their electric em gauge fluxes to the same space-time sheet, there is an electromagnetic harmonic oscillator potential contributing to the nuclear energies. The Mersenne prime M_{127} as a characterizer of the field body of nucleus is natural and it also corresponds to the space-time sheet of electron.

3. For weak forces the size of the field body would be given by electro-weak length scale $L(89)$. The size scale would also correspond to the p-adic delocalization length scale of ordinary sized nucleons and nuclei.

4. It turns out that the identification of nuclear strong interactions in terms of dark QCD with large value of $\hbar$ and color length scale scaled up to $L_c \simeq 2^{11}L(107) \simeq .5\times10^{-11}$ m (!) predicts for the nuclei same electromagnetic sizes as in the conventional theory: scaled up sizes appear only in the dark sector and characterize the size of color field body so that paradoxes are avoided. There are also reasons to believe that dark quarks are dark also with respect to electromagnetic and weak interactions so that the sizes of corresponding field bodies are scaled up by a factor $1/v_0$.

The hypothesis that the collection of primes corresponds to multi-p p-adicity rather than collection of space-time sheets implies this. For this option various field bodies could form single field body in q-adic sense with superposed p-adic fractalities much like waves of shorter wavelength scale superposed on waves of longer wavelength scale. As noticed, this might be consistent with the existence of several p-adic field bodies with respect to real topology.

Field/magnetic bodies would represent the space-time correlate for the formation of bound states. It is even possible to think that bound state entanglement corresponds to the linking of magnetic flux tubes. The contributions of say color interactions between nucleons to the binding energy would be estimated using the field magnitudes at position of exotic quarks and the hypothesis is made that these intensities correlate with the shortest distance between dark quarks although the distance along the field body is of order L_c.

This picture finds experimental support.

1. Neutron proton scattering at low energies gives however surprisingly clear evidence for the presence of the p-adic length scales $L(109)$ ($k = 109$ is prime) and $L(113)$ in nuclear physics. The scattering lengths for s and p waves are $a_s = -2.37\times10^{-14}$ m and $a_t = 5.4\times 10{-}15$ m [19]. a_s is anomalously large and the standard explanation is that deuteron almost allows singlet wave bound state. a_t is near to $L(109) = 2L(107) \simeq 5.0\times10^{-15}$ m, which is in accordance with the assumption that in triplet state neutron and proton are glued by color bond together to form structure with size or order $L(109) = 2L(107)$. a_s is of same order of magnitude as $L(113) = 2\times10^{-14}$ m so that the interpretation in terms of the $k = 113$ space-time sheet is suggestive.

2. Neutron halos at distance of about 2.5×10^{-14} m longer than even $L(113) = 2\times10^{-14}$ m are difficult to understand in the standard nuclear physics framework and provide support for the large value of L_c. They could be understood in terms of delocalization of quarks in the length scale $L(113)$ and color charges in the length scale of L_c. For instance, the nucleus in the center could be color charged and neutron halo would be analogous to a colored matter around the central halo.

8.1.2 TGD based view about dark matter

TGD suggests an explanation of dark matter as a macroscopically quantum coherent phase residing at larger space-time sheets [J6].

1. TGD suggests that $\hbar$ is dynamical and possesses a spectrum expressible in terms of generalized Beraha numbers $B_r = 4cos^2(\pi/r)$, where $r > 3$ is a rational number [A8, J6]. Just above $r = 3$ arbitrarily large values of $\hbar$ and thus also macroscopic quantum phases are possible. The criterion for transition to large $\hbar$ phase is the failure of perturbative expansion so that Mother Nature takes care of the problems of theoretician. A good guess is that the criticality condition reads as $Q_1Q_2\alpha \simeq 1$ where Q_i are gauge charges and α gauge coupling strength. This leads to universal properties of the large $\hbar$ phase. For instance, $\hbar$ is scaled in the transition to dark phase by a harmonic or subharmonic of parameter $1/v_0 \simeq 2^{11}$ which is essentially the ratio of CP_2 length scale and Planck length [D7, J6]. The criticality condition can be applied also to dark matter itself and entire hierarchy of dark matters is predicted corresponding to the spectrum of values of $\hbar$.

2. The particles of dark matter can also carry phase carry complex conformal weights but the net conformal weights for blocks of this kind of dark matter would be real. This implies macroscopic quantum coherence. It is not absolutely necessary that $\hbar$ is large for this phase.

3. From the point of view of nuclear physics application of this hypothesis is to QCD. The prediction is that the electromagnetic Compton sizes of dark quarks are scaled from $L(107)$ to about $2^{11}L(107) = L(129) = 2L(127)$, which is larger than the p-adic electromagnetic size of electron! The classical scattering cross sections are not changed but changes the geometric sizes of dark quarks, hadrons, and nuclei. The original hypothesis that ordinary valence quarks are dark whereas sea quarks correspond to ordinary value of $\hbar$ is taken as a starting point. In accordance with the earlier model, nucleons in atomic nuclei are assumed to be accompanied by color bonds connecting exotic quark and anti-quark characterized p-adic length scale $L(127)$ with ordinary value of $\hbar$ and having thus scaled down mass of order MeV. The strong binding would be due the color bonds having exotic quark and anti-quark at their ends.

4. Quantum classical correspondence suggests that classical long ranged electro-weak gauge fields serve as classical space-time correlates for dark electro-weak gauge bosons, which are massless. This hypothesis could explain the special properties of bio-matter, in particular the chiral selection as resulting from the coupling to dark Z^0 quanta. Long range weak forces present in TGD counterpart of Higgs=0 phase should allow to understand the differences between biochemistry and the chemistry of dead matter.

   The basic implication of the new view is that the earlier view about nuclear physics applies now to dark nuclear physics and large parity breaking effects and contribution of Z^0 force to scattering and interaction energy are not anymore a nuisance.

5. For ordinary condensed matter quarks and leptons Z^0 charge are screened in electro-weak length scale whereas in dark matter $k = 89$ electro-weak space-time sheet have suffered a phase transition to a p-adic topology with a larger value of k. Gaussian Mersennes, in particular those associated with $k = 113, 151, 157, 163, 167$ are excellent candidates in this respect. The particles of this exotic phase of matter would have complex conformal weights closely related to the zeros of Riemann Zeta. The simplest possibility is that they correspond to a single non-trivial zero of Zeta and there is infinite hierarchy of particles of this kind.

   In dark matter phase weak gauge fluxes could be feeded to say $k = k_Z = 169$ space-time sheet corresponding to neutrino Compton length and having size of cell. For this scenario to make sense it is essential that p-adic thermodynamics predicts for dark quarks and leptons essentially the same masses as for their ordinary counterparts [F3].

8.1.3 The identification of long range classical weak gauge fields as correlates for dark massless weak bosons

Long ranged electro-weak gauge fields are unavoidably present when the dimension D of the CP_2 projection of the space-time sheet is larger than 2. Classical color gauge fields are non-vanishing for all non-vacuum extremals. This poses deep interpretational problems. If ordinary quarks and leptons are assumed to carry weak charges feeded to larger space-time sheets within electro-weak length scale, large hadronic, nuclear, and atomic parity breaking effects, large contributions of the classical Z^0 force to Rutherford scattering, and strong isotopic effects, are expected. If weak charges are screened within electro-weak length scale, the question about the interpretation of long ranged classical weak fields remains.

During years I have discussed several solutions to these problems.

Option I: The trivial solution of the constraints is that Z^0 charges are neutralized at electro-weak length scale. The problem is that this option leaves open the interpretation of classical long ranged electro-weak gauge fields unavoidably present in all length scales when the dimension for the CP_2 projection of the space-time surface satisfies $D > 2$.

Option II: Second option involves several variants but the basic assumption is that nuclei or even quarks feed their Z^0 charges to a space-time sheet with size of order neutrino Compton length. The large parity breaking effects in hadronic, atomic, and nuclear length scales is not the only difficulty. The scattering of electrons, neutrons and protons in the classical long range Z^0 force contributes to the

Rutherford cross section and it is very difficult to see how neutrino screening could make these effects small enough. Strong isotopic effects in condensed matter due to the classical Z^0 interaction energy are expected. It is far from clear whether all these constraints can be satisfied by any assumptions about the structure of topological condensate.

Option III: During 2005 (27 years after the birth of TGD!) third option solving the problems emerged based on the progress in the understanding of the basic mathematics behind TGD.

In ordinary phase the Z^0 charges of elementary particles are indeed neutralized in intermediate boson length scale so that the problems related to the parity breaking, the large contributions of classical Z^0 force to Rutherford scattering, and large isotopic effects in condensed matter, trivialize.

Classical electro-weak gauge fields in macroscopic length scales are identified as space-time correlates for the gauge fields created by dark matter, which corresponds to a macroscopically quantum coherent phase for which elementary particles possess complex conformal weights such that the net conformal weight of the system is real.

In this phase $U(2)_{ew}$ symmetry is not broken below the scaled up weak scale except for fermions so that gauge bosons are massless below this length scale whereas fermion masses are essentially the same as for ordinary matter. By charge screening gauge bosons look massive in length scales much longer than the relevant p-adic length scale. The large parity breaking effects in living matter (chiral selection for bio-molecules) support the view that dark matter is what makes living matter living.

Classical long ranged color gauge fields always present for non-vacuum extremals are interpreted as space-time correlates of gluon fields associated with dark copies of hadron physics. It seems that this picture is indeed what TGD predicts.

8.1.4 Dark color force as a space-time correlate for the strong nuclear force?

Color confinement suggests a basic application of the basic criteria for the transition to large $\hbar$ phase. The obvious guess is that valence quarks are dark [J6, F9]. Dark matter phase for quarks does not change the lowest order classical strong interaction cross sections but reduces dramatically higher order perturbative corrections and resolves the problems created by the large value of QCD coupling strength in the hadronic phase.

The challenge is to understand the strong binding solely in terms of dark QCD with large value of $\hbar$ reducing color coupling strength of valence quarks to $v_0 \simeq 2^{-11}$. The best manner to introduce the basic ideas is as a series of not so frequently asked questions and answers.

Rubber band model of strong nuclear force as starting point

The first question is what is the vision for nuclear strong interaction that one can start from. The sticky toffee model of Chris Illert [21] is based on the paradox created by the fact alpha particles can tunnel from the nucleus but that the reversal of this process in nuclear collisions does not occur. Illert proposes a classical model for the tunnelling of alpha particles from nucleus based on dynamical electromagnetic charge. Illert is forced to assume that virtual pions inside nuclei have considerably larger size than predicted by QCD and the model. Strikingly, the model favors fractional alpha particle charges at the nuclear surface. The TGD based interpretation would be based on the identification of the rubber bands of Illert as long color bonds having exotic light quark and anti-quark at their ends and connecting escaping alpha particle to the mother nucleus. The challenge is to give meaning to the attribute "exotic".

How the darkness of valence quarks can be consistent with the known sizes of nuclei?

The assumption about darkness of valence quarks in the sense of of large $\hbar$ ($\hbar_s = \hbar/v_0$) is very natural if one takes the basic criterion for darkness seriously. The obvious question is how the dark color force can bind the nucleons to nuclei of ordinary size if the strength of color force is v_0 and color sizes of valence quarks are about $L(129)$?

It seems also obvious that $L(107)$ in some sense defines the size for nucleons, and somehow this should be consistent with scaled up size $L(k_{eff} = 129)$ implied by the valence quarks with large $\hbar$. The proposal of [J6, F9] inspired by RHIC findings [35] is that valence quarks are dark in the sense of having large value of $\hbar$ and thus correspond to $k_{eff} = 129$ whereas sea quarks correspond to ordinary value for $\hbar$ and give rise to the QCD size $\sim L(107)$ of nucleon.

If one assumes that the typical distances between sea quark space-time sheets of nucleons is obtained by scaling down the size scale of valence quarks, the size scale of nuclei comes out correctly.

Valence quarks and exotic quarks cannot be identical

The hypothesis is that nucleons contain or there are associated with them pairs of exotic quarks and flux tubes of color field bodies of size $\sim L(129)$ connecting the exotic quark and anti-quark in separate nuclei. Nucleons would be structures with the size of ordinary nucleus formed as densely packed structures of size $L(129)$ identifiable as the size of color magnetic body.

The masses of exotic quarks must be however small so that they must differ from valence quarks. The simplest possibility is that exotic quarks are not dark but p-adically scaled down versions of sea quarks with ordinary value of $\hbar$ having $k = 127$ so that masses are scaled down by a factor 2^{-10}.

Energetic considerations favor the option that exotic quarks associate with nucleons via the $k_{eff} = 111$ space-time sheets containing nucleons and dark quarks. Encouragingly, the assumption that nucleons topologically condense at the weak $k_{eff} = 111$ space-time sheet of size $L(111) \simeq 10^{-14}$ m of exotic quarks predicts essentially correctly the mass number of the highest known super-massive nucleus. Neutron halos are outside this radius and can be understood in terms color Coulombic binding by dark gluons. Tetraneutron can be identified as alpha particle containing two negatively charged color bonds.

What determines the binding energy per nucleon?

The binding energies per nucleon for $A \geq 4$ to not vary too much from 7 MeV but the lighter nuclei have anomalously small binding energies. The color bond defined by a color magnetic flux tube of length $\sim L(k = 127)$ or $\sim L(k_{eff} = 129)$ connecting exotic quark and anti-quark in separate nucleons with scaled down masses $m_q(dark) \sim xm_q$, with $x = 2^{-10}$ for option for $k = 127$, is a good candidate in this respect. Color magnetic spin-spin interaction would give the dominant contribution to the interaction energy as in the case of hadrons. This interaction energy is expected to depend on exotic quark pair only. The large zero point kinetic energy of light nuclei topologically condensed at $k_{eff} = 111$ space-time sheet having possible identification as the dark variant of $k = 89$ weak space-time sheet explains why the binding energies of D and ^{3}He are anomalously small.

What can one assume about the color bonds?

Can one allow only quark anti-quark type color bonds? Can one allow the bonds to be also electromagnetically charged as the earlier model for tetra-neutron suggests (tetra-neutron would be alpha particle containing two negatively charged color bonds so that the problems with the Fermi statistics are circumvented). Can one apply Fermi statistics simultaneously to exotic quarks and anti-quarks and dark valence quarks?

Option I: Assume that exotic and dark valence quarks are identical in the sense of Fermi statistics. This assumption sounds somewhat non-convincing but is favored by p-adic mass calculations supporting the view that the p-adic mass scale of hadronic quarks can vary. If this hypothesis holds true at least effectively, very few color bonds from a given nucleon are allowed by statistics and there are good reasons to argue that nucleons are arranged to highly tangled string like structures filling nuclear volume with two nucleons being connected by color bonds having of length of order $L(129)$. The organization into closed strings is also favored by the conservation of magnetic flux.

The notion of nuclear string is strongly supported by the resulting model explaining the nuclear binding energies per nucleon. It is essential that nucleons form what might be called nuclear strings rather than more general tangles. Attractive p-p and n-n bonds must correspond to colored ρ_0 type bonds with spin one and attractive p-n type bonds to color singlet pion type bonds. The quantitative estimates for the spin-spin interaction energy of the lightest nuclei lead to more precise estimates for the lengths of color bonds. The resulting net color quantum numbers must be compensated by dark gluon condensate, the existence of which is suggested by RHIC experiments [35]. This option is strongly favored by the estimate of nuclear binding energies.

Option II: If Fermi statistics is not assumed to apply in the proposed manner, then color magnetic flux tubes bonds between any pair of nucleons are possible. The identification of color isospin as strong isospin still effective removes color degree of freedom. As many as 8 color tubes can leave the nucleus if exotic quarks and anti-quarks are in the same orbital state and a cubic lattice like structure would become possible. This picture would be consistent with the idea that in ordinary field theory all particle pairs contribute to the interaction energy. The large scale of the magnetic flux tubes would suggest that the contributions cannot depend much on particle pair. The behavior of the binding energies favors strongly the idea of nuclear string and reduces this option to the first one.

What is the origin of strong force and strong isospin?

Here the answer is motivated by the geometry of CP_2 allowing to identify the holonomy group of electro-weak spinor connection as U(2) subgroup of color group. Strong isospin group SU(2) is identified as subgroup of isotropy group U(2) for space-time surfaces in a sub-theory defined by $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere of CP_2 and second factor of $U(1) \times U(1)$ subgroup of the holonomies for the induced Abelian gauge fields corresponds to strong isospin component I_3. The extremely tight correlations between various classical fields lead to the hypothesis that the strong isospin identifiable as color isospin I_3 of exotic quarks at the ends of color bonds attached to a given nucleon is identical with the weak isospin of the nucleon. Note that this does not require that exotic and valence quarks are identical particles in the sense of Fermi statistics.

Does the model explain the strong spin orbit coupling ($L \cdot S$ force)? This force can be identified as an effect due to the motion of fermion string containing the effectively color charged nucleons in the color magnetic field $v \times E$ induced by the motion of string in the color electric field at the dark $k = 107$ space-time sheet.

How the phenomenological shell model with harmonic oscillator potential emerges?

Nucleus can be seen as a collection of of long color magnetic flux tubes glued to nucleons with the mediation of exotic quarks and anti-quarks. If nuclei form closed string, as one expects in the case of Fermi statistics constraint, also this string defines a closed string or possibly a collection of linked and knotted closed strings. If Fermi statistics constraint is not applied, the nuclear strings form a more complex knotted and linked tangle. The stringy space-time sheets would be the color magnetic flux tubes connecting exotic quarks belonging to different nucleons.

The color bonds between the nucleons are indeed strings connecting them and the averaged interaction between neighboring nucleons in the nuclear string gives in the lowest order approximation 3-D harmonic oscillator potential although strings have $D = 2$ transversal degrees of freedom. Even in the case that nucleons for nuclear strings and thus have only two bonds to neighbors the average force around equilibrium position is expected to be a harmonic force in a good approximation. The nuclear wave functions fix the restrictions of stringy wave functionals to the positions of nucleons at the nuclear strings. Using M-theory language, nucleons would represent branes connected by color magnetic flux tubes representing strings whose ends co-move with branes.

Which nuclei are the most stable ones and what is the origin of magic numbers?

$P = N$ closed strings correspond to energy minima and their deformations obtained by adding or subtracting nucleons in general correspond to smaller binding energy per nucleon. Thus the observed strong correlation between P and N finds a natural explanation unlike in the harmonic oscillator model. For large values of A the generation of dark gluon condensate and corresponding color Coulombic binding energy favors the surplus of neutrons and the generation of neutron halos. The model explains also the spectrum of light nuclei, in particular the absence of pp, nn, ppp, and nnn nuclei.

In the standard framework spin-orbit coupling explains the magic nuclei and color Coulombic force gives rise to this kind of force in the same manner as in atomic physics context. Besides the standard magic numbers there are also non-standard ones (such as $Z, N = 6, 12$) if the maximum of binding energy is taken as a definition of magic, there are also other magic numbers than the standard ones. Hence can consider also alternative explanations for magic numbers. The geometric view about nucleus suggests that the five Platonic regular solids might defined favor nuclear configurations and it indeed turns that they explain non-standard magic numbers for light nuclei.

New magic nuclei might be obtained by linking strings representing doubly magic nuclei. An entire hierarchy of linkings becomes possible and could explain the new magic numbers 14, 16, 30, 32 discovered for neutrons [23]. Linking of the nuclear strings could be rather stable by Pauli Exclusion

Principle. For instance, ^{16}O would corresponds to linked ^4He and ^{12}C nuclei. Higher magic numbers $28, 50, ...$ allow partitions to sums of lower magic numbers which encourages to consider the geometric interpretation as linked nuclei. p-Adic length scale hypothesis in turn suggest the existence of magic numbers coming as powers of 2^3.

What about the description of nuclear reactions?

The identification of nuclei as linked and knotted strings filling the nuclear volume for constant nuclear density leads to a topological description for the nuclear reactions with simplest reactions corresponding to fusion and fission of closed nuclear strings. The microscopic description is in terms of nucleon collisions in which exotic quarks and anti-quarks are re-shared between nucleons and also new pairs are created. The distinction to ordinary string model is that the topological reactions for strings can occur only when the points at which where they are attached to nucleons collide.

The old fashioned description of the nuclear strong force is based on the meson exchange picture. The perturbation theory based on the exchange of pions doesn't however make sense in practice. In the hadronic string model this description would be replaced by hadronic string diagrams. The description of nuclear scattering in terms of nuclear strings allows phenomenological interpretation in terms of stringy diagrams but color bonds between nucleons do not correspond to meson exchanges but are something genuinely new.

8.1.5 Tritium beta decay anomaly

The proposed model explains the anomaly associated with the tritium beta decay. What has been observed [26, 27] is that the spectrum intensity of electrons has a narrow bump near the endpoint energy. Also the maximum energy E_0 of electrons is shifted downwards.

I have considered two explanations for the anomaly. The original models are based on TGD variants of original models [28, 29] involving belt of dark neutrinos or antineutrinos along the orbit of Earth. Only recently (towards the end of year 2008) I realized that nuclear string model provides much more elegant explanation of the anomaly and has also the potential to explain much more general anomalies [74, 42, 43].

8.1.6 Cold fusion and Trojan horse mechanism

Cold fusion [62] has not been taken seriously by the physics community but the situation has begun to change gradually. There is an increasing evidence for the occurrence of nuclear transmutations of heavier elements besides the production of ^4He and ^3H whereas the production rate of ^{3}He and neutrons is very low. These characteristics are not consistent with the standard nuclear physics predictions. Also Coulomb wall and the absence of gamma rays and the lack of a mechanism transferring nuclear energy to the electrolyte have been used as an argument against cold fusion.

An additional piece to the puzzle came when Ditmire *et al* [48] observed that the spectrum of electron energies in laser induced explosions of ion clusters extends up to energies of order MeV (rather than 10^2 eV!): this suggests that strong interactions are involved.

The possibility of charged color bonds explaining tetra-neutron allows to construct a model explaining both the observations of Ditmire *et al* and cold fusion and nuclear transmutations. 'Trojan horse mechanism' allows to circumvent the Coulomb wall, and explains various selection rules and the absence of gamma rays, and also provides a mechanism for the heating of electrolyte.

8.2 Model for the nucleus based on exotic quarks

The challenge is to understand the strong binding solely in terms of the color bonds and large value of $\hbar$ for valence quarks reducing color coupling strength to v_0 and scaling there sizes to $L(107)/v_0 = L(129)$. There are many questions to be answered. How exotic quarks with scaled down masses differ from dark valence quarks? How the model can be consistent with the known nuclear radii of nuclei if valence quarks have Compton length of order $L(129)$?

8.2.1 The notion of color bond

The basic notion is that of color bond having exotic quark and anti-quark at its ends. Color bonds connecting nucleons make them effectively color charged so that nuclei can be regarded as color bound states of nucleons glued together using color bonds.

The motivation for the notion of color bond comes from the hypothesis that valence quarks are in large $\hbar$ phase, and also from the ideas inspired by the work of Chris Illert [21] suggesting that long virtual pions act as "rubber bands" connecting nucleons to each other. There are indications that the quark distribution functions for the nucleons inside nuclei differ from those for free nucleons [18, 29]. QCD based estimates show that color van der Waals force is not involved [29]. The contribution of the quark pairs associated with color bonds is a possible explanation for this phenomenon.

8.2.2 Are the quarks associated with color bonds dark or p-adically scaled down quarks?

What seems clear is that color bonds with light quark and antiquark, to be referred as exotic quarks in the sequel, at their ends could explain strong nuclear force. Concerning the identification of the exotic quarks there are frustratingly many options. In lack of deeper understanding, the only manner to proceed is to try to make a detailed comparison of various alternatives in hope of identifying a unique internally consistent option.

The basic observation is that if four-momentum is conserved in the phase transition to the dark phase, the masses of quarks in large $\hbar$ phase should not differ from those in ordinary phase, which means that Compton lengths and p-adic length scale are scaled up by a factor $1/v_0$. This assumption explains elegantly cold fusion and many other anomalies [F9, J6]. The quarks at the ends of color bonds must however have scaled down masses to not affect too much the masses of nuclei. This option would also allow to identify valence and possibly also sea quarks as dark quarks in accordance with the general criterion for the transition to dark phase as proposed in the model for RHIC events [35, J6].

Exotic quarks must be light. Hence there should be some difference between exotic and valence quarks. This leaves two options to consider.

Are the exotic quarks p-adically scaled down versions of ordinary quarks with ordinary value of $\hbar$?

Exotic quarks could simply correspond to longer p-adic length scale, say M_{127} and thus having masses scaled down by a factor 2^{-10} but ordinary value of $\hbar$. One can also consider the possibility that they correspond to a QCD associated with M_{127} as proposed earlier. They could also correspond to their own weak length scale and weak bosons. This would resolve the objections against new elementary particles coming from the decay widths of intermediate gauge bosons even without assumption about the loss of asymptotic freedom implying that the QCD in question effectively exists only in finite length scale range.

p-Adic mass calculations indeed support the view to that hadronic quarks appear as several scaled up variants and there is no reason to assume that also scaled down variants could not appear. This hypothesis leads to correct order of magnitude estimates for the color magnetic spin-spin interaction energy.

For this option valence (and possibly also sea) quarks could be dark and have color sizes of order $L(k_{eff} = 129)$ as suggested by the criterion $\alpha_s Q_c^2 \simeq 1$ for color confinement as a transition to a dark phase.

Do exotic quarks correspond to large $\hbar$ and reduced c?

If valence quarks are dark one can wonder why not also exotic quarks are dark and whether there exists a mechanism reducing their masses by a factor v_0.

If one questions the assumption that $\hbar$ is a fundamental constant, sooner or later also the question "What about c?" pops up. There are indeed motivations for expecting that c has a discrete spectrum in a well-defined sense. TGD predicts an infinite variety of warped vacuum extremals defining imbeddings of M^4 to $M^4 \times CP_2$ with $g_{tt} = \sqrt{1 - R^2\omega^2}, g_{ij} = -\delta_{ij}$, and if common M^4 time coordinate is used for them the maximal signal velocity is for them given by $c_\#/c = \sqrt{1 - R^2\omega^2}$.

Physically this means that the time taken for light to travel between point A and B depends on what space-time sheet the light travels even in the case that gravitational and gauge fields are absent. The fact that the analog of Bohr quantization occurs for the deformed vacuum extremals of Kähler action suggests that $c_{\#}$ has a discrete spectrum.

This inspires the question whether also light velocity c besides $\hbar$ is quantized in powers of v_0 so that the rest energies of dark quarks would be given by $E_0 = \hbar_s c_{\#}/L(k_{eff} = k+22) = \hbar c_{\#}/L(k)$ and scale down because of the scaling $c \to v_0 \times c$. A distinction between rest mass and rest energy should be made since rest mass is scaled up as $M \to M/v_0$. Compton time would be by a factor $1/v_0^2$ longer than the ordinary Compton time.

If c and $\hbar$ can scale up separately but in powers of v_0 (or its harmonics and sub-harmonics) it is possible to have a situation in which $\hbar c$ remains invariant because mass scale is reduced v_0 and $\hbar$ is increased by $1/v_0$. In the case of dark quarks this would mean that light would propagate with velocity $2^{-11}c$ along various space-time sheets associated with dark quarks.

This admittedly complex looking option would mean that valence quarks have large $\hbar$ but ordinary c and exotic quarks have large $\hbar$ but small c due to the warping of their space-time sheet in time direction.

8.2.3 Electro-weak properties of exotic and dark quarks

Are exotic quarks scaled down with respect to electromagnetic interactions?

The earlier models involving large $\hbar$ rely on the assumption that the transition to large $\hbar$ phase with respect to electromagnetic interactions occurs only under special conditions (models for cold fusion and structure of water represent basic examples). Hence valence quarks can be in large $\hbar$ phase only with respect to strong and possibly weak interactions.

1. For p-adically scaled down exotic quarks also the electromagnetic space-time sheet should correspond to scaled up value of k since $k = 113$ would give too large contribution to the quark mass. It is not clear whether both em and color space-time sheets can correspond to $k = 127$ or whether one must have $k_{em} = 131$.

2. For exotic quarks with large $\hbar$ and small c the situation can be different $k = 107$ contribution to quark mass is scaled down by v_0 factor: $m_q(dark) = v_0 m_q \sim .05$ MeV. Since $k = 113$ contributes a considerable fraction to hadron mass, one can argue that also the $k = 113$ contribution to the mass must be scaled down so that dark quarks would be also electromagnetically dark. If so, the size of $k = 113$ dark electromagnetic field body would be of order atomic size and nuclei would represent in their structure also atomic length scale.

Are exotic and dark quarks scaled down with respect to weak interactions?

What about darkness of exotic and dark quarks with respect to weak interactions? The qualitative behavior of the binding energies of $A \leq 4$ nuclei can be understood if they possess zero point kinetic energy associated with space-time sheet with size characterized by $L(k = 111 = 3 \times 37) \simeq 10^{-14}$ m. Also the maximal mass number of super-heavy nuclei without neutron halo is predicted correctly. $k_{eff} = 111$ happens to correspond to the scaled weak length scale M_{89} which raises the possibility that dark quarks correspond to large value of $\hbar$ with respect to weak interactions. This could be the case for dark valence quarks and both identifications of exotic quarks.

1. For $k = 127$ quarks with dark weak interactions no large parity breaking effects are induced neither below mass scale m_W.

2. For large $\hbar$-small c option the scale invariance of gauge interactions would mean that the masses of the corresponding weak bosons are of order 50 MeV but the weak interaction rates of are scaled down by a factor v_0^2 since the ratios m_q/m_W invariant under the transition to dark phase appear in the rates: this at energy scale smaller than $v_0 m_w$. This disfavors this option.

8.2.4 How the statistics of exotic and ordinary quarks relate to each other?

Exotic and ordinary quarks should be identical or in some sense effectively identical in order that nuclear string picture would result.

Can one regard exotic quarks and ordinary quarks as identical fermions?

The first guess would be that this is not the case. One must be however cautious. The fact that p-adically scaled up variants of quarks appear in the model of hadrons suggested by p-adic mass calculations, suggests that the scaled up versions must be regarded as identical fermions. Since also the scaling of $\hbar$ induces only a scaling up of length scale, one might argue that this conclusion holds true quite generally.

Identity is also favored by a physical argument. If identity holds true, Fermi statistics forces the nucleons to form closed nuclear strings to maximize their binding energies. The notion nuclear string explains nicely the behavior nuclear binding energies per nucleon and also suggests that linking and knotting could define mechanisms for nuclear binding.

Could dark quarks and ordinary quarks be only effectively identical?

The idea of regarding quarks and dark quarks as identical fermions does not sound convincing, and one can ask the idea could make sense in some effective sense only.

1. The effective identity follows from a model for matter antimatter symmetry assuming that ordinary quarks form strongly correlated pairs with dark anti-quarks so that nucleons would be accompanied by dark antinucleons and quarks and dark quarks would be effectively identical. This option looks however rather science fictive and involves un-necessarily strong assumption.

2. A weaker hypothesis is inspired by the model of topological condensation based on # (/worm-hole/ topological sum) contacts [F6]. # contact can be modelled as a CP_2 type extremal with Euclidian signature of induced metric forming topological sum with the two space-time sheets having Minkowskian signature of induced metric. # contact is thus accompanied by two light-like 3-D causal horizons at which the metric determinant vanishes. These causal horizons carry of quantum numbers and are identified as partons. If the contact is passive in the sense that it mediates only gauge fluxes, the quantum numbers of the two partons cancel each other. This can be true also for four-momentum in the case that time orientations of the space-time sheets are opposite.

   This kind of # contacts between $k_{eff} = 129$ and $k = 127$ space-time sheets would force effective identity of $k = 127$ and $k_{eff} = 129$ quarks. The implication would be that in many-sheeted sense nucleons inside nuclei would have ordinary quantum numbers whereas in single sheeted point sense they would carry quantum numbers of quark or anti-quark.

8.3 Model of strong nuclear force based on color bonds between exotic quarks

In this section the color bond model of strong nuclear force is developed in more detail.

8.3.1 A model for color bonds in terms of color flux tubes

Simple model for color bond

Consider next a simple model for color bond.

1. The first guess would be that the color bond has quantum numbers of neutral pion so that also the pair of nucleons connected by a color bond would behave like a pion. This gives attractive color magnetic interaction energy and an attractive identification is as p-n bond.

2. Also the bonds with identical spins and identical color charges at the ends of the bond yield an attractive color magnetic spin-spin interaction energy. This kind of bonds would be responsible for p-p and n-n pairing. In this case color magnetic energy is however by a factor 1/3 smaller and could explain the non-existence of pp and nn bound states. An even number of neutral ρ type bonds could be allowed without anomalous contribution to the spin. High spin nuclei could contain many ρ type bonds so that antimatter would play important role in the physics of heavy nuclei.

3. A further generalization by allowing also electromagnetically charged color bonds with em quantum numbers of pion and ρ would explain tetra-neutron [59, 60] as alpha particle (pnpn) with two π_- type color bonds. This would predict a rich variety of exotic nuclei. Long color bonds connecting quark and anti-quark attached to different nucleons would also allow to understand the observation of Chris Illert [21] that the classical description of quantum tunnelling suggests that nucleons at the surface of nucleus have charges which are fractional.

This picture would suggest that the color isospin of the quark at the end of the bond equals to the weak isospin of the nucleon and is also identifiable as the strong isospin of the nucleon inside nucleus. To achieve an overall color neutrality the presence a dark gluon condensate compensating for the net color charge of colored bonds must be assumed. This could also compensate the net spin of the colored bonds.

The surplus of neutrons in nuclei would tend to create a non-vanishing color isospin which could be cancelled by the dark gluon condensate. The results of RHIC experiment [35] can be understood in TGD framework as a generation of a highly tangled string like structure containing large number of p-p and n-n type bonds and thus also dark gluon condensate neutralizing the net color charge. This would suggest that in a good approximation the nuclei could be seen as tangled string like structures formed from protons and neutrons. If the distances between nuclei are indeed what standard nuclear physics suggests, kind of nuclear strings would be in question.

Simple model for color magnetic flux tubes

Color magnetic flux tubes carrying also color electric fields would define the color magnetic body of the nucleus having size of order $L(129)$. Dark quarks would have also weak and electromagnetic field bodies with sizes $L(111)$ and $L(135)$. The color magnetic body codes information about nucleus itself but also has independent degrees of freedom, in particular those associated with linking and knotting of the flux tubes (braiding plays a key role in the models of topological quantum computation [E9]).

Color flux tubes carry a non-trivial color magnetic flux and one can wonder whether the color flux tubes can end of whether they form closed circuits. Since CP_2 geometry allows homological magnetic charges, color magnetic flux tubes could have ends with quarks and anti-quark at them acting as sources of the color magnetic field. The model for binding energies however favors closed strings. In the general case the color magnetic flux tubes would have a complex sub-manifold of CP_2 with boundary as a CP_2 projection.

The spin-spin interaction energies depend crucially on the value of the color magnetic field strength experienced by the exotic quark at the end of color flux tube, and one can at least try make educated guesses about it. The conservation of the color magnetic flux gives the condition $g_sB \propto 1/S$, where S is the area of the cross section of the tube. $S \geq L^2(107)$ is the first guess for the area if valence quarks are ordinary. $S \geq L^2(k_{eff} = 129)$ is the natural guess if valence quarks are dark.

The quantization of the color magnetic flux using the scaled up value of $\hbar$ would give $\int g_sBdS = n/v_0$ implying $g_sB \simeq n/v_0S$. When applied to $S \sim L^2(107)$ the quantization condition would give quite too large estimate for the spin-spin interaction energy. For $S \sim L^2(129)$ the scale of the interaction energy would come out correctly. For $k = 127$ option $S \sim L^2(127)$ is forced by the quantization condition.

This observation favors strongly dark valence quarks for both options. The magnetic flux of exotic quarks would be feeded to flux tubes of transverse area $\sim L^2(k)$, $k = 127$ or $k = 129$, coupling naturally with the color magnetic flux tubes of valence quarks with size $L(129)$.

A further constraint could come from the requirement that the flux tubes is such that locally the magnetic field looks like a dipole field. This would mean that the flux tube would become thicker at larger distances roughly as $S(r) \propto r^3$. An alternative restriction would come from the requirement that the energy of the color magnetic flux tube is same irrespective of its cross section at dark quark position. This would give $S \propto L$ where L is the length of the flux tube.

Quantum classical correspondence requires color bonds

Non-vacuum extremals are always accompanied by a non-vanishing classical electro-weak and color gauge fields. This is an obvious challenge for quantum classical correspondence. The presence of a suitable configuration of color bonds with dark quarks at their ends starting from nucleon gives hopes of resolving this interpretational problem. Dark quarks and anti-quarks would serve as sources of the color and weak electric gauge fluxes and quarks and nucleons would create the classical em field.

The requirement that classical Abelian gauge fluxes are equal to the quantum charges would pose very strong conditions on the physical states. For instance, quantization condition for Weinberg angle is expected to appear. The fact that classical fluxes are inversely proportional to the inverse of the corresponding gauge coupling strength $1/\alpha_i$ gives additional flexibility and with a proper choice of gauge coupling strengths the conditions might be satisfied and space-time description would also code for the values of gauge coupling strengths. Color bonds should be present in all length scales for non-vacuum extremals encouraging the hypothesis about the p-adic hierarchy of dark QCD type phases.

Identification of dark quarks and valence quarks as identical fermions forces the organization of nucleons to nuclear strings?

Quantum classical correspondence in strong form gives strong constraints on the construction. The model explaining the nuclear binding energies per nucleon strongly favors the option in which nucleons arrange to form closed nuclear strings. If dark quarks and ordinary valence quarks can be regarded as identical fermions this hypothesis follows as a prediction. Therefore this hypothesis, which admittedly looks ad hoc and might make sense only effectively (see the discussion below), deserves a detailed consideration.

Fermi statistics implies that the quark at the end of the color bond must be in a spin state which is different from the spin state of the nucleon (spin of d quark in the case of p=uud and u quark in the case of n) to allow local S-wave. For anti-quarks there are no constraints. Only d (u) quark with spin opposite to that of p (n) can be associated with p (n) end of the color bond. Hence at most five different bonds can begin from a given nucleon. In the case of proton $p_\downarrow$ they are give by $d_\uparrow\overline{d}_\downarrow$, $\overline{q}_\downarrow q_\uparrow$, $q = u, d$.

Only two bonds between given nuclei are possible as following examples demonstrate.

1. $p_\downarrow - n_\uparrow$: $d_\uparrow\overline{d}_\downarrow$, $\overline{u}_\downarrow u_\uparrow$.
2. $p_\downarrow - p_\uparrow$: $d_\uparrow\overline{d}_\uparrow$,$\overline{d}_\uparrow d_\uparrow$.

The experimentation with the rules in case of neutral color bonds supports the view that although branchings are possible, they do not allow more than $A = Z + N$ bonds. One example is 6 nucleon state with p at center connected by 5 bonds to p+ 4n at periphery and an additional bond connecting peripheral p and n. This kind of configuration could be considered as one possible configuration in the case of ^{6}Li and ^{6}He. It would seem that there is always a closed string structure with A bonds maximizing the color magnetic binding energy. The allowance of also charged color bonds makes possible to understand tetra-neutron as alpha particle with two charged color bonds.

The fact that neutron number for nuclei tends to be larger than proton number implies that the number of n-n type ρ bonds for stringy configurations is higher than p-p type bonds so that net color isospin equal equal to $I_3 = -(A - 2Z)$ is generated in case of stringy nuclei and is most naturally cancelled by a dark gluon condensate. Neutralizing gluon condensate allows neutron halo with a non-vanishing value of I_3.

8.3.2 About the energetics of color bonds

To build a more quantitative picture about the anatomy of the color bond it is necessary to consider its energetics. The assumption that in lowest order in $\hbar$ the binding energy transforms as rest energy under the p-adic scaling and scaling of $\hbar$ makes it easy to make order of magnitude estimates by scaling from the hadronic case.

Color field energy of the bond

At the microscopic level the harmonic oscillator description should correspond to the color energy associated with color bonds having u or d type quark and corresponding anti-quarks at their ends. For simplicity restrict the consideration in the sequel to electromagnetically neutral color bonds.

Besides spin-spin interaction energy and color Coulombic interaction energy there are contributions of color fields coded by the string tension $T_d = v_0 T$ of the color bond, where $T \simeq 1/\mathrm{GeV}^2$ is hadronic string tension. The energy of string with given length remains invariant in the combined scaling of $\hbar$, string tension, and length L of the color bond represented by color magnetic flux tube (which contain also color electric fields).

1. The mass of the color bonds between valence quarks assumed to have $\hbar_s = \hbar/v_0$ of length $L = xL(129)$ are given by $M(107) \sim x \times \hbar/L(107) \sim x \times .5$ GeV and correspond naturally to the energy scale of hadronic strong interactions.

2. The rest energy of the color bonds between $k = 127$ quarks with ordinary value of $\hbar$ having length $L = xL(127)$ are given by $M(127) \sim x \times \hbar/L(127) = 2^{-10}M(107)$ so that the order order of magnitude is $x \times .5$ MeV.

3. The rest energy of the color bonds between $k_{eff} = 129$ dark quarks with $c_{\#} = v_0c$ is given by the same expression. Note however that rest mass would be scaled up by a factor $1/v_0$.

The resulting picture seems to be in a dramatic conflict with the electromagnetic size of nucleus which favors the $L \sim L(107) < 2$ fm rather than $L \sim L(129)$ and which is smaller by a factor 2^{-11} and which favors also the notion of nuclear string. The resolution of the paradox is based on the notion of color magnetic body. Color bonds behave like color magnetic dipoles and bonds correspond to flux tubes of a topologically quantized dipole type color magnetic field having length of order $L(129) \simeq 5 \times 10^{-12}$ m connecting nucleons at distance $L < L(107)$.

Color magnetic spin-spin interaction energy, the structure of color bonds, and the size scale of the nucleus

Color magnetic spin-spin interaction allows to understand $\rho - \pi$ mass splitting in terms of color magnetic spin-spin interaction expected to give the dominating contribution to the nuclear binding energy. The quantitative formulation of this idea requiring consistency with p-adic mass calculations and with existing view about typical electromagnetic nuclear size scale fixed by the height of Coulomb wall leads to a rather unique picture about color magnetic bonds.

1. Questions

One can pose several questions helping to develop a detailed model for the structure of the color bond.

1. The contributions $k = 113$ and $k = 107$ space-time sheets to the mass squared are of same order of magnitude [F4]. The contributions to the mass squared add coherently inside a given space-time sheet. This requires that nucleonic space-time sheet are not directly connected by join along boundaries bonds and the assumption that color bond connect dark quarks is consistent with this. This means that it makes sense to estimate contributions to the mass squared at single nucleon level.

2. The contribution of color magnetic spin-spin interaction to the mass squared of nucleon can be regarded as coming from $k = 107$ space-time sheets as p-adic contribution but with a large value of $\hbar$. If $k = 107$ contribution would vanish, only a positive contribution to mass would be possible since the real counterpart Δm_R^2 of p-adic Δm^2 is always positive whereas $(m^2 + \Delta m^2)_R < m_R^2$ can hold true.

3. What has been said about color magnetic body and color bonds applies also to electromagnetic field body characterized by $k = 113$. The usual electromagnetic size of nucleus is defined by the relative distances of nucleons in M^4 can be much smaller than $L(113)$ so that the prediction for Coulomb wall is not reduced to the Coulomb potential at distance $L(113)$. Nucleon mass could be seen as due to p-adic thermodynamics for mass squared (or rather, conformal weight) with the real counterpart of the temperature being determined by p-adic length scale $L(113)$.

4. The model inspired by p-adic mass calculations [F4] forced the conclusion that valence quarks have join along boundaries bonds between $k = 107$ and $k = 113$ space-time sheets possibly feeding color fluxes so that closed flux loops between the two space-time sheets result. The counter intuitive conclusion was that roughly half of quark mass is contributed by the $k = 113$

space-time sheet which is by a scale factor 8 larger than the color size of quarks. If valence quarks are dark, scaled up $k = 107$ space-time sheet having $k_{eff} = 129$ becomes the larger space-time sheet, and the situation would not look so counter-intuitive anymore.

5. How the ends of the color bonds are attached to the $k = 113$ nucleon space-time sheets? The simplest assumption is that color bonds correspond to color magnetic flux tubes of length scale $L(129)$ starting at or being closely associated with $k = 107$ space-time sheets of nucleons. Hence the contribution to the mass squared would come from scaled up $k_{eff} = 129$ space-time sheet and add coherently to the dominating p-adic $k = 107$ contribution to the mass squared of nucleon.

6. If exotic quarks are $k = 127$ quarks with ordinary value of $\hbar$, one encounters the problem how their contributions can add coherently with $k_{eff} = 129$ color contribution to reduce the rest energy of nucleus. One possibility allowed by the appearance of harmonics of v_0 is that $\hbar$ is scaled up by $1/(2v_0) \simeq 2^{10}$ so that space-time sheets have same size or that p-adic additivity of mass squared is possible for effective p-adic topologies which do not differ too much from each other.

2. Estimate for color magnetic spin-spin interaction energy

Suppose the scaling invariance in the sense that the binding energies transform in the lowest order just like rest masses so that one can estimate the color magnetic spin-spin splittings from the corresponding splittings for hadrons without any detailed modelling. This hypothesis is very attractive predicts for both options that the scale of color magnetic spin-spin splitting is 2^{-n} times lower than for $\pi - rho$ system, where $n = 10$ for $n = 127$ option and $n = 11$ for $k_{eff} = 129$ option. For scaled down spin-spin interaction energy for π type bond is $E \sim .4$ MeV for $k = 127$ and $\sim .2$ MeV for $k_{eff} = 11$, which would mean that the bond is shorter than scaled up length $L(\pi)$ of color bond between valence quarks of pion.

The further assumption that color magnetic spin-spin interaction energy behaves as $\alpha_s/m_q^2L^3$, $L = x2^nL(\pi)$. This gives $E \simeq x^{-3}2^{-n}E(107)$. The value of x can be estimated from the requirement that the energy is of order few MeV. This gives $x \sim 10^{-1/3}$ for $k = 127$ option and $x \sim (20)^{-1/3}$ for $k_{eff} = 129$ option.

8.4 How the color bond model relates to the ordinary description of nuclear strong interactions?

How the notion of strong isospin emerges from the color bond model? What about shell model description based on harmonic oscillator potential? Does the model predict spin-orbit interaction? Is it possible to understand the general behavior of the nuclear binding energies, in particular the anomalously small binding energies of light nuclei? What about magic numbers? The following discussion tries to answer these questions.

8.4.1 How strong isospin emerges?

The notion of strong isospin is a crucial piece of standard nuclear physics. Could it emerge naturally in the transition to the phase involving dark quarks? Could the transition to color confined phase mean a reduction of color group as isotropy group of CP_2 type extremals representing elementary particles to $U(2)$ identifiable as strong isospin group. Could $U(1)_Y \times U(1)_{I_3}$ or $U(1)_{I_3}$ be identifiable as the Abelian holonomy group of the classical color field responsible for the selection of a preferred direction of strong isospin?

This picture would not mean breaking of the color symmetry at the configuration space level where it would rotate space-time surfaces in CP_2 like rigid bodies. Rather, the breaking would be analogous to the breaking of rotational symmetry of individual particles by particle interactions. Strong isospin would correspond to the isotropy group of the space-time surface and the preferred quantization direction to the holonomy group of the induced color gauge field. The topological condensation of quarks and gluons at hadronic and nuclear space-time surfaces would freeze the color rotational degrees of freedom apart from isotropies providing thus the appropriate description for the reduced color symmetries.

Mathematical support for the picture from classical TGD

There is mathematical support for the proposed view and closely relating to the long-standing interpretational problems of TGD.

1. CP_2 holonomy group is identifiable as U(2) subgroup of color group and well as electro-weak gauge group. Hitherto the possible physical meaning of this connection has remained poorly understood. $U(2)$ subgroup as as isotropies of space-time surfaces with $D = 2$-dimensional CP_2 projection, which belongs to a homologically non-trivial geodesic sphere S^2, and defines a sub-theory for which all induced gauge fields are Abelian and a natural selection of a preferred strong isospin direction occurs. Thus one might identify strong isospin symmetry as the $SU(2)$ subgroup of color group acting as the isotropy group of the space-time surface and strong isospin would not correspond to the group of isometries but to space-time isotropies.

2. Color isospin component of gluon field, em field and Z^0 field which corresponds to weak isospin, are proportional to each other for solutions having 2-dimensional CP_2 projection. In fact, both Z^0 and I_3 component of gluon field are proportional to the induced Kähler form with a positive coefficient. If the proposed quantum classical correspondence for color bonds holds true, this means that the signs of these charges are indeed correlated also for nucleon and quark/ anti-quark. The ratios of these charges are fixed for the extremals for which CP_2 projection is homologically non-trivial geodesic sphere S^2.

3. It is far from clear whether the classical Z^0 field can vanish for any non-vacuum extremals. If this is not the case, dark weak bosons would be unavoidable and strong isospin could be identifiable as color isospin and dark weak isospin. The predicted parity breaking effects need not be easily detectable since dark quarks would be indeed dark matter. An open question is whether some kind of duality holds true in the sense that either color field or vectorial part of Z^0 field could be used to describe the nuclear interaction. This duality brings in mind the $SO(4) \leftrightarrow SU(3)$ duality motivated by the number theoretical vision [A1, A2, A3, E2].

4. The minimal form of the quantum classical correspondence is that at least the signs of the I_3 and Y components of the color electric flux correlate with the dark quark at the end of color bond and the signs of the Z^0 field and Kähler field correlate with the sign of weak isospin and weak hyper-charge of nucleon. A stronger condition is that these classical gauge fluxes are identical with a proper choice of the values of gauge coupling strengths and that in the case of color fluxes the quark at the end of the bond determines the color gauge fluxes in the bond whereas electromagnetic would distribute freely between the bonds.

Correlation between weak isospin and color isospin

The weakest assumption motivated by this picture would be that the sign of color isospin correlates with the sign of weak isospin so that the quarks at the ends of color bonds starting from nucleon would have color isospin equal to the weak isospin of the nucleon:

$$I_{3,s} = I_{3,w} = I_{3,c} \quad .$$

This assumption would allow to interpret the attractive strong interaction between nucleons in terms of color magnetic interaction. p-n bond would be neutral π_0 type color singlet bonds. n-n and p-p bonds would have spin ± 1 and color isospin equal to strong iso-spin of $\rho_{\pm}$. Note that QCD type color singlet states invariant under $I_3 \rightarrow -I_3$ would not be possible. Color magnetic interaction mediated by the pion type color bond would be attractive for p and n since color isospins would be opposite sign but repulsive for pp and nn since color isospins would have same sign. The ρ type color bond with identical spins and color isospins I_3 would generate attractive interaction between identical nucleons. The color magnetic spin-spin interaction energy would be 3 times larger for π type bond so that the formation of deuterium as bound state of p and n and absence of pp and nn bound states might be understood.

It is not possible to exclude charged color bonds, and as will be found, their presence provides an elegant explanation for tetra-neutron [59, 60].

8.4.2 How to understand the emergence of harmonic oscillator potential and spin-orbit interaction?

Shell model based on harmonic oscillator potential and spin-orbit interaction provide rather satisfactory model of nuclei explaining among other things magic numbers.

Harmonic oscillator potential as a phenomenological description

It would be a mistake to interpret nuclear harmonic oscillator potential in terms Coulomb potential for the I_3 component of the classical gluon field having color isospin as its source. Interaction energy would have correct sign only for proton+quark/ anti-quark or neutron+quark/ anti-quark at the end of the color bond so that only neutrons or protons would experience an attractive force.

Rather, the harmonic oscillator potential codes for the presence of color Coulombic and color magnetic interaction energies and is thus only a phenomenological notion. Harmonic oscillator potential emerges indeed naturally since the nucleus can be regarded as a collection of nucleons connected by color flux tubes acting rather literally as strings. The expansion the interaction energy around equilibrium position naturally gives a collection of harmonic oscillators. The average force experience by a nucleon is expected to be radial and this justifies the introduction of external harmonic oscillator potential depending on A via the oscillator frequency.

At the deeper level the system could be seen as a tangle formed by bosonic strings represented by magnetic flux tubes connecting $k = 111$ space-time sheets containing dark quarks closely associated with nucleons. The oscillations of nucleons in harmonic oscillator potential induce the motion of dark quark space-time sheets play the role of branes in turn inducing motion of the ends of flux tubes fix the boundary values for the vibrations of the flux tubes. The average force experienced by nucleons around equilibrium configuration is expected to define radial harmonic force. This holds true even in the case of nuclear string.

In this picture $k = 111$ space-time sheets could contain the nucleons of even heaviest nuclei if the nucleon size is taken to be $2L(107)/3 \simeq 1.5$ fm. The prediction for the highest possible mass number without neutron halo, which is at radius $2.5 \times L(111)$, would be $A = 296$ assuming that nuclear radius is $R = 1.4$ fm. $A = 298$ is the mass number of the heaviest known superheavy nucleus [30] so that the prediction can be regarded as a victory of the model.

Could conformal invariance play a key role in nuclear physics?

The behavior the binding energies of $A \leq 4$ nucleons strongly suggest that nucleons are arranged to closed string like structure and have thus only two color bonds to the neighboring nucleons in the nuclear string. The thickness of the string at the positions of fermions defines the length scale cutoff defining the minimal volume taken by a single localized fermion characterized by given p-adic prime.

The conformal invariance for the sections of the string defined by color bonds is should allow a deeper formulation of the model in terms of conformal field theory. The harmonic oscillator spectrum for single particle states could be interpreted in terms of stringy mass squared formula $M^2 = M_0^2 + m_1^2 n$ which gives in good approximation

$$M = m_0 + \frac{m_1^2}{2M_0} n \ . \tag{8.4.1}$$

The force constant would be determined by M_0 which would be equal to nucleon mass.

Presumably this would bring to the mind of M-theorist nucleus as a system of A branes connected by strings. The restriction of the wave functional of the string consisting of portions connecting nucleons to each other at the junction points would be induced by the wave functions of nucleons. The bosonic excitations of the color magnetic strings would contribute to color magnetic energy of the string characterized by its string tension. This energy scale might be considerably smaller than the fermionic energy scale determined by the color magnetic spin-spin interaction.

Dark color force as the origin of spin-orbit interaction

The deviation of the magic numbers associated with protons ($Z = 2, 8, 20, 28, 40, 82$) and neutrons ($A - Z = 2, 8, 20, 28, 50, 82, 126$) from the predictions of harmonic oscillator model provided the motivation for the introduction of the spin orbit interaction V_{L-S} [29] with the following general form

$$V_{L-S}(r) = \bar{L}\cdot\bar{S}\frac{1}{r}(\frac{dV_s}{dr}+\frac{dV_I}{dr}\bar{\tau}_1\cdot\bar{\tau}_2) \ . \quad (8.4.2)$$

The interaction implies the splitting of (j,l,s) eigen states so that states $j, l = j\pm\frac{1}{2}$ have different energies. If the energy splitting is large enough, some states belonging to a higher shell come down and combine with the states of the lower shell to form a new shell with a larger magic number. This is what should happen for both proton and neutron single particle states.

The origin of spin-orbit interaction would be the classical color field created by the color isospin in p-p and n-n color bonds and dark gluons compensating the color charge. Spin-orbit interaction results in the atomic physics context from the motion of electrons in the electric field of the nucleus. The moving particle experiences in its rest frame a magnetic field $\overline{B}=\overline{v}\times\overline{E}$, which in the spherically symmetric case can by little manipulations can be cast into the form

$$\overline{B}=\frac{\overline{p}}{m}\times\frac{\overline{r}}{r}\frac{dV}{dr}=\overline{L}\frac{1}{m}\frac{1}{r}\frac{dV}{dr} \ .$$

The interaction energy is given by

$$E = -\overline{\mu}\cdot\overline{B}=-\frac{ge}{2m^2}\overline{S}\cdot\overline{L}\times\frac{1}{r}\frac{dV}{dr} \ . \quad (8.4.3)$$

Here magnetic magnetic moment is expressed in terms of spin using the standard definitions. g denotes Lande factor and equals in good approximation to $g=2$ for point like fermion.

Classical color field can be assumed to contain only I_3 component and be derivable from spherically symmetric potential. In the recent case the color bonds moving made from two quarks and moving with nuclear string experience the force.

The color magnetic moments of the quark and anti-quark are of same sign in for both ρ and π type bond so that the isospin component of the net color magnetic moment can be written as

$$\overline{\mu}_c = g\frac{g_s}{m_q(dark)}I_3\overline{S} \ . \quad (8.4.4)$$

Here g_s denotes color coupling constant, g is the Lander factor equal to $g=2$ in ideal case, and m is mass parameter. Since the color bond is color magnetic flux tube attached from its ends to dark quarks it seems that the mass parameter $m_q(dark)$ in the magnetic moment is that of dark quark and should be $m_q(dark)=v_0m_q$.

An additional factor of 2 is present because both quark and anti-quark of the bond give same contribution to the color moment of the bond. I_3 equals to the strong isospin of the nucleon to which the quark is attached and spin is opposite to the spin of this quark so that a complete correlation with the quantum numbers of the second nucleon results and one can effectively assign the spin orbit interaction with nucleons. The net interaction energy is small for spin paired states. The sign of the interaction is same for both neutrons and protons.

Using the general form of the spin orbit interaction potential in the non-relativistic limit, one can cast the $L-S$ interaction term in a the form

$$V_{L-S}(r) = \frac{16\pi\mu_c}{m_q(dark)}\bar{L}\cdot\bar{S}\frac{1}{r}\frac{dV_{I_3}}{dr} \ . \quad (8.4.5)$$

In the first order perturbation theory the energy change for (j,l,s) eigen state with $l=j+\epsilon\frac{1}{2}$, $\epsilon=\pm1$ and spherically symmetric electromagnetic gauge potential $V(r)$ [16] given by

$$\Delta E(j,l=j+\epsilon\frac{1}{2}) = \frac{4g_s^2}{m_q^2(dark)L^3}(N-P)c(l)\left[\epsilon(j+\frac{1}{2})+1\right] \ ,$$

$$c(l) = -\frac{4\pi L^3}{g_s(N-P)}\langle l|\frac{1}{r}\frac{dV_{I_3}}{dr}|l\rangle \ . \quad (8.4.4)$$

The coefficient $g_s/4\pi$ and factor $N-P$ have been extracted from the color gauge potential in the expression of $a(l)$ to get a more kinematical expression. $N-P$-proportionality is expected since the system has net nuclear color isospin proportional to $N-P$ neutralized by dark gluons which can be thought of as creating the potential in which the nuclear string moves. The constant $c(l)$ contains information about the detailed distribution of the color isospin. $c(l)$ depends also on the details of the model (the behavior of single particle radial wave function $R_{n,l}(r)$ in case of wave mechanical model and now on its analog defined by the wave function of nucleon induced by nuclear string). R denotes the nuclear charge radius.

The general order of magnitude of L is $L \sim L(129)$. What comes in mind first is the scaling $L \sim v_0^{-1}R_0$, $R_0 \sim (3/5)\times L(107) \simeq 1.5\times 10^{-15}$ m. This is not consistent with the fact that for light nuclei with $A \leq 4$ L decreases with A but conforms with the fact that spin-spin interaction energies which are very sensitive to L can depend only slightly on A so that L must be more or less independent on A. Assume $g_s^2/4\pi = .1$ and $m_q = m_u \sim .1$ GeV, $g=1$ in the formula for the color magnetic moment. By using $2\pi/L(107) \simeq .5$ GeV these assumptions lead to the estimate

$$\begin{aligned}\Delta E(j, l = j+\epsilon\frac{1}{2}) &= \frac{4g_s^2}{m_q^2 L^3(107)}(\frac{5}{3})^3 \times v_0 \times (N-P)\times c(l)\times \left[\epsilon(j+\frac{1}{2})+1\right] \\ &\simeq (\epsilon(j+\frac{1}{2})+1)\times (N-P)\times c(l)\times 2.9\ \ MeV\ \ .\end{aligned} \tag{8.4.3}$$

The splitting is predicted to be same for protons and neutrons and also the magnitude looks reasonable. If the dark gluons are at the center and create a potential which is gradually screened by the dark quark pairs, the sign of the spin-orbit interaction term is correct meaning that the contribution to binding energy is positive for $j+1/2$ state. In the case of neutron halo the unscreened remainder of the dark gluon color charge would define $1/r$ potential at the halo possibly responsible for the stability.

This estimate should be compared to the general estimate for the energy scale in the harmonic oscillator model given by $\omega_0 \simeq 41\cdot A^{-1/3}$ MeV [29] so that the general orders of magnitude make sense.

8.4.3 Binding energies and stability of light nuclei

Some examples are in order to see whether the proposed picture might have something to do with reality.

Binding energies of light nuclei

The estimate for the binding energies of light nuclei is based on the following assumptions.

1. Neglect the contribution of the string tension and dark gluon condensate to the binding energy.

2. Suppose that the number of bonds equals to A for $A \leq 4$ nuclei and that the the bonds are arranged to maximize color magnetic spin-spin interaction energy. A possible interpretation is in terms of a closed color magnetic flux tube connecting nucleons. The presence of close color magnetic flux tubes is necessary unless one allows homological color magnetic monopoles. This option favors the maximization of the number of n-p type bonds since their spin-spin interaction energy is 3 times higher than that for p-p and n-n type bonds. This is just a working hypothesis and would mean that nuclei could be seen as nuclear strings.

   The alternative interpretation is that the number bonds per nucleon is constant so that the binding energy would not depend on nucleon. The number of bonds could be quite large. Scaling the c quark mass of about 4 GeV gives gives dark mass of about 2 MeV so that two dark generations might be possible. For two dark quark generations 8+8 different quarks can appear at the ends of color flux tubes and 64 different color bonds are in principle possible (which brings in mind the idea of nuclear genetic code and TGD proposal for quantum computation utilizing braided flux tubes!). Also in this case the bond energy can depend on whether p or n is question for $P \neq N$ nuclei since p-p and n-n bonds have smaller bind energy than p-n type bonds.

3. Assume that the nucleons are topologically condensed at $k = 111$ space-time sheet with zero point kinetic energy

$$E_0(A) \sim \frac{3n}{2}\frac{\pi^2}{Am_pL^2(111)} \equiv \frac{n}{A}\times E_0(A=1) \ ,$$

where n is a numerical factor and $E_0(A=1) \simeq 23$ MeV. Let ΔE denote the color magnetic spin-spin interaction energy per nucleon for π type bond. The zero point kinetic energy is largest for $A \leq 3$ and explains why the binding energy is so small. For $n = 1$ the zero point kinetic energy would be 5.8 MeV for $A = 4$, 7.7 MeV for $A = 3$, and 11.5 MeV for $A = 2$.

With these assumptions the binding energy per bond can be written for $A \leq 4$ as

$$E = r\times \Delta E - \frac{nE_0(p)}{A^2} \ ,$$

where Δ denotes the color magnetic spin-spin interaction energy per bond. The parameter r codes for the fact that color magnetic spin-spin interaction energy depends on whether p-p or n-n type bond is in question. The values of r are $r(^4He) = 1$,$r(^3He) = 7/9$, $r(^2H) = 1$.

Estimates for n and Δ can be deduced from the binding energies of ^2H and ^{3}He . The result is $n = 1.0296$ and $\Delta E = 7.03$ MeV. The prediction for ^{4}He biding energy is 6.71 MeV which is slightly smaller than the actual energy 7.07 MeV. The value of the binding energy per nucleon is in the range 7.4-8.8 MeV for heavier nuclei which compares favorably with the prediction 7.66 MeV at the limit $A \rightarrow \infty$. The generation of dark gluon condensate and color Coulombic energy per nucleon increasing with the number of nucleons could explain the discrepancy.

(A,Z)	(2,1)	(3,1)	(3,2)	(4,2)
E_B/MeV	1.111	2.826	2.572	7.0720

Table 1. The binding energies per nucleon for the lightest nuclei.

Why certain light nuclei do not exist?

The model should also explain why some light nuclei do not exist. In the case of proton rich nuclei electromagnetic Coulomb interaction acts as un-stabilizer. For heavy nuclei with non-vanishing value of $P - N$ the positive contribution of dark gluons to the energy tends to in-stabilize the nuclei. The color Coulombic interaction energy is expected to behave as $(N - P)^2$ whereas the energy of dark gluons behaves as $|N - P|$. Hence one expects that for some critical value of $|N - P|$ color Coulombic interaction is able to compensate the contribution of dark gluon energy. One the other hand, the larger number of nn type bonds tends reduced the color magnetic spin-spin interaction energy.

1. Coulomb repulsion for pp is estimated to be .76 MeV from ^{3}He -^{3}H mass difference whereas the color magnetic binding energy would be $E_D/3 = .74$ MeV from the fact that the energy of ρ type bond is 1/3 from that for π type bond. Hence pp bound state would not be possible. The fact that nn bound state does not exist, suggests that the energy of the color neutralizing dark gluon overcomes the color Coulombic interaction energy of dark gluon and dark quarks and spin-spin interaction energy of ρ_0 type bond.

2. For ppp and nnn protons cannot be in S wave. The color magnetic bond energy per nucleon would be predicted to be $E_D = 2.233$ MeV whereas a rough order of magnitude estimate for Coulombic repulsion as

$$E_{em} = Z(Z-1)\times \left[E_B(^3H) - E_B(^3He)\right] = Z(Z-1)\times .76 \ \ MeV$$

gives $E_{em} \simeq 4.56$ MeV so that ppp bound state is not possible. nnn bound state would not be possible because three dark gluons would not be able to create high enough color Coulomb interaction energy E_c which together with color magnetic spin-spin interaction energy E_D would compensate their own negative contribution $3E_g$:

$$3E_g > E_D + E_c \ .$$

3. For pppp and nnnn Fermi statistics forces two nucleons to higher partial waves so that the states are not stable. Tetraneutron need not correspond to nnnn state in TGD framework but has more natural interpretation as an alpha particle containing two negatively charged dark quark pairs.

8.4.4 Strong correlation between proton and neutron numbers and magic numbers

The estimates for the binding energies suggest that nucleons arrange into closed nuclear strings in which nucleons are connected by long color magnetics with one dark quark anti-quark pair per nucleon. Nuclear string approach allows to understand the strong correlation between proton and neutron numbers as well as magic numbers.

Strong correlation between Z and N

$N = Z$ nuclei with maximal color magnetic spin-spin interaction energy arranged into closed nuclear strings contain only colored π type bonds between p and n and should be especially stable. The question is how to create minimum energy configurations with $N \neq Z$.

1. If only stringy configurations are allowed, the removal of the proton would create ρ type n-n bond and lead to a reduction of binding energy per nucleon. This would predict that (Z,Z) type isotopes correspond to maxima of binding energy per nucleon. The increase of the Coulombic energy disfavors the removal of neutrons and addition of protons.

2. For a given closed string structure one can always link any given proton by $\overline{u}u$ bond to neutron and by $\overline{d}d$ bonds to two protons (same for neutron). The addition of only neutron to a branch from proton gives nuclei (Z,N=Z+k), $k = 1, ..., Z$, having only π type bonds. In a similar manner nuclei with $(Z + k, Z)$, $k = 1, .., Z$, containing only π type bonds are obtained. This mechanism would predict isotopes in the ranges (Z,Z)-(Z,2Z) and (Z,Z)-(2Z,Z) with the same strong binding energy per nucleon apart from increase of the binding energy caused by the generation of dark gluon condensate which in the case of protons seems to be overcome by Coulomb repulsion. Very many of these isotopes are not observed so that this mechanism is not favored.

Consider how this picture compares with experimental facts.

1. Most $Z = N$ with $Z \leq 29$ nuclei exist and are stable against strong decays but can decay weakly. The interpretation for the absence of $Z > 29$ $Z = N$ nuclei would be in terms of Coulomb repulsion. Binding energy per nucleon is usually maximum for $N = Z$ or $N = Z + 1$ for nuclei lighter than Si. The tendency $N > Z$ for heavier nuclei could be perhaps understood in terms of the color Coulombic interaction energy of dark gluon condensate with color charges in n-n type color bonds. This would allow also to understand why for $Z = 20$ all isotopes with $(Z = 20, N > 20)$ have higher binding energy per nucleon than $(Z = 20, N = 20)$ isotope in conflict with the idea that doubly magic nucleus should have a maximal binding energy.

   The addition of neutrons to ^{40}Ca nucleus, besides increasing the binding energy per nucleon, also decreases the charge radius of the nucleus contrary to the expectation that the radius of the nucleus should be proportional to $A^{1/3}$ [29]. A possible interpretation is in terms of the color Coulombic interaction energy due to the generation of dark gluon condensate, the presence of which reduces the equilibrium charge radius of the nucleus.

2. ^{8}Be having $(Z, N) = (4, 4)$ decaying by alpha emission (to two alpha particles) is an exception to the rule. The binding energy per nucleon 7.0603 MeV of Be is slightly lower than the binding energy 7.0720 MeV of alpha particle and the pinching of the Be string to form two alpha strings could be a possible topological decay mechanism.

Magic nuclei in shell model and TGD context

Spin-spin pairing for identical nucleons in the harmonic oscillator potential is an essential element of the harmonic oscillator model explaining among other things shell structure and lowest magic numbers 2, 8, 20 but failing for higher magic numbers 28, 50, 82, 126 (the prediction is 2,8,20 and 40, 68, 82, 122). Spin-orbit coupling [22] reproduces effectively the desired shell structure by drawing some states of the higher shell to the lower shell, and it is indeed possible to reproduce the magic numbers in this manner for 3-D harmonic oscillator model.

This picture works nicely if magic nuclei are identified as nuclei which have exceptionally high abundances. ^{28}Fe, the most abundant element, is however an exception to the rule since neither Z nor N are magic in this case. The standard explanation for the stable nuclei of this kind is as endpoints of radioactive series. This explanation does not however remove the problem of understanding their large binding energy, which is after all what matters.

The surprise of recent years has been that even for neutron rich unstable nuclei 28 appears as a magic number for unstable neutrons in very neutron rich nuclei such as Si(14,28) [23] so that the notion of magic number does not seem to be so dependent on spin-orbit interactions with the nuclear environment as believed. Also new magic numbers such as N=14,16,30,32 have been discovered in the neutron sector [23]. Already the stable isotope Mg(12,14) has larger binding energy per nucleon than doubly magic Mg(12,12) and could be perhaps understood in terms of dark gluons. ^{56}Fe and ^{58}Fe correspond to N=30 and 32. The linking of two N=8 magic nuclei would give N=16 and various linkings of N=14 and N=16 nuclei would reduce the stability N = 28, 30, 32 magic nuclei to the stability of their building blocks. Perhaps these findings could provide motivations for considering whether the stringy picture might provide an alternative approach to understanding of the magic numbers.

1. The identification of magic nuclei as minima of binding energy predicts new magic numbers

The identification of the magic nuclei as minima of the binding energy as function of Z and N provides an alternative definition for magic numbers but this would predict among other things that also $Z = N = 4, 6, 12$ also correspond to doubly magic nuclei in the sense that $E_b(^8Be) = 7.0603)$, $E_b(^{12}C) = 7.677$ MeV and $E_B(^{24}Mg) = 8.2526$ are maxima for the binding energy per nucleon as a function of Z and N. For higher nuclei addition of neutrons to a doubly magic nucleus typically increases the binding energy up to some critical number of added neutrons (the generation of the dark gluon condensate would explain this in TGD framework). The maximum for the excitation energy of the first excitation seems to be the definition of magic in the shell model.

2. Platonic solids and magic numbers

The TGD picture suggest that light magic nuclei could have a different, purely geometric, interpretation in terms of five regular Platonic solids. $Z = N = 4, 6, 8, 12, 20$ could correspond to tedrahedron, octahedron (6 vertices), hexahedron (8 vertices), dodecahedron (12 vertices), and icosahedron (20) vertices. Each vertex would contain a bonded neutron and proton in the case of doubly magic nucleus. This model would predict correctly all the maxima of the binding energy per nucleon for $Z, N \leq 20$.

3. p-Adic length scale hypothesis and magic numbers

$Z = N = 8$ could be also interpreted as a maximal number of nucleons which $k = 109$ space-time sheet associated with dark quarks can contain. p-Adic length scale hypothesis would suggest that strings with length coming as p-adic length scale $L(k)$ are especially stable. Strings with thickness $L(109)$ would correspond to Z=N=2 for length $L = L(109)$, $L(k = 109 + 2n)$ would correspond to $Z = 2^{n+1}$ explaining $N = 2, 8, 16, 32$.

4. Could the linking of magic nuclei produce new magic nuclei?

Nuclear strings can become knotted and linked with fermion statistics guaranteing that the links cannot be destroyed by a 3-dimensional topological transition.

An interesting question is whether the magic numbers $N = 14, 16, 30, 32$ could be interpreted in terms of lower level magic numbers: 14=8+6,16=8+8, 30=16+40,32=16+16. This would make sense if $k = 111$ space-time sheets containing $Z, N \leq 4, 6, 8$ neutrons and protons define basic nucleon clusters forming closed nuclear strings. The linking these structures could give rise to higher magic nuclei whose stability would reduce that of the building blocks, and it would be possible to interpret magic number $Z,N = 28 = 20+8$ as linked lower level magic nuclei.

The partitions 28=20+8, 50=20+2+28 = 20+2+8+20, 82=50+2+28, 126=50+50+20+6=82+28+8+8 inspire the question whether higher doubly magic nuclei and their deformations could correspond to linked lower level magic nuclei so that a linking hierarchy would result.

Could the transition to the electromagnetically dark matter cause the absence of higher shells?

Spin orbit coupling explains the failure of the shell model as an explanation of the magic numbers. Transition to electromagnetic dark matter at critical charge number $Z = 12$ suggests an alternative explanation for the failure in the case of protons. The phase transition of Pd nuclei (Z=46) to electromagnetically dark nuclear phase inducing in turn the transition of D nuclei to dark matter phase has been proposed as an explanation for cold fusion [J6].

On basis of $Z^2\alpha_{em} \simeq 1$ criterion $Z = 12$ would correspond to the critical value for the nuclear charge causing this transition. One can argue that due to the Fermi statistics nuclear shells behave as weakly interacting units and the transition occurs for the first time for $Z = 20$ nucleus, which corresponds to Ca, one of the most important ions biologically and neurophysiologically. These necessarily completely filled structures would become structural units of nuclei at electromagnetically dark level.

An alternative interpretation is that the criterion to dark matter phase applies only to a pair of two systems and reads thus $Z_1Z_2\alpha_{em} \simeq 1$ implying that only the nuclei $Z,N \geq 40$ can perform the transition to the dark phase (what this really means is an interesting question). This would explain why Pd with $Z = 46$ has so special role in cold fusion.

Interestingly, the number of protons at $n = 2$ shell of harmonic oscillator is $Z = 12$ and thus corresponds to a critical value for em charge above which a transition to an electromagnetic dark matter phase increasing the size of the electromagnetic $k = 113$ space-time sheet of nucleus by a factor $\simeq 2^{11}$ could occur. This could explain why $n = 2$ represents the highest allowed harmonic oscillator shell with higher level structures consisting of clusters of $n < 3$ shells. Neutrons halos could however allow higher shells.

Could only the hadronic space-time sheet be scaled up for light nuclei?

The model discussed in this chapter is based on guess work and leaves a lot of room for different scenarios. One of them emerged only after a couple of months finishing the work with this chapter.

1. Is only the $\hbar$ associated with hadronic space-time sheet large?

The surprising and poorly understood conclusion from the p-adic mass calculations was that the p-adic primes characterizing light quarks u,d,s satisfy $k_q < 107$, where $k = 107$ characterizes hadronic space-time sheet [F4].

1. The interpretation of $k = 107$ space-time sheet as a hadronic space-time sheet implies that quarks topologically condense at this space-time sheet so that $k = 107$ cannot belong to the collection of primes characterizing quark.

2. Since hadron is expected to be larger than quark, quark space-time sheets should satisfy $k_q < 107$ unless $\hbar$ is large for the hadronic space-time sheet so that one has $k_{eff} = 107 + 22 = 129$. This would predict two kinds of hadrons. Low energy hadrons consists of u, d, and s quarks with $k_q < 107$ so that hadronic space-time sheet must correspond to $k_{eff} = 129$ and large value of $\hbar$. One can speak of confined phase. This allows also $k = 127$ light variants of quarks appearing in the model of atomic nucleus. The hadrons consisting of c,t,b and the p-adically scaled up variants of u,d,s having $k_q > 107$, $\hbar$ has its ordinary value in accordance with the idea about asymptotic freedom and the view that the states in question correspond to short-lived resonances.

This picture is very elegant but would mean that it would be light *hadron* rather than quark which should have large $\hbar$ and scaled up Compton length. This does not affect appreciably the model of atomic nucleus since the crucial length scales $L(127)$ and $L(129)$ are still present.

2. Under what conditions quarks correspond to large $\hbar$ phase?

What creates worries is that the scaling up of $k = 113$ quark space-time sheets of quarks forms an essential ingredient of condensed matter applications [F9] assuming also that these scaled up space-time sheets couple to scaled up $k = 113$ variants of weak bosons. Thus one must ask under what conditions $k = 113$ quarks, and more generally, all quarks can make a transition to a dark phase accompanying a simultaneous transition of hadron to a doubly dark phase.

The criterion for the transition to a large $\hbar$ phase at the level of valence quarks would require that the criticality criterion is satisfied at $k = 111$ space-time sheet and would be expressible as $Z^2\alpha_{em} = 1$ or some variant of this condition discussed above.

The scaled up $k = 127$ quark would correspond to $k = 149$, the thickness of the lipid layer of cell membrane. The scaled up hadron would correspond to $k = 151$, the thickness of cell membrane. This would mean that already the magnetic bodies of hadrons would have size of cell membrane thickness so that the formation of macroscopic quantum phases would be a necessity since the average distance between hadrons is much smaller than their Compton length.

8.4.5 A remark about stringy description of strong reactions

If nucleons are arranged into possibly linked and knotted closed nuclear strings, nuclear reactions could be described in terms of basic string diagrams for closed nuclear strings.

The simplest fusion/fission reactions $A_1 + A_2 \leftrightarrow (A_1 + A_2)$, $A_i > 2$, could correspond to reactions in which the $k = 111$ dark space-time sheets fuse or decay and re-distribution of dark quarks and anti-quarks between nucleons occurs so that system can form a new nucleus or decay to a new nuclei. This also means re-organizes the linking and knotting of the color flux tubes.

The reactions $p/n + A \rightarrow ..$ would involve the topological condensation of the nucleon to $k = 111$ space-time sheet after which it can receive quark anti-quark pair, which can be also created by dark gluon emission followed by annihilation to a dark quark pair.

8.4.6 Nuclear strings and DNA strands

Nuclear strings consisting of protons and neutrons bring in mind bit arrays. Their dark mirror counterparts in turn brings in mind the structure of DNA double strand. This idea does not look so weird once one fully accepts the hierarchies associated with TGD. The hierarchy of space-time sheets quantified by p-adic fractality, the hierarchy of infinite primes representable as a repeated second quantization of a super-symmetric arithmetic quantum field theory, the self hierarchy predicted by TGD inspired theory of consciousness, the Jones inclusion hierarchy for von Neumann factors of type II_1 appearing in quantum TGD and allowing to formulate what might be called Feynman rules for cognition, and the hierarchy of dark matters would all reflect the same reflective hierarchy.

The experience with DNA suggests that nuclear strings could form coiled tight double helices for which only transversal degrees of freedom would appear as collective degrees of freedom. DNA allows a hierarchy of coilings and DNA molecules can also link and this could happen also now. Nuclei as collections of linked nuclear strings could perhaps be said to code the electromagnetic and color field bodies and it is difficult to avoid the idea that DNA would code in the same manner field bodies at which matter condenses to form much larger structures. The hierarchy of dark matters would give rise to a hierarchy of this kind of codings.

The linking and knotting of string like structures is the key element in the model of topological quantum computation and the large value of $\hbar$ for dark matter makes it ideal for this purpose. I have already earlier proposed a model of DNA based topological quantum computation inspired by some strange numerical co-incidences [E9]. If dark matter is the essence of intelligent and intentional life at the level of molecular physics, it is difficult to see how it could not serve a similar role even at the level of elementary particle physics and provide kind of zoomed up "cognitive" representation for the ordinary matter.

The precise dark-visible correspondence might fail at the level of nuclei and nucleons because the lifetimes of the scaled down dark matter nucleons and nuclei are different from those of ordinary nucleons if dark matter is dark also with respect to weak interactions. The weak interaction rates in the lowest order are scaled up by the presence of $1/m_W^4$ factors by a factor 2^{-44} so that weak interactions are not so weak anymore. If dark electron and neutrino have their ordinary masses, dark proton and neutron would be stable. If also they appear as scaled down versions situation changes, but only a small change of the mass ratio of dark proton and neutron can make the weak decay of free dark neutron impossible kinematically and the one-to-one correspondence would make sense for stable nuclei. The beta decays of dark nuclei could however as a third order process with a considerable rate and change dramatically the weak decay rates of dark nuclei.

8.5 Neutron halos, tetra-neutron, and "sticky toffee" model of nucleus

Neutron halos and tetra neutron represent two poorly understood features of nuclear physics which all have been seen as suggesting the existence of an unknown long range force or forces.

8.5.1 Tetraneutron

There is evidence for the existence of tetra-neutrons [59]. Standard theory does not support their existence [60] so that the evidence for them came as a complete experimental surprise. Tetra-neutrons are believed to consist of 4 neutrons. In particular their lifetime, which is about 100 nanoseconds, is almost an eternity in the natural time scale of nuclear physics. The reason why the existing theory of nuclear force does not allow tetra-neutrons relates to Fermi statistics: the second pair of neutrons is necessarily in a highly energetic state so that a bound state is not possible.

Exotic quarks and charged color bonds provide perhaps the most natural explanation for tetra-neutron in TGD framework. In the model discussed hitherto only electromagnetically neutral color bonds have been considered but one can consider also charged color bonds in analogy allowing instead of neutral π and ρ also their charged companions. This would make possible to construct from two protons and neutrons the analog of alpha particle by replacing two neutral color bonds with negatively charged bonds so that one would have two $\overline{u}d$ p-n bonds and two $\overline{u}u$ p-n bonds. Statistics difficulty would be circumvented and the state would decay to four neutrons via W boson exchange between quark of charged p-n bonds and protons. The model suggests the existence of also neutral variant of deuteron.

One can consider two options according to whether the exotic quarks have large $\hbar$ but small c (Option II) or whether they are just p-adically scaled up quarks with $k = 127$ (Option I). I have considered earlier a model analogous to option II but based on the hypothesis about existence of scaled down variant of QCD associated with Mersenne prime M_{127}. The so called leptohadron physics would also be associated with M_{127} and involve colored excitations of leptons [F7] which might also represent dark matter: in this case dark valence leptons with color would correspond to $keff = 149$, which happens to correspond to the thickness of the lipid layer of cell membrane.

The notion of many-sheeted space-time predicts the possibility of fractal scaled up/down versions of QCD which, by the loss of asymptotic freedom, exist only in certain length scale range and energy range. Thus the prediction does not lead to contradictions elementary particle physics limits for the number of colored elementary particles. The scaled up dark variants of QCD like theory allow to circumvent these problems even when asymptotic freedom is assumed.

In particular, pions and other mesons could exist for $k = 127$ option as scaled down versions having much smaller masses. This lead to the earlier model of tetra-neutron as an ordinary alpha particle bound with two exotic pions with negative charges and having very small masses. This state looks like tetra-neutron and decays to neutrons weakly. The statistics problem is thus circumvented and the model makes precise quantitative predictions.

8.5.2 The formation of neutron halo and TGD

One counter argument against TGD inspired nuclear model is the short range of the nuclear forces: the introduction of the p-adic length scale $L(113) \simeq 1.6E^{-14}$ m is in conflict with this classical wisdom. There exists however direct evidence for the proposed length scale besides the evidence from the p-n low energy scattering. Some light nuclei such as ^8He, ^{11}Li and ^{11}Be possess neutron halo with radius of size $\sim 2.5E^{-14}$ m [61]. The width of the halo is rather large if the usual nuclear length scale is used as unit and the neutrons in the halo seem to behave as free particles. The short range of the nuclear forces makes it rather difficult to understand the formation of the neutron halo although the existing models can circumvent this difficulty. The proposed picture of the nucleus suggests a rather simple model for the halo.

For ordinary nuclei the densities of nucleons tend to be concentrated near the center of the nucleus. One can however consider the possibility of adding nucleons in vicinity of the boundary of the $k = 111$ space-time sheet associated with the nucleus itself. The binding force would be color interaction between the color charges of color bonds and neutralizing color charge of colored gluons in the center (or in halo itself). Neutron halo would define a separate nucleus in the sense that states could be constructed by starting from the ground state. Halo would correspond to a quantum delocalized cluster of size of alpha particle.

The case ^{11}Be provides support for the theory. Standard shell model suggests that six neutrons of ^{11}Be fill completely $1s_{\frac{1}{2}}$ and $1p_{\frac{3}{2}}$ states while $1p_{\frac{1}{2}}$ state holds one neutron so that ^{11}Be ground state has $J^{\pi} = \frac{1}{2}^{-}$ whereas experimentally ground state is known to have $J^{\pi} = \frac{1}{2}^{+}$. The system can be regarded as ^{10}Be + halo neutron. The first guess is that the state could be simply of the following form

$$|0^+\rangle \times |2s_{1/2}\rangle \ . \tag{8.5.1}$$

Color force would stabilize this state. A more general state is a superposition of higher $ns_{1/2}$ states in order to achieve more sharp localization near boundary. This increases the kinetic energy of the neutron and the small binding energy of the halo neutron about 2.5 MeV implies that the kinetic energy should be of order $5-6$ MeV. For instance, in the model described in [25] the halo neutron property and correct spin-parity for ^{11}Be can be realized if the state is superposition of form

$$\begin{aligned} |^{11}Be\rangle &= a|0^+\rangle \times |2s_{1/2}\rangle + ba|2^+\rangle \times |21d_{5/2}\rangle \ , \\ a &\simeq .74 \ , \\ b &\simeq .63 \ . \end{aligned} \tag{8.5.0}$$

The correlation between the core and and halo neutron is necessary in the model of [25] to produce bound $1/2^+$ state. The halo neutron must also rotate.

The second example is provided by two-neutron halo nuclei, such as ^{11}Li and ^{12}Be, which do not bind single neutron but bind two neutrons. This looks mysterious since free neutrons do not allow bound states. A possible explanation is that the increase of the color Coulombic interaction energy of neutron color bonds with at least N-P dark gluons makes possible binding of neutron halo to the center nucleus. The situation would be analogous to the formation of planetary system. Order of magnitude estimate for color Coulombic interaction energy of halo neutron is $E \sim (N-P)\alpha_s/L(113) \simeq (N-P) \times .8$ MeV. For $N-P=3$ the binding energy would be about 2.3 MeV and smaller than the experimental estimate 2.5 MeV. For $N-P=4$ this gives 3.2 MeV and larger than 2.5 MeV so that there is some room for the reduction of binding energy by the contribution from kinetic energy.

8.5.3 The "sticky toffee" model of Chris Illert for alpha decays

Chris Illert [21] has proposed what he calls "sticky toffee" model of alpha decay. The starting point of the work is a criticism of the wave-mechanical model for alpha decay of nuclei as occurring through tunnelling. The proposal is that tunnelling might allow a classical particle description after all. Quantum classical correspondence suggests the same in TGD framework.

The proposed description is based on the idea that the tunnelling alpha particle has abnormally small charge inside the tunnelling region. This reduces the electrostatic interaction of alpha particle with nucleus so that it can penetrate to otherwise classically non-allowed region separating it from the external world and can leak out of the parent nucleus. More quantitatively, the momentum given by $p = \sqrt{2m(E-V)}$ of alpha particle remains real during tunnelling. As the alpha particle escapes, it gradually increases its charge to its full value of 2 units possessed by the ordinary alpha particle.

What is interesting is that the model predicts the charge of the proto-alpha particle at the surface of the decaying nucleus from the knowledge of alpha particle energy, nuclear radius, and charge by using just energy conservation in Coulombic field. What is assumed that the charge of the particle is such that Coulombic energy remains equal to the alpha particle energy all the way from the nuclear surface through the Coulomb wall to the distance where alpha particle can have full charge. This is a slight idealization since it would mean that the alpha particle kinetic energy vanishes.

To my opinion, the dynamical charge of alpha particle is a manner to articulate what happens in the tunnelling. Thus the model cannot replace quantum description but only become a part of it. In particular, the successful prediction of the decay rates exponentially sensitive to the alpha particle energy cannot be deduced from a purely classical theory.

The charges at the surface of the nucleus tend to be near 1/3 and 2/3. What is amazing is that these charges correspond to the charges of the quark and anti-quark composing pion. That quarks should reveal themselves in the classical model for alpha decay is a complete surprise.

From this finding Illert concludes that during the decay the alpha particle is connected to the parent nucleus by rubber band like strings having quark and anti-quark at their ends, that is color flux tubes. These strings are interpreted as virtual pions. These strings get stretched and eventually must split since the color force between the quark and anti-quark at the ends of the string grows very strong.

This model is very attractive but has a deep problem: color forces mediate very short ranged and rapidly occurring interactions and should not be important for alpha decay which is a very slow process involving electromagnetic interactions in an essential manner. This does not diminish the pioneering value of Illert's work, just the opposite: pioneers must often have the courage to go against rationality as defined by the existing dogmas.

My earlier suggestion was that these pions serving as "rubber strings" are not ordinary pions but fractal copies of ordinary pions being much lighter and having much larger size. TGD indeed predicts the possibility of fractal copies of quantum chromo-dynamics (QCD). Thus there would exist a fractal copy of ordinary hadron physics operating in much longer length scales and having its own, much lighter, particle spectrum. The proposal was that this QCD corresponds to Mersenne prime M_{127}.

The dark QCD based on scaled up copies of ordinary quarks leads to a more elegant model in which virtual are replaced by π and ρ type color bonds, the latter being colored. Also an explanation of tetra-neutron emerges as a by-product since two pionic bonds can have negative charges. The identification of the nucleus as a nuclear string predicts the decay mechanism in which alpha particle pinches off and indeed has quarks and/or anti-quark attached to the ends of two nucleons.

Summarizing, although the model discussed in [21] does not predict tetra-neutron, it represents findings and ideas, which might be of crucial importance in the topological and geometric modelling of nuclear decays. The finding that alpha decay could be described in terms of pions, although wrong as such, opens the way to a realization that ordinary pions and thus also ordinary hadron and nuclear physics might have lighter fractal copies.

8.6 Tritium beta decay anomaly

The determination of neutrino mass from the beta decay of tritium leads to a tachyonic mass squared [26, 27]. I have considered several alternative explanations for this long standing anomaly.

1. ^{3}He nucleus resulting in the decay could be fake (tritium nucleus with one positively charged color bond making it to look like ^{3}He). The idea that slightly smaller mass of the fake ^{3}He might explain the anomaly: it however turned out that the model cannot explain the variation of the anomaly from experiment to experiment.

2. Much later I realized that also the initial ^3H nucleus could be fake (^{3}He nucleus with one negatively charged color bond). It turned out that fake tritium option can explain all aspects of the anomaly and also other anomalies related to radioactive and alpha decays of nuclei.

3. The alternative based on the assumption of dark neutrino or antineutrino belt surrounding Earth's orbit and explain satisfactorily several aspects of the anomaly but fails in its simplest form to explain the dependence of the anomaly on experiment. Since the fake tritium scenario is based only on the basic assumptions of the nuclear string model [F9] and brings in only new values of kinematical parameters it is definitely favored.

8.6.1 Tritium beta decay anomaly

A brief summary of experimental data before going to the detailed models is in order.

Is neutrino tachyonic?

Nuclear beta decay allows in principle to determine the value of the neutrino mass since the energy distribution function for electrons is sensitive to neutrino mass at the boundary of the kinematically allowed region corresponding to the situation in which final neutrino energy goes to zero [27].

The most useful quantity for measuring the neutrino mass is the so called Kurie plot for the function

$$\begin{aligned} K(E) &\equiv \left[\frac{d\Gamma/dE}{pEF(Z,E)}\right]^{1/2} \sim (E_\nu k_\nu)^{1/2} = \left[E_\nu \sqrt{E_\nu^2 - m(\nu)^2}\right]^{1/2} \quad , \\ E_\nu &= E_0 - E \ \ , \ \ E_0 = M_i - M_f - m(\nu) \ \ . \end{aligned} \tag{8.6.0}$$

Here E denotes electron energy and E_0 is its upper bound from energy and momentum conservation (for a configuration in which final state nucleon is at rest). Mass shell condition lowers the upper bound to $E \leq E_0 - m(\nu)$. For $m(\nu) = 0$ Kurie plot is straight line near its endpoint. For $m(\nu) > 0$ the end point is shifted to $E_0 - m(\nu)$ and $K(E)$ behaves as $m(\nu)^{1/2}k_\nu^{1/2}$ near the end point.

The problem is that the determination of $m(\nu)$ from this parametrization in tritium beta decay experiments gives a negative mass squared varies and is $m(\nu)^2 = -147 \pm 68 \pm 41$ eV^2 according to [27]! This behavior means that the derivative of $K(E)$ is infinite at the end point E_0 and $K(E)$ increases much faster near end point than it should. One can quite safely argue that tachyonicity gives only an ad hoc parametrization for the change of the shape of the function K deriving from some unidentified physical effect: in particular, the value of the tachyonic mass must correspond to a parameter related to new physics and need not have anything to do with neutrino mass.

More detailed experimental data

The results of Troitsk and Mainz experiments can be taken as constraints of the model. In Troitsk experiments [26] gas phase tritium is used whereas in Mainz experiments [27] liquid tritium film is used.

Troitsk experiments are described in [26]. In 1944 Troitsk experiment, the enhancement of the spectrum intensity was found to begin roughly at $V_b \simeq 7.6$ eV below E_0. The conclusion was that the rise of the spectrum intensity below 18,300 eV with respect to the standard model prediction takes place (this is illustrated in fig. 4 of [26]). No bump was claimed in this paper. In the analysis of 1996 experiment Troitsk group however concluded that the trapping of electrons gives rise to the enrichment of the low energy spectrum intensity of electrons and that when takes this effect into account, a narrow bump results.

Figure 4 of [26] demonstrates that spectrum intensity is below the theoretical value near the endpoint (right from the bump). In [26] the reduction of the spectrum intensity was assumed to be due to non-vanishing neutrino mass in [26]. The determination of $m(\nu)$ from the data near the end point assuming that beta decay is in question [26] gives $m(\nu) \sim 5$ eV.

The data can be parameterized by a parameter V_b which in the model context can be interpreted as repulsive interaction energy of antineutrinos with condensed matter suggested to explain the bump. Accordingly, the parametrization of $K(E)$ near the end point is

$$K(E) \sim (E - E_0)\theta(E - E_0) \rightarrow (E - E_0)\theta(E - E_0 + V_b) \ \ .$$

The end point is shifted to energy $E_\nu = V_b$ and $K(E)$ drops from the value V_b to zero at at this energy.

The values of V_b deduced from Troitsk and Mainz experiments are in the range $5 - 100$ eV. The value of V_b observed in Troitsk experiments using gas phase tritium [26] was of order 10 eV. In Mainz experiment [27] tritium film was used and the excess of counts around energy $V_b \simeq 100$ eV below E_0 was observed.

There is also a time variation involved with the value of V_b. In 1944 experiment [26] the bump was roughly $V_b \simeq 7.6$ eV below E_0. In 1996 experiment [26] the value of V_b was found to be $V_b \simeq 12.3$ eV [27]. Time variation was observed also in the Mainz experiment. In 'Neutrino 98' conference an oscillatory time variation for the position of the peak with a period of 1/2 years in the amplitude was reported by Troitsk group.

8.6.2 Could TGD based exotic nuclear physics explain the anomaly?

Nuclear string model explains tetra-neutron as alpha particle with two negatively charged color bonds. This inspires the question whether some fraction of decays could correspond to the decays of tritium to fake ^{3}He (tritium with one positively charged color bond) or fake tritium (^{3}He with one negatively charged color bond) to ^{3}He.

Could the decays of tritium decay to fake ^{3}He explain the anomaly?

Consider first the fake ^{3}He option. Tritium (pnn) would decay with some rate to a fake ^{3}He, call it ^{3}He$_f$, which is actually tritium nucleus containing one positively charged color bond and possessing mass slightly different than that of ^{3}He (ppn).

1. In this kind of situation the expression for the function $K(E,k)$ differs from $K(stand)$ since the upper bound E_0 for the maximal electron energy is modified:

$$\begin{aligned} E_0 &\rightarrow E_1 = M(^3\mathrm{H}) - M(^3\mathrm{He}_f) - m_\mu = M(^3\mathrm{H}) - M(^3\mathrm{He}) + \Delta M - m_\mu \ , \\ \Delta M &= M(^3\mathrm{He}) - M(^3\mathrm{He}_f) \ . \end{aligned} \quad (8.6.0)$$

   Depending on whether ^{3}He$_f$ is heavier/lighter than ^{3}He E_0 decreases/decreases. From $V_b \in [5-100]$ eV and from the TGD based prediction order $m(\overline{\nu}) \sim .27$ eV one can conclude that ΔM should be in the range 10-200 eV.

2. In the lowest approximation $K(E)$ can be written as

$$K(E) = K_0(E,E_1,k))\theta(E_1-E) \simeq (E_1-E)\theta(E_1-E) \ . \quad (8.6.1)$$

   Here $\theta(x)$ denotes step function and $K_0(E,E_0,k)$ corresponds to the massless antineutrino.

3. If a fraction p of the final state nuclei correspond to a fake ^{3}He the function $K(E)$ deduced from data is a linear combination of functions K(E,^{3}He) and K(E,^{3}He$_f$) and given by

$$\begin{aligned} K(E) &= (1-p)K(E,^3\mathrm{He}) + pK(E,^3\mathrm{He}_f) \\ &\simeq (1-p)(E_0-E)\theta(E_0-E) + p(E_1-E)\theta(E_1-E) \end{aligned} \quad (8.6.1)$$

   in the approximation $m_\nu = 0$.
   For $m(^3\mathrm{He}_f)<m(^3\mathrm{He})$ one has $E_1 > E_0$ giving

$$K(E) = (E_0-E)\theta(E_0-E) + p(E_1-E_0)\theta(E_1-E)\theta(E-E_0) \ . \quad (8.6.2)$$

   $K(E,E_0)$ is shifted upwards by a constant term $p\Delta M$ in the region $E_0 > E$. At $E = E_0$ the derivative of $K(E)$ is infinite which corresponds to the divergence of the derivative of square root function in the simpler parametrization using tachyonic mass. The prediction of the model is the presence of a tail corresponding to the region $E_0 < E < E_1$.

4. The model does not as such explain the bump near the end point of the spectrum. The decay ^{3}H→^{3}He$_f$ can be interpreted in terms of an exotic weak decay $d \rightarrow u + W^-$ of the exotic d quark at the end of color bond connecting nucleons inside ^3H. The rate for these interactions cannot differ too much from that for ordinary weak interactions and W boson must transform to its ordinary variant before the decay $W \rightarrow e + \overline{\nu}$. Either the weak decay at quark level or the phase transition could take place with a considerable rate only for low enough virtual W boson energies, say for energies for which the Compton length of massless W boson correspond to the size scale of color flux tubes predicted to be much longer than nuclear size. Is so the anomaly would be absent for higher energies and a bump would result.

5. The value of $K(E)$ at $E = E_0$ is $V_b \equiv p(E_1 - E_0)$. The variation of the fraction p could explain the observed dependence of V_b on experiment as well as its time variation. It is however difficult to understand how p could vary.

Could the decays of fake tritium to ^{3}He explain the anomaly?

Second option is that fraction p of the tritium nuclei are fake and correspond to ^{3}He nuclei with one negatively charged color bond.

1. By repeating the previous calculation exactly the same expression for $K(E)$ in the approximation $m_\nu = 0$ but with the replacement

$$\Delta M = M(^3\mathrm{He}) - M(^3\mathrm{He}_f) \quad \rightarrow \quad M(^3\mathrm{H}_f) - M(^3\mathrm{H}) \ . \tag{8.6.3}$$

2. In this case it is possible to understand the variations in the shape of $K(E)$ if the fraction of $^3\mathrm{H}_f$ varies in time and from experiment to experiment. A possible mechanism inducing this variation is a transition inducing the transformation $^3\mathrm{H}_f \rightarrow ^3\mathrm{H}$ by an exotic weak decay $d + p \rightarrow u + n$, where u and d correspond to the quarks at the ends of color flux tubes. This kind of transition could be induced by the absorption of X-rays, say artificial X-rays or X-rays from Sun. The inverse of this process in Sun could generate X rays which induce this process in resonant manner at the surface of Earth.

3. The well-known poorly understood X-ray bursts from Sun during solar flares in the wavelength range 1-8 A [44] corresponds to energies in the range 1.6-12.4 keV, 3 octaves in good approximation. This radiation could be partly due to transitions between ordinary and exotic states of nuclei rather than brehmstrahlung resulting in the acceleration of charged particles to relativistic energies. The energy range suggests the presence of three p-adic length scales: nuclear string model indeed predicts several p-adic length scales for color bonds corresponding to different mass scales for quarks at the ends of the bonds [F9]. This energy range is considerably above the energy range $5 - 100$ eV and suggests the range $[4 \times 10^{-4}, 6 \times 10^{-2}]$ for the values of p. The existence of these excitations would mean a new branch of low energy nuclear physics, which might be dubbed X-ray nuclear physics. The energy scale of for the excitation energies of exotic nuclei could corresponds to Coulomb interaction energy $\alpha_{em}m$, where m is mass scale of the exotic quark. This means energy scale of 10 keV for MeV mass scale.

4. The approximately 1/2 year period of the temporal variation would naturally correspond to the $1/R^2$ dependence of the intensity of X-ray radiation from Sun. There is evidence that the period is few hours longer than 1/2 years which supports the view that the origin of periodicity is not purely geometric but relates to the dynamics of X-ray radiation from Sun. Note that for 2 hours one would have $\Delta T/T \simeq 2^{-11}$, which defines a fundamental constant in TGD Universe and is also near to the electron proton mass ratio.

5. All nuclei could appear as similar anomalous variants. Since both weak and strong decay rates are sensitive to the binding energy, it is possible to test this prediction by finding whether nuclear decay rates show anomalous time variation.

6. The model could explain also other anomalies of radioactive reaction rates including the findings of Shnoll [74] and the unexplained fluctuations in the decay rates of ^{32}Si and ^{226}Ra reported quite recently [42] and correlating with $1/R^2$, R distance between Earth and Sun. ^{226}Ra decays by alpha emission but the sensitive dependence of alpha decay rate on binding energy means that the temporal variation of the fraction of fake ^{226}Ra isotopes could explain the variation of the decay rates. The intensity of the X-ray radiation from Sun is proportional to $1/R^2$ so that the correlation of the fluctuation with distance would emerge naturally.

7. Also a dip in the decay rates of ^{54}Mn coincident with a peak in proton and X-ray fluxes during solar flare [43] has been observed: the proposal is that neutrino flux from Sun is also enhanced during the solar flare and induces the effect. A peak in X-ray flux is a more natural explanation in TGD framework.

8. The model predicts interaction between atomic physics and nuclear physics, which might be of relevance in biology. For instance, the transitions between exotic and ordinary variants of nuclei could yield X-rays inducing atomic transitions or ionization. The wave length range 1-8 Angstroms for anomalous X-rays corresponds to the range $Z \in [11, 30]$ for ionization energies. The biologically important ions Na^+, Mg^{++}, P^-, Cl^-, K^+, Ca^{++} have $Z = (11, 15, 17, 19, 20)$. I have proposed that Na^+, Cl^-, K^+ (fermions) are actually bosonic exotic ions forming Bose-Einstein condensates at magnetic flux tubes [M2]. The exchange of W bosons between neutral Ne and A(rgon) atoms (bosons) could yield exotic bosonic variants of Na^+ (perhaps even Mg^{++}, which is boson also as ordinary ion) and Cl^- ions. Similar exchange between A atoms could yield exotic bosonic variants of Cl^- and K^+ (and even Ca^{++}, which is also boson as ordinary variant). This hypothesis is testable by measuring the nuclear weights of these ions. X-rays from Sun are not present during night time and this could relate to the night-day cycle of living organisms. Note that magnetic bodies are of size scale of Earth and even larger so that the exotic ions inside them could be subject to intense X-ray radiation. X-rays could also be dark X-rays with large Planck constant and thus with much lower frequency than ordinary X-rays so that control could be possible.

8.6.3 The model based on dark neutrinos

A common origin of the tritium beta decay anomaly was independently suggested by several groups (see [28]): a broad spike or bump like excess of counts centered $5-100$ eV below the end point energy E_0. In [28] it was suggested that a repulsive interaction of antineutrinos with condensed matter with interaction energy of order $V_b \simeq 5-100$ eV could explain the bump.

It has been pointed out by Stevenson [29] that the process in which neutrinos are absorbed from a background of electron neutrinos

$$\nu_e +^3 \mathrm{H} \rightarrow^3 \mathrm{He} + e^-$$

leads to electrons in the anomalous endpoint region. This gives an essentially constant addition to the region $E_0 - E_F < E < E_0$. The density of cosmic neutrino background is however far too small to give the required large background density of order $1/m(\nu)^3$.

The earlier -wrong- hypothesis that nuclei are Z^0 charged are consistent with both options described above as explanations of the anomaly. One can modify these models to apply also in the new framework. The problem of these models is that one is forced to make ad hoc assumptions about dynamics in long length scales. They might make sense in TGD Universe but would require experimental justification. These models in their simplest form fail also to explain the dependence of V_b on experiment and fail to provide provide insights about more general time variations of nuclear decay rates.

Neutrino belt or antineutrino belt?

The model corresponding to mechanism of [28] is that the belt consists of dark antineutrinos and the repulsive interaction energy of antineutrino with the these neutrinos explains the anomaly. The model based on dark neutrinos assumes that Earth's orbit is surrounded by a belt of dark neutrinos and that the mechanism proposed in [29] could be at work. The periodic variation of the dark neutrino density along the orbit of Earth around Sun could also explain the periodic variations of the bump.

1. The first mechanism corresponds to that suggested in [28]. The antineutrino emitted in the beta decay can transform to a dark neutrino by mixing and experiences a repulsive Z^0 force which effectively shifts the electron energy spectrum downwards. In this case the repulsive interaction energy V_b of dark anti-neutrinos with the dark antineutrinos of the solar belt would replace ΔM in the previous formula:

$$E_0 = M(^3\mathrm{H}) - M(^3\mathrm{He}) - m(\nu) \rightarrow M(^3\mathrm{H}) - M(^3\mathrm{He}) - V_b - m(\nu_d) \ . \quad (8.6.3)$$

2. Second option corresponds to the mechanism proposed in [29]. Dark neutrino transforms to ordinary one and induces by ordinary W exchange ordinary tritium beta decay. In this case the Fermi energy E_F of dark neutrino determines the width of the bump and one has $V_b = E_F$:

$$E_0 = M(^3\mathrm{H}) - M(^3\mathrm{He}) - m(\nu) \rightarrow M(^3\mathrm{H}) - M(^3\mathrm{He}) - E_F + m(\nu_d) \ . \tag{8.6.3}$$

The rate of the process would be given by the standard model and only the density of dark neutrinos and the ratio $M^2(\nu,\nu(dark))/M^2(\nu)$ appear as free parameters.

Notice that these models are simpler than the original models which assumed that the interaction of neutrinos with condensed matter carrying Z^0 charge is involved. The explanation for the dependence of V_b on experiment poses a difficulty for both models. For the antineutrino belt the repulsive interaction energy is proportional to the density of antineutrinos. For neutrino belt V_b corresponds to the Fermi energy proportional to the density of neutrinos. In both cases large variation of V_b requires a large variation of the density of antineutrinos (neutrinos) of the belt in the scale smaller than Earth size. This does not look too plausible.

Can one understand time variation of V_b?

The periodic variation of the density of neutrinos or antineutrinos in the belt should induce the variation of V_b. The ordering of the two models trying to explain this variation reflects the evolution of the general ideas about quantum TGD.

1. First model

The value of the period and the fact that maximum shift occurs when Earth is near to its position nearest to Sun suggests that the physics of solar system must be involved somehow. The simplest explanation is that gravitational acceleration tends to drive dark neutrinos (antineutrinos) as near as possible to Sun inside the belt. In thermal equilibrium with temperature T the Boltzmann factor

$$exp(-\frac{V_{gr}}{T}) = exp(-\frac{GMm(\nu_d)}{rT}) \tag{8.6.4}$$

for the dark neutrino would determine the density profile of dark neutrinos along the belt as function of the distance r to the Sun.

The existence of the dark neutrino belt conforms with the model of for the formation of solar system from dark matter with a gigantic value of Planck constant discussed in [D7, J6]. The model indeed assumes that the dark matter is located at space-time sheet surrounding the orbit of Earth. The requirement that dark neutrino density is few neutrinos per atomic volume in the belt leads to a lower bound for the mass of the belt:

$$M(belt) \simeq m(\nu_d)\frac{Vol(belt)}{a^3} > 10^{-11}M(Sun) \quad (a \simeq 10^{-10} \text{ meters}) \ . \tag{8.6.5}$$

Here it is assumed that the dark neutrino mass is same as neutrino mass, which of course is an unnecessarily strong assumption. If the belt is at rest, the time period for the variation of the tritium beta decay anomaly is exactly half year. The period seems to be few hours longer than one half year (as reported in Neutrino98 conference in Tokyo by Lobashev *et al*) [26], which suggests that belt rotates slowly relative to Earth in the same direction as Earth.

2. Second model

The model for radioactive decay rate anomalies requires that neutrinos and Earth move respect to each other and that the density of neutrinos in the laboratory volume varies along the orbit.

1. Assume first that ordinary quantum mechanics applies and neutrinos are ordinary. The simplest expectation from Equivalence Principle assuming that neutrinos and Earth move independently along geodesic lines is that the velocity is same for Earth and neutrinos. No effect results even if the density of neutrinos along the orbit varies.

2. Suppose that the neutrinos are dark in the sense of having gigantic gravitational Planck constant and are in a macroscopically quantum coherent phase delocalized along the entire orbit and described by a wave function (also neutrino Cooper pairs can be considered). If the neutrino ring is exactly circular as Bohr orbit picture suggests and contains Earth's orbit, the thickness of the ring must be at least $d = a - b$, where a and b are major and minor axis. Exact rotational symmetry implies that dark neutrinos are characterized by a phase factor characterizing the angular momentum eigen state in question (the unit of the quantized angular momentum is now very large). Thus neutrino density depends only on the transversal coordinates of the tube and vanishes at the boundary of the tube. Since the Earth's orbit is ellipse, the transversal variation of the neutrino density inside the tube induces periodic variations of the neutrino density in the detector and could explain the effects on radioactive decay rates.

Although the model might explain the time variation of V_b it does not provide any obvious explanation for beta decay rates in general and fails to explain the variation of the alpha decay rate of ^{226}Ra nor the correlation of decay rates with solar flares. Hence it is clear that the model involving only the notion of nuclear string is favored.

8.6.4 Some other apparent anomalies made possible by dark neutrinos

The appearance of dark neutrinos in the final states of beta decays allow to imagine also some other apparent anomalies.

Apparent anomaly in the inverse beta decay

For the antineutrino belt option one can consider also the possibility of an apparent anomaly in the inverse beta decay in which positron and neutrino are emitted but only electron observed. The apparent anomaly would result from the absorption of a dark antineutrino with repulsive Z^0 interaction energy with condensed matter.

In this case the value of E_0 increases

$$E_0 = M_i - M_f - m(\nu) \rightarrow \hat{E}_0 = M_i - M_f + m(\nu_d) + V_b \ , \tag{8.6.6}$$

which means that positron spectrum extends above the kinematic limit if V_b has the value predicted by the explanation of tritium beta decay anomaly.

A second anomalous situation results if the emitted neutrino transforms to a dark neutrino with negative binding energy. In this case the value of E_0 would change as

$$E_0 \rightarrow \hat{E}_0 = M_i - M_f - m(\nu_d) + V_b \ . \tag{8.6.7}$$

Apparently neutrinoless beta decay and double beta decay

Neutrinoless double beta decay (NDB) is certainly one of the most significant nuclear physics processes from the point of view of unified theories (the popular article of New Scientist [45] provides a good view of NDB and the recent rather exciting experimental situation). In the standard physics framework NDB can occur only if neutrinos are Majorana neutrinos so that neutrino number is conserved only modulo 2 meaning that neutrino and antineutrino are one and the same particle. Since no antineutrinos are emitted in the NDB, the total energy of the two electrons is larger than in the normal double beta decay, and serves as an experimental signature of the process.

There are several collaborations studying NDB. The team formed by Hans Klapdor-Kleingrothaus and colleagues from the the Max Planck Insitute for Nuclear Physics in Heidelberg have been studying this process since 1990 in Gran Sasso laboratory. The decays studied are decays of Germanium-76 isotope known to be one of the few isotopes undergoing ordinary double beta decay transforming it into Selenium. The energy of the emitted two electrons is absorbed by the surrounding Ge atoms. The total energy which is larger for NDB decay serves as a signature of the process.

Three years ago came the first paper of the Heidelberg group reporting the observation of 15 NDB decays [46]. The analysis of the experiments however received a very critical response from colleagues. The Kurchatov Institute quitted the collaboration at 2001 and represented its own analysis with the conclusion that the data do not support NDB. Three years later Heidelberg group represented 14 new candidates for NDB and a new analysis [47]. It is now admitted that the team is not obviously wrong but that there are still doubts whether the background radioactivity has been handled correctly.

In TGD Universe neutrinos are Dirac neutrinos and NDB is not possible. The possibility of dark neutrinos however allow to consider the possibility of apparently neutrinoless beta decay and double beta decay.

What would happen that the ordinary neutrino emitted in the beta decay of proton transforms into a dark neutrino by mixing. The dark neutrino would not be observed so that apparently neutrinoless beta decay would be in question. Dark neutrino has a negative interaction energy with condensed matter assuming that the explanation of tritium beta decay anomaly is correct so that electron would have an anomalously high energy. The process cannot occur if the negative energy states of the Fermi sea are filled as indeed suggested by energetic considerations.

The generalization of this process would be double beta decay involving strong interaction between decaying neutrons mediated by color bond between them and the transformation of second neutrino to dark neutrino with negative energy so that the electrons would have anomalously high energy. The same objection applies to this process as to the apparently neutrinoless beta decay.

8.7 Cold fusion and Trojan horse mechanism

The model for cold fusion has developed gradually as the understanding of quantum TGD and many-sheeted space-time has developed. Trojan horse mechanism has served as the connecting thread between various models. The last step of progress relates to the new vision about nuclear physics but it is still impossible to fix the model completely unless one poses the condition of minimality and the requirement that single mechanism is behind various anomalies.

8.7.1 Exotic quarks and charged color bonds as a common denominator of anomalous phenomena

There should exist a common denominator for anomalous behavior of water, cold fusion, the findings of Ditmire suggesting cold fusion, sono-fusion, exotic chemistries, strange properties of living matter including chiral selection, and also phenomena like low compressibility of condensed matter which standard physicist would not be worried about.

It seems that compression inducing the generation of charged color bonds between nucleons and leading to a formation of super-nuclei with atomic distances between building blocks might be the sought for common denominator. For super nuclei the repulsive weak interactions between exotic quark and anti-quark belonging to the two bonded nuclei would compensate the attractive color force so that a stable configuration of atomic size would result. Note that the weak coupling strength would be actually strong by the general criterion for transition to the large $\hbar$ phase.

The charging of color bonds would occur via W boson exchange between exotic and valence quarks with exotic W boson transforming to ordinary W via mixing.

The alternative option is a phase transition of nuclei transforming $k = 113$ em space-time sheets of valence quarks to em dark space-time sheets with a large value of $\hbar$ suggested for heavier nuclei by the general criteria. This phase transition could be avoided if the criticality forces surplus protons to transfer the electromagnetic charge of valence quarks to color bonds so that the situation reduces to the first option. In this picture standard nuclear physics would remain almost untouched and nothing new expect exotic quarks and charged color bonds is introduced.

The following examples suggest that this general picture indeed might unify a large class of phenomena.

1. The super-nuclei formed by the dark protons of water would be a basic example about this phenomenon. The occurrence of the process is plausible if also nucleons possess or can generate

closed loops with exotic quark and anti-quark at the ends of the loop belonging to the same nucleon. The fact that these protons are dark with respect to electromagnetic interactions suggests that the charge of protons is transferred to the color bonds so that the outcome is a nuclear string formed from neutrons connected by positively charged color bonds. Darkness with respect to weak interactions suggests that valence quarks are doubly dark. This would mean that the p-adic length scale of color bonds would correspond to $k_{eff} = 107 + 2 \times 22 = 151$ for $\hbar_s = n^2\hbar/v_0^2$, $n = 1$. This corresponds to the thickness of cell membrane so that the structure of water would contain information about the basic biological length scale.

2. In condensed matter the super-nuclei would form at some critical pressure when weakly charged color bonds between neighboring nuclei become possible and compensate the attractive color force. This would explain the low compressibility of condensed matter.

3. Bio-polymers in vivo might correspond to super-nuclei connected by charged color bonds whose weak charges would explain the large parity breaking involve with chiral selection. Hydrogen bond might be a basic example of a charged color bond. It could be that the value of integer n in $\hbar_s = n\hbar/v_0$ is $n = 3$ in living matter and $n = 1$ in ordinary condensed matter.

   Trojan horse mechanism might work also at the level of chemistry making possible to circumvent electronic Coulomb wall and might be an essential characteristic of the catalytic action. Note that Pd is also a powerful catalyst. $n = 1$ might however distinguish it from bio-catalysts. In separate context I have dubbed this mechanism as 'Houdini effect'.

   The reported occurrence of nuclear transmutations [69, 70] such as $^{23}Na + ^{16}O \rightarrow ^{39}K$ in living matter allowing growing cells to regenerate elements K, Mg, Ca, or Fe, could be understood as fusion of neighboring nuclei connected by charged color bond which becomes neutral by W emission so that collapse to single nucleus results in absence of the repulsive weak force. Perhaps it is someday possible to produce metabolic energy by bio-fusion or perhaps Nature has already discovered the trick!

4. In cold fusion the nuclei of target D and Pd would combine to form super-nuclei connected by charged color bonds. This would explain why the heavy loading of Pd nuclei with D (for a review of loading process see [57]) does not generate enormous pressures. Cold fusion would occur in some critical interval of loadings allowing ordinary and exotic nuclei to transform to each other. The transfer of the em charge of D to the color bond connecting D and Pd would make D effectively nn state. Together with the fact that the color bond would have length of order atomic radius would mean that the Coulomb wall of Pd and D is not felt by beam nuclei and Trojan horse mechanism would become possible. The prediction is that Coulomb wall disappears only only when deuterium or tritium target is used. If nuclei can transform to dark em phase cold fusion could occur for arbitrary target nuclei. That it is observed only for D and possibly H does not support this option.

   If valence quarks are doubly dark, their magnetic bodies have size of order $L(151) = 10$ nm, which is also the size scale of the nano-scaled Pd particles, color force would become long ranged. In sono-luminescence and son-fusion and also in nuclear transmutations similar formation of super-nuclei would occur and the collapse of super-nucleus to single nucleus could occur by the proposed mechanism.

5. In the experiments of Ditmire *et al* laser pulse induces very dense phase of Xenon atoms having $Z = 54$ which is heated to energies in which electron energies extend to MeV region and expands rapidly. $Z = 54$ means that Xe satisfies the most stringent condition of criticality for the transition to electromagnetic large $\hbar$ phase. This transition does not occur if protons feed the surplus em charge to the color bonds so that Xe nuclei also weakly charged. Assume that some fraction of Xe is in this kind of phase. The compression of Xe gas by laser pulse compresses Xe super-nuclei. If the connecting charged color bonds emit their em and weak charge by emission of W boson the super-nuclei collapse to single nucleus and nuclear fusion reactions become possible. The repulsive weak force becoming manifest in the compression generates brehmstrahlung heating the system and induces a violent explosion much like in sono-fusion.

In the sequel the experiments Ditmire *et al* and cold fusion are discussed in detail using this model.

8.7.2 The experiments of Ditmire *et al*

An important stimulation in the development of the model for cold fusion came from the observations of Ditmire *et al* [48] published in Nature. The discovery was that the energy spectrum of electrons in ionic explosions induced by the laser heating of ionic clusters extends up to energies of order MeV (rather 10^2 eV(!)): this suggests strongly a mechanism making strong interactions possible.

In the experimental arrangement of Ditmire *et al* clusters of Xenon atoms are hit by ultrashort (150 fsec), high-intensity (2×10^{16} W/cm^2) laser pulses[48]. This leads to superheating and production of high energy ions in the explosions of the superheated clusters. The highest ion energies are by 4 orders of magnitude larger than expected and of order MeV, the typical energy scale of nuclear strong interaction. The average ion energy is 45 ± 5 keV for cluster size of 6.5 nm and decreases slowly with the size of the cluster. No hot ions are produced for small clusters containing less than ~ 100 Xe atoms. It is not yet understood why the clusters explode so much more violently than molecules (producing 1 MeV ions as opposed to 100 eV ions) and small clusters. Another striking feature of the laser-cluster interaction is ionization to very high charge states, much higher than in the ionization, which can be produced by simple field ionization.

Consider first a more detailed model of the superheating as it is described in [48].

In an intensely irradiated cluster, optically and collisionally ionized electrons undergo rapid collisional heating for the short time (< ps) before the cluster disassembles in the laser field. Our recent studies of the electron energy spectra produced by the high intensity irradiation of large Xe clusters with 150 fs *laser pulse indicate that collisional heating within the cluster can produce electrons with energy up to* 3 keV*, an energy much higher than that that typical of solid target plasmas.*

A sharp peak in the measured electron energy spectrum suggested that the cluster micro-plasma exhibited a resonance in the heating by laser pulse similar to the giant resonance seen in metallic clusters. A small amount of cluster expansion during the laser pulse lowers the electron density to bring the near-infrared laser light into resonance with the free-electron plasma frequency in the cluster. This resonance greatly increases the laser electric field density within the cluster, and the laser absorption rate is enhanced, rapidly heating the electrons on a very fast (< 10 fs*) time scale to a highly non-equilibrium superheated state with mean energies of many* keV*. Charge separation of hot electrons inevitably leads to a very fast expansion of the cluster ions. This process is fundamentally different from low-intensity photo-fragmentation of cluster and far more energetic than the Coulomb explosion of small molecules.*

Authors believe (rather naturally!) that the production of hot ions is made possible by the high ion-temperatures produced by the not yet properly understood heating mechanism and suggest that this mechanism might make even table-top fusion possible.

In TGD framework the proposed general vision suggests following picture.

1. Laser pulse induces a compression of clusters of Xe atoms already containing super-nuclei with charged color bonds so that repulsive weak interaction compensated by color force in equilibrium situation becomes manifest and induces the expansion of the system much like the expansion of the bubble in sono-luminescence. The resulting brehmstrahlung heats the system.

2. The critical cluster size 6.5 nm could correspond to the p-adic length length scale $L(k_{eff} = 151) = 10$ nm for doubly dark valence quarks with $n = 1$.

3. The nuclear fusions resulting when color bonds between nuclei become neutral and induce collapse to single nucleus. Anomalously high charge states could be a byproduct of violent and very rapid fusions of neighboring color bonded Xe nuclei tearing Xe nuclei and outer electrons apart. These fusions would generate quantum coherent dark gamma ray beams transforming to ordinary gamma rays by de-coherence transition reducing the wavelength of gamma rays by a factor of 2^{-11} for $\hbar_s = \hbar/v_0$. It is also possible that dark gamma rays are absorbed by Pd super-nuclei. The wavelengths of dark gamma rays with energy of MeV would of order 2 nanometers in this case so that a coherent heating would happen in rather large volume.

8.7.3 Brief summary of cold fusion

In the following history and signatures of cold fusion are briefly summarized.

History of cold fusion

The first claim for cold fusion [58] dates back to March 23, 1989, when Pons and Fleischmann announced that nuclear fusion, producing usable amounts of heat, could be induced to take place on a table-top by electrolyzing heavy water and using electrodes made of Pd and platinum. Various laboratories all-over the world tried to reproduce the experiments. The poor reproducibility and the absence of the typical side products of nuclear fusion (gamma rays and neutrons) led soon to the conclusion (represented in the dramatic session of American Physical Society May 1, 1989) that nuclear fusion cannot explain the heat production. Main stream scientists made final conclusions about the subject of 'cold fusion' and cold fusion people became a pariah class of the scientific community.

The work with cold fusion however continued and gradually situation has changed. It became clear that nuclear reaction products, mainly ^4He, are present. Gradually also the reasons for the poor reproducibility of the experiments became better understood. A representative example about the change of the attitudes is the article of Schwinger [40] in which cold fusion is taken seriously. The article also demonstrates that the counter arguments of hot fusion people are based on the implicit assumption that hot fusion theory describes cold fusion despite the fact that the physical situations are radically different.

The development on the experimental side has been based on techniques involving the use of catalysis, nanotechnology, electrolysis, glow discharge and ultrasonic cavitation. There are now public demonstrations of cold fusion reactors, whose output energy far exceeds input energy and commercial applications are under intensive development, see for instance the home page of Russ George [39], for whom I am grateful for informing me about the recent state of cold fusion.

Rather remarkably, also the production of heavier elements has been detected [66] and this makes the explanation of the effect even more difficult in standard physics context and definitely excludes the explanations claiming that some chemical process is the source of the excess heat. The possibility of nuclear transmutation also suggests the possibility to transform ordinary nuclear wastes into non-radioactive nuclei and the first method achieving this has already been reported [61]. There are claims [69, 70] that cold fusion indeed occurs in bio-systems.

There is also some evidence for high temperature super conductivity associated with deuterium loaded palladium [66]. Good representations about the subject of cold fusion and references to the experimental work can be found at various cold fusion web-sites [62, 52, 53, 39]. Also the articles of J. Rothwell [66] and the excellent review article of E. Storms [65] are recommended.

It has become clear that cold fusion differs from hot fusion in several respects: gamma rays are not produced and the flux of neutrons is much lower than predicted by standard nuclear physics (these features are very well-come from the point of view of the technological applications). Together with the fact that Coulomb wall does not allow the occurrence of cold fusion at all in the standard physics context, this forces the conclusion that new physics must be involved.

It seems that TGD indeed could provide this new physics. The key elements of the model to be discussed are Trojan horse mechanism and coherent photon exchange action of D nuclei with Cooper pairs of the exotic super conductor formed by the D-loaded cathode material (say Palladium).

In the sequel the consideration is restricted to the case of Pd cathode: the model generalizes trivially to the case of a more general cathode material.

Signatures of cold fusion

In the following the consideration is restricted to cold fusion in which two deuterium nuclei react strongly since this is the basic reaction type studied.

In hot fusion there are three reaction types:

1) $D + D \rightarrow^4 He + \gamma$ $(23.8 MeV)$

2) $D + D \rightarrow^3 He + n$

3) $D + D \rightarrow^3 H + p$.

The rate for the process 1) predicted by standard nuclear physics is more than 10^{-3} times lower than for the processes 2) and 3) [62]. The reason is that the emission of the gamma ray involves the relatively weak electromagnetic interaction whereas the latter two processes are strong.

The most obvious objection against cold fusion is that the Coulomb wall between the nuclei makes the mentioned processes extremely improbable at room temperature. Of course, this alone implies that one should not apply the rules of hot fusion to cold fusion. Cold fusion indeed differs from hot fusion in several other aspects.

1. No gamma rays are seen.

2. The flux of energetic neutrons is much lower than expected on basis of the heat production rate an by interpolating hot fusion physics to the recent case.

   These signatures can also be (and have been!) used to claim that no real fusion process occurs. It has however become clear that the isotopes of Helium and also some tritium accumulate to the Pd target during the reaction and already now prototype reactors for which the output energy exceeds input energy have been built and commercial applications are under development, see for instance [39]. Therefore the situation has turned around. The rules of standard physics do not apply so that some new nuclear physics must be involved and it has become an exciting intellectual challenge to understand what is happening. A representative example of this attitude and an enjoyable analysis of the counter arguments against fold fusion is provided by the article 'Energy transfer in cold fusion and sono-luminescence' of Julian Schwinger [40]. This article should be contrasted with the ultra-skeptical article 'ESP and Cold Fusion: parallels in pseudoscience' of V. J. Stenger [41].

Cold fusion has also other features, which serve as valuable constraints for the model building.

1. Cold fusion is not a bulk phenomenon. It seems that fusion occurs most effectively in nano-particles of Pd and the development of the required nano-technology has made possible to produce fusion energy in controlled manner. Concerning applications this is a good news since there is no fear that the process could run out of control.

2. The ratio x of D atoms to Pd atoms in Pd particle must lie the critical range $[.85, .90]$ for the production of ^4He to occur [19]. This explains the poor repeatability of the earlier experiments and also the fact that fusion occurred sporadically.

3. Also the transmutations of Pd nuclei are observed [66].

Below a list of questions that any theory of cold fusion should be able to answer.

1. Why cold fusion is not a bulk phenomenon?

2. Why cold fusion of the light nuclei seems to occur only above the critical value $x \simeq .85$ of D concentration?

3. How fusing nuclei are able to effectively circumvent the Coulomb wall?

4. How the energy is transferred from nuclear degrees of freedom to much longer condensed matter degrees of freedom?

5. Why gamma rays are not produced, why the flux of high energy neutrons is so low and why the production of ^4He dominates (also some tritium is produced)?

6. How nuclear transmutations are possible?

8.7.4 TGD inspired model of cold fusion

The model to be discussed is based on Trojan horse mechanism and explains elegantly all those aspects of cold fusion which are in conflict with standard nuclear physics. The reaction mechanism explains also the sensitivity of the occurrence of cold fusion to small external perturbations.

Model for D-loaded Pd nano-particle

It seems that cold fusion is a critical phenomenon. The average D/Pd ratio must be in the interval $(.85, .90)$. The current must be over-critical and must flow a time longer than a critical time. The effect occurs in a small fraction of samples. Deuterium at the surface of the cathode is found to be important and activity tends to concentrate in patches. The generation of fractures leads to the loss of the anomalous energy production. Even the shaking of the sample can have the same effect. The addition of even a small amount of H_2O to the electrolyte (protons to the cathode) stops the anomalous energy production.

All these findings support the catastrophe theoretic picture according to which the decomposition into patches corresponds to criticality allowing the presence of ordinary and exotic phase whose transformation to the ordinary phases makes possible cold fusion reactions. The added ordinary protons and fractures could serve as a seed for a phase transition leading to a region where only single phase is possible.

In TGD framework Pd nano-particles correspond to space-time sheets of size of order $10^{-9}-10^{-8}$ m and fusion is restricted inside these structures. Cold fusion can be regarded as a fusion of incoming ordinary D with target D attached to the surface of Pd rather than between two free D:s as suggested by the standard nuclear physics wisdom. Thus cold fusion could be regarded as 'burning' of D associated with a finite space-time sheets so that cold fusion is not a bulk phenomenon and is very sensitive to the in-homogenities of the Pd particle. Note that this in principle makes the control of cold fusion easier.

The critical loading fraction varies in the range $.85-.90$. This value is so large that enormous pressures would be generated unless the deuterium nuclei lose their translational degrees of freedom by forming some kind of bound states with Pd nuclei. The guess is that the bound states correspond to the formation of super-nuclei with em and weakly charged color bonds connecting Pd and D nuclei. $k=113$ dark weak force, which is actually strong by the criticality condition, compensates the color force between the exotic quarks. This makes D nuclei effectively nn nuclei so that Coulomb wall does not produce difficulties.

The challenge is to understand the origin of the criticality condition for super-nucleus. The question is why the number of D nuclei per Pd nuclei varies in so narrow range for the phase transition leading to the formation of super-nuclei to occur. Catastrophe theoretic thinking suggests that a cusp catastrophe typical for phase transitions is in question. In the critical range there are two phases, exotic and ordinary phase, which can easily transform to each other. Criticality is essential for the cold fusion reactions to occur since initial state involves exotic D+Pd complex and final state involves ordinary nuclei.

The ratio x would correspond to the variable which varies in a direction transversal to cusp and whose variation therefore leads to a catastrophic jump inducing the phase transition or its reverse. x would be a pressure type variable which plays similar role also in phase transitions like liquid-gas phase transition. The critical range for x would correspond to the critical range of pressure in which liquid and gas are in equilibrium.

Second variable varies along the cusp so that the transition is possible above certain critical value. This variable presumably relates to the energetics of the transition so that transition would liberate energy above critical value of the parameter. Temperature is a natural candidate for this variable. Catastrophe theoretic model implies that for a given value x in catastrophe region both ordinary phase and exotic phase are possible. In these regions cold fusion can occur. In regions where the system is outside the catastrophe region so that system is stably in either phase, cold fusion cannot occur. This explains why Pd contains only patches were cold fusion occurs. The control variable, be it local temperature or something else, could be perhaps identified by studying the local conditions guaranteing the occurrence of cold fusion. It is indeed known that the increase of temperature favors the occurrence of cold fusion.

The behavior variable could distinguish between the two phases and could correspond to the surface density n of D nuclei bound to Pd nuclei and transformed to fake D. The potential function could be free energy minimized when the system is in constant temperature and the two phases would correspond to local minima of free energy.

Anomalous reaction kinetics of cold fusion

One can deduce a more detailed model for cold fusion from observations, which are discussed systematically in [62] and in the references discussed therein.

1. When D_2O is used as an electrolyte, the process occurs when PdD acts as a cathode but does not seem to occur when it is used as anode. This suggests that the basic reaction is between the ordinary deuterium $D = pn$ of electrolyte with the the exotic D=nn + charged color bond attached to Pd in the cathode.

2. For ordinary nuclei fusions to tritium and ^{3}He occur with approximately identical rates. The first reaction produces neutron and ^{3}He via $D+D \rightarrow n+^3He$, whereas second reaction produces proton and tritium via $D + D \rightarrow p +^3 H$. The standard nuclear physics prediction is that one neutron per each tritium nucleus should be produced. Tritium can be observed by its beta decay to ^{3}He and the ratio of neutron flux is several orders of magnitude smaller than tritium flux as found for instance by Tadahiko Mizuno and his collaborators (Mizuno describes the experimental process leading to this discovery in his book [67]). Hence the reaction producing ^{3}He cannot occur significantly in cold fusion which means a conflict with the basic predictions of the standard nuclear physics.

   The explanation is following. If D is fake D (nn+charged color bond connecting it with Pd), one expects that the production of ^{3}He is hindered since there is no proton directly available. Also in the case that the reaction n+color bond $\rightarrow p$ occurs, one expects that Coulomb wall makes the process slow.

3. The production of ^4He, which should not occur practically at all, is reported to dominate and the fraction of tritium is below .1 per cent. The explanation could be that also multiple attachments to target can occur such that D attaches to (D+Pd) by forming a charged color bond. Thus would have nnnnn state with two charged color bonds attached to Pd. This state could split from Pd and transform via exchange of two light weak bosons between exotic and valence quarks to ^4He (assuming that dark $W(113)$ can mix with $W(89)$). It is also possible that the super-nuclear string formed by Pd and D splits and emits ^4He as in ordinary alpha decay. Gamma rays need not be generated since the recoil momentum could be transferred to the Pd target like in Mössbauer effect.

4. Also more complex reactions between D and Pd and between Pd nuclei can occur. These can lead to the reactions transforming the nuclear charge of Pd and thus to nuclear transmutations.

5. The mechanism also explains why the cold fusion producing ^{3}He and neutrons does not occur using water instead of heavy water. There are reports about cold fusion also in this case [65]. If one fourth of protons in water are arranged to nuclear strings consisting of neutrons connected by positively charged color bonds as the TGD based model explaining the anomalies of water suggests [F9], these strings could attach to fake D and induce cold fusion reactions.

6. The proposed reaction mechanism explains why neutrons are not produced in amounts consistent with the anomalous energy production. The addition of water to the electrolyte however induces neutron bursts. Suppose that one fourth of protons in water forms similar dark phase being transformed to neutrons connected by positively charged color bonds, as assumed in the model of water explaining various anomalies of water [F9]. What comes in mind is that neutrons are generated when a neutron string from H_2O containing only charged color bonds attaches to D+Pd (nn + charged color bond +Pd). Neutrons of nn are connected by a neutral color bond. If charged color bonds between neutrons are energetically more favorable than neutral color bonds, nn could emit a free neutron in the process so that the outcome would be a neutron string containing only charged color bonds attached to Pd.

How objections against cold fusion are circumvented?

It has already become clear that the model allows to circumvent the basic reaction kinetic arguments against cold fusion [62].

1. Coulomb wall makes nuclear fusions impossible.

2. ^{3}He and ^3H should be produced in equal amounts. The fraction of ^4He should be smaller than 10^{-3}.

3. The claimed nuclear transmutation reactions (reported to occur also in living matter [69]) should not occur.

Consider next the objections related to energetics.

1. Gamma rays, which should be produced in most nuclear reactions such as ^4He production to guarantee momentum conservation are not observed. The explanation is that the recoil momentum goes to the macroscopic quantum phase defined by the Pd lattice as in Mössbauer effect, and eventually heats the electrolyte system. This provides the mechanism by which the liberated nuclear energy is transferred to the electrolyte difficult to imagine in standard nuclear physics framework.

2. If a nuclear reaction should occur, the immediate release of energy can not be communicated to the lattice in the time available. In the recent case the time scale is however multiplied by the factor $r = \hbar_s/\hbar$ giving scaling factor 2^{11} so that the situation changes dramatically.

8.7.5 Do nuclear reaction rates depend on environment?

Claus Rolfs and his group have found experimental evidence for the dependence of the rates of nuclear reactions on the condensed matter environment [90]. For instance, the rates for the reactions ^{50}V(p,n)^{50}Cr and ^{176}Lu(p,n) are fastest in conductors. The model explaining the findings has been tested for elements covering a large portion of the periodic table.

Debye screening of nuclear charge by electrons as an explanation for the findings

The proposed theoretical explanation [90]is that conduction electrons screen the nuclear charge or equivalently that incoming proton gets additional acceleration in the attractive Coulomb field of electrons so that the effective collision energy increases so that reaction rates below Coulomb wall increase since the thickness of the Coulomb barrier is reduced.

The resulting Debye radius

$$R_D = 69\sqrt{\frac{T}{n_{eff}\rho_a}} \ , \qquad (8.7.1)$$

where ρ_a is the density of atoms per cubic meter and T is measured in Kelvins. R_D is of order .01 Angstroms for $T = 373$ K for $n_{eff} = 1$, $a = 10^{-10}$ m. The theoretical model [88, 89] predicts that the cross section below Coulomb barrier for $X(p,n)$ collisions is enhanced by the factor

$$f(E) = \frac{E}{E+U_e}exp(\frac{\pi\eta U_e}{E}) \ . \qquad (8.7.2)$$

E is center of mass energy and η so called Sommerfeld parameter and

$$U_e \equiv U_D = 2.09\times 10^{-11}(Z(Z+1))^{1/2}\times(\frac{n_{eff}\rho_a}{T})^{1/2} \ eV \qquad (8.7.3)$$

is the screening energy defined as the Coulomb interaction energy of electron cloud responsible for Debye screening and projectile nucleus. The idea is that at R_D nuclear charge is nearly completely screened so that the energy of projectile is $E+U_e$ at this radius which means effectively higher collision energy.

The experimental findings from the study of 52 metals support the expression for the screening factor across the periodic table.

1. The linear dependence of U_e on Z and $T^{-1/2}$ dependence on temperature conforms with the prediction. Also the predicted dependence on energy has been tested [90].

2. The value of the effective number n_{eff} of screening electrons deduced from the experimental data is consistent with $n_{eff}(Hall)$ deduced from quantum Hall effect.

The model suggests that also the decay rates of nuclei, say beta and alpha decay rates, could be affected by electron screening. There is already preliminary evidence for the reduction of beta decay rate of ^{22}Na β decay rate in Pd [91], metal which is utilized also in cold fusion experiments. This might have quite far reaching technological implications. For instance, the artificial reduction of half-lives of the radioactive nuclei could allow an effective treatment of radio-active wastes. An interesting question is whether screening effect could explain cold fusion [62] and sono-fusion [68].

Electron screening and Trojan horse mechanism

These experimental findings allow to quantify the Trojan horse mechanism. The idea is that projectile nucleus enters the region of the target nucleus along a larger space-time sheet and in this manner avoids the Coulomb wall. The nuclear reaction itself occurs conventionally. In conductors the space-time sheet of conduction electrons is a natural candidate for the larger space-time sheet.

At conduction electron space-time sheet there is a constant charged density consisting of n_{eff} electrons in the atomic volume $V = 1/n_a$. This creates harmonic oscillator potential in which incoming proton accelerates towards origin. The interaction energy at radius r is given by

$$V(r) = \alpha n_{eff} \frac{r^2}{2a^3} , \tag{8.7.4}$$

where a is atomic radius.

The proton ends up to this space-time sheet by a thermal kick compensating the harmonic oscillator energy. This occurs below with a high probability below radius R for which the thermal energy $E = T/2$ of electron corresponds to the energy in the harmonic oscillator potential. This gives the condition

$$R = \sqrt{\frac{Ta}{n_{eff}\alpha}} a . \tag{8.7.5}$$

This condition is exactly of the same form as the condition given by Debye model for electron screening but has a completely different physical interpretation.

Since the proton need not travel through the nuclear Coulomb potential, it effectively gains the energy

$$E_e = Z\frac{\alpha}{R} = \frac{Z\alpha^{3/2}}{a}\sqrt{\frac{n_{eff}}{Ta}} . \tag{8.7.6}$$

which would be otherwise lost in the repulsive nuclear Coulomb potential. Note that the contribution of the thermal energy to E_e is neglected. The dependence on the parameters involved is exactly the same as in the case of Debye model. For $T = 373$ K in the ^{176}Lu experiment and $n_{eff}(Lu) = 2.2 \pm 1.2$, and $a = a_0 = .52 \times 10^{-10}$ m (Bohr radius of hydrogen as estimate for atomic radius), one has $E_e = 28.0$ keV to be compared with $U_e = 21 \pm 6$ keV of [90] ($a = 10^{-10}$m corresponds to 1.24×10^4 eV and 1 K to 10^{-4} eV). A slightly larger atomic radius allows to achieve consistency. The value of $\hbar$ does not play any role in this model since the considerations are purely classical.

An interesting question is what the model says about the decay rates of nuclei in conductors. For instance, if the proton from the decaying nucleus can enter directly to the space-time sheet of the conduction electrons, the Coulomb wall corresponds to the Coulomb interaction energy of proton with conduction electrons at atomic radius and is equal to $\alpha n_{eff}/a$ so that the decay rate should be enhanced.

Trojan horse mechanism realized in this manner does not seem to explain the basic findings about cold fusion. Trojan horse mechanism applied to deuterium projectile and D-Pd target would predict standard nuclear physics. The reported strong suppression of ^{3}He production with respect to ^3H production however requires non-standard nuclear physics and the model discussed in the previous subsection provides this physics. Both mechanisms could of course be involved.

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Chapter 9

Nuclear String Hypothesis

9.1 Introduction

Nuclear string hypothesis [F8] is one of the most dramatic almost-predictions of TGD [TGDquant]. The hypothesis in its original form assumes that nucleons inside nucleus organize to closed nuclear strings with neighboring nuclei of the string connected by exotic meson bonds consisting of color magnetic flux tube with quark and anti-quark at its ends. The lengths of flux tubes correspond to the p-adic length scale of electron and therefore the mass scale of the exotic mesons is around 1 MeV in accordance with the general scale of nuclear binding energies. The long lengths of em flux tubes increase the distance between nucleons and reduce Coulomb repulsion. A fractally scaled up variant of ordinary QCD with respect to p-adic length scale would be in question and the usual wisdom about ordinary pions and other mesons as the origin of nuclear force would be simply wrong in TGD framework as the large mass scale of ordinary pion indeed suggests. The presence of exotic light mesons in nuclei has been proposed also by Illert [21] based on evidence for charge fractionization effects in nuclear decays.

9.1.1 $A > 4$ nuclei as nuclear strings consisting of $A \leq 4$ nuclei

In the sequel a more refined version of nuclear string hypothesis is developed.

1. The first refinement of the hypothesis is that ^4He nuclei and $A < 4$ nuclei and possibly also nucleons appear as basic building blocks of nuclear strings instead of nucleons which in turn can be regarded as strings of nucleons. Large number of stable lightest isotopes of form $A = 4n$ supports the hypothesis that the number of ^4He nuclei is maximal. One can hope that even also weak decay characteristics could be reduced to those for $A < 4$ nuclei using this hypothesis.

2. One can understand the behavior of nuclear binding energies surprisingly well from the assumptions that total *strong* binding energy associated with $A \leq 4$ building blocks is *additive* for nuclear strings and that the addition of neutrons tends to reduce Coulombic energy per string length by increasing the length of the nuclear string implying increase binding energy and stabilization of the nucleus. This picture does not explain the variation of binding energy per nucleon and its maximum appearing for ^{56}Fe.

3. In TGD framework tetra-neutron [59, 60] is interpreted as a variant of alpha particle obtained by replacing two meson-like stringy bonds connecting neighboring nucleons of the nuclear string with their negatively charged variants [F8]. For heavier nuclei tetra-neutron is needed as an additional building brick and the local maxima of binding energy E_B per nucleon as function of neutron number are consistent with the presence of tetra-neutrons. The additivity of magic numbers $2, 8, 20, 28, 50, 82, 126$ predicted by nuclear string hypothesis is also consistent with experimental facts and new magic numbers are predicted [22, 23].

9.1.2 Bose-Einstein condensation of color bonds as a mechanism of nuclear binding

The attempt to understand the variation of the nuclear binding energy and its maximum for Fe leads to a quantitative model of nuclei lighter than Fe as color bound Bose-Einstein condensates of ^4He nuclei or rather, of pion like colored states associated with color flux tubes connecting ^4He nuclei. The crucial element of the model is that color contribution to the binding energy is proportional to n^2 where n is the number of color bonds. Fermi statistics explains the reduction of E_B for the nuclei heavier than Fe. Detailed estimate favors harmonic oscillator model over free nucleon model with oscillator strength having interpretation in terms of string tension.

Fractal scaling argument allows to understand ^4He and lighter nuclei as strings formed from nucleons with nucleons bound together by color bonds. Three fractally scaled variants of QCD corresponding $A > 4$ nuclei, $A = 4$ nuclei and $A < 4$ nuclei are thus involved. The binding energies of also lighter nuclei are predicted surprisingly accurately by applying simple p-adic scaling to the parameters of model for the electromagnetic and color binding energies in heavier nuclei.

9.1.3 Giant dipole resonance as de-coherence of Bose-Einstein condensate of color bonds

Giant (dipole) resonances [30, 31, 33], and so called pygmy resonances [34, 35] interpreted in terms of de-coherence of the Bose-Einstein condensates associated with $A \leq 4$ nuclei and with the nuclear string formed from $A \leq 4$ nuclei provide a unique test for the model. The key observation is that the splitting of the Bose-Einstein condensate to pieces costs a precisely defined energy due to the n^2 dependence of the total binding energy. For ^4He de-coherence the model predicts singlet line at 12.74 MeV and triplet (25.48, 27.30,29.12) MeV at ~ 27 MeV spanning 4 MeV wide range which is of the same order as the width of the giant dipole resonance for nuclei with full shells.

The de-coherence at the level of nuclear string predicts 1 MeV wide bands 1.4 MeV above the basic lines. Bands decompose to lines with precisely predicted energies. Also these contribute to the width. The predictions are in a surprisingly good agreement with experimental values. The so called pygmy resonance appearing in neutron rich nuclei can be understood as a de-coherence for $A = 3$ nuclei. A doublet (7.520,8.4600) MeV at ~ 8 MeV is predicted. At least the prediction for the position is correct.

9.1.4 Dark nuclear strings as analogs of as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code

One biological speculations [L6] inspired by the dark matter hierarchy is that genetic code as well as DNA-, RNA- and amino-acid sequences should have representation in terms of dark nuclear strings. The model for dark baryons indeed leads to an identification of these analogs and the basic numbers of genetic code including also the numbers of aminoacids coded by a given number of codons are predicted correctly. Hence it seems that genetic code is universal rather than being an accidental outcome of the biological evolution.

9.2 Some variants of the nuclear string hypothesis

The basic assumptions of the nuclear string model could be made stronger in several testable ways. One can make several alternative hypothesis.

9.2.1 Could linking of nuclear strings give rise to heavier stable nuclei?

Nuclear strings (Z_1, N_1) and (Z_2, N_2) could link to form larger nuclei $(Z_1 + Z_2, N_1 + N_2)$. If one can neglect the interactions between linked nuclei, the properties of the resulting nuclei should be determined by those of composites. Linking should however be the confining interaction forbidding the decay of the stable composite. The objection against this option is that it is difficult to characterize the constraint that strings are not allowed to touch and there is no good reason forbidding the touching.

The basic prediction would be that if the nuclei (Z_1, N_1) and (Z_2, N_2) which are stable, very long-lived, or possess exceptionally large binding energy then also the nucleus $(Z_1 + Z_2, N_1 + N_2)$ has this property. If the linked nuclear strings are essentially free then the expectation is that the half-life of a composite of unstable nuclei is that of the shorter lived nucleus. This kind of regularity would have been probably observed long time ago.

9.2.2 Nuclear strings as connected sums of shorter nuclear strings?

Nuclear strings can form connected sum of the shorter nuclear strings. Connected sum means that one deletes very short portions of nuclear string A and B and connects the resulting ends of string A and B together. In other words: A is inserted inside B or vice versa or A and B are cut to open strings and connected and closed again. This outcome would result when A and B touch each other at some point. If touching occurs at several points more complex fusion of nuclei to a larger nucleus to a composite occurs with piece of A followed by a piece of B followed... For this option there is a non-trivial interaction between strings and the properties of nuclei need not be simply additive but one might still hope that stable nuclei fuse to form stable nuclei. In particular, the prediction for the half-life based on binding by linking does not hold true anymore.

Classical picture would suggest that the two strings cannot rotate with respect to each other unless they correspond to rather simple symmetric configurations: this applies also to linked strings. If so then the relative angular momentum L of nuclear strings vanishes and total angular momentum J of the resulting nucleus satisfies $|J_1 - J_2| \leq J \leq J_1 + J_2$.

9.2.3 Is knotting of nuclear strings possible?

One can consider also the knotting of nuclear strings as a mechanism giving rise to exotic excitations of nuclear. Knots decompose to prime knots so that kind of prime nuclei identified in terms of prime knots might appear. Fractal thinking suggests an analogy with the poorly understood phenomenon of protein folding. It is known that proteins always end up to a unique highly folded configuration and one might think that also nuclear ground states correspond to unique configurations to which quantum system (also proteins would be such if dark matter is present) ends up via quantum tunnelling unlike classical system which would stick into some valley representing a state of higher energy. The spin glass degeneracy suggests an fractal landscape of ground state configurations characterized by knotting and possibly also linking.

9.3 Could nuclear strings be connected sums of alpha strings and lighter nuclear strings?

The attempt to kill the composite string model leads to a stronger formulation in which nuclear string consists of alpha particles plus a minimum number of lighter nuclei. To test the basic predictions of the model I have used the rather old tables of [25] for binding energies of stable and long-lived isotopes and more modern tables [24] for basic data about isotopes known recently.

9.3.1 Does the notion of elementary nucleus make sense?

The simplest formulation of the model assumes some minimal set of *stable* "elementary nuclei" from which more complex *stable* nuclei can be constructed.

1. If heavier nuclei are formed by *linking* then alpha particle $^4He = (Z,N) = (2,2)$ suggests itself as the lightest stable composite allowing interpretation as a closed string. For connected sum option even single nucleon n or p can appear as a composite. This option turns out to be the more plausible one.

2. In the model based on linking $^6Li = (3,3)$ and $^7Li = (3,4)$ would also act as "elementary nuclei" as well as $^9Be = (4,5)$ and $^{10}Be = (4,6)$. For the model based on connected sum these nuclei might be regarded as composites $^6Li = (3,3) = (2,2) + (1,1)$, $^7Li = (3,4) = (2,2) + (1,2)$, $^9Be = (4,5) = 2 \times (2,2) + (0,1)$ and $^{10}Be = (4,6) = (2,2) + 2 \times (1,2)$. The study of binding energies supports the connected sum option.

3. ^{10}B has total nuclear spin $J = 3$ and $^{10}B = (5,5) = (3,3)+(2,2) =^6 Li+^4 He$ makes sense if the composites can be in relative $L = 2$ state (^6Li has $J = 1$ and ^4He has $J = 0$). ^{11}B has $J = 3/2$ so that $^{11}B = (5,6) = (3,4) + (2,2) =^7 Li+^4 He$ makes sense because ^7Li has $J = 3/2$. For the model based on disjoint linking also 10^B would be also regarded as "elementary nucleus". This asymmetry disfavors the model based on linking.

9.3.2 Stable nuclei need not fuse to form stable nuclei

The question is whether the simplest model predicts stable nuclei which do not exist. In particular, are the linked ^4He composites stable? The simplest case corresponds to $^8B = (4,4) =^4 He +^4 He$ which is not stable against alpha decay. Thus stable nuclei need not fuse to form stable nuclei. On the other hand, the very instability against alpha decay suggests that ^4B can be indeed regarded as composite of two alpha particles. A good explanation for the instability against alpha decay is the exceptionally large binding energy $E = 7.07$ MeV per nucleon of alpha particle. The fact that the binding energy per nucleon for ^8Be is also exceptionally large and equal to 7.06 MeV $< E_B(^4He)$ supports the interpretation as a composite of alpha particles.

For heavier nuclei binding energy per nucleon increases and has maximum 8.78 MeV for Fe. This encourages to consider the possibility that alpha particle acts as a fundamental composite of nuclear strings with minimum number of lighter isotopes guaranteing correct neutron number. Indeed, the decomposition to a maximum number of alpha particles allows a qualitative understanding of binding energies assuming that additional contribution not larger than 1.8 MeV per nucleon is present.

The nuclei ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{36}A, and ^{40}Ca are lightest stable isotopes of form $(Z,Z) = n \times^4 He$, $n = 3, ..., 10$, for which E_B is larger than for ^4He. For the first four nuclei E_B has a local maximum as function of N. For the remaining the maximum of E_B is obtained for $(Z, Z + 1)$. $^{44}Ti = (22, 22)$ does not exist as a long-lived isotope whereas ^{45}Ti does. The addition of neutron could increase E_B by increasing the length of nuclear string and thus reducing the Coulomb interaction energy per nucleon. This mechanism would provide an explanation also for neutron halos [61].

Also the fact that stable nuclei in general have $N \geq Z$ supports the view that $N = Z$ state corresponds to string consisting of alpha particles and that $N > Z$ states are obtained by adding something between. $N < Z$ states would necessarily contain at least one stable nucleus lighter than ^4He with smaller binding energy. ^3He is the only possible candidate as the only stable nucleus with $N < Z$. ($E_B(^2H) = 1.11$ MeV and $E_B(^3He) = 2.57$ MeV). Individual nucleons are also possible in principle but not favored. This together with increase of Coulomb interaction energy per nucleon due to the greater density of em charge per string length would explain their smaller binding energy and instability.

9.3.3 Formula for binding energy per nucleon as a test for the model

The study of ^8B inspires the hypothesis that the total binding energy for the nucleus (Z_1+Z_2, N_1+N_2) is in the first approximation the sum of total binding energies of composites so that one would have for the binding energy per nucleon the prediction

$$E_B = \frac{A_1}{A_1 + A_2} \times E_{B_1} + \frac{A_2}{A_1 + A_2} \times E_{B_2}$$

in the case of 2-nucleus composite. The generalization to N-nucleus composite would be

$$E_B = \sum_k \frac{A_k}{\sum_r A_r} \times E_{B_k} \quad .$$

This prediction would apply also to the unstable composites. The increase of binding energy with the increase of nuclear weight indeed suggests a decomposition of nuclear string to a sequence alpha strings plus some minimum number of shorter strings.

The first objection is that for both Li, B, and Be which all having two stable isotopes, the lighter stable isotope has a slightly smaller binding energy contrary to the expectation based on additivity of the total binding energy. This can be however understood in terms of the reduction of Coulomb energy per string length resulting in the addition of neutron (protons have larger average distance along nuclear string along mediating the electric flux) . The reduction of Coulomb energy per unit length of nuclear string could also partially explain why one has $E_B > E_B(^4He)$ for heavier nuclei.

The composition $^6Li = (3,3) = (2,2) + (1,1)$ predicts $E_B \simeq 5.0$ MeV not too far from 5.3 MeV. The decomposition $^7Li = (3,4) = (2,2) + (1,2)$ predicts $E_B = 5.2$ MeV to be compared with 5.6 MeV so that the agreement is satisfactory. The decomposition $^8Be = (4,4) = 2\times\ ^4He$ predicts $E_B = 7.07$ MeV to be compared with the experimental value 7.06 MeV. ^9Be and ^{10}Be have $E_B = 6.46$ MeV and $E_B = 6.50$ MeV. The fact that binding energy slightly increases in addition of neutron can be understood since the addition of neutrons to ^8Be reduces the Coulomb interaction energy per unit length. Also neutron spin pairing reduces E_B. The additive formula for E_B is satisfied with an accuracy better than 1 MeV also for ^{10}B and ^{11}B.

9.3.4 Decay characteristics and binding energies as signatures of the decomposition of nuclear string

One might hope of reducing the weak decay characteristics to those of shortest unstable nuclear strings appearing in the decomposition. Alternatively, one could deduce the decomposition from the weak decay characteristics and binding energy using the previous formulas. The picture of nucleus as a string of alpha particles plus minimum number of lighter nuclei ^3He having $E_B = 2.57$ MeV, ^3H unstable against beta decay with half-life of 12.26 years and having $E_B = 2.83$ MeV, and ^2H having $E_B = 1.1$ MeV gives hopes of modelling weak decays in terms of decays for these light composites.

1. β^- decay could be seen as a signature for the presence of ^3H string and alpha decay as a signature for the presence of ^4He string.

2. β^+ decay might be interpreted as a signature for the presence of ^3He string which decays to ^3H (the mass of ^3H is only .018 MeV higher than that of ^3He). For instance, $^8B = (5,3) = (3,2)+(2,1) = {}^5Li + {}^3He$ suffers β^+ decay to $^8Be = (4,4)$ which in turn decays by alpha emission which suggests the re-arrangement to $(3,2) + (1,2) \rightarrow (2,2) + (2,2)$ maximizing binding energy.

3. Also individual nucleons can appear in the decomposition and give rise to β^- and possible also β^+ decays.

9.3.5 Are magic numbers additive?

The magic numbers $2, 8, 20, 28, 50, 82, 126$ [22] for protons and neutrons are usually regarded as a support for the harmonic oscillator model. There are also other possible explanations for magic nuclei and there are deviations from the naive predictions. One can also consider several different criteria for what it is to be magic. Binding energy is the most natural criterion but need not always mean stability. For instance $^8B = (4,4) = {}^4He + {}^4He$ has high binding energy but is unstable against alpha decay.

Nuclear string model suggests that the fusion of magic nuclear strings by connected sum yields new kind of highly stable nuclei so that also $(Z_1 + Z_2, N_1 + N_2)$ is a magic nucleus if (Z_i, N_i) is such. One has $N = 28 = 20 + 8$, $50 = 28 + 20 + 2$, and $N = 82 = 50 + 28 + 2 \times 2$. Also other magic numbers are predicted. There is evidence for them [23].

1. $^{16}O = (8,8)$ and $^{40}Ca = (20,20)$ corresponds to doubly magic nuclei and $^{60}Ni = (28,32) = (20,20) + (8,8) + {}^4n$ has a local maximum of binding energy as function of neutron number. This is not true for ^{56}Ni so that the idea of magic nucleus in neutron sector is not supported by this case. The explanation would be in terms of the reduction of E_B due to the reduction of Coulomb energy per string length as neutrons are added.

2. Also $^{80}Kr = (36,44) = (36,36) + {}^4n = (20,20) + (8,8) + (8,8) + {}^4n$ corresponds to a local maximum of binding energy per nucleon as also does $^{84}Kr = {}^{80}Kr + {}^4n$ containing two tetra-neutrons. Note however that $^{88}Zr = (40,48)$ is not a stable isotope although it can be regarded as a composite of doubly magic nucleus and of two tetra-neutrons.

9.3.6 Stable nuclei as composites of lighter nuclei and necessity of tetra-neutron?

The obvious test is to look whether stable nuclei can be constructed as composites of lighter ones. In particular, one can check whether tetra-neutron ^4n interpreted as a variant of alpha particle obtained by replacing two meson-like stringy bonds connecting neighboring nucleons of the nuclear string with their negatively charged variants is necessary for the understanding of heavier nuclei.

1. $^{48}Ca = (20, 28)$ with half-life $> 2 \times 10^{16}$ years has neutron excess of 8 units and the only reasonable interpretation seems to be as a composite of the lightest stable Ca isotope $Ca(20, 20)$, which is doubly magic nucleus and two tetra-neutrons: $^{48}Ca = (20, 28) =^{40} Ca + 2 \times^4 n$.

2. The next problematic nucleus is ^{49}Ti.

   i) $^{49}Ti = (22, 27)$ having neutron excess of 5 one cannot be expressed as a composite of lighter nuclei unless one assumes non-vanishing and large relative angular momentum for the composites. For $^{50}Ti = (22, 28)$ no decomposition can be found. The presence of tetra-neutron would reduce the situation to $^{49}Ti = (22, 27) =^{45} Ti +^4 n$. Note that ^{45}Ti is the lightest Ti isotope with relatively long half-life of 3.10 hours so that the addition of tetra-neutron would stabilize the system since Coulomb energy per length of string would be reduced.

   ii) ^{48}Ti could not involve tetra-neutron by this criterion. It indeed allows decomposition to standard nuclei is also possible as $^{48}Ti = (22, 26) =^{41} K +^7 Li$.

   iii) The heaviest stable Ti isotope would have the decomposition $^{50}Ti =^{46} Ti +^4 n$, where ^{46}Ti is the lightest stable Ti isotope.

3. The heavier stable nuclei $^{50+k}V = (23, 27 + k)$, $k = 0, 1$, $^{52+k}Cr = (24, 28 + k)$, $k = 0, 1, 2$, $^{55}Mn = (25, 30)$ and $^{56+k}Fe = (26, 30 + k)$, $k = 0, 1, 2$ would have similar interpretation. The stable isotopes $^{50}Cr = (24, 26)$ and $^{54}Fe = (26, 28)$ would not contain tetra-neutron. Also for heavier nuclei both kinds of stable states appear and tetra-neutron would explain this.

4. $^{112}Sn = (50, 62) = (50, 50) + 3 \times^4 n$, ^{116}Sn, ^{120}Sn, and ^{124}Sn are local maxima of E_B as a function of neutron number and the interpretation in terms of tetra-neutrons looks rather natural. Note that $Z = 50$ is a magic number.

Nuclear string model looks surprisingly promising and it would be interesting to compare systematically the predictions for E_B with its actual values and look whether the beta decays could be understood in terms of those of composites lighter than ^4He.

9.3.7 What are the building blocks of nuclear strings?

One can also consider several options for the more detailed structure of nuclear strings. The original model assumed that proton and neutron are basic building blocks but this model is too simple.

Option Ia)

A more detailed work in attempt to understand binding energies led to the idea that there is fractal structure involved. At the highest level the building blocks of nuclear strings are $A \leq 4$ nuclei. These nuclei in turn would be constructed as short nuclear strings of ordinary nucleons.

The basic objection against the model is the experimental absence of stable $n - n$ bound state analogous to deuteron favored by lacking Coulomb repulsion and attractive electromagnetic spin-spin interaction in spin 1 state. Same applies to tri-neutron states and possibly also tetra-neutron state. There has been however speculation about the existence of di-neutron and poly-neutron states [27, 28].

The standard explanation is that strong force couples to strong isospin and that the repulsive strong force in nn and pp states makes bound states of this kind impossible. This force, if really present, should correspond to shorter length scale than the isospin independent forces in the model under consideration. In space-time description these forces would correspond to forces mediated between nucleons along the space-time sheet of the nucleus whereas exotic color forces would be mediated along the color magnetic flux tubes having much longer length scale. Even for this option one cannot exclude exotic di-neutron obtained from deuteron by allowing color bond to carry negative em charge.

Since em charges $0, 1, -1$ are possible for color bonds, a nucleus with mass number $A > 2$ extends to a multiplet containing $3A$ exotic charge states.

Option Ib)

One might ask whether it is possible to get rid of isospin dependent strong forces and exotic charge states in the proposed framework. One can indeed consider also other explanations for the absence of genuine poly-neutrons.

1. The formation of negatively charged bonds with neutrons replaced by protons would minimize both nuclear mass and Coulomb energy although binding energy per nucleon would be reduced and the increase of neutron number in heavy nuclei would be only apparent.

2. The strongest hypothesis is that mass minimization forces protons and negatively charged color bonds to serve as the basic building bricks of all nuclei. If this were the case, deuteron would be a di-proton having negatively charged color bond. The total binding energy would be only $2.222 - 1.293 = .9290$ MeV. Di-neutron would be impossible for this option since only one color bond can be present in this state.

The small mass difference $m(^3He) - m(^3H) = .018$ MeV would have a natural interpretation as Coulomb interaction energy. Tri-neutron would be allowed. Alpha particle would consist of four protons and two negatively charged color bonds and the actual binding energy per nucleon would be by $(m_n - m_p)/2$ smaller than believed. Tetra-neutron would also consist of four protons and the binding energy per nucleon would be smaller by $m_n - m_p$ than what obtains in the standard model of nucleus. Beta decays would be basically beta decays of exotic quarks associated with color bonds.

Note that the mere assumption that the di-neutrons appearing inside nuclei have protons as building bricks means a rather large apparent binding energy this might explain why di-neutrons have not been detected. An interesting question is whether also higher n-deuteron states than ^4He consisting of strings of deuteron nuclei and other $A \leq 3$ nuclei could exist and play some role in the nuclear physics of $Z \neq N$ nuclei.

If protons are the basic building bricks, the binding energy per nucleon is replaced in the calculations with its actual value

$$E_B \rightarrow E_B - \frac{N}{A}\Delta m \ , \ \Delta m = m_n - m_p = 1.2930 \ MeV \ . \tag{9.3.1}$$

This replacement does not affect at all the parameters of the of $Z = 2n$ nuclei identified as ^4He strings.

One can of course consider also the option that nuclei containing ordinary neutrons are possible but that are unstable against beta decay to nuclei containing only protons and negatively charged bonds. This would suggest that di-neutron exists but is not appreciably produced in nuclear reactions and has not been therefore detected.

Options IIa) and IIb)

It is not clear whether the fermions at the ends of color bonds are exotic quarks or leptons. Leptopion (or electro-pion) hypothesis [F7] was inspired by the anomalous e^+e^- production in heavy ion collisions near Coulomb wall and states that electro-pions which are bound states of colored excitations of electrons with ground state mass 1.062 MeV are responsible for the effect. The model predicts that also other charged leptons have color excitations and give rise to exotic counterpart of QCD.

Also μ and τ should possess colored excitations. About fifteen years after this prediction was made, direct experimental evidence for these states finally emerges [16, 17]. The mass of the new particle, which is either scalar or pseudoscalar, is 214.4 MeV whereas muon mass is 105.6 MeV. The mass is about 1.5 per cent higher than two times muon mass. The most natural TGD inspired interpretation is as a pion like bound state of colored excitations of muon completely analogous to lepto-pion (or rather, electro-pion) [F7].

One cannot exclude the possibility that the fermion and anti-fermion at the ends of color flux tubes connecting nucleons are actually colored leptons although the working hypothesis is that they are exotic quark and anti-quark. One can of course also turn around the argument: could it be that lepto-pions are "leptonuclei", that is bound states of ordinary leptons bound by color flux tubes for a QCD in length scale considerably shorter than the p-adic length scale of lepton.

Scaling argument applied to ordinary pion mass suggests that the masses of exotic quarks at the ends of color bonds are considerably below MeV scale. One can however consider the possibility that colored electrons with mass of ordinary electron are in question in which case color bonds identifiable as colored variants of electro-pions could be assumed to contribute in the first guess the mass $m(\pi) = 1.062$ MeV per each nucleon for $A > 2$ nuclei. This implies the general replacement

$$\begin{aligned} E_B &\rightarrow E_B + m(\pi_L) - \frac{N}{A}\Delta m \text{ for } A > 2 \ , \\ E_B &\rightarrow E_B + \frac{m(\pi_L)}{2} - \frac{N}{A}\Delta m \text{ for } A = 2 \ . \end{aligned} \tag{9.3.1}$$

This option will be referred to as option IIb). One can also consider the option IIa) in which nucleons are ordinary but lepto-pion mass $m(\pi_L) = 1.062$ MeV gives the mass associated with color bond.

These options are equivalent for $N = Z = 2n$ nuclei with $A > 4$ but for $A \leq 4$ nuclei assumed to form nucleon string they options differ.

9.4 Light nuclei as color bound Bose-Einstein condensates of ^4He nuclei

The attempt to understand the variation of nuclear binding energy and its maximum for Fe leads to a model of nuclei lighter than Fe as color bound Bose-Einstein condensates of ^4He nuclei or meson-like structures associated with them. Fractal scaling argument allows to understand ^4He itself as analogous state formed from nucleons.

9.4.1 How to explain the maximum of E_B for iron?

The simplest model predicts that the binding energy per nucleon equals to $E_B(^4He)$ for all $Z = N = 2n$ nuclei. The actual binding energy grows slowly, has a maximum at ^{52}Fe, and then begins to decrease but remains above $E_B(^4He)$. The following values give representative examples for $Z = N$ nuclei.

nucleus	^4He	^8Be	^{40}Ca	^52Fe
E_B/MeV	7.0720	7.0603	8.5504	8.6104

For nuclei heavier than Fe there are no long-lived $Z = N = 2n$ isotopes and the natural reason would be alpha decay to ^{52}Fe. If tetra-neutron is what TGD suggests it to be one can guess that tetra-neutron mass is very nearly equal to the mass of the alpha particle. This would allow to regard states $N = Z + 4n$ as states as analogous to unstable states $N_1 = Z_1 = Z + 2n$ consisting of alpha particles. This gives estimate for E_B for unstable $N = Z$ states. For $^{256}Fm = (100, 156)$ one has $E_B = 7.433$ MeV which is still above $E_B(^4He) = 7.0720$ MeV. The challenge is to understand the variation of the binding energy per nucleon and its maximum for Fe.

9.4.2 Scaled up QCD with Bose-Einstein condensate of ^4He nuclei explains the growth of E_B

The first thing to come in mind is that repulsive Coulomb contribution would cause the variation of the binding energy. Since alpha particles are building blocks for $Z = N$ nuclei, ^8Be provides a test for this idea. If the difference between binding energies per nucleon for ^8Be and ^4He were due to Coulomb repulsion alone, one would have $E_c = E_B(^4He) - E_B(^8Be) = .0117$ MeV, which is of order $\alpha_{em}/L(127)$. This would conform with the idea that flux tubes mediating em interaction have length of order electron Compton length. Long flux tubes would provide the mechanism minimizing Coulomb energy. A more realistic interpretation consistent with this mechanism would be that Coulombic and color interaction energies compensate each other: this can of course occur to some degree but it seems safe to assume that Coulomb contribution is small.

The basic question is how one could understand the behavior of E_B if its variation corresponds to that for color binding energy per nucleon. The natural scale of energy is MeV and this conforms with the fact that the range of variation for color binding energy associated with $L(127)$ QCD is about 1.5 MeV. By a naive scaling the value of M_{127} pion mass is by a factor $2^{(127-107)/2} = 10^{-3}$ times smaller than that of ordinary pion and thus .14 MeV. The scaling of QCD Λ is a more reliable estimate for the binding energy scale and gives a slightly larger value but of the same order of magnitude. The total variation of E_B is large in the natural energy scale of M_{127} QCD and suggests strong non-linear effects.

In the absence of other contributions em and color contributions to E_B cancel for ^8Be. If color and Coulomb contributions on total binding energy depend roughly linearly on the number of ^4He nuclei, the cancellation to E_B should occur in a good approximation also for them. This does not happen which means that color contribution to E_B is in lowest approximation linear in n meaning n^2-dependence of the total color binding energy. This non-linear behavior suggests strongly the presence of Bose-Einstein condensate of ^4He nuclei or structures associated with them. The most natural candidates are the meson like colored strings connecting ^4He nuclei together.

The additivity of n color magnetic (and/or electric) fluxes would imply that classical field energy is n^2-fold. This does not yet imply same for binding energy unless the value of α_s is negative which it can be below confinement length scale. An alternative interpretation could be in terms of color magnetic interaction energy. The number of quarks and anti-quarks would be proportional to n as would be also the color magnetic flux so that n^2- proportionality would result also in this manner.

If the addition of single alpha particle corresponds to an addition of a constant color contribution E_s to E_B (the color binding energy per nucleon, not the total binding energy!) one has $E_B(^{52}Fe) = E_B(^4He) + 13E_s$ giving $E_s = .1834$ MeV, which conforms with the order of magnitude estimate given by M_{127} QCD.

The task is to find whether this picture could explain the behavior of E_B. The simplest formula for $E_B(Z = N = 2n)$ would be given by

$$E_B(n) = -\frac{n(n-1)}{L(A)n}k_s + nE_s \ . \tag{9.4.1}$$

Here the first term corresponds to the Coulomb interaction energy of n ^4He nuclei proportional to $n(n-1)$ and inversely proportional to the length $L(A)$ of nuclear string. Second term is color binding energy per nucleon proportional to n.

The simplest assumption is that each ^4He corresponds always to same length of nuclear string so that one has $L \propto A$ and one can write

$$E_B(n) = E_B(^4He) - \frac{n(n-1)}{n^2}E_c + nE_s \ . \tag{9.4.2}$$

The value of $E_B(^8Be) \simeq E_B(^4He)$ $(n = 2)$ gives for the unit of Coulomb energy

$$E_c = 4E_s + 2[E_B(^4He) - E_B(^8Be)] \simeq 4E_s \ . \tag{9.4.3}$$

The general formula for the binding energy reads as

$$\begin{aligned} E_B(n) &= E_B(^4He) - 2\frac{n(n-1)}{n^2}[E_B(^4He) - E_B(^8Be)] \\ &+ [-4\frac{n(n-1)}{n^2} + n]E_s \ . \end{aligned} \tag{9.4.3}$$

The condition that $E_B(^{52}Fe)$ $(n = 13)$ comes out correctly gives

$$E_s = \frac{13}{121}(E_B(^{52}Fe) - E_B(^4He)) + \frac{13\times 24}{121}[E_B(^4He) - E_B(^8Be)] \ . \tag{9.4.4}$$

This gives $E_s \simeq .1955$ MeV which conforms with M_{127} QCD estimate. For the E_c one obtains $E_c = 1.6104$ MeV and for Coulomb energy of ^4He nuclei in ^8Be one obtains $E = E_c/2 = .8052$ MeV. The order of magnitude is consistent with the mass difference of proton and neutron. The scale suggests that electromagnetic flux tubes are shorter than color flux tubes and correspond to the secondary p-adic length scale $L(2,61) = L(127)/2^{5/2}$ associated with Mersenne prime M_{61}. The scaling factor for the energy scale would be $2^{5/2} \simeq 5.657$.

The calculations have been carried out without assuming which are actual composites of ^4He nuclei (neutrons and protons plus neutral color bonds or protons and neutral and negatively charged color bonds) and assuming the masses of color bonds are negligible. As a matter fact, the mass of color bond does not affect the estimates if one uses only nuclei heavier than ^4He to estimate the parameters. The estimates above however involve ^4He so that small change on the parameters is induced.

9.4.3 Why E_B decreases for heavier nuclei?

The prediction that E_B increases as $(A/4)^2$ for $Z = N$ nuclei is unrealistic since E_B decreases slowly for $A \geq 52$ nuclei. Fermi statistics provides a convincing explanation assuming that fermions move in an effective harmonic oscillator potential due to the string tension whereas free nucleon model predicts too large size for the nucleus. The splitting of the Bose-Einstein condensate to pieces is second explanation that one can imagine but fails at the level of details.

Fermi statistics as a reason for the reduction of the binding energy

The failure of the model is at least partially due to the neglect of the Fermi statistics. For the lighter nuclei description as many boson state with few fermions is expected to work. As the length of nuclear string grows in fixed nuclear volume, the probability of self intersection increases and Fermi statistics forces the wave function for stringy configurations to wiggle which reduces binding energy.

1. For the estimation purposes consider $A = 256$ nucleus ^{256}Mv having $Z = 101$ and $E_B = 7.4241$ MeV. Assume that this unstable nucleus is nearly equivalent with a nucleus consisting of $n = 64$ ^4He nuclei ($Z = N$). Assuming single color condensate this would give the color contribution

$$E_s^{tot} = (Z/2)^2 \times E_s = 64^2 \times E_s$$

with color contribution to E_B equal to $(Z/2)E_s \simeq 12.51$ MeV.

2. Suppose that color binding energy is cancelled by the energy of nucleon identified as kinetic energy in the case of free nucleon model and as harmonic oscillator energy in the case of harmonic oscillator model.

3. The number of states with a given principal quantum number n for both free nucleons in a spherical box and harmonic oscillator model is by spherical symmetry $2n^2$ and the number of protons/neutrons for a full shell nuclei behaves as $N_1 \simeq 2n_{max}^3/3$. The estimate for the average energy per nucleon is given in the two cases as

$$\begin{aligned} \langle E\rangle_H &= 2^{-4/3} \times N^{1/3}E_0 \ , \quad E_0 = \omega_0 \ , \\ \langle E\rangle_F &= \frac{2}{5}(\frac{3}{2})^{5/3}N^{2/3}E_0 \ , \quad E_0 = \frac{\pi^2}{2m_pL^2} \ . \end{aligned} \tag{9.4.3}$$

Harmonic oscillator energy $\langle E\rangle_H$ increases as $N^{1/3}$ and $\langle E\rangle_F$ as $N^{2/3}$. Neither of these cannot win the contribution of the color binding energy increasing as N.

4. Equating this energy with the total color binding energy gives an estimate for E_0 as

$$
\begin{aligned}
E_0 &= (2/3)^{1/3} \times Z^{-4/3} \times (Z/2)^2 \times E_s \ , \\
E_0 &= \frac{5}{4}(\frac{2}{3})^{5/3} \times Z^{-5/3} \times (Z/2)^2 \times E_s \ , \\
E_s &= .1955 \ MeV \ . \qquad (9.4.2)
\end{aligned}
$$

The first case corresponds to harmonic oscillator model and second to free nucleon model.

5. For the harmonic oscillator model one obtains the estimate $E_0 = \hbar\omega_0 \simeq 2.73 \ MeV$. The general estimate for the energy scale in the harmonic oscillator model given by $\omega_0 \simeq 41 \cdot A^{-1/3}$ MeV [29] giving $\omega_0 = 6.5$ MeV for $A = 256$ (this estimate implies that harmonic oscillator energy per nucleon is approximately constant and would suggest that string tension tends to reduce as the length of string increases). Harmonic oscillator potential would have roughly twice too strong strength but the order of magnitude is correct. Color contribution to the binding energy might relate the reduction of the oscillator strength in TGD framework.

6. Free nucleon model gives the estimate $E_0 = .0626$ MeV. For the size of a $A = 256$ nucleus one obtains $L \simeq 3.8L(113) \simeq 76$ fm. This is by one order of magnitude larger that the size predicted by the standard formula $r = r_0A^{1/3}$, $r_0 = 1.25$ fm and 8 fm for $A = 256$.

   Harmonic oscillator picture is clearly favored and string tension explains the origin of the harmonic oscillator potential. Harmonic oscillator picture is expected to emerge at the limit of heavy nuclei for which nuclear string more or less fills the nuclear volume whereas for light nuclei the description in terms of bosonic ^4He nuclei should make sense. For heavy nuclei Fermi statistics at nuclear level would begin to be visible and excite vibrational modes of the nuclear string mapped to the excited states of harmonic oscillator in the shell model description.

Could upper limit for the size of ^4He Bose-Einstein condensate explain the maximum of binding energy per nucleon?

One can imagine also an alternative explanation for why E_B to decrease after $A = 52$. One might that $A = 52$ represents the largest ^4He Bose-Einstein condensate and that for heavier nuclei Bose-Einstein condensate de-coheres into two parts. Bose-Einstein condensate of $n = 13$ ^4He nuclei would the best that one can achieve.

This could explain the reduction of the binding energy and also the emergence of tetra-neutrons as well as the instability of $Z = N$ nuclei heavier than ^{52}Fe. A number theoretical interpretation related to the p-adic length scale hypothesis suggests also itself: as the size of the tangled nuclear string becomes larger than the next p-adic length scale, Bose-Einstein condensate might lose its coherence and split into two.

If one assumes that ^4He Bose-Einstein condensate has an upper size corresponding to $n = 13$, the prediction is that after $A = 52$ second Bose-Einstein condensate begins to form. E_B is obtained as the average

$$E_B(Z,N) = \frac{52}{A}E_B(^{52}Fe) + \frac{A-52}{A}E_B(^{A-52}X(Z,N)) \ .$$

The derivative

$$dE_B/dA = (52/A)[-E_B(^{52}Fe) + E_B(^{A-52}X)] + \frac{A-52}{A}dE_B(^{A-52}X(Z,N))/dA$$

is first negative but its sign must change since the nuclei consisting of two copies of ^{52}Fe) condensates have same E_B as ^{52}Fe). This is an un-physical result. This does not exclude the splitting of Bose-Einstein condensate but the dominant contribution to the reduction of E_B must be due to Fermi statistics.

9.5 What QCD binds nucleons to $A \leq 4$ nuclei?

The obvious question is whether scaled variant(s) of color force could bind nucleons to form $A \leq 4$ nuclei which in turn bind to form heavier nuclei. Since the binding energy scale for ^3He is much smaller than for ^4He one might consider the possibility that the p-adic length scale for QCD associated with ^4He is different from that for $A < 4$ nuclei.

9.5.1 The QCD associated with nuclei lighter than ^4He

It would be nice if one could understand the binding energies of also $A \leq 4$ nuclei in terms of a scaled variant of QCD applied at the level of nucleons. Here one has several options to test.

Various options to consider

Assume that neutral color bonds have negligible fermion masses at their ends: this is expected if the exotic quarks appear at the ends of color bonds and by the naive scaling of pion mass. One can also consider the possibility that the p-adic temperature for the quarks satisfies $T = 1/n \leq 1/2$ so that quarks would be massless in excellent approximation. $T = 1/n < 1$ holds true for gauge bosons and one might argue that color bonds as bosonic particles indeed have $T < 1$.

Option Ia): Building bricks are ordinary nucleons.

Option IIa): Building blocks are protons and neutral and negatively charged color bonds. This means the replacement $E_B \rightarrow E_B - \Delta m$ for $A > 2$ nuclei and $E_B \rightarrow E_B - \Delta m/2$ for $A = 2$ with $\Delta m = n_n - m_p = 1.2930$ MeV.

Options Ib and IIb are obtained by assuming that the masses of fermions at the ends of color bonds are non-negligible. Electro-pion mass $m(\pi_L) = 1.062$ MeV is a good candidate for the mass of the color bond. Option Ia allow 3 per cent accuracy for the predicted binding energies. Option IIb works satisfactorily but the errors are below 22 per cent only.

Ordinary nucleons and massless color bonds

It turns out that for the option Ia), ordinary nucleons and massless color bonds, is the most plausible candidate for $A < 4$ QCD is the secondary p-adic length scale $L(2, 59)$ associated with prime $p \simeq 2^k$, $k = 59$ with $k_{eff} = 2 \times 59 = 118$. The proper scaling of the electromagnetic p-adic length scale corresponds to a scaling factor 2^3 meaning that one has $k_{eff} = 122 \rightarrow k_{eff} - 6 = 116 = 4 \times 29$ corresponding to $L(4, 29)$.

1. *Direct p-adic scaling of the parameters*

E_s would be scaled up p-adically by a factor $2^{(127-118)/2} = 2^{9/2}$. E_c would be scaled up by a factor $2^{(122-116)/2} = 2^3$. There is also a scaling of E_c by a factor 1/4 due to the reduction of charge unit and scaling of both E_c and E_s by a factor 1/4 since the basic units are now nucleons. This gives

$$\hat{E}_s = 2^{5/2} E_s = 1.1056 \ MeV \ , \ \hat{E}_c = 2^{-1} E_c = .8056 \ MeV \ . \tag{9.5.1}$$

The value of electromagnetic energy unit is quite reasonable.

The basic formula for the binding energy reads now

$$E_B = -\frac{(n(p)(n(p)-1))}{A^2} \hat{E}_c + n\hat{E}_s \ , \tag{9.5.2}$$

where $n(p)$ is the number of protons $n = A$ holds true for $A > 2$. For deuteron one has $n = 1$ since deuteron has only single color bond. This delicacy is a crucial prediction and the model fails to work without it.

This gives

$$E_B(^2H) = \hat{E}_s \ , \ E_B(^3H) = 3\hat{E}_s \ , \ E_B(^3He) = -\frac{2}{9}\hat{E}_c + 3\hat{E}_s \ . \tag{9.5.2}$$

The predictions are given by the third row of the table below. The predicted values given are too large by about 15 per cent in the worst case.

The reduction of the value of α_s in the p-adic scaling would improve the situation. The requirement that $E_B(^3H)$ comes out correctly predicts a reduction factor .8520 for α_s. The predictions are given in the fourth row of the table below. Errors are below 15 per cent.

nucleus	^2H	^3H	^3He
$E_B(exp)/MeV$	1.111	2.826	2.572
$E_B(pred_1)/MeV$	1.106	3.317	3.138
$E_B(pred_2)/MeV$	.942	2.826	2.647

The discrepancy is 15 per cent for ^2H. By a small scaling of E_c the fit for ^3He can be made perfect. Agreement is rather good but requires that conventional strong force transmitted along nuclear space-time sheet is present and makes *nn* and *pp* states unstable. Isospin dependent strong interaction energy would be only .17 MeV in isospin singlet state which suggests that a large cancellation between scalar and vector contributions occurs. *pnn* and *ppn* could be regarded as Dn and Dp states with no strong force between D and nucleon. The contribution of isospin dependent strong force to E_B is scaled down by a factor 2/3 in $A = 3$ states from that for deuteron and is almost negligible. This option seems to allow an almost perfect fit of the binding energies. Note that one cannot exclude exotic nn-state obtained from deuteron by giving color bond negative em charge.

Other options

Consider next other options.

1. *Option IIb*

For option IIb) the basic building bricks are protons and $m(\pi) = 1.062$ is assumed. The basic objection against this option is that for protons as constituents *real* binding energies satisfy $E_B(^3He) < E_B(^3H)$ whereas Coulombic repulsion would suggest $E_B(^3He) > E_B(^3H)$ unless magnetic spin-spin interaction effects affect the situation. One can however look how good a fit one can obtain in this manner.

As found, the predictions of direct scaling are too large for $E_B(^3H)$ and $E_B(^3He)$ (slight reduction of α_s cures the situation). Since the actual binding energy increases by $m(\pi_L) - (2/3)(m_n - m_p)$ for ^3H and by $m(\pi_L) - (1/3)(m_n - m_p)$ for ^3He, it is clear that the assumption that lepto-pion mass is of order 1 MeV improves the fit. The results are given by the table below.

nucleus	^2H	^3H	^3He
$E_B(exp)/MeV$	1.111	2.826	2.572
$E_B(pred)/MeV$	.875	3.117	2.507

Here $E_B(pred)$ corresponds to the effective value of binding energy assuming that nuclei effectively consist of ordinary protons and neutrons. The discrepancies are below 22 percent.

What is troublesome that neither the scaling of α_s nor modification of E_c improves the situation for ^2H and ^3H. Moreover, magnetic spin-spin interaction energy for deuteron is expected to reduce $E_B(pred)$ further in triplet state. Thus option IIb) does not look promising.

2. *Option Ib)*

For option Ib) with $m(\pi) = 1.062$ MeV and ordinary nucleons the actual binding $E_B(act)$ energy increases by $m(\pi)$ for $A = 3$ nuclei and by $m(\pi)/2$ for deuteron. Direct scaling gives a reasonably good fit for the p-adic length scale $L(9, 13)$ with $k_{eff} = 117$ meaning $\sqrt{2}$ scaling of E_s. For deuteron the predicted E_B is too low by 30 per cent. One might argue that isospin dependent strong force between nucleons becomes important in this p-adic length scale and reduces deuteron binding energy by 30 per cent. This option is not un-necessary complex as compared to the option Ia).

nucleus	^2H	^3H	^3He
$E_B(act)/MeV$	1.642	3.880	3.634
$E_B(pred)/MeV$	1.3322	3.997	3.743

For option IIa) with $m(\pi)=0$ and protons as building blocks the fit gets worse for $A=3$ nuclei.

9.5.2 The QCD associated with ^4He

^4He must somehow differ from $A\leq 3$ nucleons. If one takes the argument based on isospin dependence strong force seriously, the reasonable looking conclusion would be that ^4He is at the space-time sheet of nucleons a bound state of two deuterons which induce no isospin dependent strong nuclear force. One could regard the system also as a closed string of four nucleons such that neighboring p and n form strong iso-spin singlets. The previous treatment applies as such.

For ^4He option Ia) with a direct scaling would predict $E_B(^4He) < 4\times \hat{E}_s = 3.720$ MeV which is by a factor of order 2 too small. The natural explanation would be that for ^4He both color and em field body correspond to the p-adic length scale $L(4,29)$ ($k_{eff}=116$) so that E_s would increase by a factor of 2 to 1.860 MeV. Somewhat surprisingly, $A\leq 3$ nuclei would have "color field bodies" by a factor 2 larger than ^4He.

1. For option Ia) this would predict $E_B(^4He) = 7.32867$ MeV to be compared with the real value 7.0720 MeV. A reduction of α_s by 3.5 per cent would explain the discrepancy. That α_s decreases in the transition sequence $k_{eff} = 127\rightarrow 118\rightarrow 116$ which is consistent with the general vision about evolution of color coupling strength.

2. If one assumes option Ib) with $m(\pi)=1.062$ MeV the actual binding energy increases to 8.13 MeV. The strong binding energy of deuteron units would give an additional .15 MeV binding energy per nucleon so that one would have $E_B(^4He) = 7.47$ MeV so that 10 per cent accuracy is achieved. Obviously this option does not work so well as Ia).

3. If one assumes option IIb), the actual binding energy would increase by .415 MeV to 7.4827 MeV which would make fit somewhat poorer. A small reduction of E_c could allow to achieve a perfect fit.

9.5.3 What about tetra-neutron?

One can estimate the value of $E_B(^4n)$ from binding energies of nuclei (Z,N) and $(Z,N{+}4)$ $(A=Z{+}N)$ as

$$E_B(^4n) = \frac{A+4}{4}[E_B(A+4) - \frac{A}{A+4}E_B(A)] \ .$$

In the table below there are some estimate for $E_B(^4n)$.

(Z,N)	(26,26)(^{52}Fe)	(50,70)(^{120}Sn)	(82,124) (^{206}Pb)
$E_B(^4n)/MeV$	6.280	7.3916	5.8031

The prediction of the above model would be $E(^4n) = 4\hat{E}_s = 3.760$ MeV for $\hat{E}_s = .940$ MeV associated with $A<4$ nuclei and $k_{eff} = 118 = 2\times 59$ associated with $A<4$ nuclei. For $k_{eff}=116$ associated with ^4He $E_s(^4n) = E_s(^4He) = 1.82$ MeV the prediction would be 7.28 MeV. 14 percent reduction of α_s would give the estimated value for of E_s for ^{52}Fe.

If tetra-neutron is ppnn bound state with two negatively charged color bonds, this estimate is not quite correct since the actual binding energy per nucleon is $E_B(^4He) - (m_n-m_p)/2$. This implies a small correction $E_B(A+4)\rightarrow E_B(A+4) - 2(m_n-m_p)/(A+4)$. The correction is negligible.

One can make also a direct estimate of ^4n binding energy assuming tetra-neutron to be ppnn bound state. If the masses of charged color bonds do not differ appreciably from those of neutral bonds (as the p-adic scaling of $\pi + -\pi^0$ mass difference of about 4.9 MeV strongly suggests) then model Ia) with $E_s = E_B(^3H)/3$ implies that the actual binding energy $E_B(^4n) = 4E_s = E_B(^3H)/3$ (see the table below). The apparent binding energy is $E_{B,app} = E_B(^4n) + (m_n-m_p)/2$. Binding energy differs dramatically from what one can imagine in more conventional models of strong interactions in which even the existence of tetra-neutron is highly questionable.

k_{eff}	2×59	4×29
$E_B(act)(^4n)/MeV$	3.7680	
$E_{B,app}(4n)/MeV$	4.4135	8.1825

The higher binding energy per nucleon for tetra-neutron might directly relate to the neutron richness of heavy nuclei in accordance with the vision that Coulomb energy is what disfavors proton rich nuclei.

According to [26], tetra-neutron might have been observed in the decay $^8He \rightarrow ^4 He + ^4 n$ and the accepted value for the mass of ^8He isotope gives the upper bound of $E(^4n) < 3.1$ MeV, which is one half of the the estimate. One can of course consider the possibility that free tetra-neutron corresponds to $L(2,59)$ and nuclear tetra-neutron corresponds to the length scale $L(4,29)$ of ^4He. Also light quarks appear as several p-adically scaled up variants in the TGD based model for low-lying hadrons and there is also evidence that neutrinos appear in several scales.

9.5.4 What could be the general mass formula?

In the proposed model nucleus consists of $A \leq 4$ nuclei. Concerning the details of the model there are several questions to be answered. Do $A \leq 3$ nuclei and $A = 4$ nuclei ^4He and tetra-neutron form separate nuclear strings carrying their own color magnetic fields as the different p-adic length scale for the corresponding "color magnetic bodies" would suggest? Or do they combine by a connected sum operation to single closed string? Is there single Bose-Einstein condensate or several ones.

Certainly the Bose-Einstein condensates associated with nucleons forming $A < 4$ nuclei are separate from those for $A = 4$ nuclei. The behavior of E_B in turn can be understood if ^4He nuclei and tetra-neutrons form separate Bose-Einstein condensates. For $Z > N$ nuclei poly-protons constructed as exotic charge states of stable $A \leq 4$ nuclei could give rise to the proton excess.

Before continuing it is appropriate to list the apparent binding energies for poly-neutrons and poly-protons.

poly-neutron	n	^2n	^3n	^4n
$E_{B,app}/MeV$	0	$E_B(^2H) + \frac{\Delta}{2}$	$E_B(^3H) + \frac{2\Delta}{3}$	$E_B(^4He) + \frac{\Delta}{2}$
poly-proton	p	^2p	^3p	^4p
$E_{B,app}/MeV$	0	$E_B(^2H) - \frac{\Delta}{2}$	$E_B(^3He) - \frac{\Delta}{3}$	$E_B(^4He) - \frac{\Delta}{2}$

For heavier nuclei $E_{B,app}(^4n)$ is smaller than $E_B(^4He) + (m_p - m_n)/2$.

The first guess for the general formula for the binding energy for nucleus (Z, N) is obtained by assuming that for maximum number of ^4He nuclei and tetra-neutrons/tetra-protons identified as ^4H nuclei with 2 negatively/positively charged color bonds are present.

1. *$N \geq Z$ nuclei*

Even-Z nuclei with $N \geq Z$ can be expressed as $(Z = 2n, N = 2(n+k) + m)$, $m = 0, 1, 2$ or 3. For $Z \leq 26$ (only single Bose-Einstein condensate) this gives for the apparent binding energy per nucleon (assuming that all neutrons are indeed neutrons) the formula

$$\begin{aligned} E_B(2n, 2(n+k)+m) &= \frac{n}{A}E_B(^4He) + \frac{k}{A}E_{B,app}(^4n) + \frac{1}{A}E_{B,app}(^mn) \\ &+ \frac{n^2+k^2}{n+k}E_s - \frac{Z(Z-1)}{A^2}E_c \ . \end{aligned} \tag{9.5.2}$$

The situation for the odd-Z nuclei $(Z, N) = (2n+1, 2(n+k)+m)$ can be reduced to that for even-Z nuclei if one can assume that the $(2n+1)^{th}$ proton combines with 2 neutrons to form ^3He nucleus so that one has still $2(k-1)+m$ neutrons combining to $A \leq 4$ poly-neutrons in above described manner.

2. *$Z \geq N$ nuclei*

For the nuclei having $Z > N$ the formation of a maximal number of ^4He nuclei leaves k excess protons. For long-lived nuclei $k \leq 2$ is satisfied. One could think of decomposing the excess protons to exotic variants of $A \leq 4$ nuclei by assuming that some charged bonds carry positive charge with an obvious generalization of the above formula.

The only differences with respect to a nucleus with neutron excess would be that the apparent binding energy is smaller than the actual one and positive charge would give rise to Coulomb interaction energy reducing the binding energy (but only very slightly). The change of the binding energy in the subtraction of single neutron from $Z = N = 2n$ nucleus is predicted to be approximately $\Delta E_B = -E_B(^4He)/A$. In the case of ^{32}S this predicts $\Delta E_B = .2209$ MeV. The real value is .2110 MeV. The fact that the general order of magnitude for the change of the binding energy as Z or N changes by one unit supports the proposed picture.

9.5.5 Nuclear strings and cold fusion

To summarize, option Ia) assuming that strong isospin dependent force acts on the nuclear space-time sheet and binds pn pairs to singlets such that the strong binding energy is very nearly zero in singlet state by the cancelation of scalar and vector contributions, is the most promising one. It predicts the existence of exotic di-,tri-, and tetra-neutron like particles and even negatively charged exotics obtained from $^2H,^3H,^3He$, and ^4He by adding negatively charged color bond. For instance, ^3H extends to a multiplet with em charges $1, 0, -1, -2$. Of course, heavy nuclei with proton neutron excess could actually be such nuclei.

The exotic states are stable under beta decay for $m(\pi) < m_e$. The simplest neutral exotic nucleus corresponds to exotic deuteron with single negatively charged color bond. Using this as target it would be possible to achieve cold fusion since Coulomb wall would be absent. The empirical evidence for cold fusion thus supports the prediction of exotic charged states.

Signatures of cold fusion

In the following the consideration is restricted to cold fusion in which two deuterium nuclei react strongly since this is the basic reaction type studied.

In hot fusion there are three reaction types:
1) $D + D \rightarrow^4 He + \gamma$ $(23.8MeV)$
2) $D + D \rightarrow^3 He + n$
3) $D + D \rightarrow^3 H + p$.
The rate for the process 1) predicted by standard nuclear physics is more than 10^{-3} times lower than for the processes 2) and 3) [62]. The reason is that the emission of the gamma ray involves the relatively weak electromagnetic interaction whereas the latter two processes are strong.

The most obvious objection against cold fusion is that the Coulomb wall between the nuclei makes the mentioned processes extremely improbable at room temperature. Of course, this alone implies that one should not apply the rules of hot fusion to cold fusion. Cold fusion indeed differs from hot fusion in several other aspects.

1. No gamma rays are seen.
2. The flux of energetic neutrons is much lower than expected on basis of the heat production rate an by interpolating hot fusion physics to the recent case.

These signatures can also be (and have been!) used to claim that no real fusion process occurs. It has however become clear that the isotopes of Helium and also some tritium accumulate to the Pd target during the reaction and already now prototype reactors for which the output energy exceeds input energy have been built and commercial applications are under development, see for instance [39]. Therefore the situation has turned around. The rules of standard physics do not apply so that some new nuclear physics must be involved and it has become an exciting intellectual challenge to understand what is happening. A representative example of this attitude and an enjoyable analysis of the counter arguments against fold fusion is provided by the article 'Energy transfer in cold fusion and sono-luminescence' of Julian Schwinger [40]. This article should be contrasted with the ultra-skeptical article 'ESP and Cold Fusion: parallels in pseudoscience' of V. J. Stenger [41].

Cold fusion has also other features, which serve as valuable constraints for the model building.

1. Cold fusion is not a bulk phenomenon. It seems that fusion occurs most effectively in nano-particles of Pd and the development of the required nano-technology has made possible to produce fusion energy in controlled manner. Concerning applications this is a good news since there is no fear that the process could run out of control.

2. The ratio x of D atoms to Pd atoms in Pd particle must lie the critical range $[.85, .90]$ for the production of ^4He to occur [19]. This explains the poor repeatability of the earlier experiments and also the fact that fusion occurred sporadically.

3. Also the transmutations of Pd nuclei are observed [66].

Below a list of questions that any theory of cold fusion should be able to answer.

1. Why cold fusion is not a bulk phenomenon?

2. Why cold fusion of the light nuclei seems to occur only above the critical value $x \simeq .85$ of D concentration?

3. How fusing nuclei are able to effectively circumvent the Coulomb wall?

4. How the energy is transferred from nuclear degrees of freedom to much longer condensed matter degrees of freedom?

5. Why gamma rays are not produced, why the flux of high energy neutrons is so low and why the production of ^4He dominates (also some tritium is produced)?

6. How nuclear transmutations are possible?

Could exotic deuterium make cold fusion possible?

One model of cold fusion has been already discussed in [F8] and the recent model is very similar to that. The basic idea is that only the neutrons of incoming and target nuclei can interact strongly, that is their space-time sheets can fuse. One might hope that neutral deuterium having single negatively charged color bond could allow to realize this mechanism.

1. Suppose that part of the deuterium in Pd catalyst corresponds to exotic deuterium with neutral nuclei so that cold fusion would occur between neutral exotic D nuclei in the target and charged incoming D nuclei and Coulomb wall in the nuclear scale would be absent.

2. The exotic variant of the ordinary $D + D$ reaction yields final states in which ^4He, ^3He and ^3H are replaced with their exotic counterparts with charge lowered by one unit. In particular, exotic ^3H is neutral and there is no Coulomb wall hindering its fusion with Pd nuclei so that nuclear transmutations can occur.

Why the neutron and gamma fluxes are low might be understood if for some reason only exotic ^3H is produced, that is the production of charged final state nuclei is suppressed. The explanation relies on Coulomb wall at the nucleon level.

1. Initial state contains one charged and one neutral color bond and final state $A = 3$ or $A = 4$ color bonds. Additional neutral color bonds must be created in the reaction (one for the production $A = 3$ final states and two for $A = 4$ final state). The process involves the creation of neural fermion pairs. The emission of one exotic gluon per bond decaying to a neutral pair is necessary to achieve this. This requires that nucleon space-time sheets fuse together. Exotic D certainly belongs to the final state nucleus since charged color bond is not expected to be split in the process.

2. The process necessarily involves a temporary fusion of nucleon space-time sheets. One can understand the selection rules if only neutron space-time sheets can fuse appreciably so that only ^3H would be produced. Here Coulomb wall at nucleon level should enter into the game.

3. Protonic space-time sheets have the same positive sign of charge always so that there is a Coulomb wall between them. This explains why the reactions producing exotic ^4He do not occur appreciably. If the quark/antiquark at the neutron end of the color bond of ordinary D has positive charge, there is Coulomb attraction between proton and corresponding negatively charged quark. Thus energy minimization implies that the neutron space-time sheet of ordinary D has positive net charge and Coulomb repulsion prevents it from fusing with the proton space-time sheet of target D. The desired selection rules would thus be due to Coulomb wall at the nucleon level.

About the phase transition transforming ordinary deuterium to exotic deuterium

The exotic deuterium at the surface of Pd target seems to form patches (for a detailed summary see [F8]). This suggests that a condensed matter phase transition involving also nuclei is involved. A possible mechanism giving rise to this kind of phase would be a local phase transition in the Pd target involving both D and Pd. In [F8] it was suggested that deuterium nuclei transform in this phase transition to "ordinary" di-neutrons connected by a charged color bond to Pd nuclei. In the recent case di-neutron could be replaced by neutral D.

The phase transition transforming neutral color bond to a negatively charged one would certainly involve the emission of W^+ boson, which must be exotic in the sense that its Compton length is of order atomic size so that it could be treated as a massless particle and the rate for the process would be of the same order of magnitude as for electro-magnetic processes. One can imagine two options.

1. Exotic W^+ boson emission generates a positively charged color bond between Pd nucleus and exotic deuteron as in the previous model.

2. The exchange of exotic W^+ bosons between ordinary D nuclei and Pd induces the transformation $Z \rightarrow Z+1$ inducing an alchemic phase transition $Pd \rightarrow Ag$. The most abundant Pd isotopes with $A = 105$ and 106 would transform to a state of same mass but chemically equivalent with the two lightest long-lived Ag isotopes. ^{106}Ag is unstable against β^+ decay to Pd and ^{105}Ag transforms to Pd via electron capture. For ^{106}Ag (^{105}Ag) the rest energy is 4 MeV (2.2 MeV) higher than for ^{106}Pd (^{105}Pd), which suggests that the resulting silver cannot be genuine.

This phase transition need not be favored energetically since the energy loaded into electrolyte could induce it. The energies should (and could in the recent scenario) correspond to energies typical for condensed matter physics. The densities of Ag and Pd are 10.49 gcm^{-3} and 12.023 gcm^{-3} so that the phase transition would expand the volume by a factor 1.0465. The porous character of Pd would allow this. The needed critical packing fraction for Pd would guarantee one D nucleus per one Pd nucleus with a sufficient accuracy.

Exotic weak bosons seem to be necessary

The proposed phase transition cannot proceed via the exchange of the ordinary W bosons. Rather, W bosons having Compton length of order atomic size are needed. These W bosons could correspond to a scaled up variant of ordinary W bosons having smaller mass, perhaps even of the order of electron mass. They could be also dark in the sense that Planck constant for them would have the value $\hbar = n\hbar_0$ implying scaling up of their Compton size by n. For $n \sim 2^{48}$ the Compton length of ordinary W boson would be of the order of atomic size so that for interactions below this length scale weak bosons would be effectively massless. p-Adically scaled up copy of weak physics with a large value of Planck constant could be in question. For instance, W bosons could correspond to the nuclear p-adic length scale $L(k = 113)$ and $n = 2^{11}$.

Few weeks after having written this chapter I learned that cold fusion is in news again: both Nature and New Scientists commented the latest results [44]. It seems that the emission of highly energetic charged particles which cannot be due to chemical reactions and could emerge from cold fusion has been demonstrated beyond doubt by Frank Cordon's team [45] using detectors known as CR-39 plastics of size scale of coin used already earlier in hot fusion research. The method is both cheap and simple. The idea is that travelling charged particles shatter the bonds of the plastic's polymers leaving pits or tracks in the plastic. Under the conditions claimed to make cold fusion possible (1 deuterium per 1 Pd nucleus making in TGD based model possible the phase transition of

D to its neutral variant by the emission of exotic dark W boson with interaction range of order atomic radius) tracks and pits appear during short period of time to the detector.

9.5.6 Strong force as a scaled and dark electro-weak force?

The fiddling with the nuclear string model has led to following conclusions.

1. Strong isospin dependent nuclear force, which does not reduce to color force, is necessary in order to eliminate polyneutron and polyproton states. This force contributes practically nothing to the energies of bound states. This can be understood as being due to the cancellation of isospin scalar and vector parts of this force for them. Only strong isospin singlets and their composites with isospin doublet (n,p) are allowed for $A \leq 4$ nuclei serving as building bricks of the nuclear strings. Only *effective* polyneutron states are allowed and they are strong isospin singlets or doublets containing charged color bonds.

2. The force could act in the length scalar of nuclear space-time sheets: $k = 113$ nuclear p-adic length scale is a good candidate for this length scale. One must be however cautious: the contribution to the energy of nuclei is so small that length scale could be much longer and perhaps same as in case of exotic color bonds. Color bonds connecting nuclei correspond to much longer p-adic length scale and appear in three p-adically scaled up variants corresponding to $A < 4$ nuclei, $A = 4$ nuclei and $A > 4$ nuclei.

3. The prediction of exotic deuterons with vanishing nuclear em charge leads to a simplification of the earlier model of cold fusion explaining its basic selection rules elegantly but requires a scaled variant of electro-weak force in the length scale of atom.

What is then this mysterious strong force? And how abundant these copies of color and electro-weak force actually are? Is there some unifying principle telling which of them are realized?

From foregoing plus TGD inspired model for quantum biology involving also dark and scaled variants of electro-weak and color forces it is becoming more and more obvious that the scaled up variants of both QCD and electro-weak physics appear in various space-time sheets of TGD Universe. This raises the following questions.

1. Could the isospin dependent strong force between nucleons be nothing but a p-adically scaled up (with respect to length scale) version of the electro-weak interactions in the p-adic length scale defined by Mersenne prime M_{89} with new length scale assigned with gluons and characterized by Mersenne prime M_{107}? Strong force would be electro-weak force but in the length scale of hadron! Or possibly in length scale of nucleus ($k_{eff} = 107 + 6 = 113$) if a dark variant of strong force with $h = nh_0 = 2^3 h_0$ is in question.

2. Why shouldn't there be a scaled up variant of electro-weak force also in the p-adic length scale of the nuclear color flux tubes?

3. Could it be that all Mersenne primes and also other preferred p-adic primes correspond to entire standard model physics including also gravitation? Could be be kind of natural selection which selects the p-adic survivors as proposed long time ago?

Positive answers to the last questions would clean the air and have quite a strong unifying power in the rather speculative and very-many-sheeted TGD Universe.

1. The prediction for new QCD type physics at M_{89} would get additional support. Perhaps also LHC provides it within the next half decade.

2. Electro-weak physics for Mersenne prime M_{127} assigned to electron and exotic quarks and color excited leptons would be predicted. This would predict the exotic quarks appearing in nuclear string model and conform with the 15 year old leptohadron hypothesis [F7]. M_{127} dark weak physics would also make possible the phase transition transforming ordinary deuterium in Pd target to exotic deuterium with vanishing nuclear charge.

The most obvious objection against this unifying vision is that hadrons decay only according to the electro-weak physics corresponding to M_{89}. If they would decay according to M_{107} weak physics, the decay rates would be much much faster since the mass scale of electro-weak bosons would be reduced by a factor 2^{-9} (this would give increase of decay rates by a factor 2^{36} from the propagator of weak boson). This is however not a problem if strong force is a dark with say $n = 8$ giving corresponding to nuclear length scale. This crazy conjecture might work if one accepts the dark Bohr rules!

9.6 Giant dipole resonance as a dynamical signature for the existence of Bose-Einstein condensates?

The basic characteristic of the Bose-Einstein condensate model is the non-linearity of the color contribution to the binding energy. The implication is that the the de-coherence of the Bose-Einstein condensate of the nuclear string consisting of ^4He nuclei costs energy. This de-coherence need not involve a splitting of nuclear strings although also this is possible. Similar de-coherence can occur for ^4He $A < 4$ nuclei. It turns out that these three de-coherence mechanisms explain quite nicely the basic aspects of giant dipole resonance (GDR) and its variants both qualitatively and quantitatively and that precise predictions for the fine structure of GDR emerge.

9.6.1 De-coherence at the level of ^4He nuclear string

The de-coherence of a nucleus having n ^4He nuclei to a nucleus containing two Bose-Einstein condensates having $n-k$ and $k > 2$ ^4He nuclei requires energy given by

$$\begin{aligned} \Delta E &= (n^2 - (n-k)^2 - k^2)E_s = 2k(n-k)E_s \ , \ \ k > 2 \ , \\ \Delta E &= (n^2 - (n-2)^2 - 1)E_s = (4n-5)E_s \ , \ \ k = 2 \ , \\ E_s &\simeq .1955 \ MeV \ . \end{aligned} \tag{9.6.-1}$$

Bose-Einstein condensate could also split into several pieces with some of them consisting of single ^4He nucleus in which case there is no contribution to the color binding energy. A more general formula for the resonance energy reads as

$$\begin{aligned} \Delta E &= (n^2 - \sum_i k^2(n_i))E_s \ , \ \ \sum_i n_i = n \ , \\ k(n_i) &= \begin{cases} n_i \ \text{for} \ n_i > 2 \ , \\ 1 \ \text{for} \ n_i = 2 \ , \\ 0 \ \text{for} \ n_i = 1 \ . \end{cases} \end{aligned} \tag{9.6.-2}$$

The table below lists the resonance energies for four manners of ^{16}O nucleus ($n = 4$) to lose its coherence.

final state	3+1	2+2	2+1+1	1+1+1+1
$\Delta E/MeV$	1.3685	2.7370	2.9325	3.1280

Rather small energies are involved. More generally, the minimum and maximum resonance energy would vary as $\Delta E_{min} = (2n-1)E_s$ and $\Delta E_{max} = n^2E_s$ (total de-coherence). For $n = n_{max} = 13$ one would have $\Delta E_{min} = 2.3640$ MeV and $\Delta E_{max} = 33.099$ MeV.

Clearly, the loss of coherence at this level is a low energy collective phenomenon but certainly testable. For nuclei with $A > 60$ one can imagine also double resonance when both coherent Bose-Einstein condensates possibly present split into pieces. For $A \geq 120$ also triple resonance is possible.

9.6.2 De-coherence inside ^4He nuclei

One can consider also the loss of coherence occurring at the level ^4He nuclei. Predictions for resonances energies and for the dependence of GR cross sections on mass number follow.

Resonance energies

For ^4He nuclei one has $E_s = 1.820$ MeV. In this case de-coherence would mean the decomposition of Bose-Einstein condensate to $n = 4 \rightarrow \sum n_i = n$ with $\Delta E = n^2 - \sum_{n_i} k^1(n_i) = 16 - \sum_{n_i} k^2(n_i)$. The table below gives the resonance energies for the four options $n \rightarrow \sum_i n_i$ for the loss of coherence.

final state	3+1	2+2	2+1+1	1+1+1+1
$\Delta E/MeV$	12.74	25.48	27.30	29.12

These energies span the range at which the cross section for $^{16}O(\gamma, xn)$ reaction has giant dipole resonances [30]. Quite generally, GDR is a broad bump with substructure beginning around 10 MeV and ranging to 30 MeV. The average position of the bump as a function of atomic number can be parameterized by the following formula

$$E(A)/MeV = 31.2A^{-1/3} + 20.6A^{-1/6} \qquad (9.6.\text{-}1)$$

given in [31]. The energy varies from 36.6 MeV for $A = 4$ (the fit is probably not good for very low values of A) to 13.75 MeV for $A = 206$. The width of GDR ranges from 4-5 MeV for closed shell nuclei up to 8 MeV for nuclei between closed shells.

The observation raises the question whether the de-coherence of Bose-Einstein condensates associated with ^4He and nuclear string could relate to GDR and its variants. If so, GR proper would be a collective phenomenon both at the level of single ^4He nucleus (main contribution to the resonance energy) and entire nucleus (width of the resonance). The killer prediction is that even ^4He should exhibit giant dipole resonance and its variants: GDR in ^4He has been reported [32].

Some tests

This hypothesis seems to survive the basic qualitative and quantitative tests.

1. The basic prediction of the model peak at 12.74 MeV and at triplet of closely located peaks at $(25.48, 27.30, 29.12)$ MeV spanning a range of about 4 MeV, which is slightly smaller than the width of GDR. According to [33] there are two peaks identified as iso-scalar GMR at $13.7 \pm .3$ MeV and iso-vector GMR at 26 ± 3 MeV. The 6 MeV uncertainty related to the position of iso-vector peak suggests that it corresponds to the triplet $(25.48, 27.30, 29.12)$ MeV whereas singlet would correspond to the iso-scalar peak. According to the interpretation represented in [33] iso-scalar *resp.* iso-vector peak would correspond to oscillations of proton and neutron densities in same *resp.* opposite phase. This interpretation can make sense in TGD framework only inside single ^4He nucleus and would apply to the transverse oscillations of ^4He string rather than radial oscillations of entire nucleus.

2. The presence of triplet structure seems to explain most of the width of iso-vector GR. The combination of GDR internal to ^4He with GDR for the entire nucleus (for which resonance energies vary from $\Delta E_{min} = (2n-1)E_s$ to $\Delta E_{max} = n^2 E_s$ $(n = A/4)$) predicts that also latter contributes to the width of GDR and give it additional fine structure. The order of magnitude for ΔE_{min} is in the range [1.3685,2.3640] MeV which is consistent with the with of GDR and predicts a band of width 1 MeV located 1.4 MeV above the basic peak.

3. The de-coherence of $A < 4$ nuclei could increase the width of the peaks for nuclei with partially filled shells: maximum and minimum values of resonance energy are $9E_s(^4He)/2 = 8.19$ MeV and $4E_s(^4He) = 7.28$ MeV for ^3He and ^3H which conforms with the upper bound 8 MeV for the width.

4. It is also possible that n ^4He nuclei simultaneously lose their coherence. If multiplet de-coherence occurs coherently it gives rise to harmonics of GDR. For de-coherent decoherence so that the emitted photons should correspond to those associated with single ^4He GDR combined with nuclear GDR. If absorption occurs for $n \leq 13$ nuclei simultaneously, one obtains a convoluted spectrum for resonant absorption energy

$$\Delta E = [16n - \sum_{j=1}^{n} \sum_{i_j} k^2(n_{i_j})]E_s \ . \quad (9.6.0)$$

The maximum value of ΔE given by $\Delta E_{max} = n \times 29.12$ MeV. For $n = 13$ this would give $\Delta E_{max} = 378.56$ MeV for the upper bound for the range of excitation energies for GDR. For heavy nuclei [31] GDR occurs in the range 30-130 MeV of excitation energies so that the order of magnitude is correct. Lower bound in turn corresponds to a total loss of coherence for single ^4He nucleus.

5. That the width of GDR increases with the excitation energy [31] is consistent with the excitation of higher GDR resonances associated with the entire nuclear string. $n \leq n_{max}$ for GDR at the level of the entire nucleus means saturation of the GDR peak with excitation energy which has been indeed observed [30].

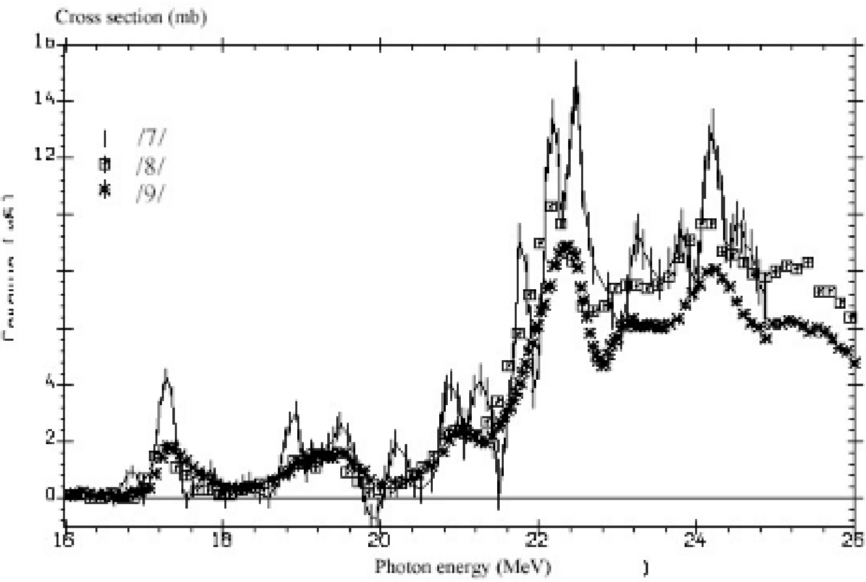

Figure 9.1: The comparison of photoneutron cross sections $^{16}O(\gamma, xn)$ obtained in one BR-experiment (Moscow State University) and two QMA experiments carried out at Saclay (France) Livermoore (USA). Figure is taken from [30] where also references to experiments can be found.

One can look whether the model might work even at the level of details. Figure 3 of [30] compares total photoneutron reaction cross sections for $^{16}O(\gamma, xn)$ in the range 16-26 MeV from some experiments so that the possible structure at 12.74 MeV is not visible in it. It is obvious that the resonance structure is more complex than predicted by the simplest model. It seems however possible to explain this.

1. The main part of the resonance is a high bump above 22 MeV spanning an interval of about 4 MeV just as the triplet at (25.48,27.30,29.12) MeV does. This suggest a shift of the predicted 3-peak structure in the range 25-30 MeV range downwards by about 3 MeV. This happens if the photo excitation inducing the de-coherence involves a dropping from a state with excitation energy of 3 MeV to the ground state. The peak structure has peaks roughly at the shifted energies but there is also an additional structure which might be understood in terms of the bands of width 1 MeV located 1.4 MeV above the basic line.

2. There are three smaller bumps below the main bump which also span a range of 4 MeV which suggests that also they correspond to a shifted variant of the basic three-peak structure. This can be understood if the photo excitation inducing de-coherence leads from an excited state with excitation energy 8.3 MeV to ground state shifting the resonance triplet (25.48, 27.30, 29.12) MeV to resonance triplet at (17.2, 19.00, 20.82) MeV.

On basis of these arguments it seems that the proposed mechanism might explain GR and its variants. The basic prediction would be the presence of singlet and triplet resonance peaks corresponding to the four manners to lose the coherence. Second signature is the precise prediction for the fine structure of resonance peaks.

Predictions for cross sections

The estimation of collision cross sections in nuclear string model would require detailed numerical models. One approach to modelling would be to treat the colliding nuclear strings as random coils with finite thickness defined by the size of $A \leq 4$ strings. The intersections of colliding strings would induce fusion reactions and self intersections fissions. Simple statistical models for the intersections based on geometric probability are possible and allow to estimate branching ratios to various channels.

In the case of GR the reduction to ^4He level means strong testable predictions for the dependence of GR cross sections on the mass number. GR involves formation of eye-glass type configuration at level of single ^4He and in the collision of nuclei with mass numbers A_1 and A_2 GR means formation of these configurations for some $A = 4$ unit associated with either nucleus. Hence the GR cross section should be in a reasonable approximation proportional to $n_1 + n_2$ where n_i are the numbers of $A = 4$ sub-units, which can be either ^4He, tetra-neutron, or possible other variants of ^4He having charged color bonds. For $Z_i = 2m_i$, $N = 2n_i$, $A_i = 4(m_i + n_i)$ nuclei one has $n_1 + n_2 = (A_1 + A_2)/4$. Also a characteristic oscillatory behavior as a function of A is expected if the number of $A = 4$ units is maximal. If GR reactions are induced by the touching of ^4He units of nuclear string implying transfer of kinetic energy between units then the GR cross sections should depend only on the energy per ^4He nucleus in cm system, which is also a strong prediction.

9.6.3 De-coherence inside $A = 3$ nuclei and pygmy resonances

For neutron rich nuclei the loss of coherence is expected to occur inside ^4He, tetra-neutron, ^3He and possibly also ^3n which might be stable in the nuclear environment. The de-coherence of tetra-neutron gives in the first approximation the same resonance energy spectrum as that for ^4He since $E_B(^4n) \sim E_B(^4He)$ roughly consistent with the previous estimates for $E_B(^4n)$ implies $E_s(^4n) \sim E_s(^4He)$.

The de-coherence inside $A = 3$ nuclei might explain the so called pygmy resonance appearing in neutron rich nuclei, which according to [34] is wide bump around $E \sim 8$ MeV. For $A = 3$ nuclei only two de-coherence transitions are possible: $3 \rightarrow 2+1$ and $3 \rightarrow 1+1+1$ and $E_s = E_B(^3H) = .940$ MeV the corresponding energies are $8E_s = 7.520$ MeV and $9 * Es = 8.4600$ MeV. Mean energy is indeed ~ 8 MeV and the separation of peaks about 1 MeV. The de-coherence at level of ^4He string might add to this 1 MeV wide bands about 1.4 MeV above the basic lines.

The figure of [35] illustrating photo-absorption cross section in ^{44}Ca and ^{48}Ca shows three peaks at 6.8, 7.3, 7.8 and 8 MeV in ^{44}Ca. The additional two peaks might be assigned with the excitation of initial or final states. This suggests also the presence of also $A = 3$ nuclear strings in ^{44}Ca besides $^H4$ and ^4n strings. Perhaps neutron halo wave function contains $^3n + n$ component besides ^4n. For ^{48}Ca these peaks are much weaker suggesting the dominance of $2{\times}^4n$ component.

9.6.4 De-coherence and the differential topology of nuclear reactions

Nuclear string model allows a topological description of nuclear decays in terms of closed string diagrams and it is interesting to look what characteristic predictions follow without going to detailed quantitative modelling of stringy collisions possibly using some variant of string models.

In the de-coherence eye-glass type singularities of the closed nuclear string appear and make possible nuclear decays.

1. At the level of ^4He sub-strings the simplest singularities correspond to $4 \rightarrow 3+1$ and $4 \rightarrow 2+2$ eye-glass singularities. The first one corresponds to low energy GR and second to one of higher

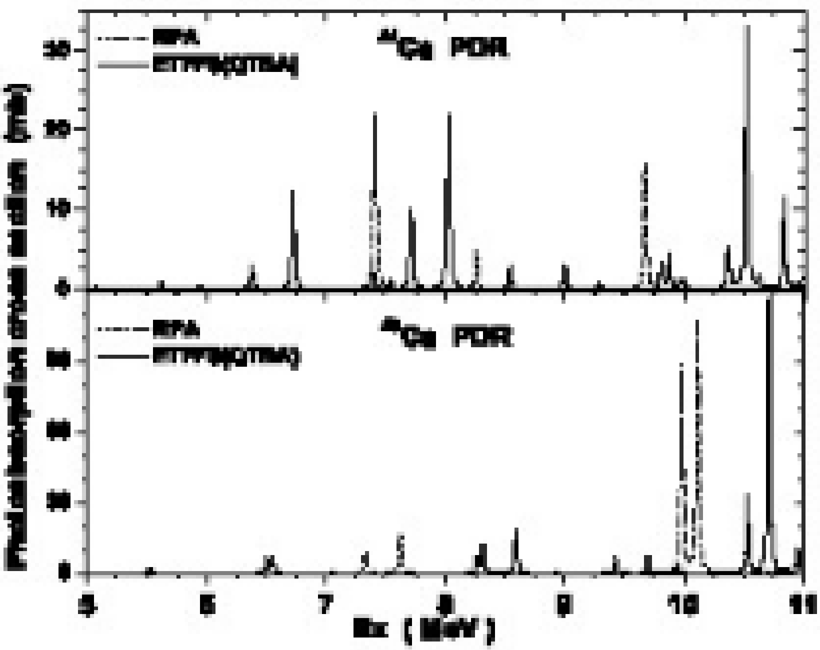

Figure 9.2: Pygmy resonances in ^{44}Ca and ^{48}Ca up to 11 MeV. Figure is taken from [35].

energy GRs. They can naturally lead to decays in which nucleon or deuteron is emitted in decay process. The singularities $4 \rightarrow 2+1+1$ *resp.* $4 \rightarrow 1+1+1+1$ correspond to eye-glasses with 3 *resp.* four lenses and mean the decay of ^4He to deuteron and two nucleons *resp.* 4 nucleons. The prediction is that the emission of deuteron requires a considerably larger excitation energy than the emission of single nucleon. For GR at level of $A = 3$ nuclei analogous considerations apply. Taking into account the possible tunnelling of the nuclear strings from the nuclear space-time sheet modifies this simple picture.

2. For GR in the scale of entire nuclei the corresponding singular configurations typically make possible the emission of alpha particle. Considerably smaller collision energies should be able to induce the emission of alpha particles than the emission of nucleons if only stringy excitations matter. The excitation energy needed for the emission of α particle is predicted to increase with A since the number n of ^4He nuclei increases with A. For instance, for $Z = N = 2n$ nuclei $n \rightarrow n-1+1$ would require the excitation energy $(2n-1)E_c = (A/2-1)E_c$, $E_c \simeq .2$ MeV. The tunnelling of the alpha particle from the nuclear space-time sheet can modify the situation.

The decay process allows a differential topological description. Quite generally, in the de-coherence process $n \rightarrow (n-k)+k$ the color magnetic flux through the closed string must be reduced from n to $n-k$ units through the first closed string and to k units through the second one. The reduction of the color color magnetic fluxes means the reduction of the total color binding energy from n^2E_c $((n-k)^2+k^2)E_c$ and the kinetic energy of the colliding nucleons should provide this energy.

Faraday's law, which is essentially a differential topological statement, requires the presence of a time dependent color electric field making possible the reduction of the color magnetic fluxes. The holonomy group of the classical color gauge field $G^A_{\alpha\beta}$ is always Abelian in TGD framework being proportional to $H^AJ_{\alpha\beta}$, where H^A are color Hamiltonians and $J_{\alpha\beta}$ is the induced Kähler form. Hence it should be possible to treat the situation in terms of the induced Kähler field alone. Obviously, the change of the Kähler (color) electric flux in the reaction corresponds to the change of (color) Kähler (color) magnetic flux. The change of color electric flux occurs naturally in a collision situation involving changing induced gauge fields.

9.7 Cold fusion, plasma electrolysis, biological transmutations, and burning salt water

The article of Kanarev and Mizuno [46] reports findings supporting the occurrence of cold fusion in NaOH and KOH hydrolysis. The situation is different from standard cold fusion where heavy water D_2O is used instead of H_2O.

One can understand the cold fusion reactions reported by Mizuno as nuclear reactions in which part of what I call dark proton string having negatively charged color bonds (essentially a zoomed up variant of ordinary nucleus with large Planck constant) suffers a phase transition to ordinary matter and experiences ordinary strong interactions with the nuclei at the cathode. In the simplest model the final state would contain only ordinary nuclear matter. The generation of plasma in plasma electrolysis can be seen as a process analogous to the positive feedback loop in ordinary nuclear reactions.

Rather encouragingly, the model allows to understand also deuterium cold fusion and leads to a solution of several other anomalies.

1. The so called lithium problem of cosmology (the observed abundance of lithium is by a factor 2.5 lower than predicted by standard cosmology [37]) can be resolved if lithium nuclei transform partially to dark lithium nuclei.

2. The so called $H_{1.5}O$ anomaly of water [36, 37, 38, 39] can be understood if 1/4 of protons of water forms dark lithium nuclei or heavier dark nuclei formed as sequences of these just as ordinary nuclei are constructed as sequences of ^4He and lighter nuclei in nuclear string model. The results force to consider the possibility that nuclear isotopes unstable as ordinary matter can be stable dark matter.

3. The mysterious behavior burning salt water [104] can be also understood in the same framework.

4. The model explains the nuclear transmutations observed in Kanarev's plasma electrolysis. Intriguingly, several biologically important ions belong to the reaction products in the case of NaOH electrolysis. This raises the question whether cold nuclear reactions occur in living matter and are responsible for generation of biologically most important ions.

9.7.1 The data

Findings of Kanarev

Kanarev has found that the volume of produced H_2 and O_2 gases is much larger than the volume resulting in the electrolysis of the water used in the process. If one knows the values of p and T one can estimate the volumes of H_2 and O_2 using the equation of state $V = nT/p$ of ideal gas. This gives

$$V(H_2;p,T) = \frac{A(H_2)}{A(H_2O)} \times \frac{M(H_20)}{m_p} = \frac{1}{9}\frac{M(H_20)}{m_p} \times \frac{T}{p} \ .$$

Here $M(H_20)$ is the total mass of the water (.272 kg for KOH and .445 kg for NaOH).

In the situation considered one should be able to produce from one liter of water 1220 liters of hydrogen and 622 liters of oxygen giving

$$V(H_2)/V(H_2O) = 1.220 \times 10^3 \ , \qquad V(O_2)/V(H_2O) = .622 \times 10^3 \ ,$$

$$r(gas) = V(H_2+O_2)/V(H_2O) = 1.844 \times 10^3 \ , \quad V(H_2)/V(O_2)) \simeq 1.96 \ .$$

$V(H_2)/V(O_2) \simeq 1.96$ is 4 per cent smaller than the prediction $V(H_2)/V(O_2) = 2$ of the ideal gas approximation.

The volumes of O_2 and H_2 are not reported separately. The table gives the total volumes of gas produced and ratios to the volume of water used.

	$M(H_2O)/kg$	$V(gas)/m^3$	$\frac{V(gas)}{V(H_2O)}$	$\frac{[V(gas)/V(H_2O)]}{r(gas)}$
KOH	.272	8.75	3.2×10^4	17.4
NaOH	.445	12.66	2.8×10^4	15.2

Table 1. The weight of water used in the electrolysis and the total volume of gas produced for KOH and NaOH electrolysis. $r(gas)$ denotes the naive prediction for the total volume of gas per water volume appearing in previous table. For KOH *resp.* NaOH the volume ratio $[V(gas)/V(H_2O)]$ is by a factor $r = 17.4$ *resp.* $r = 15.2$ higher than the naive estimate.

Findings of Mizuno

Mizuno in turn found that the Fe cathode contains Si, K, Cr, Fe, Cu for both KOH and NaOH electrolysis and in case of NaOH also Al, Sl, Ca. The fraction of these nuclei is of order one per cent. The table below gives the fractions for both KOH and NaOH.

KOH				
Element(Z,N)	Al(13,27)	Si(14,28)	Cl(17,18)	K(19,20)
%		0.94		4.50
Element(Z,N)	Ca(20,20)	Cr(24,28))	Fe(26,29)	Cu(29,34)
%		1.90	93.0	0.45
NaOH				
Element(Z,N)	Al(13,27)	Si(14,28)	Cl(17,18)	K(19,20)
%	1.10	0.55	0.20	0.60
Element(Z,N)	Ca(20,20)	Cr(24,28))	Fe(26,29)	Cu(29,34)
%	0.40	1.60	94.0	0.65

Table 2. The per cent of various nuclei in cathode for KOH and NaOH electrolysis.

The results supports the view that nuclear reactions involving new nuclear physics are involved and that part of H_2 and O_2 could be produced by nuclear reactions at the cathode.

1. For Si, K, Cr, Fe, and Cu the mechanism could be common for both $NaOH$ and KOH electrolysis and presumably involve fission of Fe nuclei. The percent of K in KOH is considerably larger than in $NaOH$ case and this is presumably due to the absorption of K^+ ions by the cathode.

2. For Al, Si, and Ca the reaction occurring only for Na should involve Na ions absorbed by the cathode and suffering cold fusion with some particles -call them just X - to be identified.

3. Cu is the only element heavier than Fe and is expected to be produced by fusion with X. Quite generally, the fractions are of order one per cent.

4. The authors suggests that the extra volume of H_2 and O_2 molecules is due to nuclear reactions in the cathode. A test for this hypothesis would be the ratio of H_2 and O_2 volumes. Large deviation from value 2 would support the hypothesis. The value near 2 would in turn support the hypothesis that the water produced by electrolysis is considerably denser than ordinary water.

9.7.2 $H_{1.5}O$ anomaly and nuclear string model

It would seem that some exotic nuclei, perhaps consisting of protons, should be involved with the cold fusion. Concerning the identification of these exotic particles there are several guidelines. $H_{1.5}O$ anomaly, anomalous production of e^+e^- pairs in heavy ion collisions, and nuclear string model.

$H_{1.5}O$ anomaly and anomalous production of electron-positron pairs in heavy ion collisions

There exists an anomaly which could be explained in terms of long open nuclear strings. The explanation of $H_{1,5}O$ anomaly [36, 37, 38, 39] discussed in [F9] as a manifestation of dark protons was one of the first applications of TGD based ideas about dark matter. The proposed explanation is that the fraction of 1/4 of protons is in attosecond time scale dark and invisible in electron scattering and neutron diffraction. Note that attosecond time scale corresponds to the time during which light travels a length of order atomic size.

A natural identification of the dark protons would be in terms of protonic strings behaving like nuclei having anomalously large size, which would be due to the anomalously large value of Planck constant. A partial neutralization by negatively charge color bonds would make these states stable.

The TGD based explanation of anomalous production of electron-positron pairs in the collisions of heavy nuclei just above the Coulomb wall [F7] is in terms of lepto-pions consisting of pairs of color octet electron and positron allowed by TGD and having mass slightly below $2m_e \simeq 1$ MeV. The strong electromagnetic fields created in collision create coherent state of leptopions decaying into electron positron pairs.

Nuclear string model

The nuclear string model describes nuclei as string like structures with nucleons connected by color magnetic flux tubes whose length is of order electron Compton length about 10^{-12} meters and even longer and thus much longer than the size scale of nuclei themselves which is below 10^{-14} meters. Color magnetic flux tubes define the color magnetic body of nucleus and each flux tube has colored fermion and antifermion at its ends. The net color of pair is non-vanishing so that color confinement binds the nucleons to the nuclear string. Nuclei can be visualized as structures analogous to plants with nucleus taking the role of seed and color magnetic body of much larger size taking the role of plant with color flux tubes however returning back to another nucleon inside nucleus.

One can imagine two basic identifications of the fermions.

1. For the first option fermions are identified as quarks. The color flux tube can have three charge states $q = +1, 0, -1$ according to whether it corresponds to $u\overline{d}, u\overline{u} + d\overline{d}$, or $\overline{u}d$ type state for quarks. This predicts a rich spectrum of exotic nuclei in which neutrons consist actually of proton plus negatively charged flux tube. The small mass difference between neutron and proton and small mass of the quarks (of order MeV) could quite well mean that these exotic nuclei are identified as ordinary nuclei. The findings of Illert [21] support the identification as quarks.

2. Lepto-hadron hypothesis [F7] encourages to consider also the possibility that color bonds have color octet electrons at their ends. This would make it easier to understand why lepto-pions are produced in the collisions of heavy nuclei.

3. One can also consider the possibility that the color bonds are superpositions of quark-antiquark pairs and colored electron-positron pairs.

Two options

One can consider two options for protonic strings. Either their correspond to open strings connected by color magnetic flux tubes or protons are dark so that giant nuclei are in question.

1. Protonic strings as open strings?

Color flux tubes connecting nucleons are long and one can ask whether it might be possible also open nuclear strings with long color flux tubes connecting widely separate nucleons even at atomic distance. These kind of structures would be favored if the ends of nuclear string are charged.

Even without assumption of large values of Planck constant for the color magnetic body and quarks the net length of flux tubes could be of the order of atomic size. Large value $\hbar$ would imply an additional scaling.

The simplest giant nuclei constructible in this manner would consist of protons connected by color magnetic flux tubes to from an open string. Stability suggest that the charge per length is not too high so that some minimum fraction of the color bonds would be negatively charged. One could speak of exotic counterparts of ordinary nuclei differing from them only in the sense that size scale is much larger. A natural assumption is that the distance between charged protonic space-time sheets along string is constant.

In the sequel the notation $X(z, n)$ will be is used for the protonic string containing net charge z and n negatively charged bonds. $a = z+n$ will denote the number of protons. z, n and a are analogous to nuclear charge Z, neutron number N, and mass number A. For open strings the charge is $z \geq 1$ and for closed strings $z \geq 0$ holds true.

This option has however problem. It is difficult imagine how the nuclear reactions could take place. One can imagine ordinary stringy diagrams in which touching of strings means that proton of protonic string and ordinary nucleus interact strongly in ordinary sense of the word. It is however difficult to imagine how entire protonic string could be absorbed into the ordinary nucleus.

2. Are protons of the protonic string dark?

Second option is that protonic strings consist of dark protons so that nuclear space-time-sheet has scale up size, perhaps of order atomic size. This means that fermionic charge is distributed in much larger volume and possibly also the fermions associated with color magnetic flux tubes have scaled up sized. The value $\hbar = 2^{11}\hbar_0$ would predict Compton length of order 10^{-12} m for nucleon and upper size of order 10^{-11} for nuclei.

Cold nuclear reactions require a transformation of dark protons to ordinary ones and this requires leakage to the sector of the imbedding space in which the ordinary nuclei reside (here the book metaphor for imbedding space is very useful). This process can take place for a neutral part of protonic string and involves a reduction of proton and fermion sizes to normal ones. The phase transition could occur first only for a neutral piece of the protonic string having charges at its ends and initiate the nuclear reaction. Part of protonic string could remain dark and remaining part could be "eaten" by the ordinary nucleus or dark protonic string could "eat" part of the ordinary nuclear string. If the leakage occurs for the entire dark proton string, the nuclear reaction itself is just ordinary nuclear reaction and is expected to give out ordinary nuclei. What is important that apart from the crucial phase transition steps in the beginning and perhaps also in the end of the reaction, the model reduces to ordinary nuclear physics and is in principle testable.

The basic question is how plasma phase resulting in electrolysis leads to the formation of dark protons. The proposal [A9] that the transition takes place with perturbative description of the plasma phase fails, might be more or less correct. Later a more detailed nuclear physics picture about the situation emerges.

3. What happens to electrons in the formation of protonic strings?

One should answer two questions.

1. What happens to the electrons of hydrogen atoms in the formation of dark protonic strings?
2. In plasma electrolysis the increase of the input voltage implies a mysterious reduction of the electron current with the simultaneous increase of the size of the plasma region near the cathode [67]. This means reduction of conductance with voltage and thus non-linear behavior. Where does electronic charge go?

Obviously the negatively charged color bond created by adding one proton to a protonic string could take the charge of electron and transform electrons as charge carriers to color bonds of dark Li isotopes which charge $Z = 3$ by gluing to existing protons sequence proton and negatively charged color bond. If the proton comes from H_2O OH^- replaces electron as a charge carrier. This would reduced the conductivity since OH^- is much heavier than electron. This kind of process and its reversal would take place in the transformation of hydrogen atoms to dark proton strings and back in atto-second time scale.

The color bond could be either $\overline{u}d$ pair or $e_8\overline{\nu}_8$ pair or quantum superposition of these. The basic vertex would involve the exchange of color octet super-canonical bosons and their neutrino counterparts. Lepton number conservation requires creation of color singlet states formed of color octet neutrinos which ar bosons and carrying lepton number -2. One color confined neutrino pair would be created for each electron pair consumed in the process and might escape the system: if this happens, the process is not reversible above the time scale defined by colored neutrino mass scale of order .1 eV which happens to be of order .1 attoseconds for ordinary neutrinos. Also ordinary nuclei could consist of nucleons connected by identical neutral color bonds (mostly).

The exchange of light counterparts of charged ρ mesons having mass of order MeV could lead to the transformation of neutral color bonds to charged ones. In deuterium cold fusion the exchange of charged ρ mesons between D and Pd nuclei could transform D nuclei to states behaving like di-neutrons so that cold fusion for D could take place. In the earlier proposal exchange of W^+ boson of scaled variant of weak interactions was proposed as a mechanism.

The formation of charged color bonds binding new dark protons to existing protonic nuclear strings or giving rise to the formation of completely new protonic strings would also increase of the rates of cold nuclear reactions.

Note that this picture leaves open the question whether the fermions associated with color bonds are quarks or electrons.

Nuclei and their dark variants must have same binding energy scale at nuclear quantum criticality

The basic question is what happens to the scale of binding energy of nuclei in the zooming up of nuclear space-time sheet. Quantum criticality requires that the binding energies scales must be same.

1. Consider first the binding energy of the nuclear strings. The highly non-trivial prediction of the nuclear string model is that the contributions of strong contact interactions at nuclear space-time sheet (having size $L < 10^{-14}$ m) to the binding energy vanish in good approximation for ground states with vanishing strong isospin. This means that the binding energy comes from the binding energy assignable to color bonds connecting nucleons together.

2. Suppose that this holds true in a good approximation also for dark nuclei for which the distances of nucleons at zoomed up nuclear space-time sheet (having originally size below 10^{-14} meters) are scaled up. As a matter fact, since the scale of binding energy for contact interactions is expected to reduce, the situation is expected to improve. Suppose that color bonds with length of order 10^{-12} m preserve their lengths. Under these assumptions the nuclear binding energy scale is not affected appreciably and one can have nuclear quantum criticality. Note that the length for the color bonds poses upper limit of order 100 for the scaling of Planck constant.

It is essential that the length of color bonds is not changed and only the size of the nuclear space-time sheet changes. If also the length and thickness of color bonds is scaled up then a naive scaling argument assuming that color binding energy related to the interaction of transforms as color Coulombic binding energy would predict that the energy scales like $1/\hbar$. The binding energies of dark nuclei would be much smaller and transformation of ordinary nuclei to dark nuclei would not take place spontaneously. Quantum criticality would not hold true and the argument explaining the transformation of ordinary Li to its dark counterpart and the model for the deuterium cold fusion would be lost.

9.7.3 A model for the observations of Mizuno

The basic objection against cold nuclear reactions is that Coulomb wall makes it impossible for the incoming nuclei to reach the range of strong interactions. In order that the particle gets to the cathode from electrolyte it should be positively charged. Positive charge however implies Coulomb wall which cannot be overcome with the low energies involved.

These two contradictory conditions can be satisfied if the electrolysis produces exotic phase of water satisfying the chemical formula $H_{1.5}O$ with 1/4 of protons in the form of almost neutral protonic strings can possess only few neutral color bonds. The neutral portions of the protonic string, which have suffered phase transition to a phase with ordinary Planck constant could get very near to the target nucleus since the charges of proton can be neutralized in the size scale of proton by the charges $\overline{u}$ and d quarks or e and $\overline{\nu}$ associated with the two bonds connecting proton to the two neighboring protons. This could make possible cold nuclear reactions.

It turns out that the model fixes protonic strings to isotopes of dark Lithium (with neutrons replaced with proton plus negatively charged color bond). What is intriguing is that the biologically most important ions (besides Na^+) Cl^-, K^+, and Ca^{++} appear at the cathode in Kanarev's plasma electrolysis actually result as outcomes of cold nuclear reactions between dark Li and Na^+.

General assumptions of the model

The general assumptions of the model are following.

1. Ordinary nuclei are nuclear strings, which can contain besides neutrons also "pseudo-neutrons" consisting of pairs of protons and negatively charged color bonds. The model for D cold fusion requires that the Pd nuclei contain also "pseudo-neutrons".

2. Reaction products resulting in the fusion of exotic protonic string transforming partially to ordinary nuclear matter (if originally in dark phase) consist of the nuclei detected in the cathode plus possibly also nuclei which form gases or noble gases and leak out from the cathode.

3. Si, K, Cr, and Cu are produced by the same mechanism in both KOH and NaOH electrolysis.

4. Al, Cl, and Ca is produced by a mechanism which must involve cold nuclear reaction between protonic string and Na ions condensed on the cathode.

5. $Cu(Z,N) = Cu(29,34)$ is the only product nucleus heavier than $Fe(26,29)$. If no other nuclei are involved and Cu is produced by cold fusion

$$X(z,n) + Fe(26,29) \rightarrow Cu(29,34) \ ,$$

the anatomy of protonic string must be

$$X(z,n) = X(3,5)$$

so that dark variant $Li(3,5)$ having charge 3 and mass number 8 would be in question. $X(3,5)$ would have 2 neutral color bonds and 5 negatively charged color bonds. To minimize Coulomb interaction the neutral color bonds must reside at the ends of the string. For quark option one would have charge $1+2/3$ at the first end and $1+1/3$ at the second end and charges of all protons between them would be neutralized. For color octet lepton color bond one would have charge 2 at the other end and zero at the other end.

For quark option the net protonic charge at the ends of the string causing repulsive interaction between the ends could make protonic string unstable against transition to dark phase in which the distance between ends is much longer even if the ends are closed within scaled up variant of the nuclear volume.

Arbitrarily long strings $X(3,n)$ having neutral bonds only at their ends are possible and their fusions lead to neutron rich isotopes of Cu nucleus decaying to the stable isotope. Hence the prediction that only Cu is produced is very general.

The simplest dark protonic strings $X(3,n)$ have quantum numbers of $Li(3,n)$. One of the hard problems of Big Bang cosmology is that the measured abundance of lithium is by a factor of about 2.5 lower than the predicted abundance [37]. The spontaneous transformation of $Li(3,n)$ isotopes to their dark variants could explain the discrepancy. Just by passing notice that Li has mood stabilizing effect [36]: the spontaneous transformation of Li^+ to its dark variant might relate to this effect.

Production mechanisms for the light nuclei common to Na and K

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These nuclei must be produced by a fission of Fe nuclei.

1. For $Si(14,14)$ production the mechanism would be cold fission of Fe nucleus to two parts in the collision with the protonic string:

$$X(3,5) + Fe(26,29) \rightarrow Si(14,14) + Al(13,14) + X(2,6) \ .$$

$X(2,6)$ represent dark or ordinary $He(2,6)$. As a noble gas He isotope would leave the cathode.

Note that arbitrarily long proton strings with two neutral bonds at their ends give neutron rich isotope of Si and exotic or ordinary isotope of He so that again the prediction is very general.

2. $K(19,20)$ is produced much more in KOH which most probably means that part of K^+ is absorbed from the electrolyte. In this case the reaction could proceed as follows:

$$X(3,5) + Fe(26,29) \rightarrow K(19,20) + Ne(7,7) + X(3,7) \ .$$

Note that the neutron number could be distributed in many manners between final states. For arbitrarily long proton string with two neutral bonds at ends higher neutron rich isotopes of K and Ne are produced. As noble gas Ne would leak out from the cathode.

Ordinary $Li(3,7)$ would decay by neutron emission to stable isotopes of Li. The temperature of the system determines whether Li boils out (1615 K under normal pressure). Li is not reported to appear in the cathode. In plasma electrolysis the temperature is in the interval $.5\times 10^4$- 10^4 C and around 10^3 C in the ordinary electrolysis so that the high temperature might explain the absence of Li. Also the in-stability of Li isotopes against transition to dark Li in electrolyte would imply the absence of Li.

3. For $Cr(24,28)$ production the simplest reaction would be

$$X(3,5)+Fe(26,29)\rightarrow Cr(24,28)+He(2,2)+X(3,4) \ .$$

Helium would leak out as noble gas. Proton string would shorten by one unit and keep its charge. $X(3,4)$ would represent the stable isotope $Li(3,4)$ or its dark counterpart and what has been said in 2) applies also now.

How to understand the difference between KOH and NOH?

One should understand why Al, Cl, and Ca are not detected in the case of KOH electrolysis.

Al, Cl, and Ca would be created in the fusion of protonic strings with $Na(11,12)$ nuclei absorbed by the cathode. With this assumption the rates are expected to be of same order of magnitude for all these processes as suggested by the one per cent order of magnitude for all fractions.

One can imagine two reaction mechanisms.

I: One could understand the production assuming only $X(3,5)$ protonic strings if the number of $X(3,5)$ strings absorbed by single Na nucleus can be $k=1,2,3$ and that nuclear fission can take place after each step with a rate which is slow as compared to the rate of absorptions involving also the phase transition to dark matter. This is however highly implausible since ordinary nuclear interactions are in question.

II: Second possibility is that the protonic strings appearing with the highest probability are obtained by fusing copies of the basic string $X(3,5)$ by using neutral color bond between the strings. The minimization of electrostatic energy requires that that neutral color bonds are equally spaced so that there are three completely neutralized protons between non-neutralized protons.

One would have thus at least the strings $X(3,5)$, $X(6,10)$, and $X(9,15)$, which correspond to dark $Li(3,5)$ and dark variants of the unstable isotopes $C(6,10)$ and $F(9,15)$. In nuclear string model also ordinary nuclei are constructed from $He(2,2)$ strings and lighter strings in completely analogous manner, and one could perhaps see the dark nuclei constructed from $Li(3,5)$ as the next level of hierarchy realized only at the level of dark matter.

The charge per nucleon would be 3/8 and the length of the string would be a multiple of 8. Interestingly, the numbers 3, 5, and 8 are subsequent Fibonacci numbers appearing very frequently also in biology (micro-tubules, sunflower patterns). The model predicts also the occurrence of cold fusions $X(z=3k,n=5k)+Fe(26,29)\rightarrow (Z,N)=(26+3k,29+5k)$. For $k=2$ this would give $Ge(32,39)$ which is stable isotope of Ge. For $k=3$ one would have $(Z,N)=(35,44)$ which is stable isotope of Br [25, 24].

Consider now detailed description of the reactions explaining the nuclei detected in the cathode.

1. $Al(13,14)$ would be produced in the reaction

$$X(3,5)+Na(11,12)\rightarrow Al(13,14)+X(1,3) \ .$$

$H(1,3)$ or its dark variant could be in question. Also the reaction $X(3,5)+Na(11,12)\rightarrow Al(13,17)+p$, where $Al(12,17)$ is an unstable isotope of Al is possible.

The full absorption of protonic string would yield $Si(14,17)$ beta-decaying to $P(15,16)$, which is stable. Either P leaks out from the cathode or full absorption does not take place appreciably.

2. $Cl(17,18)$ would be produced by the sequence

$$\begin{array}{lrcl} \textbf{I1}: & 2X(3,5)+Na(11,12) & \rightarrow & Cl(17,18)+X(0,4) \ , \\ \textbf{I2}: & X(6,10)+Na(11,12) & \rightarrow & Cl(17,18)+X(0,4) \ . \end{array}$$

$X(0,4)$ represents ordinary or dark tetra-neutron [59, 60, 26]. The instability of the transformation of tetra-neutron to dark matter could explain why its existence has remained controversial.

If the protonic string were absorbed completely, the resulting $Cl(17,22)$ - if equivalent to ordinary nucleus - would transform via beta-decays to $A(18,23)$ and then to $K(19,22)$, which is stable and detected in the target.

3. $Ca(20,20)$ would be produced in the reaction

$$\begin{array}{lrcl} \mathbf{I1}: & 3X(3,5)+Na(11,12) & \rightarrow & Ca(20,20)+X(0,7) \ , \\ \mathbf{I2}: & X(9,15)+Na(11,12) & \rightarrow & Ca(20,20)+X(0,7) \ . \end{array}$$

$X(0,7)$ would be dark counterpart of "septa-neutron". The complete absorption of nuclear string would produce $Ca(20,27)$, which (if ordinary nucleus) transforms via beta decays to $Sc(21,26)$ and then to $Ti(22,25)$, which is stable.

9.7.4 Comparison with the model of deuterium cold fusion

It is interesting to compare the model with the model for cold fusion [66, 44] reported using deuterium target and D_2O instead of water.

Earlier model

1. The model is based on the assumption that D nuclei in the target suffer a phase transition to a state in which D nuclei become neutral so that the color bond between neutron and proton becomes negatively charged: one has effectively di-neutrons.

2. The mechanism of charging of color bond must either involve weak interactions or exchange of lepto-ρ mesons already discussed briefly. The proposal is that the exchange of W bosons of scaled up version of weak physics is involved with the range of interactions given by atomic length scale. The exchange of W^+ bosons was assumed to take place between Pd and D nuclei. This mechanism could lead to the formation of negatively charged color bonds in also ordinary nuclei.

3. The neutrality of exotic D nuclei allows to overcome Coulomb wall. One can understand the reported selection rules: in particular the absence of Helium isotopes (only isotopes of H are detected). The absence of gamma rays can be understood if the resulting gamma rays are dark and leak out before a transformation to ordinary gamma rays.

Are D nuclei in Pd target dark or not?

The question whether the exotic D nuclei are dark was left pending. The recent model suggests that the answer is affirmative.

1. The basic difference between the two experiments would be that in Kanarev's experiments incoming nuclei are dark whereas in D fusion cathode contains the dark nuclei and cold nuclear reactions occur at the "dark side" and is preceded by ordinary-to-dark phase transition for incoming D.

2. D cold fusion occurs for a very restricted range of parameters characterizing target: the first parameter is doping ratio: essentially one D nucleus per one Pd nucleus is needed which would fit with the assumption that scaled up size is of the order of atom size. Temperature is second parameter. This and the fact that the situation is highly sensitive to perturbations conforms with the interpretation as a phase transition to dark matter occurring at quantum criticality.

3. The model for Kanarev's findings forces to consider the possibility that dark D nuclei combine to form longer strings and can also give rise to dark $Li(3,5)$ explaining the observed nuclear transmutations in the target.

4. In cold nuclear reactions incoming nuclei would transform to dark nuclei (the picture as a leakage between different pages of a book like structure defined by the generalized imbedding space is helpful). The reaction would take place for dark nuclei in zoomed up nuclear physics and the reaction products would be unstable against phase transition to ordinary nuclei.

5. Is it then necessary to assume that target D nuclei are transformed to neutral ones (di-neutrons effectively) in order to have cold nuclear reactions? Nuclear space-time sheets are scaled up. If nucleon space-time sheets are not scaled up, p and n are connected by color magnetic flux tubes of same length as in the case of ordinary nuclei but located at much larger nuclear space-time sheet. The classical analog for the quantal distribution of nucleon charges is even charge distribution in a sphere or radius R defined by the charge of the scaled up nucleus. The height of the Coulomb wall is $E_c = e^2/R$. If $R = a$, a the atomic radius, one has $E_c \sim .1$ keV. The wall is by a factor 10^{-4} lower than in ordinary nuclear collision so that the incoming D nucleus might overcome the Coulomb wall.

   If Coulomb wall can be overcome, all dark variants of $D+D$ reaction are possible. Helium nuclei have not been however detected, which supports the view that D in target is transformed to its neutral variant. Gamma rays would be dark and could leak out without detection which would explain the absence of gamma rays.

Nuclear quantum criticality is essential

A note about the energetics of cold nuclear reactions is in order. The nuclear quantum criticality deriving from the cancellation of the contact interaction energies between nucleons for isospin singlets and scaling up of *only* nuclear space-time sheet is an absolutely essential assumption. Otherwise dark D would have much smaller binding energy scale than the visible one, and ordinary D in the Pd target could not transform to dark "di-neutron" state. Also the transformation of incoming D to its dark variant D at cathode could not take place.

9.7.5 What happens to OH bonds in plasma electrolysis?

For an innocent novice one strange aspect of hydrolysis is how the OH bonds having energies of order 8 eV can be split in temperatures corresponding to photon energies of order .5 eV. Kanarev has suggested his own theory for how this could happen [47]. TGD suggests that OH bonds are transformed to their dark variants with scaled down bond energy and that there might be no essential difference between OH bond and hydrogen bond.

The reduction of energy of OH bonds in plasma electrolysis

Kanarev has found that in plasma electrolysis the energy of OH bonds is reduced from roughly 8 eV to about .5 eV, which corresponds to the fundamental metabolic energy quantum identifiable as the zero point kinetic energy liberate as proton drops from $k = 137$ space-time sheet to much larger space-time sheet. In pyrolysis [67] similar reduction could occur since the pyrolysis occurs above temperature about 4000 C conforming with the energy scale of hydrogen bond.

The explanation discussed in [G2] is that there is some mechanism exciting the bonds to a state with much lower bond energy. Dark matter hierarchy [A9] suggests that the excitation corresponds to the transformation of OH bond to dark bond so that the energy scale of the state is reduced.

Also in the ordinary electrolysis of water [66] the energy of OH bonds is reduced to about 3.3 eV meaning a reduction factor of order 2. The simplest interpretation would be as a transformation of OH bonds to dark OH bond with $\hbar \rightarrow 2\hbar$ (the scaling could be also by some other integer or even rational). The energy needed to transform the bond to dark bond could come from remote metabolism via the dropping of dark protons from a dark variant of some sub-atomic space-time sheet with size not smaller than the size of the atomic space-time sheet to a larger space-time sheet.

$H_{1.5}O$ anomaly suggests that 1/4 of protons of water are dark in atto-second time scale [F9] and one can imagine that both protons of water molecule can become dark under conditions defined by plasma electrolysis. Also the atomic space-time sheets and electron associated with OH bonds could become dark.

Atomic binding energies transform as $1/\hbar^2$. If the energy of hydrogen bond transforms like Coulombic interaction energy as given by the perturbative calculation, it is scaled down as $1/\hbar$ since the length of the bond scales up like $\hbar$. Effectively $\alpha_{em} \propto 1/\hbar$ is replaced by its scaled down value. For $\hbar \rightarrow 2^4\hbar_0$ the energy would scale from 8 eV to .5 eV and the standard metabolic energy quantum could induce the splitting of the dark OH bond. If 2^4 is the scale factor of $\hbar$ for dark nuclear space-time sheets, their size would be of order 10^{-3} meters. The model for cold fusion is consistent with this since what matters is different value of Planck constant for the dark nuclear space-time sheets.

There is an objection against the reduction of OH bond energy. The bonds could be split by a process in which dark nuclear reactions kick protons to $k = 133$ dark space-time sheet. In this case the maximal zero point kinetic energy liberated in the dropping back would be 8 eV and could induce breaking of OH bond. For $\hbar/\hbar_0 \geq 4$ the size of $k = 133$ dark space-time sheet would be larger than the size of $k = 137$ atomic space-time sheet.

Are hydrogen bonds dark OH bonds?

The fact that the energy of hydrogen bonds [63] is typically around .5 eV forces to ask what distinguishes hydrogen bond from dark OH bond. Could it be that the two bonds are one and the same thing so that dark OH bonds would form standard part of the standard chemistry and molecular biology? In hydrogen bond same hydrogen would be shared by the oxygen atoms of the neighboring atoms. For the first O the bond would be ordinary OH bond and for the second O its dark variant with scaled down Coulomb energy. Under conditions making possible pyrolysis and plasma electrolysis both bonds would become dark. The variation of the hydrogen bond energy could reflect the variation of the scaling factor of $\hbar$.

The concentration of the spectrum of bond energies on integer multiples of fundamental energy scale - or even better, on powers of 2 - would provide support for the identification. There is evidence for two kinds of hydrogen bonds with bond energies in ratio 1:2 [80, 79]: the TGD based model is discussed in [F9].

Mechanism transforming OH bonds to their dark counterparts

The transformation of OH bonds to dark bonds would occur both in ordinary and plasma electrolysis and only the change of Planck constant would distinguish between the two situations.

1. Whatever the mechanism transforming OH bonds to their dark counterparts is, metabolic energy is needed to achieve this. Kanarev also claims over-unity energy production [47]. Cold fusion researchers make the same claim about ordinary electrolysis. Cold nuclear reactions between Na^+ (K^+) and dark protons and dark Li could obviously serve as the primary energy source. This would provide the fundamental reason for why $NaOH$ or KOH must be present. Cold nuclear reactions would thus occur also in the ordinary electrolysis of water and provide the energy inducing the transition of OH bonds to dark ones by (say) $\hbar \rightarrow 2\hbar$ transition.

2. One can imagine several metabolic mechanisms for the visible-to-dark transformation of HO bonds. The energy spectrum of cold nuclear reactions forms a continuum whereas the energies needed to transform OH bonds to their dark variants presumably are in narrow bands. Therefore the energy liberated in cold nuclear reactions is not probably used as such. It is more plausible that standard metabolic energy quanta liberated in the dropping of protons (most naturally) to larger space-time sheets are utilized. The most important metabolic energy quanta for the dropping of proton come as $E_k = 2^{k-137}kE_0$: $E_0 = .5$ eV is liberated in the dropping of proton from atomic space-time sheet ($k = 137$) to much larger space-time sheet (the discrete spectrum of increments of the vacuum energy in the dropping approaches this energy [D8]). The energy liberated in the dark nuclear reactions would "load metabolic batteries" by kicking the dark protons to the dark variants of $k < 137$ space-time sheet (the size of dark atomic space-time sheet scales like $\hbar$). Their dropping to larger space-time sheets would liberate photons with energies near to those transforming OH bonds to hydrogen bonds.

3. A signature for the standard metabolic energy quanta would be visible light at $2eV$ and also discrete lines below it accumulating to $2eV$. Kanarev's indeed reports the presence of red light [47] as a signature for the occurrence of process.

9.7.6 A model for plasma electrolysis

Kanarev's experiments involve also other strange aspects which lead to the view that cold nuclear reactions and dark matter physics are essential aspects of not only plasma electrolysis of Kanarev but also of ordinary electrolysis and responsible for the claimed over unity energy production. Biologically important ions are produced in reactions of dark Li and Na^+ and there is very strong electric voltage over the cell membrane. This inspires the question whether cold nuclear reactions serve as a metabolic energy source in living cell and are also responsible for production of ions heavier than Na^+.

Brief description of plasma electrolysis

Electrolysis [66], pyrolysis [67], and plasma electrolysis [47, 67] of water are methods of producing free hydrogen. In pyrolysis the temperature above 4000 C leads to hydrogen and oxygen production. Oxygen production occurs also at cathode and hydrogen yield is higher than given by Faraday law for ordinary electrolysis [66].

The article of Mizuno and collaborators [67] about hydrogen production by plasma electrolysis contains a brief description of plasma electrolysis. A glow discharge occurs as the input voltage used in electrolysis is above a critical value and plasma is formed near cathode. In the arrangement of [67] plasma state is easily achieved above 140 V. If the values of temperature and current density are right, hydrogen generation in excess of Faraday's law as well as a production of oxygen at cathode (not possible in ideal electrolysis) are observed. Above 350 V the control of the process becomes difficult.

What really happens in electrolysis and plasma electrolysis?

1. Ordinary electrolysis

To understand what might happen in the plasma electrolysis consider first the ordinary electrolysis of water.

1. The arrangement involves typically the electrolyte consisting of water plus $NaOH$ or KOH without which hydrolysis is impossible for thermodynamical reasons.

2. Electronic current flows from the anode to cathode along a wire. In electrolyte there is a current of positively charged ions form anode to cathode. At the cathode the reaction $2H_2O + 2e^- \rightarrow 2H_2 + 2OH^-$ yields hydrogen molecules seen as bubbles in water. At the anode the reaction $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$ is followed by the reaction $2H^+ + 2e^- \rightarrow H_2$ and the flow of $2e^-$ to the cathode along wire. The net outcome is hydrolysis: $H_2O \rightarrow 2H_2 + 2O_2$. Note that O_2 is produced only at anode and H_2 at both anode and cathode.

2. What happens in plasma electrolysis?

In plasma electrolysis something different might happen.

1. Cold nuclear reactions should take place at cathode in presence of Na^+ ions plus dark Li and should be in equilibrium under ordinary conditions and contribute mainly to the formation of dark OH bonds. The rate of cold nuclear reactions increases with input voltage V since the currents of Na^+ and dark Li to the cathode increase. Obviously the increased rate of energy yield from dark nuclear reactions could be the real reason for the formation of plasma phase above critical voltage.

2. By previous considerations the reduction of electron current above critical voltage has interpretation as a transition in which electronic charge is transferred to negative charge of color bonds of dark proton strings. Existing protonic strings could grow longer and also new strings could be created from the ionized hydrogen resulting in the electrolysis of water. The increase of the size of the dark nuclei would mean increase of the cross sections for cold nuclear reactions. The liberated energy would ionized hydrogen atoms and give rise to a positive feedback loop somewhat like in ordinary nuclear reactions.

3. The increased energy yield in cold nuclear reactions suggests that OH bonds are transformed very effectively to dark OH bonds in the plasma region. This means that the thermal radiation can split the hydrogen bonds and induce the splitting of two water molecules to $4H$ and $2O$ and therefore production of $2H_2 + O_2$ everywhere in this kind of region. The temperature used by Kanarev corresponds to energy between .5-1 eV [47] which conforms with the fact that OH bond energy is reduced to about .5 eV. Note that the presence of anode and cathode is not absolutely necessary if cold nuclear reactions can take place in the entire electrolyte volume and generate plasma phase by positive feedback loop.

4. The prediction is that Faraday's law for hydrogen production does not hold true. O/H ratio has the value $r = O/H = 0$ for the ordinary electrolysis at cathode. $r = 1/2$ holds true if local dissociation of water molecules dominates. According to [67] r increases from electrolysis value $r = .066$ above $V = 140$ V achieving the value $r = .45$ for $V = 350$ V where the system becomes unstable. Also cold nuclear reactions could contribute to hydrogen and oxygen production and affect the value of r as suggested by the large volume of gas produced in Kanarev's experiments [46].

Over-unity energy production?

Over-unity energy production with output power 2- or even 3-fold as compared with input power has been reported from plasma electrolysis. The effectiveness is deduced from the heating of of the system. Note that Mizuno reports in [67] that 10 per cent effectiveness but this is for the storage of energy to hydrogen and does not take into account the energy going to the heating of water.

The formation of higher isotopes of Li by fusing dark protons to existing dark proton strings is a good candidate for the dominant energy production mechanism. An estimate for the energy liberate in single process $Li(3,n)+m_p+e \rightarrow Li(3,n+1)+2\nu_8$ is obtained by using energy conservation. Here $2\nu_8$ denotes color singlet bound state of two color octet excitations of neutrino.

Since e_8 and ν_8 are analogous to u and d quarks one expects that their masses are very nearly the same. This gives as the first guess $m_{\nu_8} = m_e$ and since leptopion (color bound state of color octet electrons, [F7]) has mass $m = 2m_e$ a good guess is $m(2\nu_8) = 2m_{\nu_8} = 2m_e$. The energy conservation would give

$$m(Li(3,n)) + m_p \quad = \quad m(Li(3,n+1)) + m_e + T(2\nu_8) + E(\gamma) \ . \tag{9.7.1}$$

Here $T(2\nu_8)$ is the kinetic energy of $2\nu_8$ state and E_γ is the energy of photon possibly also emitted in the process.

The process is kinematically possible if the condition

$$\Delta m \quad = \quad m(Li(3,n)) + m_p - m(Li(3,n+1) \geq m_e \ . \tag{9.7.2}$$

is satisfied. All incoming particles are approximated to be at rest, which is a good approximation taking into account that chemical energy scales are much lower than nuclear ones. For the left hand side one obtains from the mass difference of $Li(3,n=4)$ and $Li(3,5)$ isotopes the estimate $\Delta m = 1.2312$ MeV for the liberated binding energy which is considerably larger than $m_e = .51$ MeV. Hence the process is kinematically possible and $2\nu_8$ would move with a relativistic velocity $v = .81c$ and presumably leave the system without interacting with it.

The process can involve also the emission of photons and the maximal amount of energy that photon can carry out corresponds to $E = \Delta m = 1.2312$ MeV. Let us denote by $\langle E\rangle < \Delta m$ the average photonic energy emitted in the process and express it as

$$\langle E\rangle = z\Delta m \ , \quad z < 1. \tag{9.7.3}$$

One obtains an estimate for the production rate of photon energy (only this heats the system) from the incoming electron current I. If a fraction $x(V)$ of the current is transformed to negatively charged color bonds the rate for energy production becomes by a little manipulation

$$\frac{P/kW}{I/A} = x(V)z \times 3.5945 \ . \tag{9.7.4}$$

This formula allows to estimate the value of the parameter $x(V)z$ from experimental data. Since simplest Feynman graph producing also photons is obtained by adding photon line to the basic graph, one expects that z is of order fine structure constant:

$$z \sim \alpha_{em} = 1/137 \ . \tag{9.7.5}$$

The ratios of the excess power for a pair of (V, I) values should satisfy the condition

$$\frac{P(V_1)I(V_2)}{P(V_2)I(V_1)} = \frac{x(V_1)}{x(V_2)} \ . \tag{9.7.6}$$

$x(V)$ should be deducible as a function of voltage using these formulas if the model is correct.

These formulae allow to compare the predictions of the model with the experimental results of Naudin for Mizuno-Omori Cold Fusion reactor [49]. The following table gives the values of $\epsilon = x(V)z$ and ratios $x(V(n))/x(V(n_1)$ deduced from the data tabulated by Naudin [50] for the various series of experiments using the formulae above.

1. Most values of $x(V)z$ are in the range $.03 - .12$. $z = 1/137$ would give $x(V)z \leq 1/137$ so that order of magnitude is predicted correctly. One cannot over-emphasize this result.

2. Apart from some exceptions the values look rather reasonable and do not vary too much. If one neglects the exceptional values, ones has $x_{max}(V)/x_{min}(V) < 4$. $n = 1, 5, 8, 9, 29$ correspond to exceptionally small values of $x(V)$. Perhaps cold fusion is not present for some reason. The output power is smaller than input power for $n = 9$ and $n = 29$.

n	$Voltage/V$	$Current/A$	$x(V)z$	$x(V(n))/x(V(2))$
1	185	8.56	0.005	.145
2	147	2.45	0.036	1.00
3	215	2.10	0.046	1.30
4	220	9.32	0.044	1.22
5	145	1.06	0.001	.03
6	213	1.40	0.05	1.34
7	236	1.73	0.08	2.18
8	148	.83	0.01	.21
9	148	1.01	-0.00	-0.008
10	221	1.31	0.03	.87
11	279	3.03	0.05	1.46
12	200	8.58	0.03	0.89
13	199	7.03	0.07	1.91
14	215	9.78	0.04	1.07
15	207	8.34	0.03	0.74
16	247	2.19	0.06	1.69
17	260	2.20	0.02	0.55
18	257	2.08	0.03	0.71
19	195	2.95	0.06	1.59
20	198	2.62	0.07	1.98
21	182	2.40	0.05	1.26
22	212	2.27	0.06	1.74
23	259	2.13	0.12	3.22
24	260	4.83	0.04	1.05
25	209	3.53	0.04	1.16
26	230	4.99	0.10	2.79
27	231	5.46	0.09	2.53
28	233	5.16	0.10	2.85
29	155	4.60	-0.00	-0.04
30	220	4.44	0.11	2.95
31	256	5.25	0.05	1.36
32	211	3.68	0.03	.97
33	201	3.82	0.04	1.06

Table 3. The values of $x(V)z$ and $x(V(n))/x(V(1))$ deduced from the data of *Cold Fusion reaction-Experimental test results on June 25, 2003 by JL Naudin* at http://jlnlabs.online.fr/cfr/html/cfrdatas.htm.

Has living matter invented cold nuclear physics?

Intriguingly, the ions Na^+, Cl^-, K^+, Ca^{++} detected by Mizuno in the cathode in Kanarev's experiments [46] correspond to the most important biological ions. There is also a considerable evidence for the occurrence of nuclear transmutations in living matter [69, 70]. For instance, Kervran claims that it is not possible to understand where the Ca needed to form the shells of eggs comes from. A possible explanation is that dark nuclear reactions between Na^+ and dark Litium produced the needed Ca.

There is extremely strong electric field through cell membrane (resting voltage is about .06 V). The acceleration of electrons in this field could generate plasma phase and creation of dark Li nuclei via a positive feedback loop. This could mean that cold nuclear reactions serve also in living cell as a basic metabolic energy source (possibly in the dark sector) and that also biologically important ions result as products of cold nuclear reactions.

9.7.7 Comparison with the reports about biological transmutations

Kervran's book "Biological Transmutations" [69] contains a surprisingly detailed summary about his work with biological transmutations and it is interesting to find whether the proposed model could explain the findings of Kervran. TGD suggests two general mechanisms.

1. The nuclear reactions involving dark Li, C, and F predicted to be present in living matter.
2. Nuclear fusions made possible by a temporary transformation of ordinary nuclear space-time sheets to dark ones with much larger size so that Coulomb wall is reduced considerably. The nuclear reaction might proceed if it is energetically possible. Almost any reaction $A + B \rightarrow C$ is possible via this mechanism unless the nuclei are not too heavy.

Fortuitous observations

In his childhood Kervran started to wonder why hens living in a limestone poor region containing thus very little calcium in ground and receiving no calcium in their nutrition could develop the calcium required by eggs and by their own bones. He noticed that hens had the habit of eating mica, which contains silicon. Later this led to the idea that Si could somehow transmute to Ca in living matter. In the proposed model this could correspond to fusion of $Si(14,14) + \mathbf{C(6,6)} \rightarrow Ca(20,20)$ which occurs spontaneously.

Second fortuitous observation were the mysterious CO poisonings by welders working in factory. After careful studies Kervran concluded that CO must be produced endogenously and proposed that the inhaled air which had been in contact with incandescent iron induces the transformation $N_2 \rightarrow CO$ conserving both neuron and neutron number. This transformation might be understood in TGD context if the nuclear space-time sheets are part of time in dark with much larger size so that a direct contact becomes possible for nuclear space-time sheets and Coulomb wall is reduced so that the reaction can proceed with some probability if energetically possible. The thermal energy received from hot iron might help to overcome the Coulomb barrier. The mass difference $m(2N) - m(O) - m(C) = 10.45$ MeV allows this reaction to occur spontaneously.

Examples of various anomalies

Kervran discusses several plant anomalies. The ashes of plants growing in Si rich soil contain more Ca than they should: this transmutation has been already discussed. The ashes of a plant growing on Cu fibres contain no copper but 17 per cent of iron oxides in addition to other elements which could not have come from the rain water. The reaction $Cu(58) + \mathbf{Li(3,4)} \rightarrow Fe(26,32) + \mathbf{C(6,6)}$ would liberate energy of 11.5 MeV.

There are several mineral anomalies.

1. Dolomite rock is formed inside limestone rocks which would suggest the transmutation of $Ca(20,20)$ into $Mg(12,12)$. The nuclear reaction $Ca(20,20)+\mathbf{Li(3,4)} \rightarrow Mg(12,12)+Na(11,12)$ would liberate energy of 3.46 MeV. Ca emerges from Si in soil and in what Kervran refers to a "sickness of stone". The candidate reaction has been already discussed.
2. Graphite is found in siliceous rocks. Kervran proposes the reaction $Si \rightarrow C + O$. $m(Si) - m(C) - M(O) = -16.798$ MeV does not allow this reaction to proceed spontaneously but the reaction $Si + \mathbf{Li} \rightarrow C + Na$ liberates the energy 2.8880 MeV.
3. Kervran mentions the reaction $O + O \rightarrow S$ as a manner to produce sulphur from oxygen. This reaction is obviously energetically favored.

Kervran discusses the transmutations $Na \rightarrow K$ and $Na \rightarrow Ca$ occurring also in plasma electrolysis and explained by TGD based model. Further transmutations are $Na \rightarrow Mg$ and $Mg \rightarrow Ca$. $Na \rightarrow Mg$ could correspond to the reaction $Na(11,12) + \mathbf{Li(3,2)} \rightarrow Mg(12,12) + He(2,2)$ favored by the high binding energy per nucleon for ^4He (7.072 MeV). $Mg \rightarrow Ca$ would correspond to the reaction $Mg + O \rightarrow Ca$, which obviously liberates energy.

9.7.8 Are the abundances of heavier elements determined by cold fusion in interstellar medium?

According to the standard model, elements not heavier than Li were created in Big Bang. Heavier elements were produced in stars by nuclear fusion and ended up to the interstellar space in super-nova explosions and were gradually enriched in this process. Lithium problem forces to take this theoretical framework with a grain of salt.

The work of Kervran [69] suggests that cold nuclear reactions are occurring with considerable rates, not only in living matter but also in non-organic matter. Kervran indeed proposes that also the abundances of elements at Earth and planets are to high degree determined by nuclear transmutations and discusses some examples. For instance, new mechanisms for generation of O and Si would change dramatically the existing views about evolution of planets and prebiotic evolution of Earth.

Are heavier nuclei produced in the interstellar space?

TGD based model is consistent with the findings of Kervran and encourages to a consider a simple model for the generation of heavier elements in interstellar medium. The assumptions are following.

1. Dark nuclei $X(3k,n)$, that is nuclear strings of form $Li(3,n)$, $C(6,n)$, $F(9,n)$, $Mg(12,n)$, $P(15,n)$, $A(18,n)$, etc..., form as a fusion of Li strings. $n = Z$ is the most plausible value of n. There is also ^4He present but as a noble gas it need not play an important role in condensed matter phase (say interstellar dust). The presence of water necessitates that of $Li(3,n)$ if one accepts the proposed model as such.

2. The resulting nuclei are in general stable against spontaneous fission by energy conservation. The binding energy of $He(2,2)$ is however exceptionally high so that alpha decay can occur in dark nuclear reactions between $X(3k,n)$ allowed by the considerable reduction of the Coulomb wall. The induced fissions $X(3k,n) \rightarrow X(3k-2,n-2)+He(2,2)$ produces nuclei with atomic number $Z \mod 3 = 1$ such as $Be(4,5)$, $N(7,7)$, $Ne(10,10)$, $Al(13,14)$, $S(16,16)$, $K(19,20)$,... Similar nuclear reactions make possible a further alpha decay of $Z \mod 3 = 1$ nuclei to give nuclei with $Z \mod 2$ such as $B(5,6)$, $O(8,8)$, $Na(11,12)$, $Si(14,14)$, $Cl(17,18)$, $Ca(20,20)$,... so that most stable isotopes of light nuclei could result in these fissions.

3. The dark nuclear fusions of already existing nuclei can create also heavier Fe. Only the gradual decrease of the binding energy per nucleon for nuclei heavier than Fe poses restrictions on this process.

The table below allows the reader to build a more concrete view about how the heavier nuclei might be generated via the proposed mechanisms.

H(1,0)							He(2,2)
Li(3,4)	Be(4,5)	B(5,6)	C(6,6)	N(7,7)	O(8,8)	F(9,10)	Ne(10,10)
Na(11,12)	Mg(12,12)	Al(13,14)	Si(14,14)	P(15,16)	S(16,16)	Cl(17,18)	A(18,22)
K(19,20)	Ca(20,20)						

Table 4. The table gives the most abundant isotopes of stable nuclei.

The abundances of nuclei in interstellar space should not depend on time

The basic prediction of TGD inspired model is that the abundances of the nuclei in the interstellar space should not depend on time if the rates are so high that equilibrium situation is reached rapidly. The $\hbar$ increasing phase transformation of the nuclear space-time sheet determines the time scale in which equilibrium sets on. Standard model makes different prediction: the abundances of the heavier nuclei should gradually increase as the nuclei are repeatedly re-processed in stars and blown out to the interstellar space in super-nova explosion.

Amazingly, there is empirical support for this highly non-trivial prediction [55]. Quite surprisingly, the 25 measured elemental abundances (elements up to $Sn(50,70)$ (tin) and $Pb(82,124)$ (lead)) of a 12 billion years old galaxy turned out to be very nearly the same as those for Sun. For instance, oxygen abundance was 1/3 from that from that estimated for Sun. Standard model would predict that the abundances should be .01-.1 from that for Sun as measured for stars in our galaxy. The conjecture was that there must be some unknown law guaranteing that the distribution of stars of various masses is time independent. The alternative conclusion would be that heavier elements are created mostly in interstellar gas and dust.

Could also "ordinary" nuclei consist of protons and negatively charged color bonds?

The model would strongly suggest that also ordinary stable nuclei consist of protons with proton and negatively charged color bond behaving effectively like neutron. Note however that I have also consider the possibility that neutron halo consists of protons connected by negatively charged color bonds to main nucleus. The smaller mass of proton would favor it as a fundamental building block of nucleus and negatively charged color bonds would be a natural manner to minimizes Coulomb energy. The fact that neutron does not suffer a beta decay to proton in nuclear environment provided by stable nuclei would also find an explanation.

1. Ordinary shell model of nucleus would make sense in length scales in which proton plus negatively charged color bond looks like neutron.

2. The strictly nucleonic strong nuclear isospin is not vanishing for the ground state nuclei if all nucleons are protons. This assumption of the nuclear string model is crucial for quantum criticality since it implies that binding energies are not changed in the scaling of $\hbar$ if the length of the color bonds is not changed. The quarks of charged color bond however give rise to a compensating strong isospin and color bond plus proton behaves in a good approximation like neutron.

3. Beta decays might pose a problem for this model. The electrons resulting in beta decays of this kind nuclei consisting of protons should come from the beta decay of the d-quark neutralizing negatively charged color bond. The nuclei generated in high energy nuclear reactions would presumably contain genuine neutrons and suffer beta decay in which d quark is nucleonic quark. The question is whether how much the rates for these two kinds of beta decays differ and whether existing facts about beta decays could kill the model.

9.7.9 Tests and improvements

Test for the hypothesis about new physics of water

The model involves hypothesis about new physics and chemistry related to water.

1. The identification of hydrogen bond as dark OH bond could be tested. One could check whether the qualitative properties of bonds are consistent with each. One could try to find evidence for quantization of bond energies as integer multiples of same energy (possible power of two multiples).

2. $H_{1.5}O$ formula in atto-second scale should be tested further and one could look whether similar formula holds true for heavy water so that sequences of dark protons might be replaced with sequences of dark deuterons.

3. One could find whether plasma electrolysis takes place in heavy water.

Testing of the nuclear physics predictions

The model in its simplest form assumes that only dark Li, C, F, etc. are present in water. This predicts quite specific nuclear reactions in electrolyte and target and reaction product. For both target and electrolyte isotopes of nuclei with atomic number $Z + k3$ are predicted to result in cold fusion reactions if energetically possible. For a target heavier than Fe also fission reactions might take place.

The estimates for the liberated energies are obtained assuming that dark nuclei have same binding energies as ordinary ones. In some cases the liberated energy is estimated using the binding energy per nucleon for a lighter isotope. Ordinary nuclei with maximal binding energy correspond to nuclear strings having ^4He or its variants containing negatively charged color bonds as a basic structural unit. One could argue that gluing $nLi(3,5)$ or its isotope does not give rise to a ground state so that the actual energy liberated in the process is reduced so that process might be even impossible energetically. This could explain the absence of Ge from Fe cathode and the absence of Ti, Mn, and Ni in KOH plasma electrolysis [46].

Cathode: For cathode Fe and W have been used. For Fe the fusions $Fe+Li \rightarrow Cu+28.84\ \ MeV$ and $Fe + C \rightarrow Ge + 21.64\ \ MeV$ are possible energetically. Mizuno does not report the presence of

Ge in Fe target. The reduction of the binding energy of dark $C(6,10)$ by 21.64 MeV (1.35 MeV per nucleon) would make second reaction impossible but would still allow $Li + C$ and $Na + C$ fusion. Second possibility is that Ge containing negatively charged color bonds has smaller binding energy per nucleon than ordinary Ge. $W + Li \rightarrow Ir$ would liberate 8.7 MeV if binding energy of dark Li is same as of ordinary Li.

Electrolyte: Consider electrolytes containing ions X^+ with atomic number Z. If X is lighter than Fe, the isotopes of nuclei with atomic number $Z + 3k$ might be produced in fusion reactions $nLi + X$. $X = Li, K, Na$ has one electron at s-shell whereas $B, Al, Cr, ...$ has one electron at p-shell.

Reaction	$\mathbf{Li} + Li \rightarrow C$	$\mathbf{C} + Li \rightarrow F$	$\mathbf{F} + Li \rightarrow Mg$
E/MeV	27.1	24.0	31.5
	$\mathbf{Li} + Na \rightarrow Si$	$\mathbf{C} + Na \rightarrow Cl$	$\mathbf{F} + Na \rightarrow Ca$
E/MeV	34.4	30.5	33.7
	$\mathbf{Li} + K \rightarrow Ti$	$\mathbf{C} + K \rightarrow Mn$	$\mathbf{F} + K \rightarrow Ni$
E/MeV	32.2	33.6	32.7

Table 5. The estimates for the energies liberated in fusions of dark nuclei of water and the ion of electrolyte. Boldface refers to dark nuclei $Li(3,5), C(6,10)$, and $F(9,15)$.

Relationship to the model of Widom and Larsen and further tests

W. Guglinski kindly informed me about the theory of cold fusion by Widom and Larsen [54]. This theory relies on standard nuclear physics. The theory is reported to explain cold fusion reaction products nicely in terms of the transformation of electrons and protons to very low energy neutrons which can overcome the Coulomb barrier. The problem of the theory is that very high energy electrons are required since one has $Q = .78$ MeV for $e + p \rightarrow n$ and $Q = -3.0$ MeV for $e + D \rightarrow n + n$. It is difficult to understand how so energetic electrons could result in ordinary condensed matter.

Since proton plus color bond is from the point of view of nuclear physics neutron and the fusion reactions would obey ordinary nuclear physics rules, the predictions of TGD are not expected to deviate too much from those of the model of Widom and Larsen.

An important class of predictions relate to ordinary nuclear physics. Tetra-neutron could be alpha particle with two negatively charged color bonds and neutron halos could consist of protons connected to nucleus by negatively charged color bonds. This could reduce the binding energy considerably.

Cold nuclear fusion might also provide an in situ mechanism for the formation of ores. Nuclear ores in places where they should not exist but involving remnants of organic matter would be the prediction. Cold fusion has a potential for a technology allowing to generate some metals artificially.

How to optimize the energy production?

The proposed model for the plasma electrolysis suggests following improvements to the experimental arrangement.

The production of energy in process is due to three reactions: 1) $Li+p$ in plasma. 2) $Li+Fe/W...$ in target, and 3) $Li+Na/K...$ in plasma. The model suggests that 1) dominates so that basic process would occur in plasma rather than cathode.

1. Since W does not evaporate so easily, it is better material for cathode if the production of dark Li dominates energy production.

2. Cathode could be replaced with a planar electrode with fractal peaky structure generating the required strong electric fields. This could increase the effectiveness of the energy production by increasing the effective area used.

3. Since $H_2O \rightarrow OH^- + p$ is required by the generation of dark Li sequences. The energy feed must be able to follow the rapidly growing energy needs of this reaction which seems to occur as bursts.

4. The prediction is that the output power is proportional to electron current rather than input power. This suggests minimization of input power by minimizing voltage. This requires maximization of electron conductivity. Unfortunately, the transformation of electrons to OH^- ions as charge carriers reduces conductivity.

9.7.10 Burning salt water by radio-waves and cold fusion by plasma electrolysis

John Kanzius has made a strange discovery [104]: salt water in the test tube radiated by radio waves at harmonics of a frequency f=13.56 MHz burns. Temperatures about 1500 C, which correspond to .17 eV energy have been reported. One can radiate also hand but nothing happens. The original discovery of Kanzius was the finding that radio waves could be used to cure cancer by destroying the cancer cells. The proposal is that this effect might provide new energy source by liberating chemical energy in an exceptionally effective manner. The power is about 200 W so that the power used could explain the effect if it is absorbed in resonance like manner by salt water. In the following it is proposed that the cold nuclear reactions are the source of the energy.

Do radio waves of large Planck constant transform to microwaves in the process?

The energies of photons involved are very small, multiples of 5.6×10^{-8} eV and their effect should be very small since it is difficult to imagine what resonant molecular transition could cause the effect. This leads to the question whether the radio wave beam could contain a considerable fraction of dark photons for which Planck constant is larger so that the energy of photons is much larger. The underlying mechanism would be phase transition of dark photons with large Planck constant to ordinary photons with shorter wavelength coupling resonantly to some molecular degrees of freedom and inducing the heating. Microwave oven of course comes in mind immediately.

1. The fact that the effects occur at harmonics of the fundamental frequency suggests that rotational states of molecules are in question as in microwave heating. The formula for the rotational energies [49] is

$$E(l) = E_0 \times (l(l+1) \ \ , \ \ E_0 = \hbar_0^2/2\mu R^2 \ \ , \ \ \mu = m_1 m_2/(m_1+m_2) \ .$$

   Here R is molecular radius which by definition is deduced from the rotational energy spectrum. The energy inducing the transition $l \rightarrow l+1$ is $\Delta E(l) = 2E_0 \times (l+1)$.

2. $NaCl$ molecules crystallize to solid so that the rotational heating of $NaCl$ molecules cannot be in question.

3. The microwave frequency used in microwave ovens is 2.45 GHz giving for the Planck constant the estimate 180.67 equal to 180 with error of .4 per cent. The values of Planck constants for $(\hat{M^4}/G_a) \times \hat{CP_2} \hat{\times} G_b$ option (factor space of M^4 and covering space of CP_2 maximizing Planck constant for given G_a and G_b) are given by $\hbar/\hbar_0 = n_a n_b$. $n_a n_b = 4 \times 9 \times 5 = 180$ can result from the number theoretically simple values of quantum phases $exp(i2\pi/n_i)$ corresponding to polygons constructible using only ruler and compass. For instance, one could have $n_a = 2 \times 3$ and $n_b = 2 \times 3 \times 5$.

Connection with plasma electrolysis?

The burning of salt water involves also the production of O_2 and H_2 gases. Usually this happens in the electrolysis of water [66]. The arrangement involves typically electrolyte consisting of water plus $NaOH$ or KOH present also now but anode, cathode and electronic current absent. The proposed mechanism of electrolysis involving cold nuclear reactions however allows the splitting of water molecules to H_2 and O_2 even without these prerequisites.

The thermal radiation from the plasma created in the process has temperature about 1500 C which correspond to energy about .17 eV: this is not enough for splitting of bonds with energy .5 eV. The temperature in salt water could be however considerably higher.

The presence of visible light suggests that plasma phase is created as in plasma electrolysis. Dark nuclear reactions would provide the energy leading to ionization of hydrogen atoms and subsequent transformation of the electronic charge to that of charged color bonds in protonic strings. This in turn would increase the rate of cold nuclear reactions and the liberated energy would ionize more hydrogen atoms so that a positive feedback loop would result.

Cold nuclear reactions should provide the energy transforming hydrogen bonds to dark bonds with energy scaled down by a factor of about 2^{-6} from say 8 eV to .125 eV if $T = 1500C$ is accepted as temperature of water. If Planck constant is scaled up by the factor $r = 180$ suggested by the interpretation in terms of microwave heating, the scaling of the Planck constant would reduce the energy of OH bonds to about .04 eV, which happens to be slightly below the energy assignable to the cell membrane resting potential. The scaling of the size of nuclear space-time sheets of D by factor $r = 180$ is consistent with the length of color bonds of order 10^{-12} m. The role of microwave heating would be to preserve this temperature so that the electrolysis of water can continue. Note that the energy from cold nuclear reactions could partially escape as dark photons.

There are some questions to be answered.

1. Are the radio wave photons dark or does water - which is a very special kind of liquid - induce the transformation of ordinary radio wave photons to dark photons by fusing 180 radio wave massless extremals (MEs) to single ME. Does this transformation occur for all frequencies? This kind of transformation might play a key role in transforming ordinary EEG photons to dark photons and partially explain the special role of water in living systems.

2. Why the radiation does not induce a spontaneous combustion of living matter which also contains Na^+ and other ions. A possible reason is that $\hbar$ corresponds to Planck constant of dark Li which is much higher in living water. Hence the energies of dark photons do not induce microwave heating.

3. The visible light generated in the process has yellow color. The mundane explanation is that the introduction Na or its compounds into flame yields bright yellow color due to so called sodium D-lines [46] at 588.9950 and 589.5924 nm emitted in transition from 3p to 3s level. Visible light could result as dark photons from the dropping of dark protons from dark space-time sheets of size at least atomic size to larger dark space-time sheets or to ordinary space-time sheets of same size and de-cohere to ordinary light. Yellow light corresponds roughly to the rather narrow energy range .96-2.1 eV ($.59 - .63\ \mu m$). The metabolic quanta correspond to jumps to space-time sheets of increasing size give rise to the fractal series $E/eV = 2 \times (1 - 2^{-n})$ for transitions $k = 135 \rightarrow 135 + n$, $n = 1, 2, ...$ [D8]. For $n = 3, 4, 5$ the lines have energies $1.74, 1.87, 1.93$ eV and are in the visible red ($\lambda/\mu m = .71, .66, .64$). For $n > 5$ the color is yellow. In Kanarev's experiments the color is red which would mean the dominance of $n < 6$ lines: this color is regarded as a signature of the plasma electrolysis. In the burning of salt water the light is yellow [104], which allows to consider the possibility that yellow light is partially due to $n > 5$ lines. Yellow color could also result from the dropping $k = 134 \rightarrow 135$ ($n = 1$).

9.7.11 GSI anomaly

"Jester" wrote a nice blog posting titled *Hitchhikers-guide-to-ghosts-and-spooks in particle physics* summarizing quite a bundle of anomalies of particle physics and also one of nuclear physics- known as GSI anomaly. The abstract of the article *Observation of Non-Exponential Orbital Electron Capture Decays of Hydrogen-Like* ^{140}Pr *and* ^{142}Pm *Ions* [69] describing the anomaly is here.

We report on time-modulated two-body weak decays observed in the orbital electron capture of hydrogen-like ^{140}*Pr¡sup¿59+¡/sup¿ and* ^{142}*Pm¡sup¿60+¡/sup¿ ions coasting in an ion storage ring. Using non-destructive single ion, time-resolved Schottky mass spectrometry we found that the expected exponential decay is modulated in time with a modulation period of about 7 seconds for both systems. Tentatively this observation is attributed to the coherent superposition of finite mass eigenstates of the electron neutrinos from the weak decay into a two-body final state.*

This brings in mind the nuclear decay rate anomalies which I discussed earlier in the blog posting *Tritium beta decay anomaly and variations in the rates of radioactive processes* and in [F8]. These variations in decay rates are in the scale of year and decay rate variation correlates with the distance from Sun. Also solar flares seem to induce decay rate variations.

The TGD based explanation [F8] relies on nuclear string model in which nuclei are connected by color flux tubes having exotic variant quark and antiquark at their ends (TGD predicts fractal hierarchy of QCD like physics). These flux tubes can be also charged: the possible charges $\pm 1, 0$. This means a rich spectrum of exotic states and a lot of new low energy nuclear physics. The energy scale corresponds to Coulomb interaction energy $\alpha_{em} m$, where m is mass scale of the exotic quark. This means energy scale of 10 keV for MeV mass scale. The well-known poorly understood X-ray bursts from Sun during solar flares in the wavelength range 1-8 A correspond to energies in the range 1.6-12.4 keV -3 octaves in good approximation- might relate to this new nuclear physics and in turn might excite nuclei from the ground state to these excited states and the small mixture of exotic nuclei with slightly different nuclear decay rates could cause the effective variation of the decay rate. The mass scale $m \sim 1$ MeV for exotic quarks would predict Coulombic energy of order $\alpha_{em} m$ which is of order 10 keV.

The question is whether there could be a flux of X rays in time scale of 7 seconds causing the rate fluctuation by the same mechanism also in GSI experiment. For instance, could this flux relate to synchrotron radiation. I could no identify any candidate for this periodicity from the article. In any case, the prediction is what might be called X ray nuclear physics and artificial X ray irradiation of nuclei would be an easy manner to kill or prove the general hypothesis.

One can imagine also another possibility.

1. The first guess is that the transitions between ordinary and exotic states of the ion are induced by the emission of exotic W boson between nucleon and exotic quark so that the charge of the color bond is changed. In standard model the objection would be that classical W fields do not make sense in the length scale in question. The basic prediction deriving from induced field concept (classical ew gauge fields correspond to the projection of CP_2 spinor curvature to the space-time surface) is however the existence of classical long range gauge fields- both ew and color. Classical W field can induce charge entanglement in all length scales and one of the control mechanisms of TGD inspired quantum biology relies on remote control of charge densities in this manner. Also the model of cold fusion could involve similar oscillating time like entanglement allowing the bombarding nucleus to penetrate to the nucleus when proton has transformed to neuron in good approximation and charge is delocalized to the color bond having much larger size.

2. In the approximation that one has two-state system, this interaction can be modelled by using as interaction Hamiltonian hermitian non-diagonal matrix V, which can be written as $V\sigma_x$, where σ_x is Pauli sigma matrix. If this process occurs coherently in time scales longer than $\hbar/V$, an oscillation with frequency $\omega = V/\hbar$ results. Since weak interactions are in question 7 second modulation period might make sense.

The hypothesis can be tested quantitatively.

1. The weak interaction Coulomb potential energy is of form

$$\frac{V(r)}{\hbar} = \alpha_W \frac{exp(-m_W r)}{r} \ , \tag{9.7.7}$$

   where r is the distance between nucleon center of mass and the end of color flux tube and therefore of order proton Compton length r_p so that one can write

$$r = x \times r_p \ .$$

   where x should be of order unity but below it.

2. The frequency $\omega = 2\pi/\tau = V/\hbar$ must correspond to 14 seconds, twice the oscillation period of the varying reaction rate. By taking W boson Compton time t_W as time unit this condition can be written as

$$\frac{\alpha_W exp(-y)}{y} = \frac{t_W}{\tau} \ ,$$
$$y = x\frac{r_p}{r_W} = x\frac{m_W}{m_p} \simeq 80 \times x \ ,$$
$$\alpha_W = \alpha_{em}/sin^2\theta_W \ .$$

3. This gives the condition

$$\frac{exp(-y)}{y} = \frac{t_p}{\tau} \times \frac{sin^2\theta_W}{80 \times \alpha_{em}} \ . \tag{9.7.8}$$

   This allows to solve y since the left hand side is known. Feeding in proton Compton length $r_p = 1.321 \times 10^{-15}$ m and $sin^2\theta_W = .23$ one obtains that the distance between flux tube end and proton cm is $x = .6446$ times proton Compton length, which compares favorably with the guess $x \simeq 1$ but smaller than 1. One must however notice that the oscillation period is exponentially sensitive to the value of x. For instance, if the charge entanglement were between nucleons, $x > 1$ would hold true and the time scale would be enormous. Hence the simple model requires new physics and predicts correctly the period of the oscillation under very reasonable assumptions.

4. One could criticize this by saying that the masses of two states differ by amount which is of order 10 keV or so. This does not however affect the argument since the mass corresponds to the diagonal non-interaction part of the Hamiltonian contributing only rapidly oscillating phases whereas interaction potential induces oscillating mixing as is easy to see in interaction picture.

5. If one believes in the hierarchy of Planck constants and p-adically scaled variants of weak interaction physics, charge entanglement would be possible in much longer length scales and the time scale of it raises the question whether qubits could be realized using proton and neutron in quantum computation purposes. I have also proposed that charge entanglement could serve as a mechanism of bio-control allowing to induce charge density gradients from distance in turn acting as switches inducing biological functions.

So: it happened again! Again I have given a good reason for my learned critics to argue that TGD explains everything so that I am a crackpot and so on and so on. Well... after a first feeling of deep shame I dare to defend myself. In the case of standard model explanatory power has not been regarded as an argument against the theory but my case is of course different since I do not have any academic position since my fate is to live in the arctic scientific environment of Finland. And if my name were Feynman, this little argument would be an instant classic. But most theoreticians are just little opportunists building their career and this does not leave much room for intellectual honesty.

9.8 Dark nuclear strings as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code?

The minimal option is that virtual DNA sequences have flux tube connections to the lipids of the cell membrane so that their quality as hardware of tqc can be tested but that there is no virtual variant of transcription and translation machinery. One can however ask whether also virtual amino-acids could be present and whether this could provide deeper insights to the genetic code.

1. Water molecule clusters are not the only candidates for the representatives of linear molecules. An alternative candidate for the virtual variants of linear bio-molecules are dark nuclei consisting of strings of scaled up dark variants of neutral baryons bound together by color bonds having the size scale of atom, which I have introduced in the model of cold fusion and plasma electrolysis both taking place in water environment. Colored flux tubes defining braidings would generalize this picture by allowing transversal color magnetic flux tube connections between these strings.

2. Baryons consist of 3 quarks just as DNA codons consist of three nucleotides. Hence an attractive idea is that codons correspond to baryons obtained as open strings with quarks connected by two color flux tubes. The minimal option is that the flux tubes are neutral. One can also argue that the minimization of Coulomb energy allows only neutral dark baryons. The question is whether the neutral dark baryons constructed as string of 3 quarks using neutral color flux tubes could realize 64 codons and whether 20 aminoacids could be identified as equivalence classes of some equivalence relation between 64 fundamental codons in a natural manner.

The following model indeed reproduces the genetic code directly from a model of dark neutral baryons as strings of 3 quarks connected by color flux tubes.

1. Dark nuclear baryons are considered as a fundamental realization of DNA codons and constructed as open strings of 3 dark quarks connected by two colored flux tubes, which can be also charged. The analogs of DNA -, RNA -, and of amino-acid sequences would in turn correspond to sequences of dark baryons. It is assumed that the net charge of the dark baryons vanishes so that Coulomb repulsion is minimized.

2. One can classify the states of the open 3-quark string by the total charges and spins associated with 3 quarks and to the two color bonds. Total em charges of quarks vary in the range $Z_B \in \{2, 1, 0, -1\}$ and total color bond charges in the range $Z_b \in \{2, 1, 0, -1, -2\}$. Only neutral states are allowed. Total quark spin projection varies in the range $J_B = 3/2, 1/2, -1/2, -3/2$ and the total flux tube spin projection in the range $J_b = 2, 1, -1, -2$. If one takes for a given total charge assumed to be vanishing one representative from each class (J_B, J_b), one obtains $4 \times 5 = 20$ states which is the number of amino-acids. Thus genetic code might be realized at the level of baryons by mapping the neutral states with a given spin projection to single representative state with the same spin projection. The problem is to find whether one can identify the analogs of DNA, RNA and aminoacids as baryon like states.

9.8.1 States in the quark degrees of freedom

Consider first the states of dark baryons in quark degrees of freedom. These states can be constructed as representations of rotation group and strong isospin group.

1. The tensor product $2 \otimes 2 \otimes 2$ is involved in both cases. Without any additional constraints this tensor product decomposes as $4 \oplus 2 \oplus 2$: 8 states altogether. This is what one should have for DNA and RNA candidates. If one has only identical quarks uuu or ddd, one obtains only the 4-D representation corresponding to completely symmetric representation. These 4 states correspond to a candidate for amino-acids. Thus RNA and DNA should correspond to states of type uud and ddu and aminoacids to states of type uuu or ddd. What this means physically will be considered later.

2. It is known that only representations with isospin 3/2 and spin 3/2 (Δ resonance) and isospin 1/2 and spin 1/2 (proton and neutron) are realized as free baryons. Now of course a dark - possibly p-adically scaled up - variant of QCD is considered so that more general baryonic states are possible. The spin statistics problem which forced to introduce quark color strongly suggests that the construction of the codons as sequences of 3 nucleons is not a good idea.

3. Second nucleon like spin doublet - call it 2_{odd} - has wrong parity in the sense that it would require $L = 1$ ground state for two identical quarks (uu or dd pair). Dropping 2_{odd} and using only $4 \oplus 2$ for the rotation group would give degeneracies $(1, 2, 2, 1)$ and 6 states only. All the representations in $4 \oplus 2 \oplus 2_{odd}$ to get 8 states with a given quark charge and one should transform the wrong parity doublet to positive parity doublet somehow. Since open string geometry breaks rotational symmetry to a subgroup of rotations acting along the direction of the string, the attractive possibility is to add a stringy excitation with angular momentum projection $L_z = -1$ to the wrong parity doublet so that the parity comes out correctly. $L_z = -1$ orbital angular momentum for the relative motion of uu or dd quark pair in the open 3-quark string would be in question. The degeneracies for spin projection value $J_z = 3/2, ..., -3/2$ are $(1, 2, 3, 2)$. Genetic code means spin projection mapping the states in $4 \oplus 2 \oplus 2_{odd}$ to 4.

9.8.2 States in the flux tube degrees of freedom

Consider next the states in flux tube degrees of freedom.

1. The situation is analogous to a construction of mesons from quarks and antiquarks and one obtains the analogs of π meson (pion) with spin 0 and ρ meson with spin 1. States of a given charge correspond to the tensor product $2 \otimes 2 = 3 \oplus 1$ for the rotation group. Drop the singlet and take only the analog of neutral ρ meson. The physical meaning of this will be considered later.

2. Without any further constraints the tensor product $3 \otimes 3 = 5 \oplus 3 \oplus 1$ gives 8+1 states. By dropping the scalar state this gives 8 states required by DNA and RNA analogs. Bosonic statistics allows only 5 unless the two color bonds have different charges. The degeneracies of the states for DNA/RNA type realization with a given spin projection for $5 \oplus 3$ are $(1, 2, 2, 2, 1)$.

3. For aminoacids only 5 completely symmetric under the exchange of flux tubes is required and is achieved if the two color bonds have identical charges. Genetic code means the projection of the states of $5 \oplus 3$ to those of 5 with the same spin projection and same total charge.

9.8.3 Analogs of DNA,RNA, aminoacids, and of translation and transcription mechanisms

Consider next the identification of analogs of DNA, RNA and aminoacids and the baryonic realization of the genetic code, translation and transcription.

1. The analogs of DNA and RNA can be identified dark baryons with quark content *uud* and *ddu* and color bonds of different charges. There are 3 color bond pairs corresponding to charge pairs $(q_1, q_2) = (-1, 0), (-1, 1), (0, 1)$ (the order of charges does not matter). The condition that the total charge of dark baryon vanishes allows for *uud* only the bond pair $(-1, 0)$ and for *udd* only the pair $(-1, 1)$. These thus only single neutral dark baryon of type *uud resp. udd*: these would be the analogous of DNA and RNA codons. Amino-acids would correspond to either *uuu* or *ddd* with identical color bonds with charges $(-1, -1)$, $(0, 0)$, or $(1, 1)$. *uuu* with color bond charges (-1,-1) is the only neutral state. Hence only the analogs of DNA, RNA, and aminoacids are obtained, which is rather remarkable result.

2. The basic transcription and translation machinery could be realized as processes in which the analog of DNA can replicate, and can be transcribed to the analog of mRNA in turn translated to the analogs of amino-acids. In terms of flux tube connections the realization of genetic code, transcription, and translation, would mean that only dark baryons with same total quark spin and same total color bond spin can be connected by flux tubes. Charges are of course identical since they vanish.

3. Genetic code maps of $(4 \oplus 2 \oplus 2) \otimes (5 \oplus 3)$ to the states of 4×5. The most natural map takes the states with given spin to a state with the same spin so that the code is unique. This would give the degeneracies $D(k)$ as products of numbers $D_B \in \{1, 2, 3, 2\}$ and $D_b \in \{1, 2, 2, 2, 1\}$: $D = D_B \times D_b$. Only the observed degeneracies $D = 1, 2, 3, 4, 6$ are predicted. The numbers $N(k)$ of aminoacids coded by D codons would be

$$[N(1), N(2), N(3), N(4), N(6)] = [2, 7, 2, 6, 3] \ .$$

   The correct numbers for vertebrate nuclear code are $(N(1), N(2), N(3), N(4), N(6)) = (2, 9, 1, 5, 3)$. Some kind of symmetry breaking must take place and should relate to the emergence of stopping codons. If one codon in second 3-plet becomes stopping codon, the 3-plet becomes doublet. If 2 codons in 4-plet become stopping codons it also becomes doublet and one obtains the correct result $(2, 9, 1, 5, 3)$!

4. Stopping codons would most naturally correspond to the codons, which involve the $L_z = -1$ relative rotational excitation of *uu* or *dd* type quark pair. For the 3-plet the two candidates for the stopping codon state are $|1/2, -1/2\rangle \otimes \{|2, k\rangle\}$, $k = 2, -2$. The total spins are $J_z = 3/2$

and $J_z = -7/2$. The three candidates for the 4-plet from which two states are thrown out are $|1/2, -3/2\rangle \otimes \{|2, k\rangle, |1, k\rangle\}$, $k = 1, 0, -1$. The total spins are now $J_z = -1/2, -3/2, -5/2$. One guess is that the states with smallest value of J_z are dropped which would mean that $J_z = -7/2$ states in 3-plet and $J_z = -5/2$ states 4-plet become stopping codons.

9.8.4 Understanding the symmetries of the code

Quantum entanglement between quarks and color flux tubes would be essential for the baryonic realization of the genetic code whereas chemical realization could be said to be classical. Quantal aspect means that one cannot decompose to codon to letters anymore. This raises questions concerning the symmetries of the code.

1. What is the counterpart for the conjugation $ZYZ \rightarrow X_cY_cZ_c$ for the codons?
2. The conjugation of the second nucleotide Y having chemical interpretation in terms of hydrophobia-hydrophily dichotomy in biology. In DNA as tqc model it corresponds to matter-antimatter conjugation for quarks associated with flux tubes connecting DNA nucleotides to the lipids of the cell membrane. What is the interpretation in now?
3. The A-G, T-C symmetries with respect to the third nucleotide Z allow an interpretation as weak isospin symmetry in DNA as tqc model. Can one identify counterpart of this symmetry when the decomposition into individual nucleotides does not make sense?

Natural candidates for the building blocks of the analogs of these symmetries are the change of the sign of the spin direction for quarks and for flux tubes.

1. For quarks the spin projections are always non-vanishing so that the map has no fixed points. For flux tube spin the states of spin $S_z = 0$ are fixed points. The change of the sign of quark spin projection must therefore be present for both $XYZ \rightarrow X_cY_cZ_c$ and $Y \rightarrow Y_c$ but also something else might be needed. Note that without the symmetry breaking $(1, 3, 3, 1) \rightarrow (1, 2, 3, 2)$ the code table would be symmetric in the permutation of 2 first and 2 last columns of the code table induced by both full conjugation and conjugation of Y.
2. The analogs of the approximate $A - G$ and $T - C$ symmetries cannot involve the change of spin direction in neither quark nor flux tube sector. These symmetries act inside the A-G and T-C sub-2-columns of the 4-columns defining the rows of the code table. Hence this symmetry must permute the states of same spin inside 5 and 3 for flux tubes and 4 and 2 for quarks but leave 2_{odd} invariant. This guarantees that for the two non-degenerate codons coding for only single amino-acid and one of the codons inside triplet the action is trivial. Hence the baryonic analog of the approximate $A - G$ and $T - C$ symmetry would be exact symmetry and be due to the basic definition of the genetic code as a mapping states of same flux tube spin and quark spin to single representative state. The existence of full 4-columns coding for the same aminoacid would be due to the fact that states with same quark spin inside $(2, 3, 2)$ code for the same amino-acid.
3. A detailed comparison of the code table with the code table in spin representation should allow to fix their correspondence uniquely apart from permutations of n-plets and thus also the representation of the conjugations. What is clear that Y conjugation must involve the change of quark spin direction whereas Z conjugation which maps typically 2-plets to each other must involve the permutation of states with same J_z for the flux tubes. It is not quite clear what X conjugation correspond to.

9.8.5 Some comments about the physics behind the code

Consider next some particle physicist's objections against this picture.

1. The realization of the code requires the dark scaled variants of spin 3/2 baryons known as Δ resonance and the analogs (and only the analogs) of spin 1 mesons known as ρ mesons. The lifetime of these states is very short in ordinary hadron physics. Now one has a scaled up variant of hadron physics: possibly in both dark and p-adic senses with latter allowing arbitrarily small overall mass scales. Hence the lifetimes of states can be scaled up.

2. Both the absolute and relative mass differences between Δ and N *resp.* ρ and π are large in ordinary hadron physics and this makes the decays of Δ and ρ possible kinematically. This is due to color magnetic spin-spin splitting proportional to the color coupling strength $\alpha_s \sim .1$, which is large. In the recent case α_s could be considerably smaller - say of the same order of magnitude as fine structure constant 1/137 - so that the mass splittings could be so small as to make decays impossible.

3. Dark hadrons could have lower mass scale than the ordinary ones if scaled up variants of quarks in p-adic sense are in question. Note that the model for cold fusion that inspired the idea about genetic code requires that dark nuclear strings have the same mass scale as ordinary baryons. In any case, the most general option inspired by the vision about hierarchy of conscious entities extended to a hierarchy of life forms is that several dark and p-adic scaled up variants of baryons realizing genetic code are possible.

4. The heaviest objection relates to the addition of $L_z = -1$ excitation to $S_z = |1/2, \pm 1/2\rangle_{odd}$ states which transforms the degeneracies of the quark spin states from $(1,3,3,1)$ to $(1,2,3,2)$. The only reasonable answer is that the breaking of the full rotation symmetry reduces $SO(3)$ to $SO(2)$. Also the fact that the states of massless particles are labeled by the representation of $SO(2)$ might be of some relevance. The deeper level explanation in TGD framework might be as follows. The generalized imbedding space is constructed by gluing almost copies of the 8-D imbedding space with different Planck constants together along a 4-D subspace like pages of book along a common back. The construction involves symmetry breaking in both rotational and color degrees of freedom to Cartan sub-group and the interpretation is as a geometric representation for the selection of the quantization axis. Quantum TGD is indeed meant to be a geometrization of the entire quantum physics as a physics of the classical spinor fields in the "world of classical worlds" so that also the choice of measurement axis must have a geometric description.

The conclusion is that genetic code can be understand as a map of stringy baryonic states induced by the projection of all states with same spin projection to a representative state with the same spin projection. Genetic code would be realized at the level of dark nuclear physics and perhaps also at the level of ordinary nuclear physics and that biochemical representation would be only one particular higher level representation of the code. A hierarchy of dark baryon realizations corresponding to p-adic and dark matter hierarchies can be considered. Translation and transcription machinery would be realized by flux tubes connecting only states with same quark spin and flux tube spin. Charge neutrality is essential for having only the analogs of DNA, RNA and aminoacids and would guarantee the em stability of the states.

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Chapter 10

Dark Nuclear Physics and Condensed Matter

10.1 Introduction

The unavoidable presence of classical long ranged weak (and also color) gauge fields in TGD Universe has been a continual source of worries for more than two decades. The basic question has been whether electro-weak charges of elementary particles are screened in electro-weak length scale or not. The TGD based view about dark matter assumes that weak charges are indeed screened for ordinary matter in electro-weak length scale but that dark electro-weak bosons correspond to much longer symmetry breaking length scale.

The large value of $\hbar$ in dark matter phase implies that Compton lengths and -times are scaled up. In particular, the sizes of nucleons and nuclei become of order atom size so that dark nuclear physics would have direct relevance for condensed matter physics. It becomes impossible to make a reductionistic separation between nuclear physics and condensed matter physics and chemistry anymore. This view forces a profound re-consideration of the earlier ideas in nuclear and condensed physics context. It however seems that most of the earlier ideas related to the classical Z^0 force and inspired by anomaly considerations survive in a modified form.

In its original form this chapter was an attempt to concretize and develop ideas related to dark matter by using some experimental inputs with emphasis on the predicted interaction between the new nuclear physics and condensed matter. As the vision about dark matter became more coherent and the nuclear string model developed in its recent form, it became necessary to update the chapter and throw away the obsolete material. I dare hope that the recent representation is more focused than the earlier one.

10.1.1 Evidence for long range weak forces and new nuclear physics

There is a lot of experimental evidence for long range electro-weak forces, dark matter, and exotic nuclear physics giving valuable guidelines in the attempts to build a coherent theoretical scenario.

Cold fusion

Cold fusion [62] is a phenomenon involving new nuclear physics and the known selection rules give strong constraints when one tries to understand the character of dark nuclear matter. The simplest model for cold fusion found hitherto is based on the nuclear string model [F9] and will be taken as the basis of the considerations of this chapter. Also comparisons with the earlier variant of model of cold fusion [F8] will be made in the section about cold fusion.

Large parity breaking effects

Large parity breaking effects in living matter indicate the presence of long ranged weak forces, and the reported nuclear transmutations in living matter [69, 70] suggest that new nuclear physics plays a role also now. For instance, the Gaussian Mersennes $(1+i)^k-1$ for $k = 113, 151, 157163, 167$ could correspond to weak length scales and four biologically important length scales in the range 10 nm-25 μm, which seem to relate directly to the coiling hierarchy of DNA double strands. Quantum criticality of living matter against phase transitions between different values of Planck constant suggests that zeros of Riemann Zeta can appear as conformal weights of particles in living matter.

Anomalies of the physics of water

The physics of water involves a large number of anomalies and life depends in an essential manner on them. As many as 41 anomalies are discussed in the excellent web page "Water Structure and Behavior" of M. Chaplin [36]. The fact that the physics of heavy water differs much more from that of ordinary water as one might expect on basis of different masses of water molecules suggests that dark nuclear physics is involved.

1. The finding that one hydrogen atom per two water molecules remain effectively invisible in neutron and electron interactions in attosecond time scale [36, 37] suggests that water is partially dark. These findings have been questioned in [38] and thought to be erroneous in [39]. If the findings are real, dark matter phase made of super-nuclei consisting of protons connected by dark color bonds could explain them as perhaps also the clustering of water molecules predicting magic numbers of water molecules in clusters. If so, dark nuclear physics could be an essential part of condensed matter physics and biochemistry. For instance, the condensate of dark protons might be essential for understanding the properties of bio-molecules and even the physical origin of van der Waals radius of atom in van der Waals equation of state.

2. The observation that the binding energy of dark color bond for $n = 2^{11} = 1/v_0$ of the scaling of $\hbar$ corresponds to the bond energy .5 eV of hydrogen bond raises the fascinating possibility that hydrogen bonds is accompanied by a color bond between proton and oxygen nucleus. Also more general chemical bonds might be accompanied by color bonds so that dark color physics might be an essential part of molecular physics. Color bonds might be also responsible for the formation of liquid phase and thus solid state. Dark weak bonds between nuclei could be involved and might be responsible for the repulsive core of van der Waals force and be part of molecular physics too. There is evidence for two kinds of hydrogen bonds [80, 79]: a possible identification is in terms of p-adic scaling of hydrogen bonds by a factor 2. This kind of doubling is predicted by nuclear string model [F9].

3. Tedrahedral water clusters consisting of 14 water molecules would contain 8 dark protons which corresponds to a magic number for a dark nucleus consisting of protons. Icosahedral water clusters in turn consist of 20 tedrahedral clusters. This raises the question whether fractally scaled up super-nuclei could be in question. If one accepts the vision about dark matter hierarchy based in Jones inclusions to be discussed briefly later, tedrahedral and icosahedral structures of water could correspond directly to the unique genuinely 3-dimensional $G_a = E_6$ and E_8 coverings of CP_2 with $n_a = 3$ and $n_a = 5$ assignable to dark electrons. Icosahedral structures are also very abundant in living matter, mention only viruses.

Exotic chemistries

Exotic chemistries [45] in which clusters of atoms of given given type mimic the chemistry of another element. These systems behave as if nuclei would form a jellium (constant charge density) defining a harmonic oscillator potential for electrons. Magic numbers correspond to full electron shells analogous to noble gas elements. It is difficult to understand why the constant charge density approximation works so well. If nuclear protons are in large $\hbar(M^4)$ phase with $n_F = 3 \times 2^{11}$, the electromagnetic sizes of nuclei would be about 2.4 Angstroms and the approximation would be natural.

As a matter, fact nuclear string model predicts that the nuclei can have as many as $3A$ exotic charge states obtained by giving neutral color bond charge ± 1: this would give rise to quite different kind of alchemy [F9] revealing itself in cold fusion.

Free energy anomalies

The anomalies reported by free energy researchers such as over unity energy production in devices involving repeated formation and dissociation of H_2 molecules based on the original discovery of

Nobelist Irwing Langmuir [72] (see for instance [73]) suggest that part of H atoms might end up to dark matter phase liberating additional energy. The "mono-atomic" elements of Hudson suggest also dark nuclear physics [43]. There is even evidence for macroscopic transitions to dark phase [77, 78, 76].

Tritium beta decay anomaly and findings of Shnoll

Tritium beta decay anomaly [26, 27, 28, 29] suggests exotic nuclear physics related to weak interactions and that dark anti-neutrino density at the orbit of Earth around Sung oscillating with one year period is involved. This kind of remnant of dark matter would be consistent with the model for the formation of planets from dark matter. The evidence for the variation of the rates of nuclear and chemical processes correlating with astrophysical periods [74] could be understood in terms of weak fields created by dark matter and affect by astrophysical phenomena.

10.1.2 Dark rules

I have done a considerable amount of trials and errors in order to identify the basic rules allowing to understand what it means to be dark matter is and what happens in the phase transition to dark matter. It is good to try to summarize the basic rules of p-adic and dark physics allowing to avoid obvious contradictions.

The notion of field body

The notion of "field body" implied by topological field quantization is essential. There would be em, $Z0$, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

The p-adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states.*

What dark variant of elementary particle means

It is not at all clear what the notion of dark variant of elementary particle or of larger structures could mean.

1. Are only field bodies dark?

One variety of dark particle is obtained by making some of the field bodies dark by increasing the value of Planck constant. This hypothesis could be replaced with the stronger assumption that elementary particles are maximally quantum critical systems so that they are same irrespective of the value of the Planck constant. Elementary particles would be represented by partonic 2-surfaces, which belong to the universal orbifold singularities remaining invariant by all groups $G_a \times G_b$ for a given choice of quantization axes. If $G_a \times G_b$ is assumed to leave invariant the choice of the quantization axes, it must be of the form $Z_{n_a} \times Z_{n_b} \subset SO(3) \times SU(3)$. Partonic 2-surface would belong to $M^2 \times CP_2/U(1) \times U(1)$, where M^2 is spanned by the quantization axis of angular momentum and the time axis defining the rest system.

A different manner to say this is that the CP_2 type extremal representing particle would suffer multiple topological condensation on its field bodies so that there would be no separate "particle space-time sheet".

Darkness would be restricted to particle interactions. The value of the Planck constant would be assigned to a particular interaction between systems rather than system itself. This conforms with the original finding that gravitational Planck constant satisfies $\hbar = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$. Since each interaction can give rise to a hierarchy dark phases, a rich variety of partially dark phases is predicted. The standard assumption that dark matter is visible only via gravitational interactions would mean that gravitational field body would not be dark for this particular dark matter.

Complex combinations of dark field bodies become possible and the dream is that one could understand various phases of matter in terms of these combinations. All phase transitions, including the familiar liquid-gas and solid-liquid phase transitions, could have a unified description in terms of dark phase transition for an appropriate field body. At mathematical level Jones inclusions would provide this description.

The book metaphor for the interactions at space-time level is very useful in this framework. Elementary particles correspond to ordinary value of Planck constant analogous to the ordinary sheets of a book and the field bodies mediating their interactions are the same space-time sheet or at dark sheets of the book.

2. Can also elementary particles be dark?

Also dark elementary particles themselves rather than only the flux quanta could correspond to dark space-time sheet defining multiple coverings of $H/G_a \times G_b$. This would mean giving up the maximal quantum criticality hypothesis in the case of elementary particles. These sheets would be exact copies of each other. If single sheet of the covering contains topologically condensed space-time sheet, also other sheets contain its exact copy.

The question is whether these copies of space-time sheet defining classical identical systems can carry different fermionic quantum numbers or only identical fermionic quantum numbers so that the dark particle would be exotic many-fermion system allowing an apparent violation of statistics (N fermions in the same state).

Even if one allows varying number of fermions in the same state with respect to a basic copy of sheet, one ends up with the notion of N-atom in which nuclei would be ordinary but electrons would reside at the sheets of the covering. The question is whether symbolic representations essential for understanding of living matter could emerge already at molecular level via the formation of N-atoms.

Criterion for the transition to dark phase

The criterion $\alpha Q_1 Q_2 > 1$ for the transition to dark matter phase relates always to the interaction between two systems and the interpretation is that when the field strength characterizing the interaction becomes too strong, the interaction is mediated by dark space-time sheets which define $n = n(G_a) \times n(G_b)$-fold covering of $M^4 \times CP_2/G_a \times G_b$. The sharing of flux between different space-time sheets reduces the field strength associated with single sheet below the critical value.

10.1.3 Implications

Dark variants of nuclear physics

One can imagine endless variety of dark variants of ordinary nuclei and every piece of data is well-come in attempts to avoid a complete inflation of speculative ideas. The book metaphor for the extended imbedding space is useful in the attempts to imagine various exotic phases of matter. For the minimal option atomic nuclei would be ordinary whereas field bodies could be dark and analogous to n-sheeted Riemann surfaces. One can imagine that the nuclei are at the "standard" page of the book and color bonds at different page with different p-adic length scale or having different Planck constant $\hbar_{eff}$. This would give two hierarchies of nuclei with increasing size.

Color magnetic body of the structure would become a key element in understanding the nuclear binding energies, giant dipole resonances, and nuclear decays. Also other field bodies are in a key role and there seems to be a field body for every basic interaction (classical gauge fields are induced from spinor connection and only four independent field variables are involved so that this is indeed required).

Nothing prevents from generalizing the nuclear string picture so that color bonds could bind also atoms to molecules and molecules to larger structures analogous to nuclei. Even hydrogen bond might be interpreted in this manner. Molecular physics could be seen as a scaled up variant of nuclear physics in a well-defined sense. The exotic features would relate to the hierarchy of various field bodies, including color bonds, electric and weak bonds. These field bodies would play key role also in biology and replaced molecular randomness with coherence in much longer length scale.

In the attempt to make this vision quantitative the starting point is nuclear string model [F9] and the model of cold fusion based on it forcing also to conclude the scaled variants of electro-weak bosons are involved. The model of cold fusion requires the presence of a variant electro-weak interactions for which weak bosons are effectively massless below the atomic length scale. $k = 113$ p-adically scaled up variant of ordinary weak bosons which is dark and corresponds to $\hbar = n\hbar_0$, $n = 2^{11}$, is a natural option. For ordinary nuclei weak bosons could be ordinary.

Anomalies of water could be understood if one assumes that color bonds can become dark with $n = k2^{11}$, $k = 1, 3$ and if super-nuclei formed by connecting different nuclei by the color bonds are possible. Tetrahedral and icosahedral water clusters could be seen as magic super-nuclei in this framework. Color bonds could connect either proton nuclei or water molecules.

Could the notion of dark atom make sense?

One can also imagine several variants of dark atom. Book metaphor suggest one variant of dark atom.

1. Nuclei and electrons could be ordinary but classical electromagnetic interactions are mediated via dark space-time sheet "along different page of the book". The value of Planck constant would be scaled so that one would obtain a hierarchy of scaled variants of hydrogen atom. The findings of Mills [83] find an explanation in terms of a reduced Planck constant. An alternative explanation is based on the notion of quantum-hydrogen atom obtained as q-deformation of the ordinary hydrogen atom.

2. A more exotic variant if atom is obtained by assuming ordinary nuclei but dark, not totally quantum critical, electrons. Dark space-time surface is analogous to n-sheeted Riemann surface and if one assumes that each sheet could carry electron, one ends up with the notion of N-atom.

Implications of the partial darkness of condensed matter

The model for partially dark condensed matter deriving from nuclear physics allows to understand the low compressibility of the condensed matter as being due to the repulsive weak force between exotic quarks, explains large parity breaking effects in living matter, and suggests a profound modification of the notion of chemical bond having most important implications for bio-chemistry and understanding of bio-chemical evolution.

10.2 General ideas about dark matter

In the sequel general ideas about the role of dark matter in condensed matter physics are described.

10.2.1 Quantum criticality, hierarchy of dark matters, and dynamical $\hbar$

Quantum criticality is the basic characteristic of TGD Universe and quantum critical superconductors provide an excellent test bed to develop the ideas related to quantum criticality into a more concrete form.

Quantization of Planck constants and the generalization of the notion of imbedding space

The recent geometric interpretation for the quantization of Planck constants is based on Jones inclusions of hyper-finite factors of type II_1 [A9].

1. Different values of Planck constant correspond to imbedding space metrics involving scalings of M^4 *resp.* CP_2 parts of the metric deduced from the requirement that distances scale as $\hbar(M^4)$ *resp.* $\hbar(M^4)$. Denoting the Planck constants by $\hbar(M^4) = n_a\hbar_0$ and $\hbar(CP_2) = n_b\hbar_0$, one has that covariant metric of M^4 is proportional to n_b^2 and covariant metric of CP_2 to n_a^2. In Kähler action only the effective Planck constant $\hbar_{eff}/\hbar_0 = \hbar(M^4)/\hbar(CP_2)$ appears and by quantum classical correspondence same is true for Schödinger equation. Elementary particle mass spectrum is also invariant. Same applies to gravitational constant. The alternative assumption that M^4 Planck constant is proportional to n_b would imply invariance of Schrödinger equation but would not allow to explain Bohr quantization of planetary orbits and would to certain degree trivialize the theory.

2. M^4 and CP_2 Planck constants do not fully characterize a given sector $M^4_\pm \times CP_2$. Rather, the scaling factors of Planck constant given by the integer n characterizing the quantum phase $q = exp(i\pi/n)$ corresponds to the order of the maximal cyclic subgroup for the group $G \subset SU(2)$ characterizing the Jones inclusion $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors realized as subalgebras of the Clifford algebra of the "world of the classical worlds". This means that subfactor $\mathcal{N}$ gives rise to G-invariant configuration space spinors having interpretation as G-invariant fermionic states.

3. $G_b \subset SU(2) \subset SU(3)$ defines a covering of M^4_+ by CP_2 points and $G_a \subset SU(2) \subset SL(2,C)$ covering of CP_2 by M^4_+ points with fixed points defining orbifold singularities. Different sectors are glued together along CP_2 if G_b is same for them and along M^4_+ if G_a is same for them. The degrees of freedom lost by G-invariance in fermionic degrees of freedom are gained back since the discrete degrees of freedom provided by covering allow many-particle states formed from single particle states realized in G group algebra. Among other things these many-particle states make possible the notion of N-atom.

4. Phases with different values of scalings of M^4 and CP_2 Planck constants behave like dark matter with respect to each other in the sense that they do not have direct interactions except at criticality corresponding to a leakage between different sectors of imbedding space glued together along M^4 or CP_2 factors. In large $\hbar(M^4)$ phases various quantum time and length scales are scaled up which means macroscopic and macro-temporal quantum coherence. In particular, quantum energies associated with classical frequencies are scaled up by a factor n_a/n_b which is of special relevance for cyclotron energies and phonon energies (superconductivity). For large $\hbar(CP_2)$ the value of $\hbar_{eff}$ is small: this leads to interesting physics: in particular the binding energy scale of hydrogen atom increases by the factor n_b/n_a^2.

A further generalization of the notion of imbedding space?

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

1. Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2\times S^2 \subset M^4\times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4\backslash M^2$ and $\hat{CP_2} = CP_2\backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. Zero energy ontology forces to modify this picture somewhat. In zero energy ontology causal diamonds (CDs) defined as the intersections of future and past directed light-cones are loci for zero energy states containing positive and negative energy parts of state at the two light-cone boundaries. The location of CD in M^4 is arbitrary but p-adic length scale hypothesis suggests that the temporal distances between tips of CD come as powers of 2 using CP_2 size as unit. Thus M^4 is replaces by CD and $\hat{M}^4$ is replaced with $\hat{CD}$ defined in obvious manner.

3. H_4 represents a straight cosmic string inside CD. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M}:\mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M}:\mathcal{N} = 4$. Also these Jones inclusions are labeled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{CD}\times\hat{CP_2}$ implying that surfaces in $CD\times S^2$ and $(M^2\cap CD)\times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

4. The covering spaces in question would correspond to the Cartesian products $\hat{CD}_{n_a}\times\hat{CP}_{2n_b}$ of the covering spaces of $\hat{CD}$ and $\hat{CP_2}$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a}\times Z_{n_b}$. One can also consider extension by replacing $M^2\cap CD$ and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{CD}\hat{\times}G_a$ *resp.* $\hat{CP_2}\hat{\times}G_b$.

5. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds $M^2\cap CD$ or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of $M^2\cap CD$ the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

6. Also the orbifolds $\hat{CD}/G_a\times\hat{CP_2}/G_b$ can be allowed as also the spaces $\hat{CD}/G_a\times(\hat{CP_2}\hat{\times}G_b)$ and $(\hat{CD}\hat{\times}G_a)\times\hat{CP_2}/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $(M^2\cap CD)\times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2\times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in CD degrees of freedom. This is not the case. Light-likeness in $(M^2\cap CD)\times S^2$ makes sense only for surfaces $X^1\times D^2\subset(M^2\cap CD)\times S^2$, where X^1

is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $(M^2 \cap CD) \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of $(M^2 \cap CD)$ as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

2. Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M}:\mathcal{N}<4$ and it naturally corresponds to the coset spaces. For $\mathcal{M}:\mathcal{N}=4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M}:\mathcal{N}=4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$, which both are labeled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{Htimes}(G_a \times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{Htimes}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/n$ or n. If one postulates that the total number states is invariant in the transition, the increase in the number of sheets is compensated by the increase of the fundamental phase space volume proportional to $\hbar$. This would give $r(X) \rightarrow r(X)/n$ for factor space and $r(X) \rightarrow nr(X)$ for the covering space to compensate the n-fold reduction/increase of states. This would favor Option II.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in CD and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

3. A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [31] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{10.2.0}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10$ $1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [31].

The model of Laughlin [55] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [32]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are four combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this there are two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. On the following just for fun consideration option I is considered although the conservation of number of states in the phase transition changing $\hbar$ favors option II.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. The appearance of $\nu = 5/2$ has been observed [33]. The fractionized charge is $e/4$ in this case. Since $n_i > 3$ holds true if coverings are correlates for Jones inclusions, this requires to $n_b = 4$ and $n_a = 10$. n_b predicting a correct fractionization of charge. The alternative option would be $n_b = 2$ that also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [32]. $n_b = 2$ is however inconsistent with the observed fractionization of electric charge and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e\int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of

the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the identification of charge unit is rather ad hoc. Therefore this model can be taken only as a warm-up exercise. In [F12] Quantum Hall effect and charge fractionization are discussed in detail and one ends up with a rather detailed view about the delicacies of the Kähler structure of generalized imbedding space.

10.2.2 How the scaling of $\hbar$ affects physics and how to detect dark matter?

It is relatively easy to deduce the basic implications of the scaling of $\hbar$.

1. If the rate for the process is non-vanishing classically, it is not affected in the lowest order. For instance, scattering cross sections for say electron-electron scattering and e^+e^- annihilation are not affected in the lowest order since the increase of Compton length compensates for the reduction of α_{em}. Photon-photon scattering cross section, which vanishes classically and is proportional to $\alpha_{em}^4\hbar^2/E^2$, scales down as $1/\hbar^2$.

2. Higher order corrections coming as powers of the gauge coupling strength α are reduced since $\alpha = g^2/4\pi\hbar$ is reduced. Since one has $\hbar_s/\hbar = \alpha Q_1Q_2/v_0$, αQ_1Q_2 is effectively replaced with a universal coupling strength v_0. In the case of QCD the paradoxical sounding implication is that α_s would become very small.

3. The binding energy scale $E \propto \alpha_{em}^2 m_e$ of atoms scales as $1/\hbar^2$. This would suggest that a partially dark matter for which protons have a large value of $\hbar(M^4)$ does not interact appreciably with the ordinary light. Multiple coverings defined by G_a and G_b imply fractionization of various quantum numbers as $q \rightarrow q/n_a$ for CP_2 quantum numbers and as $n \rightarrow q/n_b$ for spin. One prediction is N-atom for which the $N = N(G_b)$ sheets of covering of M^4_+ can carry up to N electrons with identical quantum numbers. In this case Planck constant is scaled down by n_a/n_b so that the scale of hydrogen atom binding energy increases by $k^2 = (n_b/n_a)^2$. Mills reports this kind of scalings for $k = 2, 3, ..., 10$ [83]. Dark positive charges are however required to stabilize the electronic charge but the example of atomic nuclei suggests that N-atoms can be stable.

10.2.3 General view about dark matter hierarchy and interactions between relatively dark matters

The identification of the precise criterion characterizing dark matter phase is far from obvious. TGD actually suggests an infinite number of phases which are dark relative to each other in some sense and can transform to each other only via a phase transition which might be called de-coherence or its reversal and which should be also characterized precisely.

A possible solution of the problem comes from the general construction recipe for S-matrix. Fundamental vertices correspond to partonic 2-surfaces representing intersections of incoming and outgoing light-like partonic 3-surfaces.

1. If the characterization of the interaction vertices involves all points of partonic 2-surfaces, they must correspond to definite value of Planck constants and more precisely, definite groups G_a and G_b characterizing dark matter hierarchy. Particles of different G_b phases could not appear in the same vertex since the partons in question would correspond to vacuum extremals. Hence the phase transition changing the particles to each other analogous could not be described by a vertex and would be analogous to a de-coherence.

   The phase transition could occur at the incoming or outgoing particle lines. At space-time level the phase transition would mean essentially a leakage between different sectors of imbedding space and means that partonic 2-surface at leakage point has CP_2 projection reducing to the orbifold point invariant under G or alternatively, its $M^4_\pm$ projection corresponds to the tip of $M^4_\pm$. Relative darkness would certainly mean different groups G_a and G_b. Note that $\hbar(M^4)$ *resp.* $\hbar(CP_2)$ can be same for different groups G_a *resp.* G_b and that only the ratio of $\hbar(M^4)/\hbar(M^4)$ appears in the Kähler action.

2. One can represent a criticism against the idea that relatively dark matters cannot appear at the same interaction vertex. The point is that the construction of S-matrix for transitions transforming partonic 2-surfaces in different number fields involves only the rational (algebraic) points in the intersection of the 2-surfaces in question. This idea applies also to the case in which particles correspond to different values of Planck constant. What is only needed that all the common points correspond to the orbifold point in M^4 or CP_2 degrees of freedom and are thus intermediate between two sectors of imbedding space. In this picture phase transitions would occur through vertices and S-matrix would characterize their probabilities. It seems that this option is the correct one.

If the matrix elements for real-real transitions involve all or at least a circle of the partonic 2-surface as stringy considerations suggest [C3], then one would have clear distinction between quantum phase transitions and ordinary quantum transitions. Note however that one could understand the weakness of the quantal interactions between relatively dark matters solely from the fact that the CP_2 type extremals providing space-time correlates for particle propagators must in this case go through an intermediate state with at most point-like CP_2 projection.

At quantum level the phase transition is possible only at quantum criticality and number theoretic considerations lead to the hypothesis that super-canonical conformal weights for partons reduce to zeros of Riemann Zeta in this situation. In the general case the imaginary parts of conformal weights would be linear combinations $y = \sum_k n_k y_k$ of imaginary parts of zeros $1/2 + iy_k$ of ζ with integer coefficients.

What does one mean with dark variants of elementary particle?

It is not at all clear what one means with the dark variant of elementary particle. In this respect p-adic mass calculations provide a valuable hint. According to the p-adic mass calculations [F4], $k = 113$ characterizes electromagnetic size of u and d quarks, of nucleons, and nuclei. $k = 107$ characterizes the QCD size of hadrons. This is somewhat paradoxical situation since one would expect that quark space-time sheets would be smaller than hadronic space-time sheets.

The simplest resolution of the problem suggested by the basic characteristics of electro-weak symmetry breaking is that $k = 113$ characterizes the size of the electro-magnetic field body of the quark and that the prime characterizing p-adic mass scale labels the em field body of the particle. One can assign mass also the Z^0 body but this would be much smaller as the small scale of neutrino masses suggests. This size scale correspond to a length scale of order 10 μm, which conforms with the expectation that classical Z^0 force is important in biological length scales. The size of Z^0 body of neutrino could relate directly to the chirality selection in living matter. An interesting question is whether the Z^0 field bodies of also other elementary fermions are of this size.

If this picture is correct then dark variant of elementary particle would differ from ordinary only in the sense that its field body would be dark. This conforms with the general working hypothesis is that only field bodies can be dark.

Are particles characterized by different p-adic primes relatively dark?

Each particle is characterized by a collection of p-adic primes corresponding to the partonic 2-surfaces associate with the particle like 3-surface. Number theoretical vision supports the notion of multi-p p-adicity and the idea that elementary particles correspond to infinite primes, integers, or perhaps even rationals [E3, F6]. To infinite primes, integers, and rationals it is possible to associate a finite rational $q = m/n$ by a homomorphism. This would suggest generalization of p-adicity with q-adicity (q-adic topology does not correspond to number field) but this does not seem to be a promising idea.

The crucial observation is that one can decompose the infinite prime, call it P, to finite and infinite parts and distinguish between bosonic and fermionic finite primes of which infinite prime can be said to consist of [A8, E3, H8]. The interpretation is that bosonic and fermionic finite primes in the *infinite* part of P code for p-adic topologies of light-like partonic 3-surfaces associated with a given *real* space-time sheet whereas the primes in the *finite* part of P code for p-adic lightlike partonic 3-surfaces.

This raises two options.

1. Two space-time sheets characterized by rationals having common prime factors can be connected by a $\#_B$ contact and can interact by the exchange of particles characterized by divisors of m or n since in this case partonic 2-surface with same p-adic or effective p-adic topology can be found. This is the only possible interaction between them.

2. The number theoretic vision about the construction of S-matrix however allows to construct S-matrix also in the case that partons belong to different number fields and one ends up with a very elegant description involving only finite number of points of partonic 2-surfaces belonging to their intersection consisting of rational (algebraic points of imbedding space), which by algebraic universality could apply also to diagonal transitions. Also now the interactions mediated between propagators connecting partons with different effective p-adic topologies might be very slow so that this would give rise to relative darkness.

Interpretation of super-canonical conformal weights

Super-canonical conformal weights [B3, C2] are in general complex and define a new kind, perhaps even conserved, quantum number which could be called scaling momentum. There are strong number theoretic reasons to believe that the conformal weights are expressible in terms of zeros of Riemann Zeta.

1. Generalization of the notion of super-canonical conformal weight, p-adicization, and number theoretical universality of Riemann Zeta

It has clear that super-canonical conformal weights could actually depend on the CP_2 of the partonic 2-surface via the formula $\Delta = \zeta^{-1}(z)$, where z is the complex coordinate of the projection of the point of partonic to the geodesic sphere of CP_2 transforming linearly under $U(2) \subset SU(3)$. Note that Δ has infinite number of branches corresponding to the zeros of ζ, and the region of partonic 2-surface given branch generalizes the notion of constant conformal weight. Several branches can be associated with a given partonic 2-surface.

In the most general case Δ could be sum of $\delta M^4_\pm$ and CP_2 parts where M^4_+ part is of same form but now argument corresponds to the standard projective complex coordinate of S^2. Also now orbifold points would be introduced and the interpretation would be in terms of a selection of the quantization direction of angular momentum occurring already at the level of configuration space of 3-surfaces.

Suppose that one accepts the hypothesis of the number theoretical universality of ζ stating that the zeros $s_k = 1/2 + iy_k$ of ζ have the property that the factors $1/(1 + p^{s_k}$ are algebraic numbers for all zeros of zeta [C2, E8]. This is guaranteed if p^{iy_k} is algebraic number for any value of p and y_k. Under this assumption, p-adicization requires that the intersections of partonic 2-surfaces belonging to different number fields must correspond to points which are linear combinations of zeros of ζ with integer coefficients. Zeros of Riemann Zeta in turn correspond to orbifold points which are common to the sectors of the imbedding space characterized by different groups G_b and thus possessing different values of $\hbar(M^4)$ in general.

This means that a collection of super-canonical conformal weights can be associated with the intersection points of real parton surface with a given effective p-adic topology and that each value of conformal weight defines a number theoretic braid. Same applies to the intersections of partonic space-time sheets with different p-adic topologies. The sum of these conformal weights associated with the interaction points can be said to define the net super-canonical conformal weight of the particle. Obviously super-canonical conformal weights do not define quantum number in the standard sense of the word. In particular, the new effective quantum number does not allow an effective violation of Fermi statistics.

What is important that conformal weights associated with the quantum critical partonic 2-surface must correspond to zeros or infinite values of Riemann Zeta for quantum critical points since these points correspond to north and south poles of ζ remaining invariant under G_b.

2. Is conformal confinement needed?

The first guess was that the net value of super-canonical conformal weight is real for physical states. This would give rise to the notion of conformal confinement. It was thought that a particular kind of dark matter would correspond to a conformally confined matter with particles having complex conformal weights such that the net conformal weight is real. The proposed identification of the net super-canonical conformal weight does not support this identification.

It has also become clear that there is no strong physical reason to require the reality of conformal weights at single particle level [C2]: in zero energy ontology the reality of the net conformal weight for zero energy states is predicted in any case since all conserved quantum numbers vanish for them. Furthermore, the conjugation of the conformal weight has interpretation as generalization of phase conjugation of photons in laser physics. This means that time orientation becomes an inherent characteristic of a particle so that positive energy particles propagating in the direction of the geometric future can be distinguished from negative energy particles propagating to the direction of the geometric past.

Hierarchy of infinite primes and dark matter hierarchy

In previous consideration only the simplest infinite primes at the lowest level of hierarchy were considered. Simple infinite primes allow a symmetry changing the sign of the finite part of infinite prime. A possible interpretation in terms of phase conjugation. One can consider also more complex infinite primes at this level and a possible interpretation in terms of bound states of several particles. One can also consider infinite integers and rationals: the interpretation would be as many particle states. Rationals might correspond to states containing particles and antiparticles. At the higher levels of the hierarchy infinite primes of previous take the role of finite primes at the previous level and physically these states correspond to higher level bound states of the particles of the previous level.

Thus TGD predicts an entire hierarchy of dark matters such that the many particle states at previous level become particles at the next level. This hierarchy would provide a concrete physical identification for the hierarchy of infinite primes identifiable in terms of a repeated second quantization of an arithmetic super-symmetric QFT [E3] including both free many-particle states and their bound states. The finite primes about which infinite prime is in a well defined sense a composite of would correspond to the particles in the state forming a unit of dark matter. Particles belonging to different levels of this hierarchy would obviously correspond to different levels of dark matter hierarchy but their interactions must reduce to the fundamental partonic vertices.

10.2.4 How dark matter and visible matter interact?

The hypothesis that the value of $\hbar$ is dynamical, quantized and becomes large at the verge of a transition to a non-perturbative phase in the ordinary sense of the word has fascinating implications. In particular, dark matter, would correspond to a large value of $\hbar$ and could be responsible for the properties of the living matter. In order to test the idea experimentally, a more concrete model for the interaction of ordinary matter and dark matter must be developed and here of course experimental input and the consistency with the earlier quantum model of living matter is of considerable help.

How dark photons transform to ordinary photons?

The transitions of dark atoms naturally correspond to coherent transitions of the entire dark electron BE condensate and thus generate N_{cr} dark photons and behave thus like laser beams. Dark photons do not interact directly with the visible matter. An open question is whether even ordinary laser beams could be identified as beams of dark photons: the multiple covering property at the level of imbedding space and the fact that MEs are possible in all sectors suggests that this is not the case. Note that the transition from dark to ordinary photons implies the scaling of wave length and thus also of coherence length by a factor n_b/n_a.

Dark $\leftrightarrow$ visible transition should have also a space-time correlate. The so called topological light rays or MEs ("massless extremals") represent a crucial deviation of TGD from Maxwell's ED and have all the properties characterizing macroscopic classical coherence. Therefore MEs are excellent candidates for the space-time correlate of BE condensate of dark photons.

MEs carry in general a superposition of harmonics of some basic frequency determined by the length of ME. A natural expectation is that the frequency of classical field corresponds to the generalized de Broglie frequency of dark photon and is thus $\hbar/\hbar_s$ times lower than for ordinary photons. In completely analogous manner de Broglie wave length is scaled up by $k = \hbar_s/\hbar$. Classically the decay of dark photons to visible photons would mean that an oscillation with frequency f inside topological light ray transforms to an oscillation of frequency f/k such that the intensity of the oscillation is scaled up by a factor k. Furthermore, the ME in question could naturally decompose into $1 < N_{cr} \leq 137$

ordinary photons in the case that dark atoms are in question. Of course also MEs could decay to lower level MEs and this has an interpretation in terms of hierarchy of dark matters to be discussed next.

About the criterion for the transition increasing the value of Planck constant

An attractive assumption is that the transition to dark matter phase occurs when the interaction strength satisfies the criticality condition $Q_1Q_2\alpha \simeq 1$. A special case corresponds to self interaction with $Q_1 = Q_2$. This condition applies only to gauge interactions so that particles can be characterized by gauge charges. A more general characterization would be that transition occurs when perturbation theory ceases to converge. The criterion cannot be applied to phenomenological QFT description of strong force in terms of, say, pion exchange.

Some examples are in order to test this view.

1. Transition from perturbative phase in QCD to hadronic phase is the most obvious application. The identification of valence quarks and gluons as dark matter would predict for them QCD size ($k = 107$ space-time sheet) of about electron Compton length. This does not change the QCD cross sections in the lowest order perturbation theory but makes them excellent predictions. It also provides completely new view about how color force determines the nuclear strong force indeed manifesting itself as long ranged harmonic oscillator potential, the long range of which becomes manifest in the case of neutron halos of size of 2.5×10^{-14} m [61]. One can also understand tetraneutron in this framework. This criterion applies also in QCD plasma and explains the formation of liquid like color glass condensate detected in RHIC [35]. A possible interpretation for QCD size would be as a length of the cylindrical magnetic walls defining the magnetic body associated with u and d type valence quarks, nucleons, and nuclei. There is no need to assume that conformal weights are complex in this phase.

2. QCD size of quark must be distinguished from the electromagnetic size of quark associated with $k = 113$ space-time sheets of u and d quarks and assignable to the height of the magnetic body and defining the length scale of join along boundaries contacts feeding quark charges to $k = 113$ space-time sheets.

3. In the case of atomic nuclei the criterion would naturally apply to the electromagnetic interaction energy of two nucleon clusters inside nucleus or to to self energy ($Q^2\alpha_{em} = 1$). Quite generally, the size of the electromagnetic $k = 113$ space-time sheet would increase by a $n_F = 2^k \prod_s F_s$, where F_s are different Fermat primes (the known ones being $3, 5, 17, 257, 2^{16}+1$), in the transition to large $\hbar$ phase. Especially interesting values of n_F seem to be of form $n_F = 2^{k11}$ and possibly also $n_F = 2^{k11} \prod_s F_s$. Similar criterion would apply in the plasma phase. Note that many free energy anomalies involve the formation of cold plasma [G2].

   The criterion would give in the case of single nucleus and plasma $Z \geq 12$ if the charges are within single space-time sheet. This is consistent with cold fusion involving Palladium nuclei [62]. Since u and d quarks have $k = 113$, they both and thus both neutrons and protons could make a transition to large $\hbar$ phase. This is consistent with the selection rules of cold fusion since the production of ^3He involves a phase transition pnp$_d$ $\rightarrow$ pnp and the contraction of p_d to p is made un-probable by the Coulomb wall whereas the transition nnp$_d$ $\rightarrow$nnp producing tritium does not suffer from this restriction.

Strong and weak physics of nuclei would not be affected in the phase transition. Electromagnetic perturbative physics of nuclei would not be affected in the process in the lowest order in $\hbar$ (classical approximation) but the height of the Coulomb wall would be reduced by a factor $1/n_F$ by the increase in the electromagnetic size of the nucleus. Also Pd nuclei could make the transition and Pd nuclei could catalyze the transition in the case the deuterium nuclei.

10.2.5 Could one demonstrate the existence of large Planck constant photons using ordinary camera or even bare eyes?

If ordinary light sources generate also dark photons with same energy but with scaled up wavelength, this might have effects detectable with camera and even with bare eyes. In the following I consider in a rather light-hearted and speculative spirit two possible effects of this kind appearing in both visual perception and in photos. For crackpotters I want to make clear that I love to play with ideas to see whether they work or not, and that I am ready to accept some convincing mundane explanation of these effects and I would be happy to hear about this kind of explanations. I was not able to find any such explanation from Wikipedia using words like camera, digital camera, lense, aberrations [51].

Why light from an intense light source seems to decompose into rays?

If one also assumes that ordinary radiation fields decompose in TGD Universe into topological light rays ("massless extremals", MEs) even stronger predictions follow. If Planck constant equals to $\hbar = q \times \hbar_0$, $q = n_a/n_b$, MEs should possess Z_{n_a} as an exact discrete symmetry group acting as rotations along the direction of propagation for the induced gauge fields inside ME.

The structure of MEs should somewhat realize this symmetry and one possibility is that MEs has a wheel like structure decomposing into radial spokes with angular distance $\Delta\phi = 2\pi/n_a$ related by the symmetries in question. This brings strongly in mind phenomenon which everyone can observe anytime: the light from a bright source decomposes into radial rays as if one were seeing the profile of the light rays emitted in a plane orthogonal to the line connecting eye and the light source. The effect is especially strong if eyes are stirred. It would seem that focusing makes the effect stronger.

Could this apparent decomposition to light rays reflect directly the structure of dark MEs and could one deduce the value of n_a by just counting the number of rays in camera picture, where the phenomenon turned to be also visible? Note that the size of these wheel like MEs would be macroscopic and diffractive effects do not seem to be involved. The simplest assumption is that most of photons giving rise to the wheel like appearance are transformed to ordinary photons before their detection.

The discussions about this led to a little experimentation with camera at the summer cottage of my friend Samppa Pentikäinen, quite a magician in technical affairs. When I mentioned the decomposition of light from an intense light source to rays at the level of visual percept and wondered whether the same occurs also in camera, Samppa decided to take photos with a digital camera directed to Sun. The effect occurred also in this case and might correspond to decomposition to MEs with various values of n_a but with same quantization axis so that the effect is not smoothed out.

What was interesting was the presence of some stronger almost vertical "rays" located symmetrically near the vertical axis of the camera. In old-fashioned cameras the shutter mechanism determining the exposure time is based on the opening of the first shutter followed by closing a second shutter after the exposure time so that every point of sensor receives input for equally long time. The area of the region determining input is bounded by a vertical line. If macroscopic MEs are involved, the contribution of vertical rays is either nothing or all unlike that of other rays and this might somehow explain why their contribution is enhanced. The shutter mechanism is un-necessary in digital cameras since the time for the reset of sensors is what matters. Something in the geometry of the camera or in the reset mechanism must select vertical direction in a preferred position. For instance, the outer "aperture" of the camera had the geometry of a flattened square.

Anomalous diffraction of dark photons

Second prediction is the possibility of diffractive effects in length scales where they should not occur. A good example is the diffraction of light coming from a small aperture of radius d. The diffraction pattern is determined by the Bessel function

$$J_1(x) \ , \ x = kdsin(\theta) \ , \ k = 2\pi/\lambda.$$

There is a strong light spot in the center and light rings around whose radii increase in size as the distance of the screen from the aperture increases. Dark rings correspond to the zeros of $J_1(x)$ at $x = x_n$ and the following scaling law for the nodes holds true

$$sin(\theta_n) = x_n \frac{\lambda}{2\pi d} per.$$

For very small wavelengths the central spot is almost point-like and contains most light intensity.

If photons of visible light correspond to large Planck constant $\hbar = q \times \hbar_0$ transformed to ordinary photons in the detector (say camera film or eye), their wavelength is scaled by q, and one has

$$sin(\theta_n) \rightarrow q \times sin(\theta_n)$$

The size of the diffraction pattern for visible light is scaled up by q.

This effect might make it possible to detect dark photons with energies of visible photons and possibly present in the ordinary light.

1. What is needed is an intense light source and Sun is an excellent candidate in this respect. Dark photon beam is also needed and n dark photons with a given visible wavelength λ could result when dark photon with $\hbar = n \times q \times \hbar_0$ decays to n dark photons with same wavelength but smaller Planck constant $\hbar = q \times \hbar_0$. If this beam enters the camera or eye one has a beam of n dark photons which forms a diffraction pattern producing camera picture in the de-coherence to ordinary photons.

2. In the case of an aperture with a geometry of a circular hole, the first dark ring for ordinary visible photons would be at $sin(\theta) \simeq (\pi/36)\lambda/d$. For a distance of $r = 2$ cm between the sensor plane ("film") and effective circular hole this would mean radius of $R \simeq rsin(\theta) \simeq 1.7$ micrometers for micron wave length. The actual size of spots is of order $R \simeq 1$ mm so that the value of q would be around 1000: $q = 2^{10}$ and $q = 2^{11}$ belong to the favored values for q.

3. One can imagine also an alternative situation. If photons responsible for the spot arrive along single ME, the transversal thickness R of ME is smaller than the radius of hole, say of of order of wavelength, ME itself effectively defines the hole with radius R and the value of $sin(\theta_n)$ does not depend on the value of d for $d > R$. Even ordinary photons arriving along MEs of this kind could give rise to an anomalous diffraction pattern. Note that the transversal thickness of ME need not be fixed however. It however seems that MEs are now macroscopic.

4. A similar effect results as one looks at an intense light source: bright spots appear in the visual field as one closes the eyes. If there is some more mundane explanation (I do not doubt this!), it must apply in both cases and explain also why the spots have precisely defined color rather than being white.

5. The only mention about effects of diffractive aberration effects are colored rings around say disk like objects analogous to colors around shadow of say disk like object. The radii of these diffraction rings in this case scale like wavelengths and distance from the object.

6. Wikipedia contains an article from which one learns that the effect in question is known as lens flares [52]. The article states that flares typically manifest as several starbursts, circles, and rings across the picture and result in internal reflection and scattering from material inhomogenities in lens (such as multiple surfaces). The shape of the flares also depends on the shape of aperture. These features conform at least qualitatively with what one would expect from a diffraction if Planck constant is large enough for photons with energy of visible photon.

   The article [53] defines flares in more restrictive manner: lense flares result when *non-image* forming light enters the lens and subsequently hits the camera's film or digital sensor and produces typically polygonal shape with sides which depend on the shape of lense diaphgram. The identification as a flare applies also to the apparent decomposition to rays and this dependence indeed fits with the observations.

The experimentation of Samppa using digital camera demonstrated the appearance of colored spots in the pictures. If I have understood correctly, the sensors defining the pixels of the picture are in the focal plane and the diffraction for large Planck constant might explain the phenomenon. Since I did not have the idea about diffractive mechanism in mind, I did not check whether fainter colored rings might surround the bright spot.

1. In any case, the readily testable prediction is that zooming to bright light source by reducing the size of the aperture should increase the size and number of the colored spots. As a matter fact, experimentation demonstrated that focusing brought in large number of these spots but we did not check whether the size was increased.

2. Standard explanation predicts that the bright spots are present also with weaker illumination but with so weak intensity that they are not detected by eye. The positions of spots should also depend only on the illumination and camera. The explanation in terms of beams of large Planck constant photons predicts this if the flux of dark photons from any light source is constant.

10.2.6 Dark matter and exotic color and electro-weak interactions

The presence of classical electro-weak and color gauge fields in all length scales is an unavoidable prediction of TGD and the interpretation in terms of hierarchy of dark matters in some sense is also more or less unavoidable.

Does dark matter provide a correct interpretation of long ranged classical electro-weak gauge fields?

For two decades one of the basic interpretational challenges of TGD has been to understand how the un-avoidable presence of long range classical electro-weak gauge fields can be consistent with the small parity breaking effects in atomic and nuclear length scales. Also classical color gauge fields are predicted, and I have proposed that color qualia correspond to increments of color quantum numbers [K3]. The proposed model for screening cannot banish the unpleasant feeling that the screening cannot be complete enough to eliminate large parity breaking effects in atomic length scales so that one one must keep mind open for alternatives.

p-Adic length scale hypothesis suggests the possibility that both electro-weak gauge bosons and gluons can appear as effectively massless particles in several length scales and there indeed exists evidence that neutrinos appear in several scaled variants [24] (for TGD based model see [F3]).

This inspires the working hypothesis that long range classical electro-weak gauge and gluon fields are correlates for light or massless dark electro-weak gauge bosons and gluons.

1. In this kind of scenario ordinary quarks and leptons could be essentially identical with their standard counterparts with electro-weak charges screened in electro-weak length scale so that the problems related to the smallness of atomic parity breaking would be trivially resolved.

2. In condensed matter blobs of size larger than neutrino Compton length (about 5 μm if $k = 169$ determines the p-adic length scale of condensed matter neutrinos) the situation could be different. Also the presence of dark matter phases with sizes and neutrino Compton lengths corresponding to the length scales $L(k)$, $k = 151, 157, 163, 167$ in the range 10 nm-2.5 μm are suggested by the number theoretic considerations (these values of k correspond to so called Gaussian Mersennes [K2]). Only a fraction of the condensed matter consisting of regions of size $L(k)$ need to be in the dark phase.

3. Dark quarks and leptons would have masses essentially identical to their standard model counterparts. Only the electro-weak boson masses which are determined by a different mechanism than the dominating contribution to fermion masses [F2, F3] would be small or vanishing.

4. The large parity breaking effects in living matter would be due to the presence of dark nuclei and leptons. Later the idea that super-fluidity corresponds to Z^0 super-conductivity will be discussed: it might be that also super-fluid phase corresponds to dark neutron phase.

The basic prediction of TGD based model of dark matter as a phase with a large value of Planck constant is the scaling up of various quantal length and time scales. A simple quantitative model for condensed matter with large value of $\hbar$ predicts that $\hbar$ is by a factor $\sim 2^{11}$ determined by the ratio of CP_2 length to Planck length larger than in ordinary phase meaning that the size of dark neutrons would be of order atomic size. In this kind of situation single order parameter would characterize the behavior of dark neutrinos and neutrons and the proposed model could apply as such also in this case.

Dark photon many particle states behave like laser beams decaying to ordinary photons by de-coherence meaning a transformation of dark photons to ordinary ones. Also dark electro-weak bosons and gluons would be massless or have small masses determined by the p-adic length scale in question. The decay products of dark electro-weak gauge bosons would be ordinary electro-weak bosons decaying rapidly via virtual electro-weak gauge boson states to ordinary leptons. Topological light rays ("massless extremals") for which all classical gauge fields are massless are natural space-time correlates for the dark boson laser beams. Obviously this means that the basic difference between the chemistries of living and non-living matter would be the absence of electro-weak symmetry breaking in living matter (which does not mean that elementary fermions would be massless).

In super-canonical conformal weights are non-vanishing and can vary then Fermi statistics allows neutrinos to have same energy if their conformal weights are different so that a kind "fermionic Bose-Einstein condensate" would be in question. If both nuclear neutrons and neutrinos are in dark phase, it is possible to achieve a rather complete local cancellation of Z^0 charge density.

The model for neutrino screening was developed years before the ideas about the identification of the dark matter emerged. The generalization of the discussion to the case of dark matter option should be rather trivial and is left to the reader as well as generalization of the discussion of the effects of long range Z^0 force on bio-chemistry.

Criterion for the presence of exotic electro-weak bosons and gluons

Classical gauge fields directly are space-time correlates of quantum states. The gauge fields associated with massless extremals ("topological light rays") decompose to free part and a part having non-vanishing divergence giving rise to a light-like Abelian gauge current. Free part would correspond to Bose-Einstein condensates and current would define a coherent state of dark photons.

The dimension D of the CP_2 projection of the space-time sheet serves as a criterion for the presence of long ranged classical electro-weak and gluon fields. D also classifies the (possibly asymptotic) solutions of field equations [D1].

1. For $D = 2$ induced gauge fields are Abelian and induced Kähler form vanishes for vacuum extremals: in this case classical em and Z^0 fields are proportional to each other. The non-vanishing Kähler field implies that induced gluon fields are non-vanishing in general. This raises the question whether long ranged color fields and by quantum classical correspondence also long ranged QCD accompany non-vacuum extremals in all length scales. This makes one wonder whether color confinement is possible at all and whether scaled down variants of QCD appear in all length scales.

   The possibility to add constants to color Hamiltonians appearing in the expression of the classical color gauge fields allows to have vanishing color charges in the case of an arbitrary space-time sheet. The requirement that color quantum numbers of the generator vanish allows to add the constant only to the Hamiltonians of color hyper charge and isospin so that for $D = 2$ extremals color charges can be made vanishing. This might allow to understand how color confinement is consistent with long ranged induced Kähler field.

2. For $D \geq 3$ all classical long ranged electro-weak fields and non-Abelian color fields are present. This condition is satisfied when electric and magnetic fields are not orthogonal and the instanton density $A \wedge J$ for induced Kähler form is non-vanishing. The rather strong conclusion is that in length scales in which exotic electro-weak bosons are not present, one has $D = 2$ and gauge fields are Abelian and correspond trivially to fixed points of renormalization group realized as a hydrodynamic flow at space-time sheets [C4].

Quantum classical correspondence suggests the existence of electro-weak gauge bosons with mass scale determined by the size of the space-time sheets carrying classical long range electro-weak fields. This would mean the existence of new kind of gauge bosons.

The obvious objection is that the existence of these gauge bosons would be reflected in the decay widths of intermediate gauge bosons. The remedy of the problem is based on the notion of space-time democracy suggested strongly by the fact that the interactions between space-time sheets possessing different p-adic topologies proceed with very slow rates simply because the number of common rational (algebraic) points of partonic 2-surfaces appearing in the vertex is small.

For light exotic electro-weak bosons also the corresponding leptons and quarks would possess a large weak space-time sheet but lack the ordinary weak partonic 2-surface so that there would be no direct coupling to electro-weak gauge bosons. These space-time sheets are dark in weak sense but need not have a large value of $\hbar$. This picture implies the notion of partial darkness since any space-time sheets with different ordinary of Gaussian primes are dark with respect to each other.

Do Gaussian Mersennes define a hierarchy of dark electro-weak physics?

Gaussian Mersennes are defined as Gaussian primes of form $g_n = (1+i)^n - 1$, where n must be prime. They have norm squared $g\overline{g} = 2^n - 1$. The list of the first Gaussian Mersennes corresponds to the following values of n.

2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 113, 151, 157, 163, 167, 239, 241, 283, 353, 367, 379, 457, 997, 1367, 3041, 10141, 14699, 27529, 49207, 77291, 85237, 106693, 160423 and 203789.

The Gaussian primes $k = 113, 151, 157, 163, 167$ correspond to length scales which are of most obvious interest but in TGD framework one cannot exclude the twin prime $239, 241$ corresponds to length scales $L(k) \simeq 160$ km and 320 km. Also larger primes could be of relevant for bio-systems and consciousness. Also the secondary and higher length scales associated with $k < 113$ could be of importance and their are several length scales of this kind in the range of biologically interesting length scales. Physics and biology inspired considerations suggests that particular Gaussian primes correspond to a particular kind of exotic matter, possibly also to large $\hbar$ phase.

$k = 113$ corresponds to the electromagnetic length scale of u and d quarks and nuclear p-adic length scale. For dark matter these length scales are scaled up by a factor $\sim 2^{11}n$, where n is an integer. For $k = 113$ one obtains atomic length scale .8 A for $n = 1$. $k = 151, 153, 163, 167$ correspond to biologically important p-adic length scales varying in the range 10 nm-2.5 μm with the scaled up length scales varying in the range 2 μm- 5 mm.

On basis of biological considerations (large parity breaking in living matter) there is a temptation to assign to these length scales a scaled down copy of electro-weak physics and perhaps also of color physics. The mechanism giving rise to these states would be a phase transition transforming the ordinary $k = 89$ Mersenne of weak space-time sheets to a Gaussian Mersenne and thus increasing its size dramatically.

If given space-time sheet couples considerably only to space-time sheets characterized by same prime or Gaussian prime, the bosons of these physics do not couple directly to ordinary particles, and one avoids consistency problems due to the presence of new light particles (consider only the decay widths of intermediate gauge bosons [F5]) even in the case that the loss of asymptotic freedom is not assumed.

A question arises about the interpretation of structures of the predicted size. The strong interaction size of u and d quarks, hadrons, and nuclei is smaller than $L(k = 113) \simeq 2 \times 10^{-4}$ m for even heaviest nuclei if one accepts the formula $R \sim A^{1/3} \times 1.5 \times 10^{-15}$ m. A natural interpretation for this length scale would be as the size of the field body/magnetic body of system defined by its topologically quantized gauge fields/magnetic parts of gauge fields. The (possibly dark) p-adic length scale characterizes also the lengths of join along boundaries bonds feeding gauge fluxes from elementary particle to the space-time sheet in question. The delocalization due these join along boundaries bonds in p-adic length scale in question would determine the scale of the contribution to the mass squared of the system as predicted by p-adic thermodynamics.

10.2.7 Anti-matter and dark matter

The usual view about matter anti-matter asymmetry is that during early cosmology matter-antimatter asymmetry characterized by the relative density difference of order $r = 10^{-9}$ was somehow generated and that the observed matter corresponds to what remained in the annihilation of quarks and leptons to bosons. A possible mechanism inducing the CP asymmetry is based on the CP breaking phase of CKM matrix.

The TGD based view about energy [D4, D6] forces the conclusion that all conserved quantum numbers including the conserved inertial energy have vanishing densities in cosmological length scales. Therefore fermion numbers associated with matter and antimatter must compensate each other. Therefore the standard option is definitely excluded in TGD framework.

An early TGD based scenario explains matter antimatter asymmetry by assuming that antimatter is in vapor phase. This requires that matter and antimatter have slightly different topological evaporation rates with the relative difference of rates characterized by the parameter r. A more general scenario assumes that matter and antimatter reside at different space-time sheets. The reader can easily guess the next step. The strict non-observability of antimatter finds an elegant explanation if anti-matter is dark matter.

10.3 Dark variants of nuclear physics

The book metaphor for the extended imbedding space can be utilized as a guideline as one tries to imagine various exotic phases of matter. Atomic nuclei are assumed to be ordinary (in the sense of nuclear string model!) and only field bodies can be dark. They are analogous to n-sheeted Riemann surfaces. Nuclei can be visualized as residing at the "standard" pages of the book and dark color-/weak-/em- bonds are at different pages with different p-adic length scale or having different Planck constant $\hbar_{eff}$. This would give two hierarchies of nuclei with increasing size.

10.3.1 Constraints from the nuclear string model

In the case of exotic nuclei nuclear string model [F9] is a safe starting point. In this model nucleons are connected by color flux tubes having exotic light fermion and antifermion at their ends. Whether fermion is quark or colored excitation of lepton remains open question at this stage. The mass of the exotic fermion is much smaller than 1 MeV (p-adic temperature $T = 1/n < 1$). This model predicts large number of exotic states since color bonds, which can be regarded as colored pions, can have em charges (1,-1,0). In particular, neutral variant of deuterium is predicted and this leads to a model of cold fusion explaining its basic selection rules. The earlier model for cold fusion discussed in [F8], which served as a constraint in the earlier speculations, is not so simple than the model of [F9] .

What is important that the model requires that weak bosons for which Compton length is of order atomic size are involved. Weak bosons would behave as massless particles below the Compton and the rates for the exchanges of weak bosons would be high in the length scales considered. Weak bosons would correspond to scaled up variants of the ordinary weak bosons: scaling could be p-adic in which mass scale is reduced and weak interaction rates even above Compton length would be scaled up as $1/M_W^4$. The scaling could result also from the scaling of Planck constant in which case masses of weak bosons nor weak interaction rates in the lowest order would not be affected. If only dark scaling is involved, weak interactions would be still extremely weak above dark Compton length of weak bosons. Of course, both scalings can be imagined.

The scale of the color binding energy is $E_s = .2$ MeV for ordinary ^4He strings [D5]. $k = 151, 157, 163, 167$ define Gaussian Mersennes $G_k^{=}(1+i)^k - 1$ and excellent candidates for biologically important p-adic length scales. If M_{127} is scaled up to Gaussian Mersenne G_{167}, one obtains cell-nucleus sized (5 μm) exotic nuclei and the unit of color binding energy is still .2 eV. For p-adic length scale of order 100 μm (size of large neuron) the energy scale is still around thermal energy at room temperature.

In the case of dark color bonds it is not quite clear how the unit E_s of the color binding energy scales. If color Coulombic energy is in question, one expects $1/\hbar^2$ scaling. Rather remarkably, this scaling predicts that the unit for the energy of $A < 4$ color bond scales down to .5 eV which is the energy of hydrogen bond so that hydrogen bonds, and also other molecular bonds, might involve color bonds between proton and oxygen.

10.3.2 Constraints from the anomalous behavior of water

$H_{1.5}O$ behavior of water with respect to neutron and electron scattering is observed in attosecond time scale which corresponds to 3 Angstrom length scale, defining an excellent candidate for the size scale of exotic nuclei and Compton length of exotic weak interactions.

What happens to the invisible protons?

A possible explanation for the findings is that one fourth of protons forms neutral multi-proton states connected by possibly negatively charged color bonds of length differing sufficiently from the length of ordinary O-H bond. Although the protons are ordinary, neutron diffraction reflecting the crystal like order of water in atomic length scales would not see these poly-proton super-nuclei if they form separate closed strings.

1. For the ordinary nuclei the p-adic length scale associated with the color bonds between ^4He corresponds to M_{127}, and one can imagine exotic nuclear strings obtained by connecting two ordinary nuclei with color bonds. If second exotic nucleus is neutral (the model of cold fusion assumes that D nucleus is neutral) this could work since the Coulomb wall is absent. If the

exotic nuclei have opposite em charges, the situation improves further. New super-dense phases of condensed matter would be predicted.

If one fourth of hydrogen nuclei of water combine to form possibly neutral nuclear strings with average distance of nuclei of order $L(127)$, they are not visible in diffraction at atomic length scale because the natural length scale is shortened by a factor of order 32 but could be revealed in neutron diffraction at higher momentum exchanges. The transition between this kind of phase and ordinary nuclei would be rather dramatic event and the exchanges of exotic weak bosons with Compton lengths of order atomic size induce the formation of this kind of nuclei (this exchange is assumed in the model of cold fusion).

2. If dark color magnetic bonds are allowed, a natural distance between the building blocks of super-nuclei is given by the size scale of the color magnetic body. The size scales of dark color magnetic bodies associated with nuclear strings consisting of ^4He, ^4He and $A \leq 3$ color magnetic bodies would be $L(127+22=149) = 5$ nm, $L(118+22=140) = 2.2$ Angstrom, and $L(116+22=138) = 1.1$ Angstrom. The first scale equals to the thickness of lipid layer of cell membrane which suggests a direct connection with biology. The latter two scales correspond to molecular length scales and it is not clear why the protons of dark nuclear strings of this kind would not be observed in electron and neutron scattering. This would leave only nuclear strings formed from ^4He nuclei into consideration.

   The crucial parameter is the the unit E_s of the color binding energy. Since this parameter should correspond to color Coulombic potential it could transform like the binding energy of hydrogen atom and therefore scale as $1/hbar^2$. This would mean that $E_s = 2.2$ MeV deduced from the deuteron binding energy would scale down to .5 eV for $n = 2^{11}$. This is the energy of hydrogen bond so that hydrogen bonds might have interpretation as color bonds between nuclei. Nuclear color bonds could serve as prerequisites for the formation of bond at level of valence electrons also in the case of other bonds.

   For ^4He color bonds one would obtain $E_s = .05$ for so that the invisible protons could also belong to dark ^4He nuclear strings. The predicted $E_s = .05$ eV is very near to the energy associated with the membrane potential at the threshold for the generation of nerve pulse.

3. The third option is that color bonds have $n = 3 \times 2^{11}$ instead of $n = 2^{11}$. The color bond would be 9 times longer and probably also the distance between color bonded protons would be longer. In this case one would have $E_s = .056$ eV which is also near to the value of action potential.

The transition between the dark and ordinary nuclei would be favored by the minimization of Coulomb energy and energy differences would be small because of darkness. The transitions in which ordinary proton becomes dark and fuses to super-nuclear string or vice versa could be the basic control mechanism of bio-catalysis. Metabolic energy quantum .5 eV should relate to this transition.

Magic nuclei could have fractally scaled up variants in molecular length scale and tedrahedral and icosahedral water clusters could correspond to $A = 8$ and $A = 20$ magic nuclei with color bonds connecting nucleons belonging to different dark nuclei.

About the identification of the exotic weak physics?

The model of cold fusion requires exotic weak physics with the range of weak interaction of order atomic radius.

1. One can consider the possibility of $k = 113$ dark weak physics with $n = 2^{11}$. Weak Compton length for $k = 113$ dark weak bosons would be about 1.5 Angstrom. Above $L(135)$ weak bosons would have the mass scale $2^{-12}m_W \sim 25$ MeV and weak rates would be scaled up by 2^{48}. In [F9] it is proposed that isospin dependent strong force is nothing but a scaled variant of electro-weak force appearing as several fractally scaled up variants. Bohr radius would represent a critical transition length scale and exotic weak force could have dramatic implications for the behavior of the condensed matter in high pressures when exotic weak force would become visible.

2. Also exotic weak bosons corresponding to the ordinary value of Planck constant and to the atomic length scale $k = 137$ could be present. In this case the weak mass scale would be $2^{-24}m_W \sim 6$ eV and Compton length would be 3 Angstroms. New eV scale weak physics possibly relevant

for molecular physics would be predicted. The transitions between nuclear strings and ordinary nuclei would involve nuclear energies so that this option is not favored as an explanation of $H_{1.5}O$ anomaly.

To sum up, it would seem that the variant of ordinary nuclear physics obtained by making color bonds and weak bonds dark is the most promising approach to the $H_{1.5}O$ anomaly and cold fusion. Exotic weak bosons with Compton wave length of atomic size and the most natural assumption is that they are dark $k = 113$ weak bosons. One variant of exotic atoms is as atoms for which electromagnetic interaction between ordinary nuclei and ordinary electrons is mediated along dark topological field quanta.

10.3.3 Exotic chemistries and electromagnetic nuclear darkness

The extremely hostile and highly un-intellectual attitude of skeptics stimulates fear in anyone possessing amygdala, and I am not an exception. Therefore it was a very pleasant surprise to receive an email telling about an article published in April 16, 2005 issue of New Scientist [45]. The article gives a popular summary about the work of the research group of Walter Knight with Na atom clusters [46] and of the research group of Welford Castleman with Al atom clusters [47].

The article tells that during last two decades a growing evidence for a new kind of chemistry have been emerging. Groups of atoms seem to be able to mimic the chemical behavior of single atom. For instance, clusters of 8, 20, 40, 58 or 92 sodium atoms mimic the behavior of noble gas atoms [46]. By using oxygen to strip away electrons one by one from clusters of Al atoms it is possible to make the cluster to mimic entire series of atoms [47]. For aluminium cluster-ions made of 13, 23 and 37 atoms plus an extra electron are chemically inert.

One can imagine two explanations for the findings.

1. The nuclei are dark in the sense that the sizes of nuclear space-time sheets are scaled up implying the smoothing out of the nuclear charge.

2. Only electrons are dark in the sense of having scaled up Compton lengths so that the size of multi-electron bound states is not smaller than electron Compton length and electrons "see" multi-nuclear charge distribution.

If darkness and Compton length is assigned with the em field body, it becomes a property of interaction, and it seems impossible to distinguish between options 1) and 2).

What one means with dark nuclei and electrons?

Can the idea about dark nuclei and electrons be consistent with the minimalist picture in which only field bodies are dark? Doesn't the darkness of nucleus or electron mean that also multi-electron states with n electrons are possible?

The proper re-interpretation of the notion Compton length would allow a consistency with the minimalist scenario. If the p-adic prime labelling the particle actually labels its electromagnetic body as p-adic mass calculations for quark masses encourage to believe, Compton length corresponds to the size scale of the electromagnetic field body and the models discussed below would be consistent with the minimal scenario. Electrons indeed "see" the external charge distribution by their electromagnetic field body and field body also carries this distribution since CP_2 extremals do not carry it. One could also defend this interpretation by saying that electrons is operationally only what can be observed about it through various interactions and therefore Compton length (various Compton length like parameters) must be assigned with its field body (bodies).

Also maximal quantum criticality implies that darkness is restricted to field bodies but does not exclude the possibility that elementary particle like structures can possess non-minimal quantum criticality and thus possess multi-sheeted character.

Option I: nuclei are electromagnetically dark

The general vision about nuclear dark matter suggests that the system consists of super-nuclei analogous to ordinary nuclei such that electrons are ordinary and do not screen the Coulomb potentials of atomic nuclei.

The simplest possibility is that the electromagnetic field bodies of nuclei or quarks become dark implying delocalization of nuclear charge. The valence electrons would form a kind of mini-conductor with electrons delocalized in the volume of the cluster. The electronic analog of the nuclear shell model predicts that full electron shells define stable configurations analogous to magic nuclei. The model explains the numbers of atoms in chemically inert Al and Ca clusters and generalizes the notion of valence to the level of cluster so that the cluster would behave like single super-atom.

The electromagnetic $k = 113$ space-time sheets (em field bodies) of quarks could have scaled up size $nL(113)/v_0 = n2^{11} \times 2 \times 10^{-14}$ m, $n = 1, 2, 3$. One would have atomic size 1 Angstrom for $n = 1$. A suggestive interpretation is that the electric charge of nuclei or valence quarks assignable to their field bodies is delocalized quantum mechanically to atomic length scale. Electrons would in a good approximation experience quantum mechanically the nuclear charges as a constant background, jellium, whose effect is indeed modellable using harmonic oscillator potential.

One can test the proposed criterion for the phase transition to darkness. The unscreened electromagnetic interaction energy between a block of partially ionized nuclei with a net em charge Z with Z electrons would define the relevant parameter as $r \equiv Z^2\alpha$. For the total charge $Z \geq 12$ the condition $r \geq 1$ is satisfied. For a full shell with 8 electrons this condition is not satisfied.

Option II: Electrons are electro-magnetically dark

Since the energy spectrum of harmonic oscillator potential is invariant under the scaling of $\hbar$ accompanied by the opposite scaling of the oscillator frequency ω, one must consider also the em bodies of electrons are in large $\hbar$ phase (one can of course ask whether they could be observed in this phase!). The rule would be that the size of the bound states is larger than the scaled up electron Compton length.

The Compton wavelength of electrons would be scaled up by a factor $n2^{11}$, $n = 1, 3, 5$, where n is product of different Fermat primes, and correspond to $\sim n \times 5$ nm. The atomic cluster of this size would contain roughly $n \times 10^4(a_0/a)^3$ atoms where a is atomic volume and $a_0 = 1$ Angstrom is the natural unit.

The shell model of nucleus is in TGD framework a phenomenological description justified by nuclear string model with string tension responsible for the oscillator potential. This leads to ask whether the electrons of jellium actually form analogs of nuclear strings with electrons connected by color bonds.

10.4 Has dark matter been observed?

In this section two examples about anomalies perhaps having interpretation in terms of quantized Planck constant are discussed. The first anomaly belongs to the realm of particle physics and hence does not quite fit the title of the chapter. Second anomaly relates to nuclear physics.

10.4.1 Optical rotation of a laser beam in a magnetic field

The group of G. Cantatore has reported an optical rotation of a laser beam in a magnetic field [87]. The experimental arrangement involves a magnetic field of strength $B = 5$ Tesla. Laser beam travels 22000 times forth and back in a direction orthogonal to the magnetic field travelling 1 m during each pass through the magnet. The wavelength of the laser light is 1064 nm. A rotation of $(3.9 \pm .5) \times 10^{-12}$ rad/pass is observed.

A possible interpretation for the rotation would be that the component of photon having polarization parallel to the magnetic field mixes with QCD axion, one of the many candidates for dark matter. The mass of the axion would be about 1 meV. Mixing would imply a reduction of the corresponding polarization component and thus in the generic case induce a rotation of the polarization direction. Note that the laser beam could partially transform to axions, travel through a non-transparent wall, and appear again as ordinary photons.

The disturbing finding is that the rate for the rotation is by a factor 2.8×10^4 higher than predicted. This would have catastrophic astrophysical implications since stars would rapidly lose their energy via axion radiation.

TGD predicts the existence of a hierarchy of QCD type physics based on the predicted hierarchy of scaled up variants of quarks and also those of color excited leptons. The fact that these states are not seen in the decay widths of intermediate gauge bosons can be understood if the particles in question are dark matter with non-standard value of Planck constant and hence residing at different page of the book like structure formed by the imbedding space. I have discussed in detail the general model in case of leptohadrons consisting of colored excitation of ordinary lepton and explaining quite an impressive bundle of anomalies [F7]. Since leptopion has quantum numbers of axion and similar couplings, it is natural to propose that the claimed axion like particle -if it indeed exists- is a pion like state consisting either exotic light quarks or leptons.

Could the optical rotation be caused by a pion of a scaled down copy of ordinary QCD

The motivation for introducing axion was the large CP breaking predicted by the standard QCD. No experimental evidence has been found has been found for this breaking. The idea is to introduce a new broken U(1) gauge symmetry such that is arranged to cancel the CP violating terms predicted by QCD. Because axions interact very weakly with the ordinary matter they have been also identified as candidates for dark matter particles.

In TGD framework there is special reason to expect large CP violation analogous to that in QCD although one cannot completely exclude it. Axions are however definitely excluded. TGD predicts a hierarchy of scaled up variants of QCD and entire standard model plus their dark variants corresponding to some preferred p-adic length scales, and these scaled up variants play a key role in TGD based view about nuclear strong force [F8, F9], in the explanation of the anomalous production of e^+e^- pairs in heavy nucleus collisions near Coulomb wall [F7], high T_c superconductivity [J1, J2, J3] and also in the TGD based model of living matter [M3]. Therefore a natural question is whether the particle in question could be a pion of some scaled down variant of QCD having similar coupling to electromagnetic field. Also dark variants of this pion could be considered.

What raises optimism is that the Compton length of the scaled down quarks is of the same order as cyclotron wavelength of electron in the magnetic field in question. For the ordinary value of Planck constant this option however predicts quite too high mixing rate. This suggests that dark matter has been indeed observed in the sense that the pion corresponds to a large value of Planck constant. Here the encouraging observation is that the ratio λ_c/λ of wavelength of cyclotron photon and laser photon is $n = 2^{11}$, which corresponds to the lowest level of the biological dark matter hierarchy with levels characterized the value $\hbar(M^4_\pm) = 2^{11k}\hbar_0$, $k = 1, 2, ...$

The most plausible model is following.

1. Suppose that the photon transforms first to a dark cyclotron photon associated with electron at the lowest $n = 2^{11}$ level of the biological dark matter hierarchy. Suppose that the coupling of laser photon to dark photon can be modelled as a coefficient of the usual amplitude apart from a numerical factor of order one equal to $\alpha_{em}(n) \propto 1/n$.

2. Suppose that the coupling $g_{\pi NN}$ for the scaled down hadrons is proportional to $\alpha_s^4(n) \propto 1/n^4$ as suggested by a simple model for what happens for the nucleon and pion at quark level in the emission of pion.

Under these assumptions one can understand why only an exotic pion with mass of 1 meV couples to laser photons with wavelength $\lambda = 1$ μm in magnetic field $B = 5$ Tesla. The general prediction is that λ_c/λ must correspond to preferred values of n characterizing Fermat polygons constructible using only ruler and compass, and that the rate for the rotation of polarization depends on photon frequency and magnetic field strength in a manner not explained by the model based on the photon-axion mixing.

Scaled up variant of PCAC

Consider first briefly the scaled up variant of partially conserved axial current hypothesis (PCAC).

1. The mass of the particle would be around 1 meV. If a scaled down ordinary pion is in question, the mass ratio $m_\pi/m_A \simeq 140 \times 10^9 \sim 2^{37}$ suggests that the space-time sheet associated with gluons of this QCD is related by p-adic scale in question corresponds to $k = 107 + 2 \times 37 = 181$, which is prime and corresponds to p-adic length scale $L(181) = .327$ mm. The predicted pion mass from exact scaling would be 1.1 meV. This pion does not couple to ordinary quarks and therefore this coupling does not affect astrophysics at the level of visible matter. The parameter $\Lambda_{QCD,181}$ would be obtained by the scaling $\Lambda_{QCD}(181) = 2^{-37}\Lambda_{QCD}(107)$.

2. The interaction of pion and photons is fixed completely by the anomaly of axial current [19]

$$\langle 0|A^j_\mu(x)|\pi^k\rangle = i\delta^{jk}p_\mu f_\pi exp^{-ip\cdot x} \quad . \tag{10.4.1}$$

Here $f_\pi \simeq 93$ MeV characterizes the matrix element of axial current between vacuum and single-pion state and thus the decay rate of pion.

The form of the interaction is exactly the same as in the case of axion and given by the interaction Lagrangian

$$\begin{aligned} L &= k_{em}\pi F\wedge F \quad , \\ k_{em} &= \frac{e^2}{32\pi^2 f_\pi} \quad . \end{aligned} \tag{10.4.1}$$

The detailed arguments leading to the expression for k_{em} can be found in [19].

3. Axial current anomaly implies that the divergence of the axial current is proportional to the pion field. Writing the most general form for the matrix element of the axial current between nucleon states, this gives a relationship between pion-nucleon coupling $g_{\pi NN}$ and pion decay rate f_π:

$$\begin{aligned} \frac{g_A(0)}{f_\pi} &= \frac{g_{\pi NN}}{m_N} \quad , \\ g_A(0) &= \frac{G_A}{G_V} \quad . \end{aligned} \tag{10.4.1}$$

One has $m_N = .94$ GeV, $g^2_{\pi NN}/4\pi = 14.6$. $g_A(0) = G_A/G_V = 1.22$ is the ratio of axial and vectorial weak couplings for the fermion at zero momentum transfer. Te relationship follows from the conservation of axial current between nucleon and states that the coefficient of the term $q^\mu\overline{u}\gamma_5 u$ in the axial current matrix element between two nucleon states has a pole corresponding to the exchange of approximately massless pion. This formula generalizes trivially for the scaled up variants of QCD. The photon-axion mixing rate is proportional to $1/m_N$, where m_N is the mass of the exotic nucleon.

Comparison with the axion model

Let us compare the predictions of this model with the predictions of the axion model.

1. Axion-photon interaction Lagrangian has exactly the same form as $\pi^0\gamma\gamma$ interaction Lagrangian. The parameter f_a for the axion satisfies the condition

$$f_a \simeq \frac{\Lambda^2_{QCD}}{m_a} \quad . \tag{10.4.2}$$

Here one has $m_a \simeq 1$ meV and $\Lambda_{QCD} \simeq .2$ GeV.

2. From the fact that the rate is by a factor $r = 2.8\times 10^4$ higher than the rate expected for QCD axion with mass $m_a \simeq 1$ meV one can deduce that the mass scale of the exotic u and d quarks. The condition that the two decay rates differ by the factor $R = 2.8\times 10^4$ reads as

$$\frac{g_{A,e}(0)}{g_{\pi_e N_e N_e}} \times m_{N_e} \quad = \quad \frac{1}{\sqrt{R}} \frac{\Lambda_{QCD}^2}{m_a} \quad , \tag{10.4.3}$$

where the right hand side refers to the exotic nucleon and pion. The parameter $g_{A,e}$ can be assumed to be near to one.

Suppose first that exotic pion is not dark and that $g_{\pi_e N_e N_e} = g_{\pi NN}$ holds true. The small mass of axion implies that the right hand side is about 2.4×10^5 GeV so that m_{N_e} should be by a factor about $3.2 \times 10^6 \sim 2^{22}$ larger than m_N and corresponding quarks would roughly correspond to $k \sim 73$. This is in in contradiction with what one would expect. Basically the large decay constant of exotic pion $\propto 1/m_N$ is in conflict with the very small decay constant of axion proportional to $\propto m_a/\Lambda^2$.

Consider now various options which could cure the problem.

Option I: The first dark matter option option is that one has $\hbar = n\hbar_0$ and $g_{\pi_e N_e N_e}$ is by a factor $1/n^k \simeq 2^{-60} \simeq 10^{-18}$ smaller than $g_{\pi NN}$. The factor comes from the overall reduction factor $3.2 \times 10^6 \sim 2^{22}$ of $1/f_\pi$ and from the fact that nucleon mass scale should be reduced roughly by a factor $\sim 2^{-37}$ (just like pion mass scale).

This could be understood if the pion exchange involves the emission of k virtual gluons implying $g_{\pi_e N_e N_e} \propto \alpha_s^k \propto 1/n^k$. One virtual gluon would decay to pion and two additional exchanges are necessary since all three valence quarks of nucleon must interact: hence $k = 3$ is the minimal option. One can also argue that the quarks resulting in the decay of virtual gluon must exchange at least one gluon to become a pion. This would give $1/n^4$ behavior giving the estimate $n = 2^{15}$ assuming $g_{\pi_e N_e N_e} = g\alpha_s^4$, with g having no dependence on α_s. The higher powers of α_s in the expansion of $g_{\pi NN}$ are important for ordinary hadrons physics but small for its dark variants so that the estimate is just a rough order of magnitude estimate if even that.

Option II: One can consider also the possibility that the space-time sheet of the magnetic field is dark so that the disappearance of photons from the laser beam involves a transformation to a dark photon followed by a transformation to a dark neutral pion in the magnetic field used. This would mean that the amplitude for the process would involve an additional dimensionless factor $g_{\gamma\gamma_d} \propto \alpha_{em} \propto 1/\hbar$. This would predict $n \simeq 2^{53}$ and values of this order of magnitude are possible in the model of living matter [M3]. The smallness of this amplitude could explain the discrepancy. This option is however not very plausible.

Option III: The third option would be a combination of the first two so that the vertex would contain the factor $g_{\gamma\gamma_d} g_{\pi_e N_e N_e} = \alpha_{em} g_{\pi NN} n^{-1-k}$. For $k = 4$ one would have $n^5 \sim 2^{53}$ suggesting $n = 2^{11}$ corresponding to the lowest level in the hierarchy of preferred scaling factors $n = 2^{k11}$ of $\hbar = n\hbar_0$ in living matter. If laser photons are dark photons themselves then $g_{\pi NN} = k\alpha_s^5$ would give the same prediction. Note that the presence of higher powers of α_s in the expansion of $g_{\pi NN}$ could affect these conclusions.

Transformation of laser photons to dark cyclotron photons to exotic pions as the basic mechanism

The cyclotron wave length of electron in a magnetic field of 5 Tesla equals to $\lambda_c = 2$ mm and one has $\lambda_c/\lambda = 2^{11}$. This intriguing finding suggests that λ_c corresponds to the wavelength of dark variant of laser photon at $k = 1$ level of this hierarchy. One can therefore ask whether the basic mechanism is the transformation of the laser photon to a dark cyclotron photon with $\hbar = 2^{11}\hbar_0$ and its mixing with the $k = 181$ exotic pion.

This would predict that the effect is sensitive to the ratio λ_c/λ which should be near $n = 2^{11}$, or to a more general preferred value of n. The preferred values for the scaling factors n of $\hbar$ correspond to n-polygons constructible using ruler and compass. The values of n in question are given by $n_F = 2^k \prod_i F_{s_i}$, where the Fermat primes $F_s = 2^{2^s} + 1$ appearing in the product are distinct. The lowest Fermat primes are $3, 5, 17, 257, 2^{16} + 1$. In the model of living matter the especially favored values of $\hbar$ come as powers 2^{k11}.

Can one understand the mass scale of the exotic pion?

The model predicts preferred values for the ratio λ_c/λ and the experiments correspond to the lowest value of this ratio for biological dark matter hierarchy. In order to be taken seriously the model should also tell why just the scaled up variant of QCD with $m_\pi \simeq 1$ meV is involved.

Also this could relate somehow to the properties of the magnetic field. The frequency associated with the cyclotron photons emitted by electron in the magnetic field is $f = eB/m_e$ and for $B = 5$ Tesla the corresponding wave length is $\lambda_c = 2$ mm to be compared with $L(181) = .327$ mm. As already noticed, $\lambda_c = 2^{11}\lambda$, where $2^{11}\lambda$ is the wavelength of the dark variant of laser photon. Hence it is natural to assume that λ_c corresponds to an characteristic p-adic length scale for the exotic QCD in question.

The p-adic length scale $L(113)$ of u and d quarks is related by a factor 8 to gluon length scale $L(107)$. This would predict that exotic u and d quark correspond to $L(187) = 2.6$ mm to be compared with $\lambda_c = 2$ mm. Hence the latter scale might relate to the p-adic length scales characterizing the Compton lengths of exotic u and d quarks. The prediction would be that the mixing rate depends on magnetic field changing in a discontinuous manner for critical values of the magnetic field.

Summary

To sum up, the assumption that laser photons couple to a dark variant of an exotic pion at the first level of the biological dark matter hierarchy explains the rotation of the polarization direction if one accepts the proposed proportionality $g_{\pi NN} \propto \alpha_s^4 \propto 1/\hbar^4$ and that the transformation of the ordinary laser photon to dark photon can be modelled by a coefficient $k\alpha_{em} \propto 1/\hbar$. The model explains also why dark variants of other exotic pions are not produced.

10.4.2 Do nuclear reaction rates depend on environment?

Claus Rolfs and his group have found experimental evidence for the dependence of the rates of nuclear reactions on the condensed matter environment [90]. For instance, the rates for the reactions ^{50}V(p,n)^{50}Cr and ^{176}Lu(p,n) are fastest in conductors. The model explaining the findings has been tested for elements covering a large portion of the periodic table.

Debye screening of nuclear charge by electrons as an explanation for the findings

The proposed theoretical explanation [90]is that conduction electrons screen the nuclear charge or equivalently that incoming proton gets additional acceleration in the attractive Coulomb field of electrons so that the effective collision energy increases so that reaction rates below Coulomb wall increase since the thickness of the Coulomb barrier is reduced.

The resulting Debye radius

$$R_D = 69\sqrt{\frac{T}{n_{eff}\rho_a}} , \tag{10.4.4}$$

where ρ_a is the density of atoms per cubic meter and T is measured in Kelvins. R_D is of order .01 Angstroms for $T = 373$ K for $n_{eff} = 1$, $a = 10^{-10}$ m. The theoretical model [88, 89] predicts that the cross section below Coulomb barrier for $X(p,n)$ collisions is enhanced by the factor

$$f(E) = \frac{E}{E+U_e} exp(\frac{\pi\eta U_e}{E}) . \tag{10.4.5}$$

E is center of mass energy and η so called Sommerfeld parameter and

$$U_e \equiv U_D = 2.09\times 10^{-11}(Z(Z+1))^{1/2}\times(\frac{n_{eff}\rho_a}{T})^{1/2}\ eV \tag{10.4.6}$$

is the screening energy defined as the Coulomb interaction energy of electron cloud responsible for Debye screening and projectile nucleus. The idea is that at R_D nuclear charge is nearly completely screened so that the energy of projectile is $E+U_e$ at this radius which means effectively higher collision energy.

The experimental findings from the study of 52 metals support the expression for the screening factor across the periodic table.

1. The linear dependence of U_e on Z and $T^{-1/2}$ dependence on temperature conforms with the prediction. Also the predicted dependence on energy has been tested [90].

2. The value of the effective number n_{eff} of screening electrons deduced from the experimental data is consistent with $n_{eff}(Hall)$ deduced from quantum Hall effect.

The model suggests that also the decay rates of nuclei, say beta and alpha decay rates, could be affected by electron screening. There is already preliminary evidence for the reduction of beta decay rate of ^{22}Na β decay rate in Pd [91], metal which is utilized also in cold fusion experiments. This might have quite far reaching technological implications. For instance, the artificial reduction of half-lives of the radioactive nuclei could allow an effective treatment of radio-active wastes. An interesting question is whether screening effect could explain cold fusion [62] and sono-fusion [68]: I have proposed a different model for cold fusion based on large $\hbar$ in [F8].

Could quantization of Planck constant explain why Debye model works?

The basic objection against the Debye model is that the thermodynamical treatment of electrons as classical particles below the atomic radius is in conflict with the basic assumptions of atomic physics. On the other hand, it is not trivial to invent models reproducing the predictions of the Debye model so that it makes sense to ask whether the quantization of Planck constant predicted by TGD could explain why Debye model works.

TGD predicts that Planck constant is quantized in integer multiples: $\hbar = n\hbar_0$, where $\hbar_0$ is the minimal value of Planck constant identified tentatively as the ordinary Planck constant. The preferred values for the scaling factors n of $\hbar$ correspond to n-polygons constructible using ruler and compass. The values of n in question are given by $n_F = 2^k \prod_i F_{s_i}$, where the Fermat primes $F_s = 2^{2^s} + 1$ appearing in the product are distinct. The lowest Fermat primes are $3, 5, 17, 257, 2^{16} + 1$. In the model of living matter the especially favored values of $\hbar$ come as powers 2^{k11} [M3, J6].

It is not quite obvious that ordinary nuclear physics and atomic physics should correspond to the minimum value $\hbar_0$ of Planck constant. The predictions for the favored values of n are not affected if one has $\hbar(stand) = 2^k\hbar_0$, $k \geq 0$. The non-perturbative character of strong force suggests that the Planck constant for nuclear physics is not actually the minimal one [F8]. As a matter fact, TGD based model for nucleus implies that its "color magnetic body" has size of order electron Compton length. Also valence quarks inside hadrons have been proposed to correspond to non-minimal value of Planck constant since color confinement is definitely a non-perturbative effect. Since the lowest order classical predictions for the scattering cross sections in perturbative phase do not depend on the value of the Planck constant one can consider the testing of this issue is not trivial in the case of nuclear physics where perturbative approach does not really work.

Suppose that one has $n = n_0 = 2^{k_0} > 1$ for nuclei so that their quantum sizes are of order electron Compton length or perhaps even larger. One could even consider the possibility that both nuclei and atomic electrons correspond to $n = n_0$, and that conduction electrons can make a transition to a state with $n_1 < n_0$. This transition could actually explain how the electron conductivity is reduced to a finite value. In this state electrons would have Compton length scaled down by a factor n_0/n_1.

For instance, if one has $n_0 = 2^{11k_0}$ as suggested by the model for quantum biology [M3] and by the TGD based explanation of the claimed detection of dark matter [87], the Compton length $L_e = 2.4 \times 10^{-12}$ m for electron would reduce in the transition $k_0 \rightarrow k_0 - 1$ to $L_e = 2^{-11}L_e \simeq 1.17$ fm, which is rather near to the proton Compton length since one has $m_p/m_e \simeq .94 \times 2^{11}$. It is not too difficult to believe that electrons in this state could behave like classical particles with respect to their interaction with nuclei and atoms so that Debye model would work.

The basic objection against this model is that anyonic atoms should allow more states that ordinary atoms since very space-time sheet can carry up to n electrons with identical quantum numbers in conventional sense. This should have been seen.

Electron screening and Trojan horse mechanism

An alternative mechanism is based on Trojan horse mechanism suggested as a basic mechanism of cold fusion [F8]. The idea is that projectile nucleus enters the region of the target nucleus along a larger space-time sheet and in this manner avoids the Coulomb wall. The nuclear reaction itself occurs conventionally. In conductors the space-time sheet of conduction electrons is a natural candidate for the larger space-time sheet.

At conduction electron space-time sheet there is a constant charged density consisting of n_{eff} electrons in the atomic volume $V = 1/n_a$. This creates harmonic oscillator potential in which incoming proton accelerates towards origin. The interaction energy at radius r is given by

$$V(r) = \alpha n_{eff} \frac{r^2}{2a^3} , \tag{10.4.7}$$

where a is atomic radius.

The proton ends up to this space-time sheet by a thermal kick compensating the harmonic oscillator energy. This occurs below with a high probability below radius R for which the thermal energy $E = T/2$ of electron corresponds to the energy in the harmonic oscillator potential. This gives the condition

$$R = \sqrt{\frac{Ta}{n_{eff}\alpha}} a . \tag{10.4.8}$$

This condition is exactly of the same form as the condition given by Debye model for electron screening but has a completely different physical interpretation.

Since the proton need not travel through the nuclear Coulomb potential, it effectively gains the energy

$$E_e = Z\frac{\alpha}{R} = \frac{Z\alpha^{3/2}}{a}\sqrt{\frac{n_{eff}}{Ta}} . \tag{10.4.9}$$

which would be otherwise lost in the repulsive nuclear Coulomb potential. Note that the contribution of the thermal energy to E_e is neglected. The dependence on the parameters involved is exactly the same as in the case of Debye model. For $T = 373$ K in the ^{176}Lu experiment and $n_{eff}(Lu) = 2.2 \pm 1.2$, and $a = a_0 = .52 \times 10^{-10}$ m (Bohr radius of hydrogen as estimate for atomic radius), one has $E_e = 28.0$ keV to be compared with $U_e = 21 \pm 6$ keV of [90] ($a = 10^{-10}$m corresponds to 1.24×10^4 eV and 1 K to 10^{-4} eV). A slightly larger atomic radius allows to achieve consistency. The value of $\hbar$ does not play any role in this model since the considerations are purely classical.

An interesting question is what the model says about the decay rates of nuclei in conductors. For instance, if the proton from the decaying nucleus can enter directly to the space-time sheet of the conduction electrons, the Coulomb wall corresponds to the Coulomb interaction energy of proton with conduction electrons at atomic radius and is equal to $\alpha n_{eff}/a$ so that the decay rate should be enhanced.

10.5 Water and new physics

In this section the previous ideas are applied in an attempt to understand the very special properties of water.

10.5.1 The 41 anomalies of water

The following list of 41 anomalies of water taken from [36] should convince the reader about the very special nature of water. The detailed descriptions of the anomalies can be found in [36]. As a matter fact, the number of anomalies has now grown to 63.

1. Water has unusually high melting point.
2. Water has unusually high boiling point.
3. Water has unusually high critical point.
4. Water has unusually high surface tension and can bounce.
5. Water has unusually high viscosity.
6. Water has unusually high heat of vaporization.

7. Water shrinks on melting.
8. Water has a high density that increases on heating (up to 3.984°C).
9. The number of nearest neighbors increases on melting.
10. The number of nearest neighbors increases with temperature.
11. Pressure reduces its melting point (13.35 MPa gives a melting point of -1°C)
12. Pressure reduces the temperature of maximum density.
13. D_2O and T_2O differ from H_2O in their physical properties much more than might be expected from their increased mass; e.g. they have increasing temperatures of maximum density (11.185°C and 13.4°C respectively).
14. Water shows an unusually large viscosity increase but diffusion decrease as the temperature is lowered.
15. Water's viscosity decreases with pressure (at temperatures below 33°C).
16. Water has unusually low compressibility.
17. The compressibility drops as temperature increases down to a minimum at about 46.5°C. Below this temperature, water is easier to compress as the temperature is lowered.
18. Water has a low coefficient of expansion (thermal expansivity).
19. Water's thermal expansivity reduces increasingly (becoming negative) at low temperatures.
20. The speed of sound increases with temperature (up to a maximum at 73°C).
21. Water has over twice the specific heat capacity of ice or steam.
22. The specific heat capacity (C_P and C_V) is unusually high.
23. Specific heat capacity; C_P has a minimum.
24. NMR spin-lattice relaxation time is very small at low temperatures.
25. Solutes have varying effects on properties such as density and viscosity.
26. None of its solutions even approach thermodynamic ideality; even D_2O in H_2O is not ideal.
27. X-ray diffraction shows an unusually detailed structure.
28. Supercooled water has two phases and a second critical point at about -91°C.
29. Liquid water may be supercooled, in tiny droplets, down to about -70°C. It may also be produced from glassy amorphous ice between -123°C and - 149°C and may coexist with cubic ice up to -63°C.
30. Solid water exists in a wider variety of stable (and metastable) crystal and amorphous structures than other materials.
31. Hot water may freeze faster than cold water; the Mpemba effect.
32. The refractive index of water has a maximum value at just below 0°C.
33. The solubilities of non-polar gases in water decrease with temperature to a minimum and then rise.
34. At low temperatures, the self-diffusion of water increases as the density and pressure increase.
35. The thermal conductivity of water is high and rises to a maximum at about 130°C.
36. Proton and hydroxide ion mobilities are anomalously fast in an electric field.
37. The heat of fusion of water with temperature exhibits a maximum at -17°C.
38. The dielectric constant is high and behaves anomalously with temperature.
39. Under high pressure water molecules move further away from each other with increasing pressure.
40. The electrical conductivity of water rises to a maximum at about 230°C and then falls.
41. Warm water vibrates longer than cold water.

10.5.2 The model

Networks of directed hydrogen bonds $H-O-H\cdots OH_2$ with positively charged H acting as a binding unit between negatively charged O (donor) and OH_2 (acceptor) bonds explaining clustering of water molecules can be used to explain qualitatively many of the anomalies at least qualitatively [36].

The anomaly giving evidence for anomalous nuclear physics is that the physical properties D_2O and T_2O differ much more from H_2O than one might expect on basis of increased masses of water molecules. This suggests that dark protons of various sizes are responsible for the anomalies. That heavy water in large concentrations acts as a poison is consistent with the view that the macroscopic quantum phase of dark protons is responsible for the special biological role of water.

What proton darkness could mean? One fourth of protons of water are not seen in neither electron nor neutron scattering in atto-second time scale which translates n 3 Angstrom wavelength scale suggesting that in both cases diffraction scattering is in question. Both nuclear strong interactions and magnetic scattering contribute to the diffraction which is sensitive to the intra-atomic distances. The minimal conclusion is that the protons form a separate phase with inter-proton distance sufficiently different from that between water molecules and are therefore not seen in neutron and electron diffraction in the atto-second time scale at which protons of water molecule are visible. The stronger conclusion is that they are dark with respect to nuclear strong interactions.

The previous considerations inspired by the model of nuclei as nuclear strings suggests possible explanations.

1. Hydrogen atoms form analogs of nuclear strings connected by color bonds.

2. Nuclear protons form super-nuclei connected by dark color bonds or belong to such super-nuclei (possibly consisting of ^4He nuclei). If color bonds are negatively charged, closed nuclear strings of this kind are neutral and not visible in electron scattering: this assumption is however unnecessarily strong for invisibility in diffractive scattering in atto-second time scale.

Model for super-nuclei formed from dark protons

Dark protons could form super nuclei with nucleons connected by dark color bonds with $\hbar = k2^{11}\hbar_0$. The large distance between protons would eliminate isospin dependent strong force so that multi-proton states are indeed possible. The interpretation would be that nuclear size scale is zoomed up to $k2^{11}L(113) = kL(135) \sim .49k$ Angstrom, where n is Fermat integer: $k = 1, 3, 5$ are the smallest candidates. Dark color bonds could also connect different nuclei.

The predictions of the model for bond energy depend on the transformation properties of E_s under the scaling of $\hbar$. The interpretation of E_s as color Coulombic potential energy α_s/r suggests that E_s behaves under scaling like the binding energy of hydrogen atom ($1/\hbar^2$ scaling).

1. For $k = 1$ E_s would be about .5 eV and same as the energy of hydrogen bond. This energy is same as the universal metabolic energy quantum so that the basic metabolic processes might involve transitions dark-ordinary transition for protons. This would however suggest that the length of color bond is same as that of hydrogen bond so that the protons in question would not be invisible in diffraction in atto-second time scale. The interpretation of color bonds between atoms as hydrogen bonds is much more attractive.

2. For $k = 3$ one would have $E_s \rightarrow E_s/k^2 = .056$ eV which is the nominal value for the energy associated with the cell membrane potential at the threshold for nerve pulse generation and just above the thermal energy at room temperature. There is a temptation to assign the invisible protons suggested by the $H_{1.5}O$ formula [37] with $k = 3$ hydrogen bonds. The length of hydrogen bond is 1.6-2 Angstrom. If hydrogen bond length scales as E_s as the harmonic oscillator picture suggests, the distance would scale as k^2 and would be 9 times longer for $k = 3$ bond. This would explain the invisibility of corresponding hydrogen atoms in electron and neutron diffraction.

Hydrogen bonds as color bonds between nuclei?

The original hypothesis was that there are two kinds of hydrogen bonds: dark and "ordinary". The finding that the energy of dark nuclear color bond with $n = 2^{11}$ equals to the energy of typical hydrogen bond suggests that all hydrogen bonds are associated with color bonds between nuclei. Color bond would bind the proton to electronegative nucleus and this would lead to to the formation of hydrogen bond at the level of valence electrons as hydrogen donates its electron to the electronegative atom. The electronic contribution would explain the variation of the bond energy.

If hydrogen bonds connect H-atom to O-atom to acceptor nucleus, if E_s for p-O bond is same as for p-n color bond, and if color bonds are dark with $n = k2^{11}$, where k is Fermat integer, the bond energy is $E_s = 2.2MeV/n^2$. For $k = 1$ single bond is predicted to have bond energy $E_s = .5$ eV whereas the bond energy for n-bond structure energy would be n^2 times larger. The alternative hypothesis would be that hydrogen bonds are dark color bonds between atoms having $k = 118$ and $n = 2^{11}$.

Nuclear color bonds would serve as a prerequisite for the formation of electronic parts of hydrogen bonds and could be associated also with other molecular bonds so that dark nuclear physics might be essential part of molecular physics. Dark color bonds could be also charged which brings in additional exotic effects. The long range order of hydrogen bonded liquids could due to the ordinary hydrogen bonds. An interesting question is whether nuclear color bonds could be responsible for the long range order of all liquids. If so dark nuclear physics would be also crucial for the understanding of the condensed matter.

In the case of water the presence of $k = 3$ bonds between dark protons would bring in additional long range order in length scale of order 10 Angstrom characteristic for DNA transversal scale: also hydrogen bonds play a crucial role in DNA double strand. Two kinds of bond networks could allow to understand why water is so different from other molecular liquids containing also hydrogen atoms and the long range order of water molecule clusters would reflect basically the long range order of two kinds of dark nuclei.

Two kinds of hydrogen bonds

There is experimental evidence for two different hydrogen bonds but, contrary to the original belief, this does not relate to $H_{1.5}O$ anomaly. Li and Ross represent experimental evidence for two kinds of hydrogen bonds in ice in an article published in Nature 1993 [80], and there is a popular article "Wacky Water" in New Scientist about this finding [79]. The ratio of the force constants associated with the bonds is 1:2 which suggests that binding energies scale as 2:1. This finding excludes the possibility that all hydrogen bonds are ordinary for ice. The interpretation would that these bonds correspond to two different p-adic length scales differing by scale factor 2. $A \leq 4$ nucleons indeed correspond to p-adic length scales $L(k_{eff} = 116)$ and $L(k_{eff} = 118)$. Obviously these bonds cannot be identified as the two variants of color bond discussed above. A possible interpretation for tedrahedral and icosahedral water clusters would be as magic super-nuclei and the prediction would be that binding energy behaves as $n^2 E_s$ rather than being just the sum of the binding energies of hydrogen bonds (nE_s).

The possibility to divide the bonds to two kinds of bonds in an arbitrary manner brings in a large ground state degeneracy given by $D = 16!/(8!)^2$ unless additional symmetries are assumed and give for the system spin glass like character and explain large number of different amorphous phases for ice [36]. This degeneracy would also make possible information storage and provide water with memory.

It is interesting to compare this model with the model for hexagonal ice which assumes four hydrogen bonds per water molecule: for two of them the molecule acts as a donor and for two of them as an acceptor. Each water molecule in the vertices of a tedrahedron containing 14 hydrogen atoms has a hydrogen bond to a water molecule in the interior, each of which have 3 hydrogen bonds to molecules at the middle points of the edges of the tedrahedron. This makes 16 hydrogen bonds altogether. If all of them are of first type with bonding energy $E_s = .5$ eV and if the bond network is connected one would obtain total bond energy equal to $n^2 E_s = 258 \times .5$ eV rather than only $nE_s = 16 \times .5$ eV. Bonds of second type would have no role in the model.

Tedrahedral and icosahedral clusters of water molecules and dark color bonds

Water molecules form both tedrahedral and icosahedral clusters. ^4He corresponds to tedrahedral symmetry so that tedrahedral cluster could be the condensed matter counterpart of ^4He. It the nuclear string model nuclear strings consist of maximum number of ^4He nuclei themselves closed strings in shorter length scale.

The p-adic length scales associated with ^4He nuclei and nuclear string are $k = 116$ and $k = 127$ The color bond between ^4He units has $E_s = .2$ MeV and $n = 2^{11}$ would give by scaling $E_s = .05$ eV which is the already familiar energy associated with cell membrane potential at the threshold for nerve pulse generation. This energy is in a good approximation associated also with $n = 3 \times 2^{11}$ color bonds so that the invisible hydrogen bonds might closely relate to the formation of icosahedral clusters. The binding energy associated with a string formed by n tedrahedral clusters would be $n^2 E_s$. This observation raises the question whether the neural firing is accompanied by the re-organization of strings formed by the tedrahedral clusters and possibly responsible for a representation of information and water memory.

The icosahedral model [36] for water clusters assumes that 20 tedrahedral clusters, each of them containing 14 molecules, combine to form icosahedral clusters containing 280 water molecules. Concerning the explanation of anomalies, the key observation is that icosahedral clusters have a smaller volume per water molecule than tedrahedral clusters but cannot form a lattice structure. Note that the number 20 for the dark magic dark nuclei forming the icosahedron is also a magic number.

Tedrahedral and icosahedral clusters and dark electrons

An additional dark insight to tedrahedral and icosahedral structures is based on the observation that dark matter phases correspond to large values of n_a/n_b and there large value of M^4 Planck constant. This means $N(G_a)$-fold covering of CP_2 with the order of maximal cyclic subgroup of G_a being n_a. For tedrahedron and dodecahedron one has $n_a = 3$ and $n_a = 5$ respectively so that the increase of Planck constant would be relatively small and would correspond to Fermat polygon in both cases. These two groups are the only subgroups of $SO(3)$ which correspond to genuinely 3-dimensional symmetries. Of course, $n_a = 3$ and $n_a = 5$ have nothing to do with $n_a = 2^{11}$ but it is quite possible that also these dark matter levels are involved and could be assigned with dark electrons rather than dark color bonds between nuclei.

Synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron. One can ask whether these structures could be formed around templates formed by dark matter corresponding to 120-fold covering of CP_2 points by M^4 points and having $\hbar(CP_2) = 5\hbar_0$ perhaps corresponding color confined light dark quarks. Of course, a similar covering of M^4 points by CP_2 could be involved.

It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.

10.5.3 Comments on 41 anomalies

Some clarifying general comments about the anomalies are in order. Quite generally, it seems that it is the presence of new degrees of freedom, the presence of icosahedral clusters, and macroscopic quantum coherence of dark matter, which are responsible for the peculiar properties of water.

1. Anomalies relating to the presence of icosahedral clusters

Icosahedral water clusters have a better packing ratio than tedrahedral lattice and thus correspond to a larger density. They also minimize energy but cannot cannot form a lattice [36].

1. This explains the unusually high melting point, boiling point, critical point, surface tension, viscosity, heat of vaporization, shrinking on melting, high density increasing on heating, increase of the number of nearest neighbors in melting and with temperature. It is also possible to understand why X-ray diffraction shows an unusually detailed structure.

   The presence of icosahedral clusters allows to understand why liquid water can be super-cooled, and why the distances of water molecules increase under high pressure. The spin glass degeneracy implied by dark and ordinary hydrogen bonds could explain why ice has many glassy amorphous phases. The two phases of super-cooled water could correspond to the binary degree of freedom brought in by two different hydrogen bonds. For the first phase both hydrogen atoms of a given water molecule would be either dark or ordinary. For the second phase the first hydrogen atom would be dark and second one ordinary.

   Since icosahedral clusters have lower energy than a piece of ice of same size, they tend to super-cool and this slows down the transition to the solid phase. The reason why hot water cools faster would be that the number of icosahedral clusters is smaller: if cooling is carried with a sufficient efficiency icosahedral clusters do not form.

2. Pressure can be visualized as a particle bombardment of water clusters tending to reduce their volume. The collisions with particles can induce local transitions of hexagonal lattice to icosahedral structures with a smaller specific volume and energy and induce local melting. This would explain the low compressibility of water and why pressure reduces melting point and the temperature of maximum density and viscosity.

3. The increase of temperature is expected to reduce the number of icosahedral clusters so that the effect of pressure on these clusters is not so large. This explains the increase of compressibility with temperature below 46.5°C. The fact that the collapse of icosahedral clusters opposes the usual thermal expansion is consistent with the low thermal expansivity as well as the change of sign of expansivity near melting point. Since the square of sound velocity is inversely proportional to compressibility and density, also the increase of speed of sound with temperature can be understood.

2. The presence of dark degrees of freedom and spin glass degeneracy

The presence of dark degrees of freedom and the degeneracy of dark nucleus ground states could explain the high specific heat capacity of water. The reduction of dark matter degrees of freedom for ice and steam would explain why water has over twice the specific heat capacity of ice or steam. The possibility to relax by dissipating energy to the dark matter degrees of freedom would explain the short spin-lattice relaxation time. The fact that cold water has more degrees of freedom explains why warm water vibrates longer than cold water.

Also the high thermal and electric conductivity of water could be understood. The so called Grotthuss mechanism [36, 30] explaining OH_- and H_+ mobilities (related closely to conductivities) is based on hopping of electron of OH_- and H_+ in the network formed by hydrogen bonds and generalizes to the recent case. The reduction of conductivity with temperature would be due to the storage of the transferred energy/capture of charge carriers to the water molecule clusters.

3. Macroscopic quantum coherence

The high value of dielectric constant could derive from the fact that dark nuclei and super-nuclei are quantum coherent in a rather long length scale. For curl free electric fields potential difference must be same along space-time sheets of matter and dark matter. The synchronous quantum coherent collective motion of dark protons (and possible dark electrons) in an oscillating external electric field generates dark photon laser beams (it is not clear yet whether these dark laser beams are actually ordinary laser beams) de-cohering to ordinary photons and yield a large dynamical polarization. As the temperature is lowered the effect becomes stronger.

10.5.4 Burning salt water by radio-waves and large Planck constant

This morning (Tuesday, 14 August 2007) my friend Samuli Penttinen send an email telling about strange discovery [104] by engineer John Kanzius: salt water in the test tube radiated by radiowaves at harmonics of a frequency f=13.56 MHz burns. Temperatures about 1500 K, which correspond to .15 eV energy have been reported. You can radiate also hand but nothing happens. The original discovery of Kanzius was the finding that radio waves could be used to cure cancer by destroying the cancer cells. The proposal is that this effect might provide new energy source by liberating chemical energy in an exceptionally effective manner. The power is about 200 W so that the power used could explain the effect if it is absorbed in resonance like manner by salt water.

The energies of photons involved are very small, multiples of 5.6×10^{-8} eV and their effect should be very small since it is difficult to imagine what resonant molecular transition could cause the effect. This leads to the question whether the radio wave beam could contain a considerable fraction of dark photons for which Planck constant is larger so that the energy of photons is much larger. The underlying mechanism would be phase transition of dark photons with large Planck constant to ordinary photons with shorter wavelength coupling resonantly to some molecular degrees of freedom and inducing the heating. Microwave oven of course comes in mind immediately.

1. The fact that the effects occur at harmonics of the fundamental frequency suggests that rotational states of molecules are in question as in microwave heating. Since the presence of salt seems to be essential, the first candidate for the molecule in question is NaCl but also HCl can be considered and also water molecules. NaCl makes sense if NaCl and Na^+ and Cl^- are in equilibrium. The formula for the rotational energies [49] is

$$E(l) = E_0 \times (l(l+1) \ \ , \ \ E_0 = \hbar_0^2/2\mu R^2 \ \ , \ \ \mu = m_1m_2/(m_1+m_2) \ \ .$$

Here R is molecular radius which by definition is deduced from the rotational energy spectrum. The energy inducing the transition $l \rightarrow l+1$ is $\Delta E(l) = 2E_0 \times (l+1)$.

2. By going to Wikipedia [50], one can find molecular radii of hetero-nuclear molecules such as $NaCl$ and homonuclear di-atomic molecules such as H_2. Using $E_0(H_2) = 8.0 \times 10^{-3}$ eV one obtains by scaling

$$E_0(NaCl) = (\mu(H_2/\mu(NaCl)) \times (R(H_2)/R(NaCl)^2 \ .$$

The atomic weights are $A(H) = 1, A(Na) = 23, A(Cl) = 35$.

3. A little calculation gives $f(NaCl) = 2E_0/h = 14.08$ GHz. The ratio to the radio wave frequency is $f(NaCl)/f = 1.0386 \times 10^3$ to be compared with the $\hbar/\hbar_0 = 2^{10} = 1.024 \times 10^3$. The discrepancy is 1 per cent.

   Thus dark radio wave photons could induce a rotational microwave heating of the sample and the effect could be seen as an additional dramatic support for the hierarchy of Planck constants.

4. One can consider also the possibility that energy is feeded to the rotational degrees of freedom of water molecules as in microwave oven and salt has some other function. Both mechanisms could be involved of course. The microwave frequency used in microwave ovens is 2.45 GHz giving for the Planck constant the estimate 180.67 equal to 180 with error of .4 per cent. The values of Planck constants for $(\hat{M}^4/G_a) \times \hat{CP}_2 \hat{\times} G_b$ option (factor space of M^4 and covering space of CP_2 maximizing Planck constant for given G_a and G_b) are given by $\hbar/\hbar_0 = n_a n_b$. $n_a n_b = 4 \times 9 \times 5 = 180$ can result from the number theoretically simple values of quantum phases $exp(i2\pi/n_i)$ corresponding to polygons constructible using only ruler and compass. For instance, one could have $n_a = 2 \times 3$ and $n_b = 2 \times 3 \times 5$. This option gives a slightly better agreement than NaCl option.

There are several questions to be answered.

1. Does this effect occur also for solutions of other molecules and other solutes than water? This can be tested since the rotational spectra are readily calculable from data which can be found at net.

2. Are the radio wave photons dark or does water - which is very special kind of liquid - induce the transformation of ordinary radio wave photons to dark photons by fusing 2^{10} radio wave massless extremals (MEs) to single ME. Does this transformation occur for all frequencies? This kind of transformation might play a key role in transforming ordinary EEG photons to dark photons and partially explain the special role of water in living systems.

3. Why the radiation does not induce spontaneous combustion of living matter which contains salt. And why cancer cells seem to burn: is salt concentration higher inside them? As a matter fact, there are reports about [105]. One might hope that there is a mechanism inhibiting this since otherwise military would be soon developing new horror weapons unless it is doing this already now. Is it that most of salt is ionized to Na^+ and Cl^- ions so that spontaneous combustion can be avoided? And how this relates to the sensation of spontaneous burning [106] - a very painful sensation that some part of body is burning?

4. Is the energy heating solely due to rotational excitations? It might be that also a "dropping" of ions to larger space-time sheets is induced by the process and liberates zero point kinetic energy. The dropping of proton from k=137 (k=139) atomic space-time sheet liberates about .5 eV (0.125 eV). The measured temperature corresponds to the energy .15 eV. This dropping is an essential element of remote metabolism and provides universal metabolic energy quanta. It is also involved with TGD based models of "free energy" phenomena. No perpetuum mobile is predicted since there must be a mechanism driving the dropped ions back to the original space-time sheets.

Recall that one of the empirical motivations for the hierarchy of Planck constants came from the observed quantum like effects of ELF em fields at EEG frequencies on vertebrate brain and also from the correlation of EEG with brain function and contents of consciousness difficult to understand since the energies of EEG photons are ridiculously small and should be masked by thermal noise.

In TGD based model of EEG (actually fractal hierarchy of EEGs) the values $\hbar/\hbar_0 = 2^{k11}$, $k = 1, 2, 3, ...$, of Planck constant are in a preferred role. More generally, powers of two of a given value of Planck constant are preferred, which is also in accordance with p-adic length scale hypothesis.

10.6 Connection with mono-atomic elements, cold fusion, and sonofusion?

Anomalies are treasures for a theoretician and during years I have been using quite a bundle of reported anomalies challenging the standard physics as a test bed for the TGD vision about physics. The so called mono-atomic elements, cold fusion, and sonofusion represent examples of this kind of anomalies not taken seriously by most standard physicists. In the following the possibility that dark matter as large $\hbar$ phase could allow to understand these anomalies.

Of course, I hear the angry voice of the skeptic reader blaming me for a complete lack of source criticism and the skeptic reader is right. I however want to tell him that I am not a soldier in troops of either skeptics or new-agers. My attitude is "let us for a moment assume that these findings are real..." and look for the consequences in this particular theoretical framework.

10.6.1 Mono-atomic elements as dark matter and high T_c super-conductors?

The ideas related to many-sheeted space-time began to develop for a decade ago. The stimulation came from a contact by Barry Carter who told me about so called mono-atomic elements, typically transition metals (precious metals), including Gold. According to the reports these elements, which are also called ORMEs ("orbitally rearranged monoatomic elements") or ORMUS, have following properties.

1. ORMEs were discovered and patented by David Hudson [43] are peculiar elements belonging to platinum group (platinum, palladium, rhodium, iridium, ruthenium and osmium) and to transition elements (gold, silver, copper, cobalt and nickel).

2. Instead of behaving as metals with valence bonds, ORMEs have ceramic like behavior. Their density is claimed to be much lower than the density of the metallic form.

3. They are chemically inert and poor conductors of heat and electricity. The chemical inertness of these elements have made their chemical identification very difficult.

4. One signature is the infra red line with energy of order .05 eV. There is no text book explanation for this behavior. Hudson also reports that these elements became visible in emission spectroscopy in which elements are posed in strong electric field after time which was 6 times longer than usually.

The pioneering observations of David Hudson [43] - if taken seriously - suggest an interpretation as an exotic super-conductor at room temperature having extremely low critical magnetic fields of order of magnetic field of Earth, which of course is in conflict with the standard wisdom about super-conductivity. After a decade and with an impulse coming from a different contact related to ORMEs, I decided to take a fresh look on Hudson's description for how he discovered ORMEs [43] with dark matter in my mind. From experience I can tell that the model to be proposed is probably not the final one but it is certainly the simplest one.

There are of course endless variety of models one can imagine and one must somehow constrain the choices. The key constraints used are following.

1. Only valence electrons determining the chemical properties appear in dark state and the model must be consistent with the general model of the enhanced conductivity of DNA assumed to be caused by large $\hbar$ valence electrons with $r = \hbar/\hbar_0 = n$, $n = 5, 6$ assignable with aromatic rings. $r = 6$ for valence electrons would explain the report of Hudson about anomalous emission spectroscopy.

2. This model cannot explain all data. If ORMEs are assumed to represent very simple form of living matter also the presence electrons having $\hbar/\hbar_0 = 2^{k11}$, $k = 1$, can be considered and would be associated with high T_c super-conductors whose model predicts structures with thickness of cell membrane. This would explain the claims about very low critical magnetic fields destroying the claimed superconductivity.

Below I reproduce Hudson's own description here in a somewhat shortened form and emphasize that must not forget professional skepticism concerning the claimed findings.

Basic findings of Hudson

Hudson was recovering gold and silver from old mining sources. Hudson had learned that something strange was going on with his samples. In molten lead the gold and silver recovered but when "I held the lead down, I had nothing". Hudson tells that mining community refers to this as "ghost-gold", a non-assayable, non-identifiable form of gold.

Then Hudson decided to study the strange samples using emission spectroscopy. The sample is put between carbon electrodes and arc between them ionizes elements in the sample so that they radiate at specific frequencies serving as their signatures. The analysis lasts 10-15 seconds since for longer times lower electrode is burned away. The sample was identified as Iron, Silicon, and Aluminum. Hudson spent years to eliminate Fe, Si, and Al. Also other methods such as Cummings Microscopy, Diffraction Microscopy, and Fluorescent Microscopy were applied and the final conclusion was that there was nothing left in the sample in spectroscopic sense.

After this Hudson returned to emission spectroscopy but lengthened the time of exposure to electric field by surrounding the lower Carbon electrode with Argon gas so that it could not burn. This allowed to reach exposure times up to 300 s. The sample was silent up to 90 s after which emission lines of Palladium (Pd) appeared; after 110 seconds Platinum (Pt); at 130 seconds Ruthenium (Ru); at about 140-150 seconds Rhodium; at 190 seconds Iridium; and at 220 seconds Osmium appeared. This is known as fractional vaporization.

Hudson reports the boiling temperatures for the metals in the sample having in mind the idea that the emission begins when the temperature of the sample reaches boiling temperature inspired by the observation that elements become visible in the order which is same as that for boiling temperatures.

The boiling temperatures for the elements appearing in the sample are given by the following table.

Element	Ca	Fe	Si	Al	Pd	Rh
$T_B/^oC$	1420	1535	2355	2327	>2200	2500
Element	Ru	Pt	Ir	Os	Ag	Au
$T_B/^oC$	4150	4300	> 4800	> 5300	1950	2600

Table 2. Boiling temperatures of elements appearing in the samples of Hudson.

Hudson experimented also with commercially available samples of precious metals and found that the lines appear within 15 seconds, then follows a silence until lines re-appear after 90 seconds. Note that the ratio of these time scales is 6. The presence of some exotic form of these metals suggests itself: Hudson talks about mono-atomic elements.

Hudson studied specifically what he calls mono-atomic gold and claims that it does not possess metallic properties. Hudson reports that the weight of mono-atomic gold, which appears as a white powder, is 4/9 of the weight of metallic gold. Mono-atomic gold is claimed to behave like super-conductor.

Hudson does not give a convincing justification for why his elements should be mono-atomic so that in following this attribute will be used just because it represents established convention. Hudson also claims that the nuclei of mono-atomic elements are in a high spin state. I do not understand the motivations for this statement.

Claims of Hudson about ORMEs as super conductors

The claims of Hudson that ORMES are super conductors [43] are in conflict with the conventional wisdom about super conductors.

1. The first claim is that ORMEs are super conductors with gap energy about $\Delta = .05$ eV and identifies photons with this energy resulting from the formation of Cooper pairs. This energy happens to correspond one of the absorption lines in high T_c superconductors.

2. ORMEs are claimed to be super conductors of type II with critical fields H_{c1} and H_{c2} of order of Earth's magnetic field having the nominal value $.5 \times 10^{-4}$ Tesla [43]. The estimates for the critical parameters for the ordinary super conductors suggests for electronic super conductors critical fields, which are about .1 Tesla and thus by a factor $\sim 2^{11}$ larger than the critical fields claimed by Hudson.

3. It is claimed that ORME particles can levitate even in Earth's magnetic field. The latter claim looks at first completely nonsensical. The point is that the force giving rise to the levitation is roughly the gradient of the would-be magnetic energy in the volume of levitating super conductor. The gradient of average magnetic field of Earth is of order B/R, R the radius of Earth and thus extremely small so that genuine levitation cannot be in question.

Minimal model

Consider now a possible TGD inspired model for these findings assuming for definiteness that the basic Hudson's claims are literally true.

1. In what sense mono-atomic elements could be dark matter?

The simplest option suggested by the applicability of emission spectroscopy and chemical inertness is that mono-atomic elements correspond to ordinary atoms for which valence electrons are dark electrons with large $\hbar/\hbar_0 = n_a/n_b$. Suppose that the emission spectroscopy measures the energies of dark photons from the transitions of dark electrons transforming to ordinary photons before the detection by de-coherence increasing the frequency by the factor $r = \hbar/\hbar_0$. The size of dark electrons and temporal duration of basic processes would be zoomed up by r.

Since the time scale after which emission begins is scaled up by a factor 6, there is a temptation to conclude that $r = n_a/n_b = 6$ holds true. Note that $n = 6$ corresponds to Fermat polygon and is thus preferred number theoretically in TGD based model for preferred values of $\hbar$ [A9]. The simplest possibility is that the group G_b is trivial group and $G_a = A_6$ or D_6 so that ring like structures containing six dark atoms are suggestive.

This brings in mind the model explaining the anomalous conductivity of DNA by large $\hbar$ valence electrons of aromatic rings of DNA. The zooming up of spatial sizes might make possible exotic effects and perhaps even a formation of atomic Bose-Einstein condensates of Cooper pairs. Note however that in case of DNA $r = 6$ not gives only rise to conductivity but not super-conductivity and that $r = 6$ cannot explain the claimed very low critical magnetic field destroying the super-conductivity.

2. Loss of weight

The claimed loss of weight by a factor $p \simeq 4/9$ is a very significant hint if taken seriously. The proposed model implies that the density of the partially dark phase is different from that of the ordinary phase but is not quantitative enough to predict the value of p. The most plausible reason for the loss of weight would be the reduction of density induced by the replacement of ordinary chemistry with $\hbar/\hbar_0 = n_a/n_b = 6$ chemistry for which the Compton length of valence electrons would increase by this factor.

3. Is super-conductivity possible?

The overlap criterion is favorable for super-conductivity since electron Compton lengths would be scaled up by factor $n_a = 6, n_b = 1$. For $\hbar/\hbar_0 = n_a = 6$ Fermi energy would be scaled up by $n_a^2 = 36$ and if the same occurs for the gap energy, T_c would increase by a factor 36 from that predicted by the standard BCS theory. Scaled up conventional super-conductor having $T_c \sim 10$ K would be in question (conventional super-conductors have critical temperatures below 20 K). 20 K upper bound for the critical temperature of these superconductors would allow 660 K critical temperature for their dark variants!

For large enough values of n_a the formation of Cooper pairs could be favored by the thermal instability of valence electrons. The binding energies would behave as $E = (n_b/n_a)^2 Z_{eff}^2 E_0/n^2$, where Z_{eff} is the screened nuclear charge seen by valence electrons, n the principal quantum number for the valence electron, and E_0 the ground state energy of hydrogen atom. This gives binding energy smaller than thermal energy at room temperature for $n_a/n_b > (Z_{eff}/n)\sqrt{2E_0/3T_{room}} \simeq 17.4 \times (Z_{eff}/n)$. For $n = 5$ and $Z_{eff} < 1.7$ this would give thermal instability for $n_a = 6$.

Interestingly, the reported .05 eV infrared line corresponds to the energy assignable to cell membrane voltage at criticality against nerve pulse generation, which suggests a possible connection with high T_c superconductors for which also this line appears and is identified in terms of Josephson energy. .05 eV line appears also in high T_c superconductors. This interpretation does not exclude the interpretation as gap energy. The gap energy of the corresponding BCS super-conductor would be scaled down by $1/n_a^2$ and would correspond to 14 K temperature for $n_a = 6$.

Also high T_c super-conductivity could involve the transformation of nuclei at the stripes containing the holes to dark matter and the formation of Cooper pairs could be due to the thermal instability of valence electrons of Cu atoms (having $n = 4$). The rough extrapolation for the critical temperature for cuprate superconductor would be $T_c(Cu) = (n_{Cu}/n_{Rh})^2 T_c(Rh) = (25/36)T_c(Rh)$. For $T_c(Rh) = 300$ K this would give $T_c(Cu) = 192$ K: accoding to Wikipedia cuprate perovskite has the highest known critical temperature which is 138 K. Note that quantum criticality suggests the possibility of several values of (n_a, n_b) so that several kinds of super-conductivities might be present.

ORMEs as partially dark matter, high T_c super conductors, and high T_c super-fluids

The appearance of .05 eV photon line suggest that same phenomena could be associated with ORMEs and high T_c super-conductors. The strongest conclusion would be that ORMEs are T_c super-conductors and that the only difference is that Cu having single valence electron is replaced by a heavier atom with single valence electron. In the following I shall discuss this option rather independently from the minimal model.

1. ORME super-conductivity as quantum critical high T_c superconductivity

ORMEs are claimed to be high T_c superconductors and the identification as quantum critical superconductors seems to make sense.

1. According to the model of high T_c superconductors as quantum critical systems, the properties of Cooper pairs should be more or less universal so that the observed absorption lines discussed in the section about high T_c superconductors should characterize also ORMEs. Indeed, the reported 50 meV photon line corresponds to a poorly understood absorption line in the case of high T_c cuprate super conductors having in TGD framework an interpretation as a transition in which exotic Cooper pair is excited to a higher energy state. Also Copper is a transition metal and is one of the most important trace elements in living systems [38]. Thus the Cooper pairs could be identical in both cases. ORMEs are claimed to be superconductors of type II and quantum critical superconductors are predicted to be of type II under rather general conditions.

2. The claimed extremely low value of H_c is also consistent with the high T_c superconductivity. The supra currents in the interior of flux tubes of radius of order $L_w = .2$ μm are BCS type supra currents with large $\hbar$ so that T_c is by a factor 2^{11} higher than expected and H_c is reduced by a factor $2^{-11/2}$. This indeed predicts correct order of magnitude for the critical magnetic field.

3. $r = \hbar/\hbar_0 = 2^{11}$ is considerably higher that $r = 6$ suggested by the minimum model explaining emission spectroscopic results of Hudson. Of course, several values of $\hbar$ are possible and the values $r \in \{5, 6, 2^{k11}\}$ are indeed assumed in TGD inspired model of living matter and generalize EEG [M3]. Thus internal consistency would be achieved if ORMEs are regarded as a very simple form of living matter.

4. The electronic configurations of Cu and Gold are chemically similar. Gold has electronic configuration $[Xe, 4f^{14}5d^{10}]6s$ with one valence electron in s state whereas Copper corresponds to $3d^{10}4s$ ground state configuration with one valence electron. This encourages to think that the doping by holes needed to achieve superconductivity induces the dropping of these electrons to $k = 151$ space-time sheets and gives rise to exotic Cooper pairs. Also this model assumes the phase transition of some fraction of Cu nuclei to large $\hbar$ phase and that exotic Cooper pairs appear at the boundary of ordinary and large $\hbar$ phase.

More generally, elements having one electron in s state plus full electronic shells are good candidates for doped high T_c superconductors. Both Cu and Au atoms are bosons. More generally, if the atom in question is boson, the formation of atomic Bose-Einstein condensates at Cooper pair space-time sheets is favored. Thus elements with odd value of A and Z possessing full shells plus single s wave valence electron are of special interest. The six stable elements satisfying these conditions are ^{5}Li, ^{39}K, ^{63}Cu, ^{85}Rb, ^{133}Cs, and ^{197}Au.

2. "Levitation" and loss of weight

The model of high T_c superconductivity predicts that some fraction of Cu atoms drops to the flux tube with radius $L_w = .2$ μm and behaves as a dark matter. This is expected to occur also in the case of other transition metals such as Gold. The atomic nuclei at this space-time sheet have high charges and make phase transition to large $\hbar$ phase and form Bose-Einstein condensate and superfluid behavior results. Electrons in turn form large $\hbar$ variant of BCS type superconductor. These flux tubes are predicted to be negatively charged because of the Bose-Einstein condensate of exotic Cooper pairs at the boundaries of the flux tubes having thickness $L(151)$. The average charge density equals to the doping fraction times the density of Copper atoms.

The first explanation would be in terms of super-fluid behavior completely analogous to the ability of ordinary superfluids to defy gravity. Second explanation is based on the electric field of Earth which causes an upwards directed force on negatively charged BE condensate of exotic Cooper pairs and this force could explain both the apparent levitation and partial loss of weight. The criterion for levitation is $F_e = 2eE/x \geq F_{gr} = Am_pg$, where $g \simeq 10$ m^2/s is gravitational acceleration at the surface of Earth, A is the atomic weight and m_p proton mass, E the strength of electric field, and x is the number of atoms at the space-time sheet of a given Cooper pair. The condition gives $E \geq 5 \times 10{-}10Ax$ V/m to be compared with the strength $E = 10^2 - 10^4$ V/m of the Earths' electric field.

An objection against the explanation for the effective loss of weight is that it depends on the strength of electric field which varies in a wide range whereas Hudson claims that the reduction factor is constant and equal to 4/9. A more mundane explanation would be in terms of a lower density of dark Gold. This explanation is quite plausible since there is no atomic lattice structure since nuclei and electrons form their own large $\hbar$ phases.

4. The effects on biological systems

Some monoatomic elements such as White Gold are claimed to have beneficial effects on living systems [43]. 5 per cent of brain tissue of pig by dry matter weight is claimed to be Rhodium and Iridium. Cancer cells are claimed to be transformed to healthy ones in presence of ORMEs. The model for high T_c super conductivity predicts that the flux tubes along which interior and boundary supra currents flow has same structure as neuronal axons. Even the basic length scales are very precisely the same. On basis of above considerations ORMEs are reasonable candidates for high T_c superconductors and perhaps even super fluids.

The common mechanism for high T_c, ORME- and bio- super-conductivities could explain the biological effects of ORMEs.

1. In unhealthy state superconductivity might fail at the level of cell membrane, at the level of DNA or in some longer length scales and would mean that cancer cells are not anymore able to communicate. A possible reason for a lost super conductivity or anomalously weak super conductivity is that the fraction of ORME atoms is for some reason too small in unhealthy tissue.

2. The presence of ORMEs could enhance the electronic bio- superconductivity which for some reason is not fully intact. For instance, if the lipid layers of cell membrane are, not only wormhole-, but also electronic super conductors and cancer involves the loss of electronic super-conductivity then the effect of ORMEs would be to increase the number density of Cooper pairs and make the cell membrane super conductor again. Similar mechanism might work at DNA level if DNA:s are super conductors in "active" state.

5. Is ORME super-conductivity associated with the magnetic flux tubes of dark magnetic field $B_d = 0.2$ Gauss?

The general model for the ionic super-conductivity in living matter, which has developed gradually during the last few years and will be discussed in detail later, is based on the assumption that super-conducting particles reside at the super-conducting magnetic flux tubes of Earth's magnetic field with nominal value $B_E = .5$ Gauss. It later became clear that the explanation of ELF em fields on vertebrate brain requires $B_d = .2$ Gauss rather than B_E as was erratically assumed in the original model. The interpretation was as dark magnetic field $B_d = .2$ Gauss.

The predicted radius $L_w = .2\ \mu$m is consistent with the radius of neuronal axons. For $\hbar \rightarrow n\times 2^{11}\hbar$, $n = 3$, the radius is 1.2 μm and still smaller than the radius d of flux tube of B_E of order $d = 5\ \mu$m and scales up as $d \rightarrow \sqrt{B_d/B_E}\sqrt{r}d = \sqrt{5r/2}d$ in the replacement $\hbar/\hbar_0 \rightarrow r$, $B_E \rightarrow B_d$. Consistency is achieved even for $r = 1$ and for $r = 6$ the radius corresponds to the size of large neuron. The most natural interpretation would be that these flux tubes topologically condense at the flux tubes of B_d or B_E. Both bosonic ions and the Cooper pairs of electrons or of fermionic ions can act as charge carriers so that actually a whole zoo of super-conductors is predicted. There is even some support for the view that even molecules and macromolecules can drop to the magnetic flux tubes [K6].

Consciousness related claims

If mono-atomic elements represent dark or partially dark matter with suggested properties, the claimed finding by Hudson that 5 per cent of brain tissue of pig by dry matter weight is Rhodium and Iridium might be understood.

In order to not induce un-necessary negative reactions in materialistic readers, I have purposefully left out Hudson's references to alchemy and Biblical stories. These references admittedly begin to make sense for an open minded reader if dark matter serves as a kind of elixir of life or philosopher's stone. If there exists an infinite hierarchy of conscious entities, it would not be difficult to accept that alchemists (Newton being one of them) would have had precognition about the existence of dark matter and its significance for life.

Possible implications

The existence of exotic atoms could have far reaching consequences for the understanding of bio-systems. If Hudson's claims about super-conductor like behavior are correct, the formation of exotic atoms in bio-systems could provide the needed mechanism of electronic super-conductivity. One could even argue that the formation of exotic atoms is the magic step transforming chemical evolution to biological evolution.

Equally exciting are the technological prospects. If the concept works it could be possible to manufacture exotic atoms and build room temperature super conductors and perhaps even artificial life some day. It is very probable that the process of dropping electron to the larger space-time sheet requires energy and external energy feed is necessary for the creation of artificial life. Otherwise the Earth and other planets probably have developed silicon based life for long time ago. Ca, K and Na ions have central position in the electrochemistry of cell membranes. They could actually correspond to exotic ions obtained by dropping some valence electrons from $k = 137$ atomic space-time sheet to larger space-time sheets. For instance, the $k = 149$ space-time sheet of lipid layers could be in question.

The status of ORMEs is far from certain and their explanation in terms of exotic atomic concept need not be correct. The fact is however that TGD predicts exotic atoms: if they are not observed TGD approach faces the challenge of finding a good explanation for their non-observability.

10.6.2 Connection with cold fusion?

The basic prediction of TGD is a hierarchy of fractally scaled variants of non-asymptotically free QCD like theories and that color dynamics is fundamental even for our sensory qualia (visual colors identified as increments of color quantum numbers in quantum jump). The model for ORMEs suggest that exotic protons obey QCD like theory in the size scale of atom. If this identification is correct, QCD like dynamics might be studied some day experimentally in atomic or even macroscopic length scales of order cell size and there would be no need for ultra expensive accelerators! The fact that Palladium is one of the "mono-atomic" elements used also in cold fusion experiments as a target material [66, 65] obviously puts bells ringing.

What makes possible cold fusion?

I have proposed that cold fusion might be based on Trojan horse mechanism in which incoming and target nuclei feed their em gauge fluxes to different space-time sheets so that electromagnetic Coulomb wall disappears [F8]. If part of Palladium nuclei are "partially dark", this is achieved. Another mechanism could be the de-localization of protons to a larger volume than nuclear volume induced by the increase of $\hbar_{eff}$ meaning that reaction environment would differ dramatically from that appearing in the usual nuclear reactions and the standard objections against cold fusion would not apply anymore [F8]: this delocalization could correspond to the darkness of electromagnetic field bodies of protons.

A third proposal is perhaps the most elegant and relies on the nuclear string model [F9] predicting a large number of exotic nuclei obtained by allowing the color bonds connecting nucleons to have all possible em charges $1, 0, 1$. Many ordinary heavy nuclei would be exotic in the sense that some protons would correspond to protons plus negatively charged color bonds. The exchange of an exotic weak boson between D and Pd nuclei transforming D nuclei to exotic neutral D nuclei would occur. The range of the exotic weak interaction correspond to atomic length scale meaning that it behaves as massless particle below this length scale. For instance, W boson could be $n = 2^{11}$ dark variant of $k = 113$ weak boson but also other options are possible.

How standard objections against cold fusion can be circumvented?

The following arguments against cold fusion are from an excellent review article by Storms [62].

1. Coulomb wall requires an application of higher energy. Now electromagnetic Coulomb wall disappears in both models.

2. If a nuclear reaction should occur, the immediate release of energy can not be communicated to the lattice in the time available. In the recent case the time scale is however multiplied by the factor $r = n_a$ and the situation obviously changes. For $n_a = 2^{11}$ the time scale corresponding to MeV energy becomes that corresponding to keV energy which is atomic time scale.

3. When such an energy is released under normal conditions, energetic particles are emitted along with various kinds of radiation, only a few of which are seen by various CANR (Chemically Assisted Nuclear Reactions) studies. In addition, gamma emission must accompany helium, and production of neutrons and tritium, in equal amounts, must result from any fusion reaction. None of these conditions is observed during the claimed CANR effect, no matter how carefully or how often they have been sought. The large value of $\hbar(M^4)$ implying large Compton lengths for protons making possible geometric coupling of gamma rays to condensed matter would imply that gamma rays do not leave the system. If only protons form the quantum coherent state then fusion reactions do not involve the protons of the cathode at all and production of ^3He and thus of neutrons in the fusion of D and exotic D.

4. The claimed nuclear transmutation reactions (reported to occur also in living matter [63]) are very difficult to understand in standard nuclear physics framework.

   i) The model of [F8] allows them since protons of different nuclei can re-arrange in many different manners when the dark matter state decays back to normal.

   ii) Nuclear string model [F9] allows transmutations too. For instance, neutral exotic tritium produced in the reactions can fuse with Pd and other nuclei.

5. Many attempts to calculate fusion rates based on conventional models fail to support the claimed rates within PdD (Palladium-Deuterium). The atoms are simply too far apart. This objections also fails for obvious reasons.

Mechanisms of cold fusion

One can deduce a more detailed model for cold fusion from observations, which are discussed systematically in [62] and in the references discussed therein.

1. A critical phenomenon is in question. The average D/Pd ratio must be in the interval $(.85, .90)$. The current must be over-critical and must flow a time longer than a critical time. The effect occurs in a small fraction of samples. D at the surface of the cathode is found to be important and activity tends to concentrate in patches. The generation of fractures leads to the loss of the anomalous energy production. Even the shaking of the sample can have the same effect. The addition of even a small amount of H_2O to the electrolyte (protons to the cathode) stops the anomalous energy production.

   i) These findings are consistent the view that patches correspond to a macroscopic quantum phase involving delocalized nuclear protons. The added ordinary protons and fractures could serve as a seed for a phase transition leading to the ordinary phase [F8].

   ii) An alternative interpretation is in terms of the formation of neutral exotic D and exotic Pd via exchange of exotic, possibly dark, W bosons massless below atomic length scale [F9].

2. When D_2O is used as an electrolyte, the process occurs when PdD acts as a cathode but does not seem to occur when it is used as anode. This suggests that the basic reaction is between the ordinary deuterium $D = pn$ of electrolyte with the exotic nucleus of the cathode. Denote by $\hat{p}$ the exotic proton and by $\hat{D} = n\hat{p}$ exotic deuterium at the cathode.

   For ordinary nuclei fusions to tritium and ^3He occur with approximately identical rates. The first reaction produces neutron and ^3He via $D+D \rightarrow n+^3He$, whereas second reaction produces proton and tritium by $3H$ via $D + D \rightarrow p +^3 H$. The prediction is that one neutron per each tritium nucleus should be produced. Tritium can be observed by its beta decay to ^3He and the ratio of neutron flux is several orders of magnitude smaller than tritium flux as found for instance by Tadahiko Mizuno and his collaborators (Mizuno describes the experimental process leading to this discovery in his book [67]). Hence the reaction producing ^3He cannot occur significantly in cold fusion which means a conflict with the basic predictions of the standard nuclear physics.

   i) The explanation discussed in [F8] is that the proton in the target deuterium $\hat{D}$ is in the exotic state with large Compton length and the production of ^3He occurs very slowly since $\hat{p}$ and p correspond to different space-time sheets. Since neutrons and the proton of the D from the electrolyte are in the ordinary state, Coulomb barrier is absent and tritium production can occur. The mechanism also explains why the cold fusion producing ^3He and neutrons does not occur using water instead of heavy water.

   ii) Nuclear string model [F9] model predicts that only neutral exotic tritium is produced considerably when incoming deuterium interacts with neutral exotic deuterium in the target.

3. The production of ^4He has been reported although the characteristic gamma rays have not been detected.

   i) ^4He can be produced in reactions such as $D + \hat{D} \rightarrow^4 He$ in the model of [F8].

   ii) Nuclear string model [F8] does not allow direct production of ^4He in D-D collisions.

4. Also more complex reactions between D and Pd for which protons are in exotic state can occur. These can lead to the reactions transforming the nuclear charge of Pd and thus to nuclear transmutations.

   Both model explain nuclear transmutations. In nuclear string model [F8] the resulting exotic tritium can fuse with Pd and other nuclei and produce nuclear transmutations.

   The reported occurrence of nuclear transmutation such as $^{23}Na +^{16} O \rightarrow^{39} K$ in living matter [63] allowing growing cells to regenerate elements K, Mg, Ca, or Fe, could be understood in nuclear string model if also neutral exotic charge states are possible for nuclei in living matter. The experimental signature for the exotic ions would be cyclotron energy spectrum containing besides the standard lines also lines with ions with anomalous mass number. This could be seen as a splitting of lines. For instance, exotic variants of ions such Na^+, K^+, Cl^-, Ca^{++} with anomalous mass numbers should exist. It would be easy to mis-interpret the situation unless the actual strength of the magnetic field is not checked.

5. Gamma rays, which should be produced in most nuclear reactions such as ^4He production to guarantee momentum conservation are not observed.

   i) The explanation of the model of [F8] is that the recoil momentum goes to the macroscopic quantum phase and eventually heats the electrolyte system. This provides obviously the mechanism by which the liberated nuclear energy is transferred to the electrolyte difficult to imagine in standard nuclear physics framework.

   ii) In nuclear string model [F9] ^4He is not produced at all.

6. Both models explain why neutrons are not produced in amounts consistent with the anomalous energy production. The addition of water to the electrolyte is however reported to induce neutron bursts.

   i) In the model of [F8] a possible mechanism is the production of neutrons in the phase transition $\hat{p} \rightarrow p$. $\hat{D} \rightarrow p+n$ could occur as the proton contracts back to the ordinary size in such a manner that it misses the neutron. This however requires energy of 2.23 MeV if the rest masses of $\hat{D}$ and D are same. Also $\hat{D} + \hat{D} \rightarrow n +^3 He$ could be induced by the phase transition to ordinary matter when $\hat{p}$ transformed to p does not combine with its previous neutron partner to form D but recombines with $\hat{D}$ to form $^3\hat{He} \rightarrow^3 He$ so that a free neutron is left.

   ii) Nuclear string model [F9] would suggest that the collisions of protons of water with exotic D produce neutron and ordinary D. This requires the transformation of negatively charged color bond between p and n of target D to a neutral color bond between incoming p and neutron of target.

10.6.3 Connection with sono-luminescence and sono-fusion?

Sono-luminescence [81] is a poorly understood phenomenon in which the compression of bubbles in liquid leads to very intense emission of photons and generation of temperatures which are so high that even nuclear fusion might become possible. Sono-fusion [68] is a second closely related poorly understood phenomenon. I have discussed a TGD inspired model for sono-luminescence in terms of p-adic length scale hypothesis.

In bubble compression the density of matter inside bubble might become so high that the Compton lengths associated with possibly existing conformally confined phases inside nuclei could start to overlap and a delocalized phase of protons and/or neutrons could form and em and Z^0 Coulomb walls could disappear. Cold fusion would occur and the energy produced would explain the achieved high temperatures and sono-luminescence. Thus the causal relation would be reversed from what it is usually believed to be. The same anomalies are predicted as in the case of cold fusion also now.

Bubble compression brings in mind "mini crunch" occurring also in RHIC experiments, and p-adic fractality suggests that analogy might be rather precise in that magnetic flux tubes structure carrying Bose-Einstein condensate of protons, electrons and photons might form. The intense radiation of photons might be an analog of thermal radiation from an evaporating black hole. The relevant p-adic scale is probably not smaller than 100 nm, and this would give Hagedorn temperature which is around $T_H \sim 10$ eV for ordinary Planck constant and much smaller than fusion temperature. For $\hbar_s$ the Hagedorn temperature would be scaled up to $T_H \sim rT_H$, $r = \hbar_s/\hbar$. For $r = 10^5$ temperatures allowing nuclear fusion would be achieved.

10.7 Dark atomic physics

Dark matter might be relevant also for atomic physics and in the sequel some speculations along these lines are represented. Previous considerations assumed that only field bodies can be dark. The notion of N-atom discussed below is based on more general view about dark matter not requiring that elementary particles are maximally quantum critical in the sense that elementary particle like partonic 2-surfaces remain invariant under the cyclic groups $G_a \times G_b$ leaving invariant the choice of the quantization axes. Therefore the sheets of space-time surface associated with the sheets of the multiple coverings $H \rightarrow H/G_a \times G_b$ do not co-incide and can carry fermionic quantum numbers. The minimum option is that fermion states possibly associated with different sheets are identical so that an apparent failure of Fermi statistics would result. The additional degree of freedom would correspond to an element of group algebra of $G_a \times G_b$ for a given many-fermion state. The more general option allowing different fermion quantum numbers is not consistent with quantum classical correspondence in its strongest form.

10.7.1 From naive formulas to conceptualization

I have spent a considerable amount of time on various sidetracks in attempts to understand what the quantization of Planck constant does really mean. As usual, the understanding has emerged by unconscious processing rather than by a direct attack.

Naive approach based on formulas

The whole business started from the naive generalization of various formulas for quantized energies by replacing Planck constant with is scaled value. It seems that this approach does not lead to wrong predictions, and is indeed fully supported by the basic applications of the theory. Mention only the quantization of cyclotron energies crucial for the biological applications, the quantization of hydrogen atom, etc... The necessity for conceptualization emerges when one asks what else the theory predicts besides the simple zoomed up versions of various systems.

The geometric view about the quantization of Planck constant

After the naive approach based on simple substitutions came the attempt to conceptualize by visualizing geometrically what dark atoms could look like, and the description in terms of $N(G_a) \times N(G_b)$-fold covering $H \rightarrow H/G_a \times G_b$ emerged.

Especially confusing was the question whether one should assign Planck constant to particles or to their interactions or both. It is now clear that one can assign Planck constant to both the "personal" field bodies assignable to particles and to their interactions ("relative" or interaction field bodies), and that each interaction can correspond to both kinds of field bodies. Planck constant for the relative field bodies depends on the quantum numbers of both particles as it does in the case of gravitation. The Planck constant assignable to the particle's "personal" field body makes possible generalizations like the notion of N-atom.

Each sheet of the "personal" field body corresponds to one particular Compton length characterizing one particular interaction and electromagnetic interaction would define the ordinary Compton length. The original picture was that topological condensation of CP_2 type vacuum extremal occurs at space-time sheet with size of Compton length identified usually with particle. In the new picture this space-time sheet can be identified as electromagnetic field body.

Elementary particles have light-like partonic 3-surfaces as space-time correlates. If these 3-surfaces are fully quantum critical, they belong to the intersection of all spaces $H/G_a \times G_b$ with fixed quantization axes. This space is just the 4-D subspace $M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. Partonic 2-surfaces are in general non-critical and one can assign to them a definite value of Planck constant.

There are two geodesic spheres in CP_2. Which one should choose or are both possible?

1. For the homologically non-trivial one corresponding to cosmic strings, the isometry group is $SU(2) \subset SU(3)$. The homologically trivial one S^2 corresponds to vacuum extremals and has isometry group $SO(3) \subset SU(3)$. The natural question is which one should choose. At quantum criticality the value of Planck constant is undetermined. The vacuum extremal would be a natural choice from the point of view of quantum criticality since in this case the value of Planck constant does not matter at all and one would obtain a direct connection with the vacuum degeneracy. One can of course ask whether all surfaces $M^2 \times Y^2$, Y^2 Lagrangian sub-manifold of CP_2 should be allowed: the answer seems to be "No" since in the generic case $SO(3)$ does not act as H-isometries of Y^2.

2. The choice of the homologically non-trivial geodesic sphere as a quantum critical sub-manifold would conform with the previous guess that $\mathcal{M} : \mathcal{N} = 4$ corresponds to cosmic strings. It is however questionable whether the ill-definedness of the Planck constant is consistent with the non-vacuum extremal property of cosmic strings unless one assumes that for partonic 3-surfaces $X^3 \subset M^2 \times S^2$ the effective degrees of freedom reduce to mere topological ones.

Fractionization of quantum numbers and the hierarchy of Planck constants

The original generalization of the notion of imbedding space to a union of the factor spaces $\hat{H}/G_a \times G_b$ discussed in the section "General ideas about dark matter" does not allow charge fractionization whereas the covering spaces $\hat{H}\hat{\times}(G_a \times G_b)$ allow a fractionization in a natural manner. Also hybrid cases are obtained corresponding $(\hat{M}^4\hat{\times}G_a) \times (\hat{CP}_2/G_b)$ and $(\hat{M}^4/G_a) \times (\hat{CP}_2\hat{\times}G_b)$. The simplest assumption is that Planck constant is a homomorphism from the lattice like structure of groups with product of groups defined to be the group generated by the groups.

1. $\hat{H}/G_a \times G_b$ option

The safest and indeed natural assumption motivated by Jones inclusions is that physical states in sector $H/G_a \times G_b$ are $G_a \times G_b$ invariant meaning a discrete analog of color confinement. This alone excludes fractionization and actually implies just the opposite of it.

1. For states with vanishing fermionic quantum numbers $G_a \times G_b$ invariance means that wave functions live in the base space $H/G_a \times G_b$. For instance, L_z would be a multiple of n_a defining the order of maximal cyclic subgroup of G_a. Analogous conclusion would hold true for color quantum numbers.

2. Just as in the case of ordinary spin fermionic quantum numbers (spin, electro-weak spin) necessarily correspond to the covering group of the isometry group since a state with a half-odd integer spin does not remain invariant under the subgroups of the rotation group. In particular, states with odd fermion number cannot be $G_a \times G_b$ invariant. For even fermion numbers it is possible to have many-particle states for which individual particles transform non-trivially under orbital $G_a \times G_b$ if total $G_a \times G_b$ quantum numbers in spin like degrees of freedom compensate for the orbital quantum numbers (for instance, total spin is multiple of n_a). Hence the group algebra of $G_a \times G_b$ would characterize the states in orbital degrees of freedom as indeed assumed. The earlier picture would be correct apart from the lacking assumption about overall $G_a \times G_b$ invariance.

3. The construction of these states could be carried out just like the construction of ordinary $G_a \times G_b$ invariant states in H so that the mathematical treatment of the situation involves no mystics elements. Since $G_a \times G_b$ is actually assigned with a sector $M^4_\pm \times CP_2$ with fixed quantization axes and preferred point of H, one has center of mass degrees of freedom for the position of tip of $M^4_\pm$ and a preferred point of CP_2. This gives new degrees of freedom and one would have a rich spectrum of N-electrons, N-nucleons, N-atoms, etc.... behaving effectively as elementary particles. For example, one interesting question is whether 2-electrons could be interpreted as Cooper pairs of particular kind This would require either $s_z = 0, l_z = 0$ or $s_z = 1, l_z = mn_a - 1$, $m = 0, 1, 2...$ For instance, for $n_a = 3$ (the minimal value of n_a) one could have $s_z = l, l_z = 2$ with $J_z = 3$. One can also ask whether some high spin nuclei could correspond to N-nuclei.

4. This picture is quite predictive. For instance, in the case of gravitational interactions it would mean that the spin angular momentum of an astrophysical system is a multiple of "personal" gravitational Planck constant GM^2/v_0. The value of v_0 could be deduced from this condition and is expected to be a negative power of 2. In the same manner the relative angular momentum of planet-Sun system would be a multiple of GMm/v_0 using the gravitational Planck constant as a unit. This is a strong prediction but reduces to the Bohr quantization rule for circular orbits.

2. $\hat{H}\hat{\times}(G_a \times G_b)$ option

For this option the units of orbital angular momentum and color hyper charge and isospin are naturally scaled down by the factor n_i. In the case of spin and electro-weak spin this kind of scaling would require a covering group of Abelian Cartan group. Since the first homotopy group of SU(2) vanishes there are no coverings of $SU(2)$ in the ordinary sense of the word but quantum version of $SU(2)$ is an excellent candidate for the counterpart of the covering. Also quantum variants of other Lie groups are highly suggestive on basis of ADE correspondence.

There does not seem to be any absolute need for assuming $G_a \times G_b$ singletness. If so, there would be asymmetry between coverings and factor spaces bringing in mind confined and de-confined phases.

Since coverings *resp.* factor spaces are labelled by N^{11}-valued lattice momenta *resp.* their negatives, this asymmetry would be analogous to time reversal asymmetry. Note however that all components of lattice momenta are either positive or negative and that this fits nicely with the interpretation of p-adic integers as naturals and "super-naturals". An intriguing question is whether there might be some connection with M-theory and its 4-D compactifications (dropping reflection group one obtains 7-D lattice).

3. Implications of the new picture

This picture has several important implications.

1. There is a symmetry between CP_2 and M^4 so that for a given value of Planck constant one obtains factor space with divisor group $G_a \times G_b$ and covering space with homotopy group $G_b \times G_a$. For large values of Planck constant the large Z_n symmetry acts in M^4 factor *resp.* CP_2 factor for these two options. Therefore the large Z_n symmetry in M^4 degrees of freedom, which can be challenged in some of the applications, could be replaced with large Z^n symmetry in CP_2 degrees of freedom emerging rather naturally.

2. For a large value of Planck constant it is possible to obtain a relatively small dark matter symmetry group in M^4 factor and also the small genuinely 3-dimensional symmetry groups (tedrahedral, octahedral, icosahedral groups) can act in M^4 factor as symmetries of dark matter. Hence the groups appearing as symmetries of molecular physics (aromatic rings, DNA,...) could be identified as symmetries of dark electron pairs. These symmetries appear also in longer length scales (snow flakes and even astrophysical structures). In earlier picture one had to assume symmetry breaking at the level of visible matter.

3. The notion of N-atom generalizes. The original model predicted large electronic charges suggesting instability plus large Z_n symmetry in M^4 degrees of freedom (identified as a symmetry of field body). For instance, in the case of DNA double helix this kind of large rotational symmetry is questionable. Same applies to astrophysical systems with a gigantic value of gravitational Planck constant. The change of the roles of M^4 and CP_2 and charge fractionization would resolve these problems. This would provide a support for the idea that the electronic or protonic hot spots of catalyst and substrate correspond to fractional charges summing up to a unit charge. This framework could provide a proper realization for the original vision that symbolic level of dynamics and sex emerge already at the molecular level with sequences of catalyst sites representing "words" and their conjugates (opposite molecular sexes).

10.7.2 Dark atoms and dark cyclotron states

The development of the notion of dark atom involves many side tracks which make me blush. The first naive guess was that dark atom would be obtained by simply replacing Planck constant with its scaled counterpart in the basic formulas and interpreting the results geometrically. After some obligatory twists and turns it became clear that this assumption is indeed the most plausible one. The main source of confusion has been the lack of precise view about what the hierarchy of Planck constants means at the level of imbedding space at space-time.

The assumptions of the model of dark atom

Let us briefly summarize the basic assumptions of the model.

1. The quantized values of effective Planck constant appearing in Schrödinger equation are in the set $\hbar_{eff}/\hbar_0 \in \{n_a/n_b, n_b/n_a, n_a n_b, 1/(n_a n_b)$ corresponding to the sectors $\hat{H}/G_a \times G_b$, $\hat{H}\hat{\times}(G_a \times G_b)$, $\hat{M}^4/G_a \times (\hat{CP}_2\hat{\times})G_b$, and $(\hat{M}^4\hat{\times}G_a) \times \hat{CP}_2/G_b$. Note that one can consider the replacement of the right hand side of the formula for Planck constant by its inverse, and at this stage one must just keep mind open for the options.

2. In the case of covering spaces the units of quantum numbers are replaced by $1/n_a$ and $1/n_b$, n_i the order of maximal cyclic subgroup. Both fermion number, spin, color, and electro-weak quantum numbers can fractionize. For factor spaces units are inverses of these and in this case states are $G_a \times G_b$ singlets: hence N-atoms with dark electrons in general involve many-electron

states with even number of electrons. Simplest situation corresponds to spin singlet electron pair and one cannot exclude the possibility that valence electrons are dark electrons.

3. It is assumed that the quantum critical sub-manifolds $M^2 \times S^2$ correspond to homologically trivial geodesic sphere. Note that although quantum critical parton orbits are vacuum extremals, induced electric and Z^0 fields are non-vanishing in general. This is a very important point since it makes possible electric and magnetic fluxes between different sectors of the generalized imbedding space H. For instance, nucleus and electrons can belong to different sectors of H. A helpful visualization is provided by a book with pages glued together along $M^2 \times S^2$. Both electric and magnetic flux are assumed to be conserved as it flows from a sector to another one: therefore dark electron in the covering experiences the electric charge of nucleus as scaled down by a factor $1/N(G_b)$ giving the number of sectors.

4. In the case of factor spaces 3-surface is invariant under G_i so that one has $N(G_i)$ strict copies of the particle: G_i invariance selects states with $l_z = nn_a$ and forces many electron states in order to satisfy quantization conditions in the case of spin. Here one can consider the possibility that single particle states transform according to irreducible representations of G_i although the entire state is G_i invariant.

5. In the case of covering spaces there is no need to assume that partonic 3-surface consists of $N(G_i)$ identical copies. In this case the states are naturally classified by the representations of $G_a \times G_b$ identifiable as elements of the corresponding group algebra. Apparently one has a modified statistics since $N(G_a) \times N(G_b)$ states correspond to the same state in the ordinary sense of the word. It can happen that the action of G_i in H has some isotropy subgroup. In fact, the action of D_{2n} in M^2 and S^2 reduces to the action of the corresponding cyclic group Z_n so that has $N(G_i) = n_i$.

6. One can consider quite a number of variants for the dark atom. Even nucleus could be dark (either fractionally charged or N-nucleus with charge $N(G_b)$). Second interesting possibility is atom with ordinary nucleus and dark electrons. It is also possible that only valence electrons are dark and correspond to one of the allowed varieties.

Thermal stability

The energy scale of hydrogen atom is proportional to $1/\hbar^2$. Depending on the sector of H and on the values of n_a and n_b the scale of energy can increase or be reduced. Also charge fractionization in case of covering spaces of $\hat{CP}_2$ reduces the energy scale. By the conservation of electric flux this takes place for both proton and electron so that the energy scale receives a factor $1/N(G_b)^2$. For large values of Planck constant the energy scale is reduced and thermal stability poses upper limit on the value of Planck constant if dark matter is assumed to be in thermal equilibrium with ordinary matter.

The following table lists the four possible options.

$$\begin{array}{cccc} I & II & III & IV \\ \hat{H}\hat{\times}G_a \times G_b & \hat{H}/(G_a \times G_b) & (\hat{H}/G_a)\hat{\times}G_b & (\hat{H}/G_b)\hat{\times}G_a \end{array}$$

One can also consider two options for the formula of Planck constant.

1. For $\hbar/\hbar_0 = n_a/n_b$ in case of option I and $G_b = Z_n$ thermal stability condition boils down to the condition

$$\begin{array}{ll} I: & Z \geq \frac{n_b^3}{n_a} \times x \ , \\ II: & Z \geq \frac{n_a}{n_b} \times x \ , \\ III: & Z \geq n_a n_b^3 \times x \ , \\ IV: & Z \geq \frac{1}{n_a n_b} \times x \ . \end{array} \quad x \equiv \sqrt{\frac{E_{th}}{E_1}} \ . \tag{10.7.1}$$

Here E_{th} denotes thermal energy. Note that option III maximizes Planck constant for given $G_a \times G_b$ and is therefore especially interesting. Option IV minimizes in turn minimizes it.

By replacing the formula for Planck constant with its inverse ($\hbar/\hbar_0 = n_b/n_a$ for option I) one obtains the conditions

$$\begin{array}{ll} I: & Z \geq n_b^2 n_a \times x \ , \\ II: & Z \geq \frac{n_b}{n_a} \times x \ , \\ III: & Z \geq \frac{n_b^2}{n_a} \times x \ , \\ IV: & Z \geq n_a n_b \times x \ . \end{array} \quad x \equiv \sqrt{\frac{E_{th}}{E_1}} \ . \tag{10.7.2}$$

Recall that the preferred values of n_a and n_b correspond to the number theoretically simple quantum phases $exp(i2\pi/n_i)$ expressible using only square root function and rational functions applied on rationals. n_i are given as products $2^k \times \prod_i F_i$, where F_i are distinct Fermat primes.

2. The original proposal for the hierarchy of Planck constants coming as $\hbar/\hbar_0 = \lambda = 2^{11k}$ does not allow stable hydrogen atom at room temperature. This is not a problem since this hierarchy is associated with cyclotron energies.

3. For option I with $n_a = 1$ and $n_b \in \{3, 5, 6, 12\}$ one would have $Z \geq z \in \{1, 6, 10, 81\}$. Carbon atom would satisfy the condition fors $(n_b = 5, n_a = 1)$ and $(n_b = 6, n_a = 2)$.

4. For option II with $n_b = 1$ one obtains $Z \geq n_a$ for $E_{th} \sim E_1$. What is intriguing that aromatic carbon 5- and 6-cycles, which are abundant in biology and correspond to factor space option, would satisfy this condition for $E_{th} \sim E_1$. For $n > 6$-cycles the condition would not be satisfied. Could this condition state something non-trivial about pre-biotic evolution at high temperatures?

5. For option III with $n_b = 3$ meaning charge fractionization and n_a-fold cyclic symmetry one obtains $Z \geq n_a \times 1.3$ at room temperature. For $n_b = 3$ 5-cycles with $\hbar/\hbar_0 = 15$ and 6-cycles with $\hbar/\hbar_0 = 18$ would be stable below room temperature but not higher cycles. This estimate is of course very rough since the energy scale E_1 for possibly dark delocalized free electron pairs appearing in n-cycles need not be exactly equal to E_1.

6. If one replaces the right hand side by its inverse in the expression of Planck constant the factor space option would favor the thermal stability for large values of n_a and n-cycles with large n so that this option does not look reasonable.

Is the fractionization of principal quantum number possible?

One can also consider the fractionization $n \rightarrow n/n_b$ of the principal quantum number of hydrogen analogous to that occurring for angular momentum. If one assumes that fractionization occurs only for isometry charges this option is excluded. This argument might quite well be enough to exclude this kind of fractionization.

Since s-wave states correspond to orbits which represent radial motion between two extremes, one could consider the possibility of periodic radial orbits which run to maximal radius, back to the maximum radius at the opposite side and close after N_b loops of this kind, where N_b is the order of maximal cyclic subgroup of G_b. This would be direct a counterpart for a rotational orbit which closes only after N_b full 2π rotations.

One can consider the occurrence of this phenomenon also in the case of ordinary imbedding space. At least inn this case the interpretation in terms of a transition to chaos might be appropriate. In case of generalized imbedding space one could speak about transition to chaos by period N_b-folding and suggest fractionization of the radial quantum number to n/N_b. Similar fractionization could makes sense for all orbits which are not precisely circular. This fractionization would increase the energy scale by a factor n_b^2.

In empty space fractional diagonal quantum number would mean that ordinary hydrogen atom wave functions diverge at spatial infinity. This kind of scaling is consistent with finiteness inside dark sector if the copies of sheet fuse together at at 3-surface belonging to the quantum critical manifold $M^2 \times S^2$.

Possible experimental implications

An interesting possibility is the formation of stable hydrogen bonds as a fusion of N-hydrogen atoms with $N-k$ and k electrons to give rise to a full shell of electrons possessing an exceptional stability.

1. In the case of factor space the state would be analogous to full Fermi sea or full atomic or nuclear shells. The large value of electric charge might make the state unstable. The resulting state would be invariant under $G_a \times G_b$.

2. For covering space option the total quantum numbers for the resulting state would be those of electron. The degeneracy of states is $N(G_a) \times N(G_b)$-fold corresponding to the group algebra of $G_a \times G_b$. This would mean that the full shell for states with given energy E_n would have total energy $n_a n_b E_n$.

Consider next the possible experimental implications of N-atom concept.

1. Valence electrons could transform to dark electrons in one of the four possible senses.

   i) For covering space option fractal electrons could result. Fractal electron and its conjugate would combine to form a particle with quantum numbers of electrons but with larger mass. Catalytic sites are one possible candidate for fractal electrons and catalyst activity could be understood as a strong tendency of fractal electron and its conjugate to fuse to form an ordinary electron. The anomalously high mass would be the tell-tale signature of these exotic electrons. The effective mass of electron in condensed matter is known to vary in wide limits and to exceed electron mass even by a factor of order hundred: is this really a mere standard physics effect?

   ii) For factor space option full electron shells would be the most stable states and would have have rather high fermion number but vanishing spin. Spin singlet electron pairs would define stable $G_a \times G_b$ singlets. These states might behave like Cooper pairs.

   iii) If the value of Planck constant is smaller than its standard value, the molecular bonds containing dark electrons could be stable at anomalously high temperatures. Note that the dependence of the bond energy on Planck constant need not be non-perturbative as it is for atoms. For instance, a naive application of the formulas for vibrational and rotational energies assuming that the parameters of Hamiltonian (such as vibrational energy scale) do not depend on Planck constant would suggest that large Planck constant implies thermal stability in this kind of situations.

   iv) Both fermionic a (Na^+, K^+, Cl^-) and bosonic (Ca^{++}, Mg^{++}) ions are very important in biology. Optimist would interpret this as a support for the plasmoids as predecessors of biological life. These ions are formed in some manner and the simplest manner would be transformation of valence electrons to dark electrons and subsequence delocalization.

2. The recently discovered evidence [85] that Sun has a solid surface consisting mostly of calcium-ferrite is inconsistent with the fact that photosphere has temperature 5800 K (iron melts at 1811 K and calcium-aluminium ferrite in the range 1670-1720 K at normal pressure). Metallic bonds responsible for the solid state are due to the interaction of delocalized conduction electrons with metal atoms. If the valence electrons giving rise to conduction bands have a reduced value of Planck constant, the energy scale of the valence bands would become higher and raise the melting temperature. The reduction of Planck constant seems necessary by the non-perturbative dependence of atomic binding energies on $\hbar$.

3. The claims of Mills [83] about the scaling up of the binding energy of the hydrogen ground state by a square k^2 $(k = 2, 3, 4, 5, 6, 7, 10)$ of an integer in plasma state are a challenge for the theory. The simplest explanation is that the Planck constant is reduced by factor $1/k$.

Before I had realized that $\hbar_{eff}$ satisfies the formula $\hbar_{eff}/\hbar_0 = n_a/n_b$, the presence of $k = 2$ state in spectrum was a difficult problem and I ended up with the idea that the quantum variant of Laguerre polynomials associated with quantized radial motion could explain $n = 1/2$ and also other fractional states. Later it will be found that this approach indeed predicts these quantum numbers approximately! This raises the question whether these states might appear as metastable intermediate states for hydrogen atom in the phase having $\hbar_{eff}/\hbar_0 = 1$ and $n_a = n_b > 1$. These states would be unstable against the phase transition leading to $n_b > kn_a$, $k = 2, 3, ...$.

Living matter could perhaps be understood in terms of quantum deformations of the ordinary matter, which would be characterized by the quantum phases $q = exp(i2\pi/N)$. Hence quantum groups, which have for long time suspected to have significance in elementary particle physics, might explain the mystery of living matter and predict an entire hierarchy of new forms of matter.

As demonstrated in [L5], the notion of N-atom leads to an elegant model for the lock and key mechanism of bio-catalysis as well as the understanding of the DNA replication based on the spontaneous decay and completion of fermionic $N < N(G)$-particles to $N = N(G)$-particles. Optimal candidates for the N-particles are N-hydrogen atoms associated with bio-molecules appearing as letters in the "pieces of text" labelling the molecules. Lock and key would correspond to conjugate names in the sense that N_1 and N_2 for the letters in the name and its conjugate satisfy $N_1 + N_2 = N = N(G)$: as the molecules combine, a full fermion shell represented is formed.

10.7.3 Dark cyclotron states

Dark cyclotron states have been scaled spectrum $E_n = (n_a/n_b)E_n$ and for large values of n_a one can have energies above thermal threshold. The crucial observation is that the flux of ordinary magnetic field cannot divide into $N(G)$ dark fluxes since magnetic fluxes necessarily vanish at orbifold surfaces. Hence dark magnetic field would carry total flux which is $N(G)$ times higher than the flux of ordinary magnetic field of same intensity. Fermionic analogs of Bose-Einstein condensates are possible so that each cyclotron energy $E_n = n\hbar_0\omega$ would be replaced with spectrum extending from $(n_a/n_b)E_n$ to $(n_a/n_b)N(G_b)E_n$ in the case of fractionization.

10.7.4 Could q-Laguerre equation relate to the claimed fractionation of the principal quantum number for hydrogen atom?

In the following a semiclassical model based on dark matter and hierarchy of Planck constants is developed for the fractionized principal quantum number n claimed by Mills [83] to have at least the values $n = 1/k$, $k = 2, 3, 4, 5, 6, 7, 10$. This model can explain the claimed fractionization of the principal quantum number n for hydrogen atom [83] in terms of single electron transitions for all cases. The original model could not cope with $n = 1/2$: the basic reason is that Jones inclusions are characterized by quantum phases $q = exp(i\pi/n)$, $n > 2$. Since quantum deformation of the standard quantum mechanism is involved, this motivated an attempt to understand the claimed fractionization in terms of q-analog of hydrogen atom. The safest interpretation for them would be as states which can exist in ordinary imbedding space (and also in other branches). The natural guess would be that they can occur as intermediate states in the phase transition changing $n_b/n_a = 1$ to $k = 2, 3, ...$

The Laguerre polynomials appearing in the solution of Schrödinger equation for hydrogen atom possess quantum variant, so called q-Laguerre polynomials [17], and one might hope that they would allow to realize this semiclassical picture at the level of solutions of appropriately modified Schrödinger equation and perhaps also resolve the difficulty associated with $n = 1/2$. Unfortunately, the polynomials discussed in [17] correspond to $0 < q \leq 1$ rather than complex values of $q = exp(i\pi/m)$ on circle and the extrapolation of the formulas for energy eigenvalues gives complex energies.

q-Laquerre equation for $q = exp(i\pi/m)$

The most obvious modification of the Laguerre equation for S-wave sates (which are the most interesting by semiclassical argument) in the complex case is based on the replacement

$$\begin{aligned} \partial_x &\rightarrow \frac{1}{2}(\partial_x^{q)} + \partial_x^{\overline{q})}) \\ \partial_x^{q)} f &= \frac{f(qx) - f(x)}{(q-1)x} \quad , \\ q &= exp(i\pi/m) \end{aligned} \tag{10.7.1}$$

to guarantee hermiticity. When applied to the Laguerre equation

$$x\frac{d^2L_n}{dx^2}+(1-x)\frac{dL_n}{dx}=nL_n \ , \tag{10.7.2}$$

and expanding L_n into Taylor series

$$L_n(x) = \sum_{n\geq 0} l_n x^n \ , \tag{10.7.3}$$

one obtains difference equation

$$\begin{aligned}
a_{n+1}l_{n+1} &+ b_n l_n = 0 \ , \\
a_{n+1} &= \frac{1}{4R_1^2}\left[R_{2n+1}-R_{2n}+2R_{n+1}R_1+3R_1)\right]+\frac{1}{2R_1}\left[R_{n+1}+R_1\right] \\
b_n &= \frac{R_n}{2R_1}-n^{q)}+\frac{1}{2} \ , \\
R_n &= 2cos\left[(n-1)\pi/m\right]-2cos\left[n\pi/m\right] \ .
\end{aligned} \tag{10.7.1}$$

Here $n^{q)}$ is the fractionized principal quantum number determining the energy of the q-hydrogen atom. One cannot pose the difference equation on l_0 since this together with the absence of negative powers of x would imply the vanishing of the entire solution. This is natural since for first order difference equations lowest term in the series should be chosen freely.

Polynomial solutions of q-Laquerre equation

The condition that the solution reduces to a polynomial reads as

$$b_n = 0 \tag{10.7.2}$$

and gives

$$n^{q)} = \frac{1}{2}+\frac{R_n}{2R_1} \ , \tag{10.7.3}$$

For $n = 1$ one has $n^{q)} = 1$ so that the ground state energy is not affected. At the limit $N \to \infty$ one obtains $n^{q)} \to n$ so that spectrum reduces to that for hydrogen atom. The periodicity $R_{n+2Nk} = R_n$ reflects the corresponding periodicity of the difference equation which suggests that only the values $n \leq 2m-1$ belong to the spectrum. Spectrum is actually symmetric with respect to the middle point $[N/2]$ which suggests that only $n < [m/2]$ corresponds to the physical spectrum. An analogous phenomenon occurs for representations of quantum groups [**?**]. When m increases the spectrum approaches integer valued spectrum and one has $n > 1$ so that no fractionization in the desired sense occurs for polynomial solutions.

Non-polynomial solutions of q-Laquerre equation

One might hope that non-polynomial solutions associated with some fractional values of $n^{q)}$ near to those claimed by Mills might be possible. Since the coefficients a_n and b_n are periodic, one can express the solution ansatz as

$$\begin{aligned}
L_n(x) &= P_a^{2m)}(x)\sum_k a^k x^{2mk} = P_a^{2m)}(x)\frac{1}{1-ax^{2m}} \ , \\
P_a^{2m)}(x) &= \sum_{k=0}^{2m-1} l_k x^k \ , \\
a &= \frac{l_{2m}}{l_0} \ ,
\end{aligned} \tag{10.7.2}$$

This solution behaves as $1/x$ asymptotically but has pole at $x_\infty = (1/a)^{1/2m}$ for $a > 0$.

The expression for $l_{2m}/l_0 = a$ is

$$a = \prod_{k=1}^{2m} \frac{b_{2m-k}}{a_{2m-k+1}} \quad . \tag{10.7.3}$$

This can be written more explicitly as

$$\begin{aligned} a &= (2R_1)^{2m} \prod_{k=1}^{2m} X_k \ , \\ X_k &= \frac{R_{2m-k} + (-2n^{q)} + 1)R_1}{R_{4m-2k+1} - R_{4m-2k} + 4R_{2m-k+1}R_1 + 2R_1^2 + 3R_1} \ , \\ R_n &= 2cos\left[(n-1)\pi/m\right] - 2cos\left[n\pi/m\right] \quad . \end{aligned} \tag{10.7.1}$$

This formula is a specialization of a more general formula for $n = 2m$ and resulting ratios l_n/l_0 can be used to construct $P_a^{2m)}$ with normalization $P_a^{2m)}(0) = 1$.

Results of numerical calculations

Numerical calculations demonstrate following.

1. For odd values of m one has $a < 0$ so that a a continuous spectrum of energies seems to result without any further conditions.
2. For even values of m a has a positive sign so that a pole results.

For even value of m it could happen that the polynomial $P_a^{2m)}(x)$ has a compensating zero at x_∞ so that the solution would become square integrable. The condition for reads explicitly

$$P_a^{2m)}\left((\frac{1}{a})^{\frac{1}{2m}} \right) = 0 \quad . \tag{10.7.2}$$

If $P_a^{2m)}(x)$ has zeros there are hopes of finding energy eigen values satisfying the required conditions. Laguerre polynomials and also q-Laguerre polynomials must posses maximal number of real zeros by their orthogonality implied by the hermiticity of the difference equation defining them. This suggests that also $P_a^{2m)}(x)$ possesses them if a does not deviate too much from zero. Numerical calculations demonstrate that this is the case for $n^{q)} < 1$.

For ordinary Laguerre polynomials the naive estimate for the position of the most distant zero in the units used is larger than n but not too much so. The naive expectation is that L_{2m} has largest zero somewhat above $x = 2m$ and that same holds true a small deformation of L_{2m} considered now since the value of the parameter a is indeed very small for $n^{q)} < 1$. The ratio $x_\infty/2m$ is below .2 for $m \leq 10$ so that this argument gives good hopes about zeros of desired kind.

One can check directly whether x_∞ is near to zero for the experimentally suggested candidates for $n^{q)}$. The table below summarizes the results of numerical calculations.

1. The table gives the exact eigenvalues $1/n_{q)}$ with a 4-decimal accuracy and corresponding approximations $1/n^{q)}_{\simeq} = k$ for $k = 3, ..., 10$. For a given value of m only single eigenvalue $n^{q)} < 1$ exists. If the observed anomalous spectral lines correspond to single electron transitions, the values of m for them must be different. The value of m for which $n^{q)} \simeq 1/k$ approximation is optimal is given with boldface. The value of k increases as m increases. The lowest value of m allowing the desired kind of zero of $P^{2m)}$ is $m = 18$ and for $k \in \{3, 10\}$ the allowed values are in range $18, .., 38$.

2. $n^{q)} = 1/2$ does not appear as an approximate eigenvalue so that for even values of m quantum calculation produces same disappointing result as the classical argument. Below it will be however found that $n^{q)} = 1/2$ is a universal eigenvalue for odd values of m.

m	$1/n^{q)}_{\simeq}$	$1/n^{q)}$	m	$1/n^{q)}_{\simeq}$	$1/n^{q)}$
18	**3**	2.7568	30	8	7.5762
20	**4**	3.6748	**32**	**8**	8.3086
22	5	4.5103	**34**	**9**	9.0342
24	**5**	5.3062	**36**	**10**	9.7529
26	**6**	6.0781	38	10	10.4668
28	**7**	6.8330			

Table 1. The table gives the approximations $1/n^{q})_{\simeq} = 1/k$ and corresponding exact values $1/n^{q})$ in the range $k = 3, ..., 10$ for which $P_a^{2m)}(x_\infty)$ is nearest to zero. The corresponding values of $m = 2k$ vary in the range, $k = 18, ..., 38$. For odd values of m the value of the parameter a is negative so that there is no pole. Boldface marks for the best approximation by $1/n^{q)}_{\simeq} = k$.

How to obtain $n^{q)} = 1/2$ state?

For odd values of m the quantization recipe fails and physical intuition tells that there must be some manner to carry out quantization also now. The following observations give a hunch about be the desired condition.

1. For the representations of quantum groups only the first m spins are realized [**?**]. This suggests that there should exist a symmetry relating the coefficients l_n and l_{n+m} and implying $n^{q)} = 1/2$ for odd values of m. This symmetry would remove also the double degeneracy associated with the almost integer eigenvalues of $n^{q)}$. Also other fractional states are expected on basis of physical intuition.

2. For $n^{q)} = 1/2$ the recursion formula for the coefficients l_n involves only the coefficients R_m.

3. The coefficients R_k have symmetries $R_k = R_{k+2m}$ and $R_{k+m} = -R_m$.

There is indeed this kind of symmetry. From the formula

$$\begin{aligned} \frac{l_n}{l_0} &= (2R_1)^n \prod_{k=1}^{n} X_k \ , \\ X_k &= \frac{R_{n-k} + (-2n^{q)} + 1)R_1}{[R_{2n-2k+1} - R_{n-2k} + 4R_{n-k+1}R_1 + 2R_1^2 + 3R_1} \end{aligned} \quad (10.7.2)$$

one finds that for $n^{q)} = 1/2$ the formula giving l_{n+m} in terms of l_n changes sign when n increases by one unit

$$\begin{aligned} A_{n+1} &= (-1)^m A_n \ , \\ A_n &= \prod_{k=1}^{m} \frac{b_{n+m-k}}{a_{n+m-k+1}} = \prod_{k=1}^{m} (2R_1)^m \prod_{k=1}^{m} X_{k+n} \ . \end{aligned} \quad (10.7.1)$$

The change of sign is essentially due to the symmetries $a_{n+m} = -a_n$ and $b_{n+m} = b_n$. This means that the action of translations on A_n in the space of indices n are represented by group Z_2.

This symmetry implies $a = l_{2m}/l_0 = -(l_m)(l_0)^2$ so that for $n^{q)} = 1/2$ the polynomial in question has a special form

$$
\begin{aligned}
P_a^{2m)} &= P_a^{m)}(1-Ax^m) \ , \\
A &= A_0 \ .
\end{aligned}
\qquad (10.7.1)
$$

The relationship $a = -A^2$ implies that the solution reduces to a form containing the product of m^{th} (rather than $(2m)^{th}$) order polynomial with a geometric series in x^m (rather than x^{2m}):

$$
L_{1/2}(x) = \frac{P_a^{m)}(x)}{1+Ax^m} \ . \qquad (10.7.2)
$$

Hence the n first terms indeed determine the solution completely. For even values of m one obtains similar result for $n^{q)} = 1/2$ but now A is negative so that the solution is excluded. This result also motivates the hypothesis that for the counterparts of ordinary solutions of Laguerre equation sum (even m) or difference (odd m) of solutions corresponding to n and $2m-n$ must be formed to remove the non-physical degeneracy.

This argument does not exclude the possibility that there are also other fractional values of n allowing this kind of symmetry. The condition for symmetry would read as

$$
\begin{aligned}
\prod_{k=1}^{m}(R_k+\epsilon R_1) &= \prod_{k=1}^{m}(R_k-\epsilon R_1) \ , \\
\epsilon &= (2n^{q)}-1 \ .
\end{aligned}
\qquad (10.7.2)
$$

The condition states that the odd part of the polynomial in question vanishes. Both ϵ and $-\epsilon$ solutions so that $n^{q)}$ and $1-n^{q)}$ are solutions. If one requires that the condition holds true for all values of m then the comparison of constant terms in these polynomials allows to conclude that $\epsilon = 0$ is the only universal solution. Since ϵ is free parameter, it is clear that the m:th order polynomial in question has at most m solutions which could correspond to other fractionized eigenvalues expected to be present on basis of physical intuition.

This picture generalizes also to the case of even n so that also now solutions of the form of Eq. 10.7.2 are possible. In this case the condition is

$$
\prod_{k=1}^{m}(R_k+\epsilon R_1) = -\prod_{k=1}^{m}(R_k-\epsilon R_1) \ . \qquad (10.7.3)
$$

Obviously $\epsilon = 0$ and thus $n = 1/2$ fails to be a solution to the eigenvalue equation in this case. Also now one has the spectral symmetry $n_\pm = 1/2 \pm \epsilon$.

The symmetry $R_n = (-1)^m R_{n+m-1} = (-1)^m R_{n-m-1} = (-1)^m R_{m-n+1}$ can be applied to show that the polynomials associated with ϵ and $-\epsilon$ contain both the terms $R_n - \epsilon$ and $R_n + \epsilon$ as factors except for odd m for $n = (m+1)/2$. Hence the values of n can be written for even values of m as

$$
n^{q)}(n) = \frac{1}{2} \pm \frac{R_n}{2R_1} \ , \ n = 1, ..., \frac{m}{2} \ , \qquad (10.7.4)
$$

and for odd values of m as

$$
\begin{aligned}
n_\pm^{q)}(n) &= \frac{1}{2} \pm \frac{R_n}{2R_1} \ , \ n = 1, ..., \frac{m+1}{2} - 1 \ , \\
n^{q)} &= 1/2 \ .
\end{aligned}
\qquad (10.7.4)
$$

Plus sign obviously corresponds to the solutions which reduce to polynomials and to $n^{q)} \simeq n$ for large m. The explicit expression for $n^{q)}$ reads as

$$n_{\pm}^{q)}(n) = \frac{1}{2} \pm \frac{(sin^2(\pi(n-1)/2m) - sin^2(\pi n/2m))}{2sin^2(\pi/2m)} . \quad (10.7.5)$$

At the limit of large m one has

$$n_{+}^{q)}(n) \simeq n \ , \ n_{-}^{q)}(n) \simeq 1-n \ . \quad (10.7.6)$$

so that the fractionization $n \simeq 1/k$ claimed by Mills is not obtained at this limit. The minimum for $|n^{q)}|$ satisfies $|n^{q)}| < 1$ and its smallest value $|n^{q)}| = .7071$ corresponds to $m = 4$. Thus these zeros cannot correspond to $n^{q)} \simeq 1/k$ yielded by the numerical computation for even values of m based on the requirement that the zero of $P^{2m)}$ cancels the pole of the geometric series.

Some comments

Some closing comments are in order.

1. An open question is whether there are also zeros $|n^{q)}| > 1$ satisfying $P_a^{2m)}((1/a)^{1/2m}) = 0$ for even values of m.

2. The treatment above is not completely general since only s-waves are discussed. The generalization is however a rather trivial replacement $(1-x)d/dx \rightarrow (l+1-x)d/dx$ in the Laguerre equation to get associated Laguerre equation. This modifies only the formula for a_{n+1} in the recursion for l_n so that expression for $n^{q)}$, which depends on b_n:s only, is not affected. Also the product of numerators in the formula for the parameter $a = l_{2m}/l_0$ remains invariant so that the general spectrum has the spectral symmetry $n^{q)} \rightarrow 1 - n^{q)}$. The only change to the spectrum occurs for even values of m and is due to the dependence of $x_\infty = (1/a)^{1/2m}$ on l and can be understood in the semiclassical picture. It might happen that the value of l is modified to its q counterpart corresponding to q-Legendre functions.

3. The model could partially explain the findings of Mills and $n^{q)} \simeq 1/k$ for $k > 2$ also fixes the value of corresponding m to a very high degree so that one would have direct experimental contact with generalized imbedding space, spectrum of Planck constants, and dark matter. The fact that the fractionization is only approximately correct suggests that the states in question could be possible for all sectors of imbedding space appear as intermediate states into sectors in which the spectrum of hydrogen atom is scaled by $n_b/n_a = k = 2, 3, ...$

4. The obvious question is whether q-counterparts of angular momentum eigenstates ($idf_m/d\phi = mf_m$) are needed and whether they make sense. The basic idea of construction is that the phase transition changing $\hbar$ does not involve any other modifications except fractionization of angular momentum eigenvalues and momentum eigenvalues having purely geometric origin. One can however ask whether it is possible to identify q-plane waves as ordinary plane waves. Using the definition $L_z = 1/2(\partial_u^q + \partial_{\overline{u}}^{\overline{q}})$, $u = exp(i\phi)$, one obtains $f_n = exp(in\phi)$ and eigenvalues as $n^{q)} = R_n/R_1 \rightarrow n$ for $m \rightarrow \infty$. Similar construction applies in the case of momentum components.

10.8 Dark matter, long ranged weak force, condensed matter, and chemistry

The challenge of understanding the effects of dark weak force in condensed matter and chemistry is not easy since so many options are available. The guidelines to be used are maximal conservatism, nuclear string model and model for the cold fusion, the general criterion for the transition to dark phase, and intriguing hints that dark weak force could play important role not only in biochemistry but also in ordinary condensed matter physics contrasted with the fact that isotopic independence is not visible in the physics of condensed matter and in chemistry.

10.8.1 What is the most conservative option explaining chiral selection?

Long ranged exotic weak interactions should produce parity breaking responsible for the chiral selection. The first thing that comes in mind is that ordinary quarks or nucleons suffer a phase transition in which the p-adic prime characterizing weak space-time sheets increases, perhaps to one of the Gaussian Mersennes $k = 113, 151, ...$

There are objections against this idea.

1. The criterion $\alpha_w Q_1 Q_2 \simeq 1$ for the transition to dark phase does not apply at weak space-time sheets so that ordinary quarks should not perform this transition.

2. If ordinary nucleons make the transition to the dark weak phase with $k \leq 113$, very strong Z^0 Coulombic interaction results and isotopic dependence of chiral symmetry breaking is predicted.

3. Repulsive weak interaction would provide a nice explanation for the hard core of the interaction potential in van der Waals equation for liquid phase. Isotopic dependence is again the problem.

Nuclear string model [F9] suggests a maximally conservative model for chiral selection consistent with these objections.

1. Assume that nucleons are not affected at all in the transition and that nothing happens in the transition even at the level of exotic quarks so that they must have weak space-time sheets with size at least of order atom size.

2. The weak space-time sheet of exotic quarks associated with $k = 127$ color bonds cannot correspond to $k = 89$ since this would be seen in the decay width of the ordinary electro-weak gauge bosons. The model of cold fusion requires a phase transition transforming D to its neutral variants and this phase transition can only occur via the exchange of exotic W bosons with the range of weak interactions of order atomic distance (at least). Dark variants of $k = 113$ W bosons with $n = 2^{11}$ defines one option.

3. It would be nice to have weakly charged nuclei. Weak charges should not be however too large. This is achieved if some of the color bonds containing exotic quark and anti-quark at their ends carry net em charge and thus also weak charge. This hypothesis allows to understand tetraneutron as an alpha particle containing two negatively charged color bonds and predicts entire spectroscopy of exotic nuclei containing charged color bonds [F8, F9]. Cold fusion could be understood in terms of absence of Coulomb wall in the collision of ordinary proton with neutral variant of deuteron [F9].

4. Instead of ordinary neutrinos transformed to dark neutrinos in weak sense, neutrino species associated with with weak space-time sheets would be present and participate in the weak screening together with exotic W^+ bosons and possible exotic counterparts of electrons. The Gaussian Mersennes associated with $k = 151, 157, 163, 167$ define good candidates for the space-time sheets of exotic leptons. There is experimental evidence that neutrinos appear in several mass scales [F3].

5. Also higher levels of darkness would be allowed by the standard criterion applied to say molecules. Also a hierarchy of colored dark matters could emerge as nuclei get net color charge and combine to form molecules which are color singlets.

Consider now the implications of this picture.

1. The repulsive weak interaction between exotic of quarks of color bonds with net em and weak charge could explain the hard core of the interaction potential in van der Waals equation without isotope dependence.

2. Bio-control could occur by the variation of weak screening using W^+ bosons and exotic neutrinos. The resulting parity breaking effects would be large below the length scale L_w. Chiral selection would not have isotope dependence.

10.8.2 Questions related to ordinary condensed matter and chemistry

Consider first some questions related to ordinary condensed matter and chemistry.

1. Could electromagnetic darkness relate to the properties condensed matter?

The purely electromagnetic dark phase for $k = 113$ space-time sheets without dark weak bosons implies that atomic nuclei possess field bodies of atomic size, and one can wonder how this might relate to the basic properties of condensed matter. For instance, the linking of magnetic flux tubes of field bodies of different nuclei might have some role in quantum information processing, if the general vision of [E9] about topological quantum computation in terms of linking of magnetic flux tubes is taken seriously.

2. Does repulsive weak force relate to the stability of condensed matter?

The Coulomb repulsion of electrons could be enough to explain van der Waals equation of state. One can still wonder whether the dark weak force effective below the length scale $L_w(dark)$ could have something to do with the repulsive core in van der Waals equation of state and with the sizes of atoms in condensed matter.

The low compressibility of condensed matter indeed suggests that repulsive Z^0 force between constituent molecules is present or at least appears when one tries to compress condensed matter. The long ranged weak interactions between exotic quarks associated with color bonds of condensed matter nuclei would explain this without predicting non-trivial isotopic effects in van der Waals equation. The most conservative option is that compression induces a phase transition to a phase in which nuclei contain charged color bonds and generates strong Z^0 repulsion in the length scale of atomic radius. The fact that the density of water is reduced above freezing point when pressure is increased or temperature reduced might have explanation in terms of this mechanism.

The orthodox physicist would presumably argue that the mere electromagnetic interactions allow to understand the value of the atomic radius. The following argument challenges this belief in the case of heavy atoms.

The size of atom in the absence of the classical dark weak forces can be estimated from the expression of the radius of the orbital n given by $r_n = n^2 a$, where $a = a_0/Z$ is the radius of the lowest electronic orbital, and from the fact that a given orbital contains $2n^2$ electrons. In a reasonable approximation one has $Z = 2n_{max}^3/3$ and $n_{max} = (3Z/2)^{1/3}$. In this approximation the radius of the largest orbital identifiable as the atomic radius r_Z is

$$r_Z = (\frac{3}{2})^{2/3}\frac{a_0}{Z^{1/3}} \ . \tag{10.8.1}$$

Indeed, at distances above this radius the atom looks more or less neutral since electrons screen the nuclear charge completely. This gives an estimate for the density of the condensed phase consisting of atoms with nuclear charge Z.

$$\rho = \frac{4}{9}AZ \times \frac{m_p}{a_0^3} \ . \tag{10.8.2}$$

In case of iron ($A = 55, Z = 26$) one would have $\rho \simeq 635$ kg/dm^3: the value is roughly 100 times higher actual value $\rho = 7.8$ kg/m^3 at room temperature!

Thus the radii of heavy atoms seem to be too large in the standard physics framework. The transition to a phase in which charged color bonds are present is expected to be especially probable in the case of heavy nuclei and a generation of repulsive Z^0 force might explain the radii.

3. Could the repulsive weak core relate to the stability of chemical compounds?

Could the long ranged repulsive weak force relate the typical lengths of chemical bonds? Could it even make possible valence quark approximation? Since the generation of weakly charged color bonds and even color bonds connecting different atomic nuclei does not involve isotopic dependence, one must consider the possibility that these forces might be involved even with the physics of chemical bonds.

For instance, the generation of a chemical bond might involve formation of state containing a component in which the two nuclei have generated color bonds with opposite charges creating additional attractive force. One can also consider the possibility that nuclei generate anomalous electromagnetic charge of same sign so that a repulsive weak force between atoms results. This would give rise to a hard sphere behavior essential for the notion of valence.

At least at classical level one can question the hard sphere behavior of atoms assumed implicitly in the models of molecules based on molecular orbitals and allowing to treat full electronic shells as rigid structures so that only valence electrons are dynamical and give rise to shared orbitals. One can argue that purely electromagnetic atoms/molecules do not behave like hard spheres and that all electrons should be treated like valence electrons moving in the combined Coulomb field of the two nuclei whose distance is not fixed by the molecular size.

Since electrons are very light, one could classically regard the electronic cloud as a perfectly conducting rapidly deformable shell. When atoms approach each other the electronic charge density arranges in such a manner that it minimizes the Coulombic interaction energy between nuclei by preventing the penetration of the nuclear electric field of the other atom through the electronic shell. There the encounter of atoms would be more like a collision of point nuclei surrounded by highly deformable smooth electron mattresses than that of hard spheres.

What could go wrong with this argument? Fermi statistic might change the situation and make closed electronic shells to behave like rigid charged spheres.

10.8.3 Dark-to-visible phase transition as a general mechanism of bio-control

Dark-to-visible phase transition reduces the de-Broglie wave lengths by a factor $1/n = 2^{-11}/k$ for the favored value of the scaling factor of $\hbar$ (also other values of scaling factor are of course possible). This would essentially code patterns in dark length scale to patterns of visible matter in much shorter length scale and make possible long length scales to control short length scales in a coherent manner. This phase transition could occur separately on em, weak, and color space-time sheets. For instance, the dark phase of hydrogen ions in the case of proton need not involve dark weak phase.

The hierarchy of dark matters defines naturally a control hierarchy ordered with respect to time and length scales. Dark electrons would be functional at the lowest level of the control hierarchy whereas dark neutrinos would naturally appear at the higher levels.

The strange properties of water could be understood to a great extent if a fraction of protons has made a transition large $\hbar$ phase in electromagnetic sector (as discussed, this could actually mean that the em bodies of protons have large $\hbar$). This does not require anything anomalous in the weak and colored sectors.

The criterion for the transition is that a system consisting of sub-systems with charges Z_1 and Z_2 makes a transition to dark matter phase reads as $\alpha_{em} Z_1 Z_2 \simeq 1$.

Option I: If this criterion applies to self interactions as such, it would give in the case of atomic nuclei $Z_{cr} = 12$ (Mg).

Option II: If full nuclear shells are non-interacting, as one expects on basis of Fermi statistics, the criterion could be interpreted as stating that only nuclei having $Z = 2 + 6 + 12 = 20$ (the self interaction of the full third shell would induce the transition) can make this transition [F8]. That Ca ions ($Z = 20$) satisfy this condition would conform with the fact that play a unique role in bio-chemistry and neurophysiology.

Option III: If the criterion does not apply to self interactions and only full shells interact, the condition would be that the nucleus contains nucleon clusters with charge $Z_1 = Z_2 = 20$ giving $Z_{cr} = 40$ if the critical interaction is between separate $Z = 12$ shells. TGD inspired view about nuclear physics [F8] based on dark valence quarks and $k = 127$ exotic quarks with ordinary value of $\hbar$ at the ends of long color bonds responsible for nuclear strong force suggests that nuclei could be regarded as collections of linked and knotted nuclear strings and that the linking of magic nuclei produces new especially stable nuclei.

Cold fusion with Pd catalyst [65] having $Z = 46$ could involve local transitions of Pd catalyst to $k = 113$ dark matter phase and perhaps also the transition $k = 89 \rightarrow 113$.

For option III trace elements with $Z \geq 40$ should play a key role in living matter inducing phase transitions of lighter nuclei to dark phase as the model for cold fusion suggests. There is some support for this interpretation.

1. DNA is insulator but the implantation of Rh atoms in DNA strands is known to make it super-conductor [99], perhaps even super-conductor. Dark electrons obviously define a good candidate for the current carriers.

2. The so called mono-atomic elements [43] claimed by Hudson to possess very special physical properties have explanation in terms of dark matter phase transition [J6] and have $Z \geq 44$. Interestingly, Hudson claims that mono-atomic elements have not only very special biological effects but also affect consciousness, and that 5 per cent of brain tissue of pig by dry matter weight is Rhodium ($Z = 45$) and Iridium ($Z = 77$).

10.8.4 Long ranged weak forces in chemistry and condensed matter physics

According to the model of water, one fourth of hydrogen ions would be in dark phase such that $k = 113$ space-time sheet has transformed to large $\hbar$ phase and would have size of order atomic radius. This would suggests that that the atomic size could be understood in terms of large $\hbar$ associated with $k = 113$ electromagnetic space-time sheet. Weak interactions in this phase could be normal. Quantum classical correspondence forces however to consider the possibility for which also long range weak force is present-

Exotic nuclear quarks as sources of long ranged weak force

One can a consider a copy of weak physics for which weak space-time sheets of particles have $k > 89$, say $k = 113$. This would imply strong parity breaking effects in $k = 113$ length scale. If this transition is followed by a transition of $k = 113$ space-time sheet to dark matter phase with large value of $\hbar$, the length scale $L_w(dark) = n2^{11}L(113)$ in which strong parity breaking effects occur corresponds to atomic length scale. This kind of phase could explain chiral selection in living matter and dark weak boson condensates and dark quarks and leptons might play a fundamental role in bio-control.

The criterion for the transition to the large $\hbar$ phase does not suggest that this transition could happen to ordinary quarks and leptons. Also the fact the absence of non-trivial isotopic dependence in chemistry and condensed matter supports the conservative view "once vacuum screened-always vacuum screened".

The TGD based model of atomic nuclei however involves besides dark valence quarks color bonds having $k = 127$ quarks at their ends and their weak space-time sheets cannot correspond to $k = 89$ since this would be reflected in the decay widths of weak bosons. One possibility is that the weak space-time sheet corresponds to $k = 113$, possibly with large $\hbar$.

TGD based identification of tetra-neutron is as an alpha particle containing two negatively charged color bonds [F8]. Since there is no reason to expect that tetra-neutron would be a rare exception, one expects that ordinary nuclei of condensed matter can make transition to a phase in which some color bonds are em charged and thus carry also weak charges creating long ranged weak forces and parity breaking without the un-acceptable isotopic independence. The unavoidable long ranged weak and color fields associated with non-vacuum extremals suggest even more radical possibility. The nuclear strings associated with neighboring condensed matter nuclei could fuse to single nuclear string so that nuclei would be color and weakly charged and could carry fractional em charges.

Below $L_w(dark)$ atoms whose nuclear color bonds carry net weak charges would look like Z^0 ions and condensed matter in this phase would be kind of Z^0 plasma. The weak forces could be screened by vacuum charges above the length scale $L_w(dark)$ just as they are screened usually. Dark weak bosons would have mass obtained by scaling down the intermediate gauge boson masses by a factor 2^{-12} for $k = 113$. An essential point is that the Z^0 charge density of nuclei would be constant below L_w rather than that corresponding to Z^0 charges with nuclear size. This makes Z^0 screening by particles much more easier and the question is not whether to achieve precise enough screening in say nuclear length scale but in what scale it is possible to vary the degree of screening.

Could long ranged weak forces be key players in bio-catalysis?

Bio-catalysis involves chiral selection in an essential manner which suggests that weak force is involved. This inspires the question about the underlying mechanisms controlling the assembly and de-assembly of bio-molecules.

1. Bio-catalysis and phase transition to a phase containing charged color bonds?

The considerations related to van der Waals equation and the fact that color bonds could be unstable against beta decay via the emission of light W boson nucleon suggest that nuclei could tend to develop color bonds with the same sign of Z^0 charges. Anomalous em charges could vanish if the transition involves an emission of a dark W boson charging color bond transforming to ordinary weak boson by de-coherence and absorbed by nucleon. This kind of transition could proceed spontaneously as a two-nucleon process if the nuclei are close enough as in the situation when liquid is compressed.

If so, the resulting weak forces tend to de-stabilize these molecules. The range $L_w \simeq 2.56L(89)$ gives for this force a scale about $2.56 \times L(k_{eff} = 133) \simeq 1.3n$ Angstrom if scaled directly from the Compton length of intermediate gauge boson assuming the scaling $\hbar \rightarrow n\hbar/v_0$. $n = 3$ gives the length scale of the typical chemical bond in DNA.

The molecules need not become un-stable in the phase transition to the phase containing charged color bonds. The phase transition could only reduce the binding energies of the chemical bonds and facilitate chemical reactions serving thus as a catalyst.

Dark molecules of form AH_n, where A is arbitrary atom and H_n refers to n hydrogen atoms be in the role of biological hardware since they are not affected appreciably by this kind of phase transition. The basic molecules of life are indeed molecules of type CH_n, OH_n, NH_n, which could of course be also partially dark.

2. The variation of the strength of the Z^0 force as a control mechanism

The variation of the strength of the repulsive Z^0 force could be achieved by varying the density of screening particles. To be effective this tool should allow sharp enough length scale resolution and the resolution is determined by the p-adic length scale of the screening particle. The situation is dramatically improved by the fact that the Z^0 charge density to be screened is constant below L_w. Hence a constant Z^0 charge density of screening charges is enough to achieve a complete screening. The control of the degree of Z^0 ionization rather than control of Z^0 charge density would be in question.

3. What distinguishes between ordinary condensed matter and living matter?

If weakly charged color bonds appear already in ordinary condensed matter and give rise to the repulsive core in van der Waals equation of state, one can wonder what is the real distinction between living matter and ordinary condensed matter. The difference might relate to the value of n for the transition $\hbar \rightarrow n\hbar/v_0$ for electromagnetic space-time sheets. $n = 1$ could correspond to ordinary condensed matter with L_w in the range of 1-2 Angstrom and $n = 3$ to living matter with L_w in the range 3-6 Angstrom. Water could differ from other condensed matter systems in that it would have $n = 3$ for one fourth of hydrogen ions.

A second question relates to the identification of the weak space-time sheet of exotic quarks. Can one assume that the weak space-times sheet of exotic quarks and em space-time sheet of valence quarks in dark em phase both correspond to $k = 113$ with large $\hbar$? This hypothesis can be defended: below L_w dark electro-weak symmetry is not broken so that em and weak interactions should take place at the same space-time sheet.

10.8.5 Z^0 force and van der Waals equation of state for condensed matter

Most physicists probably think that van der Waals equation of state represents those aspects of condensed matter physics which have been thoroughly understood for long time ago. Approximate isotopic independence of the basic parameters of the state equation provides support for this belief. Isotopic independence does not however exclude the role of long ranged weak forces if they are associated with exotic $k = 127$ quarks appearing in the TGD based model of nucleus [F8]. The decay widths of weak bosons require that exotic weak bosons correspond to some other p-adic length scale than $k = 89$, say $k_{eff} = 113 + 24 = 137$ for large $\hbar$ or $k = 151$ for ordinary $\hbar$. The presence of em charged color bonds in ordinary nuclei would provide them with anomalous em and weak charges and bring in long ranged weak force.

One can imagine various scenarios for how dark weak forces might enter condensed matter physics.

1. It might be energetically favorable for the ordinary condensed matter nuclei to be in a phase containing charged color bonds. By the charge independence of strong interactions this would

not considerably affect the nuclear physical properties of condensed matter nuclei. The hard core of the interaction potential in van der Waals equation could be seen as a signature of dark weak force.

2. The nuclei could be ordinary in the ordinary liquid phase (water forming a possible exception) so that long ranged weak forces need not be present. The low compressibility of the liquid phase could however be due to a phase transition of nuclei inducing charged color bonds by exotic weak decays of exotic quarks. This would induce a repulsive weak force felt in the length scale L_w of order $3-6$ Angstrom for $k = 113$ and $\hbar \rightarrow n\hbar/v_0$, $n = 3$. The dark weak force becoming visible only when liquid is compressed would explain the hard core term in van der Waals equation. The energy provided by the compression would feed in the energy making possible the phase transition not occurring spontaneously. Sono-luminescence [81] could represent a situation in which the phase transition occurs.

   The phase transition generating charged color bonds could be induced by the direct contact of the nuclear em field bodies of exotic quarks and anti-quarks with size associated with any nucleus having $A > 1$ and having field em field body with size $L \sim nL(113)/v_0$ of order atomic radius (this point is discussed in detail in the model of nuclei based on color bonds [F8]).

Both options predict isotopic independence of compressibility and essentially standard nuclear physics. The explanation for the anomalous behavior of water above its freezing point, in particular the reduction of density as the temperature is lowered or pressure increases, could be basically due to the appearance of additional color bonds in oxygen nuclei during compression.

These considerations raise the question how weak forces reveal their implicit presence in the basic argumentation leading to van der Waals equation of state. In the sequel the deduction of van der Waals discussed in more detail to make more explicit the origin of the hard core term.

1. Van der Waals equation of state

Van der Waals equation of state provides the simplest thermodynamical model for gas-liquid phase transition. The equation can be derived from thermodynamics using the following assumptions.

1. The partition function Z_N for a condensed matter system consisting of N identical particles codes the thermodynamical information and can be deduced once the Hamiltonian of the system is known.

2. It is assumed that the Hamiltonian separates into a sum of single particle Hamiltonians $H = \sum H_i = T + U = \sum T_i + \sum U_i$. Single particle Hamiltonian consists of a sum of the kinetic energy T_i, the energy associated with internal degrees of freedom (such as rotational degrees of freedom of the molecule), and the potential energy $U_i = \sum_{j\neq i} u_{ij}$.

3. The potential energy u_{ij} is assumed to depend on the relative coordinate $\overline{r}_i - \overline{r}_j$ only and to be large and positive at short distances and vanish rapidly at large distances. Also spherical symmetry can be assumed in a good approximation. Above $2r_0$, r_0 molecular radius, u is assumed to be small and negative and in this manner generate an attractive force, which can be assumed to be of electromagnetic origin.

Consider now the approximate deduction of the equation of state.

1. The partition function factors into a product of the partition function Z_N^{id} of ideal gas and a term defined by the potential energy terms in the Hamiltonian of the whole system.

$$\begin{aligned} Z &= Z_N^{id}(T) \times Q_N(T,V) \ , \\ Q_N(T,V) &= \frac{1}{V^N} \int \prod_i dV_i exp(-U/T) \ . \end{aligned} \tag{10.8.2}$$

2. The standard manner to derive an approximate form of the partition function, free energy and pressure in turn providing the equation of state is based on the so called virial expansion using the

elementary multiplicative properties of the exponential function $exp(-U/T) = \prod_{i,j} exp(-u_{ij}/T)$ appearing in Q_N. In the lowest non-trivial order one has

$$\begin{aligned} Q_N(T,V) &\simeq \frac{N^2}{V} I_2 \ , \\ I_2 &= \int dV \lambda(\bar{r}) \ , \\ \lambda(\bar{r}) &= exp(-u_{12}(\bar{r})/T) - 1 \ . \end{aligned} \tag{10.8.1}$$

The integrand in this expression is in a good approximation equal to -1 inside the sphere of radius $2r_0$ defined by the minimal distance between the molecules of radius r_0 and positive outside this sphere and approaches zero rapidly.

3. Quite generally, one can write Q_N as

$$\begin{aligned} Q_N(T,V) &\simeq 1 + N \times \frac{n}{2} \times I_2 \simeq (1 + \frac{nI_2}{2})^N \ , \\ n &= \frac{N}{V} \ . \end{aligned} \tag{10.8.1}$$

The improved approximation is dictated by the fact that free energy must be an extensive quantity. For the free energy $F = -Tln(Z)$ one obtains an approximate expression

$$F = NF^{id} - NTnI_2 \ . \tag{10.8.2}$$

For the pressure $P = -(\partial F/\partial V)_{T,N}$ one obtains

$$P = nT(1 - nI_2/2 + \cdots) \ . \tag{10.8.3}$$

4. The value of I_2 can be calculated approximately by dividing the integration region to two parts. The first part corresponds to a sphere of radius $2r_0$ (r_0 is the radius of molecule) inside which $\lambda_{12} = -1$ could be interpreted in terms of the approximate vanishing of the exponential of the interaction potential behaving like $1/r$. The second part corresponds to the exterior of the sphere of radius $2r_0$, where λ is assumed to have positive but small values so that the exponential can be approximated by the first two terms of the Taylor series with respect to u_{12} This gives

$$I_2 \simeq -\tfrac{4\pi}{3}(2r_0)^3 + \tfrac{4\pi}{T} \int drr^2 u_{12}(r) \equiv 2b - 2a/T \ . \tag{10.8.4}$$

Note that $a > 0$ implied by $u_{12} \leq 0$ holds true.

5. The resulting equation of state is

$$P + n^2 a = nT(1 + nb) \ . \tag{10.8.5}$$

This equation is second order in n and does not give the characteristic cusp catastrophe associated with the van der Waals equation.

6. The approximation

$$1 + nb \simeq \frac{1}{1 - nb} \quad (10.8.6)$$

holding true for $nb << 1$ and then extrapolating to a region where this condition does not hold true. This gives the van der Waals equation of state

$$(P + n^2 a)(1 - nb) = nT \quad (10.8.7)$$

allowing a simple description of gas-to liquid phase transition requiring that at least third power of n appears in the equation of state. The equation allows an attractive physical interpretation. $P_{in} \equiv n^2 a$ can be identified as internal pressure mainly due to the attractive van der Waals force and 1-nb tells the fraction of free volume so that $P_{tot} V_{free} = NT$ holds true.

This trick is believed to take into account the neglected higher order terms in the virial expansion. The proper justification comes from the catastrophe theory [18]. The virial expansion gives all orders in n to the right hand side of Eq. 10.8.5 and by the general theorems of catastrophe theory cusp catastrophe is the singularity associated with a state equation with two control variables a and b. What the cusp catastrophe means is that three values of n satisfy the equation of state for given values of P and T. Two of these values correspond to stable phases, liquid and gas, the lower and upper sheets of the cusp, whereas the intermediate sheet of the cusp corresponds to an unstable phase.

In TGD framework a could be interpreted as characterizing purely electromagnetic interactions above the critical radius r_0 and and b both em and long ranged interactions below r_0. The emergence of repulsive Z^0 interactions below the critical radius r_0 would serve as a physical definition for r_0. The fraction of free volume $1 - nb$ would differ from unity because repulsive dark weak forces enter in play when the number density n tends to become larger than $1/b$.

In a very optimistic mood one might provocatively claim that the classical Z^0 Coulombic force allows to understand why the hard core approximation behind van der Waals equation works and that the setting on of dark weak force provides a precise first principle definition for the notion of the molecular radius. The criticality implied by the Z^0 Coulombic force would reflect itself as the criticality of the liquid-gas phase transition. Obviously the parameter b contains very little information about the details of the Z^0 Coulombic interaction energy besides the fact that the phase transition charging some color bonds weakly occurs when molecules are at distance $r < r_0$. The calculation of the value of the parameter a should reduce to standard electromagnetic interactions between molecules.

10.8.6 Z^0 force and chemical evolution

Although long ranged weak forces manages to hide themselves very effectively, they leave some tell tale traces about its presence. The most spectacular effect is chiral selection which is extremely difficult to understand in the standard model. Also the mysterious ability of noble gases to act as anesthetes [92] could be understood as being due to dark weak forces. If a phase transition charging some color bonds of the noble gas nuclei increasing or reducing Z occurs, noble gas atoms behave chemically as ions. A discussion (somewhat obsolete now) of the mechanism can be found in [M2].

Classical Z^0 force might also make itself visible by delicate chemical effects due to the fact that the classical Z^0 charge of the hydrogen atom vanishes. Since the exotic Z^0 charges of proton and electron necessarily vanish by the absence of color bonds the prediction is that proton and electron are in a completely exceptional role in chemistry, and in biochemistry in particular. Certainly this is the case: consider only the role of proton and electron in biochemistry (say in metabolic cycles and in polymerization). Furthermore, Z^0 force seems to be the key player in the biochemical evolution in TGD Universe: molecular stability could be controlled by the possibility to generate charged color bonds and by the screening of long ranged weak forces.

Enzymatic action, known to involve chiral selection, can be based on the control of the strength of the classical Z^0 force by varying the densities of the Bose-Einstein condensates responsible for the Z^0

screening. Metabolism involves basically the chopping of the nutrient molecules to pieces and their re-assembly. The chopping into pieces could be partially achieved by weakening the screening of the classical Z^0 force locally. The sizes of the enzymes and ribozymes are rather large and vary in the range 10-20 nm. This is not easily understood in the standard chemistry context but is what one expects if $k = 151$ weak bosons are involved.

An interesting hypothesis is that chemical evolution has proceeded via a sequence of phase transitions producing dark weak bosons corresponding to Gaussian Mersennes $G_k = (1+i)^k - 1$, $k = 113, 151, ...$ as $k = 89 \rightarrow 113$ followed by $k = 113 \rightarrow 151 \rightarrow 157 \rightarrow 163 \rightarrow 167 \rightarrow ....$

10.8.7 Parity breaking effects at molecular level

The observed parity breaking effects at molecular level are large: a natural unit for molecular dipole moments is one Debye: $e10^{-10}\ m \sim eL(137)$. This scale compares favorably with the $k = 113$ weak length scale $L_w = nx$ Angstrom, $x \in [1,2]$, $n = 1,2,3$. The larger the value of n, the larger the scale of parity breaking. The breaking of the mirror symmetry appears at geometric level and this kind of symmetry breaking does not require large parity breaking at the level of physics laws. The parity breaking however takes place in a much deeper manner: only second chirality of two mirror image molecules appears in Nature and an unsolved problem is to understand this selection of the molecular chirality.

The axial part of weak forces, in particular Z^0 force, suggests a first principle explanation for the molecular parity breaking. A phase transition generating dark weak force below length scale L_w would induce axial force implying different energies for mirror images of molecule.

Mechanism of parity breaking

One can imagine two mechanisms of chiral selection. For the first mechanism the classical Z^0 interactions between the atoms of the molecule lead to a chiral selection. If equilibrium positions correspond to the minima of Z^0 Coulomb energy, the parity breaking effect, being proportional to the gradient of Z^0 scalar potential, however vanishes. Of course, the net force involves both electromagnetic and Z^0 contributions so that the equilibrium positions do not actually correspond to the minima of Z^0 Coulomb potential. Proton is an exception because of its small vectorial Z^0 charge and by the fact that it is the only nucleus not containing color bonds (assuming that self bonding does not occur).

Second mechanism is based on the presence of an external Z^0 electric field and to the fact that the energies of a chiral molecule and its mirror image in an external Z^0 electric field are different. In this case the parity breaking contributions of the individual atoms of the molecule to the energy are in general non-vanishing and lead to chiral selection. The presence of classical Z^0 electric fields in bio-matter would not be surprising since bio-matter is also ordinary electret. Spontaneous Z^0 electric polarization might be an essential element of chiral selection and lead to energy minimization. This kind of phase transition might be induced by a rather small external perturbation such as bombarding of a system containing both chiralities with neutrinos or electrons.

Detailed form of the parity breaking interaction

Consider first in more detail the form of parity breaking interaction.

1. In molecular physics the minimization of the energy for electronic configurations selects the ground state configuration for atoms in the molecule (this is essentially due to the small mass ratio m_e/m_p).

2. The parity breaking force is proportional to the axial part of weak isospin, which is of same magnitude for all particles involved. Axial force is proportional to the gradient of Z^0 scalar potential created by exotic quarks in color bonds. Axial force is also inversely proportional to the mass of the particle involved.

   The mass scale of exotic quarks is determined by $k = 127$. The hypothesis that lepto-hadrons are bound states of colored excitations of leptons predicts also $k = 127$ for their mass scale and colored electrons would have essentially the same mass as electrons. One can make only guesses about the p-adic mass scale of exotic (possibly dark) neutrinos and electrons. The maximally non-imaginative hypothesis is that the scales are same as for ordinary leptons. In this case the mass would by a factor of about 10^{-6} smaller for dark $k = 169$ neutrinos with mass about .1 eV than for exotic quarks with mass $\sim$ .1 MeV if p-adically scaled down from that of ordinary quarks [F8]. Therefore the presence of dark neutrinos could induce the dominating parity breaking effects. For this option the Z^0 binding energy would be much larger than neutrino mass for reasonable values of nuclear Z^0 charge, which would favor the Z^0 screening by neutrinos.

3. The parity breaking Z^0 interaction energies of exotic $k = 127$ quark and anti-quark at the ends of color bond are of same sign in three cases corresponding to pion type color singlet bonds $q^{\uparrow}\overline{q}^{\downarrow}$ and em and color charged bonds $u^{\uparrow}\overline{d}^{\uparrow}$ and $d^{\uparrow}\overline{u}^{\uparrow}$. Thus the parity breaking interaction does not require the presence of color charged bonds and is in principle present for all nuclei but can of course cancel in good approximation if the net spins of $k = 127$ quarks and anti-quarks do not cancel separately.

4. For Fermi sea of dark neutrinos the parity breaking effects on energy are proportional to spin and sum up to zero if the number of neutrinos is even. Note however that complete screening is not required.

Consider now a more quantitative estimate.

1. The axial part of the Z^0 force acting on neutrinos is given by

$$V_{NPC} \simeq \pm\alpha_Z Q_Z^A(\nu)Q_Z^V(\nu)\frac{1}{m(\nu)}\bar{S}\cdot\nabla V_Z(\bar{r}) \ . \tag{10.8.8}$$

2. The order of magnitude for the energy difference of a configuration and its mirror image is obtained as the difference of axial interaction energies for configurations related by reflection. Consider a particle with Z^0 charge $Q_{Z,1}$ and mass m experiencing the axial Z^0 field created by a nucleus with anomalous Z^0 charge $Q_{Z,2}$. In this case the contribution to energy difference has order of magnitude

$$|\Delta E| \sim \frac{\alpha_Z(Q_{Z,1}Q_{Z,2}}{4mL^2} \ , \tag{10.8.8}$$

where $L \leq L_w$ is the typical distance between nucleus and the particle involved.

3. Consider now various options for the parity breaking assuming first $k = 113$ dark weak matter so that L is of order of size of atom.

   i) For $k = 169$ neutrino one would have $\Delta E \sim 1$ MeV, which does not sound reasonable. If partial neutrino screening is present for $k = 113$ at all, it must involve spin pairing. As already found, neutrino screening cannot be ideal for $k = 113$ since the Fermi energy would be rather high. Partial screening favored by the negative energies of dark neutrinos cannot be however excluded since single neutrino could be shared between several constituents of, say, linear molecule. For $k = 151$ for neutrino and electron one would have $\Delta E \sim 2$ keV.

   ii) For an exotic electron with ordinary mass but $k = 113$ weak space-time sheet the order of magnitude is $\Delta E \sim 2$ eV, which corresponds to visible frequencies. For exotic quarks with mass $m \sim .1$ MeV one would have $\Delta E \sim 10$ eV. For both cases it would not be chiral selection which would thermally unstable but the dark weak phase itself, and the selection would be absolute in the temperature range were dark weak phase is possible.

   iv) For dark $W^+(113)$ bosons having mass ~ 25 MeV one would have $\Delta E \sim 10^{-2}$ eV, which corresponds to the scale of room temperature. Unfortunately, the large mass and short lifetime of $W^+(113)$ do not favor this idea.

4. Consider now $k = 151$ weak bosons. The difficulties of $W^+(113)$ option are circumvented in the case of $W(151)$ with mass of ~ 50 eV since leptonic decays become impossible. The generation of $W^+(151)$ BE condensate is also energetically favorable due to the large Z^0 binding energy. $L(151)$ corresponds to the thickness of the cell membrane and to a minimal length of DNA double strand giving rise to an integer multiple of 2π twist with integer number (10) of DNA triplets. Note however that the large $\hbar$ length scale would be $L \sim nL(151 + 22 = 173) \simeq n \times 20$ μm. The decay of the BE condensate of dark $W(151)$ bosons (with large value of $\hbar$) to non-dark $W(151)$ bosons could allowing the control of $k = 151$ length scale by $k = 173$ length scale.

   In this case one would have $\Delta E \sim 5$ keV so that chiral selection would be highly stable. This option could be realized for linear bio-molecules. Hence the Bose-Einstein condensate of screening $k = 151$ W^+ bosons possessing net spin must be considered as a candidate for a mechanism inducing chiral selection of bio-polymers. The positive charge of the W^+ condensate could relate to the negative charge characterizing bio-polymers.

   If the order parameter of W^+ condensate around the molecule is spherically symmetric, the average interaction energy vanishes so that W bosons should possess also orbital angular momentum: the simplest option is that net angular momentum vanishes. The geometric breaking of spherical and reflection symmetries of the molecule would naturally induce the needed asymmetry of the order parameter.

10.8.8 Hydrogen bond revisited

Hydrogen bond is fundamental for the physics of water and believed to relate to its anomalous expansion at freezing point and anomalous contraction in heating above freezing point. Hydrogen bond plays also a key role in the living matter. Against this background it is perhaps somewhat surprising how poorly understood the physics of the hydrogen bond is.

The special role of hydrogen bond is consistent with the suggested role of dark Z^0 force. Hydrogen bond is believed to reflect ordinary Coulomb interaction between hydrogen bound to molecule and lost its electron partially to the molecule and electronegative atom (N, O, Cl,...) which has captured partially the electron of the atom with which its bonds, say C, and which therefore looks like having positive charge. Hydrogen bonds are in a key role in the binding of DNA strands, in the generation of geometric structure of proteins and RNA molecules, and also the molecular motors are constructed from their building blocks by hydrogen bonds. The reason why could be very simple: hydrogen bonds unlike valence and ionic bonds are relatively immune to the bio-control based on the variation of the classical Z^0 force by varying the Z^0 screening.

An interesting question is whether the hydrogen bonded state A+B of atoms A and B could be in a superposition of states with A and B in the ordinary state and a state in which A/B contains positively/negatively charged color bond changing the charge numbers A and B and effectively creating ionic bond.

If the hydrogen bond corresponds to a non-vacuum extremal in necessarily carries color gauge flux. Quantum classical correspondence together with the picture about nuclei as nuclear strings with nucleons connected by long color bonds forces to ask whether the nuclear strings of hydrogen bonded atoms fuse to form single nuclear string containing long straight section connecting the nuclei. Hydrogen bonded nuclei would become both colored and weakly charged in this kind of situation and would posses also a fractional electromagnetic charge not explainable in terms of fractional quantum Hall effect. In this kind of situation the first guess is that the exotic quark pairs associated with the color bond could play the role of valence electrons and characterize both the binding energy and parity breaking possibly associated with the bond.

10.9 Long ranged weak and color forces, phonons, and sensory qualia

Phase conjugate electromagnetic waves [43, 44] correspond in TGD framework negative energy topological light rays representing signals propagating to the geometric future [G3]. Phase conjugation is known to make sense even for sound waves [44]. Since phase conjugation means time reversal and negative energies in TGD framework, the only possible conclusion seems to be that classical sound waves and photons must correspond to their own space-time sheets. Depending on the time orientation of these space-time sheets, sound waves or their phase conjugates result in the interaction of these space-time sheets with matter.

If condensed matter is partially dark in the sense that nuclei tend to combine to form super-nuclei, the question arises whether dark weak force and dark nuclear strong force are involved with the sound waves besides em forces. Topological light rays ("massless extremals", briefly MEs) carrying classical gauge fields corresponding to an Abelian subgroup of the gauge group, be it color or electro-weak gauge group, and drifting quantum jump by quantum jump in the direction of sound wave define candidates for the space-time correlates of sound waves. Also the deformations of warped imbeddings of M^4 to $M^4 \times CP_2$ with maximal signal velocity reduced to sound velocity using M^4 as standard define candidate for the space-time sheets associated with sound waves.

In plasma phase classical electric field can cause plasma waves as longitudinal oscillations of charge density. Also the notion of Z^0 plasma wave makes sense if nuclei carry anomalous Z^0 charges due to charged color bonds. Entire dark hierarchy of these waves is possible. Even the counterparts of QCD plasma waves are possible.

10.9.1 Slowly varying periodic external force as the inducer of sound waves

The basic idea is that an external force, which is constant in the length scale of atomic nuclei or molecules sets them in a harmonic motion around equilibrium point. This slowly varying force is associated with the space-time sheet serving as the space-time correlate of phonon.

The basic fact about quantum physics of harmonic oscillator is that the resulting new ground state represents a *coherent* state having interpretation as a classical state of harmonic oscillator. If the external force depends periodically on time and spatial coordinates the intensity of the parameter characterizing coherent state varies in oscillatory manner and classical sound wave results as a consequence.

10.9.2 About space-time correlates of sound waves

Z^0 MEs ("massless extremals") represent transversal classical Z^0 fields propagating with light velocity. These transversal fields are candidates for the external force generating the coherent states giving rise to sound waves. There are however two problems.

1. How it is possible that sound velocity v is below light velocity?

2. How the Lorentz force orthogonal to the direction of propagation of classical fields inside ME can give rise to longitudinal sound waves.

One can imagine two solutions to these problems.

Option I: The first solution to both problems could be as follows. Let Z^0 ME represent a wave moving in z-direction with light velocity and let sound wave propagate in the direction of x-axis with sound velocity v_s. Assume that Z^0 electric field of linearly polarized ME is in x-direction, and thus defines a longitudinal force field inducing the coherent state. Also Z^0 magnetic field is present but for non-relativistic particles it is by a factor v/c weaker than Z^0 electric force and can be thus neglected.

Z^0 ME suffers in each quantum jump a shift consisting of a shift in z-direction and a shift in x-direction. The shift in the z-direction causes an effective reduction of the phase velocity of the field pattern inside ME. The shift in the x-direction means that the Z^0 electric field of ME moves is in x-direction and causes a longitudinal force. The velocity of the shifting motion in the x-direction must be sound velocity.

The classical force field is in a correct phase if Z^0 ME shifts in z-direction with such an average velocity that the phase $\omega t - kz$ along ME at point (t, x, y, z) changes to $\omega t - kz + \omega\Delta t - k_1\Delta x$ in the shift $x \rightarrow x + \Delta x$ of the position of ME resulting in quantum jump sequence corresponding to $t \rightarrow t + \Delta t$. This requires $\Delta z = (k_1/k)\Delta x$ giving $dz/dx = c/v_s$. Hence the rays $x = v_s t$ of constant phase for sound wave correspond to the rays of constant phase $z = ct$ along ME.

In the case of transversal sound oscillations possible in solid state Z^0 MEs shift in each quantum jump in z-direction in such a manner that effective phase velocity becomes sound velocity. Z^0 MEs generate oscillating transverse electric field inducing a coherent state of phonons. I have already earlier proposed that nerve pulse propagation corresponds to a propagation of Z^0 ME in an analogous manner [M2].

Option II: By quantum classical correspondence one might argue that sound propagation should have a direct space-time correlate. There exists an infinite variety of vacuum extremals with $D = 1$-dimensional CP_2 projection having a flat induced metric. These extremals correspond to warped imbeddings $m^0 = t, s^k = s^k(t)$ of M^4 with the induced metric $g_{tt} = 1 - R^2 s_{kl}\partial_t s^k \partial_t s^l$, $g_{ij} = -\delta_{ij}$. The maximal signal velocity using the canonical imbedding of M^4 as a reference is reduced to $c_\# = \sqrt{g_{tt}}$.

$D = 2$ vacuum deformations for this kind of space-time sheets exist but the great question mark are there non-vacuum deformations which correspond to solutions of field equations. Do they represent waves propagating with $c_\#$? This could be the case since the field equations for these deformations contain a term proportional to linearized d'Alembert equation in the background metric. Could phonon space-time sheets correspond to deformations of vacuum extremals of this kind analogous to MEs with $c_\#$ identifiable as sound velocity? Could phonons correspond to 3-D light-like surfaces representing wave fronts inside deformed vacuum extremals of this kind? Could the drifting of MEs have this kind of space-time sheets as a space-time correlate?

10.9.3 A more detailed description of classical sound waves in terms of Z^0 force

The proposed rough model is the simplest description in the case of condensed matter as long as the positions of particles vary slowly in the time scale of the oscillations associated with the sound wave.

A modified description applies when harmonic forces are between neighboring atoms. In this case the modification of standard wave equation would introduce a term representing external force to the wave equation. In one-dimensional case of one-dimensional periodic lattice with lattice constant a, elastic constant k for the elastic force between nearest neighbors, and atom mass m, one would have in the continuum approximation

$$\begin{aligned} (\partial_t^2 - v_s^2\nabla^2)A &= \frac{Q_Z E_Z}{ma} \ , \\ v_s^2 &= \frac{ka^2}{m} \ . \end{aligned} \quad (10.9.0)$$

Here a denotes lattice constant.

Temporally slowing varying Z^0 force to an harmonic external force yielding coherent states of the quantized system. Velocity resonance results when the external Z^0 field pattern has effective phase velocity equal to sound velocity $E_Z = f(u_+)$, $u_\pm = x \pm v_s t$. Writing the equations in the form

$$\partial_+\partial_- A = \frac{Q_Z f(u_+)}{ma} \ , \quad (10.9.1)$$

one finds that the general solution is of form

$$A = A_+(u_+) + A_-(u_-) + u_+ \frac{Q_Z}{ma} \int du_- f(u_-) \ . \quad (10.9.2)$$

A_+ and A_- are arbitrary functions of their argument. In the absence of dissipative effects the amplitude increases without bound.

The quantization of the model is straightforward since a one-dimensional "massless" field coupled to an external source is in question with sound velocity taking the role of light velocity. The resulting asymptotic ground state is a product of coherent states for the frequencies present in the external force term. In quantum field theory this kind of state is interpreted as a maximally classical state and thus classical sound wave.

The intensity of the sound wave would be proportional to the modulus squared of the order parameter of the coherent state proportional to the Fourier transform of the classical Z^0 force. The standard classical model for sound waves would thus be only apparently correct. In TGD framework the screened dark Z^0 force gives a contribution also to the elastic forces between atoms and explains the strong repulsive potential below atomic distances implying incompressibility of condensed matter and needed in van der Waals equation of state.

Also in the hydrodynamics dark Z^0 force would take the role of an external force. Although the quantization of the Euler's equations is far from being a trivial task and perhaps not even sensible, the proposed picture is expected to be the same also in this case for small oscillations for which wave equation holds true. In TGD framework incompressible hydrodynamic flow is interpreted from the beginning in terms of dark Z^0 magnetic force [D1], and this should make possible a first principle quantization of sound waves in the case of liquid and gas phases.

1. The hydrodynamic flow occurs along the flux tubes of Z^0 magnetic field and it is quite possible that Z^0 superconductivity equivalent with super-fluidity along flux tube occurs in sufficiently short length scales. The presence of Z^0 magnetic flux tubes parallel to the flow lines is what makes possible to apply hydrodynamic description. The incompressibility inside Z^0 magnetic flux tubes is due to the fact that Z^0 magnetic field has a vanishing divergence. Alfwen waves, identifiable as transverse oscillations of magnetic flux tubes and propagating with light velocity along the flow lines should have Z^0 counterparts and might have detectable effects on the hydrodynamic flow.

2. The Beltrami condition $\nabla \times v = \alpha v$ guarantees that a coordinate varying along flow lines is globally defined and means that super-conducting order parameter defined along the flow lines can be continued to a function defined everywhere so that there is Z^0 superconductivity also in the global sense. The complex patterns of flow reduce to the generalized Beltrami property of the topologically quantized flow. Also in the case of gas phase one expects incompressibility inside the flux tubes at least.

10.9.4 Does the physics of sound provide an operational definition of the dark Z^0 force?

The somewhat surprising conclusion supported by the existence of phase conjugate sound waves is that coherent sound waves could be a direct manifestation of the dark Z^0 force directly determining the amplitude of the sound wave understood as a coherent state. Therefore the problem of defining the notion of dark Z^0 force operationally would become trivial.

The hypothesis would predict that sound intensity for a given strength of the dark Z^0 field proportional to amplitude squared is proportional to $(N/k)^2$, where N is the anomalous color charge of the oscillating nucleus, and k elastic constant for the harmonic oscillations around the equilibrium position of (say) atom.

10.9.5 Weak plasma waves and the physics of living matter

In plasma phase electromagnetic MEs, and even more so scalar wave pulses, can generate plasma waves accompanied by longitudinal electric fields. In the case of scalar wave pulses the mechanism is simple: the longitudinal electric field of the scalar wave pulse kicks electrons so that a gradient of electron density results and oscillation starts at plasma frequency $\omega_p = e\sqrt{n/m_e}$ in the case of electron. The frequencies of transversal plasma waves are above the plasma frequency.

The notion of weak plasma frequency makes sense if condensed matter can be regarded as Z^0 plasma below the weak length scale L_w with nuclei carrying anomalous weak isospin $I_{3,L}$. Let $I_{3,L}$ be equal to N using neutron's isospin $I_{3,L} = 1/2$ as a unit so that single charged color bond corresponds to $N = \pm 2$.

For a hydrodynamic flow of water of density $\rho = 1$ kg/dm^3 giving $18n(H_2O) \simeq 10^{30}/m^3$ and $m(H_2O) = 18m_p$, W and Z^0 plasma frequencies are given by

$$\begin{aligned}
\omega_p(W) &= g_W N\sqrt{n/m} \ , \\
\omega_p(Z^0) &= g_Z N\sqrt{\tfrac{1}{2} - sin^2(\theta_W)}\sqrt{n/m} = \sqrt{\frac{\frac{1}{2}-sin^2(\theta_W)}{sin^2(\theta_W)}} \times \omega_p(W) \ , \\
g_W^2 &= e^2 tan(\theta_W) \ , \quad g_Z^2 = \frac{e^2}{sin(\theta_W)cos(\theta_W)} \ , \quad sin^2(\theta_w) \simeq .23 \ .
\end{aligned} \tag{10.9.1}$$

For $N = 2$ corresponding to single color bond Z^0 plasma frequency corresponds to an energy $E \simeq .062$ eV. Note that $\omega_p(W) = 1.08\omega_p(Z^0)$ is very near to $\omega_p(Z^0)$. The two plasma frequencies are identical for $p = 1/4$.

$\omega_p(W)$ is very nearly the frequency associated with the resting potential 0.065 eV of the cell membrane [M2]. Although this result could be a sheer co-incidence, it supports the idea that Z^0 plasma vacuum-screened in atomic length scale has a fundamental role in living matter. Of course, entire hierarchy of weak plasmas are possible and more or less forced by the fact that vacuum weak fields appear in all length scales. Weak scalar wave pulses would be an ideal tool for generating plasma oscillations whereas weak MEs would generate sound and transversal plasma waves.

10.9.6 Sensory qualia and dark forces

The TGD based model of sensory qualia relies on universality hypothesis stating that the increments of various quantum numbers in quantum jump define qualia at fundamental level in all p-adic length scales. The hierarchy of dark matters would allows to realize similar qualia in all length and time scales.

Quantum classical correspondence suggest that qualia identified as the increments of quantum numbers should have space-time correlates and charged components of weak and color gauge fields are natural candidates in this respect. If this interpretation is correct, sensations of qualia would be assignable to those space-time regions for which space-time sheet has $D > 2$-dimensional CP_2 projection. MEs would not thus serve as space-time correlates for qualia but only as communication and control tools.

$D = 3$ extremals allow interpretation them as analogs of spin glass phase possible in the vicinity of magnetization-demagnetization temperature whereas $D = 2$ phase would be analogous to ferro-magnetic phase and $D = 4$ phase to de-magnetized phase [D1]. Spin glass property suggests the identification of $D = 3$ extremals as fundamental building bricks of living systems. $D = 3$ extremals have also extremely rich hidden order related to the topology of the field lines of the induced magnetic field lines. Therefore the interpretation of $D = 3$ extremals as space-time correlates of qualia is natural.

A couple of examples are in order.

1. Hearing could correspond to the increment of weak isospin or em charge (or both of them in fixed proportion) and to $D \geq 3$ weak space-time sheets. Classical W fields would serve as a space-time correlate for the basic quale associated with hearing.

2. The increments of color quantum numbers would correspond to the visual colors. The 3+3 charged components of classical gluon field would correspond to basic color-conjugate color pairs. The reduction to $U(2)$ subgroup of color group (for instance, CP_2 projection in $r = constant$ 3-sphere of CP_2) would correspond to the restriction of color vision to black-white vision. Non-vacuum extremals having $D > 2$ (also those having $D = 2$) carrying classical em fields are always accompanied by classical color fields so that the identification is not in conflict with the existing wisdom. Space-time sheets serving as correlates for color qualia would correspond to p-adic length scales associated with multiply dark gluons.

10.10 Mechanisms of Z^0 screening

10.10.1 General view about dark hierarchy

Classical color gauge fields are always present for non-vacuum extremals and non-Abelian classical weak fields always when the dimension D of the CP_2 projection of the space-time sheet satisfies $D > 2$. Quantum classical correspondence forces the conclusion that there must be a p-adic hierarchy of dark matters creating these fields in all length scales. At the level of quantum TGD the p-adic hierarchy of dark matters relates closely with the hierarchy of space-time sheets, hierarchy of infinite primes, and hierarchy of Jones inclusions for hyper-finite type II_1 factors. In TGD inspired theory of consciousness the hierarchy corresponds to the self hierarchy and hierarchy of moments of consciousness with increasing averages duration.

There already exists some guidelines about the physical realization of this hierarchy.

1. Already the p-adic mass calculations of hadron masses led to the conclusion that quarks can appear as several p-adically scaled up variants with masses of variants differing by a multiple of half-octave. There is also experimental support for the view that ordinary neutrinos can appear as several p-adically scaled up variants [25]. This forces to ask whether also electrons could appear as scaled up of scaled down variants even in the ordinary condensed matter, and whether the notion of effective mass of electron varying in wide limits could be replaced by p-adically scaled up mass. A testable prediction is atomic spectra scaled by a power of $\sqrt{2}$.

2. In the TGD based model for atomic nuclei as color bonded nucleons with the quarks/antiquarks at the ends of bonds are identified as p-adically scaled down quarks with electromagnetic space-time sheet having $k = 127$ rather than $k = 113$. Quite generally, exotic quarks and perhaps also leptons (possibly also their color excitations) with p-adically scaled down masses would be associated with the ends of join along boundaries bonds serving as correlates for the bound state formation.

3. The decay width of ordinary weak bosons force the conclusion that the weak space-time sheets associated with exotic quarks have $k \neq 89$ $k = 113$ is a good guess in this respect and would in large $\hbar$ phase correspond to a length scale of order atomic size. The model for tetra-neutron identifies tetra-neutron as alpha particle with two charge color bonds. There is no reason to assume that charged bonds could not appear also in heavier nuclei.

   Their presence would mean also that nucleus has anomalous em and weak charges. One can even consider the possibility that the nuclear strings of neighboring atoms fuse to single nuclear string with long straight portion so that nuclei become colored and possess fractional em charges. Also linking of the nuclear strings might occur.

   If this general picture forced by quantum classical correspondence is taken seriously, one begins to wonder whether even chemical bonds could involve light dark elementary fermions. These dark particles could couple to scaled down copies of both weak bosons and colored gluons.

   Chiral selection in living matter could be due to the axial part of weak interactions between exotic quarks of different nuclei. Even the low compressibility of liquid phase could be due to the Z^0 repulsion between nuclei having anomalous weak charges in condensed phase: note that no isotopic dependence is predicted as in the earlier proposal based on the assumption that ordinary quarks are Z^0 charged.

4. Besides color and electro-weak numbers dark particles can carry complex conformal weights expressible in terms of zeros of Riemann Zeta. If the conformal weight is conserved in particle reactions and given particle can correspond to only single complex conformal weight, it must be expressible in terms of conserved quantum numbers so that neutral particles have real conformal weights. In the transition to the next level of darkness the particles of previous level could receive complex conformal weights and color and weak quantum numbers.

5. Dark $\leftrightarrow$ visible phase transitions are describable as ordinary vertices in which also a scaling of $\hbar$ occurs and scales the size of the space-time sheet representing the particle.

10.10.2 Vacuum screening and screening by particles

Suppose that phase transitions generating charged color bonds and making molecules of condensed matter Z^0 charged with the same value of Z^0 charge are possible. This transition need not generate em charge since ordinary nuclear charge can be reduced in the transition. Weak charge is however generated. This kind of transition could proceed spontaneously as a two-nucleon process if the nuclei are close enough.

This raises the question about the basic mechanisms of screening of weak charges, in particular Z^0 charge. There are two basic mechanism of screening. Vacuum screening occurs automatically above weak length scale L_w and is responsible for the massivation of weak bosons. The screening by Z^0 charges of particles occurs in length scales $L \leq L_w$ in a dense weak(ly charged) plasma containing a large number of charged particles in the volume defined by L_w.

Vacuum screening

Vacuum screening occurs automatically and is based on the generation of vacuum charges which reduces the value of weak charge of particle at the weak space-time sheet associated with particle so that the flux feeded to the next sheet is reduced. This mechanism implies massivation of gauge bosons which at each space-time sheet behave classically like massless fields. It is basically the loss of coherence and correlations due to the finiteness of particle space-time sheet which implies the massivation and screening. The screening by vacuum charges makes sense only above the length scale L_w defined by the mass scale of weak bosons.

Screening by weakly charged dark particles

The screening by dark particles carrying weak charges is appropriate in weak plasma. In situation when the density of Z^0 charge is so high that L_w sized region contains large number of Z^0 charges, screening must be due to dark particles, such as dark electrons and neutrinos.

1. If ordinary atomic nuclei can make a transition to a phase in which $k = 113$ defines the weak length scale followed by a transition to dark phase with $\hbar_s = n\hbar/v_0$. For $n = 3$, the length scale L_w above which vacuum screening occurs is about nx Angstrom, where x varies in the range $[1, 2]$ and $n = 1, 2, 3, ...$ and screening by dark particles is not necessary in the densities typical to condensed matter. For $n = 3$ the L_w is in the range 3-6 A. The fact that the screening length is of the order of atomic size and length of a typical chemical bond means that dark weak force could play an important role in bio-catalysis as already discussed.

   The situation is quite different from that for Z^0 charge localized in nuclear volume. A complete screening by particles is achieved by constant density of Z^0 charge for the screening particles equal to the average Z^0 charge density of nuclei since the charge density to be screened is constant below L_w. By varying the density of screening particles the degree of Z^0 screening can be varied.

2. The hypothesis that weak bosons with complex conformal weights correspond to Gaussian Mersennes, such as the biologically highly interesting length scales $k = 151, 157, 163, 167$ varying in the biologically most interesting length scale range 10 nm-2.5 μm is worth of studying. This kind of dark particles could have ordinary value of $\hbar$ but would possess large weak size L_w. In condensed matter weak plasma phase would appear below the length scale $L(k)$ and the weak nuclear charges would be screened by dark electrons.

   Since the Z^0 charge density is constant below $L(k)$ screening by constant charge density of dark neutrinos is possible. Experimentally one cannot exclude the possibility that scaled up variants of ordinary neutrinos and their dark counterparts could appear at p-adic length scales $k = 151, ..., 167$. For instance, the model of nerve pulse relies crucially on the assumption that $k = 151$ cell membrane space-time sheet carries neutrinos [M2].

In the sequel a classical model of Z^0 screening by dark neutrinos generalizing the Debye model of ionic screening and a genuinely quantum model of screening based on the Bose-Einstein condensate of dark neutrino Cooper pairs are discussed. The Bose-Einstein condensate of sneutrinos predicted by space-time super-symmetry would be ideal for screening purposes. Super-conformal symmetries are basic symmetries of quantum TGD at the level of the "world of classical worlds" but it seems that sparticles are not predicted by quantum TGD if its recent interpretation is correct.

Different variants of Z^0 screening by particles

The model for the Z^0 screening allows to consider at least the following options.

1. Screening by a Bose-Einstein condensate

Some particles which are bosons would Bose-Einstein condense to the ground state. One can consider several options.

1. Sneutrinos, which are predicted by theories allowing space-time super-symmetry, would be nice option but there are reasons to believe that TGD does not predict them: super-symmetry would be realized only at the level of configuration space of 3-surfaces.

2. Cooper pairs of dark neutrinos is second candidate. A phonon exchange mechanism based on classical Z^0 force could allow the formation of Cooper pairs making possible neutrino super conductivity. This mechanism is discussed in some detail in [J3].

   The questionable feature of the Cooper pair option is that the density of neutrinos is so high as compared to the Compton length defined by the rest mass of the neutrino. One can ask whether it makes to sense to regard multi-neutrino state as consisting of Cooper pairs in this kind of situation.

3. The Bose-Einstein condensate of W bosons giving rise to W super-conductivity would define the third option. The simplest option is that the very process generating the charged color bonds in nuclei occurs via emission of W bosons taking also care of screening.

   For $k = 113$ dark W bosons this option is energetically problematic since the rest mass of dark W bosons with $k = 113$ is about 25 MeV and rather high and these bosons are also highly unstable. Note however that complete screening is not needed since vacuum screening occurs automatically above L_w, and W Bose-Einstein condensate could control the degree of Z^0 screening.

   For $k = 151$ W mass is ~ 50 eV and these bosons could be stable (if the masses of exotic leptons are small enough). The negative Z^0 Coulombic interaction energy with exotic quark, given roughly by $\sim 2\alpha_Z Q_Z^2(\nu)/a$, a atomic radius, is of same order of magnitude as the rest mass. Therefore the generation of $k = 151$ W Bose-Einstein BE condensate would require rather small net energy and would lead to a gain of energy for $k = 157, 163, 167$.

2. Dark neutrinos screen the Z^0 charge

For this option dark neutrinos do not form Cooper pairs and thus fill the whole Fermi sphere. For a complete screening the Fermi energy is extremely relativistic, of the order $\pi\hbar_s/a$, a atomic radius so that this option is not energetically favored despite the fact that the ground state energy is negative due to the large Z^0 interaction energy having magnitude larger than neutrino mass.

For full screening the value of the Fermi energy for dark neutrinos at level $k = k_Z$ is determined essentially by the density of anomalous isospin per nucleon. This implies that neutrinos at the top of Fermi surface are relativistic: the Fermi energy for N units of weak isospin per nucleon is given by

$$\begin{aligned} E_F &\simeq N^{1/3}\hbar_s\frac{\pi}{a} \ , \\ a &\simeq 10^{-10}\ m \end{aligned} \tag{10.10.0}$$

and does not depend on condensate level. The order of magnitude is 10^4 eV for ordinary value of $\hbar$ but $n \times 20$ MeV for $\hbar_s = n\hbar/v_0$ and of the same order of magnitude as the rest mass of dark W boson. Hence this option is not energetically much better than W boson option. As noticed, complete screening is not needed so that neutrino screening could serve control purposes.

10.10.3 A quantum model for the screening of the dark nuclear Z^0 charge

In the sequel a quantum model for the screening of dark Z^0 charge is discussed. There are several options corresponding to a screening by neutrinos, by their Cooper pairs, or by light variants of W bosons. The screening by sneutrinos predicted if the theory allows space-time super-symmetry but this does not seem to be the case in TGD.

Some relevant observations about dark neutrinos

The experimental data about neutrino mass differences suggests that neutrinos correspond to the p-adic length scale $k = 169$ and possibly also some larger p-adic primes such as $k = 173$ [25]. $k = 169$ neutrinos would have Compton length of about $L(169)$, cell size.

Neutrinos with dark $k = 113$ weak space-time sheet need of course not correspond to the same p-adic length scale as ordinary neutrinos but one can make this assumption as a convenient working hypothesis in order to get some acquaintance with the numbers involved.

A constant Z^0 charge density of dark neutrino background can in principle cancel $k = 113$ dark Z^0 charge density which is constant in length scales $L < L_w(k_{eff} = 137)$ of order atomic size. The degree of screening is the proper parameter and cannot vary considerably in length scales smaller than $L(169)$ since this would require highly energetic neutrinos.

The Fermi sea of dark neutrinos screening completely the anomalous Z^0 charge of nuclei gives rise to Fermi momentum equal to $E_F = p_F = \hbar_s n^{1/3} \simeq N^{1/3}\hbar/L(137) \simeq N^{1/3}(\hbar_s/\hbar) \times 10^4$ eV but this requires energy. Here N is the number of Z^0 charges per nucleus.

The model of Z^0 screening based on harmonic oscillator potential does not work

The density of the nuclei is so high that there is large number of nuclei within the Bohr radius, which increases by a factor n/v_0 in large $\hbar$ phase. Also the fact that Z^0 charge density is constant within L_w favors a different treatment.

The first guess is that the presence of the anomalous nuclear Z^0 charge could be treated as a harmonic oscillator potential with origin at the center of the region containing the dark phase. One might hope that this treatment makes sense if the nuclei can be regarded as forming a fixed background stabilized by electromagnetic interactions and by screening. The objection is that translational invariance is lost. It is easy to see that the treatment fails also for other reasons.

The effective potential is given by

$$\begin{aligned} V_{eff} &= \frac{E}{m}V_Z - \frac{V_Z^2}{2m_\nu} \ , \\ V_Z &= \frac{kr^2}{2} \ , \\ k &= \frac{1}{3}Q_Z^2(\nu)\hbar_s\alpha_Z N\rho_n \ , \end{aligned} \tag{10.10.-1}$$

where $\rho_n \equiv 1/a^3$ is the number density of nuclei. N is the Z^0 charge per nucleus due to the charged color bonds using $Q_Z(\nu)$ as a unit.

The presence of the relativistic correction in-stabilizes the system above some critical value of r. The maximum $V = E^2/2m_\nu$ of the effective potential at $V = E$ corresponds to

$$r = \sqrt{\frac{6Ea^3}{\hbar_s}} \times \sqrt{\frac{1}{\alpha_Z N Q_Z^2(\nu)}} \ . \tag{10.10.0}$$

For non-relativistic energies the order of magnitude for r is

$$r \sim \sqrt{v_0 m_\nu a}/\sqrt{N\alpha_Z Q_Z^2(\nu)}$$

and smaller than the atomic radius. Thus it would seem that the potential is in practice repulsive in the non-relativistic case. For negative energies the potential is repulsive everywhere. Even for relativistic energies of of order $\hbar_s/a$ at the Fermi surface one has $r \sim a/\sqrt{N\alpha_Z Q_Z^2(\nu)}$ and not much larger than atomic radius. Obviously the treatment of nuclei in the proposed manner does not work.

The model for Z^0 screening based on constant potential well

Since Z^0 charge density is constant within L_w, the safest manner to describe the system is as free dark neutrinos or neutrino Cooper pairs in a potential well characterized by the average Z^0 interaction energy of neutrino with nucleus, both idealized as balls of radius L_w carrying a constant Z^0 charge density.

By performing a time dependent gauge transformation

$$Z^0_\mu \rightarrow Z^0_\mu + \partial_\mu\Phi \ , \quad \Phi = V_Z t \times \chi \ ,$$

where χ equals to unity inside the potential well and vanishes outside, free d'Alembert equation inside potential well results and solutions can be written as standing waves, which must vanish at the boundary of the well to minimize the singularity resulting from the fact that $A_\mu A^\mu$ term gives square of delta function at boundary. The energy identified from the time dependence of the phase factor of solution is $E_0 + V_Z = \sqrt{p^2+m^2} + V_Z$ as the non-relativistic treatment would suggest. Negative energy states obviously result if Z^0 Coulomb interaction energy $E \sim \alpha_Z Q_Z^2(\nu)N/a$ is larger than neutrino mass.

Is Bose-Einstein condensate generated spontaneously?

The formation of neutrino Cooper pairs would correspond to the pairing of neutrinos of opposite spin and would be analogous to the pairing of valence electrons and nucleon pairs inside nuclei. The Bose-Einstein condensation would result basically from the energy gap between the states at the top of Fermi sphere and bound states formed via the scattering possible at the top of Fermi sphere. If the Z^0 interaction energy of neutrinos is negative and has larger magnitude than the rest mass at the bottom of Fermi sphere, it is energetically favorable to generate Fermi sea up to a positive energy for which the neutrino system vanishes. Zero energy neutrino-antineutrino pairs for which neutrino has negative energy could be created spontaneously from vacuum and the condensate could thus be generated spontaneously.

$k = 151$ W bosons could form automatically Bose-Einstein condensate. The fact that Z^0 interaction energy has larger magnitude than W boson mass favors the spontaneous occurrence of the process. If W bosons are created by the phase transition generating charged color bonds in nuclei their charge is automatically screened.

It is illustrative to recall the basic aspects of the model for Bose-Einstein condensation in the case of ordinary ideal Boson gas.

1. In the absence of the classical Z^0 force the energy spectrum of non-relativistic neutrino Cooper pairs is that for a particle in box: $E_n = k\sum_i n_i^2 \times \pi^2/mL^2(169)$, where k is a numerical factor k characterizing the geometry. The natural unit of energy is $\pi^2\hbar^2/2mL^2(169) \simeq .05$ eV.

2. The critical temperature for Bose-Einstein condensation is in recent case obtained by applying the general formula applying in the case of free boson gas with fixed particle number N in volume V:

$$T_c = \frac{2\pi\hbar_s^2}{m}(\frac{n}{2.61})^{2/3} = 2\pi\hbar_s^2 \times (\frac{A-Z}{2.61})^{2/3} \times \frac{a^2}{m} \ . \quad (10.10.1)$$

T_c is of order .1 GeV so that Bose-Einstein condensation certainly occurs. The fraction of Bose-Einstein condensed particles is given by

$$\frac{N_{BE}}{N} = 1-(\frac{T}{T_c})^{3/2} \ . \quad (10.10.2)$$

From these estimates it should be obvious that also in the recent case Bose Einstein condensation indeed can occur and that most of the bosons are in the negative energy state.

10.11 Appendix: Dark neutrino atoms

Dark neutrinos provide a possible screening mechanism for classical Z^0 force present in dark condensed matter with weak bosons in dark $k = 113$ phase. If one takes seriously recent experimental evidence [31] and the explanation of the anomalous atmospheric μ/e ratio [32] in terms of neutrino mixing one must conclude that ν_μ and ν_τ are condensed on $k = k_Z$ level and that muon and τ neutrino have suffered large mixing whereas the mixing of ν_e with remaining neutrinos is much small.

The discussion of [F3] led to the predictions for neutrino masses as a function of common condensation level. In the following table also the $k = 13^2 = 169$ level is included since it predicts exactly the best fit value for $\nu_\tau - \nu_\mu$ mass squared difference whereas $k = 167$ predicts it within 90 per cent confidence limits. $k = 169 = 13^2$ would be allowed if the physically interesting k:s are powers of primes instead of primes: this introduces only few new p-adic length scales below one meter.

k	$m(\nu_e)/eV$	$m(\nu_\mu)/eV$	$m(\nu_\tau/eV$
163	2.16	5.28	5.36
167	.54	1.32	1.34
$169 = 13^2$	.27	.66	.67

Table 2. The table gives the masses of neutrinos as predicted by p-adic mass calculations for three condensate levels.

Only $k = 167$ is allowed by the experimental constraints and p-adic length scale hypothesis in its most stringent form. It must be however emphasized that the elementary particle black hole analogy, discussed in the third part of the book, allows also $k = 169 = 13^2$ giving the best fit to the neutrino mass squared differences. Since the experimental results about electron neutrino-muon neutrino mass difference are preliminary one cannot however exclude the existence of heavy τ neutrino effecting screening of classical Z^0 force in atomic length scales. The upper bound .3 MeV of neutrino mass almost allows $k = 131$ τ neutrino with mass of .4 MeV and it is interesting to find whether $k = 131$ τ is physically acceptable alternative. It turns out that this is not the case.

10.11.1 Dark neutrino atoms in non-relativistic approximation

To get order of magnitude picture it is useful to look first the Bohr radii and ground state energies for dark neutrino atoms assuming that the non-relativistic approximation makes sense. The Bohr radius $a_\nu = \frac{1}{m_\nu \alpha_Z Q_z^2(\nu)(A-Z)}$ and ground state energy of the neutrino atom read in terms of the ordinary Bohr radius $a_0 \simeq 0.5 \cdot 10^{-10}$ m and hydrogen atom ground state energy $E_H \simeq 13.6$ eV

$$\begin{aligned}
a_\nu &= \frac{m_e}{m_\nu}\frac{\alpha_{em}}{\alpha_Z Q_Z^2(\nu)}\frac{a_0}{(A-Z)} \\
&\simeq \frac{m_e}{m_\nu} X \frac{a_0}{(A-Z)} \ , \\
E_\nu &= X^{-2}\frac{m_\nu}{m_e}(A-Z)^2 E_H \ , \\
X &= \frac{sin(\theta_W)cos(\theta_W)}{Q_Z^2(\nu)} \simeq 1.68 \ . \qquad (10.11.-2)
\end{aligned}$$

For $\nu_\tau(131)$ (see the table below) Bohr radius is $a(\nu) = 1.95a_0 = 1.05L(137)$ and quite near to the typical size of lattice cell in condensed matter systems.

ν	m	a_ν	E_0/eV	T_I/K
$\nu_\tau(131)$	0.45 MeV	$7.5E-10$ m	4.3	$.5E+4$
$\nu_{\mu,\tau}(167)$	1.32 eV	12.8 μm	$1.32E-5$	.13
$\nu_e(167)$	.45 eV	49.8μm	$.40E-5$	.04

Table 3. Table gives Bohr radius, energy of ground state and ionization temperature for ground state of neutrino atom for different neutrino species. Data are also given for $k = 131$ τ neutrino.

For dark matter densities which are of order condensed matter densities neutrino atoms are not possible. One can however consider the possibility that a block of dark matter takes the role of "super nucleus" creating a neutrino "super-atom" with Bohr radius $\propto 1/N(A-Z)$ and binding energy $\propto N^2(A-Z)^2$, where N is the number of nuclei involved.

The observation of the spectral lines of $k = k_Z$ dark neutrino atoms would be a triumph of the theory. The transitions between different energy levels can take place via photon/phonon emission/absorption and the observation of the predicted hydrogen type emission and absorption lines or their phonon counterparts would be a direct verification of the theory.

1. A possible signature of neutrino atoms is weak absorption of light at energies of order $10^{-5}\ eV$. In dipole approximation the transition amplitudes are proportional to the sum of matrix elements for electronic and nuclear dipole moment operators so that matrix elements $(m|\bar{r}(nucleus)|n)$ and $(m|\bar{r}(electron)|n)$ are involved. The coordinate vector operators $\bar{r}(nucleus)$ and $\bar{r}(electron)$ must be expressed in terms of cm coordinates and they contain a small contribution proportional $\frac{m_\nu}{M(nucleus)}\bar{r}_\nu$ as is clear from $\bar{r}(nucleus) = \bar{r}_{cm} + \frac{m_\nu}{m(nucleus)+m_\nu}\bar{r}_{12}$ and corresponding expression for electronic coordinate vector. These terms proportional to $\bar{r}_\nu$ induce transitions between different neutrino energy levels. The transition rates are by a factor $\frac{m_\nu^2}{m^2(nucleus)} \sim 10^{-18}/A^2$ (!) smaller than their atomic physics counter parts. Transition rates are also proportional to the square of the energy difference between the levels in question and this gives additional factor of order 10^{-10} for neutrino atoms so that reduction factor of order 10^{-38} results! The observation of $k = 167$ neutrino atoms requires temperature of order .1 K and very low densities (fraction of order 10^{-12} of ordinary condensed matter density) and one can conclude that the observation of $k = 167$ neutrino atoms is extremely difficult by photon emission or absorption.

2. One can also consider the possibility of observing dark neutrino atoms via phonon absorption or emission: the coupling of the neutrinos to phonons would result indirectly from the coupling of neutrinos to atomic nuclei via classical Z^0 force and from the coupling of nuclei to phonons. A rough estimate for the relevant wavelength of sound in temperature of order .1 K gives for the wavelength of the phonon associated with transitions $\lambda \sim 10^{-9}$ meters and frequency of order 10^{10} Hz.

10.11.2 A relativistic model for dark neutrino atom

The Z^0 gauge potential around nucleus is very strong and the classical estimate for the neutrino Coulombic energy has a magnitude much larger than the rest mass of neutrino. This suggests that neutrinos and their Cooper pairs could form negative energy states around nucleus.

For neutrino atoms with several neutrinos one must take into account the screening effect of neutrinos to the Z^0 Coulombic potential of the nucleon. The self consistent model is based on the relativistic counterpart of the Schrödinger equation for the order parameter describing bosons in the Z^0 Coulomb potential created by the nucleus and neutrino charge density.

Self consistent relativistic Schrödinger equation as a model for Z^0 screening

The Laplace equation for the self-consistent Z^0 Coulomb potential reads as

$$\nabla^2 V_Z = -g_Z^2 Q_Z^2(\nu)(A-Z)\delta(\bar{r}) + g_Z^2 Q_Z^2 \overline{\Psi}\partial_t \Psi \ . \qquad (10.11.-1)$$

In the lowest order approximation the solution of this equation is Coulomb interaction energy of neutrino with nucleus

$$\begin{aligned} V_Z^0 &= -\frac{k_Z}{r} \ , \\ k_Z &= \alpha_Z Q_Z^2(\nu)(A-Z) \ . \end{aligned} \qquad (10.11.-1)$$

The d′Alembert equation for the order parameter Ψ characterizing a Bose-Einstein condensate of Cooper pairs of mass m reads as

$$\left[(-i\partial_t - V_Z)^2 + \nabla^2\right]\Psi = m^2\Psi \ . \qquad (10.11.0)$$

Specializing to stationary solutions $\Psi \propto exp(iEt)$ corresponding to energy eigenstate and assuming spherically symmetric potential, one has $\Psi = R(r)Y_m^l(\theta,\phi)$.

If $|\Psi|^2$ is spherically symmetric as one can assume under rather general conditions, the model reduces to ordinary differential equations and one can solve it numerically by iterating. By writing V_Z in the form $V_Z = f_Z/r$ one can readily integrate V_Z from

$$V_Z = -\frac{k_Z}{r} + \frac{g_Z^2 Q_Z^2 E}{r}\int_0^r dr_2 \int_0^{r_2} dr_1 r_1 R^2(r_1) \ . \qquad (10.11.1)$$

Bound states

It is possible to understand the general properties of this equation by transforming in to a form which allows to use the rather precise analogy with Schrödinger equation for hydrogen atom. There are two cases to be considered: bound states and negative energy resonances.

For the bound states the appropriate representation of the equation is

$$\left[-\frac{1}{2m}(\partial_r^2 + \frac{2}{r}\partial_r + \frac{l(l+1)}{r^2}) + \frac{E}{m}V_Z - \frac{V_Z^2}{2m}\right] R = \frac{(E^2-m^2)}{2m} \times R \ . \qquad (10.11.2)$$

When the screening is not taken into account, the equation has a close resemblance with the Schrödinger equation for the hydrogen atom. The correspondences are following:

$$k_{eff} = \frac{E}{2m}k \ , \quad E_{eff} = \frac{E^2-m^2}{2m} \ , \quad l_{eff}(l_{eff}+1) = l(l+1) - k_Z^2 \ . \qquad (10.11.3)$$

In the analog of Schrödinger equation Coulombic potential energy is replaced by an effective potential energy

$$V_{eff} = \frac{E}{m}V_Z - \frac{V_Z^2}{2m} \ . \qquad (10.11.4)$$

V_{eff} is negative for large values of V_Z, vanishes for $V = -2E$, has a maximum $V_{eff}(max) = E^2/2m$ for $V = E$ and vanishes asymptotically. Therefore V_{eff} has an attractive infinitely deep well surrounded by a potential wall of height $E^2/2m$ so that tunnelling in principle becomes possible. Since V^2 term only modifies the effective value of the angular momentum, it is possible to solve the Schrödinger equation explicitly. Bound states correspond to $E < m$. Bound states are non-relativistic with a very long range m/k_Z^2 of about 10^{-4} meters and are not interesting as far as local screening of Z^0 charge is considered.

Negative energy resonances

Relativistic negative energy resonance like solutions can be localized below the atomic radius and only these are appropriate for local screening of the Z^0 charge. For these solutions it is natural to replace the mass of the Cooper pair with its energy $|E|$. With a little re-arranging the following equation analogous to Schrödinger equation for hydrogen atom

$$\left[-\frac{1}{2|E|}(\partial_r^2 + \frac{2}{r}\partial_r + \frac{l(l+1)}{r^2}) - \frac{E}{|E|}V_Z - \frac{V_Z^2}{2|E|}\right] R = \frac{(E^2-m^2)}{2|E|}R \ . \qquad (10.11.5)$$

In the lowest order approximation one can use the unscreened Z^0 Coulombic potential allowing very close analogy with the hydrogen atom. The analogy with the hydrogen atom is revealed by the replacements

$$m_{eff} = |E| \ , \quad k_{eff} = \frac{k_Z}{2} \ , \quad E_{eff} = \frac{E^2-m^2}{2|E|} \ , \quad l_{eff}(l_{eff}+1) = l(l+1) - k_Z^2 \ . \qquad (10.11.6)$$

Note that l_{eff} can be also negative and that for negative energies the Coulombic potential term represents an attractive potential although one has $E_{eff} > 0$. Thus the proper interpretation of the negative energy states are as kind of resonance states.

An upper bound on the neutron number of nucleus

The general solution for l_{eff} allows to branches

$$l_{eff} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1+4l(l+1)-4k_Z^2} \ . \quad (10.11.7)$$

The second branch allows $l_{eff} < 0$ even when the right hand side of the equation above is positive.

The condition

$$l(l+1) > k_Z^2 - \frac{1}{4} \quad (10.11.8)$$

guaranteing the reality of l_{eff} must be satisfied. This condition is automatically satisfied for $l = 0$ for nuclei satisfying $k_Z < 1/2$: this gives

$$A - Z \leq \frac{1}{2\alpha_Z Q_Z^2(\nu)} \ . \quad (10.11.9)$$

For biologically important nuclei the condition is satisfied since the lower bound is very roughly $A - Z = 60$.

For $l > 0$ solutions the neutrino perturbation of the Coulombic potential is not spherically symmetric. Hence only $l = 0$ solution allows a simple numerical treatment based on ordinary differential equations.

The behavior of the negative energy solutions near origin

One can apply standard methods used for solving the Schrödinger equation for hydrogen atom also in the recent case.

1. One can write the normalized order parameter R in the form

$$R(r) = N \times r^{l_{eff}+1} \times exp(-i\frac{r}{|r_0|}) \times w(r) \ . \quad (10.11.10)$$

   The counterpart of Bohr radius is given by

$$|r_0| = \frac{1}{\sqrt{2E_{eff}m_{eff}}} = \frac{1}{\sqrt{E^2-m^2}} \ . \quad (10.11.11)$$

   For relativistic negative energy solutions the counterpart of Bohr radius is imaginary so that the exponential represents spherical wave.

2. Negative energy solutions are slightly singular at origin as are also the solutions of the relativistic Dirac equation. The requirement that the solution is square integrable at origin gives

$$l_{eff} > -\frac{5}{2} \ , \quad (10.11.12)$$

   The behavior $R^2r^2 \propto r^{2\delta}/r$ for $|\Psi|^2$ near origin is therefore the most singular option.

A more stringent condition results if one requires that the interaction energy between neutrinos and nucleus is finite. In the lowest order the interaction energy density behaves as $r^{2l_{eff}+1}$ so that the constraint reads as

$$l_{eff} \;\; > \;\; -2 \; . \tag{10.11.13}$$

If one requires that neutrino-neutrino Coulombic interaction energy is finite one has

$$l_{eff} \;\; > \;\; -\frac{5}{4} \; . \tag{10.11.14}$$

At large distances $1/r^{1-2\delta}$ even the most singular behavior of $|\Psi|^2$ does not guarantee square integrability but in present case one is interested in non-localized solutions analogous to those characterizing conduction electrons and square integrability is not needed. From the condition

$$l_{eff}(l_{eff}+1) \;\; = \;\; l(l+1) - k_Z^2 = l(l+1) - \alpha_Z(A-Z)Q_Z^2(\nu) \tag{10.11.15}$$

it is clear l_{eff} can be negative only for $l = 0$ solution for nuclei for which the condition $A - Z < \alpha_Z Q_Z^2$ is satisfied.

The condition determining the energy eigen values

In the case of bound states the function $w(\rho)$ reduces to a polynomial. Also for the negative energies one can consider analogous solution ansatz as a representation of a negative energy resonance state.

1. The condition for the reduction to a polynomial can be deduced using standard power series expansion and reads as

$$2(k + l_{eff} + 1) \;\; = \;\; -\frac{k_{eff}}{|E_{eff} r_0|} = -k_Z \times \left[\frac{|E|m}{E^2 - m^2}\right]^{1/2} \; . \tag{10.11.16}$$

2. One can write l_{eff} in the form $l_{eff} = -l_{eff}(min) + \Delta l$, where the value of $l_{eff}(min) = -7/2, 2,$ or $-5/4$ depending on the regularity conditions at the origin so that the condition Eq. 10.11.16 gives

$$k \;\; < \;\; -l_{eff}(min) - 1 - \Delta l \geq \frac{1}{4} - \Delta l \; . \tag{10.11.17}$$

   w is at most a first order polynomial in r. The most stringent condition guaranteing the finiteness of Z^0 interaction energy allows only the solution for which $w(\rho)$ is constant.

3. The condition of Eq. 10.11.16 guaranteing the reduction of the series of w to a polynomial reduces to the form

$$1 - 2\delta \;\; = \;\; k_Z \times \left[\frac{|E|m}{E^2 - m^2}\right]^{1/2} \; . \tag{10.11.18}$$

   The solutions are

$$\begin{aligned} \frac{|E|}{m} &= \left[b \pm \sqrt{b^2-1}\right]^{1/2} \quad , \\ b &= 1 + \frac{k_Z^2}{2(1+2\delta^2)^2} \quad . \end{aligned} \tag{10.11.18}$$

Solutions are relativistic negative energy solutions but the energy is of the same order of magnitude as the rest energy so that the total energy of the Bose-Einstein condensate is relatively small. Note that the solution is scaling covariant in the sense that in the p-adic scaling $m \rightarrow 2^k m$ also energy scales in the same manner.

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Chapter 11

Super-Conductivity in Many-Sheeted Space-Time

11.1 Introduction

In this chapter various TGD based ideas related to the role of super-conductivity in bio-systems are studied. TGD inspired theory of consciousnses provides several motivations for this.

1. Supra currents and Josephson currents provide excellent tools of bio-control allowing large space-time sheets to control the smaller space-time sheets. The predicted hierarchy of dark matter phases characterized by a large value of $\hbar$ and thus possessing scaled up Compton and de Broglie wavelengths allows to have quantum control of short scales by long scales utilizing de-coherence phase transition. Quantum criticality is the basic property of TGD Universe and quantum critical super-conductivity is therefore especially natural in TGD framework. The competing phases could be ordinary and large $\hbar$ phases and supra currents would flow along the boundary between the two phases.

2. It is possible to make a tentative identification of the quantum correlates of the sensory qualia quantum number increments associated with the quantum phase transitions of various macroscopic quantum systems [K3] and various kind of Bose-Einstein condensates and super-conductors are the most relevant ones in this respect.

3. The state basis for the fermionic Fock space spanned by N creation operators can be regarded as a Boolean algebra consisting of statements about N basic statements. Hence fermionic degreees of freedom could correspond to the Boolean mind whereas bosonic degrees of freedom would correspond to sensory experiencing and emotions. The integer valued magnetic quantum numbers (a purely TGD based effect) associated with the defect regions of super conductors of type I provide a very robust information storage mechanism and in defect regions fermionic Fock basis is natural. Hence not only fermionic super-conductors but also their defects are biologically interesting [L1, M6].

11.1.1 General ideas about super-conductivity in many-sheeted space-time

The notion of many-sheeted space-time alone provides a strong motivation for developing TGD based view about superconductivity and I have developed various ideas about high T_c super-conductivity [26] in parallel with ideas about living matter as a macroscopic quantum system. A further motivation and a hope for more quantitative modelling comes from the discovery of various non-orthodox super-conductors including high T_c superconductors [26, 25, 24], heavy fermion super-conductors and ferromagnetic superconductors [18, 20, 19]. The standard BCS theory does not work for these super-conductors and the mechanism for the formation of Cooper pairs is not understood. There is experimental evidence that quantum criticality [17] is a key feature of many non-orthodox super-conductors. TGD provides a conceptual framework and bundle of ideas making it possible to develop models for non-orthodox superconductors.

Quantum criticality, hierarchy of dark matters, and dynamical $\hbar$

Quantum criticality is the basic characteristic of TGD Universe and quantum critical superconductors provide an excellent test bed to develop the ideas related to quantum criticality into a more concrete form.

The hypothesis that Planck constants in M^4 and CP_2 degrees of freedom are dynamical possessing quantized spectrum given as integer multiples of minimum value of Planck constant [A9] adds further content to the notion of quantum criticality.

Phases with different values of M^4 and CP_2 Planck constants given by $\hbar(M^4) = n_a\hbar_0$ and $\hbar(CP_2) = n_b\hbar_0$ behave like dark matter with respect to each other in the sense that they do not have direct interactions except at criticality corresponding to a leakage between different sectors of imbedding space glued together along M^4 or CP_2 factors. The scalings of M^4 and CP_2 covariant metrics are from anyonic arguments given by n_b^2 and n_a^2 so that the value of effective $\hbar$ appearing in Schrödinger equation is given by $\hbar_{eff}/\hbar_0 = n_a/n_b$ and in principle can have all positive rational values. In large $\hbar(M^4)$ phases various quantum time and length scales are scaled up which means macroscopic and macro-temporal quantum coherence.

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and seems to be especially favored in living matter [M3].

The only coupling constant strength of theory is Kähler coupling constant g_K^2 which appears in the definition of the Kähler function K characterizing the geometry of the configuration space of 3-surfaces (the "world of classical worlds"). The exponent of K defines vacuum functional analogous to the exponent of Hamiltonian in thermodynamics. The allowed value(s) of g_K^2, which is (are) analogous to critical temperature(s), is (are) determined by quantum criticality requirement. Contrary to the original hypothesis inspired by the requirement that gravitational coupling is renormalization group invariant, α_K does not seem to depend on p-adic prime whereas gravitational constant is proportional to L_p^2. The situation is saved by the assumption that gravitons correspond to the largest non-super-astrophysical Mersenne prime M_{127} so that gravitational coupling is effectively RG invariant in p-adic coupling constant evolution [C4].

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anticommutation relations of various super-conformal algebras. Only the ratio $\hbar_{eff}/\hbar_0 = n_a/n_b$ of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants [A9]. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of Planck constants coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors. For a fixed value of n_a/n_b one obtains zoomed up versions of particles with size scaled up by n_a.

A further great idea is that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces gauge coupling strength α so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

TGD actually predicts an infinite hierarchy of phases behaving like dark or partially dark matter with respect to the ordinary matter [F6] and the value of $\hbar$ is only one characterizer of these phases. These phases, especially so large $\hbar_{eff}$ phase, seem to be essential for the understanding of even ordinary hadronic, nuclear and condensed matter physics [F6, F8, F9]. This strengthens the motivations for finding whether dark matter might be involved with quantum critical super-conductivity.

Cusp catastrophe serves as a metaphor for criticality. In the recent case temperature and doping are control variables and the tip of cusp is at maximum value of T_c. Critical region correspond to the cusp catastrophe. Quantum criticality suggests the generalization of the cusp to a fractal cusp. Inside the critical lines of cusp there are further cusps which corresponds to higher levels in the hierarchy of dark matters labelled by increasing values of $\hbar$ and they correspond to a hierarchy of subtle quantum coherent dark matter phases in increasing length scales. The proposed model for high T_c super-conductivity involves only single value of Planck constant but it might be that the full description involves very many values of them.

Many-sheeted space-time concept and ideas about macroscopic quantum phases

Many-sheeted space-time leads to obvious ideas concerning the realization of macroscopic quantum phases.

1. The dropping of particles to larger space-time sheets is a highly attractive mechanism of super-conductivity. If space-time sheets are thermally isolated, the larger space-time sheets could be at extremely low temperature and super-conducting.

2. The possibility of large $\hbar$ phases allows to give up the assumption that space-time sheets characterized by different p-adic length scales are thermally isolated. The scaled up versions of a given space-time sheet corresponding to a hierarchy of values of $\hbar$ are possible such that the scale of kinetic energy and magnetic interaction energy remain same for all these space-time sheets. For instance, for scaled up variants of space-time sheet having size scale characterized by $L(151) = 10$ nm (cell membrane thickness) the critical temperature for superconductivity could be higher than room temperature.

3. The idea that wormhole contacts can form macroscopic quantum phases and that the interaction of ordinary charge carriers with the wormhole contacts feeding their gauge fluxes to larger space-time sheets could be responsible for the formation of Cooper pairs, have been around for a decade [J5]. The rather recent realization that wormhole contacts can be actually regarded as space-time correlates for Higgs particles leads also to a new view about the photon massivation in super-conductivity.

4. Quantum classical correspondence has turned out be a very powerful idea generator. For instance, one can ask what are the space-time correlates for various notions of condensed matter such as phonons, BCS Cooper pairs, holes, etc... For instance, TGD predicts the existence of negative energy space-time sheets so that ordinary particles can and must exist in negative energy states (in cosmological scales the density of inertial energy is predicted to vanish [D6]). The question is whether holes could have quite concrete representation as negative energy space-time sheets carrying negative energy particles and whether the notion of Cooper pair of holes could have this kind of space-time correlate.

11.1.2 Model for high T_c superconductivity

The model for high T_c super-conductivity relies on the notions of quantum criticality, dynamical Planck constant, and many-sheeted space-time.

These ideas lead to a concrete model for high T_c superconductors as quantum critical superconductors allowing to understand the characteristic spectral lines as characteristics of interior and boundary Cooper pairs bound together by phonon and color interaction respectively. The model for quantum critical electronic Cooper pairs generalizes to Cooper pairs of fermionic ions and for sufficiently large $\hbar_{eff}$ stability criteria, in particular thermal stability conditions, can be satisfied in a given length scale. Also high T_c superfluidity based on dropping of bosonic atoms to Cooper pair space-time sheets where they form Bose-Einstein condensate is possible.

At qualitative level the model explains various strange features of high T_c superconductors. One can understand the high value of T_c and ambivalent character of high T_c super conductors suggesting both BCS type Cooper pairs and exotic Cooper pairs with non-vanishing spin, the existence of pseudogap and scalings laws for observables above T_c, the role of stripes and doping and the existence of a critical doping, etc... An unexpected prediction is that coherence length is actually $\hbar_{eff}/\hbar_0 = 2^{11}$ times longer than the coherence length predicted by conventional theory so that type I super-conductor would be in question with stripes serving as duals for the defects of type I super-conductor in nearly critical magnetic field replaced now by ferromagnetic phase.

At quantitative level the model predicts correctly the four poorly understood photon absorption lines and the critical doping ratio from basic principles. The current carrying structures have structure locally similar to that of axon including the double layered structure of cell membrane and also the size scales are predicted to be same so that the idea that axons are high T_c superconductors is highly suggestive.

11.2 General TGD based view about super-conductivity

Today super-conductivity includes besides the traditional low temperature super-conductors many other non-orthodox ones [16]. These unorthodox super-conductors carry various attributes such cuprate, organic, dichalcogenide, heavy fermion, bismute oxide, ruthenate, antiferromagnetic and ferromagnetic. Mario Rabinowitz has proposed a simple phenomenological theory of superfluidity and super-conductivity which helps non-specialist to get a rough quantitative overall view about super-conductivity [16].

11.2.1 Basic phenomenology of super-conductivity

Basic phenomenology of super-conductivity

The transition to super-conductivity occurs at critical temperature T_c and involves a complete loss of electrical resistance. Super-conductors expel magnetic fields (Meissner effect) and when the external magnetic field exceeds a critical value H_c super-conductivity is lost either completely or partially. In the transition to super-conductivity specific heat has singularity. For long time magnetism and super-conductivity were regarded as mutually exclusive phenomena but the discovery of ferromagnetic super-conductors [18, 20] has demonstrated that reality is much more subtle.

The BCS theory developed by Bardeen, Cooper, and Schrieffer in 1957 provides a satisfactory model for low T_c super-conductivity in terms of Cooper pairs. The interactions of electrons with the crystal lattice induce electron-electron interaction binding electrons to Cooper pairs at sufficiently low temperatures. The electrons of Cooper pair are at the top of Fermi sphere (otherwise they cannot interact to form bound states) and have opposite center of mass momenta and spins. The binding creates energy gap E_{gap} determining the critical temperature T_c. The singularity of the specific heat in the transition to super-conductivity can be understood as being due to the loss of thermally excitable degrees of freedom at critical temperature so that heat capacity is reduced exponentially. BCS theory has been successful in explaining the properties of low temperature super conductors but the high temperature super-conductors discovered in 1986 and other non-orthodox superconductors discovered later remain a challenge for theorists.

The reasons why magnetic fields tend to destroy super-conductivity is easy to understand. Lorentz force induces opposite forces to the electrons of Cooper pair since the momenta are opposite. Magnetic field tends also to turn the spins in the same direction. The super-conductivity is destroyed in fields for which the interaction energy of magnetic moment of electron with field is of the same order of magnitude as gap energy $E_{gap} \sim T_c$: $e\hbar H_c/2m \sim T_c$.

If spins are parallel, the situation changes since only Lorentz force tends to destroy the Cooper pair. In high T_c super-conductors this is indeed the case: electrons are in spin triplet state ($S = 1$) and the net orbital angular momentum of Cooper pair is $L = 2$. The fact that orbital state is not $L = 0$ state makes high T_c super-conductors much more fragile to the destructive effect of impurities than conventional super-conductors (due to the magnetic exchange force between electrons responsible for magnetism). Also the Cooper pairs of ^3He superfluid are in spin triplet state but have $S = 0$.

The observation that spin triplet Cooper pairs might be possible in ferro-magnets stimulates the question whether ferromagnetism and super-conductivity might tolerate each other after all, and the answer is affirmative [20]. The article [18] provides an enjoyable summary of experimental discoveries.

Basic parameters of super-conductors from universality?

Super conductors are characterized by certain basic parameters such as critical temperature T_c and critical magnetic field H_c, densities n_c and n of Cooper pairs and conduction electrons, gap energy E_{gap}, correlation length ξ and magnetic penetration length λ. The super-conductors are highly complex systems and calculation of these parameters from BCS theory is either difficult or impossible.

It has been suggested [16] that these parameters might be more or less universal so that they would not depend on the specific properties of the interaction responsible for the formation of Cooper pairs. The motivation comes from the fact that the properties of ordinary Bose-Einstein condensates do not depend on the details of interactions. This raises the hope that these parameters might be expressible in terms of some basic parameters such as T_c and the density of conduction electrons allowing to deduce Fermi energy E_F and Fermi momentum k_F if Fermi surface is sphere. In [16] formulas for the basic parameters are indeed suggested based on this of argumentation assuming that Cooper pairs form a Bose-Einstein condensate.

1. The most important parameters are critical temperature T_c and critical magnetic field H_c in principle expressible in terms of gap energy. In [16] the expression for T_c is deduced from the condition that the de Broglie wavelength λ must satisfy in supra phase the condition

$$\lambda \geq 2d = 2(\frac{n_c}{g})^{-1/D} \tag{11.2.1}$$

guaranteing the quantum overlap of Cooper pairs. Here n_c is the density of Bose-Einstein condensate of Cooper pairs and g is the number of spin states and D the dimension of the condensate. This condition follows also from the requirement that the number of particles per energy level is larger than one (Bose-Einstein condensation).

Identifying this expression with the de Broglie wavelength $\lambda = \hbar/\sqrt{2mE}$ at thermal energy $E = (D/2)T_c$, where D is the number of degrees of freedom, one obtains

$$T_c \leq \frac{h^2}{4Dm}(\frac{n_c}{g})^{2/D} \ . \tag{11.2.2}$$

m denotes the effective mass of super current carrier and for electron it can be even 100 times the bare mass of electron. The reason is that the electron moves is somewhat like a person trying to move in a dense crowd of people, and is accompanied by a cloud of charge carriers increasing its effective inertia. In this equation one can consider the possibility that Planck constant is not the ordinary one. This obviously increases the critical temperature unless n_c is scaled down in same proportion in the phase transition to large $\hbar$ phase.

2. The density of n_c Cooper pairs can be estimated as the number of fermions in Fermi shell at E_F having width Δk deducible from kT_c. For $D=3$-dimensional spherical Fermi surface one has

$$\begin{aligned} n_c &= \frac{1}{2}\frac{4\pi k_F^2\Delta k}{\frac{4}{3}\pi k_F^3}n \ , \\ kT_c &= E_F - E(k_F-\Delta k) \simeq \frac{h^2 k_F\Delta k}{m} \ . \end{aligned} \tag{11.2.2}$$

Analogous expressions can be deduced in $D=2$- and $D=1$-dimensional cases and one has

$$n_c(D) = \frac{D}{2}\frac{T_c}{E_F}n(D) \ . \tag{11.2.3}$$

The dimensionless coefficient is expressible solely in terms of n and effective mass m. In [16] it is demonstrated that the inequality 11.2.2 replaced with equality when combined with 11.2.3 gives a satisfactory fit for 16 super-conductors used as a sample.

Note that the Planck constant appearing in E_F and T_c in Eq. 11.2.3 must correspond to ordinary Planck constant $\hbar_0$. This implies that equations 11.2.2 and 11.2.3 are consistent within orders of magnitudes. For $D = 2$, which corresponds to high T_c superconductivity, the substitution of n_c from Eq. 11.2.3 to Eq. 11.2.2 gives a consistency condition from which n_c disappears completely. The condition reads as

$$n\lambda_F^2 = \pi = 4g \ \ .$$

Obviously the equation is not completely consistent.

3. The magnetic penetration length λ is expressible in terms of density n_c of Cooper pairs as

$$\lambda^{-2} = \frac{4\pi e^2 n_c}{m_e} \ \ . \tag{11.2.4}$$

The ratio $\kappa \equiv \frac{\lambda}{\xi}$ determines the type of the super conductor. For $\kappa < \frac{1}{\sqrt{2}}$ one has type I super conductor with defects having negative surface energy. For $\kappa \geq \frac{1}{\sqrt{2}}$ one has type II super conductor and defects have positive surface energy. Super-conductors of type I this results in complex stripe like flux patterns maximizing their area near criticality. The super-conductors of type II have $\kappa > 1/\sqrt{2}$ and the surface energy is positive so that the flux penetrates as flux quanta minimizing their area at lower critical value H_{c_1} of magnetic field and completely at higher critical value H_{c_2} of magnetic field. The flux quanta contain a core of size ξ carrying quantized magnetic flux.

4. Quantum coherence length ξ can be roughly interpreted as the size of the Cooper pair or as the size of the region where it is sensible to speak about the phase of wave function of Cooper pair. For larger separations the phases of wave functions are un-correlated. The values of ξ vary in the range $10^3 - 10^4$ Angstrom for low T_c super-conductors and in the range $5 - 20$ Angstrom for high T_c super-conductors (assuming that they correspond to ordinary $\hbar$!) the ratio of these coherence lengths varies in the range $[50 - 2000]$, with upper bound corresponding to $n_F = 2^{11}$ for $\hbar$. This would give range $1-2$ microns for the coherence lengths of high T_c super-conductors with lowest values of coherence lengths corresponding to the highest values of coherence lengths for low temperatures super conductors.

Uncertainty Principle $\delta E\delta t = \hbar/2$ using $\delta E = E_{gap} \equiv 2\Delta$, $\delta t = \xi/v_F$, gives an order of magnitude estimate for ξ differing only by a numerical factor from the result of a rigorous calculation given by

$$\xi \ = \ \frac{4\hbar v_F}{E_{gap}} \ \ . \tag{11.2.5}$$

E_{gap} is apart from a numerical constant equal to T_c: $E_{gap} = nT_c$. Using the expression for v_F and T_c in terms of the density of electrons, one can express also ξ in terms of density of electrons.

For instance, BCS theory predicts $n = 3.52$ for metallic super-conductors and $n = 8$ holds true for cuprates [16]. For cuprates one obtains $\xi = 2n^{-1/3}$ [16]. This expression can be criticized since cuprates are Mott insulators and it is not at all clear whether a description as Fermi gas makes sense. The fact that high T_c super-conductivity involves breakdown of anti-ferromagnetic order might justify the use of Fermi gas description for conducting holes resulting in the doping.

For large $\hbar$ the value of ξ would scale up dramatically if deduced theoretically from experimental data using this kind of expression. If the estimates for ξ are deduced from v_F and T_c purely calculationally as seems to be the case, the actual coherence lengths would be scaled up by a factor $\hbar/\hbar_0 = n_F$ if high T_c super-conductors correspond to large $\hbar$ phase. As also found that this would also allow to understand the high critical temperature.

11.2.2 Universality of parameters in TGD framework

Universality idea conforms with quantum criticality of TGD Universe. The possibility to express everything in terms of density of critical temperature coding for the dynamics of Cooper pair formation and the density charge carriers would make it also easy to understand how p-adic scalings and transitions to large $\hbar$ phase affect the basic parameters. The possible problem is that the replacement of inequality of Eq. 11.2.2 with equality need not be sensible for large $\hbar$ phases. It will be found that in many-sheeted space-time T_c does not directly correspond to the gap energy and the universality of critical temperature follows from the p-adic length scale hypothesis.

The effective of p-adic scaling on the parameters of super-conductors

1. The behavior of the basic parameters under p-adic scaling and scaling of Planck constant

p-Adic fractality expresses as $n \propto 1/L^3(k)$ would allow to deduce the behavior of the various parameters as function of the p-adic length scale and naive scaling laws would result. For instance, E_{gap} and T_c would scale as $1/L^2(k)$ if one assumes that the density n of particles at larger space-time sheets scales p-adically as $1/L^3(k)$. The basic implication would be that the density of Cooper pairs and thus also T_c would be reduced very rapidly as a function of the p-adic length scale. Without thermal isolation between these space-time sheets and hight temperature space-time sheets there would not be much hopes about high T_c super-conductivity.

In the scaling of Planck constant basic length scales scale up and the overlap criterion for super-conductivity becomes easy to satisfy unless the density of electrons is reduced too dramatically. As found, also the critical temperature scales up so that there are excellent hopes of obtain high T_c super-conductor in this manner. The claimed short correlation lengths are not a problem since they are calculational quantities. As a matter fact, the

2. Could gap energies be universal?

Suppose that the super-conducting electrons are at a space-time sheet corresponding to some p-adic length scale. They can leak to either larger or smaller space-time sheets via the formation of join along boundaries bonds. The energy E_J associated with the formation of a join along boundaries bond connecting two space-time sheets characterized by k_1 and k_2 mediating transfer of Cooper pair to smaller space-time sheet defines a potential barrier so that for thermal energies below this energy no join along boundaries bonds are formed to smaller space-time sheets. The gap energy deduced from T_c would not necessarily correspond in this case to the binding energy of Cooper pair but to the energy $E_J > E_{gap}$ of the join along boundaries bond.

One can imagine two options for E_J in the approximation that the interaction energy of Cooper pair with surroundings is neglected.

Option I: The formation of JAB is a process completely independent from the flow of Cooper pair through it and thermal photons are responsible for it. In this case the order of magnitude for E_J would naturally correspond to $\hbar/L(k_1)$. Cell size $L(167) = 2.5$ μm would correspond to $E_J \sim .4$ eV which does not make sense.

Option II: One cannot separate the flow of the Cooper pair through the JAB from its formation involving the localization to smaller space-time sheet requiring thermal photon to provide the difference of zero point kinetic energies. E_J would naturally correspond to the difference $\Delta E_0 = E_0(k_1) - E_0(k_2)$ of zero point kinetic energies $E_0(k) = D\pi^2\hbar^2/4mL^2(k)$ of the Cooper pair, where D is the effective dimensionality of the sheets. The reason why JABs inducing the flow $k_1 \rightarrow k_2$ of charge carriers are not formed spontaneously must be that charge carriers at k_1 space-time sheet are in a potential well. This option seems to work although it is certainly oversimplified since it neglects the interaction energy of Cooper pairs with other particles and wormhole throats behaving effectively like particles.

If E_J given as difference of zero point kinetic energies, determines the critical temperature rather than E_{gap}, universality of the critical temperature as a difference of zero point kinetic energies is predicted. In this kind of situation the mechanism binding electrons to Cooper pairs is not relevant for what is observed as long as it produces binding energy and energy gap between ground state and first excited state larger than the thermal energy at the space-time sheet in question. This temperature is expected to scale as zero point kinetic energy. As already found, the work of Rabinowitz [16] seems to support this kind of scaling law.

3. Critical temperatures for low and high T_c super conductors

Consider now critical temperatures for low and high T_c electronic super-conductors for option II assuming $D = 3$.

1. For low T_c super conductors and for the transition $k_2 = 167 \rightarrow k_1 = 163$ this would give $\Delta E_0 = E_0(163) \sim 6 \times 10^{-6}$ eV, which corresponds to $T_c \sim .06$ K. For $k_2 = 163 \rightarrow 157$ this would give $\Delta E \sim 1.9 \times 10^{-4}$ eV corresponding to 1.9 K. These orders of magnitude look rather reasonable since the coherence length ξ expected to satisfy $\xi \leq L(k_2)$, varies in the range $.1 - 1$ μm for low T_c super conductors.

2. For high T_c super-conductors with ξ in the range $5 - 20$ Angstrom, $E_J \sim 10^{-2}$ eV would give $k_1 = 149$, which would suggest that high T_c super-conductors correspond to $k = 151$ and $\xi \ll L(k_2 = 151) = 10$ nm (cell membrane thickness). In this case $\Delta << E_J$ is quite possible so that high T_c super-conductivity would be due to thermal isolation rather than a large value of energy gap. This provides a considerable flexibility concerning the modelling of mechanisms of Cooper pair formation.

4. $E_J < E_{gap}$ case as a transition to partial super-conductivity

For $E_J < E_{gap}$ the transition at $T_c \simeq E_J$ does not imply complete loss of resistivity since the Cooper pairs can flow to smaller space-time sheets and back without being destroyed and this is expected to induce dissipative effects. Some super-conductors such as $ZrZn_2$ ferromagnet do not lose their resistivity completely and the anomaly of specific heat is absent [18]. The mundane explanation is that super-conductivity exists only in clusters.

The effect of the scaling of $\hbar$ to the parameters of BCS super-conductor

It is of interest to study the behavior of the various parameters in the transition to the possibly existing large $\hbar$ variant of super-conducting electrons. Also small scalings of $\hbar$ are possible and the considerations to follow generalize trivially to this case. Under what conditions the behavior of the various parameters in the transition to large $\hbar$ phase is dictated by simple scaling laws?

1. Scaling of T_c and E_{gap}

T_c and E_{gap} remain invariant if E_{gap} corresponds to a purely classical interaction energy remaining invariant under the scaling of $\hbar$. This is not the case for BCS super-conductors for which the gap energy Δ has the following expression.

$$\begin{aligned}
\Delta &= \hbar\omega_c exp(-1/X) \ , \\
X &= n(E_F)U_0 = \frac{3}{2}N(E_F)\frac{U_0}{E_F} \ , \\
n(E_F) &= \frac{3}{2}\frac{N(E_F)}{E_F} \ . \\
\omega_c &= \omega_D = (6\pi^2)^{1/3}c_s n_n^{1/3} \ . \qquad (11.2.3)
\end{aligned}$$

Here ω_c is the width of energy region near E_F for which "phonon" exchange interaction is effective. n_n denotes the density of nuclei and c_s denotes sound velocity.

$N(E_F)$ is the total number of electrons at the super-conducting space-time sheet. U_0 would be the parameter characterizing the interaction strength of of electrons of Cooper pair and should not depend on $\hbar$. For a structure of size $L \sim 1$ μ m one would have $X \sim n_a 10^{12}\frac{U_0}{E_F}$, n_a being the number of exotic electrons per atom, so that rather weak interaction energy U_0 can give rise to $\Delta \sim \omega_c$.

The expression of ω_c reduces to Debye frequency ω_D in BCS theory of ordinary super conductivity. If c_s is proportional to thermal velocity $\sqrt{T_c/m}$ at criticality and if n_n remains invariant in the scaling of $\hbar$, Debye energy scales up as $\hbar$. This can imply that $\Delta > E_F$ condition making scaling non-sensible unless one has $\Delta \ll E_F$ holding true for low T_c super-conductors. This kind of situation would *not* require large $\hbar$ phase for electrons. What would be needed that nuclei and phonon space-time sheets correspond to large $\hbar$ phase.

What one can hope is that Δ scales as $\hbar$ so that high T_c superconductor would result and the scaled up T_c would be above room temperature for $T_c > .15$ K. If electron is in ordinary phase X is automatically invariant in the scaling of $\hbar$. If not, the invariance reduces to the invariance of U_0 and E_F under the scaling of $\hbar$. If n scales like $1/\hbar^D$, E_F and thus X remain invariant. U_0 as a simplified parametrization for the interaction potential expressible as a tree level Feynman diagram is expected to be in a good approximation independent of $\hbar$.

It will be found that high in high T_c super-conductors, which seem to be quantum critical, a high T_c variant of phonon mediated superconductivity and exotic superconductivity could be competing. This would suggest that the phonon mediated superconductivity corresponds to a large $\hbar$ phase for nuclei scaling ω_D and T_c by a factor $\simeq 2^{11}$.

Since the total number $N(E_F)$ of electrons at larger space-time sheet behaves as $N(E_F) \propto E_F^{D/2}$, where D is the effective dimension of the system, the quantity $1/X \propto E_F/n(E_F)$ appearing in the expressions of the gap energy behaves as $1/X \propto E_F^{-D/2+1}$. This means that at the limit of vanishing electron density $D = 3$ gap energy goes exponentially to zero, for $D = 2$ it is constant, and for $D = 1$ it goes zero at the limit of small electron number so that the formula for gap energy reduces to $\Delta \simeq \omega_c$. These observations suggests that the super-conductivity in question should be 2- or 1-dimensional phenomenon as in case of magnetic walls and flux tubes.

2. Scaling of ξ and λ

If n_c for high T_c super-conductor scales as $1/\hbar^D$ one would have $\lambda \propto \hbar^{D/2}$. High T_c property however suggests that the scaling is weaker. ξ would scale as $\hbar$ for given v_F and T_c. For $D = 2$ case the this would suggest that high T_c super-conductors are of type I rather than type II as they would be for ordinary $\hbar$. This conforms with the quantum criticality which would be counterpart of critical behavior of super-conductors of type I in nearly critical magnetic field.

3. Scaling of H_c and B

The critical magnetization is given by

$$H_c(T) = \frac{\Phi_0}{\sqrt{8}\pi\xi(T)\lambda(T)} \quad (11.2.4)$$

where Φ_0 is the flux quantum of magnetic field proportional to $\hbar$. For $D = 2$ and $n_c \propto \hbar^{-2}$ $H_c(T)$ would not depend on the value of $\hbar$. For the more physical dependence $n_c \propto \hbar^{-2+\epsilon}$ one would have $H_c(T) \propto \hbar^{-\epsilon}$. Hence the strength of the critical magnetization would be reduced by a factor $2^{-11\epsilon}$ in the transition to the large $\hbar$ phase with $n_F = 2^{-11}$.

Magnetic flux quantization condition is replaced by

$$\int 2eBdS = n\hbar 2\pi \ . \quad (11.2.5)$$

B denotes the magnetic field inside super-conductor different from its value outside the super-conductor. By the quantization of flux for the non-super-conducting core of radius ξ in the case of super-conductors of type II $eB = \hbar/\xi^2$ holds true so that B would become very strong since the thickness of flux tube would remain unchanged in the scaling.

11.2.3 Quantum criticality and super-conductivity

The notion of quantum criticality has been already discussed in introduction. An interesting prediction of the quantum criticality of entire Universe also gives naturally rise to a hierarchy of macroscopic quantum phases since the quantum fluctuations at criticality at a given level can give rise to higher level macroscopic quantum phases at the next level. A metaphor for this is a fractal cusp catastrophe for which the lines corresponding to the boundaries of cusp region reveal new cusp catastrophes corresponding to quantum critical systems characterized by an increasing length scale of quantum fluctuations.

Dark matter hierarchy could correspond to this kind of hierarchy of phases and long ranged quantum slow fluctuations would correspond to space-time sheets with increasing values of $\hbar$ and size.

Evolution as the emergence of modules from which higher structures serving as modules at the next level would correspond to this hierarchy. Mandelbrot fractal with inversion analogous to a transformation permuting the interior and exterior of sphere with zooming revealing new worlds in Mandelbrot fractal replaced with its inverse would be a good metaphor for what quantum criticality would mean in TGD framework.

How the quantum criticality of superconductors relates to TGD quantum criticality

There is empirical support that super-conductivity in high T_c super-conductors and ferromagnetic systems [18, 19] is made possible by quantum criticality [17]. In the experimental situation quantum criticality means that at sufficiently low temperatures quantum rather than thermal fluctuations are able to induce phase transitions. Quantum criticality manifests itself as fractality and simple scaling laws for various physical observables like resistance in a finite temperature range and also above the critical temperature. This distinguishes sharply between quantum critical super conductivity from BCS type super-conductivity. Quantum critical super-conductivity also exists in a finite temperature range and involves the competition between two phases.

The absolute quantum criticality of the TGD Universe maps to the quantum criticality of subsystems, which is broken by finite temperature effects bringing dissipation and freezing of quantum fluctuations above length and time scales determined by the temperature so that scaling laws hold true only in a finite temperature range.

Reader has probably already asked what quantum criticality precisely means. What are the phases which compete? An interesting hypothesis is that quantum criticality actually corresponds to criticality with respect to the phase transition changing the value of Planck constant so that the competing phases would correspond to different values of $\hbar$. This hypothesis seems to work in the case of high T_c super-conductivity. The prediction is that quantum criticality sets on at some critical temperature $T_{c_1} > T_c$ meaning the emergence of exotic Cooper pairs which are however unstable against decay to ordinary electrons so that the super-conductivity in question gives rise to ordinary conductivity in time scales longer than the lifetime of exotic Cooper pair dictated by temperature. These exotic Cooper pairs can also transform to BCS type Cooper pairs which are stable below T_c.

Scaling up of de Broglie wave lengths and criterion for quantum overlap

Compton lengths and de Broglie wavelengths are scaled up by an integer n, whose preferred values correspond to $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s}+1$ are distinct Fermat primes. In particular, $n_F = 2^{k11}$ seem to be favored. The scaling up means that the overlap condition $\lambda \geq 2d$ for the formation of Bose-Einstein condensate can be satisfied and the formation of Cooper pairs becomes possible. Thus a hierarchy of large $\hbar$ super-conductivities would be associated with to the dark variants of ordinary particles having essentially same masses as the ordinary particles.

Unless one assumes fractionization, the invariance of $E_F \propto \hbar_{eff}^2 n^{2/3}$ in $\hbar$ increasing transition would require that the density of Cooper pairs in large $\hbar$ phase is scaled down by an appropriate factor. This means that supra current intensities, which are certainly measurable quantities, are also scaled down. Of course, it could happen that E_F is scaled up and this would conform with the scaling of the gap energy.

Possible implications of charge and spin fractionization

Masses as given by representations of super conformal algebras and p-adic thermodynamics are invariant under changes of the Planck constants. The original assumption that Poincare quantum numbers are invariant in Planck constant changing quantum transition is however too strong and conflicts with the model explaining quantization of planetary orbits in terms of gigantic value of $\hbar_{eff}$ [D7, J6]. What happens is spin fractionization with unit of spin replaced with n_a/n_b and fractionization of color and presumably of also electro-weak charges with unit given by n_b/n_a. For instance, n_a/n_b fractionization would happen for angular momentum quantum number m, for the integer n characterizing the Bohr orbits of atom, harmonic oscillator, and integers labelling the states of particle in box.

The fractionization can be understood in terms of multiple covering of M^4 by symmetry related CP_2 points formed in the phase transition increasing $\hbar$ [A9]. The covering is characterized by $G_b \subset SU(2) \subset SU(3)$ and fixed points correspond to orbifold points. The copies of imbedding space with different G are glued with each other along M^4 factors at orbifold point, representing origin of CP_2.

An interesting implication of spin fractionization is that for n_a and $n_b = 1$ the unit of spin would become n_a standard units. This might be interpreted by saying that minimum size of a Bose Einstein condensate consisting of spin 1 Cooper pairs is $n_b/2$ Cooper pairs with spin 1. On the other hand charge could be fractionized to e/n_b in this case. A possible interpretation is that electron is delocalized to n_a separate G_a related sheets of the M^4 covering of CP_2 projection such that each of them carries a fractional charge e/n_a. Geometrically this would correspond to a ring consisting of n_a discrete points.

Quantum critical super-conductors in TGD framework

For quantum critical super-conductivity in heavy fermions systems, a small variation of pressure near quantum criticality can destroy ferromagnetic (anti-ferromagnetic) order so that Curie (Neel) temperature goes to zero. The prevailing spin fluctuation theory [21] assumes that these transitions are induced by long ranged and slow spin fluctuations at critical pressure P_c. These fluctuations make and break Cooper pairs so that the idea of super-conductivity restricted around critical point is indeed conceivable.

Heavy fermion systems, such as cerium-indium alloy $CeIn_3$ are very sensitive to pressures and a tiny variation of density can drastically modify the low temperature properties of the systems. Also other systems of this kind, such as $CeCu_2Ge_2$, $CeIn_3$, $CePd_2Si_2$ are known [18, 20]. In these cases super-conductivity appears around anti-ferromagnetic quantum critical point.

The last experimental breakthrough in quantum critical super-conductivity was made in Grenoble [19]. URhGe alloy becomes super-conducting at $T_c = .280$ K, loses its super-conductivity at $H_c = 2$ Tesla, and becomes again super-conducting at $H_c = 12$ Tesla and loses its super-conductivity again at $H = 13$ Tesla. The interpretation is in terms of a phase transition changing the magnetic order inducing the long range spin fluctuations.

TGD based models of atomic nucleus [F8] and condensed matter [F9] assume that weak gauge bosons with Compton length of order atomic radius play an essential role in the nuclear and condensed matter physics. The assumption that condensed matter nuclei possess anomalous weak charges explains the repulsive core of potential in van der Waals equation and the very low compressibility of condensed matter phase as well as various anomalous properties of water phase, provide a mechanism of cold fusion and sono-fusion, etc. [F9, J6]. The pressure sensitivity of these systems would directly reflect the physics of exotic quarks and electro-weak gauge bosons. A possible mechanism behind the phase transition to super-conductivity could be the scaling up of the sizes of the space-time sheets of nuclei.

Also the electrons of Cooper pair (and only these) could make a transition to large $\hbar$ phase. This transition would induce quantum overlap having geometric overlap as a space-time correlate. The formation of join along boundaries bonds between neighboring atoms would be part of the mechanism. For instance, the criticality condition $4n^2\alpha = 1$ for BE condensate of n Cooper pairs would give $n = 6$ for the size of a higher level quantum unit possibly formed formed from Cooper pairs. If one does not assume invariance of energies obtained by fractionization of principal quantum number, this transition has dramatic effects on the spectrum of atomic binding energies scaling as $1/\hbar^2$ and practically universal spectrum of atomic energies would result [J6] not depending much on nuclear charge. It seems that this prediction is non-physical.

Quantum critical super-conductors resemble superconductors of type I with $\lambda \ll \xi$ for which defects near thermodynamical criticality are complex structures looking locally like stripes of thickness λ. These structure are however dynamical in super-conducting phase. Quite generally, long range quantum fluctuations due to the presence of two competing phases would manifest as complex dynamical structures consisting of stripes and their boundaries. These patterns are dynamical rather than static as in the case of ordinary spin glass phase so that quantum spin glass or 4-D spin glass is a more appropriate term.

The breaking of classical non-determinism for vacuum extremals indeed makes possible space-time correlates for quantum non-determinism and this makes TGD Universe a 4-dimensional quantum spin glass. The model for high T_c super-conductors leads to the conclusion that the boundaries between the two phases are the carriers of the supra currents. Wormhole contacts appear naturally at boundaries and the mere assumption that $q\overline{q}$ type wormhole contacts feed the em gauge flux of electrons from the space-time sheet of Cooper pair to a larger space-time sheet predicts correctly the properties of high T_c Cooper pairs.

Could quantum criticality make possible new kinds of high T_c super-conductors?

The transition to large $\hbar$ phase increases various length scales by n/v_0 and makes possible long range correlations even at high temperatures. Hence the question is whether large $\hbar$ phase could correspond to ordinary high T_c super-conductivity. If this were the case in the case of ordinary high T_c super-conductors, the actual value of coherence length ξ would vary in the range $5-20$ Angstrom scaled up by a factor n/v_0 to $n-40n$ μm to be compared with the range $.2-2$ μm for low T_c super-conductors. The density of Cooper pairs would be scaled down by an immensely small factor $2^{-33}/n^3$ from its value deduced from Fermi energy so that neither high T_c nor ordinary super-conductors can correspond to larger $\hbar$ phase for electrons.

Large $\hbar$ phase for some nuclei might be involved and make possible large space-time sheets of size at least of order of ξ at which conduction electrons forming Cooper pairs would topologically condense like quarks around hadronic space-time sheets (in [F9] a model of water as a partially dark matter with one fourth of hydrogen ions in large $\hbar$ phase is developed).

Consider for a moment the science fictive possibility that super conducting electrons for some quantum critical super-conductors to be discovered or already discovered correspond to large $\hbar$ phase with $\hbar(k) = n_F\hbar_0$ keeping in mind that this affects only quantum corrections in perturbative approach but not the lowest order classical predictions of quantum theory. For $n_F = n/v_0 \simeq n2^{k11}$ with $k = 1, n = 1$ the size of magnetic body would be $L(149) = 5$ nm, the thickness of the lipid layer of cell membrane. For $k = 2, n = 1$ the size would be $L(171) = 10$ μm, cell size. If the density of Cooper pairs is of same order of magnitude as in case of ordinary super conductors, the critical temperature is scaled up by 2^{k11}. Already for $k = 1$ the critical temperature of 1 K would be scaled up to $4n^2 \times 10^6$ K if n_c is not changed. This assumption is not consistent with the assumption that Fermi energy remains non-relativistic. For $n = 1$ $T_c = 400$ K would be achieved for $n_c \rightarrow 10^{-6}n_c$, which looks rather reasonable since Fermi energy transforms as $E_F \rightarrow 8 \times 10^3 E_F$ and remains non-relativistic. H_c would scale down as $1/\hbar$ and for $H_c = .1$ Tesla the scaled down critical field would be $H_c = .5 \times 10^{-4}$ Tesla, which corresponds to the nominal value of the Earth's magnetic field.

Quantum critical super-conductors become especially interesting if one accepts the identification of living matter as ordinary matter quantum controlled by macroscopically quantum coherent dark matter. One of the basic hypothesis of TGD inspired theory of living matter is that the magnetic flux tubes of the Earth's magnetic field carry a super-conducting phase and the spin triplet Cooper pairs of electrons in large $\hbar$ phase might realize this dream. That the value of Earth's magnetic field is near to its critical value could have also biological implications.

11.2.4 Space-time description of the mechanisms of super-conductivity

The application of ideas about dark matter to nuclear physics and condensed matter suggests that dark color and weak forces should be an essential element of chemistry and condensed matter physics. The continual discovery of new super-conductors, in particular of quantum critical superconductors, suggests that super-conductivity is not well understood. Hence super-conductivity provides an obvious test for these ideas. In particular, the idea that wormhole contacts regarded as parton pairs living at two space-time sheets simultaneously, provides an attractive universal mechanism for the formation of Cooper pairs and is not so far-fetched as it might sound first.

Leading questions

It is good to begin with a series of leading questions.

1. The work of Rabinowitch [16] suggests that that the basic parameters of super-conductors might be rather universal and depend on T_c and conduction electron density only and be to a high degree independent of the mechanism of super-conductivity. This is in a sharp contrast to the complexity of even BCS model with its somewhat misty description of the phonon exchange mechanism.
   Questions: Could this mean that there exists a simple universal description of various kinds of super-conductivities? Could this mechanism involve large $\hbar$ phase for nuclei in case of quantum critical super-conductivity? Could wormhole contacts or their Bose-Einstein condensate play some role. Are the Cooper pairs of quantum critical super-conductors at the boundaries of the competing phases?

2. The effective masses of electrons in ferromagnetic super-conductors are in the range of 10-100 electron masses [18] and this forces to question the idea that ordinary Cooper pairs are current carriers. Quantum classical correspondence requires that bound states involve formation of join along boundaries bonds between bound particles. In the case of Cooper pairs in ordinary superconductors the length of join along boundaries bonds between electrons should be of order $10^3 - 10^4$ Angstroms. This looks rather strange.
   Questions: Could quantum classical correspondence help to identify the mechanism giving rise to Cooper pairs? The simplest model of pair is as a space-time sheet with size of order ξ so that the electrons are "outside" the background space-time. Could the Coulomb interaction energy of electrons with positively charged wormhole throats carrying parton numbers and feeding em gauge flux to the large space-time sheet be responsible for the gap energy? Could wormhole throats carry also quark quantum numbers and form color singlet like structures connected by long color flux tubes so that color force would be ultimately responsible for the stability of Cooper pair? In case of single electron condensed to single space-time sheet the em flux could be indeed feeded by u and $\overline{d}$ type wormhole contacts to larger space-time sheet. Or could electrons be free-travellers bound to structures involving also other particles?

3. Quantum classical correspondence forces to ask for the space-time correlates for the existing quantum description of phonons.
   Questions: What are the space-time sheets associated with phonons? Could the microscopic description of phonons in atomic length scales rely on the oscillations of wormhole contact Bose-Einstein condensates at the boundaries of nucleon space-time sheets with size scale of order atom size? Could the dark weak length scale which is of order atomic size replace lattice constant in the expression of sound velocity? What is the space-time correlate for sound velocity?

4. The new super-conductors possess relatively complex chemistry and lattice structure.
   Questions: Could it be that complex chemistry and lattice structure makes possible something very simple which is a transition to dark nuclear phase so that size of dark quarks involved would be scaled up to $L(k \rightarrow k+22 \rightarrow k+44)$, say $k = 113 \rightarrow 135 \rightarrow 157$), and the size of hadronic space-time sheets would be scaled up as $k = 107 \rightarrow 129 \rightarrow 151$? Could it be that also other p-adic primes are possible as suggested by the p-adic mass calculations of hadron masses predicting that hadronic quarks can correspond to several values of k? Could it be that the Gaussian Mersennes $(1+i)^k - 1$, $k = 151, 157, 163, 167$ spanning the p-adic length scale range 10 nm-2.5 μm correspond to p-adic length especially relevant for super-conductivity.

Photon massivation, coherent states of Cooper pairs, and wormhole contacts

The existence of wormhole contacts have been one of the most exotic predictions of TGD. The realization that wormhole contacts can be regarded as parton-antiparton pairs with parton and antiparton assignable to the light-like causal horizons accompanying wormhole contacts, and that Higgs particle corresponds to wormhole contact [F2], opens the doors for more concrete models of also super-conductivity involving massivation of photons.

The formation of a coherent state of wormhole contacts would be the counterpart for the vacuum expectation value of Higgs. The notions of coherent states of Cooper pairs and of charged Higgs challenge the conservation of electromagnetic charge. The following argument however suggests that coherent states of wormhole contacts form only a part of the description of ordinary super-conductivity. The basic observation is that wormhole contacts with vanishing fermion number define space-time correlates for Higgs type particle with fermion and antifermion numbers at light-like throats of the contact.

The ideas that a genuine Higgs type photon massivation is involved with super-conductivity and that coherent states of Cooper pairs really make sense are somewhat questionable since the conservation of charge and fermion number is lost. A further questionable feature is that a quantum superposition of many-particle states with widely different masses would be in question. The interpretational problems could be resolved elegantly in zero energy ontology [C3] in which the total conserved quantum numbers of quantum state are vanishing. In this picture the energy, fermion number, and total charge of any positive energy state are compensated by opposite quantum numbers of the negative energy state in geometric future. This makes possible to speak about superpositions of Cooper pairs and charged Higgs bosons separately in positive energy sector.

Rather remarkably, if this picture is taken seriously, super-conductivity can be seen as providing a direct support for both the hierarchy of scaled variants of standard model physics and for the zero energy ontology.

Phonon exchange mechanism

Sound waves correspond to density variations of condensed matter. If dark gluons and exotic weak bosons with weak scale of order atomic radius explain the low compressibility of condensed matter [F9] then these forces should be essential for the description of what happens for sound waves below the atomic length scale. In particular, the lattice length appearing in Debye frequency should be expressible in terms of dark weak length scale.

Quantum classical correspondence requires that phonons should have identification as space-time sheets and that sound velocity is coded in the geometry of the space-time sheet. This interpretation of course makes sense only if the space-time sheet of phonon is in contact with atoms so that atomic oscillations induce oscillations of the induced gauge fields inside it.

The obvious objection against this picture is that one can imagine the possibility of free phonons analogous to photons connecting nuclei with say distance of micrometer and having no contact with the nuclei in between. One can of course turn the situation around and ask whether free phonons are the hen and lattice oscillations the egg. Could free photons exist and induce resonant oscillations of atomic nuclei if their velocity is consistent with the sound velocity deducible from the lattice constant and elastic constant for the interactions between atoms?

The existence of warped vacuum extremals, and in general the huge vacuum degeneracy of field equations, suggest how this space-time representation of phonons might occur. The simplest warped extremal corresponds to the mapping $M^4 \rightarrow CP_2$ defined as $\Phi = \omega m^0$, where Φ is coordinate of the geodesic circle of CP_2 with other coordinates being constant. The induced metric is $g_{m^0m^0} = 1 - R^2\omega^2/4$, $g_{ij} = -\delta_{ij}$. Light velocity with respect to M^4 coordinates, which are physically preferred coordinates, is reduced to $v = \sqrt{1 - R^2\omega^2/4}$. The crazy guess would be that the reduced signal velocity could have interpretation as sound velocity with the previous prerequisites.

For small perturbations of vacuum extremals the term coming from the variation with respect to the induced metric vanishes, and the only contribution comes from the variation of the induced Kähler form. As a consequence, the field equations reduce to empty space Maxwell's equations $j^\alpha_K = 0$ for the induced Kähler form in the induced metric of determined by vacuum extremal in the lowest non-trivial order. This means that the maximal signal velocity is in general reduced and the reduction can be very large as the case of warped vacuum extremals demonstrates. The longitudinal Kähler electric field associated with phonons would serve as a correlate for the longitudinal sound waves.

In higher orders the solution develops a non-vanishing Kähler current j^α_K and this relates naturally to the fact that the phonon exchange involves dissipation. In the case of the simplest warped vacuum extremals the relevant parameter for the perturbation theory is ωR which is near to unity so that perturbative effects can be quite sizable if the phonons are representable in the proposed manner. The non-vanishing of the vacuum Lorentz force $j^\alpha_K J_{\alpha\beta}$ serves as a space-time correlate for the presence of dissipative effects. For the known solutions of field equations the Lorentz force vanishes and the interpretation is that they represent asymptotic self-organization patters. Phonons would be different and represent transient phenomena.

If this interpretation is correct, the phonon mechanism for the formation of Cooper pairs could have a description in terms of the topological condensation of electrons at space-time sheets representing phonons connecting atomic nuclei. The essential point would be that electrons of Cooper pair would be outside the space-time in well-defined sense. Also now wormhole contacts would be involved but the Coulomb interaction energy of delocalized electrons with charged wormhole throats would be negligible as compared to the interaction energy with nuclei.

Space-time correlate for quantum critical superconductivity

The series of leading questions has probably given reader a hunch about what the mechanism of super-conductivity could be in the quantum critical case.

1. Exotic Cooper pair as a pair of space-time sheets of scaled up electrons feeding their gauge fluxes to a larger space-time sheet via $q\overline{q}$ type wormhole contacts

Quantum critical electronic super-conductivity requires new kind of Cooper pairs which are responsible for supra currents in the temperature range $[T_c, T_{c_1}]$ inside stripe like regions (flux tubes). These Cooper pairs are quantum critical against decay to ordinary electrons so that in time scale characterizing quantum criticality so that super-conductivity is reduced to conductivity whose temperature dependence is characterized by scaling laws. Below T_c large $\hbar$ variants of BCS Cooper pairs are good candidates for supra current carriers and would result from exotic Cooper pairs. A model for the exotic Cooper pairs is considered in the sequel. Boundary plays an essential role in that the Cooper pairs at boundary must be in quantum critical phase also below T_c since otherwise the transformation of ordinary electrons to large $\hbar$ BCS type Cooper pairs and vice versa is not possible.

If wormhole contact for large $\hbar$ electron corresponds to e^+e^- pairs, one ends up with a stability problem since the annihilation of electron and e^+ at wormhole throat can lead to the disappearance of the space-time sheet. If there are two wormhole contacts corresponding to quark anti-quark pairs the situation changes. The requirement that the net charge of wormhole throats is +2e implies $u\overline{d}$ configuration for upper wormhole throats and its conjugate for the lower wormhole throats. If the wormhole throats of each electron carry net color quantum numbers the binding of electrons by color confining force would guarantee the stability of the exotic Cooper pair. This would require that wormhole throats form a color singlet not reducible to product of pion type $u\overline{d}$ type color singlets.

BCS type Cooper pair results when both electrons end up at same space-time sheet of exotic Cooper pair via a join along boundaries bond. This hopping would also drag the wormhole contacts with it and the second space-time sheet could contract. These Cooper pairs can in principle transform to pairs involving only two join along boundaries contacts carrying e^+e^- pairs at their throats. For these Cooper pairs case the binding of electrons would be due to phonon mechanism.

2. General comments

Some general comments about the model are in order.

1. High T_c super conductors are Mott insulators and antiferromagnets in their ground state, which would suggest that the notion of non-interacting Fermi gas crucial for BCS type description is not useful. Situation is however not so simple if antiferromagnetic phase and magnetically disordered phase with large $\hbar$ for nuclei compete at quantum criticality. Large $\hbar$ makes possible high T_c variant of BCS type superconductivity in magnetically disordered phase in interior of rivulets but it is possible to get to this phase only via a phase consisting of exotic Cooper pairs and this is possible only in finite temperature range below T_c.

2. For both exotic and phonon mediated super-conductivity Cooper pair can be said to be outside the space-time sheet containing matter. Assuming a complete delocalization in the exotic case, the interaction energy is the expectation value of the sum of kinetic and Coulombic interaction energies between electrons and between electrons and wormhole throats. In the case of phonon space-time sheets situation is different due to the much larger size of Cooper pair space-time sheet so that Coulomb interaction with wormhole throats provides the dominating contribution to the binding energy.

3. The explicit model for high T_c super-conductivity relies on quantum criticality involving long ranged quantum fluctuations. The mechanism seems could apply in all cases where quantum critical fluctuations can be said to be carriers of supra currents and exotic super-conductivity vanishes when either phase dominates completely. In the case of high T_c super-conductors quantum criticality corresponds to a quite wide temperature range, which provides support for the quantum criticality of TGD Universe.

11.2.5 Super-conductivity at magnetic flux tubes

Super-conductivity at magnetic flux tubes of magnetic flux quanta is one the basic hypothesis of the TGD based model of living matter. There is also evidence for magnetically mediated super-conductivity in extremely pure samples [22]. The magnetic coupling was only observed at lattice densities close to the critical density at which long-range magnetic order is suppressed. Quantum criticality suggests that the super-conductivity appears at the boundaries of two competing phases and that Cooper pairs correspond to space-time sheets feeding their em gauge charge via $q\overline{q}$ type wormhole contacts to larger space-time sheet.

Almost the same model as in the case of high T_c and quantum critical super-conductivity applies to magnetic flux tubes. Now the flux quantum contains BE condensate of exotic Cooper pairs interacting with wormhole contacts feeding the gauge flux of Cooper pairs from the magnetic flux quantum to a larger space-time sheet. The interaction of spin 1 Cooper pairs with the magnetic field of flux quantum orients their spins in the same direction. Large value of $\hbar$ guarantees thermal stability even in the case that different space-time sheets are not thermally isolated.

Superconductors at the flux quanta of the Earth's magnetic field

Magnetic flux tubes and magnetic walls are the most natural candidates for super-conducting structures with spin triplet Cooper pairs. Indeed, experimental evidence relating to the interaction of ELF em radiation with living matter suggests that bio-super-conductors are effectively 1- or 2-dimensional. $D \leq 2$-dimensionality is guaranteed by the presence of the flux tubes or flux walls of, say, the magnetic field of Earth in which charge carries form bound states and the system is equivalent with a harmonic oscillator in transversal degrees of freedom.

The effect of Earth's magnetic field is completely negligible at the atomic space-time sheets and cannot make super conductor 1-dimensional. At cellular sized space-time sheets magnetic field makes possible the confinement of the electron Cooper pairs in harmonic oscillator states. The critical temperature is however extremely low for ordinary value of $\hbar$ and either thermal isolation between space-time sheets or large value of $\hbar$ can save the situation.

An essential element of the picture is that topological quantization of the magnetic flux tubes occurs. In fact, the flux tubes of Earth's magnetic field have thickness of order cell size from the quantization of magnetic flux. The observations about the effects of ELF em fields on bio-matter [42, 41] suggest that similar mechanism is at work also for ions and in fact give very strong support for bio-super conductivity based on the proposed mechanism.

Energy gaps for superconducting magnetic flux tubes and walls

Besides the formation of Cooper pairs also Bose-Einstein condensation to the ground state occurs and the stability of Bose-Einstein condensate requires an energy gap which must be larger than the temperature at the magnetic flux tube.

There are several energies to be considered.

1. The Coulombic binding energy of Cooper pairs with the wormhole contacts feeding the em flux from magnetic flux tube to a larger space-time sheet defines an energy gap which is expected to be of order $E_g = \alpha/L(k)$ giving $E_g \sim 10^{-3}$ eV for $L(167) = 2.5$ μm giving a rough estimate for the thickness of the magnetic flux tube of the Earth's magnetic field $B = .5 \times 10^{-4}$ Tesla.

2. In longitudinal degrees of freedom of the flux tube Cooper pairs can be described as particles in a one-dimensional box and the gap is characterized by the length L of the magnetic flux tube and the value of $\hbar$. In longitudinal degrees of freedom the difference between $n = 2$ and $n = 1$ states is given by $E_0(k_2) = 3h^2/4m_eL^2(k_2)$. Translational energy gap $E_g = 3E_0(k_2) = 3h^2/4m_eL^2(k_2)$ is smaller than the effective energy gap $E_0(k_1) - E_0(k_2) = h^2/4m_eL^2(k_1) - h^2/4m_eL^2(k_2)$ for $k_1 > k_2 + 2$ and identical with it for $k_1 = k_2 + 2$. For $L(k_2 = 151)$ the zero point kinetic energy is given by $E_0(151) = 20.8$ meV so that E_g corresponds roughly to a temperature of 180 K. For magnetic walls the corresponding temperature would be scaled by a factor of two to 360 K and is above room temperature.

3. Second troublesome energy gap relates to the interaction energy with the magnetic field. The magnetic interaction energy E_m of Cooper pair with the magnetic field consists of cyclotron term $E_c = n\hbar eB/m_e$ and spin-interaction term which is present only for spin triplet case and is given by $E_s = \pm\hbar eB/m_e$ depending on the orientation of the net spin with magnetic field. In the magnetic field $B_{end} = 2B_E/5 = .2$ Gauss ($B_E = .5$ Gauss is the nominal value of the Earth's magnetic field) explaining the effects of ELF em fields on vertebrate brain, this energy scale is $\sim 10^{-9}$ eV for ordinary value of $\hbar$ and $\sim 2n \times 10^{-6}$ eV for $\hbar = n2^{11} \times \hbar(1)$. At the next level of dark hierarchy the energy would be $4n^2 \times 10^{-3}$ eV and would still correspond to a temperature $4n^2$ K.

The smallness of translational and magnetic energy gaps in the case of Cooper pairs at Earth's magnetic field could be seen as a serious obstacle.

1. Thermal isolation between different space-time sheets provides one possible resolution of the problem. The stability of the Bose-Einstein condensation is guaranteed by the thermal isolation of space-time if the temperature at the magnetic flux tube is below E_m. This can be achieved in all length scales if the temperature scales as the zero point kinetic energy in transversal degrees of freedom since it scales in the same manner as magnetic interaction energy.

2. The transition to large $\hbar$ phase could provide a more elegant way out of the difficulty. The criterion for a sequence of transitions to a large $\hbar$ phase could be easily satisfied if there is a large number of charge Cooper pairs at the magnetic flux tube. Kinetic energy gap remains invariant if the length of the flux tube scales as $\hbar$. If magnetic flux is quantized as a multiple of $\hbar$ and flux tube thickness scales as $\hbar^2$, B must scale as $1/\hbar$ so that also magnetic energy remains invariant under the scaling. This would allow to have stability without assuming low temperature at magnetic flux tubes.

11.3 TGD based model for high T_c super conductors

The model of exotic Cooper pairs has been already described and since high T_c superconductors are quantum critical, they provide an attractive application of the model.

11.3.1 Some properties of high T_c super conductors

Quite generally, high T_c super-conductors are cuprates with CuO layers carrying the supra current. The highest known critical temperature for high T_c superconductors is 164 K and is achieved under huge pressure of 3.1×10^5 atm for LaBaCuO. High T_c super-conductors are known to be super conductors of type II.

This is however a theoretical deduction following from the assumption that the value of Planck constant is ordinary. For $\hbar = 2^{11}\hbar_0$ ξ would be scaled up accordingly and type I super-conductor would be in question. These super-conductors are characterized by very complex patterns of penetrating magnetic field near criticality since the surface area of the magnetic defects is maximized. For high T_c super-conductors the ferromagnetic phase could be regarded as an analogous defect and would indeed have very complex structure. Since quantum criticality would be in question the stripe structure would fluctuate with time too in accordance with 4-D spin glass character.

The mechanism of high T_c super conductivity is still poorly understood [33, 40]. It is agreed that electronic Cooper pairs are charge carriers. It is widely accepted that electrons are in relative d-wave state rather than in s-wave (see [37] and the references mentioned in [33]). Cooper pairs are believed to be in spin triplet state and electrons combine to form $L = 2$ angular momentum state. The usual phonon exchange mechanism does not generate the attractive interaction between the members of the Cooper pair having spin. There is also a considerable evidence for BCS type Cooper pairs and two kinds of Cooper pairs could be present.

High T_c super conductors have spin glass like character [32]. High T_c superconductors have anomalous properties also above T_c suggesting quantum criticality implying fractal scaling of various observable quantities such as resistivity. At high temperatures cuprates are anti-ferromagnets and Mott insulators meaning freezing of the electrons. Superconductivity and conductivity is known to occur along dynamical stripes which are antiferromagnetic defects.

These findings encourage to consider the interpretation in terms of quantum criticality in which some new form of super conductivity which is not based on quasiparticles is involved. This superconductivity is assignable with the quantum fluctuations destroying antiferromagnetic order and replacing it with magnetically disordered phase possibly allowing phonon induced super-conductivity.

The doping of the super-conductor with electron holes is essential for high T_c superconductivity and the there is a critical doping fraction $p = .14$ at which T_c is highest. There is considerable evidence that holes gather on one-dimensional stripes with thickness of order few atom sizes and lengths in the range 1-10 nm [40], which are fluctuating in time scale of 10^{-12} seconds. These stripes are also present in non-conductong and non-superconducting state but in this case they do not fluctuate. One interpretation for the fluctuations is as oscillations analogous to acoustic wave and essential for the binding of Cooper pairs. Quantum criticality suggests an alternative interpretation.

T_c is inversely proportional to the distance L between the stripes. One interpretation is in terms of generalization of the Debye frequency to 2-dimensional case. One could also consider phonons with wavelength equal to the distance between the stripes. A further interpretation would be that full super-conductivity requires delocalization of electrons also with respect to stripes so that T_c would be proportional to the hopping probability of electron between neighboring stripes expected to be proportional to $1/L$ [40]. Later a TGD based interpretation will be discussed.

From free fermion gas to Fermi liquids to quantum critical systems

The article of Jan Zaanen [24] gives an excellent non-technical discussion of various features of high T_c super-conductors distinguishing them from BCS super-conductors. After having constructed a color flux tube model of Cooper pairs I found it especially amusing to learn that the analogy of high T_c super-conductivity as a quantum critical phenomenon involving formation of dynamical stripes to QCD in the vicinity of the transition to the confined phase leading to the generation of string like hadronic objects was emphasized also by Zaanen.

BCS super-conductor behaves in a good approximation like quantum gas of non-interacting electrons. This approximation works well for long ranged interactions and the reason is Fermi statistics plus the fact that Fermi energy is much larger than Coulomb interaction energy at atomic length scales.

For strongly interacting fermions the description as Fermi liquid (a notion introduced by Landau) has been dominating phenomenological approach. ^{3}He provides a basic example of Fermi liquid and already here a paradox is encountered since low temperature collective physics is that of Fermi gas without interactions with effective masses of atoms about 6 times heavier than those of real atoms whereas short distance physics is that of a classical fluid at high temperatures meaning a highly correlated collective behavior.

Many-sheeted space-time provides a possible explanation of the paradox. Space-time sheets containing join along boundaries blocks of ^3He atoms behave like gas whereas the ^3He atoms inside these blocks form a liquid. An interesting question is whether the ^3He atoms combine to form larger units with same spin as ^3He atom or whether the increase of effective mass by a factor of order six means that $\hbar$ as a unit of spin is increased by this factor forcing the basic units to consist of Bose-Einstein condensate of 3 Cooper pairs.

High Tc super conductors are neither Fermi gases nor Fermi liquids. Cuprate superconductors correspond at high temperatures to doped Mott insulators for which Coulomb interactions dominate meaning that electrons are localized and frozen. Electron spin can however move and the system can be regarded as an anti-ferromagnet. CuO planes are separated by highly oxidic layers and become super-conducting when doped. The charge transfer between the two kinds of layers is what controls the degree of doping. Doping induces somehow a delocalization of charge carriers accompanied by a local melting of anti-ferromagnet.

Collective behavior emerges for high enough doping. Highest T_c results with 15 per cent doping by holes. Current flows along electron stripes. Stripes themselves are dynamical and this is essential for both conductivity and superconductivity. For completely static stripes super-conductivity disappears and quasi-insulating electron crystal results.

Dynamical stripes appear in mesoscopic time and length scales corresponding to 1-10 nm length scale and picosecond time scale. The stripes are in a well-defined sense dual to the magnetized stipe like structures in type I super-conductor near criticality, which suggests type I super-conductivity: as found large $\hbar$ Cooper pairs would make it possible. The stripes are anti-ferromagnetic defects at which neighboring spins fail to be antiparallel. It has been found that stripes are a very general phenomenon appearing in insulators, metals, and superconducting compounds [39].

Quantum criticality is present also above T_c

Also the physics of Mott insulators above T_c reflects quantum criticality. Typically scaling laws hold true for observables. In particular, resistivity increases linearly rather than transforming from T^2 behavior to constant as would be implied by quasi-particles as current carriers. The appearance of so called pseudo-gap [23] at $T_{c1} > T_c$ conforms with this interpretation. In particular, the fact pseudo-gap is non-vanishing already at T_{c_1} and stays constant rather than starting from zero as for quasi-particles conforms with the flux tube interpretation.

Results from optical measurements and neutron scattering

Optical measurements and neutron scattering have provided especially valuable microscopic information about high T_c superconductors allowing to fix the details of TGD based quantitative model.

Optical measurements of copper oxides in non-super-conducting state have demonstrated that optical conductivity $\sigma(\omega)$ is surprisingly featureless as a function of photon frequency. Below the critical temperature there is however a sharp absorption onset at energy of about 50 meV [34]. The origin of this special feature has been a longstanding puzzle. It has been proposed that this absorption onset corresponds to a direct generation of an electron-hole pair. Momentum conservation implies that the threshold for this process is $E_g + E$, where E is the energy of the 'gluon' which binds electrons of Cooper pair together. In case of ordinary super-conductivity E would be phonon energy.

Soon after measurements, it was proposed that in absence of lattice excitations photon must generate two electron-hole pairs such that electrons possess opposite momenta [34]. Hence the energy of the photon would be $2E_g$. Calculations however predicted soft rather than sharp onset of absorption since pairs of electron-hole pairs have continuous energy spectrum. There is something wrong with this picture.

Second peculiar characteristic [35, 31, 30] of high T_c super conductors is resonant neutron scattering at excitation energy $E_w = 41$ meV of super conductor. This scattering occurs only below the critical temperature, in spin-flip channel and for for favored momentum exchange $(\pi/a, \pi/a))$, where a denotes the size of the lattice cube [35, 31, 30]. The transferred energy is concentrated in a remarkably narrow range around E_w rather than forming a continuum.

In [27] is is suggested that e-e resonance with spin one gives rise to this excitation. This resonance is assumed to play the same role as phonon in ordinary super conductivity and ee resonance is treated like phonon. It is found that one can understand the dependence of the second derivative of the photon conductivity $\sigma(\omega)$ on frequency and that consistency with neutron scattering data is achieved. The second derivative of $\sigma(\omega)$ peaks near 68 meV and assuming $E = E_g + E_w$ they found nearly perfect match using $E_g = 27$ meV. This would suggest that the energy of the excitations generating the binding between the members of the Cooper pair is indeed 41 meV, that two electron-hole pairs and excitation of the super conductor are generated in photon absorption above threshold, and that the gap energy of the Cooper pair is 27 meV. Of course, the theory of Carbotte *et al* does not force the 'gluon' to be triplet excitation of electron pair: also other possibilities can be considered.

11.3.2 Vision about high T_c superconductivity

The following general view about high T_c super-conductivity as quantum critical phenomenon suggests itself.

Interpretation of critical temperatures

The two critical temperatures T_c and $T_{c_1} > T_c$ are interpreted as critical temperatures. T_{c_1} is the temperature for the formation of a quantum critical phase consisting of ordinary electrons and exotic Cooper pairs with large value of Planck constant. Quantum criticality of exotic Cooper pairs prevails for temperatures below T_{c_1} in the case that one has conductivity. For completely static stripes there is no conductivity. The absence of fluctuations suggests the loss of quantum criticality. One interpretation could be that exotic Cooper pairs are there but there can be no conductivity since the necessary transition of incoming ordinary electrons to large $\hbar$ dark electrons and back is not possible. T_c is the temperature at which BCS type Cooper pairs with large Planck constant become possible and exotic Cooper pairs can decay to the ordinary Cooper pairs.

Model for exotic and BCS type Cooper pairs

Exotic Cooper pair is modelled as a pair of large $\hbar$ electrons with zoomed up size at space-time sheets X^4_c topologically condensed to the background space-time sheet Y^4 of condensed matter system. The Coulombic binding energy of charged particles with the quarks and antiquarks assignable to the two wormhole throats feeding the em gauge flux to Y^4 could be responsible for the energy gap. Color force would bind the two space-time sheets to exotic Cooper pair.

Electrons of exotic Cooper pair can also end up a to same space-time sheet and possibly but not necessarily feed their em fluxes via two wormhole contacts carrying electron-positron pairs. In this case they are bound by the usual phonon interaction and form ordinary Cooper pair with large value of Planck constant.

The origin of the large $\hbar$ electrons must somehow relate to the breaking of antiferromagnetic phase by stripes. The neighboring electrons in stripe possess parallel spins and could therefore form a pair transforming to a large $\hbar$ Cooper pair bound by color force. This mechanism would be the TGD counterpart for the mechanism allowing the superconducting phases at different stripes to fuse to a single super-conducting phase at longer length scales.

Various lattice effects such as superconductivity-induced phonon shifts and broadenings, isotope effects in T_c, the penetration depth, infrared and photoemission spectra have been observed in the cuprates [25]. This would support the view that quantum criticality involves the competition between exotic and large $\hbar$ variant of BCS type super-conductivity and the proposed mechanism transforming exotic Cooper pair to BCS type pairs. The loss of antiferromagnetic order for higher dopings would make possible BCS type phonon induced super-conductivity with spin singlet Cooper pairs.

What is the value of $\hbar$?

The observed stripes would carry large $\hbar_{eff}$ electrons attracted to them by hole charge. The basic question concerns the value of $\hbar_{eff}$ which in the general case is given by $\hbar_{eff} = n_a/n_b$ where n_i is the order of the maximal cyclic subgroup of G_i.

1. The thickness of stripes is few atomic sizes and the first guess is that scaled up electrons have atomic size. The requirement that the integer n_a defining the value of M^4 Planck constant correspond to a n-polygon constructible using only ruler and compass gives strong constraints. An even stronger requirement would be that subgroup $G_a \subset SU(2)$ characterizes the Jones inclusion involved and thus the covering of CP_2 by M^4 points, corresponds to exceptional group via McKay correspondence, leaves only one possibility: $N(G_b) = 120$ which corresponds to E_8 Dynkin diagram having Z_5 as maximal cyclic subgroup and involving Golden Mean. The p-adic length scale of electron would be scaled up: $L(127) \rightarrow 5L(127) \simeq L(127+12) = L(139) \simeq 1.6$ Angstrom. This picture is not consistent with the model involving cell membrane length scale and the appearance of 50 meV energy scale which can be interpreted in terms of Josephson energy for cell membrane at criticality for nerve pulse generation is too intriguing signal to be dismissed.

2. The length of stripes is in the range 1-10 nm and defines second length scale in the system. If the Compton wavelength of scaled up electron corresponds to this length then $n_a = n_F = 2^{11}$ whose powers are encountered in the quantum model of living matter would suggest itself, and would predict the effective p-adic length scale electron to be $L(127+22) = L(149) = 5$ nm, the thickness of the lipid layer of the cell membrane which brings in mind cell membrane and bio-superconductivity. It will be found that simple stability arguments favor this size scale for scaled up electrons and size $L(151)$ for the exotic Cooper pairs. The minimum option is that only the exotic Cooper pairs making possible super-conductivity above T_c and broken by quantum criticality against transition to ordinary electron need have size of order $L(151) = 10$ nm.

3. The coherence length for high T_c super conductors is reported to 5-20 Angstroms. The naive interpretation would be as the size of BCS type Cooper pair which would suggest that scaled up electrons have at most atomic size. There is however a loophole involved. The estimate for coherence length in terms of gap energy is given by $\xi = \frac{4\hbar v_F}{E_{gap}}$. If coherence length is estimated from the gap energy, as it seems to be the case, then the scaling up of Planck constant would increase coherence length by a factor n_F and give coherence length in the range $1-4$ μm.

4. The dependence $T_c \propto 1/L$, where L is the distance between stripes is a challenge for the model since it would seem to suggest that stripe-stripe interaction is important for the energy gap of BCS type Cooper pairs. One can however understand this formula solely in terms of 2-dimensional character of high T_c super-conductors. To see this, consider generalization of the 3-D formula

$$E_{gap} = \hbar\omega_c exp(-1/X)$$
$$\omega_D = (6\pi^2)^{1/3} c_s n_n^{1/3}$$

for the gap energy to 2-dimensional case. Since only the nuclei inside stripes contribute to high T_c super-conductivity it is natural to replace 3-dimensional formula for Debye frequency in 2-dimensional case with

$$\omega_D = k c_s n_h^{1/2} \ ,$$

where n_h is the 2-dimensional density of holes and k a numerical constant. Since one has $n_h \propto 1/L^2$ this indeed predicts $E_{gap} \propto 1/L$.

Quantum criticality below T_{c_1}

Exotic Cooper pairs would be present below the higher critical temperature T_{c_1} associated with high T_c super-conductors and start to transform to BCS type Cooper pairs at T_c. Also the reverse process occurs. In the intermediate temperature range they would be unstable against transition changing the value of Planck constant to ordinary ones and this instability would break the exotic super-conductivity to ordinary conductivity with resistance obeying scaling law as a function of temperature typical for quantum critical systems. The complete stability of stripes would indicate that the exotic Cooper pairs are present but conductivity is not possible since ordinary electrons entering to the system cannot transformed to exotic Cooper pairs.

Why doping by holes is necessary?

In high T_c super-conductivity doping by holes plays a crucial role. What is known that holes gather to the stripes and that there is a critical doping at which T_c is maximum. Cusp catastrophe as a general model for phase transition suggests that that super-conductivity is possible only in finite range for the hole concentration. This is indeed the case.

The holes form a positive charge density and this inspires the idea that Coulomb attraction between exotic Cooper pairs of electrons and holes leads to the formation of stripes. Stripes provide also electrons with parallel spins which can transform to exotic large $\hbar$ Cooper pairs at quantum criticality with respect to $\hbar$.

One should also understand the upper limit for the hole concentration.

1. The first explanation is that super-conductivity is not preserved above critical hole concentration due to the loss of fractal stripe structure. Part of the explanation could be that beyond critical hole concentrations it is not possible to arrange the stripes to a fractal lattice formed by a lattice of "super-stripes" which are lattices of stripes of thickness $L(151)$ containing the observed stripes such that super-stripes have separation $d \geq L(151)$. Doping fraction p gives an estimate for the distance d between super-stripes as $d = xL(151)$, $x = r/p - 1$, where r is the fraction of atoms belonging to stripe inside super-stripe and p is doping fraction. $x = 2/5$ and $p = .15$ gives $d = 5L(151)/3$. Note that ideal fractality would require $x/(1+x) = r$ giving $r \simeq p/2$.

2. One could also consider the possibility that large $\hbar$ BCS super-conductivity is not lost above critical hole concentration but is useless since the transformation of ordinary current carrying electrons to large $\hbar$ exotic Cooper pairs would not be possible. Thus a quantum critical interface allowing to transform ordinary current to supra current is necessary.

Zeros of Riemann ζ and quantum critical super conductors

A long standing heuristic hypothesis has been that the radial conformal weights Δ assignable to the functions $(r_M/r_0)^{\Delta}$ of the radial lightlike coordinate r_M of $\delta M^4_+/-$ of lightcone boundary in super-canonical algebra consisting of functions in $\delta M^4_\pm \times CP_2$ are expressible as linear combinations of zeros of Riemann Zeta. Quantum classical correspondence in turn inspires the hypothesis that these conformal weights can be mapped to the points of a geodesic sphere of CP_2 playing the role of conformal heavenly sphere.

The arguments of [C2] suggest that radial conformal weight Δ in fact depends on the point of geodesic sphere S^2 in CP_2 and is given in terms of the inverse $\zeta^{-1}(z)$ of Riemann ζ having the natural complex coordinate z of S^2 as argument. This implies a mapping of the radial conformal weights to the points of the geodesic sphere CP_2. Linear combinations of zeros correspond to algebraic points in the intersections of real and p-adic space-time sheets and are thus in a unique role from the point of view of p-adicization. This if one believes the basic conjecture that the numbers p^s, p prime and s zero of Riemann Zeta are algebraic numbers.

Zeros of Riemann Zeta have been for long time speculated to closely relate to fractal and critical systems. If the proposed general ansatz for super-canonical radial conformal weights holds true, these speculations find a mathematical justification.

Geometrically the transition changing the value of $\hbar(M^4)$ correspond to a leakage of partonic 2-surfaces between different copies of $M^4 \times CP_2$ with same CP_2 factor and thus same value of $\hbar(CP_2)$ but different scaling factor of CP_2 metric. M^4 metrics have the same scaling factor given by n_b^2.

Critical 2-surfaces can be regarded as belonging to either factor which means that points of critical 2-surfaces must correspond to the CP_2 orbifold points, in particular, $z = \xi^1/\xi^2 = 0$ and $z = \xi^1/\xi^2 = \infty$ remaining invariant under the group $G \subset SU(2) \subset SU(3)$ defining the Jones inclusion, that is the north and south poles of homologically non-trivial geodesic sphere $S^2 \subset CP_2$ playing the role of heavenly sphere for super-canonical conformal weights. If the hypothesis $\Delta = \zeta^{-1}(z)$ is accepted, the radial conformal weight corresponds to a zero of Riemann Zeta: $\Delta = s_k$ at quantum criticality.

At quantum level a necessary prerequisite for the transition to occur is that radial conformal weights, which are conserved quantum numbers for the partonic time evolution, satisfy the constraint $\Delta = s_k$. The partonic 2-surfaces appearing in the vertices defining S-matrix elements for the phase transitions in question need not be of the required kind. It is enough that $\Delta = s_k$ condition allows their evolution to any sector of H in question. An analogous argument applies also to the phase transitions changing CP_2 Planck constant: in this case however leakage occurs through a partonic 2-surface having single point as M^4 projection (the tip of $M^4_\pm$).

Quantum criticality for high temperature super-conductivity could provide an application for this vision. The super conducting stripe like regions are assumed to carry Cooper pairs with a large value of M^4 Planck constant corresponding to $n_a = 2^{11}$. The boundary region of the stripe is assumed to carry Cooper pairs in critical phase so that super-canonical conformal weights of electrons should satisfy $\Delta = s_k$ in this region. If the members of Cooper pair have conjugate conformal weights, the reality of super-canonical conformal weight is guaranteed. The model predicts that the critical region has thickness $L(151)$ whereas scaled electron with $n = 2^{11}$ effectively correspond to $L(127+22) = L(149)$, the thickness of the lipid layer of cell membrane. This picture would suggests that the formation and stability of the critical region is essential for the formation of phase characterized by high T_c super-conductivity with large value of Planck constant and forces temperature to a finite critical interval. In this framework surface super-conductivity would be critical and interior super-conductivity stable.

These observations in turn lead to the hypothesis that cell interior corresponds to a phase with large M^4 Planck constant $\hbar(M^4) = 2^{11}\hbar_0$ and cell membrane to a quantum critical region where the above mentioned condition $\Delta = s_k$ is satisfied. Thus it would seem that the possibility of ordinary electron pairs to transform to large $\hbar$ Cooper pairs is essential in living matter and that the transition takes place as the electron pairs traverse cell membrane. The quantum criticality of cell membrane might prevail only in a narrow temperature range around T=37 C. Note that critical temperature range can also depend on the group G having C_n, $n = 2^{11}$ cyclic group as maximal cyclic group (C_n and D_n are the options).

11.3.3 A detailed model for the exotic Cooper pair

Qualitative aspects of the model

High T_c superconductivity suggests that the Cooper pairs are stripe like structures of length 1-10 nm. The length of color magnetic flux tube is characterized by the p-adic length scale in question and $L(151) = nm$ is highly suggestive for high T_c superconductors.

These observations inspire the following model.

1. The space-time sheet of the exotic Cooper pair is obtained in the following manner. Take two cylindrical space-time sheets which have radius of order $L(149)$. One could of course argue that flux tubes can have this radius only along CuO plane and must flattened in the direction

orthogonal to the super-conducting plane with thickness of few atomic units in this direction. The assumption about flattening leads however to a very large electronic zero point kinetic energy. Furthermore, in the absence of flattening supra phases belonging to different CuO planes combine to form single quantum coherent phase so that coherence length can be longer than the thickness of CuO layer also in orthogonal direction.

2. Assume that the cylinders they contain electrons with u wormhole throat at top and $\overline{d}$ wormhole throat at bottom feeding the em gauge flux to the larger space-time sheet. Connect these parallel flux tubes with color magnetic bonds. If the $u\overline{d}$ states associated with the flux tubes are not in color singlet states, color confinement between wormhole quarks binds the electronic space-time sheets together and electrons are "free-travellers". These exotic Cooper pairs are energy minima for electrons are in large $\hbar$ phase if the electron kinetic energy remains invariant in $\hbar$ changing phase transition. This is achieved by fractionization of quantum numbers characterizing the kinetic energy of electron.

3. If the flux tubes carry magnetic flux electron spins are parallel to the magnetic field in minimum energy state. If the magnetic flux rotates around the resulting singlet sheeted structure the spin directions of electrons are opposite and only $S = 0$ state is possible as a minimum energy state since putting electrons to the same flux tube would give rise to a repulsive Coulomb interaction and also Fermi statistics would tend to increase the energy.

4. The homological magnetic monopoles made possible by the topology of CP_2 allows the electrons to feed their magnetic fluxes to a larger space-time sheet via u throat where it returns back via $\overline{d}$ throat. A 2-sheeted monopole field is in question. The directions of the magnetic fluxes for the two electrons are independent. By connecting the flux tubes by color bonds one obtains color bound electrons. In this kind of situation it is possible to have $S = 1$ state even when electrons are at different flux tubes portions so that energies are degenerate in various cases. The resulting four combinations give $S_z = \pm 1$ states and two $S_z = 0$ states which means spin triplet and singlet. Interestingly, the first 23 year old model of color confinement was based on the identification of color hyper charge as homological charge. In the recent conceptual framework the the space-time correlate for color hyper charge Y of quark could be homological magnetic charge $Q_m = 3Y$ so that color confinement for quarks would have purely homological interpretation at space-time level.

5. One can also understand how electrons of Cooper pair can have angular momentum ($L = 2$ in case of high T_c Cooper pairs and $L = 0$ in case of ^{3}He Cooper pairs) as well as correlation between angular momentum and spin. The generation of radial color electric field determined by the mechanical equilibrium condition $E + v \times B = 0$ inside give portion of flux tube implies that electrons rotate in same direction with velocity v. A non-vanishing radial vacuum E requires that flux tube portion contains cylindrical hole inside it. Without hole only $v = 0$ is possible. Assume that the directions of radial E and thus v can be freely chosen inside the vertical portions of flux tube. Assume that also $v = 0$ is possible in either or both portions. This allows to realize L_z values corresponding to $L = 0, 1, 2$ states.

6. Since quarks in this model appear only as parton pairs associated with wormhole contacts, one expects that the corresponding p-adic mass scale is automatically determined by the relevant p-adic length scale, which would be $L(151)$ in case of high T_c superconductors. This would mean that the mass scale of inertial mass of wormhole contact would be 10^2 eV even in the case that p-adic temperature is $T_p = 1$. For $T_p = 2$ the masses would be extremely small. The fact that the effective masses of electrons can be as high as $100m_e$ [18] means that the mass of wormhole contact does not pose strong constraints on the effective mass of the Cooper pair.

7. The decay of Cooper pair results if electrons are thrown out from 2e space-time sheet. The gap energy would be simply the net binding energy of the system. This assumption can make sense for high T_c super-conductors but does not conform with the proportionality of the gap energy to Debye frequency $\omega_D = v_s/a$ in the case of ordinary super-conductors for which phonon space-time sheets should replace color flux tubes.

8. Both the assumption that electrons condensed at $k = 149$ space-time sheets result from scaled up large $\hbar$ electrons and minimization of energy imply the the scales $L(149)$ and $L(151)$ for the

space-time sheets involved so that there is remarkable internal consistency. The model explains the spins of the exotic Cooper pairs and their angular momenta. The dark BSC type Cooper pairs are expected to have $S = 0$ and $L = 0$.

Quantitative definition of the model

There are several poorly understood energies involved with high T_c super-conductors below T_c. These are $E_g = 27$ meV, $E_1 = 50$ meV, $E_w = 41$ meV, and $E_2 = 68$ meV. These numbers allow to fix the wormholy model for quantum critical super-conductors to a high degree.

Consider now a quantitative definition of the model.

1. p-Adic length scale hypothesis combined with the ideas about high T_c super-conductivity in living matter plus the fact that the stripe like defects in high T_c superconductors have lengths 1-10 nm suggests that the length scales $L(151) = 10$ nm corresponding to cell membrane thickness and $L(149) = 5$ nm corresponding to the thickness of its lipid layer are the most important p-adic length scales. Of course, also $L(145 = 5 \times 29) = 1.25$ nm could be important. $L(151)$ would be associated with the structure consisting of two flux tubes connected by color bonds.

2. The kicking of electrons from $k = 151$ to $k = 149$ space-time sheet should define one possible excitation of the system. For wormhole contacts kicking of electron to smaller space-time sheet is accompanied by the kicking of wormhole contacts from the pair $(151, 157)$ to a pair $(149, 151)$ of smaller space-time sheets. This can be achieved via a flow along JABs $157 \rightarrow 151$ and $151 \rightarrow 149$. Also the dropping of electrons from color flux tube to larger space-time sheet defines a possible transition.

3. Assume that given electrons reside inside electronic flux tubes connected having u and $\overline{d}$ at their ends and connected by color bonds. Assume that electrons are completely delocalized and consider also the configuration in which both electrons are in the same electronic flux tube. The total energy of the system is the sum of zero point kinetic energies of electrons plus attractive Coulomb interaction energies with u and $\overline{d}$ plus a repulsive interaction energy between electrons which contributes only when electrons are in the same flux tube. Minimum energy state is obviously the one in which electrons are at different flux tubes.

   By effective one-dimensionality the Coulomb potential can be written as $V(z) = \alpha Qz/S$, where S is the thickness of the flux tube. It is assumed that S scales $L(k)^2/y$, $y > 1$, so that Coulomb potential scales as $1/L(k)$. The average values of Coulomb potential for electron quark interaction ($Q(u) = 2/3$ and $Q(\overline{d}) = 1/3$) and ee interaction are

$$
\begin{aligned}
V_{eq} &= \frac{y}{2}V(k) \ , \\
V_{ee} &= \frac{y}{3}V(k) \ , \\
V(k) &= \frac{\alpha}{L(k)} \ .
\end{aligned}
\tag{11.3.-1}
$$

   One can introduce a multiplicative parameter x to zero point kinetic energy to take into account the possibility that electrons are not in the minimum of kinetic energy. The color interactions of wormhole throats can of course affect the situation.

   With these assumptions the estimate for the energy of the 2e space-time sheet is

$$
\begin{aligned}
E_{2e}(k) &= 2xT(k) - 2V_{eq} + \epsilon V_{ee} = 2xT(k) - y(1 - \frac{\epsilon}{3})V(k) \ , \\
T(k) &= \frac{D}{2}\frac{\pi^2}{2m_eL^2(k)} \ , \\
V(k) &= \frac{\alpha}{L(k)} \ .
\end{aligned}
\tag{11.3.-2}
$$

Here $\epsilon = 1/0$ corresponds to the situation in which electrons are/are not in the same flux tube. One has $x \geq 1$ and $x = 1$ corresponds to the minimum of electron's kinetic energy. If the maximum area of the tube is $\pi L(151)^2$, one should have $y \leq \pi$. The effective dimension is $D = 1$ for flux tube. $k = 151$ and $k = 149$ define the most interesting p-adic length scales now.

4. By p-adic scaling one has

$$E_{2e}(k) = 2^{151-k} \times 2xT(151) - 2^{(151-k)/2} \times y(1 - \frac{\epsilon}{3})V(151) \ . \tag{11.3.-1}$$

The general form of the binding energy implies that it has maximum for some value of k and the maximum turns out to correspond to $k = 151$ with a rather reasonable choice of parameters x and y.

One could also require a stability against the transition $151 \rightarrow 149$. Here a difficulty is posed by the fact that color interaction energy of wormhole contacts probably also changes. One can however neglect this difficulty and look what one obtains. In this approximation stability condition reads as

$$E_{2e}(149) - E_{2e}(151) = 6xT(151) - y(1 - \frac{\epsilon}{3})V(151) > 0 \ . \tag{11.3.0}$$

One obtains

$$\frac{y}{x} \leq \frac{6T(151)}{V(151)} = \frac{6}{\alpha}\frac{\pi^2}{2m_e L(151)} \simeq 3.54 \ . \tag{11.3.1}$$

For $k > 151$ the binding energy decreases so fast that maximum of the binding energies at $k = 151$ might be guaranteed by rather reasonable conditions on parameters.

5. The general formula λ is expected to make sense and gives rather large λ. The BCS formula for ξ need not make sense since the notion of free electron gas does not apply. A good guess is that longitudinal ξ is given by the height $L(151) = 10$ nm of the stripe. Transversal ξ, which is in the range 4-20 Angstroms, would correspond to the thickness of the color magnetic flux tube containing electrons. Hence the scale for ξ should be smaller than the thickness of the stripe.

Estimation of the parameters of the model

It turns out to be possible to understand the energies E_2, E_1, E_w and E_g in terms of transitions possible for wormhole contact option. The values of the parameters x and y can be fitted from the following conditions.

1. The largest energy $E_2 = 68$ meV is identified as the binding energy in the situation in which electrons are at different flux tubes. Hence one has $E_{2e}(\epsilon = 0) = -E_2$ giving

$$-2xT(151) + yV(151) = E_2 \ . \tag{11.3.2}$$

The peak in photo-absorption cross section would correspond to the dropping of both electrons from the flux tube to a much larger space-time sheet.

2. The energy $E_g = 27$ meV is identified as the binding energy in the situation that electrons are at the same flux tube so that E_g represents the energy needed to kick electrons to a much larger space-time sheet. This gives

$$-2xT(151) + \frac{2}{3}yV(151) = E_g . \quad (11.3.3)$$

3. E_w corresponds to the difference $E_2 - E_g$ and has an interpretation as the energy needed to induce a transition from state with $\epsilon = 0$ (electrons at different flux tubes) to the state with $\epsilon = 1$ (electrons at the same flux tube).

$$E_{2e}(151, \epsilon = 1) - E(2e)(151, \epsilon = 0) = \frac{y}{3}V(151) = E_w . \quad (11.3.3)$$

This condition allows to fix the value of the parameter y as

$$y = \frac{3E_w}{V(151)} . \quad (11.3.4)$$

Condition 1) fixes the value of the parameter x as

$$x = \frac{E_w}{T(151)} . \quad (11.3.5)$$

Using $V(151) \simeq 144$ meV and $T(151) = 20.8$ meV this gives $y = .8539 < \pi$ and $x = 1.97$. The area of the color flux tube is .27 per cent about $S_{max} = \pi L^2(151)$ so that its radius equals in a good approximation $L(149)$, which looks rather large as compared to the estimated thickness of the visible stripe. $x = 1.97$ means that the electron's kinetic energy is roughly twice the minimal one. $y/x = .43$ satisfies the bound $y/x < 6T(151)/V(151) = .87$ guaranteing that the binding energy is maximum for $k = 151$. This result is rather remarkable.

4. The model should explain also the energy $E_1 \simeq 50$ meV at which sharp photon absorption sets on. The basic observation is that for neuronal membrane 50 mV corresponds to the critical voltage for the generation of nerve pulse. In super-conductor model of cell membrane 50 meV is identified as the energy of Josephson photon emitted or absorbed when Cooper pair moves from cell interior to exterior of vice versa. Thus 50 meV energy *might* correspond to the energy of Josephson photon and kick BCS type Cooper pair between the two layers of the double-layered super stripe.

   Note that 50 meV corresponds to a thermal energy of 3-D system at T= 333 K (60 C). This is not far from 37 C, which would also suggest that high T_c super-conductivity is possible at room temperatures. In the case of cell membrane quantum criticality could among other things make possible the kicking of the large $\hbar$ BCS type Cooper pairs between lipid layers of cell membrane. If so, neurons would be quantum critical only during nerve pulse generation.

One can consider also alternative explanation. 50 meV is not much higher than 41 meV so that it could relate to the $\epsilon = 0 \rightarrow 1$ transition. Recoil effects are negligible. Perhaps $m = 1$ rotational excitation of electron of 2e system residing at the same flux tube and having energy $E = 9$ meV is in question. This excitation would receive the spin of photon. The energy scale of electronic rotational excitations is $\hbar^2/2m_eL^2(149) \sim 8.4$ meV if the radius of the flux tube is $L(149)$.

To sum up, the model allows to understand the four energies assuming natural values for adjustable parameters and predicts that $k = 151$ corresponds to stable Cooper pairs. It seems that the model could apply to a large class of quantum critical super-conductors and scaled up electrons might be involved with all condensed matter phenomena involving stripes.

Model for the resonance in neutron scattering

The resonance in neutron scattering is usually understood as a resonance in the scattering from the modification of the lattice induced by the formation of stripes and this scattering gives the crucial information about cross-like structure of Fermi surface of holes suggesting crossed stripes. One can also consider the possibility that the scattering is on exotic Cooper pairs which could always accompany stripes but as such need not give rise to super-conductivity or not even conductivity unless they are in quantum critical state.

Consider now the TGD based model for neutron scattering based on the proposed model for Cooper pairs.

1. Neutrons couple naturally to the magnetic field accompanying color magnetic field at the space-time sheet of Cooper pair by magnetic moment coupling. As found, $E_w = 41$ meV can be interpreted as the energy needed to induce the $\epsilon = 0 \rightarrow 1$ transition. Spin flip necessarily occurs if the electron is kicked between the vertical flux tubes.

2. Resonance would result from the coherent coupling to the wormhole BE condensate making scattering rate proportional to N^2, where N denotes the number of wormhole contacts, which is actually identical with the total number of super conducting electrons. Therefore the prediction of the TGD based model is very similar to the prediction of [27]. The absence of the resonance above critical temperature suggests that exotic Cooper pairs are not present above T_c. The presence of quantum criticality also above T_c suggests that Cooper pairs decay to wormholy space-time sheets containing single electron plus wormholy pion $u\bar{d}$ responsible for the ordinary conductivity. The transition is possible also for these space-time sheets but they do not form Bose-Einstein condensate so that the resonance in neutron scattering is predicted to be much weaker for temperatures above the critical temperature. For overcritical doping the resonance should be absent if exotic Cooper pairs are possible only at the boundaries of two phases disappearing at critical doping.

3. The momentum transfer associated with the resonance is located around the momentum $(\pi/a, \pi/a)$ in resiprocal lattice [28], where a denotes the length for the side of the lattice cell. The only possible conclusion is that in the scattering neutron momentum is transferred to the lattice whereas the remaining small momentum is transferred to the momentum of wormhole BE condensate. Thus the situation is analogous to that occurring in Mössbauer effect.

What is the origin of picosecond time scale

The model should also predict correctly the picosecond and 1-10 nm length scales. Quantum criticality suggests that picosecond time scale relates directly to the 10 nm length scale via p-adic length scale hypothesis. $L(151) = 10$ nm defining the size for color flux tubes containing electrons of Cooper pair and lower limit for the distance between predicted super-stripes would correspond to a p-adic time scale $T(151) \sim 10^{-16}/3$ seconds for ordinary Planck constant. For $\hbar = 2^{22}\hbar_0$ this time scale would be scaled up to about $.15n$ picoseconds. This kind of length scale corresponds for electron to $n_F = 2^{22}$ rather than $n_F = 2^{11}$. One could however argue that by the very definition of quantum quantum criticality several values of n_F must be involved. The quantum model of EEG indeed assumes this kind of hierarchy [M3]. Note that $n_F = 3 \times 2^{12}$ would give picosecond scale as also (157).

Just for fun one can also consider the possibility that this time scale is due to the large $\hbar$ phase for nuclei and hadrons. Large $\hbar$ for nuclei and quarks would means gigantic Compton lengths and makes possible macroscopic quantum phase competing with ordinary phase. If one accepts TGD based model for atomic nuclei where $k = 129$ corresponds to the size of the magnetic body of ordinary nuclei [F8], the super-stripes could involve also the color magnetic bodies of dark hadrons. The size of color magnetic body for ordinary hadrons is $L(k_{eff} = 107 + 22 = 129)$ and therefore $L(k_{eff} = 129 + 22 = 151)$ for dark hadrons. This of course forces the question whether the nuclei along stripes correspond to dark nuclei. Large $\hbar$ phase for hadrons means also scaling up of the basic purely hadronic time scales. Notice that neutral pion lifetime $\sim 2 \times 10^{-16}$ seconds would be scaled up by a factor 2^{11} to .2 picoseconds.

Why copper and what about other elements?

The properties of copper are somehow crucial for high T_c superconductivity since cuprates are the only known high T_c superconductors. Copper corresponds to $3d^{10}4s$ ground state configuration with one valence electron. This encourages to think that the doping by holes needed to achieve superconductivity induces the dropping of these electrons to $k = 151$ space-time sheets and gives rise to Cooper pairs.

More generally, elements having one electron in s state plus full electronic shells are good candidates for doped high T_c superconductors. If the atom in question is also a boson the formation of atomic Bose-Einstein condensates at Cooper pair space-time sheets is favored. Superfluid would be in question. Thus elements with odd value of A and Z possessing full shells plus single s wave valence electron are of special interest. The six stable elements satisfying these conditions are ^{5}Li, ^{39}K, ^{63}Cu, ^{85}Rb, ^{133}Cs, and ^{197}Au. Partially dark Au for which dark nuclei form a superfluid could correspond to what Hudson calls White Gold [43] and the model for high T_c superconductivity indeed explains the properties of White Gold.

11.3.4 Speculations

21-Micrometer mystery

21 micrometer radiation from certain red giant stars have perplexed astronomers for more than a decade. Emission forms a wide band (with width about 4 micrometers) in the infrared spectrum which suggests that it comes form a large complex molecule or a solid or simple molecules found around starts. Small molecules are ruled out since they produce narrow emission lines. The feature can be only observed in very precise evolutionary state, in the transition between red giant phase and planetary nebular state, in which star blows off dust that is rich in carbon compounds. There is no generally accepted explanation for 21-micrometer radiation.

One can consider several explanations based on p-adic length scale hypothesis and some explanations might relate to the wormhole based super-conductivity.

1. 21 micrometers corresponds to the photon energy of 59 meV which is quite near to the zero point point kinetic energy 61.5 meV of proton Cooper pair at $k = 139$ space-time sheet estimated from the formula

$$\Delta E(2m_p, 139) = \frac{1}{2}\frac{\pi^2}{(2m_p)L(169)^2} = \frac{1}{8}\Delta E(m_p, 137) \simeq 61.5\ meV \quad .$$

   Here the binding energy of the Cooper pair tending to reduce this estimate is neglected, and this estimate makes sense only apart from a numerical factor of order unity. This energy is liberated when a Cooper pair of protons at $k = 139$ space-time sheet drops to the magnetic flux tube of Earth's magnetic field (or some other sufficiently large space-time sheet). This energy is rather near to the threshold value about 55 meV of the membrane potential. This observation and the presence of the carbon compounds leads to ask whether bio-superconductors and perhaps even some primitive forms of life might be involved.

2. 21 micrometer radiation could also result when electrons at $k = 151$ space-time sheet drop to a large enough space-time sheet and liberate their zero point kinetic energy. Scaling argument gives for the zero point kinetic energy of electron at $k = 151$ space-time sheet the value $\Delta(e, 151) \simeq$ 57.5 meV which is also quite near to the observed value. If electron is bound to wormhole with quantum numbers of $\bar{d}$ Coulombic binding energy changes the situation.

3. A possible explanation is as radiation associated with the transition to high T_c super conducting phase. There are two sources of photons. Radiation could perhaps result from the de-excitations of wormhole BE condensate by photon emission. $\lambda = 20.48$ micrometers is precisely what one expects if the space-time sheet corresponds to $p \simeq 2^k$, $k = 173$ and assumes that excitation energies are given as multiples of $E_w(k) = 2\pi/L(k)$. This predicts excitation energy $E_w(173) \simeq$ 61.5 meV. Unfortunately, this radiation should correspond to a sharp emission line and cannot explain the wide spectrum.

Ionic high T_c superconductivity and high T_c super-fluidity

The model of electronic superconductivity generalizes to the case of fermionic ions in almost trivial manner. The stability condition determining the p-adic length scale in question is obtained by replacing electron mass with the mass Am_p of ion and electron charge with the charge Ze of the ion. The expression of binding energy as sum of kinetic energy and Coulombic interaction energy has the general form

$$T_e + V_{ee} + V_{eq} = \frac{a_e}{L^2(k)} - \frac{b_e}{L(k)} \ , \tag{11.3.6}$$

and gives maximum binding energy for

$$L = \frac{2a_e}{b_e} \simeq L(151) \ . \tag{11.3.7}$$

The replacement of electrons with ions of charge Z induces the replacements

$$\begin{aligned} a_e &\rightarrow \frac{m_e}{Am_p} a_e \ , \\ b_e &\rightarrow Z^2 b_e \ , \\ L &\rightarrow \frac{m_e}{AZ^2 m_p} L_e \simeq \frac{1}{AZ^2} L(129) \ . \end{aligned} \tag{11.3.6}$$

This scale would be too short for ordinary value of $\hbar$ but if the nuclei are in large $\hbar$ phase, L is scaled up by a factor $\simeq n \times 2^{11}$ to $L(k_{eff}) = nL(k+22)$. This gives

$$L(k) \simeq \frac{n}{AZ^2} L(151) \ . \tag{11.3.7}$$

This length scale is above $L(137)$ for $AZ^2 < 2^7 n = 128n$: $n = 3$ allows all physical values of A. If $L(135)$ is taken as lower bound, one has $AZ^2 < 2^9 n$ and $n = 1$ is enough.

Second constraint comes from the requirement that the gap temperature defined by the stability against transition $k \rightarrow k-2$ is above room temperature.

$$3 \times \frac{\pi^2 \hbar^2}{2Am_p L^2(k)} \simeq 2^{-k+137} \frac{.5}{A} \quad \text{eV} \geq T_{room} \simeq .03 \quad \text{eV} \ . \tag{11.3.8}$$

Since the critical temperature scales as zero point kinetic energy, it is scaled down by a factor m_e/Am_p. $k \geq 137$ would give $A \leq 16$, $k = 135$ would give $A \leq 64$, and $k = 131$ allows all values of A.

The Bose-Einstein condensates of bosonic atoms giving rise to high T_c super fluidity are also possible in principle. The mechanism would be the dropping of atoms to the space-time sheets of electronic Cooper pairs. Thermal stability is achieved if nuclei are in doubly dark nuclear phase and electrons correspond to large $\hbar$ phase. Electronic Cooper pairs would correspond to $k_{eff} = 151+22 = 173$ space-time sheets with size about 20 μm. This is also the size scale of the Bohr radius of dark atoms [J6]. The claimed properties of so called ORMEs [43] make them a possible candidate for this kind of phase.

Are living systems high T_c superconductors?

The idea about cells and axons as superconductors has been one of the main driving forces in development of the vision about many-sheeted space-time. Despite this the realization that the supra currents in high T_c superconductors flow along structure similar to axon and having same crucial length scales came as a surprise. Axonal radius which is typically of order $r = .5$ μm. $\lambda = 2^{11}$ would predict $r = .2$ μm. The fact that water is liquid could explain why the radius differs from that predicted in case of high T_c superconductors.

Interestingly, Cu is one of the biologically most important trace elements [38]. For instance, copper is found in a variety of enzymes, including the copper centers of cytochrome c-oxidase, the Cu-Zn containing enzyme superoxide dismutase, and copper is the central metal in the oxygen carrying pigment hemocyanin. The blood of the horseshoe crab, Limulus polyphemus uses copper rather than iron for oxygen transport. Hence there are excellent reasons to ask whether living matter might be able to build high T_c superconductors based on copper oxide.

Neuronal axon as a geometric model for current carrying "rivers"

Neuronal axons, which are bounded by cell membranes of thickness $L(151)$ consisting of two lipid layers of thickness $L(149)$ are high T_c superconductors (this was not the starting point but something which popped out naturally). The interior of this structure is in large $\hbar$ nuclear phase, which is partially dark. Since the thickness of the tube should be smaller than the quantum size of the dark nuclei, a lower limit for the the radius r of the corresponding nuclear space-time sheets is obtained by scaling up the weak length scale $L_w(113) = 2^{(11-89)/2}L_w(89)$ defined by W boson Compton length by a factor 2^{22} to doubly dark weak length scale $L_w = 2^{22}L_w(113) = .2\ \mu$m.

These flux tubes with radius $r > L_w$ define "rivers" along which conduction electrons and various kinds of Cooper pairs flow. Scaled up electrons have size $L(k_{eff} = 149)$ corresponding to 5 nm, the thickness of the lipid layer of cell membrane. The observed quantum fluctuating stripes of length 1-10 nm might relate very closely to scaled up electrons with Compton length 5 nm, perhaps actually representing zoomed up electrons!

According to the model of dark Cooper pairs the $k = 149$ flux tubes at which electrons are condensed should be hollow. What comes in mind first is that a cylinder with radius $L(149)$ is in question having a hollow interior with say atomic radius.

The original assumption that exotic *resp.* BCS type Cooper pairs reside at boundaries *resp.* interior of the super-conducting rivulet. It would however seem that the most natural option is that the hollow cylindrical shells carry all supra currents and there are no Cooper pairs in the interior. If exotic Cooper pairs reside only at the boundary of the rivulet or the Cooper pairs at boundary remain critical against exotic-BCS transition also below T_c, the time dependent fluctuations of the shapes of stripes accompanying high T_c super-conductivity can be understood as being induced by the fluctuations of membrane like structures. Quantum criticality at some part of the boundary is necessary in order to transform ordinary electron currents to super currents at the ends of rivulets. In biology this quantum criticality would correspond to that of cell membrane.

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Chapter 12

Quantum Hall effect and Hierarchy of Planck Constants

12.1 Introduction

Quantum Hall effect [29, 30, 35] occurs in 2-dimensional systems, typically a slab carrying a longitudinal voltage V causing longitudinal current j. A magnetic field orthogonal to the slab generates a transversal current component j_T by Lorentz force. j_T is proportional to the voltage V along the slab and the dimensionless coefficient is known as transversal conductivity. Classically the coefficients is proportional ne/B, where n is 2-dimensional electron density and should have a continuous spectrum. The finding that came as surprise was that the change of the coefficient as a function of parameters like magnetic field strength and temperature occurred as discrete steps of same size. In integer quantum Hall effect the coefficient is quantized to $2\nu\alpha$, $\alpha = e^2/4\pi$, such that ν is integer.

Later came the finding that also smaller steps corresponding to the filling fraction $\nu = 1/3$ of the basic step were present and could be understood if the charge of electron would have been replaced with $\nu = 1/3$ of its ordinary value. Later also QH effect with wide large range of filling fractions of form $\nu = k/m$ was observed.

The model explaining the QH effect is based on pseudo particles known as anyons [39, 30]. According to the general argument of [28] anyons have fractional charge νe. Also the TGD based model for fractionization to be discussed later suggests that the anyon charge should be νe quite generally. The braid statistics of anyon is believed to be fractional so that anyons are neither bosons nor fermions. Non-fractional statistics is absolutely essential for the vacuum degeneracy used to represent logical qubits.

In the case of Abelian anyons the gauge potential corresponds to the vector potential of the divergence free velocity field or equivalently of incompressible anyon current. For non-Abelian anyons the field theory defined by Chern-Simons action is free field theory and in well-defined sense trivial although it defines knot invariants. For non-Abelian anyons situation would be different. They would carry non-Abelian gauge charges possibly related to a symmetry breaking to a discrete subgroup H of gauge group [39] each of them defining an incompressible hydrodynamical flow. According to [18] the anyons associated with the filling fraction $\nu = 5/2$ are a good candidate for non-Abelian anyons and in this case the charge of electron is reduced to $Q = e/4$ rather than being $Q = \nu e$ [36]. This finding favors non-Abelian models [35].

Non-Abelian anyons [38, 30] are always created in pairs since they carry a conserved topological charge. In the model of [18] this charge should have values in 4-element group Z_4 so that it is conserved only modulo 4 so that charges +2 and -2 are equivalent as are also charges 3 and -1. The state of n anyon pairs created from vacuum can be show to possess 2^{n-1}-dimensional vacuum degeneracy [37]. When two anyons fuse the 2^{n-1}-dimensional state space decomposes to 2^{n-2}-dimensional tensor factors corresponding to anyon Cooper pairs with topological charges 2 and 0. The topological "spin" is ideal for representing logical qubits. Since free topological charges are not possible the notion of physical qubit does not make sense (note the analogy with quarks). The measurement of topological qubit reduces to a measurement of whether anyon Cooper pair has vanishing topological charge or not.

Topological quantum computation is perhaps the most promising application of anyons [17, 18, 19, 20, 21, 22, 23].

I have already earlier proposed the explanation of FQHE, anyons, and fractionization of quantum numbers in terms of hierarchy of Planck constants realized as a generalization of the imbedding space $H = M^4 \times CP_2$ to a book like structure [A9]. The book like structure applies separately to CP_2 and to causal diamonds ($CD \subset M^4$) defined as intersections of future and past directed light-cones. The pages of the Big Book correspond to singular coverings and factor spaces of CD (CP_2) glued along 2-D subspace of CD (CP_2) and are labeled by the values of Planck constants assignable to CD and CP_2 and appearing in Lie algebra commutation relations. The observed Planck constant $\hbar$, whose square defines the scale of M^4 metric corresponds to the ratio of these Planck constants. The key observation is that fractional filling factor results if $\hbar$ is scaled up by a rational number.

In this chapter I try to formulate more precisely this idea. The outcome is a rather detailed view about anyons on one hand, and about the Kähler structure of the generalized imbedding space on the other hand.

1. Fundamental role is played by the assumption that the Kähler gauge potential of CP_2 contains a gauge part with no physical implications in the context of gauge theories but contributing to physics in TGD framework since $U(1)$ gauge transformations are representations of symplectic transformations of CP_2. Also in the case of CD it makes also sense to speak about Kähler gauge potential. The gauge part codes for Planck constants of CD and CP_2 and leads to the identification of anyons as states associated with partonic 2-surfaces surrounding the tip of CD and fractionization of quantum numbers. Explicit formulas relating fractionized charges to the coefficients characterizing the gauge parts of Kähler gauge potentials of CD and CP_2 are proposed based on some empirical input.

2. One important implication is that Poincare and Lorentz invariance are broken inside given CD although they remain exact symmetries at the level of the geometry of world of classical worlds (WCW). The interpretation is as a breaking of symmetries forced by the selection of quantization axis.

3. Anyons would basically correspond to matter at 2-dimensional "partonic" surfaces of macroscopic size surrounding the tip of the light-cone boundary of CD and could be regarded as gigantic elementary particle states with very large quantum numbers and by charge fractionization confined around the tip of CD. Charge fractionization and anyons would be basic characteristic of dark matter (dark only in relative sense). Hence it is not surprising that anyons would have applications going far beyond condensed matter physics. Anyonic dark matter concentrated at 2-dimensional surfaces would play key key role in the the physics of stars and black holes, and also in the formation of planetary system via the condensation of the ordinary matter around dark matter. This assumption was the basic starting point leading to the discovery of the hierarchy of Planck constants [A9]. In living matter membrane like structures would represent a key example of anyonic systems as the model of DNA as topological quantum computer indeed assumes [L5].

4. One of the basic questions has been whether TGD forces the hierarchy of Planck constants realized in terms of generalized imbedding space or not. The condition that the choice of quantization axes has a geometric correlate at the imbedding space level motivated by quantum classical correspondence of course forces the hierarchy: this has been clear from the beginning. It is now clear that also the first principle description of anyons requires the hierarchy in TGD Universe. The hierarchy reveals also new light to the huge vacuum degeneracy of TGD and reduces it dramatically at pages for which CD corresponds to a non-trivial covering or factor space, which suggests that mathematical existence of the theory necessitates the hierarchy of Planck constants. Also the proposed manifestation of Equivalence Principle at the level of symplectic fusion algebras as a duality between descriptions relying on the symplectic structures of CD and CP_2[C4] forces the hierarchy of Planck constants.

The first sections of the chapter contain summary about theories of quantum Hall effect appearing already in [E9]. Second section is a slightly modified version of the description of the generalized imbedding space, which has appeared already in [A9, E9, L5] and containing brief description of how to understand QHE in this framework. The third section represents the basic new results about the Kähler structure of generalized imbedding space and represents the resulting model of QHE.

12.2 About theories of quantum Hall effect

The most elegant models of quantum Hall effect are in terms of anyons regarded as singularities due to the symmetry breaking of gauge group G down to a finite sub-group H, which can be also non-Abelian. Concerning the description of the dynamics of topological degrees of freedom topological quantum field theories based on Chern-Simons action are the most promising approach.

12.2.1 Quantum Hall effect as a spontaneous symmetry breaking down to a discrete subgroup of the gauge group

The system exhibiting quantum Hall effect is effectively 2-dimensional. Fractional statistics suggests that topological defects, anyons, allowing a description in terms of the representations of the homotopy group of $((R^2)^n - D)/S_n$. The gauge theory description would be in terms of spontaneous symmetry breaking of the gauge group G to a finite subgroup H by a Higgs mechanism [39, 30]. This would make all gauge degrees of freedom massive and leave only topological degrees of freedom. What is unexpected that also non-Abelian topological degrees of freedom are in principle possible. Quantum Hall effect is Abelian or non-Abelian depending on whether the group H has this property.

In the symmetry breaking $G \to H$ the non-Abelian gauge fluxes defined as non-integrable phase factors $Pexp(i \oint A_\mu dx^\mu)$ around large circles (surrounding singularities (so that field approaches a pure gauge configuration) are elements of the first homotopy group of G/H, which is H in the case that H is discrete group and G is simple. An idealized manner to model the situation [30] is to assume that the connection is pure gauge and defined by an H-valued function which is many-valued such that the values for different branches are related by a gauge transformation in H. In the general case a gauge transformation of a non-trivial gauge field by a multi-valued element of the gauge group would give rise to a similar situation.

One can characterize a given topological singularity magnetically by an element in conjugacy class C of H representing the transformation of H induced by a 2π rotation around singularity. The elements of C define states in given magnetic representation. Electrically the particles are characterized by an irreducible representations of the subgroup of $H_C \subset H$ which commutes with an arbitrarily chosen element of the conjugacy class C.

The action of $h(B)$ resulting on particle A when it makes a closed turn around B reduces in magnetic degrees of freedom to translation in conjugacy class combined with the action of element of H_C in electric degrees of freedom. Closed paths correspond to elements of the braid group $B_n(X^2)$ identifiable as the mapping class group of the punctured 2-surface X^2 and this means that symmetry breaking $G \to H$ defines a representation of the braid group. The construction of these representations is discussed in [30] and leads naturally via the group algebra of H to the so called quantum double $D(H)$ of H, which is a quasi-triangular Hopf algebra allowing non-trivial representations of braid group.

Anyons could be singularities of gauge fields, perhaps even non-Abelian gauge fields, and the latter ones could be modelled by these representations. In particular, braid operations could be represented using anyons.

12.2.2 Witten-Chern-Simons action and topological quantum field theories

The Wess-Zumino-Witten action used to model 2-dimensional critical systems consists of a 2-dimensional conformally invariant term for the chiral field having values in group G combined with 2+1-dimensional term defined as the integral of Chern-Simons 3-form over a 3-space containing 2-D space as its boundary. This term is purely topological and identifiable as winding number for the map from 3-dimensional space to G. The coefficient of this term is integer k in suitable normalization. k gives the value of central extension of the Kac-Moody algebra defined by the theory.

One can couple the chiral field $g(x)$ to gauge potential defined for some subgroup of G_1 of G. If the G_1 coincides with G, the chiral field can be gauged away by a suitable gauge transformation and the theory becomes purely topological Witten-Chern-Simons theory. Pure gauge field configuration represented either as flat gauge fields with non-trivial holonomy over homotopically non-trivial paths or as multi-valued gauge group elements however remain and the remaining degrees of freedom correspond to the topological degrees of freedom.

Witten-Chern-Simons theories are labelled by a positive integer k giving the value of central extension of the Kac-Moody algebra defined by the theory. The connection with Wess-Zumino-Witten theory come from the fact that the highest weight states associated with the representations of the Kac-Moody algebra of WZW theory are in one-one correspondence with the representations R_i possible for Wilson loops in the topological quantum field theory.

In the Abelian case case 2+1-dimensional Chern-Simons action density is essentially the inner product $A \wedge dA$ of the vector potential and magnetic field known as helicity density and the theory in question is a free field theory. In the non-Abelian case the action is defined by the 3-form

$$\frac{k}{4\pi} Tr\left(A \wedge (dA + \frac{2}{3} A \wedge A)\right)$$

and contains also interaction term so that the field theory defined by the exponential of the interaction term is non-trivial.

In topological quantum field theory the usual n-point correlation functions defined by the functional integral are replaced by the functional averages for $Diff^3$ invariant quantities defined in terms of non-integrable phase factors defined by ordered exponentials over closed loops. One can consider arbitrary number of loops which can be knotted, linked, and braided. These quantities define both knot and 3-manifold invariants (the functional integral for zero link in particular). The perturbative calculation of the quantum averages leads directly to the Gaussian linking numbers and infinite number of perturbative link and not invariants.

The experience gained from topological quantum field theories defined by Chern-Simons action has led to a very elegant and surprisingly simple category theoretical approach to the topological quantum field theory [27, 25] allowing to assign invariants to knots, links, braids, and tangles and also to 3-manifolds for which braids as morphisms are replaced with cobordisms. The so called modular Hopf algebras, in particular quantum groups $Sl(2)_q$ with q a root of unity, are in key role in this approach. Also the connection between links and 3-manifolds can be understood since closed, oriented, 3-manifolds can be constructed from each other by surgery based on links [**?**].

Witten's article [26] "Quantum Field Theory and the Jones Polynomial" is full of ingenious constructions, and for a physicist it is the easiest and certainly highly enjoyable manner to learn about knots and 3-manifolds. For these reasons a little bit more detailed sum up is perhaps in order.

1. Witten discusses first the quantization of Chern-Simons action at the weak coupling limit $k \to \infty$. First it is shown how the functional integration around flat connections defines a topological invariant for 3-manifolds in the case of a trivial Wilson loop. Next a canonical quantization is performed in the case $X^3 = \Sigma^2 \times R^1$: in the Coulomb gauge $A_3 = 0$ the action reduces to a sum of $n = dim(G)$ Abelian Chern-Simons actions with a non-linear constraint expressing the vanishing of the gauge field. The configuration space consists thus of flat non-Abelian connections, which are characterized by their holonomy groups and allows Kähler manifold structure.

2. Perhaps the most elegant quantal element of the approach is the decomposition of the 3-manifold to two pieces glued together along 2-manifold implying the decomposition of the functional integral to a product of functional integrals over the pieces. This together with the basic properties of Hilbert of complex numbers (to which the partition functions defined by the functional integrals over the two pieces belong) allows almost a miracle like deduction of the basic results about the behavior of 3-manifold and link invariants under a connected sum, and leads to the crucial skein relations allowing to calculate the invariants by decomposing the link step by step to a union of unknotted, unlinked Wilson loops, which can be calculated exactly for $SU(N)$. The decomposition by skein relations gives rise to a partition function like representation of invariants and allows to understand the connection between knot theory and statistical physics [24]. A direct relationship with conformal field theories and Wess-Zumino-Witten model emerges via Wilson loops associated with the highest weight representations for Kac Moody algebras.

3. A similar decomposition procedure applies also to the calculation of 3-manifold invariants using link surgery to transform 3-manifolds to each other, with 3-manifold invariants being defined as

Wilson loops associated with the homology generators of these (solid) tori using representations R_i appearing as highest weight representations of the loop algebra of torus. Surgery operations are represented as mapping class group operations acting in the Hilbert space defined by the invariants for representations R_i for the original 3-manifold. The outcome is explicit formulas for the invariants of trivial knots and 3-manifold invariant of S^3 for $G = SU(N)$, in terms of which more complex invariants are expressible.

4. For $SU(N)$ the invariants are expressible as functions of the phase $q = exp(i2\pi/(k + N))$ associated with quantum groups [**?**]. Note that for $SU(2)$ and $k = 3$, the invariants are expressible in terms of Golden Ratio. The central charge $k = 3$ is in a special position since it gives rise to $k + 1 = 4$-vertex representing naturally 2-gate physically. Witten-Chern-Simons theories define universal unitary modular functors characterizing quantum computations [19].

12.2.3 Chern-Simons action for anyons

In the case of quantum Hall effect the Chern-Simons action has been deduced from a model of electrons as a 2-dimensional incompressible fluid [29]. Incompressibility requires that the electron current has a vanishing divergence, which makes it analogous to a magnetic field. The expressibility of the current as a curl of a vector potential b, and a detailed study of the interaction Lagrangian leads to the identification of an Abelian Chern-Simons for b as a low energy effective action. This action is Abelian, whereas the anyonic realization of quantum computation would suggest a non-Abelian Chern-Simons action.

Non-Abelian Chern-Simons action could result in the symmetry breaking of a non-Abelian gauge group G, most naturally electro-weak gauge group, to a non-Abelian discrete subgroup H [39] so that states would be labelled by representations of H and anyons would be characterized magnetically H-valued non-Abelian magnetic fluxes each of them defining its own incompressible hydro-dynamical flow. As will be found, TGD predicts a non-Abelian Chern-Simons term associated with electroweak long range classical fields.

12.2.4 Topological quantum computation using braids and anyons

By the general mathematical results braids are able to code all quantum logic operations [23]. In particular, braids allow to realize any quantum circuit consisting of single particle gates acting on qubits and two particle gates acting on pairs of qubits. The coding of braid requires a classical computation which can be done in polynomial time. The coding requires that each dancer is able to remember its dancing history by coding it into its own state.

The general ideas are following.

1. The ground states of anyonic system characterize the logical qubits, One assumes non-Abelian anyons with Z_4 -valued topological charge so that a system of n anyon pairs created from vacuum allows 2^{n-1}-fold anyon degeneracy [37]. The system is decomposed into blocks containing one anyonic Cooper pair with $Q_T \in \{2, 0\}$ and two anyons with such topological charges that the net topological charge vanishes. One can say that the states $(0, 1 - 1)$ and $(0, -1, +1))$ represent logical qubit 0 whereas the states $(2, -1, -1)$ and $(2, +1, +1)$ represent logical qubit 1. This would suggest 2^2-fold degeneracy but actually the degeneracy is 2-fold.

   Free physical qubits are not possible and at least four particles are indeed necessarily in order to represent logical qubit. The reason is that the conservation of Z^4 charge would not allow mixing of qubits 1 and 0, in particular the Hadamard 1-gate generating square root of qubit would break the conservation of topological charge. The square root of qubit can be generated only if 2 units of topological charge is transferred between anyon and anyon Cooper pair. Thus qubits can be represented as entangled states of anyon Cooper pair and anyon and the fourth anyon is needed to achieve vanishing total topological charge in the batch.

2. In the initial state of the system the anyonic Cooper pairs have $Q_T = 0$ and the two anyons have opposite topological charges inside each block. The initial state codes no information unlike in ordinary computation but the information is represented by the braid. Of course, also more general configurations are possible. Anyons are assumed to evolve like free particles except during swap operations and their time evolution is described by single particle Hamiltonians.

Free particle approximation fails when the anyons are too near to each other as during braid operations. The space of logical qubits is realized as k-code defined by the 2^{n-1} ground states, which are stable against local single particle perturbations for $k = 3$ Witten-Chern-Simons action. In the more general case the stability against n-particle perturbations with $n < [k/2]$ is achieved but the gates would become $[k/2]$-particle gates (for $k = 5$ this would give 6-particle vertices).

3. Anyonic system provides a unitary modular functor as the S-matrix associated with the anyon system whose time evolution is fixed by the pre-existing braid structure. What this means that the S-matrices associated with the braids can be multiplied and thus a unitary representation for the group formed by braids results. The vacuum degeneracy of anyon system makes this representation non-trivial. By the NP complexity of braids it is possible to code any quantum logic operation by a particular braid [21]. There exists a powerful approximation theorem allowing to achieve this coding classically in polynomial time [23]. From the properties of the R-matrices inducing gate operations it is indeed clear that two gates can be realized. The Hadamard 1-gate could be realized as 2-gate in the system formed by anyon Cooper pair and anyon.

4. In [18] the time evolution is regarded as a discrete sequence of modifications of single anyon Hamiltonians induced by swaps [20]. If the modifications define a closed loop in the space of Hamiltonians the resulting unitary operators define a representation of braid group in a dense discrete sub-group of $U(2^n)$. The swap operation is 2-local operation acting like a 2-gate and induces quantum logical operation modifying also single particle Hamiltonians. What is important that this modification maps the space of the ground states to a new one and only if the modifications correspond to a closed loop the final state is in the same code space as the initial state. What time evolution does is to affect the topological charges of anyon Cooper pairs representing qubits inside the 4-anyon batches defined by the braids.

   In quantum field theory the analog but not equivalent of this description would be following. Quite generally, a given particle in the final state has suffered a unitary transformation, which is an ordered product consisting of two kinds of unitary operators. Unitary single particle operators $U_n = Pexp(i\int_{t_n}^{t_{n+1}} H_0 dt)$ are analogs of operators describing single qubit gate and play the role of anyon propagators during no-swap periods. Two-particle unitary operators $U_{swap} = Pexp(i\int H_{swap}dt)$ are analogous to four-particle interactions and describe the effect of braid operations inducing entanglement of states having opposite values of topological charge but conserving the net topological charge of the anyon pair. This entanglement is completely analogous to spin entanglement. In particular, the braid operation mixes different states of the anyon. The unitary time development operator generating entangled state of anyons and defined by the braid structure represents the operation performed by the quantum circuit and the quantum measurement in the final state selects a particular final state.

5. Formally the computation halts with a measurement of the topological charge of the left-most anyon Cooper pair when the outcome is just single bit. If decay occurs with sufficiently high probability it is concluded that the value of the computed bit is 0, otherwise 1.

12.3 A generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is described. This view has developed much before the original version of this chapter was written.

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

12.3.1 Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. H_4 represents a straight cosmic string. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labelled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{M}^4 \times \hat{CP}_2$ implying that surfaces in $M^4 \times S^2$ and $M^2 \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

3. The covering spaces in question would correspond to the Cartesian products $\hat{M}^4{}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{M}^4$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing M^2 and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{M}^4 \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$.

4. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

5. Also the orbifolds $\hat{M}^4/G_a \times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{M}^4/G_a \times (\hat{CP}_2 \hat{\times} G_b)$ and $(\hat{M}^4 \hat{\times} G_a) \times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

12.3.2 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M}:\mathcal{N}<4$ and it naturally corresponds to the coset spaces. For $\mathcal{M}:\mathcal{N}=4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M}:\mathcal{N}=4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{M}^2\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$, which both are labelled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

   By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{H times}(G_a \times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{H times}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/m$ or m. Partonic 2-surface (wormhole throat) is highly analogous to black hole horizon and this led already years ago the notion of elementary particle horizon and generalization of the area law for black-holes [E5]. The $1/\hbar$-proportionality of the black hole entropy measuring the number of states associated with black hole motivates the hypothesis that the number of states associated with single sheet of the covering proportional to $1/\hbar$ so that the total number states should remain invariant in the transition changing Planck constant. Since the number of states is obviously proportional to the number m of sheets in the covering, this is achieved for $\hbar(X) \propto 1/m$ giving $r(X) \rightarrow r(X)/n$ for factor space and $r(X) \rightarrow nr(X)$ for the covering space. Option II would be selected.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in M^4 and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

12.3.3 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [31] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} , \\ \nu &= \frac{n}{m} . \end{aligned} \tag{12.3.0}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10$ $1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [31].

The model of Laughlin [29] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [32]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are $2 \times 2 = 4$ combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this one can consider two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. In the following a model based on option II for which the number of states is conserved in the phase transition changing $\hbar$.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [31, 33]. The fractionized charge is $e/4$ in the latter case [36, 35]. Since $n_i > 3$ holds true if coverings and factor spaces are correlates for Jones inclusions, this requires $n_a = 4$ and $n_b = 8$ for $\nu = 1/2$ and $n_b = 4$ and $n_a = 10$ for $\nu = 5/2$. Correct fractionization of charge is predicted. For $n_b = 2$ also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [32]. $n_b = 2$ is inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e \int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of

the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the assumption about charge fractionization -although consistent with fractionization for $\nu = 5/2$, is rather adhoc. Therefore the model can be taken as a warm-up exercise only.

12.4 Quantum Hall effect, charge fractionization, and hierarchy of Planck constants

In this section the most recent view about the relationship between dark matter hierarchy and quantum Hall effect is discussed. This discussion leads to a more realistic view about FQHE allowing to formulate precisely the conditions under which anyons emerge, describes the fractionization of electric and magnetic charges in terms of the delicacies of the Kähler gauge potential of generalized imbedding space, and relates the TGD based model to the original model of Laughlin. The discussion allows also to sharpen the vision about the formulation of quantum TGD itself.

12.4.1 Quantum Hall effect

Recall first the basic facts. Quantum Hall effect (QHE) [29, 30, 31] is an essentially 2-dimensional phenomenon and occurs at the end of current carrying region for the current flowing transversally along the end of the wire in external magnetic field along the wire. For quantum Hall effect transversal Hall conductance characterizing the 2-dimensional current flow is dimensionless and quantized and given by

$$\sigma_{xy} = 2\nu\alpha_{em} \ ,$$

ν is so called filling factor telling the number of filled Landau levels in the magnetic field. In the case of integer quantum Hall effect (IQHE) ν is integer valued. For fractional quantum Hall effect (FQHE) ν is rational number. Laughlin introduced his many-electron wave wave function predicting fractional quantum Hall effect for filling fractions $\nu = 1/m$ [29]. The further attempts to understand FQHE led to the notion of anyon by Wilzeck [30]. Anyon has been compared to a vortex like excitation of a dense 2-D electron plasma formed by the current carriers. ν is inversely proportional to the magnetic flux and the fractional filling factor can be also understood in terms of fractional magnetic flux.

The starting point of the quantum field theoretical models is the effective 2-dimensionality of the system implying that the projective representations for the permutation group of n objects are representations of braid group allowing fractional statistics. This is due to the non-trivial first homotopy group of 2-dimensional manifold containing punctures. Quantum field theoretical models allow to assign to the anyon like states also magnetic charge, fractional spin, and fractional electric charge.

Topological quantum computation [17, 18, E9, L5] is one of the most fascinating applications of FQHE. It relies on the notion of braids with strands representing the orbits of of anyons. The unitary time evolution operator coding for topological computation is a representation of the element of the element of braid group represented by the time evolution of the braid. It is essential that the group involved is non-Abelian so that the system remembers the order of elementary braiding operations (exchange of neighboring strands). There is experimental evidence that $\nu = 5/2$ anyons possessing fractional charge $Q = e/4$ are non-Abelian [36, 35].

During last year I have been developing a model for DNA as topological quantum computer [L5]. Therefore it is of considerable interest to find whether TGD could provide a first principle description of anyons and related phenomena. The introduction of a hierarchy of Planck constants realized in terms of generalized imbedding space with a book like structure is an excellent candidate in this respect [A9]. As a rule the encounters between real world and quantum TGD have led to a more precise quantitative articulation of basic notions of quantum TGD and the same might happen also now.

12.4.2 TGD description of QHE

The proportionality $\sigma_{xy} \propto \alpha_{em} \propto 1/\hbar$ suggests an explanation of FQHE [30, 29, 31] in terms of the hierarchy of Planck constants. Perhaps filling factors and magnetic fluxes are actually integer valued but the value of Planck constant defining the unit of magnetic flux is changed from its standard value - to its rational multiple in the most general case. The killer test for the hypothesis is to find whether higher order perturbative QED corrections in powers of α_{em} are reduced from those predicted by QED in QHE phase. The proposed general principle governing the transition to large $\hbar$ phase is states that Nature loves lazy theoreticians: if perturbation theory fails to converge, a phase transition increasing Planck constant occurs and guarantees the convergence. Geometrically the phase transition corresponds to the leakage of 3-surface from a given 8-D page to another one in the Big Book having singular coverings and factor spaces of $M^4 \times CP_2$ as pages.

Chern-Simons action for Kähler gauge potential (equivalently for induced classical color gauge field proportional to the Kähler form) defines TGD as almost topological QFT. This alone strongly suggests the emergence of quantum groups and fractionalization of quantum numbers [?]. The challenge is to figure out the details and see whether this framework is consistent with what is known about QHE. At least the following questions pop up immediately in mind.

1. What the effective 2-dimensionality of the system exhibiting QHE corresponds in TGD framework?
2. What happens in the phase transition leading to the phase exhibiting QHE effect?
3. What are the counterparts anyons? How the fractional electric and magnetic charges emerge at classical and quantum level.
4. The Chern-Simons action associated with the induced Kähler gauge potential is Abelian: is this consistent with the non-Abelian character of braiding matrix?

12.4.3 Quantum TGD almost topological QFT

The statement that TGD is almost topological QFT means following conjectures.

1. In TGD the fundamental physical object is light-like 3-surface X^3 connecting the light-cone boundaries of $CD \times CP_2 \subset M^4 \times CP_2$ (intersection of future and past directed light-cones) but by conformal invariance in the light-like direction of X^3 physics is locally 2-dimensional in the sense that one can regard this surface as an orbit of 2-D parton as long as one restricts to finite region of X^3. Physics at X^3 remains 3-D in discretized sense (quantum states are of course quantum superpositions of different light-like 3-surfaces).
2. At the fundamental level quantum TGD can be formulated in terms of the fermionic counterpart of Chern-Simons action fort the Kähler gauge potential associated with Kähler form of CP_2. The Dirac determinant associated with the modified Dirac action defines the vacuum functional of the theory. Dirac determinant is defined as a finite product of the values of generalizes eigenvalues (functions) of the modified Dirac operator at points defined by the strands of so called number theoretic braids which by number theoretic arguments are unique [E1, E2].
3. Vacuum functional equals to the exponent of Kähler action for a preferred extremal $X^4(X^3)$ of Kähler action, which plays the role of Bohr orbit and allows to realize 4-D general coordinate invariance. The boundary conditions of 4-D dynamics fixing $X^4(X^3)$ are fixed by the requirement that the tangent space of X^4 contains a preferred Minkowski plane $M^2 \subset M^4$ at each point. This plane can be interpreted as the plane of non-physical polarizations.
4. "Number theoretic compactification" states that space-time surfaces can be regarded as 4-surfaces of either hyper-octonionic M^8 or $M^4 \times CP_2$ (hyper-octonions corresponds to a subspace of complexified octonions with Minkowskian signature of metric). The surfaces of M^8 are hyper-quaternionic in the sense that each tangent plane is hyper-quaternionic and contains (this is essential for number theoretic compactification) the preferred hyper-complex plane M^2 defined by hyper-octonionic real unit and preferred imaginary unit. The preferred extrema of Kähler action should correspond hyper-quaternionic 4-surfaces of M^8 having preferred M^2 as a tangent space at each point.

These 'must-be-trues' are of course highly non-trivial un-proven conjectures. If one gives up conjecture about the reduction of entire 4-D dynamics to that for almost topological fermions at 3-D light-like surfaces, one must assume separately that vacuum functional is exponent of Kähler function for a preferred extremal.

12.4.4 Constraints to the Kähler structure of generalized imbedding space from charge fractionization

In the following the notion of generalized imbedding space is discussed. The new element is more precise formulation of the Kähler structure by allowing Kähler gauge potential to have what looks formally as gauge parts in both M^4 and CP_2 and of no physical significance on gauge theory context. In TGD framework the gauge parts have deep physical significance since symplectic transformations act as symmetries of Kähler and Chern-Simons-Kähler action only in the case of vacuum extremals.

Hierarchy of Planck constants and book like structure of imbedding space

TGD leads to a description for the hierarchy of Planck constants in terms of the generalization of the Cartesian factors of the imbedding space $H = M^4 \times CP_2$ to book like structures. To be more precise, the generalization takes place for any region $CD \times CP_2 \subset H$, where CD corresponds to a causal diamond defined as an intersection of future and past directed light-cones of M^4. CDs play key role in the formulation of quantum TGD in zero energy ontology in which the light-like boundaries of CD connected by light-like 3-surfaces can be said to be carriers of positive and negative energy parts of zero energy states. They are also crucial for TGD inspired theory of consciousness, in particular for understanding the relationship between experienced and geometric time [16].

Both CD and CP_2 are replaced with a book like structure consisting in the most general case of singular coverings and factor spaces associated with them. A simple geometric argument identifying the square of Planck constant as scaling factor of the covariant metric tensor of M^4 (or actually CD) leads to the identification of Planck constant as the ratio $\hbar/\hbar_0 = q(M^4)/q(CP_2)$, where $q(X) = N$ holds true for the covering of X and $q(X) = 1/N$ holds true for the factor space. N is the order of the maximal cyclic subgroup of the covering/divisor group $G \subset SO(3)$. The order of G can be thus larger than N. As a consequence, the spectrum of Planck constants is in principle rational-valued. $\hbar_0$ is unique since it corresponds to the unit of rational numbers. The field structure has far reaching implications for the understanding of phase transitions changing the value of Planck constant.

The hierarchy of Planck constants relates closely to quantum measurement theory. The selection of quantization axis has a direct correlate at the level of imbedding space geometry. This means breaking of isometries of H for a given CD with preferred choice time axis (rest frame) and quantization axis of spin. For CP_2 the choice of the quantization axes of color hyper charge and isospin imply symmetry breaking $SU(3) \rightarrow U(2) \rightarrow U(1) \times U(1)$. The "world of classical worlds" (WCW) is union over all Poincare and color translates of given $CD \times CP_2$ so that these symmetries are not lost at the level of WCW although the loss can happen at the level of quantum states.

Non-vanishing of Poincare quantum numbers requires CP_2 Kähler gauge potential to have M^4 part

Since Kähler action gives rise to conserved Poincare quantum numbers as Noether charges, the natural expectation is that Poincare quantum numbers make sense as Noether charges for Chern-Simons action. The problem is that Poincare quantum numbers vanish for standard Kähler gauge potential of CP_2 since it has no M^4 part.

The way out of the difficulty relies on the delicacies of CP_2 Kähler structure.

1. One can give up the strict Cartesian product property and assume that CP_2 Kähler gauge potential has M^4 part which is pure gauge and without physical meaning in gauge theory context. In TGD framework the situation is different. The reason is that $U(1)$ gauge transformations are induced by the symplectic transformations of CP_2 and correspond to genuine dynamical symmetries acting as isometries of WCW. They act as symmetries of Kähler action only in the case of vacuum extremals and relate closely to the spin glass degeneracy of Kähler action with the counterpart of spin glass energy landscape defined by small deformations of vacuum extremals of Kähler action. This vacuum degeneracy has been one of the most fruitful challenges of TGD.

2. Requiring Lorentz invariance one can write the non-vanishing pure gauge M^4 component of Kähler gauge potential as

$$A_a = constant \ . \tag{12.4.1}$$

Here a denotes the light-cone proper time. It is of course possible that also other components are present as it indeed turns out. Using standard formula for Noether current one finds that four-momentum is non-vanishing because of the term $A_a\partial_\alpha a$ in Chern-Simons-Kähler action. From $\partial_\alpha a = m^k m_{kl}\partial_\alpha m^l/a$ momentum current T^{k0} at given point of X^3 is proportional to the average 4-velocity with respect to the tip of light-cone: $T^{k0} \propto m^k/a$. Therefore the motion in the average sense is analogous to cosmic expansion. This is natural since the structure of CD corresponding to particular quantization axes breaks Poincare symmetry.

3. $A_a = constant$ guarantees the conservation of mass squared in the case of CP_2 type extremals at least and implies that mass squared is non-vanishing. Four-momentum is also proportional to the Kähler magnetic flux over the partonic 2-surface X^2 and X^2 must be homologically non-trivial for the net value of four-momentum to be non-vanishing. X^2 could correspond to the end of cosmic string in 4-D picture. Homological non-triviality does not seem to be necessary in the case of super-symmetric counterpart of Dirac action since Kähler flux is multiplied by the fermionic bilinear so that the outcome is more general than Kähler magnetic flux.

The M^4 part of CP_2 Kähler gauge potential for the generalized imbedding space

The non-triviality of A_a transforms topological QFT to an almost topological one, but says nothing about the covering- and factor space sectors of generalized imbedding space- the pages of the book like structure defined by the generalized imbedding space. The interpretation in terms of quantum measurement theory suggests that Lorentz symmetry and color symmetry are broken to Cartan sub-groups defining quantization axes. If anyons correspond to large $\hbar$ phase, the Kähler gauge potential of CP_2 should contain in these sectors additional gauge parts in both M^4 and CP_2 responsible for charge fractionization, magnetic monopoles, and other anyonic effects.

The basic prerequisite for anyonic effects is that fundamental group is non-trivial and for M^4 the emergence of M^2 as the intersection of sheets of the singular covering implies this for the complement of M^2. In the case of CP_2 the homologically trivial geodesic S^2 is common to the coverings and factors spaces and implies the non-triviality of the fundamental group.

Let $(u = m^0 + r_M), v = m^0 - r_M, \theta, \phi)$ define light-like spherical coordinates for $M^4_\pm$. Here m^k are linear M^4 time coordinates and r_M is radial M^4 coordinate. Denote the light-cone proper time by $a = \sqrt{uv}$. The origin of coordinates lies at the either tip of CD. Coordinates are not global so that the patches assignable to positive and negative energy parts of the zero energy state must be used.

The fixing of the rest system, that is the direction of time axis, reduces Lorentz invariance to $SO(3)$. This allows A to have an additional part

$$A_u = \frac{k_1}{u^2} \ . \tag{12.4.2}$$

The functional form of A_u will be deduced in the sequel from the conservation of anyonic charges. The fixing of the direction of the spin quantization axis reduces the symmetry to $SO(2)$ and allows introduction of a further gauge component

$$A_\phi = \frac{k_2}{u^2} \ . \tag{12.4.3}$$

Clearly one has a hierarchical breaking of symmetry: Poincare group $\rightarrow$ Lorentz group$\rightarrow$ rotation group $SO(3) \rightarrow SO(2)$. Globally the symmetry is not broken since WCW is a union over all possible choices of quantization for each CDs with all possible positions of lower tip are allowed. p-Adic length scale hypothesis results if the temporal distance between upper and lower tips is quantized in multiples 2^n. The hierarchy of Planck constants however implies that distance are quantized as rational multiples of basic distance scale.

How fractional electric and magnetic charges emerge from M^4 gauge part of CP_2 Kähler gauge potential?

The Maxwell field defined by the induced CP_2 Kähler form plays fundamental role in the construction of quantum TGD. Kähler gauge potential of CP_2 contributes directly to the classical electromagnetic gauge potential. Its coupling to $M^4 \times CP_2$ spinors is different for quarks and leptons representing different conserved chiralities of H spinors and it explains different electromagnetic charges of quarks and leptons as well as different color trialities. Also classical color gauge field is proportional to Kähler form. Therefore one might hope that the gauge parts of Kähler gauge potential might contain a lot of interesting physics.

The following series of arguments try to demonstrate following three results.

1. The anomalous contribution to the Kähler gauge potential induces anomalous electric and magnetic Kähler charges and therefore also em, Z^0, and color gauge charges.
2. Anyons can be characterized as 2-surfaces surrounding the tip of CD.
3. In sectors corresponding to the non-standard value of $\hbar$ the vacuum degeneracy of Kähler and Chern-Simons actions is dramatically reduced.

Note that in this section the consideration is restricted to the gauge parts of CP_2 Kähler gauge potential in $CD \subset M^4$. Also the gauge parts in CP_2 are possible and the Kähler potential assignable to the contact structure of CD must be considered separately.

1. The gauge part of Kähler gauge potential vanishes outside CD so that it is discontinuous at light-like boundary in the direction of the light like vector defined as the gradient of $v = t - r$. This means that for partonic 2-surfaces surrounding the tip of light-cone both Kähler electric and magnetic fluxes are non-vanishing and determined by $K_i(u)$, $i = 1, 2$. By requiring that the anomalous Kähler charge is time independent, one obtains $K_1(u) = k_1/u^2$. This means that the Kähler electric gauge field has a delta function like singularity at the light-like boundares of CD which becomes carrier of Kähler charge from the view point of complement of CD. This suggests that if one has N elementary particles at partonic 2-surface X^2 surrounding the tip of CD (wormhole throats of elementary particles are condensed to X^2), the charges of particles are effectively fractionized:
$$q \rightarrow q + \frac{Q_A}{N} \ . \tag{12.4.4}$$
2. In the case of $A_\phi = constant$ anomalous magnetic charge results since the flux expressible as line integral $\int A_\phi d\phi$ is non-vanishing because the poles of S^2 act effectively as magnetic charges. The punctures at the poles are the correlate for the selection of the quantization axes of spin. $K_2(u) = k_2/u^2$ follows from the conservation of magnetic charge. In the case of ordinary magnetic monopole spin becomes half-odd integer valued and analogous result holds also now. The minimal coupling to the gauge part of A_ϕ defining the covariant derivative D_ϕ together with covariant constancy condition implies that spin receives a fractional part for $k_2 \neq 0$ and spin fractionization results.
3. One can see the situation also differently. The 2-D partons at the ends of light-like 3-surfaces at light-like boundaries of CD interact like particles with anomalous gauge charges but the interaction is now in light-like direction. The anomalous charges indeed characterize Chern-Simons action. For $k_1 = k_2 = 0$ corresponding to $\hbar/\hbar_0 = 1$ one has Lorentz invariance and only cosmic string like objects seem to remain to the spectrum of the theory (they dominate the very early TGD based cosmology [D6]).
4. Quite generally, anyonic states can be assigned with partonic 2-surfaces surrounding surrounding the tip of CD since the fractional contribution to the gauge charge vanishes otherwise.
5. Kähler gauge potential appears in the expression of the em charge so that a fractionization of electric and magnetic em and Z^0 charges results but there is no fractionization of the weak charge. The components of the classical color gauge field are of form $G^A \propto H^A J$, H^A the

Hamiltonian of color isometry and J Kähler form. The assumption that the singular part of G^A is induced from that for J implies anomalous electric and magnetic color gauge charges located at boundaries of CD. These charges should make sense as fluxes since the $SU(3)$ holonomy is Abelian.

6. A_u contributes to the four-momentum density a term proportional to the four-vector $\partial u/\partial m^k$ which in vector notation looks like $(1, \overline{r}_M/r_M)$: thus the direction of 3-momentum rends to be same as for A_a. In the approximation that the M^4 coordinates for partonic 2-surface are constant (excellent approximation at elementary particle level) this contribution to the four-momentum is massless unlike for A_a. If the variation of the projection of A_u in Chern-Simons action is responsible for the four-momentum X^2 must carry non-vanishing homological charge for Chern-Simons action but not for its fermionic counterpart. If the variation of the projection of the singular part J_{uv} is responsible for the momentum the CP_2 projection can be 1-dimensional so that the vacuum degeneracy is reduced and the homological non-triviality in CP_2 is replaced with homological triviliaty in CD with the line connecting the tips of CD removed.

7. For $(k_1, k_2) = (0, 0)$ all space-time surfaces for which CP_2 projection is Lagrange manifold of CP_2 (generally 2-dimensional sub-manifold having vanishing induced Kähler form) are vacuum extremals For $(k_1, k_2) \neq (0, 0)$ and for partonic 2-surfaces surrounding the tip of the light-cone, the situation changes since also partonic 2-surfaces which have 1-D CP_2 projection can carry non-vanishing Kähler, em, and color charges, and even four-momentum. If M^4 projection is 2-D, the anomalous part of Kähler form contributing to the charges is completely in M^4 and the variation of of A_α in Chern-Simons action gives rise to color currents. Four-momentum can be non-vanishing even when CP_2 projection is zero-dimensional since the variation of A_a gives rise to it when X^2 surrounds the tip of CD. Hence the hierarchy of Planck constants removes partially the vacuum degeneracy. This correlation conforms with the general idea that both the vacuum degeneracy and the hierarchy of Planck constants relate closely to quantum criticality. Perhaps the hierarchy of Planck constants accompanied by the anyonic gauge parts of A makes possible to have mathematically well-define theory.

Coverings and factor spaces of CP_2 and anyonic gauge part of Kähler gauge potential in CP_2?

Nothing about possible coverings and factor spaces of CP_2 has been said above. In principle they could contribute to CP_2 Kähler gauge potential an anomalous part and would form a representation for the hierarchy of Planck constants in CP_2 degrees of freedom.

1. If Kähler gauge potential has also anyonic CP_2 part, it should fix the choice of quantization axes for color charges. Thus the anomalous components could be of form $A_{I_3} = k(I_3)$ and $A_Y = k(Y_3)$ where the angle variables vary along flow lines of I_3 and Y. Singularity would emerge both at the origin and at the 2-sphere $r = \infty$ analogous to the North pole of S^2, at which the second angle variable becomes redundant.

2. These terms would give to the anomalous Kähler magnetic charge a contribution completely analogous to that coming from A_ϕ. Also color charges would receive similar contribution.

How the values of the anomalous charges relate to the parameters characterizing the page of the Big Book?

One should be able to relate the anomalous parameters characterizing anomalous gauge potentials to the parameters n_a, n_b characterizing the coverings of CD and CP_2. Consider first various manners to understand charge fractionization.

1. The hypothesis at the end of previous section states that for n_b-fold covering of CP_2 the fractionized electric charge equals to e/n_b. This predicts charge fractionization correctly for $\nu = 5/2 = 10/4$ [36]. This simple argument could apply also to other charges. The interpretation would be that when elementary particle becomes anyonic, its charge is shared between n_b sheets of the covering of CP_2. In the case of factor space the singular factor space would appear as n_b copies meaning the presence n_b particles behaving like single particle. Charge fractionization would be only apparent in this picture.

2. This global representation of the fractionization of Kähler charge might be enough. One can however ask whether also a local representation could exist in the sense that the coupling of fermions to the gauge parts of Kähler gauge potential would represent charge fractionization at single particle level in terms of phase factors analogous to plane waves. If charge fractionization is only apparent, the total anomalous Kähler charge assignable to particles should be compensated by the total anomalous Kähler charge associated with A_u. This gives a constraint between k_1 and parameter $k(Y)$.

3. Similar argument for the Kähler magnetic charge gives a constraint between k_2 and $k(Y)$ implying $k_1 = k_2$ consistent with assumption that also the anyonic part of Kähler form is self dual. In the simplest situation $k_1 = k_2 = Nq_K k(Y)$, where N is the number of identical particles at the anyonic space-time sheet. In more general case one would have $k_1 = k_2 \sum_i N_i q_{K,i} k(Y)$. If the anyonic space-time sheet does not contain the tip of CD in its interior, the total anomalous Kähler charge associated with the fermions at it must vanish.

4. Both em and Z^0 fields contain a part proportional to Kähler form so that total anomalous gauge charges defined as fluxes should be equal to those defined as sums of elementary particle contributions.

5. Anomalous color isospin and hypercharge and corresponding magnetic charges would have also representations as color gauge fluxes by using $Q^A \propto H^A J$ restricted to Cartan algebra of color group. The couplings to the anomalous gauge parts of Kähler gauge potential in CP_2 would give rise to anomalous color charges at single particle level, and also now the condition that the total anomalous charges assignable to particles compensates that assignable to the singular part of color gauge potential is natural. Thus quite a number of consistency conditions emerge.

The foregoing discussion relates to the gauge part of Kähler gauge potential assigned to CP_2 degrees of freedom. Analogous discussion applies to the M^4 part.

1. Covariant constancy conditions appear also in Minkowski degrees of freedom and correlate the value of anomalous Poincare charges to anomalous Kähler charge. Anomalous Kähler charge k_1 gives via covariant constancy condition for induced spinors contribution to four-momentum analogous to Coulomb interaction energy with Kähler charge k_1: at point like limit the contribution is light-like. In the similar manner $k_2 = k_1$ gives rise to anomalous orbital spin via the covariant constancy condition $D_\phi \Psi = (\partial_\phi + A_\phi)\Psi = 0$ equating A_ϕ with the fractional contribution to spin. Thus both anomalous four-momentum and spin fractionization effect reflects the total anomalous Kähler charge.

2. The values of $k_1 = k_2$ should correlate directly with the order of the maximal cyclic subgroup Z_{n_a} associated with the covering/factor space of CD. For covering one should should have $k_2 = n/n_a$ since the rotation by $N \times 2\pi$ is identity transformation. For the factor space one should have $k_2 = nn_a$ since the states must remain invariant under rotations by multiples of $2\pi/N$ and spin unit becomes n_a. This picture is consistent with the scaling up of the spin unit with $\hbar/\hbar_0$. Since k_1 must be also an integer multiple of $1/n_b$, k_1 should be inversely proportional to a common factor of n_a and n_b.

That classical color hyper charge and isospin correspond to electro-weak charges is an old idea which I have not been able to kill. It is discussed also in [C4] from the point of view of symplectic fusion algebras.

1. Quark color is not a spin like quantum number but corresponds to CP_2 partial waves in cm degrees of freedom of partonic 2-surface. Hence it should not relate to the classical color charges associated with classical color gauge field or with the modes of induced spinor fields at space-time sheet. These nodes can also carry color hyper charge and isospin in the sense that they are proportional to space-time projections of phase factors representing states with constant Y and I_3 (being completely analogous to angular momentum eigen states on circle).

2. In the construction of symmetric spaces the holonomy group of the spinor connection is identified as a subgroup of the isometry group. Therefore electro-weak gauge group $U(2)_{ew}$ would

correspond to $U(2) \subset SU(3)$ defining color quantization axis. If so, the phase factors assignable to the induced spinor fields could indeed represent the electromagnetic and weak charges of the particle and one would have $Y = Y_{ew}$ and $I_3 = I_{3,ew}$. Also electro-weak quantum numbers, which are spin-like, would have geometric representation as phase factors of spinors.

3. This kind of multiple representation emerges also via number theoretical compactication [E2] meaning that space-time surfaces can be regarded either as surfaces in hyper-octonionic space $M^8 = M^4 \times E^4$ or $M^4 \times CP_2$. In M^8 electro-weak quantum numbers are represented as particle waves and color is spin like quantum number.

Again a word of caution is in order since the formula for charge fractionization is supported only by its success in $\nu = 5/2$ case. Also the proposed formulas are only heuristic guesses.

What about Kähler gauge potential for CD?

One can assign also to light-cone boundary- metrically equivalent with S^2, symplectic (or more precisely contact-) structure. This structure can be extended to a pseudo-symplectic structure in the entire CD. The structure is not global and one must introduce two patches corresponding to the two light-cone boundaries of CD.

This symplectic structure plays a key role in the construction of symplectic fusion algebra [C4]. In TGD framework Equivalence Principle is realized in terms generalized coset construction for the super-canonical conformal algebra assignable to the light-cone boundary and super-Kac-Moody algebra assignable to the light-like 3-surfaces. The cautious proposal of [C4] is that at the level of fusion algebra Equivalence Principle means the possibility to use either the symplectic fusion algebra of light-cone boundary for light-cone defined by S^2 Kähler form or the symplectic fusion algebra for light-cone boundry defined by CP_2 Kähler form.

The vacuum degeneracy of Kähler action requiring that CP_2 projection of the partonic 2-surface is non-trivial would at first seem to exclude this option. Anomalous gauge charges however remove this vacuum degeneracy for $k_1 \neq 0$ so that there are no obvious reasons excluding this manifestation of Equivalence Principle.

The Kähler gauge potential of the degenerate Kähler form assignable to the light-like boundary (basically to the $r_M = constant$ sphere S^2) and also to CD and identifiable as the Kähler form of S^2 defining its signed area can indeed contain gauge part with a structure similar that for CP_2 Kähler gauge potential and involving three rational valued constants corresponding to gauge parts A_a, A_u, and A_ϕ. The TGD based realization of the Equivalence Principle suggests that the constants associated with the two Kähler forms are identical or at least proportional to each other. One could perhaps even say that the hierarchy of Planck constants and dark matter are necessary to realize Equivalence Principle in TGD framework.

Concrete picture about gluing of different sectors of the generalized imbedding space

Intuitively the scaling of Planck constant scales up quantum lengths, in particular the size of CD. This looks trivial but one one must describe precisely what is involved to check internal consistency and also to understand how to model the quantum phase transitions changing Planck constant.

The first manner to understand the situation is to consider CD with a fixed range of M^4 coordinates. The scaling up of the covariant Kähler metric of CD by $r^2 = (\hbar/\hbar_0)^2$ scales up the size of CD by r. Another manner to see the situation is by scaling up the linear M^4 coordinates by r for the larger CD so that M^4 metric becomes same for both CDs. The smaller CD is glued to the larger one isometrically together along $(M^2 \cap CD) \subset CD$ anywhere in the interior of the larger CD. What happens is non-trivial for the following reasons.

1. The singular coverings and factor spaces are different and M^4 scaling is not a symmetry of the Kähler action so that the preferred extrema in the two cases do not relate by a simple scaling. The interpretation is in terms of the coding of the radiative corrections in powers of $\hbar$ to the shape of the preferred extremals. This becomes clear from the representation of Kähler action in which M^4 coordinates have the same range for two CDs but M^4 metric differs by r^2 factor.

2. In common M^4 coordinates the M^4 gauge part A_a of CP_2 Kähler potential for the larger CD differs by a factor $1/r$ from that for the smaller CD. This guarantees the invariance of four-momentum assignable to Chern-Simons action in the phase transition changing $\hbar$. The resulting discontinuity of A_a at M^2 is analogous to a static voltage difference between the two CDs and M^2 could be seen as an analog of Josephson junction. In absence of dissipation (expected in quantum criticality) the Kähler voltage could generate oscillatory fermion, em, and Z^0 Josephson currents between the two CDs. Fermion current would flow in opposite directions for fermions and antifermions and also for quarks and leptons since Kähler gauge potential couples to quarks and leptons with opposite signs. In presence of dissipation fermionic currents would be ohmic and could force quarks and leptons and matter and antimatter to different pages of the Big Book. Quarks inside hadrons could have nonstandard value of Planck constant.

3. The discontinuities of A_u and A_ϕ allow to assign electric and magnetic Kähler point charges $Q_K^{e/m}$ with $M^1 \subset M^2$ and having sign opposite to those assignable with $\delta CD \times CP_2$. It should be possible to identify physically M^2, the line E^1 representing quantization axis of angular momentum, and the position of Q_K.

12.4.5 In what kind of situations do anyons emerge?

Charge fractionization is a fundamental piece of quantum TGD and should be extremely general phenomenon and the basic characteristic of dark matter known to contribute 95 per cent to the matter of Universe.

1. In TGD framework scaling $\hbar = m\hbar_0$ implies the scaling of the unit of angular momentum for m-fold covering of CD only if the many particle state is Z_m singlet. Z_m singletness for many particle states allows of course non-singletness for single particle states. For factor spaces of CD the scaling $\hbar \rightarrow \hbar/m$ is compensated by the scaling $l \rightarrow ml$ for $L_z = l\hbar$ guaranteing invariance under rotations by multiples of $2\pi/m$. Again one can pose the invariance condition on many-particle states but not to individual particles so that genuine physical effect is in question.

2. There is analogy with Z_3-singletness holding true for many quark states and one cannot completely exclude the possibility that quarks are actually fractionally charged leptons with $m = 3$-covering of CP_2 reducing the value of Planck constant [A8, A9] so that quarks would be anyonic dark matter with smaller Planck constant and the impossibility to observe quarks directly would reduce to the impossibility for them to exist at our space-time sheet. Confinement would in this picture relate to the fractionization requiring that the 2-surface associated with quark must surround the tip of CD. Whether this option really works remains an open question. In any case, TGD anyons are quite generally confined around the tip of CD.

3. Quite generally, one expects that dark matter and its anyonic forms emerge in situations where the density of plasma like state of matter is very high so that N-fold cover of CD reduces the density of matter by $1/N$ factor at given sheet of covering and thus also the repulsive Coulomb energy. Plasma state resulting in QHE is one examples of this. The interiors of neutron stars and black hole like structures are extreme examples of this, and I have proposed that black holes are dark matter with a gigantic value of gravitational Planck constant implying that black hole entropy -which is proportional to $1/\hbar$ - is of same order of magnitude as the entropy assignable to the spin of elementary particle. The confinement of matter inside black hole could have interpretation in terms of macroscopic anyonic 2-surfaces containing the topologically condensed elementary particles. This conforms with the TGD inspired model for the final state of star [D4] inspiring the conjecture that even ordinary stars could posses onion like structure with thin layers with radii given by p-adic length scale hypothesis.

   The idea about hierarchy of Planck constants was inspired by the finding that planetary orbits can be regarded as Bohr orbits [40, A9]: the explanation was that visible matter has condensed around dark matter at spherical cells or tubular structures around planetary orbits. This led to the proposal that planetary system has formed through this kind of condensation process around spherical shells or flux tubes surrounding planetary orbits and containing dark matter.

   The question why dark matter would concentrate around flux tubes surrounding planetary orbits was not answered. The answer could be that dark matter is anyonic matter at partonic 2-surfaces whose light-like orbits define the basic geometric objects of quantum TGD. These partonic 2-surfaces could contain a central spherical anyonic 2-surface connected by radial flux tubes to flux tubes surrounding the orbits of planets and other massive objects of solar system to form connected anyonic surfaces analogous to elementary particles.

If factor spaces appear in M^4 degrees of freedom, they give rise to $Z_n \subset G_a$ symmetries. In astrophysical systems the large value of $\hbar$ necessarily requires a large value of n_a for CD coverings as the considerations of [D8] - in particular the model for graviton dark graviton emission and detection - forces to conclude. The same conclusion follows also from the absence of evidence for exact orbifold type symmetries in M^4 degrees of freedom for dark matter in astrophysical scales.

4. The model of DNA as topological quantum computer [L5] assumes that DNA nucleotides are connected by magnetic flux tubes to the lipids of the cell membrane. In this case, p-adically scaled down u and d quarks and their antiquarks are assumed to be associated with the ends of the flux tubes and provide a representation of DNA nucleotides. Quantum Hall states would be associated with partonic 2-surfaces assignable to the lipid layers of the cell and nuclear membranes and also endoplasmic reticulum filling the cell interior and making it macroscopic quantum system and explaining also its stability. The entire system formed in this manner would be single extremely complex anyonic surface and the coherent behavior of living system would result from the fusion of anyonic 2-surfaces associated with cells to larger anyonic surfaces giving rise to organs and organisms and maybe even larger macroscopically quantum coherent connected systems.

   In living matter one must consider the possibility that small values of n_a correspond to factor spaces of CD (consider as example aromatic cycles with Z_n symmetry with $n = 5$ or $n = 6$ appearing in the key molecules of life). Large $\hbar$ would require CP_2 factor spaces with a large value of n_b so that the integers characterizing the charges of anyonic particles would be shifted by a large integer. This is not in accordance with naive ideas about stability. One can also argue that various anomalous effects such as IQHE with ν equal to an integer multiple of n_b would have been observed in living matter.

   A more attractive option is that both CD and CP_2 are replaced with singular coverings. Spin and charge fractionization takes place but the effects are small if both n_a, n_b, and n_a/n_b are large. An interesting possibility is that the ends of the flux tubes assumed to connect DNA nucleotides to lipids of various membranes carry instead of u, d and their anti-quarks fractionally charged electrons and neutrinos and their anti-particles having $n_b = 3$ and large value of n_a. Systems such as snowflakes could correspond to large $\hbar$ zoom ups of molecular systems having subgroup of rotation group as a symmetry group in the standard sense of the word.

   The model of graviton de-coherence constructed in [D8] allows to conclude that the fractionization of Planck constant has interpretation as a transition to chaos in the sense that fundamental frequencies are replaced with its sub-harmonics corresponding to the divisor of $\hbar/\hbar_0 = r/s$. The more digits are needed to represent r/s, the higher the complexity of the system. Period doubling bifurcations leading to chaos represent a special case of this. Living matter is indeed a system at the boundary of chaos (or rather, complexity) and order and larger values of n_b would give rise to the complexity having as a signature weak charge and spin fractionization effects.

5. Coverings alone are enough to produce rational number valued spectrum for $\hbar$, and one must keep in mind that the applications of theory do not allow to decide whether only singular factor spaces are really needed.

12.4.6 What happens in QHE?

This picture suggest following description for what would happens in QHE in TGD Universe.

1. Light-like 3-surfaces - locally random light-like orbits of partonic 2-surfaces- are identifiable as very tiny wormhole throats in the case of elementary particles. This is the case for electrons in particular. Partonic surfaces can be also large, even macroscopic, and the size scales up in the scaling of Planck constant. To avoid confusion, it must be emphasized that light-likeness is with respect to the induced metric and does not imply expansion with light velocity in Minkowski

space since the contribution to the induced metric implying light-likeness typically comes from CP_2 degrees of freedom. Strong classical gravitational fields are present near the wormhole throat. Second important point is that regions of space-time surface with Euclidian signature of the induced metric are implied: CP_2 type extremals representing elementary particles and having light-like random curve as CP_2 projection represents basic example of this. Hence rather exotic gravitational physics is predicted to manifest itself in everyday length scales.

2. The simplest identification for what happens in the phase transition to quantum Hall phase is that the end of wire carrying the Hall current corresponds to a partonic 2-surface having a macroscopic size. The electrons in the current correspond to similar 2-surfaces but with size of elementary particle for the ordinary value of Planck constant. As the electrons meet the end of the wire, the tiny wormhole throats of electrons suffer topological condensation to the boundary. One can say that one very large elementary particle having very high electron number is formed.

3. The end of the wire forms part of a spherical surface surrounding the tip of the CD involved so that electrons can become carriers of anomalous electric and magnetic charges.

4. Chern- Simons action for Kähler gauge potential is Abelian. This raises the question whether the representations of the number theoretical braid group are also Abelian. Since there is evidence for non-Abelian anyons, one might argue that this means a failure of the proposed approach. There are however may reasons to expect that braid group representations are non-Abelian. The action is for induced Kähler form rather than primary Maxwell field, $U(1)$ gauge symmetry is transformed to a dynamical symmetry (symplectic transformations of CP_2 representing isometries of WCW and definitely non-Abelian), and the particles of the theory belong to the representations of electro-weak and color gauge groups naturally defining the representations of braid group.

5. The finite subgroups of $SU(2)$ defining covering and factor groups are in the general case non-commutative subgroups of $SU(2)$ since the hierarchies of coverings and factors spaces are assumed to correspond to the two hierarchy of Jones inclusions to which one can assign ADE Lie algebras by McKay correspondence. The ADE Lie algebras define effective gauge symmetries having interpretation in terms of finite measurement resolution described in terms of Jones inclusion so that extremely rich structures are expected.

6. The proposed model allows charge and spin fractionization also for IQHE since $\hbar/\hbar_0 = 1$ holds true for $n_a = n_b$. There is also infinite number of anyonic states predicting a given value of ν ($(n_a, n_b) \rightarrow k(n_a, n_b)$ symmetry).

An interesting challenge is to relate concrete models of QHE to the proposed description. Here only some comments about Laughlin's wave function are made.

1. In the description provided by Lauglin wave function FQHE results from a minimization of Coulomb energy. In TGD framework the tunneling to the page of H with m sheets of covering has the same effect since the density of electrons is reduced by $1/m$ factor.

2. The formula $\nu \propto e^2 N_e/e \int BdS$ with scaling up of magnetic flux by $\hbar/\hbar_0 = m$ implies effective fractional filling factor. The scaling up of magnetic flux results from the presence of m sheets carrying magnetic field with same strength. Since the N_e electrons are shared between m sheets, the filling factor is fractional when one restricts the consideration to single sheet as one indeed does.

3. Laughlin wave function makes sense for $\nu = 1/m$, m odd, and is m:th power of the many electron wave function for IQHE and expressible as the product $\prod_{i<j}(z_i - z_j)^m$, where z represents complex coordinate for the anyonic plane. The relative orbital angular momenta of electrons satisfy $L_z \geq m$ if the value of Planck constant is standard. If Laughlin wave function makes sense also in TGD framework, then m:th power implies that many-electron wave function is singlet with respect to Z_m acting in covering and the value of relative angular momentum indeed satisfies $L_z \geq m\hbar_0$ just as in Laughlin's theory.

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Appendix A

Appendix

A-1 Basic properties of CP_2

A-1.1 CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-1.1}$$

Here λ is any non-zero complex number. Note that CP_2 can also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [2] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-1.1}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi+\Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi-\Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-1.1}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number 3, Pontryagin number 3 and second Betti number $b = 1$.

A-1.2 Metric and Kähler structures of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i\bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5. The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-1.2}$$

where the Hermitian, in fact Kähler, metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-1.3}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-1.3}$$

The representation of the metric is given by

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-1.4}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-1.3}$$

The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-1.4}$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , \quad e^1 = \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} \ , \quad e^3 = \frac{r\sigma_3}{F} \ . \end{aligned} \tag{A-1.5}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , & e^1 &= \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \tag{A-1.5}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = dr^2/F^2 + (r^2/4F^2)(d\Psi + cos\Theta d\Phi)^2 + (r^2/4F)(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-1.5}$$

The vierbein connection satisfying the defining relation

$$de^A = -V_B^A \wedge e^B \ , \tag{A-1.6}$$

is given by

$$
\begin{array}{lllllll}
V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\
V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\
V_{03} & = & (r-\frac{1}{r})e^3 \ , & V_{12} & = & (2r+\frac{1}{r})e^3 \ .
\end{array} \tag{A-1.7}
$$

The representation of the covariantly constant curvature tensor is given by

$$
\begin{array}{lllllll}
R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\
R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\
R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ .
\end{array} \tag{A-1.8}
$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$
J \ = \ -ig_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-1.9}
$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$
J^k_{\ r}J^{rl} \ = \ -s^{kl} \ . \tag{A-1.10}
$$

The form J is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling). Locally one has therefore

$$
J \ = \ dB \ , \tag{A-1.11}
$$

where B is the so called Kähler potential, which is not defined globally since J describes magnetic monopole.

It should be noticed that the magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$
\begin{array}{lll}
B & = & 2re^3 \ , \\
J & = & 2(e^0\wedge e^3 + e^1\wedge e^2) = \frac{r}{F^2}dr\wedge(d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta d\Phi \ .
\end{array} \tag{A-1.10}
$$

The vielbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1,1).

Useful coordinates for CP_2 are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$
\begin{array}{lll}
B & = & \sum_{k=1,2} P_k dQ_k \ , \\
J & = & \sum_{k=1,2} dP_k\wedge dQ_k \ .
\end{array} \tag{A-1.10}
$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$
\begin{array}{lll}
P_1 & = & -\frac{1}{1+r^2} \ , \\
P_2 & = & \frac{r^2 cos\Theta}{2(1+r^2)} \ , \\
Q_1 & = & \Psi \ , \\
Q_2 & = & \Phi \ .
\end{array} \tag{A-1.8}
$$

A-1.3 Spinors in CP_2

CP_2 doesn't allow spinor structure in the conventional sense [5]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallelly propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the nonallowed -1- factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

A-1.4 Geodesic submanifolds of CP_2

Geodesic submanifolds are defined as submanifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [3] a general characterization of the geodesic submanifolds for an arbitrary symmetric space G/H is given. Geodesic submanifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-1.9}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic submanifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2 Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [4] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \qquad \text{(A-2.0)}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \qquad \text{(A-2.1)}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ , \end{aligned} \qquad \text{(A-2.2)}$$

and

$$B = 2re^3 \ , \qquad \text{(A-2.3)}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ^0_3 and Σ^1_2 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I^1_L + 2V_{13}I^2_L \ , \qquad \text{(A-2.4)}$$

where one have defined

$$
\begin{aligned}
I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\
I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ .
\end{aligned} \tag{A-2.4}
$$

A_{ch} is clearly left handed so that one can perform the identification

$$
W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-2.5}
$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$
\begin{aligned}
X &= re^3 \ , \\
Y &= \frac{e^3}{r} \ ,
\end{aligned} \tag{A-2.5}
$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$
\begin{aligned}
\bar{\gamma} &= aX + bY \ , \\
\bar{Z}^0 &= cX + dY \ ,
\end{aligned} \tag{A-2.5}
$$

where the normalization condition

$$
ad - bc = 1 \ ,
$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$
\begin{aligned}
A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\
&+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ .
\end{aligned} \tag{A-2.4}
$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5 \Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$
c = -d \ . \tag{A-2.5}
$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$
A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.6}
$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$
\begin{aligned}
Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\
I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ .
\end{aligned} \tag{A-2.6}
$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX+bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X-Y) \ . \end{aligned} \tag{A-2.6}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-2.7}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \tag{A-2.8}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0\wedge e^3 + e^1\wedge e^2) \ , \\ R_{12} &= 2(e^0\wedge e^3 + 2e^1\wedge e^2) \ , \\ J &= 2(e^0\wedge e^3 + e^1\wedge e^2) \ , \end{aligned} \tag{A-2.7}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.8}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-2.8}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-2.9}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-2.9}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$
\begin{aligned}
X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\
K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ ,
\end{aligned} \tag{A-2.9}
$$

In the general case the value of the coefficient K is given by

$$
K = \sum_i \left[-\frac{(18 + 2n_i^2)sin^2\theta_W}{9} \right] \ , \tag{A-2.10}
$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18 + n_i^2))} \ . \tag{A-2.11}
$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-2.12}
$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [6].

A-2.1 Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:
a) Symmetries must be realized as purely geometric transformations.
b) Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [1].

The action of the reflection P on spinors of is given by

$$
\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-2.13}
$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$
\begin{aligned}
m^k &\rightarrow T(M^k) \ , \\
\xi^k &\rightarrow \bar{\xi}^k \ , \\
\Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ .
\end{aligned} \tag{A-2.12}
$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$
\begin{aligned}
\xi^k &\rightarrow \bar{\xi}^k \ , \\
\Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ .
\end{aligned} \tag{A-2.12}
$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Space-time surfaces with vanishing em, Z^0, Kähler, or W fields

In the sequel it is shown that space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy.

A-3.1 Em neutral space-times

Em and Z^0 neutral spacetimes are especially interesting space-times as far as applications of TGD are considered. Consider first the electromagnetically neutral space-times. Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-3.0}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3 + 2p)\frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3 + p)\frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-3.0}$$

where Θ_W denotes Weinberg angle.

The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3 + 2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3 + p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-3.0}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1 - X}} \ , \\ X &= D\left[|\frac{(k + u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1 + r_0^2} \ , \ \epsilon = \frac{3 + p}{3 + 2p} \ , \end{aligned} \tag{A-3.-1}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u + k) \times [\frac{1 + r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k + 1 \ ,$$

where $sign(x)$ denotes the sign of x.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \end{aligned} \quad \text{(A-3.-1)}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which the electromagnetically neutral imbeddings become ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteing the absence of edges are satisfied.

The vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \quad \text{(A-3.0)}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

The expression for the Kähler form and Z^0 field of the electromagnetically neutral space-time surface will be needed in sequel and is given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \quad \text{(A-3.0)}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

The effective form of the CP_2 metric is given by

$$\begin{aligned} ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \end{aligned} \quad \text{(A-3.-1)}$$

and is useful in the construction of electromagnetically neutral imbedding of, say Schwartchild metric. Note however that in general these imbeddings are not extremals of Kähler action.

A-3.2 Space-times with vanishing Z^0 or Kähler fields

The results just derived generalize to the Z^0 neutral case as such. The only modification is the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F}du \wedge d\Phi$ is useful.

Also the generalization to the case of vacuum extremals is straightforward and corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u}du \wedge d\Phi = (k+u)du \wedge d\Phi \ , \\ r &= \sqrt{\frac{X}{1-X}} \ , \ X = D\vert k+u\vert \ , \\ \gamma &= -\frac{p}{2}Z^0 \ . \end{aligned} \tag{A-3.-2}$$

For vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

A-3.3 Induced gauge fields for space-times for which CP$_2$ projection is a geodesic sphere

For space-time sheets for which CP$_2$ projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \ .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

For homologically trivial geodesic sphere a standard representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP$_2$ projection color rotations and weak symmetries commute.

A-4 Second variation of the Kähler action

The Kähler action is apart from a multiplicative constant defined by the Lagrangian density

$$L = J^{\alpha\beta}J_{\alpha\beta}\sqrt{g} \ , \tag{A-4.1}$$

and depends on the imbedding space coordinates only through the induced metric and Kähler form. In order to calculate the second variation of the Kähler action one can use "covariantization" trick made possible by the covariant constancy of the imbedding space metric and Kähler form. Calculate second variation by treating components of the metric and Kähler form as a constant so that the action depends effectively only on the derivatives of the imbedding space coordinates and replace ordinary derivatives of the deformation with the covariant derivatives in the resulting expression for the second variation.

$$\begin{aligned} \partial_\alpha \delta h^k &\rightarrow D_\alpha \delta h^k \\ &= \partial_\alpha \delta h^k + \{ {}^{\ k}_{l\ m} \} \partial_\alpha h^m \delta h^l \ . \end{aligned} \tag{A-4.1}$$

The first variation of the Maxwell term is given by the expression

$$\delta_1 L = 2[T^{\alpha\beta}\delta_1 g_{\alpha\beta} + J^{\alpha\beta}\delta_1 J_{\alpha\beta}]\sqrt{g} \ , \qquad \text{(A-4.2)}$$

where the canonical energy momentum tensor $T^{\alpha\beta}$ is given by

$$T^{\alpha\beta} = J^{\alpha\nu}J^{\beta}_{\ \nu} - (1/4)g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \qquad \text{(A-4.3)}$$

and is traceless by Weyl invariance.

Second variation is obtained by differentiating first variation and decomposes into three terms

$$\delta_2 L = \delta_2^a L + \delta_2^b L + \delta_2^c L \ . \qquad \text{(A-4.4)}$$

The first term is given by the expression

$$\begin{aligned} \delta_2^a L &= [T^{\alpha\beta}\delta_2 g_{\alpha\beta} + J^{\alpha\beta}\delta_2 J_{\alpha\beta} \\ &+ (T^{\alpha\beta}\delta_1 g_{\alpha\beta} + J^{\alpha\beta}\delta_1 J_{\alpha\beta})g^{\mu\nu}\delta_1 g_{\mu\nu}]\sqrt{g} \ . \end{aligned} \qquad \text{(A-4.4)}$$

The second term is given by

$$\begin{aligned} \delta_2^b L &= [(\partial T^{\alpha\beta}/\partial g_{\mu\nu})\delta_1 g_{\alpha\beta}\delta_1 g_{\mu\nu} \\ &+ 2(\partial T^{\alpha\beta}/\partial J_{\mu\nu})\delta_1 g_{\alpha\beta}\delta_1 J_{\mu\nu}]\sqrt{g} \ . \end{aligned} \qquad \text{(A-4.4)}$$

The partial derivatives of the energy momentum tensor appearing in the expression are given by

$$\begin{aligned} \partial T^{\alpha\beta}/\partial g_{\mu\nu} &= -g^{\alpha\mu}T^{\beta\nu} + K^{\alpha\nu}g^{\beta\mu} - \frac{1}{2}K^{\mu\nu}g^{\alpha\beta} + J^{\alpha\nu}J^{\beta\mu} \ , \\ K^{\alpha\beta} &= J^{\alpha\nu}J^{\beta}_{\ \nu} \ . \end{aligned} \qquad \text{(A-4.4)}$$

$$\partial T^{\alpha\beta}/\partial J_{\mu\nu} = 2[g^{\alpha\mu}J^{\beta\nu} - g^{\alpha\beta}J^{\mu\nu}/4] \ . \qquad \text{(A-4.5)}$$

The third term is given by the expression

$$\begin{aligned} \delta_2^c L &= [(\partial J^{\alpha\beta}/\partial J_{\mu\nu})\delta_1 J_{\alpha\beta}\delta_1 J_{\mu\nu}]\sqrt{g} \ , \\ \partial J^{\alpha\beta}/\partial J_{\mu\nu} &= g^{\alpha\mu}g^{\beta\nu} \ . \end{aligned} \qquad \text{(A-4.5)}$$

Expressing the first term in terms of the coordinate variations one obtains

$$\delta_2^a L = 2[T^{\alpha\beta}h^{\perp}_{kl} + J^{\alpha\beta}J^{\perp}_{kl}]D_\alpha\delta_1 h^k D_\beta\delta_1 h^l\sqrt{g} \ , \qquad \text{(A-4.6)}$$

where $h^{\perp}_{kl}$ and $J^{\perp}_{kl}$ are the projections of the imbedding space metric and Kähler form to the orthogonal complement of the tangent space of X^4

$$\begin{aligned} h^{\perp}_{kl} &= h_{kl} - g^{\mu\nu}h_{kr}h_{ls}\partial_\mu h^r\partial_\nu h^s \ , \\ J^{\perp}_{kl} &= h^{\perp}_{kr}h^{\perp}_{ls}J^{rs} \ , \end{aligned} \qquad \text{(A-4.6)}$$

so that $\delta_2^a L$ vanishes for four-dimensional $Diff$ deformations parallel to X^4. This term vanishes also, when the induced Kähler form vanishes.

The contribution of the second term to the second variation is given by the expression

$$\begin{aligned}\delta_2^b L &= 4[(-g^{\alpha\mu}T^{\beta\nu} + K^{\alpha\nu}g^{\beta\mu} - \frac{1}{2}K^{\mu\nu}g^{\alpha\beta} + J^{\alpha\nu}J^{\beta\mu})h_{kr}h_{ls} \\ &+ 2(g^{\alpha\mu}J^{\beta\nu} - g^{\alpha\beta}J^{\mu\nu}/4)h_{ks}J_{lr}]\partial_\alpha h^k \partial_\beta h^l D_\mu \delta_1 h^r D_\nu \delta_1 h^s \sqrt{g} \ .\end{aligned} \tag{A-4.5}$$

Also this term is non-vanishing only provided the induced Kähler field is nontrivial.

The third term is given by the expression

$$\delta_2^c L = [g^{\alpha\mu}g^{\beta\nu}J_{kr}J_{ls}]\partial_\alpha h^k \partial_\beta h^l D_\mu \delta_1 h^r D_\nu \delta_1 h^s \sqrt{g} \ . \tag{A-4.6}$$

This term is the only term, which is nontrivial for the vacuum extremals with vanishing Kähler field and also in this case the variation is nontrivial for CP_2 coordinates only.

The second variation for the Kähler Lagrangian can be written in the following general form

$$\delta^2 L_{intX^4} = I_{kl}^{\alpha\beta} D_\alpha \delta h^k D_\beta \delta h^l \ , \tag{A-4.6}$$

where the general expressions for the tensor $I_{kl}^{\alpha\beta}$ reads as

$$I_{kl}^{\alpha\beta} = \partial_{\partial_\alpha h^k} \partial_{\partial_\beta h^l} L \ . \tag{A-4.6}$$

The explicit expression for the tensors $I_{kl}^{\alpha\beta}$ can be read from the expressions for $\delta L_2^i, i = a, b, c$ and $\delta_2 L_{CS}$ respectively.

The general form of the variational equations satisfied by the second variation in the interior of X^4 reads as

$$D_\alpha(I_{kl}^{\alpha\beta} D_\beta \delta h^l) = 0 \ . \tag{A-4.7}$$

On the boundary the variational equations read

$$I_{kl}^{n\beta} D_\beta \delta h^l = 0 \ . \tag{A-4.8}$$

These equations are satisfied on a dynamically generated boundary only. These equations are not satisfied on the intersection of the four-surface with the surfaces $a = \sqrt{(m^0)^2 - r_M^2} \to \infty$ and $a = 0$ (light cone boundary).

The expression for the second variation of the action reduces to a mere boundary term resulting from the intersections of the four-surface with $a \to \infty$ and $a = 0$ surfaces, when X^4 corresponds to a submanifold of light cone and reads

$$\delta^2 S = \int_{a=0}^{a=\infty} I_{kl}^{n\beta} \delta h^k D_\beta \delta h^l d^3x. \tag{A-4.8}$$

The general expressions for the tensor I suggests that only non-vanishing contribution to the second variation comes from the boundary of the light cone.

A-5 p-Adic numbers

p-Adic numbers (p is prime: 2,3,5,...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [8]. p-Adic numbers are representable as power expansion of the prime number p of form:

$$x = \sum_{k\geq k_0} x(k)p^k, \; x(k) = 0, ..., p-1 \; . \tag{A-5.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \; . \tag{A-5.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \; , \tag{A-5.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x, y) = |x - y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x, z) \leq max\{d(x, y), d(y, z)\} \; . \tag{A-5.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x, y) \leq D \; . \tag{A-5.5}$$

This division of the metric space into classes has following properties:

a) Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

b) Distances of points x and y inside single class are smaller than distances between different classes.

c) Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [10]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

A-6 Canonical correspondence between p-adic and real numbers

There exists a natural continuous map $Id : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$
\begin{aligned}
y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\
y_k &\in \{0, 1, .., p-1\} \ .
\end{aligned}
\qquad \text{(A-6.0)}
$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$
\begin{aligned}
x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\
x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ .
\end{aligned}
\qquad \text{(A-6.-1)}
$$

The p-adic images associated with these expansions are different

$$
\begin{aligned}
y_1 &= \sum_{k=N_0}^{N} x_k p^{k} \ , \\
y_2 &= \sum_{k=N_0}^{N-1} x_k p^{k} + (x_N - 1)p^{N} + (p-1)p^{N+1} \sum_{k=0,..} p^{k} \\
&= y_1 + (x_N - 1)p^N - p^{N+1} \ ,
\end{aligned}
\qquad \text{(A-6.-2)}
$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

What about the p-adic counterpart of the negative real numbers? It seems that in the applications this correspondence is not needed since canonical identification is used only in the direction $R_p \rightarrow R$ to map the predictions of p-adic probability calculus and statistics to real numbers (in particular, p-adic entanglement entropy must be mapped to its real counterpart). This means that also the inverse of the canonical identification is not needed in the applications. At tge space time level the p-adics and reals relate via common rationals. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs.

The topology induced by the canonical identification t map in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see Fig. A-6) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the

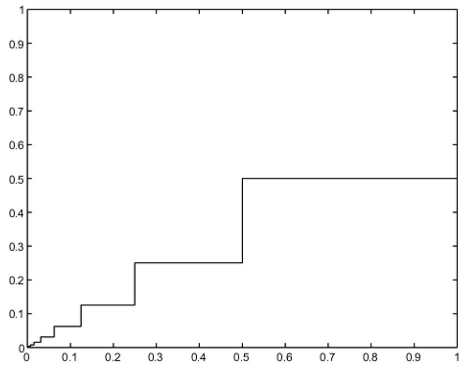

Figure A.1: The real norm induced by canonical identification from 2-adic norm.

real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. Canonical identification makes also possible to understand the connection between p-adic and real probabilities. These suggests that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p|y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \qquad \text{(A-6.-2)}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p|y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \qquad \text{(A-6.-2)}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \qquad \text{(A-6.-1)}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some nonlinear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

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